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BCL

-TR-71-2802

REPORT NO. 3.4

TITLE: NATURAL NUMBERS

AUTHOR: Gotthard Gunther

DATE: November 1, 1970

SPONSOR: AFOSR 68-1391, AFOSR 70-1865

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BIOLOGICAL COMPUTER LABORATORY
DEPARTMENT OF ELECTRICAL ENGINEERING, UNIVERSITY OF ILLINOIS, URBANA, ILLINOIS

46

UNCLASSIFIED
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) UNIVERSITY OF ILLINOIS BIOLOGICAL COMPUTER LABORATORY URBANA, ILLINOIS 61801	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP	
3. REPORT TITLE NATURAL NUMBERS IN TRANSCLASSIC SYSTEMS		
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Scientific Interim		
5. AUTHOR(S) (First name, middle initial, last name) GOTTHARD GUNTHER		
6. REPORT DATE 1 November 1970	7a. TOTAL NO. OF PAGES 42	7b. NO. OF REFS none
8a. CONTRACT OR GRANT NO AFOSR 70-1865 b. PROJECT NO 9768 c. 61102F d. 681305	9a. ORIGINATOR'S REPORT NUMBER(S) UILU-ENG-70-299 BCL Report 3.4 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) AFOSR - TR - 71 - 2802	
10. DISTRIBUTION STATEMENT Approved for public release; distribution unlimited		
11. SUPPLEMENTARY NOTES TECH, OTHER	12. SPONSORING MILITARY ACTIVITY AF Office of Scientific Research (NE) 1400 Wilson Boulevard Arlington, Virginia 22209	

13. ABSTRACT

The reason why natural numbers follow the unilinear sequence prescribed by the Peano axioms is that these axioms presuppose that natural numbers are mapped unto the classic two-valued system of logic. If the same numbers are mapped unto transclassic systems of logic which contain an indefinite number of values, any natural number selected may have several immediate predecessors and an indefinite number of immediate successors. The number of predecessors or successors depends on whether the structural basis of counting is provided by proto-, deutero-, or trito-structure. The paper shows the sequences which the numbers follow in each of these cases and their "horizontal" structural relations.

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NATURAL NUMBERS IN

TRANSCLASSIC SYSTEMS

by

Gotthard Gunther

Sponsored jointly by

Air Force Office of Scientific Research
Directorate of Information Sciences
United States Air Force

Grant [redacted]-AFOSR 68-1391
and
Grant [redacted]-AFOSR 70-1865

BIOLOGICAL COMPUTER LABORATORY
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URBANA-CHAMPAIGN
ILLINOIS

UILU-ENG-70-299

This paper is dedicated to the memory of

Warren Sturgis McCulloch.

The ideas expressed in the first part are to a great extent the result of a nightly session this author had with him toward the end of February 1969. May this paper stand as an expression of the author's deep and lasting indebtedness to a great scholar and a great man.

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PART I. MATHEMATICO - PHILOSOPHICAL PROLEGOMENA

Part II of this essay was written before Part I and offered to the Third Annual Symposium of the American Society for Cybernetics as a topic of discussion. However, owing to unforeseen circumstances, the paper was not presented at the Symposium. This turned out to be a blessing in disguise. In order to conform to the time limit for oral presentation Part II was written in a highly condensed manner and there was no opportunity to elaborate on the general epistemological aspect which served as the starting point for the intended confrontation between natural numbers and structural systems of higher complexity than our traditional logic offers. We are determined to make up for this omission in Part I because we believe that the theoretical goal of Part II will be better understood if the present author clarifies his attitude toward the basic concept of organism and its mathematical treatment in cybernetic research.

To begin with let us state that what this author has been doing for many years runs counter to the professed aims in cybernetic research. When Norbert Wiener defined cybernetics as research into "the essential unity of the set of problems centering about communication, control, and statistical mechanics, whether in the machine or living tissue," then such investigations as the present one, based on trans-classic theories of logic and ontology, run on exactly the opposite track. Theories of trans-classic logic and many-valued ontologies are introduced for the very purpose of showing <u>not</u> the "essential unity" but the <u>essential differences</u> in the concept of a machine and what Wiener called the living tissue. This is crass heresy in the High Church of Cybernetics. So far, the thesis of the essential ontological difference between what is traditionally called a machine and what is called a living organism has been largely ignored and this policy of leaving out of sight the intrinsic conceptual distinction between Life and an inanimate mechanism received a strong impulse when Norbert Wiener published his book Cybernetics. But Wiener's work starts, as is well known, with a chapter called "Newtonian and Bergsonian Time." This chapter already contains - despite the intent of its author - elements which cast heavy doubt on the theory of the essential unity of the set of problems which involve the machine as well as the living tissue. It is highly significant that Wiener's confrontation of Newtonian and Bergsonian time was not seriously followed up in subsequent research, and, in fact, this author remembers that one reviewer remarked about Wiener's book that he considered the essay on Newtonian and Bergsonian time quite superfluous and that Wiener might as well have started the presentation of his case with Chapter II which discusses Groups and Statistical Mechanics. In this paper exactly the opposite position will be taken and we want to point out that Wiener's distinction in the concept of Time leads to two different conceptual interpretations of the phenomenon which is called living tissue or organism or more generally Life.

It is a very old insight, gained already in Greek philosophy, that the phenomenon of Life is "dialectical." Plato explains what is meant by this term. If something has dialectical character it is impossible to find for it a theory in principle capable of full formalization. It will be necessary to develop two complementary theories for it which are, in a certain way, incompatible with each other (e.g., the corpuscular and wave interpretation of Matter) but which are both necessary for an exhaustive description of the phenomenon.

Present cybernetics, so far, has completely disregarded the dialectical aspects of the organism and concentrated in an entirely undialectic fashion on only one of the complementary aspects which the phenomenon of living tissue or of an organism incorporates. In the following pages we shall sketchily describe both of the two aspects accentuating their incompatibility and we shall begin with the one that has been one-sidedly favored by present day cybernetics.

An organism has, in the prevailing tradition, been described as something embodying a higher and more integrated unity than a mere classic mechanism which is a loose and partly arbitrary aggregation of anorganic parts. It has been said that an organism is dominated by the principle of superadditivity, meaning that it is essentially more than the sum of all its parts. This theory has sometimes been labelled as holism and the Oxford Dictionary defines the term as referring to a "tendency in nature to form wholes that are more than the sum of the parts by creative evolution." This provokes at once the question: If this is true for the evolutive time dimension of organisms (which corresponds roughly to Newtonian time), what about the complementary emanative time dimension? The epistemological assumption of holism seems to exclude that emanative developments will also drift towards superadditivity. On the other hand, one cannot have evolution without emanation and vice versa. This is structurally impossible. At any rate, the traditional concept of organism considers organic systems as things which result from a tendency to integrative togetherness with a mutual inhesion of parts tending to obliterate the identity of the part in favor of the reinforced identity of what is called the whole.

It should be pointed out that this holistic concept of organism which stresses the aspect of unity is in a significant agreement with the tradition of classic ontology. It will be useful to remind the reader that classic ontology is essentially monism: It considers the ultimate substratum of all reality as one-valued. It follows for this type of philosophy that the deeper we understand a phenomenon the more we will strive to see in it unity, continuity, homogeneity, and harmony. This is the classic scientific way into the depth dimension of Reality . . . leading ultimately into the coincidentia oppositorum of Nicholas of Cues. The concept of universal Being which unites and reconciles the opposites is nothing if not the expression of the belief that the whole Universe represents to us the aspect of an unbroken context of objective existence. It is inevitable that in this climate of thinking the organism appears as a marvel of integration and of a prestabilized harmony. Life, for the classic thinker, is the blessed state in which the parts of the Universe "know" each other in the sense in which the Bible uses the term in Chapter IV of Genesis: "And Adam knew his wife." The coincidentia oppositorum is maximal superadditivity.

The one-sidedness of this epistemological attitude is revealed when we look at one of the highest triumphs of cybernetics. Thanks to an early paper of Warren McCulloch and Walter Pitts, we know that any property of an organism which we can define in finite and non-ambiguous terms can be repeated and duplicated in a non-organic "classic" machine. And let us not forget that all our non-organic hardware has grown out of theories based on classic mathematics, two-valued logic and one-valued ontology. With this reminder let us have a look at the consequences of the McCulloch/Pitts discovery.

It has been said by philosophic pragmatism that we can only understand what we make. But we might also say, and with equal right, that if we can build a machine which displays - let us say - all possible behavioral traits of memory, then we have dispensed from a technical viewpoint with the necessity to understand what memory is in a living person. We know that in a living person memory establishes personal identity lasting through a given span of time; but no cyberneticist has ever seriously asserted that, if we have designed "memory" into a piece of hardware, we have infused the latter with a sense of personal identity. It is also nonsensical to think that, with the present methods of improving the memory functions of computers, we can even approach the total role the Platonic Anamnesis plays in a living system. Cybernetics as a technical discipline does not even aim at repeating an organism itself in hardware, it only aims at repeating observable behavioral traits of organisms. It is totally indifferent to the problem of whether such behavioral traits occur in an animated biological system or in an inanimate classic mechanism - and whether such difference of locality may produce a different hermeneutic significance for otherwise totally identical traits. The most important epistemological lesson that can be learned from the McCulloch-Pitts paper is that it defines - quite unintentionally but sharply - the boundary between cybernetics as a basic hermeneutic science which wants to understand the phenomenon of Life and cybernetics as a science of sophisticated mechanics which reaches for the technical know-how to imitate the results which organisms achieve within the objectively observable section of Reality. On the other hand, the process by dint of which memory contributes to the establishment of self-referential personal identity within a living person does not belong to the field of the objectively observable, which means we cannot imitate it. Being technically imitable and being on principle observable are, epistemologically speaking, synonymous terms. Thus the McCulloch-Pitts paper indirectly opens up a field of as yet untouched hermeneutic cybernetics where this novel discipline does not want to repeat and imitate living systems as a feat of hardware or even software engineering, but where we strive to understand what is left after the engineer has done his work.

