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COMPUTER PROGRAM FOR SOLVING MATHEMATICAL
PROGRAMS WITH NONLINEAR PROGRAMS IN THE
CONSTRAINTS

Jerome Bracken
James T. McGill

March 1972



INSTITUTE FOR DEFENSE ANALYSES
PROGRAM ANALYSIS DIVISION

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ABSTRACT

This paper documents a computer program to be used in solving nonlinear programming problems with nonlinear programming problems in the constraints. The program, named `INSUMT`, is used with the standard program, named `SUMT`, which implements the sequential unconstrained minimization technique for nonlinear programming. `SUMT` calls `INSUMT` when it is necessary to solve a nonlinear program in a constraint. The `INSUMT` program, together with a fairly complete example of its use, is included in the documentation.

Theory and applications of the models which can be solved using this program are treated in two companion papers.

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INTRODUCTION

Reference [2] formulates a class of mathematical programs with optimization problems in the constraints. Reference [3] gives models of a number of defense problems which can be approached using the techniques. The present paper documents a computer program for solving mathematical programs with nonlinear programming problems in the constraints.

Fiacco and McCormick in Reference [4] present theory and computational aspects of the sequential unconstrained minimization technique (SUMT) for nonlinear programming. Applications of nonlinear programming to a number of practical problems are presented in Reference [1]. The initial computer program for SUMT is documented in Reference [5]. The most recent computer program for SUMT is documented in Reference [6]. It handles equality constraints, and includes routines for computation of first and second partial derivatives using function values. Various unconstrained minimization methods are available in the SUMT program, requiring function values and first partial derivatives or requiring function values and first and second partial derivatives. Numerical differentiation routines facilitate the use of various unconstrained minimization methods. These are necessary in solving mathematical programs with nonlinear programs in the constraints, since the partial derivatives of the constraints cannot be stated explicitly.

The present paper is designed to be a supplement to Reference [6] and to be used with it.

PROBLEM DESCRIPTION AND SUMMARY OF COMPUTATIONAL PROCEDURE

One of the mathematical programs considered in Reference [2] is to choose vectors $x = (x_1, \dots, x_n)$ and $v^i = (v_1^i, \dots, v_{k_i}^i)$ for $i = 1, \dots, m$ to

$$\begin{array}{l} \text{minimize } f(x) \\ x \in X \end{array}$$

subject to

$$h_i(x) = \min_{v^i \in V^i} g^i(x, v^i) \geq 0, \quad i = 1, 2, \dots, m.$$

It is shown there that if $g^i(x, v^i)$ is concave in x on a convex set X for every $v^i \in V^i$, then $h_i(x)$ is concave on X where the convex set X may be defined by inequality and/or equality constraints. If, in addition, $f(x)$ is a convex function on X , then the mathematical program is convex.

To outline the computational technique, it is useful to differentiate between the "outside program,"

$$\begin{array}{l} \min f(x) \\ x \in X \end{array}$$

subject to

$$h_i(x) \geq 0, \quad i = 1, 2, \dots, m,$$

and the "with inside program,"

$$\min_{v^i \in V^i} g^i(x, v^i).$$

When convenient the distinction among the m inside programs will be dropped, considering only the generic problem:

$$\min_{v \in V} g(x, v) .$$

The constraint functions $h_i(x)$ in the outside problem are implicit in that their values depend upon the solution of the inside problem, which in turn depends upon the value of x . Thus, a solution technique for the overall problem must not rely on an explicit functional form for $h_i(x)$.

The computer program described in this paper, called INSUMT, is based on SUMT. The standard SUMT program is used for the outside problem. The new INSUMT program is used to solve the inside problem. SUMT and INSUMT are iterative routines. Let x^k denote the value of x for iteration k of the outside problem and v^ℓ denote iteration ℓ for the inside problem. The solution procedure is initialized by the user supplying x^0 and v^0 .

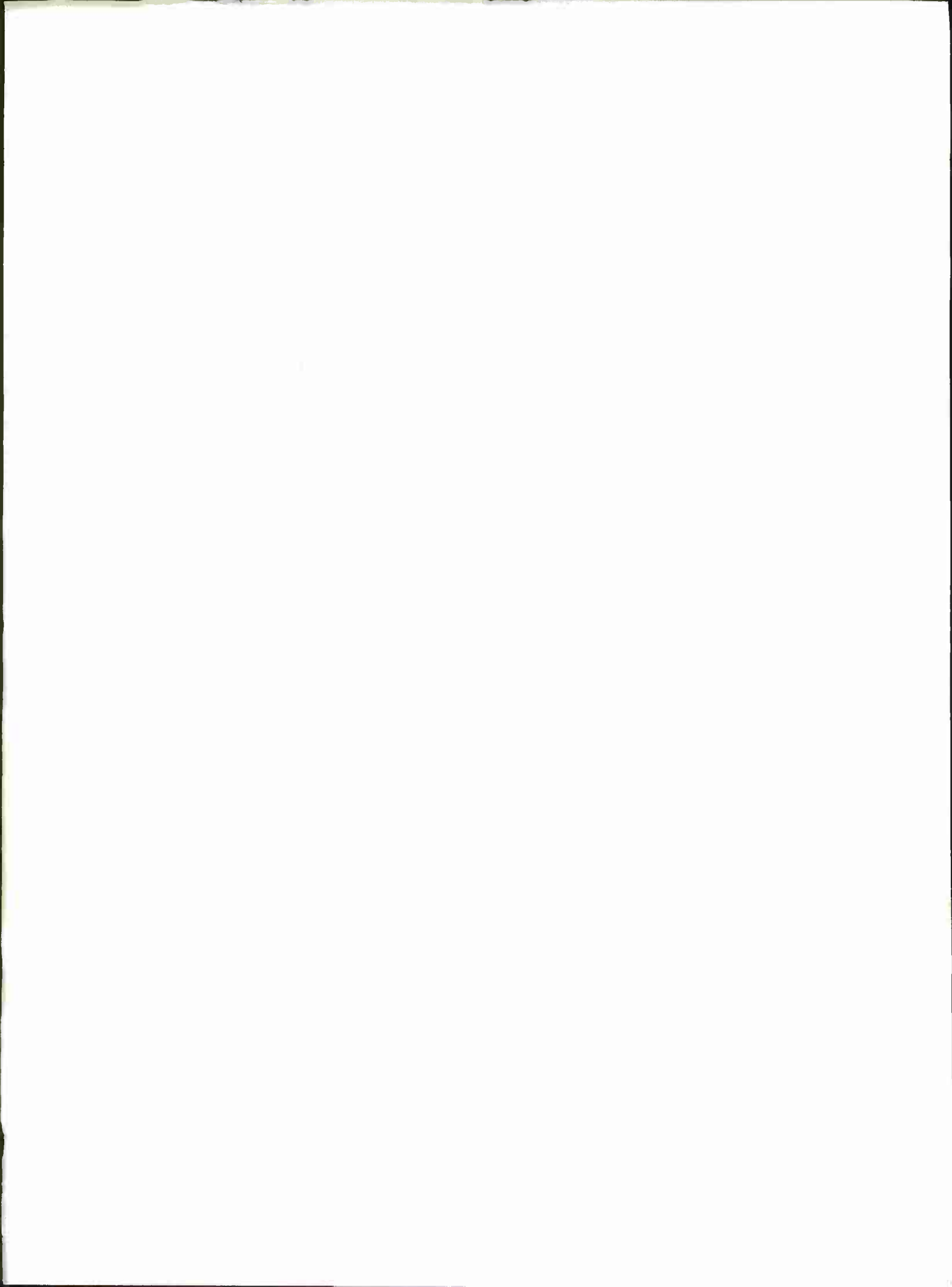
During iteration k for the outside problem, the inside problem is to choose $v \in V$ to minimize $g(x^k, v)$. At iteration ℓ of the inside problem, there is a value v^ℓ . This value is then used by INSUMT to generate a new value $v^{\ell+1}$. Continuing in this manner, the inside problem is solved, yielding $v^*(x^k)$ such that

$$g(x^k, v^*(x^k)) = \min_{v \in V} g(x^k, v) .$$

Control then passes to the outside problem which generates a new solution value x^{k+1} . The process is repeated until the sequence $\{x^k\}$ converges.

The routine INSUMT resides in core with SUMT and may be called to evaluate more than one constraint of the outside problem. For each such constraint, user-supplied subroutines of INSUMT provide information concerning the functional form of $g^i(x, v^i)$ and of the constraint set V^i . User-supplied subroutines for SUMT give analogous information for $f(x)$ and X .

The SUMT and INSUMT programs iteratively generate values of the solution variable by optimizing an unconstrained penalty function. Specifically, iteration $\ell + 1$ generates $v^{\ell+1}$ from v^{ℓ} and from the functional forms of the objective function of the inside program and the constraint set. The computation calls for the first, and sometimes second, partial derivatives of the relevant functions. These may be specified by the user. However, for the outside problem, explicit derivatives of $h_i(x)$ cannot be given. SUMT uses numerical differentiation in this case to approximate the derivatives to be used in generating x^{k+1} from x^k .



DESCRIPTION OF COMPUTER PROGRAM

The program is modular in structure. It consists basically of the SUMT program (subroutines MAIN through XMOVE), one set of user-supplied subroutines containing function evaluations for the outside mathematical program (READIN, RESTNT, GRAD1 and MATRIX), the INSUMT program (subroutines SUB through XMOVES), and one set of user-supplied subroutines containing function evaluations for the one or more inside mathematical programs (READIX, RESTNX, GRAD1S and MATRXX).

The workings of the SUMT subroutines MAIN through XMOVE are described in Reference [6]. The user-supplied subroutines for the outside program perform the following tasks when used with INSUMT.

READIN reads in the parameters used in evaluating the objective function and constraints.

RESTNT evaluates the objective function and constraints of the outside program for a value of x . Except where constraints contain inside mathematical programs, the functions are evaluated for the current value of x . Otherwise, RESTNT calls SUB, identifying the inside mathematical program to be solved for the current value of x , and SUB returns the value $v^*(x)$. RESTNT then evaluates $h(x) = g(x, v^*(x))$.

GRAD1 evaluates the first partial derivatives of the objective function and constraints in two ways. Where the constraints do not contain inside mathematical programs, for the current x the first partial derivatives are evaluated either explicitly or using numerical differentiation. Otherwise, GRAD1 uses DIFF1 to numerically differentiate the constraints which include inside mathematical programs.

MATRIX evaluates the second partial derivatives of the objective function and constraints of the outside mathematical program in two

ways. Where the constraints do not contain inside mathematical programs, for the current x the second partial derivatives are evaluated either explicitly or using numerical differentiation. Otherwise, MATRIX uses DIFF2 to numerically differentiate the constraints which include inside mathematical programs.

The INSUMT program consists of subroutines SUB through XMOVES. Three basic changes are made in SUMT to result in INSUMT.

First, all subroutine names are changed by simply adding an S to the end except where this results in more than six letters, in which case the last two letters are replaced by X (e.g., OPT becomes OPTS and RESTNT becomes RESTNX). All calls to subroutines within INSUMT are changed to include the revised names of the called subroutines. SUMT calls INSUMT only by RESTNT calling SUB. After SUB is called INSUMT calls only the subroutines within INSUMT until SUB returns to RESTNT.

Second, all labeled COMMON arrays are changed by adding an S to the end, except where this results in more than six letters, in which case the last two letters are replaced by X (e.g., SHARE becomes SHARES and CONPAR becomes CONPAX).

Third, subroutine MAIN is modified to obtain subroutine SUB by changing the name from MAIN to SUB, and by modifying the subroutine to read in data for the inside mathematical programs the first time each problem is solved and to save the data in an array for subsequent solutions of the problems.

User-supplied subroutines READIX, RESTNX, GRADIS and MATRXX are called by INSUMT. Depending on a parameter denoting the inside mathematical program being solved, the subroutines calculate appropriate function values, first partial derivatives, or second partial derivatives.

In the first several runs of a program, it is necessary to observe the intermediate points of the solution of the inside programs. RESTNT thus prints out which inside program is being solved, and INSUMT prints out the intermediate points. After it

is established that the inside programs are being successfully solved, printing of their points is suppressed by deleting the print statements from RESTNT and by modifying BODYD, CONVRX, ESTIMS, FEASS, INVERX, OPTS, OUTPDX, PUNCHS, TECHECX and TIMECS. The modifications are described later.

EXAMPLE PROBLEM

An example problem is used to describe the use of INSUMT with SUMT.

The problem is to choose $x_1, \dots, x_n$ to

$$\text{minimize } x_1 + \dots + x_n$$

subject to

$$\left[\begin{array}{l} \text{minimum} \\ v_1 + \dots + v_n \leq n \end{array} x_1(v_1 - 2)^2 + \dots + x_n(v_n - 2)^2 \right] \geq r_1$$

$$\left[\begin{array}{l} \text{minimum} \\ v_1^2 + \dots + v_n^2 \leq n \end{array} x_1^{.5} (v_1 - 2)^2 + \dots + x_n^{.5} (v_n - 2)^2 \right] \geq r_2 .$$

To illustrate the problem, take $n = 4$, $r_1 = 4$, $r_2 = 4$ and let the starting point be $x_1 = x_2 = x_3 = x_4 = 4$ for the outside program and $v_1 = v_2 = v_3 = v_4 = .5$ for both inside programs. The value of the objective function of the outside program is

$$x_1 + x_2 + x_3 + x_4 = 16 .$$

The objective function of the first inside program is

$$4(.5 - 2)^2 + 4(.5 - 2)^2 + 4(.5 - 2)^2 + 4(.5 - 2)^2 = 36 ,$$

and since $.5 + .5 + .5 + .5 = 2 \leq 4$ the first inside program starting point is feasible. For the second inside program the objective

function is

$$2(.5 - 2)^2 + 2(.5 - 2)^2 + 2(.5 - 2)^2 + 2(.5 - 2)^2 = 18 ,$$

and since $.5^2 + .5^2 + .5^2 + .5^2 = 1 \leq 4$ the second inside program starting point is feasible. Since $36 > r_1 (= 4)$, $18 > r_2 (= 4)$, the three starting points, one outside and two inside, provide a feasible point for all three programs.

The solution to the example is $x_1 = x_2 = x_3 = x_4 = 1$, with $v_1 = v_2 = v_3 = v_4 = 1$ in both inside programs, yielding

$$x_1 + x_2 + x_3 + x_4 = 1 + 1 + 1 + 1 = 4$$

and

$$\left[\begin{array}{l} 1(1 - 2)^2 + 1(1 - 2)^2 + 1(1 - 2)^2 + 1(1 - 2)^2 = 4 \\ \text{s.t.} \\ (1 + 1 + 1 + 1 = 4) \leq 4 \end{array} \right] \geq 4$$

$$\left[\begin{array}{l} 1^2(1 - 2)^2 + 1^2(1 - 2)^2 + 1^2(1 - 2)^2 + 1^2(1 - 2)^2 = 4 \\ \text{s.t.} \\ (1^2 + 1^2 + 1^2 + 1^2 = 4) \leq 4 \end{array} \right] \geq 4 .$$

COMPUTER PROGRAM INCLUDING USER-SUPPLIED SUBROUTINES FOR EXAMPLE PROBLEM

In this section the user-supplied subroutines for the outside program are presented, followed by the INSUMT program, followed by the user-supplied subroutines for the inside programs. The SUMT program is not supplied, being documented in Reference [6].

RESTNT contains print statements for each call for solution and end of solution of an inside program. The INSUMT subroutines SUB, BODYIS, CHCKEX, CONVRX, ESTIMS, FEASS, INVERX, OPTS, OUTPUS, PUNCHS, TCHECX and TIMECS contain print statements for the points of the inside programs. Thus the program as listed prints all intermediate points. However, the changes necessary to suppress printing are indicated in heavy markings directly on the listing. Simply remove the boxed statements, and replace them by the statements written, if any. Printing is done by SUB only the first time it is called, so SUB is not modified. Printing is done by CHCKEX only if control cards dictate, so it is not modified. All other print statements are removed to suppress printing of inside programs.

It should be noted that all data of the example problem are contained in the set of user-supplied subroutines for the outside program (READIN, RESTNT, GRAD1, and MATRIX) and the set of user-supplied subroutines for the inside programs (READIX, RESTNX, GRAD1S and MATRXX). For most problems READIN should be used to read data for outside and inside programs, since READIX is called each time an inside program is solved, and it should not be used.

Control cards are read in the following order: Outside program, first inside program, second inside program, and so on if there are

more inside programs. SUB saves the control card data, and inside program starting points, and provides these data each time an inside program is solved. Dimensions on PARS are presently (2,47), including up to 20 variables in the inside program. This would have to be changed for more than 2 inside programs or 20 variables.

User-Supplied Subroutines for Outside Program

```
000002      SUBROUTINE READIN
000002      9999 CONTINUE
000002      RETURN
000003      END
```

```

000005      SUBROUTINE RESTNT(IN,VAL)
000005      COMMON/PROB/ISP
000005      COMMON/OUT/NOUT,XT(100),V(100)
000005      COMMON/SHARE/ X(100), DEL(100), A(100,100),N,M, MN,NP1,NM1
10000070
000005      FN=N
000005      VAL=0.
000006      IF(IN)100,100,200
000007      100 DO 150 J=1,N
000010      150 VAL = VAL + X(J)
000012      GO TO 9999
000016      200 IF(IN-1)300,300,400
000020      300 ISP=1
000021      NOUT=N
000023      DO 305 J=1,N
000024      IF(X(J)=0.)304,304,305
000027      304 X(J)=1.E-08
000031      305 XT(J)=X(J)
000036      WRITE(6,307)
000041      307 FORMAT(11H1 PROBLEM A)
000041      CALL SUB
000042      WRITE(6,308)
000046      308 FORMAT(18H1 PROBLEM A SOLVED)
000046      DO 320 J=1,N
000051      320 VAL = VAL + X(J)*(V(J)-2.)**2
000057      VAL=VAL-4.
000061      GO TO 9999
000061      400 ISP=2
000062      NOUT=N
000064      DO 405 J=1,N
000065      IF(X(J)=0.)404,404,405
000070      404 X(J)=1.E-08
000072      405 XT(J)=X(J)
000076      WRITE(6,407)
000102      407 FORMAT(11H1 PROBLEM B)
000102      CALL SUB
000103      WRITE(6,408)
000107      408 FORMAT(18H1 PROBLEM B SOLVED)
000107      DO 420 J=1,N
000112      420 VAL = VAL + X(J)**.5 * (V(J)-2.)**2
000124      VAL=VAL-4.
000126      GO TO 9999
000126      9999 CONTINUE
000126      RETURN
000127      END

```

```

000003      SUBROUTINE GRAD1(IN)
000003      COMMON/SHARE/ X(100), DEL(100), A(100,100),N,M, MN,NP1,NM1      10000070
000005      DO 50 J=1,N
000005      50 DEL(J)=0
000010      IF(IN) 100,100,200
000011      100 DO 150 J=1,N
000013      150 DEL(J) = 1.
000017      GO TO 9999
000017      200 CALL DIFF1(IN)
000020      GO TO 9999
000022      9999 RETURN
000023      END

```

```

000005      SUBROUTINE MATRIX(IN,L)
000005      COMMON/SHARE/ X(100), DEL(100), A(100,100),N,M, MN,NP1,NM1      10000070
000005      IF(IN)100,100,200
000006      100 L=L+1
000007          GO TO 9999
000010      200 CALL DIFF2(IN)
000011          GO TO 9999
000013      9999 CONTINUE
000013          RETURN
000014          END

