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# CALCULATION OF GROUND THAWING ALLOWING FOR WATER SEEPAGE

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## CALCULATION OF GROUND THAWING ALLOWING FOR WATER SEEPAGE

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As is known, the nature of the heat exchange of the filtration flow with frozen soil depends significantly on the filtering properties of this soil in a frozen state [1]. We should differentiate two possible cases: the soil pores are filled with ice or not (open pores).

**Open Pores (Fig. 1).** On the soil surface, there is maintained a layer of water  $H$  with temperature  $T_1$ . The  $y$ -axis is directed downward; the origin of coordinates is on the soil surface. Up to the time  $\tau$ , the frozen soil has thawed to depth  $\xi$ . The filtration factor and the porosity of thawed soil ( $0 \leq y \leq \xi$ ) is signified by  $K_1$ ;  $m_1$ , while that of frozen soil is indicated by ( $\xi \leq y \leq \infty$ ) -  $k_2$ ;  $m_2$ .

At the time moment  $\tau$ , the water has seeped to depth  $y_0$ . Let us assume that during the filtration of water, in the frozen soils, the plugging of pores will not occur. Let us further assume that at each time moment  $\tau$ , the temperature distribution in the thawed ground corresponds to the established state. Moreover, the velocity  $v$  of water's velocity will depend only on time.

The determination of function  $\xi = \xi(\tau)$  reduces to a solution of the system of equations for heat conductivity and filtration:

$$a \frac{\partial^2 t}{\partial y^2} - v \frac{\partial t}{\partial y} = 0 \quad 0 \leq y \leq \xi \quad (1)$$

$$\frac{dy_0}{d\tau} = \frac{k_2}{m_2} \frac{H + y_0}{y_0 + \left(\frac{k_2}{k_1} - 1\right) \xi} \quad (2)$$

$$v = m_2 \frac{dy_0}{d\tau} \quad (3)$$

$$\lambda_r \frac{\partial t}{\partial y} \Big|_{y=\bar{y}} = Gw \frac{d\bar{y}}{d\tau} \quad (4)$$

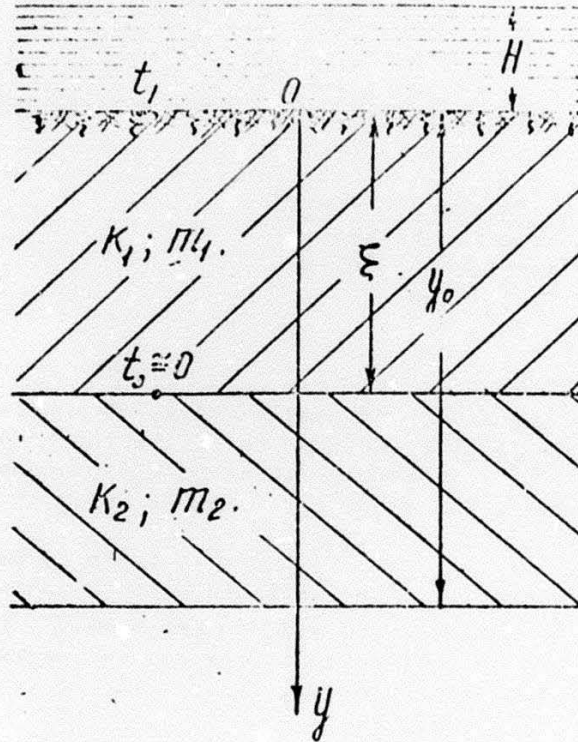


Figure 1.

The boundary conditions of the problem:

$$\begin{aligned} \tau = 0; & \quad y_0 = \bar{y}; \\ \tau = 0; & \quad \frac{dy_0}{d\tau} = 0; \\ y = 0; & \quad t = t_1; \\ y = \bar{y}; & \quad t = t_3 \approx 0; \\ y > \bar{y}; & \quad \frac{dt}{dy} = 0 \end{aligned}$$

The following notations are assumed:

$$a = \frac{\lambda_r}{c\tau}, \text{ m}^2/\text{hr}$$

where  $\lambda_T$  = the heat conductance coefficient of thawed soil, kcal/m.hr $^{\circ}$ C;  
 $C \cdot \gamma$  = heat capacity and volumetric weight of water;  $v$  = velocity of water  
 movement, m/hr; and  $t$  = soil temperature,  $^{\circ}$ C.

