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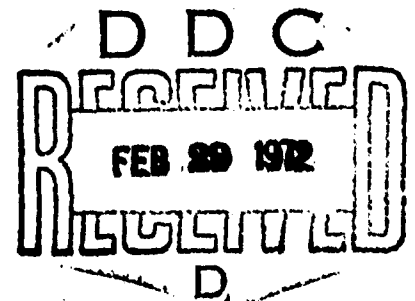
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24 January 1972

**FAST ENVELOPE ALGORITHMS
USING LINEAR COMBINATIONS OF QUADRATURE SAMPLES**

James K. Beard

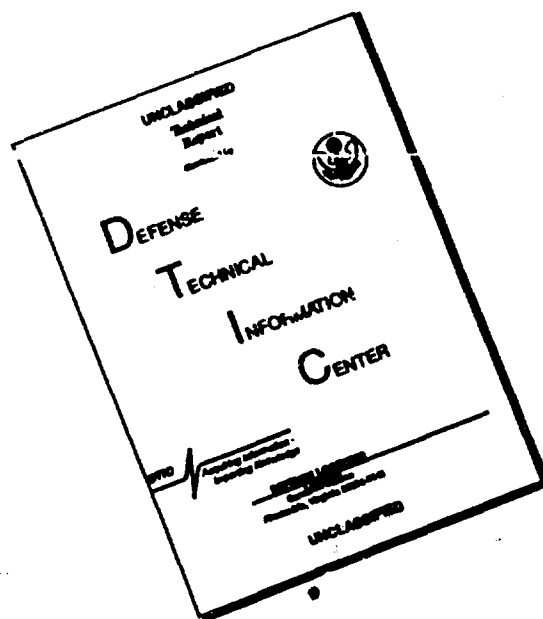
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Contract N00024-71-C-1185
Proj. Ser. No. SF 11121106 and 11121104, Task 8103

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KEY WORDS

Envelope

Quadrature sampling

Real-time

Approximation

LINK A

LINK B

LINK C

HOLE

WT

ROLE

WT

	ROLE
1. The first person to see the victim.	Witness
2. The person who called the police.	Witness
3. The person who found the victim.	Witness
4. The person who saw the suspect.	Witness
5. The person who saw the suspect flee.	Witness
6. The person who saw the suspect's car.	Witness
7. The person who saw the suspect's license plate.	Witness
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WT

ABSTRACT

An algorithm for computing samples of a signal envelope from quadrature samples is presented. The method is well suited to either simple special purpose hardware or fast software implementation. Accuracy and speed can be traded off without varying the basic form of the algorithm.

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1.0 Introduction

In many digital signal processing systems, it is necessary to convert an array of quadrature samples to an envelope array, a process equivalent to rectifying in analog systems. For each quadrature sample, the quantity to be computed is

$$r = \sqrt{x_s^2 + y_s^2} \quad , \quad (1-1)$$

where x_s and y_s are the in phase and quadrature components of the signal.

In practical systems, x_s and y_s are carried as integers and the output must also be an integer. Also, it is desirable to avoid overflow problems associated with squaring, and performing a square root is often prohibitive due to computation time-data rate tradeoffs. A simple estimate of r which avoids squaring and square roots, but which does not cause errors that would affect the performance of the system, is therefore required.

This report describes this goal by using linear combinations of

$$x = \max(|x_s|, |y_s|), \quad y = \min(|x_s|, |y_s|) \quad (1-2)$$

of the form

$$\hat{r} = ax + by \quad (1-3)$$

to compute an approximation or estimate of r as defined by Eq. (1-1). Considerable attention is devoted to the implementation by simple hardware or software oriented algorithms.

The evaluation of approximations of the type in Eq. (1-3) offers many conceptual difficulties. In order to minimize the complexity of the concepts and mathematics required to analyze the approximations, they are presented in terms of a new set of variables. For example, in the (x_s, y_s) or (x, y) plane, r as given by Eq. (1-1) is no single point or curve but is a set of circles centered at the origin. In the $(x_s/r, y_s/r)$ or $(x/r, y/r)$ plane, r becomes a single circle of radius one and centered at the origin, called the unit circle. However, in this plane, linear combinations of x and y , such as $\hat{r}$ as given by Eq. (1-3), are circles passing through the origin. However, the complex variable w , where

$$w = \frac{x + jy}{\hat{r}} = (x/\hat{r}, y/\hat{r}) \quad (1-4)$$

and $\hat{r}$ is a linear combination of x and y as given by Eq. (1-3), offers the following advantages: (a) in the w plane, where curves are plotted as $\text{Re}(w)$ versus $\text{Im}(w)$, $\hat{r}$ as given by Eq. (1-3) is a straight line, and (b) r as given by Eq. (1-1) is the unit circle (a circle of radius one centered at the origin) as in the $(x/r, y/r)$ plane. In the w plane, it becomes obvious that the optimal estimates of r are polygons approximating the unit circle, and other concepts are equally simplified. The most important of these concepts are (a) the use of (x, y) as given by Eq. (1-2) allows consideration of only half the first quadrant rather than the entire unit circle, (b) the use of n straight lines in this region results in an $8n$ sided polygon approximating the unit circle in the w plane, and (c) the use of $\hat{r} = x$ (i.e., $b=0$ in (Eq. 1-3)) for small y results in an $8n-4$ sided polygon in the w plane.

The notations used throughout this report are as follows.

<u>Quantity</u>	<u>Definition</u>
x_s	In phase sample.
y_s	Quadrature sample.
x	Largest of absolute values of x_s, y_s .
y	Smallest of absolute values of x_s, y_s .
r	True value of envelope.
$\hat{r}$	Estimate of envelope.
e	Peak absolute error of estimate.
s	rms error of estimate.
b	Multiplicative bias of estimate.

2.0 Basis of Solutions

2.1 Assumptions

The hardware capabilities used in the algorithms presented are (a) absolute value, (b) shifting, (c) magnitude comparison between two positive integers, and (d) addition-subtraction. Multiplication, in the form of shift-adds, is used, but general purpose multiplication hardware is not required.

The properties of the input assumed in the development of the algorithms are as follows. If a variable θ is defined by

$$\theta_s = \tan^{-1}(y_s/x_s) + \pi h(x_s, y_s) \quad , \quad (2-1)$$

the function $h(x_s, y_s)$ being defined by

$$h(x_s, y_s) \begin{cases} = 0 & , & x_s \geq 0 \\ = +1 & , & x_s < 0 & , & y_s \geq 0 \\ = -1 & , & x_s < 0 & , & y_s \leq 0 \end{cases} \quad (2-2)$$

the probability density function of θ is

$$p(\theta_s) \begin{cases} = \frac{1}{2\pi} & , & -\pi < \theta \leq \pi \\ = 0 & , & \text{otherwise} \end{cases} ; \quad (2-3)$$

i.e., the signal phase θ_s is uniformly distributed over $[-\pi, \pi]$.
 If the absolute value of both samples is taken and x is arbitrarily taken to be the largest, then

$$p(\theta) \begin{cases} = \frac{4}{\pi} & , \quad 0 \leq \theta \leq \frac{\pi}{4} \\ = 0 & , \quad \text{otherwise} \end{cases} \quad (2-4)$$

i.e., the modified signal phase θ is uniformly distributed over $[0, \pi/4]$.

