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**Research in Numerical Analysis Techniques
for Fog Model Simulation
Russell C. Serbagi**

**Digital Programming Services, Inc.
60 Hickory Drive
Waltham, Massachusetts 02154**

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FINAL REPORT

Period Covered: 13 January 1969 through 31 September 1971

November 15, 1971

Contract Monitor: Bernard A. Silverman

Meteorology Laboratory

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**Prepared
for
AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
BEDFORD, MASSACHUSETTS 01730**

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ABSTRACT

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1.0 Introduction

The problems that were considered during the extent of this contract were to develop stable numerical techniques to solve the non-linear partial differential equations governing the behavior of warm fog and cumulus clouds. These techniques must be valid for several forms of the same general equations. These different forms would be characterized by the simplifying assumptions that were made to reduce the general equations used for a particular model. Much of the development involved numerical experiments to determine the sensitivity of the models to the numerical procedures. A major portion of the development time was spent in conference with the Air Force to insure that the physical processes were accurately represented by the models.

This final report will deal with the most successful experiments only. These experiments often involved the generation of computer programs to validate the numerical procedures and the simplifying assumptions used in applying them. The contract considered three major models, the equations for which were provided by the contract monitor.

- 1) A two dimensional Eulerian model of the modification of warm fog by hygroscopic particle seeding.
- 2) A one dimensional warm cumulus model involving detailed calculations of the condensation and stochastic coalescence processes.
- 3) A three dimensional Eulerian model of warm fog modification by the addition of heat at the ground level from multiple point and line sources.

The verification of the models considered above can be made by collecting data during the actual experiments. Section 3 deals with the techniques of reducing the data on various

recording media collected by the AFCRL C-130 aircraft.
The techniques of data reduction are presented in the
form of procedures that also represent computer programs
for the PDP-8I resident of AFCRL.

2.0 FOG & CLOUD MODEL SIMULATION

2.1 Equations

The general equations used to approximate the warm fog and cloud models are represented by the diffusive quantities of momentum, temperature, vapor and particles. The equations of motion are given as:

$$u_t = -uu_x - vu_y - wu_z + (K_m u_x)_x - \frac{1}{\rho} P_x \quad (1)$$

$$v_t = -uv_x - vv_y - wv_z + (K_m v_y)_y - \frac{1}{\rho} P_y \quad (2)$$

$$w_t = -uw_x - vw_y - ww_z + (K_m w_z)_z + B \quad (3)$$

where u , v , and w represent the velocities in the x , y and z directions respectively; K_m the turbulent diffusion coefficient for momentum, B the buoyancy term, and P the pressure.

The temperature equation is:

$$\theta_t = -u\theta_x - v\theta_y - w\theta_z + \frac{LG}{C_p \rho} \left(\frac{1000}{P}\right)^{2/7} + (K_H \theta_x)_x + (K_H \theta_y)_y + (K_H \theta_z)_z \quad (4)$$

where θ is the potential temperature, G is the condensed water, L is the latent heat, C_p is the specific heat and K_H is the heat diffusion coefficient.

The mixing ratio equation is:

$$M_t = -uM_x - vM_y - wM_z - \frac{G}{\rho} + (K_E M_x)_x + (K_E M_y)_y + (K_E M_z)_z \quad (5)$$

where M is the mixing ratio; K_E is the vapor diffusion coefficient.

The density equation (conservation of mass) is:

$$\rho_t = -(\rho u)_x - (\rho v)_y - (\rho w)_z \quad (6)$$

The equation governing the local rate of change of particles is:

$$f_t = -(uf)_x - (vf)_y - (w+u_z)f_z + (K\rho f_x)_x + (K\rho f_y)_y + (K\rho f_z)_z - \left(\frac{dX}{dt}\right)_x + I_{\text{gain}} - I_{\text{loss}} \quad (7)$$

where f represents the particle density. The particle density is a function of the system spatial coordinates and a function of:

- the mass of water on the particle (X)
- the mass of the particle's nucleus (Y)
- the stochastic coalescence, I_{gain} (rate of particle increase) and I_{loss} (rate of particle decrease)

The droplet growth equation, derived by Silverman (1) is given as:

$$\frac{dX}{dt} = 4\pi r D' F_1 \left[\bar{\rho} - \frac{\rho_{sv}}{\bar{s}} \right] \left[1 + \frac{\rho_{sv} D' L^2 J F_1}{\bar{s} R_v T^2 k F_2} \right]^{-1} \quad (8)$$

$\bar{\rho}$ = ambient vapor density $\Sigma X \rho$

where

$$T = \theta \left(\frac{1000}{\bar{p}} \right)^{-2/7} \quad (9)$$

$$r = (.75(X+Y)/(\pi \bar{\rho}_s))^{1/3} \quad (10)$$

$$\rho_{sv} = \frac{1323.438e^{(17.2693882(T-273.16)/(T-35.86))}}{(1.0090606T - 3.5 \times 10^{-5} T^2)} \quad (11)$$

and $\bar{\rho}_s$, the particle density is given as

$$\bar{\rho}_s = \left\{ \begin{array}{l} 1 + .71 \frac{Y}{X+Y} + .21745 \left(\frac{Y}{X+Y} \right)^2, \quad \frac{Y}{X} \leq s_{\max} \\ (X+Y)\rho_{SH}\rho_H / \left[\rho_{SH}Y + \{\rho_H + s_{\max}(\rho_H - \rho_{SH})\} X \right], \quad \frac{Y}{X} > s_{\max} \end{array} \right. \quad (12)$$

$$\left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \quad (13)$$

where ρ_{SH} , ρ_H and s_{\max} are given quantities.

F_1 and F_2 are the ventilation coefficients for mass and heat transfer respectively.

