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ALLOCATION OF REPAIR TIMES AND FAILURE RATES WITH LIFE CYCLE COST CONSIDERATIONS

RESEARCH REPORT

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ABSTRACT

This paper develops a cost-based procedure for allocating repair times and failure rates to the various subsystems of a system. The allocation problem is handled as a cost minimization problem, first with no constraints, and second, with the constraint of meeting a system availability requirement. The Lagrange multiplier technique is employed to obtain the solution. An example problem is stated and solved with the aid of a computer.

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CHAPTER I

INTRODUCTION

New system requirements are more and more being directed toward the attainment of adequate overall system characteristics such as reduction of maintenance costs and general support as well as high performance requirements. As system requirements become more involved and more complex, it becomes increasingly necessary to speak of system characteristics in definite quantitative terms.

In recent years such terms as capability, dependability and cost effectiveness have been quantified in an attempt to describe system performance. However, the most common term used, and the one most specified in government contracts is inherent availability.

Blanchard and Lowery [2]* define inherent availability as " . . . the probability that a system or equipment, when used under stated conditions, without consideration for any scheduled or preventive action, in an ideal support environment (ie., available tools, spares, manpower, data, etc.), shall operate satisfactorily at a given point in time. It excludes ready time, preventive maintenance downtime, logistics time, and waiting or administrative downtime." Inherent availability can be

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*Numbers in brackets refer to list of references at the end of the paper.

expressed as

$$A_I = \frac{MTBF}{MTBF + MTTR}$$

Here MTBF is the mean time between failures and MTTR is the mean time to repair.

In order to co-ordinate the efforts of different groups concerned with different system characteristics, and to eliminate the hazards of guesswork in achieving the overall system requirement, it is necessary to establish a procedure for determining the detailed specifications for the various components that make up a system. The process of assigning availability parameters to individual components to insure the attainment of the system availability goal is termed availability allocation.

The problem that is solved in this paper assumes that an availability requirement has been determined based on operational need, and the paper is concerned with allocation of parameters to achieve that requirement.

The allocation of system availability involves solving the inequality

$$f(A_1, A_2, A_3, \ldots A_n) \geq A'$$

where A' is the system availability requirement, A_i is the allocated availability for subsystem i, and f is the functional relationship between unit and system availability.* The above inequality has an infinite number of solutions if no restrictions are placed on the allocation. The problem is to establish a solution procedure which yields

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*All availabilities referred to throughout the paper are assumed to be inherent availabilities.

a unique or limited number of solutions. The approach used in this paper is to minimize the system life cycle cost subject to the constraint of meeting the system availability goal by the method of Lagrange multipliers. Mean repair times and failure rates are first allocated on a minimum cost basis with the constraint inactive, and second, with the constraint active.

The solution of this problem for the availability parameters MTBF and MTTR is accomplished under the assumptions of constant failure rates (exponential time-to-failure distributions) and independently failing components. Also, a series configuration is assumed in which all subsystems are necessary for the system to function. The limitations of these assumptions are discussed in Chapter III.

A general solution procedure is followed to the extent feasible. Therefore, a solution set of equations is expressed under the assumptions listed above. This set of equations may be used with any applicable cost functions. Specific cost functions are considered for detailed analysis and solution in this paper. However, by substituting any differentiable cost functions into the solution equations of Chapter III, a set of 2n + 1 equations in 2n + 1 unknowns could be obtained, where n is the number of subsystems being considered. One is still faced with the problem of solving the set of simultaneous equations for the allocated failure rates and repair times, but this presents no serious drawback since computerized numerical techniques are available for the solution of such systems of equations. The methods used in this paper should be helpful in providing solution techniques for similar cases.

The costs considered in this paper are research and development, production and procurement, sparing, and maintenance costs. Those elements of life cycle cost not dependent on failure rate or repair time are not considered in this paper, while it is assumed that other costs not mentioned above which may be dependent on failure rate or repair time can be included in one of the above categories. This will be discussed further in Chapter IV.

Development and production costs are assumed negatively correlated with both failure rate and repair time. That is, high costs are associated with low failure rates or repair times while relatively low costs are associated with high failure rates and repair times. The reason for separating development cost and production cost will be discussed in Chapter III. Sparing costs are assumed to be proportional to the procurement cost of spares times the expected number of failures over the life cycle while the maintenance cost is assumed to have one component proportional to the number of hours of maintenance required over the life cycle, and another component proportional to the number of failures which occur during the life cycle. The development of this model is given in Chapter IV.

There are basically two situations in which it would be desirable to allocate availability. The first case is that during the conceptual stage of system development when no design has been formalized, an allocation based on estimated costs of reliability and maintainability development can give the optimal set of component parameters that should be established as system goals. Second, is the case where a particular design has been proposed or developed that falls short of the system availability requirement. The solution given in this paper is applicable to either case.

In Chapter III the general allocation technique is described and limitations are discussed. The specific cost functions are developed in Chapter IV and the solution to this case is given in the form of a computer program in the Appendix. In Chapter V an example problem is stated and solved and application of the computer program is discussed. In Chapter VI some estimates of the sensitivity of failure rates and repair times to the various cost factors are obtained by repeated computer runs. The summary, conclusions, and recommendations for further study are given in Chapter VII.

A brief literature survey is given in Chapter II, discussing pertinent references to reliability-maintainability allocation and life cycle cost models.

CHAPTER II

LITERATURE REVIEW

Although a number of reliability allocation techniques exist, rather limited work has been carried out in the field of maintainability allocation and reliability-maintainability allocation. The obvious reason for this discrepancy is that there simply has not been enough time in the brief history of maintainability for these developments to occur.

A brief description of the current reliability-maintainability trade-off and allocation techniques will help establish a background for the development of the paper.

A basic allocation technique is given in the Martin-Marietta prepared AMCP-705-1 [6] which calculates repair time by summing each component repair time multiplied by its expected fraction of occurrence,

$$MTTR_s = \sum_{i=1}^{n} \left[MTTR_i \frac{\lambda_i}{\sum_{i=1}^{n} \lambda_i} \right]$$

where $MTTR_s$ is the mean system repair time, $MTTR_i$ is the mean repair time for subsystem i, and λ_i is the failure rate of subsystem i. Thus, given a system availability requirement and subsystem failure rates, the system repair time goal can be determined. If estimated subsystem repair times are inadequate to meet the availability requirement, then general recommendations are given as to which subsystems or components to improve. Ideally, subsystems exhibiting the higher failure rates require a low repair time, while those with low failure rates can have higher repair times. However, no quantitative relationships have been developed, and no attempt is made to determine the effect of the changes on system cost.

Elmstad [3] presents an availability allocation method which is based on optimizing the cost effectiveness ratio with checks so that repair time and failure rate extremes are not exceeded. He also presents a design trade-off analysis which is graphically solved for specific effectiveness models.

Blanchard and Lowery [2] present a reliability-maintainability trade-off technique which consists of evaluating the total system cost savings for each of a limited number of design alternatives.

Goldman and Slattery [4] present a reliability-maintainability allocation method in which isoavailability functions are defined and failure rates and repair times are allocated on the basis of incremental cost analysis. However, no provision is made for allocating a specific availability to a subsystem.

Nye [8] has developed a cost based allocation technique which considers maintenance, sparing, production and development costs; however, it is not possible to allocate failure rates and repair times simultaneously. Either failure rates are fixed and repair times allocated by a trial and error solution procedure or repair times are fixed and failure rates are allocated.

Messer [7] has attacked the problem by assuming that differentiable cost functions exist or can be defined which give the changes in system development costs as functions of repair times and failure rates. He then minimizes this cost function subject to an availability constraint. This paper is basically an extension of Messer's paper to include production, sparing and maintenance costs.

Although numerous articles have been written on life cycle costing in recent years, relatively few life cycle cost models have been proposed. A recent paper by Koch [5] defines basic cost categories that must be included in such a model and develops a model from which the life cycle cost of fixed place equipment can be obtained. However, as stated before, this paper is concerned only with those elements of life cycle cost which are dependent on repair times and failure rates.

A general model for the cost based allocation problem is developed in Chapter III in which the method of Lagrange multipliers is used to formulate the solution.

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CHAPTER III

DEVELOPMENT OF THE MODEL

As previously stated the inherent availability of any system can be expressed as

$$A_I = \frac{MTBF}{MTBF + MTTR}. \quad (1)$$

If one considers a series system made up of components having constant failure rates, then the system failure rate can be written as the sum of the individual failure rates if all components fail independently [1]. Thus,

$$\lambda_s = \lambda_1 + \lambda_2 + \cdots + \lambda_n \quad (2)$$

and

$$MTBF = \frac{1}{\sum_{i=1}^{n} \lambda_i} \quad (3)$$

where λ_s is the system failure rate and λ_i is the failure rate of subsystem i.

The system repair time can be thought of as the weighted mean of the components' repair times (M_i) where the weighting factors are the ratios of the component failure rates to the total failure rate. Thus, if each component's repair time is multiplied by its expected fraction of occurrence and the results are summed the expected system repair time is obtained as

$$\text{MTTR} = \frac{\lambda_1}{\lambda_s}M_1 + \frac{\lambda_2}{\lambda_s}M_2 + \dots + \frac{\lambda_n}{\lambda_s}M_n$$
$$= \frac{1}{\lambda_s} \sum_{i=1}^{n} \lambda_i M_i = \frac{1}{\sum\limits_{i=1} \lambda_i} \sum_{i=1}^{n} \lambda_i M_i. \tag{4}$$

It should be noted here that the expression for MTTR assumes that all M_i's are mutually exclusive which implies that when a failure occurs in one subsystem all subsystems are turned off so that no other failure can occur while the system is being repaired.

Substituting the expressions for MTBF and MTTR into Equation 1 yields

$$A = \frac{1}{1 + \Sigma\, \lambda_i M_i}^{*} \tag{5}$$

for a system in which all subsystems are necessary for proper system operation, failures occur independently of each other and all components exhibit constant failure rates. The limitations of these assumptions are discussed below.

The assumption of constant failure rates or equivalently, an exponential time to failure density, dictates that the failure rate be constant throughout the life of the equipment. In addition to being necessary to solve analytically many system relationships, most electronic equipment appears to fit this assumption reasonably well for a long period of time after the burn in stage. For systems that exhibit wear-out, or increasing failure rate, well designed and implemented preventive maintenance policies will reduce or eliminate systematic failures and provide systems that exhibit fairly constant failure rates

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[*] All summations are from 1 to n unless otherwise stated.

over long periods of time. Thus, it seems reasonable that the exponential time to failure density would not only be a good, relatively straightforward approximation for a system, but would probably be quite realistic when considering the total life of a large group of operating systems.

