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WEAPON-MOUNT INTERFACE STUDY TRIPOD MOUNT M3

U.S. ARMY WEAPONS COMMAND

TECHNICAL REPORT

April 1972

RESEARCH DIRECTORATE

WEAPONS LABORATORY AT ROCK ISLAND

RESEARCH, DEVELOPMENT AND ENGINEERING DIRECTORATE

U. S. ARMY WEAPONS COMMAND

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A mathematical model of the M3 Tripod Mount for the M2 Machine Gun was developed by the Research Directorate, Weapons Laboratory at Rock Island to study the dynamic response of the tripod when it is subjected to force excitation due to a firing machine gun. This model was used to construct a digital computer program capable of predicting tripod motions and stresses for any reasonable firing situation. The program output consists of time histories of parameters that describe the motion of the gun barrel, and maximum stresses in the tripod legs. The program input comprises elevation and azimuth angles of the gun, properties of soil at the emplacement site of the weapon, degree of tripod leg extension, and a forcing function generated from the output of the USAWECOM computer program for the M2 gun.

The Finite-Element Method is the basis for obtaining the solution. By use of this method, the tripod is modeled as a three-dimensional elastic structure whose mathematical model is four interconnected beam-columns that, for the purposes of solution, are divided into nineteen finite-difference elements. The mathematical model of the soil is a linearly elastic half-space with point loading at the surface. The system of the resulting equations of motion was solved by the Gauss-Seidel Iterative Technique.

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FOREWORD

This report was prepared by R. P. Vaitys, A. N. Henzi, and S. J. Lis, General American Research Division, Niles, Illinois, under Contract DAAF03-70-C-0032.

This work was conducted under the direction of the Research Directorate, Weapons Laboratory at Rock Island, U. S. Army Weapons Command, with Catherine M. Robinder and Robert H. Coberly as project engineers.

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SECTION 1

INTRODUCTION

The objective of the work effort reported herein was the development of a mathematical model of the Tripod Mount M3 that is compatible with the mathematical model of Machine Gun M2, this latter being a product of USAWECOM. The modeled system includes both the tripod and the ground on which the tripod is emplaced. The purpose of the modeling is to develop analytic means of predicting the tripod's dynamic response to a force excitation arising from the firing of the gun.

Once developed, the mathematical model of the tripod was then used to construct a digital computer program for the purpose of generating numerical data on the dynamic response sought. The resulting computer program is intended to serve as an analytic tool for studying the effects on gun operation of such variables as the degree of leg extension, barrel attitude (elevation and azimuth), and the type of soil at the emplacement site.

Although the computer program as listed in Appendix B strictly applies only to the Tripod Mount M3, it is general enough so that it can be quickly converted to a program for a whole class of other machine-gun mounts. All that is necessary for the conversion is a change of certain internal constants.

The computer program is based on mathematical models of the tripod and of the soil that supports the machine gun during firing. The tripod is idealized as a three-dimensional structure consisting of three beam-columns (the two rear legs and the front leg), and one beam (the traverse bar). The finite-element approach was employed, which means that for the purposes of solution the tripod was divided into 19 beam-column elements that were treated as clamped one to another, except at three junction points and at the three feet where the tripod is in contact with the ground. Each such finite element was considered as possessing six degrees of freedom at each end — three rotations and three translations. This finite-difference treatment takes into account the flexural, torsional and compressional properties of the tripod legs, and inertial effects of both the tripod mass and the machine-gun mass, as discussed in more detail in subsequent sections. Linear elasticity is assumed to hold everywhere and for all times.

The soil model employed is somewhat limited as it represents soil behavior in the linear and elastic range only. It is known that when this particular weapon is fired on certain types of soil, the ground reaction forces exceed the soil's bearing capacity, and the tripod feet will sink into the ground as the gun continues firing. However, the current state-of-the-art of soil mechanics is such that it is not even remotely possible to make analytic predictions of plastic soil displacements due to loads dynamically applied by odd-shaped buried bodies. Therefore the computer program incorporates the only presently-available elastic model of soil that was developed primarily for analyzing foundations for vibrating machinery.