The shortcomings of the present one-sided trend of cybernetic research make themselves felt especially when we consider the fact that cybernetic methods have made deep inroads into the Humanities, investigating, for example, problems of literature with statistical methods. It stands to reason that the cyberneticist will play his games merely on the surface of the Humanities if he ignores the hermeneutic approach of understanding what is already known in a merely factual sense. The traditional hermeneutic methods, however, fail because their incompatibility with the requirements of exact algorithms as they are now used in cybernetics is too great. What is urgently required is an algorithmetization of hermeneutics. Such algorithmetization would fall within the domain of transcendental logic. But - although cybernetics so far has completely ignored transcendental logic - the great irony is that it has willingly adopted a fateful prejudice to which the instigators of transcendental logic (Kant, Fichte, and Hegel) paid homage. It is the seemingly unshakeable prejudice that hermeneutical processes are entirely incapable of formalization.

Unfortunately, we cannot raise the demand for hermeneutic understanding in addition to factual knowing without having the dialectical issue emerge again. It must be understood that hermeneutics has no field for its interpreting activity unless a problem has been formulated in such a way that any attempt at its solution forces us to develop mutually incompatible but strictly complementary theories.

A dialectic theory of organism, based on the principle of conceptual complementarity, has not yet been developed. We have pointed out that cybernetics, up to now, has favored an entirely one-sided non-dialectical concept of organism, obtaining remarkable but equally one-sided results. These results have already had a considerable impact on present society. But since their one-sidedness and concomitant social ambivalence has not yet been clearly recognized, computer theory has, sociologically speaking, proved a limited boon but to a much greater degree a calamity.

We have characterized the holistic viewpoint of organism as one where everything seems to gravitate toward a center of self-referential integrative identity with its relative reconciliation of the opposites which produces a precariously balanced superadditive unity. We shall now describe the complementary aspect of living systems which reveals itself in the intrinsic dishomology of incompatible elements straining away from each other and drifting towards dissolution of the wholeness which the other aspect shows us. Both tendencies coexist in living systems and are equally characteristic of them. Their difference is closely related to the fact that each living system has an evolutionary as well as an emanative history.

So far cybernetics has looked on living systems overwhelmingly from the viewpoint of evolution and, seen from here, the development of these systems seems - as we pointed out - to tend toward higher and more integrated forms of unity and wholeness, implementing the principle of a "transcendental" superadditivity. On the other hand, if we look at the phenomenon Life as a result of emanation, exactly the opposite type of properties seems to govern the development. Emanatively speaking, the development of systems of higher and higher organic complexity seems to accentuate a tendency towards internal disunity and disintegration. Seen from here the development seems to be guided by a principle which we might call that of super-subtractivity. In the decay of a dying system more is lost than the sum of its parts. Moreover, organic systems incorporating an unusually high complexity are capable of an intensity of dissension and disharmony which cannot possibly develop in systems of lower organization, because there is in the latter not sufficient structural richness to entertain such pitch of dissonance and discongruity as may originate in highly complex systems with intricate mediative functions - which are subject to failure. The richer a structure the more it displays incompatible properties which not only resist unification, but positively favor, by the ever-increasing amplitude of their negations, the disjointive character of the system. In other words, the stronger the trend toward superadditivity, the stronger the complementary trend towards a "super-subtractivity." This second tendency is dominated by a universal structural property which does not occur explicitly in classic one- or two-valued systems, but makes its entrance at once when we proceed to the most elementary form of trans-classic systems which possesses three values.

This author has pointed out in a former publication that all logical values in a three-valued system arrange themselves in a dichotomy of acceptance and rejection values. The acceptance values, of course, enforce through their activity the holistic tendencies of the system. The rejection values, on the other hand, work in the opposite direction. Their activity tries to pull the system apart. But the rejection values are only comparatively weak symptoms of a deeper structural feature that we have christened in Part II with the name of "discontexturality." What we mean by this term is explained in Part II by the effect discontexturality exerts on one of the simplest structural phenomena we know, namely the unilinear sequence of natural numbers. We shall notice that discontexturality dissolves the conceptual unity of any given natural number. It infuses into the general concept of natural numerosity a dialectic ambiguity. This brings us into conflict with the traditional logical theory of natural numbers where a number is considered a predicate of a predicate in the extended predicate calculus of classic logic.

The logical ambiguity and amphibology of natural numbers raises an interesting issue about the future development of mathematics with regard to its application to cybernetics. How far - we must ask - are the present mathematical theories geared toward furthering the viewpoint of holism? And what should be done to make mathematics a tool for effective investigation of the phenomenon of discontexturality? One thing is certain: Our traditional mathematical theories and practical procedures are basically derived from the duumvirate of one-valued ontology and two-valued logic. It is highly significant that the theory of many-valued ontologies which is an unavoidable consequence of discontexturality is not even mentioned in the foundation theory of mathematics. The use of many-valued logic is mostly declined because - so it is argued - it leads to unresolvable aporias.

It is not yet recognized that aporias engendered by many-valuedness of formal logic are the safest indications that we have finally arrived at the point where the dialectic structure of modern science emerges. It is characteristic for the traditional monistic and strictly non-dialectic tendency of science that these aporias are merely evaluated as subjective errors of thinking and taken as a signal for retreat from established classic methods. During the last decades enormous efforts were made to remove such aporias in the interest of a philosophic holism which aims at mapping the Universe as a system of total consistency within the contexturality of one-valued Being. On the other hand, it is also a well-known fact that all efforts to construct a scientific theory which views our Universe as an unbroken context of one-valued objective existence have failed. But let us say that such efforts have not failed because our subjective technique of reasoning was faulty (as is generally assumed); they have failed - even if impeccable reasoning was applied - because the Universe we live in does not present itself to us as an unbroken context of objective existence. It was not recognized that the emergence of unavoidable contradictions, aporias, antinomies and paradoxes in logic as well as in mathematics was not the negative symptom of a subjective failure but a positive index that our logical and mathematical reasoning had entered a new theoretical dimension with novel laws for which the classic tradition of human rationality - although impeccable in itself - provided hardly any antecedents. Contra-classic historical antecedents beholden to this mysterious dimension were available, but they belonged within the history of the human mind into the dubious and slightly disreputable side-show where Pythagorean mystical number speculations, gnosticism, the arithmetical games of the Kabbalah and Lullianism eked out their neglected and scurrilous existence. It was Schelling who made a valiant attempt to rehabilitate this tradition in which especially the paradoxes were well at home. Schelling did not succeed - in fact, he acquired only for himself a doubtful reputation. What his opponents did not see was that one had to make a distinction between a legitimate thought and its sometimes doubtful method of application. There can be no doubt that the whole success of exact Western Science was due to the fact that, since the times of the Greeks, a most rigorous process of eliminating and discarding highly legitimate problems was going on. Only such problems were selected for investigation where suitable means could be found to treat them in a controllable and rational manner - derived from the principles of classic logic.

Book M of Aristotle's Metaphysics is an excellent example of how questions were eliminated from number theory for which there could be no intelligent answer expected on the level of classic tradition. No wonder the discarded questions and problems degenerated in the course of history and were discussed in a manner which removed them farther and farther from the grasp of responsible science. But let us repeat: Their original banishment is in no way prejudicial to their seriousness, depth, and validity. One of the most important of the problems involved was the Pythagorean conception of an Arithmetica Universalis as Helmut Hasse has called it. By this term he meant the idea of a complete arithmetization of ontology all the way down to the individual object. The guiding idea of the Pythagoreans was that if any two things relate to each other in the manner of numbers, then they are themselves numbers in an ontological disguise. The idea of the Arithmetica Universalis was quickly discarded, because the scientific tradition founded by the Greeks implied that scientific reasoning was only entitled to interpret a universe from which all traces of subjectivity had been removed. On the other hand, it was entirely impossible to deal with the problem of an Arithmetica Universalis without raising the question: What is the difference between a subjectless Universe and a Universe which harbors subjectivity?

But the difference between a subjectless universe of straight objective contexturality and a universe animated by subjectivity is, on the greatest scale, equivalent to the distinction between the holistic and the discontexturalistic viewpoint which we derive from the idea of organism. A modern novelist (Franz Werfel) lets one of the figures in a utopian novel answer the question: What is the shape of the universe? The terse reply is "The whole has the shape of Man." The universe Werfel's questioner referred to is undoubtedly the universe gifted with subjectivity of which the complex organism is a structural replica. This universe was <u>not</u> the object of theoretical science as the Greeks conceived it and handed it down to us. Their enormous instinct for abstract theory made them aware that initial scientific progress could only be achieved if the object of investigation was sufficiently simplified to satisfy very elementary and basic methods of research. This led them to the ontological reduction from the animated universe to the subjectless concept of Reality. Such reduction permitted them to retain as an ultimate metaphysical perspective the holistic viewpoint by postulating the total unity of the primordial substratum, but forced them to exclude the anti-holistic viewpoint of discontexturalism from ontology. The latter viewpoint was implied in the philosophy of Heraclitus, a philosophy that significantly exerted no lasting influence on the evolution of scientific methods.