A solution to this system of equations for  $y_0$ ;  $v_1 t$ ;  $\xi$  as a function of  
 $\tau$  does not appear possible. However, with an adequate degree of approxi-  
 mation, we can recommend the following procedure for determining  $\xi = \xi(\tau)$ .

The solution to Eq. (1) has the form:

$$t = \frac{t_2 - t_1}{e^{\frac{v\tau}{a}} - 1} (e^{\frac{v\tau}{a}} - 1) + t_1 \quad (5)$$

The solution of Eq. (2) at  $\xi = \text{const}$  has the form [2]:

$$\frac{y_0 - \varphi}{H} - (1 - A) \ln \frac{y_0 + H}{H + \varphi} = \frac{K_2 \tau}{m_2 H}; \quad V = K_2 \frac{H + y_0}{y_0 + \left(\frac{K_2}{K_1} - 1\right) \varphi} \quad (6)$$

where  $A = (K_2/K_1 - 1) \xi/H$ .

If we assume that  $K_1 = K_2$ ;  $m_1 = m_2$ , Eq. (6) will acquire a simpler  
 form [2]:

$$\frac{K_2 \tau}{m_2} = y_0 + H \ln \frac{H}{H + y_0}; \quad V = \frac{K_2}{m_2} \left( \frac{H + y_0}{y_0} \right); \quad (7)$$

Substituting Eq. (5) into the Stefan condition (4), we derive

$$\frac{v \cdot \lambda_T (t_2 - t_1)}{a (e^{\frac{v\tau}{a}} - 1)} e^{\frac{v\tau}{a}} = \sigma W \frac{d\varphi}{d\tau}$$

After integration of this expression, we derive:

$$\frac{v \cdot \lambda_T (t_2 - t_1) \tau}{a \cdot \sigma \cdot W} = \varphi + \frac{a}{v} \left( e^{-\frac{v\tau}{a}} - 1 \right) \quad (8)$$

Assigning the time value  $\tau_1$ , based on Eq. (7), we find the corresponding  
 values for  $y_0$  and  $v_1$ . Then substituting the  $\tau_1$  and  $v_1$ -values into Eq. (8),  
 we find the value.

This method of calculating  $\xi$  will be sufficiently accurate because:

$$\left( \frac{K_2}{K_1} - 1 \right) \varphi \ll y_0; \quad \frac{d\varphi}{d\tau} \ll \frac{dy_0}{d\tau}$$

For the following time value  $\tau_2$ , the values of  $y_{02}$ ,  $v_2$  will be found with Eq.  
 (6), substituting into this formula  $\tau = \tau_2$ ,  $\xi = \xi_1$ . Further, substituting  $\tau =$   
 $\tau_2$  and  $v = v_2$  into (8) or (9), we find the value for  $\xi_2$ , and so forth.  
 At  $|v \xi/a| > 3$ , Eq. (8) can be written:

$$\varphi = \frac{\alpha}{v} + \frac{\lambda_r(t_1 - t_2)\tau \cdot v}{G \cdot W \cdot \alpha}; \mu \quad (9)$$

**Closed Pores.** In a case of closed pores, the necessity originates of calculating the conduct of heat exchange between the filtration flow and the subjacent permafrost soils (Fig. 2).

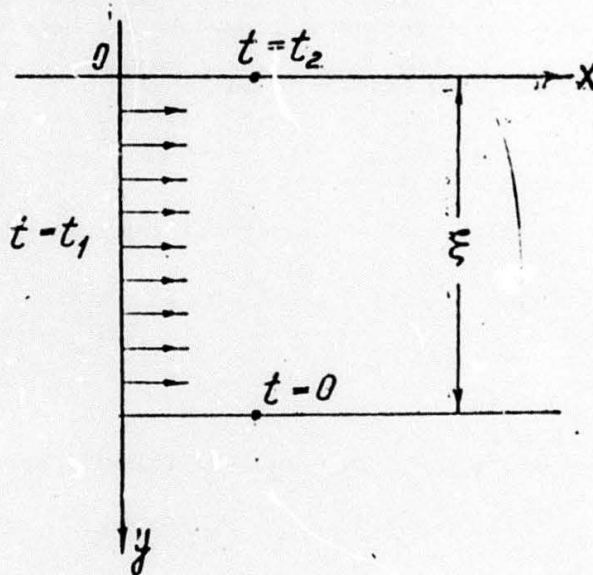


Figure 2.

