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ANALYSIS OF MULTI-WIRE STRANDS IN TENSION AND COMBINED TENSION AND TORSION

by

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ABSTRACT

Geometrical relationship of a deformed and undeformed wire strand is derived through the use of differential geometry. In conjunction with balance laws of forces and moments, the governing equations for computing the strains, elongations and end-rotations of a strand with free or fixed ends and with tensile and torsional loadings are derived. The analytical results are compared favorably with the experimental data conducted on strands of steel and epoxy.

INTRODUCTION

Strands and wire ropes have been used extensively in the construction and the industry for many years. Recently, deep ocean research requires very long cables to tow the instrumentation packages on the free ends. Due to the structure of the strand or wire rope a phenomenon called the "end-rotation" takes place even in so-called "non-rotating" ropes. The end-rotation not only produces undesirable characteristics such as kinking and "birdcaging" but also has been known to produce a considerable redistribution of loads among the wires in the strands and wire ropes, causing failure or breakage at a load as low as 60 percent of the counterpart of a fixed-ended one. A truly non-rotating rope or strand can only be designed after the detailed response of the strand and ropes to the external loads and environments is clearly understood. These response characteristics include elongation, bending and twisting stresses, end-rotation and many others. The existing method of analysis is based on some simplifying assumptions and is not adequate in the evaluation of end-rotation. A more complete analytical method is clearly needed and should have a far-reaching benefit to the ocean technology. The present paper deals with such a method. For simplicity, the scope of this paper is confined to the strands under a tensile load and a combined tensile and twisting. The geometrical relations in this paper are exact but the structural mechanics is of strength of materials type, rather than theory of elasticity approach. The results are compared with the available experimental data and are shown to be in good agreement.

LITERATURE REVIEW

There have been few systematic investigations conducted by engineering laboratories on strands and ropes. As a rule, manufacturers from time to time conduct tests for quality control purposes. However, their results are rarely available for critical, comparative study by researchers. The earliest investigation, to the author's knowledge, was conducted by the National Bureau of Standards at the request of Isthmian Canal Commission of the Canal Zone [1]. There included in its report is a discussion of the stress analysis of strands and ropes. It was asserted that the tensile stresses in the wires of different pitch angles vary in direct proportion to the sine squared of the pitch angle. The derivation of this result was later given by Hruska [2] who attributed it to Dreher [3] in a later article [4]. In the derivation, the following assumptions, in addition to the usual assumptions in Mechanics of Materials, were apparently implied:

(1) The nominal diameter of the strand or rope remains unchanged during tension.

(2) The stretch or elongation is small compared with the total length of the wire.

(3) The end-rotation of the strand or rope caused by the direct tension is small.

From this relationship, it was concluded that the core wire sustained a slightly higher direct tensile stress than the helical wires. He did not consider the flexural and torsional stresses in the helical wires, although he was apparently aware of their existence.

Attributed also to Dreher, Hruska discussed the origin mechanism and consequences of the unlaying torsion [4] . He presented a method to compute the torsional moment caused by the axial loading in strands and ropes, and discovered an error in the relation between the applied load and the reactive force in a helical wire in Dreher's derivation. He corrected this error in his article, and found that the numerical results were not affected by more than a few percent as long as the lay angle is small.

He stipulated that the torsional moment would cause a considerable twisting of the core and helical wires and also some bending of the helical wires; however, he did not show how these effects can be computed. He did not show how to determine the amount of end-rotation due to a tensile load, but he gave an expression to compute the redistribution of loads in the wires due to a known amount of end-rotation. In the derivation the same assumptions listed above were employed and the method is subjected to the same limitation. It was shown that end-rotation tends to shift a significant portion of the load from the helical wires to the core wire.

Similar calculations were carried out by Bert and Stein [5] for strands and by Gibson et al [6] for wire ropes. The latter reference included also measurements of torsional moments which showed a reasonable agreement with the analytical results as suggested by Hruska.

Hruska [7] also derived the expression for the contact forces between the core and helical wires. The basis of his analysis was the "hoop-tension" formula, as modified for the consideration of the open coil with a moderate pitch angle. It was used by Leissa [8] to compute the contact stresses for parallel cylinders. Starkey and Cress [9] extended the analysis for the case of cross wires.

ANALYSIS

1. Geometrical Consideration:

A typical strand consists of one or more layers helical wires wound around a straight core wire. To study the deformation and interaction of the wires, we need to know some of the important geometrical properties of a helix, including curvature and torsion.

The parametric representation of a helix is as follows:

$$x = R \cos t$$

$$y = R \sin t$$

$$z = (R \tan \alpha)\, t$$

where the parameter t is identified to be the polar angle. It can be verified that the above expressions indeed represent a helix. For instance, eliminating t between the first two expressions above, we obtain

$$x^2 + y^2 = R^2$$

which is the projection of the helix on x-y plane; noting also that

$$p = 2\pi R \tan \alpha$$

we can write

$$z = \frac{p}{2\pi} t;$$

then, when t increases by one revolution, while x and y would remain unchanged, z would increase by one pitch, p, as it should.

Now, the rate of change of the radius vector r is

$$\frac{d\vec{r}}{dt} = -R \sin t\, \hat{i} + R \cos t\, \hat{j} + R \tan \alpha\, \hat{k}$$

of which the magnitude is

$$\frac{ds}{dt} = \left| \frac{d\vec{r}}{dt} \right| = \frac{R}{\cos \alpha}$$

From the ratio of the above two expressions we obtain the expressions for unit tangent as follows:

$$\hat{T} = \frac{d\vec{r}}{ds} = -\cos\alpha \sin t\, \hat{i} + \cos\alpha \cos t\, \hat{j} + \sin\alpha\, \hat{k}$$

In a similar manner we obtain

$$\left| \frac{d\hat{T}}{ds} \right| = \frac{\cos^2\alpha}{R} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (1)$$

By virtue of Frenet-Serret Formulas [10] we know that $\frac{d\hat{T}}{ds}$ is the curvature c and also, the principal unit normal $\hat{N}$ is expressed by

$$\hat{N} = \frac{1}{c} \frac{d\hat{T}}{ds} = -\cos t\, \hat{i} - \sin t\, \hat{j}$$

Using the property of cross-product of two vectors we obtain the expression for binormal as follows:

$$\hat{B} = \hat{T} \times \hat{N} = \sin\alpha \sin t\, \hat{i} - \sin\alpha \cos t\, \hat{j} + \cos\alpha\, \hat{k}$$