The series assumption is somewhat restrictive in that no provision can be made for operation in a degraded mode. By judicious definition of components or units that make up a subsystem one could obtain reasonable approximations to a series system, for example, two units in parallel could be combined as one unit using the combined failure rates as the resultant unit failure rate. It should be pointed out, however, that a parallel combination of elements with constant failure rates does not result in a unit with a constant failure rate, and therefore, the constant failure rate assumption must be reanalyzed.

The assumption of independently failing subsystems or units is usually made in reliability discussions. This assumption is subject to some question since, in most mechanical or electrical systems, failure of one component can damage or degrade other components. However, by thorough failure mode analysis some of the difficulty could be removed by combining in the mean time to repair the time required to repair all items which failed or were damaged by the initial chance failure. This assumption, like the others, however, does somewhat restrict application of the model developed in this paper. It is also assumed that when a failure occurs the entire system is shut off while the failed subsystem is being repaired. The reasons for this restriction are that it simplifies formulation of the availability constraint, and it enables determination of the total operating time for each subsystem. If each subsystem operates the same amount of time, determination of the total operating time is straightforward when there is an availability constraint. However, if this assumption is not made the operating time for subsystem i is a function of λ_i and M_i. The effect of ignoring this assumption would also result in an erroneous expression for availability, Equation 5, since it would be equivalent to ignoring the probability that two or more subsystems are being repaired at the same time. Therefore, this assumption also restricts applicability of the model developed in this paper, and full consideration should be given to the assumptions whenever the model is applied to an allocation problem.

The allocation of availability parameters to the various units or subsystems of a complex system involves making decisions as to which components to consider for availability improvement and to which characteristics, repair times and/or failure rates, improvement effort should be applied. The remainder of this chapter discusses the basic development of these decision criteria.

For cases 1 and 2 of the following development it is assumed that a design has been proposed that falls short of the system availability goal. That is $A < A'$, where A' is the system availability requirement. In case 1 it is assumed that the reliability of the system is at its highest achievable level and only repair times can be improved. Then, in case 2 the opposite is considered, that is, only the failure rates can be improved. In case 3 the first 2 cases are combined to give the overall optimal set of improvements.

The components of life cycle cost considered in this paper are

1) $d_{fi}(\lambda_i)$, where d_{fi} defines the functional relationship for the research and development costs of changing the failure rate λ_i,*
2) $p_{fi}(\lambda_i)$, where p_{fi} defines the functional relationship for the production and procurement costs of changing the failure rate λ_i,
3) $d_{mi}(M_i)$, which represents the research and development costs associated with changing repair time M_i,
4) $p_{mi}(M_i)$, which represents the production and procurement costs associated with changing repair time M_i,
5) $s_i(\lambda_i, M_i)$, the sparing costs associated with subsystems i, and
6) $f_i(\lambda_i, M_i)$, the maintenance costs associated with subsystem i.

Here, the cost of maintenance manuals and test equipment should be included in production costs if they are dependent on repair times and failure rates. Other components of life cycle cost exist which may be dependent on repair times and failure rates; however, it is felt that they can be included in the above costs with no loss of generality to the model. However, one question arises as to whether the operating costs should be included. Operating costs, per se, are relatively independent of repair times and failure rates. The cost of "lost operation", or decline in profits due to downtime, can be included in

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*It is assumed here that it is possible to change the failure rate of a subsystem without affecting its repair time and vice versa.

maintenance costs, but it should not be included if the total system operating time is fixed. In other words, if a system is designed for a ten year life span with a .95 availability the total operating time is made independent of the individual repair times and failure rates.

Case 1: Reliability Fixed at the Highest Achievable Level

From Equation 5

$$\frac{1}{1 + \Sigma\overset{\circ}{\lambda}_i\overset{\circ}{M}_i} < A' \tag{6}$$

where $\overset{\circ}{\lambda}_i$ and $\overset{\circ}{M}_i$ represent the presently achieved values of failure rate i and repair time i respectively.

Let M_i be the allocated repair time for subsystem i so that

$$\frac{1}{1 + \Sigma\overset{\circ}{\lambda}_i M_i} = A'. \tag{7}$$

Rewriting, Equation 7 yields

$$\Sigma\overset{\circ}{\lambda}_i M_i = \frac{1}{A'} - 1. \tag{8}$$

Using the cost components previously defined the total cost* becomes

$$\begin{aligned} \text{T.C.} = \Sigma[&d_{fi}(\overset{\circ}{\lambda}_i) + p_{fi}\,(\overset{\circ}{\lambda}_i) + d_{mi}\,(M_i) + p_{mi}\,(M_i) \\ &+ s_i(\overset{\circ}{\lambda}_i,M_i) + f_i(\overset{\circ}{\lambda}_i,M_i)]. \end{aligned} \tag{9}$$

No definite functional relationships for the cost components are considered in this chapter.

Thus, the allocation problem is expressed as the problem of minimizing the total cost of availability improvement (in this case, repair time improvement) subject to the constraint of meeting the system

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*This does not represent the total life cycle cost, but that portion of it which is of interest in allocating repair times and failure rates.

availability requirement. Thus, the problem can be stated as

$$\text{minimize T.C.} = \Sigma[d_{fi}(\overset{\circ}{\lambda}_i) + p_{fi}(\overset{\circ}{\lambda}_i) + d_{mi}(M_i) + p_{mi}(M_i) + s_i(\overset{\circ}{\lambda}_i, M_i) + f_i(\overset{\circ}{\lambda}_i, M_i)]$$

subject to

$$\Sigma\overset{\circ}{\lambda}_i M_i = \frac{1}{A'} - 1.$$

If the constraint equation is written to be equal to zero, that is

$$\Sigma\overset{\circ}{\lambda}_i M_i - \frac{1}{A'} + 1 = 0, \tag{10}$$

the Lagrangian can be written as

$$\Lambda = \Sigma[d_{fi}(\overset{\circ}{\lambda}_i) + p_{fi}(\overset{\circ}{\lambda}_i) + d_{mi}(M_i) + p_{mi}(M_i) + s_i(\overset{\circ}{\lambda}_i, M_i) + f_i(\overset{\circ}{\lambda}_i, M_i)] + Q[\Sigma(\overset{\circ}{\lambda}_i M_i) - \frac{1}{A'} + 1] \tag{11}$$

where Q is the Lagrange multiplier. Taking derivatives with respect to M_i and Q and setting them equal to zero yields

$$\frac{\partial \Lambda}{\partial M_i} = \frac{\partial d_{mi}(M_i)}{\partial M_i} + \frac{\partial p_{mi}(M_i)}{\partial M_i} + \frac{\partial s_i(\overset{\circ}{\lambda}_i, M_i)}{\partial M_i} + \frac{\partial f_i(\overset{\circ}{\lambda}_i, M_i)}{\partial M_i} + Q\overset{\circ}{\lambda}_i = 0 \tag{12}$$

for $i = 1, 2, \ldots n$, and

$$\frac{\partial \Lambda}{\partial Q} = \Sigma\overset{\circ}{\lambda}_i M_i - \frac{1}{A'} + 1 = 0. \tag{13}$$

The above set of equations contains n + 1 unknowns $M_1, M_2, \ldots, M_n$, and Q. Since there are n + 1 equations an unique solution could be obtained if definite forms of the cost functions are inserted.

Case 2: Maintainability Fixed at the Highest Achievable Level

In this case the availability requirement is expressed as a function of the achieved repair times and the allocated failure rates,

$$\frac{1}{1 + \Sigma\lambda_i\overset{\circ}{M}_i} = A' \tag{14}$$

where λ_i is the allocated failure rate for subsystem i. The Lagrangian can now be written as

$$\begin{aligned}\Lambda = \Sigma[d_{fi}(\lambda_i) + p_{fi}(\lambda_i) + s_i(\lambda_i,\overset{\circ}{M}_i) + f_i(\lambda_i,\overset{\circ}{M}_i)] \\ + Q(\Sigma\lambda_i\overset{\circ}{M}_i - \frac{1}{A'} + 1).\end{aligned} \tag{15}$$

Differentiating with respect to λ_i and Q yields

$$\begin{aligned}\frac{\partial\Lambda}{\partial\lambda_i} = \frac{\partial d_{fi}(\lambda_i)}{\partial\lambda_i} + \frac{\partial p_{fi}(\lambda_i)}{\partial\lambda_i} + \frac{\partial s_i(\lambda_i,\overset{\circ}{M}_i)}{\partial\lambda_i} \\ + \frac{\partial f_i(\lambda_i,\overset{\circ}{M}_i)}{\partial\lambda_i} + Q\overset{\circ}{M}_i = 0\end{aligned} \tag{16}$$

for i = 1, 2, ..., n, and

$$\frac{\partial\Delta}{\partial Q} = \Sigma\lambda_i\overset{\circ}{M}_i - \frac{1}{A'} + 1 = 0. \tag{17}$$

The above set of equations also contains n + 1 unknowns λ_1, λ_2, ..., λ_n and Q, and therefore could be solved if definite forms of the functions are inserted. The third case is considered next in which both failure rates and repair times can be improved.

