

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified.)

1. ORIGINATING ACTIVITY (Corporate author)

Aerospace Medical Research Laboratory
Aerospace Medical Div, Air Force Systems Command
Wright-Patterson Air Force Base, Ohio 45433

2. REPORT SECURITY CLASSIFICATION

Unclassified

3. GROUP

N/A

4. REPORT TITLE

NONLINEAR LUMPED PARAMETER MATHEMATICAL MODEL OF DYNAMIC RESPONSE
OF THE HUMAN BODY.

5. DESCRIPTIVE NOTES (Type of report and inclusive dates)

6. AUTHOR(S) (First name, middle initial, last name)

Gordon R. Hopkins.

7. REPORT DATE

December 1971

7a. TOTAL NO. OF PAGES

28 23

7b. NO. OF REFS

9

8. CONTRACT OR GRANT NO.

PROJECT NO. 7231

9. ORIGINATOR'S REPORT NUMBER(S)

AMRL-TR-71-29

Paper No. 25

10. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

11. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited

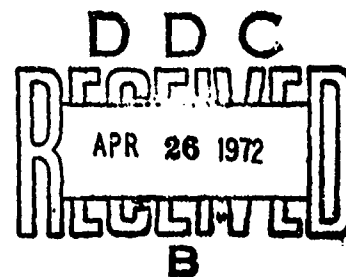
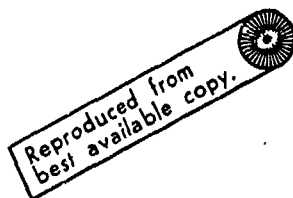
12. SUPPLEMENTARY NOTES

13. SPONSORING MILITARY ACTIVITY

Aerospace Medical Research Laboratory
Aerospace Medical Div, Air Force Systems
Command, Wright-Patterson AFB, OH 45433

14. ABSTRACT

The Symposium on Biodynamics Models and Their Applications took place in Dayton, Ohio, on 26-28 October 1970 under the sponsorship of the National Academy of Sciences - National Research Council, Committee on Hearing, Bioacoustics, and Biomechanics; the National Aeronautics and Space Administration; and the Aerospace Medical Research Laboratory, Aerospace Medical Division, United States Air Force. Most technical areas discussed included application of biodynamic models for the establishment of environmental exposure limits, models for interpretation of animal, dummy, and operational experiments, mechanical characterization of living tissues and isolated organs, models to describe man's response to impact, blast, and acoustic energy, and performance in biodynamic environments.



DD FORM 1473

Reproduced by
NATIONAL TECHNICAL
INFORMATION SERVICE
Springfield, Va. 22151

Security Classification

PAPER NO. 25

NONLINEAR LUMPED PARAMETER MATHEMATICAL MODEL
OF DYNAMIC RESPONSE OF THE HUMAN BODY

Gordon R. Hopkins

West Virginia University

ABSTRACT

Two nonlinear models of man's dynamic response to low frequency vibration are discussed. The first model uses linear spring and damper elements but accounts for the nonlinear geometry of visceral mass motion. This model adequately reproduces both the input mechanical impedance and vibration transmission characteristics for a seated human subject.

The second model includes the nonlinear effects of the lungs. The influence of this nonlinearity on the dynamic response is discussed and compared to experimental results from tests on animals.

INTRODUCTION

Over the past two decades considerable attention has been paid to the effects of vibration and impact on the human body. As humans are exposed to the implements of modern technology these mechanical stimuli reach magnitudes that can cause physiological damage ranging from mild discomfort to death.

The inaugural research effort in the study of man's dynamic response was to establish human tolerance limits to vibration and impact. Once

these were established, the attention of investigators turned toward the more perplexing problem of understanding the injury mechanisms that set the tolerance.

Using human subjects as test specimens to gather information about the internal dynamics of the body has not always been a gratifying experience. Yet, animal substitutes in most cases have proved equally disappointing. The problems associated with trying to glean information about the dynamics of the internal organs and structural components of the human body is complicated by the necessity of having to gather data without damage to the subject. This limits the types of instruments that may be used as well as severity of the test environment. In effect, investigators have been gathering data about internal dynamics of the human body by sensing only their external effects.

Useful data have been gathered in this manner however. For instance, most of our understanding of the body's dynamic response to low frequency vibration has been based on measurements such as driving point mechanical impedance, external strain, vibration transmission, and cineradiographs.

In order to make the best use of these data, mathematical models have often been employed in the effort to expand our understanding of the human body as a dynamical system.

This report discusses one such model. A nonlinear lumped parameter model that explains phenomena observed during low frequency vibration of a seated man, such as the second and third spinal resonances.

BACKGROUND INFORMATION

The rationale of the model is best discussed by first briefly reviewing the experimental data on which it is based.

Mechanical impedance has proved to be a valuable and well used tool in studying the vibration response of biological systems. With its use as a parameter, the dynamic response of a human subject to vibration can be studied as a "black box" much as one would an unknown electrical circuit with input electrical impedance. Although it has limitations, this approach yields much information about man's dynamic response to sinusoidal vibrations by showing system resonances.

