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GENERALIZED MATHEMATICAL MODEL OF A MACHINE GUN TRIPOD MOUNT

U.S. ARMY WEAPONS COMMAND

TECHNICAL REPORT

April 1972

RESEARCH DIRECTORATE

WEAPONS LABORATORY AT ROCK ISLAND

RESEARCH, DEVELOPMENT AND ENGINEERING DIRECTORATE

U. S. ARMY WEAPONS COMMAND

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An investigation was conducted by the Research Directorate, Weapons Laboratory at Rock Island to determine the structural dynamic characteristics of the M122 Tripod Mount with enough generality to be applicable to other tripod mounts of the same general configuration. The M122 Tripod Mount was mathematically modeled as a system of equations. This model is compatible with the mathematical model of the weapon developed by the USAWECOM to produce a combined weapon-mount mathematical model. Two models of the weapon-mount system were developed. The first was a linear description in which the mount was modeled as a spring with three degrees of freedom. The second was a nonlinear model where the forward leg of the tripod mount could lift off the ground as the mount is rotated about its aft two foot-pads. (U) (Moulton, P. B. and Holland, R. S.)

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FOREWORD

This report was prepared by P. B. Moulton and R. S. Holland, Kinetic Systems, Incorporated, Boston, Massachusetts, under Contract DAAF03-70-C-0037.

This work was conducted under the direction of the Research Directorate, Weapons Laboratory at Rock Island, U. S. Army Weapons Command, with Catherine M. Robinder and Robert H. Coberly as project engineers.

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INTRODUCTION

Experience with conventional small bore automatic weapons has shown that the weapon performance is greatly affected by variations in the stiffness of the weapon support. In the past this effect has been evaluated by actual test firings of the weapon-mount system. But this process is too expensive and time consuming to be useful to evaluate proposed systems since the new weapon-mount system must first be manufactured and then tested. It is therefore necessary to develop a method to aid in the design and evaluation of small bore automatic weapon systems.

The U. S. Army Weapons Command has investigated modeling the weapon-mount system mathematically. This approach allows the system's parameters to be varied so that the performance of the weapon system can be optimized. The modeling concept takes advantage of modern digital computor techniques such as trade-off studies to assist in weapon design without the requirement to construct and test actual hardware.

The study reported herein is to develop the mathematical model of the weapon mount. A general system of equations has been developed for tripod mounts similar in construction to the M122. A few such mounts are the M3, M12, M74 and M1917A2. The general equations have been developed for both a linear and a nonlinear system. In the linear representation the tripod mount acts as a set of orthogonal springs having flexibility in all three axes. The nonlinear model of the mount allows the front leg of the tripod frame to lift off the ground and the weapon-mount system to pivot about its aft two foot pads. The coefficients of the general equations have been evaluated for the M122 Tripod Mount and are presented in Section 3 of this report.

A brief introduction to the flexibility method of matrix analysis has been included as an appendix to the report. This will aid as a review to the matrix manipulations used throughout the report.

<u>Summary of General Tripod Equations.</u>

The initial phase of the investigation was to review the physical parameter of various tripod mounts such as the firing rate, weapon to mount weight ratio and the geometry. The M2, M3, M122, M74 and M191782 were typical tripod mounts used for the comparison. An estimate was made as to the first resonant frequency of the weapon using the mount as a spring. The initial investigation indicated that the first resonant frequency of the weapon-mount system was much higher than the firing rate. It was therefore possible to model the tripod mount as a system of springs without any intermediate mass points since the two frequencies were far enough apart that they were virtually decoupled.

The mathematical model describes the mount as a linear elastic system in terms of the three force displacement coordinates as shown in Figure 1.1. These coordinates at the weapon-mount interface are weapon coordinates and move with the weapon in elevation (θ) and azimuth (γ). The elastic properties of the tripod mount are presented in matrix format in terms of these three coordinates. The terms in the flexibility matrix are functions of elevation, azimuth and geometry of the weapon-mount system.

The flexibility of the tripod mount was obtained by using the flexibility method of structural analysis. The general concepts of the flexibility method applicable to this study are presented in Appendix A. The elements of the flexibility matrix are obtained by the following three steps:

Step 1. The tripod mount is divided into three substructures shown in Figure 1.1 as the main frame, the pintle, and the traverse and elevating components. The structural characteristics of each substructure are expressed in terms of their individual flexibility matrices:

$[\alpha_m]$	Mount frame flexibility matrix.
$[\alpha_p]$	Pintle flexibility matrix.
$[\alpha_t]$	Traverse and elevating components flexibility matrix.

These three unconnected matrices are written as one system flexibility matrix $\lceil\alpha\rfloor$.

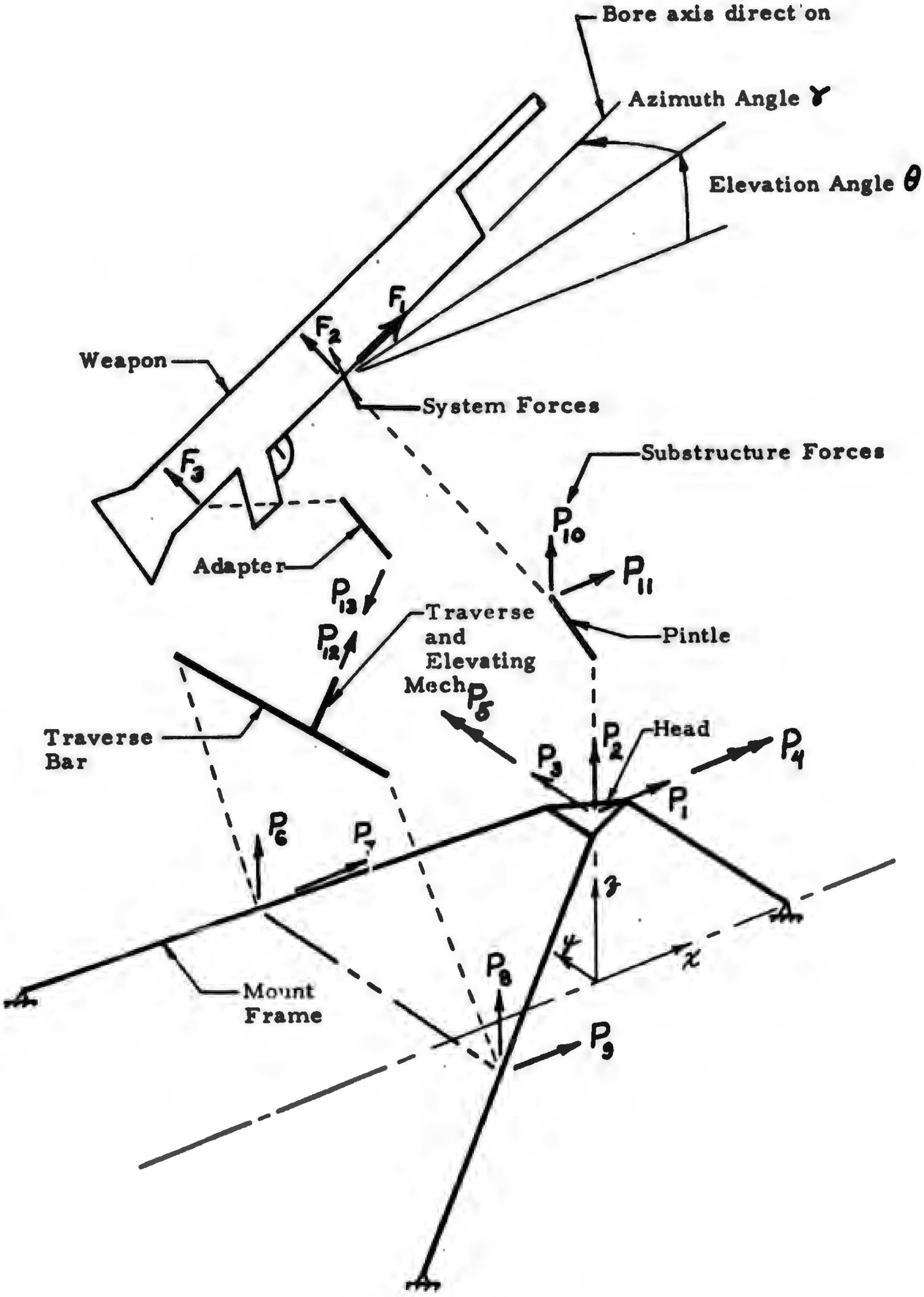


Figure 1.1 Weapon-Mount Coordinate System

$$
\left[\alpha\right] = \begin{bmatrix} [\alpha_m] & & \\ & [\alpha_p] & \\ & & [\alpha_t] \end{bmatrix} = \left[\begin{array}{ccccccccc|cc|cc}
\alpha_{11} & \alpha_{12} & 0 & 0 & \alpha_{15} & \alpha_{16} & \alpha_{17} & \alpha_{18} & \alpha_{19} & & & & \\
 & \alpha_{22} & 0 & 0 & \alpha_{25} & \alpha_{26} & \alpha_{27} & \alpha_{28} & \alpha_{29} & & & & \\
 & & \alpha_{33} & \alpha_{34} & 0 & \alpha_{36} & \alpha_{37} & -\alpha_{36} & -\alpha_{37} & & & & \\
 & & & \alpha_{44} & 0 & \alpha_{46} & \alpha_{47} & -\alpha_{46} & -\alpha_{47} & & 0 & & 0 \\
 & & & & \alpha_{55} & \alpha_{56} & \alpha_{57} & \alpha_{58} & \alpha_{59} & & & & \\
 & & & & & \alpha_{66} & \alpha_{67} & \alpha_{68} & \alpha_{69} & & & & \\
 & & & & & & \alpha_{77} & \alpha_{78} & \alpha_{79} & & & & \\
 & & & & & & & \alpha_{88} & \alpha_{89} & & & & \\
 & & & & & & & & \alpha_{99} & & & & \\
\hline
 & \text{SYM.} & & & & & & & & \alpha_{10,10} & \alpha_{10,11} & & 0 \\
 & & & & & & & & & & \alpha_{11,11} & & \\
\hline
 & & & & & & & & & & & \alpha_{12,12} & 0 \\
 & & & & & & & & & & & & \alpha_{13,13}
\end{array}\right] \qquad (1.1)
$$

The zero terms ($\alpha_{1,3} = 0$, etc.) and the repeated terms ($\alpha_{18} = \alpha_{56}$, etc.) of the mount substructure flexibility occurred due to the symmetry of the mount about the X-Z plane.

Step 2. The internal substructure forces (P_1---P_{13}) and the applied forces (F_1, F_2, F_3) are related by the transformation

$$
\{P\} = [b]\{F\} \qquad (1.2)
$$

where [b] is the force transformation matrix which is obtained from the systems' equations of equilibrium. The development of [b] for the tripod mount is presented in Section 2.2 of this report.

Step 3. The flexibility matrix [a] of the tripod mount is obtained from the following congruent transformation:

$$
[a] = [b]^T [\alpha] [b] \qquad (1.3)
$$

This transformation is developed in Appendix A. The numerical evaluation of the coefficients are presented in Section 2.3 of this report. The tripod mount flexibility has the following matrix form.

$$[a] = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{bmatrix}$$

The individual flexibility coefficients are given below in Equation 1.4. These coefficients are presented in terms of the force transformation elements and the flexibility matrix elements (α_{ij}).

$$\begin{aligned} a_{1,1} = {} & \alpha_{1,1} b_{1,1}^{2} + 2\alpha_{1,2} b_{1,1} b_{2,1} + \alpha_{2,2} b_{2,1}^{2} + 2\alpha_{1,5} b_{5,1} b_{1,1} + 2\alpha_{2,5} b_{2,1} b_{5,1} + \alpha_{3,3} b_{3,1}^{2} \\ & + 2\alpha_{3,4} b_{3,1} b_{4,1} + \alpha_{4,4} b_{4,1}^{2} + \alpha_{5,5} b_{5,1}^{2} + \alpha_{10,10} b_{10,1}^{2} + 2\alpha_{10,11} b_{10,1} b_{11,1} + \alpha_{11,11} b_{11,1}^{2} \end{aligned}$$

$$\begin{aligned} a_{1,2} = {} & \alpha_{1,1} b_{1,1} b_{1,2} + \alpha_{1,2} (b_{1,2} b_{2,1} + b_{1,1} b_{2,2}) + \alpha_{2,2} b_{2,1}^{2} + \alpha_{3,3} b_{3,1} b_{3,2} + \alpha_{3,4} (b_{4,1} b_{3,2} \\ & + b_{3,1} b_{4,2}) + \alpha_{4,4} b_{4,1} b_{4,2} + \alpha_{1,5} (b_{5,1} b_{1,2} + b_{1,1} b_{5,2}) + \alpha_{2,5} (b_{2,1} b_{5,2} + b_{5,1} b_{2,2}) \\ & + \alpha_{5,5} b_{5,1} b_{5,2} + \alpha_{10,10} b_{10,1} b_{10,2} + \alpha_{10,11} (b_{10,2} b_{11,1} + b_{10,1} b_{11,2}) + \alpha_{11,11} b_{11,1} b_{11,2} \end{aligned}$$

$$\begin{aligned} a_{1,3} = {} & \alpha_{1,1} b_{1,1} b_{1,3} + \alpha_{1,2} (b_{1,1} b_{2,3} + b_{2,1} b_{1,3}) + \alpha_{2,2} b_{2,1} b_{2,3} + \alpha_{3,3} b_{3,1} b_{3,3} \\ & + \alpha_{3,4} (b_{4,1} b_{3,3} + b_{3,1} b_{4,3}) + \alpha_{4,4} b_{4,1} b_{4,3} + \alpha_{1,5} (b_{5,1} b_{1,3} + b_{1,1} b_{5,3}) \\ & + \alpha_{1,6} b_{1,1} (b_{6,3} + b_{8,3}) + \alpha_{1,7} b_{1,1} (b_{7,3} + b_{9,3}) + \alpha_{2,6} b_{2,1} (b_{6,3} + b_{8,3}) \\ & + \alpha_{2,7} b_{2,1} (b_{7,3} + b_{9,3}) + \alpha_{3,6} b_{3,1} (b_{6,3} - b_{8,3}) + \alpha_{3,7} b_{3,1} (b_{7,3} - b_{9,3}) \\ & + \alpha_{4,6} b_{4,1} (b_{6,3} - b_{8,3}) + \alpha_{4,7} b_{4,1} (b_{7,3} - b_{9,3}) + \alpha_{5,6} b_{5,1} (b_{6,3} + b_{8,3}) \\ & + \alpha_{5,7} b_{5,1} (b_{7,3} + b_{9,3}) + \alpha_{2,5} (b_{2,1} b_{5,3} + b_{5,1} b_{2,3}) + \alpha_{5,5} b_{5,1} b_{5,3} \\ & + \alpha_{10,10} b_{10,1} b_{10,3} + \alpha_{10,11} (b_{10,3} b_{11,1} + b_{10,1} b_{11,3}) + \alpha_{11,11} b_{11,1} b_{11,3} \end{aligned}$$

$$\begin{aligned} a_{2,2} = {} & \alpha_{1,1} b_{1,2}^{2} + 2\alpha_{1,2} b_{1,2} b_{2,2} + \alpha_{2,2} b_{2,2}^{2} + \alpha_{3,3} b_{3,2}^{2} + 2\alpha_{3,4} b_{3,2} b_{4,2} \\ & + \alpha_{4,4} b_{4,2}^{2} + 2\alpha_{1,5} b_{5,2} b_{1,2} + 2\alpha_{2,5} b_{5,2} b_{2,2} + \alpha_{5,5} b_{5,2}^{2} \\ & + \alpha_{10,10} b_{10,2}^{2} + 2\alpha_{10,11} b_{10,2} b_{11,2} + \alpha_{11,11} b_{11,2}^{2} \end{aligned}$$

$$
\begin{aligned}
a_{2,3} = {} & \alpha_{1,1} b_{1,2} b_{1,3} + \alpha_{1,2}(b_{1,3} b_{2,2} + b_{1,2} b_{2,3}) + \alpha_{2,2} b_{2,2} b_{2,3} + \alpha_{3,3} b_{3,3} b_{3,2} \\
& + \alpha_{3,4}(b_{3,3} b_{4,2} + b_{4,3} b_{3,2}) + \alpha_{4,4} b_{4,2} b_{4,3} + \alpha_{1,5}(b_{1,3} b_{5,2} + b_{1,2} b_{5,3}) \\
& + \alpha_{1,6} b_{1,2}(b_{6,3} + b_{8,3}) + \alpha_{1,7} b_{1,2}(b_{7,3} + b_{9,3}) + \alpha_{2,5}(b_{2,2} b_{5,3} \\
& + b_{2,3} b_{5,2}) + \alpha_{2,6} b_{2,2}(b_{6,3} + b_{8,3}) + \alpha_{2,7} b_{2,2}(b_{7,3} + b_{9,3}) \\
& + \alpha_{3,6} b_{3,2}(b_{6,3} - b_{8,3}) + \alpha_{3,7} b_{3,2}(b_{7,3} - b_{9,3}) + \alpha_{4,6} b_{4,2}(b_{6,3} \\
& - b_{8,3}) + \alpha_{4,7} b_{4,2}(b_{7,3} - b_{9,3}) + \alpha_{5,5} b_{5,2} b_{5,3} \\
& + \alpha_{5,6} b_{5,2}(b_{6,3} + b_{8,3}) + \alpha_{5,7} b_{5,2}(b_{7,3} + b_{9,3}) \\
& + \alpha_{10,10} b_{10,2} b_{10,3} + \alpha_{10,11}(b_{10,3} b_{11,2} + b_{10,2} b_{11,3}) + \alpha_{11,11} b_{11,2} b_{11,3}
\end{aligned}
$$

$$
\begin{aligned}
a_{3,3} = {} & \alpha_{1,1} b_{1,3}^2 + 2\alpha_{1,2} b_{1,3} b_{2,3} + \alpha_{2,2} b_{2,3}^2 + \alpha_{3,3} b_{3,3}^2 + 2\alpha_{3,4} b_{3,3} b_{4,3} \\
& + \alpha_{4,4} b_{4,3}^2 + 2\alpha_{1,5} b_{1,3} b_{5,3} + 2\alpha_{1,6} b_{1,3}(b_{6,3} + b_{8,3}) + 2\alpha_{1,7} b_{1,3}(b_{7,3} \\
& + b_{9,3}) + 2\alpha_{2,5} b_{2,3} b_{5,3} + 2\alpha_{2,6} b_{2,3}(b_{6,3} + b_{8,3}) \\
& + 2\alpha_{2,7} b_{2,3}(b_{7,3} + b_{9,3}) + 2\alpha_{3,6} b_{3,3}(b_{6,3} - b_{8,3}) \\
& + 2\alpha_{3,7} b_{3,3}(b_{7,3} - b_{9,3}) + 2\alpha_{4,6} b_{4,3}(b_{6,3} - b_{8,3}) + 2\alpha_{4,7} b_{4,3}(b_{7,3} - b_{9,3}) \\
& + \alpha_{5,5} b_{5,3}^2 + 2\alpha_{5,6} b_{5,3}(b_{6,3} + b_{8,3}) + 2\alpha_{5,7} b_{5,3}(b_{7,3} + b_{9,3}) \\
& + \alpha_{6,6}(b_{6,3}^2 + b_{8,3}^2) + 2\alpha_{6,7}(b_{6,3} b_{7,3} + b_{8,3} b_{9,3}) + 2\alpha_{6,8} b_{6,3} b_{8,3} \\
& + 2\alpha_{6,9}(b_{6,3} b_{9,3} + b_{7,3} b_{8,3}) + \alpha_{7,7}(b_{7,3}^2 + b_{9,3}^2) + 2\alpha_{7,9} b_{7,3} b_{9,3} \\
& + \alpha_{10,10} b_{10,3}^2 + 2\alpha_{10,11} b_{10,3} b_{11,3} + \alpha_{11,11} b_{11,3}^2 + \alpha_{12,12} b_{12,3}^2 \\
& + \alpha_{13,13} b_{13,3}^2
\end{aligned}
\qquad (1.4)
$$

The lower triangular terms of [a] are symmetric with their transpose terms, e.g., $a_{2,1} = a_{1,2}$, etc. The force transformation terms ($b_{i,j}$) in the above equation are presented on the following page. The generalized mount dimensions used in these relations are shown in Figure 1.2.

$$
\begin{aligned}
b_{1,1} &= \cos\theta \cos\gamma & b_{1,2} &= -\sin\theta \cos\gamma & b_{1,3} &= \xi_3 \cos\gamma\left(\cos\theta - \frac{l_4}{l_3}\sin\theta\right) \\
b_{2,1} &= \sin\theta & b_{2,2} &= \cos\theta & b_{2,3} &= \xi_3\left(\sin\theta + \frac{l_4}{l_3}\cos\theta\right) \\
b_{3,1} &= \sin\gamma \cos\theta & b_{3,2} &= -\sin\theta \sin\gamma & b_{3,3} &= \xi_3 \sin\gamma\left(\cos\theta - \frac{l_4}{l_3}\sin\theta\right) \\
b_{4,1} &= -\xi_1 \sin\gamma & b_{4,2} &= -\xi_2 \sin\gamma & b_{4,3} &= -\xi_3 \sin\gamma\left(\xi_1 + \frac{l_4}{l_3}\xi_2\right) \\
b_{5,1} &= \xi_1 \cos\gamma & b_{5,2} &= \xi_2 \cos\gamma & b_{5,3} &= \xi_3 \cos\gamma\left(\xi_1 + \frac{l_4}{l_3}\xi_2\right) \\
b_{10,1} &= \sin\theta & b_{10,2} &= \cos\theta & b_{6,3} &= \xi_4 \xi_5 \cos\psi \\
b_{11,1} &= \cos\theta & b_{11,2} &= -\sin\theta & b_{7,3} &= \xi_4 \xi_5 \sin\psi \cos\gamma \\
& & & & b_{8,3} &= \xi_4 \xi_6 \cos\psi \\
& & & & b_{9,3} &= \xi_4 \xi_6 \sin\psi \cos\gamma \\
& & & & b_{10,3} &= \xi_3\left(\sin\theta + \frac{l_4}{l_3}\cos\theta\right) \\
& & & & b_{11,3} &= \xi_3\left(\cos\theta - \frac{l_4}{l_3}\sin\theta\right) \\
& & & & b_{12,3} &= \xi_4 \\
& & & & b_{13,3} &= \xi_4
\end{aligned}
\qquad (1.5)
$$

where:

$$\xi_1 = l_1 \sin\theta + l_2 \cos\theta$$

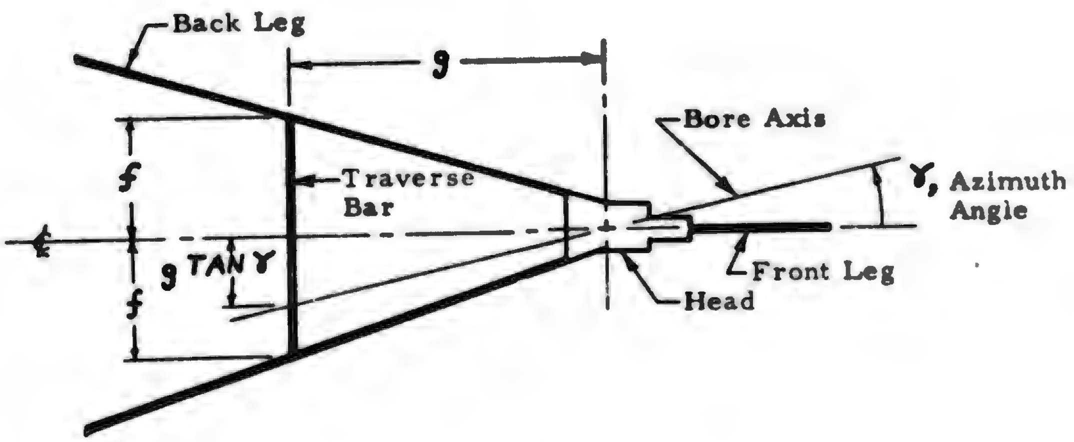
$$\xi_2 = l_1 \cos\theta - l_2 \sin\theta$$

$$\xi_3 = \frac{\sin(\psi+\theta)}{\cos(\psi+\theta) - \frac{l_4}{l_3}\sin(\psi+\theta)}$$

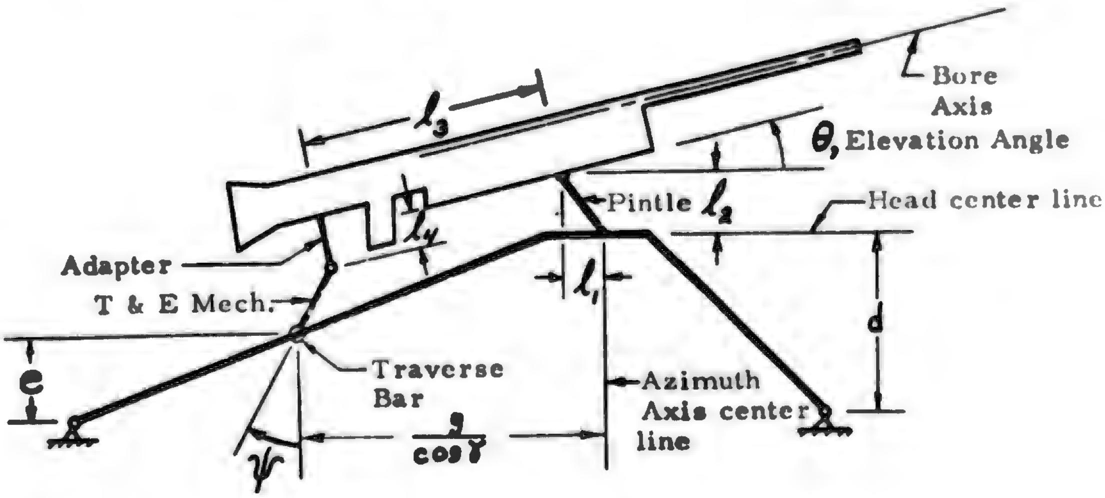
$$\xi_4 = \frac{1}{\cos(\psi+\theta) - \frac{l_4}{l_3}\sin(\psi+\theta)}$$

$$\xi_5 = \frac{f - g \tan\gamma}{2f}$$

$$\xi_6 = \frac{f + g \tan\gamma}{2f}$$



a) Top View



b) Side View in Bore Axis Plane

Figure 1.2 Generalized Tripod Mount Dimensions

1.2 Summary of M122 Tripod Mount Analyses

The flexbility of the M122 tripod mount is computed in Section 3 of this report. Equation 1.4 was used for this computation with the weapon in the reference position, i.e., $\theta = 0^\circ$, $\gamma = 0^\circ$. Two different boundary conditions were selected for this evaluation. The first condition is with all three legs of the mount pinned at the mount-ground interface. The second condition allowed the forward leg of the mount to be free in the forward (x) direction. These two boundary conditions give an approximate upper and lower bound of the mount flexibility as a function of the ground stiffness.

Pinned

$$[a] = \begin{bmatrix} 43.82 & 27.49 & 30.78 \\ 27.49 & 20.15 & 22.32 \\ 30.78 & 22.32 & 271.07 \end{bmatrix} \times 10^{-6} \text{ (in/lbs)} \qquad (1.6)$$

Front leg released

$$[a] = \begin{bmatrix} 55.77 & -36.21 & -19.88 \\ -36.21 & 359.86 & 292.50 \\ -19.88 & 292.50 & 485.95 \end{bmatrix} \times 10^{-6} \text{ (in/lbs)} \qquad (1.7)$$

The stiffness of the M122 Tripod Mount in the bore axis direction for a single degree of freedom system has been computed for the two different boundary conditions. The stiffness in the bore axis direction is the inverse of the $\alpha_{1,1}$ term in the flexibility matrix. For the $\theta = 0^\circ$, $\gamma = 0^\circ$ condition the bore axis stiffness is 18,150 and 23,200 lbs/in. Figure 1.3 shows the affect of elevation and azimuth angle on the bore axis stiffness.

Both the results of Figure 1.3 and the comparison between Equations 1.6 and 1.7 indicate that the flexibility of the tripod mount is strongly in-

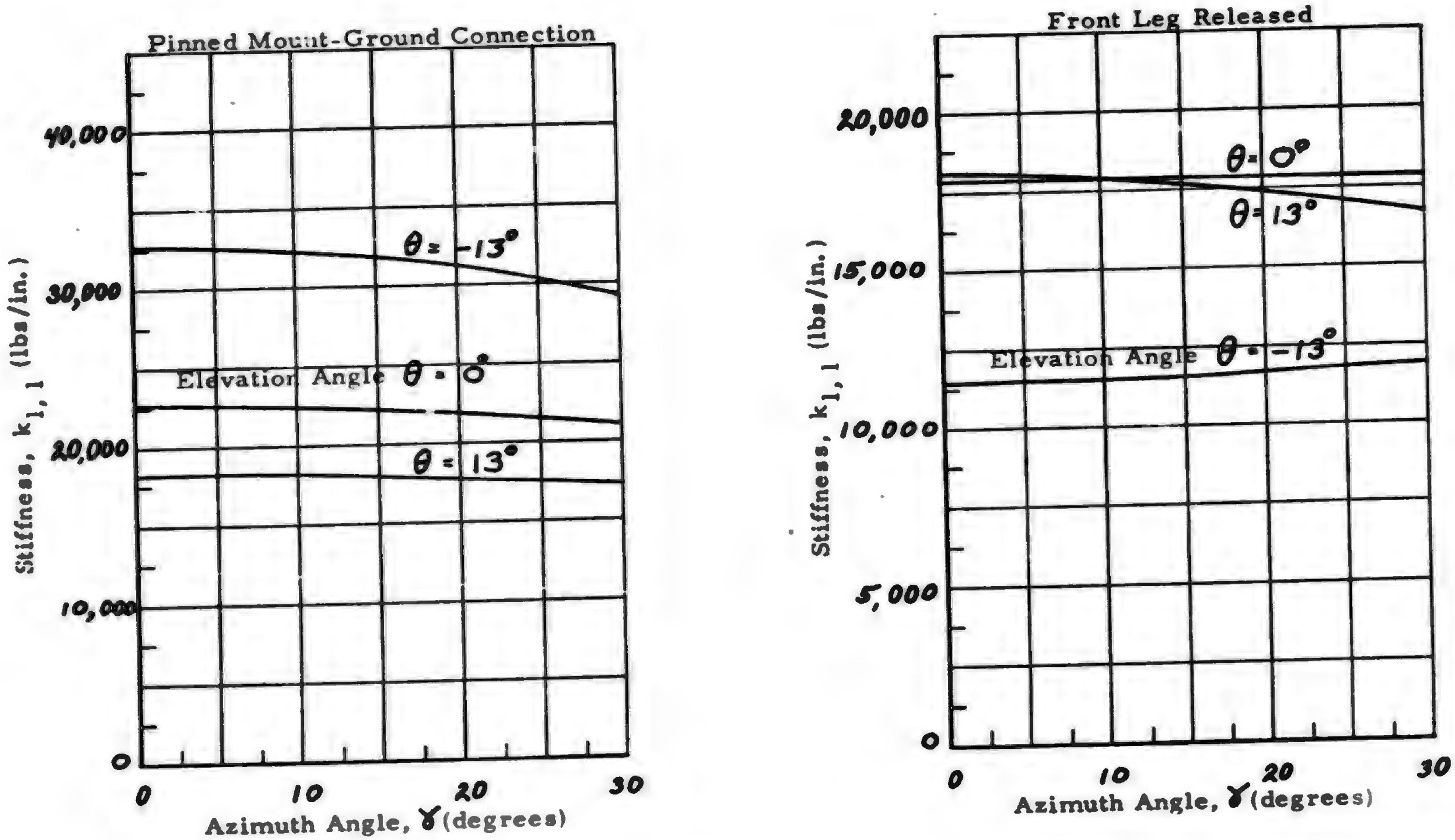


Figure 1. 3 Structural Stiffness of the M122 Tripod Mount for a Single Degree of Freedom System in the Bore Axis Direction.

fluenced by the earth-mount interaction. The earth's flexibility increases the flexibility of the tripod mount. The effect of this influence is not as great in the bore axis direction as in the vertical direction. This can be seen by comparing Equations 1.6 and 1.7 where the $\alpha_{1,1}$ term represents the displacement in the bore axis direction due to a force in that direction and $\alpha_{2,2}$ represents the displacement in the x_2 direction due to the force F_2.

Section 2
METHOD OF ANALYSIS

The method of obtaining the flexibility matrix of the weapon-mount system is developed using a generalized model of a tripod mount. The result of this method is a 3x3 matrix describing the flexibility at the mount-weapon interface. Figure 2.1 shows three weapon coordinates; x_1, x_2 and x_3. These weapon coordinates move with the weapon both in elevation and traverse angle. Coordinate x_1 is parallel with the bore axis at the weapon attachment to the pintle. Coordinates x_2 and x_3 are perpendicular to the bore axis and are in the vertical plane. Coordinate x_2 is at the pintle and coordinate x_3 at the adapter. As shown in Figure 2.1 the mount-weapon interface forces F_1, F_2 and F_3 correspond to the three weapon coordinates x_1, x_2 and x_3. The reference direction, when the elevation angle $\theta = 0°$ and the traverse angle $\gamma = 0°$, is with the bore axis in the horizontal plane pointing along the direction of the forward tripod mount leg. It should be noted that throughout the analysis, the foot pads of the mount are assumed to be supported in a horizontal plane.

The flexibility method was selected to develop the mount deformation relations since it facilitates direct evaluation of the various deformation characteristics in the final tripod mount flexibility equations. The result of the analysis is the mount flexibility matrix [a] in terms of the three interface coordinates, x_1, x_2 and x_3.

The analysis of the weapon mount is accomplished by dividing the mount structure into three basic substructures which are

1. The Mount Frame
2. The Pintle
3. The Traverse and Elevating (T & E) Components.

These three substructures are shown in Figure 2.2.

