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PULSE PAIR ESTIMATION OF
DOPPLER SPECTRUM PARAMETERS

By

Fredrick C. Benham
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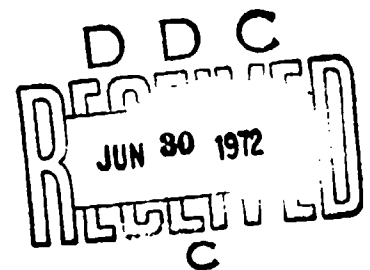
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TABLE OF CONTENTS

<u>Section No.</u>		<u>Page</u>
1.0	INTRODUCTION	1-1
1.1	Scope	1-1
1.2	Summary	1-1
1.3	Technical Background	1-4
2.0	THEORY OF PULSE PAIR ESTIMATION	2-1
2.1	The Pulse Pair Estimators	2-1
2.2	Estimation Accuracy	2-3
2.3	Hardware Implications	2-8
2.4	Waveform Flexibility	2-8
2.5	Summary	2-14
3.0	EXPERIMENTAL EVALUATION	3-1
3.1	Porcupine Radar Weather Return	3-1
3.1.1	Estimation of Mean and Spread by Conventional Spectral Analysis	3-1
3.1.2	Estimates of Mean and Spread by Pulse Pair Processing	3-4
3.1.3	Comparison of the Estimators	3-4
3.2	Simulated Radar Return	3-8
3.2.1	Purpose, Capabilities and Advantages	3-8
3.2.2	Generating Sequences of Simulated Doppler Radar Target Return with Controllable Parameters	3-10
3.2.3	Performance of Pulse Pair Estimators	3-18
3.2.4	Performance of Spectral Analysis Estimator	3-28
3.2.5	Comparison of the Width Estimators	3-34
4.0	HARDWARE REALIZATION	4-1
4.1	General Description of the Pulse-Pair Processor	4-1
4.2	Technical Description of the Pulse-Pair Processor	4-2
4.3	System Design	4-3
4.3.1	IF Amplifier - Phase Detector	4-3
4.3.2	Digitizer	4-7
4.3.3	Correlator	4-7
4.3.4	Digital Integrators	4-13
4.3.5	Function Generators	4-13
4.4	Performance	4-14
4.4.1	Comparison with the CMF	4-14
4.4.2	Comparison with Analog Spectrum Analyzers	4-16
4.4.3	FFT Processing	4-16
4.5	Conclusions	4-17
5.0	CONCLUSIONS AND RECOMMENDATIONS	5-1
References		

TABLE OF CONTENTS (Cont'd.)

<u>APPENDIX</u>	<u>TITLE</u>
A	Analysis of Accuracy of Spectral Parameter Estimates by Pulse Pair Estimation
B	Mathematical Derivations Relating to the Generation of Simulated Doppler Return
C	Fortran Main Programs
C-1	DOPGEN
C-2	PPSTAT
C-3	SPEC
D	Fortran Subroutines
D-1	FORT
D-2	PAIR
D-3	UNPK
D-4	CDCC
D-5	BESI
D-6	STAR
D-7	GNRN
E	Spectral Analysis Plots of Typical Sequences of Simulated Doppler Radar Target Return

LIST OF ILLUSTRATIONS

<u>Figure No.</u>	<u>Title</u>	<u>Page</u>
1	Spectral Density Parameters	2-1a
1a	Standard Deviation of Pulse Pairs Center Frequency Estimate ($\hat{f}_0$) vs Spectral Width (wT) with per Pulse S/N a Parameter, for Uniform Pulse Spacing	2-4
2	Standard Deviation of Pulse Pairs Width Estimate ($\hat{wT}$) vs True Spectral Width (wT) with Per Pulse S/N a Parameter, for Uniform Pulse Spacing	2-5
3	Accuracy of Pulse Pairs Estimate of Mean Velocity vs Spectral Width, for Independent Pairs (from Rummler, Ref. (4))	2-6
4	Fractional Accuracy of Pulse Pairs Estimate of Spectral Width vs True Spectral Width for Independent Pairs (from Rummler, Ref. (4))	2-7
5	Pulse Pair Waveforms	2-9
6	Tradeoff Characteristic of Pulse Pair Mean Frequency Measurement for Fixed Radar Energy $\mathcal{E}$	2-11
7	Tradeoff Characteristic of Pulse Pair Width Measurement for Fixed Radar Energy $\mathcal{E}$	2-12
8	Typical Data Spectrum for $N = 1024$	3-2
9	Overall Block Diagram of the Software Simulation System for Investigating and Comparing the Performance of the Pulse Pair and Spectrum Analyzer Estimators of Spectral Parameters	3-9
10	Block Diagram of Simulated Doppler Return Generator	3-13
11	Flowchart of DOPGEN	3-15
12	FFT Processed Simulated Doppler Spectrum of Weather Data	3-17

LIST OF ILLUSTRATIONS (Cont.)

<u>Figure No.</u>	<u>Title</u>	<u>Page</u>
13	Flowchart of PPSTAT	3-19
14	Theoretical Accuracy of Pulse Pair Estimators of Mean Frequency ($\hat{f}_0$) vs Number of Pulse Pairs Compared with Simulation Results	3-21
15	Theoretical Accuracy of Pulse Pair Estimates of Mean Frequency ($\hat{f}_0$) vs Spectral Width (w) compared with Simulation Results	3-22
16	Theoretical Accuracy of Pulse Pair Estimates of Spectral Width ($\hat{w}$) vs Number of Samples Compared with Simulation Results	3-23
17	Theoretical Accuracy of Pulse Pair Estimates of Spectral Width ($\hat{w}$) vs Actual Spectral Width (w) Compared with Simulation Results	3-24
18	Theoretical Accuracy of Pulse Pair Estimates of Spectral Width ($\hat{w}$) vs S/N Compared with Simulation Results (for $f_0 = 300$ Hz; $w = 78$ Hz)	3-25
19	Theoretical Accuracy of Pulse Pair Estimates of Spectral Width ($\hat{w}$) vs S/N Compared with Simulation Results (for $f_0 = 600$ Hz; $w = 156$ Hz)	3-26
20	Theoretical Accuracy of Pulse Pair Estimates of Spectral Width ($\hat{w}$) vs S/N Compared with Simulation Results (for $f_0 = 900$ Hz; $w = 234$ Hz)	3-27
21	Flowchart of SPEC	3-29
22	Spectral Analysis Width Estimates ($\hat{w}$) vs. Threshold Level for Simulation Input Data ($f_0 = 300$ Hz; $w = 78$ Hz)	3-31
23	Spectral Analysis Width Estimates ($\hat{w}$) vs. Threshold Level for Simulation Input Data ($f_0 = 600$ Hz; $w = 165$ Hz)	3-32

LIST OF ILLUSTRATIONS (Cont.)

<u>Figure No.</u>	<u>Title</u>	<u>Page</u>
24	Spectral Analysis Width Estimates ($\hat{w}$) vs. Threshold Level for Simulation Input Data ($f_0 = 900$ Hz; $w = 234$ Hz)	3-33
25	Pulse Pair Processor Signal Flow Diagram	4-4
26	PPP Block Diagram	4-5
27	Porcupine IF Amplifier	4-7
28	FFT Complex Multiplier	4-9

LIST OF TABLES

<u>Table No.</u>	<u>Title</u>	<u>Page</u>
1	Comparison of Pulse-Pair $\hat{f}_0$ with Spectral Analysis $\hat{f}_0$ for $T = 20:2:30$	3-5
2	Comparison of Pulse-Pair $\hat{w}$ with Spectral Analysis $\hat{w}$ for $T = 20:2:30$	3-6
3	List of Recorded Sequences of Simulated Doppler Return Generated, with Parameters as shown. (PRF = 3300 pps in all cases)	3-16
4	Comparison of Pulse Pair and Spectrum Analysis; Estimates of Spectral Width	3-35
5	Peak Signal-to-Noise Improvement of Spectrum Analyzer	3-36
6	Comparison of Pulse Pair and Spectrum Analyzer Derived Width Estimates (Spectrum Analyzer Threshold Set 5 dB above Average Background Noise)	3-37
7	Proposed Design Goals for Pulse Pair Processor	4-15

1.0 INTRODUCTION

1.1 Scope

This is the Final Report on Contract F19628-71-C-0126, which is devoted to the study and investigation of the pulse pair measurement technique for determination of the first and second moments of Doppler spectra of radar backscatter from atmospheric phenomena. This technique promises to greatly simplify and thereby reduce the costs of deriving quantitative data from a pulse Doppler weather radar. Initial efforts in the application of the pulse pair technique to weather radar signals were conducted under a portion of a prior contract with the Weather Radar Branch of the AFCLRL Meteorology Laboratory (F19628-68-C-0345). The emphasis on the present contract has been to extend the theory and to experimentally verify the performance of the pulse pair estimating technique to include the effects of non-ideal conditions, such as noise, that are encountered in an operational environment with real equipment; and to prepare a design specification for a real time pulse pair signal processor that could be utilized in weather radar research.

In the interests of overall continuity and perspective, and to facilitate understanding, this report provides a unified, self-sufficient presentation of the theory and experimental work to date, including some of the early results from the previous contract.

1.2 Summary

Major steps forward that were taken during the present contract include (1) establishing a theoretical foundation for quantitatively predicting the performance capabilities of the pulse pair estimators under various combinations of conditions, and optimizing the design parameters of the radar-processor system; and (2) experimentally verifying the theory by hundreds of quantitatively controlled performance tests conducted with the aid of the simulation system developed for this purpose.

These studies have demonstrated analytically, the feasibility of implementing a real time signal processor, using the pulse pair computing algorithm as its central core, to obtain quantitative estimates of mean velocity and velocity spread under a wide variety of conditions and for a wide variety of radars. This report describes the initial concepts and features of a processor enabling experimental confirmation of its predicted performance, in a form

suitable for performing meteorological research with Doppler radars as well. With the instrument proposed here it will be possible to collect and reduce data under a wide variety of conditions economically. This will make it possible to establish a broad data base from which the operational utility of a Doppler weather radar may be inferred. The proposed signal processor will then provide a benchmark for sizing and designing a signal processor for operational use.

The key features available in a pulse pair signal processor are

1. quantitative (digital) estimates of spectral mean, spectral width and target reflectivity in real time
2. large numbers of contiguous range gates may be processed simultaneously (up to 1024 cells in the proposed implementation)
3. selectable dwell time may be used to adjust the block length of the signal processor to the radar scanning requirement and to the meteorological conditions (integration of up to 1024 samples per range cell to be provided in proposed signal processor)
4. range resolution may be obtained (resolution equivalent to 1 ft (.5 μ s) in the proposed implementation)
5. predictable performance based on self contained data (confidence bounds on all estimates are computable based on signal processor derived S/N data)
6. sliding window operation is feasible (estimates based on the most recent N seconds of data)
7. the storage required consists essentially of three complex words per range cell (last sample, summary crosscorrelation data, summary power data) while the computing power required is to be able to calculate 2 complex multiplies and adds at the input data rate
8. additional waveform flexibility (non uniform p. r. f. and pulse widths-- limited frequency agility)
9. sensitivity comparable to spectrum analyzers
10. independent AGC for each range bin to improve overall dynamic range

as well as the features common to all digital signal processors, namely

11. non critical adjustments
12. long term, drift free, stable performance without calibration
13. flexibility in interfacing with a variety of radars.

The digital outputs of the processor can be displayed, recorded, or remoted for further display or processing.

To fully utilize the processor capabilities it must operate in conjunction with a coherent radar receiver that has a wide dynamic range and is able to accept AGC control voltages from the processor at a range cell-to-range cell rate. To insure compatibility, it is proposed to incorporate in the processor a self-contained receiver with the required characteristics that will accept I.F. and reference signals from the radar, gain control voltages from the processor, and deliver the required in-phase and quadrature complex video signals to the processor. Since the processor measures signal amplitudes to form the AGC control voltage for each range cell, this information is available as a digital output. Together with the large useful receiver dynamic range, as an added benefit the processor amplitude output in conjunction with range information can also be utilized for other purposes, such as the calculation of target reflectivity.

It should be noted that although there are other methods of estimating spectral mean, the pulse pair technique provides quantitative estimates of this parameter in digital form, and is the only known method for estimating spectral width in real time.

The balance of this report is devoted primarily to the technical aspects of the pulse pair program. The technical background is presented in the succeeding portion of this Introduction; the theory of Pulse Pair Estimation, including original analyses performed on the present contract, appears in Section 2; Section 3 deals with the methods and results of Experimental Evaluations of the pulse pair technique, including that performed with real radar data on the prior contract and that utilizing simulated radar return on the present contract; Section 4 describes the proposed Hardware Realization of a Pulse Pair Processor capable of operating on real radar data in real time; and Section 5 presents the conclusions that have been drawn and the Recommendations that have been formulated as a result of the Study and Investigation.

Detailed Analyses, Computer Programs, and Spectral Plots are included in the Appendices in support of the main text of this report.

1.3 Technical Background

A commonly occurring problem in radar measurements is to estimate the spectral mean and variance of a unimodal Gaussian signal imbedded in white Gaussian noise. The classical technique used is to form spectral estimates from the time sequence, and then form estimates of the spectral mean and variance by standard formulae. It is not difficult to show that these techniques are not optimum and result in signal processor dependent bias errors which affect the system sensitivity.

When both mean and variance are unknown, the optimum spectral parameter estimates can only be obtained implicitly from the solution of a transcendental equation in which the form of the underlying spectrum is known. To overcome these defects, various investigators have resorted to threshold tests to reject the noisy frequencies, in order to apply classical mean and variance estimators to the residue after thresholding.

This report describes a new technique, known as pulse pair analysis, which bypasses the stage of spectrum analysis, and directly estimates spectral mean and variance. Not only does this technique result in a less complex set of calculations to be made by the processor, but it has a form which is based on an optimum estimation theory and as such is relatively free of arbitrary choices. Furthermore, the technique is robust with respect to underlying spectral shape, and permits more flexibility in the choice of radar waveform.

Its major disadvantage with respect to classical spectral parameter estimation is the fact that the technique does not indicate the presence of multi-peaked spectra. When the full spectral analysis is available clutter regions may be identified and possibly rejected, and separated non-standard spectral shapes may be detected. Pulse pair analysis, being a spectral shape independent estimation technique, requires a clutter rejection device preceding it (where necessary), and yields only indirect evidence of peculiar spectrum conditions (abnormally broad widths) for indicating the presence of multiple lobed spectra.

As for sensitivity, simulation evidence, to be presented in a later section, indicates that the sensitivity of the pulse pair estimation is comparable to that of the shape independent spectrum processing techniques.

2.0 THEORY OF PULSE PAIR ESTIMATION

2.1 The Pulse Pair Estimators

It may be shown⁽¹⁾ that the optimum estimate of the spectral mean and variance of a sequence of pairs of measurements which are uncorrelated is given by

$$\text{Arg} \left[\frac{1}{\hat{W}} \int S \left(\frac{f - \hat{f}}{\hat{W}} \right) e^{-j2\pi f T} df \right] = \arg X \quad (2-1)$$

$$\left| \frac{1}{\hat{W}} \int S \left(\frac{f - \hat{f}}{\hat{W}} \right) e^{-j2\pi f T} df \right| = \frac{|X|}{Y - \sigma_N} \quad (2-2)$$

where

$$X = \frac{1}{N} \sum_{k=1}^N r_{1k} r_{2k}^* \quad (2-3)$$

$$Y = \frac{1}{2N} \sum_{k=1}^N \left[|r_{1k}|^2 + |r_{2k}|^2 \right] \quad (2-4)$$

r_{ik} is the observed signal, $S(f)$ is the underlying spectral density function, and σ_N is the mean equivalent noise cross-section (see Figure 1).

It is possible to rewrite these equations in a more illuminating form by defining

$$R(t) = \int S(f) e^{j2\pi f t} dt \quad (2-5)$$

We get

$$\arg \left\{ e^{j\pi \hat{f} T} R(\hat{W}T) \right\} = \arg X \quad (2-6)$$

$$|R(\hat{W}T)| = \frac{|X|}{Y - \sigma_N} \quad (2-7)$$

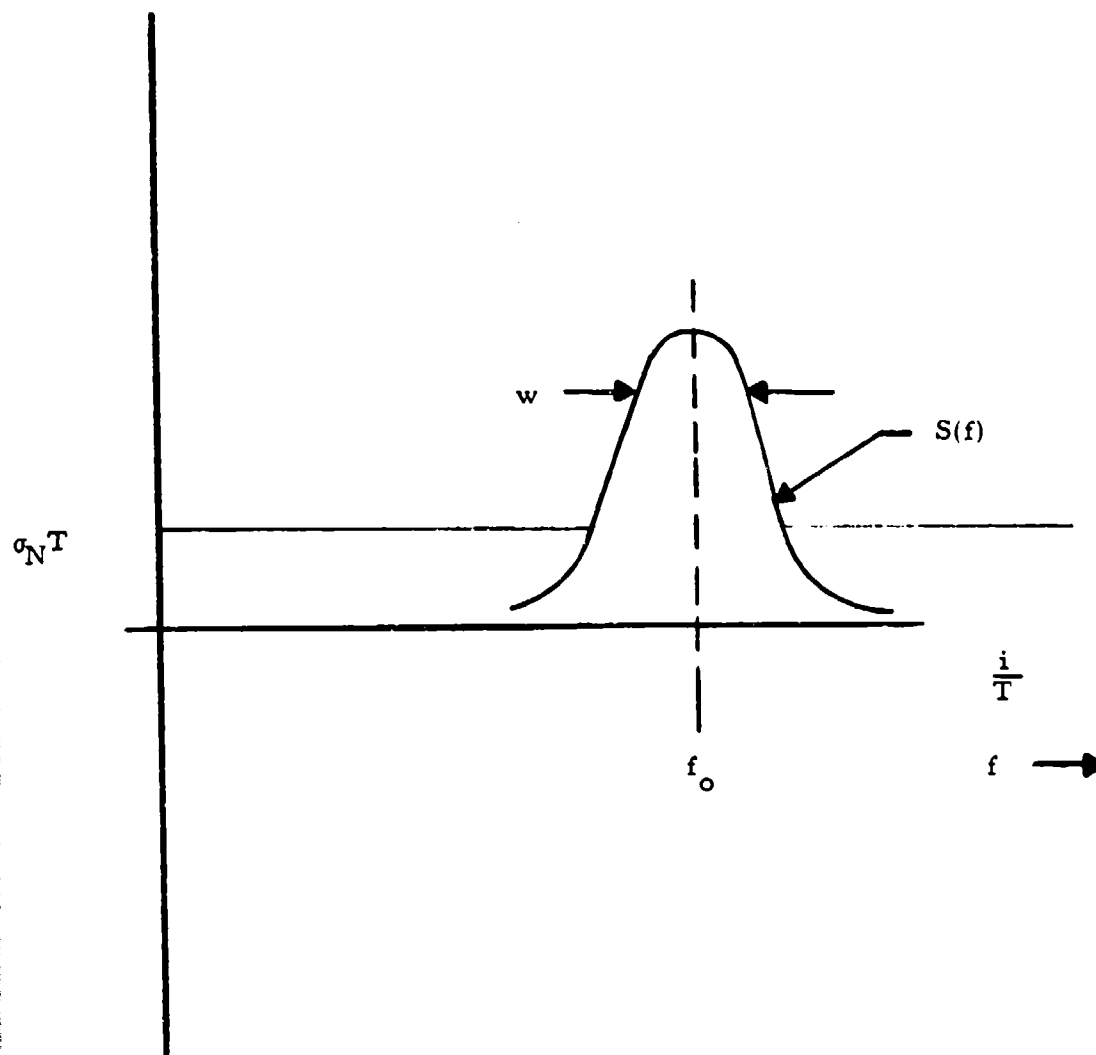


Figure 1. Spectral Density Parameters

Thus the latter equation alone determines $\hat{w}$, and furthermore, if $R(T)$ is real, the first equation is independent of $\hat{w}$.

Essentially this result says that if the underlying spectral density function of the signal is characterized by a spectrum whose shape is invariant to frequency shift and broadening in the specified manner, then these two parameters may be estimated from the pair wise measurement directly by estimating the complex correlation function of the data.

It is important to note that for small T

$$R(\hat{w}T) \cong R(0) - 1/2 \hat{w}^2 R''(0) (2\pi T)^2 \quad (2-8)$$

(No first order term is present because $S(f)$ by definition has mean zero.) Also by definition $R(0)$ and $R''(0)$ are both unity (the first to have unit energy, the second to define the parameter $\hat{w}$, i. e., unit variance for the underlying signal shape). As a result, in this case, the estimator of spectral width parameter is given by

$$\hat{w}^2 = \frac{2}{(2\pi T)^2} \left[1 - \frac{|X|}{Y - \sigma_N} \right] \quad (2-9)$$

and the estimator of the spectral mean is given by

$$\hat{f} = \frac{1}{2\pi T} \arg X. \quad (2-10)$$

These are the pulse pair spectral parameter estimators suggested by Rummmler. (2)(3)(4)

The approximations made in arriving at these results indicate that no difficulty in utilizing these estimators should arise beyond that expected from sampling considerations. Thus frequencies in excess of $\frac{1}{T}$ will be ambiguously folded by the mean estimator, and the variance estimator will be degraded when the underlying spectral width begins to approach the sampling frequency.

It is also important to note that the explicit estimators do not depend directly on any particular spectral shape. This is not true of estimators of these same parameters based on spectral measurements.

2.2 Estimation Accuracy

Since (2-9) and (2-10) are explicit functions of the data, it is straightforward to analyze the expected performance of the estimators. The analysis is complicated by the nonlinearity of the estimator functions.

Appendix A gives an approximate analysis of the performance of both the mean and the width estimators. It is shown there that the estimator is unbiased and that when the per pulse signal to noise ratio is useful, the variance of these estimators is given by

$$2M \text{ Var } (2\pi \hat{f}_0 T) = e^{(2\pi w T)^2} \left[\frac{N}{S} \left(\frac{N}{S} + 2 \left(1 - e^{-2(2\pi w T)^2} \right) \right) + 2\pi w T \sqrt{\pi} \right] \quad (2-11)$$

and

$$2M (2\pi w T)^2 \text{ Var } (2\pi \hat{w} T) = \frac{N}{S} \left[\frac{N}{S} \left(1 + 2e^{-(2\pi w T)^2} \right) + 2 \left(1 - e^{-(2\pi w T)^2} \right) \right] \quad (2-12)$$

$$+ \frac{3}{8} (2\pi w T)^3 \sqrt{\pi}$$

where

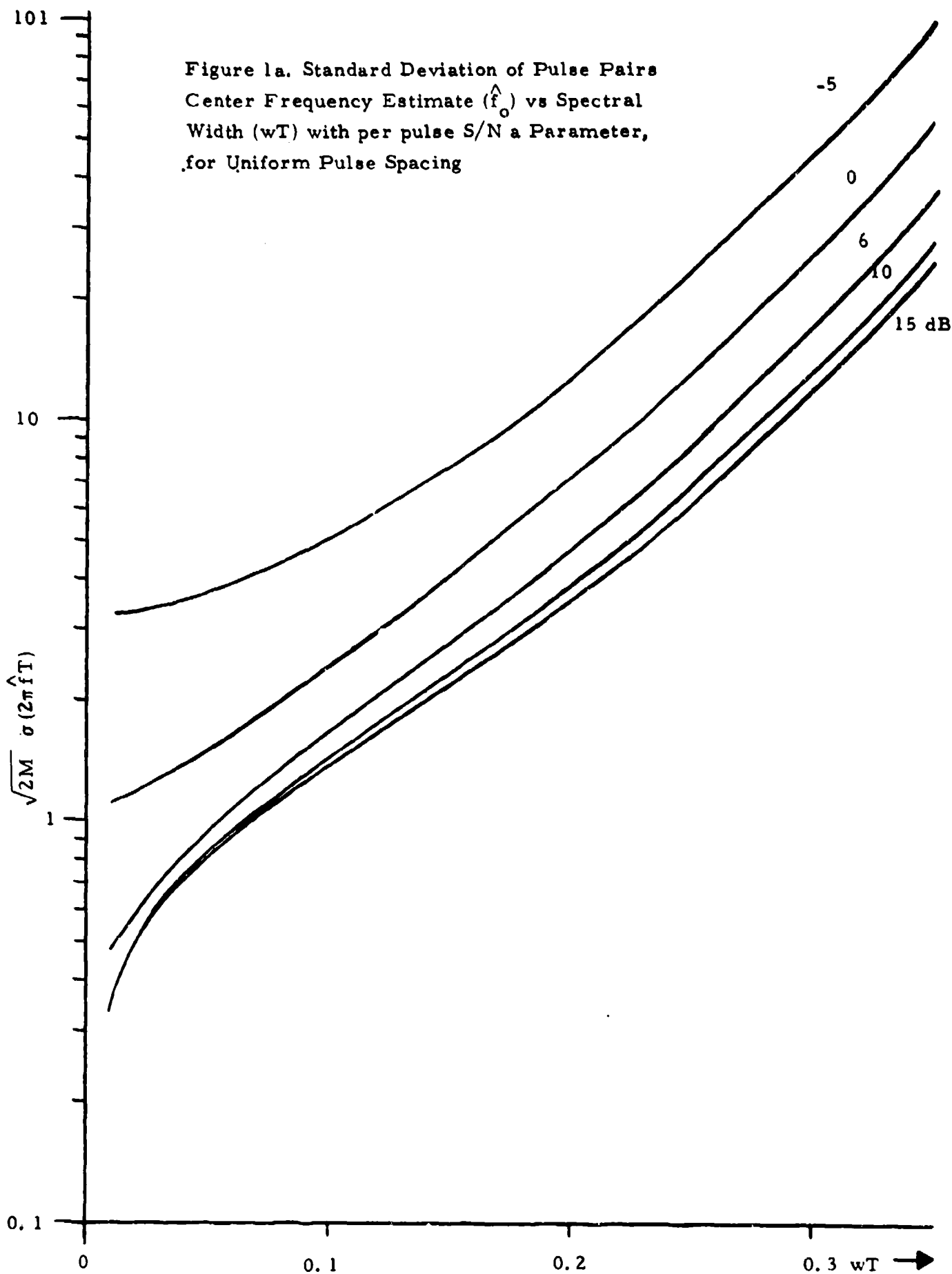
M = number of pairs in the estimate ($\approx$ number of data points in the sample)

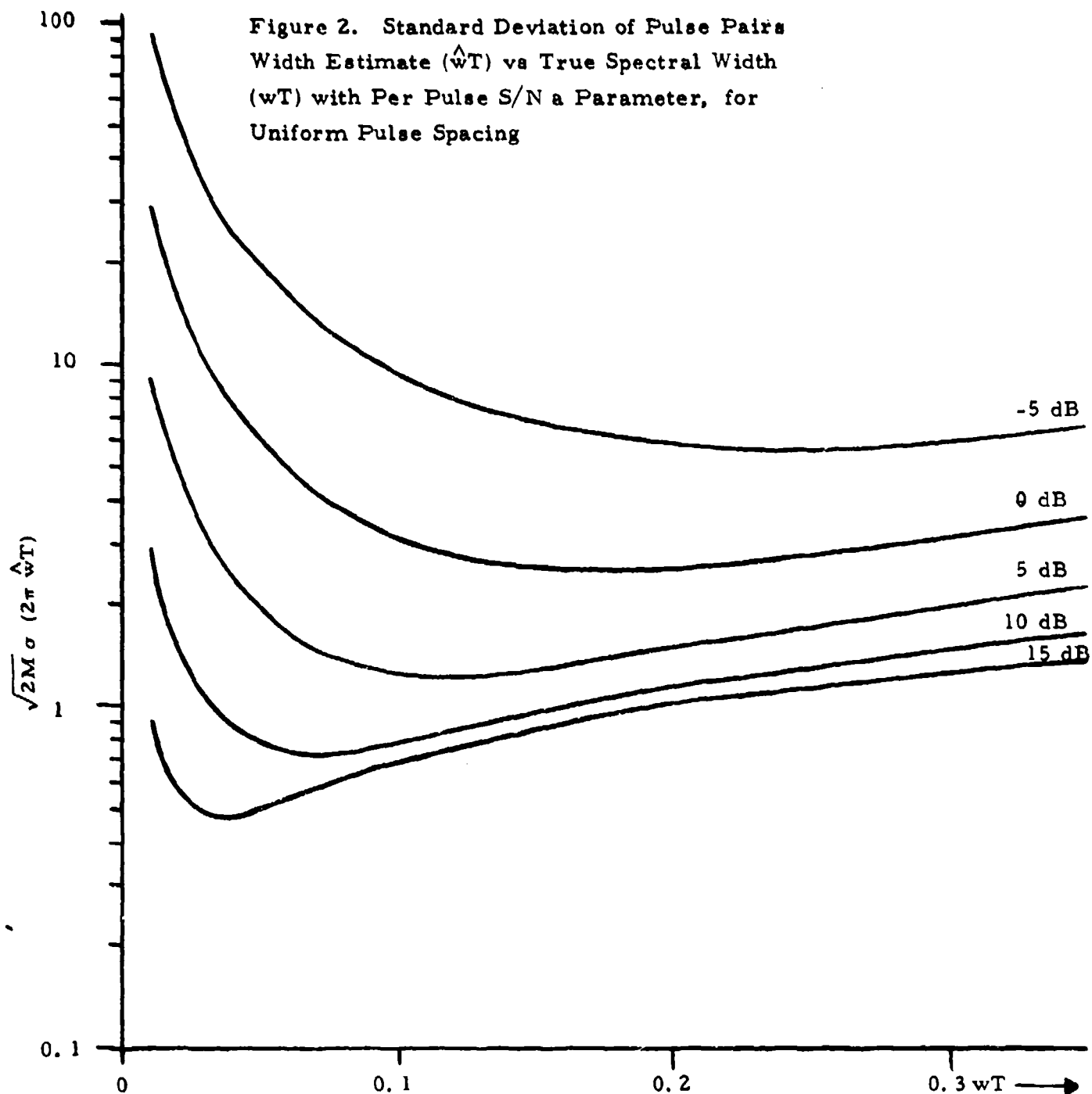
N/S = per pulse noise to signal ratio

These formulae were derived for Gaussian random variables with Gaussian shaped spectra. More general formulae are available in Appendix A.

The general formulae were derived for a uniform train of samples and account for all correlations in the data.

Figures 1a and 2 show plots of these functions. Comparable figures⁽⁴⁾ derived for the case of independent pulse pair measurements, (i.e., from a nonuniform pulse train, with successive pairs spaced so far apart that the echoes are completely decorrelated) are shown in Figures 3 and 4. Comparison of these figures, which show negligible differences at useful signal to noise ratios, are indication again of the robustness of the technique.





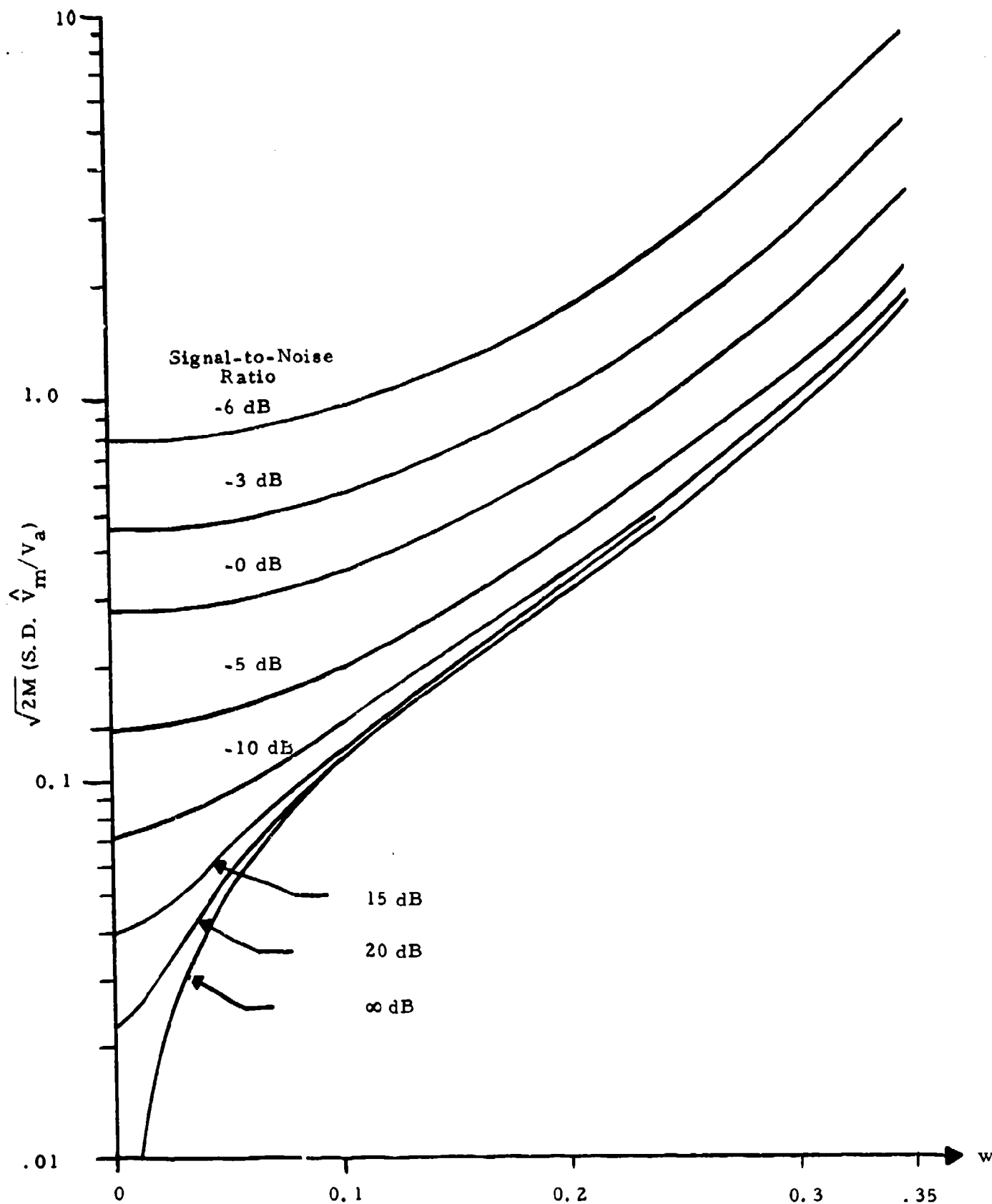


Figure 3. Accuracy of Pulse Pairs Estimate of Mean Velocity vs. Spectral Width, for Independent Pairs (from Rummler, Ref. (4))

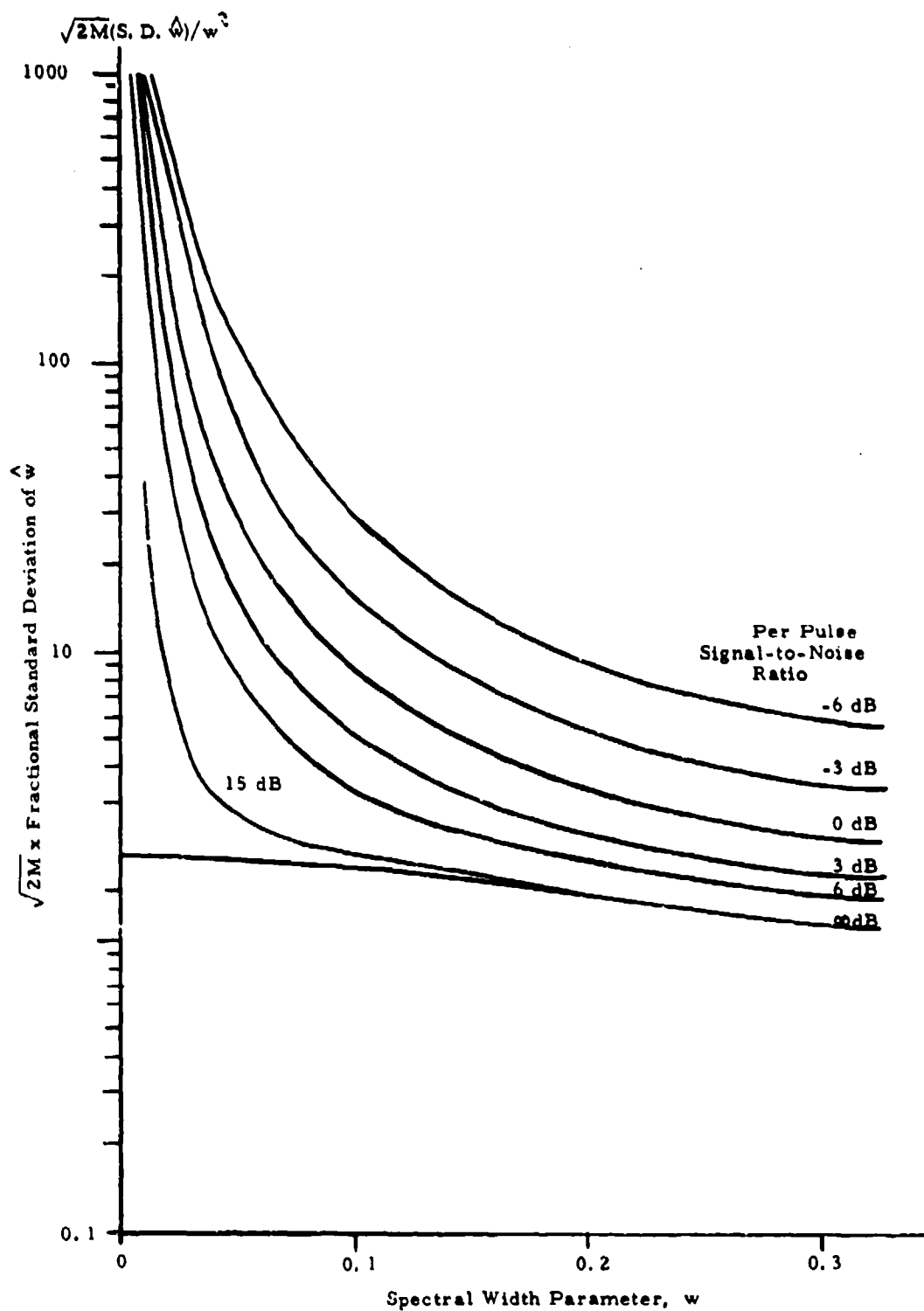


Figure 4. Fractional Accuracy of Pulse Pairs Estimate of Spectral Width vs. True Spectral Width for Independent Pairs (from Rummler, Ref. (4))

2.3 Hardware Implications

The pulse pair technique calculation in essence requires 2 complex pair multiplies and adds for each new data point in order to form X and Y as indicated in (2-3) and (2-4). The conversion of these to mean and variance estimates is an operation which need not be carried out at the input data rate. To bring this into sharp focus, a digital spectrum analyzer capable of processing up to 1024 complex samples at a single range, would require 5 times the number of comparable arithmetic operations and up to 500 times the data storage of a comparable pulse pair processor.