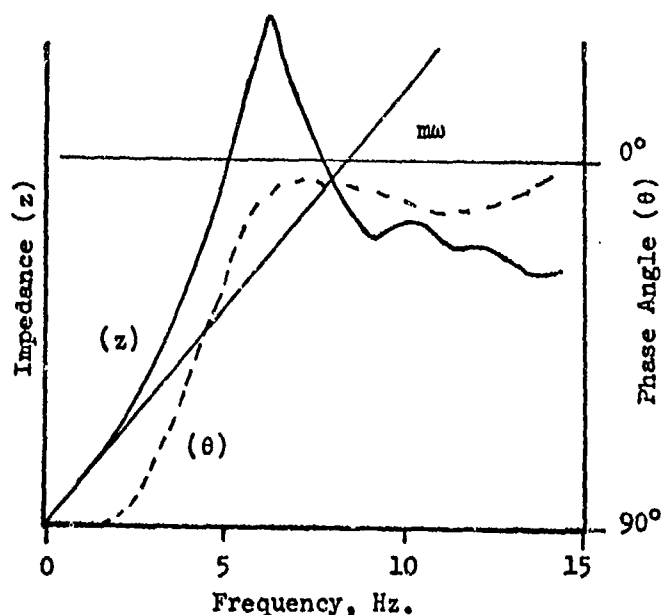


Fig. 1 Mechanical Impedance and Phase Angle for Seated Subject⁽¹⁾

Figure 1 based on the work of Coermann et al.^{1,2} illustrates the mechanical impedance of a typical subject to low frequency sinusoidal vibration. The predominant features of this curve are the peaks at 5, 11, and 14 Hz. These peaks suggest points of resonance. As von Gierke³ suggests, the first peak is due to the abdominal resonance, and the second and third due to some action of the upper torso on the vertebral column.

A second, and equally useful tool used in vibration analysis is vibration transmission, or transmissibility. Again Coermann¹ shows us, and is substantiated by the work of Pradko⁴ who used random vibration instead of sinusoidal, that the vibration response of a seated human subject can be characterized by transmissibility. Figure 2 shows this result.

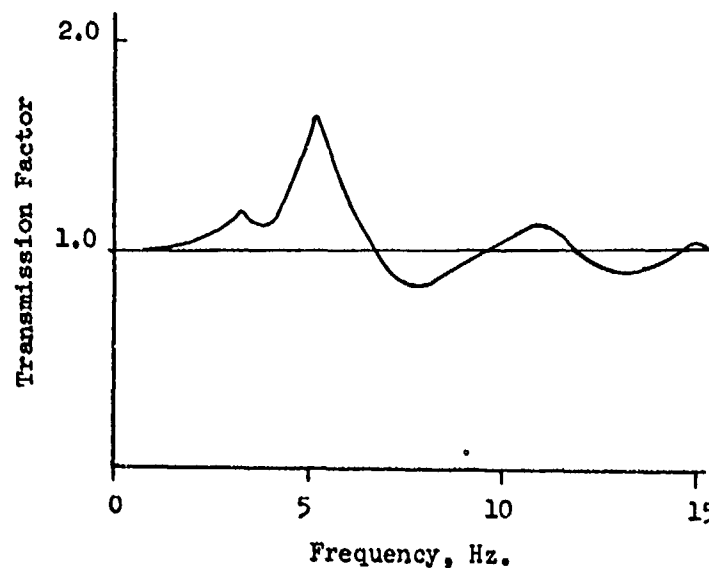


Fig. 2 The Transmission of Vibrations from the Seat to the Head of a Seated Human Subject⁽¹⁾

7
The transmission factor for a seated subject shows three peaks nominally at 5, 11, and 14 Hz. As with the mechanical impedance, it is hypothesized that these peaks are the result of resonant systems in the body. One should note that the peak corresponding to the abdominal resonance transmits the largest amplitude vibrations to the head.

Another noteworthy set of parameters that characterize the body's dynamic response to low frequency vibration are the strain measurements made by Clark et al.⁵ and shown in Figure 3. It is observed from these measurements that the first peak for the upper and lower abdominal strain occur at approximately 1 Hz apart. The implication here is that the visceral mass works against a different spring when it moves toward the head than it does when it moves toward the pelvis.

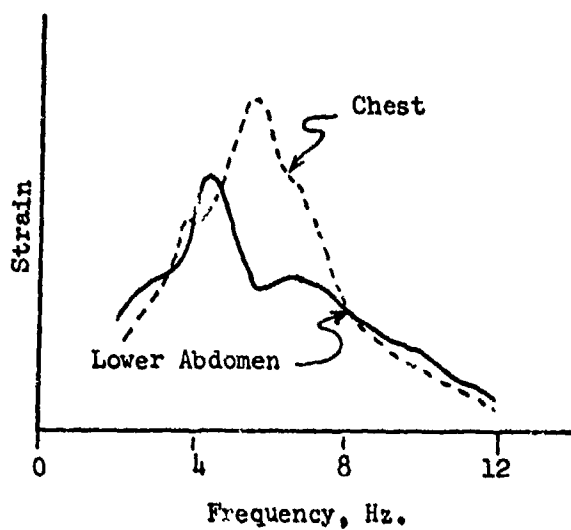


Fig. 3 Strain for Seated Subject During Low Frequency Vibration⁽⁵⁾

The final set of measurements to be considered are those made by White et al.⁶ of colon pressure during low frequency vibration. As anticipated, these measurements show the effect of the abdominal resonance in agreement with the lower abdominal strain shown in Figure 3. If the second and third peaks of Figures 1 and 2 are due to vertebral column resonances as hypothesized, then the effect on colon pressure would be minimal. As can be seen in Figure 4, this is precisely the case.