Case 3: Repair Times and Failure Rates can be Improved

The availability requirement, expressed as a function of both allocated repair times and allocated failure rates, becomes

$$A' = \frac{1}{1 + \Sigma\lambda_i M_i}. \tag{18}$$

The total cost is now expressed as:

$$T.C. = \Sigma[d_{fi}(\lambda_i) + p_{fi}(\lambda_i) + d_{mi}(M_i) + p_{mi}(M_i) + s_i(\lambda_i,M_i) + f_i(\lambda_i,M_i)] \tag{19}$$

and the Lagrangian for the general case becomes

$$\begin{aligned} \Lambda = \Sigma[d_{fi}(\lambda_i) + p_{fi}(\lambda_i) + d_{mi}(M_i) + p_{mi}(M_i) + s_i(\lambda_i,M_i) \\ f_i(\lambda_i,M_i)] + Q[\Sigma(\lambda_i M_i) - \frac{1}{A'} +1]. \end{aligned} \tag{20}$$

Differentiating with respect to λ_i, M_i and Q yields

$$\begin{aligned} \frac{\partial\Lambda}{\partial\lambda_i} = \frac{\partial d_{fi}(\lambda_i)}{\partial\lambda_i} + \frac{\partial p_{fi}(\lambda_i)}{\partial\lambda_i} + \frac{\partial s_i(\lambda_i,M_i)}{\partial\lambda_i} \\ + \frac{\partial f_i(\lambda_i,M_i)}{\partial\lambda_i} + QM_i = 0. \end{aligned} \tag{21}$$

for $i = 1, 2, \ldots, n$

$$\begin{aligned} \frac{\partial\Lambda}{\partial M_i} = \frac{\partial d_{mi}(M_i)}{\partial M_i} + \frac{\partial p_{mi}(M_i)}{\partial M_i} + \frac{\partial s_i(\lambda_i,M_i)}{\partial M_i} \\ + \frac{\partial f_i(\lambda_i,M_i)}{\partial M_i} + Q\lambda_i = 0. \end{aligned} \tag{22}$$

for $i = 1, 2, \ldots, n$

and

$$\frac{\partial\Lambda}{\partial Q} = \Sigma\lambda_i M_i - \frac{1}{A'} + 1 = 0. \tag{23}$$

The above set of equations contains 2n + 1 unknowns $\lambda_1, \lambda_2, \ldots, \lambda_n, M_1, M_2, \ldots, M_n$ and Q. Since there are 2n + 1 equations, a unique solution should be obtained if definite forms of the cost functions are inserted. It should be pointed out that this set of equations is general in that any set of differentiable cost functions could be inserted and solutions obtained. However, one should keep in mind the assumptions involved in the development.

In Chapter IV specific cost functions are discussed and analyzed. The method of solution used may provide guide lines for solutions to other cost functions.

CHAPTER IV

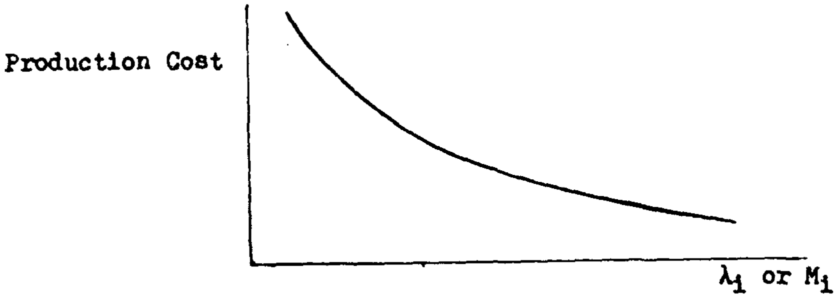
A SOLUTION TO THE ALLOCATION PROBLEM

In this chapter cost considerations associated with availability allocation are discussed, and specific cost functions are considered for solution according to the set of solution cquations obtained in Chapter III.

The objective of an allocation program should be to allocate system parameters to the individual units or subsystems in such a way as to minimize the costs of owning a system while meeting system operational performance requirements. For an availability allocation problem, the costs of concern are those associated with failure rates and repair times. These costs can generally be assigned to the following categories:

1. Development costs.
2. Production costs.
3. Inventory costs.
4. Downtime costs.

As stated in Chapter I, this paper is not concerned with establishing an availability requirement. It is assumed that an availability goal has been established and the problem is to achieve that goal at minimum cost. Functions describing the production costs of a system versus failure rate or repair time should have the characteristics of the curve shown in Figure 1.



λ_i allocated failure rate

M_i allocated repair time

Figure 1. Production Cost Versus Repair Time or Failure Rate

A function of this type suggests that production costs vary inversely with failure rate and repair time or that high costs are associated with low failure rates and repair times and low costs with high failure rates and repair times, and that the cost of producing a unit with zero repair time or failure rate is infinite since it is impossible.

The function used to describe production costs for this paper is

$$p_i(\lambda_i, M_i) = \frac{(PF)_i}{\lambda_i} + \frac{(PM)_i}{M_i} \qquad (23)$$

where

(1) $p_i(\lambda_i, M_i)$ is the cost in dollars of producing subsystem i with repair time M_i and failure rate λ_i,

(2) $(PF)_i$ is the cost factor associated with the difficulty of producing subsystem i with failure rate λ_i, and

(3) $(PM)_i$ is the cost factor associated with the difficulty of producing subsystem i with repair time M_i.

The cost associated with downtime can be separated into two general categories. The first is that which is proportional to the length of time the system is down. This term would include such factors as lost output during repair time and manpower costs to effect repairs. The second term can be described as a maintenance set up cost which is proportional to total number of system failures, which would include system shut down and start up costs and other factors not dependent on the duration of system downtime. Thus, the downtime cost function can be written

$$f_i(\lambda_i, M_i) = F_i K \lambda_i M_i + (SU)_i K \lambda_i \qquad (24)$$

where

(1) $f_i(\lambda_i, M_i)$ is the cost in dollars of maintaining subsystem i with repair time M_i and failure rate λ_i,

(2) F_i is the cost factor representing the cost per hour of downtime for subsystem i,

(3) $(SU)_i$ is the cost factor representing the fixed cost per failure for subsystem i, and

(4) K is the total system operating time, in hours, over the life cycle of the system.

The inventory costs are associated with the cost of procuring spares, and the number of spares needed. For example, a system with a high failure rate would require many more spares than a system with a low failure rate. Although for very low failure rate items the sparing cost would not be a linear function of the failure rate, for a large number of systems, with adequate inventory policies, the sparing cost would generally be a linear function of the failure rate. Since production costs were assumed to be inversely proportional to failure rates and repair times, it would be reasonable to expect that the cost of spares would also be inversely proportional to repair times and failure rates. Thus, the sparing cost function assumed in this paper is

$$s_i(\lambda_i, M_i) = \frac{S_i K \lambda_i}{\lambda_i M_i} = \frac{S_i K}{M_i} = s_i(M_i) \qquad (25)$$

where

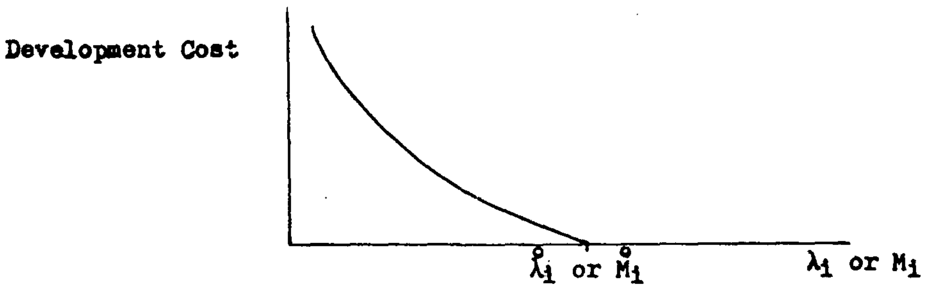
(1) $s_i(M_i)$ is the cost in dollars of sparing subsystem i with repair time M_i,

(2) S_i is the cost factor associated with sparing subsystem i, and

(3) K is, as before, the total system operating time, in hours, over the life cycle.

At this point, the problem of what to do with depot maintenance costs should be discussed. First it should be noted that the repair time, as discussed to this point, is the time required to bring the system back into operating condition at the operational, or lower levels of maintenance. It may or may not have anything to do with depot maintenance time which could be termed mean time to overhaul or rebuild. If the cost to rebuild a unit one time at the depot level is independent of, say, remove and replace time and reliability at the organizational level, then depot maintenance costs could generally be included in maintenance set up costs. However, if the line replaceable unit is made larger as a means of decreasing repair time, depot maintenance costs would depend on MTTR as previously defined. For example, in one radio set the line replaceable unit may be an IF strip, and in the second case, the entire receiver. In the first case the depot would rebuild the IF strip, and in the second case, the entire receiver. In this paper it is assumed that the second case applies, and that depot maintenance costs are inversely proportional to both repair time and failure rate. Depot maintenance costs are, therefore, included in the sparing costs.

Functions describing the development cost of increasing the availability from a level below the requirement to the required level should have many of the characteristics of the production cost functions, except that if the optimal allocation indicated a failure rate or repair time greater than that already achieved no saving in development cost would be realized in purposely increasing a failure rate or repair time. Therefore, the development cost functions chosen for this paper have the form of the curve in Figure 2.



λ_i	allocated failure rate
$\mathring{\lambda}_i$	achieved failure rate
M_i	allocated repair time
$\mathring{M}_i$	achieved repair time

Figure 2. Development Cost Versus Repair Time or Failure Rate

This curve represents a characteristic relationship between development cost and repair time or failure rate and further, it indicates that cost is not increased if a failure rate or repair time is increased. Development funds already spent on the project are not included in the model since the purpose of the allocation is to find the optimal set of repair times and failure rates given an achieved set that falls short of the availability goal.

The functions used to describe development costs for improvement of subsystem i are

$$d_{fi}(\lambda_i) = \frac{(DF)_i}{\lambda_i} - \frac{(DF)_i}{\mathring{\lambda}_i} \qquad \lambda_i < \mathring{\lambda}_i, \tag{26}$$

$$d_{fi}(\lambda_i) = 0 \; ; \; \lambda_i \geq \mathring{\lambda}_i, \tag{27}$$

$$d_{mi}(M_i) = \frac{(DM)_i}{M_i} - \frac{(DM)_i}{\mathring{M}_i} \; ; \; M_i < \mathring{M}_i, \tag{28}$$

and $$d_{mi}(M_i) = 0 \; ; \; M_i \geq \mathring{M}_i, \tag{29}$$

where

(1) $d_{fi}(\lambda_i)$ is the cost of improving the achieved failure rate $\mathring{\lambda}_i$ to λ_i,

(2) $d_{mi}(M_i)$ is the cost of improving the achieved repair time $\mathring{M}_i$ to M_i,

(3) $(DF)_i$ is the cost factor associated with the difficulty of decreasing the failure rate of subsystem i, and

(4) $(DM)_i$ is the cost factor associated with the difficulty of decreasing the repair time of subsystem i.

