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TABLES OF THE HYPERGEOMETRIC DISTRIBUTION FUNCTIONS

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FOREWORD

Tables and graphs based on the hypergeometric distribution are presented for use in determining the confidence interval of the sample estimate of the number of defectives in a finite population. Similarly, the sample size can be determined which would give a certain quality level as the lower bound for a selected confidence level. The hypergeometric distribution is particularly suited for small populations (less than 1,000) where a saving in the sample size is desired even at the expense of some loss in precision of the estimate.

The tables of point and cumulative probabilities are tabulations of selected sample and population combinations. The selected sample sizes range from 4 to 40 and the population, from 50 to 1,000.

For those that have access to an IBM 1401 Model B-4, 8K memory, the computer program is included as Appendix D.

The authors wish to express their appreciation to A. Ohta and K. Thornton for editing and assembly of the tables and to J. Mitchell for supervising the computer tabulation.

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INTRODUCTION

In quality evaluation, the basic question is: "What is the quality level of the stockpile in question?" More often this question is put in the following form: "How large must the sample be to give a certain level of assurance that the stockpile is no worse than X% defective if no defectives are observed in the sample?" In this latter form, the requirement is not for a precise estimate of the stockpile quality level but rather some assurance that the quality is not below a specified level. In this situation, the implication is that there is some willingness to sacrifice some precision if a reduction in the sample size required can be realized.

Most approaches until recently have been based on the binomial distribution. But in cases where the stockpile is small, say less than 500, and the unit item cost high, the binomial has not been a very satisfactory model. As is normally the case in quality evaluation where the populations are small and sampling is without replacement, it appeared that the hypergeometric distribution was the more realistic model to use, but until the advent of the modern-day computer, the formidable task of calculating the probabilities on a desk calculator prevented its use.

Of primary concern to the Oahu Laboratory is coping with small stockpiles of high unit cost weapons. What is desired is a method whereby stockpiles of extremely high or low quality (percent operability) can be readily detected using a minimal size sample. For stockpiles falling in between, additional samples must be tested if greater precision in the quality estimates is desired. As a result, a study was made of a two-stage sampling method based on the hypergeometric distribution and using 95% operability at the 90% confidence level as the lower bound for "good" stockpiles.

APPROACH TO THE PROBLEM

Based on the hypergeometric distribution two main mathematical approaches are proposed.

Approach I

The first approach may be stated in this mathematical form

$$(1) \qquad P(D|S,N,M) = \frac{\binom{M}{D}\binom{N-M}{S-D}}{\binom{N}{S}} = \frac{\left[\frac{M!}{(M-D)!\ D!}\right]\left[\frac{(N-M)!}{(N-M-S+D)!\ (S-D)!}\right]}{\frac{N!}{(N-S)!\ S!}}$$

where: S = Sample size
N = Population size
D = Sample defectives
M = Population defectives

and obviously

S - D = Sample Non-defectives
N - M = Population Non-defectives

Equation (1) states that the probability of obtaining D defectives in a sample of size S, given M defectives in a population of N items, is equal to the number of ways of drawing D out of M items times the number of ways of drawing S-D out of N-M items divided by the number of ways of drawing S out of N items. In stockpile quality estimation it is desired to find the confidence interval for the number of defectives in a finite population. Since M is not known, an upper bound on the true M is sought. Call this bound M_u. First assume that $M_u = M_1$. Then look up in an appropriate table (reference (4)) the sum of the probabilities of drawing D or less defectives in the sample. If this sum is less than α, the significance level (e.g. .10), the proper M_u should be less than M_1. Then choose $M_u = M_2$, $M_2 < M_1$ and repeat the above procedure until

$$\sum_{D_i=0}^{D} P(D_i|S,N,M_u-1) \leq \alpha \leq \sum_{D_i=0}^{D} P(D_i|S,N,M_u)$$

Then select M_u-1 or M_u as the upper bound depending on which corresponding sum is closer to α.

The maximum likelihood value is given as $\hat{M} \leq \frac{D}{S}(N+1)$ (reference 1, p. 294) or more completely $\frac{D}{S}(N+1) - 1 \leq \hat{M} \leq \frac{D}{S}(N+1)$ (reference 3, p. 3). References for this approach are found in (1), (2), and (3).

Approach II

The approach followed is to assume that populations with M defectives where M ranges from D to (N-S+D) are tested randomly. In this case $P(D|S,N,M_i)$ is related to $P(D|S,N,M_D)$, $P(D|S,N,M_{D+1})$, ... $P(D|S,N,M_{N-S+D})$. The probability of the observed sample coming from a population with M_i defectives is given as:

$$(2) \qquad P(M_i \mid D,S,N) = \frac{P(D|S,N,M_i)}{\sum_{M=D}^{N-S+D} P(D|S,N,M)} \qquad \text{where, } D \leq M_i \leq N\text{-}S\text{+}D$$

Equation (2) states that if, from a population of size N, a sample S is drawn and D defectives are observed, the probability of M_i defectives in the population is equal to the ratio of the probability of that set of M_i, N, S, and D to the total sets of N, M_j, S, D where M_j is allowed to range from D to (N-S+D). The number of defectives in the population cannot be less than the number of defectives observed in the sample nor greater than the difference between the total population N, and (S-D) (sample non-defectives).

The denominator $\sum_{M_j=D}^{N-S+D} P(D|S,N,M_j)$ can be shown equal to $\frac{N+1}{S+1}$, a constant. This makes it valid to tabulate P(D S, N, M) instead of P(M S, N, D) for checking purposes. In the form of equation (1), equation (2) becomes the derived equation:

$$(3) \qquad P(M \mid D,S,N) = \frac{\binom{M}{D}\binom{N-M}{S-D}}{\binom{N+1}{S+1}} = \frac{S+1}{N+1} \quad P(D|S,N,M)$$

For brevity, let $P(M_i | D, S, N) = P(M)$. Then these recurrence relationships are very useful for computational purposes:

(4) $$\frac{P(M)}{P(M+1)} = \frac{(M-D+1)\ (N-M)}{(M+1)\ (N-M-S+D)}$$

(5) $$\frac{P(M)}{P(M-1)} = \frac{(M)\ (N-M-S+D+1)}{(M-D)\ (N-M+1)}$$

From (4) and (5) it can be seen that $P(M) > P(M+1)$, where they exist, as long as $(M-D+1)\ (N-M) > (M+1)\ (N-M-S+D)$ and similarly $P(M) > P(M-1)$ as long as $(M)\ (N-M-S+D+1) > (M-D)\ (N-M+1)$. It follows that the maximum likelihood integer $\hat{M}$ may be expressed as:

(6) $$\frac{D}{S}(N+1) - 1 \leq \hat{M} \leq \frac{D}{S}(N+1)$$

There will be two $\hat{M}$ values where the extreme right and left expressions are: (a) integers and (b) exist. Since equations (4), (5) and (6) show that the probabilities decrease from the maximum likelihood value, serial computations should start with $\hat{M}$ as shown below in equations (7a) and (7b). (Note: $\hat{M}$ differs from that in [Reference 1] and [Reference 3].)

(7a) $$P(\hat{M}) \geq P(\hat{M}-1) \geq P(\hat{M}-2) \geq \ldots\ P(\hat{M}-k)$$

(7b) $$P(\hat{M}) \geq P(\hat{M}+1) \geq P(\hat{M}+2) \geq \ldots\ P(\hat{M}+i)$$

(7c) $$\sum_{R}^{T} P(M_j) = 1 - \alpha$$

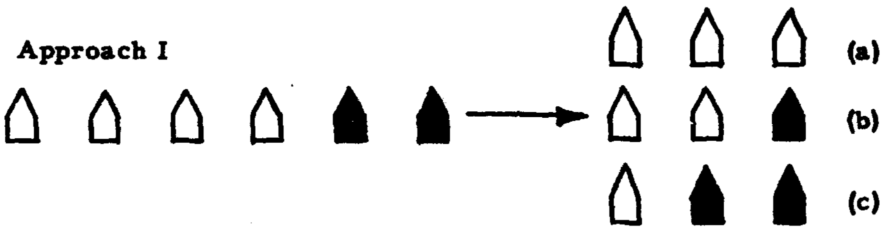
If $P(\hat{M}+i) \geq P(\hat{M}-k)$, $P(\hat{M}+i)$ is added to $\sum P(M_j)$ which starts with $P(\hat{M})$; otherwise $P(\hat{M}-k)$ is added. Equation (7c) states that R to T is the range of M defectives in the population when the sum of all the higher probabilities is equal to $1-\alpha$, the confidence level.

$$(8) \qquad \sum_{D}^{L} P(M_j) = 1-\alpha ,$$

L being determined by computations. Equation (8) gives the upper bound of M_j or the so-called one-tail test for small values of D. For $D = 0$ and $D = S$, the one-tail and two-tail tests coincide. For most values of D, the probabilities form an asymmetric as well as a discrete distribution. The asymmetries are illustrated in figure 1 where curved lines are drawn through point probabilities for $N = 50$, $S = 8$, and $D = 0$ to 4.

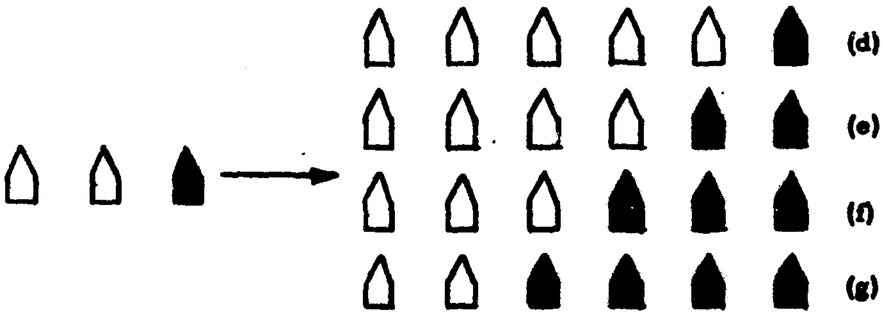
Approaches I and II may perhaps be better illustrated by the use of black and white missiles.