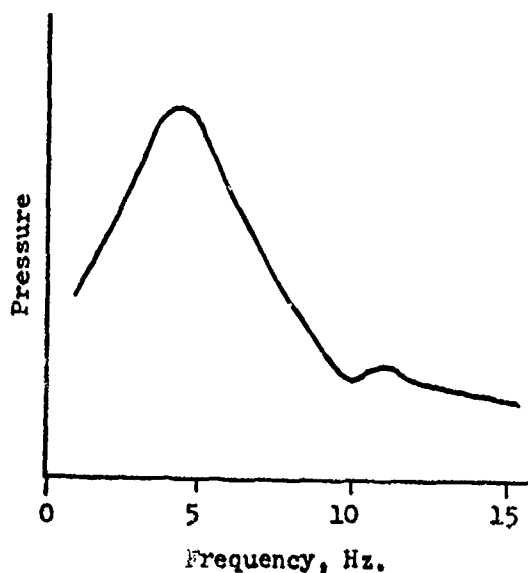


Fig. 4 The "Mean" Colon Pressure During Low Frequency Vibration for a Seated Subject⁽⁶⁾

In summary, these experimental measurements require a model to:

1. Have a similar impedance and phase versus frequency relationship as that for a human subject.
2. Demonstrate the same vibration transmission characteristics as does a human subject.

- 9
3. Corroborate the organ motions as inferred from the measurements of strain and colon pressure.

DEFINITION OF MODEL

It has been demonstrated by Hopkins⁷, Suggs², and others by the construction of mathematical, electric analog, and mechanical models that an uncoupled two degree of freedom system as shown schematically in Figure 5 can adequately simulate the input mechanical impedance of the human body over the 0-15 Hz range. For purposes where only the input impedance is necessary, such as coupling with other machinery for their evaluation, this type of model is sufficient. However, for inquiring into the internal dynamics of the human body, such a model is not satisfactory. It lacks sufficient anatomical similarity to predict injury.

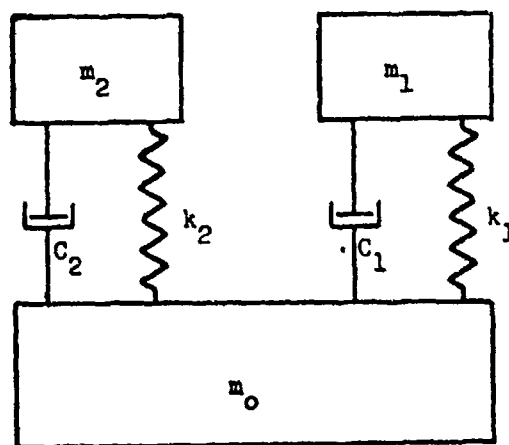


Fig. 5 Uncoupled Two Degree of Freedom Model

There is no way to use this type of model to study the transmission of vibration to various parts of the body or to corroborate such data as transmissibility, strain and internal pressures. This deficiency leads one to consider a coupled mass linear model, such as shown in Figure 6, that could accommodate these considerations.

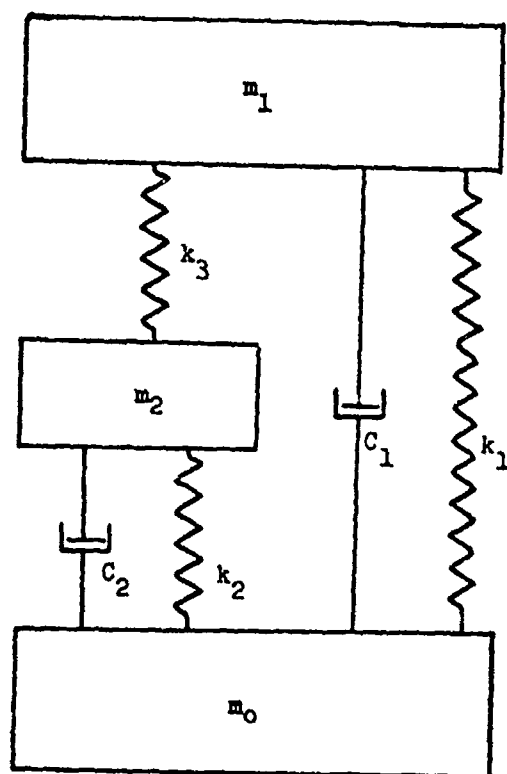


Fig. 6 Linear Coupled Mass Model

The linear coupled mass offers enough geometric similarity to the human body to be able to relate components to their anatomical counterparts. For instance, k_1 and C_1 could represent the vertebral column, m_2 the upper torso mass, m_3 the visceral mass, m_1 the lower torso mass and so on. However, this model does not adequately simulate the input impedance.