We place the origin of coordinates on the soil surface. The positive y-axis is directed downward. On the soil surface, temperature  $t = t_2$ . At time moment  $\tau$ , the depth of thawing  $\xi$ ; at this depth,  $t = 0$ . The direction of filtration flow is along the X-axis. The original water temperature at  $X = 0$  equals  $t = t_1$ .

A similar problem has been solved by V. G. Gol'dtman [1] for determining the effective heat conductivity factor  $\lambda_{ef}$  (in the direction of the Y-axis) and also the thawing depth of frozen ground:

$$\left. \begin{aligned} \alpha \frac{\partial^2 t}{\partial y^2} &= \frac{\partial t}{\partial \tau}; & 0 \leq y \leq \xi \\ t(y, 0) &= t_1; \\ \frac{\partial t}{\partial y} \Big|_{y=0} &= 0; \\ t(\xi, \tau) &= 0 \end{aligned} \right\} \quad (10)$$

As is evident from the diagram (10), in a calculation of the conductive heat exchange, we incorrectly assigned the boundary condition at the upper surface of filtration flow--  $\partial t / \partial y|_{y=0} = 0$ . In reality, on the upper surface of filtration flow, we should not assume the absence of a heat flow but a quite definite temperature value  $t = t_2$  equalling the soil temperature under natural

conditions at the occurrence depth of the filtration flow's upper surface (in Fig. 2,  $y = 0$ ). In addition, we did not take into account the conductive transport of heat along the filtration flow. A more precise mathematical formulation of the stipulated problem has the form:

$$\left. \begin{aligned} \frac{\partial^2 t}{\partial y^2} - \frac{v}{a} \frac{\partial t}{\partial x} &= 0; \quad 0 \leq y \leq \varphi \\ y=0, t=t_2; \\ y=\varphi, t=0; \\ x=0; 0 < y < \varphi; t=t_1 \end{aligned} \right\} \quad (11)$$

where  $v$  = velocity of water movement, m/hr;

$$a = \frac{\lambda_{ef}}{c \gamma}, \text{ m}^2/\text{hr}$$

$\lambda_{ef}$  = effective heat conductivity factor in direction of  $y$ -axis;  
 $c$  and  $\gamma$  = heat capacity and volumetric weight of water, respectively.