By a similar procedure as before we obtain

$$\frac{d\hat{B}}{ds} = \frac{\sin\alpha \cos\alpha}{R} (\cos t\, \hat{i} + \sin t\, \hat{j})$$

Again, by Frenet-Serret Formulas we obtain the expression of torsion as follows:

$$b = -\frac{d\hat{B}}{ds} \cdot \hat{N} = \frac{\sin\alpha \cos\alpha}{R} \quad \dots\dots\dots\dots\dots\dots\dots \quad (2)$$

2. Change of geometry due to elongation and rotation

A. Inextensible helix subjected to a pure elongation (no rotation)

In the present case, the following conditions hold:

$$S = S', \quad n = n', \quad L' = L + \delta$$

Since $L = S \sin\alpha$ and $L' = S' \sin\alpha'$, we have,

$$\sin\alpha' = \frac{L'}{L} \sin\alpha \quad \dots\dots\dots\dots\dots\dots\dots\dots \quad (3A)$$

or

$$\sin\alpha' = (1 + \frac{\delta}{L}) \sin\alpha \quad \dots\dots\dots\dots\dots\dots\dots \quad (3B)$$

On the other hand, since $2\pi n R = S \cos\alpha$ and $2\pi n' R' = S' \cos\alpha'$, we obtain,

$$R' = \frac{\cos\alpha'}{\cos\alpha} R \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

so that $$c' = \frac{\cos^2\alpha'}{R'} = \frac{\cos\alpha}{R}\sqrt{1 - (1 + \frac{\delta}{L})^2 \sin^2\alpha} \quad \ldots\ldots\ldots(5)$$

$$b' = \frac{\cos\alpha' \sin\alpha'}{R'} = \frac{\cos\alpha \sin\alpha}{R} (1 + \frac{\delta}{L}) \quad \ldots\ldots(6)$$

B. Inextensible helix subjected to a pure rotation, i.e., to rotate an angle ϕ without elongation. In this case, the following conditions hold:

$$S = S', \quad L = L'; \quad \frac{t'}{t} = 1 - \frac{\phi}{2\pi n} = 1 - \frac{\phi R \tan\alpha}{L}$$

From Eq. (3A), we conclude,

$$\alpha' = \alpha$$

However, since $z = z'$, we have,

$$(R' \tan\alpha') t' = (R \tan\alpha) t$$

or $$R' = \frac{R}{1 - \frac{\phi R \tan\alpha}{L}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

so that $$c' = (1 - \frac{\phi R \tan\alpha}{L}) \frac{\cos^2\alpha}{R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

$$b' = (1 - \frac{\phi R \tan\alpha}{L}) \frac{\cos\alpha \sin\alpha}{R} \quad \ldots\ldots\ldots\ldots\ldots(9)$$

C. Inextensible helix subjected to a combination of elongation and rotation.

We note from the above that the elongation tends to decrease the radius of the helix and the unlaying rotation tends to increase it. It is of interest to study the net effect of the combination of the two.

We shall apply the two deformations sequentially. First, let the helix undergo a pure elongation δ, and let R and R' be the initial and final radii, respectively. We recall, from Eq. (4), that the following relation holds between these radii:

$$R_1 = \frac{\cos \alpha_1}{\cos \alpha} R$$

where, as per Eq. (3B), $\cos \alpha_1 = \sqrt{1 - (1 + \frac{\delta}{L})^2 \sin^2 \alpha}$ ....(10)

Subsequently, the helix undergoes a pure rotation ϕ while the radius increases from R_1 to R'. The radii are related by an expression similar to Eq. (7), as follows:

$$R' = \frac{R_1}{1 - \frac{\phi R_1 \tan \alpha_1}{L_1}} = \frac{R_1}{1 - \frac{\phi R \tan \alpha}{L}}$$

In general, R and R' need not be the same, and we can relate them by:

$$R' = K_1 R$$

where K_1 is a constant factor, depending on certain practical considerations to be discussed later. Combining the above expressions by eliminating R_1 and R', we obtain:

$$K_1 R = \frac{\cos \alpha'}{\cos \alpha} \frac{R}{1 - \frac{\phi R \tan \alpha}{L}}$$

Substituting Eq. (10) into the above and rearranging, we have:

$$\frac{\delta}{L} = \frac{1}{\sin \alpha} \sqrt{1 - K_1^2 (\cos \alpha - \frac{\phi R \sin \alpha}{L})^2} - 1 \quad .....(11)$$

The final curvature and torsion are, respectively,

$$c' = \frac{\cos^2 \alpha'}{R'} = \frac{1 - (1 + \frac{\delta}{L})^2 \sin^2 \alpha}{K_1 R} \quad ....(12)$$

$$b' = \frac{\cos \alpha' \sin \alpha'}{R'} = \frac{(1 + \frac{\delta}{L}) \sin \alpha \sqrt{1 - (1 + \frac{\delta}{L})^2 \sin^2 \alpha}}{K_1 R} \quad ....(13)$$

D. Extensible helix with rigid, frictionless core with fixed pitch angle, subjected to an elongation.

In the present case, the conditions are:

$$S' = S + \Delta S, \quad L' = L + \delta \,, \quad \alpha' = \alpha \,, \quad R' = R$$

We deduce easily that

$$\delta = (\Delta S) \sin \alpha$$
$$= \epsilon_h L \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

where $\epsilon_h = \frac{\Delta S}{S}$

This elongation must be accompanied by a rotation in laying direction of the following amount:

$$\Delta \theta = \frac{\delta}{R \tan \alpha} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

Note that since the radius and pitch of the helix remains unchanged, there is no torsion or bending in the wire in this type of deformation.

E. Extensible helix with a rigid, frictionless core subjected to a pure elongation.

This case is equivalent to the combination of case D and an additional pure rotation, equal to $\Delta\theta$ in unlaying direction. It can be shown that:

$$\sin \alpha' = \frac{1 + \frac{\delta}{L}}{\left[(1 + \frac{\delta}{L})^2 + K_1^2 \cot^2 \alpha\right]^{1/2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

and, noting that:

$$\frac{S'}{S} = (1 + \frac{\delta}{L}) \frac{\sin \alpha}{\sin \alpha'} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (16A)$$

we have,

$$\epsilon_h = \frac{S' - S}{S} = \sin \alpha \sqrt{(1 + \frac{\delta}{L})^2 + K_1^2 \cot^2 \alpha} - 1 \quad \ldots\ldots (17)$$

The final curvature and torsion are, respectively,

$$c' = \frac{\cos^2\alpha'}{R'} = \frac{\cot^2\alpha}{R\left[(1+\frac{\delta}{L})^2 + K_1^2\cot^2\alpha\right]} \quad \ldots\ldots\ldots\ldots\ldots (18)$$

$$b' = \frac{\cos\alpha'\sin\alpha'}{R'} = \frac{(1+\frac{\delta}{L})\cot\alpha}{R\left[(1+\frac{\delta}{L})^2 + K_1^2\cot^2\alpha\right]} \quad \ldots\ldots\ldots (19)$$

In the above expressions, a coefficient K_1 was added to each of the cotangent terms, so that reduction of radii of strand may be taken into account, if needed.