```


INSUMT

```

SUBROUTINE SUB
C
C      MARCH 1971
C
C MAIN IS THE PROGRAM THAT INITIATES THE SUMT ALGORITHM. THE INPUT OF
C PARAMETERS, OPTIONS, AND STARTING POINT IS DONE IN MAIN. AFTER THE
C SOLUTION OF ONE NLP PROBLEM MAIN LOOKS FOR DATA FOR ANOTHER NLP PROB.
C
000002 COMMON/PROR/ISP
000002 COMMON/OUT/NOUT,XT(100),V(100)
000002 COMMON/IN/W(100)
C
000002 COMMON/SHARES/X(100), DEL(100), A(100,100),N,M, MN,NPI,NMI
000002 COMMON /EQALS/H, HI, MZ
000002 COMMON /OPTNSS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
000002 COMMON /VALUES/F,G,P,RSIGMA, RJ(200), RH0
000002 COMMON/CRSTS/OELX(100), DELX0(100), RHOIN,RATIO, EPSI, THETA0,
1 RSIG1, GI, XI(100), X2(100), X3(100), XR2(100), XR1(100), PR1,
2 PR2,P1, F1, RJ(200), DOTI, PGRAD(100), DIAG(100),
3 PREV3,AOELX, NTCTR, NUMINI, NPHASE, NSATIS
000002 COMMON/EXPDPX / NEXOP1, NEXOP2, XEP1, XEP2
C
000002 DIMENSION IFTS(2),PARS(2,47)
000002 DATA IFTS/0,0/
C
000002 DO 5 J=1,NOUT
000004 5 W(J)=XT(J)
000010 IF (IFTS(1,ISP)) 10,10,20
C
C      PARAMETER CARD
000012 10 REAO (5,50) EPSI,RHOIN,THETA0,RATIO,TMMAX,M,N,MZ
C
C      INITIAL X VECTOR CARD FORMAT
000036 REAO (5,60) (X(I),I=1,N)
000051 NTCTR=0
000052 NPI=N+1
000054 NMI=N-1
C SUBROUTINE READIN IS UNDER PROGRAMMER CONTROL
000055 CALL REAO1X
C OPTION CARD FOLLOWS PROGRAMMERS DATA
000056 REAO (5,80) NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
000106 WRITE (6,110)
000112 WRITE (6,120) N,M,MZ,TMMAX,RHOIN,RATIO,EPsi,THETA0
000136 WRITE (6,130)
000142 WRITE (6,80) NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
C--READ TOLERANCES
000172 READ (5,60) XEP1, XEP2
000202 WRITE (6,90)
000206 WRITE (6,70) XEP1, XEP2
C--READ SECOND OPTION CARD
000216 REAO (5,80) NEXOP1,NEXOP2
000226 WRITE (6,100)
000232 WRITE (6,80) NEXOP1,NEXOP2
000242 PARS (ISP, 1) = EPSI
000244 PARS (ISP, 2) = RHOIN
000246 PARS (ISP, 3) = THETA0
000247 PARS (ISP, 4) = RATIO
000251 PARS (ISP, 5) = TMMAX

```

```

000252      PARS (ISP, 6) = M
000254      PARS (ISP, 7) = N
000256      PARS (ISP, 8) = MZ
000260      DO 4 K=1,20
000262      L=K*8
000264      4 PARS (ISP,L) = X(K)
000271      PARS (ISP,29) = NT1
000273      PARS (ISP,30) = NT2
000275      PARS (ISP,31) = NT3
000277      PARS (ISP,32) = NT4
000301      PARS (ISP,33) = NT5
000303      PARS (ISP,34) = NT6
000305      PARS (ISP,35) = NT7
000307      PARS (ISP,36) = NT8
000311      PARS (ISP,37) = NT9
000313      PARS (ISP,38) = NT10
000315      PARS (ISP,39) = NT11
000317      PARS (ISP,40) = XEP1
000320      PARS (ISP,41) = XEP2
000322      PARS (ISP,42) = XEP3
000323      PARS (ISP,43) = NEXOP1
000325      PARS (ISP,44) = NEXOP2
000327      PARS (ISP,45) = NEXOP3
000331      PARS (ISP,46) = NEXOP4
000333      PARS (ISP,47) = NEXOP5
000335      IFTS(ISP)=1
000337      GO TO 25

```

C

```

000337      20 EPSI = PARS(ISP,I)
000341      RHOIN= PARS(ISP,2)
000343      THETA0 = PARS(ISP,3)
000344      RATIO = PARS(ISP,4)
000346      TMMAX = PARS(ISP,5)
000347      M = PARS(ISP,6)
000351      N = PARS(ISP,7)
000353      MZ= PARS(ISP,8)
000355      DO 21 K=1,20
000357      L = K*8
000361      21 X(K) = PARS(ISP,L)
000366      NT1 = PARS(ISP,29)
000370      NT2 = PARS(ISP,30)
000372      NT3 = PARS(ISP,31)
000374      NT4 = PARS(ISP,32)
000376      NT5 = PARS(ISP,33)
000400      NT6 = PARS(ISP,34)
000402      NT7 = PARS(ISP,35)
000404      NT8 = PARS(ISP,36)
000406      NT9 = PARS(ISP,37)
000410      NT10 = PARS(ISP,38)
000412      NT11 = PARS(ISP,39)
000414      XEP1 = PARS(ISP,40)
000415      XEP2 = PARS(ISP,41)
000417      XEP3 = PARS(ISP,42)
000420      NEXOP1=PARS(ISP,43)
000422      NEXOP2=PARS(ISP,44)
000424      NEXOP3=PARS(ISP,45)
000426      NEXOP4=PARS(ISP,46)
000430      NEXOP5=PARS(ISP,47)

```

000432	GO TO 25	
000433	C 25 CALL SETS(TMMAX)	
000435	CALL TIMECS	
000436	NPHASE=4	000470
	C --- JUST TO GET AN INITIAL PRINTOUT	000480
000437	CALL EVALUS	
000440	P0=0.0	000500
000441	G=0.0	000510
000442	H=0.0	000520
000443	RSIGMA=0.0	000530
000444	CALL OUTPUX (2)	
000445	CALL STORES	
000446	IF (NEXOP1.GT.1) CALL CHCKEX	
000452	IF (NEXOP1.EQ.3) STOP 01072	000570
000456	IF (NEXOP1.EQ.5) STOP 01104	000580
000462	CALL FEASS	
	C NPHASE=5 IS USED TO INDICATE NO FEASIBLE POINT EXIST	000600
000463	GO TO (30,30,30,30,10), NPHASE	000610
000474	30 NPHASE=2	000620
000475	NTCTR=0	000630
000476	CALL BODY5	
000477	DO 35 J=1,N	
000501	35 V(J)=X(J)	
000505	RETURN	
	C	000670
	C PARAMETER CARD	000680
000505	50 FORMAT (5E12.0,3I4)	000690
	C INITIAL X VECTOR CARD FORMAT	000700
000505	60 FORMAT (6E12.6)	000710
000505	70 FORMAT (6E20.7)	000720
	C OPTION CARD FORMAT	000730
000505	80 FORMAT (10I7)	000750
000505	90 FORMAT (13H0 TOLERANCES)	000760
000505	100 FORMAT (26H0 SECOND SET OF OPTIONS)	000770
000505	110 FORMAT (56H1 NONLINEAR PROGRAMMING ROUTINE-SUMT VERSION 4 SUB	
	I)	000790
000505	120 FORMAT (1H0,5X,2HN=13,6X,2HM=13,6X,3HMZ=13//8X,10HMAX. TIME=E14.7,	000800
	14X,2HR=E14.7,4X,6HRATIO=E14.7,6X,8HEPSILON=E14.7,4X,6HTHETA=E14.7)	000810
000505	130 FORMAT (18H0 OPTIONS SELECTED)	000820
000505	END	000830

```

SUBROUTINE BODY5
C
C      OCTOBER 1970
C
C BODY COORDINATES THE FLOW AMONG THE SUBROUTINES THAT ACTUALLY DO THE
C CALCULATIONS REQUIRED BY THE VARIOUS PARTS OF THE ALGORITHM.
COMMON/SHARES/X(100), OEL(100), A(100,100), N,M, MN,NP1,NM1
COMMON /OPTNSS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
COMMON /VALUES/F,G,P,RSIGMA, RJ(200), RHO
COMMON/CRSTS/OELX(100), OELX0(100), RHOIN,RATIO, EPSI, THETA0,
1 RSIG1, G1, X1(100), X2(100), X3(100), XR2(100), XR1(100), PR1,
2 PR2,P1, F1, RJ1(200), OOTT, PGRAS(100), DIAG(100),
3 PREV3, ADELX, NTCTR, NUMINI, NPHASE, NSATIS
COMMON/CONPAX / NF1, NF2,NF3
NF2=2
NF3=2
MN=0
NUMINI=0
C OPTION OF GETTING INITIAL RHO
CALL RHOCOX
CALL EVALUS
10 CALL XMOVES
GO TO (30,20), NT3
20 CALL TIMECS
CALL OUTPUX (1)
GO TO 40
30 CALL TCHECX
C IN FEASIBILITY PHASE 4 MEANS FEAS ACHIEVED
40 GO TO (50,50,50,200), NSATIS
50 CALL CONVRX (N1)
GO TO (60,10,130), N1
C MINIMUM ACHIEVED IF N1=1
60 GO TO (70,80), NT3
70 CALL TIMECS
CALL OUTPUX (1)
C --- NUMBER OF MINIMA ACHIEVED INCREASED BY 1
80 NUMINI=NUMINI+1
MN=0
GO TO (190,90,90), NPHASE
90 CALL ESTIMS
C FINAL MIGHT HAVE BEEN CALLED BY ESTIM_CONVERGED IF N2=1
GO TO (100,110,120), NT4
C NT4=1 FINAL CONVERGENCE ON 0 ORDER ESTIMATES, NT4=2 CONVERGE ON FIRS
C ORDER ESTIMATES, NT4=3 CONVERGE ON SECONO ORDER ESTIMATES.
100 CALL FINALS(NF1)
GO TO (130,140), NF1
110 GO TO (130,140), NF2
120 GO TO (130,140), NF3
130 RETURN
140 RHO=RHO/RATIO
C USING PREVIOUSLY COMPUTED VALUES FOR F, AND RJ P IS RECOMPUTED WITH THE
C NEW VALUE OF RHO.
CALL PEVALX
C A VECTOR IS LEFT IN DELX(1) BY ESTIM
IF (NUMINI-2) 10,150,150
150 GO TO (10,190,160), NT7
160 CALL GRAOS(2)

```

000144	CALL OPTS		001420
000145	GO TO (180,170), NT3		
000153	170 WRITE (6,210)	170 CONTINUE	
000157	CALL OUTPUX (1)		001450
000161	180 GO TO 50		001460
000162	190 IF (G) 90,90,200		001470
000164	200 RETURN		001480
	C --- DUAL VALUE GREATER THAN 0 MEANS NO FEASIBLE POINT EXISTS		001490
	C		001500
000165	210 FORMAT (6X,30HMOVED ON EXTRAPOLATION VECTOR)		001510
000165	END		

	SUBROUTINE CHCKEX	
C		001530
C	MARCH 1971	001540
C		001550
C	CHCKEX COMPUTES AND LIST THE FIRST PARTIAL DERIVATIVES USING GRAD1	001560
C	AND THEN USING NUMERICAL DIFFERENCING (DIFF1). IF REQUESTED THE	001570
C	SECOND PARTIAL DERIVATIVES ARE COMPUTED AND LISTED USING MATRIX AND	001580
C	DIFF2.	001590
000002	COMMON/SHARES/X(100), DEL(100), A(100,100),N,M, MN,NP1,NM1	
000002	COMMON /EQALS/H, H1, MZ	
000002	MMZ=1+M+MZ	001630
000005	DO 5 J=1,N	001640
000006	DEL(J)=1.2345678	001650
000010	5 CONTINUE	001660
000012	DO 10 I=1,MMZ	001670
000013	IN=I-1	001680
000015	WRITE (6,170) IN	001690
000022	CALL GRAD1S(IN)	
000024	WRITE (6,180) (DEL(J),J=1,N)	001710
000037	CALL DIFF1S(IN)	
000041	WRITE (6,182) (DEL(J),J=1,N)	001730
000054	10 CONTINUE	001740
	C...SOMETIMES ONLY FIRST DERIVATIVES ARE TO BE CHECKED	001750
000057	IF (NEXOP1.LT.4) GO TO 160	001760
000061	WRITE (6,190)	001770
000065	DO 150 I=1,MMZ	001780
000067	IN=I-1	001790
000071	WRITE (6,170) IN	001800
000076	IT=2	001810
000077	DO 30 K=1,N	001820
000101	DO 20 J=1,N	001830
000102	20 A(K,J)=0.	001840
000110	30 CONTINUE	001850
000112	CALL MATRX(X(IN,IT)	
000114	IF (IT.EQ.1) GO TO 150	001870
000116	DO 50 K=2,N	001880
000120	KM1=K-1	001890
000122	DO 40 J=1,KM1	001900
000123	IF (A(K,J).EQ.0.0) GO TO 40	001910
000126	NEXOP1=5	
000127	WRITE (6,210) K,J	001930
000137	GO TO 60	001940
000140	40 CONTINUE	001950
000143	50 CONTINUE	001960
000145	60 DO 90 K=1,N	001970
000147	DO 70 J=K,N	001980
000150	IF (A(K,J).NE.0.0) GO TO 80	001990
000153	70 CONTINUE	002000
000156	WRITE (6,220) K	002010
000163	GO TO 90	002020
000164	80 WRITE (6,200) K,(A(K,J),J=1,N)	002030
000203	90 CONTINUE	002040
000206	DO 110 K=1,N	002050
000207	DO 100 J=1,N	002060
000210	100 A(K,J)=0.	002070
000216	110 CONTINUE	002080
000220	WRITE(6,115) IN	002090

		SUBROUTINE CONVRX (N1)	
	C		002330
	C	OCTOBER 1970	002340
	C		002350
	C	AFTER EACH ITERATION OF THE ALGORITHM TO LOCATE THE MINIMUM OF THE	002360
	C	PENALTY FUNCTION, CONVRG DETERMINES IF THE CURRENT POINT IS CLOSE	002370
	C	ENOUGH TO THE POINT GIVING THE MINIMUM VALUE OF THE P FUNCTION.	002380
	C	N1 SET EQUAL TO 1 IF MINIMUM HAS BEEN FOUND.	002390
	C	N1 SET EQUAL TO 2 IF MINIMUM HAS NOT BEEN FOUND AND TIME IS NOT UP	002400
	C	N1 SET EQUAL TO 3 OTHERWISE	002410
000003	C	DOIT SET EQUAL TO (DEL P) (INVERSE (DEL (DEL P))) (DEL P) IN OPT	002420
000003		COMMON/SHARES/X(100), OEL(100), A(100,100), N,M, MN,NP1,NM1	
000003		COMMON /OPTNSS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10	
000003		COMMON /VALUES/F,G,P0,RSIGMA, RJ(200), RHO	
		COMMON /CRSTS/OELX(100), DELX0(100), RHOIN,RATIO, EPSI, THETA0,	
	1	RSIG1, G1, X1(100), X2(100), X3(100), XR2(100), XR1(100), PR1,	
	2	PR2,P1, F1, RJ1(200), DOIT, PGRAO(100), DIAG(100),	
	3	PREV3, ADELX, NTCTR, NUMINI, NPHASE, NSATIS	
000003		COMMON/EXPORX / NEXOP1, NEXOP2, XEP1, XEP2	
000003		COMMON /TSWS/NSWW	
000003		N1=2	002520
000004		IF (MN.LE.1) Q1=P0	002530
000011		GO TO (10,20,30), NT9	002540
000020	10	IF (ABS(DOIT).LT.EPSI) GO TO 70	002550
000024		GO TO 40	002560
000024	20	IF (ABS(DOIT).LT.(P1-P0)/5.0) GO TO 70	002570
000032		GO TO 40	002580
000032	30	IF (ADELX.LT.EPSI) GO TO 70	002590
000035	40	GO TO (50,60), NSWW	002600
000043	50	IF (MN.LE.1) RETURN	002610
000047		IF (P0.XEP2.LT. Q1) GO TO 75	002620
000053		WRITE (6,80)	
000056		GO TO 70	002640
000060	60	CALL PUNCHS	
000061		WRITE (6,90)	
000065		N1=3	002670
	C		002680
000067	C	FOUND THE MINIMUM TO THE SUBPROBLEM.	002690
		RETURN	002700
000070	70	N1=1	002710
000071	75	Q1 = P0	002720
000073		RETURN	002730
	C		002740
000073	80	FORMAT (100H APPARENTLY ROUNDOFF ERRORS PREVENT A MORE ACCURATE DE	002750
		TERMINATION OF THE MINIMUM OF THIS SUBPROBLEM.)	002760
000073	90	FORMAT (48H**** TIME IS UP, CALLING EXIT FROM CONVRG. ****)	002770
000073		END	002780

	SUBROUTINE DIFF15(IN)	
C		002800
C	FEBUARY 1971	002810
C		002820
C	DIFF1 COMPUTES THE FIRST DERIVATIVES BY NUMERICAL DIFFERENCING.	002830
C		002840
C	--USER CAN CALL FOR DIFFERENCING OF SELECTED FUNCTIONS	002850
000003	COMMON/SHARES/X(100), DEL(100), A(100,100), N,M, MN,NP1,NM1	
000003	COMMON/EXPOPX / NEXOP1, NEXOP2, XEP1, XEP2	
000003	COMMON/ STIRXS, XSTR(100) , xSSS(100), DDLL(100)	
000003	DO 10 J=1,N	002890
000005	10 XSTR(J)=X(J)	002900
000011	DO 30 J=1,N	002910
000012	IF (J.EQ.1) GO TO 20	002920
000014	JM1=J-1	002930
000015	X(JM1)=XSTR(JM1)	002940
000017	20 X(J)=XSTR(J)+XEP1	002950
000022	CALL RESTNX (IN,ZZ2)	
000024	X(J)=XSTR(J)-XEP1	002970
000027	CALL RESTNX (IN,ZZ1)	
000032	30 DEL(J)=(ZZ2-ZZ1)/(2.*XEP1)	002990
000042	X(N)=XSTR(N)	003000
000043	RETURN	003010
000044	END	003020

	SUBROUTINE DIFF2S(IN)	
C		003040
C	OCTOBER 1970	003050
C		003060
C	DIFF2 COMPUTES THE SECOND DERIVATIVES BY NUMERICAL DIFFERENCING.	003070
C		003080
000003	COMMON/SHARES/X(100), OEL(100), A(100,100),N,M, MN,NP1,NM1	
000003	COMMON/EXPOPX / NEXOP1, NEXOP2, XEP1, XEP2	
000003	COMMON/ STIRXS/XSTR(100) , XSSS(100), ODLL(100)	
000003	DO 10 J=1,N	003120
000005	10 XSSS(J)=X(J)	003130
000011	DO 50 J=1,N	003140
000012	IF (J.EQ.1) GO TO 20	003150
000014	JM1=J-1	003160
000015	X(JM1)=XSSS(JM1)	003170
000017	20 X(J)=XSSS(J)+XEP1	003180
000022	CALL GRAD1S(IN)	
000023	DO 30 I=1,N	003200
000026	30 ODLL(I)=OEL(I)	003210
000032	X(J)=XSSS(J)-XEP1	003220
000035	CALL GRAD1S(IN)	
000036	DO 40 I=J,N	003240
000041	40 A(J,I)=(ODLL(I)-OEL(I))/(2.*XEP1)	003250
000057	50 CONTINUE	003260
000061	X(N)=XSSS(N)	003270
000062	RETURN	003280
000063	END	003290

```

SURROUTINE ESTIMS
C
C          OCTOBER 1970
C
C ESTIM PERFORMS THE COMPUTATIONS TO ESTIMATE THE LAGRANGE MULTIPLIERS
C AND MAKE THE FIRST- AND SECOND-ORDER ESTIMATES OF THE FINAL SOLUTION
C OF THE PROBLEM.
000002 COMMON/SHARES/X(100), OEL(100), A(100,100),N,M, MN,NP1,NM1
000002 COMMON /EQALS/H, H1, MZ
000002 COMMON /OPTNSS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
000002 COMMON /VALUES/F,G,P0,RSIGMA, RJ(200), RHO
000002 COMMON/CRSTS/OELX(100), DELX(100), RHOIN,RATIO, EPSI, THETA0,
1 RSIG1, G1, X1(100), X2(100), X3(100), XR2(100), XR1(100), PR1,
2 PR2,P1, F1, R1(200), DOTT, PGRAD(100), DIAG(100),
3 PREV3,ADELA, NTCTR, NUMINI, NPHASE, NSATIS
000002 COMMON/CONPAX / NF1, NF2,NF3
000002 CALL STORES
000003 Z10=RATIO**2
000005 Z9=RATIO
000006 Z1=1.0/Z9+1.0/Z10
000011 Z2=Z1+1./Z9**3
000014 Z3=1./Z9**3
000016 Z4=Z10+Z9
000017 Z5=Z9**3
000020 Z6=1.0/((Z10-1.0)*(Z9-1.0))
000024 Z7=1./Z9
000026 Z8=1./(Z9-1.)
000030 RQ=1.0/RHO
000032 IF (NUMINI-2) 150,80,10
000034 10 WRITE (6,340)
000040 P0=(PR2-Z4*PR1+Z5*P1)*Z6
000047 G=(RATIO*G1-GR1)/(RATIO-1.)
000053 DO 20 I=1,N
000055 20 X(I)=(XR2(I)-Z4*XR1(I)+Z5*X1(I))*Z6
000066 NP=NPHASE
000070 NPHASE=4
000071 CALL EVALUS
000072 NPHASE=NP
000074 CALL OUTPUX (2)
C CHECK TO SEE IF ESTIMATES HAVE CONVERGED
000075 GO TO (70,30,70), NPHASE
000103 30 DO 50 J=1,M
000106 IF (RJ(J)) 40,50,50
000110 IF (THETA0+RJ(J)) 70,50,50
000113 50 CONTINUE
000116 GO TO (70,70,60), NT4
000125 60 CALL FINALS(NF3)
000127 70 CONTINUE
000127 80 WRITE (6,340)
000133 G=(RATIO*G1-GR1)/(RATIO-1.)
000140 P0=(Z9*P1-P1)*Z8
000144 DO 90 I=1,N
000145 90 X(I)=(Z9*X1(I)-XR1(I))*Z8
000153 NP=NPHASE
000155 NPHASE=4
000156 CALL EVALUS
000157 NPHASE=NP