The flexibilities (α's) of each of these substructures are determined in terms of the substructure's coordinates (δ's) and their forces (P's). Each

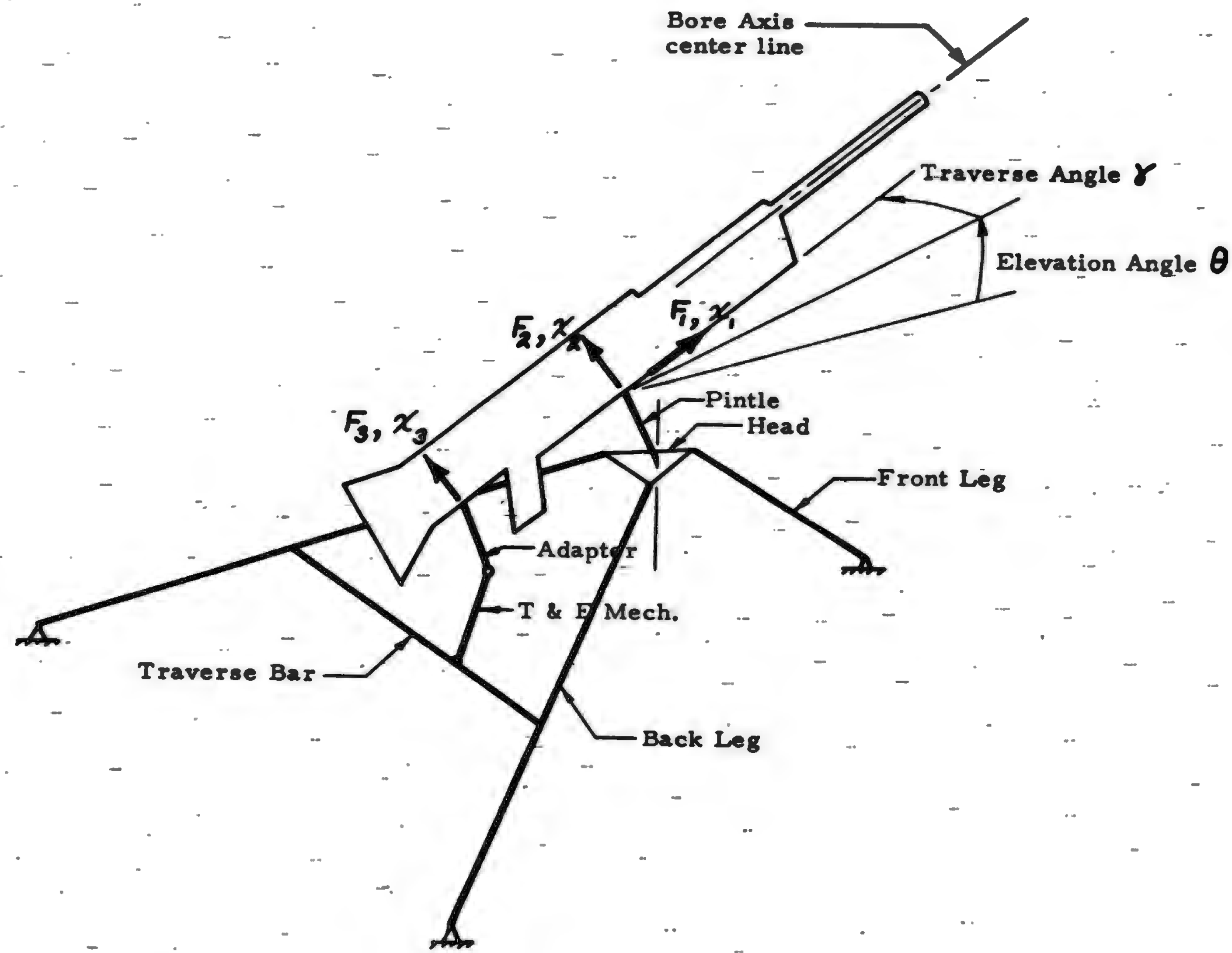


Figure 2.1 Weapon Coordinate System at the Tripod Mount Interface

substructure flexibility includes all of its pertinent structural elements. The three mount matrices are defined as:

1) $[\alpha_m]$ mount frame
2) $[\alpha_p]$ pintle
3) $[\alpha_t(\theta, \gamma)]$ traverse and elevating components

The coordinate numbering and positive sense for these substructures is shown in Figure 2.2

The three substructure matrices can be assembled into a single flexibility matrix as follows:

$$[\alpha] = \begin{bmatrix} [\alpha_m] & & \\ & [\alpha_p] & \\ & & [\alpha_t] \end{bmatrix} \tag{2.1}$$

For a given elevation and traverse angle, a force transformation relating weapon forces (F_1, F_2, F_3) and substructure forces (P_1---P_{13}) can be defined as follows:

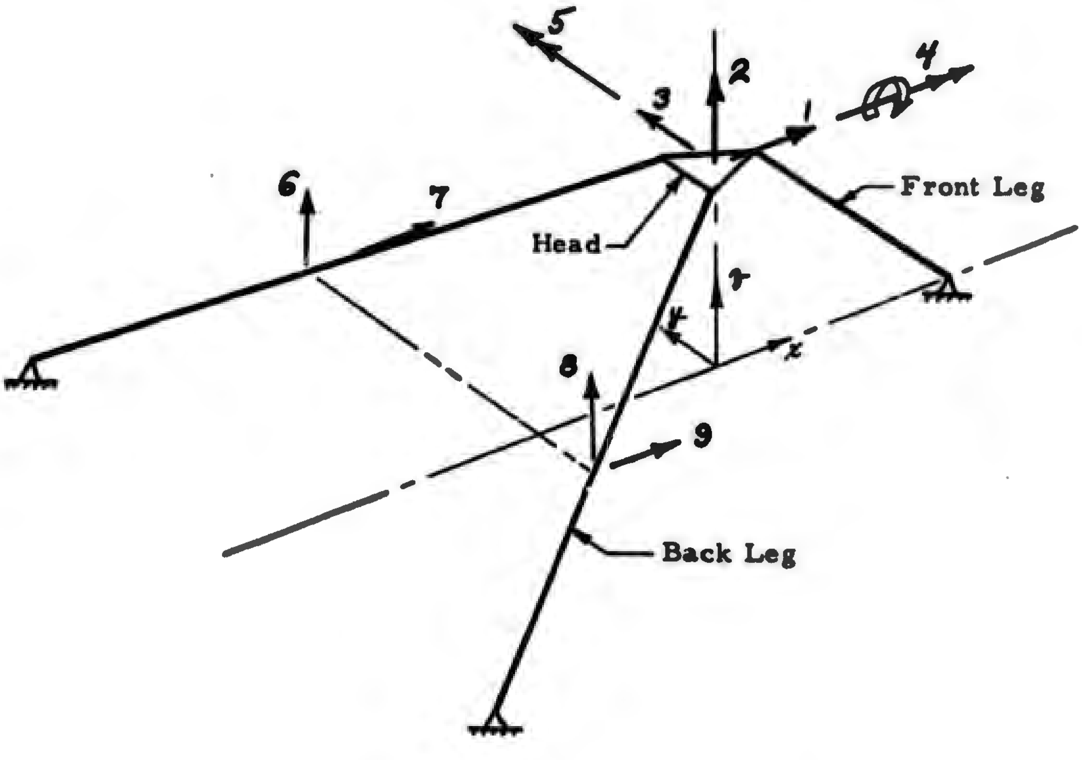
$$\{P\} = [b]\{F\} \tag{2.2}$$

where [b] is the force transformation between the weapon forces $\{F\}$ and the substructure forces $\{P\}$. The force transformation can be partitioned into three sections corresponding to each of the three substructures as follows:

$$[b] = \begin{bmatrix} b_m \\ \hline b_p \\ \hline b_t \end{bmatrix} \begin{matrix} \text{mount frame} \\ \text{pintle} \\ \text{traverse and elevation components} \end{matrix} \tag{2.3}$$

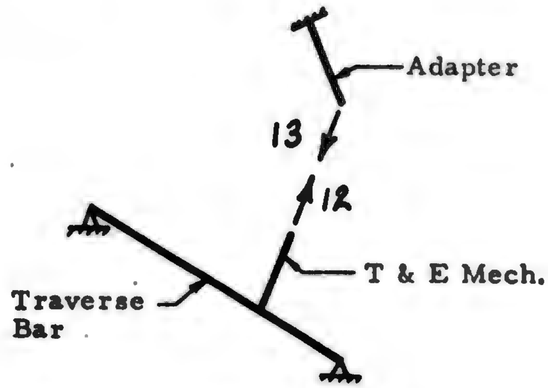
The weapon-mount flexibility matrix is finally obtained by a congruent transformation of the subsystem's flexibility matrix using the force transformation matrix.

$$[a] = [b]^T[\alpha][b] \tag{2.4}$$



a) Mount Frame

b) Pintle



c) Traverse and Elevating Components

Figure 2.2 Coordinate System for the Three Substructures of the Tripod Mount

This can be rewritten in simplified form since the $[\neg\alpha\_]$ matrix is a diagonal matrix.

$$[a] = \sum_{i=m,p,t} [b_i]^T [\alpha_i][b_i] \tag{2.5}$$

2.1 Tripod Mount Geometry

The geometric relationships of the mount and the weapon are defined in Figure 2.3. This figure shows the geometry and connectivity of the mount and mount-weapon interface. Lengths ℓ_3 and ℓ_4 define the relative positions of the adapter hinged support point with respect to the pintle. The over-all pintle dimensions with respect to the intersection of the pintle rotation axis and the center of the head are defined by ℓ_1 and ℓ_2. The pintle support point on the mount frame and the traverse bar are defined by dimensions d, e, f and g. The weapon mount system is assumed to be hinged at three locations in the bore axis plane: 1) at the pintle-weapon junction, 2) at the adapter - T & E mechanism junction and 3) at the T & E mechanism-traverse bar junction. The hinged connections at the top and bottom T & E mechanism are significant because it allows only axial load along the line of action of the two hinges. Consequently the reactions of the weapon due to the pintle and the adapter are determinant.

The geometry assumed for the analysis deviates from the actual structure in that the actual T & E mechanism is clamped so that it always remains perpendicular to the axis of the traverse bar, and in the assumed geometry this mechanism remains in the vertical plane through the bore axis. This results in a slight error of the T & E loads and deflections, but the over-all loads and reaction in the remaining portions of the system are still correct as shown in Figure 2.4.

The traverse and elevating mechanism angle is defined from the mount and weapon geometry and the weapon position as shown in Figure 2.5. The mechanism slope is:

$$x_1 = \frac{g}{\cos\gamma} - \ell_3 \cos\theta + \ell_4 \sin\theta - \ell_1$$

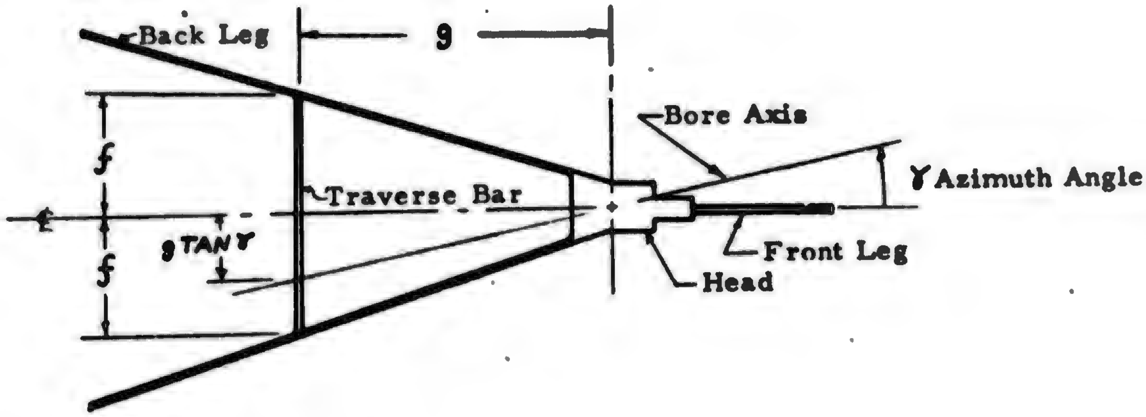
and

$$x_2 = d - e + \ell_2 - \ell_3 \sin\theta - \ell_4 \cos\theta \tag{2.6}$$

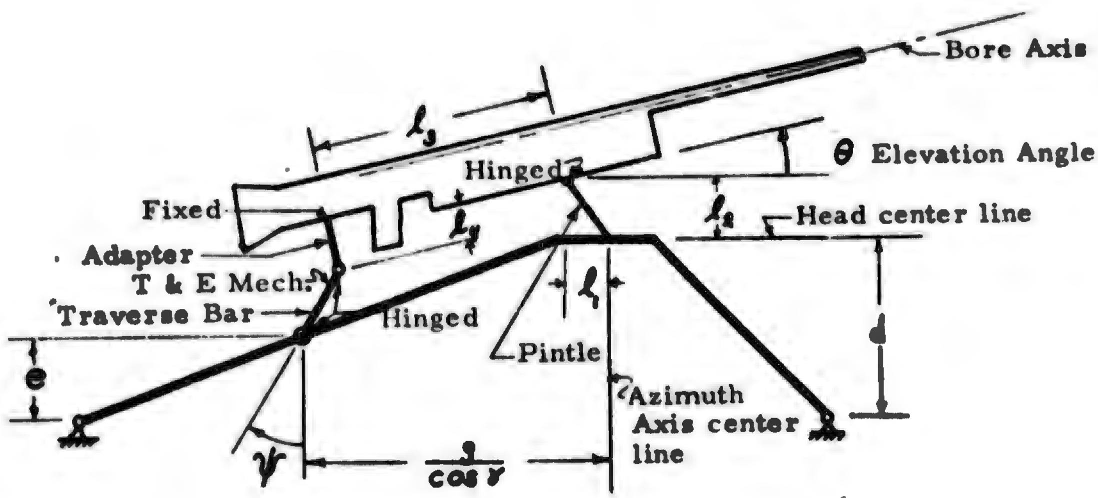
where

$$\psi = \arctan \frac{x_1}{x_2} \tag{2.7}$$

and the length of the mechanism L is equal to $\sqrt{x_1^2 + x_2^2}$. (2.8)

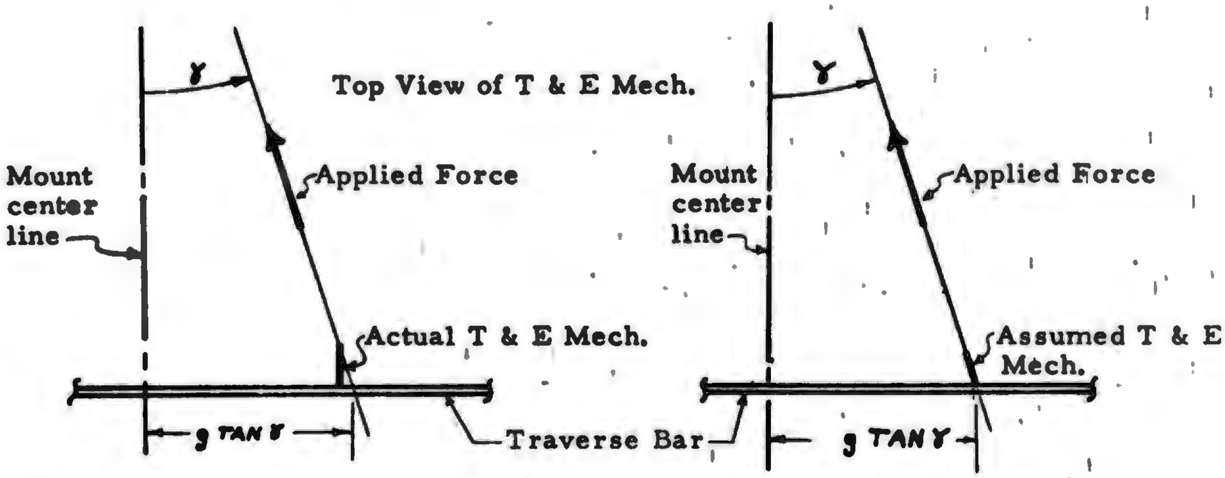


a) Top View



b) Side View in Bore Axis Plane

Figure 2. 3 Geometry of Generalized Tripod Mount

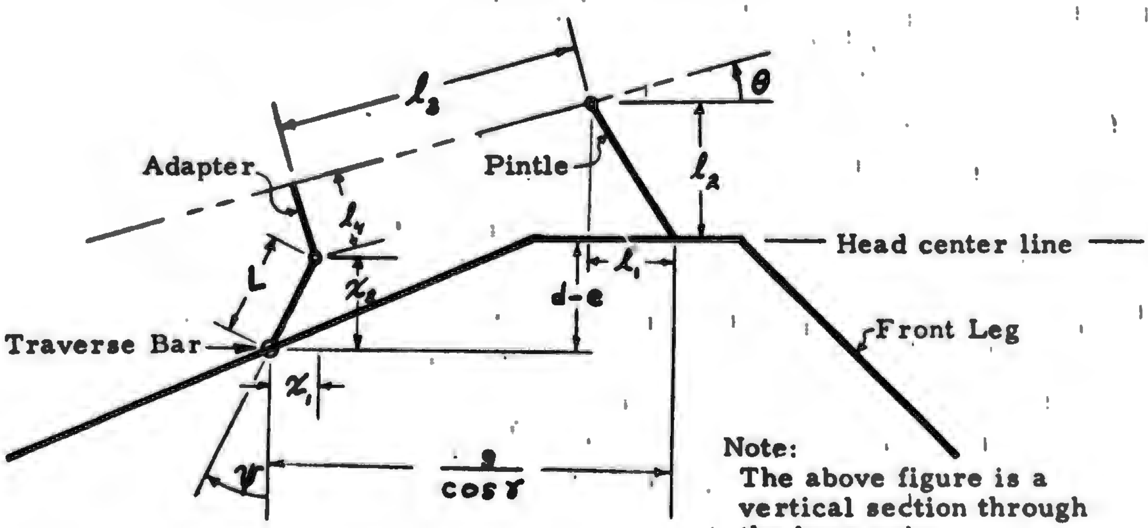


a) Actual Geometry

b) Assumed Geometry

Note: The assumed geometry of the T & E Mech. applies the same loads on the Traverse Bar as the actual geometry.

Figure 2.4 Traverse and Elevating Mechanism Actual Geometry and Assumed Geometry.



Note:
The above figure is a vertical section through the bore axis.

Figure 2.5 Traverse and Elevating Mechanism Angle γ.

2.2 Force Transformation Matrix

The terms of the force transformation matrix [b] presented in Equation 2.2 are developed in this section. The ij th term of the transformation is defined as follows:

$b_{i,j}$ = value of the i th substructure force, (P_i), due to the application of a unit force at the j th system coordinate (F_j = 1.0) with all other system forces zero.

The terms of the [b] matrix are found by applying a unit force at each system coordinate, one at a time and computing the resulting internal forces at the substructure coordinates (P_1 - - - P_{13}).

Computation of $b_{i,1}$ Terms

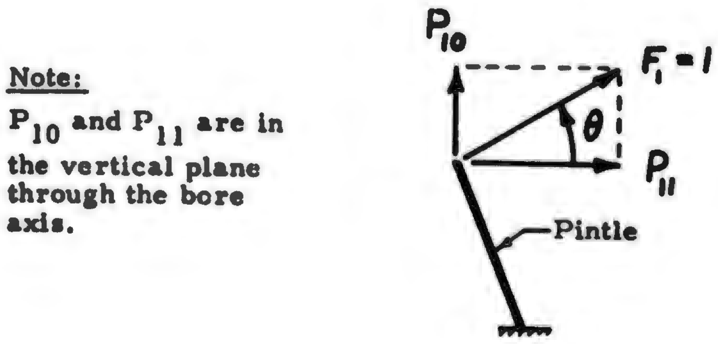
To compute the first column of the [b] force transformation matrix, apply $F_1 = 1$ and let $F_2 = F_3 = 0$. Referring to Figure 2.1, it is recognized that a force applied at the top of the pintle is resisted entirely by the pintle and the mount head; no load exists in the T & E components.

Figure 2.6 shows the reduction of the substructure forces resulting from $F_1 = 1$. The substructure coordinates are obtained from Figure 2.2. The pintle forces, Figure 2.6a, are found by resolving $F_1 = 1$ into the P_{10} and P_{11} directions:

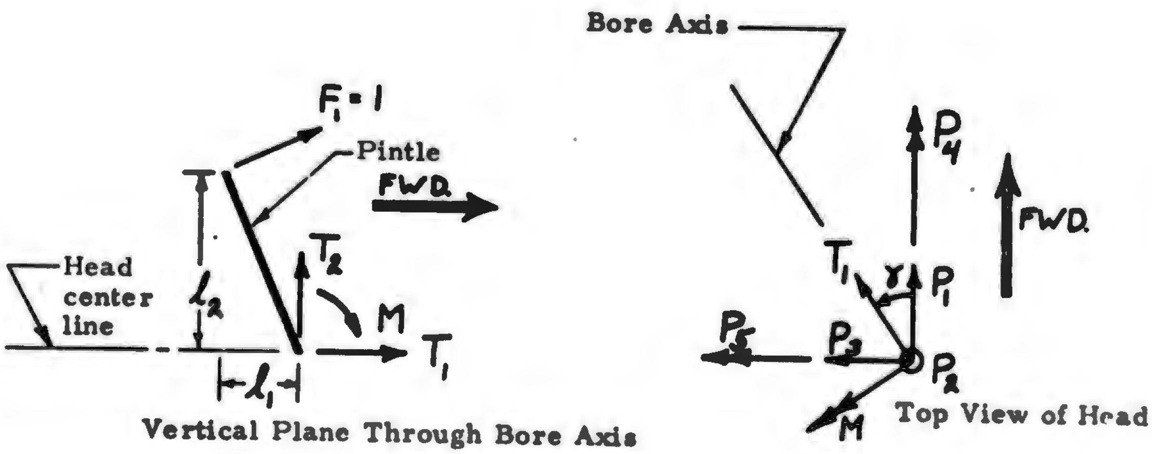
$$P_{10} = \sin\theta = b_{10,1}$$
$$P_{11} = \cos\theta = b_{11,1} \qquad (2.9)$$

The forces on the head are found by first resolving forces in the vertical plane of the bore axis to the center line of the head, see Figure 2.6b, this yields

$$T_1 = \cos\theta$$
$$T_2 = \sin\theta$$
$$M = \ell_1 \sin\theta + \ell_2 \cos\theta \qquad (2.10)$$



a) Pintle Substructure Forces



b) Mount Frame Substructure Forces

Figure 2.6 Resolution of Forces due to a Unit Load at F_1

Then the forces from Equation 2.10 are resolved into the frame forces as follows:

$$
\begin{aligned}
P_1 &= T_1 \cos\gamma = \cos\gamma\cos\theta &&= b_{1,1}\\
P_2 &= T_2 = \sin\theta &&= b_{2,1}\\
P_3 &= T_1 \sin\gamma = \sin\gamma\cos\theta &&= b_{3,1}\\
P_4 &= -M\sin\gamma = -\sin\gamma(\ell_1\sin\theta + \ell_2\cos\theta) &&= b_{4,1}\\
P_5 &= M\cos\gamma = \cos\gamma(\ell_1\sin\theta + \ell_2\cos\theta) &&= b_{5,1}
\end{aligned}
\tag{2.11}
$$

All other tripod mount forces are zero, i.e., $P_6 = P_7 = P_8 = P_9 = P_{12} = P_{13} = 0$, and therefore their corresponding force transformation terms $b_{6,1} = b_{7,1} = b_{8,1} = b_{9,1} = b_{12,1} = b_{13,1} = 0$.

Computation of $b_{i,2}$ Terms

Following the same procedure used to find Equation 2.11 the substructure forces due to $F_2 = 1$ are as follows:

$$
\begin{aligned}
P_1 &= -\sin\theta\cos\gamma &&= b_{1,2}\\
P_2 &= \cos\theta &&= b_{2,2}\\
P_3 &= -\sin\theta\sin\gamma &&= b_{3,2}\\
P_4 &= -\sin\gamma(\ell_1\cos\theta - \ell_2\sin\theta) &&= b_{4,2}\\
P_5 &= \cos\gamma(\ell_1\cos\theta - \ell_2\sin\theta) &&= b_{5,2}\\
P_{10} &= \cos\theta &&= b_{10,2}\\
P_{11} &= -\sin\theta &&= b_{11,2}
\end{aligned}
\tag{2.12}
$$

As before, all other tripod mount forces and the corresponding force transformation terms $b_{i,2}$ are zero.

Computation of $b_{i,3}$ Terms

The computation of the $b_{i,3}$ terms is performed by applying a unit load for F_3. This case however differs from the previous two cases because application of F_3 results in reactions through both the T & E components and the pintle. The applied load, reactions and geometry are shown in Figure 2.7. First the static reactions R_1 and R_2 due to $F_3 = 1$ will be calculated and then the P_{13} and P_{12} forces will be calculated.

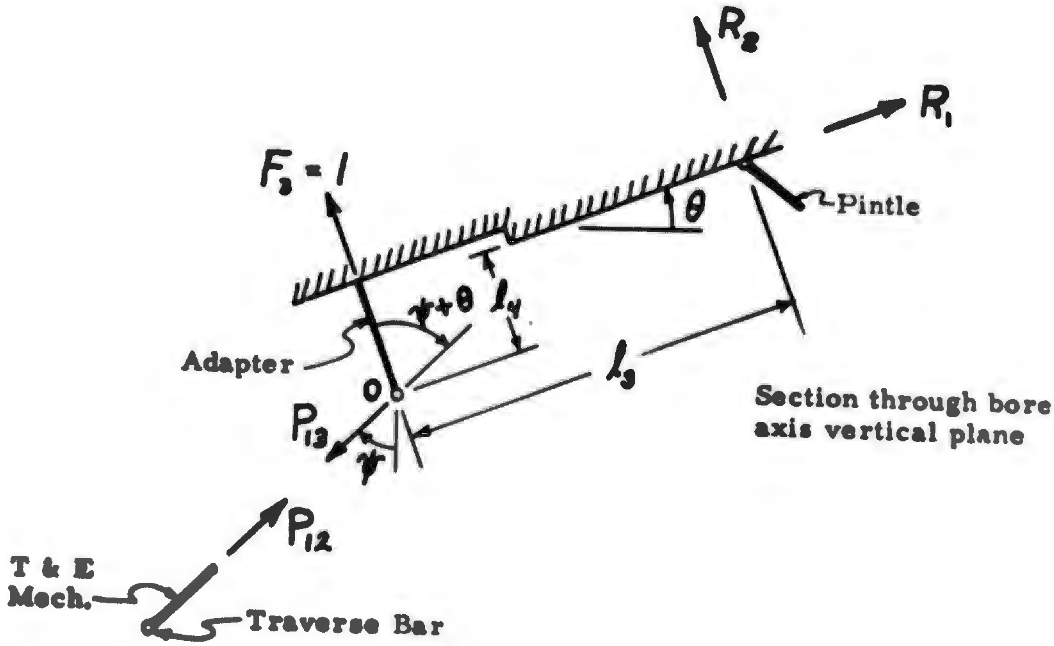


Figure 2.7 Weapon Reactions For $F_3 = 1$

Reaction R_1 is obtained by summing the forces in the R_1 direction.

$$R_1 = P_{13} \sin(\psi + \theta) \tag{2.13}$$

Equilibrium of forces in the R_2 direction yields

$$R_2 + 1 = P_{13} \cos(\psi + \theta) \tag{2.14}$$

Equilibrium of moments about O yields

$$\ell_3 R_2 = \ell_4 R_1$$

$$R_2 = \frac{\ell_4}{\ell_3} R_1 \qquad (2.15)$$

Substitution of (2.15) into (2.14) produces

$$\frac{\ell_4}{\ell_3} R_1 + 1 = P_{13} \cos(\gamma + \theta)$$

therefore

$$R_1 = \frac{\ell_3}{\ell_4}\left(P_{13} \cos(\gamma + \theta) - 1\right) \qquad (2.16)$$

Substracting (2.13) from (2.16) yields

$$0 = \frac{\ell_3}{\ell_4}\left(P_{13} \cos(\gamma + \theta) - 1\right) - P_{13} \sin(\gamma + \theta)$$

Therefore, P_{13} - which is the $b_{13,3}$ term - equals:

$$P_{13} = \frac{1}{\cos(\gamma+\theta) - \frac{\ell_4}{\ell_3} \sin(\gamma+\theta)} = b_{13,3} \qquad (2.17)$$

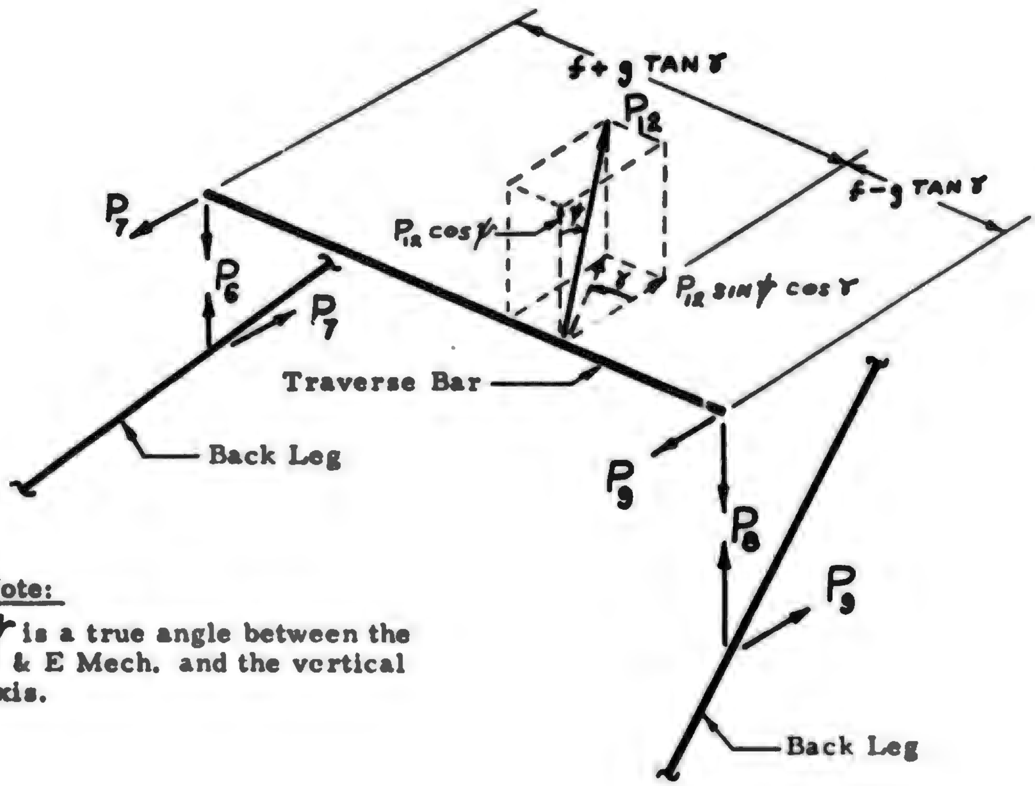
R_1 can now be found using Equation (2.16): This reaction will be needed to compute the pintle reaction.

$$R_1 = \frac{\sin(\gamma+\theta)}{\cos(\gamma+\theta) - \frac{\ell_4}{\ell_3} \sin(\gamma+\theta)} \qquad (2.18)$$

Since P_{13} and P_{12} are colinear (as shown in Figure 2. 7), $P_{12} = P_{13}$, therefore:

$$b_{12,3} = b_{13,3} \qquad (2.19)$$

The mount forces P_6 to P_9 due to the T & E components are shown in Figure 2. 8. P_{12} is resolved into its vertical and horizontal directions as shown. P_6 and P_8 are obtained from the equilibrium of the traverse



Note:

ψ is a true angle between the T & E Mech. and the vertical axis.

Figure 2.8 Free Body Diagram of the Traverse Bar

bar in the vertical direction.

$$P_6 = P_{12} \cos\psi \frac{f - g \tan\gamma}{2f}$$

$$P_8 = P_{12} \cos\psi \frac{f + g \tan\gamma}{2f} \qquad (2.20a)$$

Similarly P_7 and P_9 are obtained from the equilibrium in the horizontal direction.

$$P_7 = P_{12} \sin\psi \cos\gamma \frac{f - g \tan\gamma}{2f}$$

$$P_9 = P_{12} \sin\psi \cos\gamma \frac{f + g \tan\gamma}{2f} \qquad (2.20b)$$

where $P_{12} = P_{13}$ and the equation for P_{13} was given in Equation 2.17.

The pintle reactions are now obtained using R_1 and R_2. The substructure forces were previously defined for $F_1 = 1$ and $F_2 = 1$. Since R_1 and R_2 have been defined to be colinear with F_1 and F_2, and have the same positive direction; the substructure forces in the pintle and mount frame head become

$$P_j = R_1 b_{j,1} + R_2 b_{j,2} \qquad (2.21)$$

from Equation 2.15, Equation 2.21 becomes

$$P_j = R_1 \left(b_{j,1} + \frac{l_4}{l_3} b_{j,2}\right): \quad j = 1,2,3,4,5,10,11 \qquad (2.22)$$

The b's in the above equation are defined by Equations 2.9, 2.11 and 2.12.

The substructure forces (P_1 - - - - P_{13}) resulting from $F_3 = 1$ are presented below as Equation 2.23. In this equation the following should be noted:

P_1 through P_5 are obtained by substituting Equations 2.11, 2.12 and 2.18 into Equation 2.22.

P_6 through P_9 are obtained by substituting Equation 2.17 into Equation 2.20.

P_{10} and P_{11} are obtained by substituting Equations 2.9, 2.12 and 2.18 into Equation 2.22.