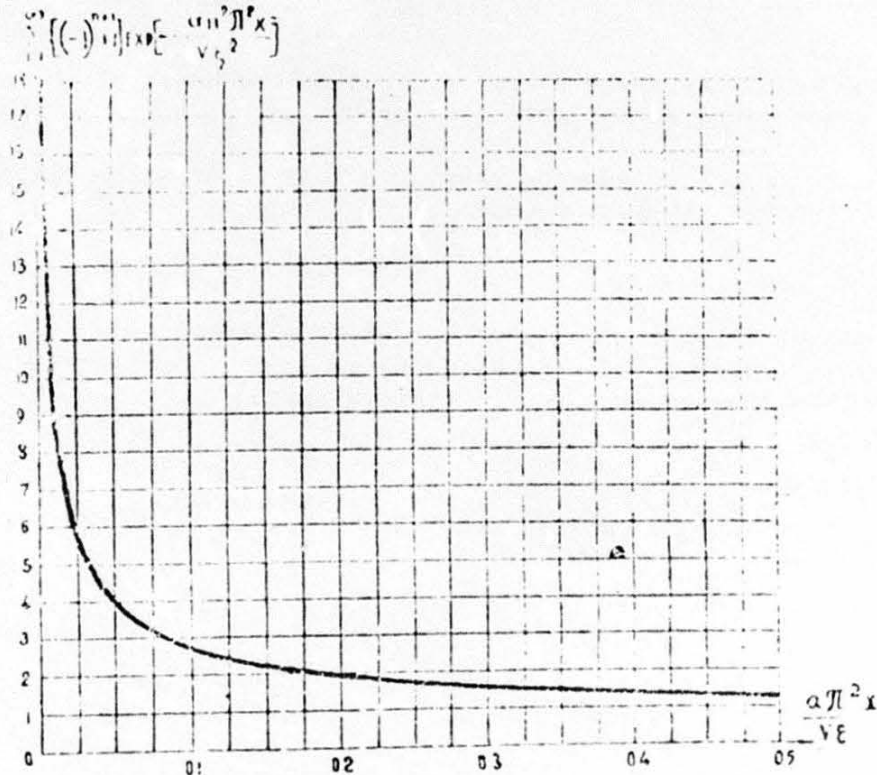


Figure 3.

A solution to the problem has the form:

$$t = t_2 \left(1 - \frac{y}{\varphi}\right) + \frac{2t_1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1} + 1}{n} \sin\left(\frac{n\pi y}{\varphi}\right) \exp\left[-\frac{\alpha n^2 t_1 y}{V\varphi^2}\right] \quad (12)$$

Let us determine the heat flow at the lower boundary  $y = \xi$ :

$$q|_{y=\varphi} = \frac{\lambda_0 + t_1}{\varphi} + \frac{2\lambda_0 t_1}{\varphi} \sum_{n=1}^{\infty} [(-1)^{n+1} + 1] \exp\left[-\frac{\alpha n^2 t_1 \xi}{V\varphi^2}\right] \quad (13)$$

The value of the sum:

$$\sum_{n=1}^{\infty} [(-1)^{n+1} + 1] \exp\left[-\frac{\alpha n^2 t_1 \xi}{V\varphi^2}\right]$$

where

$$\kappa = \frac{\alpha t_1^2}{V\varphi}$$

is represented in Fig. 3. As is obvious from the figure, the value for the sum can be represented in the form:

$$\sum_{n=1}^{\infty} [(-1)^{n+1} + 1] \exp\left[-\frac{\alpha n^2 t_1 \xi}{V\varphi^2}\right] = A \exp\left[-\frac{B\alpha t_1^2 \xi}{V\varphi^2}\right] + C; \quad (14)$$

where the constants A, B and C are selected on the basis of the curve (Fig. 3) in the known range of variations in criterion  $\gamma = \frac{\alpha t_1^2 \xi}{V\varphi^2}$ . If the water is

filtered by the complete section through a soil layer which has thawed, the velocity of water's movement equals:

$$V = \frac{Q}{L\gamma\varphi} \quad \text{at } t_2 = t_n \quad (15)$$

where Q = output of water, kg/hour; L = width of flow, m; and  $t_n$  = soil temperature on surface, °C. If the upper surface of filtration flow occurs at depth  $l$  from the day surface, it follows that:

$$t_2 = \frac{t_n(\varphi - l)}{\varphi} \quad ; \quad V = \frac{Q}{L\gamma(\varphi - l)} \quad (16)$$

The effective heat conductivity factor according to V. G. Gol'dtman constitutes a function of velocity:

$$\lambda_{3, \varphi} = \lambda + D \frac{Q}{L\gamma\varphi} \quad (17)$$

where  $\lambda$  = heat conductivity factor of water-saturated thawed coarse-grained soil; D = coefficient of heat dispersion. According to V. G. Gol'dtman, for the Magadan region, D = 7. Let us analyze the case of a complete section. Substituting (14), (15) and (17) into (13), we derive:

$$q|_{y=\varphi} = \left(\lambda + \frac{DQ}{L\gamma\varphi}\right) \frac{1}{\varphi} \left\{ t_2 + 2t_1 \left[ A \exp\left(-\frac{B\alpha t_1^2 \xi L\gamma}{Q\varphi}\right) + C \right] \right\} \quad (18)$$

