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**UPPER PERCENTAGE POINTS OF THE
INDIVIDUAL ROOTS OF THE WISHART MATRIX**

D. S. CLEMM

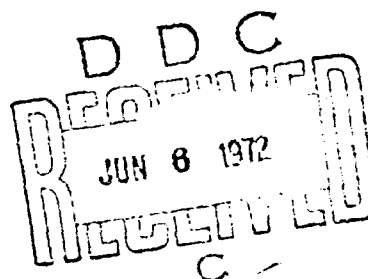
A. K. CHATTOPADHYAY

P. R. KRISHNAIAH

APPLIED MATHEMATICS RESEARCH LABORATORY

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34

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A. K. CHATTOPADHYAY
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**AEROSPACE RESEARCH LABORATORIES
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
WRIGHT-PATTERSON AIR FORCE BASE, OHIO**

FOREWORD

This report was prepared for the Applied Mathematics Research Laboratory, Aerospace Research Laboratories by D. S. Clemm, A. K. Chattopadhyay and P. R. Krishnaiah under Project 7071, "Research in Applied Mathematics". The work of A. K. Chattopadhyay was performed at the Aerospace Research Laboratories while in the capacity of Technology Incorporated Visiting Research Associate under Contract F 33615-71-C-1463, T. I. Project No. 4262B.

In this report, the authors gave exact percentage points of the smallest and intermediate roots of the Wishart matrix.

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ABSTRACT

Let $S: p \times p$ be distributed as central Wishart matrix with n degrees of freedom and let $E(S) = I_p$ where I_p is an identity matrix. Also, let $\theta_1 < \dots < \theta_p$ be the roots of S . In this report, the authors gave exact upper 10%, 5%, 2.5% and 1% points of the distributions of θ_i ($i=1,2,\dots,p-1$) for $p=2(1)10$ and $n=(p+1)(1)20(2)30(5)50$.

TABLE OF CONTENTS

Section	Page
1. Introduction	1
2. Cumulative Distribution Function of the Individual Roots	1
3. References	5
Tables	

1. INTRODUCTION

The marginal distribution of the individual roots of the Wishart matrix are useful in testing certain statistical hypotheses. Pillai and Chang [7] constructed tables for the upper percentage points of the largest root of the Wishart matrix. Davis [2] gave a recurrence relation for the marginal densities of the individual roots by using results in [1]. Krishnaiah and Waikar [5] gave exact expressions for the cumulative distribution functions (c.d.f.'s) of intermediate roots. In this paper, we give exact upper 10%, 5%, 2.5% and 1% points of the smallest and intermediate roots of the Wishart matrix.

2. CUMULATIVE DISTRIBUTION FUNCTIONS OF THE INDIVIDUAL ROOTS

Let $S: p \times p$ be distributed as Wishart matrix with n degrees of freedom and let $E(S) = n I_p$, where I_p is the p th order identity matrix. Also, let $\theta_1 < \dots < \theta_p$ be the eigenvalues of S . Then the joint density of $\theta_1 < \dots < \theta_p$ is given by

$$f(\theta_1, \dots, \theta_p) = k(p, n) \prod_{i=1}^p [\theta_i^r \exp(-\frac{\theta_i}{2})] \prod_{i>j} \pi(\theta_i - \theta_j) \quad 0 < \theta_1 < \dots < \theta_p < \infty \quad (2.1)$$

where

$$r = (n-p-1)/2, \quad \text{and}$$

$$k(p, n) = \pi^{p/2} \left(\frac{1}{2}\right)^{\frac{np}{2}} / \prod_{i=1}^p [\Gamma((n+1-i)/2) \Gamma((p+1-i)/2)]. \quad (2.1a).$$

The following exact expression for the c.d.f. of the intermediate root θ_s ($1 \leq s \leq p-1$) was given in Krishnaiah and Waikar [5]:

$$P[\theta_s < x] = P[\theta_{s+1} < x] + k(p, n) \int_1^{\infty} \rho(\psi; s, \{k_1, \dots, k_s\}, 0, x) \cdot \rho(\psi; p-s, \{t_1, \dots, t_{p-s}\}, x, \infty), \quad (2.2)$$

where $\{k_1, \dots, k_s\}$ is a subset of integers $\{0, 1, \dots, p-1\}$ such that $k_1 < \dots < k_s$ and $t_1 < \dots < t_{p-s}$ is the subset complementary to $k_1 < \dots < k_s$ while $\sum_1$ denotes the summation over $\binom{p}{s}$ possible subsets $k_1 < \dots < k_s$. Further $\psi(y) = \exp\left(-\frac{y}{2}\right)$ and the sign inside $\sum_1$ is positive or negative according as $s(s+3)/2 + \sum_{i=1}^s k_i$ is even or odd. The function $\rho(\cdot)$ is defined by

$$\rho(\psi; p, \{k_1, \dots, k_p\}, L, U) = \Delta(\psi; 2m, \{k_1, \dots, k_{2m}\}, L, U) \text{ when } p = 2m \quad (2.3)$$

and

$$\rho(\psi; p, \{k_1, \dots, k_p\}, L, U) = \sum_{i=1}^{2m+1} (-1)^{i+1} F_{k_i}(L, U) \cdot G_i(\psi; 2m+1, \{k_1, \dots, k_{2m+1}\}, L, U) \text{ when } p = 2m+1 \quad (2.4)$$

where

$$\Delta(\psi; 2m, \{k_1, \dots, k_{2m}\}, L, U) = \left| (f_{k_i}^{k_j}(L, U))_{i,j=1, \dots, 2m} \right|^{1/2},$$

$$G_t(\psi; 2m+1, \{k_1, \dots, k_{2m+1}\}, L, U) = \left| (f_{k_i}^{k_j}(L, U))_{i,j=1, \dots, t-1, t+1, \dots, 2m+1} \right|^{1/2}$$

for $t=1, \dots, 2m+1$ while $G_1(\psi; 1, k_1, L, U)=1$. Further

$$f_s^t(L, U) = F_s^t(L, U) - F_t^s(L, U),$$

$$F_s^t(L, U) = \int_L^U F_s(L, \theta) \theta^t \psi(\theta) d\theta,$$

$$F_s(L, \theta) = \int_L^0 x^s \psi(x) dx.$$

Also it is known (see[3,4]), that

$$P[\theta_1 \leq x] = 1 - k(p, n) \rho(\psi; 1, r, c, \infty). \quad (2.5)$$

Using (2.2) and (2.5), we have constructed tables for the exact values of x for $p=2(1)10$, $n=(p+1)(1)20(2)30(5)50$, $k=1(1)(p-1)$ and $\alpha=0.01, 0.025, 0.05, 0.10$ where

$$P[\theta_s \leq x] = 1 - \alpha.$$

The values are given up to four decimal places and they may differ from actual values by at most one unit in the last decimal. As a check for the accuracy of the tables we have used the programs to compute the values of

$$P[x \leq \theta_1] + P[\theta_1 \leq x \leq \theta_2] + \dots + P[\theta_{p-1} \leq x \leq \theta_p] + P[\theta_p \leq x]$$

for some x and found them to differ from 1 in the 12th decimal only.