The dimensions of all cost factors are shown below

$$\$ = \frac{PF_i}{1/hr.} \quad ; PF_i = \$/hr.$$

$$\$ = \frac{PM_i}{hr.} \quad ; PM_i = (\$)(hr.)$$

$$\$ = F_i(hr.) \quad ; F_i = \$/hr.$$

$$\$ = \frac{SU_i hr.}{hr.} \quad ; SU_i = \$$$

$$\$ = \frac{S_i hr.}{hr.} \quad ; S_i = \$$$

$$\$ = \frac{DF_i}{1/hr.} \quad ; DF_i = \$/hr.$$

$$\$ = \frac{DM_i}{hr.} \quad ; DM_i = (\$)(hr.)$$

Information for determining the hourly maintenance cost must, of course, come from the using organization. It could be determined reasonably from past data and should include training costs if the technician turnover rate is high enough to require that new technicians must be trained throughout the system life cycle. Maintenance set up costs will be primarily dependent on the location of maintenance technicians, tools and test equipment relative to the operating system, and the cost of system shut off and start up.

As implied by Equation 25, the cost per failure is given by

$$\$/\text{failure} = \frac{S_i}{\lambda_i M_i}$$

which includes the cost of procuring the spare and carrying it in inventory. Suppose subsystem i has the following characteristics,

$$\lambda_i = .001/hr.$$

$$M_i = 1 \text{ hr.}$$

and it has been determined that the average inventory cost per failure is

$$\$/\text{failure} = \$25$$

then S_i is given by

$$S_i = \$25(.001) = \$.025.$$

The above cost factors, hourly maintenance, maintenance set up and sparing costs, should all be weighted by an expected value inflation factor and a present worth interest factor. They may also be dependent on the location and/or environment of the operating system.

The value of development and production cost factors may be dependent on the level of availability already achieved or predicted for a given design. For example, a proposed system design may have a predicted availability of .90 and it is determined that a .95 availability is necessary. In this case, it may be necessary to reevaluate the development and production cost factors since different technologies may be needed for such an improvement. In any case, the total development cost per system will be inversely proportional to the number of systems produced, whereas production cost per system will be relatively independent of the number of systems produced if production set up costs are included in development costs rather than production costs.

Since the mean time to repair is typically much less than the mean time between failures the failure rate development cost factor should be much less than the repair time development cost factor if development efforts required, for the same percentage improvement, are similar. Thus, given a subsystem for which it has been determined that to decrease λ_i and M_i by one half would cost $10,000 each, and the parameters of the subsystem are as given above, that is,

$$\lambda_i = .001/\text{hr.}$$

$$M_i = 1 \text{ hr.}$$

then the cost associated with each would be

$$\$10{,}000 = \frac{DF_i}{.0005}$$

$$\$10{,}000 = \frac{DM_i}{.5}$$

or,

$$DF_i = \$5/\text{hr.}$$

$$DM_i = \$5{,}000/\text{hr.}$$

Production cost factors would be determined in a similar manner, however the reason for production costs varying with repair time and failure rate would be due to such factors as more expensive materials, more elaborate production techniques, or tighter quality control.

All cost factors are constants that are determined for each subsystem based on existing data, past experience, familiarity with the equipment, and engineering judgment. The importance of careful estimates for these factors cannot be overemphasized.

If the functions previously discussed are substituted into the total cost equation obtained in Chapter III, the following equation is obtained:

$$\text{T.C.} = \Sigma\Big[\frac{DF_i + PF_i}{\lambda_i} + \frac{DM_i + PM_i}{M_i} + \frac{S_iK}{M_i} + SU_iK\lambda_i + F_iK\lambda_iM_i\Big] + Q(\Sigma\lambda_iM_i - \frac{1}{A'} + 1) - \frac{DF_i}{\check{\lambda}_i} - \frac{DM_i}{\check{M}_i} \quad . \qquad (30)$$

Differentiating with respect to λ_i, M_i and Q yields

$$\frac{\partial\, \text{T.C.}}{\partial \lambda_i} = \frac{-(DF_i + PF_i)}{\lambda_i^2} + SU_iK + F_iKM_i + QM_i = 0 \qquad (31)$$

for i = 1, 2, ..., n

$$\frac{\partial T.C.}{\partial M_i} = \frac{-(DM_i + PM_i + S_iK)}{M_i^2} + F_iK\lambda_i + Q\lambda_i \qquad (32)$$

for i = 1, 2, ..., n and

$$\frac{\partial T.C.}{\partial Q} = \Sigma\lambda_iM_i - \frac{1}{A'} + 1 = 0 \qquad (33)$$

The solution to this set of equations represents the set of improvements in subsystem repair times and failure rates which minimize the improvement cost while achieving the availability goal.

Separating the variables λ_i and M_i yields

$$\begin{aligned}(SU_iK)^2\lambda_i^4 - (F_iK + Q)(DM_i + PM_i + S_iK)\lambda_i^2 \\ - 2\,SU_iK(DF_i + PF_i)\lambda_i^2 + (DF_i + PF_i)^2 = 0 \qquad (34)\end{aligned}$$

for i = 1, 2, ..., n

$$\begin{aligned}(F_iK + Q)^2(DF_i + PF_i)M_i^4 - (DM_i + PM_i + S_iK)^2(F_iK + Q)M_i \\ - (DM_i + PM_i + S_iK)^2SU_iK = 0 \qquad (35)\end{aligned}$$

for i = 1, 2, ... n

Although general solutions to fourth order polynomials exist, it is unlikely that they would add any insight to the solution in this case. A computer program is listed in the Appendix which was developed to solve the equations by the Newton-Raphson technique, first with the constraint inactive, and second, with the constraint active.

If the hourly maintenance cost for each subsystem is the same, Equation 30 can be written as

$$T.C. = \Sigma\left[\frac{DF_i + PF_i}{\lambda_i} + \frac{DM_i + PM_i + S_iK}{M_i} + SU_iK\lambda_i\right]$$

$$+ F\ K\ \Sigma\ \lambda_iM_i + Q(\Sigma\lambda_iM_i - \frac{1}{A'} + 1) - \frac{DF_i}{\check{\lambda}_i} - \frac{DM_i}{\check{M}_i}\ .$$

where F is equal to F_i, a constant for all i.

The above equation can also be written

$$T.C. = \Sigma\left[\frac{DF_i + PF_i}{\lambda_i} + \frac{DM_i + PM_i + S_iK}{M_i} + SU_iK\lambda_i\right]$$

$$+ F\ K(\frac{1}{A'} - 1) - \frac{DF_i}{\check{\lambda}_i} - \frac{DM_i}{\check{M}_i} \qquad (36)$$

since, when the constraint is active

$$\Sigma\lambda_iM_i = \frac{1}{A'} - 1.$$

Here it is seen that under the conditions of identical hourly maintenance costs for each subsystem, and an active availability constraint, the optimal values of λ_i and M_i are independent of hourly maintenance costs. The reasoning is that an active availability constraint implies that a fixed amount of maintenance will be performed and if all hourly maintenance costs the same it makes no difference on which subsystem it is performed.

A further simplification can be made if maintenance set-up costs are negligible. This would imply that the maintenance technicians and necessary repair equipment are in close proximity to the operating system and also that shut-down and start-up costs are negligible. If this is the case Equation 36 can be written as

$$T.C. = \Sigma\left[\frac{DF_i + PF_i}{\lambda_i} + \frac{DM_i + PM_i}{M_i} + \frac{S_iK}{M_i}\right]$$

$$+ FK\Sigma\lambda_iM_i + Q(\Sigma\lambda_iM_i - \frac{1}{A'} + 1) - \frac{DF_i}{\check{\lambda}_i} - \frac{DM_i}{\check{M}_i}\ . \qquad (37)$$

Differentiating now with respect to λ_i, M_i, and Q yields

$$\frac{\partial T.C.}{\partial \lambda_i} \quad \frac{-(DF_i + PF_i)}{\lambda_i^2} + (FK + Q)M_i = 0 \tag{38}$$

for i = 1, 2, ..., n

$$\frac{\partial T.C.}{\partial M_i} = \frac{-(DM_i + PM_i + S_iK)}{M_i^2} + (FK + Q)\lambda_i = 0 \tag{39}$$

for i = 1, 2, ..., n

and

$$\frac{\partial T.C.}{\partial Q} = \Sigma\lambda_i M_i - \frac{1}{A'} + 1 = 0.$$

Solving for λ_i, M_i, and Q yields

$$\lambda_i = \left[\frac{(DF_i + PF_i)^2}{(FK + Q)(DM_i + PM_i + S_iK)}\right]^{1/3}, \tag{40}$$

$$M_i = \left[\frac{(DM_i + PM_i + S_iK)^2}{(FK + Q)(DF_i + PF_i)}\right]^{1/3}$$

for i = 1, 2, ..., n

and

$$Q = \left[\frac{\frac{1}{A'} - 1}{(DF_i + PF_i)(DM_i + PM_i + S_iK)}\right]^{2/3} - FK \tag{42}$$

Although the assumption that the hourly maintenance cost is the same for each subsystem may apply to a large variety of systems, the assumption that maintenance set-up costs are zero is quite restrictive. Here again, all assumptions must be carefully investigated before choosing a model.

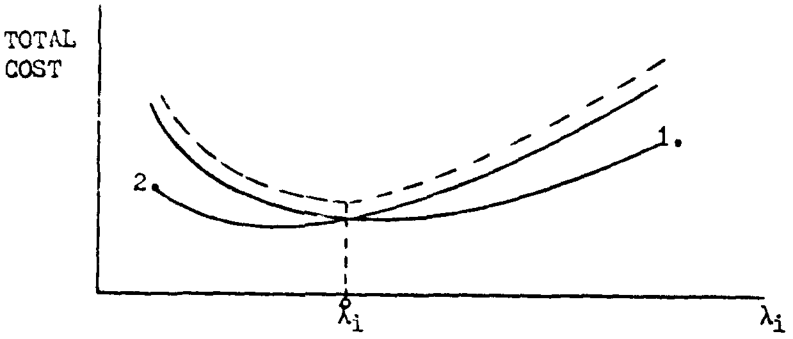
The computer program listed in the Appendix will solve all cases discussed to this point. If hourly maintenance costs are constant for all subsystems they are punched the same on all data cards, and if maintenance set up costs are zero they are read in as zero.