Let us formulate the Stefan equation at the boundary of phase transitions:

$$\left(\lambda + \frac{DQ}{Lr\varphi}\right) \frac{1}{\varphi} \left[ t_2 + 2t_1 \left( A \exp \left[ -\frac{B\alpha\pi^2 x L r}{Q\varphi} \right] + C \right) \right] = Gw \frac{d\varphi}{d\tau} \quad (19)$$

where  $G$  = heat of phase transitions kcal/k<sup>2</sup>; and  $W$  = volumetric iciness of frozen soil, k<sup>2</sup>/m<sup>3</sup>. Dividing the variables, we derive:

$$\frac{\tau}{Gw} = \int_0^{\varphi} \frac{\varphi \cdot d\varphi}{\left(\lambda + \frac{DQ}{Lr\varphi}\right) \left[ t_2 + 2t_1 \left( A \exp \left[ -\frac{B\alpha\pi^2 x L r}{Q\varphi} \right] + C \right) \right]} \quad (20)$$

Substituting the "a" value into (20), we get:

$$\frac{\tau}{Gw} = \int_0^{\varphi} \frac{\varphi \cdot d\varphi}{\left(\lambda + \frac{DQ}{Lr\varphi}\right) \left[ t_2 + 2t_1 \left( A \exp \left[ -\frac{B\pi^2 x (\lambda L r \varphi + DQ)}{Q\varphi^2 r c} \right] + C \right) \right]} \quad (20)$$

For the low  $x$ -values, the function  $\xi = \xi(x_1, \tau)$  can be determined by a numerical integration of Eq. (20). For the high values, Eq. (20) will assume the form:

$$\frac{t_2 \tau}{Gw} = \int_0^{\varphi} \frac{\varphi \cdot d\varphi}{\lambda + \frac{DQ}{Lr\varphi}} \quad (21)$$

After integration and transformations, we obtain the following expression for estimating the thawing depth at high  $X$ -values:

$$\frac{\lambda t_2 \tau}{Gw} = \frac{\varphi^2}{2} - \frac{DQ}{\lambda L r} \varphi + \left( \frac{DQ}{\lambda L r} \right)^2 \ln \frac{DQ + \lambda L r \varphi}{DQ} \quad (22)$$

The system (10) can be utilized for estimating the thawing rate of two lateral sides of a vertical talik crevice in which a filtration flow will move, or the base and top of a water-bearing layer bounded by frozen layers. The mathematical formulation of such a problem has the form:

$$\left. \begin{aligned} \lambda_{\phi} \frac{\partial^2 t}{\partial y^2} + \lambda \frac{\partial^2 t}{\partial x^2} - C \gamma^v \frac{\partial t}{\partial x} &= 0 \\ x > 0; y = \pm \varphi; t = 0; \\ x = 0; y < \pm \varphi; t = t_1 \end{aligned} \right\} \quad (23)$$

Moreover, in the given case  $\xi$  is half of the thawed layer,  $M$ . We match the origin of coordinates with the middle of the thawed soil layer. The solution of problem (23) has the form:

$$t = t_1 \sum_{n=1}^{\infty} \frac{2}{\xi} (-1)^{n-1} \cos \left( \xi_n \frac{y}{\varphi} \right) \exp \left\{ \left[ \frac{\lambda_{\phi} + v}{2\lambda\alpha} \right]^2 + \frac{\lambda_{\phi}}{\lambda \xi^2} \xi_n^2 \right\} x \quad (24)$$

where  $\xi_n = (2n-1) \pi / 2$ ;  $n = 1, 2, 3, \dots$ . We determine the heat flow at the boundary of phase transitions:

$$q \Big|_{y=\varphi} = 2\lambda_{\phi} t_1 \frac{1}{\varphi} \sum_{n=1}^{\infty} \exp \left\{ \left[ \frac{\lambda_{\phi} + v}{2\lambda\alpha} - \sqrt{\left( \frac{\lambda_{\phi} + v}{2\lambda\alpha} \right)^2 + \frac{\lambda_{\phi} (2n-1)^2 \pi^2}{4\lambda \cdot \xi^2}} \right] x \right\} \quad (25)$$

Now let us solve a similar problem under the condition that we disregard the conductive heat transfer in the direction of the main filtration flow ( $X$ -axis) as compared with the convective flow in this same direction.