2.2 Coordinate Systems

The most obvious coordinate system for consideration of the problem at hand, namely, the x_s, y_s plane, has the fundamental drawback that in this system r is a family of circles rather than a single locus. Therefore, the x_s, y_s plane coordinate system is not used in this report.

The $y/x, r/x$ plane constrains the envelope to the locus

$$r/x = \sqrt{1+(y/x)^2} \quad , \quad 0 \leq y/x \leq 1 \quad ,$$

but the weakness of this system is that y/x has the probability density

$$p(y/x) = \frac{4/\pi}{1+(y/x)^2} \quad , \quad 0 \leq y/x \leq 1 \quad ,$$

a form which complicates evaluation of b and s . Also, this coordinate system is related to optimal estimates by complicated relationships.

The $x/r, y/r$ plane constrains the envelope value to the unit circle. Estimates of the form $\hat{r} = ax + by$ in normalized form,

$$\hat{r}/r = a \cos\theta + b \sin\theta, \quad \cos\theta = x/r, \quad \sin\theta = y/r, \quad (2-5)$$

are circles passing through the origin with diameter $(a^2 + b^2)^{1/2}$ and centered at $(a/2, b/2)$, as shown in Fig. 1. The angle of maximum $\hat{r}$, ψ , is

$$\psi = \tan^{-1}(b/a), \quad \hat{r} \text{ at maximum} \quad (2-6)$$

The θ of Eq. (2-5) is the same as that of Eq. (2-4). This is the coordinate system in which the solution will be presented.

Another coordinate system of importance is the w plane, where w is the complex variable

$$w = r/\hat{r} (\cos\theta + j \sin\theta) = u + jv = \frac{x}{ax+by} + j \frac{y}{ax+by} \quad (2-7)$$

The variable w is the complex conjugate of the reciprocal of $\hat{r}/r$ expressed as a complex variable.

$$\frac{1}{w} = \frac{\hat{r}}{x + jy} = \frac{\hat{r}}{r} \exp(-j\theta)$$

The true envelope is again the unit circle, but Eq. (2-5) in the w plane is a straight line,

$$au + bv = 1, \quad (2-8)$$

with slope $-a/b$ (or, $\tan(\psi + \pi/2)$), with closest approach to the origin $(a^2 + b^2)^{-1/2}$. The line corresponding to the circle in Fig. 1 is shown in Fig. 2. The angle ψ is the angle of minimum $|w|$.

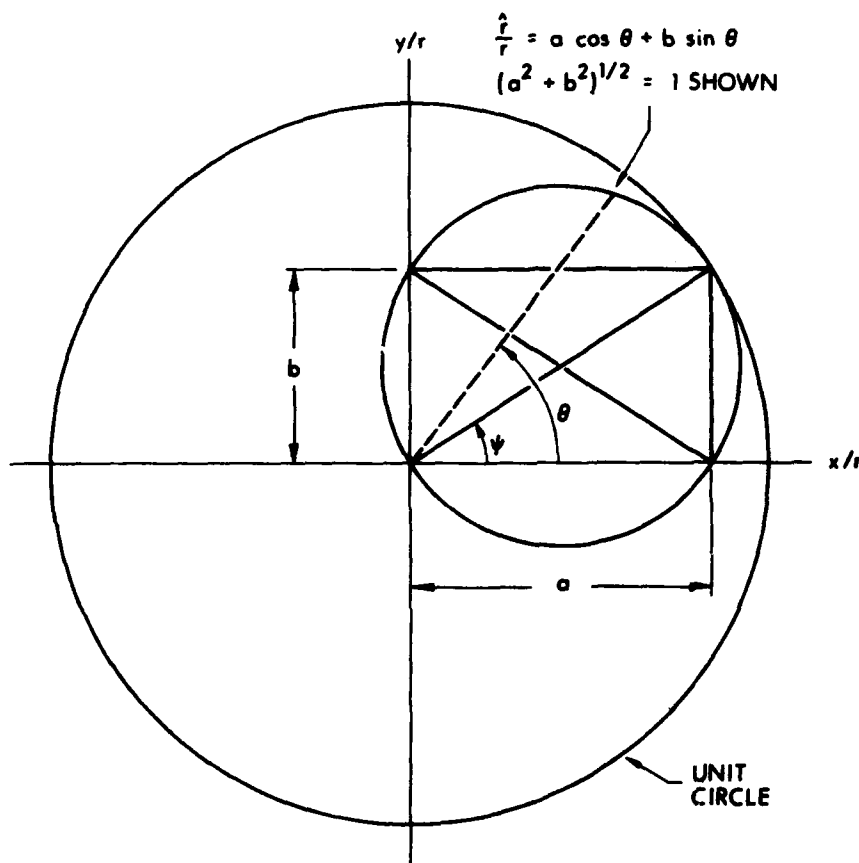


FIGURE 1
 x/r, y/r COORDINATE SYSTEM

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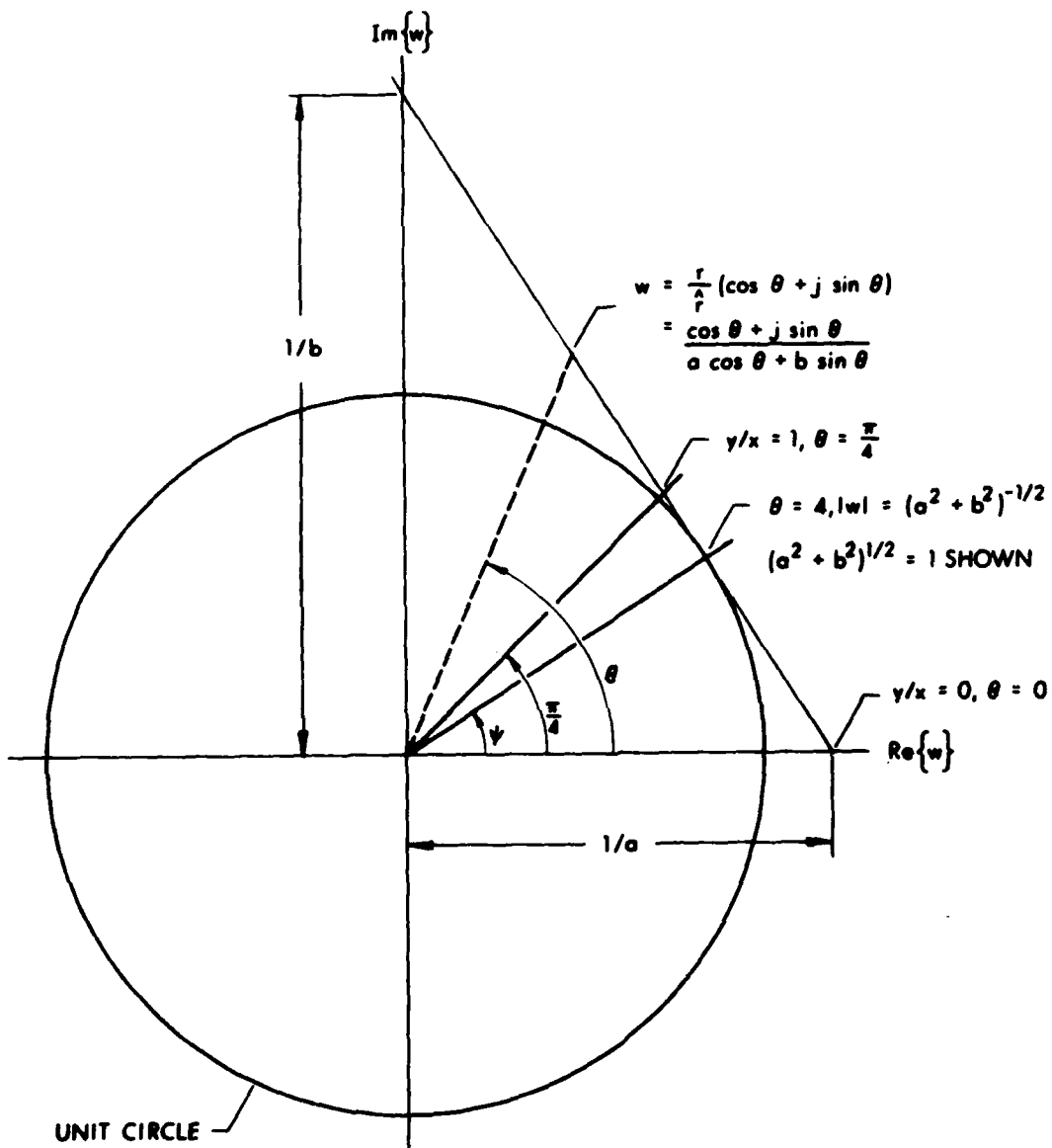