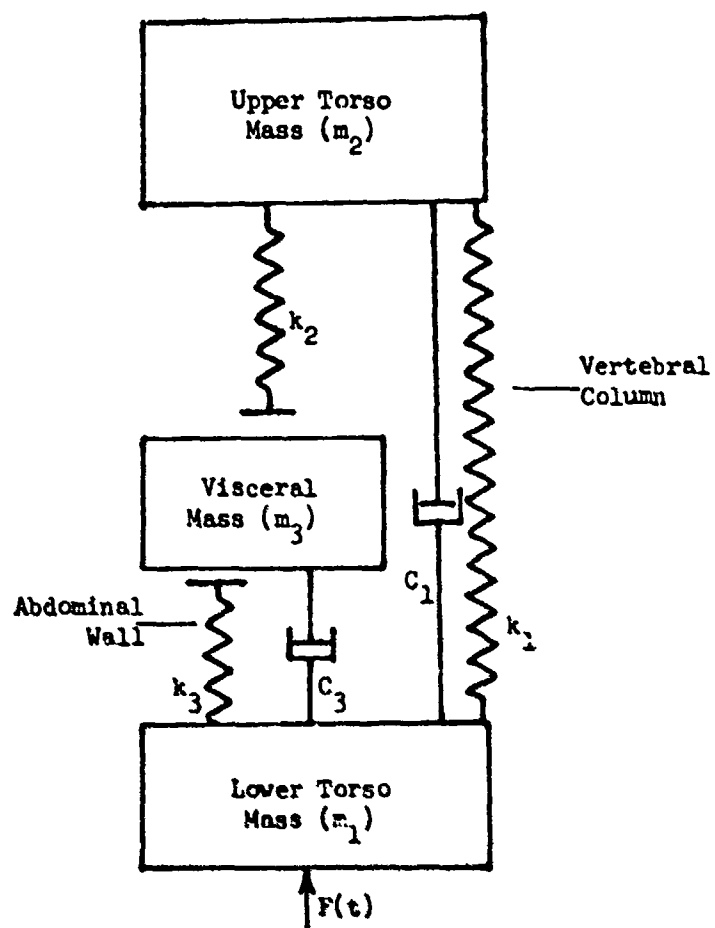


Fig. 7 Nonlinear Geometry Model of Human Body

If one examines the anatomy of the human body in more detail, it may be observed that the visceral organs are not tethered but are supported in the abdominal cavity by the abdominal muscle wall, the pelvis, and the diaphragm. When the visceral mass moves during vibration it exhibits a nonlinear geometry. As the visceral mass moves upward toward the head, it compresses the lungs but does not put tension on the abdominal wall or pelvis since it is not tethered to either. On the other hand, as the mass moves toward the pelvis, it does not put tension on the diaphragm and in turn the lungs.

The simplest model that can accommodate this nonlinear geometry is one with linear elements constructed such that the springs are not rigidly attached to the mass that represents the visceral mass.

This model is shown schematically in Figure 7 where the linear spring, k_1 , and damper, C_1 , represent the vertebral column, k_2 , the abdominal wall, k_3 , the lungs and C_2 , the frictional damping on the internal movement of the organs.

DYNAMIC BEHAVIOR OF NONLINEAR GEOMETRIC MODEL

A mathematical model based on the schematic shown in Figure 7 has been formulated and interpreted as a computer model using IBM's Continuous System Modeling Program. The dynamic behavior of this model is shown in Figures 8, 9, and 10.