F. Extensible helix with a rigid, frictionless core subjected to a combined elongation and rotation.

In this final case, the cotangent term in case E must be reduced by $R\phi/L$ to take account of the effect of end-rotation, i.e.,

$$\sin\alpha' = \frac{1+\frac{\delta}{L}}{\left[(1+\frac{\delta}{L})^2 + K_1^2(\cot\alpha - \frac{R\phi}{L})^2\right]^{1/2}} \quad \ldots\ldots\ldots (20)$$

$$\epsilon_h = \sin\alpha\sqrt{(1+\frac{\delta}{L})^2 + K_1^2(\cot\alpha - \frac{R\phi}{L})^2} - 1 \quad \ldots\ldots (21)$$

and the final curvature and torsion are

$$c' = \frac{K_1(\cot\alpha - \frac{R\phi}{L})^2}{R\left[(1+\frac{\delta}{L})^2 + K_1^2(\cot\alpha - \frac{R\phi}{L})^2\right]} \quad \ldots\ldots\ldots\ldots (22)$$

$$b' = \frac{(1+\frac{\delta}{L})(\cot\alpha - \frac{R\phi}{L})}{R\left[(1+\frac{\delta}{L})^2 + K_1^2(\cot\alpha - \frac{R\phi}{L})^2\right]} \quad \ldots\ldots\ldots\ldots (23)$$

We remark in passing that it is easily verified that Case F can be constructed by the superposition of Case C on the Case D; that is to say, Case F can be decomposed into a combination of Cases D and C above. We shall use the latter statement later to simplify the analysis.

G. Discussion.

The results shown above appear to be reasonable and degenerate into special cases in a proper manner. Case F and the associated expressions reduce to Case E by setting $\phi = 0$ and reduce to Case D by setting $\phi = -\Delta\theta$ and using Eq. (15). By setting $\epsilon_h = 0$ in Eq. (21) and solving for δ/L, we obtain Eq. (11). Also, setting $S' = S$ in Eq. (16A), we obtain Eq. (3B). Based on the consistency of these results we should have considerable confidence on these results.

It is evident that all these expressions are exact, geometrical relations. In the limiting case, from Eq. (4), we note that R' vanishes with $\cos\alpha'$ as α' approaching $\pi/2$, while the helix becomes a straight line and the curvature c' also vanishes as expected. In the following paragraphs, we shall show that they include nearly all previously known results as special cases.

Setting $K_1 = 1$ in Eq. (21) and making a binomial expansion of the radical to linear terms, we obtain,

$$\epsilon_h \sim \sin^2\alpha \left(\epsilon_c - \frac{R\,\phi}{L\,\tan\alpha} \right), \text{ with } \epsilon_c = \frac{\delta}{L} \quad \ldots\ldots\ldots (24)$$

which was obtained by Hruska [4] by an elementary method, valid for small elongation and rotation. If further, we restrict the deformation to pure elongation then $\phi = 0$ and the above expression becomes,

$$\epsilon_h \sim \epsilon_c \sin^2\alpha$$

which is the sine square law asserted by Griffith and Bragg [1] and rediscovered by Hruska [2]. It was used without modification in more recent literature [6,8,9].

When the constant K_1 is set to be 1 Eq. (11) becomes

$$\frac{\delta}{L} = \sqrt{1 + 2\frac{\phi R}{L \tan\alpha} - \frac{\phi^2 R^2}{L^2}} - 1$$

then, for small ϕ, $\frac{\phi^2 R^2}{L^2}$ can be neglected. We expand the remaining terms in the radical through binomial expansion and keeping only the linear term, and obtain

$$\delta \sim \frac{\phi R}{\tan\alpha}$$

which was obtained by Hruska [4] , by an elementary method.

In Eqs. (5) or (12), if we neglect $\frac{\delta}{L}$, then the final curvature C' becomes $\cos^2\alpha/R$, which is the curvature used in computing the "normal forces" in strands by Hruska [7] . Leissa [8] used the same relationship to calculate the contact stresses.

3. Analysis of the response of strands due to end loads.

We shall first discuss the behavior of a strand initially straight subjected to an axial tension. Two cases are included: first, a strand undergoes a pure elongation without end-rotation; second, a strand undergoes an elongation but is free to rotate until a natural equilibrium condition is reached. The first case is intended to simulate the test condition when the strands are gripped by non-rotating heads in a universal testing machine. The second case resembles a strand loaded by a hanging weight or by a swivel crosshead.

A. Fixed-ended strand in tension.

The first case follows the geometrical relationship as discussed in Case 2E. Setting $K_1 = 1$ in Eq.(17) we have

$$\frac{P_h}{EA_h} = \sin\alpha\sqrt{\left(1 + \frac{P_c}{EA_c}\right)^2 + K_1^2\cot^2\alpha} - 1 \quad \text{...........(25)}$$

in which, the stress-strain relations, $\epsilon_h = P_h/EA_h$ and $\delta/L = P_c/EA_c$ are used.

In addition, the total tensile load P must be resisted by the resultant in the core wire and the vertical components of the resultants in the helical wires. Since the geometry is significantly changed by the applied load, we shall use the geometrical parameters, in particular the pitch angle, at the equilibrium or final condition of the strand. The law of balance of forces requires that,

$$P_c + NP_h \sin\alpha' = P \quad \text{........................(26)}$$

From Eqs. (25) and (26), P_c and P_h can be solved. From the former and using the stress-strain relation we can also compute δ. Then, using Eqs. (1), (2), (18), and (19), we can compute the changes of curvature and torsion from which, the bending and torsional moment can be determined by the following:

$$M_b = E\,I_h\,(c - c') \quad \text{..........................(27A)}$$

$$M_t = G\,J_h\,(b - b') \quad \text{..........................(27B)}$$

The flexural and shearing stresses in the helical wires can be determined directly from these results.