This efficiency in signal processing makes it possible to consider implementing a real time signal processor which can obtain spectral mean and variance estimates in real time with selectable dwell time per gate of up to a 1000 complex samples per dwell, with as many as 1000 range cells processed simultaneously. Available technology currently permits these calculations to be made at a 2 MHz data rate, thereby permitting range resolution 0.5 μ s (i. e., 250 ft.) and even higher thruput rate processors are being developed now.

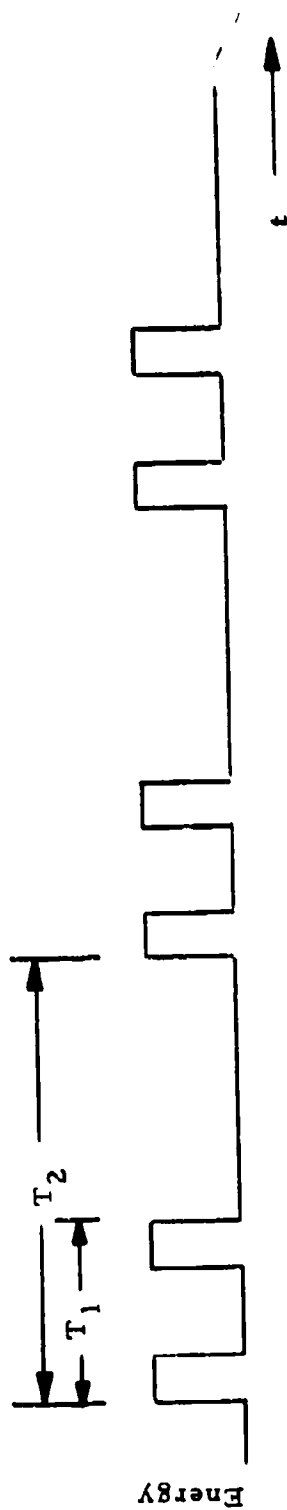
In a later section we describe a candidate signal processor implementing this technique to the above performance specifications. That section details the flexibility and performance which may be expected of a relatively modest signal processor.

2.4 Waveform Flexibility

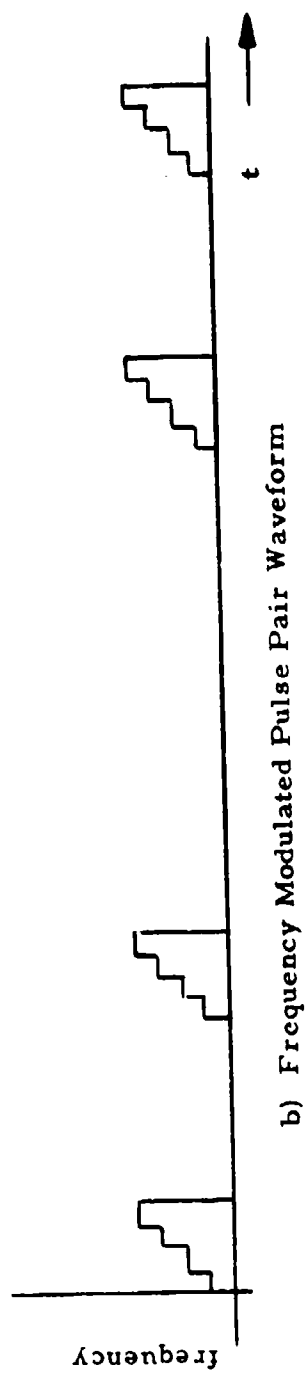
The restriction of the processor to consecutive pairs makes possible a great deal of waveform flexibility which can be used to advantage to overcome certain inherent radar limitations. Prime among these are the range-Doppler coupling which makes the unambiguous Doppler interval of the radar vary inversely with its unambiguous range interval. For weather radars, this amounts to a reduction of the Doppler analysis band of the radar as its range is extended.

The pulse pair technique makes it possible to use the waveform shown in Figure 5a to change the range-Doppler coupling relationship to

$$f_d \leq \frac{1}{T_2} \quad (2-13)$$



a) Simple Pair Spaced Waveform



b) Frequency Modulated Pulse Pair Waveform

Figure 5. Pulse Pair Waveforms

This makes it possible to increase the Doppler coverage band on relatively small cells located at greater distances from the radar. Selection of T_1 and T_2 permits the radar to operate with an unambiguous Doppler spread reciprocally related to the dimension of an isolated cell. In weather radars such a technique might permit the unambiguous Doppler measurement of such conditions as severe convective cells and possibly local CAT conditions at low angles and long ranges.

The second waveform possibility, shown in Figure 5b, is available with both equally and unequally spaced pairs. The accuracy equations, (2-11) and (2-12), reveal that a key performance parameter is the per pulse signal to noise ratio. The equations indicate that there is an optimum S/N per pulse which minimizes the measurement errors for a fixed radar energy. That is, if S/N and M are related by the equation

$$\mathcal{E} = M (S/N) \quad (2-14)$$

(which indicates that a fixed energy is available to the measurement that may be distributed in any way between the number of pairs processed and the S/N used per pulse) then either equation (2-11) or (2-12) has a minimum at a definite value of M.

In particular, it may be shown that for the measurement of mean velocity, the optimum per pulse S/N is given by

$$(S/N)_{\text{opt}} = (2\pi wT \sqrt{\pi})^{-1/2} \quad (2-15)$$

and at that point the measurement variance is given by

$$\text{Var } (2\pi \hat{f}_o T)_{\text{opt}} = \frac{e}{\mathcal{E}} \frac{(2\pi wT)^2}{\left[1 - e^{-2(2\pi wT)^2} + \left(2\pi wT \sqrt{\pi} \right)^{1/2} \right]} \quad (2-16)$$

Figures 6 and 7 indicate the sensitivity of this condition to nonoptimum conditions for mean Doppler and width measurements respectively. It is important to note that minimum error for both measurements does not occur at the same value of signal to noise ratio. Indeed, the curves show that substantially more per pulse S/N is required to minimize width errors.

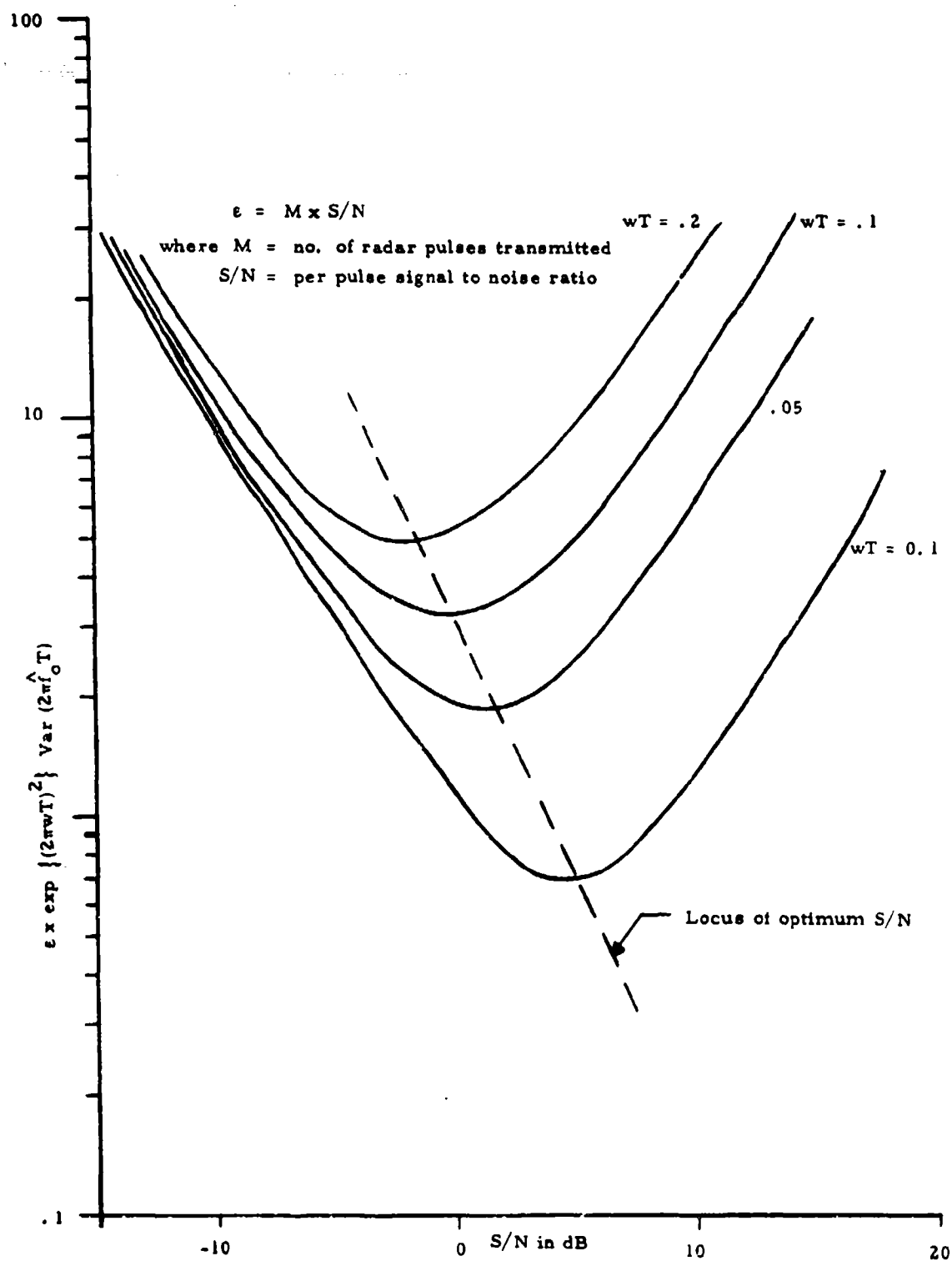


Figure 6. Tradeoff Characteristic of Pulse Pair Mean Frequency Measurement for Fixed Radar Energy ϵ

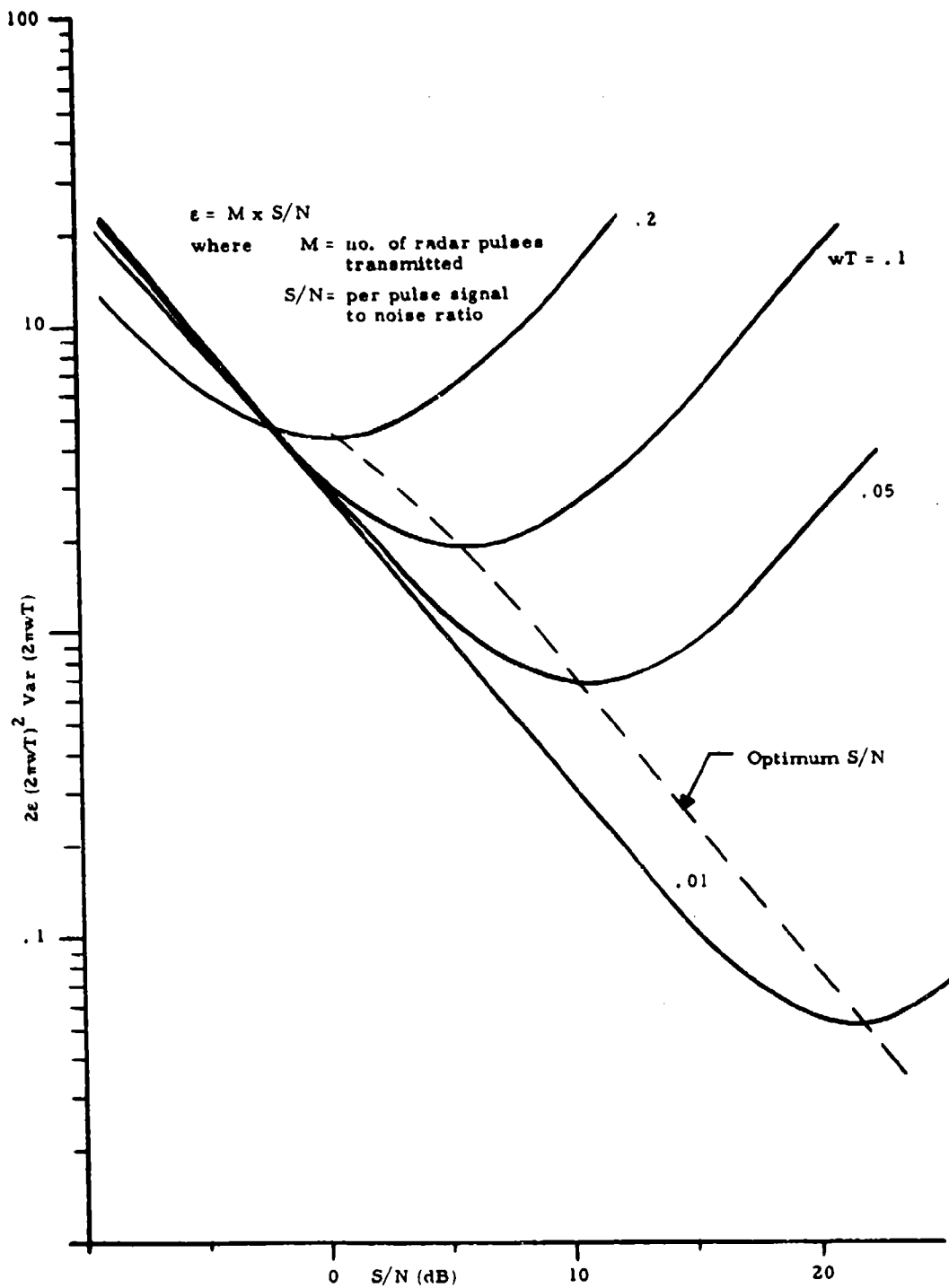


Figure 7. Tradeoff Characteristic of Pulse Pair Width Measurement for Fixed Radar Energy ϵ

The tradeoff characteristics may be used to establish the benefits to be obtained by the frequency modulation waveform of Figure 5b as follows. Suppose that the radar and meteorological parameters are such that only normalized widths of 0.05 or greater are of interest. Then Figure 7 indicates that the data should be collected at the signal to noise ratio of 7.5 dB per pulse. Suppose then that it were required to make that measurement to an accuracy of 10%. Then

$$\frac{\sigma_w^2}{w^2} = \frac{\sigma_{2\pi \hat{w}T}^2}{(2\pi wT)^2} = 0.01 \quad (2-17)$$

For $wT = 0.05$, Figure 7 indicates that

$$2\epsilon (2\pi wT)^2 \sigma_{2\pi \hat{w}T}^2 = 0.8 \quad (2-18)$$

so that

$$\epsilon = \frac{0.4}{(2\pi wT)^4 \frac{\sigma_w^2}{w^2}} \quad (2-19)$$

$$= 36 \text{ dB}$$

Thus, at least $M = (36 - 7.5) \text{ dB}$

$= 930$ pulses must be processed to achieve the requisite accuracy.

If the peak power in the pulse were such that 13.5 dB per pulse were available for the measurement, this theory suggests that transmission of four frequency coded subpulses each with 7.5 dB per pulse signal to noise ratio for a total transmission of 250 groups of four might result in better overall accuracy than would the same length of uncoded transmission. The improvement in performance would be negligible at $wT = .05$ for the conditions given, but at $wT = 0.1$ the frequency modulation would improve the accuracy by 40% $= (\sqrt{2}-1)$.

There is an important caveat which must be considered in this regard; namely, that additional receivers may be required to implement the frequency modulation system for extended targets.

2.5 Summary

The pulse pair technology offers new potential for Doppler weather radar systems. It permits the economic implementation of a real time signal processor which can produce prodigious data reduction, reducing the enormous amount of data produced by such a radar to more manageable dimensions. It does so without a sacrifice in sensitivity in cases of interest.

It makes it possible to consider new radar modulation techniques permitting the radar to collect the data more efficiently. Thus a frequency and pulse pair spacing agile radar, which has merit in both increasing the data rate (number of pulse pairs available) and suppressing some of the range-Doppler ambiguities, can be processed without penalty in a pulse pair analyzer.

The pulse pair technique produces estimates which have predictable accuracy and are free of arbitrary parameter selection (such as the threshold level in the spectrum analyzer technique). In a digital processor, the block length of the analysis interval may be varied on command so that the processor can easily be properly matched to the radar operating conditions (p. r. f. and scan rate).

If data rates were not a consideration, it is clear from the structure of the processor, that it could be implemented as a sliding window processor, so that running averages of the most recent N1 pulse pair data could be used to obtain continuously updated mean and variance estimates.

In short, it appears that the pulse pair processing technique offers an efficient means of automating pulse Doppler weather radar processing with a technique which is accurate, sensitive and flexible. The equipment used in conjunction with clutter cancelling techniques should permit operation in a significant background of ground clutter.

Its ability to signal the presence of unusual spectral shape is perhaps its ultimate limitation.

3.0 EXPERIMENTAL EVALUATION

This section describes several efforts to date to experimentally verify the theoretically predicted accuracy capabilities of the pulse pair estimators of spectral mean and spread and to compare their performance with other methods of estimation using identical input data. The initial experiment, which was conducted on the previous contract, compared the results of pulse pair and conventional spectral analysis processing of real Porcupine Radar weather return^(5, 6). The limitations encountered with this approach to evaluation motivated the more extensive and precise experimental evaluations accomplished on the present contract using simulated radar return. Although all of the experiments to date have involved non-real time pulse pair processing of the tape recorded radar output data, real time processing is both feasible and advantageous; a proposed real time hardware processor is described in Section 4.

3.1 Porcupine Radar Weather Return

A sequence of 10240 consecutive raw complex video samples (both in-phase and quadrature) were tape recorded from the output of the Porcupine weather radar during a light rain. The PRF was 3300 pps, the antenna elevation angle was 40°, and the data was obtained from the range cell with the maximum S/N. The analog data was then quantized for subsequent processing on a digital computer by the "pulse pair" method, with conventional spectral analysis as a control. The latter was required since the true spectral parameters of the radar doppler return from the rain were unknown.

3.1.1 Estimates of Mean and Spread by Conventional Spectral Analysis

The recorded data was processed by conventional spectral analysis in three different block lengths: one block of 8192 complex samples; two blocks of 4096 samples; and ten blocks of 1024 samples. A typical plot of the amplitude of the spectral components of one of the ten blocks of 1024 samples (representing approximately 1 second of weather data) as analyzed by a digital computer programmed to execute a Fast Fourier Transform is shown in Figure 8, "Typical Data Spectrum for a Sequence of 1024 Samples." The ragged appearance of the spectrum is due partly to noise and partly to the distributed nature of the weather target.

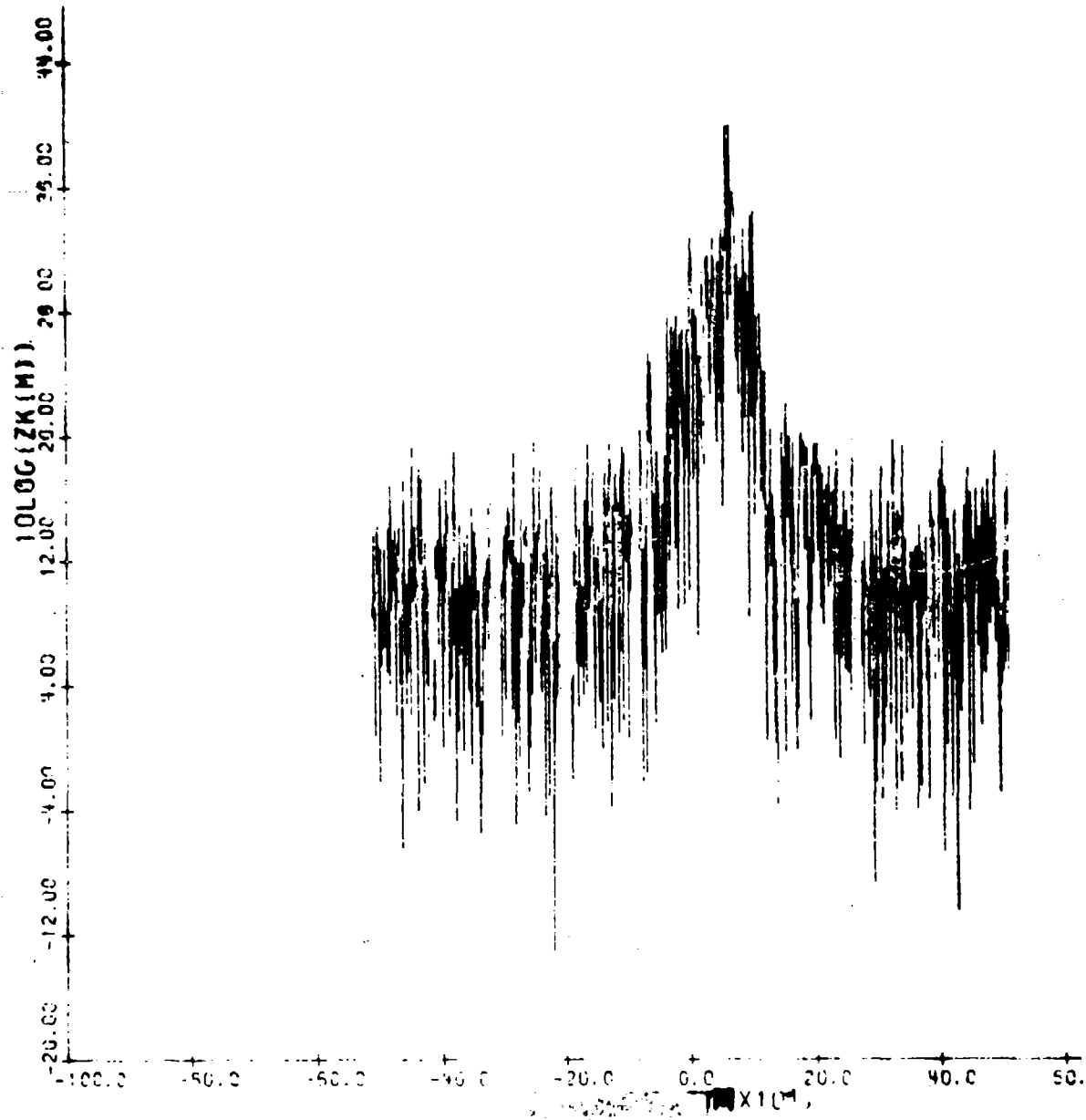


Figure 8. Typical Data Spectrum for $N = 1024$

The computer was programmed as follows to estimate the first moment ($\hat{f}_0$) of the doppler spectrum and the second moment ($\hat{w}^2$) about $\hat{f}_0$.

Letting $z(n) = x(n) + jy(n)$ denote the n^{th} sample in a block of length N , the spectral amplitudes

$$P(k) = \left| \frac{1}{N} \sum_{n=0}^{N-1} a(n) z(n) e^{-j2\pi kn/N} \right|^2, \quad k = 0, \dots, N-1 \quad (3-1)$$

are computed, where the $a(n)$ are 60 dB Dolph Chebyshev weight coefficients for a block of length N .

The unambiguous frequency interval is then shifted from $0 \leq k \leq N-1$ to $-\frac{N}{2} + 1 \leq m \leq \frac{N}{2} - 1$ by the transformation

$$Z(m) = \begin{cases} P(m-1), & 1 \leq m \leq N/2 \\ P(m+N), & -\frac{N}{2} + 1 \leq m \leq 0. \end{cases} \quad (3-2)$$

as shown in Figure (4).

The spectral mean and width could be calculated in accordance with

$$\hat{f}_0 = \frac{\text{PRF}}{N} \bar{m}, \quad \text{where } \bar{m} = \frac{\sum m Z(m)}{\sum Z(m)} \quad (3-3)$$

and

$$\hat{w} = \frac{\text{PRF}}{N} \left(\frac{\sum (m - \bar{m})^2 Z(m)}{\sum Z(m)} \right)^{1/2} \quad (3-4)$$

However, these classical algorithms for the mean and width of the distribution make no provision for the effects of noise, although it is evident from Figure 8 that portions of the spectral plots are predominantly noise, which would tend to obscure and degrade the estimates of the spectral parameters. Neither is there a developed theory for the effects of noise on their accuracy for spectra derived data. Nevertheless, despite the lack of guidance from theory, it is necessary to employ some additional (though not necessarily optimum) technique together with the spectral analysis estimators to improve their accuracy so that they may serve as standards of comparison for the pulse pair estimators. A common

method is to select a threshold level referenced to the noise level or to the peak of the spectrum, and to use only those spectral components whose amplitudes exceed the threshold in the computation of spectral mean and width as a special set, which falls within the general class of spectral analysis estimators. Such a preliminary computational step, prior to the above calculation of the mean and width, has been introduced into the spectral analysis estimator computer program.

Rather than restrict the results of the spectral analysis estimates to a single, possibly poor choice of threshold, the estimates for $\hat{f}_0$ and $\hat{w}$ were computed for each block of data over a range of assumed noise thresholds from 20 dB to 30 dB below the spectral peak, and are shown in Tables 1 and 2 respectively. It will be noted that the estimates of $\hat{f}_0$ vary with threshold level within each block over a range of from 5.5% to 12.9% of the nominal value, depending upon the specific block.

The spectral analysis estimates of $\hat{w}$ are even more sensitive to the value of threshold. They vary with threshold level within each block over a much larger range of from 51% to 68% depending upon the specific block. The potential problem of using such measurements as a standard of comparison for evaluating the performance of the pulse pair estimator is noted here in passing, and will be discussed further later on.

3.1.2 Estimates of Mean and Spread by Pulse Pair Processing

The identical Porcupine radar data processed by spectral analysis were also processed by the pulse pair technique. The ten blocks of digitized sequences of 1024 samples were inputted to a computer programmed to implement the pulse pair estimators of equations (2.9) and (2.10). These results are also shown in Table 1 and 2 respectively. The estimate of $\hat{w}$ includes a correction for noise, the level of which was deduced from the experimental data by assuming that the peaked portion of the spectrum was composed of signal plus noise, while the flat skirts were noise alone.

3.1.3 Comparison of the Estimators

Numerical comparisons of the pulse pair and spectral analysis estimates of $\hat{f}_0$ and $\hat{w}$ obtained from the Porcupine weather return data may be made from the results in Tables 1 and 2.

Table 1. Comparison of Pulse-Pair $\hat{f}_0$ with Spectral Analysis $\hat{f}_0$ for T=20:2:30

Block Nos.	Block-length, N	Pulse-Pair $\hat{f}_0$	Spectral Analysis $\hat{f}_0$					
			T=20	T=22	T=24	T=26	T=28	T=30
1	1024	178.27	189.45	185.44	180.25	177.57	173.42	172.94
2	1024	182.95	181.68	180.12	175.40	166.78	159.57	155.96
3	1024	175.80	169.22	167.03	161.68	157.44	155.76	154.49
4	1024	184.06	166.70	167.21	159.46	154.13	153.01	151.73
5	1024	199.08	192.13	187.70	183.82	179.05	175.49	174.44
6	1024	208.83	220.03	215.70	210.29	201.63	198.43	196.65
7	1024	196.50	208.18	204.82	202.96	197.26	193.72	190.70
8	1024	203.98	220.10	219.21	215.03	211.20	211.50	208.87
9	1024	181.00	179.55	167.99	161.42	161.42	158.72	156.72
10	1024	186.10	177.47	179.18	178.66	175.63	170.07	167.96
1-4	4096	180.26	179.80	178.40	177.29	172.37	167.84	164.60
5-8	4096	201.90	201.88	199.02	192.94	189.17	184.55	182.22
1-8	8192	190.78	192.89	191.16	189.38	183.51	179.17	174.62

Table 2. Comparison of Pulse-Pair $\hat{w}$ with Spectral Analysis $\hat{w}$ for T=20:2:30

Block Nos.	Block-length, N	Pulse Pair $\hat{w}$	Spectral Analysis $\hat{w}$					
			T=20	T=22	T=24	T=26	T=28	T=30
1	1024	213.46	153.23	224.27	274.52	313.05	333.53	341.41
2	1024	206.24	100.78	128.19	191.50	251.93	288.23	310.62
3	1024	188.96	128.02	178.45	227.94	266.36	294.83	307.83
4	1024	208.71	163.90	225.26	279.31	308.01	326.38	337.02
5	1024	222.75	154.17	207.06	259.84	295.05	316.47	324.52
6	1024	224.16	150.87	209.01	271.00	322.57	348.36	359.21
7	1024	206.15	139.04	173.37	222.04	263.00	296.26	310.12
8	1024	199.50	109.49	116.40	166.44	220.44	260.65	284.81
9	1024	246.70	162.39	224.70	282.27	321.15	351.33	363.45
10	1024	227.75	114.42	169.82	220.21	263.83	297.03	316.04
1-4	4096	204.33	102.65	110.25	130.38	182.89	325.84	274.93
5-8	4096	213.29	125.67	153.12	212.89	265.84	303.07	322.57
1-8	8192	209.19	110.65	120.26	151.47	203.84	255.55	294.68

These tables show the values of $\hat{f}_0$ and $\hat{w}_0$ as estimated block by block by pulse pairs and for each threshold level for the spectral analysis method. The blocks of 1024 samples have been numbered sequentially to identify the data from which the estimates were derived. The different estimates for a given block of data appear on a horizontal line alongside that block.

As was noted above, the spectral analysis estimate of $\hat{f}_0$ varies with the threshold, T . It agrees best with the pulse pair estimate of $\hat{f}_0$ when $T = 20$ dB, for which case the agreement is within better than 1% when the block length, N , is 4096 or greater. Since the spectral analysis estimates of $\hat{f}_0$, which were used as the control, do not vary too widely, it was generally concluded that the pulse pair estimate of $\hat{f}_0$ was definitely satisfactory under the conditions for which the data was taken.

On the other hand, the spectral analysis estimates of doppler spread, $\hat{w}$, are quite sensitive to the value of T . Varying as they do over a range of about 60%, they do not provide a satisfactory standard of comparison for evaluating the performance of the pulse pair estimates of $\hat{w}$. Without a knowledge of the true doppler spread of the weather radar return, it was impossible to draw any conclusions as to the effectiveness of the pulse pair method for estimating $\hat{w}$ with this particular data.

It appeared clear from the above results, that real radar weather return of opportunity did not provide a promising source of data from which the behavior of the pulse pair estimators could be quantitatively explored. Not only is there no assurance of obtaining data with the range of spectral and S/N parameters required for a valid exploration, but the accuracy with which the true values of the parameters embedded in the data (which are needed as a basis for comparison) can be established is in itself a function of the parameters. Since we cannot reasonably evaluate one unknown against another unknown, the alternate approach of using simulated radar return with prescribed parameters was indicated.

3.2 Simulated Radar Return

3.2.1 Purpose, Capabilities and Advantages

In view of the limitations encountered in attempting to evaluate the pulse-pair estimators by means of real Porcupine Radar weather return data, (see 3.1 above) more extensive experimental verification of their theoretically predicted performance was undertaken on the present contract using simulated radar weather return.

The use of simulation techniques to generate the input data for evaluating the performance of the estimators provides many advantages which overcome the principal limitations of the use of real radar data for evaluation purposes. Primarily, it makes possible the generation of statistically significant numbers of simulated complex radar samples with prescribed spectral distributions (mean frequency and width) and S/N. This permits the generation of data tapes with controllable parameters made to order and in sufficient quantities per case to suit the evaluation needs. Since the true parameters are known in advance, a direct and precise determination of estimator performance accuracy can be made at all times. The ensemble of precise results can then be analyzed by the computer to determine the statistical behavior of the estimators against the synthetic distributed weather targets as a function of the controllable parameters, number of samples entering into the estimate, etc., and the results compared with the theory.

The particular cases chosen for the simulation tests were tailored to facilitate comparison with the theoretical performance predictions derived in Section 2. A PRF of 3300, the highest available for the Porcupine Radar was used for all cases. A spectral mean of 300 Hz and width of 78 Hz were selected for the initial cases. These values are equivalent to a weather target velocity mean and spread of approximately 8 meters per second and 2 meters per second respectively, as observed with the Porcupine Radar. To vary the S/N parameter, two extreme values of S/N were chosen, including 0 dB and ∞ dB (no noise present) and two cases closer to the poor S/N condition. To conveniently vary the theoretically significant wT parameter, several multiples of the initial mean and spread were selected to yield target velocity distributions with 16 meters mean, 4 meters spread; and 24 meters mean, 6 meters spread. Each of these cases was varied over the range of four S/N values established initially.

An overall block diagram of the software simulation system utilized for investigating the pulse pair technique appears in Figure 9. The system is capable of generating sequences of simulated radar doppler return signals with controllable parameters; estimating the mean and width of the spectral distributions by both pulse pair and spectral analysis techniques; performing statistical analyses of estimator accuracy; and plotting and/or printing out the results.

The simulation system is programmed in FORTRAN and was run on the CDC 6600 Computer at AFCRL. To conveniently suit the capacity and capabilities of the computer, the simulation system was divided into separable steps, which could be run at different times. To facilitate such division into steps, the sequences of simulated radar doppler return which were to be used as input data to

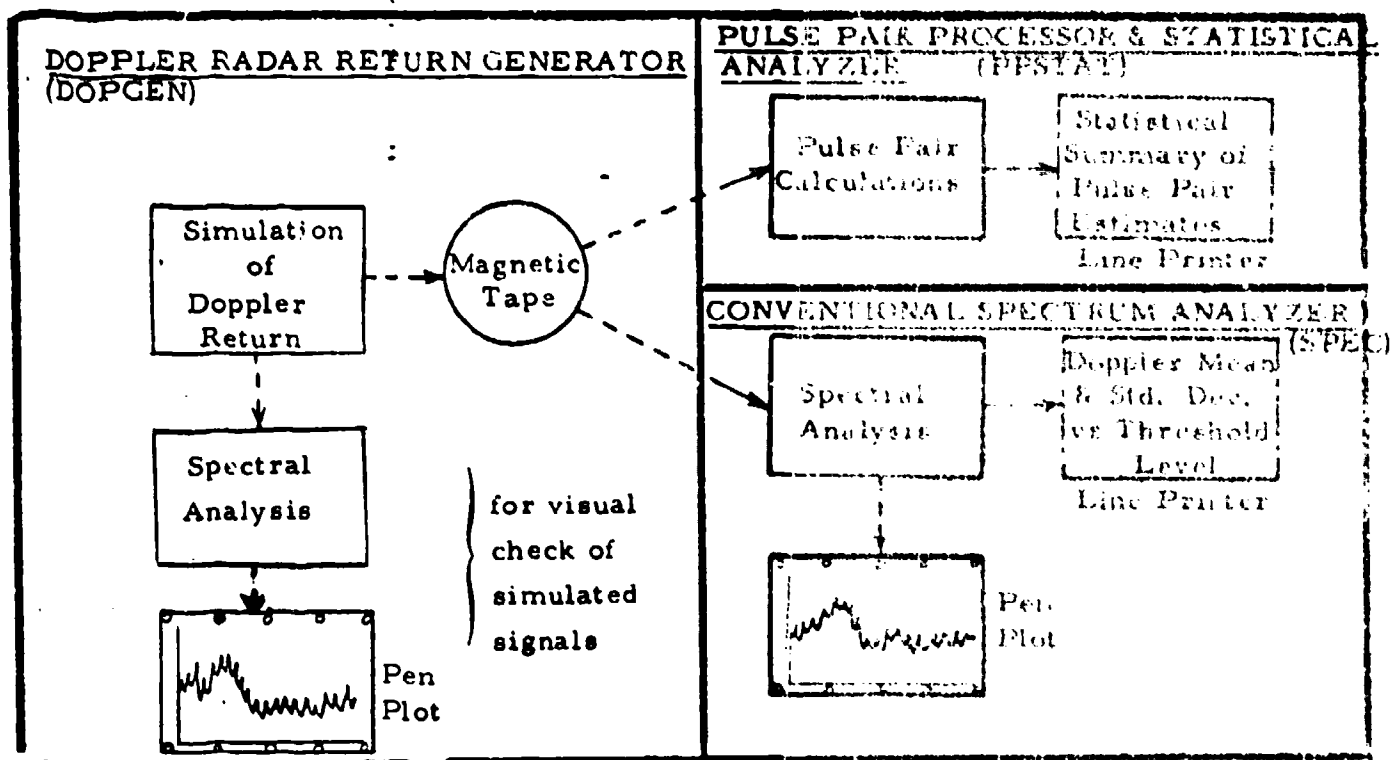


Figure 9. Overall Block Diagram of the Software Simulation System for Investigating and Comparing the Performance of the Pulse Pair and Spectrum Analyzer Estimators of Spectral Parameters

the signal processors under investigation were recorded on magnetic tape for later processing and reference. A library of such input data tapes was generated to provide desired numbers of statistically independent sequences of simulated return, with desired combinations of spectral parameters and S/N. The signal processing schemes, and modifications thereto, could thereby be statistically evaluated and compared on the basis of identical input data. Provision was also made to visually verify the validity of the simulated data; designated sequences of returns from among those generated could be selected for spectral analysis by conventional FFT and automatically plotted for visual check.

The forthcoming sub-sections deal in greater detail with the generation of the simulated doppler returns; the comparative performance of the pulse pair and spectrum analysis estimators against the simulated input data; and a comparison of pulse pair estimator performance as experimentally determined by simulation with that predicted from theory. Excellent correlation between theory and experiment are shown.

3.2.2 Generating Sequences of Simulated Doppler Radar Target Return with Controllable Parameters

In the real Porcupine Radar case, a uniform PRF was used to generate sequences of uniformly spaced coherent echoes reflected from distributed meteorological targets as observed in a fixed range gate. The uniformly spaced echoes, in the form of digitized complex numbers derived from the phase detected coherent video output, were then processed two at a time to yield the pulse pair estimates of the target spectral parameters.

The object of the simulation program is to generate similar sequences of uniformly spaced complex numbers equivalent to radar echoes reflected from a distributed target with a given radial velocity distribution, as sampled at a given uniform PRF and with a given output S/N. The problem is mathematically equivalent to that of numerically generating a sequence of sampled values of a stationary Gaussian process having a specified correlation function or power spectrum. The method employed is adapted from Levin⁽⁷⁾.