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The entries in the following table are the values of x for different values of N, p, S and α where

$$P[\theta_S \leq x] = (1-\alpha),$$

$$f(\theta_1, \dots, \theta_p) = k(p, N) \prod_{i=1}^p [\theta_i^r \exp(-\theta_i/2)] \prod_{i>j} (\theta_i - \theta_j)$$

$$0 < \theta_1 < \dots < \theta_S < \dots < \theta_p < \infty,$$

$r = (N-p-1)/2$, and $k(p, N)$ is given by Eq. (2.1a).

TABLE 1
UPPER PERCENTAGE POINTS OF THE INDIVIDUAL ROOTS

		P= 2		S=1				P= 3		S=1	
N	α	.01	.025	.05	.10	N	α	.01	.025	.05	.10
3						4					
4	4.61	3.69	3.00	2.30		5	3.07	2.46	2.00	1.54	
5	6.09	5.04	4.23	3.40		6	4.21	3.49	2.94	2.36	
6	7.49	6.33	5.42	4.43		7	5.33	4.51	3.88	3.21	
7	8.83	7.58	6.59	5.55		8	6.42	5.53	4.82	4.07	
8	10.15	8.81	7.74	6.61		9	7.51	6.54	5.76	4.94	
9	11.44	10.01	8.88	7.67		10	8.59	7.55	6.71	5.82	
10	12.70	11.21	10.01	8.72			9.66	8.55	7.66	6.70	
	13.95	12.39	11.12	9.75							
11	15.19	13.56	12.24	10.81		11	10.72	9.56	8.62	7.59	
12	16.41	14.72	13.34	11.85		12	11.79	10.56	9.57	8.49	
13	17.63	15.87	14.45	12.89		13	12.84	11.57	10.53	9.39	
14	18.83	17.02	15.54	13.93		14	13.90	12.57	11.49	10.30	
15	20.03	18.16	16.64	14.97		15	14.95	13.58	12.44	11.20	
16	21.22	19.30	17.73	16.00		16	16.01	14.58	13.41	12.12	
17	22.41	20.43	18.81	17.04		17	17.05	15.58	14.37	13.03	
18	23.58	21.56	19.90	18.07		18	18.10	16.58	15.33	13.95	
19	24.76	22.68	20.98	19.11		19	19.15	17.59	16.29	14.86	
20	25.93	23.81	22.06	20.14		20	20.19	18.59	17.26	15.79	
22	28.25	26.04	24.21	22.20		22	22.28	20.59	19.19	17.63	
24	30.56	28.26	26.36	24.25		24	24.36	22.59	21.12	19.49	
26	32.85	30.47	28.50	26.31		26	26.44	24.60	23.06	21.35	
28	35.14	32.67	30.65	28.35		28	28.51	26.60	25.00	23.22	
30	37.41	34.87	32.76	30.41		30	30.58	28.60	26.94	25.09	
35	43.05	40.33	38.06	35.51		35	35.75	33.60	31.80	29.78	
40	48.65	45.76	43.35	40.64		40	40.90	38.61	36.58	34.50	
45	54.21	51.16	48.61	45.75		45	46.05	43.61	41.55	39.23	
50	59.74	56.54	53.86	50.84		50	51.18	48.61	46.44	43.98	

TABLE 1 (CONTINUED)

		P= 3			S=2			P= 4			S=1		
N	α	.01	.025	.05	.10	N	α	.01	.025	.05	.10		
4	8.41		7.22	6.30	5.32	5		2.30	1.84	1.50	1.15		
5	10.05		8.77	7.75	6.68	6		3.23	2.68	2.26	1.82		
6	11.60		10.24	9.15	7.99	7		4.16	3.53	3.04	2.52		
7	13.11		11.57	10.51	9.27	8		5.09	4.39	3.83	3.24		
8	14.57		13.06	11.84	10.53	9		6.02	5.25	4.64	3.98		
9	16.00		14.42	13.15	11.77	10		6.96	6.13	5.46	4.74		
10	17.40		15.76	14.43	12.99								
11	18.78		17.08	15.71	14.21	11		7.89	7.01	6.29	5.51		
12	20.14		18.39	16.96	15.41	12		8.83	7.89	7.12	6.29		
13	21.49		19.68	18.21	16.60	13		9.77	8.78	7.97	7.08		
14	22.92		20.96	19.44	17.78	14		10.71	9.67	8.82	7.93		
15	24.14		22.23	20.67	18.95	15		11.66	10.57	9.67	8.59		
16	25.45		23.49	21.89	20.13	16		12.60	11.47	10.54	9.51		
17	26.74		24.74	23.10	21.29	17		13.55	12.37	11.40	10.33		
18	28.03		25.98	24.30	22.45	18		14.50	13.28	12.27	11.15		
19	29.31		27.22	25.50	23.61	19		15.45	14.19	13.14	11.93		
20	30.58		28.45	26.69	24.76	20		16.40	15.10	14.02	12.82		
22	33.10	30.89		29.06	27.05	22		18.31	16.93	15.78	14.50		
24	35.60	33.31	33.31	31.41	29.32	24		20.22	18.77	17.56	15.20		
26	38.08	35.71	35.71	33.75	31.59	26		22.13	20.61	19.34	17.92		
28	40.53	38.10	38.10	36.08	33.84	28		24.05	22.46	21.13	19.64		
30	42.98	40.47	40.47	38.39	36.09	30		25.97	24.31	22.93	21.37		
35	49.01	46.34	46.34	44.13	41.65	35		30.78	28.97	27.45	25.74		
40	54.98	52.16	52.16	49.81	47.19	40		35.60	33.65	32.01	30.16		
45	60.88	57.92	57.92	55.45	52.69	45		40.43	38.36	36.60	34.61		
50	65.74	63.64	63.64	61.05	58.16	50		45.28	43.07	41.21	39.09		