```

10 CONTINUE

80 CONTINUE

000161	CALL OUTPUX (2)	003880
000162	C CHECK TO SEE IF ESTIMATES HAVE CONVERGED	003890
000171	GO TO (140,100,140), NPHASE	003900
000171	100 DO 120 J=1,M	003910
000173	IF (RJ(J)) 110,120,120	003920
000175	110 IF (RJ(J)+THETA0) 140,120,120	003930
000200	120 CONTINUE	003940
000203	GO TO (140,130,140), NT4	
000212	130 CALL FINALS(NF2)	003960
000214	140 CONTINUE	
000214	150 WRITE (6,350)	003980
000220	IF (M) 180,180,160	003990
000222	DO 170 J=1,M	004000
000224	170 RJ(J)=RHO/RJ1(J)	004010
000230	180 IF (MZ) 210,210,190	004020
000232	190 DO 200 J=1,MZ	004030
000234	MNJ=M,J	004040
000236	200 RJ(MNJ)=2.*RJ1(MNJ)*RQ	004050
000244	210 GO TO (220,240), NT2	004060
000252	220 DO 230 I=1,N	004070
000254	230 X(I)=RHO/X1(I)	
000260	240 CALL OUTPUX (2)	
000262	CALL REJECX	
000263	IF (NUMINI-2) 280,300,250	004100
000266	250 GO TO (280,310,260), NT7	004110
	C SECONO ORDER MOVE FOR NEXT MINIMUM	004120
000275	260 DO 270 I=1,N	004130
000277	270 DELX(I)=Z1*X1(I)-Z2*XR1(I)+Z3*XR2(I)	004140
000310	280 PR2=PR1	004150
000312	GR2=GR1	004160
000313	PR1=PR1	004170
000314	GR1=GR1	004180
000315	DO 290 I=1,N	004190
000317	XR2(I)=XR1(I)	004200
000321	290 XR1(I)=X1(I)	004210
000324	RETURN	004220
000325	300 GO TO (280,310,310), NT7	004230
000334	310 DO 320 I=1,N	004240
000336	320 DELX(I)=(X1(I)-XR1(I))*Z7	004250
000343	GO TO 280	004260
	C	004270
000344	330 FORMAT (/26H0 2ND ORDER ESTIMATES)	004280
000344	340 FORMAT (/26H0 1ST ORDER ESTIMATES)	004290
000344	350 FORMAT (/25H0 LAGRANGE MULTIPLIERS)	004300
000344	ENO	004310

150 CONTINUE

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SUBROUTINE EVALUS
C
C          OCTOBER 1970
C
C IN THE NORMAL PHASE EVALU CALLS THE USER-SUPPLIED ROUTINES TO EVALUATE
C THE OBJECTIVE FUNCTION AND THE CONSTRAINT FUNCTIONS AT THE CURRENT
C POINT. IN THE FEASIBILITY PHASE THIS ROUTINE PUTS THE NEGATIVE SUM OF
C THE VIOLATED CONSTRAINTS IN LOCATION F.
000002 COMMON/SHARES/X(100), OEL(100), A(100,100),N,M, MN,NP1,NM1
000002 COMMON/EGALS/H, H1, MZ
000002 COMMON/DPNRS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
000002 COMMON/VALUES/F,G,P0,RSIGMA, RJ(200), RHO
000002 COMMON/CRSTS/DELX(100), OELX(100), RHOIN,RATIO, EPSI, THETA0,
1 RSIG1, G1, X1(100), X2(100), X3(100), XR2(100), XR1(100), PR1,
2 PR2, P1, F1, RJ1(200), ODTT, PGRAO(100), OIAG(100),
3 PREV3,AOELX, NTCTR, NUMINI, NPHASE, NSATIS
000002 H=0.0
000003 RSIGMA=0.0
000004 F=0.0
000005 NSATIS=2
000006 GO TO (10,100,190,200), NPHASE
C
C   =1 FEASIBILITY
C   =2 NORMAL
C   =3 GUESS
C   =4 ALL FUNCTIONS ARE TO BE EVALUATED
C FEASIBILITY
000016 10 GO TO (20,40), NT2
C NON-NEGATIVES INCLUDED
000024 20 DO 30 I=1,N
000026 IF (X(I)) 260,260,30
000030 30 RSIGMA=RSIGMA-RHO*ALOG(X(I))
000040 40 IF (M.EQ.0) GO TO 90
000041 DO 80 J=1,M
000043 CALL RESTNX (J,RJ(J))
000045 IF (RJ1(J).LE.0.0) GO TO 50
000047 IF (RJ(J).GT.0.0) GO TO 60
C VIOLATION OF A PREVIOUSLY SATISFIED CONSTRAINT
000052 GO TO 260
000052 50 IF (RJ(J).GT.0.0) GO TO 70
C ALL VIOLATED CONSTRAINTS ADDED INTO OBJECTIVE FUNCTION
000055 F=F-RJ(J)
000057 GO TO 80
000057 60 RSIGMA=RSIGMA-RHO*ALOG(RJ(J))
000064 GO TO 80
C INDICATES SATISFACTION OF CONSTRAINT(1ORMORE)
000065 70 NSATIS=1
000066 RSIGMA=RSIGMA-RHO*ALOG(RJ(J))
000073 80 CONTINUE
000076 90 CONTINUE
C EQUALITIES NOT COMPUTED IN FEAS. PHASE
000076 P0=F+RSIGMA
000100 G=F-RHO*FLOAT(M)
000105 IF (NT2.EQ.1) G=G-RHO*FLOAT(N)
000112 RETURN
C REGULAR PHASE
000113 100 GO TO (110,130), NT2
C NON NEGATIVITIES INCLUDED

