2340

NRL Memorandum Report 2340
NRL Computer Bulletin 25

A Fortran Subroutine to Produce Combinations of n Distinct Things Taken m at a Time

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July 1971

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Washington, D.C.

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CONTENTS

ABSTRACT

A Fortran subroutine to generate all or particular combinations of n things taken m at a time is herein described. The procedure establishes and tabulates a certain one-to-one correspondence between the integers from 1 to ${}_nC_m$ and the individual combinations of n distinct "things" taken m at a time, where

$$ {}_nC_m = \frac{n!}{m!(n-m)!} $$

In all cases, the set of "things" treated by the subroutine is the set of integers {1,...,n}.

PROBLEM STATUS

This is a final report on this phase of the problem.

AUTHORIZATION

NRL General and Administrative Function 78-1601

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A Fortran Subroutine to Produce Combinations of n Distinct Things Taken m at a Time

1.0 IDENTIFICATION

1.1 Title

Combinations Subroutine

1.2 Identification Name

GO-NRL-CMBN

1.3 Classification Code

GO - Statistical Analysis and Probability, General

1.4 RCC Identification Name

G0001R00

1.5 Entry Points

CMBN

1.6 Programming Language

Language: 3600/3800 FØRTRAN
Routine Type: Subroutine
Operating System: Drum Scope 2.0

1.7 Computer and Configuration

CDC 3800

1.8 Contributor or Programmer

James P. Grimes, Code 7816JG, Research Computation Center, Mathematics and Information Sciences Division

1.9 Contributing Organization

NRL - Naval Research Laboratory, Washington, D. C. 20390

1.10 Program Availability

1.10.1 Submittal: Program write-up, Fortran source deck, source listing

1.10.2 On File: RCC Program Library

1.11 Verification

The subroutine was checked for the following values of N, M, and NN; (see Section 3.2 for definition of N, M, and NN).

N	M	NN
17	15	0
17	16	0
20	20	0
101	100	2
0	0	1
20	20	1
19	0	0
1	1	0
1	1	1
6	7	0
0	1	0

1.12 Date

1 July 1971

2.0 PURPOSE

2.1 Description of the Routine

To generate all or particular combinations of the integers from 1 to n taken a given number at a time.

This procedure establishes and tabulates a certain one-to-one correspondence between the integers from 1 to ${}_nC_m$ and the individual combinations of n distinct "things" taken m at a time. Here

$${}_nC_m = \frac{n!}{m!(n-m)!}$$

In all cases, the set of "things" treated by the subroutine is the set of integers $\{1,\ldots,n\}$.

The type of correspondence referred to may be illustrated for the case n=5, and m=3, as follows:

Index	Combination
1	1,2,3
2	1,2,4
3	1,2,5
4	1,3,4
5	1,3,5
6	1,4,5
7	2,3,4
8	2,3,5
9	2,4,5
10	3,4,5

2.2 Problem Background

In many phases of investigative mathematics, statistics, probability and related areas, the need often arises to generate and tabulate all or particular combinations of n distinct "things" taken m at a time. A computerized algorithm is the only practical way to handle this problem, especially as the number of combinations and the number to be taken at a time become relatively large. Previously, this type of computerized procedure was unavailable to NRL's scientific community.

3.0 USAGE

3.1 Calling Sequence or Operational Procedure

The calling sequence for the subroutine is:
CALL CMBN (N, M, NN, MM)
All four parameters are in CDC 3600/3800 integer format.

3.2 Arguments, Parameters, and/or Initial Conditions

Input parameters:
N and M are the n and m described in Section 2.1, i.e., as in ${}_nC_m$.

NN is an input integer which establishes the correspondence referred to in Section 2.1.
If NN=0, the correspondence is set by the subroutine and all combinations from 1 to ${}_nC_m$ are generated,

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indexed, and printed out by the subroutine. Otherwise, if NN = k, k = 1,...,${}_nC_m$, only that combination which corresponds to k is generated, indexed, and printed out as demonstrated in Section 2.1. MM is a one-dimensional input array which must be dimensioned in the calling program and whose length must be at least 4(m+1).