TABLE 1 (CONTINUED)

		p = 4				S = 3				
		S = 2				S = 3				
N	α	p = 4				S = 3				
		.01	.025	.05	.10	N	.01	.025	.05	.10
5	5.03	5.20	4.54	3.85	12.21	5	12.21	10.83	9.74	8.56
6	7.37	6.46	5.73	4.95	13.92	6	13.92	12.47	11.31	10.05
7	8.67	7.58	6.89	6.04	15.57	7	15.57	14.05	12.82	11.49
8	9.94	8.88	8.03	7.11	17.16	8	17.16	15.57	14.29	12.90
9	11.19	10.07	9.16	8.13	18.71	9	18.71	17.06	15.73	14.27
10	12.42	11.24	10.28	9.24	20.23	10	20.23	18.52	17.13	15.52
11	13.63	12.40	11.40	10.30	21.72	11	21.72	19.96	18.52	16.96
12	14.84	13.55	12.51	11.35	23.18	12	23.18	21.37	19.89	18.27
13	15.03	14.70	13.61	12.41	24.62	13	24.62	22.76	21.24	19.57
14	17.22	15.84	14.70	13.45	26.05	14	26.05	24.14	22.57	20.86
15	18.39	16.97	15.80	14.51	27.46	15	27.46	25.50	23.90	22.14
16	19.57	18.10	16.89	15.55	28.95	16	28.95	26.85	25.21	23.40
17	20.73	19.22	17.97	16.60	30.23	17	30.23	28.19	26.51	24.66
18	21.89	20.34	19.06	17.64	31.60	18	31.60	29.52	27.80	25.91
19	23.04	21.45	20.14	18.68	32.96	19	32.96	30.83	29.08	27.15
20	24.19	22.56	21.22	19.72	34.31	20	34.31	32.14	30.36	28.39
22	26.48	24.75	23.37	21.60	36.98	22	36.98	34.74	32.89	30.84
24	28.76	26.98	25.51	23.87	39.62	24	39.62	37.30	35.39	33.27
26	31.02	29.18	27.65	25.94	42.23	26	42.23	39.85	37.87	35.69
28	33.27	31.36	29.78	28.01	44.82	28	44.82	42.37	40.34	38.03
30	35.51	33.54	31.90	30.07	47.38	30	47.38	44.87	42.78	40.47
35	41.08	38.97	37.20	35.22	53.72	35	53.72	51.05	48.83	46.37
40	46.60	44.36	42.47	40.35	59.95	40	59.95	57.15	54.81	52.20
45	52.10	49.72	47.73	45.43	66.12	45	66.12	63.18	60.73	57.99
50	57.56	55.07	52.97	50.61	72.21	50	72.21	69.15	66.59	63.73

TABLE 1 (CONTINUED)

P= 5			P= 5			P= 5			P= 5		
S=1			S=1			S=1			S=2		
N	α	α	N	α	α	N	α	α	N	α	α
	.01	.025		.01	.025		.01	.025		.01	.025
6	1.54	1.48	6	.92	4.08	6	4.73	4.08	6	3.03	3.57
7	2.63	2.18	7	1.43	1.20	7	5.87	4.73	7	3.96	4.57
8	3.42	2.91	8	2.08	1.64	8	6.99	5.15	8	4.89	5.57
9	4.23	3.65	9	2.70	2.50	9	8.10	6.20	9	5.82	6.56
10	5.05	4.41	10	3.35	3.19	10	9.19	7.25	10	6.75	7.55
11	5.88	5.18		4.02	3.90			8.29			
12	6.71	5.96	11	4.70	4.62	11	10.28	9.32	11	7.70	8.54
13	7.55	6.76	12	5.40	5.36	12	11.36	10.36	12	8.64	9.53
14	8.40	7.56	13	6.11	6.11	13	12.44	11.39	13	9.58	10.52
15	9.25	8.37	14	6.84	6.87	14	13.51	12.41	14	10.52	11.51
16	10.11	9.18	15	7.57	7.64	15	14.58	13.44	15	11.47	12.50
17	10.98	10.00	16	8.31	8.41	16	15.65	14.47	16	12.42	13.49
18	11.84	10.83	17	9.05	9.20	17	16.71	15.49	17	13.37	14.48
19	12.71	11.66	18	9.82	9.99	18	17.77	16.51	18	14.32	15.47
20	13.59	12.50	19	10.59	10.79	19	18.83	17.53	19	15.27	16.46
			20	11.59	11.59	20	19.89	18.55	20	16.22	17.45
22	15.35	14.18		12.13	13.21		21.99	20.59	22	17.14	19.43
24	17.11	15.88	22	13.70	14.85	22	24.10	22.63	24	20.05	21.41
25	18.89	17.59	24	15.28	16.50	24	25.20	24.67	25	21.97	23.39
29	20.68	19.31	26	16.83	18.17	26	28.29	26.70	29	23.89	25.37
30	22.47	21.05	28	18.50	19.85	28	30.38	28.73	30	25.81	27.36
35	26.89	25.42		22.60	24.10		35.59	33.81			
40	31.54	29.83	35	26.76	28.39	35	40.78	38.88	35	30.63	32.31
45	36.11	33.29	40	30.98	32.74	40	45.96	43.94	40	35.46	37.27
50	40.72	38.77	50	35.24	37.12	50	51.13	49.00	50	40.31	42.23
										45.16	47.19

TABLE 1 (CONTINUED)

p= 5 S= 3						p= 5 S= 4					
N	α	.01	.025	.05	.10	N	α	.01	.025	.05	.10
6	9.16		8.16	7.36	6.49	6	16.02		14.50	13.26	11.93
7	10.62		9.55	8.68	7.75	7	17.79		16.19	14.90	13.50
8	12.03		10.89	9.98	8.93	8	19.49		17.83	16.49	15.03
9	13.40		12.21	11.25	10.19	9	21.14		19.43	18.03	16.51
10	14.75		13.51	12.50	11.39	10	22.75		20.99	19.54	17.97
11	16.08		14.79	13.73	12.57	11	24.33		22.51	21.03	19.40
12	17.39		16.05	14.95	13.74	12	25.88		24.01	22.48	20.81
13	18.69		17.30	16.16	14.91	13	27.41		25.49	23.92	22.20
14	19.97		18.54	17.36	16.07	14	28.91		26.95	25.34	23.57
15	21.25		19.77	18.56	17.22	15	30.39		28.39	26.74	24.93
16	22.51		20.99	19.74	18.36	16	31.86		29.81	28.13	26.28
17	23.76		22.21	20.92	19.50	17	33.31		31.22	29.51	27.61
18	25.00		23.41	22.10	20.64	18	34.75		32.62	30.87	28.94
19	26.24		24.61	23.26	21.77	19	36.17		34.01	32.22	30.25
20	27.47		25.81	24.43	22.90	20	37.58		35.38	33.57	31.56
22	29.91		28.18	26.74	25.14	22	40.38		38.10	36.23	34.15
24	32.33		30.53	29.04	27.37	24	43.14		40.79	38.85	36.71
26	34.73		32.87	31.32	29.59	26	45.86		43.45	41.46	39.24
28	37.12		35.20	33.59	31.81	28	48.56		46.08	44.03	41.76
30	39.49		37.51	35.86	34.01	30	51.23		48.69	46.59	44.25
35	45.36		43.24	41.48	39.49	35	57.81		55.13	52.90	50.42
40	51.17		48.93	47.05	44.94	40	64.27		61.46	59.12	56.51
45	56.93		54.56	52.58	50.36	45	70.65		67.71	65.26	62.53
50	62.64		60.17	58.09	55.75	50	76.95		73.89	71.34	68.49