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000121	110	DO 120 I=1,N	004890
000123		IF (X(I)) < 60.260, I20	004900
000125	120	RSIGMA=RSIGMA-RHO*ALOG(X(I))	004910
000135	130	IF (M.EQ.0) GO TO 150	004920
000136		DO 140 J=1,M	004930
000140		CALL RESTNX (J,RJ(J))	
000142		IF (RJ(J).LE.0.0) GO TO 260	004950
000144		RSIGMA=RSIGMA-RHO*ALOG(RJ(J))	004960
000151	140	CONTINUE	004970
		C EVALUATE AND ADD IN EQUALITY CONSTRAINTS	004980
000154	150	CONTINUE	004990
000154		CALL RESTNX (0,F)	
000156		IF (MZ) 180,180,180	005010
000160	160	DO 170 I=1,MZ	005020
000162		J=1+M	005030
000164		CALL RESTNX (J,RJ(J))	
		C ADD INTO THIRD TERM OF P FUNCTION	005050
000166		H=H+(RJ(J))**2	005060
000171	170	CONTINUE	005070
000174		H=H/RHO	005080
000175	180	P0=RSIGMA+H	005090
000177		P0=F+P0	005100
000201		G=2.*H-RHO*FLOAT(M)	005110
000205		G=G+F	005120
000207		IF (NT2.EQ.1) G=G-RHO*FLOAT(N)	005130
		C DUAL VALUE	005140
000214		RETURN	005150
		C GUESS PHASE NOT CONEO	005160
000215	190	RETURN	005170
		C== STRAIGHT FUNCTION EVALUATION (MAIN+FEAS ONLY)	005180
000216	200	CONTINUE	005190
000216		IF (M.EQ.0) GO TO 220	005200
000217		DO 210 I=1,M	005210
000221		CALL RESTNX (I,RJ(I))	
000223	210	CONTINUE	005230
000226	220	CALL RESTNX (0,F)	
		C EQUALITY CONSTRAINTS	005250
000230		IF (MZ) 250,250,230	005260
000232	230	DO 240 I=1,MZ	005270
000234		KZ=M+I	005280
000236	240	CALL RESTNX (KZ,RJ(KZ))	
000243	250	RETURN	005300
		C CONSTRAINTS VIOLATED NOT SO BEFORE	005310
000244	260	NSATIS=3	005320
000245		P0=10.E35	005330
000247		RETURN	005340
000247		END	005350

```

SUBROUTINE FEASS
C
C          OCTOBER 1970
C
C FEAS DETERMINES WHETHER THE STARTING POINT IS FEASIBLE. IF IT IS NOT,
C FEAS LOOKS FOR A FEASIBLE ONE. IF NONE EXISTS, A MESSAGE IS PRINTED
C AND CONTROL RETURNS TO MAIN.
000002 COMMON/SHARES/X(100), OEL(100), A(100,100), N,M, MN,NP1,NM1
000002 COMMON /OPTNSS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
000002 COMMON /VALUES/F,G,P,RSIGMA, RJ(200), RHO
000002 COMMON/CRSTS/DELX(100), OELX(100), RHOIN,RATIO, EPSI, THETA0,
1 RSIG1, G1, X1(100), X2(100), X3(100), XR2(100), XR1(100), PR1,
2 PR2,P1, F1, RJ1(200), OOTI, PGRAO(100), DIAG(100),
3 PREV3,ADELX, NCTR, NUMINI, NPHASE, NSATIS
000002 NPHASE=1
000003 GO TO (10,50), NT2
000011 10 NFIX=1
000012 00 30 I=1,N
000014 IF (X(I)) 20,20,30
000016 20 NFIX=2
000017 X(I)=1.E-05
000021 30 CONTINUE
000024 GO TO (50,40), NFIX
000032 40 NPHASE=NFIX
000033 CALL EVALUS
C JUST GET ALL CONSTRAINTS EVALUATED
000034 NPHASE=1
000035 WRITE (6,130)
000041 CALL OUTPUX (2)
000043 50 IF (M) 90,90,60
000045 60 DO 70 I=1,M
000047 IF (RJ(I)) 100,100,70
000051 70 CONTINUE
000054 80 CALL TIMECS
000055 WRITE (6,140)
000061 G=0.0
000062 CALL RESTNX (0,F)
000064 CALL OUTPUX (2)
000066 90 RETURN
000067 100 CALL BODY5
000070 00 110 I=1,M
000072 IF (RJ(I)) 120,120,110
000074 110 CONTINUE
000077 GO TO 80
000077 120 WRITE (6,150)
C TO INDICATE TO MAIN TO START ON NEXT PROBLEM.
000103 NPHASE=5
000104 GO TO 90
C
000105 130 FORMAT (1H0,2X,48HMADE VIOLATED NON-NEGATIVITYIES SLIGHTLY POSITIVE
1)
000105 140 FORMAT (51H0*****THE FEASIBLE STARTING POINT TO BE USED IS ...)
000105 150 FORMAT (3X,49HTHIS PROBLEM POSSESSES NO FEASIBLE STARTING POINT, W
1ILL LOOK FOR DATA FOR NEXT PROBLEM. )
000105 ENO

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SUBROUTINE FINALS(N2)		
C		005920
C	OCTOBER 1970	005930
C		005940
C	FINAL CONTAINS THE TESTS USED TO DETERMINE WHETHER A POINT SATISFIES	005950
C	THE FINAL CONVERGENCE CRITERION CHOSEN TO DETERMINE IF THE NLP	005960
C	PROBLEM HAS BEEN SOLVED.	005970
C	N2 SET EQUAL TO 1 IF CONVERGENCE CRITERION IS SATISFIED.	005980
C	N2 SET EQUAL TO 2 OTHERWISE.	005990
000003	COMMON/SHARES/X(100), DEL(100), A(100,100), N,M, MN,NP1,NM1	
000003	COMMON /OPTNSS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10	
000003	COMMON /VALUES,F,G,P,RSIGMA, RJ(200), RHO	
000003	COMMON/CRSTS/DELX(100), DELX0(100), RHOIN,RATIO, EPSI, THETA0,	
	1 RSIG1, G1, X1(100), X2(100), X3(100), XR2(100), XRI(100), PRI,	
	2 PR2,P1, F1, RJ1(200), DOT1, PGRAD(100), DIAG(100),	
	3 PREV3,ADELX, NTCTR, NUMINI, NPHASF, NSATIS	
	GO TO (10,20,30), NT5	006070
000012	10 EPSIL=ABS(F/G-1.)	006080
000016	IF (EPSIL-THETA0) 50,50,70	006090
000021	20 IF (ABS(RSIGMA)-THETA0) 50,50,70	006100
000025	30 IF (NUMINI-1) 50,40,40	006110
000030	40 PEST=PRI-(PRI-P0)/(1.-1./SQRT(RATIO))	006120
000040	EPSIL=ABS(PEST/G-1.)	006130
000043	IF (EPSIL-THETA0) 50,70,70	006140
000046	50 N2=1	006150
000047	GO TO (80,60), NT6	006160
000055	60 CALL PUNCHS	
000056	GO TO 80	006180
000060	70 N2=2	006190
000061	80 RETURN	006200
000062	END	006210

	SUBROUTINE GRAOS(IS)	
	C	006230
	C OCTOBER 1970	006240
	C	006250
	C GRAD COMPUTES THE GRADIENT OF THE PENALTY FUNCTION AND THE OUTER	006260
	C PRODUCT FACTORS OF THE MATRIX OF SECOND PARTIALS OF P.	006270
	C IF (IS=1) ACCUM. MATRIX OF 2ND PARTIALS IF (IS=2) COUNT	006280
000003	COMMON/SHARES/X(100), DEL(100), A(100,100), N,M, MN,NP1,NM1	
000003	COMMON /EQALS/H, M1, M2	
000003	COMMON /OPTNSS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10	
000003	COMMON /VALUES/F,G,P0,RSIGMA, RJ(200), RHO	
000003	COMMON/CRSTS/OELX(100), OELX0(100), RHOIN,RATIO, EPSI, THETA0,	
	1 RSIG1, G1, X1(100), X2(100), X3(100), XR2(100), XRI(100), PR1,	
	2 PR2, P1, F1, RJ1(200), DOTT, PGRAD(100), OIAG(100),	
	3 PREV3, AOELX, NTCTR, NUMINI, NPHASE, NSATIS	
000003	GO TO (10,30), IS	006370
000011	10 DO 20 I=1,N	006380
000013	DO 20 J=1,I	006390
000014	A(I,J)=0.	006400
000024	30 DO 40 J=1,N	006410
000026	40 OELX0(I)=0.	006420
	C THIS SECTION WORKS CORRECTLY IN FEASIBILITY PHASE AS WELL AS NORMAL PH	006430
000031	GO TO (50,80), NT2	006440
000037	50 DO 70 I=1,N	006450
000041	OELX0(I)=-RHO/X(I)	006460
000044	GO TO (60,70), IS	006470
000051	60 A(I,I)=(-OELX0(I)/X(I))	006480
000056	70 CONTINUE	006490
000061	80 CONTINUE	006500
000061	IF (M.LE.0) GO TO 180	006510
000063	DO 170 K=1,M	006520
000064	CALL GRAOIS(K)	
000065	IF (RJ(K).GT.0.0) GO TO 110	006540
	C ALL VIOLATED CONSTRAINT GRAOS ADDED TO OBJ. FUNCTION	006550
000071	DO 100 I=1,N	006560
000072	IF (OEL(I)) 90,100,90	006570
000073	90 OELX0(I)=OELX0(I)-OEL(I)	006580
000076	100 CONTINUE	006590
000101	GO TO 170	006600
000101	110 TT=RHO/RJ(K)	006610
000104	DO 160 I=1,N	006620
000105	IF (DEL(I)) 120,160,120	006630
	C IF OEL(I)=0 SKIP ALL THE FOLLOWING COMPUTATION INVOLVING * BY OEL(I)	006640
000106	120 T=TT*OEL(I)	006650
000111	DELX0(I)=DELX0(I)-T	006660
000113	GO TO (130,160), IS	006670
000120	130 T=T/RJ(K)	006680
000122	DO 150 JJ=1,I	006690
000124	IF (OEL(JJ)) 140,150,140	006700
000125	A(I,JJ)=A(I,JJ)+T*OEL(JJ)	006710
000133	150 CONTINUE	006720
000136	160 CONTINUE	006730
000141	170 CONTINUE	006740
	C EQUALITY CHANGES FOR GRAD	006750
000144	180 IF (M2.LE.0) GO TO 250	006760
000146	GO TO (250,190,250), NPHASE	006770
000155	190 RQ=2./RHO	006780

000157	DO 240 J=1,MZ	006790
000161	K=M*J	006800
000163	CALL GRADIS(K)	
000164	TT=RQ*RJ(K)	006820
000167	DO 230 I=1,N	006830
000171	IF (DEL(I),EQ,0,0) GO TO 230	006840
000172	DELX0(I)=DELX0(I)+DEL(I)*TT	006850
000175	GO TO (200,230), IS	006860
000203	200 T=RQ*DEL(I)	006870
000206	DO 220 JJ=I,I	006880
000207	IF (DEL(JJ)) 210*220*210	006890
000210	210 A(I,JJ)=A(I,JJ)+T*DEL(JJ)	006900
000216	220 CONTINUE	006910
000221	230 CONTINUE	006920
000224	240 CONTINUE	006930
000226	250 GO TO (260,280), IS	006940
000234	260 DO 270 I=1,N	006950
000236	270 DIAG(I)=A(I,I)	006960
000246	280 GO TO (290,330,290), NPHASE	006970
	C LEAVES NEGATIVE GRADIENT IN DELP	006980
000255	290 DO 300 I=1,N	006990
000257	300 DELX0(I)=-DELX0(I)	007000
000263	310 ADELX=0.	007010
000264	DO 320 I=1,N	007020
000266	320 ADELX=ADELX+DELX0(I)**2	007030
000273	ADELX=SQRT(ADELX)	007040
000275	RETURN	007050
000275	330 CALL GRADIS(0)	
000277	DO 340 I=1,N	007070
000302	340 DELX0(I)=-DELX0(I)-DEL(I)	007080
	C LEAVES THE NFG. GRAD OF P IN DELX0	007090
000307	GO TO 310	007100
000307	END	007110

		SUBROUTINE INVERX(NSME)	007130
	C		007140
	C	OCTOBER 1970	007150
	C		007160
	C	INVERS SOLVES THE SET OF EQUATIONS FOR THE MOVE-VECTOR USING THE	007170
	C	CROUT PROCEDURE. IF THE MATRIX IS NOT POSITIVE DEFINITE, A DIFFERENT	007180
	C	METHOD IS USED.	007190
	C		007200
	C	*****PERFORMING A L-U DECOMPOSITION OF THE MATRIX A, TAKING ADVANAGE OF	007210
	C	*****THE SYMMETRY OF THE A MATRIX.	007220
	C	*****IF A NON-POSITIVE PIVOT CANDIDATE IS GENERATED, THEN MCCORMICK'S	007230
	C	*****PROCEDURE IS USED(SEE PP. 167-168 IN FIACCO AND MCCORMICK).	007240
	C		007250
	C	*****IF NSME =1 WORKING WITH A NEW A MATRIX. IF NSME= 2 USING PREVIOUS	007260
	C	*****A MATRIX, BUT HAVE A NEW RIGHT-HAND-SIDE.	007270
	C		007280
000003	C	*****NINV IS THE NUMBER OF NON-POSITIVE PIVOT CANDIDATES GENERATED.	
000003		COMMON/SHARES/X(100), DEL(100), A(100,100), N,M, MN,NP1,NM1	
000003		COMMON /OPTNSS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10	
		COMMON/CRST5/DELX(100), OELX0(100), RHOIN,RATIO, EPSI, THETA0,	
		1 RSIG1, G1, X1(100), X2(100), X3(100), XR2(100), XR1(100), PR1,	
		2 PR2,P1, F1, RJ1(200), DOTT, PGRAD(100), DIAG(100),	
		3 PREV3,ADELX, NTCTR, NUMINI, NPHASE, NSATIS	
000003		COMMON/EXPORX / NEXOP1, NEXOP2, XEP1, XEP2	
000003		DIMENSION B(100)	007360
000003		EQUIVALENCE (B,DELX)	007370
000003		GO TO(20, 170), NSME	007380
000011	20	NINV=0	007390
000012		IF (A(1,1)) 40,30,50	007400
000014	30	NINV=1	007410
000015		GO TO 70	007420
000016	40	NINV=1	007430
000017	50	A(1,1)=1./A(1,1)	007440
000021		DO 60 I=2,N	007450
000022	60	A(1,I)=A(1,I)*A(1,1)	007460
000030	70	DO 160 J=2,N	007470
000032		JM1=J-1	007480
000034		T=0.	007490
000035		DO 90 I=1,JM1	007500
000036		IF (A(I,J)) 80,90,80	007510
000041	80	T=T+A(J,I)*A(I,J)	007520
000051	90	CONTINUE	007530
000054		A(J,J)=A(J,J)-T	007540
000060		IF (A(J,J)) 110,100,120	007550
000064	100	NINV=NINV+1	007560
000066		GO TO 170	007570
000066	110	NINV=NINV+1	007580
000070	120	A(J,J)=1./A(J,J)	007590
000074		IF (J.EQ.N) GO TO 170	007600
000076		JP1=J+1	007610
000100		DO 150 L=JP1*N	007620
000101		T=0.	007630
000102		DO 140 I=1,JM1	007640
000104		IF (A(I,J)) 130,140,130	007650
000107	130	T=T+A(L,I)*A(I,J)	007660
000117	140	CONTINUE	007670
000122		A(L,J)=A(L,J)-T	007680

000127		A(J,L)=A(L,J)*A(J,J)		007690
000136	150	CONTINUE		007700
000140	160	CONTINUE		007710
000142	170	CONTINUE		007720
000142		IF (NINV) 180,180,290		007730
000144	180	B(1)=B(1)*A(1,1)		007740
000146		DO 210 J=2,N		007750
000147		T=0.		007760
000150		JM1=J-1		007770
000152		DO 200 I=1,JM1		007780
000153		IF (A(J,I)) 190,200,190		007790
000156	190	T=T+A(J,I)*B(I)		007800
000164	200	CONTINUE		007810
000167		B(J)=(B(J)-T)*A(J,J)		007820
000174	210	CONTINUE		007830
000177		DO 240 I=1,NM1		007840
000200		NMK=N-I		007850
000202		DO 230 J=1,I		007860
000203		L=NM1-J		007870
000205		IF (A(NMK,L)) 220,230,220		007880
000210	220	B(NMK)=B(NMK)-A(NMK*L)*B(L)		007890
000216	230	CONTINUE		007900
000221	240	CONTINUE		007910
000223	250	GO TO (280,260), NT3		007920
000231	260	WRITE (6,430)	260 CONTINUE	
000235		WRITE (6,420) (DELX(I),I=1,N)		
000250	270	WRITE (6,440)	270 CONTINUE	
000254		WRITE (6,420) (DELX(I),I=1,N)		
000267	280	RETURN		007970
	C--	COMPUTE ORTHOGONAL MOVE		007980
000270	290	CONTINUE		007990
000270		DO 350 II=1,N		008000
000273		I=N-II+1		008010
000276		IF (A(I,I)) 310,300,320		008020
000302	300	H(I)=0.0		008030
000304		GO TO 350		008040
000304	310	B(I)=1.0		008050
000306		GO TO 330		008060
000307	320	B(I)=0.0		008070
000311	330	IP1=I+1		008080
000313		IF (IP1.GT.N) GO TO 350		008090
000316		DO 340 J=IP1,N		008100
000317	340	H(I)=B(I)-A(I,J)*B(J)		008110
000331	350	CONTINUE		008120
000334		GO TO 360		008130
	C--	CHECK MAYBE DO DIFF FOR P.S.D.		008140
000334	360	ZC2=0.0		008150
000335		DO 370 I=1,N		008160
000337	370	ZC2=ZC2+DELX(I)*B(I)		008170
000344		IF (ZC2) 380,400,400		008180
000345	380	DO 390 I=1,N		008190
000347	390	B(I)=-B(I)		008200
000353	400	WRITE (6,450)		
	C MCC	ZANGWILL ONE MOD		008220
000357		IF (NEXOP2.NE.2) GO TO 250		008230
000362		DO 410 K=1,N		008240
000364	410	B(K)=B(K)+DELX(K)		008250
000371		GO TO 250		008260

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000371      C
000371      420  FORMAT (7E17.8)
000371      430  FORMAT (1H0,6X,12HDEL P VECTOR)
000371      440  FORMAT (1H0,6X,24HSECOND ORDER MOVE VECTOR)
000371      450  FORMAT (1H0,6X,15HORTHOGONAL MOVE)
000371      END

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008270
008280
008290
008300
008310
008320

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	SUBROUTINE OPTS		
C			008340
C	MARCH 1971		008350
C			008360
C	OPT LOOKS FOR A MINIMUM ALONG THE SEARCH VECTOR USING THE GOLOEN		008370
C	SECTION SEARCH METHOD.		008380
000002	COMMON/SHARES/X(100), OEL(100), A(100,100),N,M, MN,NP1,NM1		
000002	COMMON /VALDES/F,G,P0,RSIGMA, RJ(200), RHO		
000002	COMMON/CRSTS/DELX(100), OELX(100), RHO1,N,RAT10, EPS1, THETA0,		
	1 RSIG1, G1, X1(100), X2(100), X3(100), XR2(100), XR1(100), PR1,		
	2 PR2,P1, F1, RJ1(200), DOTT, PGRAO(100), DIAG(100),		
	3 PREV3,AOELX, NTCTR, NUMIN1, NPHASE, NSAT1S		
000002	KSW=1		008450
000003	N405=1		008460
000004	P31=P0		008470
000006	ISW=1		008480
000007	DOTT=0.		008490
000010	DO 10 J=1,N		008500
000011	10 DOTT=0DOTT+OELX(J)*DELX(J)		008510
000016	GO TO 40		008520
000017	DO 30 I=1,N		008530
000021	30 OELX(I)=-DELX(I)		008540
000025	40 CONTINUE		008550
000025	N404=0		008560
000026	MN=MN+1		008570
	C MN IS NOW NUMB. OF POINTS AFTER MIN ACHIEVED		008580
000030	NTCTR=NTCTR+1		008590
000031	DO 50 I=1,N		008600
000032	50 X2(I)=X(I)		008610
000036	PX1=P0		008620
000037	N401=0		008630
000040	60 N401=N401+1		008640
000042	DO 70 I=1,N		008650
000043	70 X(I)=X2(I)+OELX(I)		008660
000050	CALL EVALUS		
	C 1 MEANS SATIS.OF CONSTRAINT NT*PREV. 2MEANS NOCHANGE 3MEANS VIOLATION		008680
	C IF POINT IS NOT FEASIBLE GIVE IT AN ARBITRARILY HIGH VALUE		008690
000051	GO TO (540,90,R0), NSAT1S		008700
000060	80 PX2=10.E35		008710
000062	P0=10.E35		008720
000063	GO TO 100		008730
000064	CONTINUE		008740
000064	PX2=P0		008750
000066	IF (PX1-PX2) 100,100,150		008760
000070	100 IF (N401-2) 130,110,110		008770
000073	110 DO 120 I=1,N		008780
000075	120 X1(I)=X(I)		008790
000101	P1=PX2		008800
000102	GO TO 430		008810
	C ONLY ONE POINT SO FAR COMPUTED		008820
000103	130 DO 140 I=1,N		008830
000105	140 X3(I)=X2(I)		008840
000111	PREV3=PX1		008850
000112	GO TO 180		008860
000113	150 DO 160 I=1,N		008870
000115	X3(I)=X2(I)		008880
000117	X2(I)=X(I)		008890

000120	160	DELX(I)=1.61803399*DELX(I)	008900
000124		PREV3=PX1	008910
000126		PX1=PX2	008920
000127		GO TO 60	008930
		C GOLDEN SECTION SEARCH METHOD.	008940
		C R VECTOR GOES TO X1(I)	008950
000127	170	P0=1.E36	008960
000131		N404=N404+1	008970
000132	180	DO 190 I=1,N	008980
000134	190	X1(I)=X(I)	008990
000140		P1=P0	009000
000141		DO 200 I=1,N	009010
000143		X(I)=.38196601*(X1(I)-X3(I))+X3(I)	009020
000150	200	X2(I)=X(I)	009030
000153		CALL EVALUS	
000154		GO TO (540,270,210), NSATIS	009050
000163	210	IF (N404.LT.30) GO TO 170	009060
000166	211	CONTINUE	009070
		C THERE IS NO REFERENCE TO 211, THE ABOVE STATEMENT IS A DUMMY STATEMENT	009071
		C--IT IS POSSIBLE NO FEASIBLE POINT EXIST, IF NOT TRY MOVING ON DELX0.	009080
		C-- IF IT IS NOT POSSIBLE TO MOVE ON DELX0 THEN WE MUST BE AT A	009090
		C-- SOLUTION OF NLP PROBLEM.	009100
000166		IF (N404.GT.100) GO TO 240	009110
000172	220	DO 230 I=1,N	009120
000174		IF (ABS(ABS(X3(I)/X1(I))-1.).GT.1.E-7) GO TO 170	009130
000203	230	CONTINUE	009140
000205	240	GO TO (250,260), N405	009150
000213	250	N405=2	009160
		C--TRY TO MOVE ON GRAOIENT.	009170
000214		NTCTR=NTCTR-1	009180
000216		MN=MN-1	009190
000217		GO TO 20	009200
000217	260	WRITE (6,580)	
000223		CALL TIMECS	
000224		CALL OUTPUX(I)	
000226		CALL REJECA	
000227		STOP 22042	009250
		C	009260
000231	270	CONTINUE	009270
000231		N404=0	009280
000232		PX1=P0	009290
000234		DO 280 I=1,N	009300
000235	280	X(I)=.38196601*(X1(I)-X2(I))+X2(I)	009310
000244		CALL EVALUS	
000245		GO TO (540,290,220), NSATIS	009330
000254	290	PX2=P0	009340
000256		N401=1	009350
000257	300	N401=N401+1	009360
000261		IF (N401_25) 340,310,310	009370
000263	310	KSW=2	009380
000264		IF (N401_40) 320,460,460	009390
000267	320	DO 330 I=1,N	009400
000271		IF (ABS(X2(I)/X(I)-1.0).GE.1.E-7) GO TO 340	009410
000276	330	CONTINUE	009420
000301		GO TO 460	009430
000301	340	IF (ABS(PX1/PX2-1.).LE.1.E-7) GO TO 460	009440
000306		IF (PX1-PX2) 350,460,400	009450
		C FROM LEFTORIGHT X3(I)(PREV3)X2(I)(PX1)X(I)PX2 X1(I)P1	009460

000310	350	DO 360 I=I,N	009470
000312	360	X1(I)=X(I)	009480
		C THROW AWAY RIGHT PART	009490
000316		P1=PX2	009500
000317		DO 370 I=1,N	009510
		C POINTXP1 BECOMES XP2	009520
000321	370	X(I)=.38196001*(X1(I)-X3(I))+X3(I)	009530
		C TEMPORARILY IN X STORAGE	009540
000330		CALL EVALUS	
000331		GO TO (540,380,170), NSATIS	009560
000340	380	CONTINUE	009570
000340		PX2=PX1	009580
		C SWITCH VECTORS TO PROPER POSITION	009590
000342		PX1=P0	009600
000343		DO 390 I=I,N	009610
000344		XX=X2(I)	009620
000346		X2(I)=X(I)	009630
000347	390	X(I)=XX	009640
000353		GO TO 300	009650
		C LEFT SIDE TOSSED AWAY	009660
		C-- CHANGES FOR NONUNIMODAL FN	009670
		C-- GO TO THROW AWAY RIGHT IN THIS CASE INIT VAL LT FIB PT	009680
000353	400	IF (PREV3-PX2) 350,350,410	009690
000356	410	DO 420 I=1,N	009700
000360		X3(I)=X2(I)	009710
000362	420	X2(I)=X(I)	009720
000365		PREV3=PX1	009730
000367		PX1=PX2	009740
000370	430	DO 440 I=I,N	009750
000372	440	X(I)=.38196601*(X1(I)-X2(I))+X2(I)	009760
000401		CALL EVALUS	
000402		GO TO (540,450,170), NSATIS	009780
000411	450	CONTINUE	009790
000411		PX2=P0	009800
000413		GO TO 300	009810