By performing their ontological reduction, focussing on holism, but discarding all motive of discontexturality in their metaphysics, the Greeks avoided the difficult confrontation between formal logic and mathematics on one side and the theory of dialectics on the other. It should be added that we moderns have followed faithfully this Greek tradition up to the present day. On the side of philosophy a first attempt to defect from the Greek tradition was made in Kant's Critique of Pure Reason, especially in his doctrine of Transcendental Dialectics. Fichte, Hegel, and Schelling followed in his path, but mathematics and empirical sciences have up to now avoided the issue completely. Their methodical ideal is still the radical objectivization of their subject-matter.

However, since the advent of cybernetics it has become impossible to dodge the problem of subjectivity any longer. And since cybernetics cannot progress without the proper mathematical tools, mathematical foundation theory is confronted with the demand to forge new instruments capable of dealing with the peculiar properties of self-referential systems. This calls for a revision of the traditional logical and ontological basis of mathematics. The mathematics of our day still starts from the assumption of neo-Platonic henism and its vulgar derivative, called monism. But henism leads inevitably to holism. It follows that our present mathematical reasoning is not yet geared to the idea of a Universe which reveals itself to us only as a dialectic union of holism and discontexturalism. It has been said before that such dialectic union demands a specific logic and concomitant arithmetic to deal with the contradictory, but also complementary, aspects which the appearance of Life in a physical Universe displays.

On the one hand, living systems must be considered as contexts of objectivity infused with a subjectivity that is progressively objectivizable. On the other hand, living systems must also be regarded as contexts of subjectivity which have been generated by a gradual subjectivization of the natural objects. The difficulty is that both processes are not exactly inverse to each other. A subtle asymmetry is involved. Thus the confluence of the objectivization of the subject and the subjectivization of the object produces intricate structures which are nowadays not even remotely understood because we have not yet developed any mathematical tools for a progressive formalization of dialectic logic. These tools are so far missing because we do not yet possess a dialectic theory of natural numbers.

A first attempt to dialectically analyze the concept of natural numbers was made by Plato. But since it ran counter to the general trend of Greek science the problem was practically forgotten till modern times when mathematics was confronted with the problem of many-valued logic. But even now very little progress has been made because there is no general agreement about what many-valued logic really is. In fact, two concepts of many-valued logic exist side by side: In 1921 Emil L. Post introduced a triadic classification of "positive," "negative," and "mixed" for the traditional functions of two-valued logic. In other words: Any additional values are here considered as "mixtures" of positive and negative. This means that the original ontological conception of two-valuedness is retained and any additional values are only second-order derivatives from the two classic values. But there exists also (without giving up the theory of "mixed" values) the possibility of interpreting any additional values which are added to positive and negative as different in kind and not derived from a mingling of positive and negative. In the first case a theory of many-valuedness results that is symmetrical but not completely formalizable. Any radical formalization of it makes the original two-valuedness reappear. The second interpretation of many-valuedness leads to an asymmetrical system where radical formalization does not mean retrogression to two-valuedness. If we introduce Post's concept of many-valuedness, there is no need for a revision of the classic ontological foundation of mathematics; no problem of dialectics is involved. On the other hand, if we confront mathematics with the second interpretation of many-valuedness, we obtain an additional richness of ontologically interpretable structure which requires a new mapping; it is then reasonable to ask whether the theory of real numbers could be derived by an analogue method from rational numbers - and so on down the hierarchy of numerical types to the natural numbers at the bottom. What is still missing is the postulated connection between the rational and the real numbers. If this connection were found we would be entitled to say that all higher types of numbers are nothing else but very sophisticated aspects of natural numbers and their properties - properties derived from classic systems as well as from trans-classic ones.

As a reason for the difficulty of finding the missing link, the argument is usually given that, if we start discussing real numbers, we get involved with the problem of infinity. For this very reason it seems impossible to interpret the calculus of real numbers as a calculus with any finite sets of rational numbers. However, the argument - although correct as far as it goes - does not satisfy a logician who takes trans-classic systems into consider.tion, because so far mathematical foundation theory has made no distinction between concepts of infinity, relating to a subjectless universe and those relating to a universe endowed with the property of self-reference. What is infinite per se in the first universe may be treated as finite in the second. Seen from here, the otherwise divergent approaches to the theory of real numbers of Weierstrass and Cantor on one side and Dedekind on the other seem to be almost identical. In both approaches the set used for the definition of a real number must be infinite. But if one reads the relevant papers of Cantor as well as Dedekind's "Stetigkeit und irrationale Zahlen," one cannot escape the impression that these famous mathematicians are only concerned with the behavior of numbers in a subjectless universe.

Part II tries to show how natural numbers behave in a universe that embodies self-reference. However, there is a fundamental distinction between the idea of a self-referential universe as it was conceived in a former mythical philosophy of nature, as, for example, in Fechner's "Weltseele," or, if we want to go back to the most ancient Scriptures of mankind, as in the saying of the Chhāndogya Upanishad "Self is all this," and the idea of self-referentiality as we conceive it here. In the mystical philosophy of nature it was assumed that the universe was self-referential as a whole - because no distinction was made between auto-referentiality and self-referentiality. This led, if a living system was considered to be a (complete or incomplete) structural replica of the universe, automatically to the holistic interpretation of an organism. In contra-distinction to this tradition we maintain, however, that, although the universe as a whole may be considered to be auto-referential, it can have the property of self-reference only in preferred ontological locations of suitably high complexity of structure. It is this distinction that has led us to the dialectical antithesis of holism and discontexturalism with regard to the interpretation of living systems. In the evolutive striving towards Life, mechanisms developed more and more holistic aspects and followed a principle of super-additivity. But all Life is condemned to decay and Death, which shows up in the theory of structures as the property of discontexturality and a tendency toward super-subtractivity.

In the following Part II we have made a first tentative effort to lay a foundation for a theory of mathematics for living systems. However, owing to the external circumstances which dictated the composition of Part II, we have confined ourselves to the utilization of the discontextural and not the super-subtractive motive in discussing the behavior of natural numbers in trans-classic systems.

PART II. THE MAPPING OF NATURAL NUMBERS ONTO KENOGRAMMATIC STRUCTURES

In Lewis Carroll's profound fairy tale THROUGH THE LOOKING GLASS there is a scene where Tweedledum and Tweedledee have led Alice to the Red King who is sleeping in the woods: "'He's dreaming now,' said Tweedledee: 'and what do you think he is dreaming about?'
Alice said 'Nobody can guess that.'
'Why, about you!' Tweedledee exclaimed, clapping his hands triumphantly. 'And if he left off dreaming about you, where do you suppose you'd be?'
'Where I am now, of course,' said Alice.
'Not you!' Tweedledee retorted contemptuously. 'You'd be nowhere. Why, You're only a sort of thing in his dream!'
'If that there king was to wake,' added Tweedledum, you'd go out - bang - just like a candle!'
'I shouldn't!' Alice exclaimed indignantly. Besides, if I'm only a sort of thing in his dream, what are you, I should like to know?'
'Ditto,' said Tweedledum.
'Ditto, ditto!' cried Tweedledee.
He shouted this so loud that Alice couldn't help saying 'Hush! you'll be waking him, I'm afraid, if you make so much noise.'
'Well, it's no use your talking about waking him 'said Tweedledum, 'when you're only one of the things in his dream. You know very well you're not real.'"

Lewis Carroll is talking about one of the most ancient, most fundamental and most unsolved problems of Philosophy. It is the problem of the relation between Reality as a Prototype and its re-appearance in the Image. The Image is an iteration of the Prototype; it repeats or maps the contexturality of the Prototype and, as a mere repetition, it is structurally identical with what it has mapped. What distinguishes the Image from the Prototype and establishes logically their identies is their mutual discontexturality. No matter how loud the discourse between Alice and the Tweedle brothers may get, it will not wake the Red King, because the existence or mode of Reality of Alice and the Twins is discontextural with the physical body of the King who is - or seems at least - to be lying in front of them in the grass. The chain of cause and effect as well as that of logical reason and consequence always ends within the limits of a given contexturality.

We shall equate the specific context of existence to which the body of the King belongs with an ontological locus and say that the body of the Red King occupies a different ontological locus from the one in which Alice - who has stepped through the looking glass - exists. What Tweedledum tries to explain to Alice is that the events occurring within a given contexturality will not carry over into a different one.

However, if neither the chain of cause and effect nor the linkage between reason and consequence will carry over from one ontological locus to another which belongs to a different contexturality, then any counting process will also be confined to the contextural limits within which it originates. In other words: for a trans-classic logic which derives from a plurality of ontologies and ontological loci there is not just one sequence of natural numbers. There are many such sequences, all obeying the so-called Peano axioms but separated from each other by their mutual discontexturality.

Our traditional scientific methods use a logic that is based on the contraposition of two ontological loci which refer to the metaphysical designations of Substance or Being on one side and Negativity or Nothingness on the other. Their discontexturality is obvious. No chain of events, started within the ontological locus of Being will carry over into and continue within Nothingness. The same goes, of course, for the process of counting. It is trivial to say that we shall stop counting if there is nothing to count. The point we intend to make here is that our original process of counting will also stop, if we switch over from one ontological locus to another and discover that there is something to count. This will always be the case, if the other ontological locus belongs to a different contextural domain which may combine two or more ontological loci. Such a cross-over into a different contexturality would force us to start another sequence of natural numbers.