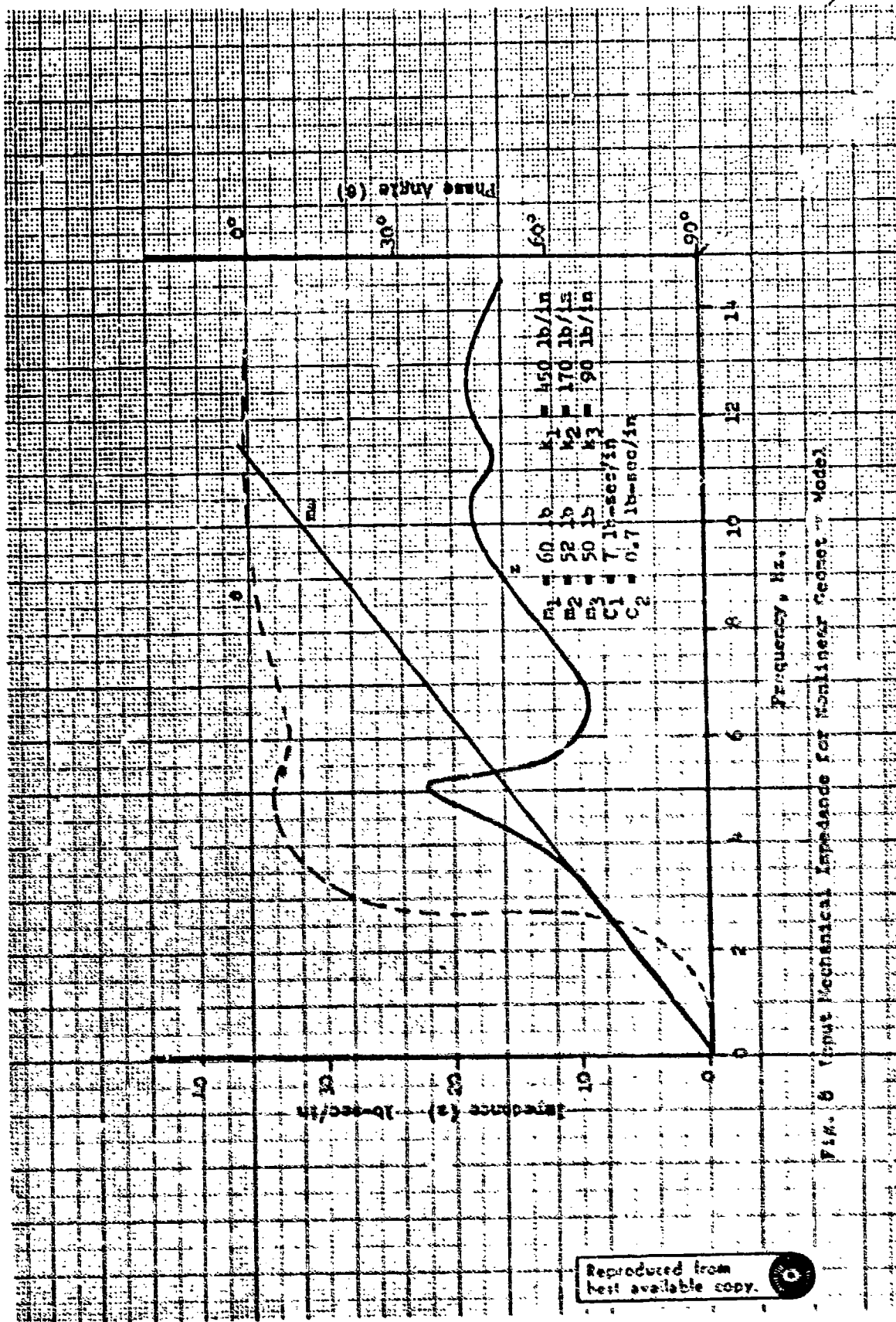


FIG. 6 Input Mechanical Impedance for Nonlinear System Model

$m_1 = 60 \text{ lb}$
 $m_2 = 52 \text{ lb}$
 $m_3 = 50 \text{ lb}$
 $k_1 = 450 \text{ lb/in}$
 $k_2 = 170 \text{ lb/in}$
 $k_3 = 90 \text{ lb/in}$
 $c_1 = 7 \text{ (lb-sec)/in}$
 $c_2 = 0.7 \text{ lb-sec/in}$

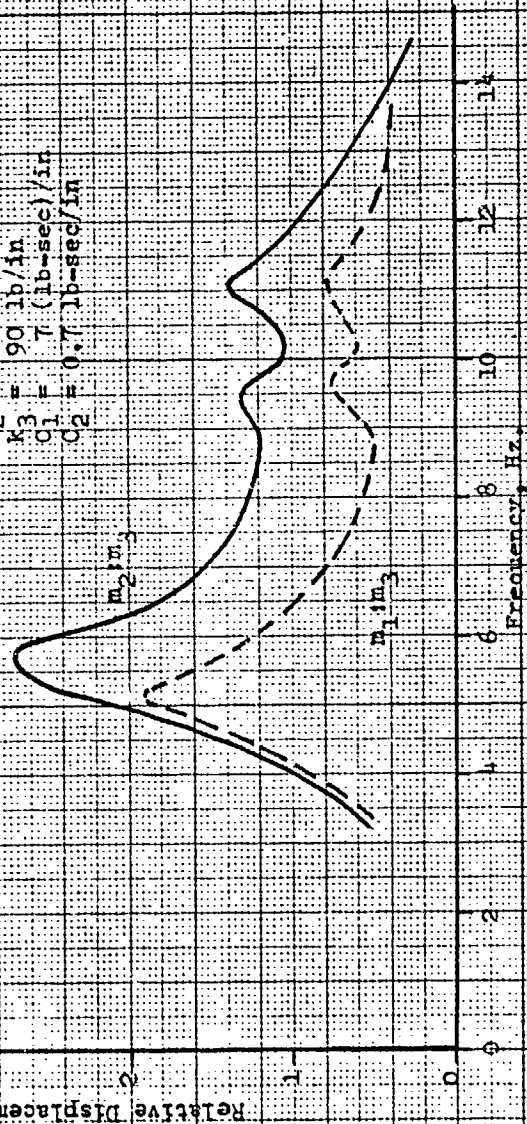


Fig. 9 Relative Displacements of Masses for Nonlinear Geometry Model

DISCUSSION OF NONLINEAR GEOMETRY MODEL

Comparison of Figures 1 and 8 shows that this two degree of freedom model with nonlinear geometry adequately reproduces the mechanical impedance and phase angle characteristics of the human body; even to displaying the second and third resonant peaks. Comparison of Figures 2 and 9 shows that the model also demonstrates the same general vibration transmission characteristics as the human body.

If one assumes that the displacements of the visceral mass can be related to the external strains on the chest and abdominal wall, then Figure 10 agrees with the strain measurements presented in Figure 3.

It is concluded that the model does satisfactorily demonstrate the same dynamic response to low frequency vibration as does a seated human for small amplitude vibrations.

NONLINEAR MODEL

Since all the data on which the above model is based was gathered from forced sinusoidal vibration with low amplitude accelerations, from 1/4 to 1/2 g., its use for the study of vibration with higher amplitude acceleration or impact is questionable. Experience with the nonlinear geometry model has shown that its dynamic response is not dependent on the amplitude of the forced vibration.