Then

$$\begin{aligned} \frac{\partial^2 t}{\partial y^2} - \frac{v}{a} \frac{\partial t}{\partial x} &= 0; \\ x > 0; y &= \pm \varphi; t = 0; \\ x = 0; y < \pm \varphi, t &= t_1 \end{aligned}$$

where  $2\xi$  = depth of thawed soil layer, m. The solution to the problem has the form:

$$t = t_1 \sum_{n=1}^{\infty} \frac{4}{(2n-1)\pi} (-1)^{n-1} \cos \left[ \frac{(2n-1)\pi y}{2\varphi} \right] \exp \left[ -\frac{a\xi^2 x}{v\varphi^2} \right] \quad (26)$$

Let us determine the heat flow at the boundary of the phase transitions

$$q \Big|_{y=\varphi} = 2\lambda_{\phi} t_1 \frac{1}{\varphi} \sum_{n=1}^{\infty} \exp \left[ -\frac{a(2n-1)^2 \pi^2 x}{4v\varphi^2} \right] \quad (27)$$

Let us formulate the Stefan condition for estimating the thawing rate of frozen grounds:

$$\frac{2\lambda_{\phi} t_1}{\varphi} \sum_{n=1}^{\infty} \exp \left[ -\frac{a(2n-1)^2 \pi^2 x}{4v\varphi^2} \right] = \sigma W \frac{d\varphi}{dt} \quad (23)$$

The velocity of water movement:

$$V = \frac{Q}{\rho L \gamma \varphi}; \quad (29)$$

By analogy with (17), the coefficient of effective heat conductivity  $\lambda_{ef}$  has the form:

$$\lambda_{\phi} = \lambda + D \frac{Q}{2L\gamma\varphi}; \quad (30)$$

Coefficient "a"

$$a = \frac{2\lambda L \gamma \varphi + DQ}{2 \cdot L \cdot \gamma^2 \varphi \cdot c} \quad (31)$$

Substituting (29)-(31) into (28), after transformation, we derive:

$$\frac{2t_1 \pi}{\sigma W} = \int_{\varphi_0}^{\varphi} \frac{\varphi \cdot d\varphi}{\lambda_{\phi} \sum_{n=1}^{\infty} \exp \left[ -\frac{(2n-1)^2 \pi^2 (2\lambda L \gamma \varphi + DQ)}{4a\gamma c \varphi^2} x \right]} \quad (32)$$

where  $2\xi_0$  = initial soil depth, m. The estimation of the depth of thawed soil  $2\xi$  can be conducted by a numerical integration of Eq. (32).

On a calculation of acicular hydro-thawing. The mathematical formulation of the problem has the form:

$$\begin{aligned} \frac{\partial^2 t}{\partial z^2} + \frac{1}{z} \frac{\partial t}{\partial z} + \frac{\lambda}{\lambda_{\phi}} \frac{\partial^2 t}{\partial x^2} - \frac{v\gamma}{\lambda_{\phi}} \frac{\partial t}{\partial x} &= 0; \\ x=0; z < \varphi; t &= t_1; \\ 0 < x < h; z = \varphi; t &= 0 \end{aligned} \quad (33)$$

The following notations have been adopted:  $h$  = depth of needle's submergence, m;  $t_1$  = initial water temperature;  $v$  = velocity of water flow, m/hr;  $a = \lambda_{ef}/c\gamma$ ;  $c$  and  $\gamma$  = heat capacity and weight of water by volume;  $r$  = radius, m;  $\xi$  = radius of halo of soil's thawing around needle, m;  $\lambda_{ef}$  = effective coefficient of heat conductivity, kcal/m,hr, °C; and  $\lambda$  = heat conductivity coefficient of water-saturated thawed ground, kcal/m,hr, °C.