Since classic logic has at its disposal for objective descriptions of Reality only a single ontological locus, there will be no question of different contexturalities. Everything belongs to the same ontological context or it is just nothing. Being is one-valued - it just is. That is all there is to it. That our classic logic is two-valued is entirely due to the fact that it represents a mapping process. You may have something that is one-valued, but you cannot map something with one value. But it should not be forgotten that the second value plays only a supporting and assisting role. Ontologically speaking it designates nothing. One-valued Being is auto-referential. It refers to nothing outside its own contexturality. Simply because there is none. Auto-reference is reference between different elements belonging to the same ontological locus. It follows that a scientific world concept coupled with a two-valued mapping process is forced to exclude from scientific inquiry all such phenomena as do not belong to the contexturality of one-valued Being. On the other hand, it is evident that the very process of thought is hetero-referential. Thought refers to and maps something that is not Thought and that does not belong to Thought contexturality. The belief that Thought-context and objective Being are contexturally identical is Magic. The magic formula of a sorcerer would have a physical effect only if formula and object belonged to the same ontological locus. But a formula is only a hetero-referential image of something that is and it partakes qua image in a different contexturality.

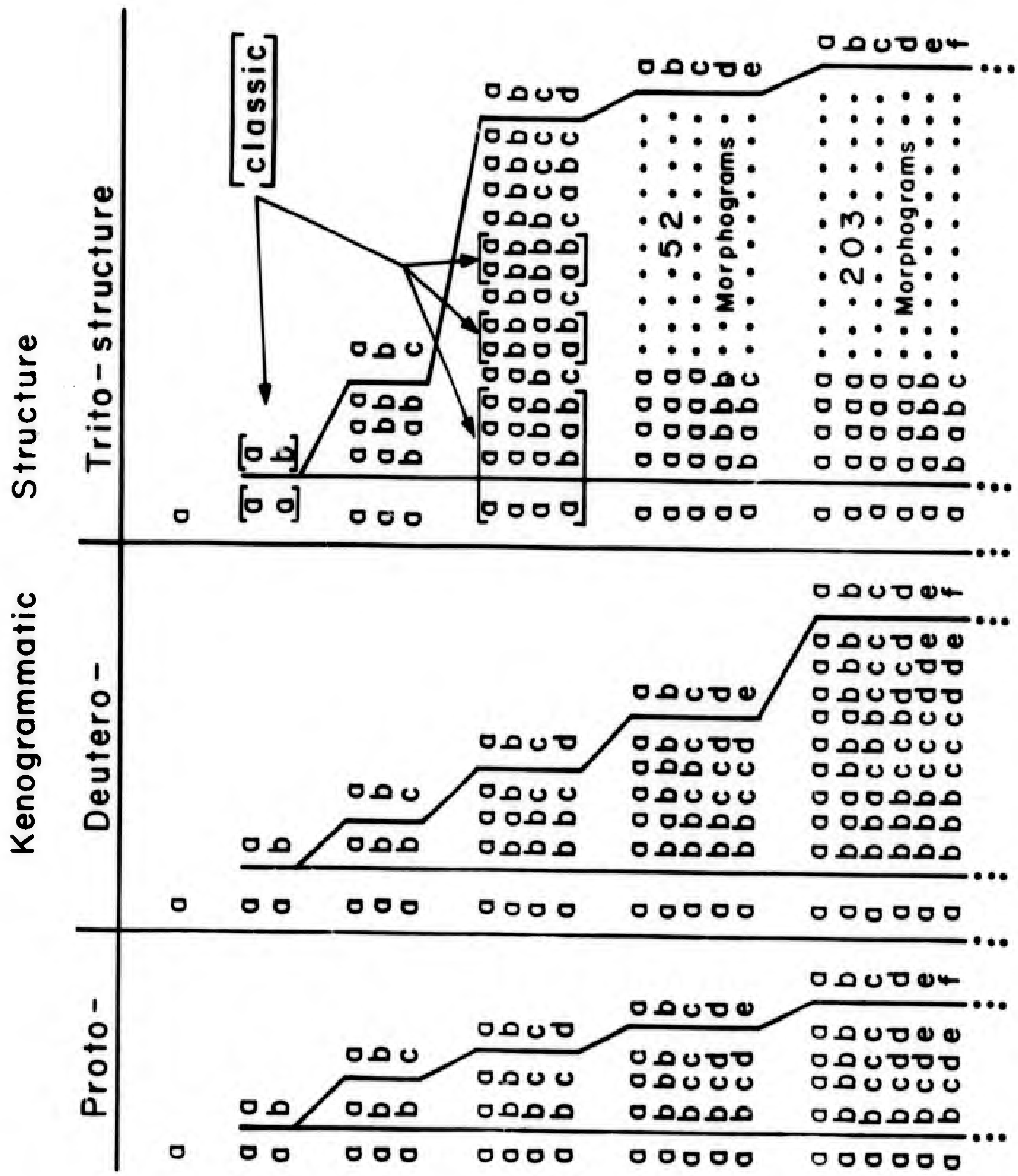
Unfortunately, we possess hetero-referentiality where the reference carries from one locus to another merely as a "subjective" vehicle for scientific inquiry. This vehicle is our traditional two-valued logic. We are not yet in possession of an ontology supporting a concept of Being where "Being" would not only refer to something that just is but to one that, at the same time, has an image, because it is capable of sustaining "objective" processes of hetero-reference. But if we talk about entities capable of hetero-referential actions we refer in fact to living systems and imply a trans-classic concept of Existence which can accomodate discontexturality. However, the integration of discontextural elements that characterizes living bodies does not exhaust itself in the simple distinction-between Prototype and its hetero-referential Image (including the image-making process). The phenomenon fills more than two ontological loci. In fact, the number of ontological loci involved in the existence and the activities of an organism is practically inexhaustible, and we can only point out that Time, e.g., requires its own ontological loci and so does Subjective Self-reference and also the juxtaposition of the I as the subjective ego and the Thou as its objective counterpart. We leave the question open as to how many ontological loci are required to establish all these separate contexturalities. That a contexturality is identical with a single ontological locus is true only for special cases of considerable simplicity. It is our contention that organisms encompass an indefinite number of ontological loci and, concomitant with this, different contexturalities. If we combine this assertion with the insight that a system of natural numbers is always confined to the specific contexturality within which it originates, then we will be forced to the conclusion that we need a special theory of natural numbers for the phenomenon of living matter. The issue does not exist in traditional two-valued mathematics because the latter has only one ontological theme, namely that of auto-referential Being. And the auto-referential sequence of natural numbers is defined by the well-known axioms of Peano. The key-axiom is: no two numbers have the same successor. Wherever this axiom is valid we shall henceforth speak of a Peano sequence.

Since the structural properties of hetero- and self-reference can only be described in a trans-classic system of logic it will be necessary for a mathematical theory of living systems to map the natural numbers onto the basic logical elements of trans-classic logic. These elements, however, are not the values but the kenograms, i.e. empty places which merely indicate structure and which may or may not be occupied by values. Either a single or a collection of kenograms may represent an ontological locus. If an ontological locus coincides with a single kenogram we shall say that the resulting system has auto-referential contexturality. No two ontological loci may have the same number of kenograms. The supply of kenograms is infinite and, for the purpose of mapping numbers onto them, they may be composed in sequences of any required length. This affords us two choices of composing kenogrammatic sequences: we may either repeat a single kenogram until the predetermined length of a sequence is reached, or we may fill the sequence with kenograms of different shape. (We shall use as symbols for kenograms the small letters of the Latin alphabet.) Kenogrammatic sequences of constantly increasing length, added vertically to each other, and those of equal length, joined horizontally, form what shall be called a kenogrammatic structure which can appear in three degrees of differentiation. We shall call them

proto-structure
deutero-structure
trito-structure.

In proto-structure (see Table I) only one kenogram is iteratable and the placing of the symbols is irrelevant. In deutero-structure any symbol is repeatable, provided there is room for repetition; the placing still remains irrelevant. What distinguishes trito-structure is that now the placing of symbols becomes important.