Krause and Lange⁹ have shown with experiments on pigs that mechanical impedance is nonlinear with respect to the acceleration amplitude of the forced vibration. Their results are reproduced in Figure 12.

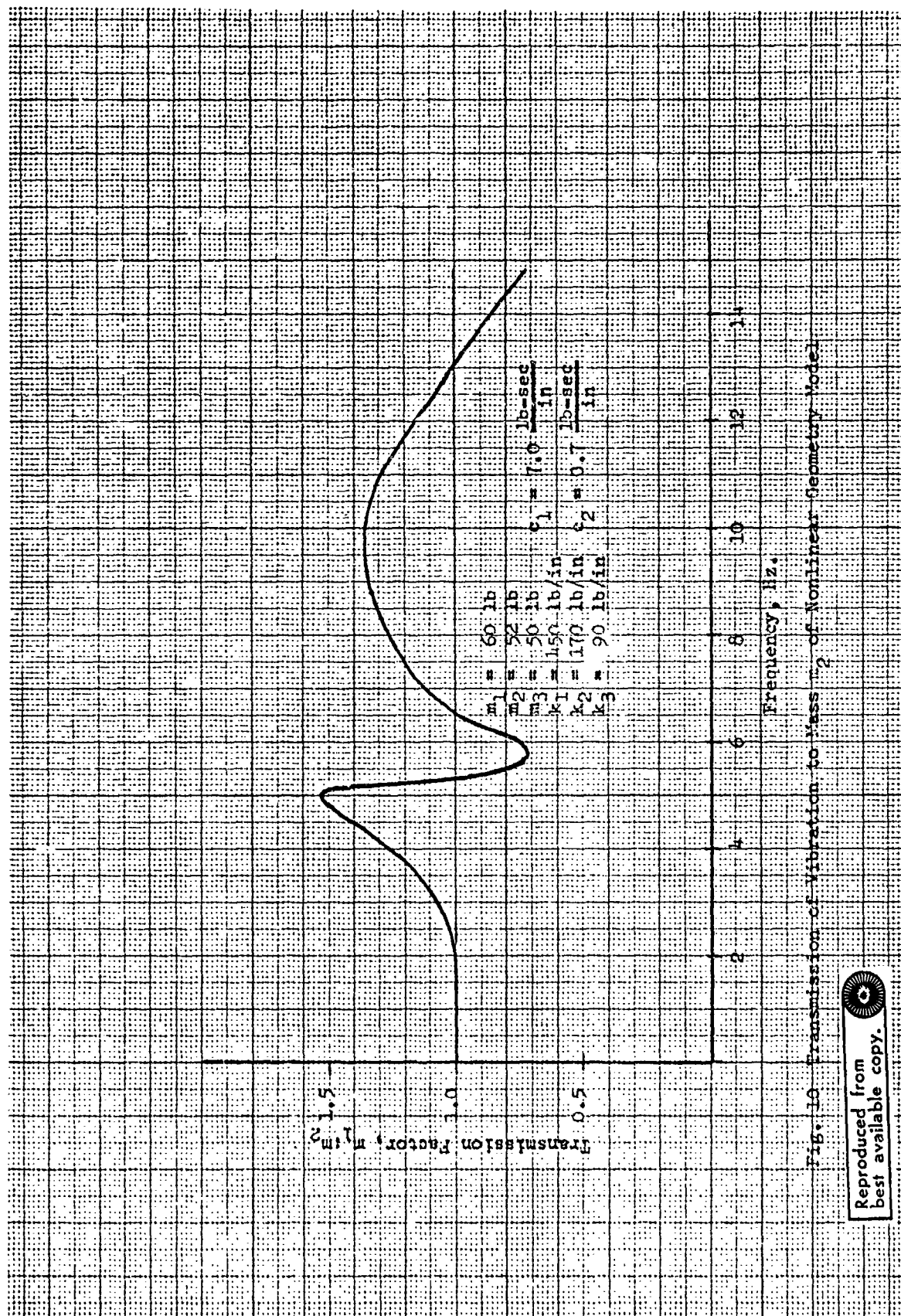


Fig. 10 Transmission of Vibration to Mass m_2 of Nonlinear Geometry Model

Reproduced from
best available copy.

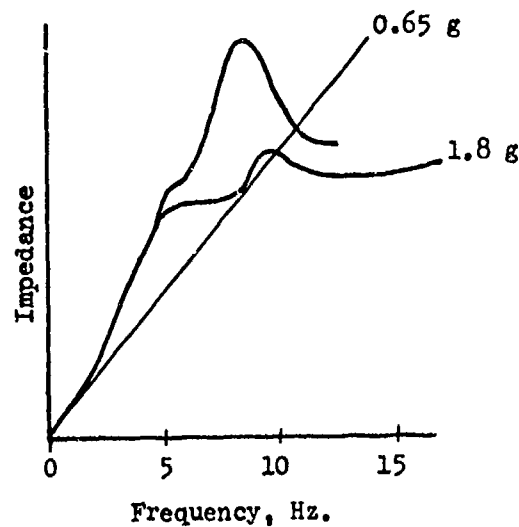


Fig. 12 Impedance of 70-lb Pig⁽⁹⁾

If the nonlinear geometry model is slightly sophisticated to include the nonlinear effects of the lungs by modeling them as a piston in a cylinder with an orifice, then the nonlinear behavior of biological systems observed by Krause can be simulated.