TABLE 1 (CONTINUED)

P=6			S=1			P=6			S=2		
N	α	.01	.025	.05	.10	N	α	.01	.025	.05	.10
7	1.54	1.23	1.00	1.00	.77	7	3.90	3.90	3.37	2.95	2.50
8	2.21	1.84	1.55	1.25	1.25	8	4.90	4.90	4.30	3.82	3.31
9	2.91	2.47	2.13	1.77	1.77	9	5.88	5.88	5.22	4.69	4.12
10	3.63	3.13	2.74	2.32	2.32	10	6.86	6.86	6.15	5.57	4.95
11	4.36	3.81	3.37	2.90	2.90	11	7.84	7.84	7.08	6.46	5.78
12	5.10	4.50	4.02	3.43	3.43	12	8.82	8.82	8.01	7.35	6.62
13	5.85	5.21	4.68	4.11	4.11	13	9.80	9.80	8.94	8.24	7.47
14	6.52	5.93	5.36	4.75	4.75	14	10.77	10.77	9.88	9.14	8.33
15	7.20	6.66	6.06	5.40	5.40	15	11.75	11.75	10.81	10.04	9.19
16	8.18	7.40	6.76	6.06	6.06	16	12.73	12.73	11.75	10.95	10.05
17	8.37	8.15	7.47	6.73	6.73	17	13.71	13.71	12.69	11.85	10.92
18	9.76	8.30	8.19	7.41	7.41	18	14.69	14.69	13.63	12.76	11.80
19	10.56	9.67	8.93	8.10	8.10	19	15.67	15.67	14.58	13.68	12.67
20	11.37	10.44	9.66	8.81	8.81	20	16.65	16.65	15.52	14.59	13.55
22	13.00	11.99	11.16	10.23	10.23	22	18.61	18.61	17.42	16.43	15.32
24	14.64	13.57	12.68	11.63	11.63	24	20.57	20.57	19.31	18.27	17.10
26	16.30	15.17	14.22	13.16	13.16	26	22.53	22.53	21.22	20.12	18.99
28	17.93	16.73	15.78	14.66	14.66	28	24.49	24.49	23.12	21.98	20.69
30	19.66	18.41	17.36	16.17	16.17	30	26.45	26.45	25.03	23.84	22.50
35	23.92	22.53	21.36	20.03	20.03	35	31.37	31.37	29.81	28.51	27.04
40	28.27	26.72	25.43	23.98	23.98	40	36.29	36.29	34.61	33.20	31.51
45	32.55	30.95	29.56	27.98	27.98	45	41.21	41.21	39.42	37.92	36.21
50	36.98	35.23	33.74	32.05	32.05	50	46.13	46.13	44.24	42.65	40.84

TABLE 1 (CONTINUED)

		P= 6			S=3			P= 6			S=4		
		α			α			α			α		
		N			N			N			N		
		.01			.05			.025			.05		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10		
		.10			.10			.10			.10</		

TABLE 1 (CONTINUED)

		P= 6			S=5			P= 7			S= 1		
N	α	.01	.025	.05	.10	N	α	.01	.025	.05	.10		
7	19.86		18.20	16.85	15.38	8	1.32		1.05	.86	.56		
8	21.65		19.94	18.54	17.01	9	1.91		1.59	1.34	1.03		
9	23.39		21.62	20.18	18.60	10	2.54		2.16	1.85	1.54		
10	25.03		23.27	21.77	20.14								
11	26.75		24.87	23.34	21.65	11	3.18		2.75	2.40	2.03		
12	28.37		26.45	24.88	23.15	12	3.84		3.36	2.97	2.55		
13	29.97		28.00	26.39	24.62	13	4.51		3.99	3.56	3.10		
14	31.54		29.53	27.88	26.07	14	5.20		4.63	4.17	3.66		
15	33.09		31.04	29.35	27.50	15	5.91		5.29	4.79	4.24		
16	34.62		32.53	30.81	26.91	16	6.62		5.96	5.43	4.84		
17	36.13		34.00	32.25	30.31	17	7.34		6.65	6.07	5.45		
18	37.63		35.45	33.67	31.70	18	8.07		7.34	6.73	6.07		
19	39.11		36.91	35.09	33.08	19	8.81		8.04	7.41	6.70		
20	40.58		38.34	36.49	34.45	20	9.56		8.75	8.09	7.35		
22	43.48		41.17	39.26	37.15	22	11.07		10.20	9.47	8.66		
24	46.34		43.97	42.00	39.82	24	12.50		11.67	10.89	10.01		
26	49.17		46.73	44.71	42.47	26	14.16		13.16	12.33	11.39		
28	51.96		49.46	47.39	45.08	28	15.73		14.68	13.79	12.79		
30	54.72		52.16	50.04	47.68	30	17.32		16.21	15.27	14.22		
35	61.52		58.82	56.58	54.08	35	21.35		20.11	19.05	17.86		
40	68.18		65.36	63.00	60.33	40	25.45		24.08	22.92	21.61		
45	74.75		71.90	69.34	66.60	45	29.51		28.13	26.86	25.43		
50	81.23		78.16	75.61	72.75	50	33.82		32.22	30.86	29.32		

TABLE 1 (CONTINUED)