		C THE INTERIOR POINTS NOW GIVE EQUAL VALUE FOR P. COMPUTE MIDPOINT.	009820
000413	460	DO 470 I=I,N	009830
000415		OELX0(I)=X(I)	009840
000417		X(I)=(OELX0(I)+X2(I))*0.5	009850
000422	470	CONTINUE	009860
000424		CALL EVALUS	
000425		GO TO (480,490), KSW	009880
000433	480	IF (ABS(P0/PX1-1.)>.1E-7) GO TO 520	009890
000441	490	GO TO (500,510), ISW	009900
000447	500	IF (P0.LT.P31) GO TO 510	009910
000452		ISW=2	009920
		C IF P-FUNCTION DIONOT GO DOWN TRY NEG VECT.	009930
000453		GO TO 20	009940
000453	510	RETURN	009950
000454	520	DO 530 I=I,N	009960
000456	530	X(I)=OELX0(I)	009970
000462		GO TO 350	009980
		C ARE WE NOW IN FEASIBILITY PHASE	009990
000462	540	DO 550 I=I,M	010000
000464		IF (RJ(I)) 560,560,550	010010
000466	550	CONTINUE	010020
000471		NSATIS=4	010030
000472		RETURN	010040

	C---	PROBLEM HAS BECOME FEASIBLE	010050
	C---	P - FUNCTION CHANGES IF A CONSTRAINT BECOMES FEASIBLE	010060
000472	560	MN=0	010070
000473		DO 570 I=1,M	010080
000475	570	RJ1(I)=RJ(I)	010090
000501		RETURN	010100
	C		010110
000501	580	FORMAT (80M OPT CAN'T FIND A FEASIBLE POINT, THAT GIVES A LOWER V	010120
		ALUE OF THE P-FUNCTION.)	010130
000501		END	010140

SUBROUTINE OUTPUX (K)			
C			010160
C	OCTOBER 1970		010170
C			010180
C	OUTPUT PRINTS OUT INFORMATION ON THE RESULTS OF EACH ITERATION AND THE		010190
C	SOLUTION ESTIMATES AND THE ESTIMATES OF THE LAGRANGE MULTIPLIERS		010200
000003	COMMON/SHARES/X(100), DEL(100), A(100*100),N,M, MN,NP1,NM1		
000003	COMMON /EQALS/H, H1, MZ		
000003	COMMON /OPTNSS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NTR,NT9,NT10		
000003	COMMON /VALUES/F,G,P0,RSIGMA, RJ(200), RHO		
000003	COMMON/CRSTS/DELX(100), OELX0(100), RHO1N,RATIO, EPS1, THETA0,		
	1 RSIG1, G1, X1(100), X2(100), X3(100), XR2(100), XR1(100), PR1,		
	2 PR2,P1, F1, RJ1(200), DOTT, PGRA0(100), DIAG(100),		
	3 PREV3,ADELX, NTCTR, NUMINI, NPHASE, NSATIS		
000003	NZ=M*MZ		010290
000005	GO TO (10,20), K		010300
000013	10 WRITE (6,60)		010310
000017	WRITE (6,70) NTCTR,DOTT,RHO,ADELX,NPHASE		010320
000035	20 WRITE (6,80) F,P0,G,RSIGMA,M		010330
000053	WRITE (6,90) (X(I),I=1,N)		010340
000066	WRITE (6,110)		010350
000072	GO TO (30,40), NT2		010360
000101	30 WRITE (6,120)		010370
000105	WRITE (6,100) (RJ(I),I=1,NZ)		010380
000120	GO TO 50		010390
000122	40 WRITE (6,100) (RJ(I),I=1,NZ)		010400
000135	50 RETURN		010410
C			010420
000136	60 FORMAT (50H0*****)		010430
000136	70 FORMAT (10X,6HP0INT=I4,6X,6H DOTT=E15.7,6X,4HRHO=E15.7,6X,10HMAGNI		010440
	1TUNE=E15.7,6X,6HPHASE=I2)		010450
000136	80 FORMAT (8X,2HF=E15.7,5X,2HP=E15.7,5X,2HG=E15.7,5X,7HRSIGMA=E15.7,5		010460
	1X,2HH=E15.7,		010470
000136	90 FORMAT (6X,25HTHE CURRENT VALUE OF X IS/(6E20.7))		010480
000136	100 FORMAT (6E20.7)		010490
000136	110 FORMAT (6X,21HTHE CONSTRAINT VALUES)		010500
000136	120 FORMAT (28X,34HNOT INCLUDING THE NON-NEGATIVITIES)		010510
000136	END		

SUBROUTINE OUTPUX (K)

9999 CONTINUE

RETURN

END

	SUBROUTINE PEVALX		
C			010540
C	OCTOBER 1970		010550
C			010560
C	PEVALU COMPUTES THE VALUE OF THE PENALTY FUNCTION AND THE VALUE OF THE		010570
C	DUAL USING PREVIOUSLY COMPUTED VALUES FOR F, AND RJ.		010580
000002	COMMON/SHARES/X(100), OEL(100), A(100,100), N,M, MN,NP1,NM1		
000002	COMMON /EQUALS/H, M1, M2		
000002	COMMON /OPTNSS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10		
000002	COMMON /VALUES/F,G,P0,RSIGMA, RJ(200), RHO		
000002	COMMON/CRSTS/OELX(100), OELX0(100), RHO1N,RAT10, EPS1, THETA0,		
	1 RSIG1, G1, X1(100), X2(100), X3(100), XR2(100), XR1(100), PR1,		
	2 PR2,P1, F1, RJ1(200), OOT1, PGRA0(100), OIAG(100),		
	3 PREV3, AOELX, NTCTR, NUMIN1, NPHASE, NSAT1S		
000002	H=0.0		010670
000003	RSIGMA=0.0		010680
	C NONNEGS IF INCLUDED ARE ADDED TO P--ARE POS, IN ALL PHASES		010690
000004	GO TO (10,30), NT2		010700
000012	10 DO 20 J=1,N		010710
000014	20 RSIGMA=RSIGMA-RHO*ALOG(X(1))		010720
000024	30 GO TO (40,50,150), NPHASE		010730
	C OBJECTIVE FUNCTION - SIGMA VIOL. CONSTS.		010740
000033	40 F=0.0		010750
000034	50 IF (M) 100,100,60		010760
000036	60 DO 90 J=1,M		010770
000040	IF (RJ(J)) 80,80,70		010780
000042	70 RSIGMA=RSIGMA-RHO*ALOG(RJ(J))		010790
000047	GO TO 90		010800
000050	80 F=F-RJ(J)		010810
000053	90 CONTINUE		010820
	C EQUALITIES NOT ADDED IN FEAS. PHASE		010830
000056	100 CONTINUE		010840
000056	IF (M2) 140,140,110		010850
000060	110 GO TO (140,120,150), NPHASE		010860
000067	120 DO 130 J=1,M2		010870
000071	K=M,1		010880
000073	130 H=H+RJ(K)**2		010890
000100	H=H/RHO		010900
000101	140 HS=H+RSIGMA		010910
000103	P0=F+HS		010920
000105	HMS=2.*H-RHO*FLOAT(M)		010930
000111	G=F+HMS		010940
000113	IF (NT2, EQ, 1) G=G-RHO*FLOAT(N)		010950
000120	150 RETURN		010960
000121	END		010970

	SUBROUTINE PUNCHS	
C		010990
C	OCTOBER 1970	011000
C		011010
C	THIS SUBROUTINE PUNCHES THE STOPPING POINTS AND ASSOCIATED PARAMETERS	011020
C	SO THAT ANOTHER RUN MAY BE MADE STARTING WHERE THE CURRENT ONE	011030
C	STOPPED	011040
C	THIS ROUTINE IS CALLED IF NT6#2:	011050
000002	COMMON/SHARES/X(100), DEL(100), A(100,100), N,M, MN,NP1,NM1	
000002	COMMON /EQALS/M, H1, MZ	
000002	COMMON /OPTNSS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10	
000002	COMMON /VALUES/F,G,P0,RSIGMA, RJ(200), RHO	
000002	COMMON/CRSTS/DELX(100), OELX0(100), RHOIN,RATIO, EPSI, THETA0,	
	1 RSIG1, G1, X1(100), X2(100), X3(100), XR2(100), XR1(100), PR1,	
	2 PR2,P1, F1, RJ1(200), OOTI, PGRA0(100), OIAG(100),	
	3 PREV3, AOELX, NTCTR, NUMIN1, NPHASE, NSATIS	
000002	COMMON/EXPXPX / NEXOP1, NEXOP2, XEP1, XEP2	
000002	T=60.0	011150
000004	WRITE (7,10) EPSI,RHO,THETA0,RATIO,T,M,N,MZ	
C	TMMAX.IS SET TO 60. SECONDS	011170
000027	NT1=3	011190
000030	WRITE (7,20) (X(I),I=1,N)	
C	SET RHO OPTION SO THIS VALUE OF RHO WILL BE USE FOR THE RESTART.	011200
000043	WRITE (7,30) NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10	
000073	WRITE (7,20) XEP1, XEP2	
000103	WRITE (7,30) NEXOP1,NEXOP2	
000113	RETURN	011240
C		011250
000114	10 FORMAT (5E12.5,3I4)	011260
000114	20 FORMAT (6E12.5)	011270
000114	30 FORMAT (10I7)	011280
000114	END	011290

		SUBROUTINE REJECX	
C			011310
C		OCTOBER 1970	011320
C			011330
C		REJECT RETURNS THE STORED VALUES OF THE OBJECTIVE FUNCTION, THE	011340
C		CONSTRAINT FUNCTIONS AND THE PENALTY FUNCTION TO THEIR NORMAL LOCATION	011350
000002		COMMON/SHARES/X(100), DEL(100), A(100,100),N,M, MN,NP1,NM1	
000002		COMMON /EQALS/H, H1, MZ	
000002		COMMON /VALUES/F,G,P0,RSIGMA, RJ(200), RHO	
000002		COMMON/CRSTS/DELX(100), DELX0(100), RHO1N,RATIO, EPS1, THETA0,	
		1 RSIG1, G1, X1(100), X2(100), X3(100), XR2(100), XR1(100), PR1,	
		2 PR2,P1, F1, RJ1(200), DOTT, PGRAD(100), DIAG(100),	
		3 PREV3, ADELX, NTCTR, NUM1N1, NPHASE, NSATIS	
000002		DO 10 I=1,N	011430
000004	10	X(I)=X1(I)	011440
000010		MMZ=M+MZ	011450
000012		DO 20 J=1,MMZ	011460
000013	20	RJ(J)=RJ1(J)	011470
000017		P0=P1	011480
000020		RSIGMA=RSIG1	011490
000022		G=G1	011500
000023		F=F1	011510
000025		H=H1	011520
000026		RETURN	011530
000027		END	011540

		SUBROUTINE RHOCOX	
	C		011560
	C	OCTOBER 1970	011570
	C		011580
	C	SUBROUTINE TO COMPUTE INITIAL RHO VALUE	011590
	C	CONTROLLED BY COL. 7 ON OPTION CARO	011600
000002		COMMON/SHARES/X(100), DEL(100), A(100,100), N,M, MN,NP1,NM1	
000002		COMMON /OPTNSS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10	
000002		COMMON /VALUES/F,G,P0,RSIGMA, RJ(200), RHO	
000002		COMMON/CRSTS/OELX(100), OELX0(100), RHOIN,RATIO, EPS1, THETA0,	
	1	RSIG1, G1, X1(100), X2(100), X3(100), XR2(100), XR1(100), PR1,	
	2	PR2,P1, F1, RJ(200), OOT1, PGRAO(100), OIAG(100),	
	3	PREV3, AOELX, NTCTR, NUMINI, NP HASE, NSATIS	
000002		GO TO (110,50,10,190), NT1	011680
000012	10	RHO=RHOIN	011690
000014	20	IF (RHO) 30,30,40	011700
000016	30	RHO=1.	011710
000020	40	RETURN	011720
000021	50	NPARI=1	011730
000022	60	RHO=1.	011740
	C	2 MEANS RHO WHICH MINIMIZES GRADIENT MAG.	011750
000024		CALL GRADS(4)	
000025		OO 70 I=1,N	011770
000027	70	PGRAO(1)=OELX0(1)	011780
000033		RHO=2.	011790
000034		CALL GRADS(2)	
000036		OO 80 I=1,N	011810
000040		OELX0(1)=OELX0(1)-PGRAO(1)	011820
000042	80	PGRAO(1)=PGRAO(1)-OELX0(1)	011830
000046		GO TO (90,130), NPARI	011840
000054	90	OOT1=0.	011850
000055		OOT2=0.	011860
000056		OO 100 I=1,N	011870
000057		OOT1=OOT1+OELX0(1)*PGRAO(1)	011880
000062	100	OOT2=OOT2+OELX0(1)**2	011890
000067		RHO=ABS(OOT1/OOT2)	011900
000071		GO TO 20	011910
	C	3 MEANS COMPUTE RHO SO AS TO MINIMIZE OEL P(/DDP/1.)DEL P	011920
000072	110	NPARI=1	011930
000073	120	NPARI=2	011940
	C	USE OF AND OR SUBROUTINE	011950
000074		GO TO 60	011960
000075	130	RHO=1.	011970
	C	ASSUME SIGMA TERM IS CONS10. GRTER IHAN F TERM	011980
000077		CALL SECORX (2)	
000100		OO 140 I=1,N	012000
000102	140	OELX(1)=PGRAO(1)	012010
000106		CALL INVERX (1)	
000107		OO 150 I=1,N	012030
000111		X1(1)=OELX(1)	012040
000113	150	OELX(1)=OELX0(1)	012050
000116		CALL SECORX (2)	
000120		CALL INVERX (1)	
000122		OO 160 I=1,N	012080
000124	160	XR2(1)=OELX(1)	012090
000130		GO TO (170,200), NPARI	012100
000136	170	OOT1=0.	012110

000137		DOT2=0.	012120
000140		DO 180 I=1,N	012130
000141		DOT1=DOT1+PGRAH(I)*X1(I)	012140
000144	180	DOT2=DOT2+DELXN(I)*XR2(I)	012150
000151		RHO=SQRT(ABS(DOT1/DOT2))	012160
000156		GO TO 20	012170
000156	190	NPAR2=2	012180
	C	RHO MINIMIZES 2ND ORDER MOVE	012190
000157		GO TO 120	012200
	C	USES INTERNAL SUB. TO COM /DDP/-1 OF AND /DDP/- OR	012210
000160	200	DOT1=0.0	012220
000161		DOT2=0.0	012230
000162		DO 210 I=1,N	012240
000163		DOT1=X1(I)**2+DOT1	012250
000166	210	DOT2=X1(I)*XR2(I)+DOT2	012260
000172		RHO=ABS(DOT1/DOT2)	012270
000175		GO TO 20	012280
000175		END	012290

SUBROUTINE SECORX(IS)		012310
C		012320
C	OCTOBER 1970	012330
C	SECOND EVALUATES THE MATRIX OF SECOND PARTIALS OF THE PENALTY	012340
C	FUNCTION.	012350
C		012360
C	(1) MEANS DONT COMPUTE GRAD, OUTER PRODUCT (IN SECOND)	012370
000003	COMMON/SHARES/X(100), DEL(100), A(100,100),N,M, MN,NP1,NM1	
000003	COMMON/EQUALS/H, H1, MZ	
000003	COMMON/OPTNSS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10	
000003	COMMON/VALUES/F,G,0,RSIGMA, RJ(200), RHO	
000003	COMMON/CRSTS/DELX(100), DELX0(100), RHOIN,RATIO, EPS1, THETA0,	
	1 RSIG1, G1, X1(100), X2(100), X3(100), XR2(100), XR1(100), PR1,	
	2 PR2,P1, F1, RJ1(200), DOTT, PGRAD(100), DIAG(100),	
	3 PREV3,ADDELX, NTCTR, NUMIN1, NPHASE, NSATIS	
000003	DO 10 I=1,N	012460
000005	DO 10 J=1,N	012470
000006	10 A(I,J)=0.	012480
000017	GO TO (230,20), IS	012490
	CGRAD. TERM NOT PREV. COMPUTED	012500
000024	20 DO 30 I=1,N	012510
000026	DO 30 J=1,I	012520
000027	A(I,J)=0.0	012530
000033	30 CONTINUE	012540
000037	GO TO (40,60), NT2	012550
000045	40 DO 50 I=1,N	012560
000047	50 A(I,I)=RHO/X(1)**2	012570
000060	60 CONTINUE	012580
000060	IF (M.LE.0) GO TO 130	012590
000062	DO 120 IN=1,M	012600
000063	IF (RJ(IN)) 120,120,70	012610
000065	70 CALL GRAD1S(IN)	
000067	TT = RHO/RJ(IN)**2	012630
000072	DO 110 I=1,N	012640
000074	IF (DEL(I)) 80,110,80	012650
000075	80 T=TT*DEL(I)	012660
000100	DO 100 J=1,I	012670
000101	IF (DEL(J)) 90,100,90	012680
000102	90 A(I,J)=A(I,J)+T*DEL(J)	012690
000110	100 CONTINUE	012700
000113	110 CONTINUE	012710
000116	120 CONTINUE	012720
	C EQUALITY CONSTRAINTS	012730
000121	130 IF (MZ) 210,210,140	012740
000123	140 GO TO (210,150,230), NPHASE	012750
000132	150 RQ=2./RHO	012760
000134	DO 200 JJ=1,MZ	012770
000136	IN=M+JJ	012780
000140	CALL GRAD1S(IN)	
000141	DO 190 I=1,N	012800
000144	IF (DEL(I)) 160,190,160	012810
000145	160 T=RQ*DEL(I)	012820
000150	DO 180 J=1,I	012830
000151	IF (DEL(J)) 170,180,170	012840
000152	170 A(I,J)=A(I,J)+T*DEL(J)	012850
000160	180 CONTINUE	012860

000163	190	CONTINUE	012870
000166	200	CONTINUE	012880
000170	210	DO 220 I=1,N	012890
000172		OIAG(I)=A(I,I)	012900
000176	220	A(I,I)=0.	012910
		C READY NOW FOR MATRIX OF 2ND PARTIALS OF RESTRAINTS	012920
000203	230	GO TO (240,510,520), NT10	012930
000212	240	IF (M ₁ LE.0) GO TO 340	012940
000214		DO 330 I=1,M	012950
000215		LORN=2	012960
		C CONSTRAINT ASSUMED NONLINEAR	012970
000216		CALL MATRXX (IN,LORN)	
000220		IF (LORN.LT.2) GO TO 330	012990
000224		IF (RJ(IN).GT.0.0) GO TO 280	013000
000227		DO 260 I=2,N	013010
000230		IM1=I-1	013020
000232		DO 260 J=1,IM1	013030
000233		IF (A(J,I)) 250,260,250	013040
000236	250	A(I,J)=A(I,J)-A(J,I)	013050
000245		A(J,I)=0.	013060
000247	260	CONTINUE	013070
000254		DO 270 I=1,N	013080
000256		DIAG(I)=DIAG(I)-A(I,I)	013090
000262	270	A(I,I)=0.0	013100
000270		GO TO 330	013110
000270	280	T=-RHO/RJ(IN)	013120
000273		DO 300 I=2,N	013130
000274		IM1=I-1	013140
000276		DO 300 J=1,IM1	013150
000277		IF (A(J,I)) 290,300,290	013160
000302	290	A(I,J)=A(I,J)+T*A(J,I)	013170
000312		A(J,I)=0.	013180
000315	300	CONTINUE	013190
000322		DO 320 I=1,N	013200
000324		IF (A(I,I)) 310,320,310	013210
000327	310	OIAG(I)=OIAG(I)+T*A(I,I)	013220
000335		A(I,I)=0.	013230
000340	320	CONTINUE	013240
000343	330	CONTINUE	013250
000346	340	CONTINUE	013260
000346		GO TO (520,350,520), NPHASE	013270
000355	350	IF (MZ.EQ.0) GO TO 420	013280
		EQUALITY SECOND PARTIALS HERE	013290
000356		IF (NT10.GE.2) GO TO 420	013300
000361		DO 410 I=1,MZ	013310
000362		IN=M+I	013320
000364		LORN=2	013330
000365		CALL MATRXX (IN,LORN)	
000367		IF (LORN.LT.2) GO TO 410	013350
000373		T=2*RJ(IN)/RHO	013360
000376		DO 380 I=2,N	013370
000377		IM1=I-1	013380
000401		DO 370 J=1,IM1	013390
000402		IF (A(J,I)) 360,370,360	013400
000405	360	A(I,J)=A(I,J)+T*A(J,I)	013410
000415		A(J,I)=0.0	013420
000420	370	CONTINUE	013430
000423	380	CONTINUE	013440

000425		DO 400 I=1,N	013450
000427		IF (A(I,I)) 390,400,390	013460
000432	390	DIAG(I)=DIAG(I)+T*A(I,I)	013470
000440		A(I,I)=0.0	013480
000443	400	CONTINUE	013490
000446	410	CONTINUE	013500
		C GET MATRIX OF 2ND PARTIALS OF ORJECTIVE FUNCTION	013510
000451	420	LLL=2	013520
000452		CALL MATRXX(0,LLL)	
000454		IF (LLL.LT.5) GO TO 490	013540
000460		DO 440 I=2,N	013550
000461		IMI=I-I	013560
000463		DO 440 J=I,IMI	013570
000464		IF (A(J,I)) 430,440,430	013580
000467	430	A(I,J)=A(I,J)+A(J,I)	013590
000476	440	A(J,I)=A(I,J)	013600
000511		DO 470 I=I,N	013610
000513		IF (A(I,I)) 450,460,450	013620
000516	450	A(I,I)=DIAG(I)+A(I,I)	013630
000522		GO TO 470	013640
000523	460	A(I,I)=DIAG(I)	013650
000527	470	CONTINUE	013660
000532	480	RETURN	013670
000533	490	DO 500 I=I,N	013680
000535		A(I,I)=DIAG(I)	013690
000541		DO 500 J=I,N	013700
000542	500	A(I,J)=A(J,I)	013710
000556		GO TO 480	013720
000556	510	GO TO (520,350,350), NPHASE	013730
000565	520	DO 530 I=2,N	013740
000567		IMI=I-1	013750
000571		DO 530 J=1,IMI	013760
000572	530	A(J,I)=A(I,J)	013770
000606		DO 540 I=1,N	013780
000607	540	A(I,I)=DIAG(I)	013790
000616		GO TO 480	013800
000617		END	013810

	SUBROUTINE SETS(TMMAX)	
	C	013830
	C FEBUARY 1971	013840
	C	013850
000003	C SET STORES THE TIME AT WHICH THE PROBLEM IS BEGUN	013860
000003	COMMON /TSWS/NSWW	
	COMMON /TMXS/TMO*EXT*EXT90	
	C	013890
	C SECOND GIVES JOB CPU EXECUTION TIME IN 1/1000 OF A SECOND	013900
	C	013910
000003	CALL SECOND (TMO)	013920
000005	EXT=TMMAX+TMO	013930
000007	EXT90= TMO + 0.90*TMMAX	013940
000012	NSWW=1	013950
000013	RETURN	013960
000014	END	013970

SURROUTINE STORES		
C		013990
C	OCTOBER 1970	014000
C		014010
C	STORE STORES THE VALUES OF THE CURRENT POINT AND THE ASSOCIATED	014020
C	VALUES OF THE FUNCTIONS IN A TEMPORARY AREA.	014030
000002	COMMON/SHARES/X(100), DEL(100), A(100,100),N,M, MN,NP1,NM1	
000002	COMMON /EQALS/H, H1, MZ	
000002	COMMON /VALUES/F,G,P0,RSIGMA, RJ(200), RHO	
000002	COMMON/CRSTS/DELX(100), DELX0(100), RHOIN,RATIO, EPSI, THETA0,	
	1 RSIG1, G1, X1(100), X2(100), X3(100), XR2(100), XR1(100), PR1,	
	2 PR2,P1, F1, RJ1(200), DOTT, PGRAD(100), DIAG(100),	
	3 PREV3,ADELX, NTCTR, NUMINI, NPHASE, NSATIS	
000002	DO 10 I=1,N	014110
000004	10 X1(I)=X(I)	014120
000010	MMZ=M+MZ	014130
000012	DO 20 J=1,MMZ	014140
000013	20 RJ1(J)=RJ(J)	014150
000017	P1=P0	014160
000020	F1=F	014170
000022	G1=G	014180
000023	RSIG1=RSIGMA	014190
000025	H1=H	014200
000026	RETURN	014210
000027	END	014220

	SUBROUTINE TCHFCX	
C		014240
C	FEBRUARY 1971	014250
C		014260
C	TCHECK CHECKS THE NUMBER OF SECONDS THAT HAVE ELAPSED SINCE THE START	014270
C	OF THE PROBLEM. IF THE SOLUTION IS TAKING LONGER THAN 90 PER-CENT	014280
C	OF THE ESTIMATED MAXIMUM TIME, A SWITCH IS SET TO GIVE MORE OUTPUT,	014290
000002	COMMON /OPTNSS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10	
000002	COMMON /TSWS/NSWW	
000002	COMMON /TMXS,TMO,EXT,EXT90	
000002	CALL SECOND (SECS)	014330
000004	IF (SECS.LT.EXT90) RETURN	014340
C	GETTING CLOSE TO EXCEEDING THE TIME LIMIT SET OUTPUT OPTION TO GIVE	014350
C	MORE OUTPUT.	014360
000007	NT3=2	014370
000010	X=SECS - TMO	014380
000012	WRITE(6,10) X	
000020	10 FORMAT (6X,5H TIME=,F9.3,8H SECONDS)	014400
000020	IF (SECS .GT. EXT) NSWW=2	014410
000024	CALL OUTPUX (1)	
000026	RETURN	014430
000027	END	014440

	SUBROUTINE TIMECS	
C		014460
C	FEBUARY 1971	014470
C		014480
C	TIMEC CHECKS THE NUMBER OF SECONDS THAT HAVE ELAPSED SINCE THE START	014490
C	OF THE PROBLEM. IT PRINTS THIS NUMBER. IF THE SOLUTION IS TAKING	014500
C	TIMEC CHECKS THE NUMBER OF SECONDS THAT HAVE ELAPSED SINCE THE START	014490
C	OF THE PROBLEM. IT PRINTS THIS NUMBER. IF THE SOLUTION IS TAKING	014500
C	LONGER THAN THE ESTIMATED MAXIMUM TIME, A SWITCH IS SET TO TERMINATE	014510
C	THE RUN.	014520
000002	COMMON /TSWS/NSWW	
000002	COMMON /TMXS/TMO,EXT,EXT90	
C		014550
C	SECOND GIVES JOB CPU EXECUTION TIME IN 1/1000 OF A SECOND	014560
C		014570
000002	CALL SECOND (SECS)	014580
000004	X=SECS-TMO	014590
000006	WRITE (6,20) X	
000014	IF (SECS.LT.EXT) GO TO 10	014610
000017	NSWW=2	014620
000020	10 RETURN	014630
C		014640
000021	20 FORMAT (6X,5HTIME=F9.3,8H SECONDS)	014650
000021	END	014660