We have computed the elongation of the core wire and combined strains in the helical wires in a typical 5/16 - in. seven- wire strand at different load levels. The results are plotted in Figs. (1) and (2). Also shown are the experimental results by Dr. Durelli and his associates [11]. It is seen that the results are in good agreement.

B. Free - ended strands in tension.

For the second case, the geometrical relationship must be replaced by Eq. (21), which, similar to Eq. (25), should be expressed in terms of resultant forces in the wires. Furthermore, Eq. (26), which states the forces must be in balance, obviously, remains to be true in the present case.

In the present case, the end - rotation ϕ is also an unknown; we must look for an additional equation, which according to Hruska [4] should be the balance law for moments. It is not obvious, however, how a moment is produced by an axial load in the vertical direction. The present writer believes that here is the difficulty confronting the analysts and is the origin of the controversy between Dreher's and Hruska's approaches [3, 4].

To obviate this difficulty, we make the following proposition. We assume that the core wire and the helical wires to be disconnected at the ends and reach their equilibrium positions through the following two steps:

(1) The strand is assumed to be extensible and we impose on the end of each helical wire a tangential force F such that the resultant force on the helical wire is kept tangential to the helical curve ; then,

(2) the strand is assumed to be inextensible and we apply an equal and opposite forces to neutralize the forces F, such that whole strand is brought to the final equilibrium position, by a combination of elongation and rotation, similar to Case 2C.

We have already asserted the validity of this decomposition. We shall explain presently how this decomposition simplifies the analysis.

Step (1) restricts the helical wires to have the same radius and pitch angle. We recall from Case 2D that for this type of deformation an elongation and a small rotation are produced but the wires are not subjected to any bending or torsion. Step (2) produces a twisting moment in the strand and the radius and the pitch of the strand are changed, resulting a considerable bending and twisting in the helical wires as well as twisting in the core wire. By this decomposition process, we can conclude that a pure extension of helical wires produces no bending and twisting and the twisting moment explained in step (2) is responsible to the bending and twisting of the wires.

Balance of moments: The applied twisting moment on the strand must be equal to the sum of the twisting moment in the core wire and the vertical components of the bending and twisting moments in the helical wires. Using Eqs. (27), we obtain,

$$K_1 N P_h R \cos \alpha' = \frac{G J_c \phi}{L} + N \left[E I_h (c - c') \sin \alpha' + G J_h (b - b') \cos \alpha' \right] \quad \text{.........(28)}$$

where c, c', b, b', and $\sin \alpha'$ are given by Eqs. (1), (22), (2), (23), and (20), respectively. We shall now discuss the meaning of K_1. Close examination of steel strands reveals that there is usually a small gap between the core and the helical wires in the unstressed condition. This gap is closed due to the reduction in radii caused by the tensile load as explained in Case 2A. $K_1 = 1$ only if the strand is tight initially. It was found that a very small gap initially of this kind may produce a considerable increase in elongation and end - rotation of the strand.

We can, in principle, solve for P_c, P_h and ϕ from Eqs. (21), (26), and (28). Due to the non - linear and implicit nature of these equation, a tedious trial-and-error procedure must be used.

Figs. (3), (4), and (5), show the results computed for a 5/16-in. seven wire strand, which agree well with the experimental results. Now we study the effect of a combined tension and torsion.

C. Strands subjected to a combined tension and torsion.

If a twisting moment, M_o, is applied in addition to the tensile load, Eqs. (21) and (26) remain to be valid but a term M_o must be added to the left-hand side of Eq. (28). M_o is positive if it acts in the unlaying direction; otherwise, it is negative. The solution of this problem is the same as that discussed in the last section. Fig. (6) shows, for a 5/16-in. strand, the relation of the end-rotation ϕ vs. M_o for $P = 5000$ lb. When $M_o = 0$, the result degenerate into that for a free-ended case; when $\phi=0$, the result degenerates into that for a fixed-ended case. Fig. (7) shows the non-dimensional plot for ϕ vs. M_o corresponding to both the steel strand and an epoxy strand of 1½-in. in diameter under various loadings. We see that the agreement between the analytical and the experimental results is good for engineering purposes.

CONCLUSION

The method presented herein appears to be reasonable and the elongations, strains and end-rotations calculated from it agree well with the experimental data made in strands of two different materials under two different end conditions and loading conditions. Extension of this method to be applicable to wire ropes and synthetic fiber ropes is highly desirable.

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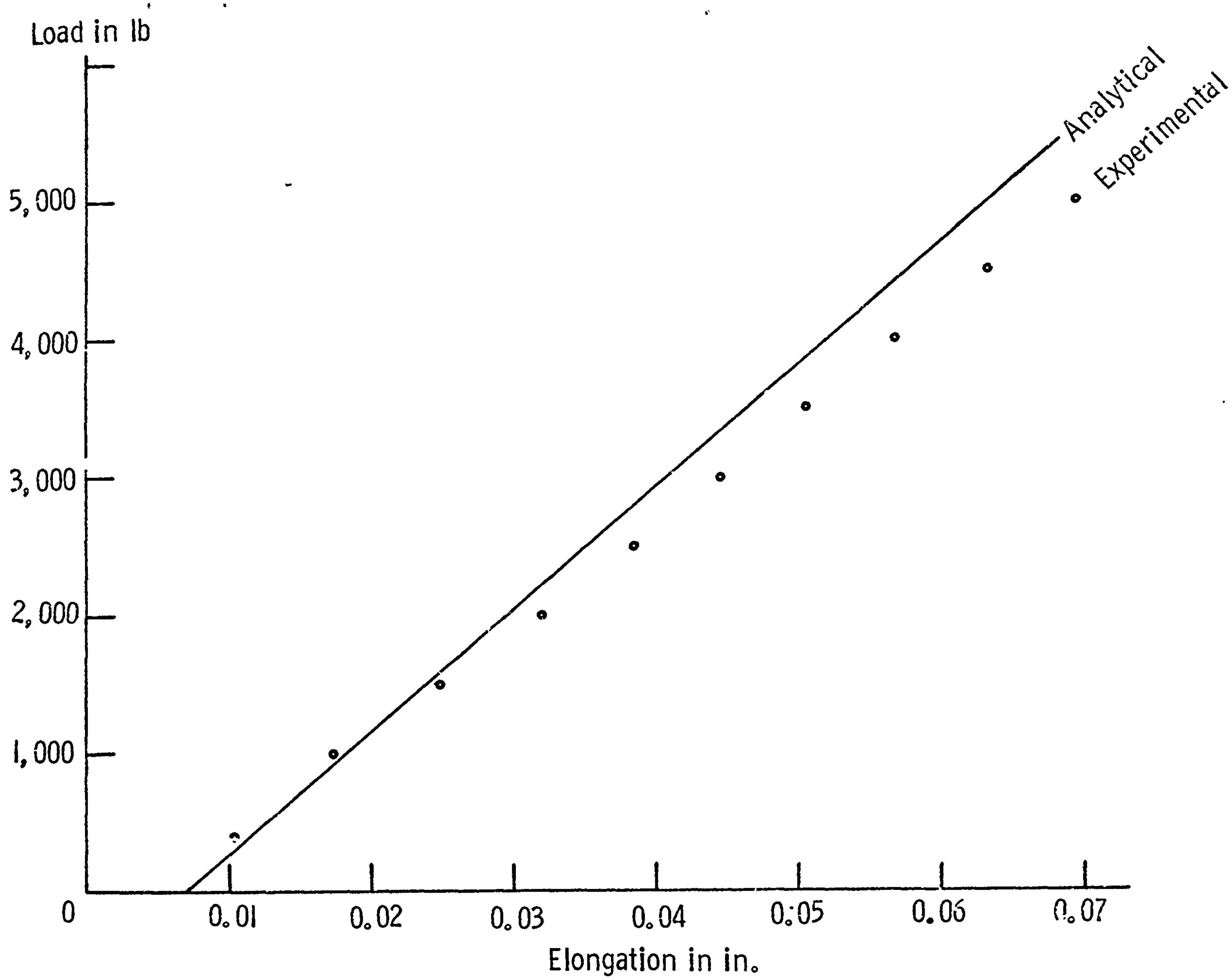