Approach I



There are six missiles in the population - four white and two black. The three possible outcomes in a sample of three are labeled (a), (b), and (c); the chance or probability of drawing any one is determined by equation (1).

Approach II. In stockpile quality evaluation it is desirable to take approach II which is the more realistic statistical model. In this case the population is estimated from a known sample.



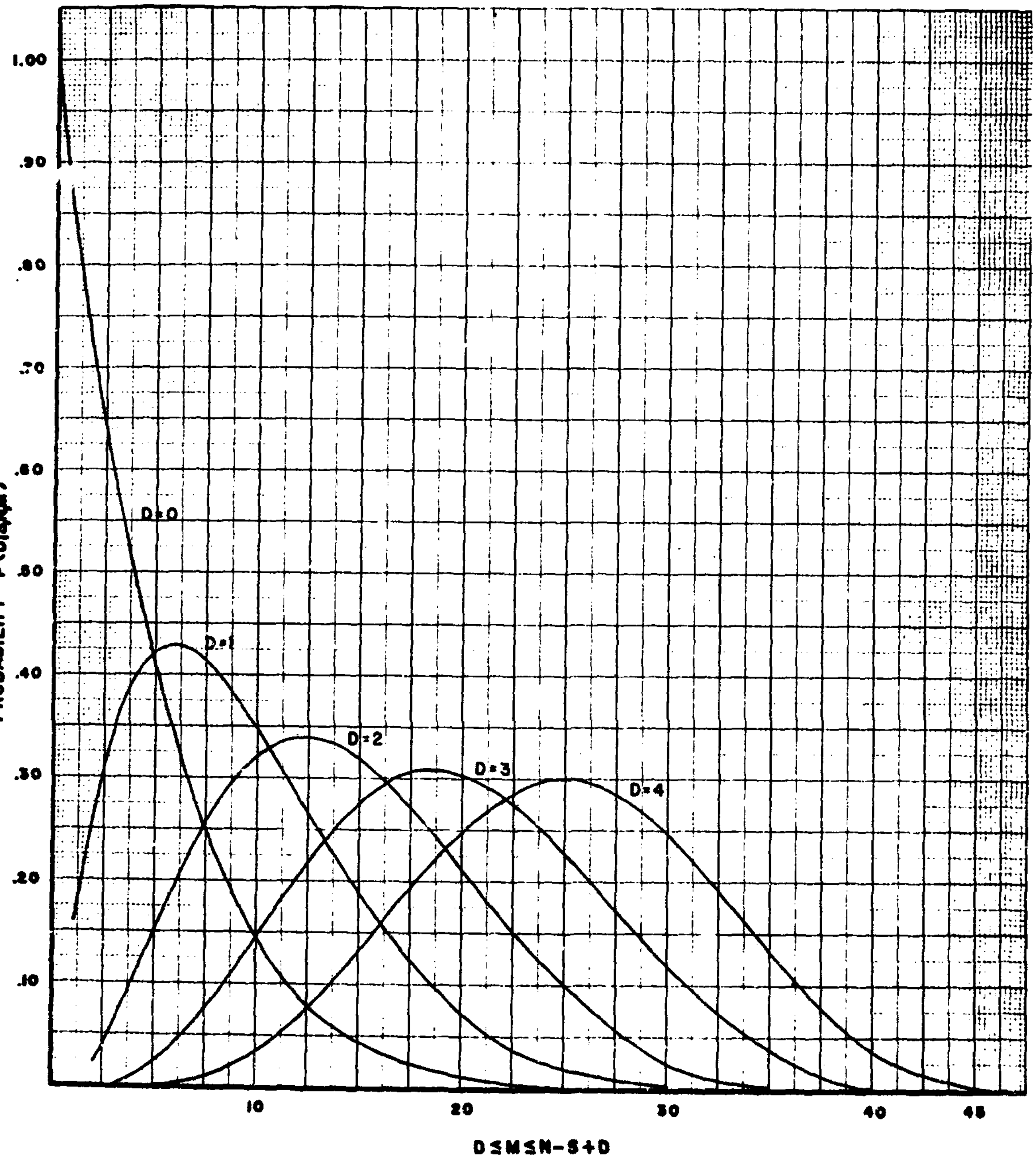


FIGURE 1. CHANGE IN SHAPE OF GENERALIZED POINT PROBABILITY CURVES DUE TO THE NUMBER OF DEFECTIVES PRESENT IN A SAMPLE.

Here, from a population of six, is a sample of three wherein two white and one black missiles were noted. The four possible populations from which this sample could have been drawn are: (d), (e), (f), and (g). The chances or probability of any one of the populations being the one from which this particular sample is drawn is determined by equation (3).

Symmetries

Since $\frac{S+1}{N+1}\ P(D|S,N,M) = P(M|S,N,D)$ by equation (3), symmetries given by Lieberman and Owen (Reference 4) for $P(D|S,N,M)$ apply in many instances where S and N are fixed.

(9) $$P(M = M_1 | D_1, S, N) = P(M = N-M_1 | S, S-D_1, N)$$

(10) $$\sum_{M=A}^{B} P(M|S, D = D_1, N) = \sum_{M=N-A}^{N-B} P(M|S, D = S-D_1, N)$$

Equations (9) and (10) show that tables need only involve half the sample size.

Population defectives M given and sample size S unknown

(11) $$P(M|D,S,N) = P(S|D,M,N)$$

(12) $$\sum_{S=D}^{N-M+D} P(S|D,M,N) = \frac{N+1}{M+1}$$

Equations (11) and (12) give the basic equations for the problem of sample size estimation when N, D, and M are known. Any table for M defectives may be used by interchanging S for M.

Two Stage Sampling

Out of a total allowable sample of size S from a population N, an initial sample S_1 is tested and D_1 defectives are found. By means of a table similar to Appendix C, it is found that x defectives gives the desired percent operability and y defectives ($y > x$) do not. If $D_1 \leq x$ then the lot is accepted. If $D_1 \geq y$, the lot is rejected. If $x < D_1 < y$, then the remainder of S, called S_2, is sequentially tested. If at any time a total of $D_1 + C$ defectives are found, the lot is rejected, since the total sample S would have at least $D_1 + C$ defectives. If the total sample S is tested and $(D_1 + C - 1)$ defectives are found, the lot is accepted. $D_1 + C$ is determined from the probability table for N, S, and M, the number of defectives that will be tolerated.

Best Sample Size S for a Given D

For some values of D sample defectives, a large sample size S is required to reach the desired percent operability for a given confidence level. In these cases, the percent operability should perhaps be lowered to the point where the additional sample units give less than some pre-selected gain value in percent operability. This is illustrated in figure 2, which is patterned after the graphs in Appendix B.

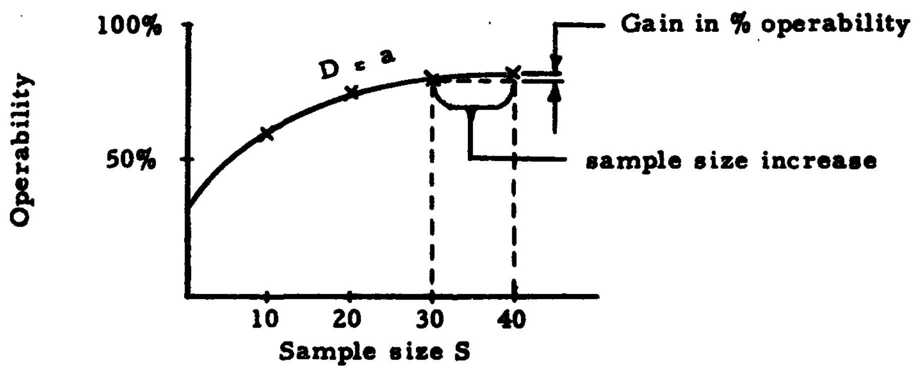


FIGURE 2. SAMPLE SIZE VERSUS OPERABILITY.

Here for D = a, an additional sample of 10, from 30 to 40 will result in a very small gain in percent operability and therefore, S = 30 may be the better choice of sample size.

RESULTS

Tables and Graphs

The table and graphs in Appendices A and B are useful as "quick look" references. The table gives the range for population defectives for certain fixed confidence levels. The graphs give different population lines in terms of percent operability versus fixed sample sizes. Smooth curves are drawn through interpolated points. Only the lower percent operability values are plotted for clarity.

Print-out of Probabilities

A sample of the computer print-out of probabilities is given in Appendix C. Point probabilities and sums for $P(D|S, N, M)$ were tabulated instead of $P(M|D, S, N)$ for purposes of checking with other tables.

The abbreviations are:

S = Sample size
N = Population size
D = Sample defective
M = Population defective
P = $P(D|S, N, M)$

$$\text{SUM} = \sum_{M=R}^{T} P(D|S, N, M)$$ where the sum contains the highest probabilities from R to T

$$\text{CONF} = \frac{\sum_{R}^{T} P(D|S, N, M)}{\sum_{M=D}^{N-S+D} P(D|S, N, M)}$$

$$\text{INTERVAL} = \begin{cases} \text{Lower } M \\ \dfrac{\text{Lower } M}{N} \\ \text{Upper } M \\ \dfrac{\text{Upper } M}{N} \end{cases}$$

LEFT SUM = Sum of decreasing probabilities to the left of maximum likelihood

RIGHT SUM = Sum of decreasing probabilities to the right of maximum likelihood

A one-tail test is possible from this type of table. Assume $1-\alpha$ is the confidence level, $Q = \sum_{M=D}^{N-S+D} P(D|S, N, M)$, and

$$R = \sum_{M=\hat{M}+1}^{N-S+D} P(D|S, N, M).$$

Compute αQ and subtract from R. Then trace back in the "Right sum" column until a value just exceeding $R - \alpha Q$ is found. The value of upper M in the same row is the one-tail upper bound on the population defectives.