```

SUBROUTINE XMOVES
C
C          MARCH 1971
C
C XMOVE DETERMINES THE VECTOR ALONG WHICH THE SEARCH FOR A MINIMUM IS
C USING DPT.
000002      COMMON/SHARES/X(100), DFL(100), A(100,100),N,M, MN,NP1,NM1
000002      COMMON/CRSTS/DFLX(100), DELX0(100), RMDIN,RAT10, EPS1, THETA0,
1      RSIG1, GI,      X1(100), X2(100), X3(100), XR2(100), XRI(100), PR1,
2      PR2,PI, FI, RJ1(200), DOT1, PGRAD(100), OIAG(100),
3      PRFV3, ADELX, NTCTR, NUMIN1, NPHASE, NSATIS
000002      COMMON/EXPDX / NEXP1, NEXP2, KEP1, KEP2
000002      COMMON /XVFS/SIG(100),YY(100),XXX(100),DELL(100)
C--NEXP2 DETERMINES HOW MOVE IS TO BE MADE
C NEXP2 = 1 USE MODIFIED NEWTON RAPHSON METHOD.
C         = 2 USE MODIFIED NEWTON RAPHSON METHOD, BUT ADD DELX0 TO
C         ORTHOGONAL MOVE VECTOR IF HESSIAN IS INDEFINITE.
C         = 3 USE STEEPEST DESCENT METHOD.
C         = 4 USE MCCORMICK'S MODIFICATION OF THE FLETCHER-POWELL
C           METHOD.
000002      GO TO (10,10,10,30), NEXP2
C--NEWTON -RAPH WITH WHATEVER METHOD IS IN INVERSE
000012      10 CALL GRADS(1)
C--ONE (1) MEANS ACCUMULATE MATRIX OF SECOND PARTIAL DERIVATIVES
000014      CALL SECORX (1)
000016      DO 20 I=1,N
000020      20 DELX(I)=DELX0(I)
000024      CALL INVERX(1)
C IF A NONPOSITIVE PIVOT IS ENCOUNTERED IN INVERSE AN ATTEMPT IS MADE TO
C COMPUTE A VECTOR HAVING A POSITIVE DOT PRODUCT WITH A NEGATIVE
C EIGENVECTOR AND THE NEGATIVE OF DEL P.
000025      CALL STORES
000026      CALL OPTS
000027      RETURN
C--F-P-D-MCC MOVE
000030      30 CALL GRAOS(2)
C--MN IS NO. OF MOVES FOR THIS VALUE OF RHO
000032      IF (MN,NE,0) GO TO 70
000033      40 IREP=0
000034      IT=0
C--SET INITIAL GUESS INVERS MATRIX OF SECOND PARTIAL DERIVATIVES
C-- USE PARTIAL INVERSE IF KNOWN
000035      DO 50 I=1,N
000036      DO 50 J=1,N
000037      50 A(I,J)=0.0
000050      DO 60 I=1,N
000051      60 A(I,1)=1.0
000060      DO 80 I=1,N
000062      80 DELX(1)=DELX0(1)
000066      IF (IREP,GT,N) GO TO 40
000071      IF (IT,EQ,0) GO TO 130
000072      DO 90 I=1,N
000073      SIG(I)=X(1)-XXX(I)
000076      90 YY(1)=DELL(1)-DELX0(1)
C--NEGATIVE GRADIENT STORED AND COMPUTED
C--COMPUTE HY
000102      DO 100 I=1,N

```

000104	DELX(1)=0.0	015180
000105	DO 100 J=1,N	015190
000107	100 DELX(I)=DELX(1)+A(I,J)*YY(J)	015200
	C-- COMPUTE Y(SIG-HY)-1	015210
000122	ZCON=0.0	015220
000123	DO 110 I=1,N	015230
000124	110 ZCON=ZCON+YY(I)*(SIG(I)-DELX(1))	015240
000132	IF (ZCON.EQ.0.0) GO TO 130	015250
000133	IREP=IREP+1	015260
000135	ZC=1./ZCON	015270
	C-- UPDATE H MATRIX USING MCC FORMULA WHEN SCALAR NE 0	015280
000136	DO 120 I=1,N	015290
000140	T1=ZC*(SIG(1)-DELX(1))	015300
000143	DO 120 J=1,N	015310
000144	120 A(I,J)=A(I,J)+T1*(-OELX(J)+SIG(J))	015320
000153	A(J,I)=A(I,J)	015330
	C-- STORE CURRENT POINT AND CURRENT GRADIENT (NEG)	015340
000165	130 DO 140 I=1,N	015350
000167	XXX(I)=X(I)	015360
000171	140 DELL(I)=OELX0(I)	015370
000174	DO 150 I=1,N	015380
000176	DELX(1)=0.0	015390
000177	DO 150 J=1,N	015400
000201	150 DELX(I)=OELX(1)+A(I,J)*OELX0(J)	015410
000214	ZC1=0.0	015420
000215	DO 160 I=1,N	015430
000216	160 ZC1=OELX(1)**2+ZC1	015440
000223	ZC1=SQRT(ZC1)	015450
000225	DO 170 I=1,N	015460
000226	170 DELX(1)=OELX(1)/ZC1	015470
000232	CALL STORES	
000233	CALL OPTS	
000234	IT=IT+1	015500
000236	RETURN	015510
000236	180 CONTINUE	015520
	C STEEPEST DESCENT	015530
000236	CALL GRADS(4)	
000240	DO 190 I=1,N	015550
000242	190 OELX(I)=OELX0(I)	015560
000246	CALL STORES	
000247	CALL OPTS	
000250	RETURN	015590
000251	END	015600

User-Supplied Subroutines for Inside Programs

	SUBROUTINE READIX
000002	9999 CONTINUE
000002	RETURN
000003	END

```

SUBROUTINE RESTNX(IN,VAL)
000005      COMMON/PROB/ISP
000005      COMMON/IN/W(100)
000005      COMMON/SHARES/X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
000005      IF (ISP-1) 1000,1000,2000
000007 1000 FN=N
000011      VAL=0.
000011      IF (IN) 1100,1100,1200
000013 1100 DO 1150 J=1,N
000015 1150 VAL=VAL+W(J)*(X(J)-2.):**2
000023      GO TO 9999
000024 1200 VAL=FN
000025      DO 1250 J=1,N
000027 1250 VAL=VAL-X(J)
000033      GO TO 9999
000033 2000 FN=N
000035      VAL=0.
000035      IF (IN) 2100,2100,2200
000037 2100 DO 2150 J=1,N
000041 2150 VAL = VAL + W(J)**.5 * (X(J)-2.):**2
000053      GO TO 9999
000054 2200 VAL=FN
000055      DO 2250 J=1,N
000057 2250 VAL=VAL-X(J)**2
000063      GO TO 9999
000064 9999 CONTINUE
000064      RETURN
000065      END

```

```

SURROUTINE GRADIS(IN)
COMMON/PROB/ISP
COMMON/IN/W(100)
COMMON/SHARES/X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
DO 500 J=1,N
500 DEL(J)=0.
IF(ISP=1) 1000,1000,2000
1000 IF(IN) 1100,1100,1200
1100 DO 1150 J=1,N
1150 DEL(J) = 2. * W(J) * (X(J)-2.)
GO TO 9999
1200 DO 1250 J=1,N
1250 DEL(J)=-1.
GO TO 9999
2000 IF(IN) 2100,2100,2200
2100 DO 2150 J=1,N
2150 DEL(J) = 2. * W(J) **.5 * (X(J)-2.)
GO TO 9999
2200 DO 2250 J=1,N
2250 DEL(J) =-2. * X(J)
GO TO 9999
9999 CONTINUE
RETURN
END
000003
000003
000003
000003
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000063

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000005      SUBROUTINE MATRXX (IN,L)
000005      COMMON/PROB/ISP
000005      COMMON/IN/W(100)
000005      COMMON/SHARES/X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
000005      IF (ISP=1,1000,1000,2000
000007      1000 IF (IN) 1100,1100,1200
000011      1100 DO 1150 J=1,N
000013      1150 A(J,J) = 2.*W(J)
000023      GO TO 9999
000023      1200 CONTINUE
000023      GO TO 9999
000024      2000 IF (IN) 2100,2100,2200
000026      2100 DO 2150 J=1,N
000030      2150 A(J,J) = 2.*W(J)**.5
000044      GO TO 9999
000045      2200 DO 2250 J=1,N
000047      2250 A(J,J) = -2.
000056      GO TO 9999
000056      9999 CONTINUE
000056      RETURN
000057      END

```


USER-SUPPLIED INFORMATION CARDS AND RESULTS PRINTING OUT ALL POINTS OF INSIDE PROGRAMS

It is useful in debugging the overall program and in increasing solution efficiency to print out the solutions of the inside programs, preferably all points. The starting points of the inside programs should be feasible, which is dependent upon the starting point of the outside program. The starting point of the outside program should also be feasible. The solutions of the inside program for the starting point of the outside program should be examined. Also, several points of the outside program, and the corresponding inside program solutions, should be examined.

The SUB computer program, leaving in the statements for printing of points of the inside program, provides the opportunity to accomplish the above, when used with the user-supplied information cards given here.

User-supplied information cards are: parameter card, initial starting point, option card, tolerance card and second option card. Three sets of user-supplied information cards are necessary for the example problem.

The outside program has, on the parameter card, information that $EPSI = 1.E-05$, $RHOIN = 10.$, $THETAO = 1.E-05$, $RATIO = 10.$, $TMMAX = 360.$, $M = 2$, $N = 4$, $MZ = 0$. The initial starting point is $x_1 = x_2 = x_3 = x_4 = 4$. The option card indicates that all points of the outside program are to be printed out (Option 3 = 2), and that there is at least one nonlinear constraint (Option 10 = 1). The tolerance card indicates that for numerical differentiation and control of P-function minimization the tolerances are .001. Finally, the second option card indicates that the problem is to be solved without checking first derivatives (Option 1 = 1), and that steepest descent is used to minimize the P-function (Option 2 = 3).

The inside program information cards are similar, with the following exceptions. Each problem has one constraint ($M = 1$). The starting points are $v_1 = v_2 = v_3 = v_4 = .5$. The first option cards indicate that all subproblem solutions of the inside programs are to be printed out (Option 3 = 1). The first option cards also indicate that the first inside program has all linear constraints (Option 10 = 2) and the second inside program has at least one non-linear constraint (Option 10 = 1). The second option cards for both inside programs indicate that the second-order method, called generalized Newton-Raphson, is used to minimize the P-function (Option 2 = 1).

The printout of results gives initial information for the outside program.

It indicates that Problem A is being solved. It gives initial information for Problem A, noting that SUB is used. It solves Problem A for the initial starting point of the outside program, requiring 16 points, and indicates that Problem A is solved.

It indicates that Problem B is being solved. It gives initial information for Problem B, noting that SUB is used. It solves Problem B for the initial starting point of the outside program, requiring 16 points, and indicates that Problem B is solved.

It prints out information for the outside program, namely the starting point values of the constraints found by solving Problem A and Problem B, and the feasible starting point to be used in the overall procedure.

It then prints that Problem A is being solved. The initial information is not printed this time, however, it having been bypassed in SUB after the first time. Again, Problem A is solved. The printout is given up to point 4 of this solution. The remainder, not included here, is similar for more points of the outside program.

User-Supplied Information Cards

1.E-05	10.	1.E-05	10.	360.	2	4	0
4.	4.	4.	4.				
3	2	1	1	1	1	1	
.001	.001						
1							
3							
1.E-05	100.	1.E-05	10.	180.	1	4	0
.5	.5	.5	.5				
3	1	1	1	1	1	2	
.001	.001						
1							
1							
1.E-05	100.	1.E-05	10.	180.	1	4	0
.5	.5	.5	.5				
3	1	1	1	1	1	1	
.001	.001						
1							
1							

Printout of Results

NONLINEAR PROGRAMMING ROUTINE-SUMT VERSION 4 3/22/71

N= 4 M= 2 MZ= 0

MAX. TIME= 3.6000000E+02 R= 1.0000000E+01 RATIO= 1.0000000E+01 EPSILON= 1.0000000E-05 THETA= 1.0000000E-05

OPTIONS SELECTED

3	1	2	1	1	1	1	1
---	---	---	---	---	---	---	---

TOLERANCES

1.0000000E-03 1.0000000E-03

SECOND SET OF OPTIONS

TIME= 3 .025 SECONDS


```

NONLINEAR PROGRAMMING ROUTINE-SUHT VERSION 4      SUB
N= 4      M= 1      WZ= 0

MAX. TIME= 1.8000000E+02      P= 1.0000000E+02      RATIO= 1.0000000E+01      EPSILON= 1.0000000E-05      THETA= 1.0000000E-05

OPTIONS SELECTED
3 1 1 1 1 1 1 1 2

TOLERANCES
1.0000000E-03      1.0000000E-03

SECOND SET OF OPTIONS
TIME= 0.000 SECONDS
F= 3.6000000E+01      P= 0.      G= 0.      R51GMA= 0.      M= 0.
THE CURRENT VALUE OF X IS
5.0000000E-01      5.0000000E-01      5.0000000E-01      5.0000000E-01
THE CONSTRAINT VALUES
2.0000000E+00      NOT INCLUDING THE NON-NEGATIVITY
TIME= .014 SECONDS

*****THE FEASIBLE STARTING POINT TO BE USED IS ...
F= 3.6000000E+01      P= 0.      G= 0.      R51GMA= 0.      M= 0.
THE CURRENT VALUE OF X IS
5.0000000E-01      5.0000000E-01      5.0000000E-01      5.0000000E-01
THE CONSTRAINT VALUES
2.0000000E+00      NOT INCLUDING THE NON-NEGATIVITY
TIME= .073 SECONDS

*****
POINT= 2      DOTY= 6.4890716E-07      RHO= 1.0000000E+02      MAGNITUDE= 2.3681602E-02      PHASE= 2
F= 2.2595486E+01      P= 1.3438514E+02      G= -4.7740451E+02      R51GMA= 1.1178965E+02      M= 0.
THE CURRENT VALUE OF X IS
1.2320851E+02      1.2320851E+02      1.2320851E+02      1.2320851E+02
THE CONSTRAINT VALUES
8.1163225E-01      8.1163225E-01      8.1163225E-01      8.1163225E-01
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITY
7.5347099E-01

LAGRANGE MULTIPLIERS
F= 2.2595486E+01      P= 1.3438514E+02      G= -4.7740451E+02      R51GMA= 1.1178965E+02      M= 0.
THE CURRENT VALUE OF X IS
1.2320851E+02      1.2320851E+02      1.2320851E+02      1.2320851E+02
THE CONSTRAINT VALUES
1.3271911E+02      NOT INCLUDING THE NON-NEGATIVITY
TIME= .145 SECONDS

*****
POINT= 4      DOTY= 1.3778905E-07      RHO= 1.0000000E+01      MAGNITUDE= 5.0772586E-03      PHASE= 2
F= 2.0166114E+01      P= 3.2520721E+01      G= -2.9833886E+01      R51GMA= 1.2354607E+01      M= 0.
THE CURRENT VALUE OF X IS
8.7733258E-01      8.7733258E-01      8.7733258E-01      8.7733258E-01
THE CONSTRAINT VALUES
4.9866666E-01      NOT INCLUDING THE NON-NEGATIVITY

1ST ORDER ESTIMATES
F= 1.9904710E+01      P= 2.1202452E+01      G= 1.9896184E+01      R51GMA= 0.      M= 0.
THE CURRENT VALUE OF X IS

```

8.8463262E-01 8.8463262E-01 8.8463262E-01 8.8463262E-01
 THE CONSTRAINT VALUES NOT INCLUDING THE NON-NEGATIVITIES
 4.6146952E-01

LAGRANGE MULTIPLIERS
 F= 1.9904710E+01 P= 2.1202452E+01 G= 1.9896184E+01 RSIGMA= 0. H= 0.
 THE CURRENT VALUE OF X IS
 1.1398186E+01 1.1398186E+01 1.1398186E+01 1.1398186E+01
 THE CONSTRAINT VALUES NOT INCLUDING THE NON-NEGATIVITIES
 2.0380310E+01
 TIME= .233 SECONDS

 POINT= 6 DDT= 3.6201352E-07 RHO= 1.0000000E+00 MAGNITUDE= 1.1263842E-02 PHASE= 2
 F= 1.6876951E+01 P= 1.9210771E+01 G= 1.1876951E+01 RSIGMA= 2.3338206E+00 H= 0.
 THE CURRENT VALUE OF X IS
 9.7296084E-01 9.7296084E-01 9.7296084E-01 9.7296084E-01
 THE CONSTRAINT VALUES NOT INCLUDING THE NON-NEGATIVITIES
 1.0815663E-01

2ND ORDER ESTIMATES
 F= 1.6497058E+01 P= 1.7696832E+01 G= 1.6511488E+01 RSIGMA= 0. H= 0.
 THE CURRENT VALUE OF X IS
 9.8458573E-01 9.8458573E-01 9.8458573E-01 9.8458573E-01
 THE CONSTRAINT VALUES NOT INCLUDING THE NON-NEGATIVITIES
 6.1657062E-02

1ST ORDER ESTIMATES
 F= 1.6529552E+01 P= 1.7731808E+01 G= 1.6511488E+01 RSIGMA= 0. H= 0.
 THE CURRENT VALUE OF X IS
 9.8358620E-01 9.8358620E-01 9.8358620E-01 9.8358620E-01
 THE CONSTRAINT VALUES NOT INCLUDING THE NON-NEGATIVITIES
 6.5655187E-02

LAGRANGE MULTIPLIERS
 F= 1.6529552E+01 P= 1.7731808E+01 G= 1.6511488E+01 RSIGMA= 0. H= 0.
 THE CURRENT VALUE OF X IS
 1.0277906E+00 1.0277906E+00 1.0277906E+00 1.0277906E+00
 THE CONSTRAINT VALUES NOT INCLUDING THE NON-NEGATIVITIES
 9.2458498E+00
 TIME= .335 SECONDS

 POINT= 8 DDT= 5.5412871E-08 RHO= 1.0000000E-01 MAGNITUDE= 1.2123883E-02 PHASE= 2
 F= 1.6098637E+01 P= 1.6539599E+01 G= 1.5598637E+01 RSIGMA= 4.4096222E-01 H= 0.
 THE CURRENT VALUE OF X IS
 9.9692234E-01 9.9692234E-01 9.9692234E-01 9.9692234E-01
 THE CONSTRAINT VALUES NOT INCLUDING THE NON-NEGATIVITIES
 1.2310625E-02

2ND ORDER ESTIMATES
 F= 1.6008118E+01 P= 1.6227761E+01 G= 1.6012157E+01 RSIGMA= 0. H= 0.
 THE CURRENT VALUE OF X IS
 9.9974633E-01 9.9974633E-01 9.9974633E-01 9.9974633E-01

```

THE CONSTRAINT VALUES      NOT INCLUDING THE NON-NEGATIVITYS
1.0146632E-03

1ST ORDER ESTIMATES
F= 1.6013291E-01          P= 1.6242802E-01      G= 1.6012157E-01      RSIGMA= 0.      H= 0.
THE CURRENT VALUE OF X IS
9.9958473E-01          9.9958473E-01      9.9958473E-01
THE CONSTRAINT VALUES
1.6610684E-03      NOT INCLUDING THE NON-NEGATIVITYS

LAGRANGE MULTIPLIERS
F= 1.6013291E-01          P= 1.6242802E-01      G= 1.6012157E-01      RSIGMA= 0.      H= 0.
THE CURRENT VALUE OF X IS
1.0030872E-01          1.0030872E-01      1.0030872E-01
THE CONSTRAINT VALUES
8.1230644E-00      NOT INCLUDING THE NON-NEGATIVITYS
TIME= .433 SECONDS

*****
POINT= 10      ODTT= 5.8714668E-09      RHO= 1.0000000E-02      MAGNITUDE= 1.2290563E-02      PHASE= 2
F= 1.6009988E-01          P= 1.6076860E-01      G= 1.5959988E-01      RSIGMA= 6.6871855E-02      H= 0.
THE CURRENT VALUE OF X IS
9.9968791E-01          9.9968791E-01      9.9968791E-01
THE CONSTRAINT VALUES
1.2483446E-03      NOT INCLUDING THE NON-NEGATIVITYS