P_{12} and P_{13} are defined by Equations 2.17 and 2.19.

$$P_1 = P_{13} \sin(\psi+\theta)\left(\cos\theta - \frac{\ell_4}{\ell_3}\sin\theta\right)\cos\gamma = b_{1,3}$$

$$P_2 = P_{13} \sin(\psi+\theta)\left(\sin\theta + \frac{\ell_4}{\ell_3}\cos\theta\right) = b_{2,3}$$

$$P_3 = P_{13} \sin(\psi+\theta)\left(\cos\theta - \frac{\ell_4}{\ell_3}\sin\theta\right)\sin\gamma = b_{3,3}$$

$$P_4 = -P_{13} \sin(\psi+\theta)\left[\ell_1\sin\theta + \ell_2\cos\theta + \frac{\ell_4}{\ell_3}(\ell_1\cos\theta - \ell_2\sin\theta)\right]\sin\gamma = b_{4,3}$$

$$P_5 = P_{13} \sin(\psi+\theta)\left[\ell_1\sin\theta + \ell_2\cos\theta + \frac{\ell_4}{\ell_3}(\ell_1\cos\theta - \ell_2\sin\theta)\right]\cos\gamma = b_{5,3}$$

$$P_6 = P_{13}\cos\psi\,\frac{f - g\tan\gamma}{2f} = b_{6,3}$$

$$P_7 = P_{13}\sin\psi\cos\gamma\,\frac{f - g\tan\gamma}{2f} = b_{7,3}$$

$$P_8 = P_{13}\cos\psi\,\frac{f + g\tan\gamma}{2f} = b_{8,3}$$

$$P_9 = P_{13}\sin\psi\cos\gamma\,\frac{f + g\tan\gamma}{2f} = b_{9,3}$$

$$P_{10} = P_{13}\sin(\psi+\theta)\left(\sin\theta + \frac{\ell_4}{\ell_3}\cos\theta\right) = b_{10,3}$$

$$P_{11} = P_{13}\sin(\psi+\theta)\left(\cos\theta - \frac{\ell_4}{\ell_3}\sin\theta\right) = b_{11,3}$$

$$P_{12} = P_{13} = b_{12,3}$$

$$P_{13} = \frac{1}{\cos(\psi+\theta) - \frac{\ell_4}{\ell_3}\sin(\psi+\theta)} = b_{13,3} \qquad (2.23)$$

2.3 General Form of Substructure and Force Transformation Matrices

The assembled force transformation terms given in Section 2.2 are presented in matrix notation in Equation 2.24.

$$
\left[\, b \,\right] = \begin{bmatrix} [b_m] \\ \hline [b_p] \\ \hline [b_e] \end{bmatrix} = \begin{bmatrix}
b_{1,1} & b_{1,2} & b_{1,3} \\
b_{2,1} & b_{2,2} & b_{2,3} \\
b_{3,1} & b_{3,2} & b_{3,3} \\
b_{4,1} & b_{4,2} & b_{4,3} \\
b_{5,1} & b_{5,2} & b_{5,3} \\
0 & 0 & b_{6,3} \\
0 & 0 & b_{7,3} \\
0 & 0 & b_{8,3} \\
0 & 0 & b_{9,3} \\
\hline
b_{10,1} & b_{10,2} & b_{10,3} \\
b_{11,1} & b_{11,2} & b_{11,3} \\
\hline
0 & 0 & b_{12,3} \\
0 & 0 & b_{13,3}
\end{bmatrix} \qquad (2.24)
$$

The terms of this transformation were developed in Section 2.2. At that time it was shown that the zero terms in the mount transformation exist because the T & E components were unloaded when unit loads were applied at the pintle. The general form of the substructure flexibility matrix $[\alpha]$ is

$$
[\alpha] = \begin{bmatrix} [\alpha_m] & & \\ & [\alpha_p] & \\ & & [\alpha_e] \end{bmatrix} = \left[\begin{array}{ccccccccc|cc|cc}
\alpha_{1,1} & \alpha_{1,2} & 0 & 0 & \alpha_{1,5} & \alpha_{1,6} & \alpha_{1,7} & \alpha_{1,6} & \alpha_{1,7} & & & & \\
 & \alpha_{2,2} & 0 & 0 & \alpha_{2,5} & \alpha_{2,6} & \alpha_{2,7} & \alpha_{2,6} & \alpha_{2,7} & & & & \\
 & & \alpha_{3,3} & \alpha_{3,4} & 0 & \alpha_{3,6} & \alpha_{3,7} & -\alpha_{3,6} & -\alpha_{3,7} & & & & \\
 & & & \alpha_{4,4} & 0 & \alpha_{4,6} & \alpha_{4,7} & -\alpha_{4,6} & -\alpha_{4,7} & & & & \\
 & & & & \alpha_{5,5} & \alpha_{5,6} & \alpha_{5,7} & \alpha_{5,6} & \alpha_{5,7} & & 0 & & 0 \\
 & & & & & \alpha_{6,6} & \alpha_{6,7} & \alpha_{6,8} & \alpha_{6,9} & & & & \\
 & & & & & & \alpha_{7,7} & \alpha_{6,9} & \alpha_{7,9} & & & & \\
 & & & & & & & \alpha_{6,6} & \alpha_{6,7} & & & & \\
 & & & & & & & & \alpha_{7,7} & & & & \\
\hline
 & & & & & & & & & \alpha_{10,10} & \alpha_{10,11} & & 0 \\
 & & & & & & & & & & \alpha_{11,11} & & \\
\hline
 & \text{SYM.} & & & & & & & & & & \alpha_{12,12} & 0 \\
 & & & & & & & & & & & & \alpha_{13,13}
\end{array}\right] \qquad (2.25)
$$

The zero terms in the mount frame flexibility matrix [α_m], exist because the frame is symmetric in the plane formed by the substructure coordinates 1 and 2 as shown in Figure 2.2. This results in no coupling between some coordinates, for example a horizontal force at 1 produces no transverse motion of 3, therefore $\alpha_{1,3} = 0$. The [α_m] matrix has repeated terms, due to the symmetry of the frame and the symmetric location of the coordinates: 6 and 8; and 7 and 9; e.g., $\alpha_{1,8} = \alpha_{1,6}$ and $\alpha_{1,9} = \alpha_{1,7}$. The terms in the substructure flexibility matrix remain in symbolic form for the general mount equations. When the equations are applied to a specific mount the $\alpha_{i,j}$'s can be given their numeric value.

A general approach to determine the mount frame flexibility, [α_m] is presented in Appendix B.

2.4 Evaluation of the Tripod Mount Flexibility Matrix

The tripod mount flexibility can be obtained using the flexibility and transformation matrices defined in the previous section. Equation 2.5 gives the matrix manipulations required to compute [a].

$$[a] = \sum_{i=m,p,t} [a^i] \qquad (2.26)$$

where

$$[a^i] = [b_i]^T [\alpha_i] [b_i]$$

The fl·xibility matrix [a^i] of each of the three substructures, and their co· ·oution to the total tripod mount flexibility [a] is evaluated in the remaining portion of Section 2.4.

Contribution of Mount Frame

The $a_{j,k}$ th term of [a^m] is given by the double summation

$$a^m_{j,k} = \sum_{l=1}^{9} \sum_{i=1}^{9} b_{i,j} \, \alpha_{i,l} \, b_{l,k} \qquad (j = 1,2,3 \; ; \; k = 1,2,3) \qquad (2.27)$$

By expanding Equation 2.27 we obtain a general mount flexibility term.

$$
\begin{aligned}
a^{m}_{j,k} = {} & \alpha_{1,1} b_{1,j} b_{1,k} + \alpha_{2,2} b_{2,j} b_{2,k} + \alpha_{3,3} b_{3,j} b_{3,k} + \alpha_{4,4} b_{4,j} b_{4,k} \\
& + \alpha_{5,5} b_{5,j} b_{5,k} + \alpha_{6,6}(b_{6,j} b_{6,k} + b_{8,j} b_{8,k}) \\
& + \alpha_{7,7}(b_{7,j} b_{7,k} + b_{9,j} b_{9,k}) + \alpha_{1,2}(b_{1,j} b_{2,k} + b_{2,j} b_{1,k}) \\
& + \alpha_{1,5}(b_{1,j} b_{5,k} + b_{5,j} b_{1,k}) + \alpha_{1,6}[b_{1,j}(b_{6,k} + b_{8,k}) \\
& + b_{1,k}(b_{6,j} + b_{8,j})] + \alpha_{1,7}[b_{1,j}(b_{7,k} + b_{9,k}) + b_{1,k}(b_{7,j} + b_{9,j})] \\
& + \alpha_{2,5}(b_{2,j} b_{5,k} + b_{5,j} b_{2,k}) + \alpha_{2,6}[b_{2,j}(b_{6,k} + b_{8,k}) \\
& + b_{2,k}(b_{6,j} + b_{8,j})] + \alpha_{2,7}[b_{2,j}(b_{7,k} + b_{9,k}) + b_{2,k}(b_{7,j} + b_{9,j})] \\
& + \alpha_{3,4}(b_{3,j} b_{4,k} + b_{4,j} b_{3,k}) + \alpha_{3,6}[b_{3,j}(b_{6,k} - b_{8,k}) + b_{3,k}(b_{6,j} - b_{8,j})] \\
& + \alpha_{3,7}[b_{3,j}(b_{7,k} - b_{9,k}) + b_{3,k}(b_{7,j} - b_{9,j})] \\
& + \alpha_{4,6}[b_{4,j}(b_{6,k} - b_{8,k}) + b_{4,k}(b_{6,j} - b_{8,j})] \\
& + \alpha_{4,7}[b_{4,j}(b_{7,k} - b_{9,k}) + b_{4,k}(b_{7,j} - b_{9,j})] \\
& + \alpha_{5,6}[b_{5,j}(b_{6,k} + b_{8,k}) + b_{5,k}(b_{6,j} + b_{8,j})] \\
& + \alpha_{5,7}[b_{5,j}(b_{7,k} + b_{9,k}) + b_{5,k}(b_{7,j} + b_{9,j})] \\
& + \alpha_{6,7}(b_{6,j} b_{7,k} + b_{7,j} b_{6,k} + b_{8,j} b_{9,k} + b_{9,j} b_{8,k}) + \alpha_{6,8}(b_{6,j} b_{8,k} + b_{8,j} b_{6,k}) \\
& + \alpha_{6,9}[b_{6,j} b_{9,k} + b_{9,j} b_{6,k} + b_{7,j} b_{8,k} + b_{8,j} b_{7,k}] \\
& + \alpha_{7,9}[b_{7,j} b_{9,k} + b_{9,j} b_{7,k}]
\end{aligned}
\tag{2.28}
$$

The superscript, m, denotes the mount frame, thus $a^m_{j,k}$ is the contribution of the mount frame flexibility to the j, k th term of the system flexibility matrix. The $a^m_{j,k}$ terms of [a] are evaluated using Equation 2.28. Accounting for zero terms in the $[b_m]$ matrix as given in Equation 2.24 the results are as follows:

$$
\begin{aligned}
a^m_{1,1} = {} & \alpha_{1,1} b^2_{1,1} + 2\alpha_{1,2} b_{1,1} b_{2,1} + \alpha_{2,2} b^2_{2,1} + 2\alpha_{1,5} b_{5,1} b_{1,1} \\
& + 2\alpha_{2,5} b_{2,1} b_{5,1} + \alpha_{3,3} b^2_{3,1} + 2\alpha_{3,4} b_{3,1} b_{4,1} + \alpha_{4,4} b^2_{4,1} \\
& + \alpha_{5,5} b^2_{5,1}
\end{aligned}
$$

$$
\begin{aligned}
a^m_{1,2} = {} & \alpha_{1,1} b_{1,1} b_{1,2} + \alpha_{1,2} (b_{1,2} b_{2,1} + b_{1,1} b_{2,2}) \\
& + \alpha_{2,2} b_{2,1} b_{2,2} + \alpha_{3,3} b_{3,1} b_{3,2} \\
& + \alpha_{3,4} (b_{4,1} b_{3,2} + b_{3,1} b_{4,2}) + \alpha_{4,4} b_{4,1} b_{4,2} \\
& + \alpha_{1,5} (b_{5,1} b_{1,2} + b_{1,1} b_{5,2}) \\
& + \alpha_{2,5} (b_{2,1} b_{5,2} + b_{5,1} b_{2,2}) + \alpha_{5,5} b_{5,1} b_{5,2}
\end{aligned}
$$

$$
\begin{aligned}
a^m_{1,3} = {} & \alpha_{1,1} b_{1,1} b_{1,3} + \alpha_{1,2} (b_{1,1} b_{2,3} + b_{2,1} b_{1,3}) \\
& + \alpha_{2,2} b_{2,1} b_{2,3} + \alpha_{3,3} b_{3,1} b_{3,3} \\
& + \alpha_{3,4} (b_{4,1} b_{3,3} + b_{3,1} b_{4,3}) + \alpha_{4,4} b_{4,1} b_{4,3}
\end{aligned}
$$

$$
\begin{aligned}
&+ \alpha_{1,5}(b_{5,1} b_{1,3} + b_{1,1} b_{5,3}) + \alpha_{1,6} b_{1,1}(b_{6,3} + b_{8,3}) \\
&+ \alpha_{1,7} b_{1,1}(b_{7,3} + b_{9,3}) + \alpha_{2,6} b_{2,1}(b_{6,3} + b_{8,3}) \\
&+ \alpha_{2,7} b_{2,1}(b_{7,3} + b_{9,3}) + \alpha_{3,6} b_{3,1}(b_{6,3} - b_{8,3}) \\
&+ \alpha_{3,7} b_{3,1}(b_{7,3} - b_{9,3}) + \alpha_{4,6} b_{4,1}(b_{6,3} - b_{8,3}) \\
&+ \alpha_{4,7} b_{4,1}(b_{7,3} - b_{9,3}) + \alpha_{5,6} b_{5,1}(b_{6,3} + b_{8,3}) \\
&+ \alpha_{5,7} b_{5,1}(b_{7,3} + b_{9,3}) + \alpha_{2,5}(b_{2,1} b_{5,3} \\
&+ b_{5,1} b_{2,3}) + \alpha_{5,5} b_{5,1} b_{5,3}
\end{aligned}
$$

$$
\begin{aligned}
a^{m}_{2,2} = {} & \alpha_{1,1} b_{1,2}^{2} + 2\alpha_{1,2} b_{1,2} b_{2,2} + \alpha_{2,2} b_{2,2}^{2} + \alpha_{3,3} b_{3,2}^{2} \\
&+ 2\alpha_{3,4} b_{3,2} b_{4,2} + \alpha_{4,4} b_{4,2}^{2} + 2\alpha_{1,5} b_{5,2} b_{1,2} \\
&+ 2\alpha_{2,5} b_{5,2} b_{2,2} + \alpha_{5,5} b_{5,2}^{2}
\end{aligned}
$$

$$
\begin{aligned}
a^{m}_{2,3} = {} & \alpha_{1,1} b_{1,2} b_{1,3} + \alpha_{1,2}(b_{1,3} b_{2,2} + b_{1,2} b_{2,3}) + \alpha_{2,2} b_{2,2} b_{2,3} \\
&+ \alpha_{3,3} b_{3,3} b_{3,2} + \alpha_{3,4}(b_{3,3} b_{4,2} + b_{4,3} b_{3,2}) + \alpha_{4,4} b_{4,2} b_{4,3} \\
&+ \alpha_{1,5}(b_{1,3} b_{5,2} + b_{1,2} b_{5,3}) + \alpha_{1,6} b_{1,2}(b_{6,3} + b_{8,3}) \\
&+ \alpha_{1,7} b_{1,2}(b_{7,3} + b_{9,3}) + \alpha_{2,5}(b_{2,2} b_{5,3} + b_{2,3} b_{5,2}) \\
&+ \alpha_{2,6} b_{2,2}(b_{6,3} + b_{8,3}) + \alpha_{2,7} b_{2,2}(b_{7,3} + b_{9,3}) \\
&+ \alpha_{3,6} b_{3,2}(b_{6,3} - b_{8,3}) + \alpha_{3,7} b_{3,2}(b_{7,3} - b_{9,3}) \\
&+ \alpha_{4,6} b_{4,2}(b_{6,3} - b_{8,3}) + \alpha_{4,7} b_{4,2}(b_{7,3} - b_{9,3}) \\
&+ \alpha_{5,5} b_{5,2} b_{5,3} + \alpha_{5,6} b_{5,2}(b_{6,3} + b_{8,3}) \\
&+ \alpha_{5,7} b_{5,2}(b_{7,3} + b_{9,3})
\end{aligned}
$$

$$
\begin{aligned}
a^m_{3,3} = {} & \alpha_{1,1} b^2_{1,3} + 2\alpha_{1,2} b_{1,3} b_{2,3} + \alpha_{2,2} b^2_{2,3} + \alpha_{3,3} b^2_{3,3} \\
& + 2\alpha_{3,4} b_{3,3} b_{4,3} + \alpha_{4,4} b^2_{4,3} + 2\alpha_{1,5} b_{1,3} b_{5,3} \\
& + 2\alpha_{1,6} b_{1,3}(b_{6,3} + b_{8,3}) + 2\alpha_{1,7} b_{1,3}(b_{7,3} + b_{9,3}) \\
& + 2\alpha_{2,5} b_{2,3} b_{5,3} + 2\alpha_{2,6} b_{2,3}(b_{6,3} + b_{8,3}) \\
& + 2\alpha_{2,7} b_{2,3}(b_{7,3} + b_{9,3}) + 2\alpha_{3,6} b_{3,3}(b_{6,3} - b_{8,3}) \\
& + 2\alpha_{3,7} b_{3,3}(b_{7,3} - b_{9,3}) \\
& + 2\alpha_{4,6} b_{4,3}(b_{6,3} - b_{8,3}) + 2\alpha_{4,7} b_{4,3}(b_{7,3} - b_{9,3}) \\
& + \alpha_{5,5} b^2_{5,3} + 2\alpha_{5,6} b_{5,3}(b_{6,3} + b_{8,3}) \\
& + 2\alpha_{5,7} b_{5,3}(b_{7,3} + b_{9,3}) + \alpha_{6,6}(b^2_{6,3} + b^2_{8,3}) \\
& + 2\alpha_{6,7}(b_{6,3} b_{7,3} + b_{8,3} b_{9,3}) + 2\alpha_{6,8} b_{6,3} b_{8,3} \\
& + 2\alpha_{6,9}(b_{6,3} b_{9,3} + b_{7,3} b_{8,3}) + \alpha_{7,7}(b^2_{7,3} + b^2_{9,3}) \\
& + 2\alpha_{7,9} b_{7,3} b_{9,3}
\end{aligned}
\qquad (2.29)
$$

<u>Contribution of the Pintle</u>

This substructure's contribution to [a] is, from Equation 2.26

$$
\left[a^p\right] = \begin{bmatrix} b_{10,1} & b_{11,1} \\ b_{10,2} & b_{11,2} \\ b_{10,3} & b_{11,3} \end{bmatrix} \begin{bmatrix} \alpha_{10,10} & \alpha_{10,11} \\ \alpha_{11,10} & \alpha_{11,11} \end{bmatrix} \begin{bmatrix} b_{10,1} & b_{10,2} & b_{10,3} \\ b_{11,1} & b_{11,2} & b_{11,3} \end{bmatrix} \qquad (2.30)
$$

Premultplying $[\alpha_p][b_p]$ by $[b_p]^t$ produces

$$
\begin{bmatrix} b_{10,1}\alpha_{10,10} + b_{11,1}\alpha_{10,11} & b_{10,1}\alpha_{10,11} + b_{11,1}\alpha_{11,11} \\ b_{10,2}\alpha_{10,10} + b_{11,2}\alpha_{10,11} & b_{10,2}\alpha_{10,11} + b_{11,2}\alpha_{11,11} \\ b_{10,3}\alpha_{10,10} + b_{11,3}\alpha_{10,11} & b_{10,3}\alpha_{10,11} + b_{11,3}\alpha_{11,11} \end{bmatrix} \begin{bmatrix} b_{10,1} & b_{10,2} & b_{10,3} \\ b_{11,1} & b_{11,2} & b_{11,3} \end{bmatrix}
$$

Completing the congruent transformation yields

$$a^p_{1,1} = \alpha_{10,10}\, b^2_{10,1} + 2\alpha_{10,11}\, b_{10,1} b_{11,1} + \alpha_{11,11}\, b^2_{11,1}$$

$$a^p_{1,2} = \alpha_{10,10}\, b_{10,1} b_{10,2} + \alpha_{10,11}(b_{10,2} b_{11,1} + b_{10,1} b_{11,2}) + \alpha_{11,11}\, b_{11,1} b_{11,2}$$

$$a^p_{1,3} = \alpha_{10,10}\, b_{10,1} b_{10,3} + \alpha_{10,11}(b_{10,3} b_{11,1} + b_{10,1} b_{11,3}) + \alpha_{11,11}\, b_{11,1} b_{11,3}$$

$$a^p_{2,2} = \alpha_{10,10}\, b^2_{10,2} + 2\alpha_{10,11}\, b_{10,2} b_{11,2} + \alpha_{11,11}\, b^2_{11,2}$$

$$a^p_{2,3} = \alpha_{10,10}\, b_{10,2} b_{10,3} + \alpha_{10,11}(b_{10,3} b_{11,2} + b_{10,2} b_{11,3}) + \alpha_{11,11}\, b_{11,2} b_{11,3}$$

$$a^p_{3,3} = \alpha_{10,10}\, b^2_{10,3} + 2\alpha_{10,11}\, b_{10,3} b_{11,3} + \alpha_{11,11}\, b^2_{11,3} \qquad (2.31)$$

<u>Contribution of Traverse and Elevating Components</u>

This substructure's contribution to [a] is given by

$$[a^t] = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ b_{12,3} & b_{13,3} \end{bmatrix} \begin{bmatrix} \alpha_{12,12} & 0 \\ 0 & \alpha_{13,13} \end{bmatrix} \begin{bmatrix} 0 & 0 & b_{12,3} \\ 0 & 0 & b_{13,3} \end{bmatrix} \qquad (2.32)$$

Evaluating this congruent transformation yields

$$a^t_{3,3} = \alpha_{12,12}\, b^2_{12,3} + \alpha_{13,13}\, b^2_{13,3} \qquad (2.33)$$

The remaining elements in the $[a^t]$ matrix are zero because of the zero terms in $[b_t]$, e. g.,

$$a^t_{1,1} = a^t_{1,2} = a^t_{1,3} = a^t_{2,2} = a^t_{2,3} = 0$$

<u>Total Tripod Mount Flexibility Matrix.</u>

The tripod mount flexibility matrix is given by summing the contributions from the three substructures.

$$[a] = [a^m] + [a^p] + [a^t] \qquad (2.34)$$

where: $[a^m]$ is given by Equation 2.29

$[a^P]$ is given by Equation 2.31

$[a^t]$ is given by Equation 2.33

After summing Equations 2.29, 2.31 and 2.33, the total flexibility expressions are given in Equation 1.4. The individual terms of the matrix are

$$[a] = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{bmatrix} \quad (2.35)$$

The stiffness matrix of the mount is found by inverting Equation 2.35.

$$[K] = [a]^{-1} \quad (2.36)$$

For the case of a weapon mathematical model with one main gun degree of freedom in the bore axis direction, the spring stiffness is simply

$$k_{1,1} = \frac{1}{a_{1,1}}. \quad (2.37)$$

This corresponds to the coefficient C_2 (with proper consideration for sign) of the mount force expression given in USAWECOM Tech Rpt. 70-114 entitled "Mathematical Model of the Stoner 5.56 MM Medium Machine Gun, XM207", (Reference 1).

2.5 Transformation to Other Weapon Coordinates

The flexibility matrix derived above describes the mount flexibility in terms of the three mount-weapon interface coordinates x_1, x_2, x_3. In some cases, other coordinates may be used in the dynamic analysis, for example, it may be desirable to use the main gun center of gravity displacements and rotation to describe the system's motion. In such a case it is necessary to transform the mount flexibility to correspond to the new coordinates. This is accomplished by first developing the force transformation between the mount interface forces $\{F\}$ and the main gun C.G. forces $\{F_{mg}\}$.

$$\{F\} = [b_{mg}]\{F_{mg}\} \tag{2.38}$$

where $[b_{mg}]$ the force transformation matrix is determined by the weapon geometry shown in Figure 2.9. The interface and C.G. forces are:

$$\{F\} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} \qquad \{F_{mg}\} = \begin{Bmatrix} F_x \\ F_y \\ M \end{Bmatrix}_{mg}$$

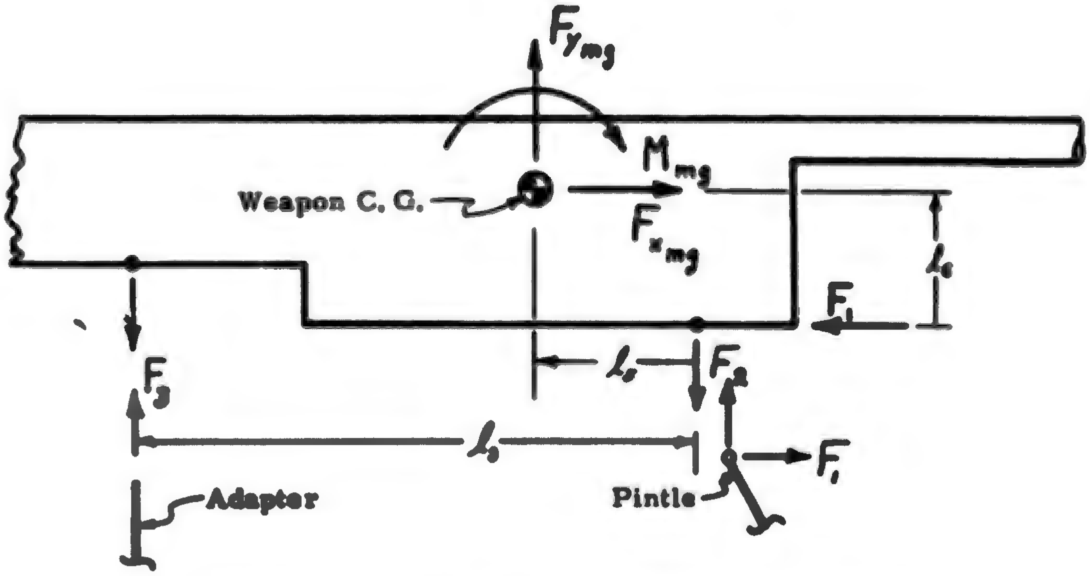


Figure 2.9 Weapon Center of Gravity Coordinates

The main gun force transformation $[b_{mg}]$ is evaluated by successive application of unit forces at the center of gravity coordinates.

To obtain the first column in the $[b_{mg}]$ matrix, let $F_{x_{mg}} = 1$. The horizontal equilibrium yields

$$F_1 = 1 \tag{2.39}$$

Vertical equilibrium yields

$$F_3 = -F_2 \tag{2.40}$$

Equilibrium of moments about the C. G. yields

$$\ell_6 + \ell_5 F_2 - (\ell_3 - \ell_5) F_3 = 0$$

Using Equation 2.40 this becomes

$$\ell_6 + \ell_5 F_2 + \ell_3 F_2 - \ell_5 F_2 = 0$$

therefore

$$F_2 = -\frac{\ell_6}{\ell_3} \tag{2.41}$$

To obtain the second column in the $[b_{mg}]$ matrix, let $F_{y_{mg}} = 1$. Horizontal equilibrium yields $F_1 = 0$. Equilibrium of moments about F_3 yields

$$(1)(\ell_3 - \ell_5) = \ell_3 F_2$$

therefore

$$F_2 = 1 - \frac{\ell_5}{\ell_3} \tag{2.42}$$

Equilibrium of moments about F_2 yields

$$(1)\,\ell_5 = \ell_3 F_3$$

therefore

$$F_3 = \frac{\ell_5}{\ell_3} \tag{2.43}$$

To obtain the third column of $[b_{mg}]$, let $M_{mg} = 1$. The horizontal equilibrium yields $F_1 = 0$. The vertical equilibrium yields $F_2 = -F_3$. The sum of the moments about the point where the adapter attaches to the weapon (where $M_{mg} = 1$ and $F_1 = 0$) yields

$$1 + F_2 \ell_3 = 0$$

$$F_2 = -\frac{1}{\ell_3} \tag{2.44}$$

From the three loading conditions evaluated above, the force transformation matrix $[b_{mg}]$ becomes

$$[b_{mg}] = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{\ell_c}{\ell_s} & \left(1-\frac{\ell_c}{\ell_s}\right) & -\frac{1}{\ell_s} \\ \frac{\ell_c}{\ell_s} & \frac{\ell_c}{\ell_s} & \frac{1}{\ell_s} \end{bmatrix} \qquad (2.45)$$

The flexibility, in terms of the main gun C. G. coordinates is then

$$[a_{mg}] = [b_{mg}]^T [a] [b_{mg}] \qquad (2.46)$$

2.6 Tripod Mount Damping

The effect of damping in a dynamic model is most important when the system is excited at or close to one of its resonant frequencies. In the case of the tripod mount the weapon firing rate of 550 rounds per minute (9.2 Hz) are much lower than the fundamental frequency of the weapon-mount systems. Therefore, the effect of damping on the system's response should be minimal. If it is desired to include damping, it can best be represented as equivalent viscous damping. For the dynamic model described in Reference 1 the damping is introduced through the mount force coefficient C_3 which is defined as

$$C_3 = 2\zeta\sqrt{m\,k_\psi} \qquad (2.47)$$

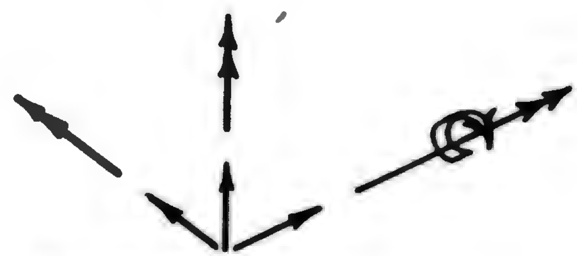
where ζ is the percent of critical viscous damping and is approximately .03 for the tripod mount. m is the combined mass of the weapon and mount and k_ψ is given by Equation 2.37.

Section 3
EVALUATION OF THE FLEXIBILITY MATRIX FOR THE M122 TRIPOD MOUNT

This section presents the application of the tripod mount flexibility equations developed in Section 2 by computing the flexibility matrix for the M122 Tripod Mount. The analysis is performed, as described in Section 1, as follows: first, compute the three substructure flexibility matrices; second, evaluate the force transformation relations for the various weapon orientation; third, combine the substructure flexibilities and transformations to obtain the total flexibility of the weapon-mount system. The general configuration of the M122 Tripod Mount is obtained from the USAWECOM Drawing No. F 7790723.

3.1 M122 Mount Frame Substructure

The elements of the flexibility matrix for the M122 Tripod Mount Frame are evaluated herein. The flexibility of the frame is described in terms of its nine coordinates which interface with the pintle and the traversing mechanism as shown in Figure 2.2. The numerical evaluation of the frame was performed using the ICES computer program - STRUDL II (Reference 2). The idealized frame used for the analysis is shown in Figure 3.1. The frame has members of uniform cross section, and all members have rigid connections. The frame dimensions were determined from USAWECOM drawings which are listed as they are referenced. Included as an appendix to this report is a general analytical approach for computing the flexibility matrix of an indeterminate frame such as the Tripod Mount Frame. This appendix is included, in the event no standard frame programs are available to the reader such as the one referenced above.



Positive Sign Convention

Reference Drawings

Item	USAWECOM Dwg. No.
Front Leg	B5139991 C6108195
Rear Leg	B6103205 D5559337
Head	D5559331 D5559336
Sleeve	D6108206

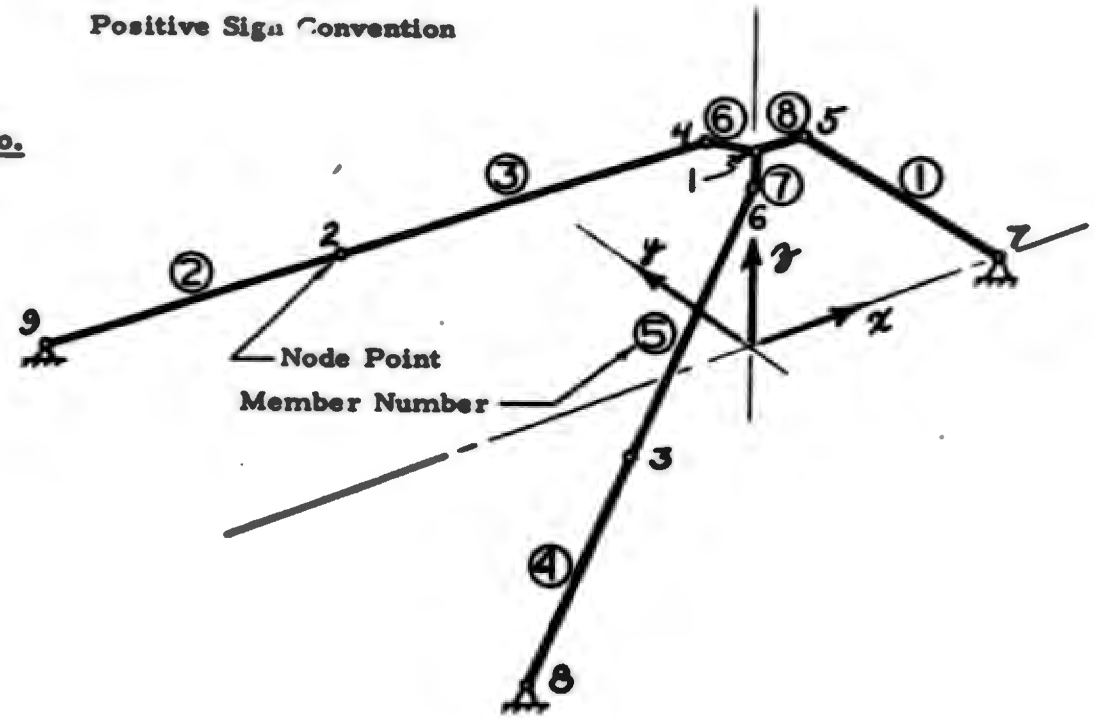


Figure 3.1 Structural Idealization of the Tripod Mount Frame

In order to determine the importance of the earth's flexibility on the frame, two cases were evaluated.

Case 1. All three legs pinned at the frame-ground junction so that the ends can rotate but cannot translate.

Case 2. The front leg released in the fore-aft direction with all other boundary conditions remaining the same as for the above case. The forward direction is shown in Figure 3.1 as the x direction.

The following nodal geometry was computed from the referenced drawings:

Table 3.1 Frame Joint Coordinates

Joint No.	x (in.)	y (in.)	z (in.)
1	0.0	0.0	7.2
2	-11.243	7.139	3.338
3	-11.243	-7.139	3.338
4	-1.39	1.03	7.2
5	1.07	0.0	7.2
6	-1.39	-1.03	7.2
7	8.27	0.0	0.0
8	-19.76	-12.42	0.0
9	-19.76	12.42	0.0

The cross sectional properties of members (1) to (8) are presented on the following page. These properties are used in the subsequent analysis.

Members ① to ⑤:

$$A = \frac{\pi}{4}(D_o^2 - D_i^2) = \frac{\pi}{4}(1^2 - 0.8^2) = 0.282 \text{ in.}^2$$

$$I = \frac{\pi}{64}(D_o^4 - D_i^4) = \frac{\pi}{64}(1^4 - 0.8^4) = .029 \text{ in.}^4$$

$$J = 2I = .058 \text{ in.}^4$$

Members ⑥ to ⑧:

These members represent the head which is about ten times stiffer than the legs, therefore assume $A=2.8 \text{ in}^2$, $I=0.29 \text{ in}^4$, $J=0.58 \text{ in}^4$.

The results of the two STRUDL-II computer analyses are presented on the following page. The coordinates of the flexibility matrix have been previously presented in Figure 2.2. Note that the results are normalized to the Modulus of Elasticity of the frame. The flexibility coefficients, $\alpha_{1,3} = \alpha_{1,4} = \alpha_{2,3} = \alpha_{2,4} = \alpha_{3,5} = \alpha_{4,5} = 0$, as presented in Section 2.3.