The quantity \bar{s} found in equation (8) is given as

$$\bar{s} = (1+s)\phi \left(1 + \frac{s}{2}\right) \quad (14)$$

where $s = .018016 \gamma n$

n is given and

$$\gamma = \min\left(\frac{Y}{X}, s_{\max}\right) 1000/m \quad (15)$$

where m is the molecular weight of the solute and

$$\phi = \left\{ \begin{array}{ll} .899 + (.033954 + .004802\gamma)\gamma & \gamma \geq .5 \\ .921 & .3 < \gamma < .5 \\ .94 - \sqrt{\gamma} / 31.4 & \gamma \leq .3 \end{array} \right. \quad (16)$$

D' , the modified coefficient of molecular diffusion is given as

$$D' = (.0015T - .18374)r / (r + .000486622) \quad (17)$$

The latent heat, L , is given as

$$L = 750.2696 - .56T \quad (18)$$

k , the coefficient of thermal conductivity is given as

$$k = (1.15628 \times 10^{-5} + .17 \times 10^{-6}T)r / (r + 1.929 \times 10^{-5}) \quad (19)$$

The gas constant for water vapor is

$$R_v = 4.615 \times 10^{-6} \quad (20)$$

and J is a conversion constant 4.18684×10^7 .

The diffusion coefficients K_m, K_H, K_e , and K_p are evaluated as functions of the Richardson number, a quantity that measures the degree of air turbulence. The Richardson number, R_i , is evaluated as follows:

$$\bar{R}_i = 9.8\theta_z / (\theta u_z) \quad (21)$$

and

$$R_i = \left\{ \begin{array}{ll} .24 & \bar{R}_i > .25 \\ -2. & \bar{R}_i < -2. \\ \bar{R}_i & -2. \leq \bar{R}_i \leq .25 \end{array} \right\} \quad (22)$$

Two quantities used to directly calculate the diffusion coefficients are ϕ and α which are functions of R_i . For the unstable condition ($R_i < 0$),

$$\phi = (1 - 15R_i)^{-1/4} \quad (23)$$

$$\alpha = 1.35(1 - 9R_i)^{1/4} \phi \quad (24)$$

For the stable condition, $R_i \geq 0$,

$$\phi = 1 + 4.7\zeta \quad (25)$$

$$\alpha = \phi / (.74 + 4.7\zeta) \quad (26)$$

where

$$\zeta = \frac{.74 - 9.4R_i - \sqrt{4.9R_i - .5476}}{44.18R_i - 8.4} \quad (27)$$

Having ϕ and α , the diffusion coefficients are

$$K_m = .1225 \zeta^2 |u_z / \phi|^2 \quad (28)$$

$$K_H = K_e = K_p = \alpha K_m \quad (29)$$

The buoyancy term in equation (3) is

$$B = 9.8 \left\{ \frac{\theta_v - \theta_{vi}}{\theta_{vi}} - \frac{Q}{\rho} \right\} \quad (30)$$

where θ_v the local virtual potential temperature is given by

$$\theta_v = (1 + .61m)\theta \quad (31)$$

θ_{vi} is the ambient virtual temperature.

The water content is given by

$$Q = \int_0^{\infty} \int_0^{\infty} x f dx dy \quad (32)$$

The condensation term in equation (5) is

$$G = \int_0^{\infty} \int_0^{\infty} X f_t dX dY \quad (33)$$

where X is the mass of the water, Y is the mass of the nucleus, and f is the particle density.

The stochastic coalescence is given by two integral expressions:

$$I_{g+h}^{\{X, Y, x, y, z\}} = \int_0^X \int_0^Y f(X_c, Y_c, x, y, z) V(X_c, Y_c / X', Y', x, y, z) f(X', Y', x, y, z) dX' dY' \quad (34)$$

$$I_{0s}^{\{X, Y, x, y, z\}} = \int_0^X \int_0^Y f(X, Y, x, y, z) V(X, Y / X', Y', x, y, z) f(X', Y', x, y, z) dX' dY' \quad (35)$$

where $X_c = X - X'$, $Y_c = Y - Y'$, V is the collection kernel, and the primed variables are the parameters of the particle being captured.

2.1.1 Two Dimensional Eulerian Warm Fog Seeding Model

The system of equations governing this model are⁽³⁾:

$$M_t = -\frac{G}{\rho} + KM_{xx} + KM_{zz} \quad (1)$$

$$f_t = (V_t f)_z + K(f_{xx} + f_{zz}) + \left(\frac{dX}{dt} f\right)_x \quad (2)$$

where

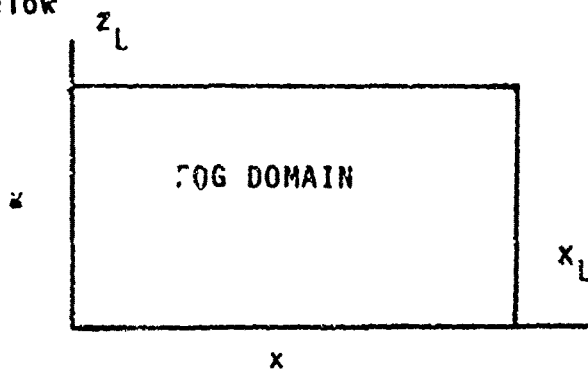
$$\rho = (2506.612M^{.714})/\theta \quad (3)$$

and θ is given.

These equations are reduced from the general equations given in section 2.1, and are valid only under the following assumptions:

- a) $W_t = 0$
- b) $\theta_t = 0$, temperature variations due to condensations processes are considered small.
- c) $K_p = K_m = K_H = K$
- d) and since θ is constant over the entire domain, then $\frac{\partial K}{\partial x} = \frac{\partial K}{\partial z} = 0$
- e) no coalescence

A schematic of the space domain is represented in the figure below



The boundary conditions are:

- a) No diffusion across the top, left and bottom boundaries and symmetry about the vertical axis.

$$\frac{\partial M}{\partial z}(x, z_L) = \frac{\partial M}{\partial z}(x, 0) = \frac{\partial M}{\partial z}(0, z) = 0 \quad (4)$$

$$\frac{\partial f}{\partial z}(x, z_L) = \frac{\partial f}{\partial z}(x, 0) = \frac{\partial z}{\partial z}(0, z) = 0 \quad (5)$$

- b) The boundary at the right is sufficiently far enough away from the seeding area that the boundary conditions are:

$$M(x_L, z) = M_I(z) \quad (6)$$

$$f(x_L, z) = f_I \quad (7)$$

where M_I and f_I are the initial conditions.