FIGURE 2
THE w PLANE

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3.0 Optimal Estimates

Optimal estimates, in the context of this report, are taken to mean the use of n straight lines in the w plane (i.e., simple linear combinations of x_s and y_s) to approximate the unit circle with minimum rms error s_n . The multiplicative bias b_n is irrelevant to signal processing considerations. In the w plane,

$$w = u + jv \quad , \quad v/u = y/x = \tan\theta \quad ,$$

and θ is uniformly distributed, so an optimal fit for a fixed number of lines will be a regular polygon, independent of the loss or error criterion. The use of the absolute values x and y , $x \geq y \geq 0$, instead of x_s and y_s restricts θ to the range 0 to $\pi/4$, and n linear combinations of x and y are equivalent, in general, to $8n$ linear combinations of x_s and y_s . If the line at $\theta=0$ in the w plane is vertical ($\hat{r}=cx$), or if the line at $\theta=\pi/4$ in the w plane has a slope of -1 ($\hat{r}=c(x+y)$), the lines in the w plane will be parallel to tangents to the unit circle at those points and the polygon will have $8n-4$ or $8n-8$ sides. Thus, the $\hat{r}=c(x+y)$ case is to be avoided; however, the simplicity of $\hat{r}=cx$ near $\theta=0$ makes this case worthy of consideration. Accordingly, only the $8n$ and $8n-4$ cases are of practical importance. These two cases are shown in Figs. 3 and 4 for $n=2$.

The parameters e , s , and b will be computed below for the $8n$ and $8n-4$ sided polygon cases. The subscript p will be used in the $8n-4$ case equations to prevent confusion.