These two cases show a two order of magnitude increase in some of the flexibilities by releasing just one tripod leg boundary constraint. The largest increase occurs for the tripod head coordinates in the fore & aft and vertical directions, e.g., $\alpha_{1,1}$, $\alpha_{1,2}$ and α_{22}, because when the mount is pinned the loads applied at the head are resisted mainly as a three dimensional truss. When the front leg is released the mount must then resist the loads by bending of the legs, thereby resulting in much larger deformations. The tripod mount displacements resulting from forces P_3 or P_4 applied to the head do not change, because these loads do not require a front leg reaction in the x direction to resist the load. However, these would change if any of the rear leg reactions were released. The flexibility at the traverse bar attachment points do not change as severely as the head because the traverse bar forces (6 through 9) are already resisted by the bending of the legs.

The results of these comparisons indicate that to correctly model the tripod frame it is necessary to include the effect of the tripod mount supporting medium. Therefore, to properly idealize the ground flexibility, Case 1- All three frame legs are pinned at the mount-ground junction

$$[\alpha_m] = \frac{1}{E} \begin{bmatrix} 63.199 & 1.432 & 0 & 0 & -38.486 & -164.64 & 85.98 & -161.14 & 85.98 \\ & 79.564 & 0 & 0 & 24.30 & 151.74 & -36.30 & 151.74 & -36.30 \\ & & 181.02 & 36.247 & 0 & 103.87 & 68.56 & -103.87 & -68.56 \\ & & & 75.589 & 0 & 205.88 & 35.49 & -205.88 & -35.49 \\ & & & & 58.624 & 263.55 & -101.82 & 263.55 & -101.82 \\ & & & & & 5234.69 & -1516.47 & 649.95 & -552.32 \\ & & & & & & 1447.96 & -552.32 & -115.45 \\ & & & & & & & 5234.69 & -1516.47 \\ & \text{SYM.} & & & & & & & 1447.96 \end{bmatrix} \qquad (3.1)$$

Case 2- The front leg of the frame is free to translate in the fore-aft direction, the other two frame legs are pinned at the mount-ground junction.

$$[\alpha_m] = \frac{1}{E} \begin{bmatrix} 1536.77 & -3452.77 & 0 & 0 & -227.81 & -3194.41 & 1282.18 & -3194.41 & 1282.18 \\ & 8176.58 & 0 & 0 & 468.09 & 7255.00 & -2840.30 & 7255.00 & -2840.30 \\ & & 181.02 & 36.25 & 0 & 103.87 & 68.56 & -103.87 & -68.56 \\ & & & 75.59 & 0 & 205.88 & 35.49 & -205.88 & -35.49 \\ & & & & 82.95 & 652.87 & -255.50 & 652.87 & -255.50 \\ & & & & & 11466.17 & -3976.33 & 6881.43 & -3012.18 \\ & & & & & & 2418.99 & -3012.18 & 855.58 \\ & & & & & & & 11466.17 & -3976.33 \\ & \text{SYM.} & & & & & & & 2418.99 \end{bmatrix} \qquad (3.2)$$

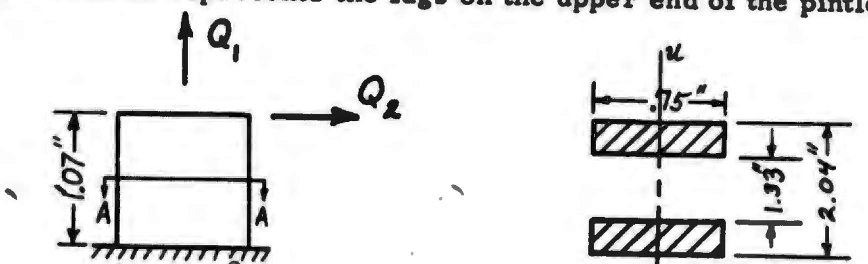
three orthoginal springs in the horizontal plane should be attached to the bottom of each of the three legs. This additional flexibility can be easily incorporated into the structural model of the frame substructure.

3.2 M122 Mount Pintle Substructure

The pintle flexibility matrix, $[\alpha_p]$, will be determined in terms of its two coordinates δ_{10} and δ_{11} and their corresponding forces P_{10} and P_{11}. This matrix describes the pintle flexibility from the weapon attachment point to the center of the head. For the M122 mount, the pintle is idealized using the three elements shown on the following page.

Element ① Flexibility

This element represents the lugs on the upper end of the pintle.



Area = 0.53 in.^{2}
Inertia = .025 in.^{4}

SECTION A-A

The flexibility matrix of element ① is a diagonal matrix since there is no cross coupling between the shear and axial coordinates.

$$[\bar{\alpha}_{①}] = \begin{bmatrix} \bar{\alpha}_{11} & 0 \\ 0 & \bar{\alpha}_{22} \end{bmatrix}$$

$$= \frac{1}{E} \begin{bmatrix} \frac{l_1}{A} & 0 \\ 0 & \left(\frac{l_1^3}{3I} + \frac{k\, l_1}{0.385A} \right) \end{bmatrix} \qquad (3.3)$$

Where the factor of 0.385 in the shear term comes from the relation $\frac{G}{E} = \frac{1}{2(1+\nu)} = \frac{1}{2(1+0.3)} = 0.385$.

Note that $\bar{\alpha}_{22}$ includes deformation due to bending and shear because the element is of a low aspect ratio (k is shear shape factor).

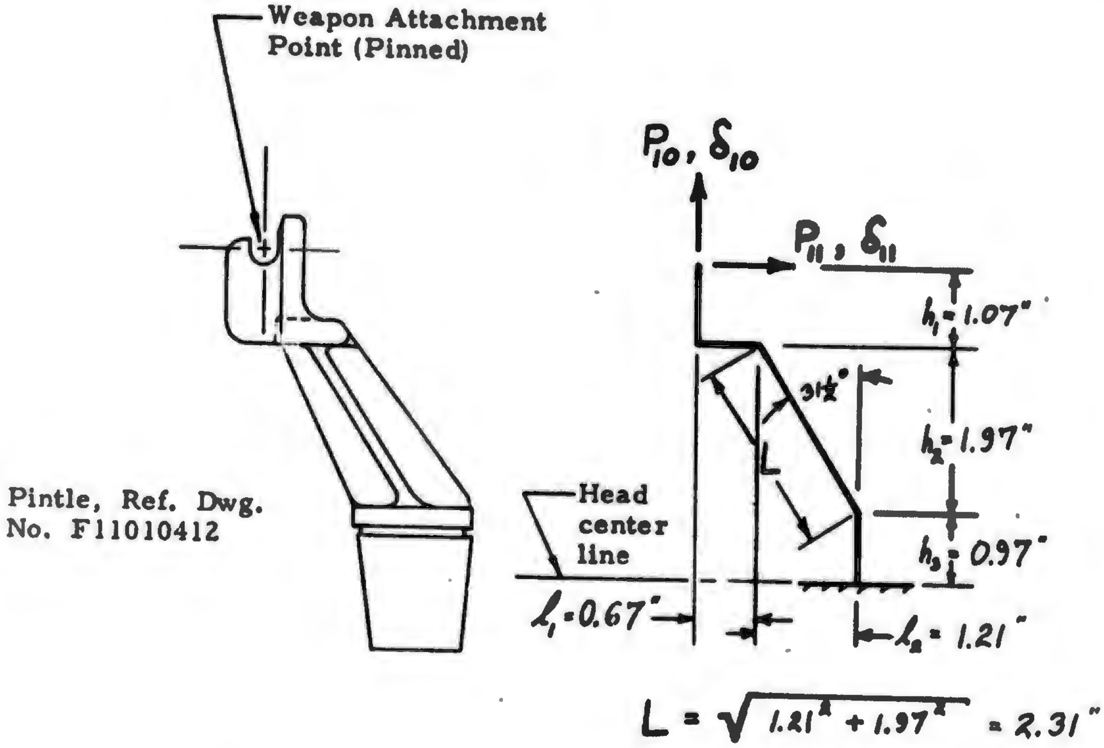


Figure 3.2 Pintle Structural Model

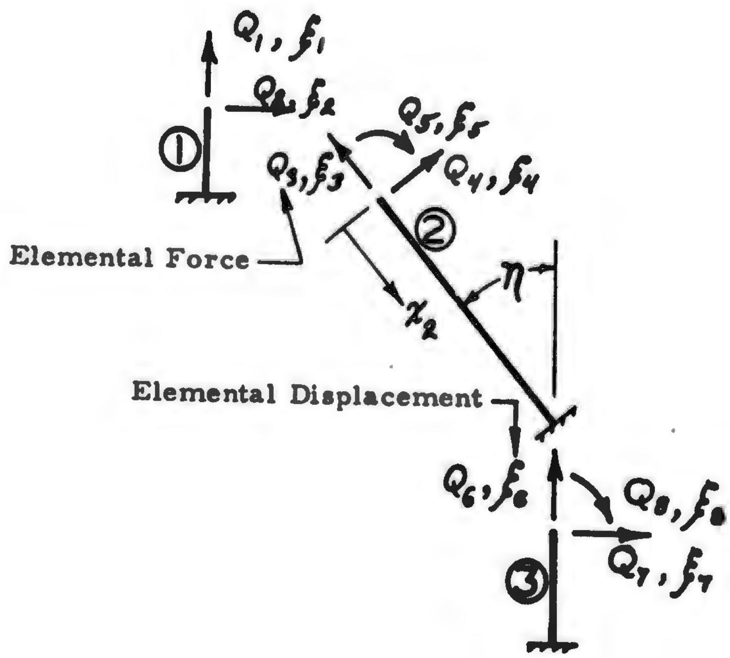


Figure 3.3 Pintle Structural Elements

Substituting the element dimensions into Equation 3. 3 gives

$$\left[\bar{\alpha}_{①}\right] = \frac{1}{E}\begin{bmatrix} \frac{1.07}{0.53} & 0 \\ 0 & \frac{(1.07)^3}{3(.025)} + \frac{1.5(1.07)}{0.385(0.53)} \end{bmatrix}$$

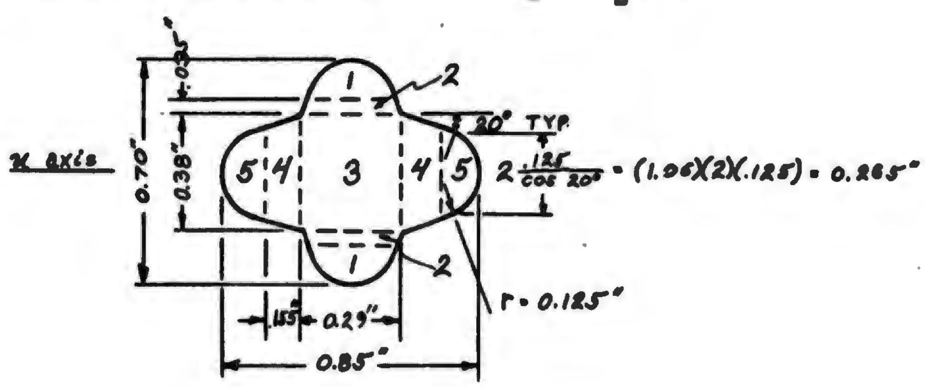
Then preforming the calculations gives

$$\left[\bar{\alpha}_{①}\right] = \frac{1}{E}\begin{bmatrix} 2.02 & 0 \\ 0 & 24.1 \end{bmatrix} \qquad (3.4)$$

<u>Element ② Flexibility</u>

This element represents the tapered body of the pintle. The element is assumed to have a varying area and moment of inertia.

Cross section at the top of element ② ($x_2 = 0$).

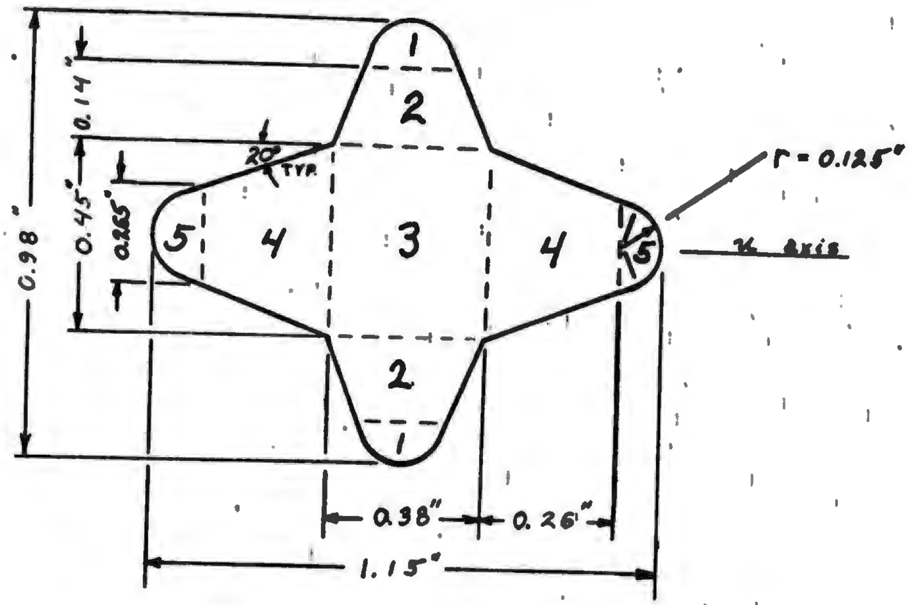


	Area		u Axis Area Inertia	
1	$2\frac{\pi(.125)^2}{2}$	$= .0491$	$(.0491)(0.278)^2$	$= .00379$ *
2	$2\left[.035\left(\frac{0.29+0.265}{2}\right)\right]$	$= .0194$	$(0.194)(0.205)^2$	$= .00082$ **
3	$(0.38)(0.29)$	$= 0.110$	$\frac{(0.29)(0.38)^3}{12}$	$= .00133$
4	$2(0.155)\left(\frac{0.38+0.265}{2}\right)$	$= 0.100$	$\frac{2(0.155)(0.265+0.38)(0.38^2+0.265^2)}{48}$	$= .00089$ **
5	$2\frac{\pi(0.125)^2}{2}$	$= .0491$	$\frac{\pi(0.125)^4}{4}$	$= .00019$
		$0.327\ in.^2$		$.00702\ in.^4$

* Includes only the transfer term, Ad^2

** Obtained from Reference 3

Cross section at the middle of element ② (x_2=1.15 in)

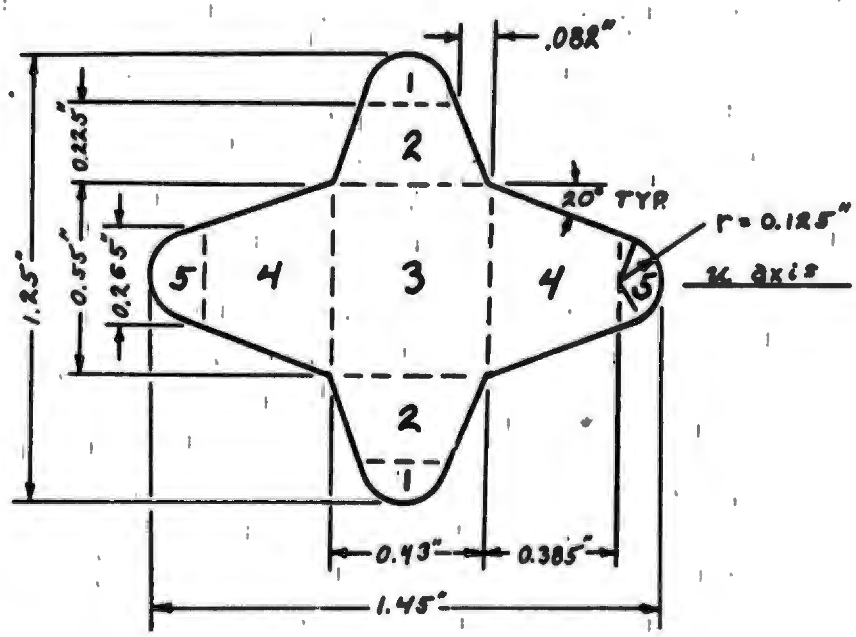


	Area		u Axis Area Inertia	
1	$2\frac{\pi}{2}(0.125)^2$	$= .0491$	$.0491(0.418)^2$	$= .00858$ *
2	$2\left[0.14\frac{(0.38+0.265)}{2}\right]$	$= .0903$	$.0903(0.285)^2$	$= .00733$ **
3	$(0.38)(0.45)$	$= 0.171$	$0.38\frac{(0.45)^3}{12}$	$= .00289$
4	$2\left[0.26\frac{(0.45+0.265)}{2}\right]$	$= 0.186$	$\frac{2(0.26)(0.45+0.265)(0.45^2+0.265^2)}{48}$	$= .00211$ **
5	$2\frac{\pi}{2}(0.125)^2$	$= .0491$	$\frac{\pi(0.125)^4}{4}$	$= .0002$
		$0.546\ in^2$		$.0182\ in^4$

* Includes only the transfer term, Ad^2

** Obtained from Reference 3

Cross section at the bottom of element ② (x_2 = 2.31 in.)



	Area		u Axis Area Inertia	
1	$2\frac{\pi}{2}(0.125)^2$	$= .0491$	$.0491(0.553)^2$	$= .0150$ *
2	$2(0.225)\left(\frac{0.43+0.265}{2}\right)$	$= 0.156$	$\frac{2(0.225)^3(0.43+.265)[2(0.265)+.082]}{12(0.265+.082)} + 0.156(0.275)^2$	$= .0120$ **
3	$(0.55)(0.43)$	$= 0.236$	$\frac{0.43(0.55)^3}{12}$	$= .0059$
4	$2(0.385)\left(\frac{0.55+0.265}{2}\right)$	$= 0.314$	$\frac{2(.385)(0.55+0.265)(0.55^2+0.265^2)}{48}$	$= .0049$ **
5	$2\frac{\pi}{2}(0.125)^2$	$= .0491$	$\frac{\pi}{4}(0.125)^4$	$.0002$
		0.804 in.^{2}		$.0380$ in.^{4}

* Includes only the transfer term, Ad^2

** Obtained from Reference 3

Based on the properties of these three sections, we can express the area and moment of inertia as a linear function of its length x_2.

$$A(x_2) = 0.33 + 0.206\, x_2 \tag{3.5}$$

and

$$I(x_2) = .0070 + .0134\, x_2 \tag{3.6}$$

The flexibility of element ② has the general form shown in Equation 3.7, since there is no cross coupling between the axial coordinate (3) and the shear and moment coordinates (4 and 5).

$$\left[\bar{\alpha}_{②}\right] = \begin{bmatrix} \bar{\alpha}_{33} & 0 & 0 \\ 0 & \bar{\alpha}_{44} & \bar{\alpha}_{45} \\ 0 & \bar{\alpha}_{5,4} & \bar{\alpha}_{55} \end{bmatrix} \tag{3.7}$$

The strain energy approach is used to evaluate the terms of this matrix. Consider the element axial force Q_3; the strain energy is

$$U_{Q_3} = \frac{1}{2}\int_0^{2.31} \frac{Q_3^2\, dx_2}{AE} = \frac{Q_3^2}{2E}\int_0^{2.31} \frac{dx_2}{0.33 + 0.206\, x_2}$$

$$= \frac{Q_3^2}{2E}\left(\frac{1}{0.206}\right)(\ln 0.804 - \ln 0.33) = \frac{Q_3^2}{2E}(4.32) \tag{3.8}$$

then

$$\bar{\alpha}_{3,3} = \frac{\partial^2 U}{\partial Q_3^2} = \frac{1}{E}(4.32) \tag{3.9}$$

The remaining terms are obtained by consideration of the bending strain energy. The moment in the element is

$$M(x_2) = Q_5 + Q_4 x \tag{3.10}$$

then

$$U = \frac{1}{2}\int_0^{2.31} \frac{M^2}{EI}\, dx_2 = \frac{1}{2E}\int_0^{2.31} \frac{x^2 Q_4^2 + 2Q_4 Q_5 x + Q_5^2}{.0070 + .0134\, x_2}\, dx_2 \tag{3.11}$$

Expanding and integrating Equation 3.11 gives

$$U = \frac{1}{2E}\Big[Q_4^2 \frac{1}{.0134^3}\Big\langle \frac{1}{2}(.038)^2 - 2(.0070)(.038) + .0070^2 \ln .038$$

$$+ 1.5(.0070)^2 - (.0070)^2 \ln .0070\Big\rangle$$

$$+ 2 Q_4 Q_5 \frac{1}{.0134^2}\Big\langle .038 - .0070 \ln .038 - .0070 + .0070 \ln .0070\Big\rangle$$

$$+ Q_5^2 \frac{1}{.0134}\Big\langle \ln .038 - \ln .0070\Big\rangle\Big]$$

$$= \frac{1}{2E}\Big[143.96\, Q_4^2 + 2\,(106.69) Q_4 Q_5 + 126.24\, Q_5^2\Big] \quad (3.12)$$

The flexibility terms are given as

$$\overline{\alpha}_{4,4} = \frac{\partial^2 U}{\partial Q_4^2} = \frac{1}{E}(143.96)$$

$$\overline{\alpha}_{4,5} = \overline{\alpha}_{5,4} = \frac{1}{2}\frac{\partial^2 U}{\partial Q_4\, \partial Q_5} = \frac{1}{E}(106.69)$$

$$\overline{\alpha}_{5,5} = \frac{\partial^2 U}{\partial Q_5^2} = \frac{1}{E}(126.24) \quad (3.13)$$

Substituting Equations 3.9 and 3.13 into 3.7 gives the flexibility matrix of element ② :

$$\left[\overline{\alpha}_{②}\right] = \frac{1}{E}\begin{bmatrix} 4.32 & 0 & 0 \\ 0 & 143.96 & 106.69 \\ 0 & 106.69 & 126.24 \end{bmatrix} \quad (3.14)$$

Element ③ Flexibility

This element has several abrupt changes in its cross section. For simplicity, an average radius of 0.62 in. was used and the element was assumed to be of constant cross section. The area and inertia of element ③ is:

$$A_3 = \pi (0.62)^2 = 1.21 \text{ in.}^2 \qquad I_3 = \frac{\pi (0.62)^4}{4} = 0.116 \text{ in.}^4$$

The flexibility matrix of this element has the same form as element ② since there is no cross coupling between the axial coordinate (6) and the shear and mount coordinates (7 and 8).

$$\left[\bar{\alpha}_{③}\right] = \begin{bmatrix} \bar{\alpha}_{6,6} & 0 & 0 \\ 0 & \bar{\alpha}_{7,7} & \bar{\alpha}_{7,8} \\ 0 & \bar{\alpha}_{8,7} & \bar{\alpha}_{8,8} \end{bmatrix} \qquad (3.15)$$

The terms are given by

$$\bar{\alpha}_{6,6} = \frac{h_3}{EA_3} = \frac{1}{E}\,\frac{0.97}{1.21} = 0.80\frac{1}{E}$$

$$\bar{\alpha}_{7,7} = \frac{h_3^3}{E\,3I_3} = \frac{(0.97)^3}{E(3)(0.116)} = 2.62\frac{1}{E}$$

$$\bar{\alpha}_{7,8} = \bar{\alpha}_{8,7} = \frac{h_3^2}{E\,2I_3} = \frac{(0.97)^2}{E(2)(0.116)} = 4.05\frac{1}{E}$$

$$\bar{\alpha}_{8,8} = \frac{h_3}{EI_3} = \frac{(0.97)}{E(0.116)} = 8.36\frac{1}{E} \qquad (3.16)$$

Substituting Equation 3.16 into Equation 3.15 gives

$$\left[\bar{\alpha}_{③}\right] = \frac{1}{E}\begin{bmatrix} 0.80 & 0 & 0 \\ 0 & 2.62 & 4.05 \\ 0 & 4.05 & 8.36 \end{bmatrix} \qquad (3.17)$$

Combining the Three Elements into the Pintle Substructure

Now the three element flexibility matrices can be combined to create the pintle flexibility matrix. Equation 3.18 defines the flexibility of the three uncoupled elements of the pintle.

$$\left[\diagdown \bar{\alpha} \diagdown\right] = \begin{bmatrix} [\bar{\alpha}_{①}] & 0 & 0 \\ 0 & [\bar{\alpha}_{②}] & 0 \\ 0 & 0 & [\bar{\alpha}_{③}] \end{bmatrix} \qquad (3.18)$$

The force transformation between the pintle substructure forces P_{10} and P_{11} and the element forces Q_1 ---- Q_8 are given by

$$\{Q\} = [b]\{P\} \qquad (3.19)$$

where [b] is the force transformation matrix. The pintle substructure flexibility matrix is: $[\alpha_p] = [b]^T[\diagdown\bar{\alpha}\diagdown][b]$ (3.20)

Due to the diagonal form of $[\diagdown\alpha\diagdown]$, $[\alpha_p]$ can be expressed more compactly by partitioning the force transformation matrix [b].

$$\begin{Bmatrix} Q_{①} \\ \hdashline Q_{②} \\ \hdashline Q_{③} \end{Bmatrix} \quad \begin{bmatrix} b_{①} \\ \hdashline b_{②} \\ \hdashline b_{③} \end{bmatrix} \begin{Bmatrix} P \end{Bmatrix} \qquad (3.21)$$

then

$$[\alpha_p] = \sum_{i=①,②,③} [b_i]^T [\bar{\alpha}_i] [b_i] \qquad (3.22)$$

Thus the contribution of each element to the total deformation can be separately computed and then superimposed at the last step.

The force transformation matrix [b] is obtained by defining the internal pintle forces (Q's) due to unit pintle substructure forces (P's).

Let $P_{10} = 1$; then from Figures 3.2 and 3.3

$$
\begin{aligned}
Q_1 &= 1 \\
Q_2 &= 0 \\
Q_3 &= \cos \eta \\
Q_4 &= \sin \eta \\
Q_5 &= \ell_1 \\
Q_6 &= 1 \\
Q_7 &= 0 \\
Q_8 &= \ell_1 + \ell_2
\end{aligned}
\qquad (3.23)
$$

Let $P_{11} = 1$; then from Figures 3.2 and 3.3

$$
\begin{aligned}
Q_1 &= 0 \\
Q_2 &= 1 \\
Q_3 &= -\sin \eta \\
Q_4 &= \cos \eta \\
Q_5 &= h_1 \\
Q_6 &= 0 \\
Q_7 &= 1 \\
Q_8 &= h_1 + h_2
\end{aligned}
\qquad (3.24)
$$

The force transformation matrix [b] is defined as the internal forces due to unit system force. Therefore, Equation 3.21 can be written using Equations 3.23 and 3.24

$$\begin{Bmatrix} Q_1 \\ Q_2 \\ \hdashline Q_3 \\ Q_4 \\ Q_5 \\ \hdashline Q_6 \\ Q_7 \\ Q_8 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \hdashline C & -S \\ S & C \\ 0.67 & 1.07 \\ \hdashline 1 & 0 \\ 0 & 1 \\ 1.88 & 3.04 \end{bmatrix} \begin{Bmatrix} P_{10} \\ P_{11} \end{Bmatrix} \qquad (3.25)$$

where C = Cos (31°30') = .85264

S = Sin (31°30') = .5225

Evaluation of Equation 3.22 for element ①:

$$\frac{1}{E}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2.02 & 0 \\ 0 & 24.1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \frac{1}{E}\begin{bmatrix} 2.02 & 0 \\ 0 & 24.1 \end{bmatrix} \qquad (3.26)$$

Evaluation of Equation 3.22 for element ② :

$$\frac{1}{E}\begin{bmatrix} C & S & 0.67 \\ -S & C & 1.07 \end{bmatrix}\begin{bmatrix} 4.32 & 0 & 0 \\ 0 & 143.96 & 106.69 \\ 0 & 106.69 & 126.24 \end{bmatrix}\begin{bmatrix} C & -S \\ S & C \\ 0.67 & 1.07 \end{bmatrix}$$

$$= \frac{1}{E}\begin{bmatrix} 173.81 & 273.31 \\ 273.31 & 445.04 \end{bmatrix} \qquad (3.27)$$

Evaluation of Equation 3.22 for element ③ :

$$\frac{1}{E}\begin{bmatrix} 1 & 0 & 1.88 \\ 0 & 1 & 3.04 \end{bmatrix}\begin{bmatrix} 0.80 & 0 & 0 \\ 0 & 2.62 & 4.05 \\ 0 & 4.05 & 8.36 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1.88 & 3.04 \end{bmatrix}$$

$$= \frac{1}{E}\begin{bmatrix} 30.35 & 55.39 \\ 55.39 & 104.50 \end{bmatrix} \qquad (3.28)$$

The total substructure flexibility $[\alpha_p]$ is found by summing the contributions of the three elements. Adding Equations 3.26, 3.27 and 3.28 produces the pintle flexibility matrix.

$$[\alpha_p] = \begin{bmatrix} \alpha_{10,10} & \alpha_{10,11} \\ \alpha_{11,10} & \alpha_{11,11} \end{bmatrix}$$

$$[\alpha_p] = \frac{1}{E}\begin{bmatrix} 206.18 & 328.70 \\ 328.70 & 573.64 \end{bmatrix} \qquad (3.29)$$

3.3 M122 Mount Traverse and Elevating Components Substructure

The computation of the flexibility matrix, $[a^t]$, of the T & E components substructure is determined for the two coordinates δ_{12} and δ_{13} and their corresponding forces P_{12} and P_{13}. These two forces are equal, but were defined separately in order to facilitate computation of the flexibilities of the elements within the substructure. The matrix, $[a^t]$, describes the deformation characteristics of the adapter, traverse and elevating mechanism, and the traverse bar shown in Figure 3.4.

Adapter Flexibility

The adapter flexibility, $\alpha_{13,13}$ will be computed using strain energy. The adapter configuration is shown in Figure 3.5. The adapter is divided into four elements Ⓐ, Ⓑ, Ⓒ, Ⓓ and the strain energy is computed for each element in terms of P' and P''. The total strain

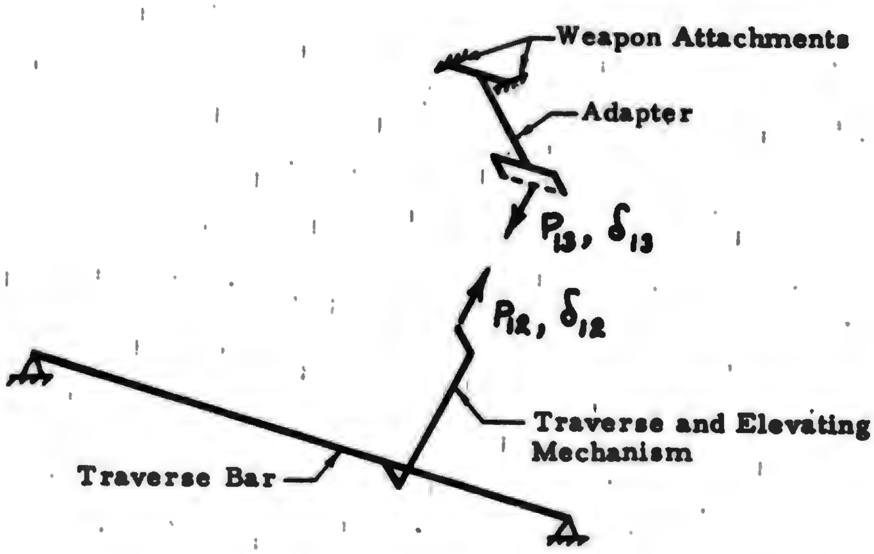


Figure 3.4 The Traverse and Elevating Components

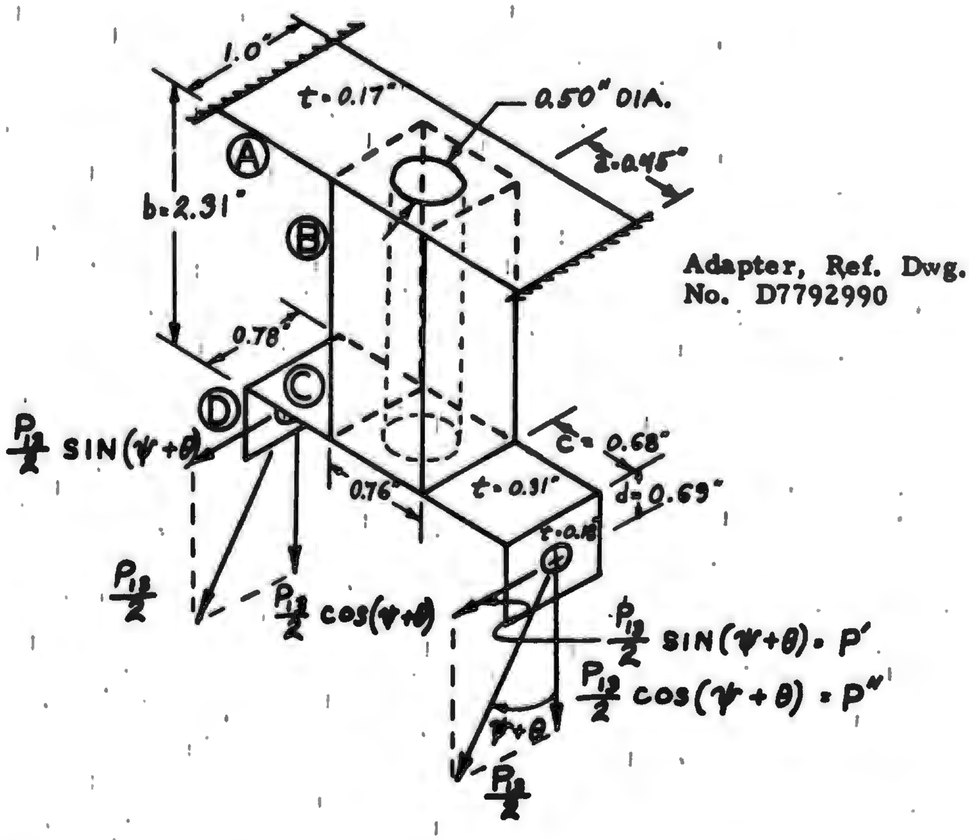
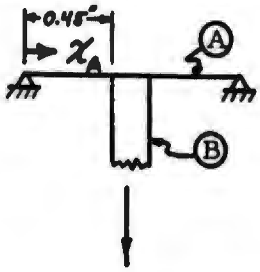


Figure 3.5 Dimensions and Geometry of the Adapter

energy is the sum of the contribution from each part.

Strain Energy of Element Ⓐ

Element Ⓐ is assumed to be pinned at the weapon interface and fixed to element Ⓑ .