Program for the Hypergeometric Series

The symbolic language program developed for the IBM 1401 computer, Mod B4, is given in Appendix D. Accuracy in computation of factorials was mainly accomplished by having the decision to multiply by a number from the numerator or divide by a number from the denominator depend upon the number of leading zeros resulting from the previous calculation. Individual probabilities were then usually accurate to ten places and sums of probabilities to eight.

The abbreviations used in the print-out in Appendix D are:

PG LIN	=	Page and line identification
CT	=	Count for instruction or reserved storage
OP	=	Operation instruction
A OPERAND	=	A or I address of instruction
B OPERAND	=	B address of instruction
D	=	D character modification of the basic instruction

REFERENCES

[1] Deming, William Edwards, Some Theory of Sampling, New York, John Wiley and Sons, 1950, p 294

[2] Katz, Leo, Confidence Intervals for the Number Showing a Certain Characteristic in a Population When Sampling is Without Replacement Journal of the American Statistical Association v. 48 (1953), 256-261

[3] Odell, Patrick L., Tables and Graphs for Determining an Upper Confidence Bound on the Number of Defectives in a Finite Population (U)

U. S. Naval Nuclear Ordnance Evaluation Unit, Albuquerque, New Mexico, 1960, p. 1

[4] Lieberman, G. J. and Owen, D. B., Tables of the Hypergeometric Probability Distribution, Stanford University Press, 1961.

APPENDIX A: TABLES OF POPULATION DEFECTIVES FOR CERTAIN CONFIDENCE LEVELS.

N	S	D	90% Confidence Levels		95% Confidence Levels	
			M_L	M_U	M_L	M_U
50	8	0	0	10	0	13
		1	1	17	1	20
		2	4	24	4	27
		3	9	31	7	33
		4	14	37	12	38
	10	0	0	8	0	10
		1	1	14	1	17
		2	3	20	3	23
		3	7	26	6	28
		4	11	31	9	32
	12	0	0	7	0	9
		1	1	12	1	14
		2	3	17	2	19
		3	6	22	5	24
		4	9	26	8	28
60	8	0	0	12	0	16
		1	1	21	1	24
		2	5	30	4	33
		3	10	37	9	40
		4	16	44	14	46
	10	0	0	10	0	13
		1	1	17	1	20
		2	4	25	3	27
		3	8	31	7	34
		4	12	37	11	40
	12	0	0	8	0	11
		1	1	15	1	17
		2	3	21	3	24
		3	7	27	6	29
		4	10	32	9	34
70	8	0	0	15	0	18
		1	1	25	1	29
		2	6	36	5	39
		3	12	44	10	47
		4	18	51	16	54
	10	0	0	12	0	15
		1	1	20	1	24
		2	5	29	4	33
		3	9	36	8	40
		4	14	43	12	46
	12	0	0	10	0	13
		1	1	17	1	20
		2	4	25	3	28
		3	7	31	6	34
		4	12	38	10	40
80	8	0	0	17	0	21
		1	2	29	1	33
		2	7	40	5	44
		3	13	50	11	54
		4	21	59	18	62
	10	0	0	14	0	18
		1	1	24	1	28
		2	5	33	4	37
		3	10	42	8	46
		4	16	50	14	53
	12	0	0	12	0	15
		1	1	20	1	24
		2	4	28	3	32
		3	8	36	7	39
		4	13	43	11	46
90	8	0	0	19	0	24
		1	2	33	1	37
		2	7	45	6	50
		3	15	57	12	60
		4	24	67	20	70
	10	0	0	16	0	20
		1	1	27	1	31
		2	6	38	4	42
		3	11	47	9	51
		4	18	56	16	60
	12	0	0	13	0	17
		1	1	23	1	27
		2	5	32	4	36
		3	9	40	8	44
		4	15	49	13	54
100	8	0	0	21	0	27
		1	2	36	1	42
		2	8	50	6	55
		3	17	63	13	67
		4	26	74	23	78
	10	0	0	18	0	22
		1	2	31	1	35
		2	6	42	5	47
		3	13	53	10	57
		4	20	63	17	67
	12	0	0	15	0	19
		1	1	25	1	30
		2	5	36	4	40
		3	10	45	9	50
		4	16	54	14	58
120	8	0	0	26	0	33
		1	2	44	1	50
		2	10	61	7	67
		3	19	76	16	81
		4	31	89	27	94
	10	0	0	21	0	27
		1	2	37	1	42
		2	7	50	6	57
		3	15	64	12	69
		4	24	76	20	80
	12	0	0	18	0	23
		1	1	31	1	36
		2	6	43	5	49
		3	12	55	10	60
		4	19	65	16	70
	15	0	0	15	0	19
		1	1	25	1	30
		2	5	36	4	41
		3	10	46	8	50
		4	15	54	12	59
	17	0	0	13	0	17
		1	1	22	1	27
		2	4	32	3	36
		3	8	40	7	45
		4	13	48	11	52
140	8	0	0	30	0	38
		1	3	52	2	60
		2	11	71	8	78
		3	22	90	19	94
		4	36	104	31	109
	10	0	0	25	0	32
		1	2	43	1	50
		2	8	59	6	66
		3	17	74	14	81
		4	27	88	23	94
	12	0	0	21	0	27
		1	2	37	1	43
		2	7	51	5	57
		3	14	64	11	70
		4	22	76	19	82
	15	0	0	17	0	22
		1	1	30	1	35
		2	6	42	4	47
		3	11	53	9	58
		4	17	63	15	69
	17	0	0	15	0	20
		1	1	26	1	31
		2	5	37	4	43
		3	10	48	8	52
		4	15	56	12	62
160	8	0	0	35	0	44
		1	3	59	2	68
		2	12	81	9	89
		3	26	101	21	109
		4	41	119	36	128
	10	0	0	29	0	37
		1	2	49	1	57
		2	10	68	7	76
		3	20	86	16	93
		4	31	101	27	108
160	12	0	0	25	0	31
		1	2	42	1	49
		2	8	59	6	66
		3	16	74	13	81
		4	25	87	21	94
	15	0	0	20	0	26
		1	2	35	1	40
		2	6	48	5	55
		3	13	61	10	67
		4	20	73	17	79
	17	0	0	18	0	23
		1	1	30	1	36
		2	5	43	4	49
		3	11	56	9	60
		4	17	65	15	71
180	8	0	0	39	0	50
		1	3	66	2	77
		2	14	92	11	101
		3	29	114	23	122
		4	47	134	39	140
	10	0	0	33	0	41
		1	2	56	2	65
		2	11	77	8	86
		3	22	96	18	104
		4	36	114	30	122
	12	0	0	28	0	35
		1	2	47	1	55
		2	9	66	7	74
		3	18	83	15	91
		4	29	99	24	106
	15	0	0	23	0	29
		1	2	39	1	46
		2	7	54	5	62
		3	14	69	11	75
		4	22	82	19	89
	17	0	0	20	0	26
		1	2	35	1	41
		2	6	49	5	55
		3	12	61	10	68
		4	19	73	16	80
	20	0	0	17	0	22
		1	1	30	1	35
		2	5	42	4	48
		3	10	53	8	59
		4	16	63	14	70
200	8	0	0	44	0	56
		1	3	74	2	86
		2	16	103	11	112
		3	33	127	26	136
		4	51	148	43	156
	10	0	0	37	0	46
		1	3	62	2	72
		2	13	86	9	95
		3	26	107	20	116
		4	39	127	33	135
	12	0	0	31	0	40
		1	2	52	1	62
		2	10	74	7	82
		3	20	93	16	101
		4	32	110	26	118
	15	0	0	25	0	33
		1	2	44	1	51
		2	8	61	6	69
		3	15	76	12	86
		4	24	91	21	99
	17	0	0	23	0	29
		1	2	39	1	46
		2	7	55	5	61
		3	14	69	11	76
		4	22	85	18	89
	20	0	0	19	0	25
		1	1	33	1	39
		2	6	47	4	53
		3	11	59	9	66
		4	18	71	15	77

APPENDIX A - CONTINUED:

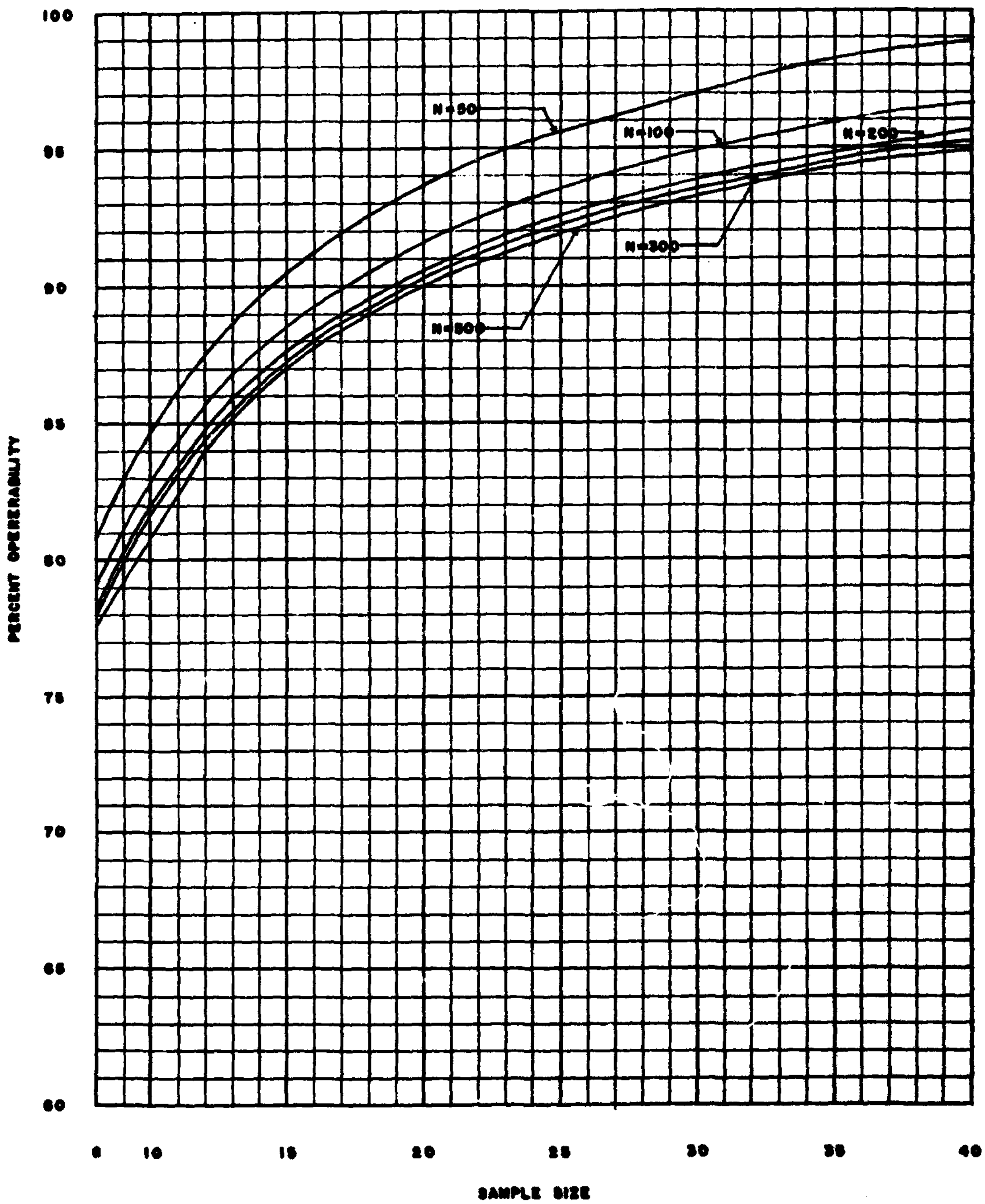
N	S	D	95% Confidence Levels M_L	95% Confidence Levels M_U	99% Confidence Levels M_L	99% Confidence Levels M_U
225	8	0	0	50	0	62
		1	4	84	2	96
		2	17	114	13	126
		3	36	143	29	153
		4	58	168	49	176
	10	0	0	41	0	52
		1	3	70	2	81
		2	13	96	10	107
		3	27	120	22	131
		4	44	143	37	152
	12	0	0	35	0	45
		1	2	59	2	70
		2	11	83	8	93
		3	22	104	18	114
		4	35	123	30	133
	15	0	0	29	0	37
		1	2	49	1	58
		2	8	68	6	77
		3	17	86	14	95
		4	28	103	23	112
	17	0	0	26	0	33
		1	2	44	1	52
		2	7	61	6	70
		3	15	77	12	86
		4	24	92	20	101
	20	0	0	22	0	28
		1	2	38	1	45
		2	6	52	5	60
		3	13	67	10	74
		4	20	80	17	88
	25	0	0	18	0	23
		1	1	30	1	36
		2	5	43	4	49
		3	10	54	8	60
		4	16	65	14	72
250	8	0	0	58	0	69
		1	4	93	3	107
		2	19	127	14	140
		3	40	159	32	170
		4	64	186	54	196
	10	0	0	46	0	58
		1	3	77	2	90
		2	15	108	11	119
		3	30	134	24	145
		4	49	159	41	169
	12	0	0	39	0	50
		1	3	67	2	76
		2	12	91	9	104
		3	24	116	20	127
		4	39	137	33	148
	15	0	0	32	0	41
		1	2	55	1	64
		2	9	76	7	86
		3	19	96	15	106
		4	30	114	26	124
	17	0	0	29	0	37
		1	2	49	1	58
		2	8	68	6	77
		3	17	86	13	96
		4	27	103	23	112
	20	0	0	25	0	32
		1	2	42	1	50
		2	7	59	[illegible]	67
		3	14	76	11	83
		4	23	89	19	98
	25	0	0	20	0	26
		1	1	34	1	40
		2	6	48	4	56
		3	11	60	9	68
		4	18	72	15	80
275	8	0	0	66	0	77
		1	5	103	3	118
		2	21	140	16	155
		3	43	174	35	187
		4	70	200	59	215
275	10	0	0	51	0	64
		1	4	86	2	99
		2	16	118	12	132
		3	33	147	27	160
		4	53	174	44	186
	12	0	0	43	0	55
		1	3	74	2	86
		2	13	102	10	114
		3	27	128	22	140
		4	43	152	36	163
	15	0	0	35	0	45
		1	2	60	1	71
		2	10	84	8	95
		3	21	106	17	117
		4	32	126	28	137
	17	0	0	32	0	40
		1	2	54	1	64
		2	9	75	7	86
		3	18	95	15	105
		4	29	113	24	123
	20	0	0	27	0	35
		1	2	47	1	55
		2	8	65	6	74
		3	15	82	12	91
		4	24	98	20	107
	25	0	0	22	0	28
		1	1	37	1	45
		2	6	53	5	61
		3	12	67	10	75
		4	19	80	16	88
	30	0	0	18	0	24
		1	1	31	1	38
		2	5	44	4	51
		3	10	56	8	63
		4	16	68	14	75
300	8	0	0	67	0	84
		1	5	112	3	129
		2	23	154	17	169
		3	47	190	38	204
		4	77	224	65	236
	10	0	0	55	0	70
		1	4	94	2	108
		2	18	129	13	146
		3	36	161	29	175
		4	58	190	49	203
	12	0	0	47	0	60
		1	3	80	2	96
		2	14	111	10	124
		3	29	139	25	158
		4	47	166	39	170
	15	0	0	39	0	50
		1	2	66	2	78
		2	11	92	8	106
		3	23	116	18	127
		4	36	138	30	148
	17	0	0	36	0	44
		1	2	59	1	69
		2	10	83	7	98
		3	20	104	16	115
		4	32	126	26	128
	20	0	0	30	0	38
		1	2	51	1	60
		2	8	71	6	81
		3	17	90	13	99
		4	26	107	22	118
	25	0	0	24	0	31
		1	2	42	1	49
		2	7	58	5	66
		3	13	73	11	82
		4	21	88	17	96
	30	0	0	20	0	26
		1	1	36	1	42
		2	6	49	4	55
		3	11	61	9	69
		4	17	74	14	82
350	8	0	0	78	0	98
		1	6	131	3	150
		2	26	179	20	198
		3	55	222	46	239
		4	89	261	76	275
	10	0	0	65	0	82
		1	4	109	3	127
		2	20	150	15	168
		3	42	186	34	205
		4	67	222	57	237
	12	0	0	55	0	70
		1	4	96	2	110
		2	17	130	13	146
		3	36	165	27	170
		4	56	195	46	200
	15	0	0	46	0	58
		1	3	75	2	95
		2	13	108	9	121
		3	26	136	21	149
		4	42	161	35	175
	17	0	0	41	0	52
		1	2	69	2	88
		2	11	96	8	109
		3	23	121	18	134
		4	37	146	31	158
	20	0	0	35	0	46
		1	2	60	1	70
		2	9	85	7	96
		3	19	108	15	114
		4	31	126	26	135
	25	0	0	28	0	36
		1	2	49	1	57
		2	8	69	6	78
		3	15	88	12	96
		4	26	102	20	113
	30	0	0	24	0	30
		1	2	41	1	48
		2	6	57	5	66
		3	13	72	10	81
		4	20	87	17	96
	35	0	0	20	0	26
		1	1	35	1	42
		2	5	49	4	56
		3	11	62	9	70
		4	17	75	14	83
400	8	0	0	89	0	112
		1	7	[illegible]	6	172
		2	30	206	22	225
		3	63	255	51	273
		4	101	298	86	314
	10	0	0	74	0	96
		1	5	120	3	148
		2	23	172	17	198
		3	48	215	39	233
		4	77	254	64	271
	12	0	0	64	0	81
		1	4	107	2	128
		2	19	149	14	167
		3	39	187	31	204
		4	62	221	52	238
	15	0	0	52	0	67
		1	3	89	2	106
		2	15	122	11	139
		3	30	155	24	171
		4	48	186	40	200
	17	0	0	47	0	60
		1	3	80	2	95
		2	13	111	9	126
		3	26	139	21	156
		4	42	166	35	180
	20	0	0	40	0	51
		1	2	68	1	81
		2	11	95	8	109
		3	22	120	17	130
		4	35	140	29	150

APPENDIX A - CONTINUED:

N	S	D	95% Confidence Levels M_L	95% Confidence Levels M_U	99% Confidence Levels M_L	99% Confidence Levels M_U
400	25	0	0	38	0	42
		1	2	56	1	66
		2	9	70	6	89
		3	17	90	14	110
		4	28	118	23	130
	30	0	0	27	0	35
		1	2	47	1	56
		2	7	65	5	75
		3	15	83	12	93
		4	23	100	19	110
	35	0	0	23	0	30
		1	1	40	1	48
		2	6	56	5	65
		3	12	71	10	80
		4	20	86	16	96
	40	0	0	20	0	26
		1	1	36	1	42
		2	5	49	4	57
		3	11	63	9	71
		4	17	75	14	84
500	5	0	0	100	0	126
		1	7	160	4	193
		2	36	231	25	256
		3	70	286	57	307
		4	114	336	96	383
	10	0	0	86	0	106
		1	6	141	3	164
		2	26	196	19	216
		3	54	249	43	263
		4	86	306	72	308
	12	0	0	72	0	91
		1	4	121	3	142
		2	21	160	15	187
		3	43	209	35	230
		4	70	249	58	268
	15	0	0	59	0	75
		1	4	100	2	118
		2	16	138	12	157
		3	34	175	27	193
		4	54	208	46	226
	17	0	0	53	0	67
		1	3	89	2	106
		2	14	124	10	141
		3	29	156	23	173
		4	47	187	39	204
	20	0	0	45	0	56
		1	3	78	2	95
		2	12	108	9	123
		3	25	136	20	151
		4	39	162	32	177
	25	0	0	37	0	47
		1	3	63	1	76
		2	9	87	7	100
		3	19	110	15	123
		4	31	133	25	146
	30	0	0	31	0	40
		1	2	53	1	63
		2	8	74	6	85
		3	16	93	13	109
		4	25	112	21	126
	35	0	0	26	0	34
		1	2	46	1	56
		2	7	64	5	73
		3	14	81	11	91
		4	22	97	18	107
	40	0	0	23	0	30
		1	1	39	1	48
		2	6	56	4	64
		3	12	71	10	80
		4	19	86	16	95
600	5	0	0	112	0	140
		1	8	187	5	216
		2	38	267	28	283
		3	78	358	63	361
		4	127	396	107	399
600	10	0	0	93	0	118
		1	6	157	4	183
		2	29	216	21	240
		3	60	270	48	293
		4	96	318	80	339
	12	0	0	80	0	101
		1	5	135	3	158
		2	23	186	17	209
		3	46	233	38	255
		4	77	276	64	297
	15	0	0	66	0	84
		1	4	111	2	131
		2	18	156	13	174
		3	37	193	30	214
		4	60	231	50	251
	17	0	0	59	0	75
		1	3	99	2	118
		2	16	139	11	156
		3	32	174	26	193
		4	52	206	43	226
	20	0	0	50	0	65
		1	3	86	2	102
		2	13	119	10	137
		3	27	151	22	168
		4	43	180	36	198
	25	0	0	41	0	53
		1	2	70	1	83
		2	10	97	8	112
		3	21	123	17	138
		4	34	147	28	162
	30	0	0	34	0	44
		1	2	59	1	70
		2	9	82	6	94
		3	16	104	14	117
		4	28	125	23	138
	35	0	0	29	0	38
		1	2	51	1	60
		2	8	71	6	82
		3	15	90	12	101
		4	24	108	20	120
	40	0	0	26	0	33
		1	2	45	1	53
		2	7	63	5	72
		3	13	79	11	90
		4	21	95	17	105
700	5	0	0	157	0	197
		1	11	262	6	302
		2	52	369	39	396
		3	109	446	88	478
		4	177	523	149	580
	10	0	0	131	0	168
		1	8	219	5	256
		2	40	303	29	337
		3	83	378	66	418
		4	134	466	112	476
	12	0	0	112	0	143
		1	7	189	4	222
		2	38	261	23	293
		3	67	327	55	356
		4	108	380	90	410
	15	0	0	92	0	118
		1	5	156	3	186
		2	25	217	18	248
		3	58	272	41	300
		4	83	306	69	342
	17	0	0	83	0	106
		1	5	141	3	166
		2	22	190	16	221
		3	46	246	36	271
		4	72	291	60	318
	20	0	0	71	0	91
		1	4	121	3	143
		2	11	168	13	191
		3	38	212	30	236
		4	60	253	50	278
700	25	0	0	58	0	74
		1	3	99	2	118
		2	14	137	10	157
		3	30	174	23	196
		4	47	207	39	229
	30	0	0	49	0	63
		1	3	84	2	100
		2	12	116	9	133
		3	25	147	19	164
		4	39	176	32	196
	35	0	0	42	0	54
		1	2	71	1	85
		2	10	100	7	118
		3	21	127	17	143
		4	33	152	27	168
	40	0	0	37	0	47
		1	2	63	1	75
		2	9	88	7	102
		3	18	111	15	126
		4	29	134	24	149
1000	5	0	0	225	0	282
		1	16	375	9	432
		2	75	516	56	546
		3	156	638	125	686
		4	253	746	213	787
	10	0	0	188	0	237
		1	12	315	7	356
		2	57	433	41	482
		3	118	540	96	586
		4	191	638	160	681
	12	0	0	161	0	204
		1	10	271	5	317
		2	46	374	33	419
		3	95	468	75	512
		4	153	555	127	597
	15	0	0	133	0	169
		1	7	224	[illegible]	264
		2	35	318	25	350
		3	74	390	50	430
		4	118	463	90	503
	17	0	0	119	0	152
		1	6	201	4	230
		2	31	279	22	316
		3	64	350	50	387
		4	103	417	85	456
	20	0	0	102	0	131
		1	5	173	3	206
		2	26	241	18	276
		3	54	304	42	330
		4	86	363	70	397
	25	0	0	83	0	107
		1	4	142	2	168
		2	20	197	14	223
		3	42	248	33	278
		4	67	297	55	308
	30	0	0	70	0	90
		1	4	120	2	143
		2	17	167	12	191
		3	35	211	27	236
		4	56	252	46	278
	35	0	0	60	0	78
		1	3	103	2	130
		2	14	144	10	168
		3	29	182	23	208
		4	47	218	38	248
	40	0	0	53	0	69
		1	3	91	2	109
		2	12	126	9	146
		3	26	161	20	182
		4	41	190	34	206

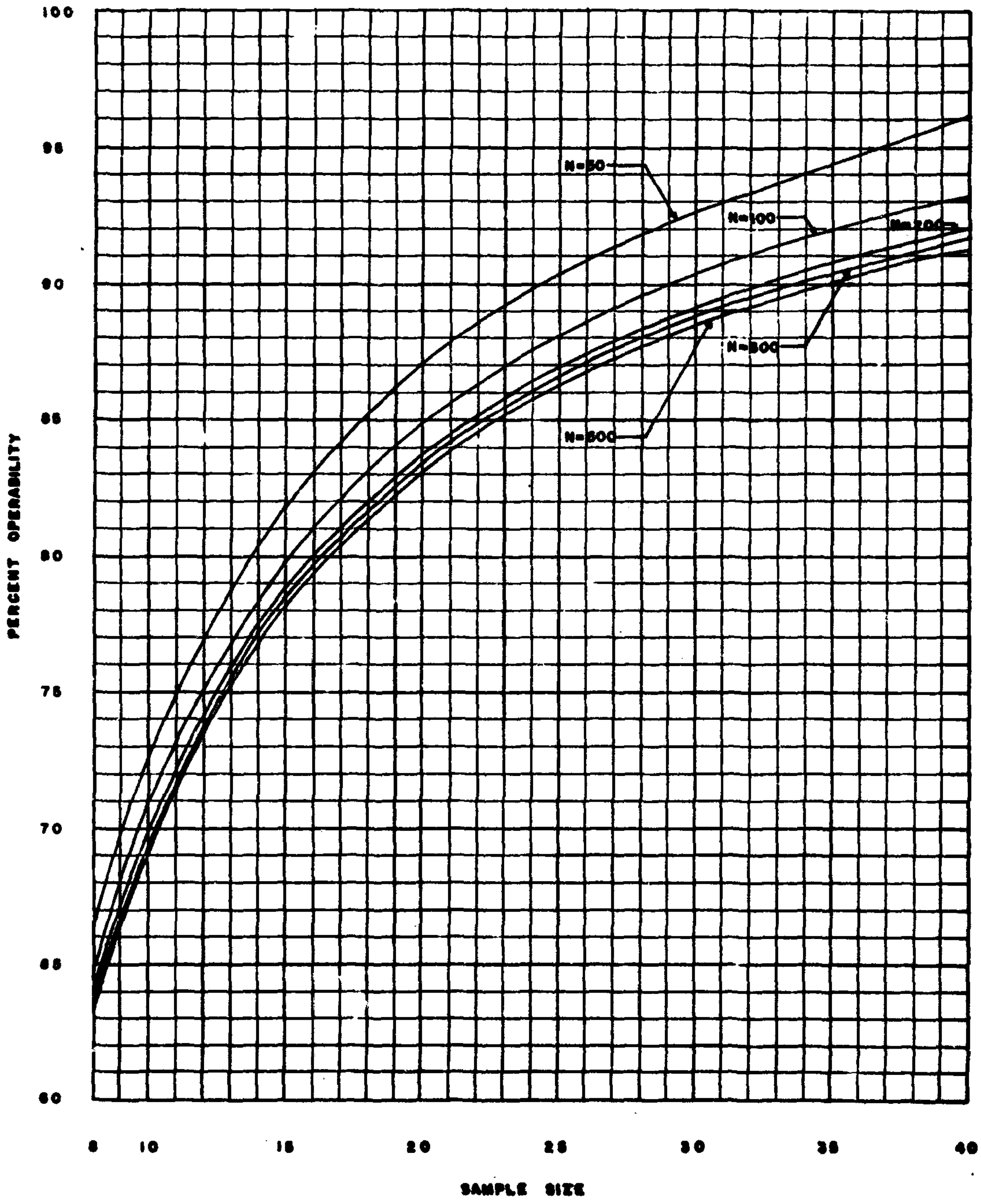
NOTE: WHERE CONFIDENCE LEVELS DO NOT FALL ON 95% OR 99%, THE NEXT HIGHEST LEVELS ARE SELECTED.