The computer program was constructed by using the Gauss-Seidel Iteration Method to solve the many simultaneous equations of motion for displacements at nineteen discrete points of the tripod. This information is then further processed to produce an output consisting of time-histories of gun-barrel elevation angle and displacement of the feet, and time histories of peak stresses within each one of the twelve elements that can be expected to be more severely loaded than others.

The inputs required to run this program fall into two categories: geometrical data and the forcing function. Most of the required geometrical data are built into the program as internal constants; the only external inputs are the lengths of the finite elements since they depend on the degree of extension of the tripod legs. The forcing function that must be specified consists of two time-dependent force vectors and one moment. The computer program will run on any arbitrarily-selected forcing function. However, to achieve any degree of realism in the predicted dynamic motions and stresses, the forcing function must represent the interaction forces and moments between the tripod and the machine gun. One of the forces is the pintle pin reaction, the other -- elevating-mechanism reaction; the moment is applied at the pintle, and it acts about an axis parallel to the barrel centerline.

The computer output shown in Appendix C was obtained by using a forcing function generated from output of a particular run on the USAWECOM's computer program for Machine Gun M2.

The subsequent sections of this report are concerned with the description of the mathematical models of the tripod and the soil (Section 2); details of input and output are given in Section 3; equations of motion for individual finite elements of tripod, as well as for soil, are listed in Section 4; and Section 5 explains how the equations of motion are assembled into a system of equations, and how this system is programmed for solution on a digital computer.

Appendix A is an operations manual for the computer program, giving instructions on how to prepare the inputs and how to interpret the output. Appendix B contains the complete listing of program in FORTRAN for the IBM 1130 digital computer. Appendix C presents a sample of output format, and finally, Appendix D presents a few typical derivations of equations listed in Section 4.

SECTION 2

DESCRIPTION OF THE MATHEMATICAL MODEL

This section describes the mathematical model that simulates the visco-elastic system supporting the machine gun in a firing situation. This system consists of the tripod mount and the soil underneath it. Each of these two subsystems is discussed separately.

2.1 Tripod Model

The Tripod Mount M3, shown in Figure 1, for the purposes of analysis is somewhat idealized in that it is assumed to consist of segmentally uniform beams and concentrated masses. The distribution of these masses is shown in Figure 2 which also indicates the division of the beam parts of the tripod into finite elements that are essential to the specified method of solution. Figure 2 also shows the global coordinate system employed in the analysis; the coordinate axes are assumed to be fixed to the ground such that the XZ-plane coincides with the surface of the local terrain (which may or may not be horizontal). The Y-axis -- which again may or may not be vertical -- initially (i.e. before the gun is fired on the tripod in this particular location) passes through the leg confluence point 4.

The heavy, numbered lines in Figure 2 represent the centerlines of the tubular legs and of the traverse bar. The numbers without parentheses are used to label the so-called nodal points which indicate the beginning and the end of any particular finite element. The numbers in parentheses are used to identify particular finite elements. Thus we see that the tripod model was postulated to consist of 19 elements; actually the number of elements into which any given structural system may be divided is rather arbitrary as it depends only on two considerations:

a) the capacity of digital computer available,
b) the requirement that all elements be not shorter than four diameters (Ref. 2).

In order to achieve the highest accuracy it is preferable, if the computer capacity permits it, to divide a structural system into the largest number of elements consistent with requirement (b) above. This is what was more or less done in segmenting the subject tripod, except that some elements, as we shall see later, turned out to be shorter than four diameters.