The bending strain energy due to the vertical component of P_{13} is $\frac{1}{2}\int_0^{\ell}\frac{2M^2}{EI_A}\,dx_A$ where $M = \frac{x_A P''}{2}$. The twisting strain energy due to the horizontal component of P_{13} is $\frac{1}{2}\int_0^{\ell}\frac{2T^2}{GJ_A}\,dx_A$ where $T = \frac{P'}{2}(b+d)$. The section properties of element Ⓐ are as follows:

$$I_A = \frac{bh^3}{12} = \frac{1\,(0.17)^3}{12} = .00041 \text{ in.}^4$$

$$J_A = \frac{1}{3}bh^3 = .00164 \text{ in.}^4$$

The strain energy of element Ⓐ is

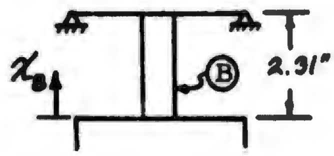
$$U_A = \frac{1}{2}\int_0^{\ell=.45} 2\,\frac{\left(\frac{x_A P''}{2}\right)^2}{EI_A}\,dx_A + \frac{1}{2}\int_0^{\ell=.45} 2\,\frac{\left[\frac{P'}{2}(b+d)\right]^2}{GJ_A}\,dx_A$$

$$= \frac{1}{2}\,\frac{P''^2}{2}\,\frac{(.45)^3}{3E\,(.00041)} + \frac{1}{2}\,\frac{P'^2}{2}\,\frac{(3)^2\,(.45)}{0.385E\,(.00164)}$$

$$= \frac{1}{2}\,\frac{37}{E}\,P''^2 + \frac{1}{2}\,\frac{3207}{E}\,P'^2 \qquad (3.30)$$

Strain Energy of Element Ⓑ

Element Ⓑ is assumed to be a solid square block with a circular hole down its center



The bending strain energy due to P' is $\frac{1}{2}\int_0^b \frac{M^2}{EI_B}\,dx_B$ where $M = P'(d + x_B)$.
The axial strain energy due to P'' is $\frac{1}{2}\int_0^b \frac{P^2}{EA_B}\,dx_B$ where $P = P''$.
The section properties of element Ⓑ are as follows:

$$A_B = (0.76)(0.78) - \frac{\pi}{4}(0.5)^2 = 0.396 \text{ in.}^2$$

$$I_B = \frac{(0.76)(0.78)^3}{12} - \frac{1}{4}\pi(0.25)^4 = .0270 \text{ in.}^4$$

The strain energy of element Ⓑ is

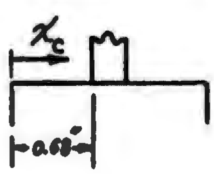
$$U_B = \frac{1}{2}\frac{P'^2}{EI_B}\int_0^{b=2.31}(d + x_B)^2\,dx_B + \frac{1}{2}\frac{P''^2}{EA_B}\int_0^{b=2.31} dx_B$$

$$= \frac{1}{2}\frac{P'^2}{E(.027)}\left[0.69^2(2.31) + 0.69(2.31)^2 + \frac{1}{3}(2.31)^3\right] + \frac{1}{2}P''^2\frac{2.31}{0.396E}$$

$$= \frac{1}{2}\frac{329}{E}P'^2 + \frac{1}{2}\frac{6}{E}P''^2 \qquad (3.31)$$

Strain Energy of Element Ⓒ

The x_c dimension of element Ⓒ is shown below



The bending strain energy due to P' is $\frac{1}{2}\int_0^c \frac{2M^2}{EI_c} dx_c$ where $M = \frac{1}{2} P' x_c$.

The bending strain energy due to P'' is $\frac{1}{2}\int_0^c \frac{2M_1^2}{EI_{c_1}} dx_c$ where $M_1 = \frac{1}{2} P'' x_c$.

The twisting strain energy due to P' is $\frac{1}{2}\int_0^c \frac{2T^2}{GJ_c} dx_c$ where $T = \frac{1}{2} P' d$.

The section properties of element Ⓒ are as follows:

$$I_c = \frac{(0.31)(0.78)^3}{12} = .0123 \text{ in.}^4$$

$$I_{c_1} = \frac{(0.78)(0.31)^3}{12} = .00194 \text{ in.}^4$$

$$J_c = \frac{1}{3}(.78)(.31)^3 = .00775 \text{ in.}^4$$

The strain energy of element Ⓒ is:

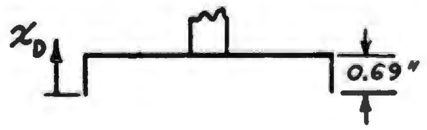
$$U_c = \frac{1}{2}\frac{P'^2}{2EI_c}\int_0^{c=.68} x_c^2\, dx_c + \frac{1}{2}\frac{P''^2}{2EI_{c_1}}\int_0^{c=.68} x_c^2\, dx_c + \frac{1}{2}\frac{P'^2 d}{2EI_c}\int_0^{c=.68} dx_c$$

$$= \frac{1}{2}\frac{P'^2}{2}\frac{(0.68)^3}{3E(.0123)} + \frac{1}{2}\frac{P''^2}{2}\frac{(0.68)^3}{3E(.00194)} + \frac{1}{2}\frac{P'^2}{2}\frac{(0.68)(0.69)^2}{0.385E(.00775)}$$

$$= \frac{1}{2}\frac{58}{E}P'^2 + \frac{1}{2}\frac{27}{E}P''^2 \qquad (3.32)$$

Strain Energy of Element Ⓓ

The x_d dimension of element Ⓓ is shown below



The bending strain energy due to P' is $\frac{1}{2}\int_0^d \frac{2M^2}{EI_D} dx_d$ where $M = \frac{1}{2} P' x_d$.

The axial strain energy due to P'' is $\frac{1}{2}\int_0^d \frac{2P^2}{EA_D} dx_d$ where $P = \frac{1}{2} P''$.

The section properties of element Ⓓ are as follows:

$$A_D = (0.18)(0.78) = 0.141 \text{ in.}^2$$

$$I_D = \frac{(0.18)(0.78)^3}{12} = .00712 \text{ in.}^4$$

The strain energy of element Ⓓ is:

$$U_D = \frac{1}{2}\frac{P'^2}{2EI_D}\int_0^{d=.69} x_d^2\,dx_d + \frac{1}{2}\frac{P''^2}{2EA_D}\int_0^{d=.69} dx_d$$

$$= \frac{1}{2}\frac{P'^2}{2}\frac{(0.69)^3}{3E(.00712)} + \frac{1}{2}\frac{P''^2}{2}\frac{(0.69)}{E(0.141)}$$

$$= \frac{1}{2}\frac{8}{E}P'^2 + \frac{1}{2}\frac{2}{E}P''^2 \qquad (3.33)$$

The total strain energy of the adapter is

$$U = U_A + U_B + U_C + U_D \qquad (3.34)$$

substituting Equations 3.30 to 3.33 into 3.34 gives

$$U = \frac{1}{2}\frac{3602}{E}P'^2 + \frac{1}{2}\frac{72}{E}P''^2$$

Substituting $P' = \text{SIN}(\psi+\theta)P_{13}$ and $P'' = \text{COS}(\psi+\theta)P_{13}$ into this equation, we obtain

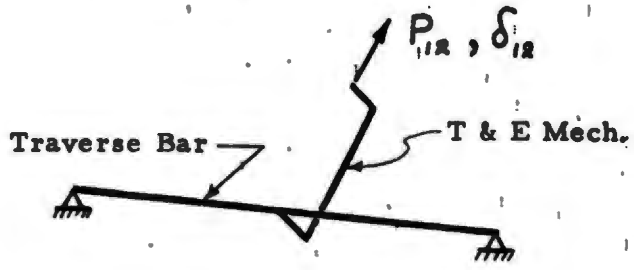
$$U = \frac{1}{2}\frac{3602}{E}\sin^2(\psi+\theta)P_{13}^2 + \frac{1}{2}\frac{72}{E}\cos^2(\psi+\theta)P_{13}^2$$

The flexibility influence coefficient is the second derivative of the strain energy $\alpha_{13,13} = \frac{\partial^2 U}{\partial P_{13}^2}$, therefore the flexibility of the adapter is

$$\alpha_{13,13} = \frac{3602}{E}\sin^2(\psi+\theta) + \frac{72}{E}\cos^2(\psi+\theta) \qquad (3.35)$$

<u>Traversing and Elevating Mechanism and Traverse Bar Flexibility</u>

This component is comprised of two elements: the T & E mechanism whose flexibility is a function of the length of the mechanism; and the traverse bar, whose flexibility varies with the aximuth angle.



Since these elements act in series; the total flexibility of the two parts is the sum of the individual flexibilities:

$$\alpha_{12,12} = \alpha_T + \alpha_B \tag{3.36}$$

where α_T is the flexibility of the T & E mechanism, and α_B is the flexibility of the Traverse Bar.

<u>Traverse Bar Flexibility</u>

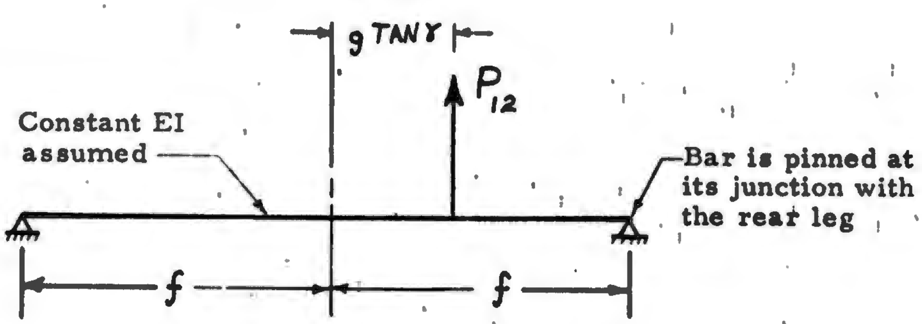


Figure 3. 6 Traverse Bar Structual Model

The deflection of the traverse bar at the unit load is

$$\alpha_B = \frac{(f - g\tan\gamma)^2 (f + g\tan\gamma)^2}{3EI(2f)} = \frac{(f^2 - g^2\tan^2\gamma)^2}{6EIf} \tag{3.37}$$

From USAWECOM Drawing No. C7115350 the diameter of the traverse bar is 0. 75 in. The inertia of the bar is therefore $I = \frac{\pi}{64}(.75)^4 = .0155$ in.^{4}

The other dimensions were presented in Table 3.1 and are: f=7.14 in.; g=11.24 in. Equation 3.37 becomes

$$\alpha_B = \frac{\left[7.14^2 - 11.24^2 \, TAN^2 \gamma\right]^2}{6\,(.0155)(7.14)\,E} = \frac{1}{E}\left(62.56 - 155.04\ TAN^2 \gamma\right)^2 \tag{3.38}$$

which is the flexibility of the traverse bar as a function of azimuth angle γ. Table 3.2 shows how the flexibility of the traverse bar is affected by variations in azimuth angle.

Table 3.2 Traverse Bar Flexibility for Various Azimuth Angles

Azimuth Angle (γ)	Flexibility (α_B) (lbs/in.)
0°	3,914/E
6°	3,703/E
13°	2,948/E
20°	1,766/E
26°	660/E

Traverse and Elevating Mechanism Flexibility

The flexibility in the T & E Mech. is due to the bending in the screw assembly. Since the length of the mechanism varies with weapon elevation and azimuth angle so does its flexibility. The flexibility will be computed as a function of the mechanism length. For any given weapon attitude the screw assembly length can be computed and the corresponding flexibility obtained directly. Figure 3.7 shows the components of this mechanism.

From USAWECOM Drawing Nos. F7791119 and F7790723 the length of the mechanism as shown in Figure 3.8 when the elevation and azimuth angles are zero is 4.99 in. This is used as the reference length. The flexibility for the reference length is as follows:

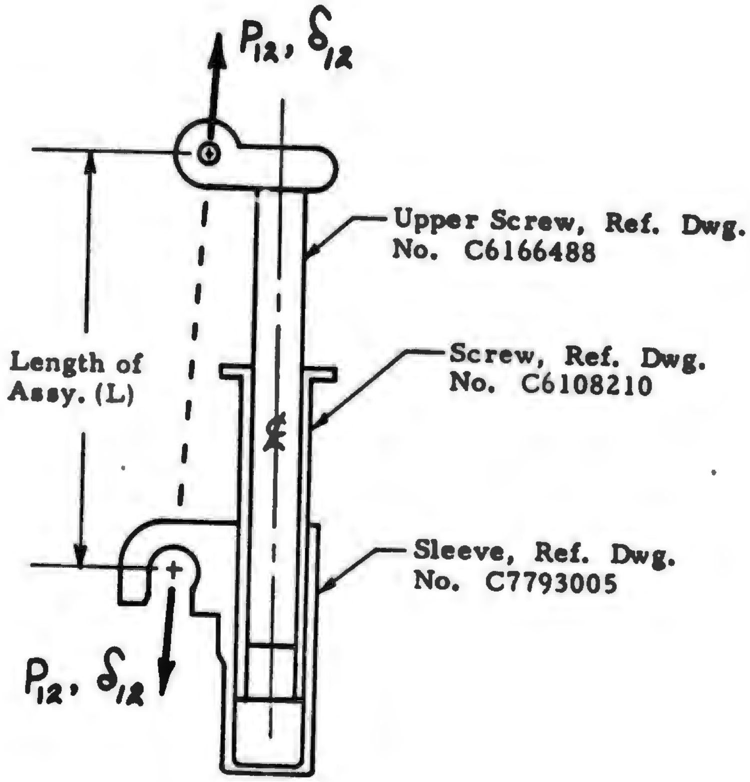


Figure 3.7 The Traverse and Elevating Mechanism

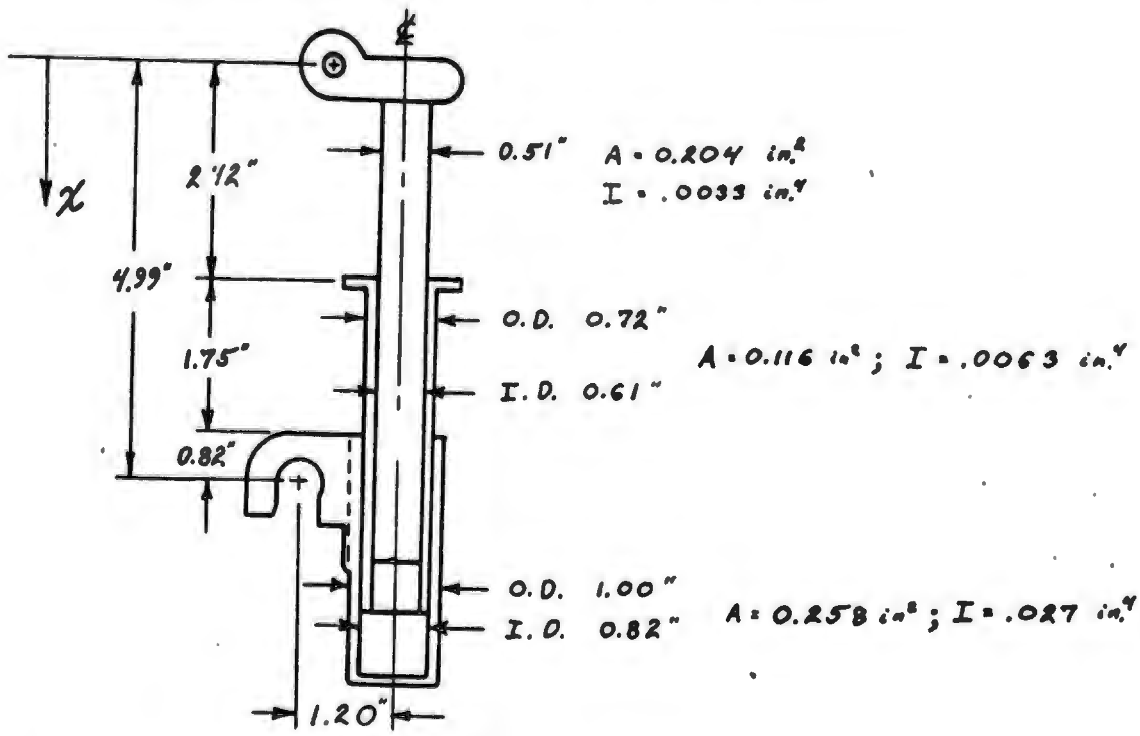


Figure 3.8 Dimensions and Geometry of the Traverse and Elevating Mechanism

The strain energy in the mechanism is given by

$$U = \frac{1}{2}\int_0^L \frac{M^2}{EI}\,dx + \frac{1}{2}\int_0^L \frac{P^2}{AE}\,dx \tag{3.39}$$

where M(x) and P are (from geometry shown in Figure 3.8).

$$M(x) = \left(0.87 + \frac{1.20 - 0.87}{4.99}x\right)P_{12}$$

and $\quad P = P_{12}$

Substituting M(x) and P into Equation 3.39 gives

$$U = \frac{1}{2}\frac{P_{12}^2}{E}\Bigg[\int_0^{2.42} \frac{0.757 + 0.115x + .00435x^2}{.0033}\,dx + \int_{2.42}^{4.17} \frac{0.757 + 0.115x + .00435x^2}{.0033 + .0063}\,dx + \int_{4.17}^{4.99} \frac{0.757 + 0.115x + .00435x^2}{.027 + .0063}\,dx + \frac{2.42}{0.204} + \frac{1.75}{0.204 + 0.116} + \frac{0.82}{0.116 + 0.258}\Bigg] \tag{3.40}$$

Evaluating Equation 3.40

$$U = \frac{1}{2}\frac{P_{12}^2}{E}(932)$$

Then

$$\alpha_T = \frac{\partial^2 U}{\partial P_{12}^2} = \frac{932}{E}$$

This same computation was performed for other mechanism lengths and the results are shown in Table 3.3. As seen from this table the flexibility of the T & E Mech. is close to being a linear function of the length, L. Using the end points from Table 3.3 we obtain

$$\alpha_T(L) = [521 + 206(L - 2.99)]\frac{1}{E}$$

$$\alpha_T(L) = (206L - 95)\frac{1}{E} \tag{3.41}$$

Table 3.3 Traverse and Elevating Mechanism Flexibility for Various Mechanism Lengths

Mechanism Length (in.)	Flexibility (α_T) (lbs/in.)
2.99	521/E
3.99	727/E
4.49	831/E
4.99	932/E
5.49	1,035/E
5.99	1,138/E
6.99	1,346/E

The total flexibility of the T & E components $\alpha_{12,12}$ is given by Equation 3.36 as the sum of the flexibility of the T & E Mech. (α_T) and the flexibility of the Traverse Bar (α_B).

$$\alpha_{12,12} = \alpha_T + \alpha_B$$

where α_B is given by Equation 3.38 and α_T is given by Equation 3.41. Substituting these two equations into Equation 3.36 produces the flexibility of the M122 Mount Traverse and Elevating Component Substructure.

$$\alpha_{12,12} = \frac{1}{E}\left[206\,L - 95 + \left(62.56 - 155.04\,\tan^2\gamma\right)^2\right] \qquad (3.42)$$

where L is computed by Equation 2.8 and γ is the azimuth angle of the weapon.

3.4 Flexibility of the M122 Tripod Mount

The flexibility matrix of the M122 Tripod Mount can now be computed by combining the individual flexibility matrices of the three substructures.

These substructure flexibility matrices are combined using the force transformation matrix [b], as was shown in Section 2, to produce an equation for each element of the Tripod Mount Flexibility Matrix. This matrix was presented in Equation 2.35 as

$$[a] = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{bmatrix}$$

Since the matrix is symmetric ($a_{ij} = a_{ji}$), only the upper triangular portion needs to be computed to completely define the system's structural characteristics. The individual terms of [a] were given by Equation 2.34.

$$[a] = [a^m] + [a^p] + [a^t]$$

Since the flexibility matrix of the weapon mount is a function of elevation and azimuth angle and is comprised of many terms; only two examples will be illustrated. The first example is the variation of the $a_{1,1}$ term of the flexibility matrix as a function of elevation and azimuth. This term corresponds to the flexibility of the mount in the bore axis direction. The second example is the flexibility matrix for the complete mount for the reference position, e.g., zero elevation and azimuth angles.

Figure 3.9 presents the M122 Tripod Mount geometry used for the numerical evaluation in this section.

<u>M122 Mount Flexibility in Weapon Bore Axis Direction for Varying Elevation and Azimuth Angles</u>

The $a_{1,1}$ term will be considered in detail because it corresponds to the mount stiffness term in the machine gun equations of motion in USAWECOM Technical Rpt. 70-114; Reference 1. From Equation 2.34 the $a_{1,1}$ term is

$$a_{1,1} = a_{1,1}^m + a_{1,1}^p + a_{1,1}^t \qquad (3.43)$$

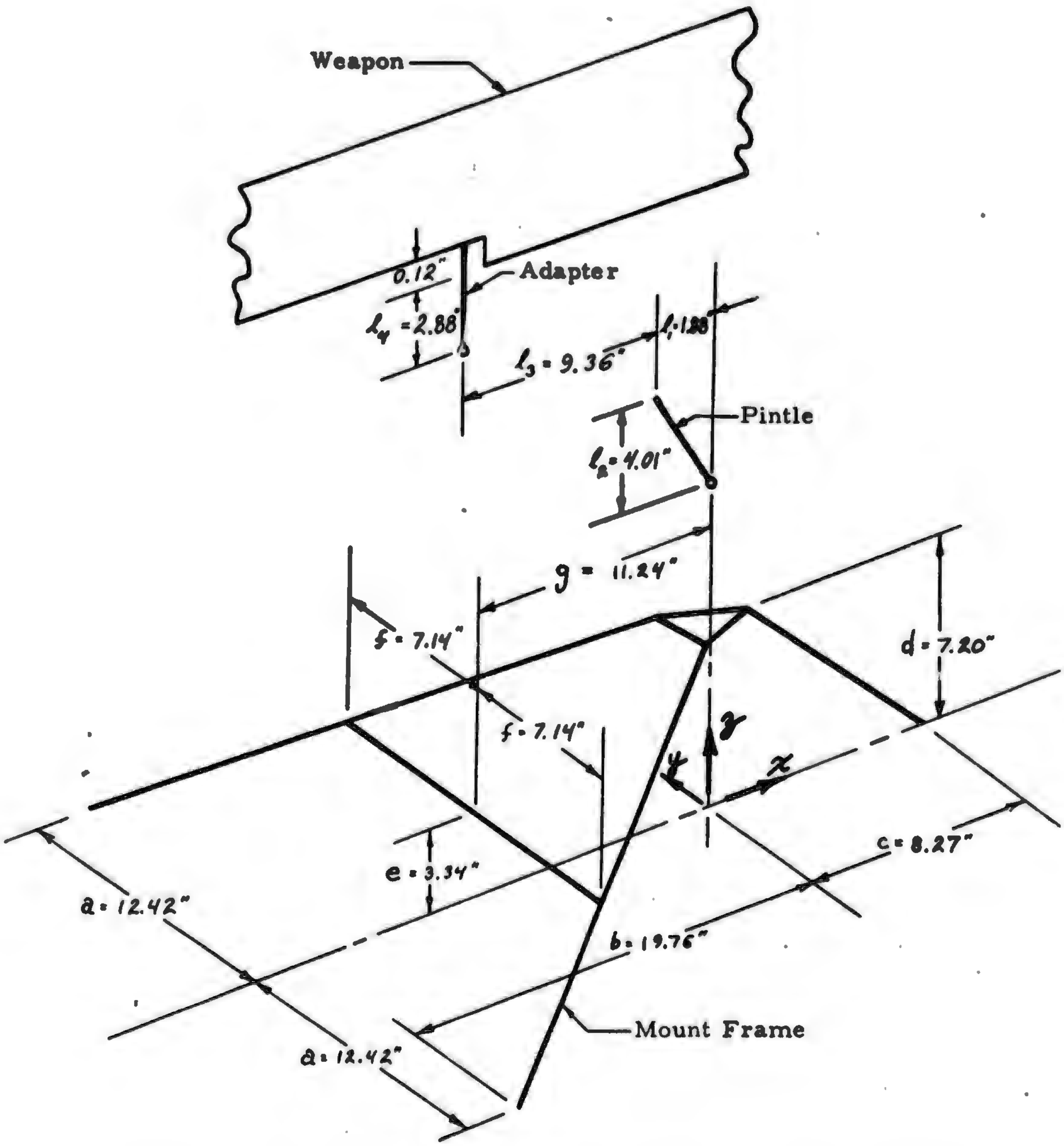


Figure 3.9 Dimensions and Geometry of the M122 Tripod Mount

Substituting the $a_{1,1}$ term for each of the three substructures (Equations 2.29, 2.31 and 2.33) gives

$$\begin{aligned} a_{1,1} = {} & \alpha_{1,1} b_{1,1}^2 + 2\alpha_{1,2} b_{1,1} b_{2,1} + \alpha_{2,2} b_{2,1}^2 + 2\alpha_{1,5} b_{5,1} b_{1,1} \\ & + 2\alpha_{2,5} b_{2,1} b_{5,1} + \alpha_{3,3} b_{3,1}^2 + 2\alpha_{3,4} b_{3,1} b_{4,1} + \alpha_{4,4} b_{4,1}^2 \\ & + \alpha_{5,5} b_{5,1}^2 + \alpha_{10,10} b_{10,1}^2 + 2\alpha_{10,11} b_{10,1} b_{11,1} + \alpha_{11,11} b_{11,1}^2 \end{aligned} \qquad (3.44)$$

The substructure flexibilities (α 's) have been computed in Sections 3.1, 3.2 and 3.3. The variation in stiffness due to elevation (θ) and azimuth angle (γ) is specified through the (b's). Substituting the b's from Equations 2.9, 2.11 and 2.12 into Equation 3.44 produces the general expression for the flexibility of the Tripod Mount in the weapon bore axis direction.

$$\begin{aligned} a_{1,1} = {} & \alpha_{1,1} \cos^2\theta \cos^2\gamma + 2\alpha_{1,2} \cos\theta \sin\theta \cos\gamma \\ & + \alpha_{2,2} \sin^2\theta + 2\alpha_{1,5} \cos^2\gamma \cos\theta (l_1 \sin\theta + l_2 \cos\theta) \\ & + 2\alpha_{2,5} \sin\theta \cos\gamma (l_1 \sin\theta + l_2 \cos\theta) + \alpha_{3,3} \sin^2\gamma \cos^2\theta \\ & - 2\alpha_{3,4} \cos\theta \sin^2\gamma (l_1 \sin\theta + l_2 \cos\theta) + \alpha_{4,4} \sin^2\gamma (l_1 \sin\theta + l_2 \cos\theta)^2 \\ & + \alpha_{5,5} \cos^2\gamma (l_1 \sin\theta + l_2 \cos\theta)^2 + \alpha_{10,10} \sin^2\theta \\ & + 2\alpha_{10,11} \sin\theta \cos\theta + \alpha_{11,11} \cos^2\theta \end{aligned} \qquad (3.45)$$

Equation 3.45 has been evaluated for several values of elevation and azimuth orientation.

The α coefficients in Equation 3.45 have been evaluated for two different cases of mount-ground boundary conditions. The first case assumes all three foot pads pinned at the ground junction. The second case assumes the forward leg is released in the fore-aft (x) direction and the forward leg is free to move along the ground.

Tables 3.4 and 3.5 present the computation of $a_{1,1}$ for varying elevation and azimuth angles. The individual terms in this summation are presented in order to indicate the contribution of the various element flexibilities to the system's flexibility. Table 3.4 contains the flexibilities for the mount with all three foot pads pinned to the ground, and Table 3.5 contains the flexibilities for the mount with the forward leg released. Comparison of the computed flexibilities shows the effect of azimuth angular variation on the bore axis flexibility is small compared to the effect of elevation angular variation.

For a single degree of freedom system, the stiffness is the reciprocal of the flexibility. Tables 3.4 and 3.5 also present the stiffness $k_{1,1}$ corresponding to each $a_{1,1}$. The stiffnesses computed in these tables were presented in graphical form in Figure 1.3. The pinned mount is always stiffer than the mount with the front leg released in the forward direction. The mount stiffness variation with elevation is more regular for the pin ended mount than for the released mount because of the small coupling terms $\alpha_{1,2}$ in the mount frame substructure flexibility matrix. The comparison of these two cases demonstrate the necessity of incorporating the mount and its supporting medium into a single model.

M122 Tripod Mount Flexibility Matrix for Zero Elevation and Azimuth Angles

The complete mount flexibility matrix $[\,a\,]$ has been computed for the weapon in the reference position ($\theta = 0^\circ$ and $\gamma = 0^\circ$). Both the pin ended and the front leg released mount frame flexibilities are used.

First, several preliminary computations are required. The angle of the T & E Mech. must be computed. From Equation 2.6

$$x_1 = \frac{g}{\cos\gamma} - l_3\cos\theta + l_4\sin\theta - l_1$$

and

$$x_2 = d - e + l_2 + l_3\sin\theta - l_4\cos\theta$$

Table 3.4 Numerical Values of the Flexibility and Stiffness of the M122 Tripod Mount in the Bore Axis Direction as a Function of Elevation and Azimuth Angular Variations. The Mount Frame is Pinned at the Mount-Ground Interface.

TERMS		$\theta = -13°$			$\theta = 0°$			$\theta = 13°$		
Constants	$f(\theta,\gamma)$	$\gamma = 0°$	$\gamma = 13°$	$\gamma = 26°$	$\gamma = 0°$	$\gamma = 13°$	$\gamma = 26°$	$\gamma = 0°$	$\gamma = 13°$	$\gamma = 26°$
$\alpha_{2,2}+\alpha_{10,10} = 285.74$	$\sin^2\theta$	14.45	14.45	14.45	0	0	0	14.45	14.45	14.45
$2\alpha_{10,11} = 657.40$	$\sin\theta\cos\theta$	-144.09	-144.09	-144.09	0	0	0	144.09	144.09	144.09
$\alpha_{11,11} = 573.64$	$\cos^2\theta$	544.61	544.61	544.61	573.64	573.64	573.64	544.61	544.61	544.61
$\alpha_{1,1} = 63.20$	$\cos^2\theta\cos^2\gamma$	60.02	56.97	48.48	63.20	60.02	51.03	60.02	56.97	48.48
$2\alpha_{1,2} = 2.86$	$\cos\theta\sin\theta\cos\gamma$	-0.63	-0.61	-0.56	0	0	0	0.63	0.61	0.56
$2\alpha_{1,5} = -76.98$	$\xi_1\cos^2\gamma\cos\theta$	-261.32	-248.10	-211.10	-308.69	-293.09	-249.37	-324.79	-308.37	-262.38
$2\alpha_{2,5} = 48.60$	$\xi_1\sin\theta\cos\gamma$	-38.09	-37.11	-34.23	0	0	0	47.33	46.11	42.54
$\alpha_{3,3} = 181.02$	$\sin^2\gamma\cos^2\theta$	0	8.70	33.03	0	9.16	34.79	0	8.70	33.03
$-2\alpha_{3,4} = -72.50$	$\xi_1\sin^2\gamma\cos\theta$	0	-12.45	-47.30	0	-14.71	-55.88	0	-15.48	-58.78
$\alpha_{4,4} = 75.59$	$\xi_1^2\sin^2\gamma$	0	46.43	176.32	0	61.51	233.58	0	71.72	272.35
$\alpha_{5,5} = 58.62$	$\xi_1^2\cos^2\gamma$	711.54	675.53	574.80	942.62	894.97	761.47	1099.06	1043.51	887.86
$a_{1,1}$		886.5	904.3	954.4	1270.7	1291.5	1349.2	1585.4	1606.9	1666.8
$k_{1,1}$ (lbs/in.)		32,710.	32,070	30,380	22,820.	22,450.	21,490.	18,290.	18,050.	17,400.

Notes: $\xi_1 = l_1 \sin\theta + l_2 \cos\theta = 1.88 \sin\theta + 4.01 \cos\theta$

All flexibility terms are normalized by E; the stiffness terms include E (29×10^6 psi for Steel).

Table 3.5 Numerical Values of the Flexibility and Stiffness of the M122 Tripod Mount in the Bore Axis Direction as a Function of Elevation and Azimuth Angular Variations. The Mount Frame Aft Two Legs are Pinned at the Mount-Ground Interface and the Forward Leg is Released in the Fore-Aft Direction.

TERMS		$\theta = -13°$			$\theta = 0°$			$\theta = 13°$		
Constants	$f(\theta,\gamma)$	$\gamma = 0°$	$\gamma = 13°$	$\gamma = 26°$	$\gamma = 0°$	$\gamma = 13°$	$\gamma = 26°$	$\gamma = 0°$	$\gamma = 13°$	$\gamma = 26°$
$\alpha_{2,2}+\alpha_{10,10} = 8{,}382.8$	$\sin^2\theta$	424.19	424.19	424.19	0	0	0	424.19	424.19	424.19
$2\alpha_{10,11} = 657.40$	$\sin\theta\cos\theta$	-144.09	-144.09	-144.09	0	0	0	144.09	144.09	144.09
$\alpha_{11,11} = 573.64$	$\cos^2\theta$	544.61	544.61	544.61	573.64	573.64	573.64	544.61	544.61	544.61
$\alpha_{1,1} = 1536.77$	$\cos^2\theta\cos^2\gamma$	1459.00	1385.17	1178.62	1536.77	1459.00	1241.44	1459.00	1385.17	1178.62
$2\alpha_{1,2} = -6905.54$	$\cos\theta\sin\theta\cos\gamma$	1513.59	1474.79	1360.40	0	0	0	-1513.59	-1474.79	-1360.40
$2\alpha_{1,5} = -455.62$	$\xi_1\cos^2\gamma\cos\theta$	-1546.70	-1468.42	-1249.45	-1827.04	-1734.58	-1475.92	-1922.27	-1824.98	-1552.85
$2\alpha_{2,5} = 936.18$	$\xi_1\sin\theta\cos\gamma$	-733.71	-714.90	-659.45	0	0	0	911.87	888.50	819.58
$\alpha_{3,3} = 181.02$	$\sin^2\gamma\cos^2\theta$	0	8.70	33.03	0	9.16	34.79	0	8.70	33.03
$-2\alpha_{3,4} = -72.50$	$\xi_1\cos\theta\sin^2\gamma$	0	-12.45	-47.30	0	-14.71	-55.88	0	-15.48	-58.78
$\alpha_{4,4} = 75.59$	$\xi_1^2\sin^2\gamma$	0	46.43	176.32	0	61.51	233.58	0	71.72	272.35
$\alpha_{5,5} = 82.95$	$\xi_1^2\cos^2\gamma$	1006.87	955.91	813.37	1333.84	1266.34	1077.51	1555.22	1476.51	1256.34
$a_{1,1}$		2523.8	2499.9	2430.2	1617.2	1620.3	1629.1	1603.1	1628.2	1700.8
$k_{1,1}$ (lbs/in.)		11,490.	11,600.	11,930.	17,930.	17,900.	17,800.	18,090	17,810	17,050

Notes: $\xi_1 = l_1 \sin\theta + l_2\cos\theta = 1.88\sin\theta + 4.01\cos\theta$

All flexibility terms are normalized by E; the stiffness terms include E (29×10^6 psi for Steel).