A. Theory

The following parameters of the desired sequence must be controllable:

<u>Term</u>	<u>Symbol</u>	<u>Units</u>
MEAN DOPPLER	f_o	Hz
SECOND MOMENT	$m_2 = w^2$	Hz ²
SAMPLING RATE	PRF	Hz
S/N RATIO (per pulse)	B	dB

1. Spectral Distribution - An approximation to the desired spectral distribution of the radar video data is achieved by playing Gaussian noise of zero mean and unity variance through a filter consisting of two cascaded, low-pass networks. The transfer function of the filter is given by:

$$H(s) = \frac{a^2}{(s + a)^2} \quad \text{where } a = 2\pi w \text{ and } s = \text{complex frequency (rad/sec)} \quad (3-5)$$

From this expression it is possible to derive the second moment and half power point of the filter. The second moment is given by:

$$m_2 = \frac{\left(\frac{1}{2\pi}\right)^2 \int_{-j\infty}^{+j\infty} H(s) H(-s) (-s^2) ds}{\int_{-j\infty}^{+j\infty} H(s) H(-s) ds} = w^2 \quad (3-6)$$

The half power point (-3dB) occurs at

$$\omega_3 = a [\sqrt{2} - 1]^{1/2} \quad (3-7)$$

By Z-transform theory, the recursion relation for the equivalent digital filter is given by the impulse invariant transformation; namely, if

$$H(s) = \frac{a^2}{(s+a)^2}$$

then,

$$H(Z) = \frac{Z^{-1} a^2 T e^{-aT}}{(1 - e^{-wT} Z^{-1})^2} ; \text{ where } T = \frac{1}{\text{PRF}} \quad (3-8)$$

The resulting recursion relation is then

$$y_n = x_{n-1} a^2 T e^{-aT} + 2 e^{-aT} y_{n-1} - e^{-2aT} y_{n-2} \quad (3-9)$$

Simulation of radar quadrature data is achieved as illustrated in

Figure 10. The sequences $\{x_1, x_2\}$ are two independent Gaussian sequences which are then played through the same filter, $H(Z)$, specified by (3-8) and (3-9).

Complex multiplication of the output sequences $\{y_1, y_2\}$ by $e^{j2\pi f_0 nT}$ shifts the mean frequency from zero to f_0 .

Transient effects of the filter are minimized by discarding a number of initial terms. This number has been set at three times the fractional bandwidth of the filter (with respect to the PRF) and it is given by:

$$\text{No. of discarded initial terms} = \frac{3 \cdot (\text{PRF}) \pi}{w} \quad (3-10)$$

2. Signal to Noise Ratio - When Gaussian noise is added to sequences $\{y_1^i, y_2^i\}$ (See Figure 10), the per-pulse signal to noise ratio (S/N) is specified by:

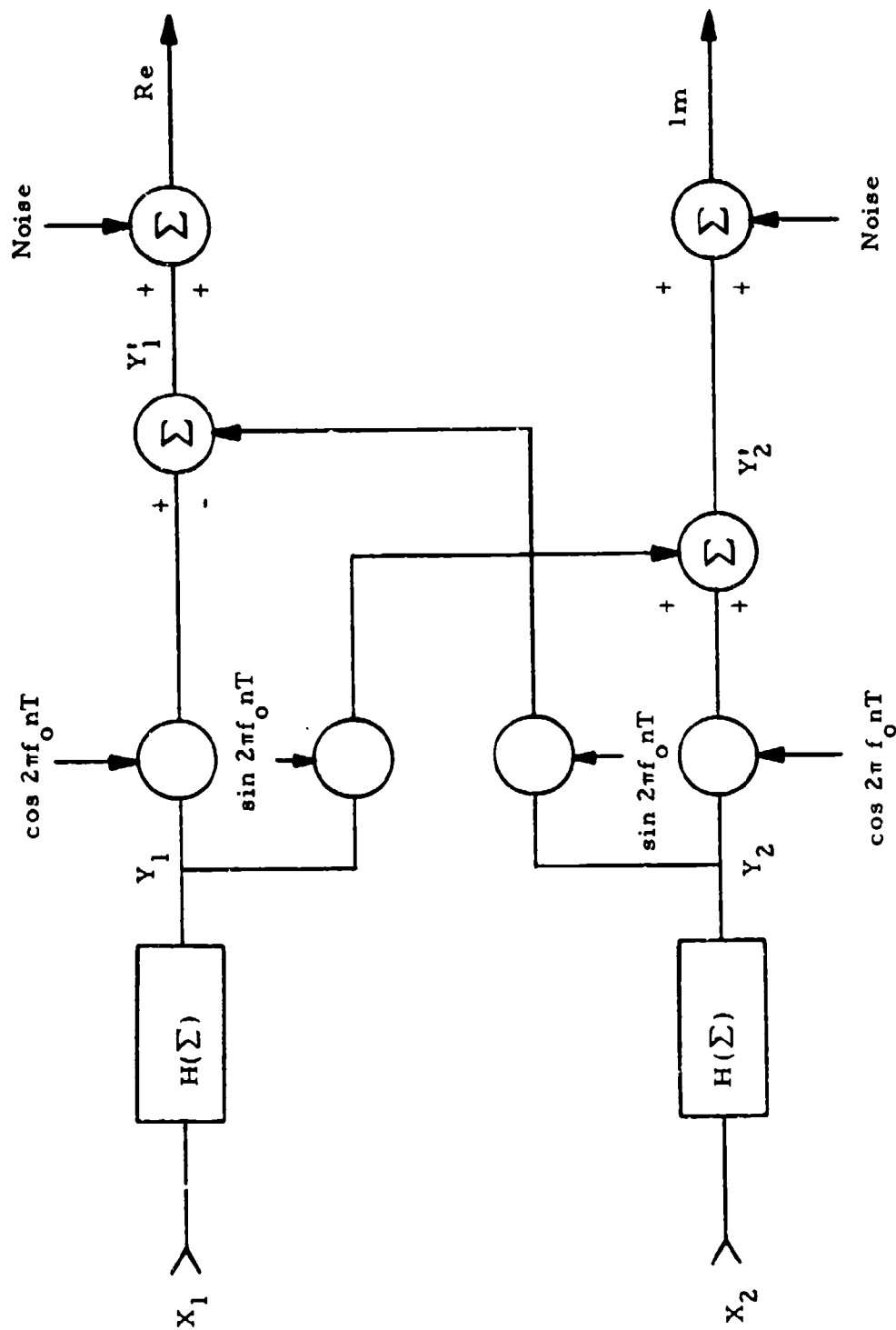


Figure 10. Block Diagram of Simulated Doppler Return Generator

$$S/N = \frac{a^4 T^2 (3e^{-aT} + e^{aT})}{\sigma_N^2 (e^{aT} - e^{-aT})} = B \text{ [dB]} \quad (3-11)$$

where σ_N^2 is the variance of the added, zero mean, Gaussian noise. A given S/N ratio is then achieved by specifying σ_N^2 as

$$\sigma_N^2 = \frac{a^4 T^2 (3e^{-aT} + e^{aT}) 10^{-B/10}}{(e^{aT} - e^{-aT})} \quad (3-12)$$

where B is the desired S/N ratio in dB.

Mathematical derivations relating to the above processes appear in Appendix B.

B. Description of Computer Program

The program developed for generating the simulated doppler target return on the AFCRL CDC 6600 is called DOPGEN. The flowchart for DOPGEN is shown in Figure 11, and the program statements coded in FORTRAN appear in Appendix C1.

Input to the program is in the form of data cards on which are specified:

- (1) mean Doppler frequency;
- (2) standard deviation of Doppler frequency;
- (3) per-pulse signal-to-noise ratio;
- (4) FFT plot control; and,
- (5) number of blocks (N) of 1024 complex samples.

Upon reading the first data card, the program generates 1024 complex samples which simulate Doppler return (a PRF of 3300 is used throughout). The data block is then written on magnetic tape along with twelve ancillary words containing information about the data. This procedure continues until N independent data blocks are written with the specified Doppler mean and standard deviation.* A second data card can be used to change the mean and the width of the Doppler spectrum or to change the S/N ratio. If no more data blocks

* Since each data block occupies about 3.5 ft. of magnetic tape, 685 data blocks will fit on a standard 2400 ft. long tape.

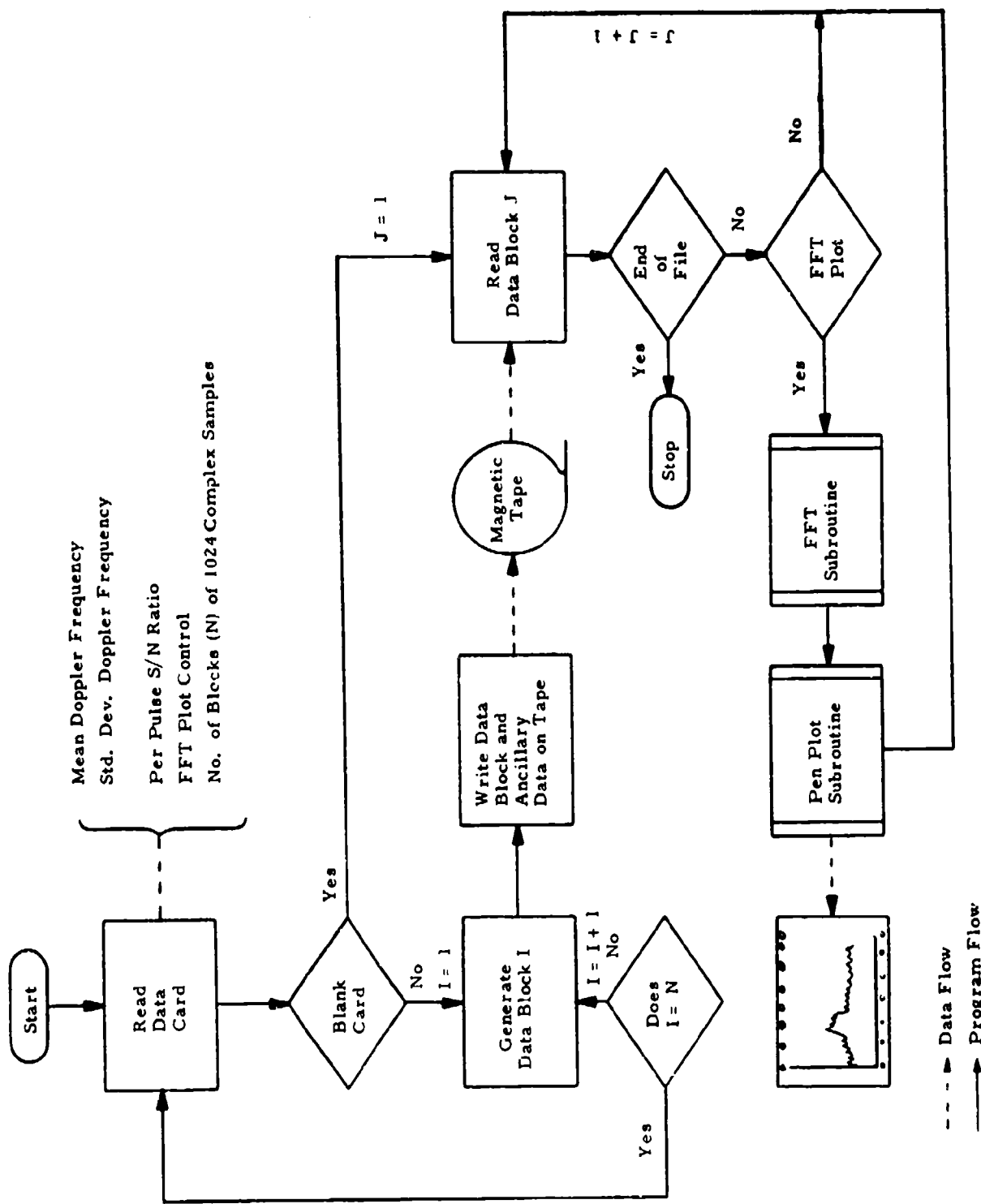


Figure 11. Flowchart of DOPGEN

are to be generated, as indicated by a blank data card, the program rewinds the tape and reads each block to verify the readability of the tape. In addition, for those data blocks on which the FFT plot control parameter (one of the ancillary words) is set to one, the program performs an FFT on the data and plots the transformed data on a Calcomp Plotter (available at AFCRL). This is used as a check to verify the validity of the simulated data - a sample plot is shown in Figure 12.

A library of 390 separate sequences of 1024 complex samples was generated by the DOPGEN simulation program and recorded on magnetic tape for use as input data. Each sequence of 1024 samples constitutes a data block. Three pairs of mean doppler and doppler standard deviation were run, each at four conditions of S/N; a PRF of 3300 pps, representative of the Porcupine Radar, was used in all cases. Thirty independent data blocks with specific parameters as shown in Table 3 were generated for each combination of parameters. A sample spectral analysis plot for each of the twelve combinations is presented in Appendix E.

Block No.	f_o Mean Doppler (Hz)	w Std. Dev. Doppler (Hz)	S/N (dB)	Tape
41-70	300	78	5	RPK07
101-130	600	156	∞	RPK08
131-160	600	156	0	RPK08
161-190	600	156	5	RPK08
191-220	600	156	15	RPK08
221-250	900	234	∞	RPK08
251-280	900	234	0	RPK08
281-310	900	234	5	RPK08
311-340	900	234	15	RPK08
341-370	300	78	∞	RPK09
371-400	300	78	0	RPK09
401-430	300	78	5	RPK09
431-460	300	78	15	RPK09

Table 3. List of Recorded Sequences of Simulated Doppler Return Generated, with Parameters as shown.
(PRF = 3300 pps in all cases)

PRF 3300 pps
 Number of Samples 1024
 Cycles per Sample Point 3.22
 Doppler Mean Frequency 300 Hz
 Doppler Standard Deviation. 78 Hz
 S/N 5 db

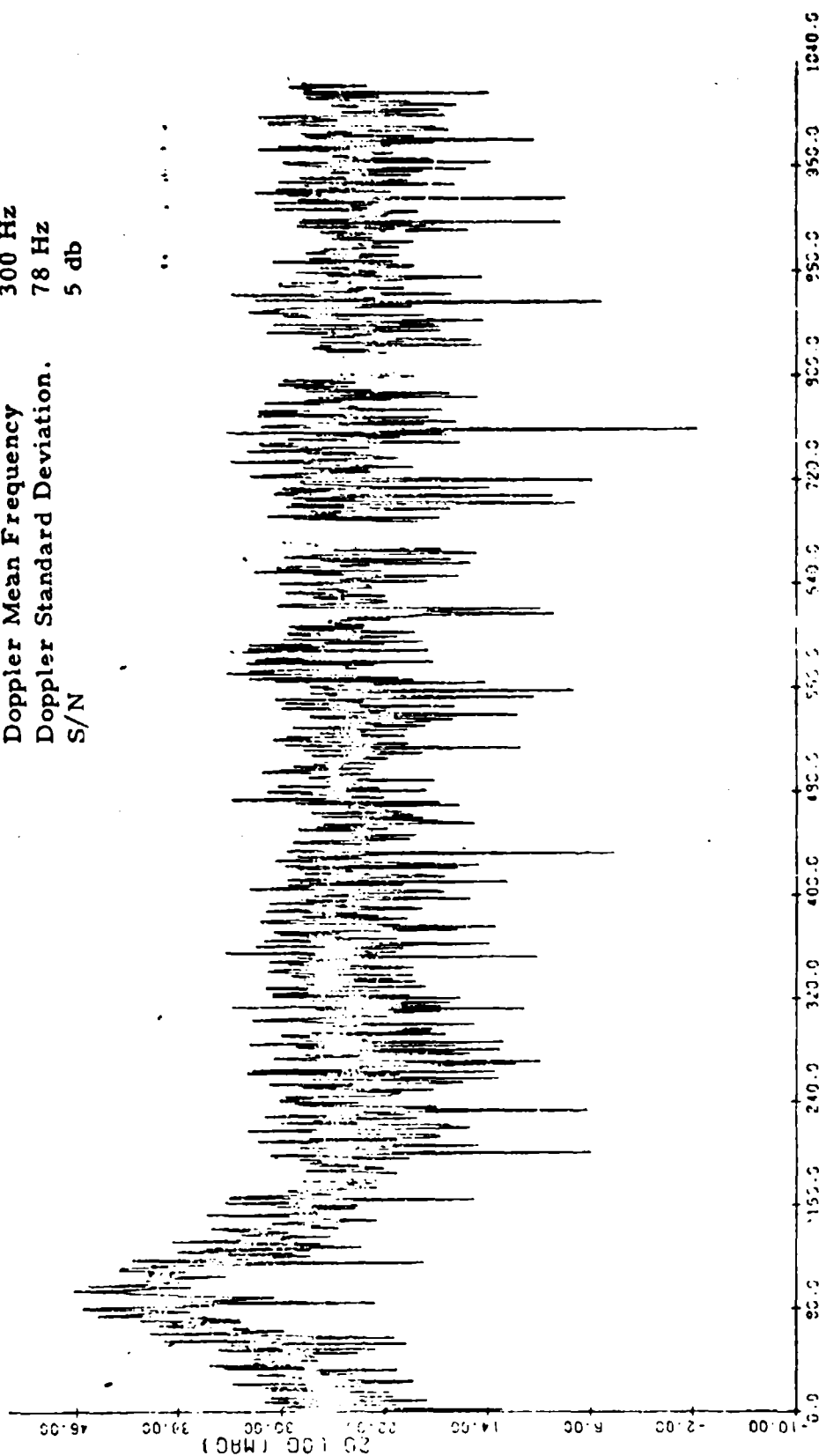


Figure 12. FFT Processed Simulated Doppler Spectrum of Weather Data

3.2.3 Performance of Pulse Pair Estimators

A. Computer Program

As shown in Figure 9 of the overall simulation system, the tape recorded sequences of simulated doppler return are processed by the pulse pair algorithms to yield estimates of spectral mean and width as a function of the number of pulse pairs entering into the estimate. The resulting estimates are then compared with the known given spectral parameters of the input data and the performance of the pulse pair estimators is statistically analyzed and summarized to produce printouts of estimator accuracy versus number of pulse pair samples for each simulated test condition.

The program for pulse pair processing and statistical evaluation is called PPSTAT. A flowchart of PPSTAT appears in Figure 13, and the coded FORTRAN statements for this program may be found in Appendix C-2. As shown in the flowchart, before reading the magnetic tape generated by DOPGEN, PPSTAT reads from a data card two data block numbers (the numbers are assigned by DOPGEN and are contained in the ancillary data associated with each block). Pulse Pair estimates are then made on consecutive data blocks starting with the first block number and continuing to the data block with the second block number. The estimates are made on samples of size 16, 32, 64, 128, 256, 512, and 1024. After operating on the variable sample sizes from each data block, the program performs a statistical summary of all the Pulse Pair estimates to yield estimator accuracy versus sample size.

B. Results

The hundreds of pulse pair estimates of spectral mean and width produced, analyzed, and printed out by the simulation system are presented in a manner intended to facilitate comparison with theoretically predicted performance capabilities. The close correlation shown between theory and experiment in the Figures that follow confirms the validity of the theory under the practical conditions of noise background, correlation between pulse pairs, etc. Analytic predictions of pulse pair estimator performance for particular sets of conditions can, therefore, be made with confidence.

The following comparisons between pulse pair experimental data and theory have been calculated and plotted from the results of the simulation tests.

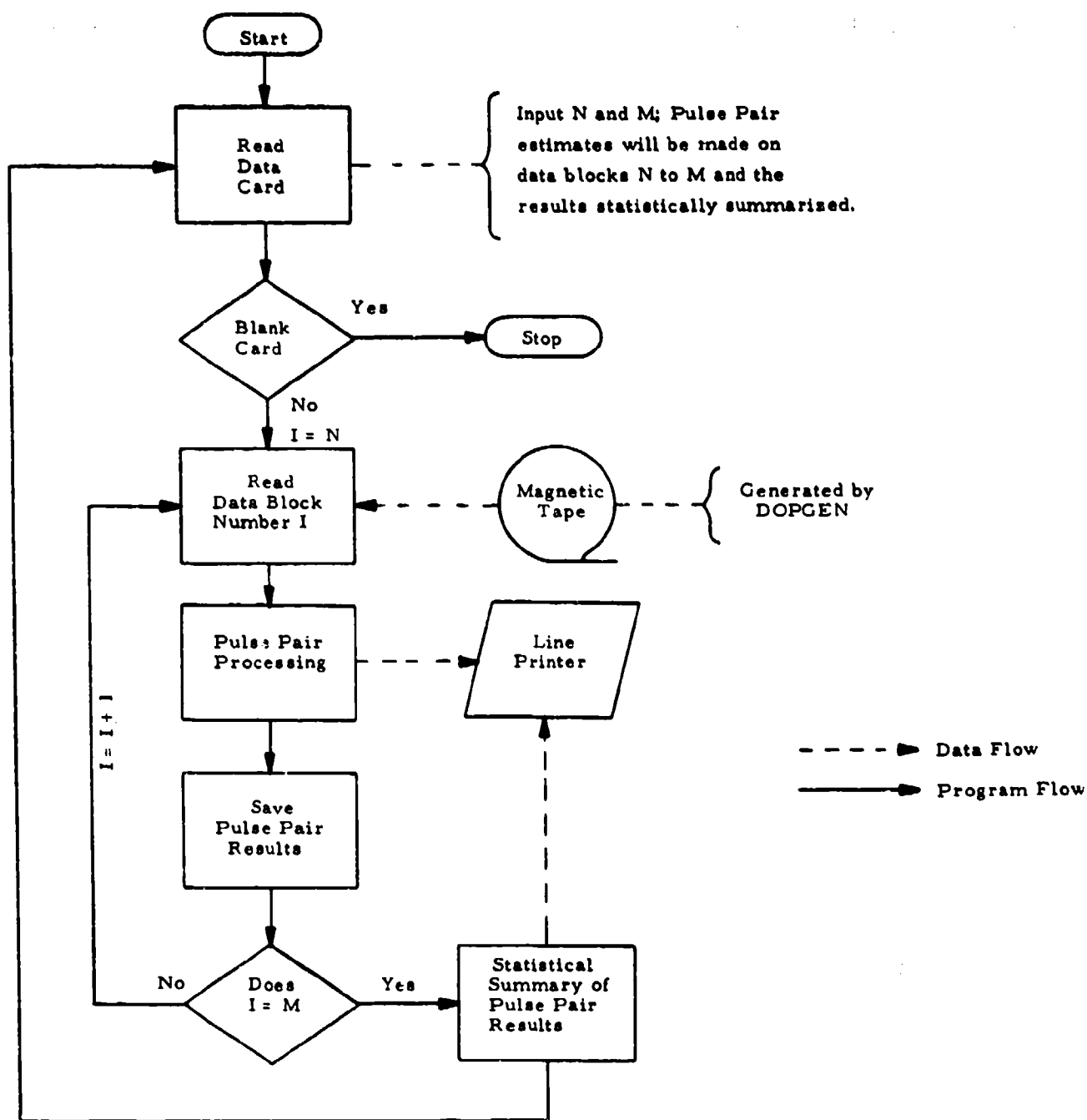


Figure 13. Flowchart of PPSTAT

Figure**Data Compared**

- | | |
|----|---|
| 14 | Accuracy of Mean Frequency Estimates vs Number of Pulse Pairs |
| 15 | Accuracy of Mean Frequency Estimates vs Spectral Width |
| 16 | Accuracy of Spectral Width Estimates vs Number of Pulse Pairs |
| 17 | Accuracy of Spectral Width Estimates vs Actual Spectral Width |
| 18 | Accuracy of Spectral Width Estimates vs Signal-to-Noise Ratio
(for $f_0 = 300$ Hz; $w = 78$ Hz) |
| 19 | Accuracy of Spectral Width Estimates vs Signal-to-Noise Ratio
(for $f_0 = 600$ Hz; $w = 156$ Hz) |
| 20 | Accuracy of Spectral Width Estimates vs Signal-to-Noise Ratio
(for $f_0 = 900$ Hz; $w = 234$ Hz) |

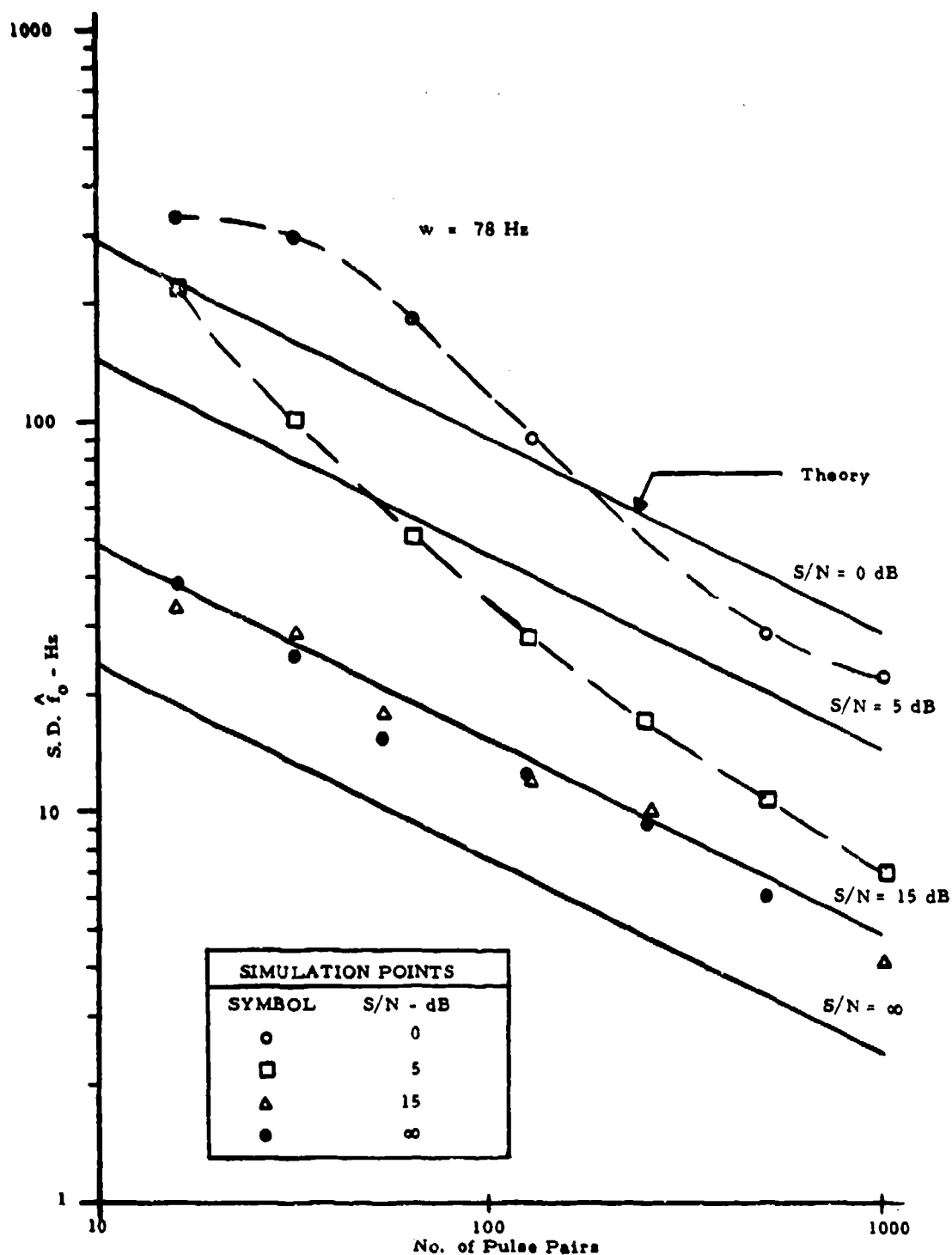


Figure 14. Theoretical Accuracy of Pulse Pair Estimates of Mean Frequency ($\hat{f}_0$) vs Number of Pulse Pairs Compared with Simulation Results

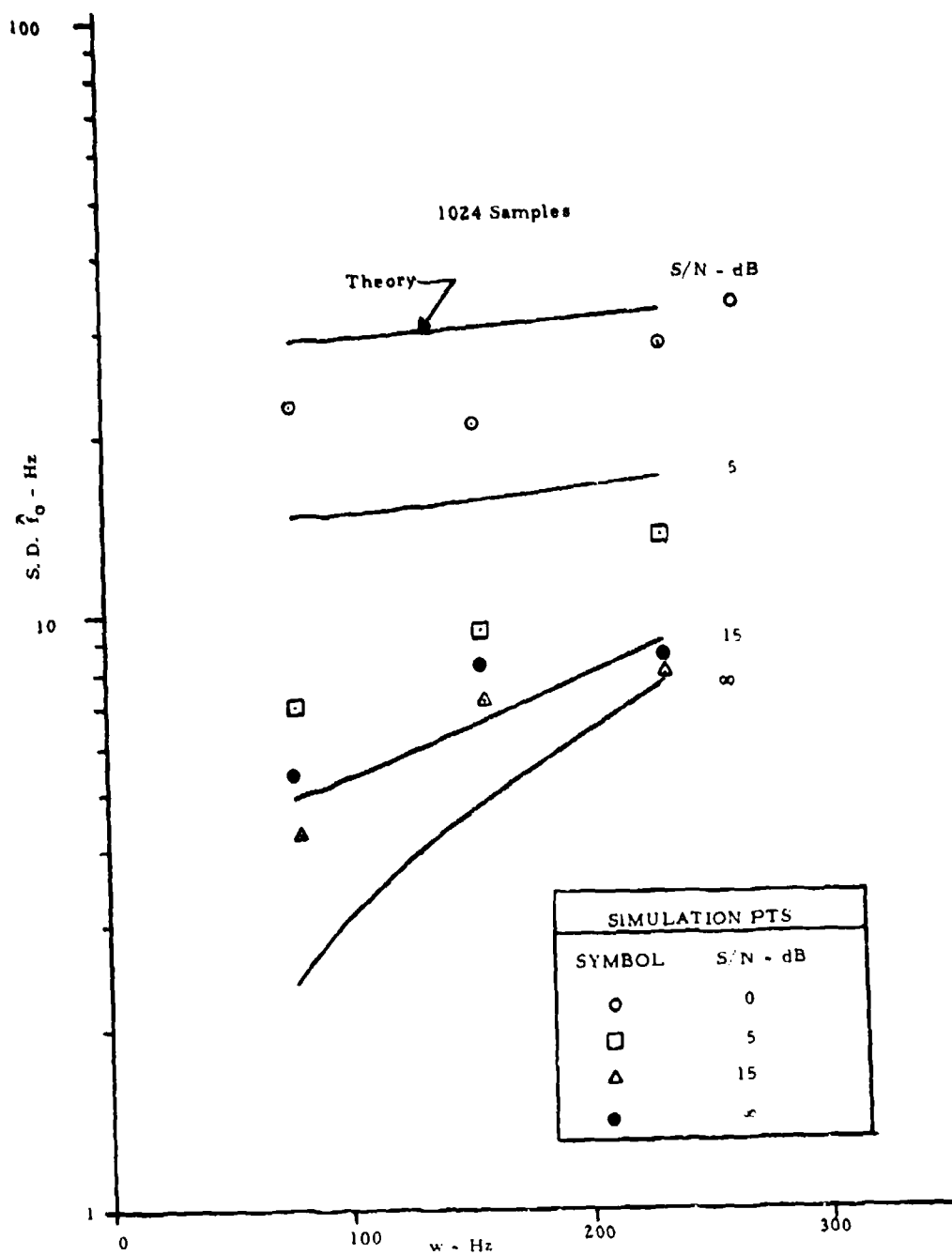


Figure 15. Theoretical Accuracy of Pulse Pair Estimates of Mean Frequency ($\hat{f}_0$) vs Spectral Width (w) with Simulation Results

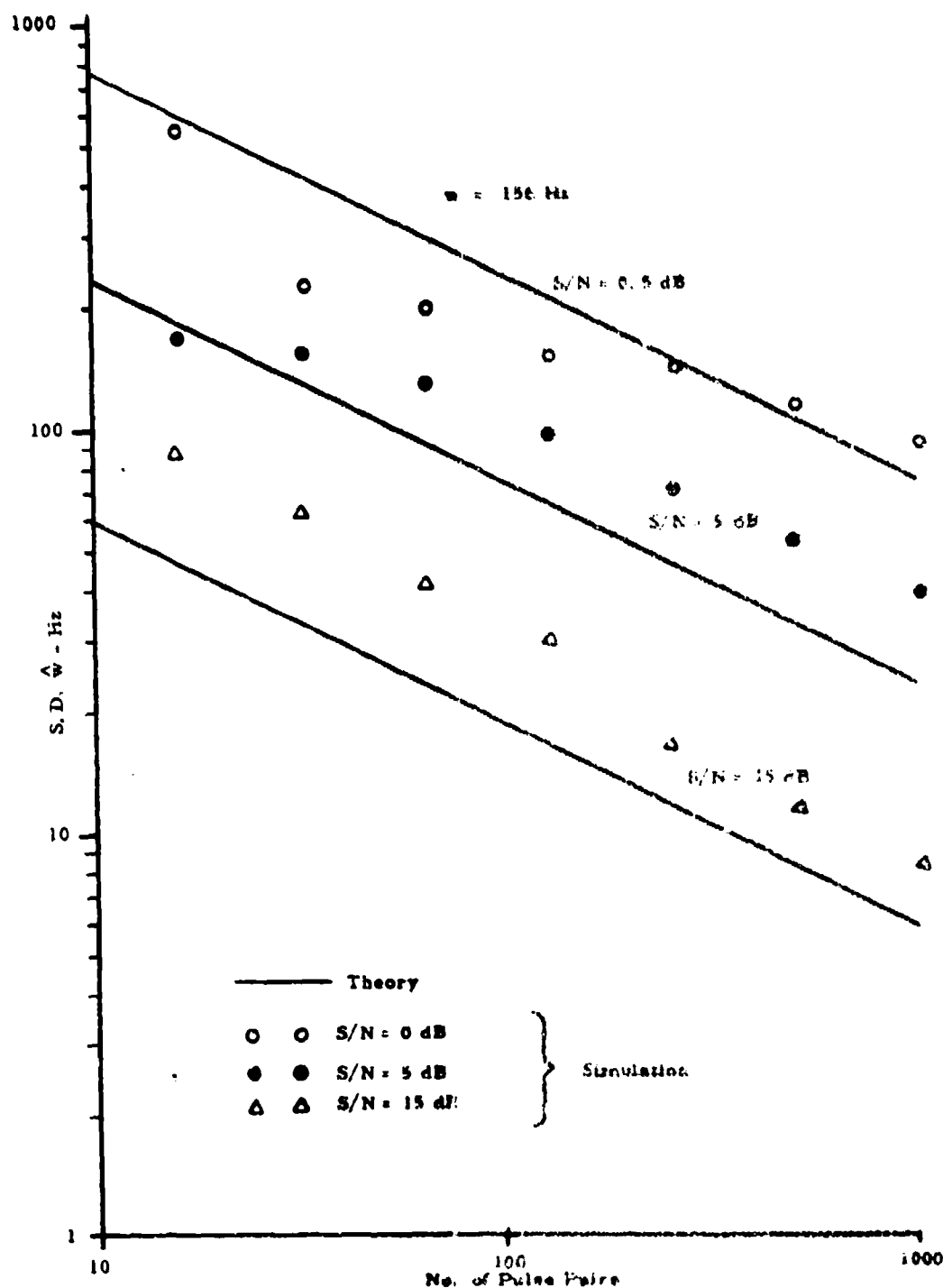


Figure 16. Theoretical Accuracy of Pulse Pair Estimates of Spectral Width (w) vs Number of Samples Compared with Simulation Results

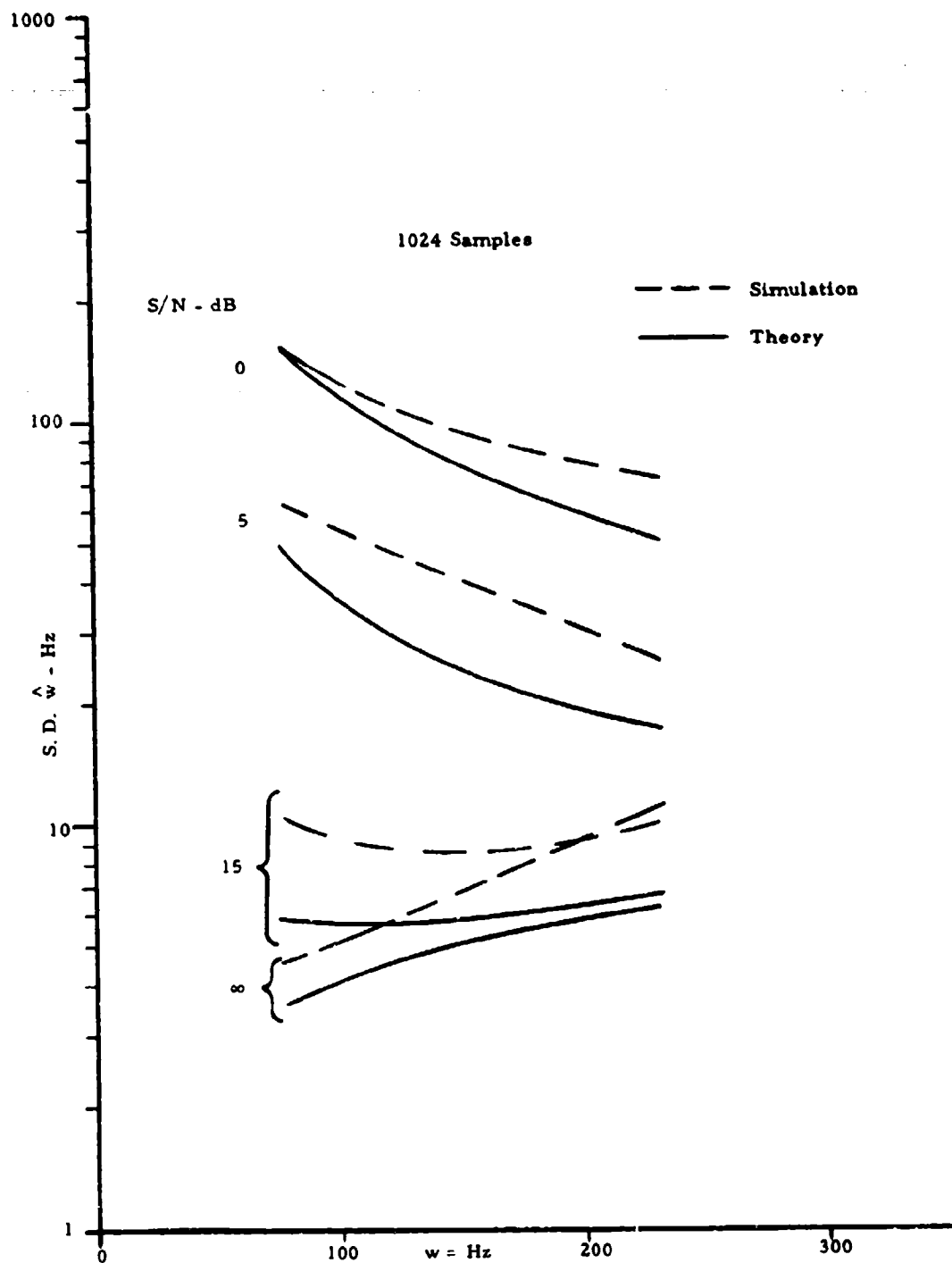


Figure 17. Theoretical Accuracy of Pulse Pair Estimates of Spectral Width ($\hat{w}$) vs Actual Spectral Width (w) Compared with Simulation Results