Fig. 1 Load vs Elongation in a 5/16-in. Strand with Fixed-End

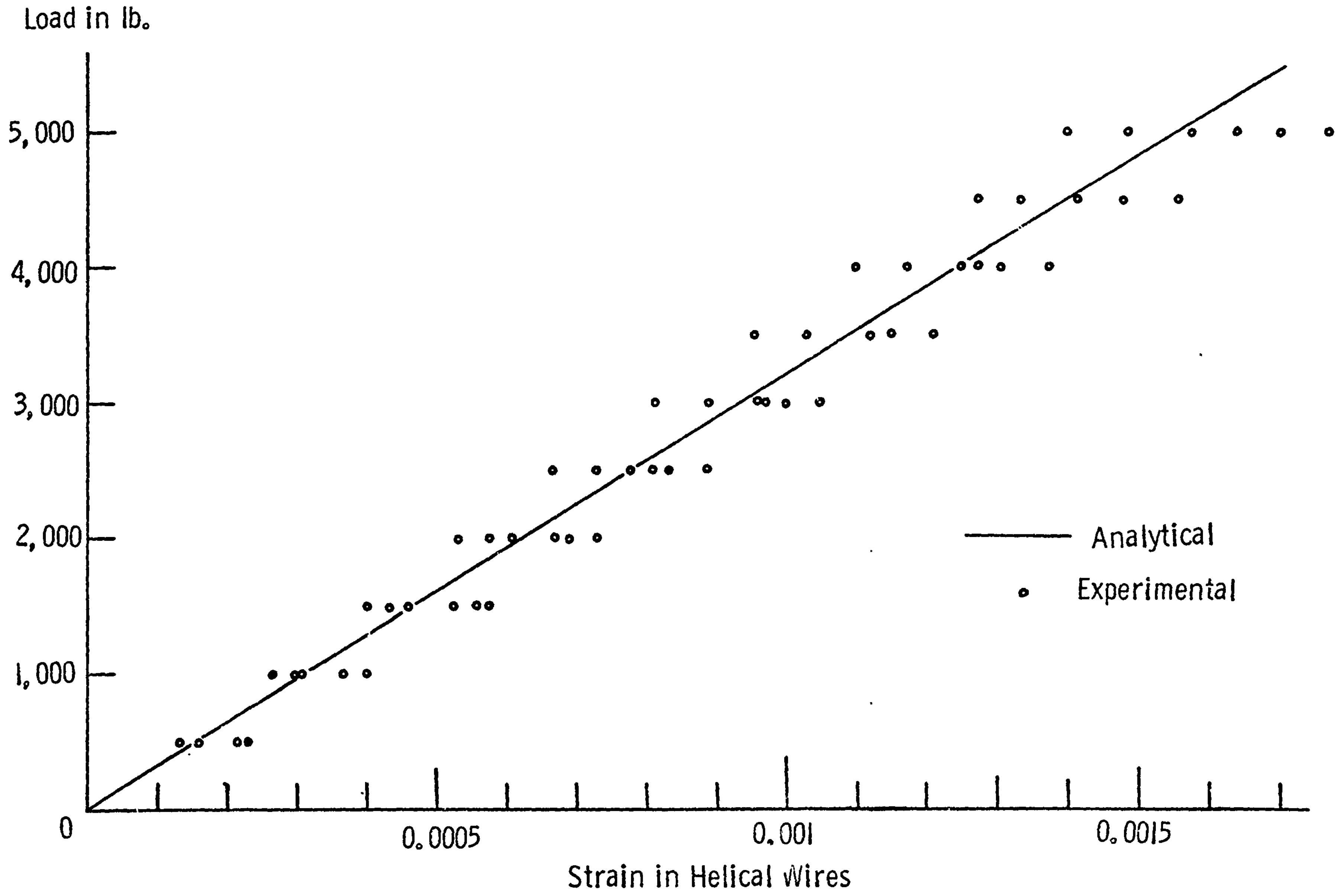

Fig. 2 Load vs Strain in Helical Wires of 5/16-in. Strand with Fixed-End