For the reference case, $\theta = 0^\circ$ and $\gamma = 0^\circ$. The other dimensions in the above equation are given in Figure 3.9, therefore

$$\chi_1 = 11.24 - 9.36 + 0 - 1.88 = 0$$

$$\chi_2 = 7.20 - 3.34 + 4.01 + 0 - 2.88 = 4.99$$

The equation for the T & E Mech. angle ψ is

$$\psi = \text{ARCTAN}\,\frac{\chi_1}{\chi_2} = \text{ARCTAN}\ 0 = 0^\circ$$

and the length of the T & E Mech. is, from Equation 2.8:

$$L = \left[\chi_1^2 + \chi_2^2\right]^{1/2} = \left[0 + 4.99^2\right]^{1/2} = 4.99 \text{ in.}$$

The flexibility of the T & E Components which are functions of θ, γ and ψ can now be defined. From Equation 3.35

$$\alpha_{13,13} = \frac{3602}{E} \sin^2(\psi + \theta) + \frac{72}{E} \cos^2(\psi + \theta)$$

$$= \frac{72}{E}$$

From Equation 3.42

$$\alpha_{12,12} = \frac{1}{E}\left[206L - 95 + (62.56 - 155.0 \tan^2\gamma)^2\right]$$

$$= \frac{1}{E}\left[206L - 95 + (62.56 - 0)^2\right]$$

$$= \frac{1}{E}\left[4{,}846.7\right]$$

The force transformations for this case are:

$b_{1,1} = 1$	$b_{1,2} = 0$	$b_{1,3} = 0$
$b_{2,1} = 0$	$b_{2,2} = 1$	$b_{2,3} = 0$
$b_{3,1} = 0$	$b_{3,2} = 0$	$b_{3,3} = 0$
$b_{4,1} = 0$	$b_{4,2} = 0$	$b_{4,3} = 0$
$b_{5,1} = 4.01$	$b_{5,2} = 1.88$	$b_{5,3} = 0$
		$b_{6,3} = 1/2$
$b_{10,1} = 0$	$b_{10,2} = 1$	$b_{7,3} = 0$
$b_{11,1} = 1$	$b_{11,2} = 0$	$b_{8,3} = 1/2$
		$b_{9,3} = 0$
		$b_{10,3} = 0$
		$b_{11,3} = 0$
		$b_{12,3} = 1$
		$b_{13,3} = 1$

where $b_{i,1}$ are computed from Equations 2.9 and 2.11, $b_{i,2}$ from Equation 2.12 and $b_{i,3}$ from Equation 2.23.

The mount flexibility terms are obtained from Equation 1.4. Substituting the force transformation values given above into these equations produces the flexibility terms of the [a] matrix for the reference position, $\theta = 0^\circ$ and $\gamma = 0^\circ$.

$$a_{1,1} = \alpha_{1,1} + 2(4.01)\alpha_{1,5} + 4.01^2\alpha_{5,5} + \alpha_{11,11}$$

$$a_{1,2} = \alpha_{1,2} + 1.88\,\alpha_{1,5} + 4.01\,\alpha_{2,5} + 4.01\,(1.88)\alpha_{5,5} + \alpha_{10,11}$$

$$a_{1,3} = \alpha_{1,6} + 4.01\,\alpha_{5,6}$$

$$a_{2,2} = \alpha_{2,2} + 2(1.88)\,\alpha_{2,5} + 1.88^2\,\alpha_{5,5} + \alpha_{10,10}$$

$$a_{2,3} = \alpha_{2,6} + 1.88\,\alpha_{5,6}$$

$$a_{3,3} = \frac{1}{2}\,\alpha_{6,6} + \frac{1}{2}\,\alpha_{6,8} + \alpha_{12,12} + \alpha_{13,13} \qquad (3.46)$$

Substituting the substructure flexibilities (α 's) computed in Sections 3.1, 3.2 and 3.3 into Equation 3.46 produces the tripod mount flexibilities. All computations include the mount Young's Modulus for Steel of 29 x 10^6 psi.

<u>M122 Tripod Mount With All Feet Pinned</u>

$$a_{1,1} = \frac{1}{29\times 10^6}\left[63.20 + 2(4.01)(-38.49) + 4.01^2(58.62) + 573.64\right]$$

$$= 43.82 \times 10^{-6}\ \ (\text{in.}/\text{lb})$$

$$a_{1,2} = \frac{1}{29\times 10^6}\left[1.432 + 1.88(-38.49) + 4.01(24.30) + 4.01(1.88)(58.62) + 328.70\right]$$

$$= 27.49 \times 10^{-6}\ \ (\text{in.}/\text{lb})$$

$$a_{1,3} = \frac{1}{29\times 10^6}\left[-164.14 + 4.01(263.55)\right] = 30.78 \times 10^{-6}\ (\text{in}/\text{lb})$$

$$a_{2,2} = \frac{1}{29\times 10^6}\left[79.56 + 2(1.88)(24.30) + 1.88^2(58.62) + 206.18\right] = 20.15 \times 10^{-6}\ \ (\text{in.}/\text{lb})$$

$$a_{2,3} = \frac{1}{29\times10^6}\left[151.74 + 1.88(263.55)\right]$$

$$= 22.32 \times 10^{-6} \text{ (in./lb)}$$

$$a_{3,3} = \frac{1}{29\times10^6}\left[\tfrac{1}{2}(5{,}234.7) + \tfrac{1}{2}(649.95) + 4{,}846.7 + 72.0\right]$$

$$= 271.07 \times 10^{-6} \text{ (in./lb)}$$

In matrix form

$$[a] = \begin{bmatrix} 43.82 & 27.49 & 30.78 \\ 27.49 & 20.15 & 22.32 \\ 30.78 & 22.32 & 271.07 \end{bmatrix} \times 10^{-6} \text{ (in./lb)} \qquad (3,47)$$

M122 Tripod Mount With Front Leg Restraint Released

$$a_{1,1} = \frac{1}{29\times10^6}\left[1{,}536.8 + 2(4.01)(-227.81) + 4.01^2(82.95) + 573.64\right] = 55.77\times10^{-6} \text{ (in./lb)}$$

$$a_{1,2} = \frac{1}{29\times10^6}\left[-3{,}452.8 + 1.88(-227.81) + 4.01(468.09) + 4.01(1.88)(82.95) + 328.70\right] = -36.21\times10^{-6} \text{ (in./lb)}$$

$$a_{1,3} = \frac{1}{29\times10^6}\left[-3{,}194.4 + 4.01(652.87)\right] = -19.88\times10^{-6} \text{ (in./lb)}$$

$$a_{2f} = \frac{1}{29\times10^6}\left[8{,}176.6 + 2(1.88)(468.09) + 1.88^2(82.95) + 206.18\right] = 359.86\times10^{-6} \text{ (in./lb)}$$

$$a_{2,3} = \frac{1}{29 \times 10^6}\left[7{,}255.0 + 1.88\,(652.87)\right] = 292.50 \times 10^{-6}\ (\text{in./lb})$$

$$a_{3,3} = \frac{1}{29 \times 10^6}\left[\tfrac{1}{2}(11{,}466.2) + \tfrac{1}{2}(6{,}881.4) + 4{,}846.7 + 72.0\right]$$

$$= 485.95 \times 10^{-6}\ (\text{in./lb})$$

In matrix form

$$[a] = \begin{bmatrix} 55.77 & -36.21 & -19.88 \\ -36.21 & 359.86 & 292.50 \\ -19.88 & 292.50 & 485.95 \end{bmatrix} \qquad (3.48)$$

The mount flexibility matrix for these two cases verify that the mount flexibility is strongly influenced by the mount frame boundary conditions. The mount with front leg released is more flexible than the mount with all legs pinned. Coordinate 2 has the largest increase in flexibility because the released frame must resist a vertical force at the head by frame bending whereas the pinned mount resists the same loading as a truss with axial loads.

Section 4

Mathematical Model of the Tripod Mount with Forward Leg Lift Off

The mount flexibility equations developed in Section 2 of this report describe a linear structural system where the three feet of the tripod mount were constrained to remain in the horizontal plane. This linear representation of the mount is insufficient to completely describe the system, since under certain firing conditions the forward leg of the tripod mount will lift off the ground. During firing of the weapon, the inertia and/or applied loads of the machine gun can become large enough to cause a tensile force in the front mount leg. This tensile force would have to be resisted by an equal and opposite tensile force through the front leg foot and in turn across the foot-ground interface. This is not possible since the class of mounts under consideration can transmit only compressive forces across the mount-ground interface. Therefore, when the summation of the forces produces a tensile force in the front leg, the linear mount model is no longer applicable and a new mathematical model is required.

Developed in this section is a mathematical model of the weapon-mount system which allows the front leg of the mount to lift off the ground and allows the weapon-mount system to rotate about an axis through the feet of the two back legs. This model is termed a nonlinear model since it incorporates into the mathematical model a nonlinear force deflection relation for the forward tripod leg. This section also presents the following: 1) the conditions under which the linear and nonlinear mount equations are to be applied, 2) the equations of motion for both the linear and nonlinear systems, and 3) the flexibility for the nonlinear tripod mount model.

The mathematical model of the tripod mount developed herein assumes that the equations of motion will be solved via a digital computer program utilizing a step-by-step numerical integration technique. This type of computer program is required since it is necessary to determine at each time step of the integration whether or not the front leg is in contact with the ground. When the leg is in contact with the ground the linear mathematical model developed in Section 2 is used to describe the response of the system. When the front leg is not in contact with the ground, the nonlinear model is used.

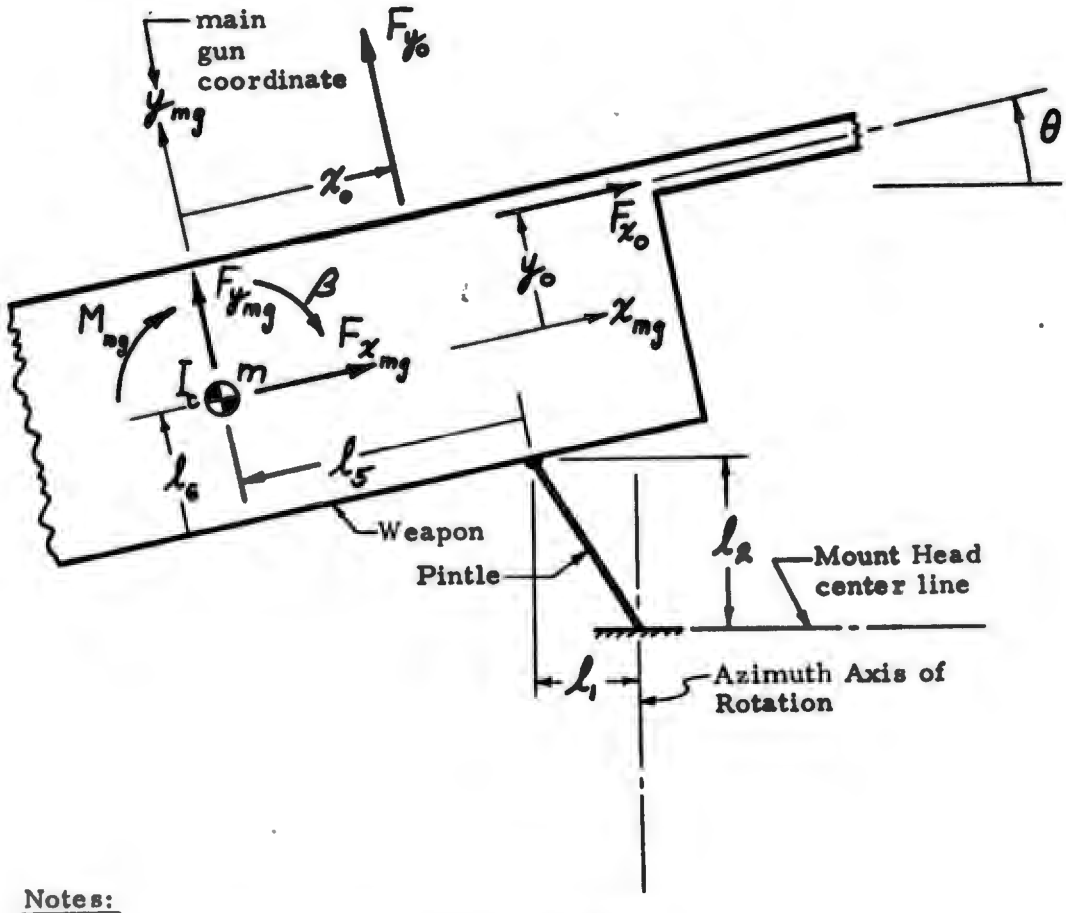
Initially the system is at rest and the front leg is subjected to a compressive load due to the force of gravity. As the analysis progresses and the forces are developed due to weapon firing, the condition of the mount must be checked to determine if, or when lift off occurs. The condition of lift off can be defined by either of the following two methods:

1. Front Leg Axial Force - The axial force in the front leg can be computed by using the three following quantities: the weapon displacements; a transformation to the front leg element displacements; and the force-deflection relation of the front leg.
2. External Equilibrium - Summation of moments due to the weapon inertial forces and gravitational forces about the axis through the two back feet.

The latter approach has been adapted for the analysis since it is the more direct of the two methods.

Figure 4.1 presents the coordinates and forces acting on the weapon. This figure shows the degrees of freedom of the main gun in the bore axis direction and the forces due to the operating parts of the weapon. The actual programmed solution of the weapon model would include the coordinates of the operating parts and their forces. For the present analysis these are grouped together as two forces and termed "operating parts forces".

The equations of motion of this system prior to front leg lift off are presented below in matrix form where m is the mass of the weapon plus an effective mass of the mount. The stiffness matrix of the mount is obtained by inverting the main gun flexibility matrix defined by Equation 2.46 in Section 2,e.g., $[k]=[a_{mg}]^{-1}$. Since the frequency of the excitation is considerably less than the fundamental



Notes:

Section is Vertical Plane through Bore Axis

F_{x_o} and F_{y_o} are operating part forces

$F_{x_{mg}}$ and $F_{y_{mg}}$ are C.G. forces

M_{mg} is moment about C.G.

I_c is mass moment of inertia of system about C.G.

m is mass of weapon plus effective mass of mount

Figure 4.1 Weapon Coordinates and Applied Forces for Linear Model frequency of the weapon-mount, the damping terms have been omitted.

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_c \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\beta} \end{Bmatrix}_{mg} + \begin{bmatrix} k_{1,1} & k_{1,2} & k_{1,3} \\ k_{2,1} & k_{2,2} & k_{2,3} \\ k_{3,1} & k_{3,2} & k_{3,3} \end{bmatrix} \begin{Bmatrix} x \\ y \\ \beta \end{Bmatrix}_{mg} = \begin{Bmatrix} F_{x_0} \\ F_{y_0} \\ F_{x_0} y_0 - F_{y_0} x_0 \end{Bmatrix} \tag{4.1}$$

At each time step of the integration process, the system is checked to determine if the front leg has lifted off the ground. This is accomplished by the summation of moments due to the external forces which are the reactions of the operating parts (F_{x_0} and F_{y_0}) and the system's weight (W); and the inertial forces about the 0-0 axis as shown in Figure 4.2. As shown in this figure, the moment is M_0 and is positive for a vertical force down at the C.G.

When the moment M_0 is positive, the system equilibrium is maintained by a vertical compressive (upward) reaction force acting on the foot of the forward leg. When this moment becomes negative a vertical tensile force would be required at the forward foot. Since the ground-foot interface cannot transmit tensile forces, the instant that the mount becomes zero is the critical condition and defines when lift off occurs. The computation of the moment for this condition in terms of the applied forces is performed as follows:

The horizontal and vertical position of the weapon's center of gravity in the bore axis plane are obtained from Figure 4.2 as

$$x_3 = \ell_1 + \ell_5 \cos\theta + \ell_6 \sin\theta$$
$$x_4 = \ell_2 - \ell_5 \sin\theta + \ell_6 \cos\theta$$

The center of gravity with respect to the axis of rotation is given by

$$\begin{aligned} x_c &= b - x_3 \cos\gamma \\ &= b - \cos\gamma(\ell_1 + \ell_5 \cos\theta + \ell_6 \sin\theta) \\ z_c &= d + x_4 \\ &= d + \ell_2 - \ell_5 \sin\theta + \ell_6 \cos\theta \end{aligned} \tag{4.2}$$

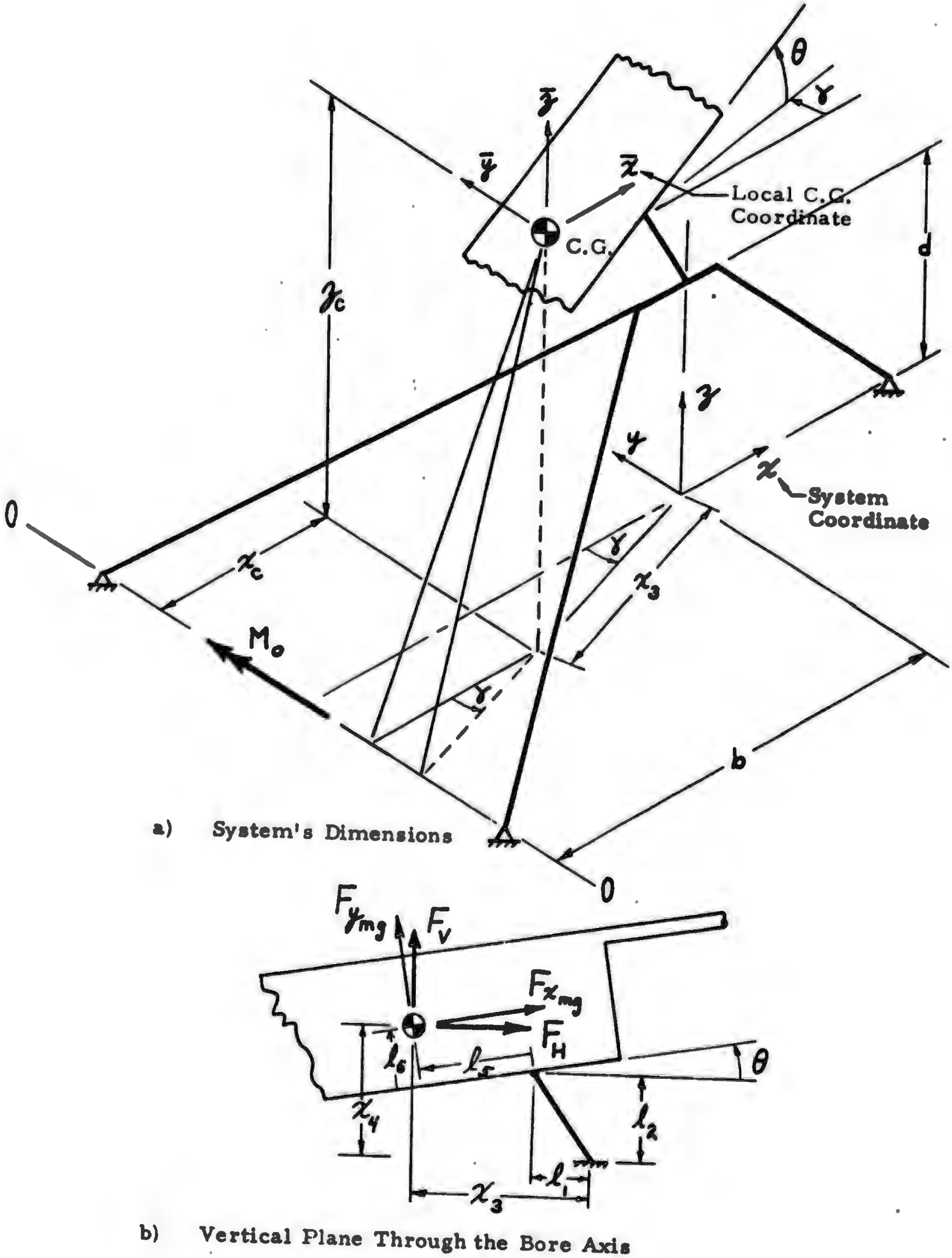


Figure 4.2 Generalized Tripod Mount Dimensions for the Linear Model

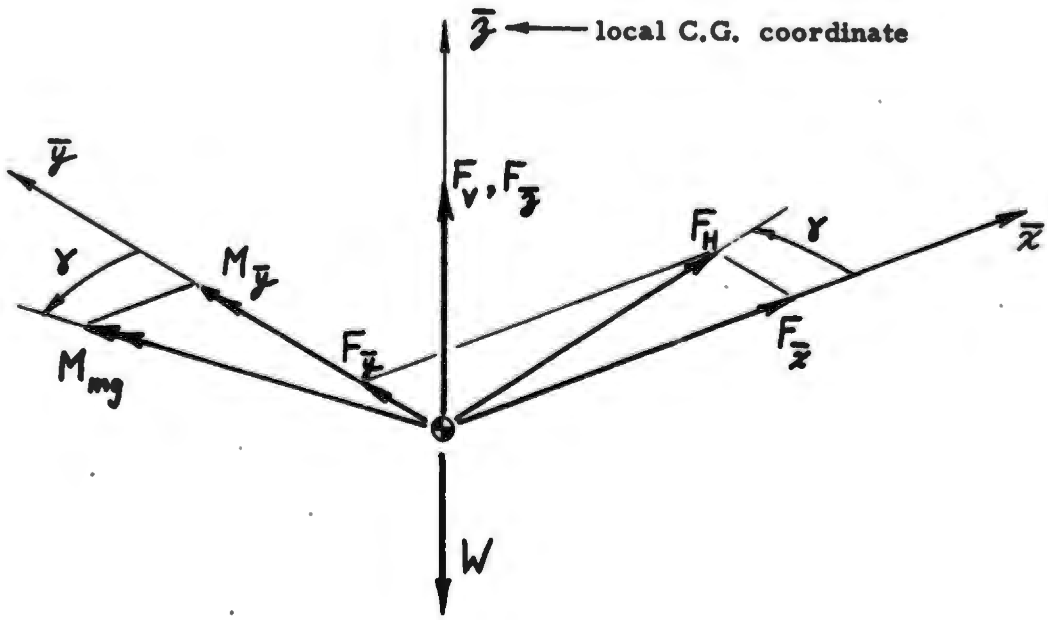
The main gun forces and moments are obtained from Figure 4.1 as follows:

$$F_{x_{mg}} = F_{x_o} - m\ddot{x}_{mg}$$
$$F_{y_{mg}} = F_{y_o} - m\ddot{y}_{mg}$$
$$M_{mg} = F_{x_o} y_o - F_{y_o} x_o - I_c \ddot{\beta} \qquad (4.3)$$

The external and inertial forces of the model must be resolved into vertical and horizontal forces in the bore axis plane as shown in Figure 4.2b.

$$F_V = F_{x_{mg}} \sin\theta + F_{y_{mg}} \cos\theta$$
$$F_H = F_{x_{mg}} \cos\theta - F_{y_{mg}} \sin\theta \qquad (4.4)$$

These forces and the moment M_{mg} are rotated to the local $\bar{x}$, $\bar{y}$, $\bar{z}$ axes at the center of gravity of the weapon as shown below



The C. G. forces and moment are

$$F_{\bar{x}} = F_H \cos\gamma$$
$$F_{\bar{y}} = F_H \sin\gamma$$
$$F_{\bar{z}} = F_V$$
$$M_{\bar{y}} = M_{mg} \cos\gamma \qquad (4.5)$$

The moment of these forces about the 0-0 axis from Figure 4.2 is

$$M_o = x_c(W - F_{\bar{z}}) + z_c F_{\bar{x}} + M_{\bar{y}} \qquad (4.6)$$

Substituting Equations 4.3, 4.4 and 4.5 into 4.6 yields M_o in terms of the external, inertial and gravitational forces.

$$\begin{aligned} M_o = {} & x_c W + F_{x_o}\left[z_c \cos\gamma\cos\theta + y_o\cos\gamma - x_c \sin\theta\right] \\ & - F_{y_o}\left[z_c \cos\gamma\sin\theta + x_o\cos\gamma + x_c\cos\theta\right] \\ & + \ddot{x}_{mg}\, m\left[-z_c\cos\gamma\cos\theta + x_c\sin\theta\right] \\ & + \ddot{y}_{mg}\, m\left[z_c\cos\gamma\sin\theta + x_c\cos\theta\right] \\ & - \ddot{\beta}\, I_c \cos\gamma \end{aligned} \qquad (4.7)$$

As discussed previously, when M_o becomes zero the front leg is in the incipient lift off condition.

When the front leg lifts off the ground, a new set of equations are required to describe the system's response; since the system now has a rigid body degree of freedom, allowing the weapon-mount system to rotate about the 0-0 axis. The only elastic deformation which can occur, must be in the plane formed by the 0-0 axis and the weapon C. G. The forces perpendicular to this plane cause rigid body rotation of the system about the 0-0 axis. The coordinates for this condition are shown in Figure 4. 3. The equations of motion for the nonlinear system are shown below

$$\begin{bmatrix} m & 0 \\ 0 & \bar{I}_o \end{bmatrix} \begin{Bmatrix} \ddot{x}_r \\ \ddot{\beta}_o \end{Bmatrix} + \begin{bmatrix} k_r & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} x_r \\ \beta_o \end{Bmatrix} = \begin{Bmatrix} F_r \\ \bar{M}_o \end{Bmatrix} \tag{4.8}$$

where $\bar{I}_o$ is the mass moment of inertia of the system about the 0-0 axis, i.e., $m\, r_c^2 + I_c \cos^2\gamma$

$\bar{M}_o$ is the moment of external and gravity forces about the 0-0 axis. From Equation 4. 7, $\bar{M}_o$ is

$$\begin{aligned} \bar{M}_o = x_c W + F_{x_o}\left[z_c \cos\gamma \cos\theta + y_o \cos\gamma - x_c \sin\theta \right] \\ - F_{y_o}\left[z_c \cos\gamma \sin\theta + x_o \cos\gamma + x_c \cos\theta \right] \end{aligned} \tag{4.9}$$

and F_r is the component of the forces which lie in the C. G., 0-0 plane, along the line of action in the radial direction.

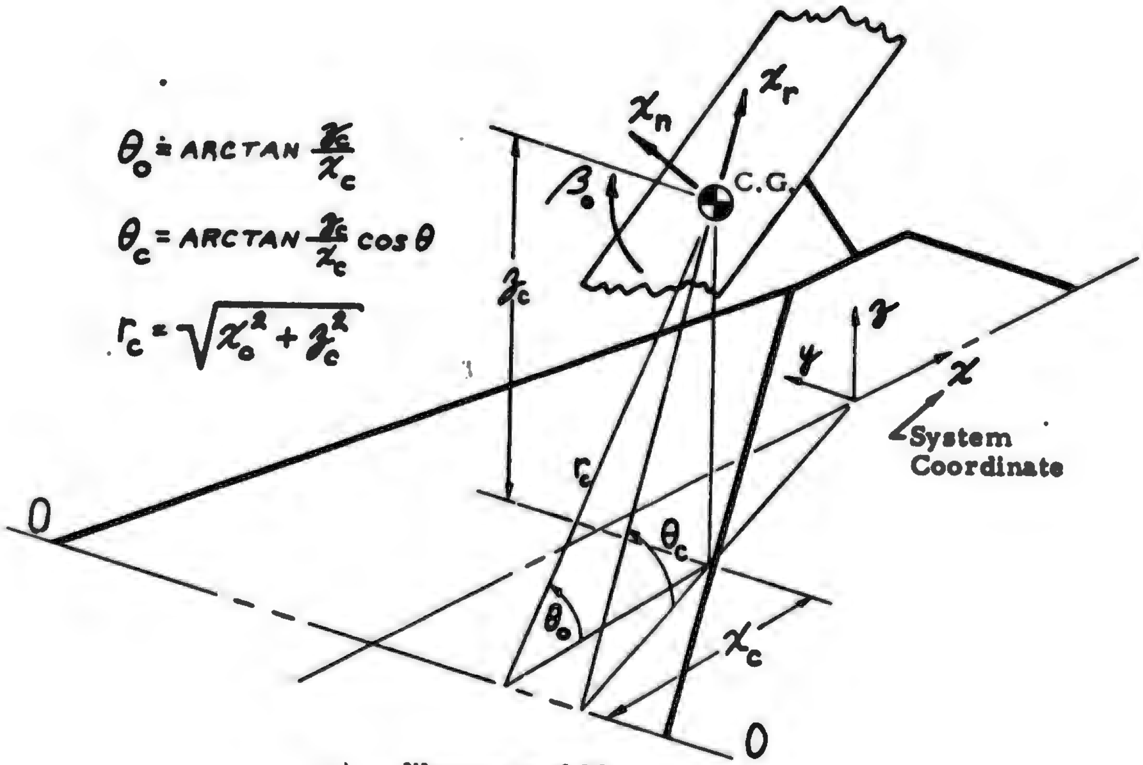
F_r is computed by resolving F_{x_o} and F_{y_o} into the radial direction. Since F_x, F_y and F_r are all in the vertical plane through the bore axis, the definition of F_r given by a rotation transformation is

$$F_r = F_{x_{mg}} \cos(\theta_c - \theta) + F_{y_{mg}} \sin(\theta_c - \theta) \tag{4.10}$$

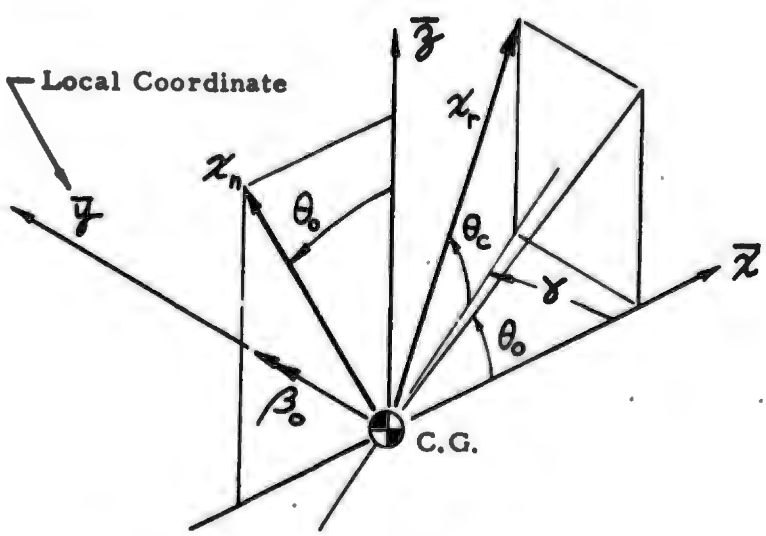
The displacements computed from the nonlinear system's equations of motion, Equation 4. 8, are related to the weapon's C. G. coordinates by a transformation similar to that given for the forces.

As shown in Figure 4. 3b the displacement normal to the C. G., 0-0 plane x_n is related to the rotation about the $\bar{y}$ axis by the radius of rotation r_c.

$$x_n = -r_c \beta_o \tag{4.11}$$



a) Weapon and Mount



b) Weapon Center of Gravity

Figure 4.3 Tripod Mount Coordinates for Nonlinear Model

Resolving x_r the radial displacement into the weapon's main gun coordinates x_{mg} and y_{mg} yields

$$x_{mg} = x_r \cos(\theta_c - \theta)$$
$$y_{mg} = x_r \sin(\theta_c - \theta) \tag{4.12}$$

Similarly, resolving x_n into local coordinates yields

$$x_{mg} = x_n \left[-\sin\theta_o \cos\gamma \cos\theta + \cos\theta_o \sin\theta \right]$$
$$y_{mg} = x_n \left[\cos\theta_o \cos\theta + \sin\theta_o \cos\gamma \sin\theta \right] \tag{4.13}$$

The rotation of β_o into the bore axis direction produces

$$\beta_{mg} = \beta_o \cos\gamma \tag{4.14}$$

Combining Equations 4.11, 4.12, 4.13 and 4.13 defines the transformation from the x_r and β_o coordinates to the main gun coordinates x_{mg}, y_{mg} and β_{mg}.

$$\begin{Bmatrix} x \\ y \\ \beta \end{Bmatrix}_{mg} = \begin{bmatrix} \cos(\theta_c-\theta) & r_c \sin\theta_o \cos\gamma\cos\theta - \cos\theta_o \sin\theta \\ \sin(\theta_c-\theta) & -r_c \cos\theta_o \cos\theta + \sin\theta_o \cos\gamma \sin\theta \\ 0 & \cos\gamma \end{bmatrix} \begin{Bmatrix} x_r \\ \beta_o \end{Bmatrix} \tag{4.15}$$

This transformation also relates the accelerations of the corresponding coordinates.

The nonlinear equations of motion are used as long as the rotation β_o of the mount is negative. When β_o becomes equal to or greater than zero the analysis reverts back to the linear system of equations. The numerical integration then proceeds as described for the linear equations of motion.

4.1 Tripod Mount Radial Stiffness

The radial stiffness k_r used in Equation 4.8 will be obtained by first deriving a_r, and then inverting it to yield k_r. The radial mount flexibility a_r cannot be obtained using the mount flexibility matrix previously derived in Section 2 since the front leg of the mount has lifted off the ground. In order to determine a_r it is necessary to assume that the horizontal reaction force in the y direction is equally divided between the two back foot pads. With this assumption the mount flexibility can be easily determined using the flexibility method since the remaining reactions are given from equilibrium relations.