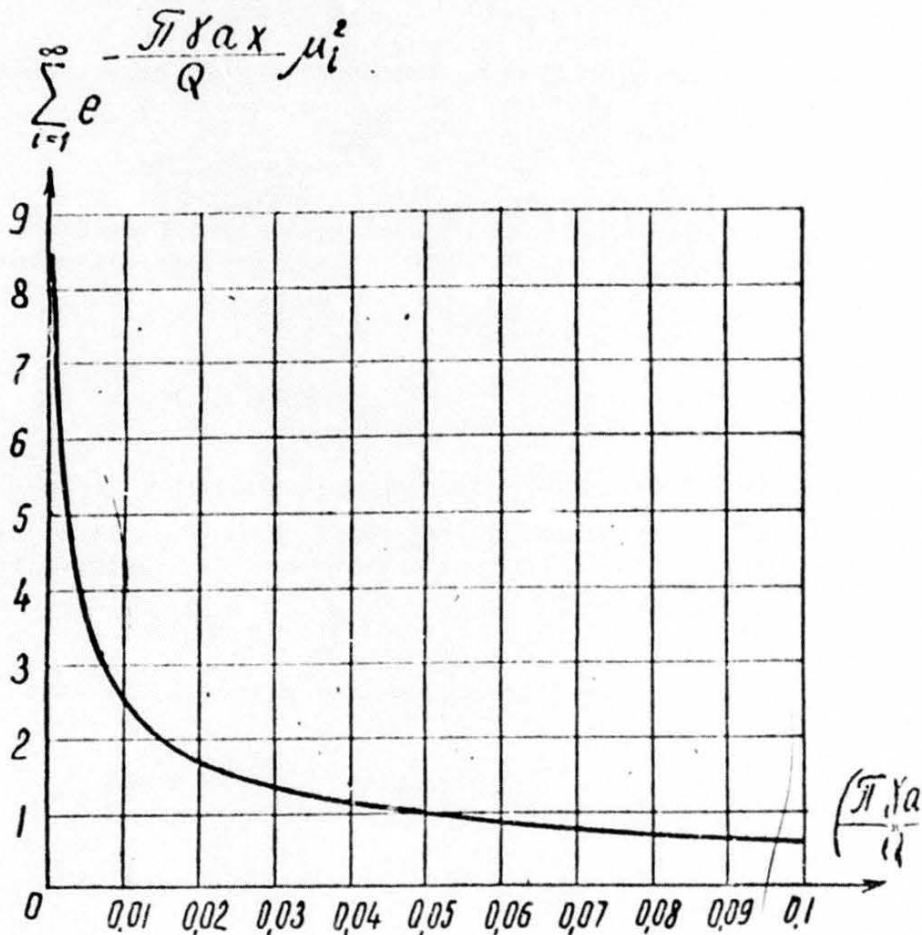


Figure 4.

The solution of problem (33) has the form:

$$t = t_1 \sum_{i=1}^{\infty} \frac{2}{\mu_i J_1(\mu_i)} J_0\left(\mu_i \frac{r}{\xi}\right) \exp\left\{-\left[\frac{v \lambda_{ef} \lambda_{ef}}{2a\lambda} - \sqrt{\left(\frac{v \lambda_{ef} \lambda_{ef}}{2a\lambda}\right)^2 + \frac{\mu_i^2 \lambda_{ef}}{\lambda}}\right] \frac{y}{\xi}\right\} \quad (34)$$

where  $\mu_i$  = roots of equation  $J_0(h) = 0$ .

$$\begin{array}{llll} \mu_1 = 2.405; & \mu_2 = 8.094; & \mu_3 = 11.8; & \mu_4 = 14.9; \\ \mu_5 = 18.1; & \mu_6 = 21.2; & \mu_7 = 24.35; & \mu_8 = 27.52. \end{array}$$

$J_0(\mu)$  and  $J_1(\mu)$  = the Bessel functions of the first type, of zeroth and first orders, respectively. We determine velocity  $v$  at assigned output  $Q$  kg/hr:

$$v = Q / \pi \xi^2 \gamma \quad (35)$$

We then find:

$$\frac{\lambda_{\text{eff}}}{\lambda} = 1 + \frac{DQ}{\lambda \gamma \xi^2}; \quad \alpha = \frac{\lambda \gamma \xi^2 + DQ}{s \cdot \gamma^2 \cdot \xi^2 C} \quad (36)$$

We find the value for the heat flow to the boundary of phase transitions (with consideration of Eqs. (35) and (36))

$$q|_{z=\xi} = \frac{2\lambda_{\text{eff}} \xi}{\gamma} \sum_{i=1}^{\infty} \exp\left\{ \left[ \frac{\alpha}{2\gamma \xi \alpha \gamma} \left( 1 + \frac{DQ}{\lambda \gamma \xi^2} \right) - \sqrt{\left[ \frac{\alpha}{2\gamma \xi \alpha \gamma} \left( 1 + \frac{DQ}{\lambda \gamma \xi^2} \right) \right]^2 + \mu_i \left( 1 + \frac{DQ}{\lambda \gamma \xi^2} \right)} \right] \frac{x}{\xi} \right\} \quad (37)$$