P = 7			S = 2			P = 7			S = 3		
N	α	.01	.025	.05	.10	N	α	.01	.025	.05	.10
3	3.33		2.87	2.51	2.14	8	6.24		5.57	5.03	4.45
3	4.20		3.69	3.28	2.84	9	7.36		6.64	6.05	5.42
10	5.02		4.52	4.06	3.57	10	8.47		7.70	7.07	6.38
11	5.97		5.35	4.85	4.31	11	9.57		8.75	8.08	7.34
12	6.85		6.19	5.65	5.07	12	10.66		9.80	9.09	8.31
13	7.75		7.04	6.46	5.83	13	11.75		10.34	10.09	9.27
14	8.54		7.89	7.28	6.61	14	12.83		11.88	11.10	10.23
15	9.54		8.75	8.10	7.39	15	13.90		12.91	12.10	11.20
16	10.44		9.51	8.93	8.19	16	14.97		13.95	13.10	12.16
17	11.34		10.48	9.77	8.98	17	16.04		14.98	14.10	13.13
18	12.25		11.35	10.61	9.79	18	17.11		16.01	15.10	14.16
19	13.16		12.23	11.45	10.59	19	18.17		17.04	16.11	15.06
20	14.07		13.10	12.30	11.41	20	19.23		18.07	17.11	16.03
22	15.30		14.67	14.01	13.06	22	21.35		20.13	19.11	17.97
24	17.73		16.64	15.73	14.72	24	23.66		22.18	21.11	19.91
26	19.53		18.43	17.47	16.40	26	25.56		24.22	23.11	21.85
28	21.42		20.22	19.22	18.09	28	27.66		26.27	25.11	23.80
30	23.28		22.02	20.97	19.79	30	29.75		28.31	27.10	25.75
35	27.93		26.55	25.39	24.08	35	34.98		33.41	32.10	30.62
40	32.51		31.12	29.86	28.43	40	40.18		38.50	37.09	35.50
45	37.31		35.71	34.35	32.82	45	45.37		43.59	42.09	40.39
50	42.02		40.32	38.88	37.24	50	50.55		48.67	47.08	45.29

TABLE 1 (CONTINUED)

		P= 7			S=4			P= 7			S=5		
		α			α			α			α		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.05		
		.01			.025			.025			.		

TABLE 1 (CONTINUED)

		P = 7 S = 5			P = 8 S = 1						
N	α	.01	.025	.35	.10	N	α	.01	.025	.05	.10
8	23.70		21.93	20.44	18.90	9		1.15	.92	.75	.53
9	25.52		23.70	22.20	20.57	10		1.59	1.40	1.18	.95
10	27.29		25.42	23.88	22.20						
11	29.02		27.10	25.52	23.73	11		2.25	1.91	1.64	1.37
12	30.71		28.74	27.13	25.35	12		2.83	2.44	2.14	1.81
13	32.37		30.36	28.71	26.83	13		3.43	3.00	2.66	2.29
14	34.01		31.95	30.27	28.40	14		4.05	3.58	3.20	2.78
15	35.62		33.52	31.80	29.90	15		4.69	4.17	3.75	3.30
16	37.20		35.07	33.32	31.38	16		5.34	4.78	4.33	3.84
17	38.77		36.60	34.81	32.84	17		6.00	5.41	4.92	4.39
18	40.32		38.12	36.30	34.28	18		6.67	6.04	5.52	4.95
19	41.86		39.62	37.77	35.72	19		7.35	6.69	6.14	5.53
20	43.38		41.10	39.22	37.14	20		8.04	7.34	6.77	6.13
22	46.38		44.04	42.10	39.95	22		9.45	8.58	8.05	7.34
24	49.33		46.92	44.93	42.72	24		10.83	10.05	9.37	8.50
26	52.24		49.78	47.73	45.46	26		12.34	11.45	10.71	9.88
28	55.12		52.53	50.50	48.17	28		13.82	12.88	12.09	11.20
30	57.97		55.38	53.24	50.85	30		15.32	14.32	13.48	12.54
35	64.96		62.24	59.98	57.46	35		19.15	18.02	17.07	15.99
40	71.81		68.96	66.60	63.96	40		23.05	21.81	20.75	19.55
45	78.54		75.58	73.11	70.36	45		27.03	25.67	24.52	23.20
50	85.14		82.10	79.54	76.63	50		31.07	29.60	28.35	25.93

TABLE 1 (CONTINUED)

P = .0				S = 2				P = .0				S = 3			
N		α		.01	.025	.05	.10	N		α		.01	.025	.05	.10
9	3	2.90	2.50		2.19		1.85	9	5.40	4.82	4.36		3.46		
10	10	3.69	3.24		2.88		2.50	10	6.41	5.79	5.28		4.73		
11	11	4.43	3.99		3.58		3.15	11	7.42	6.74	6.20		5.60		
12	12	5.29	4.74		4.30		3.83	12	8.42	7.70	7.11		6.47		
13	13	6.16	5.51		5.03		4.51	13	9.42	8.66	8.04		7.35		
14	14	6.92	6.29		5.78		5.22	14	10.41	9.62	8.96		8.24		
15	15	7.74	7.08		6.53		5.93	15	11.41	10.57	9.89		9.12		
16	16	8.57	7.87		7.29		6.65	16	12.40	11.53	10.81		10.01		
17	17	9.41	8.67		8.06		7.33	17	13.40	12.49	11.74		10.91		
18	18	10.25	9.48		8.84		8.13	18	14.39	13.45	12.67		11.81		
19	19	11.10	10.29		9.62		8.88	19	15.38	14.41	13.61		12.71		
20	20	11.95	11.11		10.41		9.64	20	16.38	15.38	14.54		13.51		
22	22	13.56	12.76		12.01		11.17	22	18.36	17.30	16.42		15.43		
24	24	15.38	14.42		13.62		12.73	24	20.35	19.23	18.29		17.25		
26	26	17.12	16.10		15.25		14.30	26	22.33	21.16	20.18		19.08		
28	28	18.86	17.79		16.90		15.89	28	24.32	23.09	22.07		20.91		
30	30	20.62	19.50		18.56		17.50	30	26.30	25.03	23.96		22.76		
35	35	25.04	23.80		22.75		21.58	35	31.27	29.87	28.70		27.38		
40	40	29.50	28.15		27.01		25.72	40	36.23	34.73	33.47		32.04		
45	45	34.00	32.54		31.31		29.92	45	41.19	39.59	38.24		36.71		
50	50	38.52	36.97		35.66		34.16	50	46.16	44.46	43.03		41.41		

TABLE 1 (CONTINUED)