Output parameters:
There are no output parameters as such since all output is performed by the subroutine. However, after execution of the subroutine for each value of NN, whether supplied by the user or the subroutine itself, the m integers comprising the output combination occupy the first m locations of the array MM. The subroutine prints out this combination and the MM array is reused for the next combination. This procedure is repeated until all combinations (NN = 0) or the particular combination (NN = k, k = 1,...,${}_nC_m$) is generated and printed out.

3.2.1 Special Provisions and Restrictions

Apparently, the basic algorithm imposes no restrictions on the parameters. However, there are computer-imposed limitations, such as storage capacity, word size, etc. Nevertheless, to facilitate the output, n, m, and ${}_nC_m$ have been limited to 10^5. Of course, $n \geq m$.
For those cases in which $m \geq 16$, CMBN uses logarithms for some of the intermediate steps; this fact results in appreciably longer computation time for these cases. For the most part, computation time depends on the size of m and ${}_nC_m$.

3.3 Space Required (Decimal and Octal)

3.3.1 Unique Storage: 1475 octal (829 decimal) locations in CMBN

The dimension of the input array MM is user supplied.

3.3.2 Common Blocks: None

3.3.3 Temporary Storage: None

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3.4 Messages and Instructions to the Operator

None

3.5 Error Returns, Messages, and Codes

None

3.6 Informative Messages to the User

The message "INVALID INPUT PARAMETER(S) N,M≤0 OR M>N" will be printed, if the input parameters are invalid.

3.7 Input

See the "Input parameters" described in Section 3.2

3.8 Output

See "Output parameters" in Section 3.2

3.9 Formats

None

3.10 External Routines and Symbols

None

3.11 Timing

Running time is primarily dependent on the size of m and ${}_nC_m$. Some examples of running time on the CDC-3800 computer follow: ${}_7C_4$, 24 seconds; ${}_9C_3$, 24 seconds; ${}_{15}C_5$, 72 seconds; ${}_{20}C_{17}$, 91 seconds.

3.12 Accuracy

The procedure has been tested by the author for values of m up to 100. Apparently, it continues to be valid for higher values of m, but computation time greatly increases.

3.13 Cautions to Users

3.13.1 MM is an input array and must be dimensioned in the main program. Its size must be at least 4(m+1). The array name in the main program may be any integer variable.

3.13.2 Since the value of ${}_nC_m$ can become very large, the user should make sure he does not exceed storage capacity or machine word size using integer format. To facilitate the output by the subroutine n, m, and ${}_nC_m$ must be less than 10^5.

3.13.3 The user should consider running time and the amount of output produced for large values of m and ${}_nC_m$.

3.13.4 Of course, n and m must be positive and $n \geqslant m$.

3.14 Program Deck Structure

$^7_9$JØB card

$^7_9$FTN card

main program
CMBN subroutine

SCØPE

$^7_9$LØAD card

$^7_9$RUN card

$^{77}_{88}$(end-of-file card)

3.15 References - Literature - Appendices

Narec Bulletin #120, Neliac Bulletin #58, by Peter C. Ryan, formerly Code 6410, NRL.

4.0 METHOD OR ALGORITHM

The procedure used in the subroutine is a modification and generalization of an algorithm used in a NELIAC-N computer function by Peter C. Ryan, formerly Code 6410, NRL. For further information or details, see reference listed in Section 3.15.