Let us examine a case when the value of conductive heat flow along the X-axis can be disregarded. Then Eqs. (34) and (37) will acquire a simpler form:

$$t = t_1 \sum_{i=1}^{\infty} \left[ \frac{2}{\mu_i J_1(\mu_i)} \right]_0 (\mu_i \frac{2}{\xi}) \exp\left[ -\frac{\alpha \mu_i^2 x}{\gamma \xi^2} \right]; \quad (38)$$

$$q|_{z=\xi} = \frac{2t_1}{\xi} \left( \lambda + \frac{DQ}{\gamma \xi^2} \right) \sum_{i=1}^{\infty} \exp\left[ -\frac{\pi \alpha \mu_i^2 x}{Q} \right]; \quad (39)$$

The value of the sum  $\sum_{i=1}^{\infty} \exp\left[ -\frac{\pi \alpha \mu_i^2 x}{Q} \right]$  depending on parameter  $\pi \gamma \alpha x / Q$  has been presented in Fig. 4. As is apparent from the figure, in a fixed range of variation in parameter  $\pi \gamma \alpha x / Q$ , the sum's value can be presented in the form:

$$\sum_{i=1}^{\infty} \exp\left[ -\frac{\pi \alpha \mu_i^2 x}{Q} \right] = A \exp\left[ -\frac{B \pi \gamma \alpha x}{Q} \right] + C, \quad (40)$$

where A, B and C = parameters of the curve (see Fig. 4). Substituting (36) and (40) into (39), we obtain:

$$q = \frac{2t_1}{\xi} \left( \lambda + \frac{DQ}{\gamma \xi^2} \right) \left\{ A \exp\left[ -\frac{B(\lambda \gamma \xi^2 + DQ)}{Q \gamma C \xi^2} x \right] + C \right\} \quad (41)$$

Let us write the Stefan equation for estimating the thawing halo of frozen ground around a needle  $\xi$ ,  $(x_1, T)$ :

$$\frac{2t_1 \tau}{\gamma} \left( \lambda + \frac{DQ}{\gamma \xi^2} \right) \left\{ A \exp\left[ -\frac{B(\lambda \gamma \xi^2 + DQ)}{Q \gamma C \xi^2} x \right] + C \right\} = G W \frac{d\xi}{dt} \quad (42)$$

Separating the variables, we get the following equation:

$$\frac{2t_1 \tau}{G W} \int_{z_{ur}}^{\xi} \frac{\xi \cdot d\xi}{\left( \lambda + \frac{DQ}{\gamma \xi^2} \right) \left\{ A \exp\left[ -\frac{B(\lambda \gamma \xi^2 + DQ)}{Q \gamma C \xi^2} x \right] + C \right\}} \quad (43)$$

where  $z_{ur}$  = needle's radius, m. If we assume  $a = a_{cp}$ , i.e. adopt the average value of the conventional temperature conductivity factor for the entire period

of hydro-thawing, Eq. (43) will assume the form:

$$\frac{2t_1}{\sigma_w} \left[ A e^{-\frac{B \gamma \alpha_{cp} x}{a}} + C \right] \tau = \int_{z_{ur}}^{\varphi} \frac{\varphi \cdot d\varphi}{\lambda + \frac{DQ}{\gamma \pi \varphi^2}} \quad (44)$$

Integrating Eq. (44), we get the following expression for determining the thawing halo of the soil around a needle:

$$\frac{4\lambda t_1}{\sigma_w} \left[ A e^{-\frac{B \gamma \alpha_{cp} x}{a}} + C \right] \tau = \varphi^2 - z_{ur}^2 - \frac{DQ}{\lambda \pi \gamma} \ln \frac{\lambda \gamma \pi \varphi^2 + DQ}{\lambda \gamma \pi z_{ur}^2 + DQ} \quad (45)$$

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