The initial conditions over the entire domain are:

$$M(x, z) = M_I(z) \quad (8)$$

$$f(x, z) = f_I \quad (9)$$

2.1.2 One Dimensional Warm Cumulus Cloud Model

The reduced set of equations for this model are given as:

$$w_t = -w w_z + K w_{zz} + B + E w \quad (1)$$

$$\theta_t = -w \theta_z + \frac{L G}{C_p \rho} \left(\frac{1000}{P} \right)^{2/7} + K \theta_{zz} + E \delta \theta \quad (2)$$

$$M_t = -w M_z - \frac{G}{\rho} + K M_{zz} + E \delta M \quad (3)$$

$$f_t = -(w + v_t) f_z + K f_{zz} - \left(\frac{dx}{dt} \right)_x + I_{gain} - I_{loss} + E f \quad (4)$$

where E , the dynamic entrainment term satisfying two dimensional continuity is given as:

$$E = \begin{cases} 0 & E' \geq 0 \\ \frac{E'}{R} & E' < 0 \end{cases} \quad (5)$$

where

$$\frac{E}{R} = \left[\frac{\theta_t}{\theta} + \frac{.61 M_t}{1 + .61 M} - w (\log p)_z - w_z \right] \left[\frac{\theta_e}{\theta} + \frac{\theta}{\theta_e} - 1 \right]^{-1} \quad (6)$$

where

θ_e is the temperature of the environment

$\delta M = M - M_I$, M_I the ambient mixing ratio

$\delta \theta = \theta - \theta_I$, θ_I the ambient temperature

and

$$\begin{aligned} \rho &= (2506.612 \text{ M}^{714})/\theta \\ P &= R\theta/\rho \end{aligned} \quad (7)$$

The assumption that reduce the general set equations to those above are:

$$\begin{aligned} \text{a) } & K_M = K_H = K_\rho = K \\ \text{b) } & \frac{\partial K}{\partial z} = 0 \end{aligned}$$

The initial conditions for the system are

$$\begin{aligned} M(z) &= M_I(z) \\ \theta(z) &= \theta_I(z) \\ f(z) &= f_I \end{aligned}$$

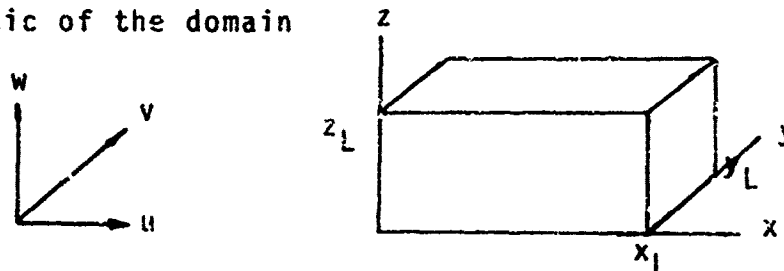
The boundary conditions for this model are those imposed by equation (4) in section 2.1.3 and the bottom boundary.

$$\begin{aligned} f(0) &= f_I \\ M(0) &= M_I \end{aligned}$$

2.1.3 Three Dimensional Eulerian Model of Warm Fog

The three dimensional Eulerian model of warm fog modification by the addition of heat sources at the ground level are identical to the general set in section 2.1 with the assumption that the effects of stochastic coalescence are negligible. (ie $I_{\text{gain}} = I_{\text{loss}} = 0$)

The domain of the equations is taken as rectangular and therefore, in a three-dimensional system, there are six boundary conditions to consider. The figure below is a schematic of the domain



The conditions are selected such that the mass is flowing into the left and front boundaries, and flowing out of the right, back and top boundaries. The bottom boundary is considered the ground. The boundaries in which mass is flowing out have the boundary conditions:

right face,

$$C(x_1, y, z, t) = \frac{C(x_L - \delta x, y, z, t)^2}{C(x_L - 2\delta x, y, z, t)} \quad (1)$$

where C is vector of

$$\begin{pmatrix} u \\ v \\ w \\ \theta \\ M \\ p \\ f \end{pmatrix} \quad (2)$$

back face,

$$C(x, y_L, z, t) = \frac{C(x, y_L - \delta y, z, t)^2}{C(x, y_L - 2\delta y, z, t)} \quad (3)$$

top face,

$$C(x, y, z_L, t) = \frac{C(x, y, z_L - \delta z, t)^2}{C(x, y, z_L - 2\delta z, t)} \quad (4)$$

The left, front and bottom faces are maintained at the initial conditions of the system. The initial conditions are given as some function of z and are chosen such that the velocities are positive to ensure the proper flow out of the back, right and top faces and in the front and left faces. The velocity at the bottom face is zero for all time. Stated more precisely,

$$C(0, y, z, t) = C_I(z) \quad (5)$$

$$C(x, 0, z, t) = C_I(z) \quad (6)$$

$$C(x, y, 0, t) = C_I(0) \quad (7)$$

where $C_I(z)$ is the initial conditions for the entire system.

2.2 Method of Calculation

The equations presented for the three models are of the same general nature and are integrated numerically by an explicit finite difference scheme to be discussed later. An important consideration of the method is the different time steps used in the transport and condensation processes. The reason for the different time steps in the procedure are only economic relative to computer time. That is, if all the equations were to be integrated with a time step necessary to integrate the condensation processes, a large amount of computer time would be necessary. If the condensation processes would be allowed to go on independently during the time step of the transport process, much computer time could be saved. The reason for this is due to the time saved in storing and fetching quantities at the adjacent nodes in the network. The condensation process is independent of any spatial considerations during one particular time step and therefore, independent of neighboring nodal processes. The integration process used in the time domain is merely a first term Taylor series expansion

$$f(t+\delta t) = f(t) + \delta t \left(\frac{df}{dt} \right) \quad (1)$$

where δt is chosen so as to maintain numerical stability.