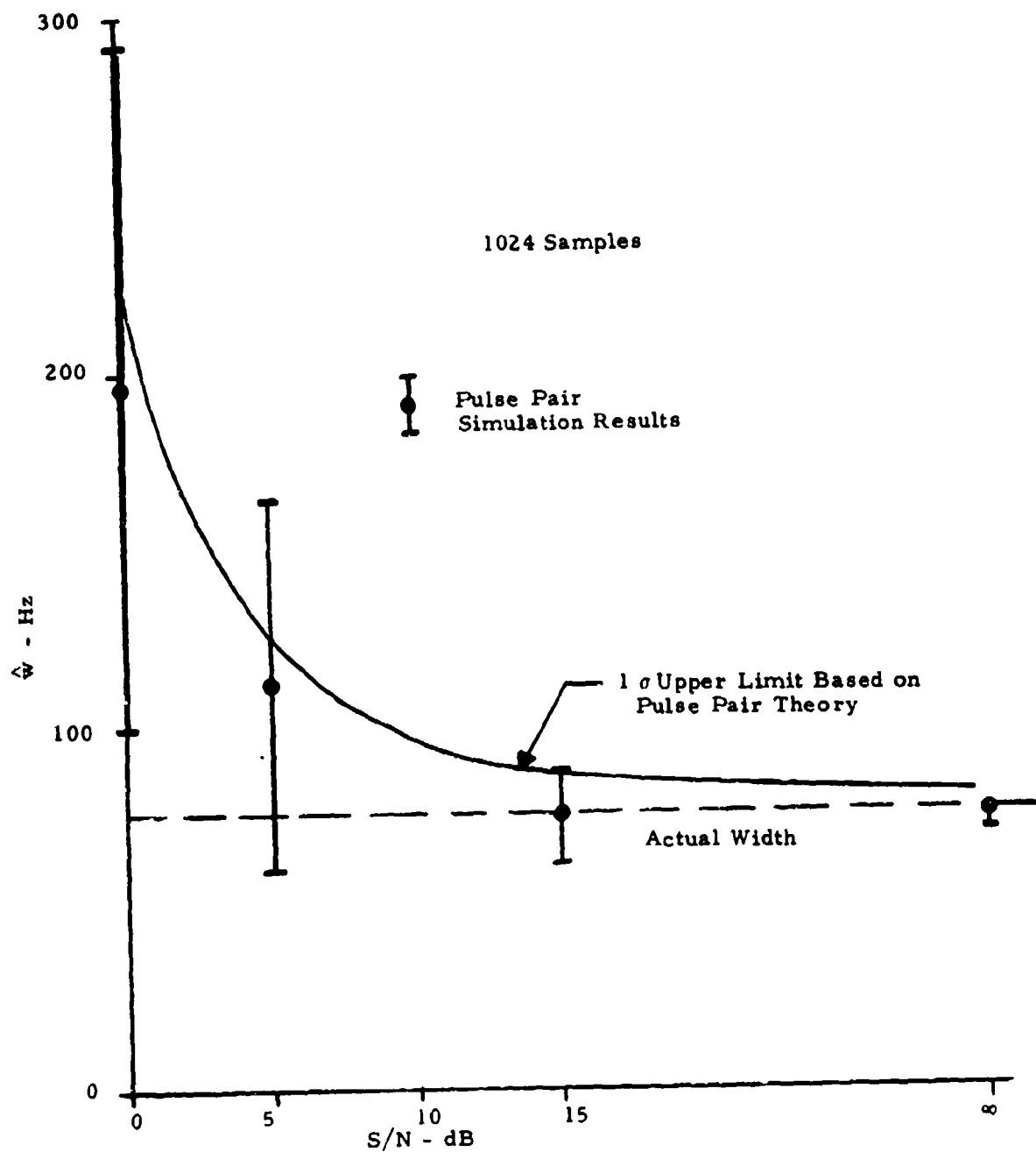


Figure 18. Theoretical Accuracy of Pulse Pair Estimates of Spectral Width ($\hat{w}$) vs S/N Compared with Simulation Results (for $f_o = 300$ Hz; $w = 78$ Hz)

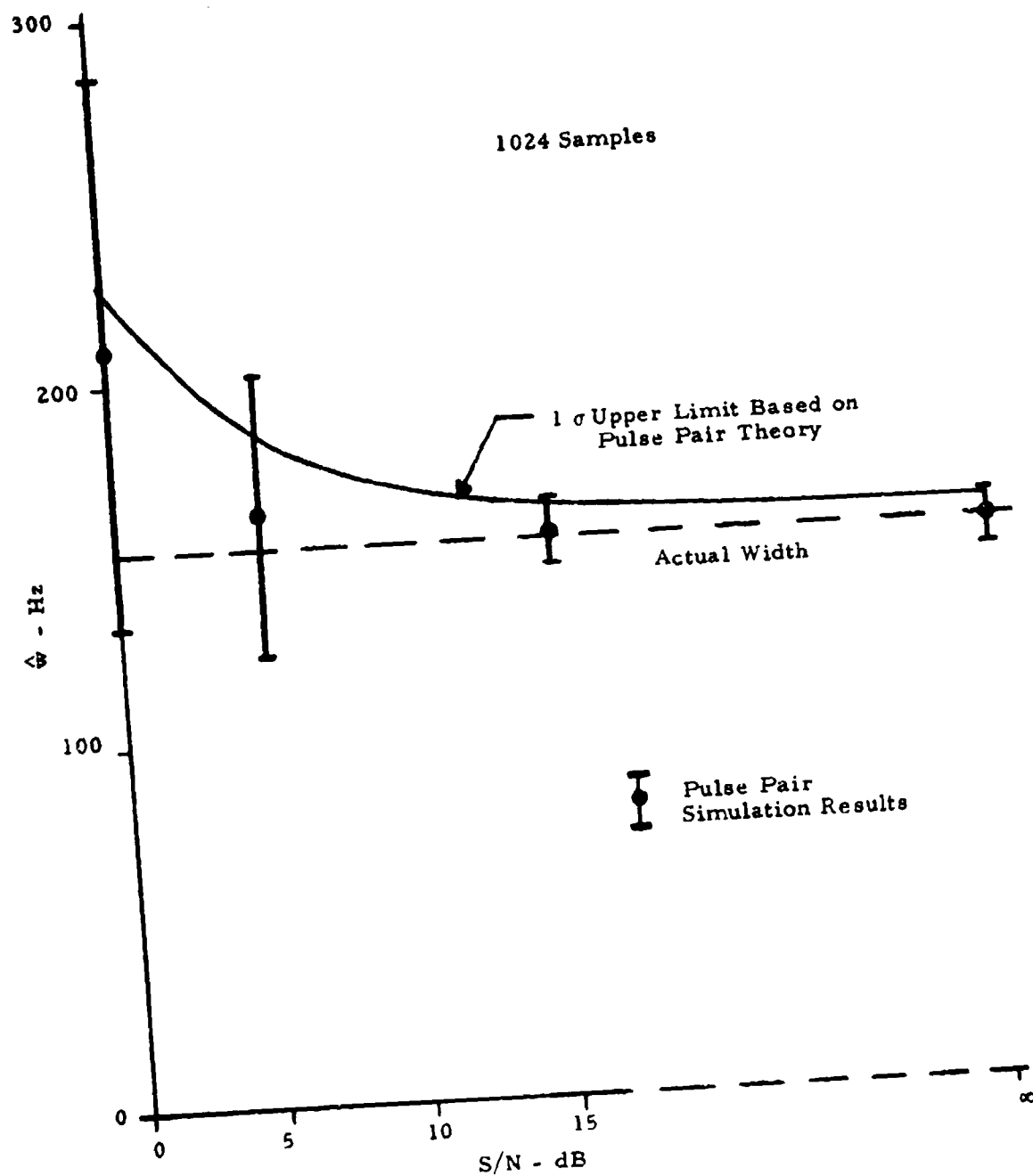


Figure 19. Theoretical Accuracy of Pulse Pair Estimates of Spectral Width (Δw) vs S/N Compared with Simulation Results (for $f_o = 600$ Hz; $w = 156$ Hz)

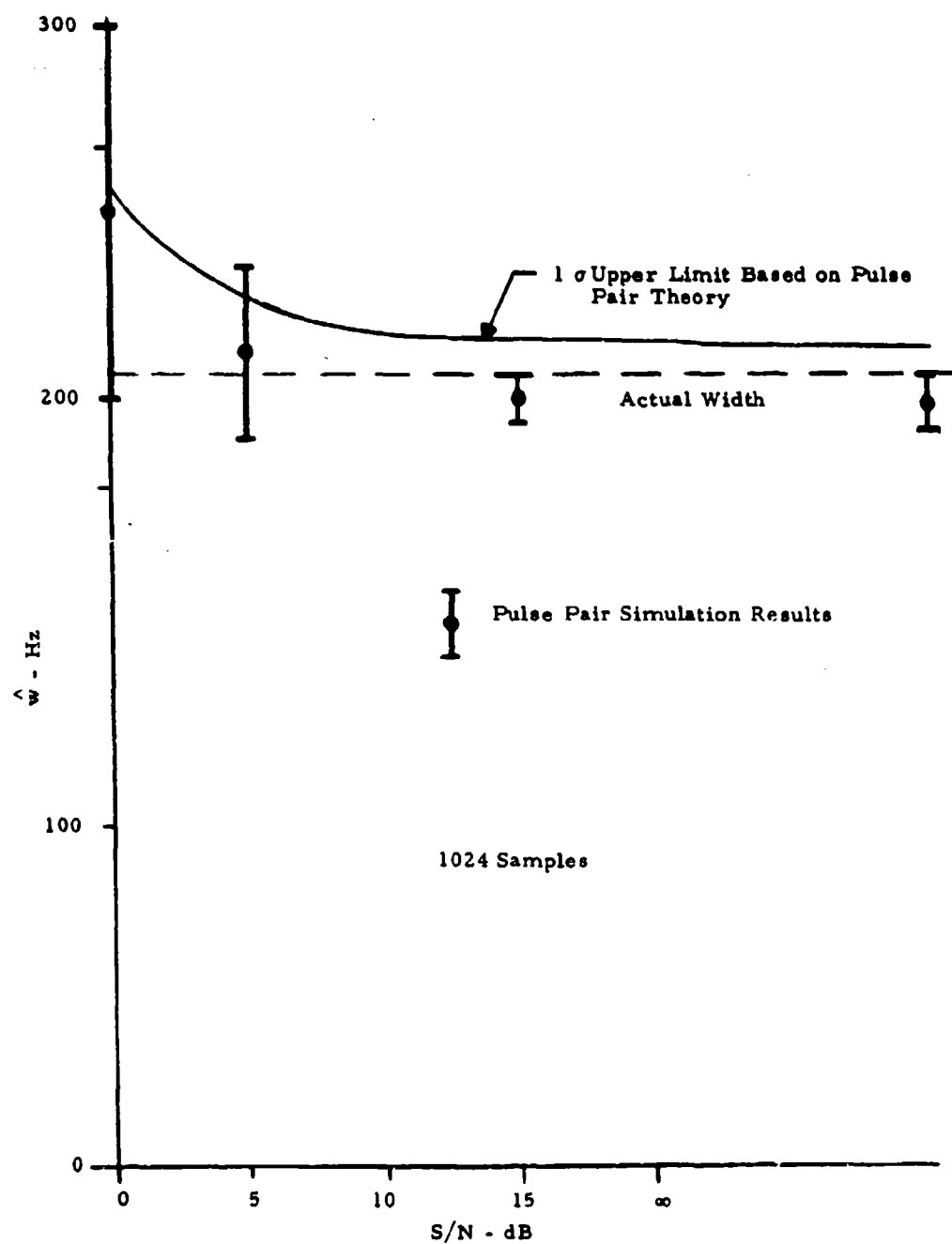


Figure 20. Theoretical Accuracy of Pulse Pair Estimates of Spectral Width ($\hat{w}$) vs S/N Compared with Simulation Results (for $f_0 = 900$ Hz; $w = 234$ Hz)

3.2.4 Performance of Spectral Analysis Estimator

A. Computer Program

In order to provide a standard against which to compare the performance of the pulse pair estimators under varying conditions of signal to noise, the simulated doppler return generated by DOPGEN was also analyzed by the conventional spectral analysis method. As in the case of real Porcupine radar data (Section 3.1.1 above), the spectral parameter estimates were obtained in two steps. First, a spectral analysis is performed on an entire data block to yield its spectrum; then the mean and spectral width parameters are calculated from all spectral components above a given threshold level, for a range of threshold levels. The resulting estimates are printed out as functions of threshold level. This spectral analysis program is designated SPEC. As shown in the flowchart, Figure 21, the Spectral Analysis program searches the tape generated by DOPGEN and reads the data block whose number is specified on the input data card. The 1024 complex samples are then weighted using 60 dB Dolph-Chebyshev weighting coefficients and input into the FFT sub-routine. If desired, the FFT output will be plotted. Next, the mean and standard deviation are calculated based on all FFT output points above a specified threshold level. The threshold level is automatically incremented in steps of 2 dB starting at 2 dB below the maximum FFT output point and continuing to 40 dB below the maximum point. The mean and standard deviation values are printed on the line printer as a function of threshold level. The above procedure is repeated for each input data card until a blank card is encountered. SPEC coded in FORTRAN appears in Appendix C3.

B. Results

It was concluded in the discussion of results with Porcupine Radar input data (Section 3.1 above) that the spectral analysis estimator of $\hat{f}_0$ was definitely satisfactory. Therefore, no further analysis and evaluation of the performance of this estimator was done with the results of the simulation tests.

On the other hand, there was interest in how well the thresholded spectral analysis estimator of spectral width ($\hat{w}$) would show up against the simulated input data.

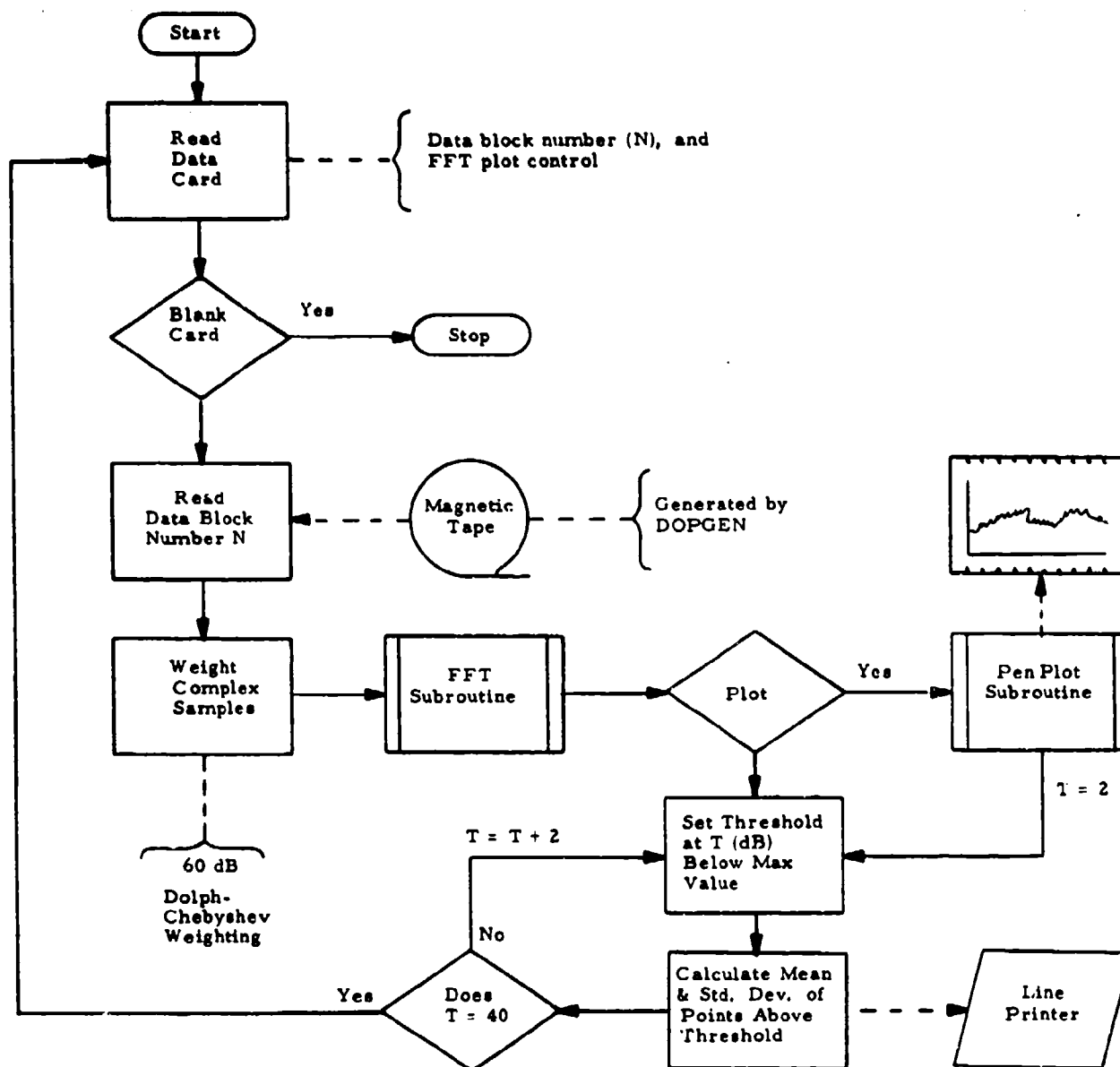


Figure 21. Flowchart of SPEC

Typical results are plotted in Figures 22, 23, and 24. Each figure depicts the width estimates produced by a single block of samples for different combinations of input data parameters, f_0 , w and S/N , as a function of threshold level. As was indicated earlier, no claim is made that the thresholded estimator of Eq.(3-4) is optimum. It is, however, straightforward and readily implementable by computer. To preclude possible bias effects from aliasing, the spectral width was calculated about the true mean, f_0 . Despite these precautions, the width estimates again are shown to be very sensitive to the value of the threshold. Only in the cases of very high S/N do the estimates asymptotically approach the true width.

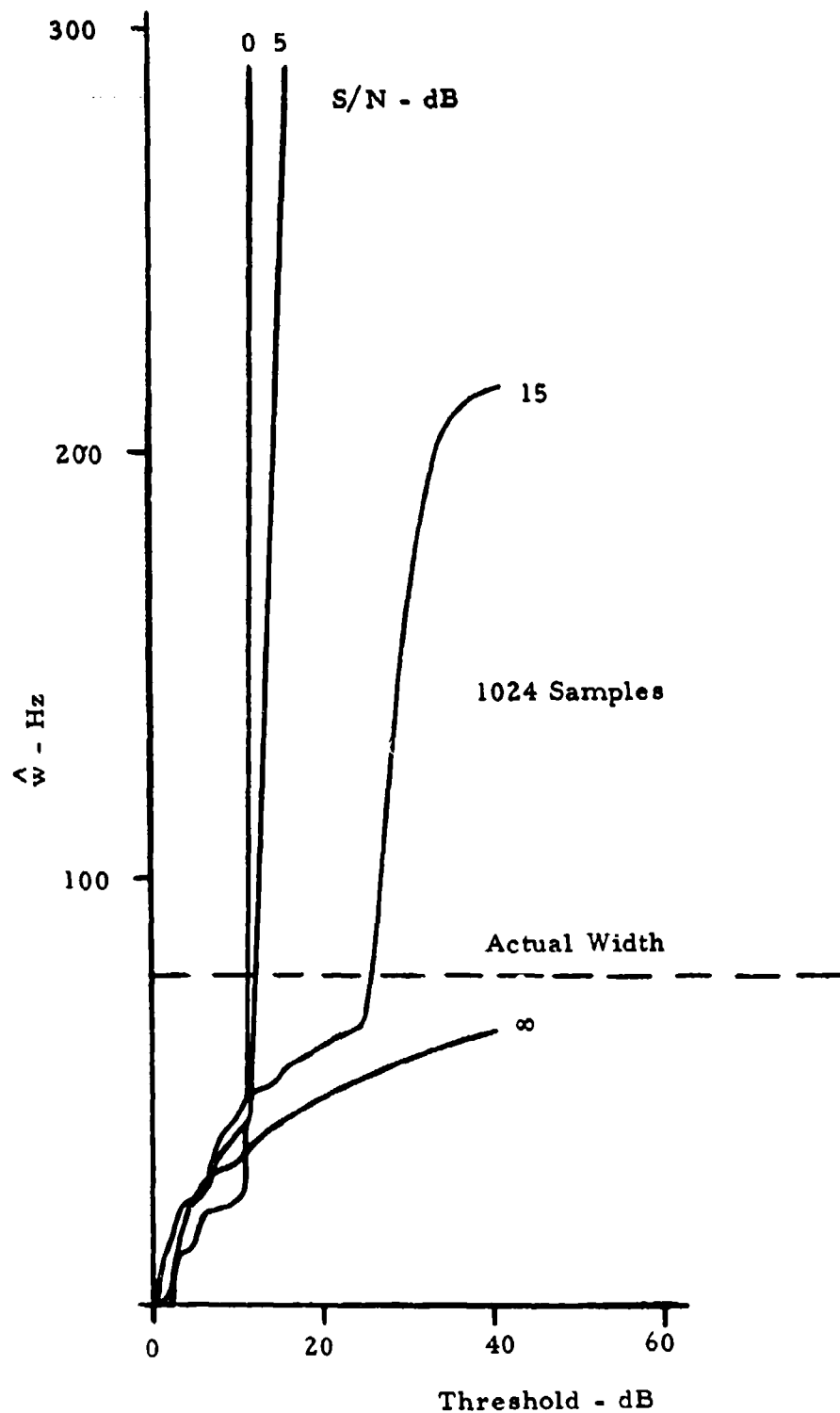


Figure 22. Spectral Analysis Width Estimates ($\hat{w}$) vs Threshold Level for Simulated Input Data ($f_0 = 300$ Hz; $w = 78$ Hz)

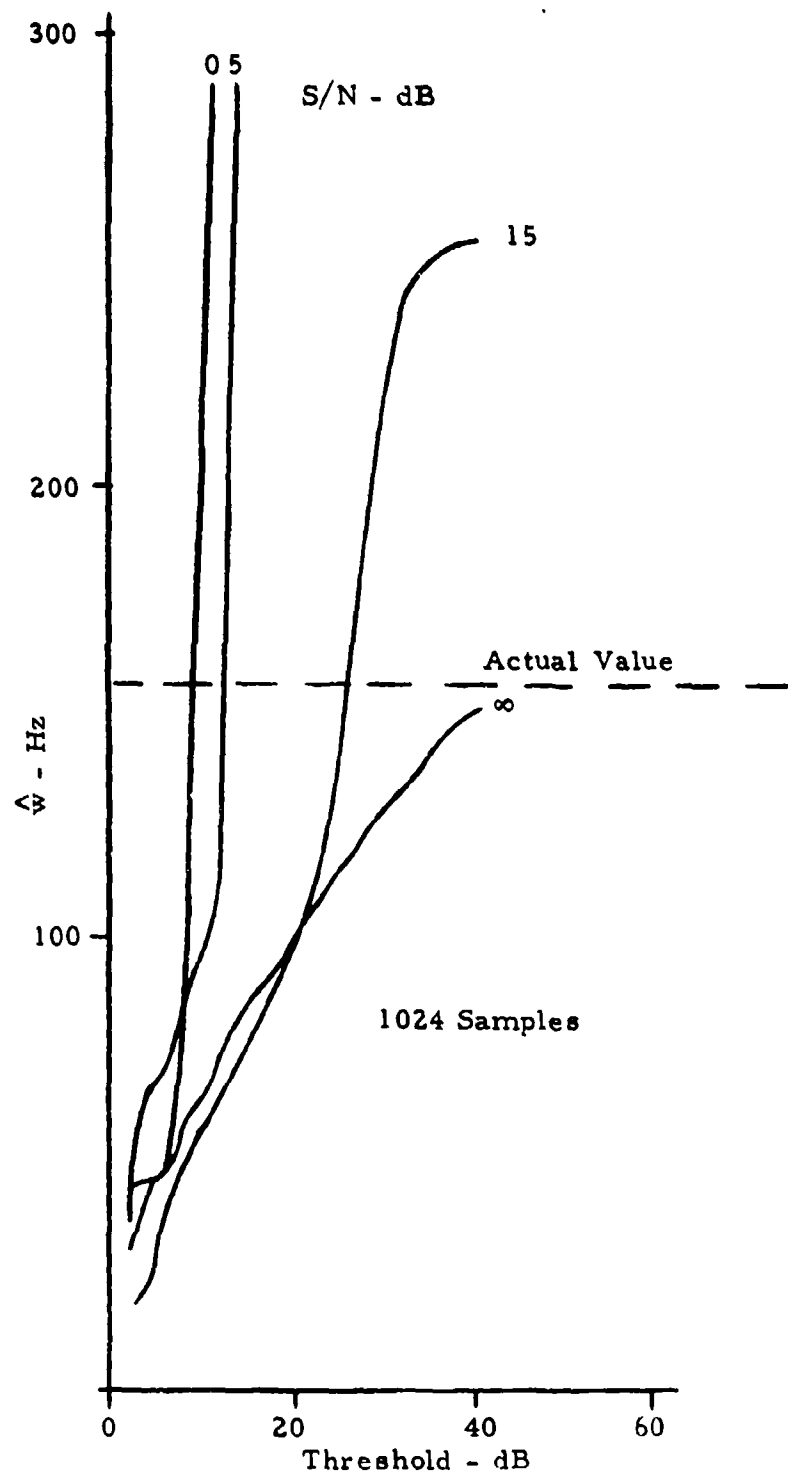


Figure 23. Spectral Analysis Width Estimates ($\hat{w}$) vs Threshold Level for Simulated Input Data ($f_o = 600$ Hz; $w = 156$ Hz)

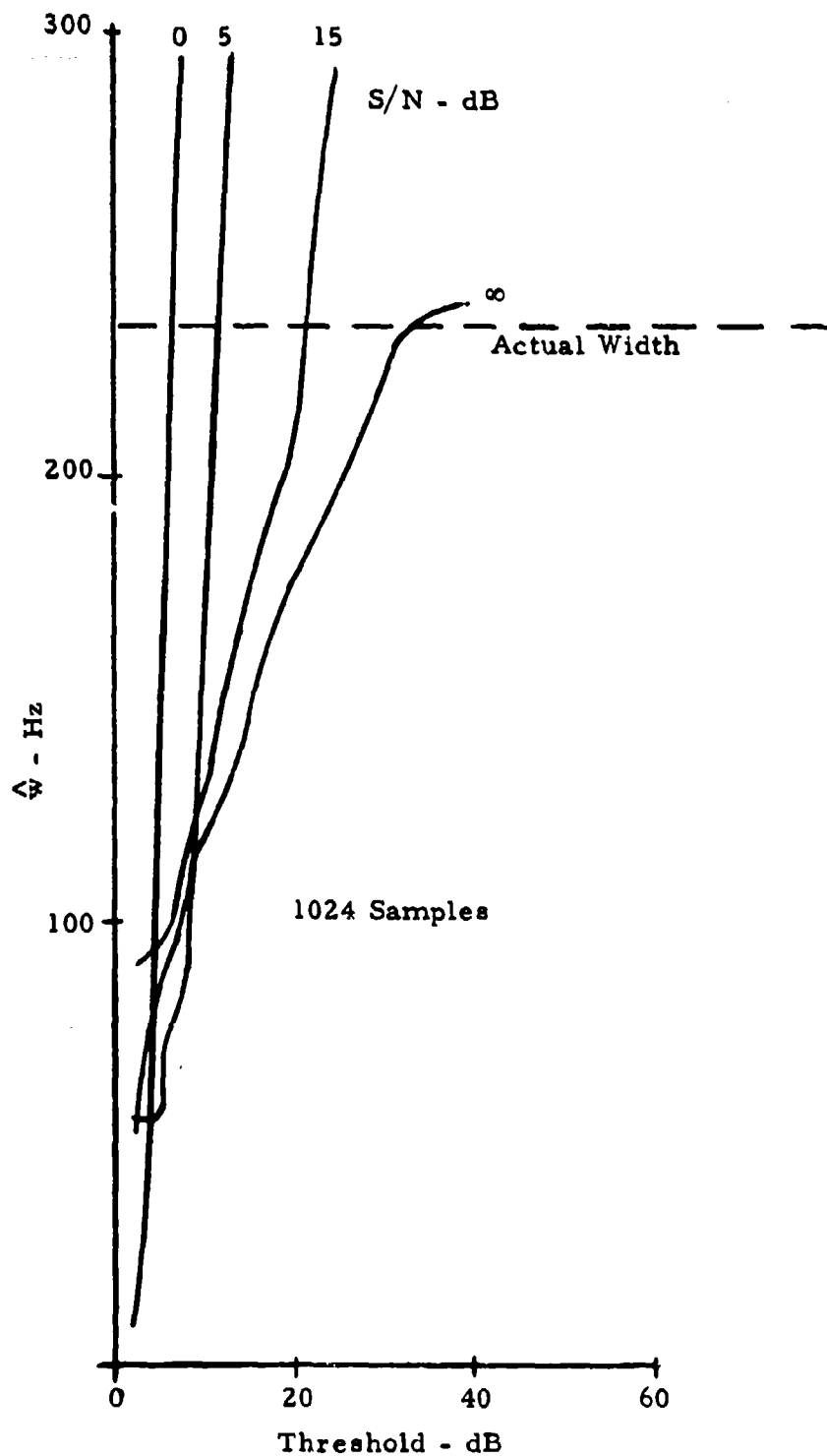


Figure 24. Spectral Analysis Width Estimates ($\hat{w}$) vs. Threshold Level for Simulated Input Data ($f_0 = 900$ Hz; $w = 234$ Hz)

3.2.5 Comparison of the Width Estimators

The accuracy achieved by the pulse pair width estimator in the simulation tests is compared in Table 4 with the accuracy predicted from theory, and the estimates yielded by the spectral analysis estimator. The results are shown for all 12 cases of f_o , w and S/N . (It should be noted that the pulse pair data is presented in the form of the standard deviation of the estimates from the true value, based on a statistical analysis of 20 estimates on as many independent blocks of simulated data for each case shown; whereas the spectrum analysis results are for a single block of samples for each case, as a function of threshold level.)

Table 4 and Figures 18-20 show that the pulse pair width estimates obtained with the 1024 samples and S/N in excess of 5 dB per pulse produces width errors of less than 3 dB for all cases tested. Translated into meteorological terms, for the Porcupine radar, actual widths of .2 m/sec were estimated at .3 m/sec with the equivalent at a 1/3 sec look (at 3300 pulses/sec) at a signal-to-noise ratio of 5 dB.

For a signal which is a Gaussian random variate with spectral density function

$$S(f) = \frac{\sigma_s^2}{\sqrt{2\pi} w} e^{-\frac{f^2}{2w^2}}$$

imbedded in white noise with spectral density coefficient N_o , the ratio of the peak spectral value to the rms noise background level in the frequency domain is given by

$$R_s = \frac{S(0)}{N_o} = \frac{\sigma_s^2}{\sqrt{2\pi} w N_o}$$

If this datum is sampled at the rate $1/T$, the per pulse signal-to-noise ratio is given by

$$\begin{aligned} S/N &= \frac{T}{N_o} \int c(f) df \\ &= \frac{T \sigma_s^2}{N_o} \end{aligned}$$

Table 4. Comparison Between Pulse Pair & Spectrum Analysis; Estimates of Spectral Width

Actual Parameters of Input Data		ESTIMATES									
		Pulse Pair Standard Deviation for 20 Blocks		$\hat{w}$ (Hz) by Spectrum Analysis (for 1 Block)							
		Theory	Simulation	Threshold Level -dB							
				8	12	16	20	24	28	32	
Case	f_o (Hz)	w (Hz)	S/N (dB)	$\hat{w}$ (Hz)	S.D. $\hat{w}$ (Hz)	23	263	634	722	748	752
1	300	78	0	110.	153.	23	263	634	722	748	752
2	300	78	5	46.	62.	37	79	369	559	617	630
3	300	78	15	8.7	10.4	38	51	57	62	66	131
4	300	78	∞	3.05	4.6	33	39	45	49	54	56
5	600	156	0	70.	92.	86	399	608	686	699	702
6	600	156	5	29.	39.	88	133	380	506	545	556
7	600	156	15	6.7	8.6	51	66	82	98	123	239
8	600	156	∞	4.5	7.0	61	75	88	102	113	133
9	900	234	0	51.	71.	422	655	752	767	772	773
10	900	234	5	21.	25.	93	264	464	562	586	592
11	900	234	15	8.5	10.0	118	151	184	216	292	336
12	900	234	∞	7.1	11.3	107	129	155	177	193	233

1024 Samples Per Block

thus

$$D \hat{=} R_i/S/N = \frac{1}{\sqrt{2\pi} WT} .$$

Thus the spectral peak output of the simulated signals exceeds the input signal-to-noise ratio by approximately the values shown in Table 5.

Table 5

Peak Signal-to-Noise Improvement of Spectrum Analyzer

W (Hz)	D in dB
78	12.3
156	9.3
234	6.3

This calculation shows that the spectrum analyzer output reduction to mean and variance may be expected to perform extremely poorly when the threshold is set at approximately $D + S/N$ (in dB) below the peak output. These points correspond approximately to the break points shown on Figures 22 to 24. Assuming that the spectrum analyzer threshold had been set 5 dB above the noise background level (high enough to assure that the noise peaks would not disturb the measurement), Figures 22 to 24 indicate that the measured widths would be as shown in Table 6.

It should again be noted that the data shown on Figures 22 - 24 represents only a single spectral sample and that the thresholds shown are measured with respect to the sample peak value. Table 6 attempts to incorporate this factor by using the breakpoints of the measurement, as the threshold level. These are at approximately the threshold shown in Table 6.

In summary, the spectrum analyzer measurements appear to consistently underestimate the width parameter while the pulse pair technique overestimates it. The simulation indicates that the accuracies available for basically the same data conditions are roughly comparable for the cases tested. Figures 22 - 24 indicate a marked sensitivity to threshold settings which is not directly evident in the pulse pair measurements. It is important to note that the pulse pair measurement of width uses an estimate of the noise background level also and thus has a

Table 6

Comparison of Pulse Pair and
Spectrum Analyzer Derived Width Estimates

(Spectrum Analyzer Threshold Set 5 dB Above Average Background Noise)

Actual Width	S/N Threshold Spectrum Analysis Estimate			Pulse Pair Estimate
w	dB			$\hat{w}$
78	0	7.3	22	180
	5	12.3	44	112
	15	22.3	64	75
156	0	4.3	47	209
	5	9.3	92	163
	15	10.3	95	157
234	0	1.3	37	283
	5	6.3	76	240
	15	16.3	204	226

corresponding parameter uncertainty in its process. In other words, both techniques require a measurement of the noise alone background to obtain useful final results.

The conclusion to be drawn from this comparison is that the pulse pair technique shows no evident lack of sensitivity as compared with the spectrum analyzer approach in situations characteristic of the meteorological applications.

4.0 HARDWARE REALIZATION

This section describes a real time digital processor design to reduce the pulse pair technique to practice. A brief theoretical review of the signal processing algorithms to be implemented, is followed by discussions and block diagrams of the processor and its key components. Design goals for the proposed pulse pair processor are defined, and its performance compared with CMF, Analog Spectrum Analyzer, and Fast Fourier Transform processors. Finally, the design tradeoffs to be considered during the design phase are discussed.

4.1 General Description of the Pulse-Pair Processor

This processor operates on the output of a radar, and is capable of producing real-time independent estimates of the spectral mean and width of a Doppler radar signal for each range cell viewed by the radar.

The processor accepts the radar IF signal and system trigger as inputs and computes the mean frequency (mean velocity) and spectral width (velocity distribution) in each of 256, 512, 768 or 1024 range cells, selectable by a front panel switch. Each range cell has a width of 0.5, 1 or 2 microseconds selectable by a front panel switch. The processor produces both analog outputs for viewing on real time displays, and digital outputs which may be remoted or recorded for future computer processing. An internal digital automatic gain control system independently measures the signal amplitude and adjusts the gain of the processor for each range cell processed to maintain a 90 dB dynamic range.

Although the pulse pair technique will work with either coherent or non-coherent radars, a coherent receiver output is necessary in order to provide complex samples of the target return to the processor. In addition, to achieve independent spectral estimates from range cell to range cell, the receiver gain must have a wide dynamic range and be range cell-to-cell controllable. Accordingly, the receiver requirements must be included as part of the processor considerations; how much of an existing radar receiver can be used as is and how much must be modified or incorporated as part of the processor depends upon the characteristics of the particular radar whose output is to be processed. The receiver for the Pulse Pair Processor described below is interfaced to match the Porcupine Radar.