P = .01		P = .025		P = .05		P = .10		P = .01		P = .025		P = .05		P = .10	
N	α	N	α	N	α	N	α	N	α	N	α	N	α	N	α
3	8.76	7.99	7.35	6.66	9	13.21	12.18	11.34	10.41	12.18	11.34	10.41	9.57	8.76	7.99
10	9.98	9.16	8.48	7.74	10	14.63	13.55	12.67	11.70	13.55	12.67	11.70	10.83	10.00	9.16
11	11.19	10.31	9.60	8.81	11	16.02	14.90	13.98	12.96	14.90	13.98	12.96	12.18	11.34	10.41
12	12.38	11.46	10.71	9.87	12	17.39	16.23	15.27	14.21	16.23	15.27	14.21	13.55	12.67	11.70
13	13.55	12.60	11.81	10.93	13	18.74	17.54	16.55	15.45	17.54	16.55	15.45	14.63	13.75	12.83
14	14.72	13.72	12.90	11.93	14	20.08	18.84	17.81	16.67	18.84	17.81	16.67	15.83	14.90	14.00
15	15.88	14.85	13.99	13.04	15	21.40	20.12	19.06	17.89	20.12	19.06	17.89	17.00	16.17	15.23
16	17.03	15.95	15.08	14.09	16	22.70	21.39	20.30	19.09	21.39	20.30	19.09	18.18	17.34	16.46
17	18.17	17.07	16.16	15.14	17	24.00	22.65	21.53	20.29	22.65	21.53	20.29	19.37	18.54	17.66
18	19.31	18.18	17.24	16.19	18	25.28	23.90	22.76	21.48	23.90	22.76	21.48	20.55	19.71	18.83
19	20.45	19.28	18.31	17.23	19	26.56	25.15	23.92	22.57	25.15	23.92	22.57	21.72	20.87	19.99
20	21.58	20.38	19.39	18.27	20	27.82	26.38	25.19	23.65	26.38	25.19	23.65	22.89	22.00	21.10
22	23.83	22.57	21.53	20.35	22	30.34	28.84	27.59	26.19	28.84	27.59	26.19	25.19	24.21	23.23
24	26.07	24.75	23.66	22.43	24	32.82	31.27	29.97	28.52	31.27	29.97	28.52	27.27	26.27	25.27
26	28.29	26.92	25.78	24.50	26	35.29	33.68	32.33	30.83	33.68	32.33	30.83	29.57	28.47	27.47
28	30.50	29.09	27.90	26.57	28	37.73	36.07	34.68	33.13	36.07	34.68	33.13	31.83	30.63	29.63
30	32.71	31.24	30.02	28.63	30	40.16	38.45	37.02	35.41	38.45	37.02	35.41	34.03	32.73	31.73
35	38.19	36.61	35.28	33.79	35	46.17	44.34	42.81	41.03	44.34	42.81	41.03	39.57	38.07	37.07
40	43.64	41.95	40.53	38.93	40	52.10	50.16	48.54	46.71	50.16	48.54	46.71	45.27	43.77	42.77
45	49.05	47.27	45.76	44.06	45	57.97	55.93	54.22	52.29	55.93	54.22	52.29	50.77	49.27	48.27
50	54.45	52.56	50.97	49.18	50	63.79	61.66	59.86	57.84	61.66	59.86	57.84	56.37	54.87	53.87

TABLE-1 (CONTINUED)

		P = 8				S = 6				P = 8				S = 7			
N	α	.01	.025	.05	.10	N	α	.01	.025	.05	.10	N	α	.01	.025	.05	.10
9		19.14	17.78	16.67	15.44	9		27.56	25.68	24.14	22.46	9		27.56	25.68	24.14	22.46
10		20.76	19.36	18.20	16.93	10		29.40	27.47	25.89	24.16	10		29.40	27.47	25.89	24.16
11		22.35	20.90	19.71	18.39	11		31.19	29.22	27.60	25.82	11		31.19	29.22	27.60	25.82
12		23.90	22.41	21.18	19.82	12		32.95	30.93	29.27	27.45	12		32.95	30.93	29.27	27.45
13		25.43	23.90	22.63	21.23	13		34.67	32.61	30.92	29.05	13		34.67	32.61	30.92	29.05
14		26.93	25.37	24.07	22.63	14		36.36	34.26	32.53	30.63	14		36.36	34.26	32.53	30.63
15		28.42	26.81	25.48	24.01	15		38.02	35.88	34.13	32.18	15		38.02	35.88	34.13	32.18
16		29.88	28.24	26.88	25.37	16		39.66	37.49	35.70	33.72	16		39.66	37.49	35.70	33.72
17		31.33	29.66	28.27	26.72	17		41.28	39.07	37.25	35.23	17		41.28	39.07	37.25	35.23
18		32.77	31.06	29.64	28.05	18		42.88	40.64	38.78	36.73	18		42.88	40.64	38.78	36.73
19		34.19	32.45	31.00	29.39	19		44.46	42.18	40.30	38.22	19		44.46	42.18	40.30	38.22
20		35.60	33.83	32.35	30.71	20		46.03	43.72	41.81	39.69	20		46.03	43.72	41.81	39.69
22		38.38	36.55	35.03	33.32	22		49.12	46.74	44.77	42.59	22		49.12	46.74	44.77	42.59
24		41.13	39.24	37.67	35.91	24		52.16	49.72	47.70	45.46	24		52.16	49.72	47.70	45.46
26		43.85	41.90	40.28	38.46	26		55.15	52.65	50.58	48.28	26		55.15	52.65	50.58	48.28
28		46.53	44.53	42.86	41.00	28		58.11	55.55	53.43	51.07	28		58.11	55.55	53.43	51.07
30		49.19	47.14	45.43	43.51	30		61.03	58.42	56.25	53.84	30		61.03	58.42	56.25	53.84
35		55.75	53.58	51.76	49.72	35		68.20	65.46	63.18	60.64	35		68.20	65.46	63.18	60.64
40		62.19	59.90	57.99	55.84	40		75.21	72.35	69.97	67.31	40		75.21	72.35	69.97	67.31
45		68.54	66.15	64.14	61.89	45		82.10	79.12	76.64	73.87	45		82.10	79.12	76.64	73.87
50		74.81	72.32	70.23	67.88	50		88.89	85.80	83.23	80.35	50		88.89	85.80	83.23	80.35

TABLE 1 (CONTINUED)

P = 9			S = 1			P = 9			S = 2		
N	α	.01	.025	.05	.10	N	α	.01	.025	.05	.10
10	1.02		.92	.67	.51	10		2.57	2.22	1.94	1.65
11	1.51		1.25	1.05	.85	11		3.29	2.89	2.57	2.23
12	2.02		1.71	1.48	1.23	12		4.01	3.57	3.21	2.82
13	2.55		2.20	1.93	1.63	13		4.75	4.26	3.87	3.44
14	3.11		2.72	2.40	2.07	14		5.50	4.97	4.54	4.07
15	3.68		3.25	2.90	2.53	15		6.26	5.65	5.23	4.72
16	4.27		3.80	3.42	3.01	16		7.02	6.42	5.93	5.38
17	4.87		4.37	3.95	3.50	17		7.80	7.16	6.64	6.06
18	5.45		4.95	4.50	4.02	18		8.58	7.91	7.35	6.74
19	6.12		5.54	5.07	4.55	19		9.36	8.66	8.08	7.44
20	6.75		6.15	5.65	5.09	20		10.16	9.42	8.81	8.14
22	8.06		7.39	6.63	6.21	22		11.76	10.97	10.30	9.57
24	9.40		8.67	8.06	7.38	24		13.38	12.53	11.82	11.03
25	10.77		9.98	9.32	8.58	26		15.02	14.11	13.36	12.51
28	12.17		11.32	10.61	9.82	28		16.67	15.71	14.91	14.01
30	13.59		12.69	11.93	11.08	30		18.33	17.33	16.48	15.53
35	17.22		16.19	15.33	14.35	35		22.54	21.42	20.47	19.41
40	20.95		19.81	18.84	17.74	40		26.81	25.58	24.54	23.36
45	24.76		23.51	22.44	21.23	45		31.13	29.79	28.66	27.39
50	28.64		27.29	26.13	24.81	50		35.48	34.05	32.84	31.46