The equations of motion for this model, shown in Figure 11, are:

$$m_1 \ddot{x}_1 = k_1(x_2 - x_1) + C_1(\dot{x}_2 - \dot{x}_1) + k_2(x_3 - x_1) + C_3(\dot{x}_3 - \dot{x}_1) \\ + k_3 \delta(x_3 - x_1) + F(t)$$

$$m_2 \ddot{x}_2 = -k(x_2 - x_1) - C_1(\dot{x}_2 - \dot{x}_1) + F_{\text{LUNGS}}$$

$$m_3 \ddot{x}_3 = -k_3 \delta(x_3 - x_1) - c_3(\dot{x}_3 - \dot{x}_1) - F_{\text{LUNGS}}$$

where

$$\delta(x) = \begin{cases} 0 & , \quad x > 0 \\ x & , \quad x \leq 0 \end{cases}$$

The force exerted by the lungs on the visceral mass (F_{LUNGS}) is derived from the model shown in Figure 14.

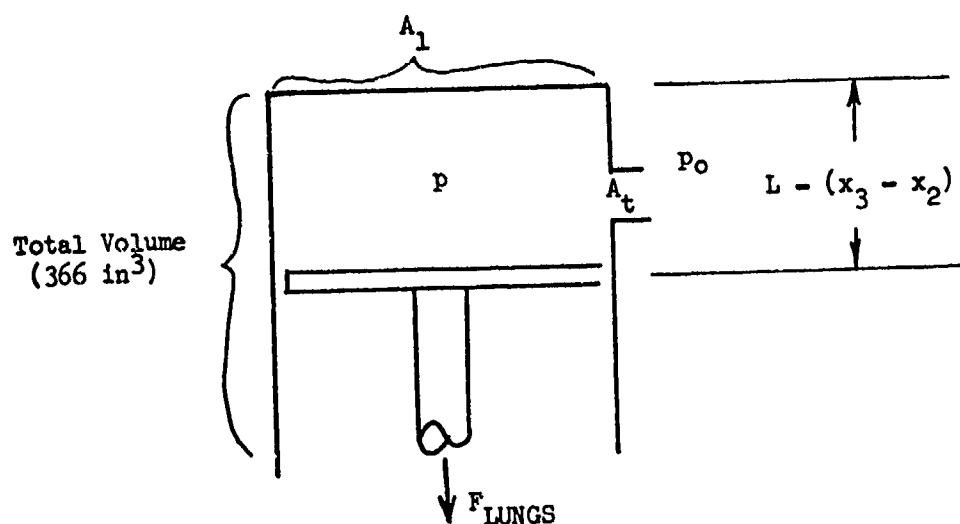


Fig. 14 Lung Model

Assuming isotropic compression, the equation of state $pV = mRT$ gives the following formulation for F_{LUNGS} :

$$p = \int \left\{ c \left(\frac{m}{V} \right)^\delta \left[\frac{\delta m}{m} - (\delta - 1) \frac{\dot{V}}{V} \right] - \frac{p \dot{V}}{V} \right\} dt$$

where

$$V = A_L [L - (x_3 - x_2)]$$

$$m = A_t \sqrt{\frac{2\delta}{\delta - 1} p_L p_L s \left(\frac{p_s}{p_L} \right)^{2/\delta} \left[1 - \left(\frac{p_s}{p_L} \right)^{\frac{\delta-1}{\delta}} \right]} \text{sign}(p - p_L)$$

where s denotes smaller of p and p_L

and L the larger of the two,

and

$$m = m_{\text{Initial}} - \int \dot{m} dt$$

Then,

$$F_{\text{LUNGS}} = \frac{\delta(x_2 - x_3)}{(x_2 - x_3)} p A_L$$

With this mathematical formulation a digital computer model was constructed using IBM's Continuous System Modeling Program.

The CSMP for this nonlinear model was subsequently used to determine the effects of different amplitude levels of vibration on the mechanical impedance. Figure 15 shows these results.

Although there are no mechanical impedance data available for humans at low frequency vibration levels as high as 3 g's., comparison with the results of Krause, Figure 12, shows that the dynamic behavior of the nonlinear model is similar to that of a pig. That is, there is a general stiffening of the system as the acceleration level is increased.