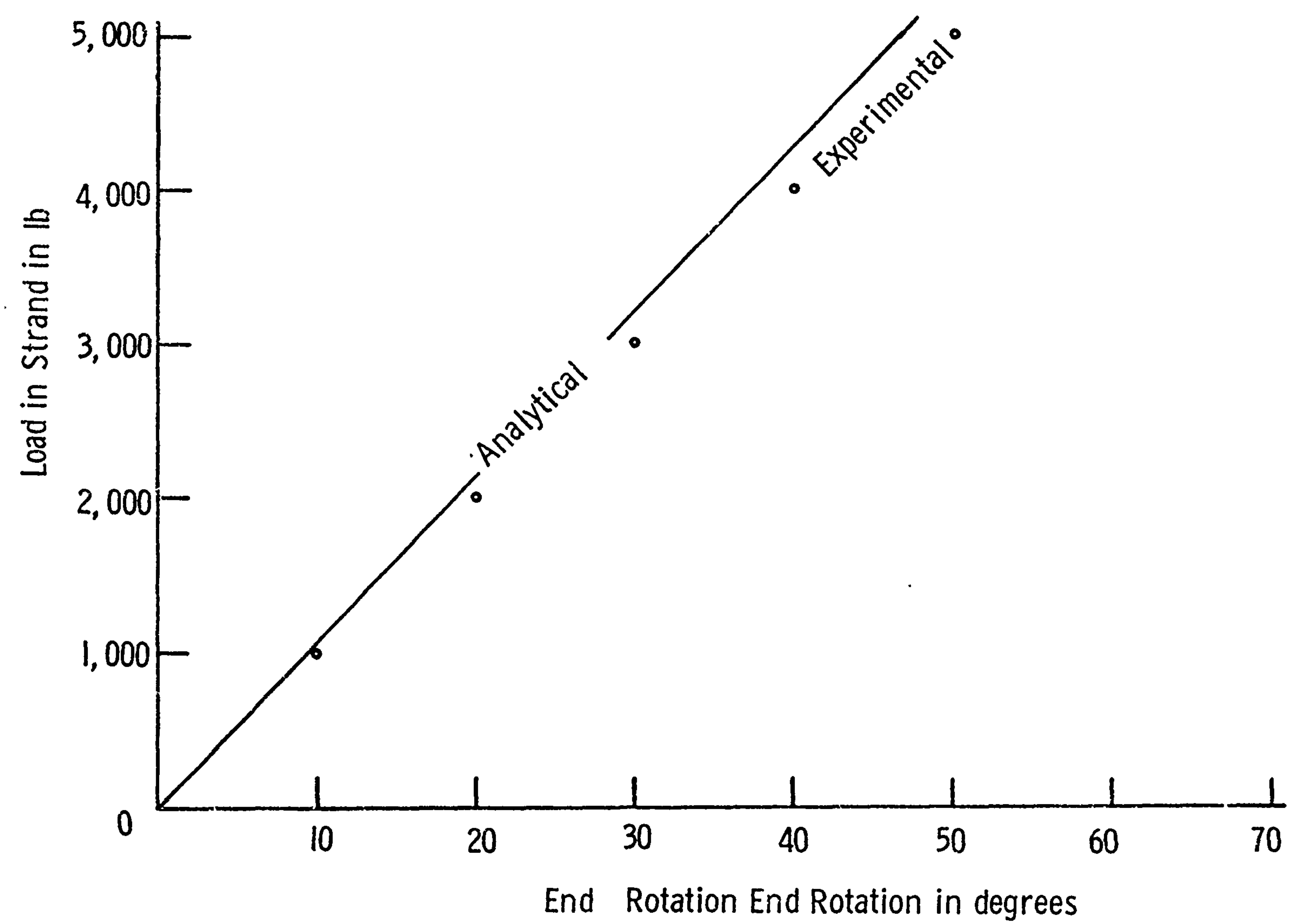

Fig. 3 Load vs. End Rotation in a 5/16-in. Strand with Free-End

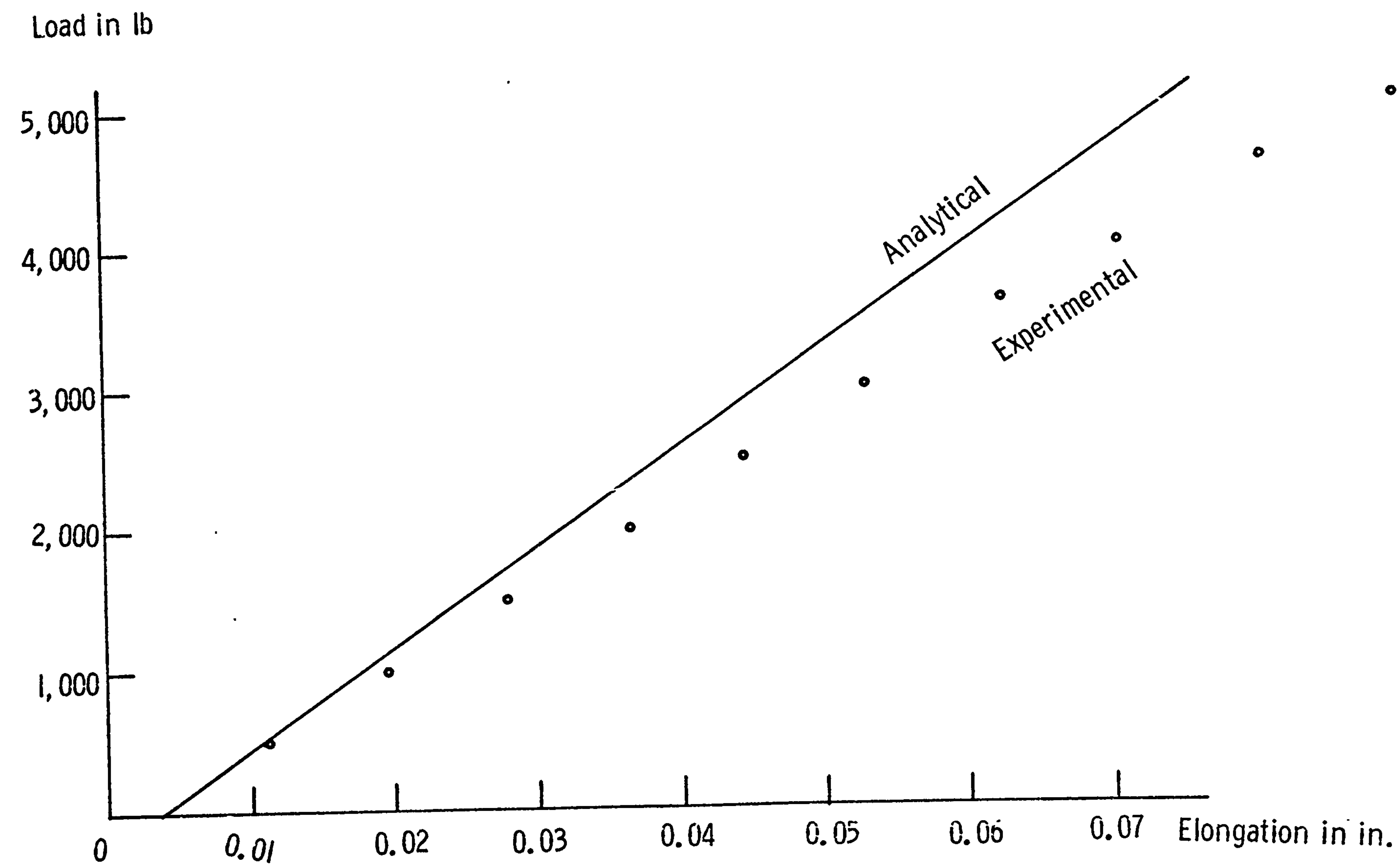

Fig. 4 Load vs. Elongation in a 5/16-in. Strand with Free-End