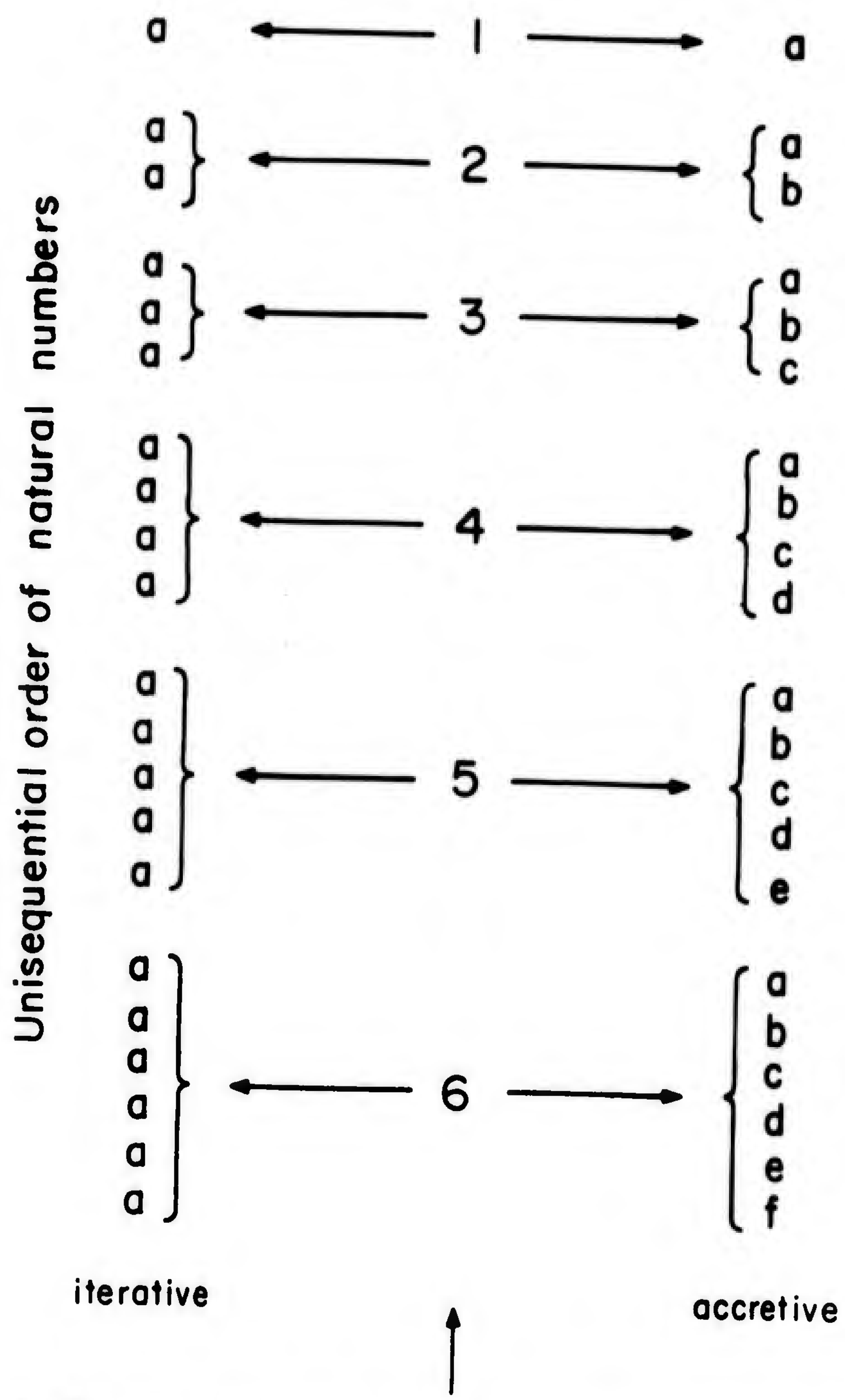
Table I


Kenogrammatic Structure
Proto-
Deutero-
Trito-structure
classic
52 Morphograms
203 Morphograms

In order to undertake the mapping of natural numbers onto this structure we shall, for the time being, only consider the kenogrammatic sequences which are outside the vertical lines of Table I. We notice that the ones on the extreme left side are always formed by writing the only available symbol again and again. We shall call this "increase by iteration." If we turn to the right side we notice that no symbol is ever repeated; we shall call this "increase by accretion." In Table II we have confronted the Peano numbers with symbol sequences originated by straight iteration and by straight accretion which constitute the boundary cases of the kenogrammatic structure. The integers in the center of the table represent the Peano numbers, and the arrows, pointing to the left and to the right, aim at the kenogrammatic sequences with which the numbers are associated.

It is now possible to make the following statements: Each set, consisting of n kenogrammatic places on the left side, belongs to the same natural numbers as the set consisting of n places on the right side. If we take, for example, the two kenogrammatic sequences on the second line, then both sets are doubletons. Equally, the set consisting of three places on both sides of the third line is a triple; i.e. each set belongs to the same natural number. Thus, kenogrammatic sequences of equal length always coincide arithmetically with the integer between them....provided they are built up by the principle of consistent iteration or accretion. There is no difference between iteration and accretion in terms of Peano numbers.

Table II



A Peano-sequence of natural numbers is indifferent to the distinction of iteration and accretion

This indifference, however, disappears at once if we introduce two or even more ontological loci. It should be remembered that, although the first ontological locus requires only one kenogram, the second - to distinguish it from the first- requires two. This means that we encounter for the first time a trans-classic logical situation when we arrive at three-place sequences, because in this case we do no longer face a simple contraposition of pure iteration or pure accretion, since the sequences

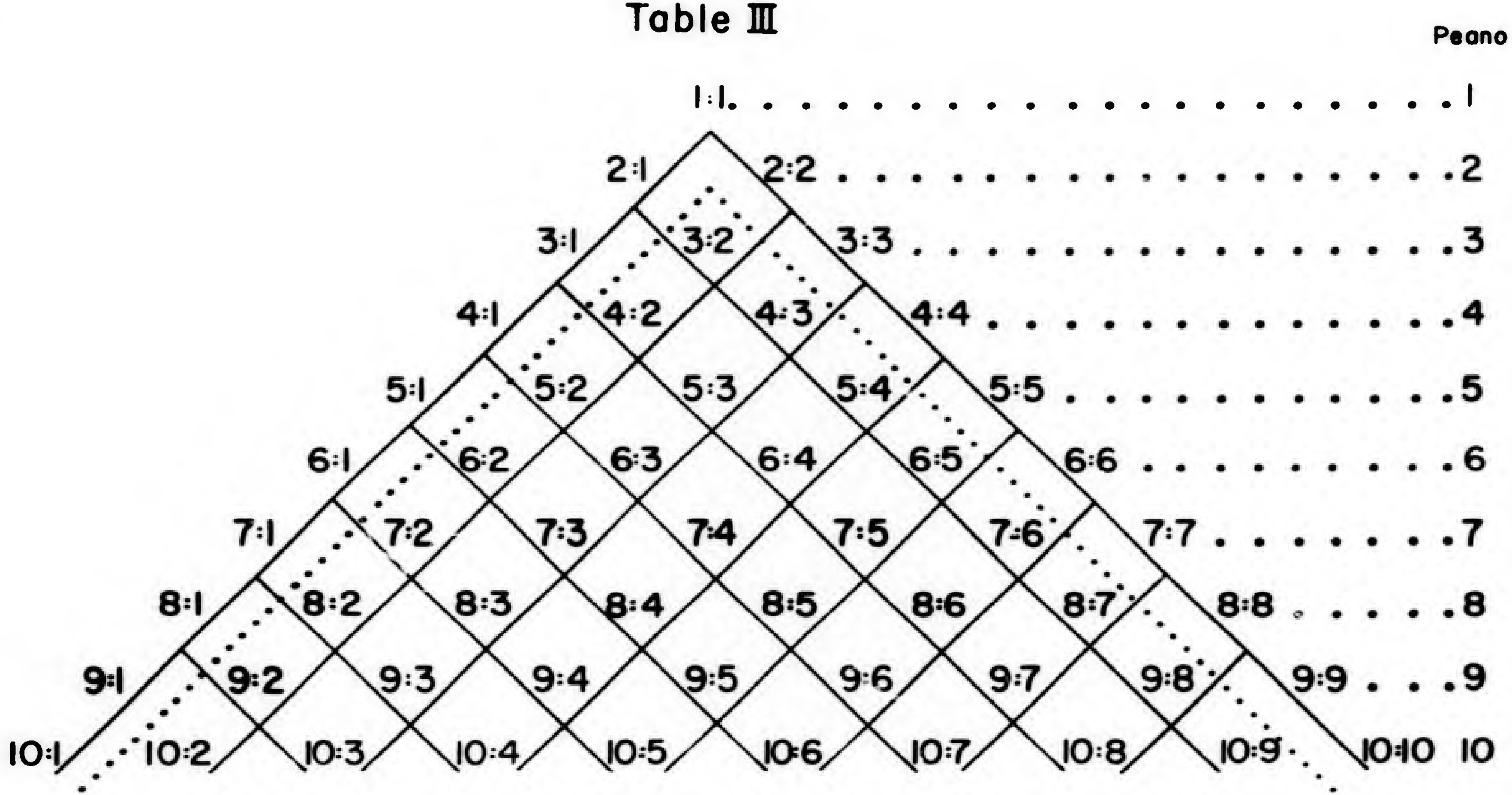
a a a
a and b are separated by a "mediative" sequence b.
a c b

It follows that, if we introduce the concept of a plurality of ontological loci which would permit the introduction of discontexturality into the theory of logic, a corresponding system of natural numbers should be devised where we would not just proceed from a given number to its Peano successor but from a predecessor number with the specified logical property X to a successor number with - let us say - the property Y.

In Table III we have given the sequential arrangement of the first ten proto-numbers with the corresponding Peano numbers on the extreme right side. Within this semi-Platonic pyramid we have separated the numbers of straight iteration and straight accretion from the "mediative" numbers by two dotted lines meeting at the top. The numeral ahead of the colon indicates the length of the kenogrammatic sequence, the second numeral gives the degree of accretion.