5.0 SOURCE LANGUAGE LISTING

```
      SUBROUTINE CMBN(N,M,NN,MM)
C     GO-NRL-CMBN,CDC 3800 COMPUTER SUBROUTINE TO PRODUCE
C     COMBINATIONS OF N DISTINCT THINGS TAKEN M AT A TIME.
C     (IN ALL CASES, THE SET OF THINGS TREATED BY THE
C     SUBROUTINE IS THE SET OF INTEGERS %1,....,N=)
C     TITLE-COMBINATIONS SUBROUTINE
C     IDENT NAME-GO-NRL-CMBN
C     LANGUAGE-FORTRAN
C     PROGRAMMER-JAMES P. GRIMES,CODE 7816JG
C                RESEARCH COMPUTATION CENTER, MISD
C                (A MODIFICATION AND GENERALIZATION OF A
C                NELIAC-N COMPUTER FUNCTION BY MR.
C                PETER C. RYAN,FORMERLY CODE 6410,NRL)
C     IDENT NUMBER-G0001R00
C     ORGANIZATION-NRL-NAVAL RESEARCH LABORATORY
C                  WASHINGTON,D.C.20390
C     DATE-2 NOVEMBER 1970
C     PURPOSE-TO GENERATE ALL OR PARTICULAR COMBINATIONS
C             OF THE INTEGERS FROM 1 TO N.
C
C     USAGE-THE CALLING SEQUENCE FOR THE SUBROUTINE IS:
C           CALL CMBN(N,M,NN,MM)
C           ALL FOUR PARAMETERS ARE IN CDC 3600/3800
C           INTEGER FORMAT.
C
C     INPUT PARAMETERS-N AND M ARE THE INPUT INTEGERS IN
C                      C(N,M), THAT IS, THE COMBINATIONS
C                      OF N THINGS TAKEN M AT A TIME.
C
C                      NN,AN INPUT INTEGER WHICH ESTABLISHES
C                      A CERTAIN ONE-TO-ONE CORRESPONDENCE
C                      BETWEEN THE INTEGERS FROM 1 TO C(N,M)
C                      AND THE INDIVIDUAL COMBINATIONS OF N
C                      THINGS TAKEN M AT A TIME.
C
C                      IF NN=0, THE CORRESPONDENCE IS SET BY
C                      THE SUBROUTINE AND ALL COMBINATIONS
C                      FROM 1 TO C(N,M) ARE GENERATED,
C                      INDEXED, AND PRINTED OUT BY THE
C                      SUBROUTINE.
C
C                      IF NN=K, K=1,....,C(N,M), ONLY THAT
C                      COMBINATION WHICH CORRESPONDS TO K
C                      IS GENERATED, INDEXED, AND PRINTED
C                      OUT.
C
C                      MM IS A ONE-DIMENSIONAL INPUT ARRAY
C                      WHICH MUST BE DIMENSIONED IN THE
C                      CALLING PROGRAM AND WHOSE LENGTH
C                      MUST BE AT LEAST 4*(M+1).
```

```
C
C     OUTPUT PARAMETERS-THERE ARE NO OUTPUT PARAMETERS
C                       AS SUCH SINCE ALL OUTPUT IS
C                       PERFORMED BY THE SUBROUTINE.
C
C     RESTRICTIONS-TO FACILITATE THE OUTPUT, N,M,AND
C                  C(N,M) HAVE BEEN LIMITED TO 10**5.
C     N AND M MUST BE POSITIVE AND N≥M.
C     COMPUTATION TIME DEPENDS, FOR THE MOST PART, ON
C     THE SIZE OF M AND C(N,M).
C     THE PROCEDURE HAS BEEN TESTED FOR VALUES OF M UP
C     TO 100.
C     CAUTION-AS M AND C(N,M) BECOME LARGE, THE RUNNING
C             TIME AND AMOUNT OF OUTPUT INCREASES