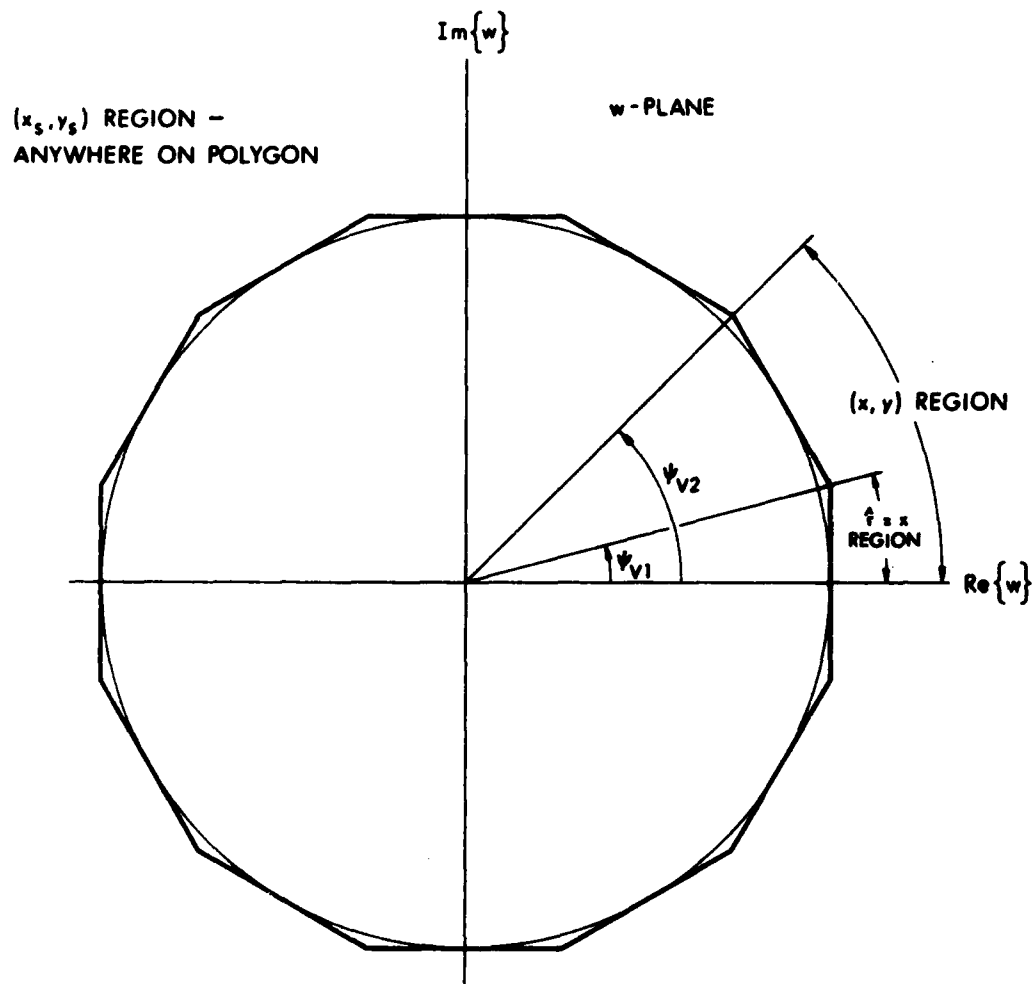


FIGURE 3
8p-4 CASE, $p = 2$, IN THE w PLANE

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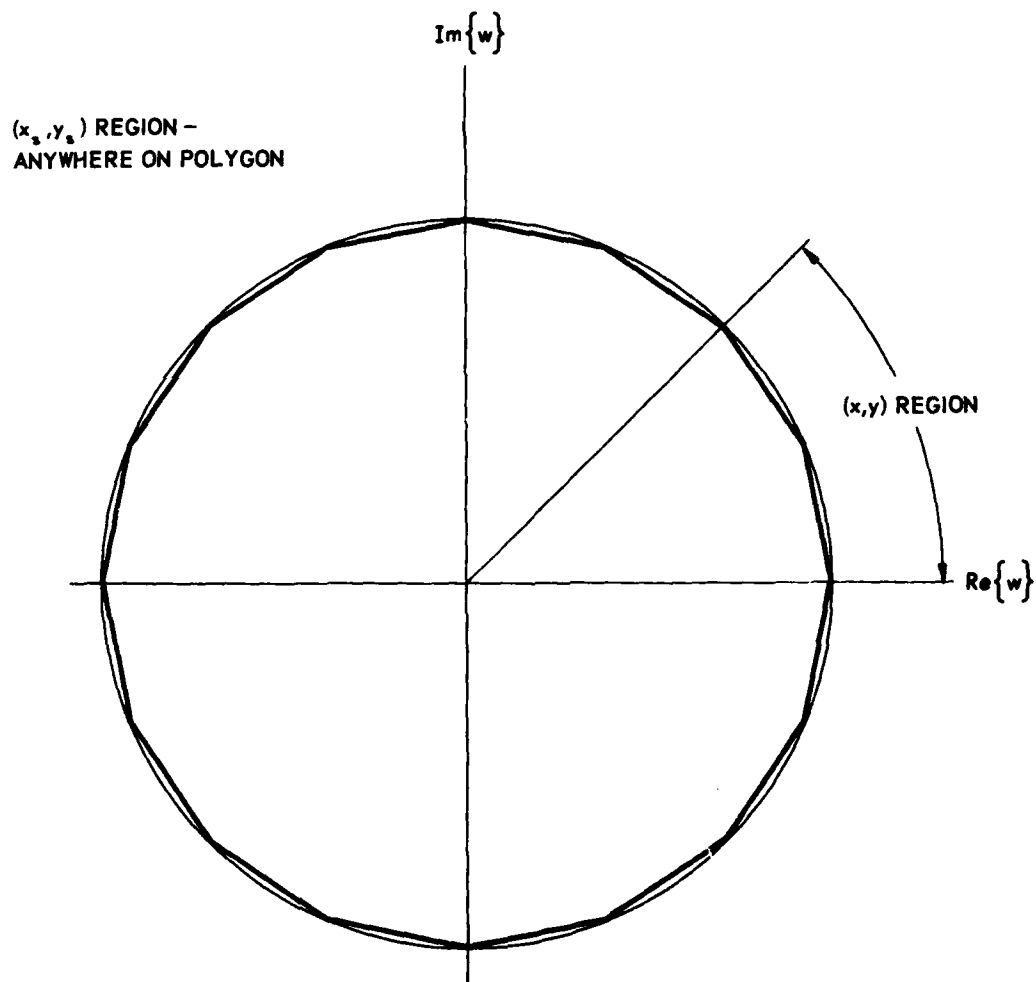


FIGURE 4
 $8n$ CASE, $n = 2$, IN THE w PLANE

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 JKB - RFO
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The sides of the regular polygons are defined by the angles of minimum $|w|$,

$$\psi_m = \frac{(m-1/2)\pi}{4n} , \quad 1 \leq m \leq n \text{ (8n case)} , \quad (3-1)$$

$$\psi_q = \frac{(q-1)\pi}{4p-2} , \quad 1 \leq q \leq p \text{ (8p-4 case)} , \quad (3-2)$$

the vertices,

$$\psi_{vm} = \frac{m\pi}{4n} , \quad 0 \leq m \leq n \text{ (8n case)} , \quad (3-3)$$

$$\psi_{vq} = \frac{(q-1/2)\pi}{4p-2} , \quad 1 \leq q \leq p \text{ (8p-4 case)} , \quad (3-4)$$

and the fact that the vertices are on the unit circle in the 8n case,

$$a_m^2 + b_m^2 = \sec^2\left(\frac{\pi}{8n}\right) , \quad (3-5)$$

and the sides are tangent to the unit circle in the 8p-4 case,

$$a_q^2 + b_q^2 = 1 . \quad (3-6)$$

Using Eqs. (3-5) and (3-6) with $b/a = \tan\psi$, where ψ is given by Eq. (3-1) and (3-2), yields

$$\begin{aligned} \hat{r}_m &= \left[x \cos(\psi_m) + y \sin(\psi_m) \right] \sec\left(\frac{\pi}{8n}\right) \\ &= r \cos(\theta - \psi_m) \sec\left(\frac{\pi}{8n}\right) , \end{aligned} \quad (3-7)$$

and

$$\begin{aligned}\hat{r}_q &= x \cos(\psi_q) + y \sin(\psi_q) \\ &= r \cos(\theta - \psi_q)\end{aligned}\quad (3-8)$$

For the $8n$ case, the peak error is at the ψ_m ,

$$e_n = \frac{\hat{r}(\psi_m)}{r} - 1 = \sec\left(\frac{\pi}{8n}\right) - 1, \quad (3-9)$$

while for the $8p-4$ case, the peak error is at the vertices,

$$e_p = 1 - \frac{\hat{r}(\psi_{vq})}{r} = 1 - \cos\left(\frac{\pi}{8p-4}\right). \quad (3-10)$$

The multiplicative bias is found by integrating $\hat{r}/r$ with respect to θ over one side of the polygon and dividing by the angle between the vertices, since θ is uniformly distributed over the sector corresponding to each side of the w plane polygon.

Thus,

$$b_n = \frac{8n}{\pi} \int_0^{\pi/(8n)} \cos\left(\theta - \frac{\pi}{8n}\right) \sec\left(\frac{\pi}{8n}\right) d\theta = \frac{\tan\left(\frac{\pi}{8n}\right)}{\frac{\pi}{8n}}, \quad (3-11)$$

and

$$b_p = \frac{8p-4}{\pi} \int_0^{\pi/(8p-4)} \cos\theta d\theta = \frac{\sin\left(\frac{\pi}{8p-4}\right)}{\frac{\pi}{8p-4}}, \quad (3-12)$$

using the obvious expedient of integrating over half a side.

The rms error is found by averaging $(\hat{x}/br)^2 - 1$ in the same way.

Thus,

$$s_n^2 = \frac{8n}{\pi} \int_0^{\pi/(8n)} \frac{\cos^2\left(\theta - \frac{\pi}{8n}\right)}{\left[\sin\left(\frac{\pi}{8n}\right)/\left(\frac{\pi}{8n}\right)\right]^2} d\theta - 1$$

(3-13)

$$= \frac{1}{2} \frac{1 + \frac{\sin\left(\frac{\pi}{4n}\right)}{\left(\frac{\pi}{4n}\right)}}{\left[\sin\left(\frac{\pi}{8n}\right)/\left(\frac{\pi}{8n}\right)\right]^2} - 1,$$

and

$$s_p^2 = \frac{8p-4}{\pi} \int_0^{\pi/(8p-4)} \frac{\cos^2(\theta)}{\left[\sin\left(\frac{\pi}{8p-4}\right)/\left(\frac{\pi}{8p-4}\right)\right]^2} d\theta - 1$$

(3-14)

$$= \frac{1}{2} \frac{1 + \frac{\sin\left(\frac{\pi}{4p-2}\right)}{\frac{\pi}{4p-2}}}{\left[\sin\left(\frac{\pi}{8p-4}\right)/\left(\frac{\pi}{8p-4}\right)\right]^2} - 1.$$

The multiplicative bias b is unimportant in most system applications, but $-10 \log_{10}(s^2)$ is the upper limit for signal-to-noise out of the system. For quadrature sampled, very narrowband noise, $20 \log_{10}(e)$ is the relative amplitude of the peak-to-peak "flutter", or amplitude modulation, generated by the envelope algorithm.