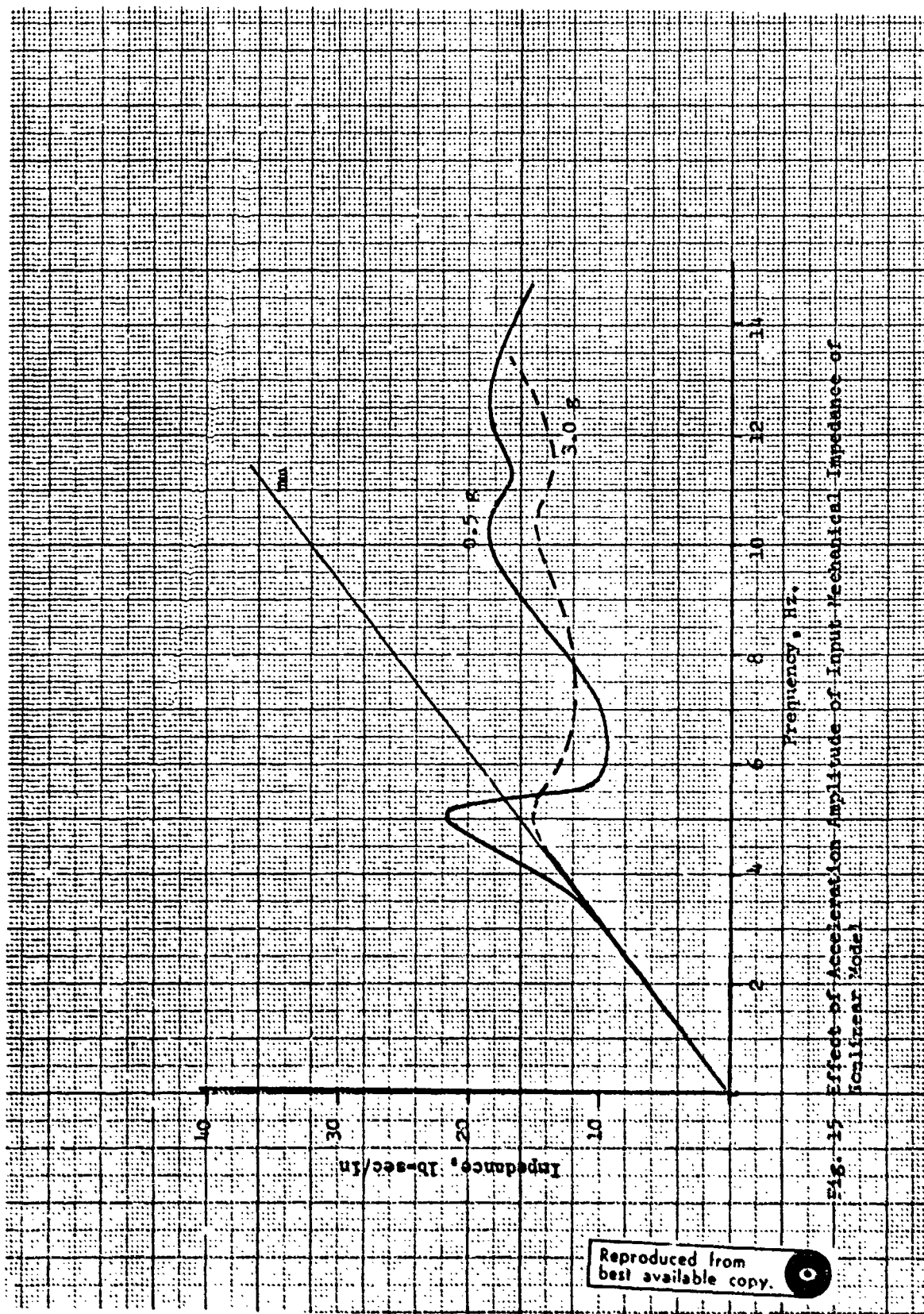


Fig. 15 Effect of Acceleration Amplitude of Input Mechanical Impedance of Nonlinear Model

Reproduced from best available copy.

CONCLUSIONS

1. The inclusion of nonlinear geometry in the two degree of freedom model of the human body's dynamic response of the human body improves the dynamic similarity.
2. The nonlinear mechanics of the lungs probably account for most of the body's nonlinear response to low frequency vibration.
3. If sufficient effort were expended to establish the governing parameters of the nonlinearity by vibration tests, nonlinear lumped parameter models could add understanding of the body mechanics during certain impact situations.

REFERENCES

1. Coermann, R. R. "The Mechanical Impedance of the Human Body in Sitting and Standing Position," Human Factors, Vol. 14, (1963), pp. 227-253.
2. Suggs, C. W., Abrams, C. F. and Stikeleather, L. F. "Application of a Damped Spring-Mass Human Vibration Simulator in Vibration Testing of Vehicle Seats," Ergonomics, Vol. 12, No. 1, (1969), pp. 79-90.
3. Goldman, D. E. and von Gierke, H. E. "Effects of Shock and Vibration on Man," Shock and Vibration Handbook, Vol. 3, McGraw-Hill, (1961).
4. Pradko, F., Lee, R.A. and Greene, J. D. "Human Vibration-Response Theory," Biomechanics Monograph, ASME, (1967), pp. 205-222.
5. Clark, W. S., Lange, K. O. and Coermann, R. R. "Deformation of the Human Body Due to Unidirectional Forced Sinusoidal Vibration," Human Factors, Vol. 19, (1963), pp. 255-279.
6. White, G. H., Lange, K. O., and Coermann, R. R. "The Effects of Simulated Buffeting on the Internal Pressure of Man," Human Factors, Vol. 14, (1963), pp. 275-290.
7. Hopkins, G. R. "The Dynamic Response of the Human Body to Low Frequency Vibration," Masters Thesis, University of Kentucky, (1961).
8. Lange, K. O. and Edwards, R. G. "Force Input and Thoraco-Abdominal Strain Resulting from Sinusoidal Motion Imposed on the Human Body," Aerospace Medicine, (May, 1970), pp. 538-543.
9. Krause, H. E. and Lange, K. O. "The Nonlinear Behavior of Biomechanical Systems," Biomechanics Monograph, ASME, (1967), pp. 57-66.