TABLE 1 (CONTINUED)

TABLE 1 (CONTINUED)

		P = .9		S = 5				P = .9		S = 5	
N	α	.01	.025	.05	.10	N	α	.01	.025	.05	.10
10		11.44	10.56	9.84	9.05	10		16.27	15.14	14.22	13.20
11		12.73	11.81	11.05	10.22	11		17.74	16.57	15.61	14.55
12		14.01	13.05	12.25	11.37	12		19.19	17.98	16.98	15.88
13		15.27	14.25	13.44	12.52	13		20.61	19.36	18.33	17.19
14		16.51	15.47	14.61	13.65	14		22.01	20.73	19.67	18.49
15		17.74	16.57	15.78	14.73	15		23.39	22.08	20.99	19.77
16		18.97	17.86	16.94	15.91	16		24.76	23.41	22.29	21.04
17		20.18	19.04	18.09	17.03	17		26.12	24.74	23.59	22.31
18		21.38	20.21	19.24	18.15	18		27.46	26.05	24.87	23.56
19		22.58	21.38	20.38	19.26	19		28.80	27.35	26.15	24.80
20		23.78	22.54	21.52	20.37	20		30.12	28.64	27.41	26.04
22		26.14	24.85	23.78	22.57	22		32.74	31.20	29.93	28.50
24		28.43	27.15	26.03	24.77	24		35.32	33.74	32.41	30.93
26		30.82	29.43	28.26	26.95	26		37.98	36.25	34.87	33.34
28		33.14	31.70	30.49	29.13	28		40.42	38.73	37.32	35.73
30		35.45	33.95	32.70	31.30	30		42.94	41.20	39.75	38.11
35		41.16	39.56	38.21	36.63	35		49.15	47.30	45.75	44.00
40		46.82	45.11	43.68	42.03	40		55.27	53.32	51.68	49.83
45		52.44	50.63	49.12	47.40	45		61.32	59.27	57.54	55.60
50		58.02	56.12	54.53	52.72	50		67.31	65.17	63.36	61.32

TABLE 1 (CONTINUED)

		P = 9			S = 7			P = 9			S = 8		
		α			α			α			α		
		.01			.05			.01			.05		
		.025			.10			.025			.10		
		N			N			N			N		
10	22.55	21.14	19.94	18.62	10	31.43	29.45	27.93	26.05	10	31.43	29.45	27.93
11	24.24	22.75	21.52	20.15	11	33.28	31.26	29.61	27.78	11	33.28	31.26	29.61
12	25.36	24.33	23.06	21.65	12	35.09	33.83	31.34	29.47	12	35.09	33.83	31.34
13	27.45	25.88	24.58	23.14	13	36.87	34.77	33.04	31.13	13	36.87	34.77	33.04
14	29.02	27.41	26.08	24.60	14	38.61	36.47	34.71	32.76	14	38.61	36.47	34.71
15	30.56	28.92	27.55	26.04	15	40.32	38.15	36.35	34.37	15	40.32	38.15	36.35
16	32.08	30.40	29.01	27.46	16	42.01	39.80	37.98	35.96	16	42.01	39.80	37.98
17	33.59	31.97	30.45	28.87	17	43.68	41.43	39.58	37.52	17	43.68	41.43	39.58
18	35.07	33.33	31.88	30.26	18	45.33	43.05	41.16	39.07	18	45.33	43.05	41.16
19	36.54	34.77	33.29	31.64	19	46.95	44.64	42.73	40.60	19	46.95	44.64	42.73
20	38.00	36.20	34.69	33.02	20	48.56	46.22	44.27	42.12	20	48.56	46.22	44.27
22	40.68	39.02	37.46	35.73	22	51.74	49.33	47.33	45.12	22	51.74	49.33	47.33
24	43.72	41.80	40.20	38.41	24	54.85	52.38	50.34	48.06	24	54.85	52.38	50.34
26	46.52	44.54	42.90	41.05	26	57.92	55.40	53.30	50.97	26	57.92	55.40	53.30
28	49.26	47.26	45.57	43.67	28	60.95	58.37	56.23	53.84	28	60.95	58.37	56.23
30	52.03	49.95	48.22	46.27	30	63.94	61.31	59.12	56.68	30	63.94	61.31	59.12
35	58.77	56.57	54.74	52.69	35	71.28	68.52	66.22	63.65	35	71.28	68.52	66.22
40	65.38	63.08	61.15	58.93	40	78.45	75.57	73.17	70.49	40	78.45	75.57	73.17
45	71.89	69.48	67.47	65.20	45	85.48	82.49	79.99	77.21	45	85.48	82.49	79.99
50	78.31	75.81	73.71	71.35	50	92.40	89.38	86.72	83.83	50	92.40	89.38	86.72

TABLE 1 (CONTINUED)

P=10			S=3			P=10			S=4		
N	α	.01	.025	.05	.10	N	α	.01	.025	.05	.10
11		4.26	3.81	3.45	3.06	11		6.94	6.24	5.75	5.21
12		5.11	4.62	4.21	3.73	12		7.96	7.22	6.59	6.11
13		5.96	5.42	4.99	4.51	13		8.88	8.20	7.63	7.02
14		6.81	6.24	5.77	5.25	14		9.89	9.17	8.58	7.92
15		7.67	7.06	6.55	6.00	15		10.90	10.14	9.52	8.83
16		8.53	7.88	7.35	6.76	16		11.91	11.12	10.46	9.74
17		9.39	8.71	8.15	7.53	17		12.91	12.09	11.41	10.65
18		10.26	9.55	8.96	8.31	18		13.92	13.06	12.35	11.56
19		11.13	10.39	9.78	9.09	19		14.92	14.04	13.30	12.48
20		12.01	11.24	10.60	9.89	20		15.92	15.01	14.25	13.40
22		13.77	12.94	12.25	11.43	22		17.92	16.96	16.15	15.24
24		15.54	14.65	13.92	13.03	24		19.92	18.90	18.05	17.09
26		17.31	16.38	15.60	14.72	26		21.92	20.85	19.96	18.95
28		19.10	18.12	17.30	16.37	28		23.92	22.80	21.86	20.81
30		20.90	19.87	19.00	18.53	30		25.92	24.75	23.78	22.67
35		25.41	24.27	23.31	22.23	35		30.91	29.64	28.57	27.36
40		29.96	28.72	27.67	26.49	40		35.30	34.53	33.37	32.06
45		34.54	33.20	32.07	30.79	45		40.89	39.43	38.19	36.73
50		39.14	37.72	36.51	35.14	50		45.88	44.33	43.01	41.52