The problem presented in equations (1) through (7) of section 2.1 can be divided into two main processes:

- a) transport processes
- b) condensation processes

The transport processes occur at a much lower rate than do the condensation processes. Therefore, to integrate both processes at the same δt would be wasteful. However, if the two processes could be integrated simultaneously but at different rates, our problem of waste would be eliminated.

For convenience, the problem can be independently divided into the two processes.

$$f_t = T + C \quad (2)$$

If the stability criterion for transport (T) is δt and the stability criterion for condensation (C) is Δt ($\Delta t < \delta t$), then the equation (1) could be rewritten as

$$f(t+\Delta t) = T\Delta t + C\delta t \quad (3)$$

Equation (3) could then be applied as many times as it takes to reach the time $t+\delta t$. During the time step δt , T is only calculated once, whereas C is recalculated at each step Δt . This amounts to a considerable savings since most of the computer time is spent in calculating T. For example, let $\delta t = 3\Delta t$, then

$$f(t+\delta t/3) = T\Delta t + C(f(t))\Delta t + f(t) \quad (4)$$

$$f(t+2\delta t/3) = T\Delta t + C(f(t+\Delta t))\Delta t + f(t+\Delta t) \quad (5)$$

$$f(t+\delta t) = T\Delta t + C(f(t+2\Delta t))\Delta t + f(t+2\Delta t) \quad (6)$$

2.3 Finite Difference Techniques

The equations for the three models discussed in earlier sections have in common many terms of the same general form. Instead of giving the complete set of finite difference equations for all models, the finite difference form for each general term will be given.

The form of time derivatives are approximated by

$$f_t = \{f(t+\Delta t) - f(t)\} / \Delta t \quad (1)$$

Other terms of the general form uv_x will be approximated by the following finite difference scheme by Crowley⁽⁴⁾

$$uv_x = 1/2u_i(v_{i+1}-v_{i-1}) / \Delta x - (F^+ - F^-) / \Delta x \quad (2)$$

and terms of the general form $(uv)_x$ are:

$$(uv)_x = (F^- - F^+) / \Delta x \quad (3)$$

where

$$F^+ = \frac{v^+}{2} \left[u_{i+1} + u_i + \frac{\Delta t}{\Delta x} v^+ (u_{i+1} - u_i) \right] \quad (4)$$

and

$$v^+ = (v_{i+1} + v_i) / 2, \quad v^- = (v_{i-1} + v_i) / 2 \quad (5)$$

The terms of the form $(Ku_x)_x$ are approximated as

$$(Ku_x)_x = \frac{(K_{i+1} + K_i)(u_{i+1} - u_i) - (K_i + K_{i-1})(u_i - u_{i-1})}{2\Delta x^2} \quad (6)$$

The equations representing particle diffusion are continuous over all X (mass classes) and over all Y (water classes). These equations are made discrete by a logarithmic transformation used by Berry.⁽²⁾ The domain of both X and Y are transformed as follows:

$$X_j = \begin{cases} 0 & j = 1 \\ X_0 e^{3(j-2)/J_0} & j > 1 \end{cases} \quad (7)$$

and

$$Y_k = Y_0 e^{3(k-1)/k_0} \quad (8)$$

Substituting equations (7) and (8) into equation (7) of section 2.1 gives

$$\begin{aligned} f_t^{j,k} = & u f_x^{j,k} - v f_y^{j,k} - (w+v_t) f_z^{j,k} + (K_p f_x^{j,k})_x \\ & + (K_p f_y^{j,k})_y + (K_p f_z^{j,k})_z \\ & - \frac{J_0}{3X_j} \cdot \frac{\partial \left(\frac{dX_j}{dt} f^{j,k} \right)}{\partial j} \end{aligned} \quad (9)$$

where $j=1, j_{\max}$ and $k=1, k_{\max}$. Equation (9) now represents a discrete set of simultaneous difference equations that span the domain of water and mass classes of interest.

2.4 Stability Criteria

The finite difference scheme used for the solution of the equations are subject to limiting steps in time and space. These constraints must be maintained throughout the entire solution to insure stability in the numerical solution of the finite difference equations. The stability criteria by Silverman⁽³⁾ are:

$$\delta t \leq \left[\frac{v}{\Delta} + \frac{2K}{\Delta^2} \right]^{-1} \quad (1)$$

where

$$\begin{aligned} v &= \max(u, v, w) \\ \Delta &= \min(\Delta x, \Delta y, \Delta z) \\ K &= \max(K_p) \end{aligned}$$

and

$$\Delta t \leq \left[\frac{J_0}{3X_j} \cdot \frac{dX}{dt} \right]^{-1} \quad (2)$$

where δt and Δt are referenced in equations (3) through (6) of section 2.3. If however, $\Delta t = \delta t$, there is no need of separating the transport and condensation processes as recommended in equations (3) through (6) of section 2.3 and thus the overall stability can be written as:

$$\delta t \leq \left[\frac{v}{\Delta} + \frac{2K}{\Delta^2} + \frac{J_0}{3} \sigma \right]^{-1} \quad (3)$$

where

$$\sigma = \min \left(\frac{1}{X} \frac{dX}{dt} \right) \quad (4)$$

3.0

PDP-8I
SOFTWARE
SYSTEM

3.1 SCH-8

Program SCH-8 makes possible the selective recovery of data from magnetic tape cassettes using the DL-622 1/4" magnetic tape reader. Provision is made for checking incoming data for accuracy of transmission and for notifying the operator, by means of teletype messages, of parity failures and program status.

Various checking features of SCH-8 make possible the recovery of data over long periods of time from specified sources which yield information subject to wide variation due to meter scale or range changing. Checks are also performed on selected data to screen out and correct erroneous data values (straddle) introduced by metering devices during range transitions.

Three averaging operations are provided for special handling of any one or all of the 18 input channels and provision is made for generation of the standard deviation of 1 or all channels during a program run.