C             APPRECIABLY.
C
C
C
C
C
      DIMENSION JSPEC(5)
      DIMENSION MM(1)
      IF(M.LE.0.OR.M.GT.N)60,70
   60 PRINT 65
   65 FORMAT(5X,*INVALID INPUT PARAMETER(S) N,M≤0 OR M>N*,3(/))
      RETURN
   70 IJK=0
      IF(NN.EQ.0)1,14
    1 IJK=1
      IF(N.EQ.1)2,3
    2 FNF=1.
      GO TO 5
    3 FNF=1.
      DO 4,I=1,N
    4 FNF=FNF*I
    5 IF(M.EQ.1)6,7
    6 FMF=1.
      GO TO 9
    7 FMF=1.
      DO 8,I=1,M
    8 FMF=FMF*I
    9 K=N-M
      IF(K.EQ.0.OR.K.EQ.1)10,11
   10 FKF=1.
      GO TO 13
   11 FKF=1.
      DO 12,I=1,K
   12 FKF=FKF*I
   13 NC=FNF/(FMF*FKF)+.5
      DO 49, II=1,NC
```

```
      NN=II
   14 MN=4*(M+1)
      DO 15,I=1,MN
   15 MM(I)=0
      IM=M+1
      IC=IM+M+1
      IN=IC+M+1
      IF(M.EQ.1)16,17
   16 MM=NN
      GO TO 45
   17 MM(IC+1)=N
      MM(IN+1)=M
      IL=0
      IF(M.GT.15)30,18
   18 M1=M-1
      DO 29,J=1,M1
      MM(IC+J+1)=MM(IC+J)-MM(IN+J)+1
      IA=MM(IC+J+1)
      IB=1
      IH=IL
      MJ=M-J
      DO 19,L=1,MJ
   19 IB=IB*L
      DO 26,I=1,IA
      ID=1
      DO 20,L=1,MJ
   20 ID=ID*(IA-I+L)
      IE=ID/IB
      IL=IH
      IH=IL+IE
      IF(IH.GT.NN)21,22
   21 MM(IN+J+1)=I
      GO TO 27
   22 IF(NN.EQ.IH)23,26
   23 MM(IN+J+1)=I
      IF=0
      DO 24,L=1,J
   24 IF=IF+MM(IN+L+1)
      MM(IN+J+2)=N-M-IF+J+1
      J2=J+2
      DO 25,L=J2,M
   25 MM(IN+L+1)=1
      GO TO 42
   26 CONTINUE
   27 IF(M-1.EQ.J)28,29
   28 MM(IN+M+1)=NN-IL
   29 CONTINUE
      GO TO 42
   30 M1=M-1
      DO 41,J=1,M1
```

```
      MM(IC+J+1)=MM(IC+J)-MM(IN+J)+1
      IA=MM(IC+J+1)
      FB=0
      IH=IL
      MJ=M-J
      DO 31,L=1,MJ
      FL=L
      FL=LOGF(FL)
   31 FB=FB+FL
      DO 38,I=1,IA
      FD=0
      DO 32,L=1,MJ
      ID=IA-I+L
      FA=ID
      FA=LOGF(FA)
   32 FD=FD+FA
      FD=FD-FB
      FD=EXP(FD)+.5
      IE=FD
      IL=IH
      IH=IL+IE
      IF(IH.GT.NN)33,34
   33 MM(IN+J+1)=I
      GO TO 39
   34 IF(NN.EQ.IH)35,38
   35 MM(IN+J+1)=I
      IF=0
      DO 36,L=1,J
   36 IF=IF+MM(IN+L+1)
      MM(IN+J+2)=N-M-IF+J+1
      J2=J+2
      DO 37,L=J2,M
   37 MM(IN+L+1)=1
      GO TO 42
   38 CONTINUE
   39 IF(M-1.EQ.J)40,41
   40 MM(IN+M+1)=NN-IL
   41 CONTINUE
   42 DO 43,K=1,M
   43 MM(IM+K+1)=MM(IM+K)+MM(IN+K+1)
      DO 44,IX=1,M
   44 MM(IX)=MM(IM+IX+1)
      IF(IJK.EQ.1)45,46
   45 IF(NC.LT.10E4) LL=5
      IF(NC.LT.10E3) LL=4
      IF(NC.LT.10E2) LL=3
      IF(NC.LT.10E1) LL=2
      IF(NC.LT.10E0) LL=1
      GO TO 47
   46 IF(NN.LT.10E4) LL=5
```

```
47 IF(N.LT.10E4) JJ=5
   IF(N.LT.10E3) JJ=4
   IF(N.LT.10E2) JJ=3
   IF(N.LT.10E1) JJ=2
   IF(N.LT.10E0) JJ=1
   JKL=5+LL
   KK=(130-LL-5)/(JJ+1)
   ENCODE(37,48,JSPEC)LL,KK,JJ,JKL,KK,JJ
48 FORMAT(5H(3X,I,I1,4H,2X,I2,2H(I,I1,6H,1X)/(,
  C I2,2HX,,I2,2H(I,I1,6H,1X))))
   PRINT JSPEC,NN,(MM(J),J=1,M)
49 CONTINUE
   END
```