The radial stiffness of the tripod mount will be derived from the structural idealization shown in Figure 4.4. The model consists of

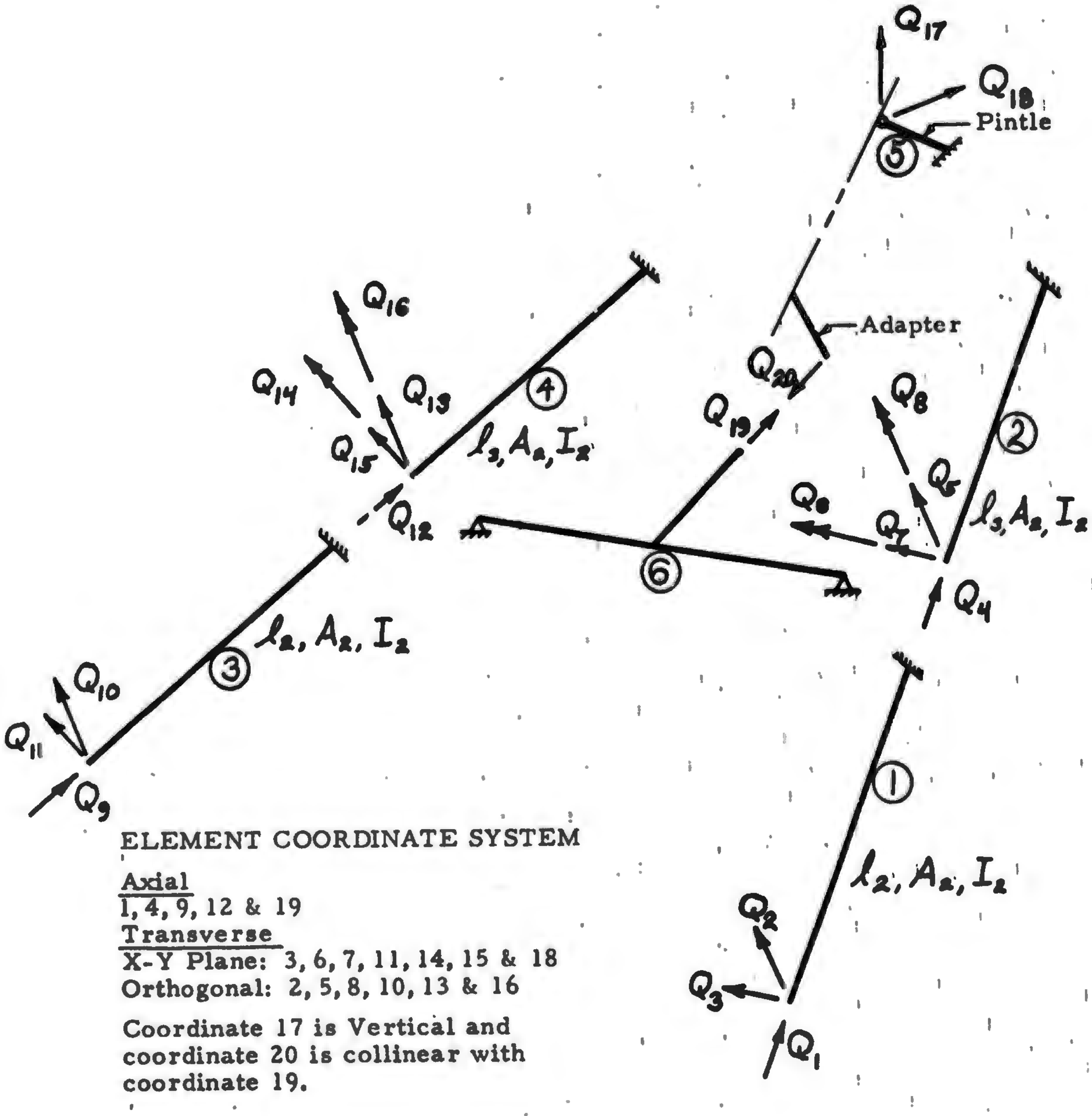


Figure 4.4 **Elements of the Nonlinear Tripod Mount**

six substructure elements whose flexibilities have been defined in other sections of this report. The flexibility of elements ①, ②, ③ and ④ are defined in Appendix B, and the flexibility of elements ⑤ and ⑥ are treated in Section 2.

The flexibility matrices of these elements, obtained from their respective sections, are shown below.

$$\left[\alpha_{①}\right]=\left[\alpha_{③}\right]=\begin{bmatrix}\alpha_{1,1} & 0 & 0\\ 0 & \alpha_{2,2} & 0\\ 0 & 0 & \alpha_{3,3}\end{bmatrix}=\frac{1}{E}\begin{bmatrix}l_2/A_2 & 0 & 0\\ 0 & l_2^3/3I_2 & 0\\ 0 & 0 & l_2^3/3I_2\end{bmatrix} \tag{4.16}$$

$$\left[\alpha_{②}\right]=\left[\alpha_{④}\right]=\begin{bmatrix}\alpha_{4,4} & 0 & 0 & 0 & 0\\ & \alpha_{5,5} & \alpha_{5,6} & 0 & 0\\ & & \alpha_{6,6} & 0 & 0\\ & & & \alpha_{5,5} & -\alpha_{5,6}\\ \text{SYM.} & & & & \alpha_{6,6}\end{bmatrix}$$

$$=\frac{1}{E}\begin{bmatrix}l_3/A_2 & 0 & 0 & 0 & 0\\ & l_3^3/3I_2 & l_3^2/2I_2 & 0 & 0\\ & & l_3/I_2 & 0 & 0\\ & & & l_3^3/3I_2 & -l_3^2/2I_2\\ \text{SYM.} & & & & l_3/I_2\end{bmatrix} \tag{4.17}$$

The dimensions and cross section properties used in Equations 4.16 and 4.17 are shown in Figure 4.4

$$\left[\alpha_{⑤}\right] = \begin{bmatrix} \alpha_{17,17} & \alpha_{17,18} \\ \alpha_{18,17} & \alpha_{18,18} \end{bmatrix} \quad (4.18)$$

$$\left[\alpha_{⑥}\right] = \begin{bmatrix} \alpha_{19,19} & 0 \\ 0 & \alpha_{20,20} \end{bmatrix} \quad (4.19)$$

The evaluation of the flexibilities of elements ⑤ and ⑥ were discussed in Section 3.

In order to compute the radial flexibility a_r, it is first necessary to develope the force transformation relating the radial force F_r to the twenty element forces, Q_1 --- Q_{20}. As shown in Figure 4.5 a unit force $F_r = 1$ is applied at the C.G. of the weapon in the vertical plane of the bore axis. The resulting internal element forces Q_i's are the desired force transformation terms. The reactions at the rear legs are shown in Figure 4.5b and c. The horizontal reaction force in the y direction is assumed to be eqully divided between the two foot pads (R_3 and R_6). This assumption is reasonable since a majority of the horizontal force in the bore axis plane is reacted by the pintle. This is due to the greater rigidity of the pintle as compared to the T & E mechanicism. The y direction component of the horizontal force on the pintle divides equally between the back two legs since the mount frame is symmetric about the X-Z plane.

From Figure 4.5 the reaction R_1 through R_6 are:

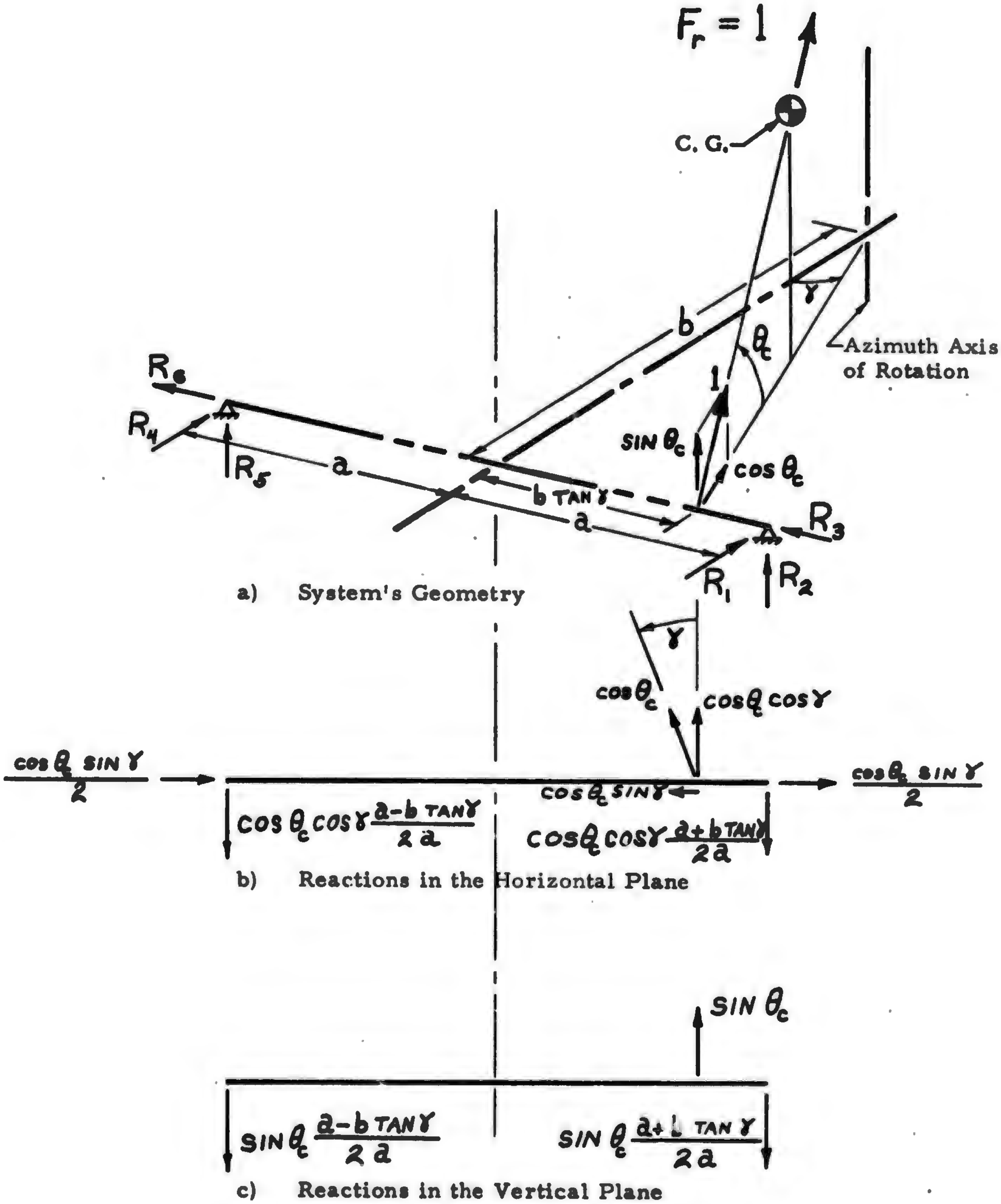


Figure 4. 5 Base Reactions for a Unit Load at F_r

$$R_1 = -\sin\theta_c \frac{a + b\tan\gamma}{2a}$$

$$R_2 = -\cos\theta_c \cos\gamma \frac{a + b\tan\gamma}{2a}$$

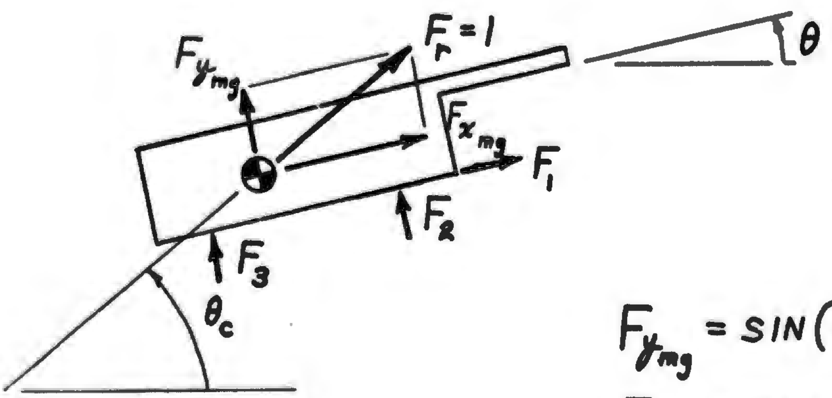
$$R_3 = -\frac{\cos\theta_c \sin\gamma}{2}$$

$$R_4 = -\cos\theta_c \cos\gamma \frac{a - b\tan\gamma}{2a}$$

$$R_5 = -\sin\theta_c \frac{a - b\tan\gamma}{2a}$$

$$R_6 = R_3 \qquad (4.20)$$

Prior to computing the force transformation matrix, it is first necessary to calculate the weapon-mount interface forces. As shown below, for a unit load at coordinate F_r, the main gun forces are:



$$F_{y_{mg}} = \sin(\theta_c - \theta)$$

$$F_{x_{mg}} = \cos(\theta_c - \theta) \qquad (4.21)$$

The force transformation from the main gun forces (Equation 4.21) into the weapon-mount interface forces is obtained by multiplying the main gun forces by the force transformation $[b_{mg}]$, Equation 2.45 in Section 2. The results of this matrix multipication are the mount forces (Equation 4.22).

$$
\begin{aligned}
F_1 &= \cos(\theta_c - \theta) \\
F_2 &= -\frac{l_6}{l_3}\cos(\theta_c - \theta) + \left(1 - \frac{l_5}{l_3}\right)\sin(\theta_c - \theta) \\
F_3 &= \frac{l_6}{l_3}\cos(\theta_c - \theta) + \frac{l_5}{l_3}\sin(\theta_c - \theta)
\end{aligned}
\tag{4.22}
$$

The pintle element forces Q_{17} and Q_{18} are obtained by using the pintle force transformation terms $b_{10,i}$ and $b_{11,i}$ from Equation 1.5 of Section 1. The pintle coordinates 10 and 11 as shown in Figure 1.1 are the same as the pintle coordinates 17 and 18 shown in Figure 4.4. The matrix multiplication to transform the local C.G. coordinates into the pintle element forces is as follows:

$$
\begin{Bmatrix} Q_{17} \\ Q_{18} \end{Bmatrix} = \begin{bmatrix} b_{10,1} & b_{10,2} & b_{10,3} \\ b_{11,1} & b_{11,2} & b_{11,3} \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}
$$

The resulting pintle elemental forces are

$$
\begin{aligned}
Q_{17} = {} & \sin\theta\cos(\theta_c - \theta) + \cos\theta\Big[-\frac{l_6}{l_3}\cos(\theta_c - \theta) \\
& + \left(1 - \frac{l_5}{l_3}\right)\sin(\theta_c - \theta)\Big] + \Big[\frac{l_6}{l_3}\cos(\theta_c - \theta) \\
& + \frac{l_5}{l_3}\sin(\theta_c - \theta)\Big]\Big[\xi_3\Big]\Big[\sin\theta + \frac{l_4}{l_3}\cos\theta\Big]
\end{aligned}
$$

$$
\begin{aligned}
Q_{18} = {} & \cos\theta\cos(\theta_c - \theta) - \sin\theta\Big[-\frac{l_6}{l_3}\cos(\theta_c - \theta) \\
& + \left(1 - \frac{l_5}{l_3}\right)\sin(\theta_c - \theta)\Big] + \Big[\frac{l_6}{l_3}\cos(\theta_c - \theta) \\
& + \frac{l_5}{l_3}\sin(\theta_c - \theta)\Big]\Big[\xi_3\Big]\Big[\cos\theta - \frac{l_4}{l_3}\sin\theta\Big]
\end{aligned}
\tag{4.23}
$$

where $\xi_3 = \dfrac{\sin(\psi + \theta)}{\cos(\psi + \theta) - \frac{l_4}{l_3}\sin(\psi + \theta)}$

The T & E mechanism forces Q_{19} and Q_{20} are obtained from the force transformation terms $b_{12,i}$ and $b_{13,i}$ in Equation 1.5. The matrix multiplication is

$$\begin{Bmatrix} Q_{19} \\ Q_{20} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & b_{12,3} \\ 0 & 0 & b_{13,3} \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

The resulting T & E mechanism elemental forces are

$$Q_{19} = Q_{20} = \xi_4 \left[\frac{l_6}{l_3} \cos(\theta_c - \theta) + \frac{l_5}{l_3} \sin(\theta_c - \theta) \right] \tag{4.24}$$

where $$\xi_4 = \frac{1}{\cos(\psi + \theta) - \frac{l_4}{l_3} \sin(\psi + \theta)}$$

The element ① forces (Q_1, Q_2 and Q_3) as shown in Figure 4.4, are obtained by rotation of the end force reactions using Equation B-4b from Appendix B, as follows:

$$\begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{Bmatrix} = \begin{bmatrix} \cos\eta_2 \cos\eta_3 & \sin\eta_3 & \sin\eta_2 \cos\eta_3 \\ -\cos\eta_2 \sin\eta_3 & \cos\eta_3 & -\sin\eta_2 \sin\eta_3 \\ -\sin\eta_2 & 0 & \cos\eta_2 \end{bmatrix} \begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} \tag{4.25}$$

where R_1, R_2 and R_3 are given by Equation 4.20.

The element ③ forces are obtained, as shown above, from Equation B-4d in Appendix B as follows:

$$\begin{Bmatrix} Q_9 \\ Q_{10} \\ Q_{11} \end{Bmatrix} = \begin{bmatrix} \cos\eta_2 \cos\eta_3 & \sin\eta_3 & -\sin\eta_2 \cos\eta_3 \\ -\cos\eta_2 \sin\eta_3 & \cos\eta_3 & \sin\eta_2 \sin\eta_3 \\ \sin\eta_2 & 0 & \cos\eta_2 \end{bmatrix} \begin{Bmatrix} R_4 \\ R_5 \\ R_6 \end{Bmatrix} \tag{4.26}$$

where R_4, R_5 and R_6 are given by Equation 4.20.

The element ② forces are given from Equation B-4c as

$$\begin{Bmatrix} Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \end{Bmatrix} = \begin{bmatrix} \cos\eta_2 \cos\eta_3 & \sin\eta_3 & \sin\eta_2 \cos\eta_3 & \sin\eta_3 & \cos\eta_2 \cos\eta_3 \\ -\cos\eta_2 \sin\eta_3 & \cos\eta_3 & -\sin\eta_2 \sin\eta_3 & \cos\eta_3 & -\cos\eta_2 \sin\eta_3 \\ -\ell_2 \cos\eta_2 \sin\eta_3 & \ell_2 \cos\eta_3 & -\ell_2 \sin\eta_2 \sin\eta_3 & 0 & 0 \\ -\sin\eta_2 & 0 & \cos\eta_2 & 0 & -\sin\eta_2 \\ \ell_2 \sin\eta_2 & 0 & -\ell_2 \cos\eta_2 & 0 & 0 \end{bmatrix} \begin{Bmatrix} R_1 \\ R_2 \\ R_3 \\ P_8 \\ P_9 \end{Bmatrix} \qquad (4.27)$$

where R_1, R_2, R_3 are given by Equation 4.20 and P_8 and P_9 are rewritten from Equation 2.20 as

$$P_8 = Q_{19} \cos\psi \frac{f + g \tan\gamma}{2f}$$

$$P_9 = Q_{19} \sin\psi \cos\gamma \frac{f + g \tan\gamma}{2f}$$

The element force Q_{19} has already been calculated and is presented in Equation 4.24.

The element ④ forces are obtained from Equation B-4e as follows:

$$\begin{Bmatrix} Q_{12} \\ Q_{13} \\ Q_{14} \\ Q_{15} \\ Q_{16} \end{Bmatrix} = \begin{bmatrix} \cos\eta_2 \cos\eta_3 & \sin\eta_3 & -\sin\eta_2 \cos\eta_3 & \sin\eta_3 & \cos\eta_2 \cos\eta_3 \\ -\cos\eta_2 \sin\eta_3 & \cos\eta_3 & \sin\eta_2 \sin\eta_3 & \cos\eta_3 & -\cos\eta_2 \sin\eta_3 \\ -\ell_2 \cos\eta_2 \sin\eta_3 & \ell_2 \cos\eta_3 & \ell_2 \sin\eta_2 \sin\eta_3 & 0 & 0 \\ \sin\eta_2 & 0 & \cos\eta_2 & 0 & \sin\eta_2 \\ -\ell_2 \sin\eta_2 & 0 & -\ell_2 \cos\eta_2 & 0 & 0 \end{bmatrix} \begin{Bmatrix} R_4 \\ R_5 \\ R_6 \\ P_6 \\ P_7 \end{Bmatrix} \qquad (4.28)$$

where R_4, R_5 and R_6 are given by Equation 4.20 and P_6 and P_7 are rewritten from Equation 2.20 as

$$P_6 = Q_{19} \cos\psi \, \frac{f - g \tan\gamma}{2f}$$

$$P_7 = Q_{19} \sin\psi \cos\gamma \, \frac{f - g \tan\gamma}{2f}$$

The radial direction flexibility is given by the congruent transformation, which is shown in Appendix A (Equation A-12) as

$$a_r = \lfloor b \rfloor \left[\diagdown \alpha \diagdown\right] \{b\} \tag{4.29}$$

where the b column matrix and α diagonal matrix are shown below.

$$\{b\} = \left\{\begin{array}{c} b_{①} \\ \hline b_{②} \\ \hline b_{③} \\ \hline b_{④} \\ \hline b_{⑤} \\ \hline b_{⑥} \end{array}\right\} = \left\{\begin{array}{c} b_1 \\ b_2 \\ b_3 \\ \hline b_4 \\ b_5 \\ b_6 \\ b_7 \\ b_8 \\ \hline b_9 \\ b_{10} \\ b_{11} \\ \hline b_{12} \\ b_{13} \\ b_{14} \\ b_{15} \\ b_{16} \\ \hline b_{17} \\ b_{18} \\ \hline b_{19} \\ b_{20} \end{array}\right\} = \left\{\begin{array}{c} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \\ Q_9 \\ Q_{10} \\ Q_{11} \\ Q_{12} \\ Q_{13} \\ Q_{14} \\ Q_{15} \\ Q_{16} \\ Q_{17} \\ Q_{18} \\ Q_{19} \\ Q_{20} \end{array}\right\}$$

where the Q's are given by Equations 4.23, 4.24, 4.25, 4.26, 4.27 and 4.28, and

$$\lceil \alpha \rfloor = \begin{bmatrix} [\alpha_{①}] & & & & & \\ & [\alpha_{②}] & & & & \\ & & [\alpha_{③}] & & & \\ & & & [\alpha_{④}] & & \\ & & & & [\alpha_{⑤}] & \\ & & & & & [\alpha_{⑥}] \end{bmatrix}$$

where the α's are given in Equations 4.16, 4.17, 4.18 and 4.19.

Due to the diagonal form of $\lceil \alpha \rfloor$, Equation 4.29 can be expressed as a summation.

$$\alpha_r = \sum_{i=①---⑥} \lfloor b_i \rfloor [\alpha_i] \{b_i\} \qquad (4.30)$$

Performing the multiplications and addition of Equation 4.30 gives the radial flexibility a_r as

$$\begin{aligned} a_r = {} & \alpha_{1,1}(b_1^2 + b_9^2) + \alpha_{2,2}(b_2^2 + b_{10}^2) + \alpha_{3,3}(b_3^2 + b_{11}^2) \\ & + \alpha_{4,4}(b_1^2 + b_{12}^2) + \alpha_{5,5}(b_5^2 + b_7^2 + b_{13}^2 + b_{15}^2) \\ & + \alpha_{6,6}(b_6^2 + b_3^2 + b_{14}^2 + b_{16}^2) + 2\alpha_{5,6}(b_5 b_6 \\ & - b_7 b_8 + b_{13} b_{14} - b_{15} b_{16}) + \alpha_{17,17} b_{17}^2 + \alpha_{18,18} b_{18}^2 \\ & + 2\alpha_{17,18} b_{17} b_{18} + \alpha_{19,19} b_{19}^2 + \alpha_{20,20} b_{20}^2 \end{aligned} \qquad (4.31)$$

The desired radial stiffness is the inverse of the radial flexibility, e.g.,

$$k_r = \frac{1}{a_r}.$$

Appendix A

Introduction to the Flexibility Methods of Matrix Analysis

A brief introduction is presented into the general concepts of the flexibility method of structural analysis. A more through presentation is contained in References 4, 5 and 6. The flexibility method can be applied to both determinate and indeterminate structures. The first section of this appendix deals with the application to determinate structures. This section illustrates how the flexibility matrix of a complex structure can be obtained from the individual flexibility matrices of its substructures. The second section of this appendix will deal with the application of the flexibility matrix to indeterminate structures. This section will show how to : 1) operate on an indeterminate structure to make it determinate, 2) obtain the flexibility matrix for the determinate structure, and 3) obtain the final flexibility matrix of the indeterminate structure by performing a force elimination operation.

Application to Determinate Structure

The structure to be analyzed is divided into elements describing its deformation properties. Two coordinate systems are selected, one for the over-all structure (external coordinates) and the other for the elements (internal coordinates). The over-all structural coordinates are selected based on consideration of the external forces and the coordinates desired for the final flexibility matrix. The element coordinates are selected based on ease of computing static displacements in the element.

The flexibility matrix for each element can be expressed as:

$$\{\delta_i\} = [\alpha_i]\{P_i\} \tag{A-1}$$

where $\{\delta_i\}$ = displacements of i th element coordinates

$[\alpha_i]$ = flexibility of i th element

$\{P_i\}$ = forces of i th element

All the element flexibility equations can be combined into one equation of unconnected elements as follows:

$$\begin{Bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_n \end{Bmatrix} = \begin{bmatrix} [\alpha_1] & & & \\ & [\alpha_2] & & \\ & & \ddots & \\ & & & [\alpha_n] \end{bmatrix} \begin{Bmatrix} P_1 \\ P_2 \\ \vdots \\ P_n \end{Bmatrix} \qquad \text{(A-2)}$$

or $\{\delta\} = [\diagdown\alpha\diagdown]\{P\}$

Using the equilibrium conditions of the structure, the system forces $\{F\}$ can be transformed to the element forces $\{P\}$ where [b] is the force transformation matrix.

$$\{P\} = [b]\{F\} \qquad \text{(A-3)}$$

Since the structure is determinate, [b] can be computed directly from the equations of equilibrium.

The flexibility matrix of the assembled structure, [a] can be determined from the elemental flexibility matrices $[\alpha_i]$ using the force transformation [b]. This is accomplished as follows: first consider the structure (assumed to be linear) is acted upon by its system forces, then the strain energy in the structure is equal to the work done by each force, F_i, moving through its corresponding displacement, x_i. The equation for the strain energy is as follows:

$$U = \frac{1}{2}\sum_{i=1}^{n} F_i x_i = \frac{1}{2}\lfloor F \rfloor \{x\} \qquad \text{(A-4)}$$

The flexibility equation of the complete structure is

$$\{x\} = [a]\{F\} \qquad \text{(A-5)}$$

Substituting $\{x\}$ from Equation A-5 into Equation A-4 yields an equation for the strain energy as follows:

$$U = \frac{1}{2}\lfloor F \rfloor [a]\{F\} \qquad \text{(A-6)}$$

Similarly, the strain energy in the i th element of the structure can be shown to have the same form as Equation A-6.

$$U = \frac{1}{2}\lfloor P_i \rfloor [\alpha_i]\{P_i\} \qquad \text{(A-7)}$$

The total strain energy for all the elements is the sum of the individual elements.

$$U = \sum_{i=1}^{n} \frac{1}{2} \lfloor P_i \rfloor [\alpha_i] \{P_i\} \qquad \text{(A-8)}$$

This can be rewritten to correspond to the matrix form of Equation A-2 which yields

$$U = \frac{1}{2} \lfloor P \rfloor \left\lceil \alpha \right\rfloor \{P\} \qquad \text{(A-9)}$$

The strain energy of the system is the same regardless of the form used to describe it. Therefore, equating Equations A-6 and A-9

$$\lfloor F \rfloor [a] \{F\} = \lfloor P \rfloor \left\lceil \alpha \right\rfloor \{P\} \qquad \text{(A-10)}$$

Substituting Equation A-3 into Equation A-10 produces the following equality

$$\lfloor F \rfloor [a] \{F\} = \lfloor F \rfloor [b]^T \left\lceil \alpha \right\rfloor [b] \{F\} \qquad \text{(A-11)}$$

Then from Equation A-11 we can evaluate the flexibility matrix [a] as follows:

$$[a] = [b]^T \left\lceil \alpha \right\rfloor [b] \qquad \text{(A-12)}$$

The matrix product in Equation A-12 is a congruent transformation which gives the assembled structure flexibility [a] in terms of the force transformation [b] and the unconnected element flexibility $\left\lceil \alpha \right\rfloor$.

<u>Application to Indeterminate Structures</u>

Here, as for the determinate case, the structure is divided into internal elements for which the force deflection relations can be established, and external forces and coordinates desired for the final flexibility matrix. The unconnected flexibility equation of the elements is defined the same as for a determinate structure.

$$\begin{Bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_n \end{Bmatrix} = \begin{bmatrix} [\alpha_1] & & & \\ & [\alpha_2] & & \\ & & \ddots & \\ & & & [\alpha_n] \end{bmatrix} \begin{Bmatrix} P_1 \\ P_2 \\ \vdots \\ P_n \end{Bmatrix}$$

or $$\{\delta\} = \left\lceil \alpha \right\rfloor \{P\}$$

Since the structure is indeterminate the force transformation matrix, [b], cannot be evaluated directly from the equations of equilibrium. The basic structure must be "cut" so that the cut structure is determinate and also kinematically stable. The redundant forces at these cuts can then be evaluated to satisfy the compatibility conditions of the structure. The cut structure is called the primary structure. The forces and displacements of the external coordinates are $\{F\}^e$ and $\{x\}^e$. The forces and displacements of the redundant coordinates are $\{F\}^r$ and $\{x\}^r$. The cut structure is determinate and the internal element forces can be easily evaluated. The element forces due to the external forces are

$$\{P\}^e = [b_e]\{F\}^e \tag{A-13}$$

The element forces due to the redundant forces are

$$\{P\}^r = [b_r]\{F\}^r \tag{A-14}$$

The actual uncut structure element forces are a superposition of the forces due to the external and the redundant forces.

$$\{P\} = \{P\}^e + \{P\}^r = [b_e]\{F\}^e + [b_r]\{F\}^r \tag{A-15}$$

Equation A-15 can be rewritten in a partitioned matrix form as follows:

$$\{P\} = \left[[b_e] \,\vdots\, [b_r]\right] \begin{Bmatrix} F^e \\ \hline F^r \end{Bmatrix} \tag{A-16}$$

Note that the redundant forces $\{F\}^r$ are still unknown. The i th column of $[b_r]$ gives the values of $\{P\}$ when $F_i^r = 1$, similarly the i th column of $[b_e]$ gives the values of $\{P\}$ when $F_i^e = 1$.

The system flexibility matrix, [a] is given from Equation A-12 as

$$[a] = [b]^T[\alpha][b]$$

For the indeterminate structure, [b], is given by Equation A-16 as

$$[b] = \left[[b_e] \,\vdots\, [b_r]\right] \tag{A-17}$$

Substituting Equation A-17 into Equation A-12 yields:

$$[a] = \left[[b_e] \,\vdots\, [b_r]\right]^T [\alpha] \left[[b_e] \,\vdots\, [b_r]\right]$$

$$[a] = \left[\begin{array}{c:c} [a]_{1,1} & [a]_{1,2} \\ \hdashline [a]_{2,1} & [a]_{2,2} \end{array}\right] \qquad \text{(A-18)}$$

where

$$[a]_{1,1} = [b_e]^T [\lceil \alpha \rfloor] [b_e]$$

$$[a]_{1,2} = [b_e]^T [\lceil \alpha \rfloor] [b_r]$$

$$[a]_{2,1} = [b_r]^T [\lceil \alpha \rfloor] [b_e]$$

$$[a]_{2,2} = [b_r]^T [\lceil \alpha \rfloor] [b_r]$$

The strain energy of the structure is given by Equation A-6

$$U = \frac{1}{2} \lfloor F \rfloor [a] \{F\}$$

where

$$\{F\} = \left\{\begin{array}{c} F_e \\ \hdashline F_r \end{array}\right\}$$

Substituting Equation A-18 into Equation A-6 produces

$$\begin{aligned} U = {} & \frac{1}{2} \lfloor F^e \rfloor [a]_{1,1} \{F^e\} \\ & + \frac{1}{2}\left(\lfloor F^r \rfloor [a]_{2,1} \{F^e\} + \lfloor F^e \rfloor [a]_{1,2} \{F^r\} \right) \\ & + \frac{1}{2} \lfloor F^r \rfloor [a]_{2,2} \{F^r\} \end{aligned} \qquad \text{(A-19)}$$

The second term of this equation can be written as $\lfloor F^r \rfloor [a]_{2,1} \{F^e\}$

because $\lfloor F^e \rfloor [a]_{1,2} \{F^r\} = \lfloor F^r \rfloor [a]_{2,1} \{F^e\}$. The actual deflections of the structure are due to the external forces $\{F\}^e$, only. Thus applying Castigliano's Second Theorem ($\frac{\partial U}{\partial P_i} = \delta_i$) gives

$$\chi_{r_i} = 0 = \frac{\partial U}{\partial F_i^r} \quad for \quad i = 1, 2, \cdots \tag{A-20}$$

Applying Equation A-20 to Equation A-12 (for $i = 1, 2 \cdots$) gives

$$[a]_{2,1}\{F^e\} + [a]_{2,2}\{F^r\} = 0 \tag{A-21}$$

Solving Equation A-21 for $\{F\}^r$ we obtain

$$\{F^r\} = -[a]_{2,2}^{-1}[a]_{2,1}\{F^e\} \tag{A-22}$$

Substituting $\{F^r\}$ into Equation A-15 yields

$$\{P\} = [b_e]\{F^e\} - [b_r][a]_{2,2}^{-1}[a]_{2,1}\{F^e\}$$

$$= \Big[[b_e] - [b_r][a]_{2,2}^{-1}[a]_{2,1}\Big]\{F^e\} \tag{A-23}$$

From Equation A-18, the flexibility equation of the structure is

$$\begin{Bmatrix} \chi_e \\ \hline \chi_r \end{Bmatrix} = \left[\begin{array}{c|c} [a]_{1,1} & [a]_{1,2} \\ \hline [a]_{2,1} & [a]_{2,2} \end{array}\right] \begin{Bmatrix} F^e \\ \hline F^r \end{Bmatrix} \tag{A-24}$$

which can be expanded as follows

$$\{\chi_e\} = [a]_{1,1}\{F^e\} + [a]_{1,2}\{F^r\}$$

and

$$\{\chi_r\} = [a]_{2,1}\{F^e\} + [a]_{2,2}\{F^r\} \tag{A-25}$$

The flexibility matrix of the actual indeterminate structure is obtained by substituting $\{F^r\}$ into $\{x_e\}$ above. Then

$$\{\chi_e\} = [a]_{1,1}\{F^e\} - [a]_{1,2}[a]_{2,2}^{-1}[a]_{2,1}\{F^e\}$$

or $$\{x_e\} = \left[[a]_{1,1} - [a]_{1,2}[a]_{2,2}^{-1}[a]_{2,1}\right]\{F^e\} \qquad \text{(A-26)}$$

The system flexibility equation is $\{x_e\} = [a]\{F_e\}$, thus from Equation A-26 the system flexibility matrix is

$$[\dot{a}] = [a]_{1,1} - [a]_{1,2}[a]_{2,2}^{-1}[a]_{2,1} \qquad \text{(A-27)}$$

This is commonly referred to as the reduced flexibility matrix.