An option code (E) is provided which, when associated with an input channel, produces a teletype listing of that channel's values while not contributing to any data averaging operation.

It should be noted that the range change checking and straddle checking for the specified channels commence with the very first scan values recovered at time of program initialization.

Any format, range change, or straddle parameter modifications must be accomplished prior to program initialization.

3.2 Metro-8 and Dead Reckoning

The programs Metro-8 (ME-8), Dead Reckoning (DR-8) operate on DL-620 meteorological data as generated by program SCH-8. ME-8 calculates true temperature, true air speed, Tacan position coordinates with altitude correction and wind direction and velocity from Doppler drift measurements. Wind level averaging of the calculated wind velocity and direction is also provided. DR-8 calculates dead reckoning coordinates with or without wind drift effects in addition to true temperature, true air speed and Tacan position. The programs are written in FOCAL for use with a PDP-8I computer system having 8K storage and a high speed reader (HSR). A discussion of the calculations mentioned above follows:

A. True Temperature and Air Speed

ME-8 and DR-8 both require calculations of true temperature T and true air speed V_a . The formulas used are:

$$T(^{\circ}\text{K}) = \frac{T_{\text{IND}} + 273.16}{1 + .1992M^2}$$

where T_{IND} is the measured (Rosemont) temperature in $^{\circ}\text{C}$ and M is the Mach number

$$V_a (\text{knots}) = \sqrt{1516.4TM^2} + 3\sqrt{T/P}$$

where P is the indicated pressure in millibars.

B. Tacan Position

ME-8 and DR-8 both calculate Tacan position as follows:

Let D_T be the direct line Tacan distance, θ_T the indicated Tacan bearing, θ_D the magnetic deviation, H the altitude of the airplane, X_0 and Y_0 the coordinates of the Tacan station. The component of the distance along the ground D_G of the airplane from the Tacan station becomes:

$$D_G = \sqrt{D_T^2 - H^2}$$

The true bearing θ corrected for magnetic deviation is:

$$\theta = \theta_T + \theta_D$$

and consequently the Tacan coordinates become:

$$X_P = X_0 + D_G \sin \theta \quad , \quad Y_P = Y_0 + D_G \cos \theta$$

Two schemes have been used to calculate the altitude H from the measured pressure P , using in both cases the U.S. standard atmosphere. Method 1 is a least squares, fourth order polynomial curve fit to the tabulated data (H in nautical miles, P in millibars).

$$H = a_0 + a_1 P + a_2 P^2 + a_3 P^3 + a_4 P^4$$

$$a_0 = 7.09398$$

$$a_1 = -.4613 \times 10^{-2}$$

$$a_2 = -.170930 \times 10^{-4}$$

$$a_3 = .254226 \times 10^{-7}$$

$$a_4 = -.108037 \times 10^{-10}$$

The RMS error in curve fitting to fourth order is ± 50.6 ft for H in the range 1780 ft (950 mb) to 23,560 ft (400 mb).

An alternative Method 2 assumes that the Standard Atmosphere obeys a power law of the form

$$H = a - bP^n, \quad n = 2/7$$

From two know points (H_1, P_1) and (H_2, P_2) one can then generate the equation:

$$H = H_1 + \left(\frac{H_2 - H_1}{P_1^n - P_2^n} \right) (P_1^n - P^n)$$

An attached calculation, along with the least squares program and calculation, indicates the approximation involved in replacing the 2/7 power by 9/32 (9/32 is equivalent to first raising a quantity to the 9 power, followed by 5 successive square roots to avoid the use of logarithm and exponential functions in FOCAL). The results for P in nautical miles are:

$$\begin{aligned} H &= .53288 + 2.29121(6.98344 - P^n) & n &= 2/7 \\ H &= .53288 + 2.39629(6.77455 - P^n) & n &= 9/32 \end{aligned}$$

For $P_1=900$ mb, $H_1=3,240$ ft, $P_2=500$ mb, $H_2=18,280$ ft both forms of the equation are good to within about 1% relative to tabulated data in the range 900 mb to about 400 mb.

C. Dead Reckoning Calculations

Consider two successive data points $i-1$ and i during which time interval $\Delta t_i = t_i - t_{i-1}$ the averaged true air speed is \bar{v}_i and the averaged true heading is $\bar{\theta}_i$. Then the incremental components of the distance traveled during Δt_i by

dead reckoning are:

$$\Delta x_i = \bar{v}_i \Delta t_i \sin \bar{\theta}_i, \quad \Delta y_i = \bar{v}_i \Delta t_i \cos \bar{\theta}_i$$

At time t_i , therefore, the dead reckoning coordinates are:

$$x_i = x_{i-1} + \Delta x_i, \quad y_i = y_{i-1} + \Delta y_i$$

and the flight path may be generated by successive integration over small time intervals.

The effect of wind is to shift by vector addition the dead reckoning path through increments Δx_i^W and Δy_i^W caused by a wind vector of velocity W_i and direction ϕ_i , where:

$$\Delta x_i^W = -W_i \Delta t_i \sin \phi_i, \quad \Delta y_i^W = -W_i \Delta t_i \cos \phi_i$$

Wind data is given in the form of (W_k, ϕ_k) for levels $k=1$ through $k=n$ corresponding to known pressures P_k (or altitude).