If, for the equations given, an allocated failure rate (λ_i) or repair time (M_i) is greater than that already achieved, the development cost factor, DF_i or DM_i respectively, is set equal to zero since a zero cost is associated with increasing a failure rate or repair time. The equations are then resolved for the same value of the Lagrange multiplier. If, upon resolving, the respective failure rate or repair time is less than that already achieved, the optimal value is identical to the value already achieved. If, however, the newly allocated failure rate (repair time) is greater than that already achieved, the optimal value is given by the value obtained with $DF_i(DM_i) = 0$. This can be illustrated as follows: the development cost for failure rate i is given by Equations 26 and 27 which are repeated here for convenience,

$$d_{fi}(\lambda_i) = \frac{DF_i}{\lambda_i} - \frac{DF_i}{\mathring{\lambda}_i} \qquad \text{for } \lambda_i < \mathring{\lambda}_i$$

$$d_{fi}(\lambda_i) = 0 \qquad \text{for } \lambda_i \geq \mathring{\lambda}_i \,.$$

The first equation intercepts the zero axis at $\lambda_i = \mathring{\lambda}_i$ but is valid only for $\lambda_i < \mathring{\lambda}_i$, thus, the development cost curve is continuous, and the total cost is as shown in Figure 3.



Curve 1, Cost curve assuming $DF_i > 0$ for all λ_i

Curve 2. Cost curve assuming $DF_i = 0$ for all λ_i

\- - - - - - Actual Cost Curve

Figure 3. Total Cost versus Failure Rate

Suppose $\mathring{\lambda}_i$ is the value already achieved. Curve 1 applies if $\lambda_i < \mathring{\lambda}_i$ while curve 2 applies where $\lambda_i > \mathring{\lambda}_i$. While the optimal value of curve 1 is to the right of $\mathring{\lambda}_i$, the optimal value of curve 2 lies to the left of $\mathring{\lambda}_i$. However, the optimal value of the total cost curve lies at $\lambda_i = \mathring{\lambda}_i$.

The case above may indicate that the failure rate or repair time in question was achieved at more expense than necessary, while a cheaper, more advantageous route to the availability requirement existed. Or, it may be the result of the various cost factors changing due to either unforeseen problems or technological advances during research and development.

It would be of interest to obtain the optimal failure rates and repair times for a system that has no specific availability requirement. Of course, in this case, hourly maintenance costs must be included, whether or not it is the same for each subsystem, since there is no constraint on the amount of maintenance to be performed. The computer program in the Appendix will obtain an optimal set of failure rates and repair times (the first set in the print-out under the heading "The Lagrange Multiplier is Zero") however, it is assumed that the system will operate a fixed number of operating hours.

This concludes Chapter IV. The solutions obtained in this chapter are applied to an example problem in Chapter V and the computer program developed to handle the calculations is discussed.

CHAPTER V

APPLICATION OF ALLOCATION TECHNIQUE

An example problem is considered in this chapter for a system which consists of only five subsystems, although the model and the computer program can be used to handle a system of any size.

To enable solution of the problem by the program listed in the Appendix the following parameters must be known:

(1) Development, production, sparing and maintenance cost factors.

(2) Required inherent availability.

(3) Acceptable availability tolerance.

(4) Expected total operating hours of the system.

(5) Predicted values of failure rates and repair times currently achieved.

Determination of the various cost factors was briefly discussed in Chapter IV, and as previously stated, this paper is not concerned with establishing an availability requirement. An availability tolerance is necessary to ensure that the computer does not iterate the solution procedure endlessly if it cannot converge exactly to the required availability due to truncation error. For example, the availability requirement may be stated as

$$A = .95 \pm .001$$

Determination of the expected total operating hours of the system may be quite difficult since it is dependent on administrative and logistics downtime which may vary with the using organization and the location of the system. Total operating time will also be dependent on the amount of preventive maintenance performed.

To illustrate how this figure could be determined, suppose that the administrative and logistics delay time is, typically, some percentage of the hourly maintenance time, and that the system is either operating or it is down for maintenance. The total calendar time can then be expressed by the following equation:

$$T = TA_o + [(1/A_I - 1)P + (1/A_I - 1)]TA_o + \alpha TA_o$$

then,

$$A_o = \frac{1}{1 + \alpha + (1/A_I - 1)(1 + P)}$$

where

A_o is the percentage of the total time that the system is operating,

$(1/A_I - 1)$ represents the amount of corrective maintenance per hour of operating time,

P is the amount of administrative and logistics delay time per hour of corrective maintenance,

A_I is the inherent availability as previously defined,

α is the amount of preventive maintenance required per hour of operating time, and

T is the total calendar time in hours.

Thus, the total operating time of the system is given by

$$\text{Operating time} = TA_o.$$

It was assumed that no administrative or logistics delay time is associated with preventive maintenance; however, the total time equation could easily be written to include such a term.

If an initial allocation is being performed, that is, no values for achieved failure rates and repair times exist, the last two terms of Equation 30, $\frac{DF_i}{\mathring{\lambda}_i}$ and $\frac{DM_i}{\mathring{M}_i}$, are zero, since $\mathring{\lambda}_i$ and $\mathring{M}_i$ are virtually infinite and some nonzero development cost will be associated with obtaining finite repair times and failure rates. When using the program listed in the Appendix it is necessary to enter unrealistically high values for the achieved failure rates and repair times when performing an initial allocation. When this is done the terms $\frac{DF_i}{\mathring{\lambda}_i}$ and $\frac{DM_i}{\mathring{M}_i}$ will be very small and can be neglected in the total cost equation.

The input format and sequence of all cost factors, and the location of the parameters discussed above is given in the Appendix.

The example problem solved is for the case in which a system has been proposed that falls short of the availability goal. Predictions for the failure rates and repair times are listed in Table 1. The repair time includes all active portions of downtime. For example, these might consist of fault isolation, removal, repair and/or replacement, and check out.

Table 1. Subsystem Repair Times and Failure Rates

Subsystem	Repair Time(hr.)	Failure Rate(failure/hr.)
1	15	.0019
2	8	.0051
3	10	.0034
4	10	.0034
5	10	.0034

Availability = .8538

The cost factors assumed for this example are listed in Table 2.

Table 2. Subsystem Cost Factors

Subsystem	DM($-hr.)	PM($-hr.)	DF($1hr.)	PF($1hr.)
1	17,000	17,000	25.00	25.00
2	12,000	12,000	18.00	13.00
3	20,000	20,000	10.00	10.00
4	20,000	20,000	5.00	5.00
5	30,000	30,000	10.00	10.00

	S($)	SU($)	F_1($1hr.)
1	.10	2.00	8.00
2	.06	2.00	6.00
3	.05	2.00	8.00
4	.08	2.00	10.00
5	.12	2.00	12.00

It is desired to achieve an availability of .95 at the least possible cost.

The optimal allocations for this system, found by Messer [7] from considering only development costs, are listed in Table 3.

Table 3. Subsystem Allocations Considering Development Costs Only

Subsystem	Repair Time(hr.)	Failure Rate(failures/hr.)
1	3.10	.00190
2	2.74	.00411
3	4.69	.00234
4	5.90	.00148
5	6.14	.00205

Availability = .9529

Comparing these results to the repair times and failure rates in Table 1, it is seen that the first failure rate has remained the same while all other parameters have decreased.

Substituting all of the cost factors listed in Table 2 into Equations 33, 34, and 35 yields the results listed in Table 4.

Table 4. Subsystem Allocations Considering All Cost Factors

Subsystem	Repair Time(hr.)	Failure Rate(failures/hr.)
1	3.9955	.002741
2	2.8416	.003988
3	4.6910	.002265
4	5.8693	.001400
5	6.0118	.001913

Availability = .9500

Comparing Table 4 and Table 3 it is seen that the first failure rate should deliberately be increased. Messer used the same development cost function that is used here, namely that the development cost for increasing a failure rate or repair time is zero, so that, if development costs only are considered, no repair time or failure rate could increase in the optimal solution. However, when all cost factors are considered it may be desirable to increase one or more failure rates or repair times. If the allocation procedure indicates that a failure rate or repair time should be increased, no savings in development costs can be realized since development funds to achieve the present levels of repair times and failure rates have already been spent. However, a savings in production costs will be realized for the case where the solution indicates an increasing failure rate, and a savings in production and sparing costs can be realized for the case of an increasing repair time.

The results of the allocation technique are guidelines to be taken into consideration, not hard and fast rules. Also, it will never be more accurate than the estimates of the cost factors involved, nor more valid than the assumptions made in deriving the model.

The total additional cost is also evaluated at each iteration of the program, however, it must be cautioned, that one should not compare the costs listed as a means of determining the "price" of additional availability. As stated before, the allocations are made under the assumption that the system will operate a fixed number of hours; therefore, if the availabilities listed are significantly different from the required availability, system lifetime must also be significantly different from the planned lifetime of the system.

Some estimates of the sensitivity of failure rates and repair times to the various cost factors are obtained in Chapter VI.

CHAPTER VI

SENSITIVITY ESTIMATION

Knowledge of the sensitivity of failure rates and repair times to the various parameters discussed earlier is important when determining how much effort should be devoted to estimating each parameter. If repair times and failure rates are relatively insensitive to one parameter as opposed to another it is obvious that more effort should be alloted to obtaining an accurate estimate of the more sensitive parameter.

In this chapter an attempt is made to obtain estimates of failure rate and repair time sensitivity by varying certain parameters and observing the effects of their variation. This example consists of a system with five subsystems having the same cost factors. They are as shown in Table 5.

The number of operating hours is

$$K = 20{,}000,$$

and the availability constraint is

$$A_I = .95 \pm .0001.$$

The results of this allocation will, of course, have all λ_i's equal and all M_i's equal. They are

$$\lambda_1 = \lambda_2 = \ldots = \lambda_5 = .00285368$$

$$M_1 = M_2 = \ldots = M_5 = 3.0872.$$

Table 5

Cost Factors for Sensitivity Estimation

Subsystem	DF_i	PF_i	DM_i	PM_i
1	10	10	10,000	10,000
2	10	10	10,000	10,000
3	10	10	10,000	10,000
4	10	10	10,000	10,000
5	10	10	10,000	10,000

Subsystem	S_i	SU_i	F_i	$\mathring{\lambda}_i$	$\mathring{M}_i$
1	.25	4.0	12.0	99999.	99999999.
2	.25	4.0	12.0	99999.	99999999.
3	.25	4.0	12.0	99999.	99999999.
4	.25	4.0	12.0	99999.	99999999.
5	.25	4.0	12.0	99999.	99999999.

The availability is

$$A_i = .9500,$$

and the total cost is

$$\text{T.C.} = \$82,711.25 \text{ per system.}$$

By recalling Equation 30,

$$\text{T.C.} = \Sigma\left[\frac{DF_i + PF_i}{\lambda_i} + \frac{DM_i + PM_i + S_iK}{M_i} + SU_iK\lambda_i + F_iK\lambda_iM\right]$$

it is obvious that the sensitivity of repair times and failure rates will not be dependent on a percentage change in, say, DF_i alone, but on the percentage change of the sum $DF_i + PF_i$. Likewise, it will be dependent on the percentage change of the sum $DM_i + PM_i + S_iK$. In the estimations that follow only one parameter at a time was varied while all others were as listed in Table 5. All variations were made on subsystem 5. The results of these estimates are shown in Table 6.