4.2 Technical Description of the Pulse Pair Processor

The PPP performs spectral mean and width estimations according to equations (4-1) and (4-3) respectively, which are equivalent to earlier equations (2-9) and (2-10). They are rewritten here in more convenient form for the discussion of hardware.

$$\hat{f}_0 = \frac{\text{PRF}}{2\pi} \phi(\overline{z_{k+1} z_k^*}) \quad (4-1)$$

$$\hat{w} = \frac{\text{PRF}}{\pi\sqrt{2}} \sqrt{1 - \frac{|\overline{z_{k+1} z_k^*}|}{|\overline{z_k}|^2 - N_0^2}} \quad (4-2)$$

where the z 's are complex samples of signal plus noise, and N_0^2 is the receiver noise power in the absence of signal. z^* denotes the complex conjugate of z . Equation (4-1) estimates mean frequency by determining the phase differences in pairs of complex samples (z_{k+1} , z_k), averaging these differences to find the mean phase difference, and scaling this mean phase difference by the PRF to form a mean frequency estimate. The phase differences averaged are weighted according to sample amplitude so that phase differences from strong returns count more than differences from weak returns. The results in Section 3 showed that as the number of samples processed increases, the accuracy of $\hat{f}_0$ improves asymptotically.

Equation (4-2) estimates spectral width $\hat{w}$ in terms of signal coherence, including a correction for noise, N_0 .

$$\alpha = \frac{|\overline{z_{k+1} z_k^*}|}{|\overline{z_k}|^2 - N_0^2} \quad (4-3)$$

which may be measured. The parameter α is well-defined for all sampled signals including those with multimodal spectra. It may be seen from equation (4-3) that in the absence of noise $\alpha = 1$ for sinusoidal signals, and that α does not depend on mean signal frequency, which may exceed the complex sampling frequency (PRF). As is usual in sampled data systems, the signal bandwidth must not exceed the PRF.

Figure 25 shows the signal flow of the PPP algorithm applied to the sampled Doppler signal from one range cell. A delay and multiplier are used to form the products $z_{k+1} z_k^*$, which are then averaged. The sample energies $|z_k|^2$ are also averaged, and function generators are used to compute $\hat{f}_0$ and $\hat{w}$.

There are a number of alternate ways of deriving the noise correction term ($\bar{N}_0^2$) required for the calculation of the width estimate. One such method is depicted in Figure 25. A measurement of receiver noise (n_0^2) (which may be made, for example, at the output of the receiver at a range in excess of that in which any signal return may be expected) is corrected for the receiver gain applicable to the particular range cell, and introduced as the noise correction ($\bar{N}_0^2$) in the width estimate ($\hat{w}$). The correction factor is obtained by calibrating the receiver. The mean power output ($\hat{p}$), which is made up of averaged measurements of signals plus noise, is delivered for use as the range cell's control signal to set the receiver AGC at its range.

Detailed design of the noise correction scheme has been deferred pending a hardware tradeoff study of alternate schemes, to be performed as part of a future hardware fabrication effort. The description of the system design which follows does not include the noise correction feature.

4.3 System Design

The PPP system, shown in Figure 26, comprises a fast AGC IF amplifier, a two-point correlator, three (or four) digital integrators, a timing generator and several function generators. The PPP accepts 30 MHz IF input and produces digital and analog $\hat{f}_0$, $\hat{w}$, range and integrated video outputs. The major subsystems of the PPP are described below.

4.3.1 IF Amplifier Phase Detector

Achievement of the processor 90 dB dynamic range requires independent gain control in each range cell processed, which implies that the AGC voltage must be a time varying signal with a bandwidth comparable to the IF bandwidth. Such a system requires an IF amplifier of unusual design and closely integrated IF amplifier and AGC loop designs. For this reason, the IF amplifier and phase detectors are included in the PPP.

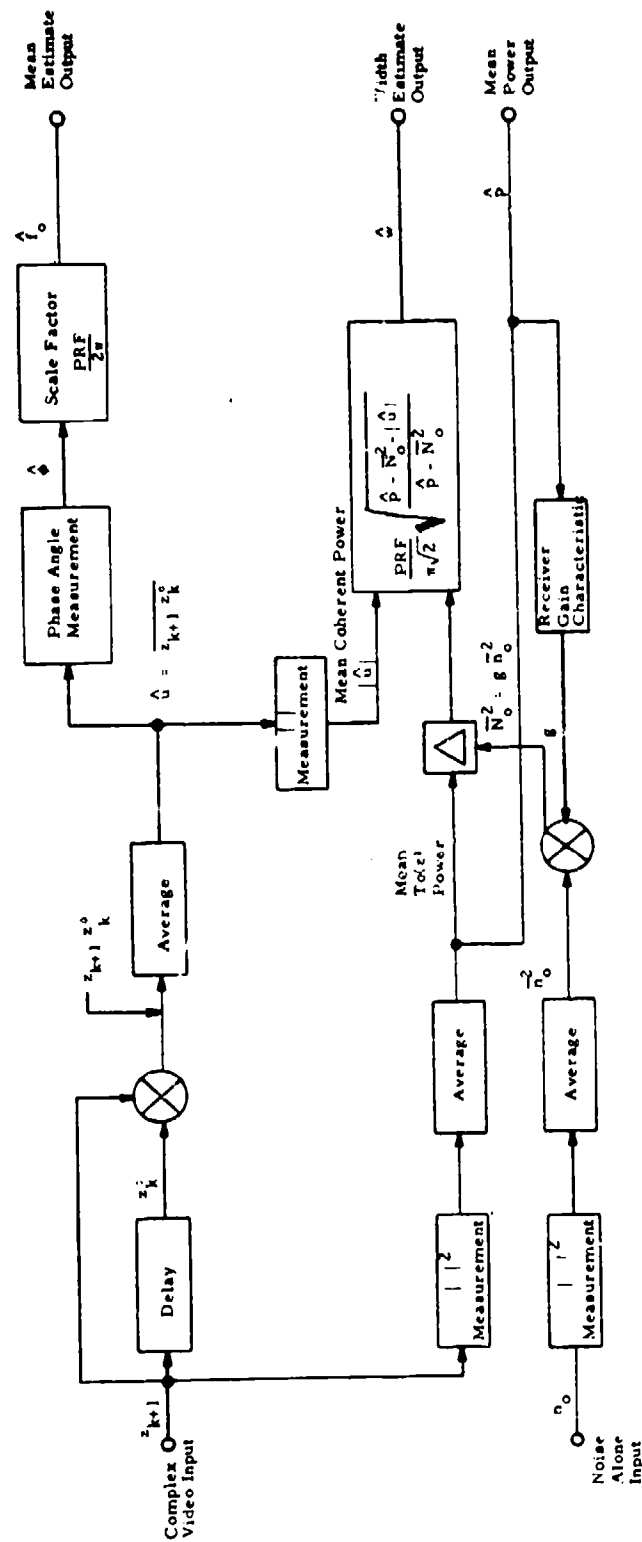
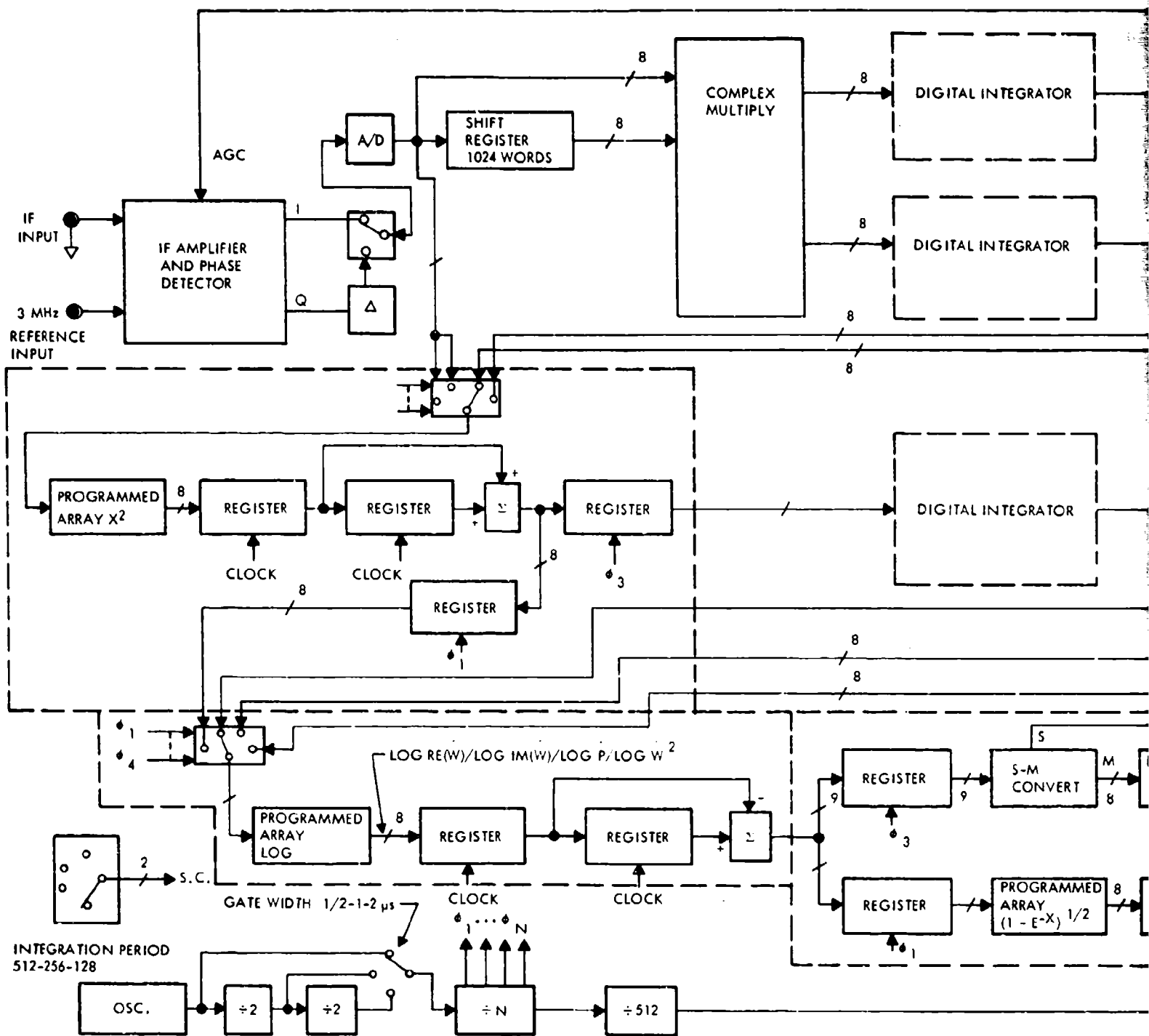


Figure 25. Pulse Pair Processor Signal Flow Diagram



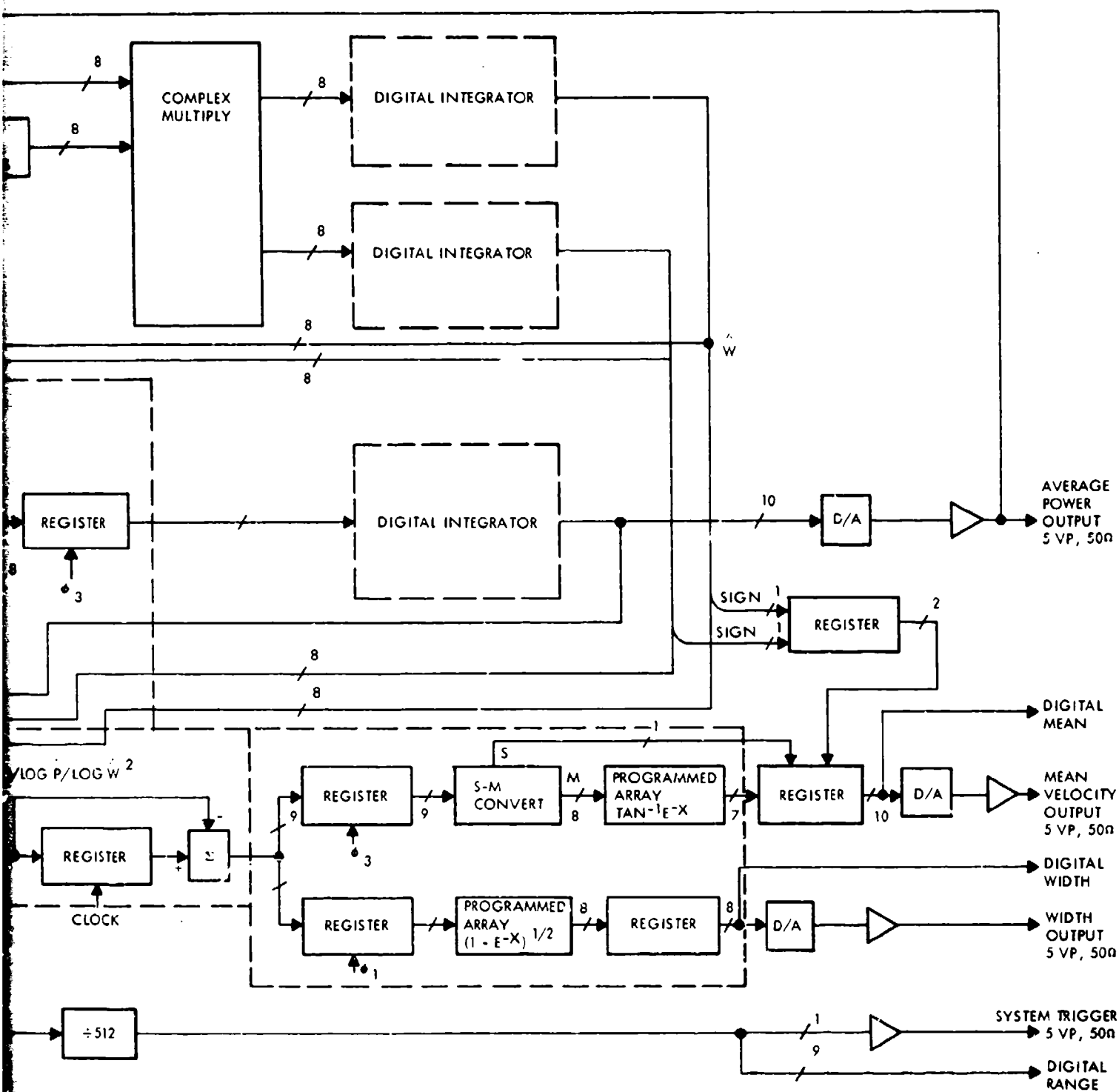


Figure 26. PPP Block Diagram

An IF amplifier-phase detector, Figure 27, was recently designed for the PORCUPINE radar which is acceptable for the PPP with slight modifications. The amplifier achieved approximately 60 dB AGC control range, with AGC applied to two out of a total of three identical gain stages. AGC response time was less than 200 ns. By adding AGC to the third gain stage, this amplifier is expected to achieve a 90 dB AGC control range without degradation of response time.

4.3.2 Digitizer

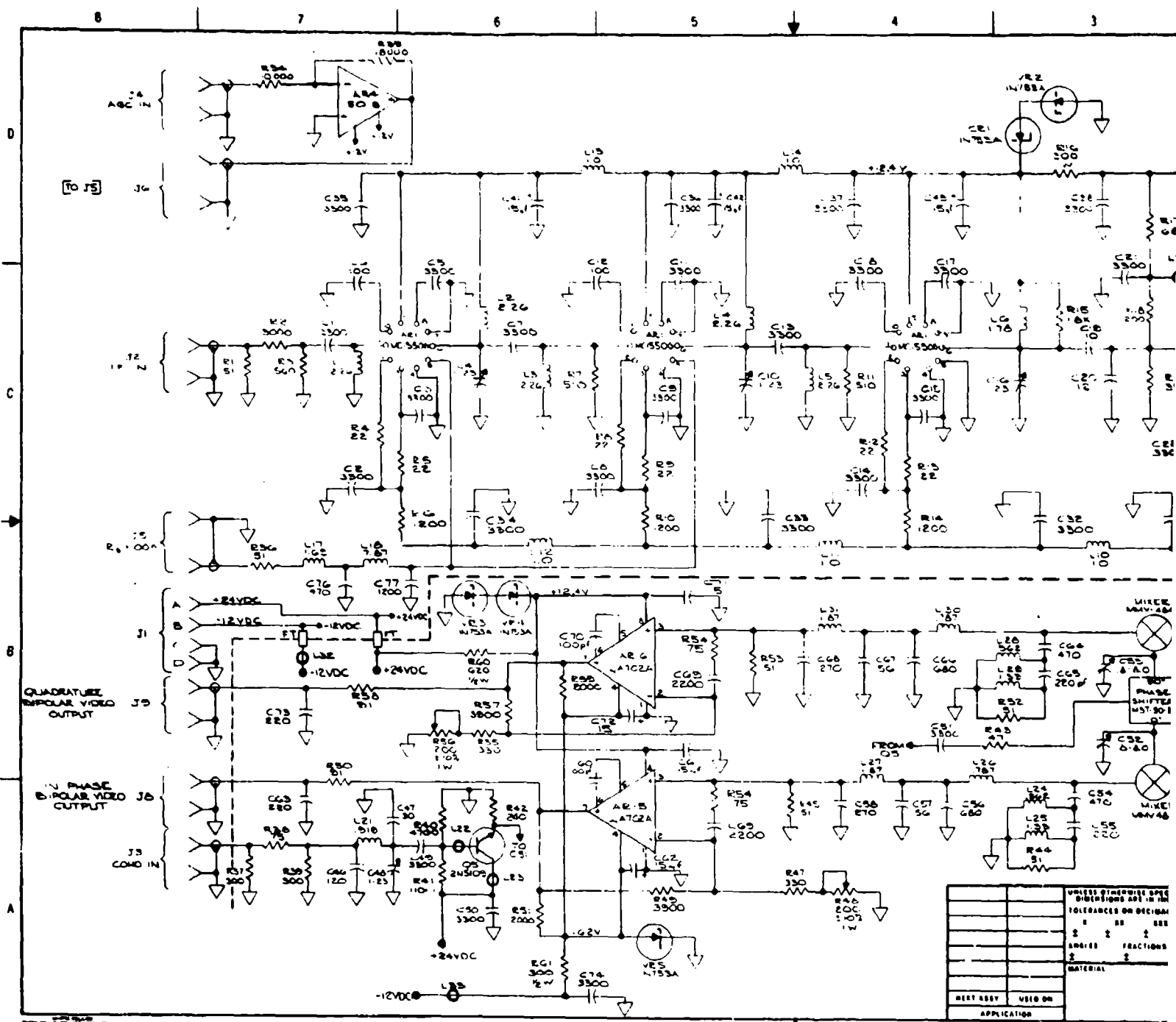
The digitizer accepts in-phase and quadrature bipolar video and produces digitized estimates of these inputs. Eight bit quantization appears more than adequate to compute $\hat{f}_0$ to the accuracy of the algorithm. A single A/D converter, such as the ILC Data Devices VADC-B, may be switched to quantize both input channels. This is preferable to the use of two simultaneous A/D converters because the single converter is less expensive and because problems of balancing the input channels are simplified by the use of a single A/D.

The suggested A/D converter can take an analog sample every 150 ns and is compatible with a range resolution of 500 ns. A 150 ns delay line must be used in one input channel to compensate for the A/D sampling delay. This A/D includes a sample and hold circuit, operates from standard +5V and $\pm 15V$ power supplies and is contained on a single 8.8" x 4.5" circuit card.

4.3.3 Correlator

The correlator accepts complex video samples z_k from the digitizer and generates products of the form $z_{k+1} z_k^*$ where the sample z_{k+1} is delayed by one range sweep time from the sample z_k . This delay is accomplished using MOS dynamic shift registers organized as 16 bits x 512 words. Recently available 1024 bit registers, such as the Intel 1404A, allow this delay to be fabricated quite compactly.

The complex multiplier which forms the $z_{k+1} z_k^*$ products will be similar to the multiplier developed for a proprietary Raytheon FFT pulse compression system. This multiplier, shown in Figure 28, employs programmed arrays and adders to perform a 24 x 24 bit complex multiply and add every 0.5 μs . Approximately 280 integrated circuits (DIP'S) are required, hence, the multiplier could be fabricated on four 72-DIP Augat panels. The PPP requires only a 16 x 16 bit complex multiplier; one circuit card could probably be saved by redesigning the FFT multiplier for the less demanding PPP requirement.



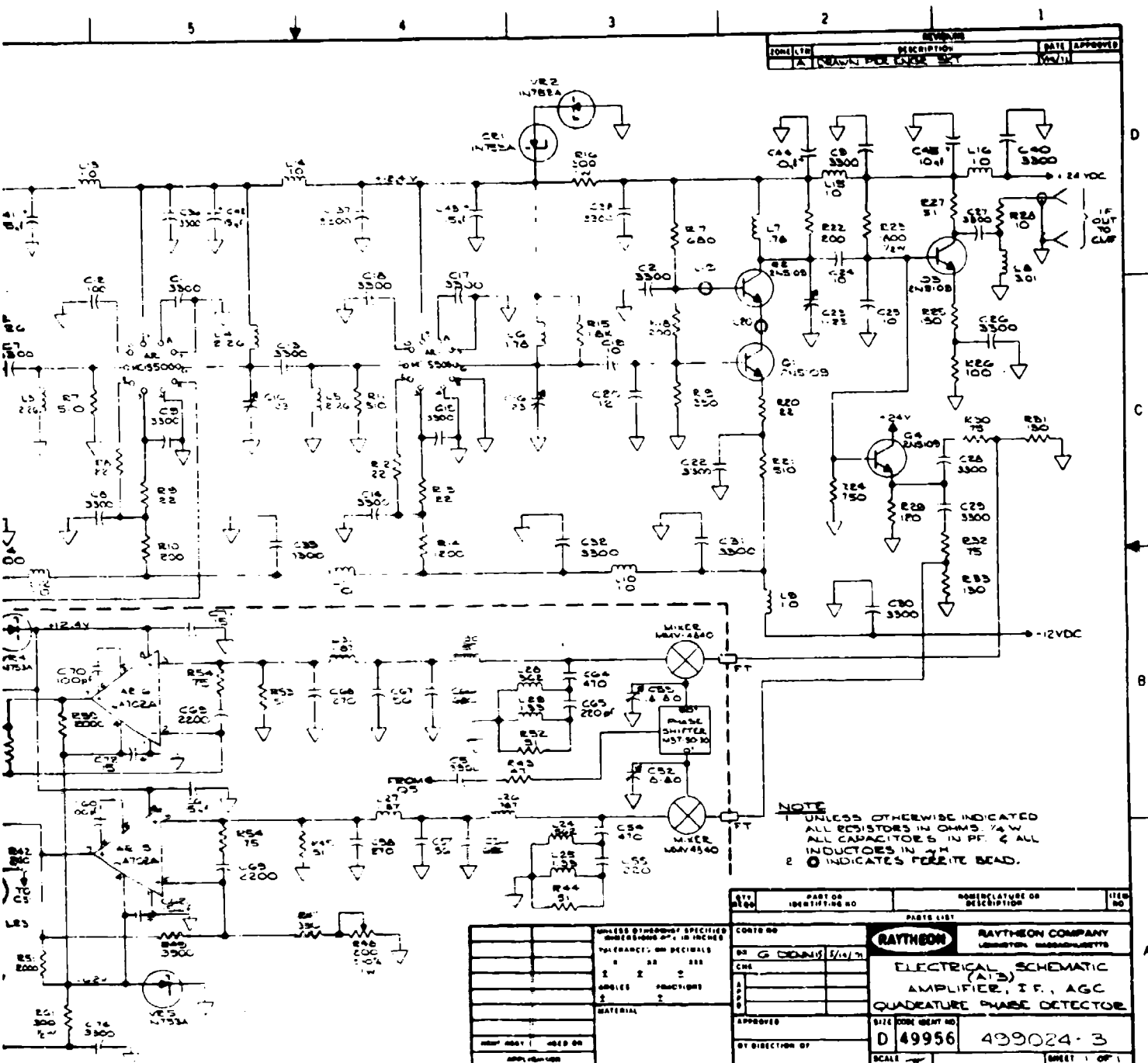
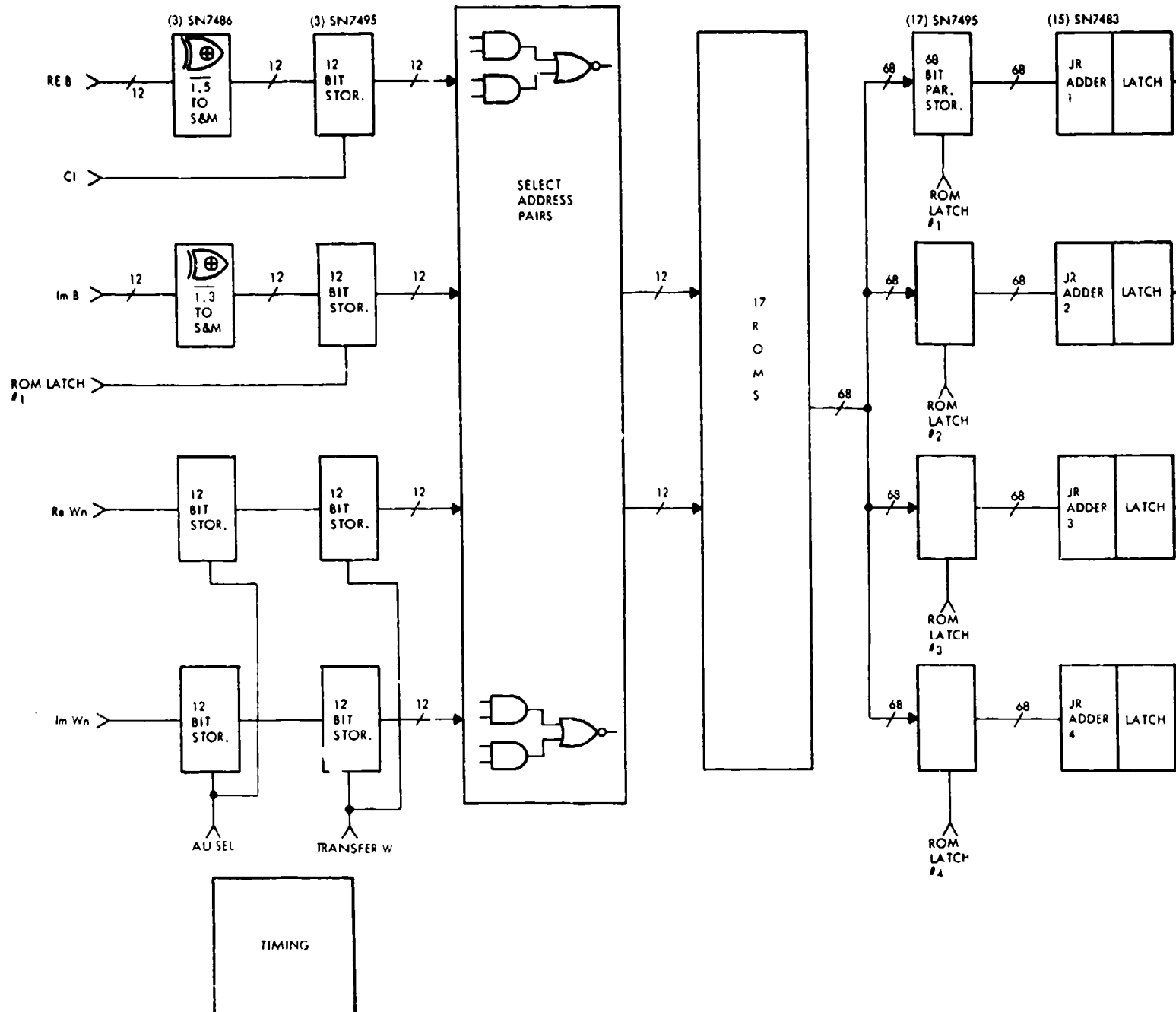


Figure 27. Porcupine IF Amplifier



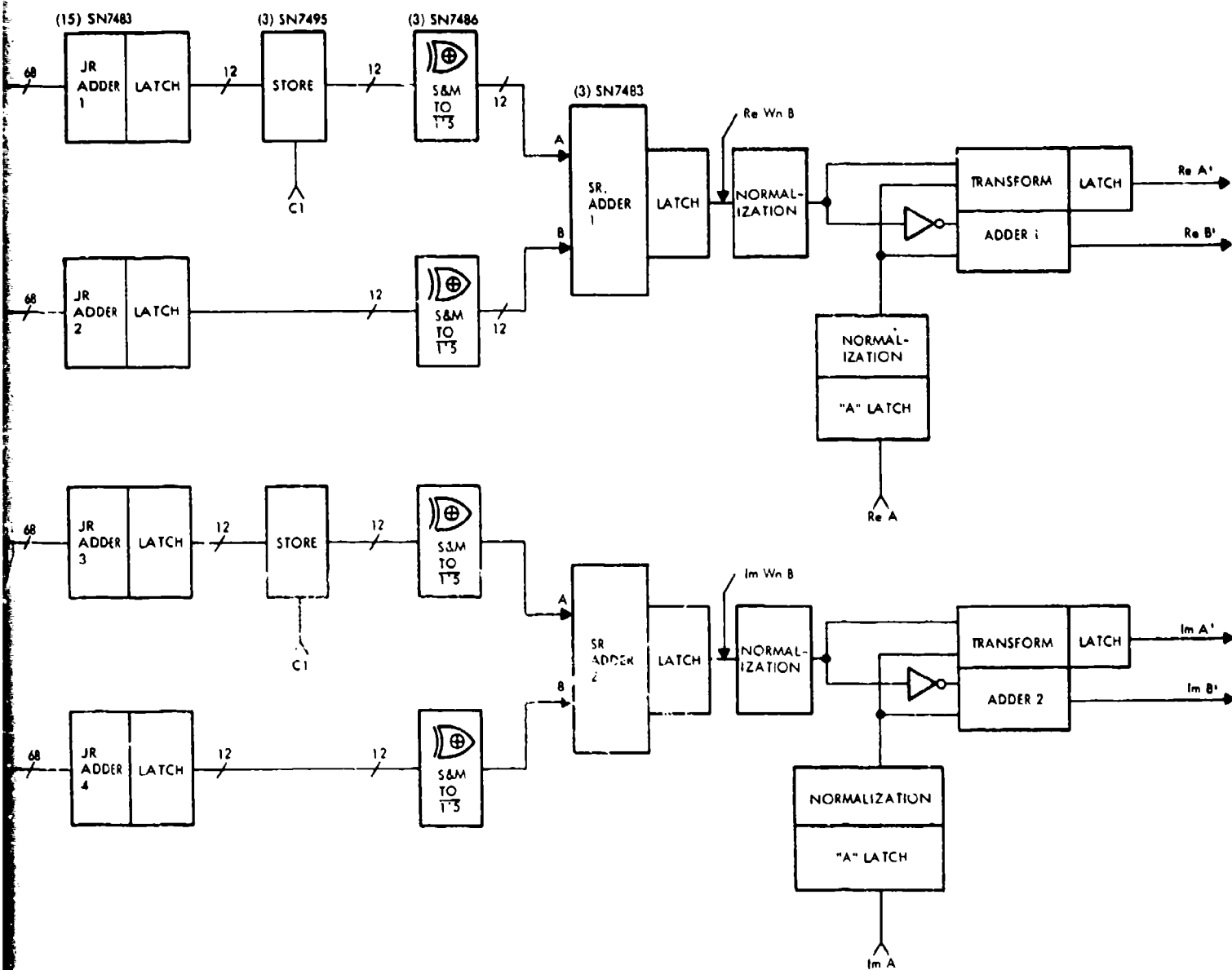


Figure 28. FFT Complex Multiplier

4.3.4 Digital Integrators

A minimum of three digital integrators are required, two to compute the real and imaginary parts of $\hat{u}$ and one to compute average power $\hat{p}$. These three integrators will be similar to an integrator recently designed for AFCRL, to minimize design costs, but the number of sweeps integrated will only be variable up to 1024 by factors of 2.

An integrated power measurement is required to operate the AGC. This output may be derived either from the power channel ($\hat{p}$) of the PPP or through a separate integrator. The integrator which produces AGC output, whether separate or power channel, must be of the exponential rather than square moving window type. Theory shows that the gain and phase of the square window integrator,

$$|g_v(\omega)| = \frac{\sqrt{2}}{\omega T} \sqrt{1 - \cos \omega T} \quad (4-4)$$

$$\phi(\omega) = \tan^{-1} \frac{\cos \omega T - 1}{\sin \omega T} \quad (4-5)$$

makes it unsuitable for use in stable feedback loops with loop gain greater than 1. A design study should determine the performance differences in PPP's employing only three exponential window integrators, and PPP's employing three square window integrators to compute $\hat{u}$ and $\hat{p}$ and one exponential integrator for AGC.

4.3.5 Function Generators

Three function generators are required for the PPP: a squared magnitude generator, a log generator and an output function generator. As shown in Figure 27, all function generators will consist of programmed arrays combined with registers, gating circuits and adders appropriate to the function computed.

Programmed arrays will contain the functions x^2 , $\ln x$, $\tan^{-1} e^{-x}$, and $(1 - e^{-x})^{1/2}$. Each programmed array will consist of two integrated circuits such as the Monolithic Memories MM6300. The MM6300 is organized as an 8-bit in, 4-bit out array which may be programmed to contain any desired bit pattern; two such circuits form an 8-bit in, 8-bit out digital function generator. Approximately 50 ns after an 8-bit input word is supplied to the two programmed arrays, an 8-bit output word is available, allowing each pair of arrays to compute, and an output register to store, up to 4 outputs during an 0.5 μ s range cell.

These arrays may be ordered preprogrammed by the manufacturer or may be programmed by the user. Some vendors (e.g., Motorola) sell programmers for their arrays. Raytheon is currently constructing a programmer for the Monolithic Memories array in connection with a high speed pulse compression FFT development; this programmer is expected to be available for use in PPP fabrication.

4.4 Performance

The detailed performance of the PPP will depend upon a number of design decisions, such as the AGC loop characteristics and the exact form of the function generators; operational considerations such as the signal characteristics and the number of pulses integrated; and the basic accuracy of the pulse pair algorithm. Analysis of these factors should be made in a design study but is beyond the scope of this hardware feasibility report.

By assuming a set of design goals for PPP hardware, subject to change in the design study, a few general comparisons may be made between the PPP and other related signal processing hardware. The design goals in Table 7 are therefore assumed for the purposes of comparison.

4.4.1 Comparison with the CMF

The Porcupine Coherent Memory Filter is a spectrum analyzer, and does not produce estimates of spectral mean and width directly. Modification of the CMF to accommodate different pulse repetition rates and pulse widths is expensive and time consuming. By contrast, the PPP directly produces both spectral mean and width estimates in digital form in real time and operates over a variety of PRR's and pulse widths. Its quantitative estimates can also be recorded or remoted for further processing, plotting or display.

Recently, the CMF has been used to generate displays from which an observer can estimate mean frequency. The PPP can generate such displays with much finer time and frequency resolution by addition of a low cost voltage-to-delay converter. In addition, the PPP mean and standard deviation outputs can be directly displayed on a PPI scope to provide Plan Mean Indicator or Plan Width Indicator displays. These displays could be contoured if desired by using existing contour generator equipment (such as digital integrators).

The following compares the resolution capabilities of the CMF and PPP

	CMF	PPP	Unit
Range Resolution	1860	150	Meters (m)
Range Elements	192	1024	-
Velocity Resolution	$0.5 \lambda f_o / 50$	$0.5 \lambda f_o / 1024$	m/s
Dynamic Range	30	90	dB
Input Data Rate	1.75	32	M Bits/s
Size	1.0	0.3	19" rack x 6'

Table 7. Proposed Design Goals for Pulse Pair Processor

DESIGN GOAL

Signal Input:	30 MHz IF, -80 dBm to +10 dBm Input Impedance 50 ohms
Reference Input:	30 MHz, 1 v p-p into 100 ohms
Trigger Input:	2 v p - 90 v p pulse 0.1 μ s - 100 μ s pulse width
Analog Outputs:	Mean frequency, standard deviation mean power and bipolar video Output Impedance - 50 ohms
Digital Outputs:	Mean frequency, standard deviation mean power and cell count buffered TTL level outputs 10, 8, and 10 bits respectively
Trigger Output:	Nominal 15 v p into 50 ohms - coincident with 1st range cell output
Range Cells Processed:	256, 512, 768 or 1024 nominal selectable by front panel switch
PRF:	300 pps - 5000 pps
Range Cell Width:	0.5 μ s, 1 μ s, or 2 μ s - selectable by front panel switch
Samples Processed:	16, 32, 64, 128, 256, 512, or 1024, selectable by front panel switch
Internal Quantization:	Digital signals shall be represented by a minimum of 8 bits at maximum input level
Dynamic Range:	Normal and quadrature bipolar video shall be represented internally by a minimum of 7 bits over an IF input range of 100 μ v rms - 1 v rms with 1024 samples processed
AGC:	The processor shall include a dynamic AGC system to provide independent gain control for each range cell processed.
Input Power:	115 v AC $\pm$ 10%, 60 Hz $\pm$ 2 Hz
Mechanical Configuration:	Mountable in a 19" rack cabinet
Ambient Temperature:	0°C - 40°C
Construction:	According to best commercial practice

4.4.2 Comparison with Analog Spectrum Analyzers

Presently available analog spectrum analyzers such as the Federal Scientific "Ubiquitous" may, like the CMF, be used to generate a display from which an observer can estimate spectral means and widths indirectly. Unlike the CMF and PPP, however, such analyzers are not presently capable of processing the entire output of a weather radar in real time, and cannot generate any real-time displays. The following compares the resolution of the PPP and Ubiquitous analyzer.

	Ubiquitous	PPP	Unit
Range Resolution	1200 ¹	150	Meters (m)
Range Elements	10	1024	-
Velocity Resolution	$0.5 \lambda f_o / 500$	$0.5 \lambda f_o / 1024$	m/s
Dynamic Range	40	90	dB
Input Data Rate	0.60 ²	32	M Bits/s
Size	0.3	0.3	19" rack x 6'

1 Analyzer, operating without external sample and hold system

2 Maximum continuous

4.4.3 Fast Fourier Transform Processing

FFT processing to compute spectral mean and width may be performed either by computer or by special purpose hardware. Computer processing requires that the weather radar signal be recorded and analyzed in non-real time, but subject to this restriction can provide essentially any desired frequency resolution. The PPP is compared below to a special purpose hardware FFT recently developed by Raytheon for pulse compression.

	FFT	PPP	Unit
Range Resolution	150	150	Meters (m)
Range Elements	2(1024)	1024	-
Velocity Resolution	$0.5 \lambda f_o / 2048$	$0.5 \lambda f_o / 1024$	m/s
Dynamic Range	60	90	dB
Input Data Rate	(32)	32	M Bits/s
Size	2(4)	0.3	19" rack x 6'

Numbers in () indicate anticipated values for a 1024 gate machine.