For a $4k$ sided polygon, the asymptotic forms for e , b , and s for large k are, from Eq. (3-9) through Eq. (3-14),

$$\begin{aligned} e &\sim \frac{1}{2} \left(\frac{\pi}{4k} \right)^2 \\ b_n &\sim 1 + \frac{1}{3} \left(\frac{\pi}{8n} \right)^2 \quad (k=2n) \\ b_p &\sim 1 - \frac{1}{6} \left(\frac{\pi}{8p-4} \right)^2 \quad (k=2p-1) \\ s &\sim \frac{1}{3\sqrt{5}} \left(\frac{\pi}{4k} \right)^2 \end{aligned} \quad (3-15)$$

Also, for large k , the peak error of $\hat{r}$ is described asymptotically by

$$1 - \frac{1}{3} e \leq \frac{\hat{r}}{br} \leq 1 + \frac{2}{3} e \quad (3-16)$$

4.0 Implementation

The simplest way to implement Eq. (3-7) or Eq. (3-8) is to compute

$$\hat{r}_1 = x + y \tan\left(\frac{\pi}{8n}\right) \quad (8n \text{ case}) \quad (4-1)$$

or

$$\hat{r}_1 = x \quad (8p-4 \text{ case}) \quad (4-2)$$

first. Then, for $1 \leq m \leq n-1$ or $1 \leq q \leq p-1$, the quantity

$$\hat{r}_{m+1} - \hat{r}_m = 2 \tan\left(\frac{\pi}{8n}\right) \left(-x \sin\left(\frac{m\pi}{4n}\right) + y \cos\left(\frac{m\pi}{4n}\right) \right) \quad (4-3)$$

or

$$\hat{r}_{q+1} - \hat{r}_q = 2 \sin\left(\frac{\pi}{8p-4}\right) \left(-x \sin\left(\frac{(q-\frac{1}{2})\pi}{4p-2}\right) + y \cos\left(\frac{(q-\frac{1}{2})\pi}{4p-2}\right) \right) \quad (4-4)$$

can be used to form $\hat{r}_2, \hat{r}_3, \dots$, as follows. Compute

$$\Delta_m = y - x \tan\left(\frac{m\pi}{4n}\right) \quad (4-5)$$

or

$$\Delta_q = y - x \tan\left(\frac{(q-\frac{1}{2})\pi}{4p-2}\right) \quad (4-6)$$

If $\Delta \leq 0$, the computation is completed; otherwise, use

$$\hat{r}_{m+1} - \hat{r}_m = 2 \tan\left(\frac{\pi}{8n}\right) \cos\left(\frac{m\pi}{4n}\right) \Delta_m \quad (4-7)$$

or

$$\hat{r}_{q+1} - \hat{r}_q = 2 \sin\left(\frac{\pi}{8p-4}\right) \cos\left(\frac{(q-\frac{1}{2})\pi}{4p-2}\right) \Delta_q \quad (4-8)$$

to update $\hat{r}_m$ or $\hat{r}_q$. If $m=n$ or $q=p$, the computation is finished; otherwise, repeat the computation of Δ and subsequent sign check.

The quantities

$$c_1(m) = \tan\left(\frac{m\pi}{4n}\right) \quad , \quad 1 \leq m \leq n-1 \quad , \quad (4-9)$$

$$c_2(m) = 2 \tan\left(\frac{\pi}{8m}\right) \cos\left(\frac{m\pi}{4n}\right) \quad , \quad 1 \leq m \leq n-1 \quad , \quad (4-10)$$

$$c_1(q) = \tan\left(\frac{(q-\frac{1}{2})\pi}{4p-2}\right) \quad , \quad 1 \leq q \leq p-1 \quad , \quad (4-11)$$

and

$$c_2(q) = 2 \sin\left(\frac{\pi}{8p-4}\right) \cos\left(\frac{(q-\frac{1}{2})\pi}{4p-2}\right) \quad , \quad 1 \leq q \leq p-1 \quad , \quad (4-12)$$

along with $\tan(\pi/8n)$, e_n , e_p , b_n , b_p , s_n , and s_p , are given in the appendix, for all n and p from 1 to 12, in both decimal and octal.

The multiplications in Eqs. (4-1), (4-5), (4-6), (4-7), and (4-8) can be accomplished within the purposes of the algorithm by using only

the top L bits of $\tan(m\pi/4n)$, c_1 and c_2 , where L is the number of zero leading bits in b or S , depending on whether narrowband flutter or signal-to-noise is the limiting factor. In the CDC 3200, if $(A)=-y$ and $(Q)=+x$, $-\Delta$ can be computed faster by several shift-add (SHQ, AQA) instructions than by a single multiply-add (MUA, ADA) execution if the number of nonzero bits among the most significant L in the multiplier is less than six. An AZJ,GE instruction can then be used to retrieve $\hat{r}$ and exit if $\Delta \leq 0$. The $p=2$ (regular 12 sided polygon) case is programmed as an example in the appendix; entering with x_s and y_s in (A) and (Q) , the execution times are 17 μsec minimum, 24 μsec average, and 34 μsec maximum, including call and return with the result in (A) . This routine is on disc file SPAM.

5.0 Appendix

5.1 Example

This is a listing of a COMPASS routine for the 12 sided polygon envelope estimate shown in Fig. 3. The peak error is $+0, -0.034_{10}$, the multiplicative bias is 0.989, and the rms relative error is 0.01_{10} so that narrowband flutter is 0.15 dB peak-to-peak and the maximum signal-to-noise is 40 dB. The execution times are 17 μ sec minimum, 24 μ sec average, and 34 μ sec maximum, including call and return. Entry is accomplished by a RTJ XY.SQRT instruction with the fixed point quadrature samples in (A) and (Q), and return is to P+1 with the envelope estimate in (A).

LISTING OF XY.SQRT

```

00000 20000024 20 0 P00024 0
00001 01077777 01 0 77777 0
00002 05400000 05 1 00000 0
00003 16477777 16 1 77777 0
00004 05500000 05 1 00000 1
00005 16577777 16 1 77777 1
00006 03700010 03 1 P00010 3
00007 13000030 13 0 00030 0
00010 41000024 41 0 P00024 0
00011 16477777 16 1 77777 0

*** AN AQF HERE WITHIN SAVE (A) FOR P.G.F. ?
00012 12477775 12 1 77775 0
00013 53040000 53 0 40000 0
00014 03200000 03 0 P00000 2
00015 12477773 12 1 77773 0
00016 53040000 53 0 40000 0
00017 16477777 16 1 77777 0
00020 13077746 13 0 77746 0
00021 20000024 20 0 P00024 0
00022 53040000 53 0 40000 0
00023 01400001 01 1 P00001 0
00024

XY.SORT
RHAT
**
0
-0
0
-0
**
24
RHAT
-0
-2
XY.SORT-1
-4
-0
-25
RHAT
XY.SORT
1
END

```

NUMMER OF LINES WITH DIAGNOSTICS

9

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5.2 Tables

Following are tables of constants to be used in choosing and implementing an envelope estimate based on a $4k$ sided polygon. For any k , the peak relative error is $+0, -EP$ for odd k or $+EP, -0$ for even k , the rms relative error is $20 \log_{10} SP$, and the multiplicative bias is BP . If L is the number of leading zero bits in BP or SP , implementation is as follows.