The wind speed and direction at the instantaneous pressure P is then found by linear interpolation of the X and Y components between levels j and $j+1$ such that $P_j \leq P_i \leq P_{j+1}$, namely:

$$\begin{aligned} W_x &= W_j(1-Z) \sin \phi_j + W_{j+1} Z \sin \phi_{j+1} \\ W_y &= W_j(1-Z) \cos \phi_j + W_{j+1} Z \cos \phi_{j+1} \\ \Delta x_i^W &= W_x \Delta t_i, \quad \Delta y_i^W = W_y \Delta t_i \end{aligned}$$

where

$$Z = (P - P_j) / (P_{j+1} - P_j)$$

D. Approximations to the Mach Number and Arccosine Function

Evaluation of the Mach number M is required for the computation of the true temperature and true air speed. In particular we calculate:

$$M^2 = 5 (1+x)^{n-1} \quad \text{where } x = \frac{\Delta P}{P}, \quad n = \frac{2}{7}$$

since in FOCAL we cannot exponentiate to a fractional power of $(2/7)$ we must instead calculate:

$$M^2 = 5 \exp\{(2/7)\ln(1+x)\} - 1$$

which requires the use of exponential and logarithm functions. To avoid the use of these functions and thus save locations for variable storage, we use the expansion:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2} x^2 + \frac{n(n-1)(n-2)}{3} x^3 + \dots$$

which for $n=2/7$ to fourth order in x yields:

$$M^2 = \frac{5x}{49} \left[14 - 5x + \frac{20}{7} x^2 - \frac{95}{49} x^3 \right]$$

This approximation is accurate to .5% for $0 \leq x \leq .5$

The arctangent function is required in evaluating the computed wind direction from Doppler drift. Using the law of cosines to calculate the interior angle θ between the wind speed vector and the air speed vector yields $\cos^{-1}(x) = \theta$.

To find θ , FOCAL uses:

$$\theta = \tan^{-1} \left(\frac{\sqrt{1-x^2}}{x} \right)$$

In an effort to eliminate the FOCAL representation of $\tan^{-1}x$ we employ the following polynomial expansion:

$$\theta = \sqrt{1-x} (a_0 + a_1x + a_2x^2 + a_3x^3) \quad 0 \leq x \leq 1$$

$$a_0 = 1.57073$$

$$a_1 = -.21211$$

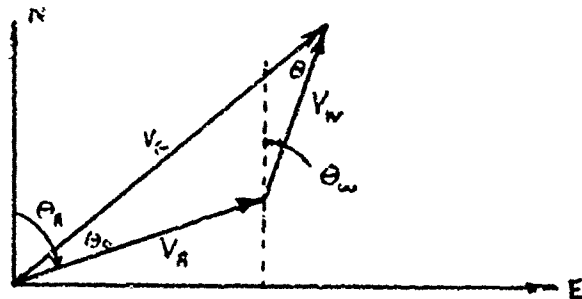
$$a_2 = .07426$$

$$a_3 = -.01873$$

which is accurate to an absolute error of 5×10^{-5} ($.03^\circ$) in the range $0 \leq x \leq 1$.

The net effect of eliminating FEXP, FLOG and FATN functions from FOCAL is to save some 255 locations, or about 51 variables.

E. Determination of Wind Speed and Direction from Doppler Drift



Let V_A be the vector velocity of the airplane through the air directed at a true angle θ_A from North, let V_G be the ground velocity vector (Doppler speed) making an angle θ_D (Doppler drift angle) with respect to V_A and let V_W be the

resulting wind velocity vector making an angle θ_W with respect to North which causes the drift. Solving the wind triangle for V_W by the law of cosines gives:

$$V_W^2 = V_G^2 + V_A^2 - 2V_A V_G \cos(\theta_D)$$

Further application of the law of cosines gives the interior angle ϕ between V_G and V_W , namely:

$$\cos(\phi) = \frac{V_G^2 + V_W^2 - V_A^2}{2V_G V_W} = \frac{V_G - V_A \cos(\theta_D)}{V_W}$$

Finally the wind direction θ_W is calculated as:

$$\theta_W = \theta_A + \theta_D + 180 \pm \phi$$

where the sign of ϕ is the same as the sign of θ_D .

F. Wind Level Averaging

ME-8 computes wind directions and velocity from Doppler drift for each time point according to section E. Because of the scatter in the calculations due to data uncertainties and unsteady flight conditions, a method of averaging the wind speed and direction over many data records has been instituted which can be used to form wind level averages for input into DR-8. In particular the components of the calculated wind are written:

$$V_{x_i} = V_W \sin \theta_W, \quad V_{y_i} = V_W \cos \theta_W$$

and added to sums S_{x_i} and S_{y_i} for pressure level i according to:

$$S_{x_i} = \sum_{k=1}^{n_i} V_{x_{ik}} \quad , \quad S_{y_i} = \sum_{k=1}^{n_i} V_{y_{ik}}$$

where $k=1,2,\dots,n_i$ is the k th entry to the sums for level i . The pressure of wind levels are $P_i=940-80i$ (mb) ($i=1,7$), i.e. 7 levels in 80 mb steps from 860 mb to 380 mb, and a set of wind components is assumed to belong to level i if the associated pressure P for which the wind components were calculated falls within a band $P_i \pm 40$ mb. At the termination of the run, the wind averages are calculated as:

$$\bar{V}_{x_i} = \frac{1}{n_i} S_{x_i} \quad , \quad \bar{V}_{y_i} = \frac{1}{n_i} S_{y_i} \quad , \quad \bar{V}_i = \sqrt{\bar{V}_{x_i}^2 + \bar{V}_{y_i}^2}$$

$$\bar{\theta}_i = \cos^{-1} (\bar{V}_{y_i} / \bar{V}_i)$$

G. Dead Reckoning Flight Curvature Correction

In earlier versions of DR-8 dead reckoning, no account of flight curvature was made other than to assume that the path was straight between time t_1 and t_2 with an average heading of ψ_2 and a straight line distance travelled of $D_s = V_2(t_2 - t_1)$ where V_2 is the true air speed calculated at t_2 . Consequently, dead reckoning coordinates were calculated as:

$$x_2 = x_1 + D_s \sin \psi_2 \quad y_2 = y_1 + D_s \cos \psi_2 \quad D_s = V_2 \Delta t$$

This amounts to having the airplane flying in straight line segments after instantaneous changes in heading angle.