Table 6

Sensitivity to Cost Factors

	$\lambda_1 - \lambda_4$	λ_5	$M_1 - M_4$	M_5	Cost 1
$DF_5+PF_5=40$	.002785	.004398	3.5933	2.8596	88,257.56
$DF_5+PF_5=10$	.002912	.001843	3.7678	4.7361	78,424.88
$DM_5+PM_5+S_5K=50,000$	.002783	.002226	3.5902	5.6773	88,084.31
$DM_5+PM_5+S_5K=12,500$	.002913	.003624	3.7699	2.3903	78,578.19
$SU_5=8$	.002856	.992797	3.6907	3.7296	82,942.69
$SU_5=2$	.002852	.002883	3.6852	3.6652	82,595.50
$F_5=24$	.002912	.002617	3.7675	3.3628	84,990.75
$F_5=6$	.992817	.003000	3.6369	3.8896	81,365.06

	Cost 2	Change in Parameter	% Change in Cost 1	% Change in Cost 2
$DF_5+PF_5=40$	83,710.86	doubled	6.7	1.2
$DF_5+PF_5=10$	83,853.56	halved	-5.2	1.4
$DM_5+PM_5+S_5K=50,000$	83,681.38	doubled	6.5	1.2
$DM_5+PM_5+S_5K=12,000$	83,791.15	halved	-5.0	1.3
$SU_5=8$	82,729.18	doubled	.28	.022
$SU_5=2$	82,711.80	halved	-.14	.00067
$F_5=24$	82,879.25	doubled	2.8	.20
$F_5=6$	82,748.78	halved	-1.6	.045

Cost 1 represents the true cost per system if the cost factor listed in the left hand column actually has the value stated, whereas Cost 2 represents the true cost per system if the cost factor in the left hand column is estimated to have the value given and has its actual value as listed in Table 5.

Although no general conclusions can be drawn from Table 6, since specific numbers were assumed, the table indicates that the optimal repair times and failure rates, as well as the optimal cost, are relatively insensitive to the maintenance cost factors, SU_i and F_i. This should be expected since, when an availability constraint exists, the total maintenance performed will be forced to be relatively small compared to the optimal unconstrained value, unless, of course, the optimal value is nearly equal to the constrained value.

The sensitivity with respect to the number of operating hours was obtained using the cost factors of Table 5, and varying K. The results are listed in Table 7.

Table 7

Sensitivity to the Number of Operating Hours

	λ_i	M_i	Cost per System
K = 20,000	.002854	3.6872	$82,711.25
K = 10,000	.003032	3.4727	$72,316.50
K = 40,000	.002577	4.0830	$102,854.60

Table 7 indicates that as the number of operating hours increase the optimal failure rates decrease while the optimal repair times increase. The reasons for this are that the total sparing cost is proportional to K and inversely proportional to M_i and the total maintenance set up cost is proportional to both K and λ_i.

Finally, the sensitivity with respect to the number of systems produced was estimated by varying the development cost factors. Reducing DF_i and DM_i by one half implies that twice as many systems as represented by Table 5 will be produced while increasing DF_i and DM_i by a factor of two implies that half as many systems will be produced. All other cost factors remained the same. The results of these allocations are shown in Table 8.

Table 8

Sensitivity to Number of Systems Produced

	λ_i	M_i	Cost per System
$DF_i = 10$ $DM_i = 10{,}000$	.00285368	3.6872	$ 82,711.25
$DF_i = 5$ $DM_i = 5{,}000$	.00275190	3.8236	$ 67,134.44
$DF_i = 20$ $DM_i = 20{,}000$	.00296773	3.5456	$113,713.80

In all cases the availability is

$$A_I = .9500$$

Table 8 shows that the optimal repair times and failure rates are also relatively insensitive to the number of systems produced; however, the availability constraint is also active here and the optimal allocations would not be expected to change significantly when all development cost factors change by the same percentage. However, as stated before, the change in λ_i or M_i is not sensitive to DF_i or DM_i per se, but to the sums $DF_i + PF_i$ and $DM_i + PM_i + S_iK$. Thus, the same percentage change in two development cost factors may cause two very different changes in the sums stated depending on the relative magnitude of the development cost factors to the other cost factors. Under these conditions the optimal allocations would not be expected to be insensitive to the number of systems produced.

In the following chapter, the results of this paper are summarized and recommendations for further study are given.

CHAPTER VII

SUMMARY AND CONCLUSIONS

A method for the cost-based allocation of repair times and failure rates has been developed. The solution is based on the minimization of life cycle cost subject to the constraint of an inherent availability requirement.

This allocation technique is applicable to systems in which all subsystems exhibit a constant failure rate and failures occur independently. The removal of these assumptions would generalize the allocation procedure and certainly make it more realistic. However, with consideration of time dependent failure rates, analytic solution techniques are usually infeasible, if possible at all.

It is also assumed that the system can be described by a series model; that is, all subsystems are necessary for proper system operation, and that when a failure occurs in one subsystem, all subsystems are turned off for the duration of maintenance downtime. Extensions to other models have not been considered in this paper, although it would appear feasible to solve the allocation problem for cases in which definite system models could be developed. The only change necessary is that of formulating the availability constraint.

The usefulness of this technique lies in its ability to determine the set of repair times and failure rate improvements that result in the minimum life cycle cost to achieve a specified availability. Another useful feature of the technique, as applied in the computer program, is that the optimal repair time and failure rate allocations are obtained for the case where the availability constraint is inactive. However, a different system lifetime is implied if the resulting availability is significantly different than that of the constraint.

It was determined that if hourly maintenance costs are identical for each subsystem then the optimal repair time and failure rate allocations are independent of hourly maintenance costs when the availability constraint is active. It should be noted, however, that this was made possible by the way in which the availability constraint was formulated.

No provision has been made in this paper to account for the effects of various spares provisioning policies, manpower levels and repair facilities on the system and subsystem repair times. Well-designed systems can have disastrous field results when accompanied by ill-planned logistics and administration. Therefore, it is necessary to consider the effects of these policies on total system operation. Even after a basic allocation program has been instigated, these factors should be taken into consideration before indicated changes are made in the system.

Specific work could also be accomplished for cases where a maximum allowable repair time exists. It is possible that a system could meet the availability requirement and still experience a failure that would result in a long downtime that could produce disastrous results. For this reason it would be desirable to have an allocation technique which would ensure that maximum allowable repair times were not exceeded. For this problem, it is necessary to consider repair time distributions so that variations about the mean can be analyzed.

It would also be desirable, in cases where high availability is not essential, to allocate repair times and failure rates subject to a total production constraint; that is, allocate the parameters so that a minimum required amount of production could be achieved over a fixed period of time at a minimum cost. Essential to this problem is formulating a production constraint for a given system configuration.

In conclusion, the results of this paper should be of interest to equipment designers and users. Where it is necessary to achieve a required inherent availability, this allocation technique will direct attention to the specific parameters that need to be improved. Also, forcing the contractor to consider all cost factors involved will provide the user with valuable information on logistics and administrative planning even if some of the cost information must come from the user.

APPENDIX

APPENDIX

COMPUTER PROGRAM FOR THE ALLOCATION PROBLEM

The purpose of the program listed here is to provide a computerized technique for solving the availability allocation problem. The program was run on the IBM 360-65; however, it could be run on the IBM 1130 if the statements in lines 80 and 81 are replaced by the following statements:

```
       IF(Q) 91, 90, 91
    90 IF(A-AA) 91, 55, 55
    91 IF(ABS(A-AA)-ACC) 55, 55, 17
```

The program requires less than 8,000 bytes of memory for a system composed of five subsystems. To rewrite the program for a larger system it is necessary to make two changes in the program. These are

(1) The variables in the DIMENSION statement, line 2, must be dimensioned as large as the number of subsystems in the system, and

(2) The number of subsystems, N, must be stated in line 5.

The significant variables of the program are listed below, followed by a program flowchart.

Variables

DF(I)	Failure rate development cost factor of ith subsystem
DM(I)	Repair time development cost factor of ith subsystem

PF(I)	Failure rate production cost factor of ith subsystem
PM(I)	Repair time production cost factor of ith subsystem
S(I)	Sparing cost factor of ith subsystem
SU(I)	Maintenance set-up cost factor for subsystem i
F(I)	Hourly maintenance cost factor for subsystem i
XF(I)	Achieved failure rate of subsystem i
XM(I)	Achieved repair time of subsystem i
AA	Required availability
A	Allocated availability
ACC	Availability tolerance
N	Number of subsystems
K	Total number of system operating hours
Q	Lagrange multiplier
XFN(I)	Allocated failure rate of subsystem i
XMN(I)	Allocated repair time of subsystem i
A4, A3, A2, A1, A0	Coefficients of failure rate polynomial
B4, B3, B2, B1, B0	Coefficients of repair time polynomial
JF(I)	Coefficient of failure rate development cost factor (value is 0 or 1)
JM(I)	Coefficient of repair time development cost factor (value is 0 or 1)

If the Newton Raphson technique does not converge an error message is printed out prior to the failure rate and repair time allocations and the allocations will be incorrect. If convergence was obtained no error message will be printed out. The first page of the printout contains the cost factors in the following order:

$DF_i \quad PF_i \quad DM_i \quad PM_i \quad S_i \quad SU_i \quad F_i$

All other pages of the printout contain, in the following order,

The value of the Lagrange multiplier,

The failure rate repair time allocations,

The inherent availability,

The total cost in dollars, and

A message that the cost is optimal for the given availability if all roots converged.

The data format is as shown on pages 53 and 54. The data input sequence is CARD 1_1, CARD 2_1, CARD 1_2, CARD 2_2, ..., CARD 1_n, CARD 2_n.