4.5 Conclusions

A hardware PPP capable of computing spectral mean and width for each of 1024 $1/2$ - μ s radar range gates appears completely feasible at this time. The processor can be fabricated primarily from available, designed components and is expected to occupy no more than 24" of 19" rack space. No other signal processing device now available can approach the dynamic range and data throughput capabilities of the PPP, except possibly the FFT, which would cost approximately 10 times more than a PPP of comparable resolution.

Table 7 gives a summary of the design goals for the proposed signal processor.

In the design phase of the PPP, detailed design tradeoffs of the following items must be accomplished.

1. The dynamic AGC loop
2. The digital function generation
3. The radar calibration system
4. The noise monitoring system

5.0 CONCLUSIONS AND RECOMMENDATIONS

The preceding sections of this report show analytically and verify by data from controlled simulation experiments that the pulse pair technique produces estimates of spectral mean and width that have predictable accuracy over a wide range of non-ideal operating conditions. Performance for unimodal spectra is insensitive to spectral shape and the degree of correlation between successive pulse pairs; the theory stands up under conditions of poor signal-to-noise ratio.

From a processor standpoint, it is shown that a flexible digital pulse pair processor capable of handling a large number of range cells simultaneously is feasible, and that it could be arranged to easily match a variety of radars and radar operating conditions, including PRFs, pulse widths and data rates. A theoretical basis for optimizing the waveform parameters of a radar for pulse pair processing is also presented.

A hardware implementation of a flexible digital pulse pair processor suitable for exploring the utility of the technique in real time processing of weather radar data is proposed. The design goals are defined, and its performance is compared with other contemporary signal processors.

The experimental work together with the simulation studies demonstrate that the pulse pair technique can yield good quality estimates of mean and variance using radar data at signal-to-noise ratios obtainable with existent Doppler weather radars. The hardware tradeoff studies as well as the comparative performance characteristics demonstrate that the pulse pair technique is not only feasible but efficient.

Based on these conclusions, the construction of a flexible experimental model of such a real-time pulse pair processor is recommended for the purpose of collecting a large data base on observable conditions. Experience gained while conducting research with the experimental model will provide the basis for defining the specifications for simpler models intended for operational use. A study should also be made to determine the best means to display, plot, and/or record the resulting data for further analysis, reduction and evaluation.

The fabrication phase for the experimental model should be preceded by a design evaluation phase, during which the final parameters for the equipment can be established from the results of tradeoff studies.

It is further recommended that some additional analysis be performed to provide theoretical guidelines for comparing pulse pair analysis to spectra data processors. In particular, although the theory of pulse pair estimation has now been established, no theoretical foundation exists at present for predicting the performance of width estimates derived from spectral analyzer data; such a theoretical foundation should be developed. Analyses are also required to properly relate and interface the pulse pair processor with particular radars, ancillary devices, and operational usages as a research tool.

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APPENDIX A

ANALYSIS OF ACCURACY OF SPECTRAL PARAMETER ESTIMATES BY PULSE PAIR ESTIMATION

H. L. Groginsky

ABSTRACT

This appendix gives a mathematical analysis of the performance of a pulse pair estimator of spectral mean and variance. It assumes that complex data (I & Q) is collected at a uniform rate from a distributed target whose underlying spectral density function is $G(f)$ and that $G(f)$ is observed only after it is mixed with white additive noise.

I. Introduction

If the signal spectrum is written as

$$G(f) = \frac{S}{W} R\left(\frac{f-f_0}{W}\right) \quad (1)$$

with $R(f)$ real and positive for all f ,

$$R(-f) = R(f), \quad (2)$$

$$\int R(f) df = 1, \quad (3)$$

$$\int f R(f) df = 0, \quad (4)$$

$$\int f^2 R(f) df = 1, \quad (5)$$

S = signal power,

and if the data r_k is given by

$$r_k = s_k + n_k \quad (6)$$

then the pulse pair estimators of f_0 and W are given by

$$\hat{f}_0 = \frac{1}{2\pi T} \tan^{-1} \left(\frac{\text{Im} \{X\}}{\text{Re} \{X\}} \right) \quad (7)$$

and

$$\hat{w}^2 = \left(\frac{1}{2\pi T} \right)^2 \left[1 - \frac{|X|}{Y - \sigma_n} \right] \quad (8)$$

where

$$X = \frac{1}{M} \sum_{n=0}^{M-1} r_k r_{k+1}^* \quad (9)$$

$$Y = \frac{1}{M} \sum_{n=0}^{M-1} |r_k|^2 \quad (10)$$

$$\sigma_n = \langle n_k n_k^* \rangle = \langle |n_k|^2 \rangle \quad (11)$$

2. Spectral Mean Estimates

Let

$$g(\tau) = \int_{-\infty}^{\infty} G(f) e^{j2\pi f\tau} df \quad (12)$$

then

$$\langle s_k s_{k+j}^* \rangle = g(jT) \quad (13)$$

$$\langle s_k s_{k+j} \rangle = 0 \quad (14)$$

$$\langle n_k n_{k+j}^* \rangle = N \delta_{kj} \quad (15)$$

$$\langle n_k n_{k+j} \rangle = 0 \quad (16)$$

where

$$s_k = s(kT).$$

and δ_{kj} is the Kronecker delta function.

Note that

$$\begin{aligned}
 g(T) &= \frac{S}{w} \int R\left(\frac{f-f_o}{w}\right) e^{j2\pi f T} df \\
 &= S e^{j2\pi f_o T} \int R\left(\frac{f-f_o}{w}\right) e^{j2\pi \frac{(f-f_o)}{w} w T} df/w \\
 &= S e^{j2\pi f_o T} \rho(wT)
 \end{aligned} \tag{17}$$

and that $\rho(wT)$ is real.

Now

$$\begin{aligned}
 \langle X \rangle &= \frac{1}{M} \sum_{k=0}^{M-1} r_k r_{k+1}^* = g(T) \\
 &= S e^{j2\pi f_o T} \rho(wT)
 \end{aligned} \tag{18}$$

Since S and ρ are real,

$$\arg \langle X \rangle = 2\pi f_o T \tag{19}$$

which shows that the mean estimator is unbiased.

Now consider

$$\tan \hat{\theta} = \frac{\text{Im}\{X\}}{\text{Re}\{X\}} \tag{20}$$

$$\text{with } \hat{\theta} = 2\pi f_o T \tag{21}$$

If $\hat{\theta}$ is to be a useful estimator, $\hat{\theta} \approx \theta_T$, where θ_T is the true mean phase change per pair. Writing

$$\hat{\theta} = \theta_T + \delta\hat{\theta}, \tag{22}$$

$$\text{Re}\{X\} = \text{Re}\{g(T)\} + \epsilon_1, \tag{23}$$

$$\text{Im}\{X\} = \text{Im}\{g(T)\} + \epsilon_2 \tag{24}$$

we have

$$\begin{aligned} \tan \theta_T &= \frac{\hat{\delta \theta}}{\cos^2 \theta_T} = \frac{\text{Im}\{g(T)\} + \epsilon_2}{\text{Re}\{g(T)\} + \epsilon_1} \\ &= \frac{\text{Im}\{g(T)\}}{\text{Re}\{g(T)\}} \left(1 + \frac{\epsilon_2}{\text{Im}\{g(T)\}}\right) \left(1 - \frac{\epsilon_1}{\text{Re}\{g(T)\}}\right) \end{aligned} \quad (25)$$

from which it follows that

$$\hat{\delta \theta} \approx \frac{\text{Re}\{g(T)\} \epsilon_2 - \text{Im}\{g(T)\} \epsilon_1}{|g(T)|^2} \quad (26)$$

$$\text{since } \cos \theta_T = \frac{\text{Re}\{g(T)\}}{|g(T)|} \quad (27)$$

Now by defining a complex error

$$\epsilon = \epsilon_1 + j\epsilon_2 \quad (28)$$

we may write

$$\begin{aligned} \hat{\delta \theta} &= \frac{(g(T) + g^*(T))(\epsilon - \epsilon^*) - (g(T) - g^*(T))(\epsilon + \epsilon^*)}{4j |g(T)|^2} \\ &= \frac{g^*(T) \epsilon - \epsilon^* g(T)}{2j |g(T)|^2} \\ &= \frac{\text{Im}(g^* \epsilon)}{|g|^2} \\ &= \text{Im} \left(\frac{\epsilon}{g(T)} \right) \end{aligned} \quad (29)$$

Now define a new complex estimator error

$$\delta \hat{\psi} = \operatorname{Re} \left(\frac{\epsilon}{g(T)} \right) \quad (30)$$

then

$$\delta \hat{\psi} + j \delta \hat{\theta} = \frac{\epsilon}{g(T)} \quad (31)$$

Thus

$$\langle (\delta \hat{\psi})^2 \rangle + \langle (\delta \hat{\theta})^2 \rangle = \left\langle \left| \frac{\epsilon}{g(T)} \right|^2 \right\rangle \quad (32)$$

and

$$\langle (\delta \hat{\psi})^2 \rangle - \langle (\delta \hat{\theta})^2 \rangle = \operatorname{Re} \left\langle \frac{\epsilon^2}{g^2(T)} \right\rangle \quad (33)$$

from which it follows that

$$\begin{aligned} \operatorname{Var} \delta \hat{\theta} &= \langle (\delta \hat{\theta})^2 \rangle = \frac{1}{2} \left[\left\langle \left| \frac{\epsilon}{g(T)} \right|^2 \right\rangle - \operatorname{Re} \left\langle \frac{\epsilon^2}{g^2(T)} \right\rangle \right] \\ &= \frac{1}{2} \operatorname{Re} \left\{ \left\langle \left| \frac{\epsilon}{g(T)} \right|^2 \right\rangle - \left\langle \frac{\epsilon^2}{g^2(T)} \right\rangle \right\} \end{aligned} \quad (34)$$

Now

$$X = g(T) + \epsilon \quad (35)$$

Therefore

$$\left\langle \left| \frac{\epsilon}{g(T)} \right|^2 \right\rangle = \frac{\operatorname{Var} |X|^2}{|g(T)|^2} \quad (36)$$

and

$$\left\langle \frac{\epsilon^2}{g^2(T)} \right\rangle = \frac{\operatorname{Var} X^2}{g^2(T)} \quad (27)$$

Note that $\operatorname{Var} X^2$ is a complex quantity defined by

$$\operatorname{Var} X^2 = \langle (X - g(T))^2 \rangle$$

$$= \langle X^2 \rangle - g^2(T) \quad (38)$$

and

$$\operatorname{Var} \delta \hat{\theta} = \frac{1}{2} \operatorname{Re} \left\{ \operatorname{Var} \frac{|X|^2}{|g(T)|^2} - \operatorname{Var} \frac{X^2}{g^2} \right\} \quad (38 \text{ bis})$$

Now we can evaluate the requisite expressions using the assumption of Gaussian statistics for the raw input data as follows:

$$\begin{aligned}
 |X| &= \frac{1}{M^2} \sum_{k=0}^{M-1} \sum_{\ell=0}^{M-1} \langle r_k r_{k+1}^* r_\ell^* r_{\ell+1} \rangle \\
 &= \frac{1}{M^2} \sum_{k=0}^{M-1} \sum_{\ell=0}^{M-1} \left[\langle r_k r_{k+1}^* \rangle \langle r_\ell^* r_{\ell+1} \rangle \right. \\
 &\quad \left. + \langle r_k r_\ell^* \rangle \langle r_{k+1}^* r_{\ell+1} \rangle \right. \\
 &\quad \left. + \langle r_k r_{\ell+1} \rangle \langle r_{k+1}^* r_\ell^* \rangle \right] \quad (39)
 \end{aligned}$$

From (13)-(16) it follows that

$$\begin{aligned}
 \text{term 1} &= |g(T)|^2 \\
 \text{term 2} &= \begin{cases} |g((\ell-k)T)|^2 & \ell \neq k \\ |S+N|^2 & \ell = k \end{cases} \\
 \text{term 3} &= 0.
 \end{aligned}$$

Note S/N is the per sample signal to noise ratio. For notational convenience we will write

$$g_n = g(nT). \quad (41)$$

Then

$$\begin{aligned}
\langle |X|^2 \rangle &= |g_1|^2 + \frac{1}{M^2} \sum_{k=0}^{M-1} \sum_{\ell=0}^{M-1} |g_{\ell-k}|^2 \\
&\quad + \frac{1}{M^2} \sum_{k=0}^{M-1} \left[(S+N)^2 - S^2 \right] \\
&= |g_1|^2 + \frac{1}{M} \left[(S+N)^2 - S^2 \right] \\
&\quad + \frac{1}{M} \sum_{n=-(M-1)}^{M-1} |g_n|^2 f_M(n)
\end{aligned} \tag{42}$$

where

$$f_M(n) = 1 - \frac{1}{M} - \frac{|n|}{M} . \tag{43}$$

Similarly

$$\begin{aligned}
\langle X^2 \rangle &= \frac{1}{M^2} \sum_{k=0}^{M-1} \sum_{\ell=0}^{M-1} \langle r_k r_{k+1}^* r_\ell r_{\ell+1}^* \rangle \\
&= \frac{1}{M^2} \sum_{k=1}^{M-1} \sum_{\ell=1}^{M-1} \left[\langle r_k r_{k+1}^* \rangle \langle r_\ell r_{\ell+1}^* \rangle \right. \\
&\quad + \langle r_k r_\ell \rangle \langle r_{k+1}^* r_{\ell+1}^* \rangle \\
&\quad \left. + \langle r_k r_{\ell+1}^* \rangle \langle r_{k+1}^* r_\ell \rangle \right]
\end{aligned} \tag{44}$$

Then (13) - (16) yields

$$\text{term 1} = g_1^2$$

$$\text{term 2} \equiv 0$$

$$\text{term 3} = \begin{cases} g_{l+1-k} g_{l-k-1}^* & l \neq k-1, k+1 \\ (S+N) g_{-2}^* & l = k-1 \\ g_2 (S+N) & l = k+1 \end{cases}$$

and substituting these in (44) yields

$$\begin{aligned} \langle x^2 \rangle &= g_1^2 + \frac{1}{M^2} \sum_{k=0}^{M-1} \sum_{l=0}^{M-1} g_{l+1-k} g_{l-k-1}^* \\ &\quad + \frac{N [g_2 + g_{-2}^*] (M-1)}{M^2} \\ &= g_1^2 + \frac{1}{M} \sum_{n=-(M-1)}^{M-1} g_{n+1} g_{n-1}^* f_M(n) \\ &\quad + \frac{N}{M} \left(1 - \frac{1}{M}\right) (g_2 + g_{-2}^*). \end{aligned} \quad (45)$$

Now substituting (44) and (45) in (38 bis) yields

$$\begin{aligned} \text{Var} \{\hat{\theta}\} &= \frac{1}{2M} \text{Re} \left\{ \frac{(S+N)^2 - S^2}{|g_1|^2} - N \left(1 - \frac{1}{M}\right) \frac{(g_2 + g_{-2}^*)}{g_1^2} \right. \\ &\quad \left. + \sum_{n=-(M-1)}^{M-1} \left(\frac{|g_n|^2}{|g_1|^2} - \frac{g_{n+1} g_{n-1}^*}{g_1^2} \right) f_M(n) \right\} \end{aligned} \quad (46)$$

For large M and small wT (46) yields approximately

$$\begin{aligned} \text{Var} \{\hat{\delta\theta}\} &= \frac{1}{2M} \left[\frac{(1 + N/S)^2 - 1}{|\rho(wT)|^2} - \frac{N}{S} \frac{2\rho(2\pi)}{|\rho(wT)|^2} \right. \\ &\quad \left. + \frac{(2\pi wT)}{|\rho(wT)|^2} \sqrt{\pi} \right] \\ &= \frac{1}{2M} \left[\frac{\frac{N}{S} \left[\frac{N}{S} + 2 \left(1 - \rho(2wT) \right) \right] + (2\pi wT) \sqrt{\pi}}{|\rho(wT)|^2} \right] \end{aligned} \quad (47)$$

Equation (47) is a general expression for predicting the variance of the pulse pair estimate of the mean Doppler frequency accounting for the correlations in the data itself. It is virtually independent of actual spectral shape.

3. Width Estimator

We have shown that the spectral width estimate is given by

$$\hat{w}^2 = \frac{2}{(2\pi T)^2} \left[1 - \frac{|X|}{Y - N} \right] = B^2 \quad (48)$$

If the estimate is useful, it must have small errors and hence

$$\begin{aligned} \bar{w}^2 + 2\bar{w} (\delta\hat{w}) &= B_T + \eta_B \\ &= B_T^2 + \left\{ - \frac{g_{1r} \epsilon_1 + g_{1Im} \epsilon^2}{|g_1| s} + \frac{|g_1| \eta}{s^2} \right\} \frac{2}{(2\pi T)^2} \end{aligned}$$

where

$$|X| \triangleq \sqrt{(g_{1r} + \epsilon_1)^2 + (g_{1Im} + \epsilon_2)^2} \quad (50)$$

$$\text{and } Y = S + N + \eta \quad (51)$$

Note that

$$g_{1r} \epsilon_1 + g_{1l} \text{Im } \epsilon_2 = \text{Re}(g^* \epsilon) \quad (52)$$

so that

$$(2\pi T)^2 \overline{w} (\delta \hat{w}) = \frac{|g_1|}{S} \left[\frac{\eta}{S} - \text{Re} \left\{ \frac{\epsilon}{g_1} \right\} \right]. \quad (53)$$

It follows that

$$(2\pi T)^4 \left(\frac{\overline{w} S}{|g_1|} \right)^2 \text{Var } \hat{w} = \text{Var}(\delta \psi) + \text{Var}\left(\frac{\eta}{S}\right) - 2 \text{cov} \left\{ (\delta \psi) \left(\frac{\eta}{S} \right) \right\} \quad (54)$$

From (32) and (33) it follows that

$$\begin{aligned} \text{Var} \{ \delta \psi \} &= \frac{1}{2} \text{Re} \left\{ \left\langle \left| \frac{\epsilon}{g_1} \right|^2 \right\rangle + \left\langle \frac{\epsilon^2}{g_1^2} \right\rangle \right\} \\ &= \frac{1}{2M} \left\{ \frac{(S+N)^2 - S^2}{|g_1|^2} + N \left(1 - \frac{1}{M}\right) \frac{(g_2 + g_{-2}^*)}{g_1^2} \right. \\ &\quad \left. + \sum_{n=-(M-1)}^{M-1} \left(\frac{|g_n|^2}{|g_1|^2} + \frac{g_{n+1} g_{n-1}^*}{g^2} \right) f_M(n) \right\}. \end{aligned} \quad (55)$$

Following the methods used earlier we have

$$\left\langle \left(\frac{\eta}{S} \right)^2 \right\rangle = \frac{1}{S^2} \left[\frac{1}{M^2} \sum_{k=0}^M \sum_{\ell=0}^M \langle |r_k|^2 |r_\ell|^2 \rangle - (S+N)^2 \right] \quad (56)$$

$$\begin{aligned}
&= \frac{1}{S^2} \left[\frac{1}{M^2} \sum_k \sum_\ell |g_{\ell-k}|^2 + \frac{(S+N)^2 - S^2}{M} \right] \\
&= \frac{1}{MS^2} \left[\sum_{n=-M}^M |g_n|^2 f_{M+1}(n) + (S+N)^2 - S^2 \right]
\end{aligned}$$

Similarly

$$\begin{aligned}
\text{Cov}(\delta\psi, \hat{N}/S) &= \frac{\text{Re}}{Sg_1} \langle (X-g_1)(Y-S-N) \rangle \\
&= \frac{1}{Sg_1} \text{Re} \left\{ \frac{1}{M^2} \sum_k \sum_\ell \langle r_k r_{k+1}^* r_\ell r_\ell^* \rangle - g_1(S+N) \right\} \\
&= \frac{1}{Sg_1 M} \left[\sum_{Mn} g_n g_{n-1}^* f_M(n) + N(g_1 + g_{-1}^*) \right] \\
&= \frac{1}{M} \left\{ \sum_n \frac{g_n g_{n-1}^*}{Sg_1} f_M(n) + \frac{N}{S} \frac{g_1 + g_{-1}^*}{g_1} \right\}
\end{aligned} \tag{57}$$

Now assembling all the pieces yields

$$\begin{aligned}
(2\pi T)^4 \left(\frac{\overline{w} S}{|g_1|} \right) \text{Var}(\hat{w}) \\
&= \frac{1}{2M} \left\{ \left[(1 + N/S)^2 - 1 \right] \left[\frac{1}{\rho_1^2} + 2 \right] \right. \\
&\quad \left. + N \frac{g_2 + g_{-2}^*}{g_1^2} - \frac{4N}{S} \left(\frac{g_1 + g_{-1}^*}{g_1} \right) \right\}
\end{aligned}$$

$$+ \sum_{n=-(M-1)}^{M-1} \left[|g_n|^2 \left(\frac{1}{|g_1|^2} + \frac{2}{S^2} \right) + \frac{g_{n+1} g_{n-1}^*}{g_1^2} - \frac{4 g_n g_{n-1}^*}{S g_1} \right] f_M^{(n)} \quad (58)$$

This is the requisite expression for the variance of the width parameter estimator.

4. Example

To reduce the equations to useful form, it is of interest to evaluate them for a Gauss shaped spectrum. In this case

$$R(f) = \frac{1}{\sqrt{2\pi}} e^{-\frac{f^2}{2}} \quad (59)$$

so that

$$\rho(t) = e^{-\frac{(2\pi t)^2}{2}} \quad (60)$$

$$G(f) = \frac{S/w}{\sqrt{2\pi}} e^{-\frac{(f-f_o)^2}{2w^2}} \quad (61)$$

$$\begin{aligned} g_n &= S e^{j2\pi f_o T} \rho(nwT) \\ &= S e^{j2\pi f_o T} e^{-\frac{(2\pi n w T)^2}{2}} \end{aligned} \quad (62)$$

$$\frac{N (g_2 + g_{-2}^*)}{g_1^2} = 2 \frac{N}{S} \frac{|\rho_2|}{|\rho_1|^2} \quad (63)$$

$$\frac{g_{n+1} g_{n-1}^*}{g_1^2} = |\rho_n|^2, \quad (64)$$

$$\frac{g_1 + g_{-1}^*}{g_1} = 2, \quad (65)$$

$$\begin{aligned} \frac{g_n g_{n-1}^*}{S g_1} &= e^{-\frac{(2\pi Tw)^2}{2}} \left[n^2 + (n-1)^2 - 1 \right] \\ &= e^{- (2\pi Tw)^2 n (n-1)} \\ &= e^{- (2\pi Tw)^2 (m+1/2) (m-1/2)} \\ &= e^{\frac{1}{4} (2\pi Tw)^2 - (2\pi mTw)^2} \end{aligned} \quad (66)$$

which leads to

$$\begin{aligned} 2M \text{ Var } \{\hat{\theta}\} &= \frac{1}{|\rho_1|^2} \left\{ (1 + N/S)^2 - 1 - 2N/S |\rho_2| \left(1 - \frac{1}{M}\right) \right. \\ &\quad \left. + \sum_{n=-(M-1)}^{M-1} |\rho_n|^2 (1 - |\rho_1|^2) f_M(n) \right\} \end{aligned} \quad (67)$$

$$\begin{aligned}
2M (2\pi T)^4 \left(\frac{w}{|p_1|} \right)^2 \text{Var } \hat{w} = & \frac{1}{|p_1|^2} \left\{ \left[(1 + N/S)^2 - 1 \right] \left[1 + 2 |p_1|^2 \right] \right. \\
& + 2 \frac{N}{S} |p_2| - \frac{8N}{S} |p_1|^2 \\
& \left. + \sum_{n=-(M-1)}^{M-1} |p_n|^2 \left[1 + 3 |p_1|^2 - 4 |p_1|^2 e^{\frac{1}{4} (2\pi Tw)^2} \right] \right\} \quad (68)
\end{aligned}$$

These expressions may be simplified as follows (for large M and $2\pi Tw \ll 1$)

$$\begin{aligned}
(1 + N/S)^2 - 1 - 2 N/S |p_2| (1 - \frac{1}{M}) &= N/S (2 + N/S) - 2 N/S |p_2| \\
&= N/S [(N/S + 2 (1 - |p_2|))] \quad (69)
\end{aligned}$$

$$\begin{aligned}
& \sum |p_n|^2 (1 - |p_1|^2) f_M(n) \\
& \approx \sum_{-\infty}^{\infty} |p_n|^2 (1 - |p_1|^2) \\
& \approx (2\pi w T)^2 \sum_{-\infty}^{\infty} e^{-(2\pi n w T)^2} \\
& \approx 2\pi w T \int_{-\infty}^{\infty} e^{-t^2} dt \\
& = 2\pi w T \sqrt{\pi} \quad (70)
\end{aligned}$$

Similarly

$$\begin{aligned}
 & \left[(1 + N/S)^2 - 1 \right] \left[1 + 2|\rho_1|^2 \right] + \frac{2N}{S} |\rho_2| - \frac{8N}{S} |\rho_1|^2 \\
 &= N/S (N/S + 2) (1 + 2|\rho_1|^2) + \frac{2N}{S} |\rho_2| - \frac{8N}{S} |\rho_1|^2 \\
 &= N/S \left[N/S (1 + 2|\rho_1|^2) + 2 + 2|\rho_2| - 4|\rho_1|^2 \right] \\
 &= N/S \left[N/S (1 + 2|\rho_1|^2) + 2(1 - |\rho_1|^2)^2 \right], \tag{71}
 \end{aligned}$$

$$\begin{aligned}
 & \sum |\rho_n|^2 \left[1 + 3|\rho_1|^2 - 4|\rho_1|^2 e^{\frac{1}{4}(2\pi wT)^2} \right] f_M(n) \\
 &= \sum |\rho_n|^2 \left[1 + 3e^{(2\pi wT)^2} - 4e^{-\frac{3}{4}(2\pi wT)^2} \right] \\
 &\approx \frac{3}{8} (2\pi wT)^4 \sum |\rho_n|^2 \\
 &\approx \frac{3}{8} (2\pi wT)^3 \sqrt{\pi}. \tag{72}
 \end{aligned}$$

As a result it is possible to reduce the variance expressions to

$$\begin{aligned}
 2 M \text{Var} \{ \hat{\theta} \} &= e^{(2\pi wT)^2} \left[\frac{N}{S} \left(\frac{N}{S} + 2 \left(1 - e^{-2(2\pi wT)^2} \right) \right) \right. \\
 &\quad \left. + 2\pi wT \sqrt{\pi} \right] \tag{73}
 \end{aligned}$$

$$2M (2\pi\omega T)^2 \text{ Var } (2\pi\hat{\omega}T) = \frac{N}{S} \left[\frac{N}{S} \left(1 + 2 e^{-(2\pi\omega T)^2} \right) + 2 \left(1 - e^{-(2\pi\omega T)^2} \right)^2 \right] \\ + \frac{3}{8} (2\pi\omega T)^3 \sqrt{\pi} \quad (74)$$

Figures 1 and 2 are essentially plots of (73) and (74) and enable the prediction of system performance for various choices of parameters.

K-Σ SEMI-LOGARITHMIC 46 5493
3 CYCLES X 70 DIVISIONS
KEUFFEL & ESSER CO

$\sigma (2\pi fT)$
BM

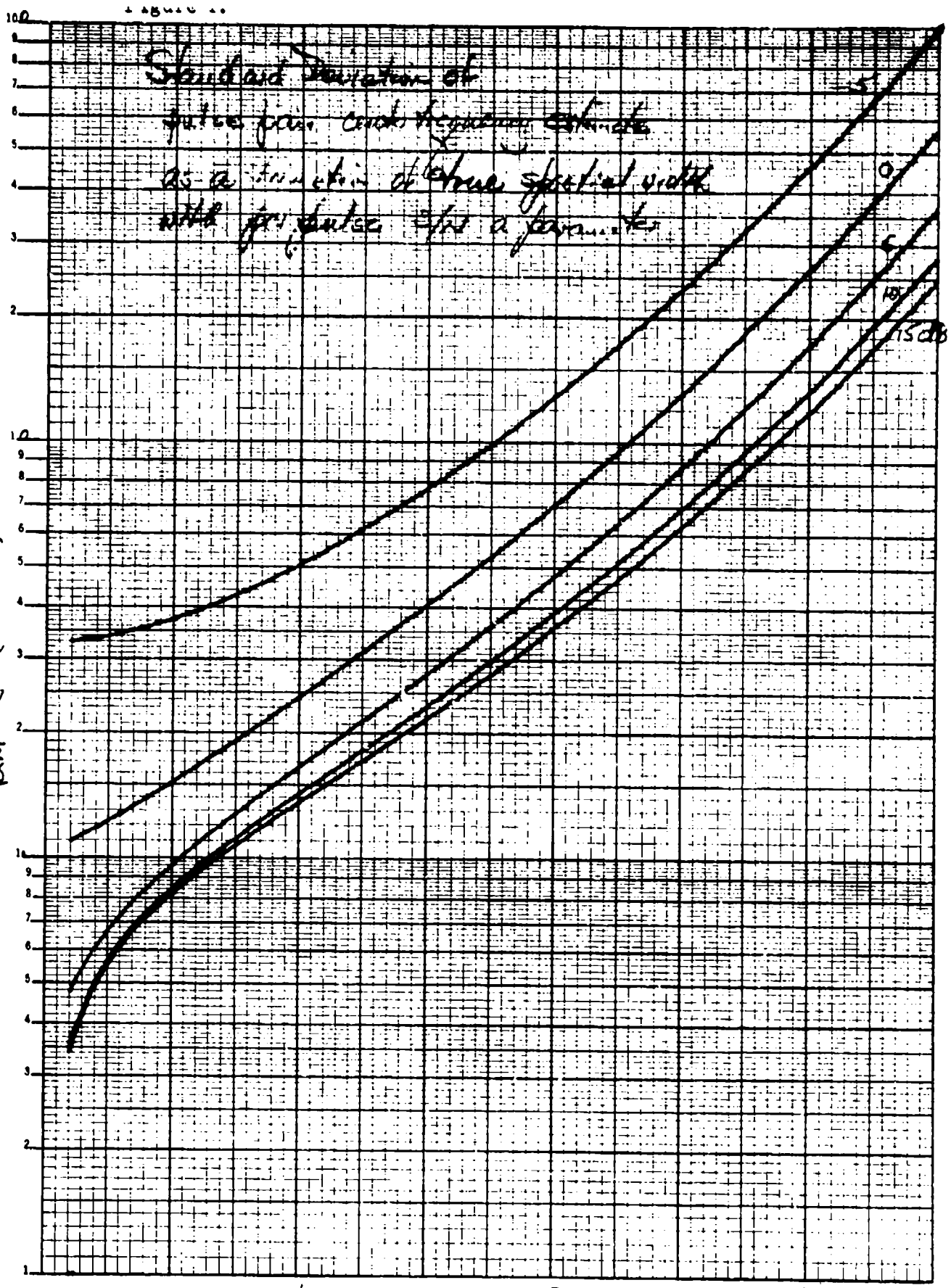
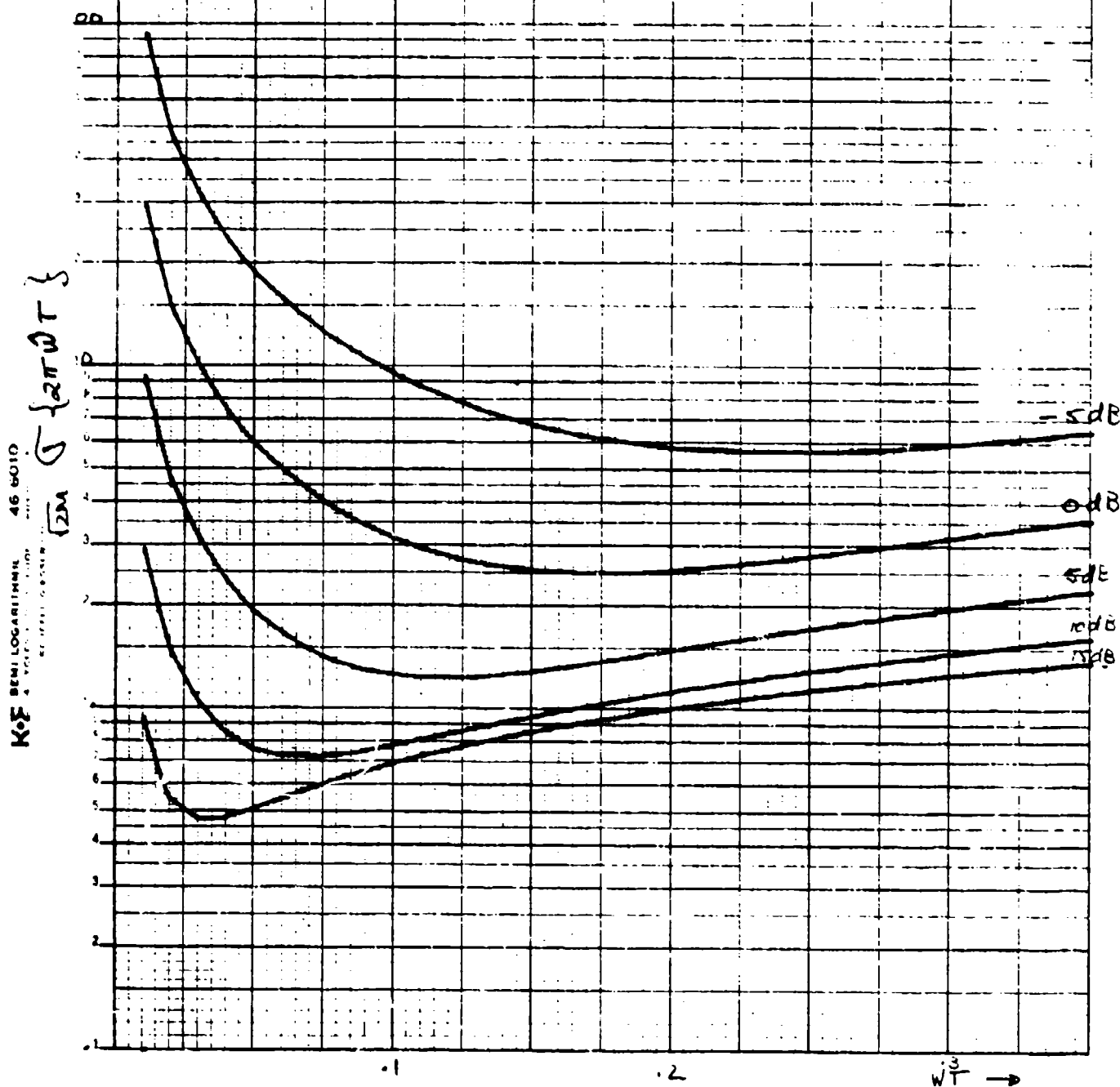


Figure 2.

Standard deviation
of pulse pair width estimate
as a function of the width
with pulse rate a parameter



APPENDIX B

MATHEMATICAL DERIVATIONS RELATING TO THE GENERATION OF SIMULATED DOPPLER RETURN

a. Second Moment:

$$m_2 = \frac{\left(\frac{1}{2\pi j}\right)^3 \int_{-j\infty}^{+j\infty} H(s) H(-s) (-s^2) ds}{\frac{1}{2\pi j} \int_{-j\infty}^{+j\infty} H(s) H(-s) ds} = w^2$$

$$\begin{aligned} \frac{1}{2\pi j} \int_{-j\infty}^{+j\infty} H(s) H(-s) ds &= \frac{1}{2\pi j} \int_{-j\infty}^{+j\infty} \frac{a^4}{(s+a)^2 (-s+a)^2} ds \\ &= a^4 \frac{d}{ds} \left[\frac{1}{(-s+a)^2} \right] \Big|_{s=-a} \\ &= \frac{1}{4} a, \text{ where } a = 2\pi w \end{aligned}$$

Similarly,

$$\frac{1}{2\pi j} \int_{-j\infty}^{+j\infty} H(s) H(-s) (-s^2) ds = \frac{1}{4} a^3$$

therefore,

$$m_1 = w^2$$

b. Half Power Points (ω_3):

$$|H(j\omega)| = \frac{a^2}{\omega^2 + a^2} = \frac{1}{\sqrt{2}}$$

$$\therefore \omega_3 = a [\sqrt{2} - 1]^{1/2}.$$

c. Recursion Relation:

$$\frac{Y(Z)}{X(Z)} = H(Z) = \frac{Z^{-1} a^2 T e^{-aT}}{(1 - 2e^{-aT} Z^{-1} + e^{-2aT} Z^{-2})}$$

$$Y(Z) = a^2 T e^{-aT} X(Z) Z^{-1} + 2e^{-aT} Y(Z) Z^{-1} - e^{-2aT} Y(Z) Z^{-2}$$

which yields

$$y_n = a^2 T e^{-aT} x_{n-1} + 2e^{-aT} y_{n-1} - e^{-2aT} y_{n-2}.$$

d. Signal to Noise:

$$= \text{Signal Power} = \frac{2}{2\pi j} \oint_1 H(Z) H(1/Z) \frac{dZ}{Z}$$

The factor 2 is due to the power from both the real and imaginary channels. The contour of integration is the unit circle. The Z-transform is used because a digital signal is the desired output from the simulator; therefore, signal power should be found in terms of the Z-transform. To continue,

$$S = \frac{2}{2\pi j} \oint_1 \frac{a^4 T^2 Z dZ}{(Z - e^{-aT})^2 (Z - e^{aT})^2}.$$

Since there is only one (second order) pole inside the unit circle at $Z = e^{-aT}$, the value of S is given by:

$$= \frac{2 \cdot 2\pi j}{2\pi j} \frac{d}{dZ} \left[\frac{a^4 T^2 Z}{(Z - e^{-aT})^2} \right] \Big|_{Z=e^{-aT}}$$

Resulting in

$$= 2a^4 T^2 \frac{(3e^{-aT} + e^{aT})}{(e^{aT} - e^{-aT})^3}$$

Noise power, N, is given by:

$$N = 2 \sigma_N^2$$

The factor 2 results from noise power in the real and imaginary channels.