A more realistic approach is to observe the change in heading $\Delta\psi = \psi_1 - \psi_2$ and assume that the flight path was a circle of radius $R = D_c / \Delta\psi$ (rad) where D_c is the arc length along the circle between points 1 and 2. D_c is just $\bar{V} \Delta t$

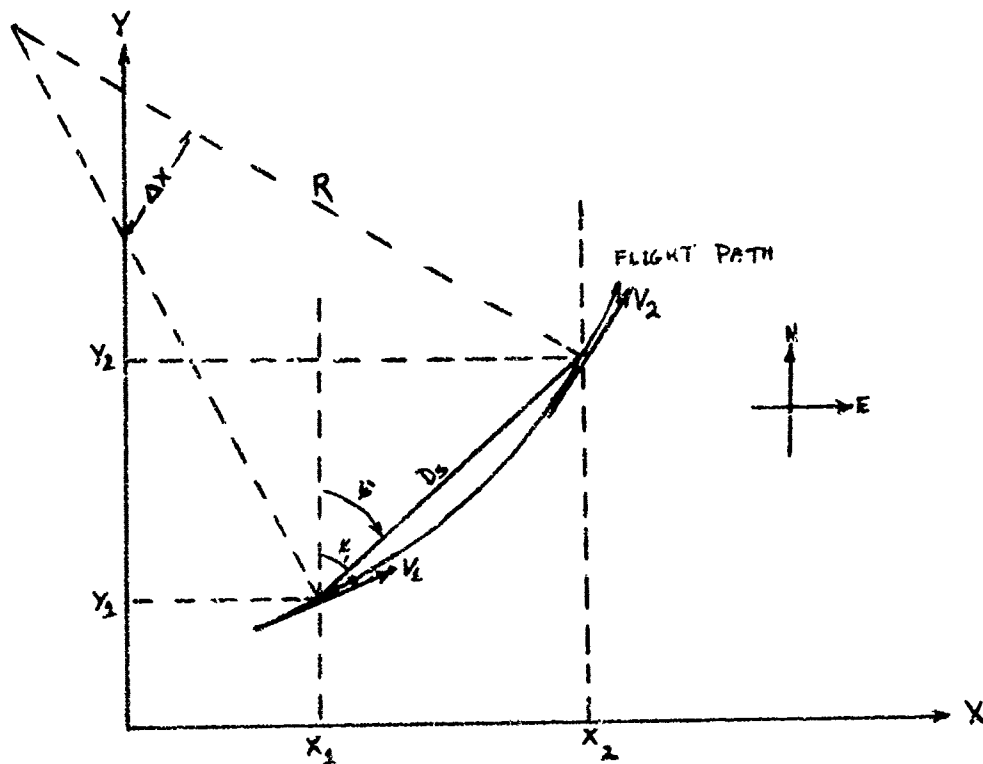
and $\bar{V}=(V_1+V_2)/2$ is the average velocity. Consequently, the straight line distance or chord length D_s between points 1 and 2 can be shown to be:

$$D_s = f D_c \quad f = \sin(\Delta\phi) / \Delta\phi \quad \Delta\phi(\text{rad}) = \pi \psi \quad (\text{deg}) / 180$$

where f is a fraction which changes from 1.0 for $\Delta\psi=0$ to .90 for $\Delta\psi=90^\circ$. Thus after a 90° turn, the straight line distance is some 10% smaller than that calculated with the old model. Finally, the dead reckoning coordinates of point 2 are now calculated as:

$$x_2 = x_1 + \frac{1}{2} f D_c [\sin\psi_1 + \sin\psi_2],$$

$$y_2 = y_1 + \frac{1}{2} f D_c [\cos\psi_1 + \cos\psi_2]$$



More sophisticated models can be devised other than assuming circular flight paths between time elements when angular changes occur, however, as a practical matter for short time intervals, the angular changes $\Delta\psi$ will be small (f close to unity, $\bar{\psi}$ close to ψ_2) and the effects of flight path curvature will be small but cumulative.

3.3 Pencil Follower Program

The Pencil Follower program written in PAL III assembly language for the PDP-8I system is designed to process raw digitized data on paper tape from the Pencil Follower and convert it to FORTRAN/FOCAL compatible format. The coordinates of each point of each curve are rotated into the reference frame of the Pencil Follower table and the data presented in the following form:

<u>Line</u>	<u>Contents</u>	
1	$N_{ID}, X_0, Y_0, \Delta X, \Delta Y$	(ID length, base frame coordinates)
2	N, ID	(No of data points, Curve ID label)
3	$X_1, Y_1, X_2, Y_2, X_3, Y_3, X_4, Y_4$	(Groups of 4 data points)
.		
.		
3	$\dots X_N, Y_N$	(More of the same)
4	N_C	(No of curves-no of type 2 lines)
5		(End of tape)

There are in general N_C groups of one type 2 line followed by a string of $N/4$ type 3 lines - 4 data points to a line.

3.4 FLIPPER

FLIPPER is a FOCAL program for converting the Pencil Follower program output data in serial form to a parallel form for use as input in other programs. In versions 1.0 and 1.1 of Flipper, the first data string appearing on the Pencil Follower output tape is treated as the master curve $(x_i^1, y_i^1, i=1, N_1)$. Each succeeding string of data $(x_j^n, y_j^n, j=1, N_n, n>1)$ is then processed by linear interpolation to produce N_1 values of y of that string corresponding to each x_i^1 value of the master string, i.e.

$$y_i^n = y_j^n + \left(\frac{y_{j+1}^n - y_j^n}{x_{j+1}^n - x_j^n} \right) (x_i^1 - x_j^n) \quad n>1; i=1, N_1$$
$$j=1, N_n$$

if

$$x_j^n < x_i^1 < x_{j+1}^n$$

Version 1.2 operates in a similar fashion except that the x_i^1 's are prescribed instead of saved from the master string. After the Pencil Follower tape is read with the HSR, an option is available to select fast output on the HSP or a typed listing on the TTY. Version 1.0 has a storage capacity of 230 data points, while versions 1.1 and 1.2 can handle over 1000 points with the use of a special FNEW function which stores variables in Field 1 of 8K FOCAL.