The "arithmetic" resulting from proto-numbers may be considered rather trivial. But the mapping of the Peano sequence onto deutero-structure shows already less trivial features. Table IV displays the

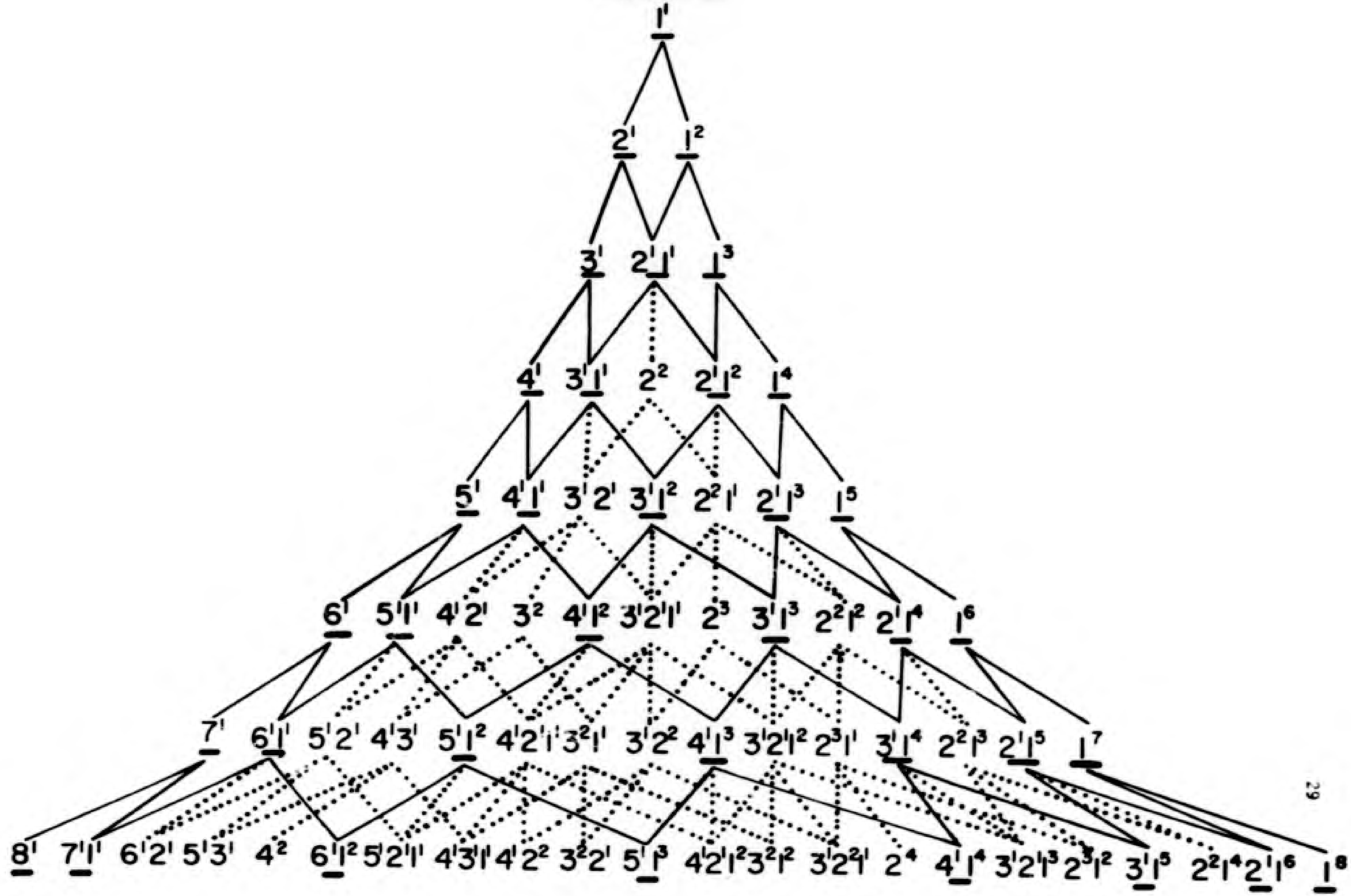
Table III


Peano
1:1 1
2:1 2:2 2
3:1 3:2 3:3 3
4:1 4:2 4:3 4:4 4
5:1 5:2 5:3 5:4 5:5 5
6:1 6:2 6:3 6:4 6:5 6:6 6
7:1 7:2 7:3 7:4 7:5 7:6 7:7 7
8:1 8:2 8:3 8:4 8:5 8:6 8:7 8:8 8
9:1 9:2 9:3 9:4 9:5 9:6 9:7 9:8 9:9 . . . 9
10:1 10:2 10:3 10:4 10:5 10:6 10:7 10:8 10:9 10:10 10

formation pattern of deutero-numbers if 1 is added, either iteratively or accretively to a predecessor number. Since in deutero-structure any kenogram is repeatable the simple notation of proto-structure will no longer satisfy our requirements. A deutero-structural sequence and its corresponding number however will be fully characterized if we count the number of iterations and indicate by superscript for how many kenograms a given sub-sequence of iterations occurs.

In Table IV the lines of succession which connect deutero-numbers with each other are partly drawn in a dotted fashion. The lines fully drawn out repeat the summation sequence (+1) for proto-structure; and the dotted lines show the additional summation sequences (+1) which are produced by deutero-structure. It should be noted that a number sequence, once it has entered a dotted line, never merges again with a continuous line. This means, deutero-structure contains two distinct patterns of successorship. As in proto-structure, the numbers representing either pure iteration or pure accretion play a separate role. No numbers which belong to the succession pattern of the dotted lines issue from them. On the other hand, every mediative number is the point of origin for a specific sequence of dotted lines. However, one should not interpret the deutero-numbers of straight iteration and straight accretion and all the other numbers in deutero-structure connected with them by continuous lines as proto-numbers which form some sort of conglomerate with bona fide deutero-numbers turning up along the dotted lines. All deutero-numbers, no matter what their characteristics, are only differentiae specificae within the various classes of proto-numbers. And what the continuous lines in Table III form can only be called a quasi-proto-structure.

Table IV



1^1
2^1 1^2
3^1 $2^1 1^1$ 1^3
4^1 $3^1 1^1$ 2^2 $2^1 1^2$ 1^4
5^1 $4^1 1^1$ $3^1 2^1$ $3^1 1^2$ $2^2 1^1$ $2^1 1^3$ 1^5
6^1 $5^1 1^1$ $4^1 2^1$ 3^2 $4^1 1^2$ $3^1 2^1 1^1$ 2^3 $3^1 1^3$ $2^2 1^2$ $2^1 1^4$ 1^6
7^1 $6^1 1^1$ $5^1 2^1$ $4^1 3^1$ $5^1 1^2$ $4^1 2^1 1^1$ $3^2 1^1$ $3^1 2^2$ $4^1 1^3$ $3^1 2^1 1^2$ $2^3 1^1$ $3^1 1^4$ $2^2 1^3$ $2^1 1^5$ 1^7
8^1 $7^1 1^1$ $6^1 2^1$ $5^1 3^1$ 4^2 $6^1 1^2$ $5^1 2^1 1^1$ $4^1 3^1 1^1$ $4^1 2^2$ $3^2 2^1$ $5^1 1^3$ $4^1 2^1 1^2$ $3^2 1^2$ $3^1 2^2 1^1$ 2^4 $4^1 1^4$ $3^1 2^1 1^3$ $2^3 1^2$ $3^1 1^5$ $2^2 1^4$ $2^1 1^6$ 1^8

So far the association of natural numbers with the general structure of trans-classic logic has been comparatively simple because the logical properties of identity and difference (or iteration and accretion) we dealt with could easily be expressed in quantities of places, symbols and partitions. If we approach trito-structure and its context of individual morphograms, the problem of associating natural numbers with logical structure becomes more intricate. The location of a symbol at a specific place in a sequence was totally irrelevant in proto- and deutero-structure. But the exact localisation of a given kenogram is precisely what distinguishes one individual morphogram from another within a given deutero-grammatic class.

A short reminder of a characteristic stipulation of our traditional methods of counting - be they binary, ternary, decimal or having any other radix or base - will be necessary. In order to be able to continue the process of counting indefinitely it is assumed that an unlimited number of empty places for writing down numerals will be at our disposal. These empty places are visualized to extend from right to left and, to represent the process of counting notationally, we fill them with the available numerals proceeding from right to left. As everybody knows, only the places which have been filled are relevant for determining the magnitude of the number. The potential infinity of empty places waiting to be filled and located left of the last place which has been filled does not contribute anything to the characterization of the number. The places on the left merely constitute an infinite background of total indeterminacy against which we do our counting.

An arithmetic notation for trito-structure requires us to drop the assumption that counting trito-numbers is done against such a backdrop of total indeterminacy. In other words: not only the places of the right side of the first numeral which takes occupancy in one of the empty places are relevant for the characterization of the number, but the empty places on the left side, still waiting to be filled, also count. This, of course, means that we must be able to enumerate them. Their availability is limited. It follows that the association of the binary method of counting with the eight classic symbol sequences or morphograms imp ies the stipulation that we use a number system with only four empty places available for occupancy by numerals. In Table V we have shown, first under a), the abstract sequence of empty places starting at the right and extending to the left into infinity; under b) the mapping of the binary notation of natural numbers onto the four and only four places available for the eight classic morphograms; under c) the first four steps of mapping a number system with the radix 3 onto four-place morphograms. Table V b) demonstrates that the mapping of a number system with the radix 2 onto the classic morphograms is feasible because there is a close structural affinity between this notational system and classic logic - which was already noted by Leibniz. But there is no similar affinity between the ternary system of counting and three-valued logic. This is evidenced in Table V c) where we have braced the second and third number together. Both numbers represent the same morphogram and consequently the same trito-number. It is here that the limitation of empty places which we stipulated makes itself felt. The two configurations which we have braced together could and would represent different numbers only if

Table V

a)

. . . . . .						
. . . . . .						

b)

0				
1				1
10			1	
11			1	1
100		1		
101		1		1
110		1	1	
111		1	1	1

c)

0				
1				1
2				2
10			1	0
. .	. . .	. . .	. . .	. . .

we assume that they belong to a system in which the number of empty places is infinite and that the places on the left side of the numerals did nothing for the characterization of the number. On the other hand, a morphogram is, as the term intends to convey, a 'Gestalt'. And it is the intrinsic character of a Gestalt that is finite. The infinite Gestalt is a contradictio in adjecto.