Therefore the S/N ratio is given by:

$$S/N = \frac{a^4 T^2 (3e^{-aT} + e^{aT})}{\sigma_N^2 (e^{aT} - e^{-aT})^3}$$

APPENDIX C
FORTRAN MAIN PROGRAMS

C-1	DOPGEN
C-2	PPSTAT
C-3	SPEC

APPENDIX C-1
DOPGEN Program Coding

DOPGEN-1

C COMMENT--SYSTEM SUPPLIED SUBROUTINES REQUIRED--DOPOLT,ENDPLT
COMMENT--USER SUPPLIED SUBROUTINES REQUIRED--GARN,FORT

REAL MAGWK
COMPLEX WK
COMPLEX Z,W,B

C DIMENSION RN(24),Z(1524),X(1024),Y(1024),YR(3),YI(3),LAB(3),
1 S(513)

C EQUIVALENCE (Z(1),Y(1)) , (Z(513),X(1))

C DATA LAB / 1CH P.L.KELLY, 10H 7025 , 10H MISC /

C NAMELIST /N1/ A,FHZ,FMU,SIG,XR,N,AT,EAT,C1,C2,C3,WNOT,WT
NAMELIST /N2/ FM2,C5,C6,SIGDB,PRF

C PI=3.141592653
T=1.0/3300.0
PRF=3300.0
XR=82593476.
FMU=0.0
SIG=1.0
N=1024
NM=10
NBLKS=0
IPRINT=1
REWIND 9

C-3

C 10 READ(5,1000)AVGDOP,SIGDOP,STON,IBLK1,IBLK2,IFFT
WRITE(6,1000)AVGDOP,SIGDOP,STON,IBLK1,IBLK2,IFFT
IF(1BLK1.EQ.0)GO TO 500

SET UP CONSTANTS

FHZ=AVGDOP
A=2.*PI*SIGDOP
CB=STON
FM2=A**2/(4.0*PI**2)
FNPK=3.0*PRF*PI/A+1.0
NN=FVPK
IF (NN.GT.500) GO TO 610

C C C

```

C6=0.0
SIGUB=0.0
AT=AT
EAT=1.0/EXP(AT)
C1=A*AT*EAT
C2=2.0*EAT
C3=1.0/EXP(2.0*AT)
C4=EXP(AT)

NOISE=0
IF (DB.GI.100.0) GO TO 5
NOISE=1
C5=10.0*(DB*0.1)
C6=A**4*T**2*(3.0*EAT+C4)/((C4-EAT)**3)
SIGDB=SQRT(C6/C5)
5 WNOT=2.0*PI*FHZ
C INITIALIZE FILTER
DO 20 J=1,2
YR(J)=0.0
20 YI(J)=0.0
IF (XR.EQ.0.0) GO TO 30
YY=RANF(XR)
XR=0.0
RMARG=0.0
30 CONTINUE

```

C-4

```

DO 700 ICT=1BLK1,1BLK2

IF(ICT.NE.1BLK1)IFFT=0

NBLKS=NBLKS+1
C GENERATE N COMPLEX NUMBERS

DO 100 I=1,N10

CALL GNRN (RN,X1,X2,SIG,FMU)

YR(3)=X1*C1+YR(2)*C2-YR(1)*C3
YI(3)=X2*C1+YI(2)*C2-YI(1)*C3

```

```

Z(I)=CMPLX(YR(3),YI(3))
C
C  UPDATE SEQUENCE
C
DO 50 J=1,2
YR(J)=YR(J+1)
50 YI(J)=YI(J+1)
C
100 CONTINUE
C
DO 105 I=1,N
105 Z(I)=Z(I+NN)
C
C  GENERATE WEIGHTED COMPLEX INPUT SEQUENCE
C
WT=WNOT*I
C
DO 110 I=1,N
FI=I-1
ARG=WT*FI
W=CMPLX(COS(ARG),SIN(ARG))
110 Z(I)=Z(I)*W
C
C  IF (NOISE.EQ.0) GO TO 200
C
DO 115 I=1,N
CALL GNRN (RN,X1,X2,SIGDB,0.0)
B=CMPLX(X1,X2)
115 Z(I)=Z(I)+B
C
200 WRITE (9) ICT,A,PRF,FM2,FHZ,FMU,SIG,XR,DB,N,MM,IFFT,
1      Z(J),J=1,N)
C
700 CONTINUE
GO TO 10
C
500 END FILE 9
REWIND 9
WRITE (6,2000)
WRITE(6,4000)NBLKS
C
510 READ (9) ICT,A,PRF,FM2,FHZ,FMU,SIG,XR,DB,N,MM,IFFT,
1      Z(J),J=1,N)
IF (EOF(9)) 600,520

```

C-6

DOPGEN.5

```

C
  YMAX=Y(I)
  DO 152 I=2,N
    IF (YMAX.LT.Y(I)) YMAX=Y(I)
  152 CONTINUE
  DO 153 I=1,N
    153 Y(I)=Y(I)/YMAX
  C
    DO 170 I=1,N
      X(I)=I-1
      170 Y(I)=20.0*ALOG10(Y(I))
      GO TO 155
  C
  C    PLOT RESULTS
  C
  180 CONTINUE
  CALL DQFLOT (LAB,X,Y,N,IH ,1,12H20 LOG (MAG),12,0,0)
  IPLOT=1
  C
  GO TO 510
  C
  600 IF(IPLOT.EQ.1)CALL ENDPLT
  REWIND 9
  CALL EXIT
  610 WRITE (6,2040)
  CALL EXIT
  C
  1000 FORMAT(3F10.0,3I10)
  2000 FORMAT (1H1/)
  2010 FORMAT (1X,4(15,2E13.5))
  2020 FORMAT (1X,4(15,E13.5,13X))
  2030 FORMAT(/1X,15,8E12.4,2X,3I5)
  2040 FORMAT (10X,13HVI TOO LARGE)
  3000 FORMAT (/4(1X,6E15.6)/10X,3E20.8)
  4000 FORMAT(1X,15,23H DATA BLOCKS GENERATED)
  END

```

APPENDIX C-2
PPSTAT Program Coding

PPSTAT-1

PROGRAM PPSTAT(INPUT,OUTPUT,TAPE9=3000,TAPE5=INPUT,TAPE6=OUTPUT)

COMMENT--SYSTEM SUPPLIED SUBROUTINES REQUIRED--NONE

COMMENT--USER SUPPLIED SUBROUTINES REQUIRED--NONE

REAL MEANE1,MEANE2

REAL MEANE3

COMPLEX Z,W,WH,SWH

DIMENSION Z(1024),D(8),NS(40)

DIMENSION E1(30,7),E2(30,7)

DIMENSION VARE4(7),VARE5(7),SIGE4(7),SIGE5(7)

5 REWIND 9

10 CONTINUE

READ(5,1100)NMAX,NSETS,NINC,IBLK1,ISUM,ISN,IRW,FNOISE

WRITE(6,1100)NMAX,NSETS,NINC,IBLK1,ISUM,ISN,IRW,FNOISE

IF (NINC.EQ.0) GO TO 300

IBLK2=IBLK1+NSETS-1

JJ=0

DO 11 I=IBLK1,IBLK2

JJ=JJ+1

11 NS(JJ)=I

WRITE(6,1000)(NS(I),I=1,NSETS)

NALKS=NSETS

NM=NMAX-1

DO 200 I=1,NSETS

NN=NS(I)

NRS=NINC

15 READ (9) ICT,(D(J),J=1,8),N,MM,IFFT,(Z(J),J=1,1024)

IF (EOF(9)) 5,20

20 WRITE(6,1000)ICT

IF (NN.NE.1) GO TO 15

SNDB=D(8)

XXX1=SNDB/10.0

SN=10.**XXX1

SIGDOP=SQRT(D(3))

WRITE(6,1500)NN,N,D(8),D(4),SIGDOP

PPSTAT-2

```

WRITE(6,1550)Z(1),Z(2)
WRITE(6,1560)Z(1023),Z(1024)
WRITE(6,2000)

C
SMH=CMPLX(0.0,0.0)
SK=REAL(Z(1))*2+AIMAG(Z(1))*2

C
KK=0
DO 30 J=1,NM

C
M=CONJG(Z(J))*Z(J+1)
SMH=SMH+M
SK=SK+REAL(Z(J+1))*2+AIMAG(Z(J+1))*2

C
IF (J.EQ.NM) GO TO 25
IF (J.EQ.NRS-1)GO TO 25
GO TO 30

C
25 CONTINUE
SNK=1.
IF (ISN.EQ.1)SNK=SN/(1.+SN)
IF (ISN.LE.1)FNOISE=0.
FJ=J
WH=SMH/FJ
SKK=(SK/(FJ+1.))*SNK-FNOISE
PI=3.141592653
FMU=1650.*ATAN(AIMAG(WH)/REAL(WH))/PI
AW=SQRT(REAL(WH))*2+AIMAG(WH)*2
UK=(1.0-AW/SKK)/(2.0*PI*2)
IF (UK.LT.0.)UK=0.
J)=3300.**2*UK
VAR=SQRT(SQ)

C
JJ=J+1

C
WRITE(6,2010)JJ,WH,FMU,SKK,UK,SQ,VAR

C
KK=KK+1
E1(1,KK)=FMU
E2(1,KK)=SQ

C
NRS=2*NRS

```

30 CONTINUE

C

200 CONTINUE

C

IF (ISUM.EQ.0)GO TO 5

C

WRITE(6,3000)

DO 270 I=1,KK

SE1=0.0

SE2=0.0

SE3=0.0

SE1SQ=0.0

SE2SQ=0.0

SE3SQ=0.0

SE4SQ=0.0

SE5SQ=0.0

VARE1=0.0

VARE2=0.0

VARE3=0.0

DO 250 J=1,NBLKS

SE1=SE1+E1(J,I)

SE2=SE2+E2(J,I)

SE3=SE3+SQRT(E2(J,I))

250 CONTINUE

MEANE1=SE1/NBLKS

MEANE2=SE2/NBLKS

MEANE3=SE3/NBLKS

IF (NBLKS.EQ.1)GO TO 260

DO 255 J=1,NBLKS

SE1SQ=SE1SQ+(E1(J,I)-MEANE1)**2

SE3SQ=SE3SQ+(SQRT(E2(J,I))-MEANE3)**2

SE4SQ=SE4SQ+(E2(J,I)-SIGDOP)**2

SE5SQ=SE5SQ+(SQRT(E2(J,I))-SIGDOP)**2

255 SE2SQ=SE2SQ+(E2(J,I)-MEANE2)**2

VARE1=SE1SQ/(NBLKS-1)

VARE2=SE2SQ/(NBLKS-1)

VARE3=SE3SQ/(NBLKS-1)

VARE4(1)=SE4SQ/NBLKS

VARE5(1)=SE5SQ/NBLKS

SIGE1=SQRT(VARE1)

SIGE2=SQRT(VARE2)

SIGE3=SQRT(VARE3)

SIGE4(1)=SQRT(VARE4(1))

SIGE5(1)=SQRT(VARE5(1))

C-11

```

260 WRITE(6,4000)MEANE1,VARE1,SIGE1,MEANE2,VARE2,SIGE2,
1      MEANE3,VARE3,SIGE3
270 CONTINUE
DO 275 I=1,KK
275 WRITE(6,5000)VARE4(I),SIGE4(I),VARE5(I),SIGE5(I)
C
IF(IRW.EQ.1)REWIND 9
GO TO 10
C
300 REWIND 9
C
1000 FORMAT(10I5)
1100 FORMAT(7I5,E15.5)
1500 FORMAT(1H,5H9LOCK,15,10X,3HN =,15,13X,5HS/N =,F5.0,
1      11X,14HMEAN DOPPLER =,F5.0,11X,13HSIG DOPPLER =,F5.0)
1550 FORMAT(//1X,9HZ( 1 ) =,11X,2E15.5,10X,9HZ( 2 ) =,11X,2E15.5)
1560 FORMAT(//1X,9HZ(1023) =,11X,2E15.5,10X,9HZ(1024) =,11X,2E15.5)
2000 FORMAT(//8X,12HWK HAT(REAL),3X,
1      12HWK HAT(IMAG),6X,7HMUKHAT ,10X,2HMK,13X,2HUK,
2      11X,7HSIGK*2,9X,4HSIGK)
2010 FORMAT(1X,15,7E15.6)
3000 FORMAT(1H-,6X,9HMEAN MEAN,7X,8HVAR MEAN,7X,8HSIG MEAN,
1      7X,8HMEAN VAR,8X,7HVAR VAR,8X,7HSIG VAR,
2      7X,8HMEAN SIG,8X,7HVAR SIG,8X,7HSIG SIG)
4000 FORMAT(1H ,9E15.5)
5000 FORMAT(//1X,60X,2E15.5,15X,2E15.5)
END

```

APPENDIX C
SPEC Program Coding

PROGRAM SPEC TRACE CDC 6600 FTM V3.0-P270+ OPT=0 03/2

PROGRAM SPEC(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE8,TAPE9)

COMMENT--SYSTEM SUPPLIED SUBROUTINES REQUIRED--DQPLOT,ENDPLT
COMMENT--USER SUPPLIED SUBROUTINES REQUIRED--COCC,BEST,FORT

C THE FOLLOWING CONTROL CARDS ARE NEEDED WHEN PLOTS ARE DESIRED
C FTM(A)
C ATTACH,P=N,P=N2PLOTS,MR=1.
C SETCORE.
C LOAD,PEV.
C REQUEST,TAPE33.
C REQUEST,TAPE8,HI.(RPK09/NORING).
C REQUEST,TAPE8,HI.(RPK08/NORING)
C LGO.
C UNLOAD,TAPE33.
C UNLOAD,TAPE9.
C UNLOAD,TAPE9.

REAL MEAN
COMPLEX Z
DIMENSION Z(1024),A(1024),X(1024),Y(1536)
DIMENSION S(513),LAB(3)

DATA LAB /104 P.L.KELLY,104 7025 ,104 MIS /

IFS=-1
M=10
N=1024
KPILOT=0
IPILOT=0
JAVG=0
IRW=0

REWIND 8
REWIND 3

CALL COCC(A)

10 READ(5,1000)IBLK,NUNIT,LIMIT1,LIMIT2,IRW,IPILOT,IBIAS
WRITE(6,1000)IBLK,NUNIT,LIMIT1,LIMIT2,IRW,IPILOT,IBIAS
IF(1IBLK.EQ.0)GO TO 11
IF(LIMIT1.EQ.0)GO TO 12
GO TO 11
12 LIMIT1=1
LIMIT2=1024
13 CONTINUE

IF(IRW.EQ.1)REWIND NUNIT

15 READ(NUNIT)F1,F2,F3,F4,F5,F6,F7,F8,I1,I2,I3,(Z(I),I=1,1024)
WRITE(5,1000)F1,F2,F3,F4,F5,F6,F7,F8,I1,I2,I3
IF(IOT.NE.1)GO TO 15

DO 20 I=1,1024

20 Z(I)=Z(I)+A(I)

PROGRAM SPEC TRACE CDC 6600 FTN V3.0-P270+ OPT=0 03/2

```

      CALL FORT(Z,4,S,IFS,IFERR)

      DO 30 I=1,1024
30  Y(I)=REAL(Z(I))**2+AIMAG(Z(I))**2

      YMAX=Y(1)
      DO 40 I=1,1024
40  IF(YMAX.LT.Y(I))YMAX=Y(I)

      IF(IPL0T.EQ.0)GO TO 75
      DO 50 I=1,1024
50  Y(I)=Y(I)/YMAX

      FFF=-1.0
      DO 60 I=1,1024
      FFF=FFF+1.0
      X(I)=FFF
60  Y(I)=10.*ALOG10(Y(I))

      CALL D02PLOT(LAB,X,Y,N,14,1,12H20 LOG (MAG),12,0,0)
      KFL0T=1

      DO 70 J=1,1024
70  Y(J)=10.*(Y(J)/10.)*YMAX
75  CONTINUE

      IF(IAVG.EQ.0)GO TO 200
      VC=0.
      DO 76 I=1,100
76  AVG=AVG+Y(I)
      AVG=AVG/100.

      DO 77 I=1,1024
77  Y(I)=Y(I)-AVG
200 IF(IBIAS.EQ.J)GO TO 230
      ISHFT=512-IBIAS
      DO 210 I=1,1024
      J1=1024+ISHFT+1-I
      K1=1024+1-I
210 Y(J1)=Y(K1)
      DO 220 I=1,ISHFT
      J2=ISHFT+1-I
      K2=1024+ISHFT+1-I
220 Y(J2)=Y(K2)
230 CONTINUE
      WRITE (6,2000)

      THR=0.

      DO 100 I=1,20
      THR=THR+2.

      YMINV=YMAX/(10.0**(THR*0.1))

```

PROGRAM SPEC TRACE CDC 6600 FTN V3.0-P270+ OPT=0 03/2

```
S1=0.
S2=0.
FM=LIMIT1-1

DO 80 J=LIMIT1,LIMIT2
FM=FM+1.
IF(Y(J).LE.YMINV)GO TO 80
YSQ=Y(J)
S1=S1+YSQ
S2=S2+FM*YSQ
80 CONTINUE

UKP=S2/S1
MEAN=(3300./1024.)*UKP

S3=0.
FM=LIMIT1-1

DO 90 J=LIMIT1,LIMIT2
FM=FM+1.
IF(Y(J).LE.YMINV)GO TO 90
YSQ=Y(J)
S3=S3+YSQ*(FM-UKP)**2
90 CONTINUE

VKP=S3/S1
SKP=SQRT(VKP)
SIG=(3300./1024.)*SKP

WRITE(6,3000)IHR,UKP,MEAN,SKP,SIG

100 CONTINUE

GO TO 10

110 CONTINUE
IF(KPLOT.EQ.1)CALL ENOPLT
REWIND 8
REWIND 9
CALL EXIT

1000 FORMAT(7I10)
2000 FORMAT(14I)
3000 FORMAT(1X,F3.0,4E15.5)

END
```

APPENDIX D
FORTRAN SUBROUTINES

D-1	FORT
D-2	PAIR
D-3	UNPK
D-4	CDCC
D-5	BESI
D-6	STAR
D-7	GNRN

APPENDIX D-1

FORT

SUBROUTINE FORT' ,M,S,IFS,IFERR)

FOURIER TRANSFORM SUBROUTINE, PROGRAMMED IN SYSTEM/360,
BASIC PROGRAMMING SUPPORT, FORTRAN IV. FORM C28-6504
THIS DECK SET UP FOR IBSYS ON IBM 7094.

DOES EITHER FOURIER SYNTHESIS, I.E., COMPUTES COMPLEX FOURIER SERIES
GIVEN A VECTOR OF N COMPLEX FOURIER AMPLITUDES, OR, GIVEN A VECTOR
OF COMPLEX DATA X DOES FOURIER ANALYSIS, COMPUTING AMPLITUDES.
A IS A COMPLEX VECTOR OF LENGTH N=2**M COMPLEX NOS. OR 2*N REAL
NUMBERS. A IS TO BE SET BY USER.

M IS AN INTEGER 0..13, SET BY USER.

SINUS IS A VECTOR SIN(J)= SIN(2*PI*J/NP), J=1,2,...,NP/4-1,
COMPUTED BY PROGRAM.

IF IS A PARAMETER TO BE SET BY USER AS FOLLOWS-

IFS=0 TO SET NP=2**M AND SET UP SINE TABLE.

```

      IF(S=1) TO SET N=NP=2**M, SET UP SIN TABLE, AND DO FOURIER
      SYNTHESIS, REPLACING THE VECTOR A BY

```

```
X(J)=SUM OVER K=0,N-1 OF A(K)*EXP(2*PI*I/N)**(J*K),
      WHERE I=SQRT(-1),
```

THE X S ARE STORED WITH RE X(J) IN CELL 2*J+1
AND IM X(J) IN CELL 2*J+2 FOR J=0,1,2,...,N-1.
THE A S ARE STORED IN THE SAME PANNER.

```

IFSF=-1 TO SET N=NP=2**M,SET UP SIN TABLE, AND OO FOURIER
ANALYSIS, TAKING THE INPUT VECTOR A AS X AND
REPLACING IT BY THE A SATISFYING THE ABOVE FOURIER SERIES.
IFSF=+2 TO OO FOURIER SYNTHESIS ONLY, WITH A PRE-COMPUTED S.
IFSF=-2 TO OO FOURIER ANALYSIS ONLY, WITH A PRE-COMPUTED S.
IFERR IS SET BY PROGRAM TO-
=0 IF NO ERROR DETECTED.

```

$\neq 1$ IF M IS OUT OF RANGE., OR, WHEN IFS=+2,-2, THE

PRE-COMPUTED TABLES NOT LARGE ENOUGH.

$= -1$ WHEN IFS $= +1, -1$, MEANS ONF. IS RECOMPUTING S TABLE UNNECESSARILY.

NOTE- AS STATED ABOVE, THE MAXIMUM VALUE OF M FOR THIS PROGRAM ON THE IBM 7094 IS 13. FOR 360 MACHINES HAVING GREATER STORAGE CAPACITY, ONE MAY INCREASE THIS LIMIT BY REPLACING 13 IN

FORT 002
FORT 003
FORT 004
FORT 005
FORT 006
FORT 007
FORT 008
FORT 009
FORT 010
FORT 011
FORT 012
FORT 013
FORT 014
FORT 015
FORT 016
FORT 017
FORT 018
FORT 019
FORT 020
FORT 021
FORT 022
FORT 023
FORT 024
FORT 025
FORT 026
FORT 027
FORT 028
FORT 029
FORT 030
FORT 031
FORT 032
FORT 033
FORT 034
FORT 035
FORT 036
FORT 037
FORT 038
FORT 039
FORT 040
FORT 041
FORT 042
FORT 043
FORT 044

FORT-2

```

C      STATEMENT 3 BELOW BY LOG2 N, WHERE N IS THE MAX. NO. OF
C      COMPLEX NUMBERS ONE CAN STORE IN HIGH-SPEED CORE. ONE MUST
C      ALSO ADD MORE DO STATEMENTS TO THE BINARY SORT ROUTINE
C      FOLLOWING STATEMENT 24 AND CHANGE THE EQUIVALENCE STATEMENTS
C      FOR THE K S.
C
      DIMENSION A(1), S(1), K(14)
      EQUIVALENCE (K(13),K1),(K(12),K2),(K(11),K3),(K(10),K4)
      EQUIVALENCE (K( 9),K5),(K( 8),K6),(K(7),K7),(K( 6),K8)
      EQUIVALENCE (K( 5),K9),(K( 4),K10),(K( 3),K11),(K( 2),K12)
      EQUIVALENCE (K( 1),K13),( K(1),N2)
      IF(M)2,2,3
      3 IF(M-13) 5,5,2
      2 IFERR=1
      1 RETURN
      5 IFERR=0
      N=2**M
      IF( IABS(IFS) - 1 ) 200,200,10
      WE ARE USING TRANSFORM ONLY. SEE IF PRE-COMPUTED
      S TABLE IS SUFFICIENTLY LARGE
      10 IF( N-NP )20,20,12
      12 IFERR=1
      GO TO 200
      C      SCRAMBLE A, BY SANDE S METHOD
      20 K(1)=2*N
      22 K(L)=K(L-1)/2
      DO 24 L=M,12
      24 K(L+1)=2
      C      NOTE EQUIVALENCE OF KL AND K(14-L)
      C      BINARY SORT-
      IJ=2
      KOPT=K1
      DO 30 J1=2,KOPT,2
      DO 30 J2=J1,K2,K1
      DO 30 J3=J2,K3,K2
      DO 30 J4=J3,K4,K3
      DO 30 J5=J4,K5,K4
      DO 30 J6=J5,K6,K5
      DO 30 J7=J6,K7,K6
      DO 30 J8=J7,K8,K7
      DO 30 J9=J8,K9,K8
      DO 30 J10=J9,K10,K9
      DO 30 J11=J10,K11,K10

```

FORT 045
 FORT 046
 FORT 047
 FORT 048
 FORT 049
 FORT 050
 FORT 051
 FORT 052
 FORT 053
 FORT 054
 FORT 055
 FORT 056
 FORT 057
 FORT 058
 FORT 059
 FORT 060
 FORT 061
 FORT 062
 FORT 063
 FORT 064
 FORT 065
 FORT 066
 FORT 067
 FORT 068
 FORT 069
 FORT 070
 FORT 071
 FORT 072
 FORT 073
 FORT 074
 FORT 075
 FORT 076

 FORT 078
 FORT 079
 FORT 080
 FORT 081
 FORT 082
 FORT 083
 FORT 084
 FORT 085
 FORT 086
 FORT 087

```

DO 30 J12=J11,K12,K11
DO 30 J1=J12,K13,K12
IF(IJ-J1)28,30,30
28 T=A(IJ-1)
A(IJ-1)=A(J1-1)
A(J1-1)=T
T=A(IJ)
A(IJ)=A(J1)
A(J1)=T
30 IJ=IJ+2
IF(IFS)32,2,36
C DOING FOURIER ANALYSIS,SO DIV. BY N AND CONJUGATE.
32 FN = N
DO 34 I=1,N
A(2*I-1) = A(2*I-1)/FN
34 A(2*I)=-A(2*I)/FN
C SPECIAL CASE- L=1
36 DO 40 I=1,N,2
T = A(2*I-1)
A(2*I-1) = T + A(2*I+1)
A(2*I+1)=T-A(2*I+1)
T=A(2*I)
A(2*I) = T + A(2*I+2)
40 A(2*I+2)= T - A(2*I+2)
IF(M-1) 2,1 ,50
SET FOR L=2
50 LEXP1=2
LEXP1=2**(L-1)
LEXP=8
LEXP=2**(L+1)
NPL = 2**M
NPL = NP* 2**L
60 DO 130 L=2,M
C SPECIAL CASE- J=0
N2OPT=N2
DO 80 I=2,N2OPT,LEXP
I1=I + LEXP1
I2=I1+ LEXP1
I3 =I2+LEXP1
T=A(I1-1)
A(I1-1) = T +A(I2-1)
A(I2-1) = T-A(I2-1)
T =A(I1)
A(I1) = T+A(I2)

```

FORT-4

FORT 131
FORT 132
FORT 133
FORT 134
FORT 135
FORT 136
FORT 137
FORT 138
FORT 139
FORT 140
FORT 141
FORT 142
FORT 143
FORT 144
FORT 145
FORT 146
FORT 147
FORT 148
FORT 149
FORT 150
FORT 151
FORT 152
FORT 153
FORT 154
FORT 155
FORT 156
FORT 157
FORT 158
FORT 159
FORT 160
FORT 161
FORT 162
FORT 163
FORT 164
FORT 165
FORT 166
FORT 167
FORT 168
FORT 169
FORT 170
FORT 171
FORT 172
FORT 173
FORT 174

```

A(I2) = I-A(I2)
T= -A(I3)
TI = A(I3-1)
A(I3-1) = A(I1-1) - T
A(I3) = A(I1) - TI
A(I1-1) = A(I1-1) + T
80 A(I1) = A(I1) + TI
IF(L-2) 120,120,90
90 KLAST=N2-LEXP
JJ=NPL
DO 110 J=4,LEXPI,2
NPJJ=NT-JJ
UR=SNPJJ)
UI=S(JJ)
ILAST=J+KLAST
DO 100 I= J,ILAST,LEXP
I1=I+LEXPI
I2=I1+LEXPI
I3=I2+LEXPI
T=A(I2-1)*UR-A(I2)*UI
TI=A(I2-1)*UI+A(I2)*UR
A(I2-1)=A(I1-1)-T
A(I2) =A(I1) - TI
A(I1-1) =A(I1-1)+T
A(I1) =A(I1)+TI
T=-A(I3-1)*UI-A(I3)*UR
TI=A(I3-1)*UR-A(I3)*UI
A(I3-1)=A(I1-1)-T
A(I3) =A(I1) -TI
A(I1-1)=A(I1-1)+T
100 A(I1) =A(I1) +TI
C END OF I LOOP
110 JJ=JJ+NPL
C END OF J LOOP
120 LEXP1=2*LEXPI
LEXP = 2*LEXP
130 NPL=NPL/2
C END OF L LOOP
140 IF(IFS)145,2,1
C DOING FOURIER ANALYSIS. REPLACE A BY CONJUGATE.
145 DO 150 I=1,N
150 A(2*I) =-A(2*I)
160 GO TO 1
RETURN

```

FORT-5

```

C      MAKE TABLE OF S(J)=SIN(2*PI*J/NP), J=1,2,...,NT-1, NT=NP/4
      200 NP=N
      MP=M
      NT=N/4
      MT=M-2
      IF(MT) 260,260-205
      205 THETA=.7853981634
      THETA=PI/2**(L+1)      FOR L=1
      210 JSTEP = NT
      JSTEP = 2*( MT-L+1 ) FOR L=1
      JDIF = NT/2
      JDIF = 2*(MT-L)      FOR L=1
      S(JDIF) = SIN(THETA)
      IF (MT-2)260,220,220
      220 DO 250 L=2,MT
      THETA = THETA/2.
      JSTEP2 = JSTEP
      JSTEP = JDIF
      JDIF = JDIF/2
      S(JDIF)=SIN(THETA)
      JCL=NT-JDIF
      S(JCL)=COS(THETA)
      JLAST=NT-JSTEP2
      IF(JLAST-JSTEP)250,230,230
      230 DO 240 J=JSTEP,JLAST,JSTEP
      JC=NT-J
      JD=J+JDIF
      240 S(JD)=S(J)*S(JCL)+S(JDIF)*S(JC)
      250 CONTINUE
      260 IF(IFS)20,1,20
      END

```

APPENDIX D-2

PAIR

PAIR-1

SUBROUTINE PAIR(Z,N,PRF,NOISE,MEAN,SIG,POWER)

REAL NOISE

REAL MEAN,MUKHAT

COMPLEX Z,WKHAT

DIMENSION Z(N)

PI=3.141592653

WKHAT=CMPLX(0.,0.)

SK=REAL(Z(1))*2+AIMAG(Z(1))*2

M=N-1

DO 10 I=1,M

WKHAT=WKHAT+CONJG(Z(I))*Z(I+1)

SK=SK+REAL(Z(I+1))*2+AIMAG(Z(I+1))*2

WKHAT=WKHAT/(N-1)

MUKHAT=(PRF/2.)*ATAN2(AIMAG(WKHAT),REAL(WKHAT))/PI

MEAN=MUKHAT

10

SK=SK/N

POWER=SK

SK=SK-NOISE

UK=(1.-SQRT(REAL(WKHAT)**2+AIMAG(WKHAT)**2))/SK/(2.*PI**2)

VAR=(PRF)**2*UK

SIG=SQRT(VAR)

RETURN

END

APPENDIX D-3

UNPK

UNPK-:

```
SUBROUTINE UNPK(X,Y)
INTEGER X,Y
DIMENSION X(200),Y(521)
X EQUALS PACKED INPUT ARRAY
Y EQUALS UNPACKED OUTPUT ARRAY
I EQUALS INDEX OF X WORD BEING UNPACKED
J EQUALS INDEX OF Y WORD BEING OBTAINED FROM X
K EQUALS POSITION IN X OF BYTE BEING EXTRACTED
I=1
J=0
K=0
10 J=J+1
K=K+1
IF(J.EQ.522)GO TO 20
Y(J)=MBYTE(X(I),K)
IF(K.LT.5)GO TO 10
K=0
I=I+1
GO TO 10
C ACCOUNT FOR NEGATIVE NUMBERS
20 DO 30 N=1,521
30 IF(Y(N).GT.2047)Y(N)=2048-Y(N)
RETURN
END
```

APPENDIX D-4

CDCC

```

C
SUBROUTINE CDCC(A)
60 DB DOLPH CHEBYSHEV WEIGHTING COEFFICIENTS
DIMENSION A(1024)
N=1024
R=1000.0
V=ALOG(R+SQRT(R*R-1.0))
CALL RESI(V,BV,IER)
FN=N-1
C=2.0/FN
M=0
DO 40 J=1,N
M=M+1
FJ=J-1
Z=FJ*C-1.0
IF(J.NE.1.AND.J.NE.N)GO TO 20
A(M)=FN/(2.0*V*BV)
GO TO 30
20 T=SQRT(1.0-Z*Z)
VTT=V*T
CALL RESI(VTT,BVT,IER)
A(M)=BVT/(T*BV)
30 CONTINUE
40 CONTINUE
RETURN
END

```

APPENDIX D-5

BESI

BESI-1

```
C
SUBROUTINE BESI(X,BI,IER)
  MODIFIED BESSEL FUNCTION OF THE FIRST KIND, FIRST ORDER
  IER=0
  BI=0.0
  IF(X.NE.0.0)GO TO 10
  GO TO 100
10 IF(X.GT.0.0)GO TO 20
  IER=1
  GO TO 100
20 TOL=1.E-6
  IF(X.GT.12.0)IER=2
  XX=X/2.0
  TERM=XX
  BI=XX
  XX=XX*XX
  DO 30 K=1,500
    IF(ABS(TERM).LT.ABS(BI*TOL))GO TO 100
    FK=K*(K+1)
    TERM=TERM*XX/FK
  30 BI=BI+TERM
  IER=3
100 RETURN
  END
```

APPENDIX D-6

STAR

STAR -1

```
SUBROUTINE STAR(N)
  DIMENSION IFORM(81)
  IF(N.LE.0)RETURN
  IF(N.GT.80)RETURN
  DO 1 I=1,81
  1 IFORM(I)=1H
  IFORM(1)=9H(1H+,51X,
    M=N+1
  DO 2 I=2,M
  2 IFORM(I)=4H1H ,
    IFORM(M)=4H1H*)
  PRINT IFORM
  RETURN
END
```

APPENDIX D-7

GNRN

GNRN-1

```

SUBROUTINE GNRN (RN,X1,X2,SIG,FMU)
DIMENSION RN(24)
: GENERATE UNIFORM RANDOM NUMBERS
DO 30 J=1,24
30 RN(J)=RANF(RNARG)

C
C
C
: GENERATE 2 NORMAL RANDOM NUMBERS

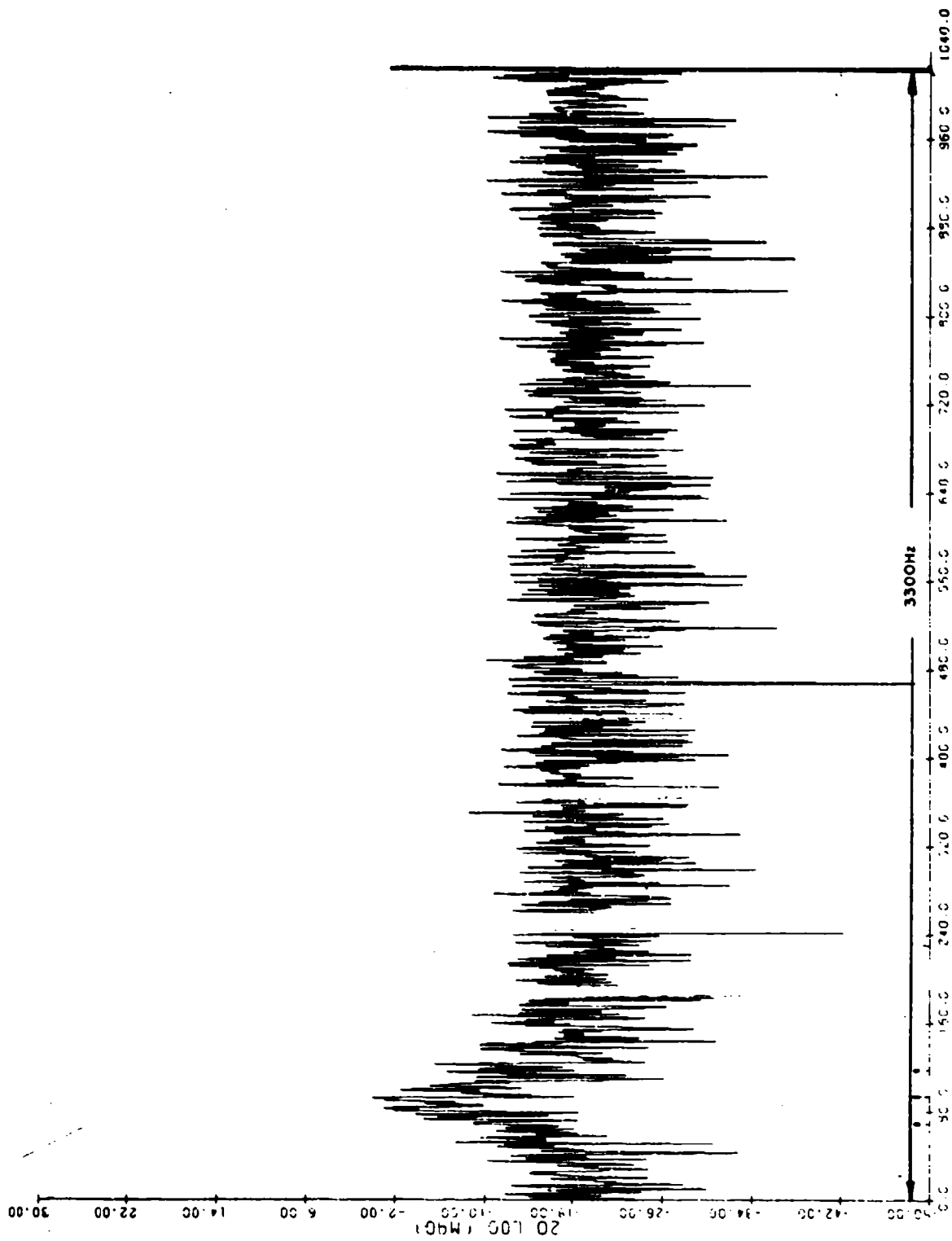
X1=RN(1)
X2=RN(13)
DO 40 J=2,12
X1=X1+RN(J)
40 X2=X2+RN(J+12)
X1=(X1-6.0)*SIG+FMU
X2=(X2-6.0)*SIG+FMU
RETURN
END
```