3.5 Fog Drop Size Distribution Program

This program is designed to operate on measurements of the cross section of water droplets on a photographic plate using the curve digitizer apparatus. The binary curve digitizer output tape is first processed by the Pencil Follower program to rotate and translate the data with reference to the curve digitizer table and produce a corrected FOCAL compatible data tape. This tape (an example of the format is attached) is now processed by the Fog Drop Size Distribution Program which calculates a mean diameter for each set of 4 edge measurements $(x_i, y_i; i=1,4)$ according to:

$$D_1 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad , \quad D_2 = \sqrt{(x_4 - x_3)^2 + (y_4 - y_3)^2}$$

$$D_{\text{mean}} = \sqrt{D_1 D_2}$$

and then produces a scaled histogram plot according to the number of classes and step size specified by the operator. The program is written in FOCAL, requiring 8K extended storage, high speed paper tape read, and no extended functions.

3.6 Vertical Velocity Program for F100

The Vertical Velocity program calculates the vertical velocity of air from data recorded on the F100 oscillograph trace. It uses angle of attack, pitch angle and relative height data reduced to the input form via the Pencil Follower and Flipper (vers 1.2) programs. A mean True Air speed is entered via the teletypewriter for each pass.

Output is via punched paper tape.

Format of output tape is:

Time (sec)

Vertical Velocity, w , (m/sec)

Angle of attack, α , (deg)

Pitch angle, θ , (deg) - with respect to θ at start
of cloud penetration

Relative height, H , (m), - with respect to a value
prior to cloud penetration.

Output is midpoint value of
3-point averaging interval.

Height change, ΔH , (m), - $\Delta H = H_2 - H_1$

Angle of attack minus pitch angle, $(\alpha - \theta)$ (deg)

Mathematical description

1. (Vertical velocity of air) - (vertical velocity of aircraft)
 $= W_{air} - W_{acft}$

$$\text{and } W_{acft} = \frac{\Delta H}{\Delta t}$$

2. $\overline{TAS} \sin \alpha - \overline{TAS} \sin \theta = W_{air} - \frac{\Delta H}{\Delta t}$
or (since $\sin \alpha = \alpha$ $\sin \theta = \theta$)

3. $(\alpha - \theta) \overline{TAS} = W_{air} - \Delta H / \Delta t$

Rewriting 3.

$$4. W_{air} = (\alpha - \theta) \overline{TAS} + \frac{\Delta H}{\Delta t}$$

Due to noise, data are smoothed over a 3-point array. The smoothing equations used are:

$$\tilde{\alpha} = \frac{1}{6} (\alpha_{i-1} + 4\alpha_i + \alpha_{i+1})$$

$$\tilde{\theta} = \frac{1}{6} (\theta_{i-1} + 4\theta_i + \theta_{i+1})$$

$$\overline{\Delta H} = \frac{1}{2} (H_{i+1} - H_{i-1})$$

$$\Delta t = (T_{i+1} - T_i), \text{ (since time increments are equal.)}$$

Due to possible precession of the Pitch Gyro, a correction term is added to the pitch angle. This term when applied to initial data, (prior to cloud penetration, i.e. when $\frac{\Delta H}{\Delta t}$ is small) forces the vertical velocity to be initially, approximately, zero. The term, TZ = $(\tilde{\alpha} - \tilde{\theta})$ is calculated using the mean of the first two data sets available. To partially offset ΔH changes, HT (1) is set equal to HT (2) for initial data sets. This is an additional improvement for the calculation setting $W_{initial} = 0$. See Section 7.0 of program.

The program version of Equation 4 is then:

$$5. W = (AB - [TB + TZ]) * TS * DR * CT + DH/DT$$

$$\text{where } AB = \tilde{\alpha}$$

$$TB = \tilde{\theta}$$

$$TS = \overline{TAS}$$

$$DH = \Delta H$$

$$DR = \frac{\pi}{180}$$

$$CT = 6080 / (3600 \times 3.28) \text{ if } \overline{TAS} \text{ is given in knots}$$
$$= 1 \text{ if } \overline{TAS} \text{ is given in m/sec}$$

3.7 Additional Software

The following additional programs were developed to provide a means of operating and evaluating the performance of various peripheral devices added to the PDP-8I.

Included are specialized diagnostic programs, hardware simulation programs, and various data handling, listing, edit, and code conversion programs.

1. CSC

Column Selector Program provides capability of selecting up to 10 columns of data from an input tape of any consistent format and reformatting them to be output on either the TTY or HSP. (operator selection)

2. LIBM

List IBM - Provides a means of quickly listing the output of the Pencil Follower which is in IBM code on the PDP-8 Teletype. (ASCII). The output format is defined by the operator. Pencil Follower function codes are interpreted as TTY carriage controls.

3. FEYE

Dump program for the DL-622. Reads buffer full of data from magnetic tape and prints contents on PDP-8 TTY.

4. THSR

Test High Speed reader. Tests for known sequence of character and sends alarm if not encountered.

5. FINTP

Routine to load core buffer from punched paper tape containing specially formatted data.

6. DUMP
Program to dump special binary packed data onto paper tape.
7. RCON
Program to unpack special binary packed data from paper tape and load into field 1 of PDP-8.
8. SIMCYP
Simulate operation of data transfer from DL-622 to CIPHER utilizing PCI and parity characters as counting triggers. Transfers 800 characters of DL-622 data.
9. DGNOS
IOT exerciser for DL-622. Provides capability for specifying delay in microseconds following "Read Initiation" IOT.
10. EDITS
Tape copy with edit program. Copies punched paper tape with character string substitution when operator specified character sequence is encountered.
11. SIMCYP (Modified)
DL-622 data transfer to CIPHER simulator. Same as SIMCYP with the following exception: Counting triggered by occurrence of PCI and parity characters. A count is maintained until '10' scans have been detected, at which time a 'data block' is transferred.
12. SSI
DL-622 recovery test program. Utilizes PCI and parity for detection of BAD SCANS. Function is to search and log bad scans. Longitudinal parity checking is basis for identification.

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