6.0 COMPARISON

No routine was available to the author for comparison.

7.0 TEST METHOD AND RESULTS

A test program that demonstrates all combinations of 7 "things" taken 4 at a time and specific combinations in accordance with the established correspondence of the individual combinations and the positive integers along with the subroutine printouts follow.

```
      PROGRAM TCMBN
      DIMENSION II(20), KARRAY(5)
      DATA (KARRAY=3,12,7,20,5)
      N=7
      M=4
      CALL CMBN(N,M,0,II)
      PRINT 1
    1 FORMAT(10(/))
      DO 2 I=1,5
      CALL CMBN(N,M,KARRAY(I),II)
    2 CONTINUE
      END
```

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1	1	2	3	4
2	1	2	3	5
3	1	2	3	6
4	1	2	3	7
5	1	2	4	5
6	1	2	4	6
7	1	2	4	7
8	1	2	5	6
9	1	2	5	7
10	1	2	6	7
11	1	3	4	5
12	1	3	4	6
13	1	3	4	7
14	1	3	5	6
15	1	3	5	7
16	1	3	6	7
17	1	4	5	6
18	1	4	5	7
19	1	4	6	7
20	1	5	6	7
21	2	3	4	5
22	2	3	4	6
23	2	3	4	7
24	2	3	5	6
25	2	3	5	7
26	2	3	6	7
27	2	4	5	6
28	2	4	5	7
29	2	4	6	7
30	2	5	6	7
31	3	4	5	6
32	3	4	5	7
33	3	4	6	7
34	3	5	6	7
35	4	5	6	7

3	1	2	3	6
12	1	3	4	6
7	1	2	4	7
20	1	5	6	7
5	1	2	4	5

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DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Naval Research Laboratory Washington, D. C. 20390	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

A FORTRAN SUBROUTINE TO PRODUCE COMBINATIONS OF n DISTINCT THINGS TAKEN m AT A TIME

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

A final report on this phase of the problem.

5. AUTHOR(S) (First name, middle initial, last name)

James P. Grimes

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
July 1971	18	1

8a. CONTRACT OR GRANT NO. NRL General and Administrative b. PROJECT NO. Function 78-1601 c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) NRL Memorandum Report 2340 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) NRL Computer Bulletin 25

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Department of the Navy (Office of Naval Research), Washington, D.C. 20360

13. ABSTRACT

A Fortran subroutine to generate all or particular combinations of n things taken m at a time is herein described. The procedure establishes and tabulates a certain one-to-one correspondence between the integers from 1 to ${}_nC_m$ and the individual combinations of n distinct "things" taken m at a time, where

$$ {}_nC_m = \frac{n!}{m!(n-m)!} . $$

In all cases, the set of "things" treated by the subroutine is the set of integers {1,...,n}.

DD FORM 1 NOV 65 1473 (PAGE 1)

S/N 0101-807-6801

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Combinations Computer Subroutine Statistical Analysis Probability One-to-One Correspondence Mathematics Parameters						