APPENDIX B GRAPH I



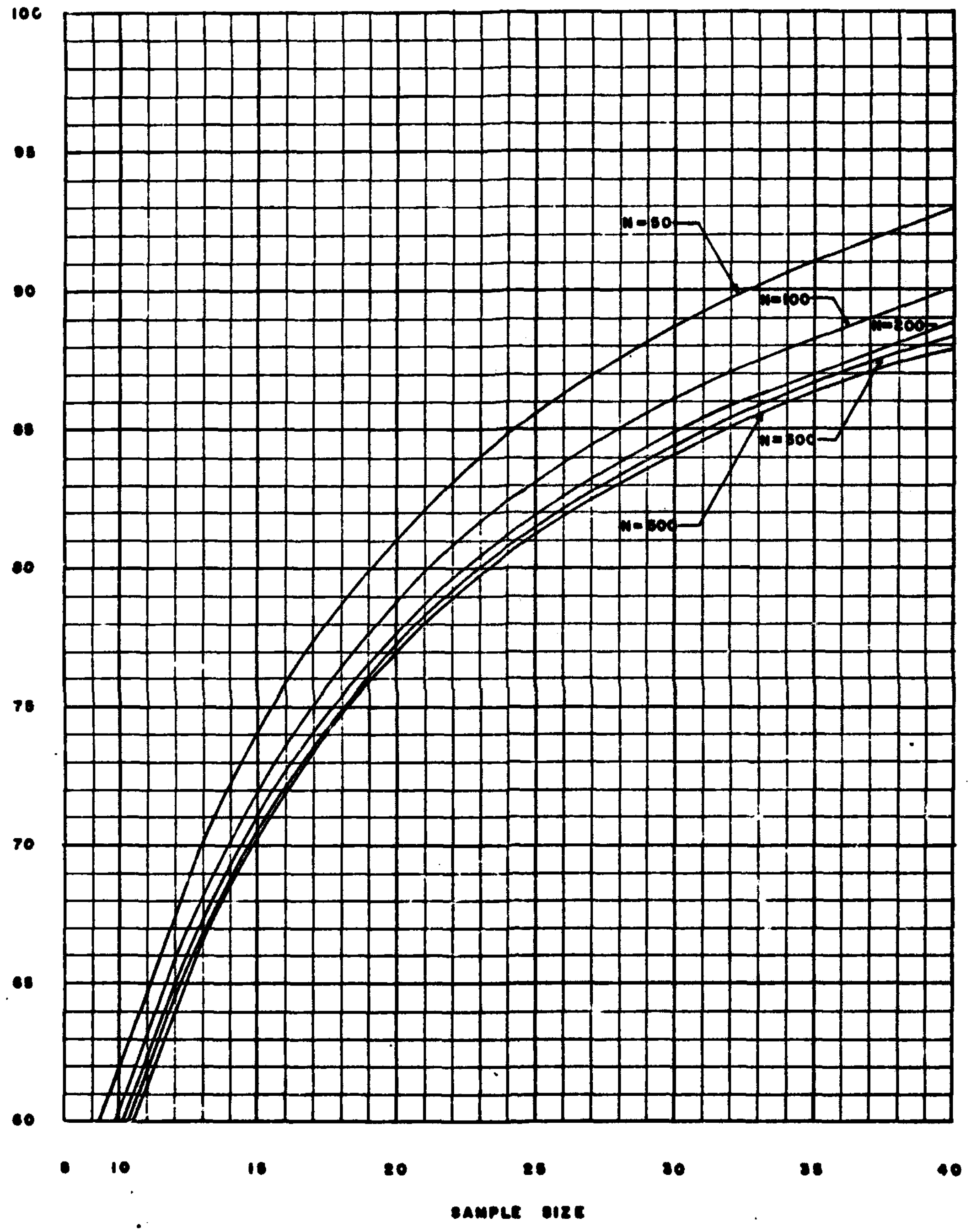
PERCENT OPERABILITY VERSUS FIXED SAMPLE SIZE
.90 CONFIDENCE LEVEL FOR NO DEFECTIVE

APPENDIX B GRAPH 2



PERCENT OPERABILITY VERSUS FIXED SAMPLE SIZE
.90 CONFIDENCE LEVEL FOR ONE DEFECTIVE

APPENDIX B GRAPH 3



PERCENT OPERABILITY VERSUS FIXED SAMPLE SIZE
.90 CONFIDENCE LEVEL FOR TWO DEFECTIVES

APPENDIX C: SAMPLE OF COMPUTER PRINT-OUT OF PROBABILITIES.

HYPERGEOMETRIC SERIES

S	N	D	N	P	SUM	CONF	INTERVAL				LEFT SUM	RIGHT SUM
4	50	0	0	0.9999999999	0.9999999999	0.0980						
4	50	0	1	0.9199999999	1.9199999999	0.1882	0	0.0000	1	0.0200	0.0000000000	0.9199999999
4	50	0	2	0.8448979591	2.7648979591	0.2710	0	0.0000	2	0.0400	0.0000000000	1.7648979591
4	50	0	3	0.7744897959	3.5393877550	0.3469	0	0.0000	3	0.0600	0.0000000000	2.5393877551
4	50	0	4	0.7085757707	4.2479635258	0.4164	0	0.0000	4	0.0800	0.0000000000	3.2479635258
4	50	0	5	0.6469604863	4.8949240121	0.4798	0	0.0000	5	0.1000	0.0000000000	3.8949240121
4	50	0	6	0.5894528875	5.4843768996	0.5376	0	0.0000	6	0.1200	0.0000000000	4.4843768996
4	50	0	7	0.5358662613	6.0202431610	0.5902	0	0.0000	7	0.1400	0.0000000000	5.0202431610
4	50	0	8	0.4860182370	6.5062613981	0.6378	0	0.0000	8	0.1600	0.0000000000	5.5062613981
4	50	0	9	0.4397307859	6.9459921841	0.6809	0	0.0000	9	0.1800	0.0000000000	5.9459921841
4	50	0	10	0.3968302214	[illegible]3428224055	0.7198	0	0.0000	10	0.2000	0.0000000000	6.3428224055
4	50	0	11	0.3571471993	7.6999696048	0.7548	0	0.0000	11	0.2200	0.0000000000	6.6999696048
4	50	0	12	0.3205167173	8.0204863221	0.7863	0	0.0000	12	0.2400	0.0000000000	7.0204863221
4	50	0	13	0.2867781155	8.3072644376	0.8144	0	0.0000	13	0.2600	0.0000000000	7.3072644376
4	50	0	14	0.2557750759	8.5630395136	0.8395	0	0.0000	14	0.2800	0.0000000000	7.5630395136
4	50	0	15	0.2273556231	8.7903951367	0.8618	0	0.0000	15	0.3000	0.0000000000	7.7903951367
4	50	0	16	0.2013721233	8.9917672600	0.8815	0	0.0000	16	0.3200	0.0000000000	7.9917672600
4	50	0	17	0.1776812852	9.1694485453	0.8989	0	0.0000	17	0.3400	0.0000000000	8.1694485453
4	50	0	18	0.1561441597	9.3255927051	0.9142	0	0.0000	18	0.3600	0.0000000000	8.3255927051
4	50	0	19	0.1366261398	9.4622188449	0.9276	0	0.0000	19	0.3800	0.0000000000	8.4622188449
4	50	0	20	0.1189969604	9.5812158054	0.9393	0	0.0000	20	0.4000	0.0000000000	8.5812158054
4	50	0	21	0.1031306990	9.6843465045	0.9494	0	0.0000	21	0.4200	0.0000000000	8.6843465045
4	50	0	22	0.0889057750	9.7732522796	0.9581	0	0.0000	22	0.4400	0.0000000000	8.7732522796
4	50	0	23	0.0762049500	9.8494572296	0.9656	0	0.0000	23	0.4600	0.0000000000	8.8494572296
4	50	0	24	0.0649153278	9.9143725575	0.9719	0	0.0000	24	0.4800	0.0000000000	8.9143725575
4	50	0	25	0.0549283543	9.9693009118	0.9773	0	0.0000	25	0.5000	0.0000000000	8.9693009118
4	50	0	26	0.0461398176	10.0154407294	0.9819	0	0.0000	26	0.5200	0.0000000000	9.0154407294
4	50	0	27	0.0384498480	10.0538905775	0.9856	0	0.0000	27	0.5400	0.0000000000	9.0538905775
4	50	0	28	0.0317629179	10.0856534954	0.9887	0	0.0000	28	0.5600	0.0000000000	9.0856534954
4	50	0	29	0.0259878419	10.1116413373	0.9913	0	0.0000	29	0.5800	0.0000000000	9.1116413373
4	50	0	30	0.0210377768	10.1326791141	0.9933	0	0.0000	30	0.6000	0.0000000000	9.1326791141
4	50	0	31	0.0168302214	10.1495093356	0.9950	0	0.0000	31	0.6200	0.0000000000	9.1495093356
4	50	0	32	0.0132870169	10.1627963525	0.9963	0	0.0000	32	0.6400	0.0000000000	9.1627963525
4	50	0	33	0.0103343465	10.1731306990	0.9973	0	0.0000	33	0.6600	0.0000000000	9.1731306990
4	50	0	34	0.0079027355	10.1810334346	0.9981	0	0.0000	34	0.6800	0.0000000000	9.1810334346
4	50	0	35	0.0059270516	10.1869604863	0.9987	0	0.0000	35	0.7000	0.0000000000	9.1869604863
4	50	0	36	0.0043465045	10.1913069908	0.9991	0	0.0000	36	0.7200	0.0000000000	9.1913069908
4	50	0	37	0.0031046461	10.1944116369	0.9994	0	0.0000	37	0.7400	0.0000000000	9.1944116369
4	50	0	38	0.0021493703	10.1965610073	0.9996	0	0.0000	38	0.7600	0.0000000000	9.1965610073
4	50	0	39	0.0014329135	10.1979939209	0.9998	0	0.0000	39	0.7800	0.0000000000	9.1979939209
4	50	0	40	0.0009118541	10.1989057750	0.9998	0	0.0000	40	0.8000	0.0000000000	9.1989057750
4	50	0	41	0.0005471124	10.1994528875	0.9999	0	0.0000	41	0.8200	0.0000000000	9.1994528875