2ND ORDER ESTIMATES
F= 1.6000021E-01          P= 1.6023249E-01      G= 1.6000139E-01      RSIGMA= 0.      H= 0.
THE CURRENT VALUE OF X IS
9.9999935E-01          9.9999935E-01      9.9999935E-01
THE CONSTRAINT VALUES
2.6177795E-06      NOT INCLUDING THE NON-NEGATIVITYS

1ST ORDER ESTIMATES
F= 1.6000154E-01          P= 1.6025445E-01      G= 1.6000139E-01      RSIGMA= 0.      H= 0.
THE CURRENT VALUE OF X IS
9.999520E-01          9.999520E-01      9.999520E-01
THE CONSTRAINT VALUES
1.9202286E-05      NOT INCLUDING THE NON-NEGATIVITYS

LAGRANGE MULTIPLIERS
F= 1.6000154E-01          P= 1.6025445E-01      G= 1.6000139E-01      RSIGMA= 0.      H= 0.
THE CURRENT VALUE OF X IS
1.0003122E-02          1.0003122E-02      1.0003122E-02
THE CONSTRAINT VALUES
8.0106089E-00      NOT INCLUDING THE NON-NEGATIVITYS
TIME= .527 SECONDS

*****
POINT= 12      ODTT= 1.9072340E-08      RHO= 1.0000000E-03      MAGNITUDE= 7.0193568E-02      PHASE= 2
F= 1.6001001E-01          P= 1.6009987E-01      G= 1.5996001E-01      RSIGMA= 8.9861545E-03      H= 0.
THE CURRENT VALUE OF X IS
9.9996871E-01          9.9996871E-01      9.9996871E-01
THE CONSTRAINT VALUES

```

NOT INCLUDING THE NON-NEGATIVITYS

1.2514601E-04

2ND ORDER ESTIMATES

F= 1.6000001E+01 P= 1.6002326E+01 G= 1.6000003E+01 RSIGMA= 0. H= 0.
THE CURRENT VALUE OF X IS
9.9999996E-01 9.9999996E-01 9.9999996E-01
THE CONSTRAINT VALUES
1.5571006E-07 NOT INCLUDING THE NON-NEGATIVITYS

1ST ORDER ESTIMATES

F= 1.6000003E+01 P= 1.6002557E+01 G= 1.6000003E+01 RSIGMA= 0. H= 0.
THE CURRENT VALUE OF X IS
9.9999991E-01 9.9999991E-01 9.9999991E-01
THE CONSTRAINT VALUES
3.4617658E-07 NOT INCLUDING THE NON-NEGATIVITYS

LAGRANGE MULTIPLIERS

F= 1.6000003E+01 P= 1.6002557E+01 G= 1.6000003E+01 RSIGMA= 0. H= 0.
THE CURRENT VALUE OF X IS
1.0000313E-03 1.0000313E-03 1.0000313E-03
THE CONSTRAINT VALUES
7.9906660E+00 NOT INCLUDING THE NON-NEGATIVITYS

TIME= .618 SECONDS

POINT= 14 ODT= 3.4340067E-07 RHO= 1.0000000E-04 MAGNITUDE= 8.8571925E-01 PHASE= 2
F= 1.6000008E+01 P= 1.6001228E+01 G= 1.5999598E+01 RSIGMA= 1.1312673E-03 H= 0.
THE CURRENT VALUE OF X IS
9.9999695E-01 9.9999695E-01 9.9999695E-01
THE CONSTRAINT VALUES
1.2217264E-05 NOT INCLUDING THE NON-NEGATIVITYS

2ND ORDER ESTIMATES

F= 1.5999997E+01 P= 1.6000233E+01 G= 1.5999997E+01 RSIGMA= 0. H= 0.
THE CURRENT VALUE OF X IS
1.0000001E+00 1.0000001E+00 1.0000001E+00
THE CONSTRAINT VALUES
-3.3720913E-07 NOT INCLUDING THE NON-NEGATIVITYS

1ST ORDER ESTIMATES

F= 1.5999997E+01 P= 1.6000256E+01 G= 1.5999997E+01 RSIGMA= 0. H= 0.
THE CURRENT VALUE OF X IS
1.0000001E+00 1.0000001E+00 1.0000001E+00
THE CONSTRAINT VALUES
-3.3037527E-07 NOT INCLUDING THE NON-NEGATIVITYS

LAGRANGE MULTIPLIERS

F= 1.5999997E+01 P= 1.6000256E+01 G= 1.5999997E+01 RSIGMA= 0. H= 0.
THE CURRENT VALUE OF X IS
1.00000031E-04 1.00000031E-04 1.00000031E-04
THE CONSTRAINT VALUES
8.1051389E+00 NOT INCLUDING THE NON-NEGATIVITYS

```

TIME= .746 SECONDS
*****
POINT= 16 ODT= 1.662793E-07 RHO= 1.000000E-05 MAGNITUDE= 1.827536E+00 PHASE= 2
F= 1.600000E+01 P= 1.6000146E+01 G= 1.5999959E+01 RSIGMA= 1.365596E-04 H= 0.
THE CURRENT VALUE OF X IS
9.9999971E-01 9.9999971E-01 9.9999971E-01
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITY
1.1729808E-06

2ND ORDER ESTIMATES
F= 1.6000000E+01 P= 1.6000023E+01 G= 1.6000000E+01 RSIGMA= 0. H= 0.
THE CURRENT VALUE OF X IS
1.0000000E+00 1.0000000E+00 1.0000000E+00 1.0000000E+00
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITY
-5.137168E-08

1ST ORDER ESTIMATES
F= 1.6000000E+01 P= 1.6000026E+01 G= 1.6000000E+01 RSIGMA= 0. H= 0.
THE CURRENT VALUE OF X IS
1.0000000E+00 1.0000000E+00 1.0000000E+00 1.0000000E+00
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITY
-5.4161688E-08

LAGRANGE MULTIPLIERS
F= 1.6000000E+01 P= 1.6000026E+01 G= 1.6000000E+01 RSIGMA= 0. H= 0.
THE CURRENT VALUE OF X IS
1.0000003E-05 1.0000003E-05 1.0000003E-05 1.0000003E-05
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITY
8.5252888E+00

```

PROBLEM A SOLVED

NONLINEAR PROGRAMMING ROUTINE-SUMT VERSION 4 SUR

N= 4 M= 1 WZ= 0
 MAX. TIME= 1.800000E+02 R= 1.000000E+02 RATIO= 1.000000E+01 EPSILON= 1.000000E-05 THETA= 1.000000E-05

OPTIONS SELECTED

3 1 1 1 1 1 1 1 1 1

TOLERANCES

1.000000E-03 1.000000E-03

SECOND SET OF OPTIONS

1
 TIME= 0.000 SECONDS
 F= 1.800000E+01 P= 0. RSIGMA= 0. H= 0.
 THE CURRENT VALUE OF X IS
 5.000000E-01
 THE CONSTRAINT VALUES NOT INCLUDING THE NON-NEGATIVES
 3.000000E+00
 TIME= .014 SECONDS

*****FEASIBLE STARTING POINT TO BE USED IS ***

F= 1.800000E+01 P= 0. G= 0. RSIGMA= 0. H= 0.
 THE CURRENT VALUE OF X IS
 5.000000E-01
 THE CONSTRAINT VALUES NOT INCLUDING THE NON-NEGATIVES
 3.000000E+00
 TIME= .112 SECONDS

CO

POINT= 2 DOTT= 6.6344853E-07 RHO= 1.000000E+02 MAGNITUDE= 2.5042075E-02 PHASE= 2
 F= 1.1108596E+01 P= 6.3481406E+01 G= -4.8889140E+02 RSIGMA= 5.2372810E+01 H= 0.
 THE CURRENT VALUE OF X IS
 8.2162208E-01
 THE CONSTRAINT VALUES NOT INCLUDING THE NON-NEGATIVES
 1.2997486E+00

LAGRANGE MULTIPLIERS

F= 1.1108596E+01 P= 6.3481406E+01 G= -4.8889140E+02 RSIGMA= 5.2372810E+01 H= 0.
 THE CURRENT VALUE OF X IS
 1.2171046E+02
 THE CONSTRAINT VALUES NOT INCLUDING THE NON-NEGATIVES
 7.6937953E+01
 TIME= .217 SECONDS

POINT= 4 DOTT= 7.4900159E-08 RHO= 1.000000E+01 MAGNITUDE= 3.2592589E-03 PHASE= 2
 F= 1.0438018E+01 P= 1.6020672E+01 G= -3.9561982E+01 RSIGMA= 5.5826542E+00 H= 0.
 THE CURRENT VALUE OF X IS
 8.5774248E-01
 THE CONSTRAINT VALUES NOT INCLUDING THE NON-NEGATIVES
 1.0571113E+00

1ST ORDER ESTIMATES

F= 1.0364798E+01 P= 1.0747257E+01 G= 1.0363509E+01 RSIGMA= 0. H= 0.
 THE CURRENT VALUE OF X IS

```

8.6175586E-01      9.6175586E-01      8.6175586E-01      8.6175586E-01
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITYIES
1.0295073E+00

LAGRANGE MULTIPLIERS
F= 1.0364798E+01      P= 1.0747257E+01      G= 1.0363509E+01      RSIGMA= 0.      H= 0.
THE CURRENT VALUE OF X 15
1.1658511E+01      1.1658511E+01      1.1658511E+01
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITYIES
9.4597417E+00
TIME= .341 SECONDS

*****
POINT= 6      ODT= 3.456549/E-10      RHO= 1.0000000E+00      MAGNITUDE= 2.0385174E+04      PHASE= 2
F= 8.7631835E+00      P= 9.9641443E+00      G= 3.7631835E+00      RSIGMA= 1.2009608E+00      H= 0.
THE CURRENT VALUE OF X 15
9.5338740E-01      9.5338740E-01      9.5338740E-01
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITYIES
3.6420989E-01

2ND ORDER ESTIMATES
F= 8.5690130E+00      P= 9.2764891E+00      G= 8.5770908E+00      RSIGMA= 0.      H= 0.
THE CURRENT VALUE OF X 15
9.6504753E-01      9.6504753E-01      9.6504753E-01
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITYIES
2.7473309E-01

1ST ORDER ESTIMATES
F= 8.5861258E+00      P= 9.2911968E+00      G= 8.5770908E+00      RSIGMA= 0.      H= 0.
THE CURRENT VALUE OF X 15
9.6401461E-01      9.6401461E-01      9.6401461E-01
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITYIES
2.8270333E-01

LAGRANGE MULTIPLIERS
F= 8.5861258E+00      P= 9.2911968E+00      G= 8.5770908E+00      RSIGMA= 0.      H= 0.
THE CURRENT VALUE OF X 15
1.0488916E+00      1.0488916E+00      1.0488916E+00
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITYIES
2.7456695E+00
TIME= .480 SECONDS

*****
POINT= 8      ODT= 1.9350286E+08      RHO= 1.0000000E-01      MAGNITUDE= 3.6524186E+03      PHASE= 2
F= 8.0969863E+00      P= 8.4026486E+00      G= 7.5969863E+00      RSIGMA= 3.0564231E-01      H= 0.
THE CURRENT VALUE OF X 15
9.9395662E-01      9.9395662E-01      9.9395662E-01
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITYIES
4.8200967E-02

2ND ORDER ESTIMATES
F= 8.0190147E+00      P= 8.2184213E+00      G= 8.0229644E+00      RSIGMA= 0.      H= 0.
THE CURRENT VALUE OF X 15
9.9881229E-01      9.9881229E-01      9.9881229E-01

```

THE CONSTRAINT VALUES
9.4960700E-03
NOT INCLUDING THE NON-NEGATIVITY

1ST ORDER ESTIMATES
F= 8.0245899E+00 P= 8.2291491E+00 G= 8.0229644E+00 RSIGMA= 0. H= 0.
THE CURRENT VALUE OF X IS
9.9846431E-01 9.9846431E-01
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITY
1.2276093E-02

LAGRANGE MULTIPLIERS
F= 8.0245899E+00 P= 8.2291491E+00 G= 8.0229644E+00 RSIGMA= 0. H= 0.
THE CURRENT VALUE OF X IS
1.0060801E-01 1.0060801E-01 1.0060801E-01
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITY
2.0746472E+00
TIME= .609 SECONDS

POINT= 10 ODT= 8.6014929E-09 RHO= 1.0000000E-02 MAGNITUDE= 7.4404491E-03 PHASE= 2
F= 8.0099660E+00 P= 8.0630144E+00 G= 7.9599660E+00 RSIGMA= 5.3048390E-02 H= 0.
THE CURRENT VALUE OF X IS
9.9937732E-01 9.9937732E-01 9.9937732E-01
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITY
4.9798896E-03

2ND ORDER ESTIMATES
F= 8.000812E+00 P= 8.0232179E+00 G= 8.0002971E+00 RSIGMA= 0. H= 0.
THE CURRENT VALUE OF X IS
9.9999493E-01 9.9999493E-01 9.9999493E-01
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITY
4.0589271E-05

1ST ORDER ESTIMATES
F= 8.0003261E+00 P= 8.0252772E+00 G= 8.0002971E+00 RSIGMA= 0. H= 0.
THE CURRENT VALUE OF X IS
9.9997962E-01 9.9997962E-01 9.9997962E-01
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITY
1.6303709E-04

LAGRANGE MULTIPLIERS
F= 8.0003261E+00 P= 8.0252772E+00 G= 8.0002971E+00 RSIGMA= 0. H= 0.
THE CURRENT VALUE OF X IS
1.0006231E-02 1.0006231E-02 1.0006231E-02
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITY
2.0080766E+00
TIME= .738 SECONDS

POINT= 12 ODT= 5.6830703E-08 RHO= 1.0000000E-03 MAGNITUDE= 5.9880157E-02 PHASE= 2
F= 8.0009973E+00 P= 8.086012E+00 G= 7.9959973E+00 RSIGMA= 7.6039334E-03 H= 0.
THE CURRENT VALUE OF X IS
9.9993767E-01 9.9993767E-01 9.9993767E-01
THE CONSTRAINT VALUES

NOT INCLUDING THE NON-NEGATIVITYS

4.986113E-04

?NO ORDER ESTIMATES

F= 7.999978E+00

THE CURRENT VALUE OF X IS

1.000001E+00

THE CONSTRAINT VALUES

-1.10S2083E-06

P= 8.0023258E+00

G= 8.0000000E+00

RSIGMA= 0.

1.0000001E+00

1.0000001E+00

H= 0.

NOT INCLUDING THE NON-NEGATIVITYS

1ST ORDER ESTIMATES

F= 8.000011E+00

THE CURRENT VALUE OF X IS

9.999993E-01

THE CONSTRAINT VALUES

5.3623128E-07

P= 8.002553E+00

G= 8.0000000E+00

RSIGMA= 0.

9.999993E-01

9.999993E-01

H= 0.

NOT INCLUDING THE NON-NEGATIVITYS

LAGRANGE MULTIPLIERS

F= 8.000011E+00

THE CURRENT VALUE OF X IS

1.0000623E-03

THE CONSTRAINT VALUES

2.00S5710E+00

P= 8.002553E+00

G= 8.0000000E+00

RSIGMA= 0.

1.0000623E-03

1.0000623E-03

H= 0.

NOT INCLUDING THE NON-NEGATIVITYS

TIME= .856 SECONDS

POINT= 14

00TT= 1.881576E-07

F= 8.000101E+00

THE CURRENT VALUE OF X IS

9.9999368E-01

THE CONSTRAINT VALUES

S.054S165E-05

P= 8.001094E+00

G= 7.999601E+00

RSIGMA= 0.

9.9999368E-01

9.9999368E-01

H= 0.

NOT INCLUDING THE NON-NEGATIVITYS

PHASE= 2

MAGNITUDE= 3.6276577E-01

RSIGMA= 9.8926485E-04

RSIGMA= 0.

?NO ORDER ESTIMATES

F= 8.000011E+00

THE CURRENT VALUE OF X IS

9.999990E-01

THE CONSTRAINT VALUES

7.607S462E-07

P= 8.0002326E+00

G= 8.000001E+00

RSIGMA= 0.

9.999990E-01

9.999990E-01

H= 0.

NOT INCLUDING THE NON-NEGATIVITYS

1ST ORDER ESTIMATES

F= 8.000011E+00

THE CURRENT VALUE OF X IS

9.999991E-01

THE CONSTRAINT VALUES

7.58S0938E-07

P= 8.0002558E+00

G= 8.000001E+00

RSIGMA= 0.

9.999991E-01

9.999991E-01

H= 0.

NOT INCLUDING THE NON-NEGATIVITYS

LAGRANGE MULTIPLIERS

F= 8.000011E+00

THE CURRENT VALUE OF X IS

1.000063E-04

THE CONSTRAINT VALUES

1.978428E+00

P= 8.0002558E+00

G= 8.000001E+00

RSIGMA= 0.

1.000063E-04

1.000063E-04

H= 0.

NOT INCLUDING THE NON-NEGATIVITYS

TIME= .969 SECON05

POINT= 16 DOTT= 4.024653E-07 HWD= 1.000000E-05 MAGNITUDE= 1.336755E+00 PHASE= 2
 F= 8.0000089E+00 P= 8.0001321E+00 G= 7.9999589E+00 RSIGMA= 1.232788E-04 H= 0.
 THE CURRENT VALUE OF X IS
 9.9999945E-01 9.9999945E-01 9.9999945E-01
 THE CONSTRAINT VALUES NOT INCLUDING THE NON-NEGATIVITIES
 4.4265993E-06

2ND ORDER ESTIMATES
 F= 7.9999986E+00 P= 8.0000233E+00 G= 7.9999986E+00 RSIGMA= 0. H= 0.
 THE CURRENT VALUE OF X IS
 1.0000001E+00 1.0000001E+00 1.0000001E+00
 THE CONSTRAINT VALUES NOT INCLUDING THE NON-NEGATIVITIES
 -7.1241136E-07

1ST ORDER ESTIMATES
 F= 7.9999986E+00 P= 8.0000257E+00 G= 7.9999986E+00 RSIGMA= 0. H= 0.
 THE CURRENT VALUE OF X IS
 1.0000001E+00 1.0000001E+00 1.0000001E+00
 THE CONSTRAINT VALUES NOT INCLUDING THE NON-NEGATIVITIES
 -6.9770213E-07

LAGRANGE MULTIPLIERS
 F= 7.9999986E+00 P= 8.0000257E+00 G= 7.9999986E+00 RSIGMA= 0. H= 0.
 THE CURRENT VALUE OF X IS
 1.0000006E-05 1.0000006E-05 1.0000006E-05
 THE CONSTRAINT VALUES NOT INCLUDING THE NON-NEGATIVITIES
 2.2590705E+00

```

PROBLEM 8 SOLVED
F= 1.6000000E+01  P= 0.  G= 0.  H= 0.
THE CURRENT VALUE OF X IS
4.0000000E+00  4.0000000E+00  4.0000000E+00
THE CONSTRAINT VALUES
1.2000000E+01  NOT INCLUDING THE NON-NEGATIVITY
TIME= 1.900 SECONDS  4.0000000E+00
*****THE FEASIBLE STARTING POINT TO BE USED IS ***
F= 1.6000000E+01  P= 0.  G= 0.  H= 0.
THE CURRENT VALUE OF X IS
4.0000000E+00  4.0000000E+00  4.0000000E+00
THE CONSTRAINT VALUES
1.2000000E+01  NOT INCLUDING THE NON-NEGATIVITY
4.0000000E+00  4.0000000E+00

```

```

PROBLEM A
TIME= 0.000 SECONDS
F= 3.600000E+01 P= 0. G= 0. RSIGMA= 0. M= 0.
THE CURRENT VALUE OF X IS
5.000000E-01 5.000000E-01 5.000000E-01
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITYES
2.000000E+00
TIME= .013 SECONDS