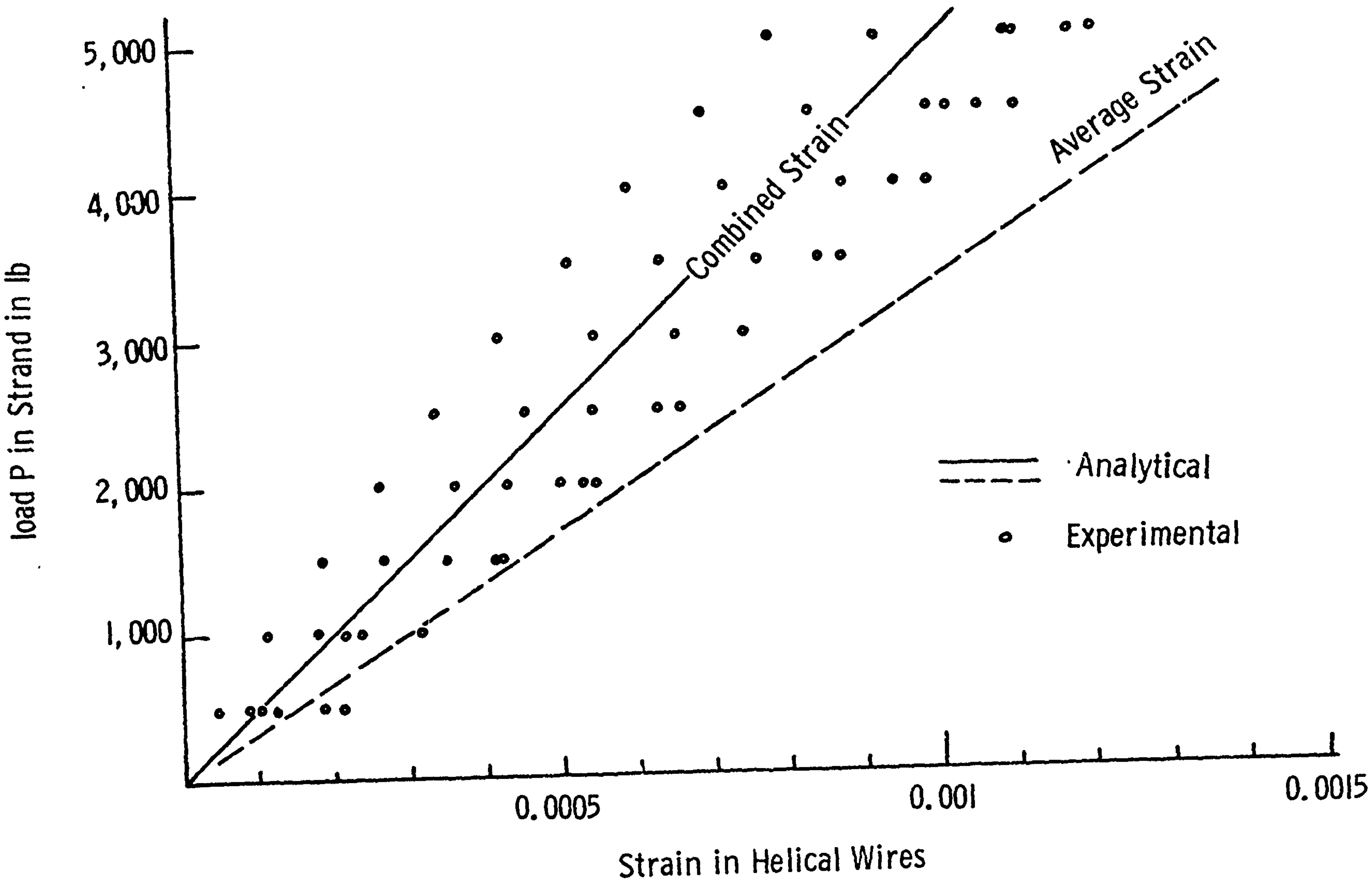

Fig. 5 Load vs. Strain in Helical Wires of a 5/16 in. Strand with Free-End

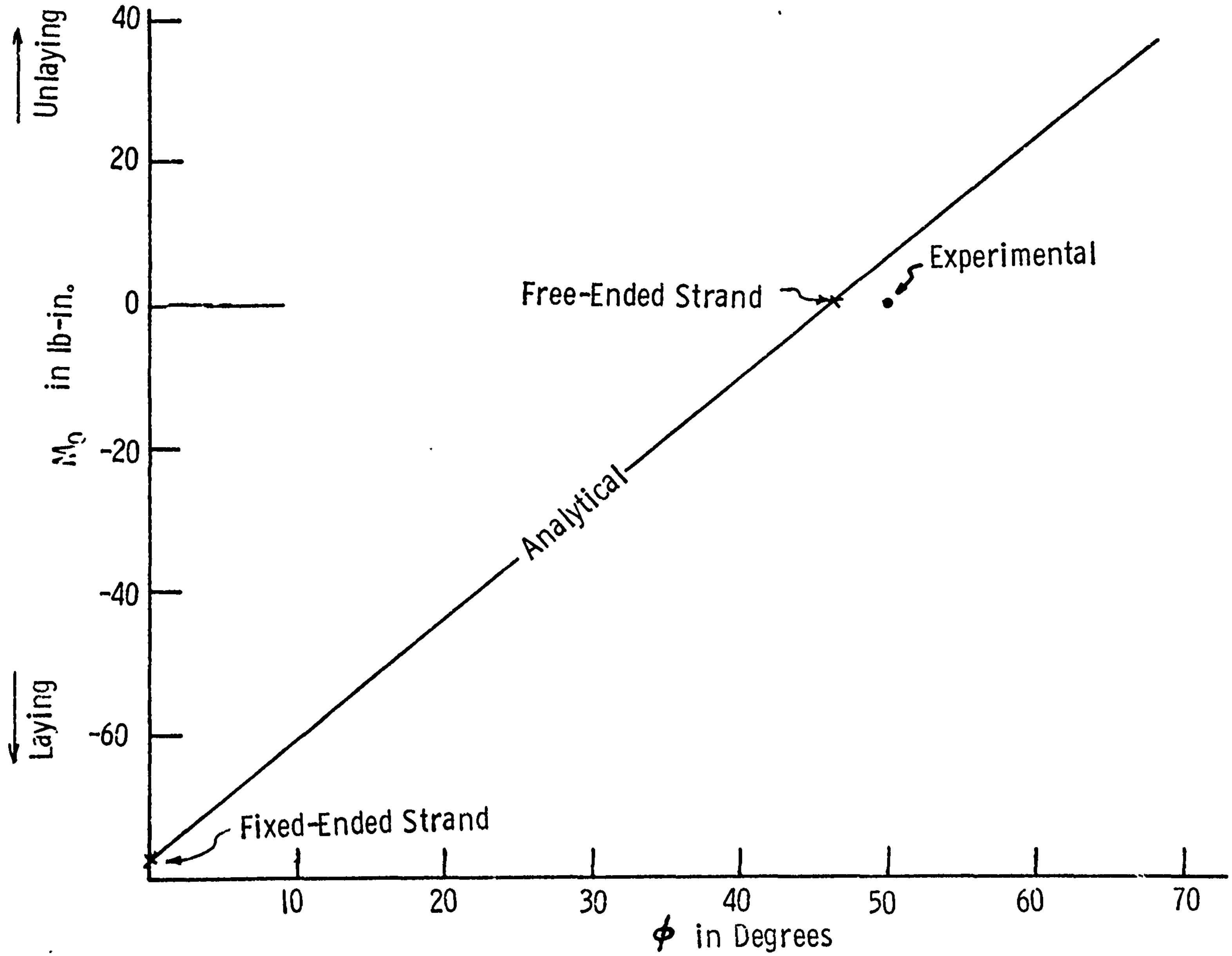

Fig. 6 Effect of Additional Moment to the