TABLE 1 (CONTINUED)

P=10						S=5						P=10						S=6					
N		α		.01		.025		.05		.10		N		α		.01		.025		.05		.10	
11	10.12				9.35		8.72		8.02			11	14.23				13.26		12.47			11.59	
12	11.31				10.50		9.83		9.09			12	15.58				14.58		13.75			12.82	
13	12.48				11.64		10.93		10.16			13	16.91				15.87		15.01			14.05	
14	13.55				12.76		12.03		11.22			14	18.23				17.15		16.26			15.26	
15	14.80				13.89		13.12		12.28			15	19.53				18.42		17.49			16.46	
16	15.95				15.00		14.21		13.33			16	20.82				19.67		18.72			17.66	
17	17.09				16.11		15.29		14.38			17	22.09				20.92		19.93			18.84	
18	18.23				17.21		16.37		15.43			18	23.36				22.15		21.14			20.02	
19	19.36				18.32		17.44		16.47			19	24.61				23.38		22.35			21.19	
20	20.46				19.41		18.52		17.51			20	25.86				24.60		23.54			22.36	
22	22.73				21.60		20.65		19.60			22	28.34				27.02		25.91			24.68	
24	24.95				23.77		22.78		21.67			24	30.79				29.42		28.27			26.98	
26	27.17				25.94		24.90		23.74			26	33.22				31.80		30.60			29.26	
28	29.37				28.09		27.02		25.81			28	35.63				34.16		32.92			31.54	
30	31.57				30.24		29.13		27.88			30	38.02				36.51		35.23			33.90	
35	37.02				35.59		34.39		33.03			35	43.95				42.32		40.96			39.42	
40	42.45				40.32		39.63		38.17			40	49.50				48.08		46.63			44.99	
45	47.84				46.22		44.85		43.30			45	55.60				53.78		52.25			50.52	
50	53.21				51.50		50.05		48.42			50	61.35				59.44		57.84			56.02	

TABLE 1 (CONTINUED)

P=10						S=7						P=10						S=8					
N		α		.01		.025		.05		.10		N		α		.01		.025		.05		.10	
11	19.40	18.19	17.20	16.09	15.09	11	26.08	24.54	23.27	21.87		11	26.08	24.54	23.27	21.87							21.87
12	20.92	19.67	18.54	17.49	16.49	12	27.76	26.18	24.88	23.44		12	27.76	26.18	24.88	23.44							23.44
13	22.41	21.12	20.06	18.87	17.87	13	29.41	27.60	26.46	24.98		13	29.41	27.60	26.46	24.98							24.98
14	23.87	22.55	21.46	20.24	19.24	14	31.03	29.38	28.02	26.50		14	31.03	29.38	28.02	26.50							26.50
15	25.32	23.96	22.84	21.58	20.58	15	32.62	30.94	29.55	27.99		15	32.62	30.94	29.55	27.99							27.99
16	26.74	25.36	24.20	22.92	21.92	16	34.20	32.48	31.06	29.47		16	34.20	32.48	31.06	29.47							29.47
17	28.15	26.74	25.56	24.24	23.24	17	35.75	34.00	32.55	30.93		17	35.75	34.00	32.55	30.93							30.93
18	29.55	28.10	26.90	25.55	24.55	18	37.29	35.51	34.03	32.38		18	37.29	35.51	34.03	32.38							32.38
19	30.94	29.46	28.23	26.85	25.85	19	38.80	37.00	35.49	33.81		19	38.80	37.00	35.49	33.81							33.81
20	32.31	30.80	29.54	28.14	27.14	20	40.31	38.47	36.94	35.23		20	40.31	38.47	36.94	35.23							35.23
22	35.02	33.46	32.16	30.70	29.70	22	43.28	41.38	39.60	38.03		22	43.28	41.38	39.60	38.03							38.03
24	37.70	36.09	34.74	33.22	32.22	24	46.20	44.25	42.62	40.80		24	46.20	44.25	42.62	40.80							40.80
26	40.35	38.69	37.29	35.73	34.73	26	49.06	47.07	45.40	43.53		26	49.06	47.07	45.40	43.53							43.53
28	42.97	41.26	39.82	38.21	37.21	28	51.92	49.87	48.15	46.23		28	51.92	49.87	48.15	46.23							46.23
30	45.57	43.81	42.33	40.67	39.67	30	54.73	52.63	50.87	48.91		30	54.73	52.63	50.87	48.91							48.91
35	51.97	50.10	48.53	46.76	45.76	35	61.65	59.43	57.58	55.49		35	61.65	59.43	57.58	55.49							55.49
40	58.27	56.30	54.64	52.77	51.77	40	68.42	66.10	64.15	61.97		40	68.42	66.10	64.15	61.97							61.97
45	64.49	62.42	60.68	58.72	57.72	45	75.08	72.66	70.63	68.34		45	75.08	72.66	70.63	68.34							68.34
50	70.63	68.47	66.66	64.61	63.61	50	81.64	79.13	77.02	74.64		50	81.64	79.13	77.02	74.64							74.64

TABLE 1 (CONTINUED)

N	α	P=10 S=9			
		.01	.025	.05	.10
11	35.30	33.24	31.55	29.63	
12	37.17	35.07	33.34	31.43	
13	38.99	36.95	35.09	33.14	
14	40.79	38.61	36.81	34.83	
15	42.55	40.33	38.51	36.49	
16	44.28	42.03	40.18	38.12	
17	45.99	43.71	41.82	39.73	
18	47.69	45.37	43.45	41.33	
19	49.35	47.00	45.06	42.91	
20	51.00	48.52	46.55	44.47	
22	54.25	51.81	49.79	47.54	
24	57.45	54.95	52.87	50.57	
26	60.59	58.03	55.91	53.55	
28	63.69	61.07	58.91	56.43	
30	66.74	64.08	61.87	59.40	
35	74.23	71.45	69.13	66.54	
40	81.55	78.64	76.23	73.53	
45	88.72	85.70	83.19	80.39	
50	95.76	92.65	90.05	87.14	