4	50	0	42	0.0003039513	10.1997568388	0.9999	0	0.0000	42	0.8400	0.0000000000	9.1997568388
4	50	0	43	0.0001519756	10.1999088145	0.9999	0	0.0000	43	0.8600	0.0000000000	9.1999088145
4	50	0	44	0.0000651324	10.1999739470	0.9999	0	0.0000	44	0.8800	0.0000000000	9.1999739470
4	50	0	45	0.0000217108	10.1999956578	0.9999	0	0.0000	45	0.9000	0.0000000000	9.1999956578
4	50	0	46	0.0000043421	10.1999999999	0.9999	0	0.0000	46	0.9200	0.0000000000	9.1999999999
4	50	1	12	0.4395657837	0.4395657837	0.0430						
4	50	1	13	0.4386018237	0.8781676074	0.0860	12	0.2400	13	0.2600	0.0000000000	0.4386018237
4	50	1	11	0.4365132435	1.3146808510	0.1288	11	0.2200	13	0.2600	0.4365132435	0.4386018237
4	50	1	14	0.4340425531	1.7487234042	0.1714	11	0.2200	14	0.2800	0.4365132435	0.8726443768
4	50	1	10	0.4290056448	2.1777290490	0.2135	10	0.2000	14	0.2800	0.8655188884	0.8726443768
4	50	1	15	0.4262917933	2.6040208423	0.2552	10	0.2000	15	0.3000	0.8655188884	1.2989361702
4	50	1	9	0.4165870603	3.0206079027	0.2961	9	0.1800	15	0.3000	1.2821059487	1.2989361702
4	50	1	16	0.4157359965	3.4363438992	0.3368	9	0.1800	16	0.3200	1.2821059487	1.7146721667
4	50	1	17	0.4027442466	3.8390881458	0.3763	9	0.1800	17	0.3400	1.2821059487	2.1174164133
4	50	1	8	0.3987841945	4.2378723404	0.4154	8	0.1600	17	0.3400	1.6808901432	2.1174164133
4	50	1	18	0.3876682587	4.6255405992	0.4534	8	0.1600	18	0.3600	1.6808901432	2.5050846721
4	50	1	7	0.3751063829	5.0006469821	0.4902	7	0.1400	18	0.3600	2.0559965262	2.5050846721
4	50	1	19	0.3708423795	5.3714893616	0.5266	7	0.1400	19	0.3800	2.0559965262	2.8759270516
4	50	1	20	0.3525835866	5.7240729483	0.5611	7	0.1400	20	0.4000	2.0559965262	3.2285106382
4	50	1	6	0.3450455927	6.0691185410	0.5950	6	0.1200	20	0.4000	2.4010421189	3.2285106382
4	50	1	21	0.3331914893	6.4023100303	0.6276	6	0.1200	21	0.4200	2.4010421189	3.5617021276
4	50	1	22	0.3129483282	6.7152583586	0.6583	6	0.1200	22	0.4400	2.4010421189	3.8746504559
4	50	1	5	0.3080764220	7.0233347807	0.6885	5	0.1000	22	0.4400	2.7091185410	3.8746504559
4	50	1	23	0.2921189792	7.3154537599	0.7172	5	0.1000	23	0.4600	2.7091185410	4.1667694311
4	50	1	24	0.2709509335	7.5864046895	0.7437	5	0.1000	24	0.4800	2.7091185410	4.4377203647
4	50	1	4	0.2636561007	7.8500607902	0.7696	4	0.0800	24	0.4800	2.9727746417	4.4377203647
4	50	1	25	0.2496743378	8.0997351280	0.7940	4	0.0800	25	0.5000	2.9727746417	4.6873947025
4	50	1	26	0.2285019539	8.3282370820	0.8164	4	0.0800	26	0.5200	2.9727746417	4.9158966565
4	50	1	3	0.2112244897	8.5394615718	0.8372	3	0.0600	26	0.5200	3.1839991315	4.9158966565
4	50	1	27	0.2076291793	8.7470907511	0.8575	3	0.0600	27	0.5400	3.1839991315	5.1235258358
4	50	1	28	0.1872340425	8.9343247937	0.8759	3	0.0600	28	0.5600	3.1839991315	5.3107598784
4	50	1	29	0.1674772036	9.1018019973	0.8923	3	0.0600	29	0.5800	3.1839991315	5.4782370820
4	50	1	2	0.1502040816	9.2520060790	0.9070	2	0.0400	29	0.5800	3.3342032131	5.4782370820
4	50	1	30	0.1485019537	9.4005080329	0.9216	2	0.0400	30	0.6000	3.3342032131	5.6267390360
4	50	1	31	0.1309342162	9.5314422492	0.9344	2	0.0400	31	0.6200	3.3342032131	5.7576732522
4	50	1	32	0.1133825445	9.6448247937	0.9455	2	0.0400	32	0.6400	3.3342032131	5.8710557967
4	50	1	33	0.0974301241	9.7417629179	0.9550	2	0.0400	33	0.6600	3.3342032131	5.9679939209
4	50	1	34	0.0826747720	9.8244376899	0.9631	2	0.0400	34	0.6800	3.3342032131	6.0506686930
4	50	1	1	0.0799999999	9.9044376899	0.9710	1	0.0200	34	0.6800	3.4142032131	6.0506686930
4	50	1	35	0.0691489361	9.9735866261	0.9778	1	0.0200	35	0.7000	3.4142032131	6.1198176291
4	50	1	36	0.0568996960	10.0304863221	0.9833	1	0.0200	36	0.7200	3.4142032131	6.1767173252
4	50	1	37	0.0459487624	10.0764350846	0.9878	1	0.0200	37	0.7400	3.4142032131	6.2226660877
4	50	1	38	0.0363044776	10.1127395622	0.9914	1	0.0200	38	0.7600	3.4142032131	6.2589705653
4	50	1	39	0.0279418150	10.1406773773	0.9941	1	0.0200	39	0.7800	3.4142032131	6.2869083803
4	50	1	40	0.0208423709	10.1615197968	0.9962	1	0.0200	40	0.8000	3.4142032131	6.3077507900
4	50	1	41	0.0149544072	10.1764741641	0.9976	1	0.0200	41	0.8200	3.4142032131	6.3227051671
4	50	1	42	0.0102127659	10.1866869300	0.9986	1	0.0200	42	0.8400	3.4142032131	6.3329179331
4	50	1	43	0.0065349544	10.1932218844	0.9993	1	0.0200	43	0.8600	3.4142032131	6.3394528875