In our traditional systems of numeral notation we are completely at liberty to choose as many numerals as we like. It is a mere matter of expediency. This is not the case in systems of trito-numbers. If the number of empty places available by stipulation is 'n' and we do not count the empty place itself as a numeral, we always require n-1 numerals - not more and not less. If we use more, our system would be structurally redundant. If we use less, it would be incomplete. However, this rule would not only permit but even require the introduction of the numeral 2 and also 3 into the four-place arrangement of Table V c). What we obviously need is an additional specific stipulation about the introduction of a new numeral. In our traditional methods of notation a new numeral will be introduced (if available) immediately after its predecessor has been written down. In the case of the trito-numbers, however, the introduction of a new numeral must be restricted by an additional rule. To illustrate what we mean we turn again to Table V c). We shall ask ourselves the question: how is it possible to introduce the numeral 2 without producing structural redundancy within the four available places. For every numeral or numerals we have written down we have reproduced these four places again and again, so that they form together an oblong area of as yet undertermined length.

Within this area we may count our empty places horizontally as well as vertically. And in order to determine a place within this area we shall say that it is the nth place, counting either from right to left or from top to bottom. It is obvious that the introduction of 2 will not cause a redundancy if the structural configuration into which 2 is placed is different from the one into which we have inserted 1. But it is obvious in Table V c) that 2 was inserted in exactly the same structural configuration (four empty places) as the numeral 1 was inserted. This can easily be avoided by carrying the number 1 from the first horizontal place to the second. This produces a new structural configuration, and we may start counting again within the vertical sequence of the places on the extreme right side. This will carry us now legitimately to the numeral 2. In other words, the numeral 2 may be and must be introduced immediately after the numeral 1 has occurred directly above and on the left side of the place in which we want to put down 2. This means we must count our numerals along two Cartesian coordinates as shown in Table VI.

The two Cartesian coordinates in Table VI are indicated by dotted lines into which we have inserted the decimal numerals up to 5. Our counting process, however, does not carry us along one of the coordinates; it follows instead the zigzagging line that starts with the 0 at the top and transports us finally to 5. Since we are at liberty to introduce as many numerals as we wish, it is possible to continue our zigzagging sequence without any limit. However, the introduction of a new numeral interrupts this apparent continuity structurally and establishes a new finite system of trito-numbers.

Table VI

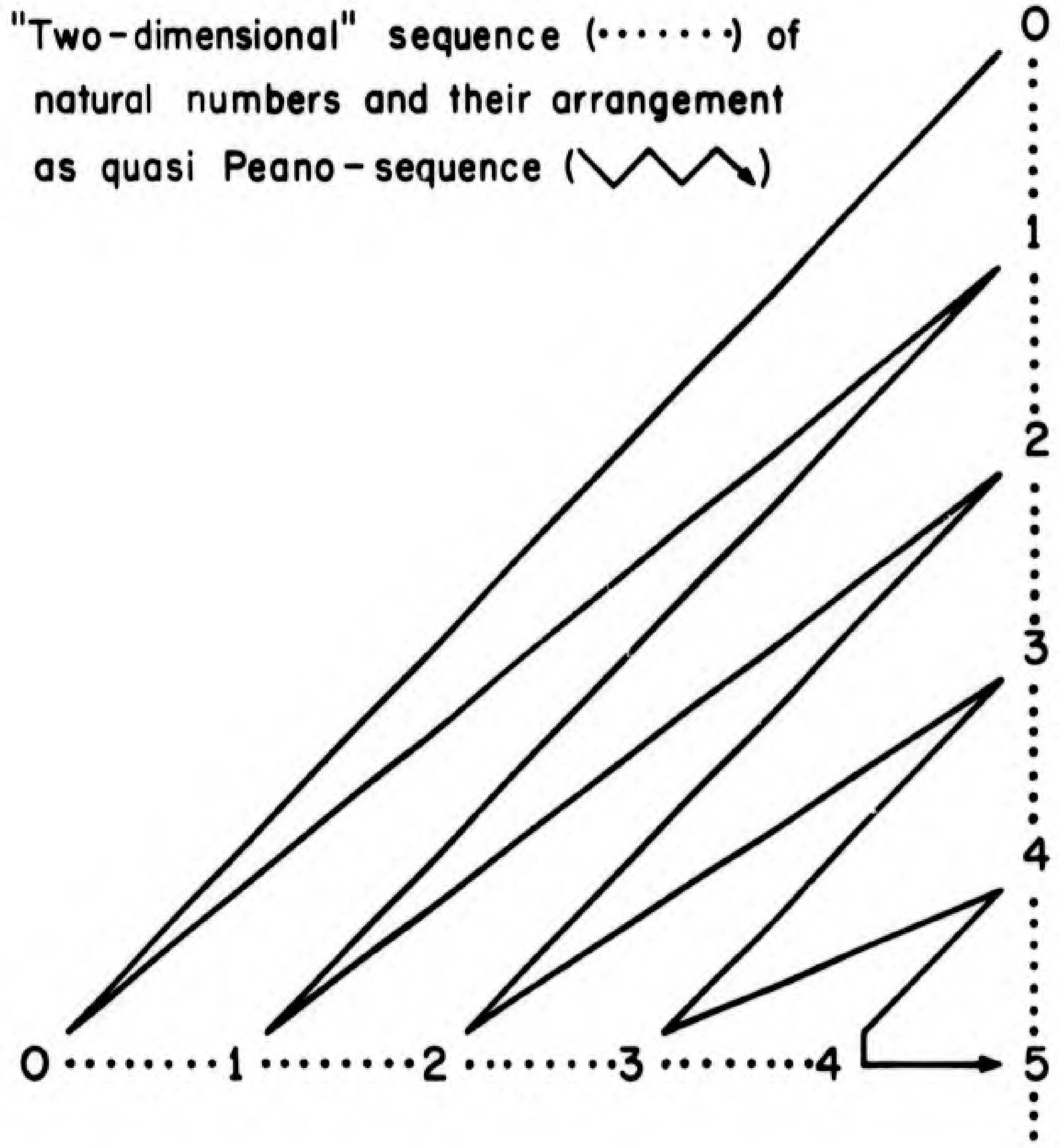

"Two-dimensional" sequence (•······•) of
natural numbers and their arrangement
as quasi Peano-sequence (╲╱╲╱╲↘)
0
1
2
3
4
5
0
1
2
3
4

Table VII presents the initial four of these finite systems. In the first column on the left side we find the morphograms to which the number systems belong. It should be noted that the morphogrammatic sequences are not written vertically, as was done in Table I, but horizontally in order to conform with the method we adopted for writing the trito-numbers. All trito-numbers are written out with the full complement of zeros which belong to a given system. The second system of trito-numbers which encompasses only two numerals is the one from which our traditional system of natural numbers issues. It is the kenogrammatic basis of a genuine Peano sequence. In the third column from the left the binary equivalents of the trito-numbers are given and in the extreme right column the decimal equivalents. In both cases we have adopted a somewhat unusual way of writing down our numbers. In order to conform with the methods of writing the trito-numbers which always start with the 0, our binary and decimal equivalents are preceded by a short sequence of dots, ending with a 0, separated from the numbers proper by a vertical line. The vertical line separates the numbers themselves from what we shall call: their place-designator. The place-designator is supposed to indicate, in the case of Table VII, that these numbers are written against a backdrop of an infinity of zeros which have to be available in order that the numbers may extend from right to left as far as it is required by their indefinitely increasing magnitude. No place-designator is required for the separate systems of trito-numbers in the second column, because they are not written against such a backdrop. Each system has its pre-determined length and width and cannot extend any further.