Find x = absolute value of largest sample, y = absolute value of smallest sample. Define an initial $\hat{r}$ by

$$\hat{r} = x, \quad k \text{ odd}; \quad \hat{r} = x + C_0 y, \quad k \text{ even} \quad (5-1)$$

Use shift-adds for the multiplication, using only the nonzero bits in the L bits of C_0 following the binary point. Then, compute

$$-\Delta = -y + x * C_1(1), \quad (5-2)$$

using shift-adds with the nonzero bits among the L most significant in $C_1(1)$ for the multiplication. If $-\Delta \geq 0$, exit with $\hat{r}$ in (A). Otherwise, update $\hat{r}$ using

$$\hat{r} = \hat{r} + \Delta * C_2(1), \quad (5-3)$$

using shift-adds with the nonzero bits among the L most significant in $C_2(1)$ for the multiplication, as before. If no C_1 's or C_2 's remain, exit with $\hat{r}$ in (A). Otherwise, repeat the operations and checks beginning with Eq. (5-2) using $C_1(2)$ and $C_2(2)$, $C_1(3)$ and $C_2(3)$, etc., until either $-\Delta \geq 0$ or there are no more C_1 's or C_2 's.

TABLES OF PARAMETERS

4 SIDED POLYGON, ERROR LIMITS +0.-EP

DECIMAL	2.9280321881E-01	EP	9.0031631615E-01	RP	9.7720810291E-02	SP	9.7720810291E-02	C0	0
OCTAL	.2257542		.7147544				.0620207		0

C1 ARRAY, DECIMAL

C1 ARRAY, OCTAL

C2 ARRAY, DECIMAL

C2 ARRAY, OCTAL

8 SIDED POLYGON, FROM LIMITS +FN, -N

DECIMAL	FN	HN	SP	CO
OCTAL	4.2392200273E-02	1.0547M61751E 00	2.3331609511E-02	4.1421346234E-01
	.0521365	1.0340317	.0137442	.3240475

C1 ARRAY, DECIMAL

C1 ARRAY, OCTAL

C2 ARRAY, DECIMAL

C2 ARRAY, OCTAL

12 SIDED POLYGON. ERROR LIMITS +0.-EP

DECIMAL	3.4074173716E-02	EP	RP	SP	CU	0
OCTAL	.0213443		9.8861592945E-01	1.0284343176E-02		0
			.7721276	.0052100		
C1 ARRAY. DECIMAL						
	2.6794919243E-01					
C1 ARRAY. OCTAL						
	.2111412					
C2 ARRAY. DECIMAL						
	5.0000000000E-01					
C2 ARRAY. OCTAL						
	.4000000					

16 SIDEN POLYGON, ERROR LIMITS +FN,-0

	EN	BN	SP	CO
DECIMAL	1.9591158227E-02	1.0130523683E-00	5.7683470148E-03	1.9891236738E-01
OCTAL	.0120176	1.0065355	.0027501	.1456575

C1 ARRAY, DECIMAL
4.1421356236E-01

C1 ARRAY, OCTAL
.3240475 .

C2 ARRAY, DECIMAL
3.6754212998E-01

C2 ARRAY, OCTAL
.2741350 .

20 SIDED POLYGON, ERROR LIMITS +0.-EP

	EP	RP	CP	C0
DECIMAL	1.2311659437E-02	9.9589273524E-01	3.6868452096E-03	0
OCTAL	.0062333	.7757132	.0017064	0
C1 ARRAY, DECIMAL				
	1.5838444032E-01	5.0952544949E-01		
C1 ARRAY, OCTAL				
	.1210574 .4047010 .			
C2 ARRAY, DECIMAL				
	3.0901699437E-01	2.7876825791E-01		
C2 ARRAY, OCTAL				
	.2361570 .2165653 .			

24 SINED POLYGON, ERROR LIMITS +EN,-0

	EN	AN	SP	CO
DECIMAL	8.6289605824E-03	1.0057509964E 00	2.5584715611E-03	1.3165249759E-01
OCTAL	.0043260	1.0027435	.0012364	.1033177
C1 ARRAY, DECIMAL				
	2.6794919243E-01	5.7735024918E-01		
C1 ARRAY, OCTAL				
	.2111412 .4474647 .			
C2 ARRAY, DECIMAL				
	2.5433309502E-01	2.2802881476E-01		
C2 ARRAY, OCTAL				
	.2021577 .1646003 .			

28 SIDED POLYGON, EPRON LIMITS +0,-EP

	EP	BP	SP	CO
DECIMAL	6.2877901073E-03	9.9790319004E-01	1.8788768504E-03	0
OCTAL	.0031602	.7767323	.0007544	0
C1 ARRAY, DECIMAL				
	1.1267293990E-01	3.4991513395E-01	6.2834164537E-01	
C1 ARRAY, OCTAL				
	.0715404 .2631201 .5015540 .			
C2 ARRAY, DECIMAL				
	2.2252093396E-01	2.1136280516E-01	1.8960606274E-01	
C2 ARRAY, OCTAL				
	.1617344 .1541574 .1410501 .			

32 SINED POLYGON, ERROR LIMITS +FN,-0

	FN	AN	SP	CO
DECIMAL	4.8385723494F-03	1.0032251966E 00	1.438115725E-04	9.8491403354F-02
OCTAL	.0023643	1.0015154	.0005711	.0623327
C1 ARRAY, DECIMAL				
	1.989123673RE-01	4.1421356236E-01	6.6817863792E-01	
C1 ARRAY, OCTAL				
	.1456575 .3240475 .5260470 .			
C2 ARRAY, DECIMAL				
	1.9319783731E-01	1.8198438338F-01	1.6378521793E-01	
C2 ARRAY, OCTAL				
	.1427255 .1351331 .1236672 .			

36 SIDED POLYGON, ERROR LIMITS +0,-EP

DECIMAL	3.8053014709E-03	9.9473124394E-01	1.1360627031E-03	CO
OCTAL	.0017454	.7772633	.0004516	
C1 ARRAY, DECIMAL	8.7488663525E-02	2.6794919243E-01	4.6630765815E-01	7.0020753820E-01
C1 ARRAY, OCTAL	.0546265	.2111412	.3565776	.5464032
C2 ARRAY, DECIMAL	1.7364817767E-01	1.6837196565E-01	1.5797985667E-01	1.4278760468E-01
C2 ARRAY, OCTAL	.1307207	.1261516	.1207054	.1110667

40 SIDED POLYGON, ERROR LIMITS +EN,-0

	EN	RN	SP	CO
DECIMAL	3.09219H4289F-03	1.0020612537F 00	9.2000359321E-03	7.4701706424E-02
OCTAL	.0014525	1.0010343	.0003612	.11502271
C1 ARRAY, DECIMAL				
	1.5838444032E-01	3.2491969623E-01	5.0952544949E-01	7.2654252794E-01
C1 ARRAY, OCTAL				
	.1210574 .2462676 .4047010 .5637726 .			
C2 ARRAY, DECIMAL				
	1.5546551643F-01	1.4969954224E-01	1.4024746849E-01	1.2734203662E-01
C2 ARRAY, OCTAL				
	.1174623 .1145127 .1076350 .1011460 .			