APPENDIX D: SYMBOLIC LANGUAGE PROGRAM FOR IBM 1401.

```
PG LIN  CT  LABEL   OP   A OPERAND        B OPERAND      D

                    CTL  4
                         * HYPERGEOMETRIC SERIES
                         * SUM FROM BOTH SIDES OF MAX LIKELIHOOD
                         * SENSE SWITCH ... TO ... PERCENT CONF.
                                 HYPERGEOMETRIC SERIES
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APPENDIX D: CONTINUED.

PG LIN	CT	LABEL	OP	A OPERAND	B OPERAND	D
12 040	7		ZS	DIV +009	DIV +010	
12 050	7		ZA	MS	MULT +004	
12 060	7		A	BCU1	MULT +004	
12 070	7		D	MULT +004	DIV +006	
12 080	7		MCW	DIV +019	RSUM	
12 090	7		ZA	CON00	MULT +001	
12 100	7		A	RSUM	MULT +022	
12 110	7		MCW	MULT +020	LEV00	
12 120	7		ZA	CON05	MULT +001	
12 130	7		A	RSUM	MULT +022	
12 140	7		MCW	MULT +020	LEV05	
12 150	7		ZA	CON90	MULT +001	
12 160	7		A	RSUM	[illegible]	
12 170	7		MCW	MULT +020	LEV90	
12 180	7		ZA	CON95	MULT +001	
12 190	7		A	RSUM	MULT +022	
13 010	7		MCW	MULT +020	LEV95	
13 020	7		ZA	CON99	MULT +001	
13 030	7		A	RSUM	MULT +022	
13 040	7		MCW	MULT +020	LEV99	
13 050	7		MCW	CONEX1	MULT +002	
13 060	7		MCW	RSUM	MULT +023	
13 070	7		MCW	MULT +020	LEVEX1	
13 080	7		MCW	CONEX2	MULT +002	
13 090	7		MCW	RSUM	MULT +023	
13 100	7		MCW	MULT +020	LEVEX2	
13 110	4		B	CONMUL		
14 010	7	CONMUL	ZA	MDAT	A	
14 020	7		S	MS	A	
14 030	7		ZA	RS	B	
14 040	7		ZA	MD16	C	
14 050	7		A	MS	C	
14 060	7		S	MMAT	C	
14 070	7		S	MS	C	
14 080	7		ZA	MS	D	
14 090	7		S	MS	D	
14 100	8		BWZ	AREAD1	A	K
14 110	8		BWZ	AREAD1	B	K
14 120	8		BWZ	AREAD1	C	K
14 130	8		BWZ	AREAD1	D	K
14 140	7		ZA	A	WP	
14 150	7		A	B	WP	
14 160	7		A	C	WP	
14 170	7		A	D	WP	
14 180	7		MZ	WP -001	WP	
14 190	7		ZA	A	MARG1	
14 200	7		A	C	MARG1	
15 010	7		ZA	C	MARG2	
15 020	7		A	D	MARG2	
15 030	7		ZA	A	MARG3	
15 040	7		A	B	MARG3	
15 050	7		ZA	B	MARG4	
15 060	7		A	D	MARG4	
15 070	7		MZ	MARG1 -001	MARG1	
15 080	7		MZ	MARG2 -001	MARG2	
15 090	7		MZ	MARG3 -001	MARG3	
15 100	7		MZ	MARG4 -001	MARG4	
15 110	7		ZA	A	MULT +003	
15 120	7		S	BCU1	MULT +003	
15 130	8		BWZ	CXB	MULT +003	B
15 140	7		MCW	BUG001	A	
15 150	7	CXB	ZA	B	MULT +003	
15 160	7		S	BCU1	MULT +003	
15 170	8		BWZ	CXC	MULT +003	B
15 180	7		MCW	BUG001	B	
15 190	7	CXC	ZA	C	MULT +003	
15 200	7		S	BCU1	MULT +003	
16 010	8		BWZ	CXD	MULT +003	B
16 020	7		MCW	BUG001	C	
16 030	7	CXD	ZA	D	MULT +003	
16 040	7		S	BCU1	MULT +003	
16 050	8		BWZ	ORDER	MULT +003	B
16 060	7		MCW	BUG001	D	
16 070	7	ORDER	ZA	[illegible]	0100	
16 080	7		MCW	[illegible]	0094	
16 090	7	MCMST1	MCW	0094	0099	
16 100	7	ADST1	A	[illegible]	0099	
16 110	7		C	MARG 1	MARG 2	T
16 120	8		B	MUMAR		T
16 130	4		B	COMP16		
16 140	7	MUMAR	MCW	MARG 2	MARGT	
16 150	7		MCW	MARG 1	MARG 2	
16 160	7		MCW	MARGT	MARG 1	
16 170	7	COMP16	C	0000	[illegible]	
16 180	5		B	ADST1		U
16 190	7		A	BCU5	0094	
16 200	7		C	0094	[illegible]	
17 010	5		B	MCMST1		U
17 020	7		ZA	MARG1	TEST20	
17 030	7		S	BCU1	TEST20	
17 040	8		BWZ	[illegible]	TEST20	B
17 050	7		ZA	BCU1	MARG1	
17 060	7	[illegible]	ZA	MARG2	TEST20	
17 070	7		S	BCU1	TEST20	
17 080	8		BWZ	[illegible]	TEST20	B
17 090	7		ZA	[illegible]	MARG2	
17 100	7	[illegible]	ZA	MARG3	TEST20	
17 110	7		S	[illegible]	TEST20	
17 120	8		BWZ	[illegible]	TEST20	B
17 130	7		ZA	[illegible]	MARG3	
17 140	7	[illegible]	ZA	[illegible]	TEST20	
17 150	7		S	[illegible]	TEST20	
17 160	8		BWZ	[illegible]	TEST20	B
17 170	7		ZA	[illegible]	MARG4	
17 180	7	[illegible]	SW	[illegible]	MONLY	
17 190	7		ZA	[illegible]	DIV +020	
17 200	7		MCW	[illegible]	DIV +010	
18 010	7		MCW	[illegible]	MARGT	
18 020	7	CONPU	C	WP	MARG1	
18 030	5		B	[illegible]		L
18 040	7		D	WP	DIV +006	
18 050	7		ZA	DIV +019	MULT +019	
18 060	8	[illegible]	BWZ	[illegible]	[illegible]	1
18 070	7		ZA	MULT +005	TEST20	
18 080	7		S	[illegible]	TEST20	
18 090	8		BWZ	[illegible]	TEST20	K
18 100	7	ZAMUL	ZA	MULT +007	DIV +025	
18 110	7		A	[illegible]	WP	
18 120	7		MZ	WP -001	WP	

PG LIN	CT	LABEL	OP	A OPERAND	B OPERAND	D
18 130	4		B	COMPM		
18 140	7	MVNEXD	ZA	A	WP	
18 150	7		MZ	WP -001	WP	
18 160	7		ZA	WP	TESTZE	
18 170	7		S	BCU1	TESTZE	
18 180	8		BWZ	MULTHU	TESTZE	K
18 190	7		ZA	B	A	
18 200	7		ZA	C	B	
18 210	7		ZA	D	C	
18 220	7		ZA	ZERO	D	
19 010	7		ZA	BCU1	MARG1	
19 020	7		MZ	MARG1 -001	MARG1	
19 030	4		B	COMPM		
19 040	7	MULTPR	ZA	MARGT	TESTZE	
19 050	7		S	BCU2	TESTZE	
19 060	8		BWZ	MVNEXN	TESTZE	K
19 070	7		M	MARGT	MULT +029	
19 080	7		MCW	MULT +025	MOCW	
19 090	7		ZA	MOCW	MULT +019	
19 100	7		S	BCU1	MARGT	
19 110	8		BWZ	MULTPR	MONLY	1
19 120	4		B	CHEZE2		
19 130	8	MULTHU	BWZ	FINISH	MNORL	1
19 140	4		SW	MONLY		
19 150	4		B	MULTPR		
19 160	7	MVNEXN	ZA	MARG3	MARGT	
19 170	7		ZA	MARG4	MARG3	
19 180	7		ZA	ZERO	MARG4	
19 190	7		ZA	ZERO	TESTZE	
19 200	7		S	MARGT	TESTZE	
20 010	8		BWZ	MULTPR	TESTZE	K
20 020	8		BWZ	FINISH	MONLY	1
20 030	4		SW	MNORL		
20 040	4		B	ZAMUL		
20 050	7	FINISH	CW	ENDN	ENDL	
20 060	7		MCW	MULT +019	PRODNN	
20 070	7		MCW	MULT +019	PRODL	
20 080	7		MCW	MULT +019	PRODR	
20 090	7		ZA	MULT +019	SUMP21	
20 100	7		ZA	ZERO	SUMPR	
20 110	7		ZA	ZERO	SUMPL	
20 120	7		LCA	LEDT21-005	0237	
20 130	7		MCE	SUMP21-005	0237	
20 140	7		LCA	LEDT21-005	0255	
20 150	7		MCE	SUMP21-005	0255	
20 160	7		MCS	MD16	0209	
20 170	7		MCS	MS	0204	
20 180	7		LCA	PCT -005	0219	
20 190	4		SW	0101		
21 010	7		MCE	MMAT	0219	
21 011	7		LCA	PCT -005	0214	
21 012	7		MCS	MS	0214	
21 020	5		B	WPIN		B
21 021	7	CONF	ZA	ZERO	DIV +027	
21 022	7		MCW	SUMP21-009	DIV +023	
21 023	7		MZ	DIV +022	DIV +023	
21 024	7		D	RSUM -009	DIV +012	
21 025	4		SW	DIV +008		
21 026	7		LCA	PCT	0264	
21 027	7		MCE	DIV +019	0264	
21 028	4		CW	DIV +008		
21 032	4		B	WPIN		
21 040	1	WPIN	B			
21 050	4		B	CTLINE		
21 080	8	PCHCH	MCW	PUNCHD		C
21 090	7		SW	PRIGHT	PLEFT	
21 100	7		ZA	MD16	LNUM1	
21 110	7		S	MS	LNUM1	
21 120	7		ZA	MD16	LNUM2	
21 130	7		S	MD16	LNUM2	
21 140	7		A	BCU1	LNUM2	
21 150	7		ZA	MD16	LDEN1	
21 160	7		ZA	MD16	LDEN2	
21 170	7		S	MD16	LDEN2	
21 180	7		S	MS	LDEN2	
21 200	7		A	MS	LDEN2	
22 010	7		A	BCU1	LDEN2	
22 020	7		ZA	MD16	[illegible]	
22 023	7		MZ	DIV +022	DIV +023	
22 030	7		A	BCU1	[illegible]	
22 040	7		ZA	MD16	[illegible]	
22 050	7		S	MD16	[illegible]	
22 060	7		A	RS	[illegible]	
22 070	7		S	RS	[illegible]	
22 080	7		ZA	MD16	[illegible]	
22 090	7		S	MS	[illegible]	
22 100	7		A	BCU1	[illegible]	
22 110	7		ZA	MD16	[illegible]	
22 120	7		S	MD16	[illegible]	
22 130	4		B	RIGHT		
22 140	4	RIGHT	CW	PLEFT		
22 150	8		BWZ	[illegible]	LASTR	1
22 160	8	VAGND	BWZ	LEFT	[illegible]	1
22 170	7		ZA	[illegible]	TESTZE	
22 180	7		S	BCU1	TESTZE	
22 190	8		BWZ	[illegible]	TESTZE	K
22 200	4		B	COMPR		
23 010	8	[illegible]	BWZ	[illegible]	LASTL	1
23 020	4		B	[illegible]		
23 021	8	[illegible]	BWZ	[illegible]	LEND	1
23 022	4		B	LEFTR		
23 030	7	[illegible]	SW	LASTR	LASTR	
23 040	4		B	[illegible]		
23 050	7	COMPR	ZA	[illegible]	TESTZE	
23 060	7		S	[illegible]	TESTZE	
23 070	8		BWZ	[illegible]	TESTZE	K
23 080	4		B	[illegible]		
23 090	7	[illegible]	SW	LASTR	LASTR	
23 100	4		B	[illegible]		
23 110	7	CONMUL2	ZA	[illegible]	MULT +004	
23 120	7		M	[illegible]	MULT +020	
23 130	7		ZA	[illegible]	DIV +020	
23 140	7		MCW	MULT +010	DIV +020	
23 150	7		MZ	DIV +019	DIV +020	
23 160	7		ZA	[illegible]	MULT +005	
23 170	7		S	[illegible]	MULT +020	
23 180	4		SW	MULT +004		
23 190	7		D	MULT +020	DIV +003	
23 200	4		CW	MULT +004		

APPENDIX D: CONTINUED.

PG LIN	CT	LABEL	OP	A OPERAND	B OPERAND	D	PG LIN	CT	LABEL	OP	A OPERAND	B OPERAND	D
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

U. S. NAVAL AMMUNITION DEPOT
OAHU, HAWAII

1804:sm
5220/NT
3 Jan 1963

From: Commanding Officer
To: Chief, Bureau of Naval Weapons (FQ-1)
Department of the Navy
Washington 25, D.C.

Subj: Tables of the Hypergeometric Distribution Functions

Encl: (1) Mathematical derivation of the tables
(2) Tables of point and cumulative probabilities for various sample and population size combinations

1. Enclosures (1) and (2) are forwarded for publication by the U. S. Government Printing Office. Enclosure (1) contains mathematical derivation of the hypergeometric distribution function, tables, graphs and IBM 1401 computer program. Enclosure (2) (forwarded under separate cover) contains two copies of the tabulations of the hypergeometric probabilities.

2. The tables, which include point and cumulative probabilities are designed primarily for use by personnel familiar with statistics to estimate stockpile quality level.

3. Copy addressees are advised that copies of the tabular presentation, enclosure (2), which are quite voluminous cannot be made available by this command. It is presumed that they will be generally available through the Bureau of Naval Weapons or the Government Printing Office if the Bureau of Naval Weapons decides to have them published.

H. H. MEEKER, JR.
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