Appendix B
Method to Determine the Flexibility Matrix for the Tripod Mount Frame

The frame of the M122 Tripod Mount is an indeterminate structure. As presented in the body of this report, the flexibility matrix for the mount frame was computed using the ICES STRUDL-II computer program. There are many computer programs available for computing the flexibility matrix of indeterminate frames. This appendix presents a method for computing the flexibility of the tripod mount frame in the event no existing computer programs are available. This method, if desired, can be programed for a digital computer.

The mount frame flexibility matrix $[\alpha_m]$ can be computed numerically in a straight forward manner using the flexibility method of structural analysis. The structural configuration for which the analysis is performed is shown in Figure B-1.

The mount flexibility will be determined in terms of the nine coordinates δ_1 - - - - δ_9 and their corresponding forces P_1 --- P_9. Pin ended connections were assumed for the mount model at the junction of the ground and the mount's three feet. Therefore only three reaction forces exist at the end of each leg. The front and back legs of the mount have both axial and transverse flexibility. All three legs are rigidly attached to the head which is assumed to be rigid because it is much stiffer than the legs.

To start the analysis, the mount is broken into its five flexible elements. The independent forces Q_1 through Q_{19} are defined for the elements as shown in Figure B-2.

The flexibility matrix for each of the elements is given below. These are for members of uniform cross section, however they can be modified for nonuniform members in which case the α's would change but the remaining steps of the analysis remain the same. All elements in Figure B-2 are circular, therefore $I_u = I_v$.

$$[\alpha_1] = \begin{bmatrix} \alpha_{11} & 0 & 0 \\ 0 & \alpha_{22} & 0 \\ 0 & 0 & \alpha_{33} \end{bmatrix} = \frac{1}{E} \begin{bmatrix} l_1/A_1 & 0 & 0 \\ 0 & l_1^3/3I_1 & 0 \\ 0 & 0 & l_1^3/3I_1 \end{bmatrix}$$

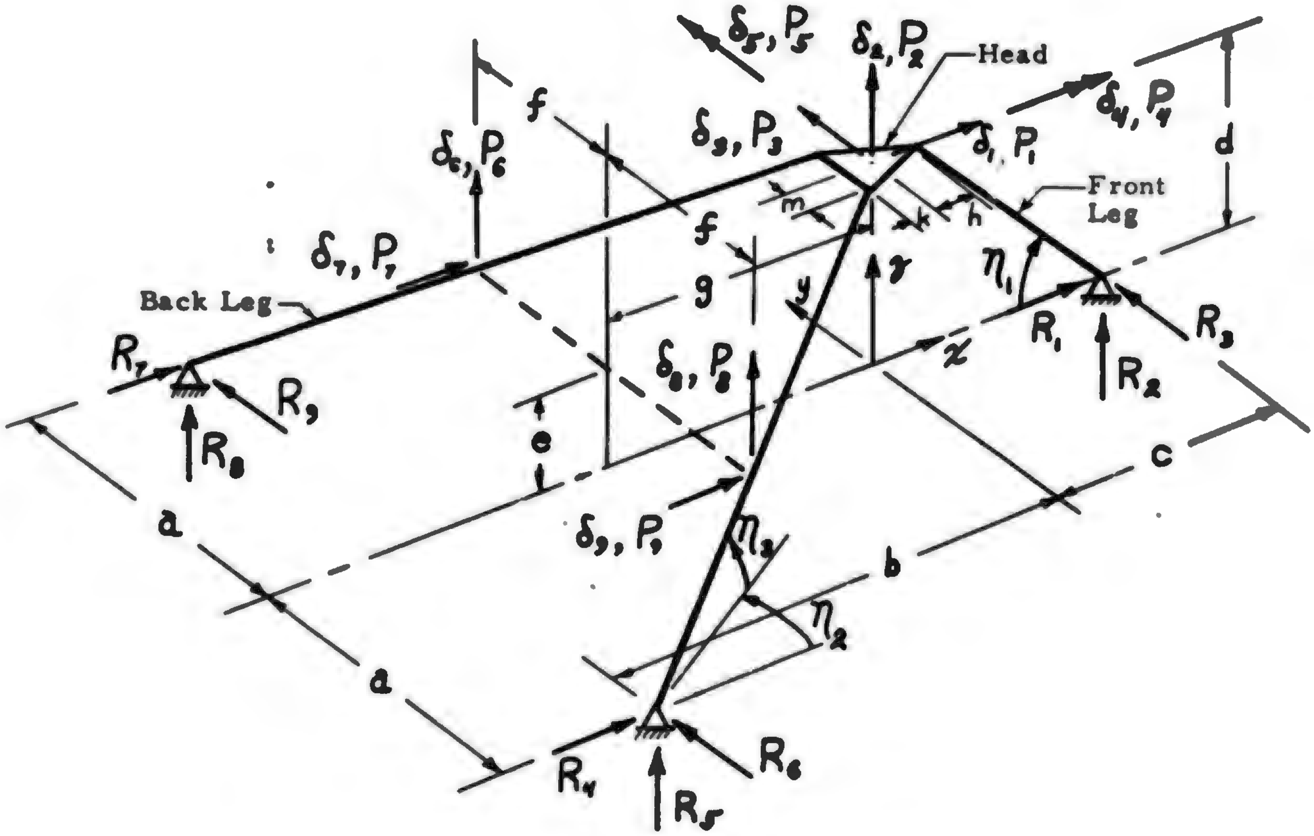


Figure B-1 Structural Model of the Tripod Mount Frame

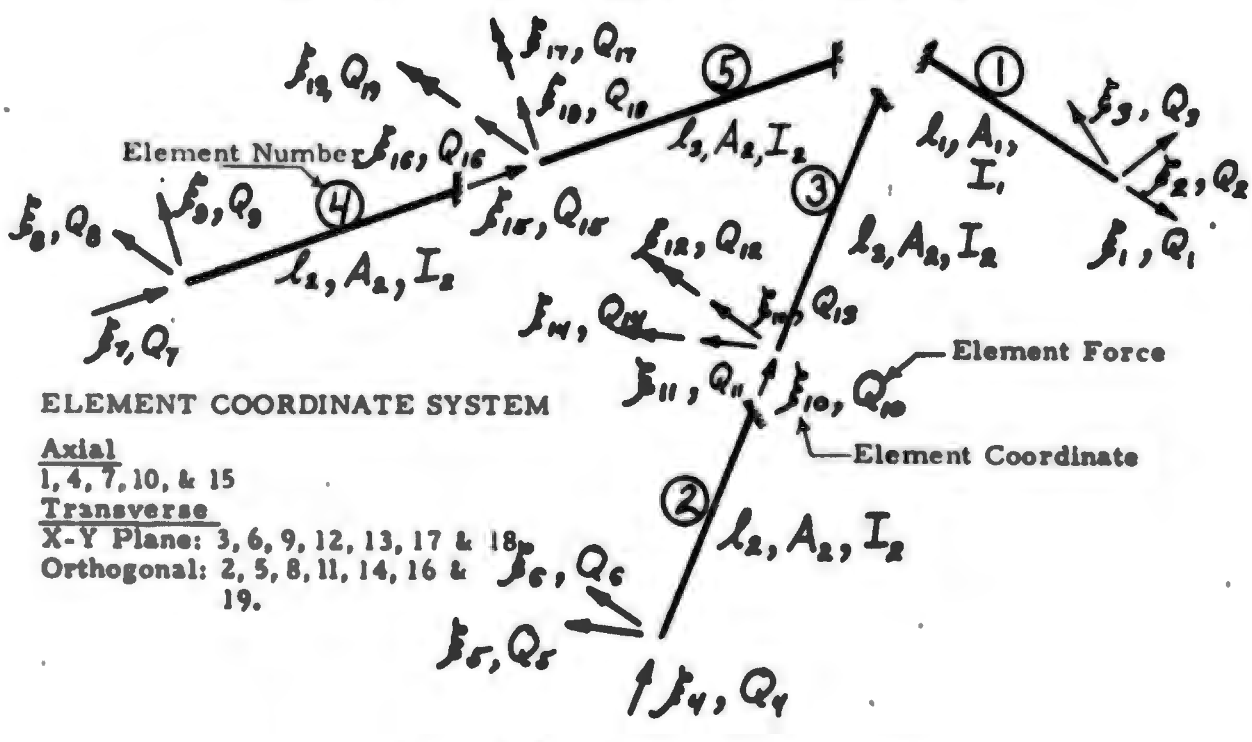


Figure B-2 Mount Frame Elements

$$
\left[\bar{\alpha}_{③}\right] = \left[\bar{\alpha}_{④}\right] = \begin{bmatrix} \bar{\alpha}_{4,4} & 0 & 0 \\ 0 & \bar{\alpha}_{5,5} & 0 \\ 0 & 0 & \bar{\alpha}_{6,6} \end{bmatrix} = \frac{1}{E} \begin{bmatrix} l_2/A_2 & 0 & 0 \\ 0 & l_2^3/3I_2 & 0 \\ 0 & 0 & l_2^3/3I_2 \end{bmatrix}
$$

$$
\left[\bar{\alpha}_{⑤}\right] = \left[\bar{\alpha}_{⑤}\right] = \begin{bmatrix} \bar{\alpha}_{7,7} & 0 & 0 & 0 & 0 \\ 0 & \bar{\alpha}_{8,8} & \bar{\alpha}_{8,9} & 0 & 0 \\ 0 & \bar{\alpha}_{8,9} & \bar{\alpha}_{9,9} & 0 & 0 \\ 0 & 0 & 0 & \bar{\alpha}_{8,8} & -\bar{\alpha}_{8,9} \\ 0 & 0 & 0 & -\bar{\alpha}_{8,9} & \bar{\alpha}_{9,9} \end{bmatrix}
$$

$$
= \frac{1}{E} \begin{bmatrix} l_2/A_2 & 0 & 0 & 0 & 0 \\ 0 & l_2^3/3I_2 & l_2^2/2I_2 & 0 & 0 \\ 0 & l_2^2/2I_2 & l_2/I_2 & 0 & 0 \\ 0 & 0 & 0 & l_2^3/3I_2 & -l_2^2/2I_2 \\ 0 & 0 & 0 & -l_2^2/2I_2 & l_2/I_2 \end{bmatrix} \qquad \text{(B-1)}
$$

In the case of a determinate structure the system's flexibility is easily determined since the relation between the system forces (P's) and the element forces (Q's) is obtained using static analysis. However, the structural model of the tripod mount frame is indeterminate to the third degree.

The indeterminate order is equal to the number of unknowns minus the number of independent equations of equilibrium. For the tripod frame there are 9 reactions (R_1 through R_9) and six equations of equilibrium (three translation and three rotation). Therefore the tripod frame is indeterminate to the third order, which makes the determination of the flexibility matrix more difficult.

The indeterminate mount structure is "cut" to remove 3 redundants while still maintaining kinematic stability. In the case of the mount, it is convenient to cut the structure at reactions R_3, R_4 and R_6 shown in Figure B-1. The resulting structure is then both determinate and stable.

Now the force transformation relations of the cut structure are given by

$$\begin{Bmatrix} Q_1 \\ \vdots \\ \vdots \\ Q_{19} \end{Bmatrix} = \left[\begin{array}{c:c} b_e & b_r \end{array} \right] \begin{Bmatrix} P_1 \\ \vdots \\ P_9 \\ \hline R_3 \\ R_4 \\ R_6 \end{Bmatrix} \qquad \text{(B-2)}$$

where the terms in the transformation are:

$b_{e_{i,j}}$ = value of i th element force (Q_i) due to $P_j = 1$ ($P_i = 0$; for all $i \neq j$)

and $b_{r_{i,j}}$ = value of i th element force (Q_i) due to $R_j = 1$ ($j = 3, 4, 6$)

These force transformations can be directly evaluated from statics since the cut structure is determinate. The structure has six unknown reactions and six equations of equilibrium. These equations of equilibrium including all the applied loads and reactions of the cut structure are:

1) Equilibrium of forces in x direction.

$$P_1 + P_7 + P_9 + R_1 + R_4 + R_7 = 0 \qquad \text{(B-3a)}$$

2) Equilibrium of forces in y direction.

$$P_3 + R_3 + R_6 + R_9 = 0 \qquad \text{(B-3b)}$$

3) Equilibrium of forces in z direction.

$$P_2 + P_6 + P_8 + R_2 + R_5 + R_8 = 0 \qquad \text{(B-3c)}$$

4) Equilibrium of moments about x axis (positive moment vector is in positive x direction).

$$-P_3 d + P_4 + P_6 f - P_8 f - R_5 a + R_8 a = 0 \qquad \text{(B-3d)}$$

5) Equilibrium of moments about y axis (positive moment vector is in positive y direction).

$$P_1 d + P_5 + P_6 g + P_7 e + P_8 g + P_9 e - R_2 c + R_5 b + R_8 b = 0 \qquad \text{(B-3e)}$$

6) Equilibrium of moments about z axis (positive moment vector is in positive z direction)

$$-P_7 f + P_8 f + R_3 c + R_4 a - R_6 b - R_7 a - R_9 b = 0 \quad \text{(B-3f)}$$

After the six reactions have been computed from Equations B-3, the element forces, $Q_{i,j}$ must be determined. This is accomplished by relating the reactions in the local x, y, z coordinates to the element forces which are oriented with respect to the axis of the leg as shown in Figure B-3:

For the forward leg

$$\begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{Bmatrix} = \begin{bmatrix} \cos\eta_1 & -\sin\eta_1 & 0 \\ \sin\eta_1 & \cos\eta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} \quad \text{(B-4a)}$$

For the right back leg, first rotate from local x, y, z coordinates through angle η_2 about Z axis which gives

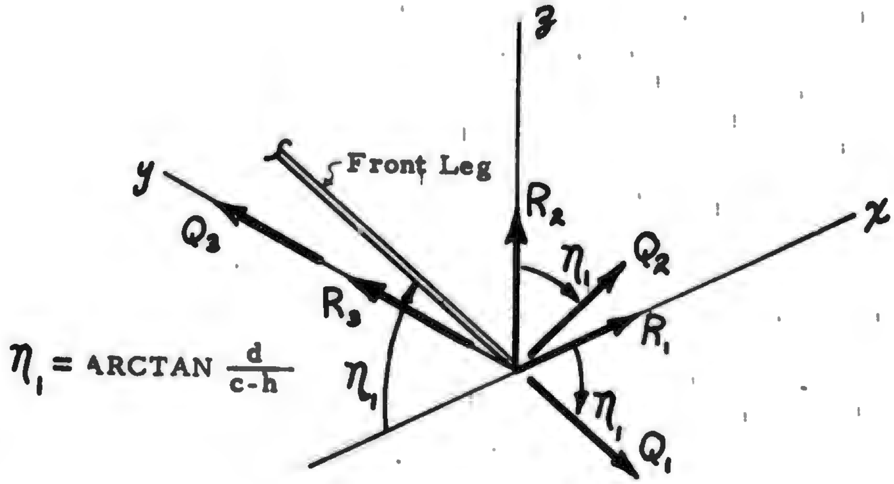
$$T = R_4 \cos\eta_2 + R_6 \sin\eta_2$$

$$Q_6 = -R_4 \sin\eta_2 + R_6 \cos\eta_2$$

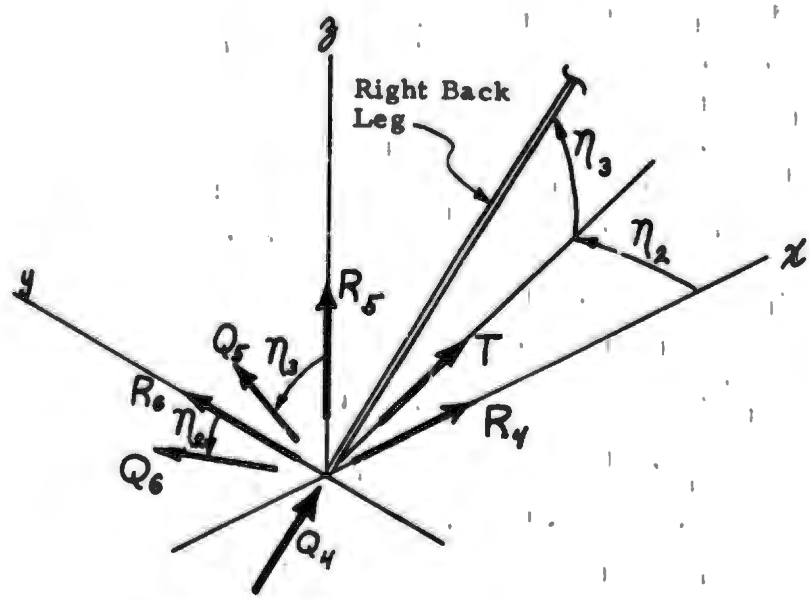
Then rotate through η_3 angle about Q_6 axis which gives

$$\begin{Bmatrix} Q_4 \\ Q_5 \\ Q_6 \end{Bmatrix} = \begin{bmatrix} \cos\eta_2\cos\eta_3 & \sin\eta_3 & \sin\eta_2\cos\eta_3 \\ -\cos\eta_2\sin\eta_3 & \cos\eta_3 & -\sin\eta_2\sin\eta_3 \\ -\sin\eta_2 & 0 & \cos\eta_2 \end{bmatrix} \begin{Bmatrix} R_4 \\ R_5 \\ R_6 \end{Bmatrix} \quad \text{(B-4b)}$$

The forces at the upper leg element can be computed from the lower leg forces plus proper rotation of P_8 and P_9



a) Forward Leg



b) Right Back Leg

Figure B-3 Angular Relation Between Mount Reactions and Element Forces

$$
\begin{aligned}
Q_{10} &= Q_4 + \sin\eta_3 P_8 + \cos\eta_3 \cos\eta_2 P_9 \\
Q_{11} &= Q_5 + \cos\eta_3 P_8 - \sin\eta_3 \cos\eta_2 P_9 \\
Q_{12} &= Q_5 \ell_2 \\
Q_{13} &= Q_6 - \sin\eta_2 P_9 \\
Q_{14} &= -Q_6 \ell_2
\end{aligned}
$$

which becomes

$$
\begin{Bmatrix} Q_{10} \\ Q_{11} \\ Q_{12} \\ Q_{13} \\ Q_{14} \end{Bmatrix} =
\begin{bmatrix}
\cos\eta_2\cos\eta_3 & \sin\eta_3 & \sin\eta_2\cos\eta_3 & \sin\eta_3 & \cos\eta_2\cos\eta_3 \\
-\cos\eta_2\sin\eta_3 & \cos\eta_3 & -\sin\eta_2\sin\eta_3 & \cos\eta_3 & -\cos\eta_2\sin\eta_3 \\
-\ell_2\cos\eta_2\sin\eta_3 & \ell_2\cos\eta_3 & -\ell_2\sin\eta_2\sin\eta_3 & 0 & 0 \\
-\sin\eta_2 & 0 & \cos\eta_2 & 0 & -\sin\eta_2 \\
\ell_2\sin\eta_2 & 0 & -\ell_2\cos\eta_2 & 0 & 0
\end{bmatrix}
\begin{Bmatrix} R_4 \\ R_5 \\ R_6 \\ P_8 \\ P_9 \end{Bmatrix}
\quad \text{(B-4c)}
$$

Following a similar sequence, the left leg element forces are given by

$$
\begin{Bmatrix} Q_7 \\ Q_8 \\ Q_9 \end{Bmatrix} =
\begin{bmatrix}
\cos\eta_2\cos\eta_3 & \sin\eta_3 & -\sin\eta_2\cos\eta_3 \\
-\cos\eta_2\sin\eta_3 & \cos\eta_3 & \sin\eta_2\sin\eta_3 \\
\sin\eta_2 & 0 & \cos\eta_2
\end{bmatrix}
\begin{Bmatrix} R_7 \\ R_8 \\ R_9 \end{Bmatrix}
\quad \text{(B-4d)}
$$

and

$$
\begin{Bmatrix} Q_{15} \\ Q_{16} \\ Q_{17} \\ Q_{18} \\ Q_{19} \end{Bmatrix} =
\begin{bmatrix}
\cos\eta_2\cos\eta_3 & \sin\eta_3 & -\sin\eta_2\cos\eta_3 & \sin\eta_3 & \cos\eta_2\cos\eta_3 \\
-\cos\eta_2\sin\eta_3 & \cos\eta_3 & \sin\eta_2\sin\eta_3 & \cos\eta_3 & -\cos\eta_2\sin\eta_3 \\
-\ell_2\cos\eta_2\sin\eta_3 & \ell_2\cos\eta_3 & \ell_2\sin\eta_2\sin\eta_3 & 0 & 0 \\
\sin\eta_2 & 0 & \cos\eta_2 & 0 & \sin\eta_2 \\
-\ell_2\sin\eta_2 & 0 & -\ell_2\cos\eta_2 & 0 & 0
\end{bmatrix}
\begin{Bmatrix} R_7 \\ R_8 \\ R_9 \\ P_6 \\ P_7 \end{Bmatrix}
\quad \text{(B-4e)}
$$

Each column of [b] is determined by a simultaneous solution of Equation B-3 and substitution of the results into Equation B-4. For example, column 1 of [b] is determined by letting $P_1=1$; P_2----$P_9=0$ and $R_3=R_4=R_6=0$. In which case Equation B-3 becomes

$$1 + R_1 + R_7 = 0$$
$$R_9 = 0$$
$$R_2 + R_5 + R_8 = 0$$
$$-R_5 + R_8 = 0$$
$$d - R_2 c + (R_5 + R_8) b = 0$$
$$R_7 a + R_9 b = 0$$

Solving these equations gives

$$P_1 = 1$$
$$R_1 = -1$$
$$R_2 = \frac{d}{c+b}$$
$$R_5 = -\frac{d}{2(c+b)}$$
$$R_7 = 0$$
$$R_8 = -\frac{d}{2(c+b)}$$
$$R_9 = 0$$

all other P's and R's are zero.

Substitution of these values into Equation B-4 gives

$$Q_1 = -\cos\eta_1 - \sin\eta_1\left(\frac{d}{c+b}\right) = b_{1,1}$$
$$Q_2 = -\sin\eta_1 + \cos\eta_1\left(\frac{d}{c+b}\right) = b_{2,1}$$
$$Q_3 = 0 = b_{3,1}$$
$$Q_4 = -\frac{d}{2(c+b)}\sin\eta_3 = b_{4,1}$$
$$Q_5 = -\frac{d}{2(c+b)}\cos\eta_3 = b_{5,1}$$

$$Q_6 = 0 = b_{6,1}$$

$$Q_7 = -\frac{d}{2(c+b)} \sin\eta_3 = b_{7,1}$$

$$Q_8 = -\frac{d}{2(c+b)} \cos\eta_3 = b_{8,1}$$

$$Q_9 = 0 = b_{9,1}$$

$$Q_{10} = -\frac{d}{2(c+b)} \sin\eta_3 = b_{10,1}$$

$$Q_{11} = -\frac{d}{2(c+b)} \cos\eta_3 = b_{11,1}$$

$$Q_{12} = -\frac{d\,\ell_2}{2(c+b)} \cos\eta_3 = b_{12,1}$$

$$Q_{13} = 0 = b_{13,1}$$

$$Q_{14} = 0 = b_{14,1}$$

$$Q_{15} = -\frac{d}{2(c+b)} \sin\eta_3 = b_{15,1}$$

$$Q_{16} = -\frac{d}{2(c+b)} \cos\eta_3 = b_{16,1}$$

$$Q_{17} = -\frac{d\,\ell_2}{2(c+b)} \cos\eta_3 = b_{17,1}$$

$$Q_{18} = 0 = b_{18,1}$$

$$Q_{19} = 0 = b_{19,1}$$

The remaining columns of [b] are determined in the same fashion. The total transformation matrix is given in Table B-1. The angles and lengths given in the table are defined below using the following outline dimensions of the mount as shown in Figure B-1: a, b, c, d, e, m, k and h.

$$\ell_1 = \left[d^2 + (c-h)^2\right]^{1/2}$$

$$C_1 = \cos\eta_1 = \frac{c-h}{\ell_1}$$

$$S_1 = \sin\eta_1 = \frac{d}{\ell_1}$$

$$C_2 = \cos\eta_2 = \frac{b-k}{\left[(b-k)^2 + (a-m)^2\right]^{1/2}}$$

$$S_2 = \sin\eta_2 = \frac{a-m}{\left[(b-k)^2 + (a-m)^2\right]^{1/2}}$$

Table B-1 Force Transformation Matrix

	$b_{i,j}$											
i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$	$j=7$	$j=8$	$j=9$	$j=10$	$j=11$	$j=12$
1	$-C_1-S_1P_4$	S_1P_5	$-\frac{b}{a}C_1$	0	$-S_1P_7$	$-S_1P_1$	$-C_1P_8-S_1P_9$	$-S_1P_3$	$-C_1P_8-S_1P_9$	$-C_1P_{10}$	$-2C_1$	0
2	$-S_1+C_1P_4$	$-C_1P_5$	$-\frac{b}{a}S_1$	0	S_1P_7	C_1P_1	$-S_1P_8+C_1P_9$	C_1P_3	$-S_1P_8+C_1P_9$	$-S_1P_{10}$	$-2S_1$	0
3	0	0	0	0	0	0	0	0	0	1	0	0
4	$-\frac{1}{2}S_3P_4$	$-S_3P_6$	$-\frac{d}{2a}S_3$	$\frac{1}{2a}S_3$	$-\frac{1}{2}S_3P_7$	S_3P_2	$-\frac{1}{2}S_3P_9$	S_3P_3	$-\frac{1}{2}S_3P_9$	0	C_2C_3	S_2C_3
5	$-\frac{1}{2}C_3P_4$	$-C_3P_6$	$-\frac{d}{2a}C_3$	$\frac{1}{2a}C_3$	$-\frac{1}{2}C_3P_7$	C_3P_2	$-\frac{1}{2}C_3P_9$	C_3P_3	$-\frac{1}{2}C_3P_9$	0	$-C_2S_3$	$-S_2S_3$
6	0	0	0	0	0	0	0	0	0	0	$-S_2$	C_2
7	$-\frac{1}{2}S_3P_4$	$-S_3P_6$	$\frac{b}{a}C_2C_3+\frac{d}{2a}S_3+S_2S_3$	$-\frac{1}{2a}S_3$	$-\frac{1}{2}S_3P_7$	S_3P_3	$\frac{f}{a}C_2C_3-\frac{1}{2}S_3P_9$	S_3P_2	$\frac{f}{a}C_2C_3-\frac{1}{2}S_3P_9$	$C_2C_3P_{10}+S_2C_3$	C_2C_3	S_2C_3
8	$-\frac{1}{2}C_3P_4$	$-C_3P_6$	$-\frac{b}{a}C_2S_3+\frac{d}{2a}C_3-S_2S_3$	$-\frac{1}{2a}C_3$	$-\frac{1}{2}C_3P_7$	C_3P_3	$-\frac{f}{a}C_2S_3-\frac{1}{2}C_3P_9$	C_3P_2	$-\frac{f}{a}C_2S_3-\frac{1}{2}C_3P_9$	$-C_2S_3P_{10}-S_2S_3$	$-C_2S_3$	$-S_2S_3$
9	0	0	$\frac{b}{a}S_2-C_2$	0	0	0	$\frac{f}{a}S_2$	0	$\frac{f}{a}S_2$	$S_2P_{10}-C_2$	S_2	C_2
10	$-\frac{1}{2}S_3P_4$	$-S_3P_6$	$-\frac{d}{2a}S_3$	$\frac{1}{2a}S_3$	$-\frac{1}{2}S_3P_7$	S_3P_3	$-\frac{1}{2}S_3P_9$	$S_3(P_3+1)$	$-\frac{1}{2}S_3P_9+C_2C_3$	0	C_2C_3	S_2C_3
11	$-\frac{1}{2}C_3P_4$	$-C_3P_6$	$-\frac{d}{2a}C_3$	$\frac{1}{2a}C_3$	$-\frac{1}{2}C_3P_7$	C_3P_3	$-\frac{1}{2}C_3P_9$	$C_3(P_3+1)$	$-\frac{1}{2}C_3P_9-C_2S_3$	0	$-C_2S_3$	$-S_2S_3$
12	$-\frac{1}{2}C_3P_4\ell_2$	$-C_3P_6\ell_2$	$-\frac{d}{2a}C_3\ell_2$	$\frac{1}{2a}C_3\ell_2$	$-\frac{1}{2}C_3P_7\ell_2$	$C_3P_3\ell_2$	$-\frac{1}{2}C_3P_9\ell_2$	$C_3P_3\ell_2$	$-\frac{1}{2}C_3P_9\ell_2$	0	$-C_2S_3\ell_2$	$-S_2S_3\ell_2$
13	0	0	0	0	0	0	0	0	$-S_2$	0	$-S_2$	C_2
14	0	0	0	0	0	0	0	0	0	0	$S_2\ell_2$	$-C_2\ell_2$
15	$-\frac{1}{2}S_3P_4$	$-S_3P_6$	$\frac{b}{a}C_2C_3+\frac{d}{2a}S_3+S_2S_3$	$-\frac{1}{2a}S_3$	$-\frac{1}{2}S_3P_7$	S_3P_3	$\frac{f}{a}C_2C_3-\frac{1}{2}S_3P_9+C_2C_3$	S_3P_2	$\frac{f}{a}C_2C_3-\frac{1}{2}S_3P_9$	$C_2C_3P_{10}+S_2C_3$	C_2C_3	S_2C_3
16	$-\frac{1}{2}C_3P_4$	$-C_3P_6$	$-\frac{b}{a}C_2C_3+\frac{d}{2a}C_3-S_2S_3$	$-\frac{1}{2a}C_3$	$-\frac{1}{2}C_3P_7$	C_3P_3	$-\frac{f}{a}C_2S_3-\frac{1}{2}C_3P_9-C_2S_3$	C_3P_2	$-\frac{f}{a}C_2S_3-\frac{1}{2}C_3P_9$	$-C_2S_3P_{10}-S_2S_3$	$-C_2S_3$	$-S_2S_3$
17	$-\frac{1}{2}C_3P_4\ell_2$	$-C_3P_6\ell_2$	$(b_{16,3})\ell_2$	$-\frac{1}{2a}C_3\ell_2$	$-\frac{1}{2}C_3P_7\ell_2$	$C_3P_3\ell_2$	$(b_{16,7})\ell_2$	$C_3P_2\ell_2$	$-(b_{16,9})\ell_2$	$-S_3(C_2P_{10}+S_2)\ell_2$	$-C_2S_3\ell_2$	$-S_2S_3\ell_2$
18	0	0	$\frac{b}{a}S_2-C_2$	0	0	0	S_2	0	$\frac{f}{a}S_2$	$S_2P_{10}-C_2$	S_2	C_2
19	0	0	$-(\frac{b}{a}S_2-C_2)\ell_2$	0	0	0	$-\frac{f}{a}S_2\ell_2$	0	$-\frac{f}{a}S_2\ell_2$	$-(S_2P_{10}-C_2)\ell_2$	$-S_2\ell_2$	$-C_2\ell_2$

The geometric parameters p_1 through p_{10} are defined on the following page.

$$C_3 = \cos\eta_3 = \frac{[(b-k)^2 + (a-m)^2]^{1/2}}{[(b-k)^2 + (a-m)^2 + d^2]^{1/2}}$$

$$S_3 = \sin\eta_3 = \frac{d}{[(b-k)^2 + (a-m)^2 + d^2]^{1/2}}$$

$$\ell_2 = \frac{e}{S_3} \qquad f = a - \ell_2 S_2 C_3$$

$$\ell_3 = \frac{d}{S_3} - \ell_2 \qquad g = b - \ell_2 C_2 C_3$$

Other geometric parameters used to define the force transformation matrix in Table B-1 are as follows:

$$p_1 = \frac{g-b}{c+b} \qquad p_5 = \frac{b}{2c+b} \qquad p_8 = \frac{a+f}{a}$$

$$p_2 = \frac{f(c+b) - a(g+c)}{2a(c+b)} \qquad p_6 = \frac{c}{2c+b} \qquad p_9 = \frac{c}{c+b}$$

$$p_3 = \frac{-a(g+c) - f(c+b)}{2a(c+b)} \qquad p_7 = \frac{1}{c+b} \qquad p_{10} = \frac{b+c}{a}$$

$$p_4 = \frac{d}{c+b}$$

Columns 1 to 9 of Table B-1 make the $[b_e]$ portion of the force transformation matrix (Equation B-2) and columns 10 to 11 form the $[b_r]$ portion of the transformation.

The individual elemental flexibility matrices are placed in one matrix $[\ulcorner\alpha\lrcorner]$:

$$[\ulcorner\bar{\alpha}\lrcorner] = \begin{bmatrix} [\bar{\alpha}_{1}] & 0 & 0 & 0 & 0 \\ & [\bar{\alpha}_{2}] & 0 & 0 & 0 \\ & & [\bar{\alpha}_{3}] & 0 & 0 \\ & & & [\bar{\alpha}_{4}] & 0 \\ \text{SYM.} & & & & [\bar{\alpha}_{5}] \end{bmatrix} \qquad \text{(B-5)}$$

Multiply the flexibility with the transformation matrix using a congruant transformation:

$$[b]^T[\alpha][b] = \begin{bmatrix} b_e^T \\ \hline b_r^T \end{bmatrix} [\ulcorner\alpha\lrcorner] \left[b_e \mid b_r \right] = \left[\begin{array}{c|c} a_{1,1} & a_{1,2} \\ \hline a_{2,1} & a_{2,2} \end{array}\right] \qquad \text{(B-6)}$$

where

$$[a_{1,1}] = [b_e^T]\left\lceil \alpha \right\rfloor[b_e]$$

$$[a_{1,2}] = [b_e^T]\left\lceil \alpha \right\rfloor[b_r]$$

$$[a_{2,1}] = [a_{1,2}]^T = [b_r^T]\left\lceil \alpha \right\rfloor[b_e]$$

$$[a_{2,2}] = [b_r^T]\left\lceil \alpha \right\rfloor[b_r]$$

Finally the desired mount flexibility is obtained from the reduced flexibility matrix which was presented in Appendix A as Equation A-27.

$$[\alpha_m] = [a_{1,1}] - [a_{1,2}][a_{2,2}^{-1}][a_{2,1}]. \qquad \text{(B-7)}$$

Summary

Numerical computation of the mount frame flexibility matrix can be obtained using the method described above.

1) Compute $\left\lceil \alpha \right\rfloor$, $[b_e]$ and $[b_r]$ from mount geometry, material properties, and cross section properties.

2) Multiply these matrices to obtain $[a_{11}]$, $[a_{12}]$, $[a_{21}]$ and $[a_{22}]$.

3) Invert $[a_{22}]$

4) Compute $[\alpha_m]$.

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