Table VII

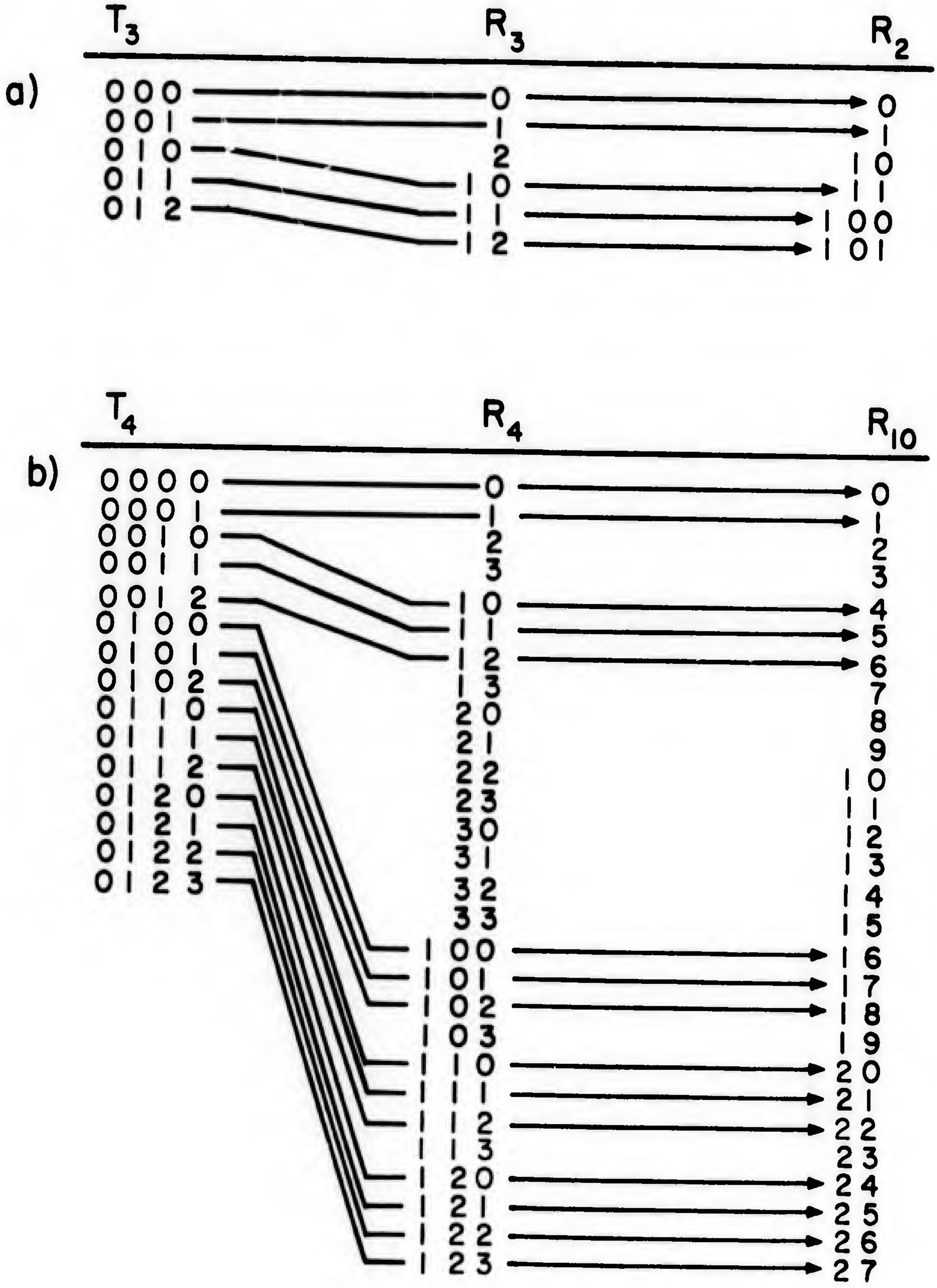
Morphograms	Trito-numbers	Binary equivalents	Decimal equivalents
a	0	···0 0	···0 0
a a	0 0	···0 0	···0 0
a b	0 1	···0 1	···0 1
a a a	0 0 0	···0 0	···0 0
a a b	0 0 1	···0 1	···0 1
a b a	0 1 0	···0 1 1	···0 3
a b b	0 1 1	···0 1 0 0	···0 4
a b c	0 1 2	···0 1 0 1	···0 5
a a a a	0 0 0 0	···0 0	···0 0
a a a b	0 0 0 1	···0 1	···0 1
a a b a	0 0 1 0	···0 1 0 0	···0 4
a a b b	0 0 1 1	···0 1 0 1	···0 5
a a b c	0 0 1 2	···0 1 1 0	···0 6
a b a a	0 1 0 0	···0 1 0 0 0 0	···0 1 6
a b a b	0 1 0 1	···0 1 0 0 0 1	···0 1 7
a b a c	0 1 0 2	···0 1 0 0 1 0	···0 1 8
a b b a	0 1 1 0	···0 1 0 1 0 0	···0 2 0
a b b b	0 1 1 1	···0 1 0 1 0 1	···0 2 1
a b b c	0 1 1 2	···0 1 0 1 1 0	···0 2 2
a b c a	0 1 2 0	···0 1 1 0 0 0	···0 2 4
a b c b	0 1 2 1	···0 1 1 0 0 1	···0 2 5
a b c c	0 1 2 2	···0 1 1 0 1 0	···0 2 6
a b c d	0 1 2 3	···0 1 1 0 1 1	···0 2 7

On the other hand, if we combine the separate systems of trito-numbers into a quasi-Peano sequence, such sequence will, under certain circumstances, i.e. if combined with other number sequences, also require a place-designator, since its extension has no limits. The adding of a place-designator is not required in classic mathematics, because the natural numbers it employs are, logically speaking, always written against this backdrop of a potential infinity of zeros. In other words, the logical place of the traditional Peano numbers cannot change, since they appear only in one ontological locus.

The situation is different in a trans-classic system. In this new dimension classic logic unfolds itself into an infinity of two-valued sub-systems, all claiming their own Peano sequences. It follows that natural numbers--running concurrently in many ontological loci--must then be written against an infinity of potential backdrops. This suggests that the place-designator, shown in Table VII, is by no means the only one.

Our last Table VIII offers an opportunity to study the changing of the binary and decimal equivalents of trito-numbers in various trito-grammatic systems. The method of finding the equivalents for any conventional number system is very simple and demonstrated in Table VIII for trito-grammatic numbers of three (T_3) and four (T_4) places. In order to find the equivalents of a conventional number system with the radix two (R_2) we confront, first, our three-place trito-numbers with the notation of a number system with the radix 3 (R_3), as we have done in Table VIII a). We connect then the numbers of (T_3)- omitting the zeros on the left- with the corresponding notation in (R_3). If we do so we

Table VIII


T3
R3
R2
a)
0 0 0
0 0 1
0 1 0
0 1 1
0 1 2
0
1
2
1 0
1 1
1 2
0
1
1 0
1 1
1 0 0
1 0 1
T4
R4
R10
b)
0 0 0 0
0 0 0 1
0 0 1 0
0 0 1 1
0 0 1 2
0 1 0 0
0 1 0 1
0 1 0 2
0 1 1 0
0 1 1 1
0 1 1 2
0 1 2 0
0 1 2 1
0 1 2 2
0 1 2 3
0
1
2
3
1 0
1 1
1 2
1 3
2 0
2 1
2 2
2 3
3 0
3 1
3 2
3 3
1 0 0
1 0 1
1 0 2
1 0 3
1 1 0
1 1 1
1 1 2
1 1 3
1 2 0
1 2 1
1 2 2
1 2 3
0
1
2
3
4
5
6
7
8
9
1 0
1 1
1 2
1 3
1 4
1 5
1 6
1 7
1 8
1 9
2 0
2 1
2 2
2 3
2 4
2 5
2 6
2 7

skip the single numeral 2 for which their is no correspondence in (T_3). Since the equivalent of 2 in a ternary system (R_3) is 10 and 2 has no equivalent in (T_3), 10 also of (R_2) has no equivalent in (T_3). Thus, no arrow pointing to an equivalence goes from (R_3) to (R_2). In Table X b) the same method is applied to find the equivalences of a system with the radix 10 (R_{10}) with four-place trito-numbers. This time, the intermediate step is, of course, provided by a quaternary system (R_4). All quarternary numbers without notational correspondence in (T_4) are omitted and the remaining arrows point to the decimal numerals that correspond to the four-place trito-numbers. —

To conclude this presentation it should be emphasized that the foregoing remarks do not imply a full-fledged theory of the behavior of natural numbers in a trans-classic system of logic. Their only aim is to draw attention to a specific arithmetical problem in the Cybernetic theory of biological systems. The mythological contra-position of body and soul is nothing but a terse expression for the background of total discontexturality against which living systems have to be analyzed with regard to their basic structure.

So far Western scientific tradition has been exclusively concerned with the theory of a universe which presents to us an aspect of unbroken contexturality. The theory of such a universe is equivalent with the theory of auto-referential objects. Their nature was explored and so exhaustively described that we have practically come to the end of this epoch of scientific inquiry. A living organism, on the other hand, is a cluster of relatively discontextural sub-systems held together by a mysterious function called self-reference and hetero-referentially linked to an environment of even greater discontexturality. In order to integrate the concept of discontexturality into logic we have introduced the theory of ontological loci. Any classic system of logic or mathematics refers to a given ontological locus; it will describe the contextural structure of such a locus more or less adequately. But its statements - valid for the locus in question - will be invalid for a different locus. To put it crudely: true statements about a physical body will not be true about the soul... and vice versa.

A philosophic theory of Cybernetics would imply that the total discontexturality between dead matter and soulful life which the classic tradition assumes may be resolved in a hierarchy of relative discontexturalities. We repeat what we stated at the beginning: our system of natural numbers is valid within the context of a given ontological locus, but it is not valid across the discontexturality which separates one ontological locus from the next. However, there is a way to connect a Peano sequence of natural numbers in one ontological locus with the Peano sequence in a different one. This connection is expressed arithmetically and with different degrees of complexity in the proto-, deutero- and trito-numbers. These number systems do not refer to the contexturality of a given ontological locus but to a universal sub-structure that connects these loci with each other. Thus these numbers have, what we shall call, an inter-ontological semantic relevance. The terms Life, Self or Soul have always been mysterious, because they refer to an inter-ontological phenomenon.

Since the classic tradition knows only a single ontology it has no theoretical means at its disposal to describe phenomena which fall, so to speak, between different ontologies. The philosophic theory on which Cybernetics may rest in the future may well be called an inter-ontology. But its description - as Kipling would say - is another story.