44 SIDED POLYGON. ERROR LIMITS +0.-EP

	EP	RP	SP	CP
DECIMAL	2.5478853786E-03	9.4915056049E-01	7.6032149523E-04	
OCTAL	.0012337	.7774413	.0003071	
C1 ARRAY, DECIMAL				
7.152141155RE-02	2.1753668456E-01	3.7298071632E-01	5.4604131170E-01	7.4850062328E-01
C1 ARRAY, OCTAL				
.0444747 .1573017	.2767565 .4274454	.5772164 .		
C2 ARRAY, DECIMAL				
1.4231483827E-01	1.3941771856E-01	1.3368245616E-01	1.2522580445E-01	1.1421991649E-01
C2 ARRAY, OCTAL				
.1106730 .1073034	.1043440 .1000732	.0723661 .		

48 SIDEN POLYGON, FRROR LJ:IIIS +FII,-0

	FN	HN	SP	CO
DECIMAL	2.1456707619F-03	1.0014303451E 00	6.388222840MF-04	6.5543462411F-02
OCTAL	.0010624	1.0005670	.0002474	.0414357
C1 ARRAY, DECIMAL				
	1.3165249758F-01	2.6794919243E-01	4.1421356236E-01	5.773502691MF-01
				7.6732698795F-01
C1 ARRAY, OCTAL				
	.1033177 .2111412	.3260475 .4474647	.6106761 .	
C2 ARRAY, DECIMAL				
	1.2996545880F-01	1.2662024695F-01	1.2110852757E-01	1.1352460770E-01
				1.0399825031F-01
C2 ARRAY, OCTAL				
	.1024255 .1006506	.0760037 .0720776	.0651764 .	

52 SINED POLYGON, ERROR LIMITS +0,-EP

	EP	RP	SP	CO
DECIMAL	1.8244457606E-03	9.9939177739E-01	5.4428658831E-04	0
OCTAL	.0007362	.7775404	.0002165	0
C1 ARRAY, DECIMAL				
6.0488856060E-02	1.8325680127E-01	3.1161260248E-01	4.5006346613E-01	6.0452107439E-01
7.8344996792E-01				
C1 ARRAY, OCTAL				
.0367606	.2374273	.3463353	.4654075	.6211006
C2 ARRAY, DECIMAL				
1.2053668025E-01	1.1877898403E-01	1.1528922275E-01	1.1011828500E-01	1.0334157468E-01
9.5057911507E-02				
C2 ARRAY, OCTAL				
.0755560	.0730163	.0703027	.0647223	.0605267

56 SIDED POLYGON, FROM LIMITS +FM,-0

	EN	HN	SP	CN
DECIMAL	1.5756637731E-03	1.0010503874E 00	4.6928529175E-03	5.6154735110E-02
OCTAL	.0006350	1.0004233	.0001730	.0346016
C1 ARRAY, DECIMAL				
1.1267293940E-01	2.2824347438E-01	3.4991513395E-01	4.8157461880E-01	6.2434164537E-01
7.9747338891E-01				
C1 ARRAY, OCTAL				
.0715404 .1666705	.2631201 .3664417	.5015540 .6302347		
C2 ARRAY, DECIMAL				
1.1161136138E-01	1.0950155431E-01	1.0601470167E-01	1.0119465265E-01	9.5102022146E-02
8.7813428536E-02				
C2 ARRAY, OCTAL				
.0711122 .0700411	.0662171 .0636375	.0605423 .0547536		

60 SIDEN POLYGON, ERROR LIMITS +0,-EP

	EP	HP	SP	CN
DECIMAL	1.3704652093E-03	9.9954313647E-01	4.0983162586E-04	n
OCTAL	.0005472	.7776102	.0001531	n
C1 ARRAY, DECIMAL				
5.2407779283E-02	1.5438444032E-01	2.6794919244E-01	3.8386403504E-01	5.0052544940E-01
6.4940759321E-01	8.0974403317E-01			
C1 ARRAY, OCTAL				
.0326523	.1210574	.21111412	.3044235	.4047010
				.5143762
				.6364700
C2 ARRAY, DECIMAL				
1.0452846327E-01	1.0338322755E-01	1.0110530356E-01	9.7719648702E-02	9.3263356923E-02
8.7785252291E-02	8.1345354067E-02			
C2 ARRAY, OCTAL				
.0654114	.0647352	.0636101	.0620205	.0576003
				.0547443
				.0515142

64 SIDED POLYGON, ERROR LIMITS +FN*-0

	FN	RN	SP	CH
DECIMAL	1.2059964356E-03	1.0000039653E 00	3.5031127439E-04	4.9126844764E-02
OCTAL	.0004741	1.0003226	.0001362	.0311162
C1 ARRAY, DECIMAL				
9.8491403356E-02	1.9491236738E-01	3.0334668361E-01	4.1421356236E-01	5.3451113594E-01
6.6817863792E-01	9.2067879082E-01			
C1 ARRAY, OCTAL				
.0623327	.2332404	.4215257	.5260570	.6441400
C2 ARRAY, DECIMAL				
9.7780581121E-02	9.6365782251E-02	9.4022928225E-02	9.0774541994E-02	8.6652026924E-02
8.1694965463E-02	7.5951136824E-02			
C2 ARRAY, OCTAL				
.0620405	.0601074	.0542732	.0516477	.0467061

68 SINED POLYGON, ERROR LIMITS +0.-FP

	FP	HP	SP	C0
DECIMAL	1.0670251504E-03	9.9964429962E-01	3.1815581454F-04	
OCTAL	.0004276	.7776426	.0001233	
C1 ARRAY, DECIMAL				
4.6232790198E-02	1.3949404272E-01	2.3519786280E-01	3.3514621934E-01	4.4154319217F-01
5.5699634508E-01	6.8501593256E-01	8.3038944041E-01		
C1 ARRAY, OCTAL				
.0275275	.1073274	.3420437	.5365647	.4511215
C2 ARRAY, DECIMAL				
9.2268359466E-02	9.1481158349E-02	8.9913472254E-02	8.7578676117E-02	8.4494489590E-02
8.0693807104E-02	7.6202473501E-02	7.1061007265E-02		
C2 ARRAY, OCTAL				
.0571735	.0566552	.0532062	.0470100	.0443042

72 SIDED POLYGON, ERROR LIMITS +EN,-0

	FN	RN	SP	CP
DECIMAL	9.5268513534E-04	1.0006351033E 00	2.8393270457E-04	4.3660942304E-02
OCTAL	.0003715	1.0002464	.0001123	.0262654
C1 ARRAY, DECIMAL				
8.7488663525E-02	1.76326498071E-01	2.6794919243E-01	3.6397023427E-01	4.6630765815E-01
5.7735026918E-01	7.0020753820E-01	8.3909963118E-01		
C1 ARRAY, OCTAL				
.0546265	.1322170	.2111412	.2722645	.3565776
			.4474647	.5464032
			.6554747	.
C2 ARRAY, DECIMAL				
8.6989599678E-02	8.5995270151E-02	8.43466464707E-02	8.2055731735E-02	7.9140505092E-02
7.5622971425E-02	7.1529901274E-02	6.6892445389E-02		
C2 ARRAY, OCTAL				
.0544236	.0540171	.0531367	.0520063	.0465601
			.0444771	.0421774
			.	.