End-Rotation Free-Ended Strand Fixed-ended Strand

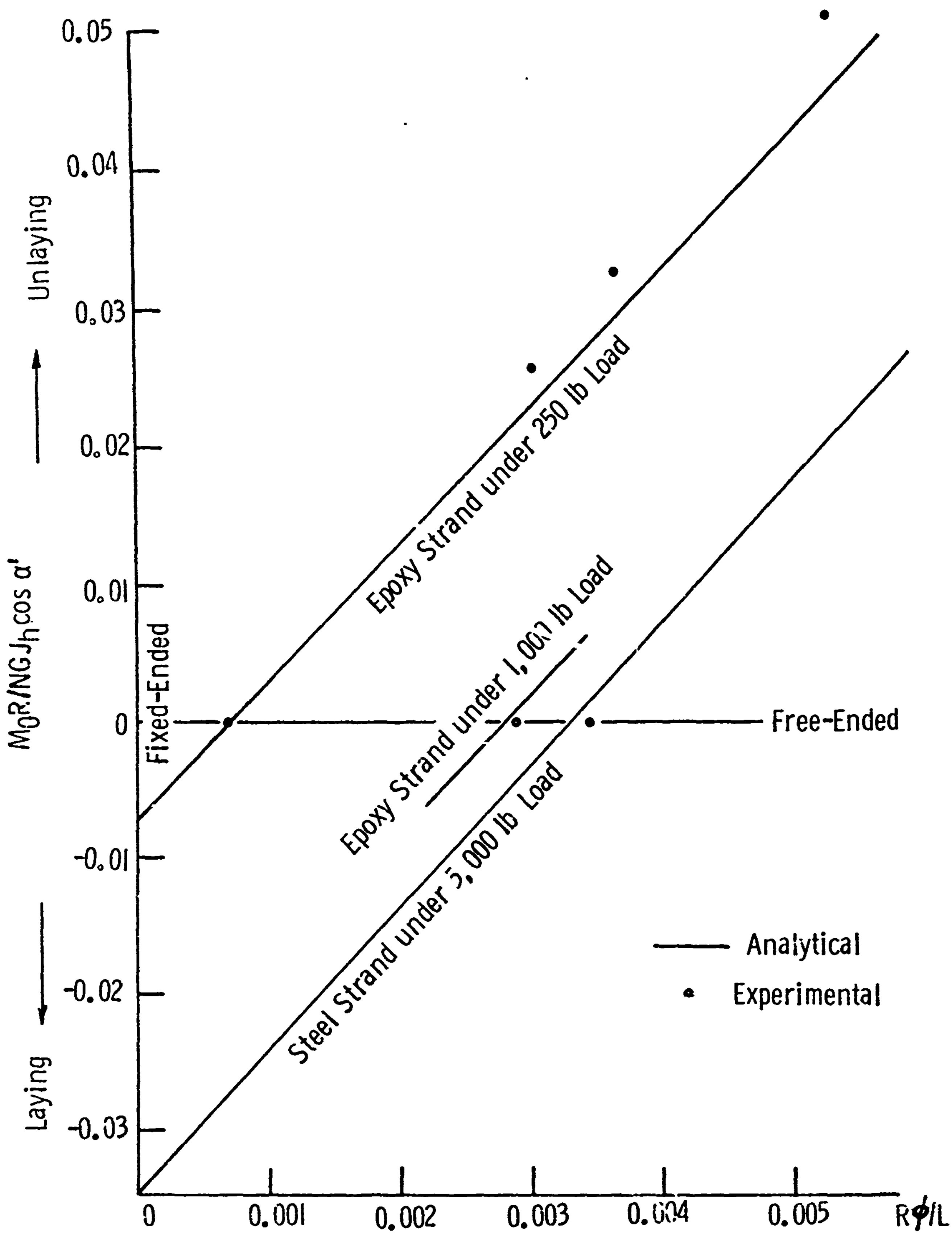

Fig. 7 Non-Dimesional Moment vs End-Rotation Coefficient in Different Strands in Combined Tension and Twisting