*****THE FEASIBLE STARTING POINT TO BE USED 15 ... G= 0. M= 0.
F= 3.600000E+01 P= 0. RSIGMA= 0.
THE CURRENT VALUE OF X IS
5.000000E-01 5.000000E-01 5.000000E-01
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITYES
2.000000E+00
TIME= .073 SECONDS

*****
POINT= 2 OOT= 6.48971E-07 RHO= 1.000000E+02 MAGNITUDE= 2.368160E-02 PHASE= 2
F= 2.259548E+01 P= 1.343851E+02 G= -4.774045E+02 RSIGMA= 1.1178965E+02 M= 0.
THE CURRENT VALUE OF X IS
1.2320851E+02 1.2320851E+02 1.2320851E+02
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITYES
1.327191E+02
TIME= .145 SECONDS

*****
POINT= 4 OOT= 1.377895E-07 RHO= 1.000000E+01 MAGNITUDE= 5.077258E-03 PHASE= 2
F= 2.016611E+01 P= 3.252072E+01 G= -2.983388E+01 RSIGMA= 1.2354607E+01 M= 0.
THE CURRENT VALUE OF X IS
8.7733258E-01 8.7733258E-01 8.7733258E-01
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITYES
4.986696E-01

15T ORDER ESTIMATES
F= 1.9904710E+01 P= 2.1202452E+01 G= 1.9896184E+01 RSIGMA= 0. M= 0.
THE CURRENT VALUE OF X IS
8.8463262E-01 8.8463262E-01 8.8463262E-01
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITYES
4.6146952E-01

LAGRANGE MULTIPLIERS
F= 1.9904710E+01 P= 2.1202452E+01 G= 1.9896184E+01 RSIGMA= 0. M= 0.
THE CURRENT VALUE OF X IS
1.1398186E+01 1.1398186E+01 1.1398186E+01
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITYES
2.0380310E+01

```


RESULTS PRINTING OUT ALL POINTS OF OUTSIDE PROGRAM

The SUMT program is modified by changing RESTNT, and the INSUMT program is modified by changing BODYIS, CONVRX, ESTIMS, FEASS, INVERX, OPTS, OUTPUS, PUNCHS, TCHECX, and TIMECS to eliminate printing of inside programs.

The user-supplied information cards are identical to those of the previous section, and are not repeated here. The printouts of the inside programs called for by their information cards are by-passed by the changes in subroutines mentioned above.

The printout of the solution prints initial information for the outside program, for Problem A, and for Problem B. From then on all of the printout pertains to the outside program. At point 17 the value of r has been reduced to $1.E-04$, and the solution has been approximately attained. The program cuts off at central processor time equals 360 seconds.

Printout of Results

NONLINEAR PROGRAMMING ROUTINE-SUMT VERSION 4 3/22/71

N# 4 M# 2 M2# 0

MAX. TIME= 3.6000000E+02 R= 1.0000000E+01 RATIO= 1.0000000E+01 EPSILON= 1.0000000E-05 THETA= 1.0000000E-05

OPTIONS SELECTED

3 1 2 1 1 1 1 1 1 1

TOLERANCES

1.0000000E-03 1.0000000E-03

SECOND SET OF OPTIONS

1 3
TIME= .027 SECONDS

```

NONLINEAR PROGRAMMING ROUTINE-SUMT VERSION 4  SUB
N= 4  M= 1  MZ= 0
MAX. TIME= 1.000000E+02  M= 1.000000E+02  RATIO= 1.000000E+01  EPSILON= 1.000000E-05  THETA= 1.000000E-05
OPTIONS SELECTED
3  1  1  1  1  1  1  2
TOLERANCES
1.000000E-03  1.000000E-03
SECOND SET OF OPTIONS
1  1

```

```

NONLINFAH PROGRAMMING ROUTINE-SUMT VERSION 4 SUR
N= A P= 1 47= 0
MAX. TIME= 1.8000000E+02 Q= 1.0000000F+02 PATIO= 1.0000000E+01 EPSILON= 1.0000000F+05 THETA= 1.0000000F+05
OPTIONS SELECTED
3 1 1 1 1 1 1 1 1 1
TOLERANCES
1.00000000F+03 1.00000000F+03
SECOND SET OF OPTIONS
1 1 F= 1.6000000E+01 P= 0. Q= 0. PSIGMA= 0. H= 0.
THE CURRENT VALUE OF X IS
A.0000000E+00 4.0000000E+00 4.0000000E+00
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITYS
1.2000000E+01 4.0000000E+00
TIME= .943 SECONDS
*****THE FEASIBLE STARTING POINT TO BE USED IS *** Q= 0. H= 0.
E= 1.6000000E+01 P= 0. RSIGMA= 0.
THE CURRENT VALUE OF X IS
A.0000000E+00 4.0000000E+00 4.0000000E+00
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITYS
1.2000000E+01 4.0000000E+00
TIME= 23.600 SECONDS
*****
POINT= 1 NOIT= 3.5011227E+01 MHO= 1.0000000E+01 MAGNITUDE= 5.0170286E+00 PHASE= 2
F= 5.7545151E+01 P= -1.1304499E+02 G= -2.4575446E+00 HSIGMA= -1.7058741E+02 H= 0.
THE CURRENT VALUE OF X IS
1.4386288E+01 1.4386288E+01 1.4386288E+01
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITYS
5.3545259E+01 1.1171714E+01
TIME= 35.111 SECONDS
*****
POINT= 2 UOIT= 5.1208486E+08 MHO= 1.0000000E+01 MAGNITUDE= 2.2624292E+04 PHASE= 2
F= 5.7542615E+01 Q= -1.1304499E+02 G= -2.4575446E+00 HSIGMA= -1.7058741E+02 H= 0.
THE CURRENT VALUE OF X IS
1.4386288E+01 1.4386288E+01 1.4386288E+01
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITYS
5.3542615E+01 1.1171354E+01
TIME= 35.111 SECONDS
*****
LAGRANGE MULTIPLIERS
F= 5.7542615E+01 P= -1.1304499E+02 G= -2.4575446E+00 PSIGMA= -1.7058741E+02 H= 0.
THE CURRENT VALUE OF X IS
6.9513940E+01 6.9513940E+01 6.9513940E+01
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITYS
1.4386288E+01 1.4386288E+01 1.4386288E+01
TIME= 66.810 SECONDS
*****
POINT= 3 UOIT= 3.2394380E+00 MHO= 1.0000000E+00 MAGNITUDE= 1.7994439E+00 PHASE= 2
F= 7.0079447E+00 P= 3.3326411E+00 G= 1.4079447E+00 HSIGMA= -4.4753076E+00 H= 0.
THE CURRENT VALUE OF X IS

```

```

1.9519472E+00 1.4519472F+00 1.9519472F+00 1.9519472F+00 PHASE= 2
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITYS
TIME= 0.232 SECONDS
*****
POINT= 5 0.01E= 2.675427E-07 MHO= 1.0000000F+00 MAGNITUDE= 5.1724489E-04 H= 0.
F= 7.808459E+00 P= 3.326410E+00 G= 1.808459E+00 RSTIMA= -4.4758249F+00 H= 0.
THE CURRENT VALUE OF X IS
1.9521145F+00 1.9521165E+00 1.9521145F+00
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITYS
3.404476E+00 1.588736E+00
*****
1ST ORDER ESTIMATES
F= 2.2424715E+00 P= 1.626348E+01 G= 2.2424715E+00 RSTIMA= 0. H= 0.
THE CURRENT VALUE OF X IS
5.1226451E-01 5.1226452F-01 5.1226452E-01 5.1226452F-01
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITYS
2.625722E-01 6.294309E-01
TIME= 107.247 SECONDS
*****
LAGRANGE MULTIPLIERS
F= 2.2424715E+00 P= 1.626348E+01 G= 2.2424715E+00 RSTIMA= 0. H= 0.
THE CURRENT VALUE OF X IS
5.1226451E-01 5.1226452F-01 5.1226452E-01 5.1226452F-01
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITYS
2.625722E-01 6.294309E-01
TIME= 107.247 SECONDS
*****
POINT= 6 0.01E= 3.239911E+00 MHO= 1.0000000F+01 MAGNITUDE= 1.799074F+00 H= 0.
F= 4.2191776E+00 P= 4.5720697E+00 G= 3.6191776E+00 RSTIMA= 3.5268405F-01 H= 0.
THE CURRENT VALUE OF X IS
1.0548459E+00 1.0548459F+00 1.0548459E+00 1.0548459F+00
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITYS
2.1438451E-01 1.0822856F-01
TIME= 124.671 SECONDS
*****
POINT= 7 0.01E= 7.620011E+07 MHO= 1.0000000F+01 MAGNITUDE= 8.7297200E-04 H= 0.
F= 4.2191776E+00 P= 4.5720699E+00 G= 3.6191776E+00 RSTIMA= 3.5268405F-01 H= 0.
THE CURRENT VALUE OF X IS
1.0547944E+00 1.0547944F+00 1.0547944E+00 1.0547944F+00
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITYS
2.1417841E-01 1.0812923F-01
*****
2ND ORDER ESTIMATES
F= 3.4359021E+00 P= 4.5930801E+00 G= 3.6203678E+00 RSTIMA= 0. H= 0.
THE CURRENT VALUE OF X IS
9.5897553E-01 9.5897552F-01 9.5897552E-01 9.5897552F-01
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITYS
-1.6409711E-01 -4.2907059F-02
*****
1ST ORDER ESTIMATES
F= 3.4359021E+00 P= 4.7047862E+00 G= 3.6203678E+00 RSTIMA= 0. H= 0.
THE CURRENT VALUE OF X IS

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9.5504196E-01	9.5509194F-01	9.5509194E-01	9.5509194F-01
THF CONSTRAINT VALUES	NOT INCLUDING THE NON-NEGATIVITY		
1.7663133E-01	-9.0466653F-02		

```

LAGRANGE MULTPLIFHS
F= 3.020367HE+00 P= 4.7097842E+00 G= 3.8203678E+00 RSIHMA= 0. H= 0.
THE CURRENT VALUE OF X IS
9.4805205E-02 9.4805206E-02 9.4805206F-02
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITYFS
4.5624932E+01 9.2482785F-01
TIME= 152.961 SECONDS

*****
POINT= 4 DOT= 3.2390600E+00 HMO= 1.000000F-02 MAGNITUDE= 1.7997389F-00 PHASE= 2
F= 4.0201711E+00 P= 4.1049827E+00 G= 3.9601711E+00 RSIHMA= 8.48116ATF-02 H= 0.
THE CURRENT VALUE OF X IS
1.0050428E+00 1.0050428E+00 1.0050428F+00
THE CONSTRAINT VALUES
NOT INCLUDING THE NON-NEGATIVITYFS
2.0171942E+02 1.0073698F-02
TIME= 176.87M SECONDS

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*****
POINT= 11      NDT= 2.660278E+06      HQD= 1.0000000F+02      MARGITUDE= 1.4310360F+03      PHASE= 2
F= 4.0201149E+00      P= 4.1049829E+00      G= 3.0601149E+00      MSIGMA= 8.4867987E-02      H= 0.
THE CURRENT VALUE OF X IS
1.0050287E+00      1.0050287E+00      1.0050287E+00
THE CONSTANT VALUES NOT INCLUDING THE NON-NEGATIVITYS
2.0115744E-02      1.0045678E-02
PNO ORIGIN ESTIMATES
F= 3.9997910E+00      P= 4.0464510E+00      G= 3.9479968E+00      MSIGMA= 0.      H= 0.
THE CURRENT VALUE OF X IS
3.9994777E-01      9.9994776E-01      9.9994776F-01
THE CONSTANT VALUES NOT INCLUDING THE NON-NEGATIVITYS
-2.0810406E-04      -1.0335564E-04

```

```

1ST ORDER ESTIMATES
  F= 3.9979964E+00      P= 4.0530843E+00      G= 3.9979964E+00      HSI64= 0.      H= 0.
THE CURRENT VALUE OF X IS
  9.9969921E+01      9.9969920F-01      9.9969920F-01
THE CONSTRAINT VALUES
  NOT INCLUDING THE NON-NEGATIVITY'S
    -2.0022956E-03      -1.0000652E-03
LAGRANGE MULTIPLIERS
  F= 3.9979964E+00      P= 4.0530843E+00      G= 3.9979964E+00      HSI64= 0.      H= 0.
THE CURRENT VALUE OF X IS
  9.99699643E+03      9.9969964E-03      9.9969964F-03
THE CONSTRAINT VALUES
  NOT INCLUDING THE NON-NEGATIVITY'S
    4.9712295E-01      9.9965294E-01
TIME= 201.330 SECONDS

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*****
POINT= 12      D01T= 3.274195E+00      H40= 1.0000000E-03      PHASE= ?
F= 4.0020147E+00      P= 4.0151190E+00      G= 3.9460147E+00      H= 0.
THE CURRENT VALUE OF X IS      M511MA= 1.3104221F-02      MAGNITUDE= 1.7992811E+00

```

1.0005037E+00 1.0005037E+00 1.0005037E+00 PHASE= 2
 THE CONSTRAINT VALUES
 NOT INCLUDING THE NON-NEGATIVITYS
 2.0156495E-03 1.0083689E-03
 TIME= 234.886 SECONDS

 POINT= 14 ODT= 2.0009282E-04 HMO= 1.0000000E-03 MAGNITUDE= 1.414417E-02
 F= 4.0020147E+00 P= 4.0151190E+00 G= 3.9940147E+00 PSIGMA= 1.3104221E-02 H= 0.
 THE CURRENT VALUE OF X IS
 1.0005037E+00 1.0005037E+00 1.0005037E+00
 THE CONSTRAINT VALUES
 NOT INCLUDING THE NON-NEGATIVITYS
 2.0156495E-03 1.0083689E-03
 APPARENTLY ROUND-OFF ERRORS PREVENT A MORE ACCURATE DETERMINATION OF THE MINIMUM OF THIS SUBPROBLEM.

2ND OTHER ESTIMATES
 F= 4.0000239E+00 P= 4.0046497E+00 G= 4.0000036E+00 RSIGMA= 0. H= 0.
 THE CURRENT VALUE OF X IS
 1.0000060E+00 1.0000060E+00 1.0000060E+00
 THE CONSTRAINT VALUES
 NOT INCLUDING THE NON-NEGATIVITYS
 2.4754009E-05 1.3768135E-05

1ST OTHER ESTIMATES
 F= 4.0000036E+00 P= 4.0051341E+00 G= 4.0000036E+00 RSIGMA= 0. H= 0.
 THE CURRENT VALUE OF X IS
 1.0000009E+00 1.0000009E+00 1.0000009E+00
 THE CONSTRAINT VALUES
 NOT INCLUDING THE NON-NEGATIVITYS
 4.4421430E-06 2.9327844E-06

LAGRANGE MULTIPLIERS
 F= 4.0000036E+00 P= 4.0051341E+00 G= 4.0000036E+00 RSIGMA= 0. H= 0.
 THE CURRENT VALUE OF X IS
 9.9949656E-04 9.9949657E-04 9.9949657E-04
 THE CONSTRAINT VALUES
 NOT INCLUDING THE NON-NEGATIVITYS
 4.9411801E-01 9.9170060E-01
 TIME= 254.309 SECONDS

 POINT= 15 ODT= 3.2450944E+00 HMO= 1.0000000E-04 MAGNITUDE= 1.801145E+00
 F= 4.0001962E+00 P= 4.0019712E+00 G= 3.9995942E+00 RSIGMA= 1.7749474E-03 H= 0.
 THE CURRENT VALUE OF X IS
 1.0000491E+00 1.0000491E+00 1.0000491E+00
 THE CONSTRAINT VALUES
 NOT INCLUDING THE NON-NEGATIVITYS
 1.9711422E-04 9.9242635E-05
 TIME= 294.894 SECONDS

.....
 POINT= 17 ODT= 5.0294919E-04 HMO= 1.0000000E-04 MAGNITUDE= 2.2627258E-02
 F= 4.0001900E+00 P= 4.0019714E+00 G= 3.9995940E+00 RSIGMA= 1.7814014E-03 H= 0.
 THE CURRENT VALUE OF X IS
 1.0000475E+00 1.0000475E+00 1.0000475E+00
 THE CONSTRAINT VALUES
 NOT INCLUDING THE NON-NEGATIVITYS
 1.9713354E-04 9.6102524E-05
 APPARENTLY ROUND-OFF ERRORS PREVENT A MORE ACCURATE DETERMINATION OF THE MINIMUM OF THIS SUBPROBLEM.

2ND OTHER ESTIMATES

F= 3.999972E+00 P= 4.0004638E+00 G= 3.999972E+00 RSTGMA= 0. H= 0.
 THE CURRENT VALUE OF X IS
 9.999967E-01 9.9999676E-01 9.9999676E-01
 THE CONSTRAINT VALUES
 NOT INCLUDING THE NON-NEGATIVITYS
 -1.2796163E-05 -5.3559433E-06

1ST ORDER ESTIMATES
 F= 3.999972E+00 P= 4.0005105E+00 G= 3.999972E+00 RSTGMA= 0. H= 0.
 THE CURRENT VALUE OF X IS
 9.9999681E-01 9.9999680E-01 9.9999680E-01
 THE CONSTRAINT VALUES
 NOT INCLUDING THE NON-NEGATIVITYS
 -1.1430379E-05 -5.2730559E-06

LAGRANGE MULTIPLIERS
 F= 3.999972E+00 P= 4.0005105E+00 G= 3.999972E+00 RSTGMA= 0. H= 0.
 THE CURRENT VALUE OF X IS
 9.9999525E-05 9.9995252E-05 9.9995252E-05
 THE CONSTRAINT VALUES
 NOT INCLUDING THE NON-NEGATIVITYS
 5.240169E-01 1.0405554E+00
 TIME= 315.447 SECONDS

 POINT= 18 DOTT= 3.2081309E+00 HMO= 1.0000000E-05 MAGNITUDE= 1.7011256E+00 PHASE= 2
 F= 4.0000147E+00 D= 4.0002418E+00 G= 3.9999597E+00 RSTGMA= 2.2209197E-04 H= 0.
 THE CURRENT VALUE OF X IS
 1.0000049E+00 1.0000049E+00 1.0000049E+00
 THE CONSTRAINT VALUES
 NOT INCLUDING THE NON-NEGATIVITYS
 2.0593770E-05 1.098132E-05

REFERENCES

1. Bracken, J. and McCormick, G.P., Selected Applications of Nonlinear Programming, Wiley, 1968.
2. Bracken, J. and McGill, J.T., "Mathematical Programs With Optimization Problems in the Constraints", Institute for Defense Analyses, Research Paper P-725, July 1971.
3. Bracken, J. and McGill, J.T., "Mathematical Programs With Optimization Problems in the Constraints: Applications to Defense Analyses", Institute for Defense Analyses, Research Paper P-784, August 1971 (DRAFT).
4. Fiacco, A.V. and McCormick, G.P., Nonlinear Programming: Sequential Unconstrained Minimization Techniques, Wiley, 1968.
5. McCormick, G.P., Mylander, W.C. and Fiacco, A.V., "Computer Program Implementing the Sequential Unconstrained Minimization Technique for Nonlinear Programming", Research Analysis Corporation, Technical Paper TP-151, April 1965.
6. Mylander, W.C., Holmes, R.L. and McCormick, G.P., "A Guide to SUMT - Version 4: The Computer Program Implementing the Sequential Unconstrained Minimization Technique for Nonlinear Programming", Research Analysis Corporation, RAC-P-63, October 1971.

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13. ABSTRACT This paper documents a computer program to be used in solving nonlinear programming problems with nonlinear programming problems in the constraints. The program, named INSUMT, is used with the standard program, named SUMT, which implements the sequential unconstrained minimization technique for nonlinear programming. SUMT calls INSUMT when it is necessary to solve a nonlinear program in a constraint. The INSUMT program, together with a fairly complete example of its use, is included in the documentation. Theory and applications of the models which can be solved using this program are treated in two companion papers.			

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14	KEY WORDS	LINK A		LINK B		LINK C	
		ROLE	WT	ROLE	WT	ROLE	WT
	OPTIMIZATION						
	NONLINEAR PROGRAMMING						
	MAX-MIN						
	TWO-SIDED OPTIMIZATION						
	GAME THEORY						
	SUMT						
	INSUMT						
	COMPUTER PROGRAM						
	ALGORITHM						

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