76 SIDED POLYGON, ERROR LIMITS +0.-EP

	FP	RP	SP	CO
DECIMAL	8.5424157439E-04	9.9971523655E-01	2.5481402449E-04	n
OCTAL	.0003377	.7776653	.0001026	n
C1 ARRAY, DECIMAL				
4.1360305942E-02	1.2464987001E-01	2.0967795045E-01	2.9771249497E-01	3.9020121084E-01
4.8807021096E-01	5.9587066271E-01	7.1398627665E-01	8.4695679648E-01	
C1 ARRAY, OCTAL				
.0251323	.0776442	.1532657	.2303335	.3076207
			.3722323	.4610537
			.5554372	.6615105
C2 ARRAY, DECIMAL				
8.2579345473E-02	8.2015244807E-02	8.0890896856E-02	7.9213982064E-02	7.6995955448E-02
7.4251968386E-02	7.1000765085E-02	6.7264554565E-02	6.3068858936E-02	
C2 ARRAY, OCTAL				
.0522175	.0517736	.0513251	.0504354	.0473300
			.0460106	.0442643
			.0423410	.0402251

80 SIDED POLYGON, FROM LIMITS +EN+0

	FN	AN	SP	CO
DECIMAL	7.7155858161E-04	1.0005143592E 00	2.2996012684E-04	3.9090107107E-02
OCTAL	.0003122	1.0002047	.0000742	.0240735
C1 ARRAY, DECIMAL				
	7.8701706824E-02	1.5838444032E-01	2.4007875908E-01	3.2691969623E-01
	5.0952544949E-01	6.1280078812E-01	7.2654252799E-01	8.5408068545E-01
C1 ARRAY, OCTAL				
	.0502271	.1210574	.1727272	.2462676
			.3240475	.4047010
			.4716020	.5637724
				.6652241
C2 ARRAY, DECIMAL				
	7.8337977444E-02	7.7612761182E-02	7.6409036446E-02	7.4734224591E-02
	7.0015483360E-02	6.7000646595E-02	6.3572728551E-02	5.9752863514E-02
C2 ARRAY, OCTAL				
	.0500677	.0475716	.0470761	.0451272
			.0462071	.0436621
			.0422337	.0404312
				.0364477

84 SIDED POLYGON, ERROR LIMITS +0,-EP

	EP	RP	SP	CP
DECIMAL	6.992951675AE-04	9.9976689070E-01	2.0852127637E-04	0
OCTAL	.0002573	.7777027	.0000665	0
C1 ARRAY, DECIMAL				
3.7417360057E-02	1.1267293990E-01	1.8921020839E-01	2.6794919243E-01	3.4991513394E-01
4.3629553996E-01	5.2851543101E-01	6.2834164536E-01	7.3803358710E-01	8.6054958507E-01
C1 ARRAY, OCTAL				
.0231206 .0715404	.1407003 .2111412	.2631201 .3373042	.4164631 .5015540	.5714771 .6704711
C2 ARRAY, DECIMAL				
7.4730093586E-02	7.4312172588E-02	7.3478667777E-02	7.2234240450E-02	7.0585849952E-02
6.8542714749E-02	6.6116260883E-02	6.3320058063E-02	6.0169743792E-02	5.6482935910E-02
C2 ARRAY, OCTAL				
.0462060 .0460304	.0454760 .0447676	.0441075 .0430600	.0416640 .0403270	.0366351 .0350131

88 SIDED POLYGON, ERROR LIMITS +EN,-0

	EN	RN	SP	CO
DECIMAL	6.3758058241F-04	1.0004250447E 00	1.9004617734F-04	3.5715090578F-02
OCIAL	.0002471	1.0001573	.0000617	.0222224
C1 ARRAY, DECIMAL				
7.1521411558E-02	1.4377829399E-01	2.1753668456E-01	2.9362669294E-01	3.7298071632F-01
4.5668469790F-01	5.4604111170F-01	6.4266097716E-01	7.4850062328E-01	8.6650493252E-01
C1 ARRAY, OCIAL				
.0444747	.1114725	.2767565	.4274454	.5772164
C2 ARRAY, DECIMAL				
7.1248185241F-02	7.0701124903F-02	6.9797777651F-02	6.8536746927E-02	6.6926498540E-02
6.4975178217E-02	6.2692769222F-02	6.0090892224E-02	5.7182805862E-02	5.3483329005F-02
C2 ARRAY, OCIAL				
.0443652	.0441463	.0430564	.0400624	.0352161
		.0412107		.0335071

92 SINED POLYEDON. ERROR LIMITS +0.-EP

	FP	HP	CP	CP	CP
DECIMAL	5.8297760552E-04	9.998056655E-01	1.7389330017E-04		0
OCTAL	.0002307	.7777150	.0000555		0
C1 ARRAY, DECIMAL					
3.4161025301E-02	1.0280311701E-01	1.7241741743E-01	2.4360333114E-01	3.1738573342E-01	
3.9434883333E-01	4.7557407186E-01	5.6227193272E-01	6.5590001106E-01	7.5432303750E-01	
8.7195478489E-01					
C1 ARRAY, OCTAL					
.0213731	.0645052	.1302162	.1746126	.2424006	.3117201
				.3633762	.4377042
					.5176442
					.6042061
					.6743416
C2 ARRAY, DECIMAL					
6.8242413363E-02	6.7924235732E-02	6.7289363953E-02	6.6340758101E-02	6.5080841014E-02	
6.3521477676E-02	6.1663947884E-02	5.9518912305E-02	5.7096372074E-02	5.4407622204E-02	
5.1465198889E-02					
C2 ARRAY, OCTAL					
.0427413	.0426157	.0423474	.0417567	.0412451	.0404136
				.0374447	.0363624
					.0351674
					.0346665
					.0332632

96 SIDED POLYGON, ERROR LIMITS +FIM--0

	EN	PN	CP	CN
DECIMAL	5.3564930606F-04	1.0003571265E 00	1.5985376781E-04	3.2736610412E-02
OCTAL	.0002143	1.0001355	.0000517	.0206056
C1 ARRAY, DECIMAL				
6.5543462813E-02	1.316524975MF-01	1.9441236738E-01	2.6794319243F-01	3.3445425886F-01
4.1421356236E-01	4.9314542601F-01	5.7735026918E-01	6.6817343788E-01	7.6732698795E-01
8.7697644297E-01				
C1 ARRAY, OCTAL				
.0414357 .1033177	.1456575 .2111412	.2556317 .3240475	.3743731 .4474647	.5260670 .6106761
C2 ARRAY, DECIMAL				
6.5333037636F-02	6.4913048342E-02	6.4215171244F-02	6.3242274924E-02	6.1998565474F-02
6.0449368647E-02	5.8721147059E-02	5.6701472503E-02	5.4430903535E-02	5.1943394434E-02
4.9225373738E-02				
C2 ARRAY, OCTAL				
.0413465 .0411705	.0407015 .0403025	.0375744 .0367607	.0360413 .0350200	.0336767 .0326605

END JOB
ELAPSED JOB TIME 00 HR 02 MIN 07 SEC

APPLIED
RESEARCH
LABORATORIES