INPUT FORMAT CARD 1_1

1-10

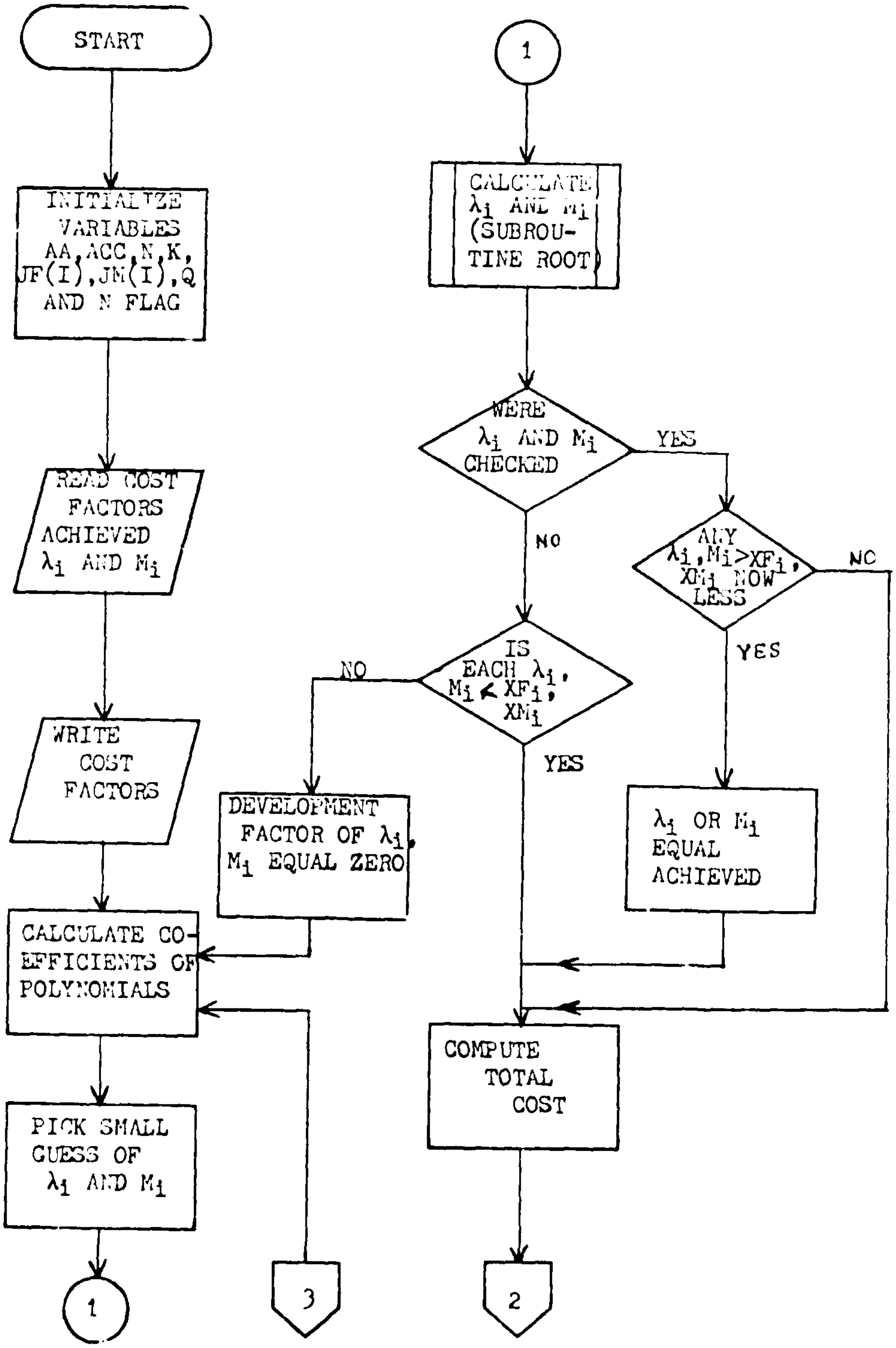
Maintenance set up cost factor

F 10.2

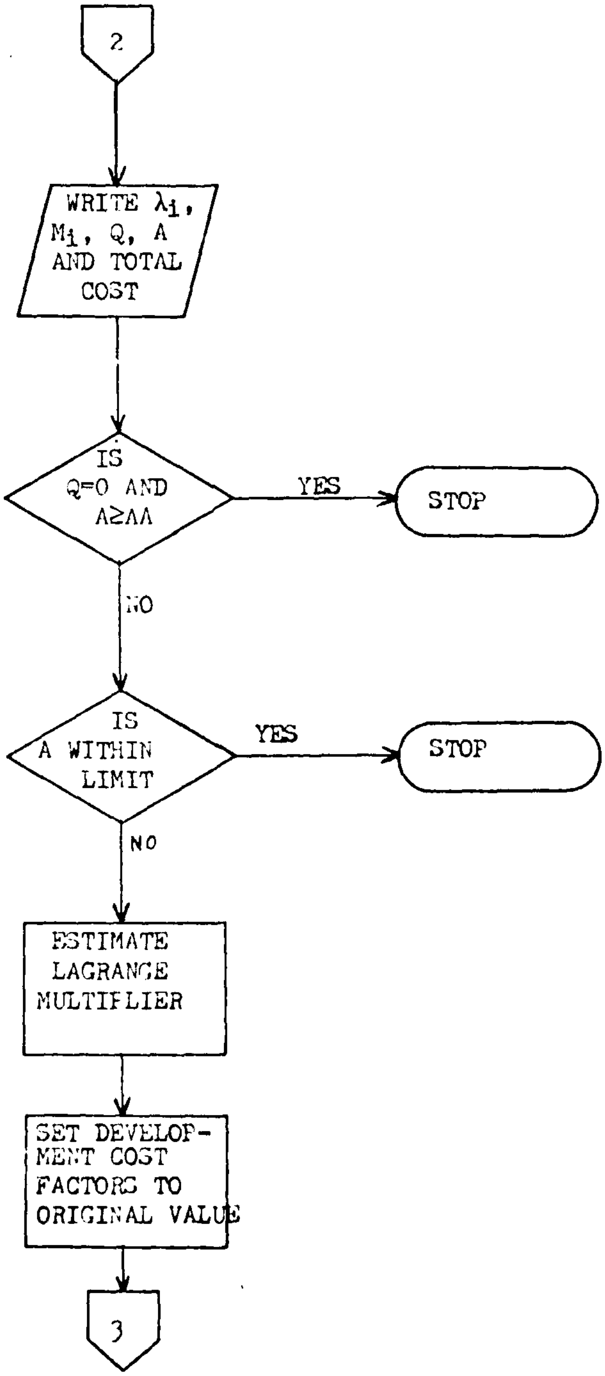
INPUT FORMAT CARD 21

71-80	Presently achieved repair time	F 10.8
61-70	Presently achieved failure rate	F 10.8
51-60	Hourly maintenance cost factor	F 10.2
41-50	Sparing Cost factor	F 10.2
31-40	Repair time production cost factor	F 10.2
21-30	Repair time development cost factor	F 10.2
11-20	Failure rate production cost factor	F 10.2
1-10	Failure rate development cost factor	F 10.2

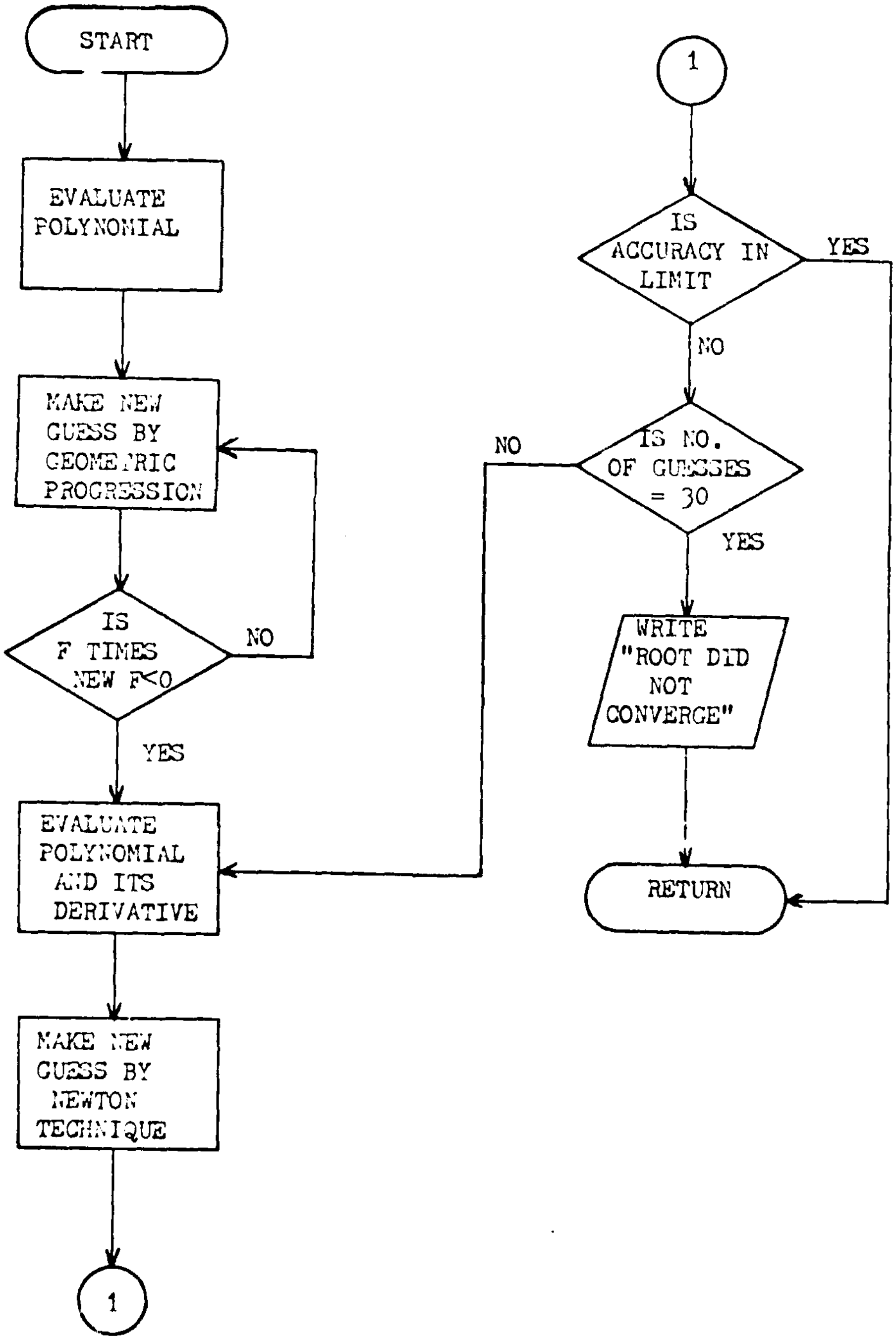
MAIN PROGRAM FLOWCHART



MAIN PROGRAM FLOWCHART (CONT.)