APPENDIX E

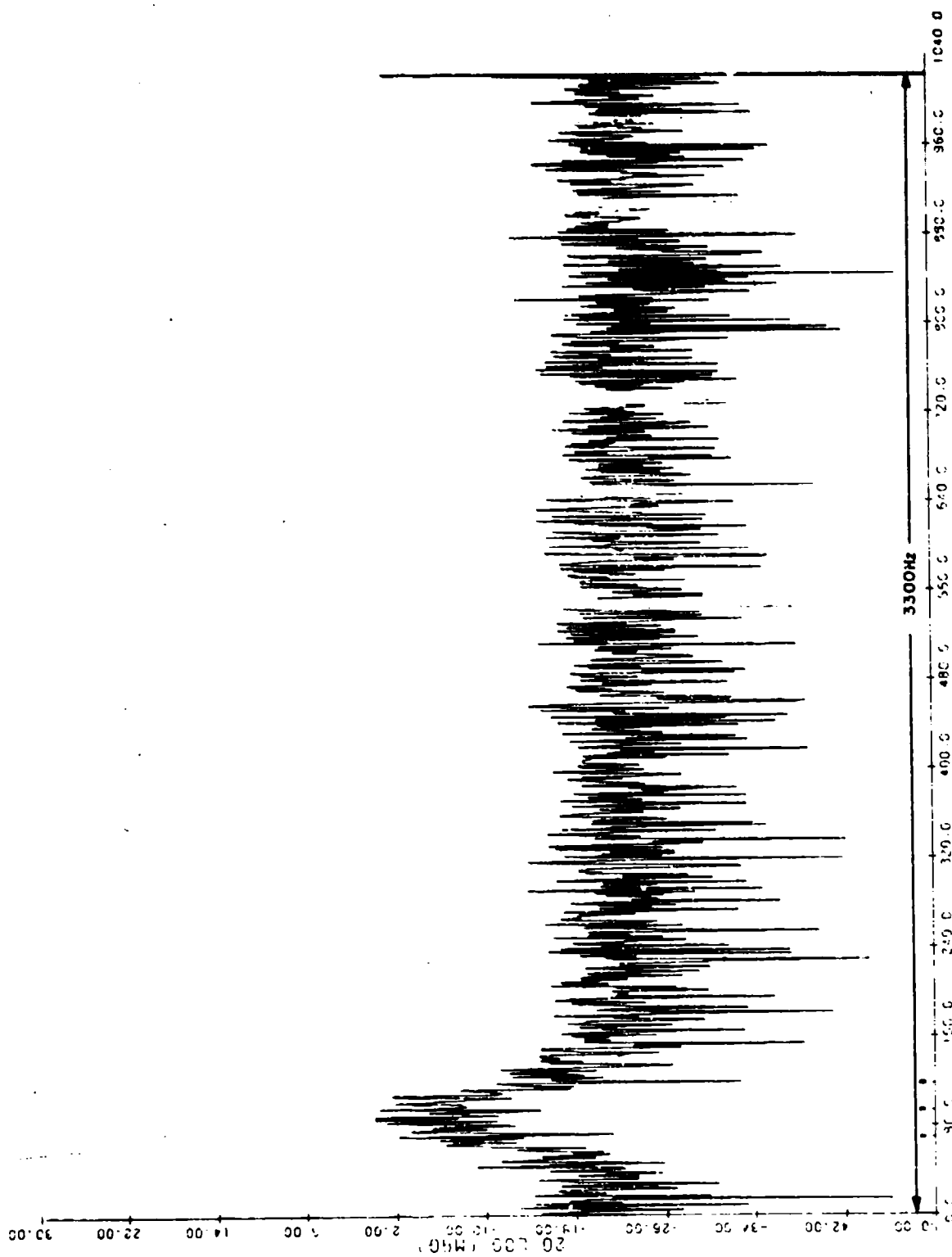
SPECTRAL ANALYSIS PLOTS OF TYPICAL SEQUENCES OF SIMULATED DOPPLER RADAR TARGET RETURN

The radar PRF is 3300 PPS in all cases. Mean, width, and single pulse S/N parameters are as listed below:

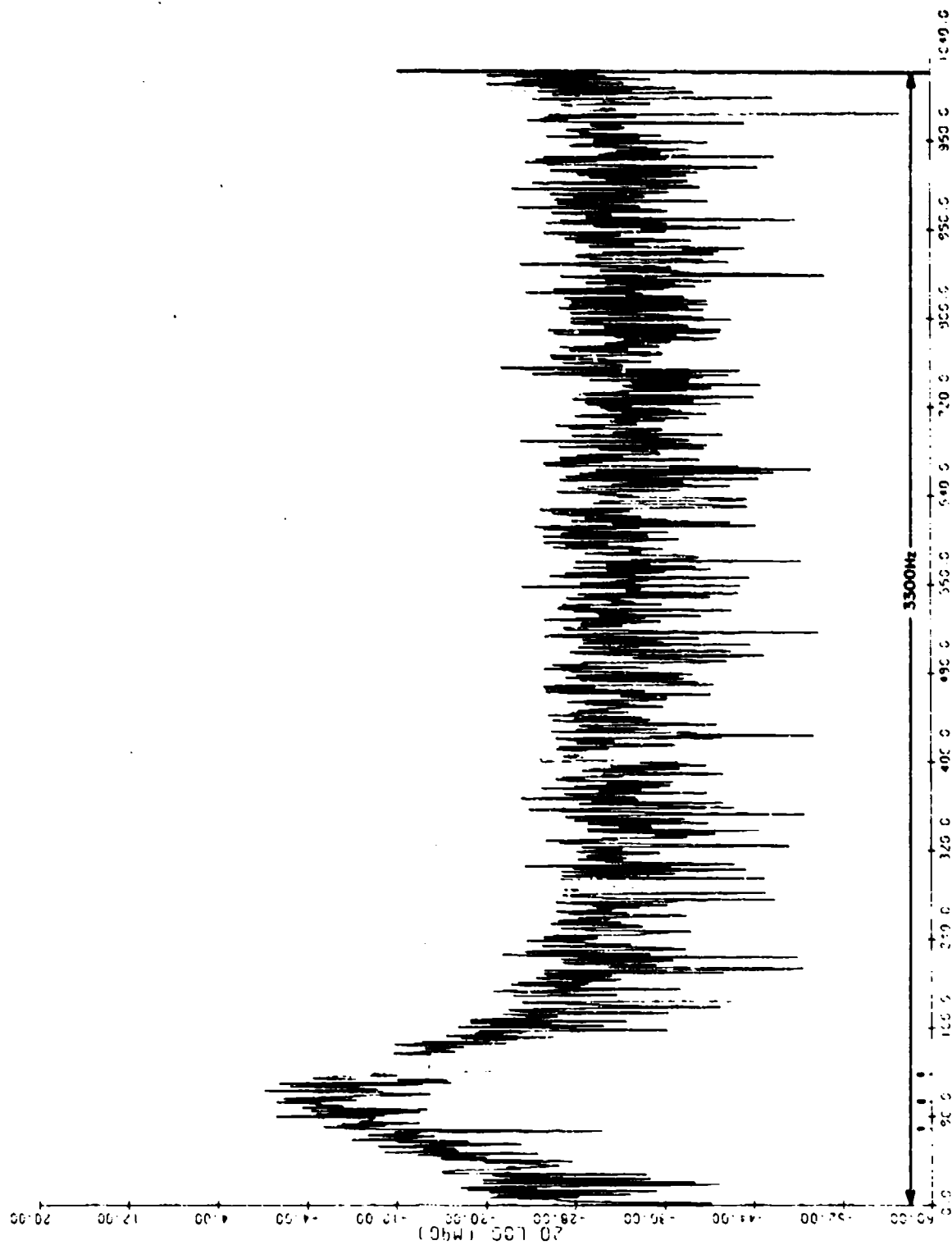
<u>Case</u>	<u>Mean (μ) Hz</u>	<u>Width (σ) Hz</u>	<u>Single-Pulse S/N dB</u>
1	300	78	0
2	300	78	5
3	300	78	15
4	300	78	∞
5	600	156	0
6	600	156	5
7	600	156	15
8	600	156	∞
9	900	234	0
10	900	234	5
11	900	234	15
12	900	234	∞



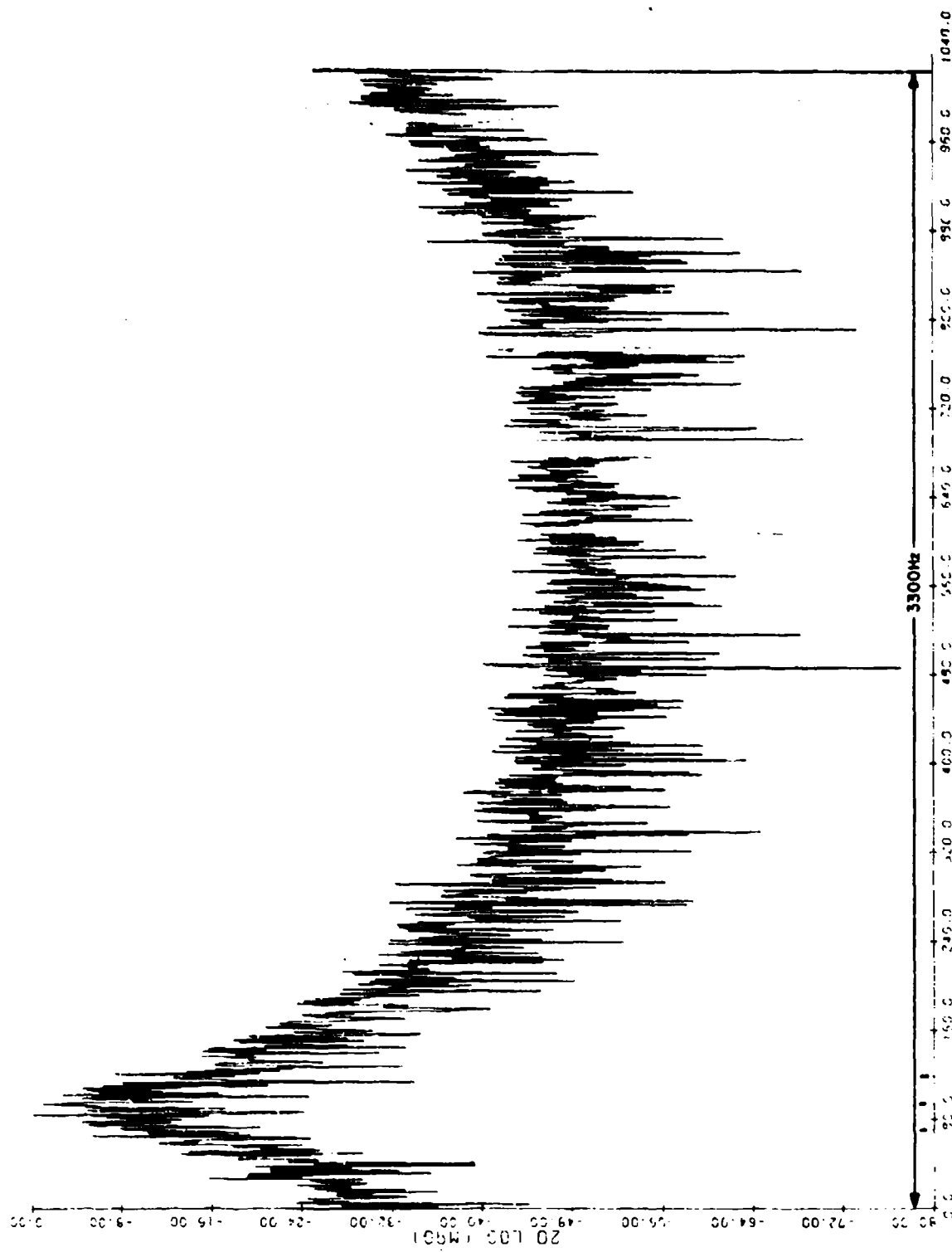
CASE 1: $\mu = 300 \text{ Hz}$, $\sigma = 78 \text{ Hz}$, $S/N = 0 \text{ dB}$



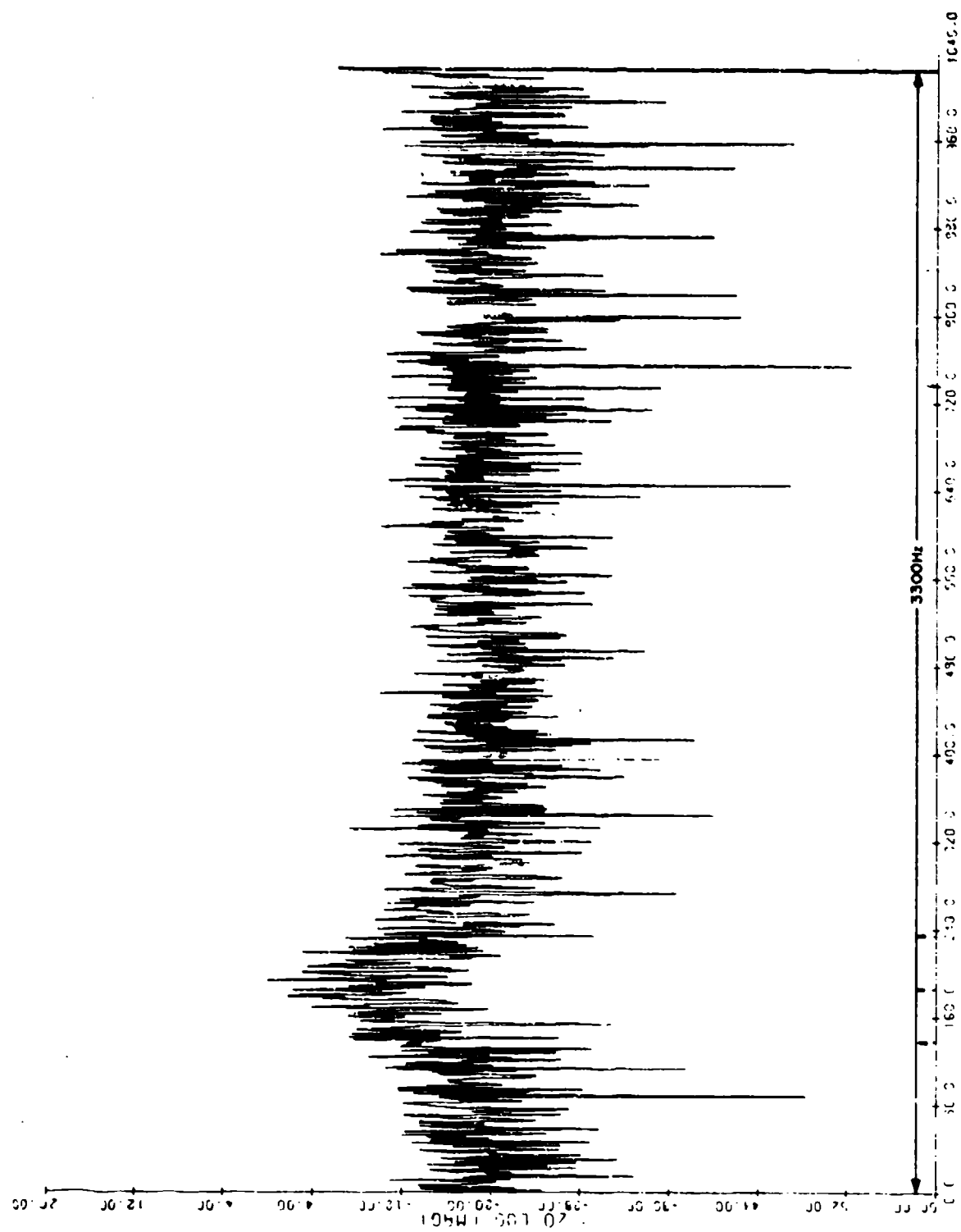
CASE 2: $\mu = 300 \text{ Hz}$, $\sigma = 78 \text{ Hz}$, $S/N = 5 \text{ dB}$



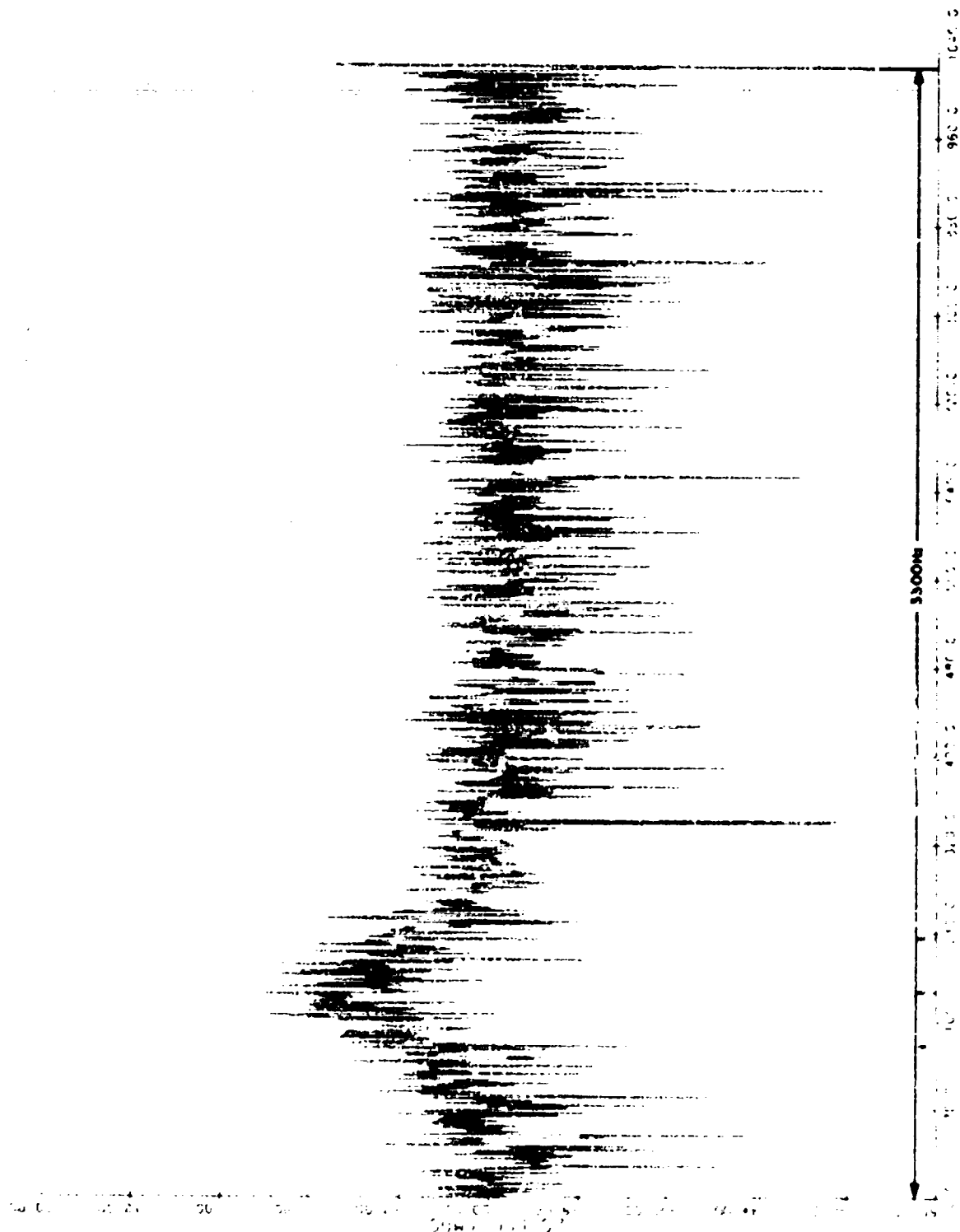
CASE 3: μ 300 Hz, σ = 78 Hz, S/N = 15 dB



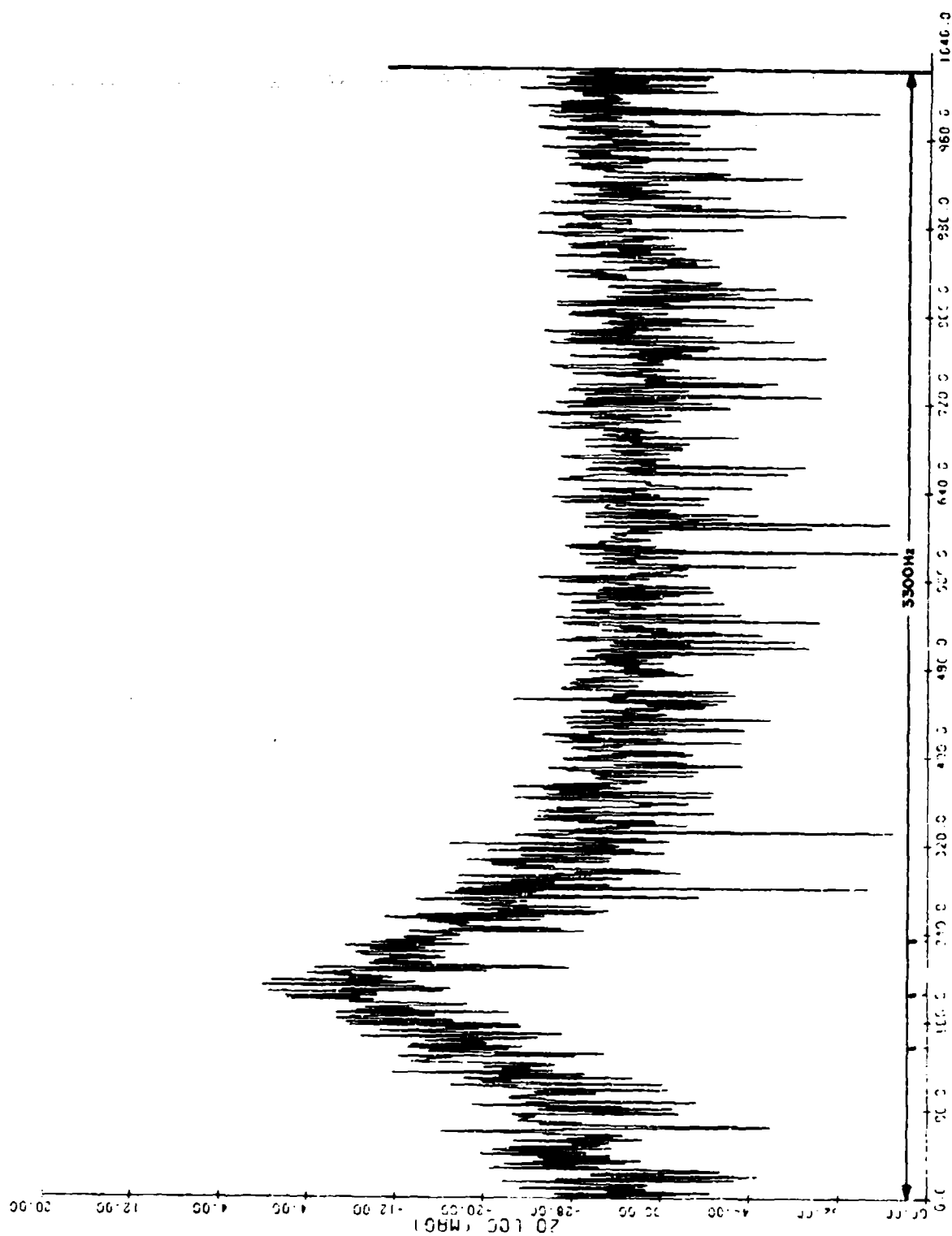
CASE 4: $\mu = 300 \text{ Hz}$, $\sigma = 78 \text{ Hz}$, $S/N = \infty$



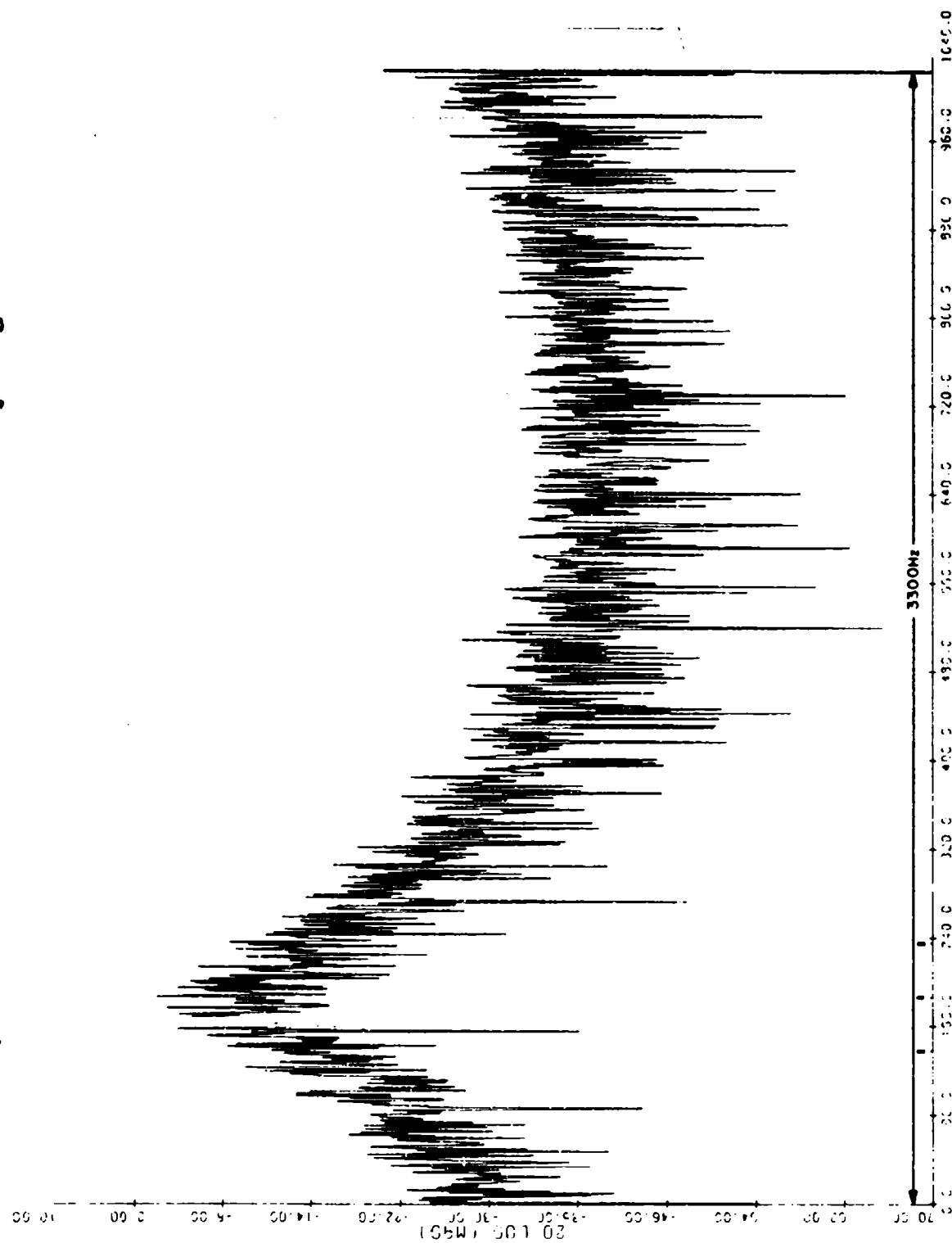
CASE 5: $\mu = 600 \text{ Hz}$, $\sigma = 156 \text{ Hz}$, $S/N = 0 \text{ dB}$



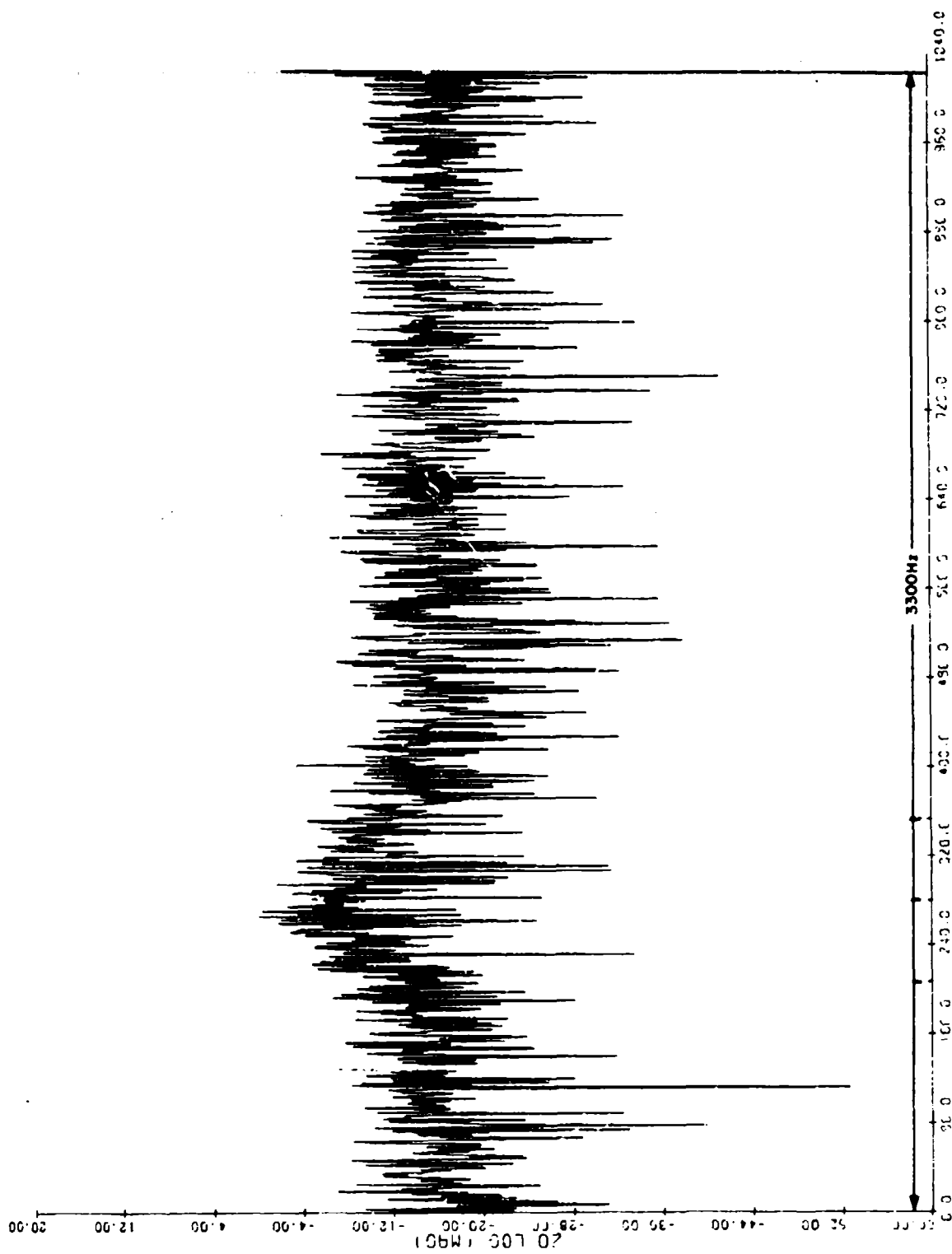
CASE 6: $\mu = 600 \text{ Hz}$, $\sigma = 156 \text{ Hz}$, $S/N = 5 \text{ dB}$



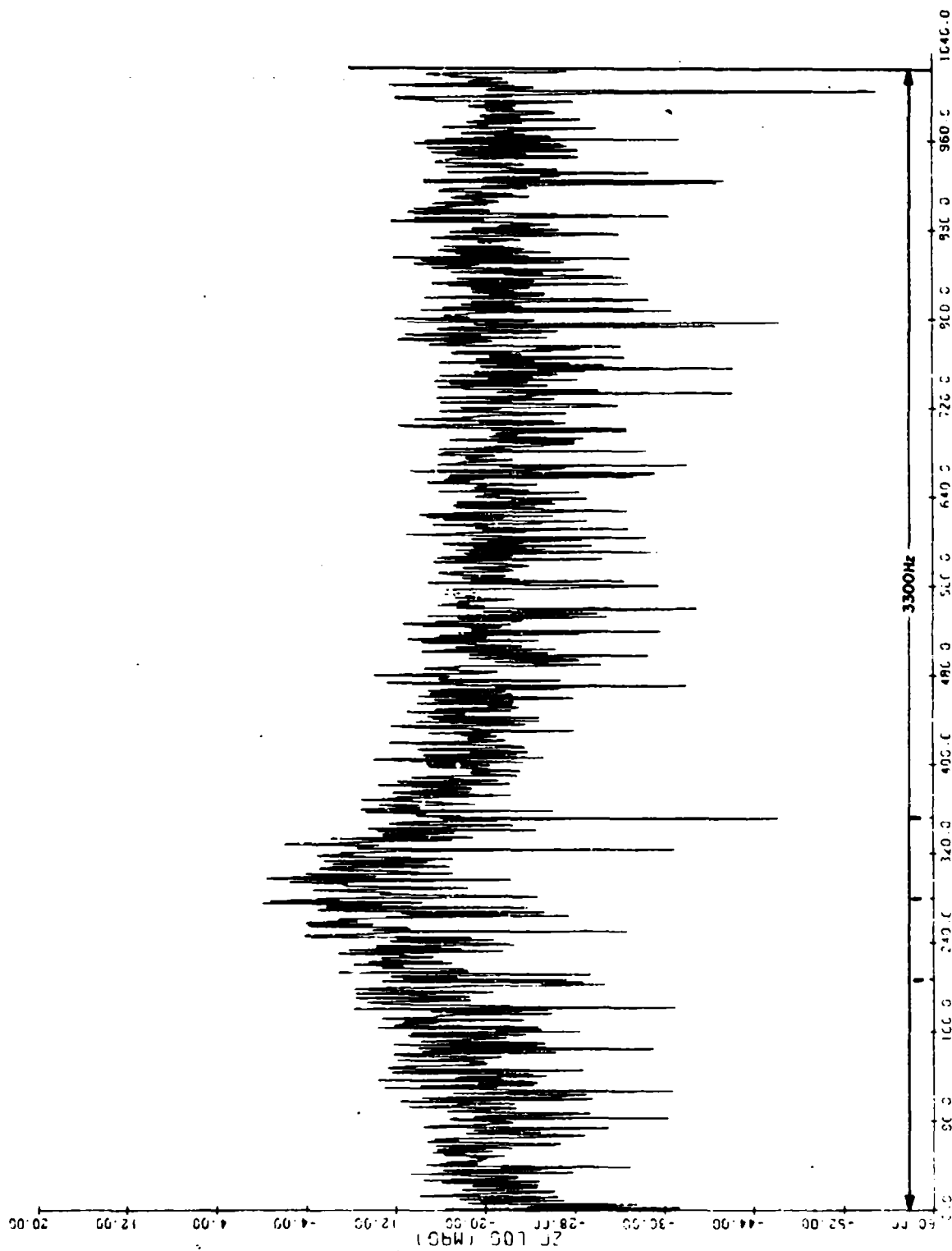
CASE 7: $\mu = 600 \text{ Hz}$, $\sigma = 156 \text{ Hz}$, $S/N = 15 \text{ dB}$



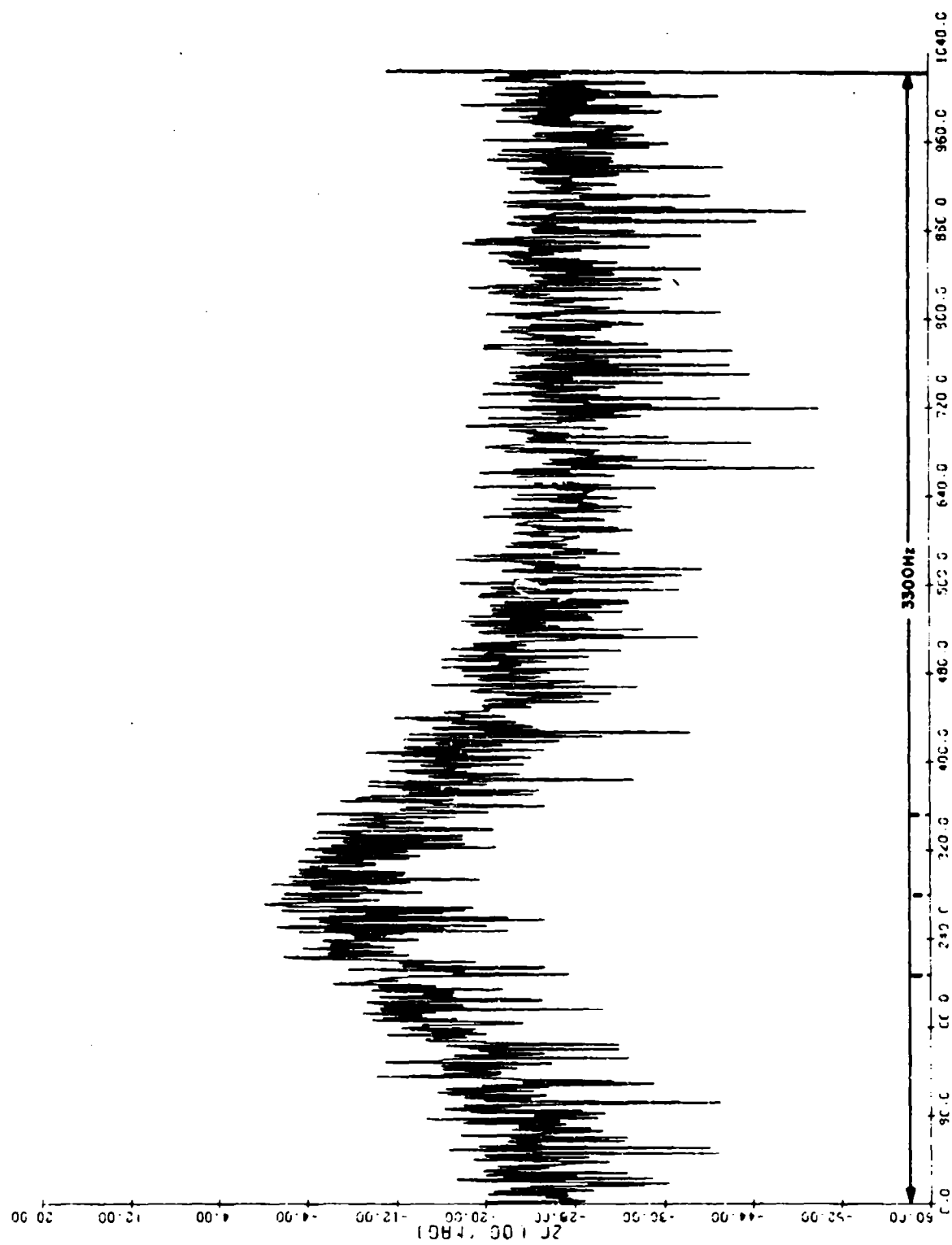
CASE 8: $\mu = 600 \text{ Hz}$, $\sigma = 156 \text{ Hz}$, $S/N = \infty$



CASE 9: $\mu = 900 \text{ Hz}$, $\sigma = 234 \text{ Hz}$, $S/N = 0 \text{ dB}$



CASE 10: $\mu = 900 \text{ Hz}$, $\sigma = 234 \text{ Hz}$, $S/N = 5 \text{ dB}$



CASE 11: $\mu = 900 \text{ Hz}$, $\sigma = 234 \text{ Hz}$, $S/N = 15 \text{ dB}$