SUBROUTINE FLOWCHART



```
//$WATFIVR JOB (42059,15-H ,*10,001,20),' F. B. HUBLER               •            72
      REAL  K ,  JF   ,  JM
      DIMENSION DF(10),PF(10),DM(10),PM(10),S (10),F(10),XF(10),XM(10),
     1XFN(10),XMN(10),JF(10),JM(10),SU(10)
      AA=.95
      ACC=.001
      N=5
      K=20000.
      C=0.
      NFLAG=1
      DO 81 I=1,N
      JF(I)=1.
   81 JM(I)=1.
      DO 1 I=1,N
      READ(5,80) SU(I)
   80 FORMAT (F10.2)
    1 READ(5,2) DF(I),PF(I),DM(I),PM(I),S (I),F(I),XF(I),XM(I)
    2 FORMAT (6F10.2,2F10.8)
      WRITE(6,86)
   86 FORMAT('1',2X,'FAILURE RATE COST FACTORS',9X,'REPAIR TIME COST
     1FACTORS',5X,'SPARING',11X,'MAINTENANCE',//6X,'DEV',11X,
     2'PROD',15X,'DEV',11X,'PROD',14X,'COST',10X,'COST FACTORS')
      WRITE(6,85)(DF(I),PF(I),DM(I),PM(I),S (I),SU(I),F(I),I=1,N)
   85 FORMAT(//3X,F12.2,3X,F12.2,3X,F12.2,3X,F12.2,3X,F12.5,3X,F12.5,3X,
     1F12.5)
   18 DO 3 I=1,N
C     COMPUTE COEFFICIENTS OF POLYNOMIALS
      A4=(SU(I)*K)**2
      A3=-(F(I)*K+C)*(JM(I)*DM(I)+PM(I)+S(I)*K)
      A2=-2.*SU(I)*K*(JF(I)*DF(I)+PF(I))
      A1=0.
      A0=(JF(I)*DF(I)+PF(I))**2
      B4=(F(I)*K+C)**2*(JF(I)*DF(I)+PF(I))
      B3=0.
      B2=0.
      B1=-(JM(I)*DM(I)+PM(I)+S(I)*K)**2*(F(I)*K+C)
      B0=-(JM(I)*DM(I)+PM(I)+S(I)*K)**2*SU(I)*K
C     FIND ROOTS OF POLYNOMIALS
      X=.000001
      CALL ROOT(X,1,A4,A3,A2,A1,A0)
      XFN(I)=X
      Y=.0001
      CALL ROOT(Y,1,B4,B3,B2,B1,B0)
      XMN(I)=Y
    3 CONTINUE
C     IF APPROPRIATE DEVELOPMENT COST FACTORS WERE SET EQUAL TO ZERO
C     CHECK TO SEE IF THE OPTIMAL SOLUTION IS THAT ALREADY ACHIEVED
      GO TO (26,30), NFLAG
C     SET APPROPRIATE DEVELOPMENT COST FACTORS EQUAL TO ZERO
   26 DO 20 I=1,N
      IF(XF(I)-XFN(I))21,22,22
   21 JF(I)=0.
      GO TO 23
   22 JF(I)=1.
   23 IF(XM(I)-XMN(I))24,25,25
   24 JM(I)=0.
      GO TO 20
   25 JM(I)=1.
   20 CONTINUE
C     IF ANY DEVELOPMENT COST FACTORS ARE ZERO FIND ROOTS OF
C     POLYNOMIALS AGAIN
```

```
      DO 40 I=1,N
      IF(JF(I)-0.)42,41,42
   42 IF(JM(I)-0.)40,41,40
   40 CONTINUE
      GO TO 50
   41 NFLAG=2
      GO TO 18
C     CHECK TO SEE IF THE OPTIMAL SOLUTION IS THAT ALREADY ACHIEVED
   30 DO 31 I=1,N
      IF(JF(I)-0.)32,33,32
   33 IF(AF(I)-AFN(I))32,32,34
   34 AFN(I)=AF(I)
   32 IF(JM(I)-0.)31,35,31
   35 IF(AM(I)-XMN(I))31,31,36
   36 XMN(I)=AM(I)
   31 CONTINUE
   50 WRITE(6,51)W
   51 FORMAT(1H1,20X,'THE LAGRANGE MULTIPLIER IS',F15.4,///20X,
     1'FAILURE RATE',20X,'REPAIR TIME')
      WRITE(6,52) (XFN(I),XMN(I),I=1,N)
   52 FORMAT(//,20X, F12.8,20X,F12.4)
C     COMPUTE AVAILABILITY
      SUM=0.
      DO 10 I=1,N
   10 SUM=SUM+XFN(I)*XMN(I)
      A=1./(1.+SUM)
      WRITE(6,53)A
   53 FORMAT(///,20X,'THE AVAILIBILITY IS',F8.4)
C     COMPUTE TOTAL COST
      COST=0.
      DO 71 I=1,N
   71 COST=COST+(JF(I)*UF(I)+PF(I))/XFN(I)+(JM(I)*UM(I)+PM(I)+S(I)*K)
     1/XMN(I)+SU(I)*K*XFN(I)+F(I)*K*XFN(I)*XMN(I)-JF(I)*OF(I)/AF(I)
     2-JM(I)*OM(I)/AM(I)
      WRITE(6,78)COST
   78 FORMAT(///10X,'THE TOTAL COST IS',F15.2,//10X,
     1'THIS COST IS OPTIMAL IF ALL ROOTS CONVERGED')
C     MAKE AVAILABILITY CHECK
      IF(G.EQ.0.0.AND.A.GT.AA)STOP
      IF(ABS(A-AA)-ACC)55,55,17
   55 STOP
C     NORMALIZE THE SUM OF FAILURE RATE REPAIR TIME PRODUCTS TO THE
C     REQUIRED VALUE AND ESTIMATE THE LAGRANGE MULTIPLIER
   17 R=(1./AA-1.)/SUM
      SR=SQRT(R)
      RN=N
      G=0.
      DO 19 I=1,N
      XFN(I)=SR*XFN(I)
      XMN(I)=SR*XMN(I)
      G=G+((JF(I)*UF(I)+PF(I))/XFN(I)**2-SU(I)*K)/(2.*RN*XMN(I))+
     1((JM(I)*UM(I)+PM(I)+S(I)*K)/XMN(I)**2)/(2.*RN*XFN(I))-F(I)*K/RN
   19 CONTINUE
      DO 57 I=1,N
      JF(I)=1.
   57 JM(I)=1.
      NFLAG=1
      GO TO 18
      END

      FUNCTION FCT(C4,C3,C2,C1,C0,Z)
```

```
      FCT=C4*Z**4+C3*Z**3+C2*Z**2+C1*Z+C0
      RETURN
      END

      SUBROUTINE ROOT(Z,I,C4,C3,C2,C1,C0)
C     FIND ZERO CROSSING INTERVAL USING GEOMETRIC PROGRESSION
      DO 4 J=1,30
      PFCT=FCT(C4,C3,C2,C1,C0,Z)
      Z=2.*Z
      IF (FCT(C4,C3,C2,C1,C0,Z)*PFCT)5,12,4
    4 CONTINUE
C     FIND ROOT USING NEWTON RAPHSON TECHNIQUE
    5 DO 6 J=1,30
      PZ=Z
      DFCT=4.*C4*Z**3+3.*C3*Z**2+2.*C2*Z+C1
      Z=PZ-FCT(C4,C3,C2,C1,C0,Z)/DFCT
C     CHECK ACCURACY ( TO 5 SIGNIFICANT FIGURES )
      IF(ABS((Z-PZ)/Z)-.00001)12,6,6
    6 CONTINUE
      WRITE(6,11)I
   11 FORMAT(//10X,'NO CONVERGENCE FOR',I2,'TH FAILURE RATE OR REPAIR
     1TIME')
   12 RETURN
      END

    //$DATA
```

```
FAILURE RATE COST FACTORS        REPAIR TIME COST    FACTORS     SPARING          MAINTENANCE
   DEV            PROD              DEV          PROD             COST          COST FACTORS

        10.00          10.00      30000.00      30000.00       0.12000       2.00000      12.00000

         5.00           5.00      20000.00      20000.00       0.08000       2.00000      10.00000

        10.00          10.00      20000.00      20000.00       0.05000       2.00000       8.00000

        18.00          18.00      12000.00      12000.00       0.06000       2.00000       6.00000

        25.00          25.00      17000.00      17000.00       0.10000       2.00000       8.00000
```

```
THE LAGRANGE MULTIPLIER IS        0.0000

FAILURE RATE                 REPAIR TIME

 0.00295431                       9.3812

 0.00225936                       9.5949

 0.00340000                      10.2480

 0.00510000                       6.7570

 0.00465858                       6.9497

THE AVAILIBILITY IS  0.8686

THE TOTAL COST IS       59497.71
THIS COST IS OPTIMAL IF ALL ROOTS CONVERGED
```

```
THE LAGRANGE MULTIPLIER IS    609727.8000

FAILURE RATE                  REPAIR TIME

  0.00195128                       6.1347

  0.00142938                       5.9952

  0.00231467                       4.7969

  0.00407953                       2.9095

  0.00280170                       4.0858

THE AVAILIBILITY IS  0.9479

THE TOTAL COST IS       76369.94
THIS COST IS OPTIMAL IF ALL ROOTS CONVERGED
```

```
            THE LAGRANGE MULTIPLIER IS    662659.0000

            FAILURE RATE                  REPAIR TIME

              0.00191275                      6.0118

              0.00135964                      5.8693

              0.00226482                      4.6910

              0.00398764                      2.8416

              0.00274126                      3.9955

            THE AVAILIBILITY IS  0.9500

        THE TOTAL COST IS       77855.81
        THIS COST IS OPTIMAL IF ALL ROOTS CONVERGED
CORE USAGE       OBJECT CODE=    7536 BYTES,ARRAY AREA=     524 BYTES,TOTAL AREA AVAILABLE=  133216  BYTES
COMPILE TIME=     1.41 SEC,EXECUTION TIME=     4.45 SEC,   TAMU/WATFIV - VER 1 LEV 2  AUGUST 1970     DATE=  71/066
$NO JOB  //SDATA
```

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