Now a few comments about identifying the nodal points on the actual structure of the tripod. Points 1, 16 and 19 are the points of intersection of the leg centerlines with the top surface of their respective

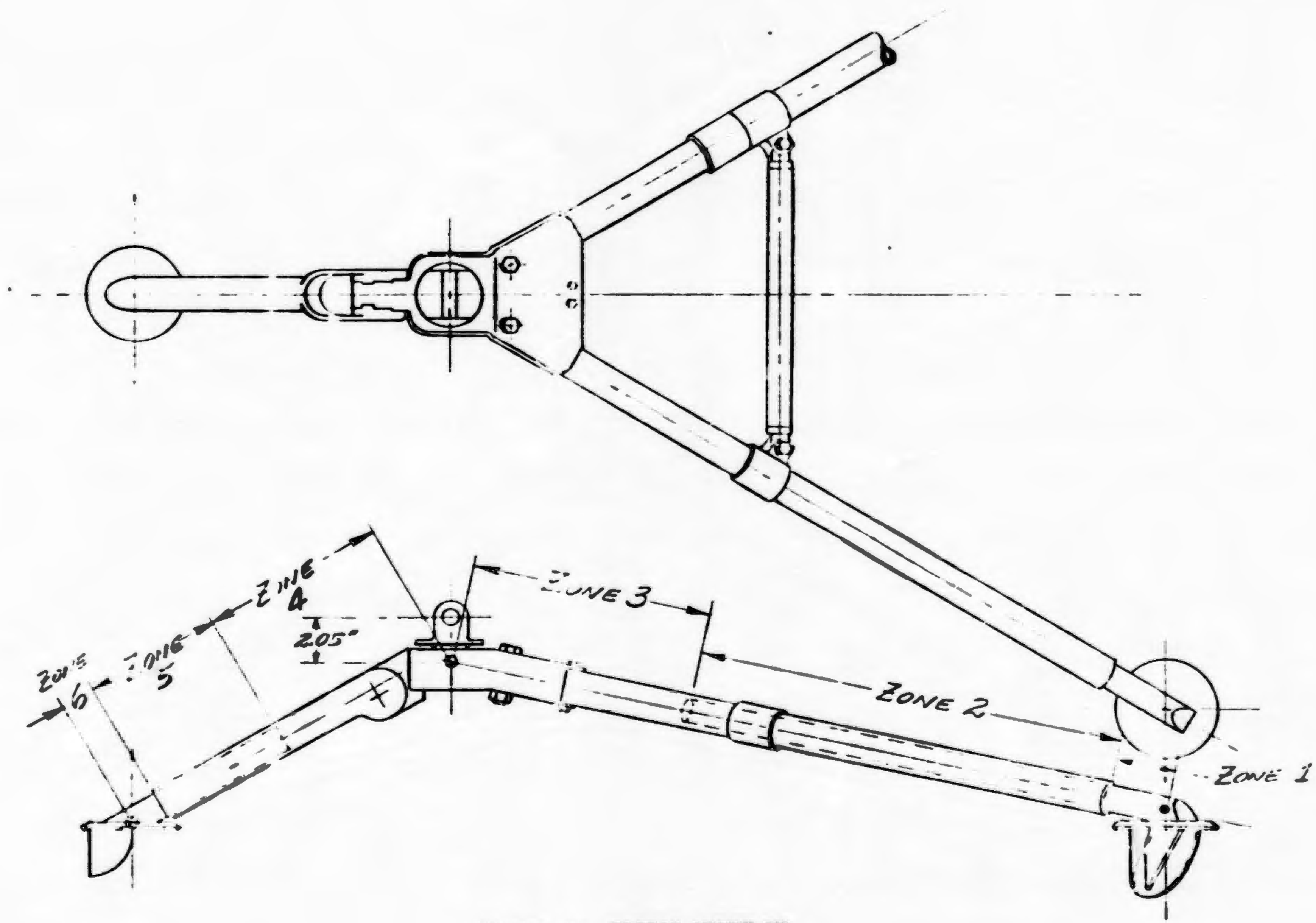


Figure 1: TRIPOD MOUNT M3

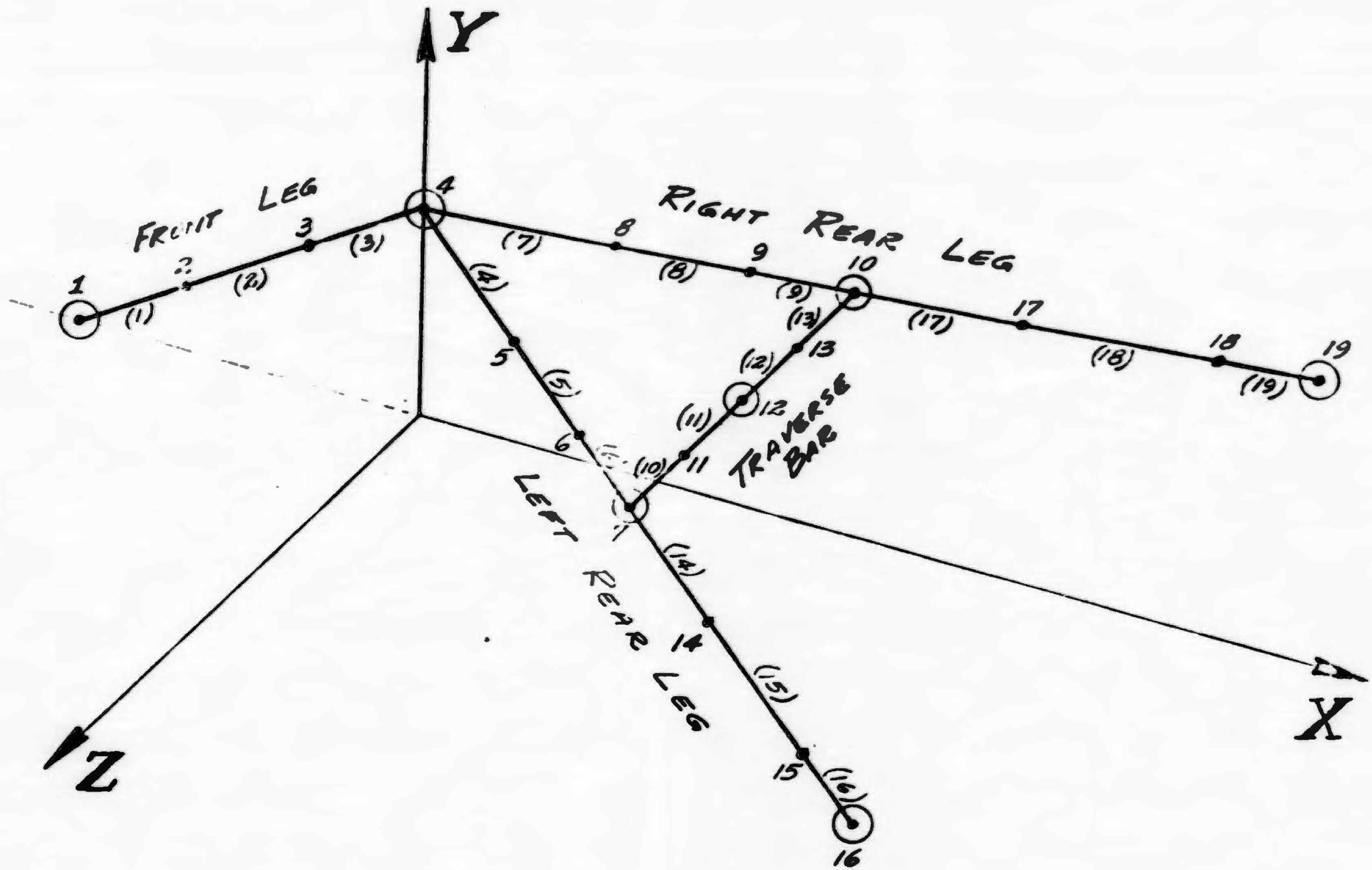


Figure 2: FINITE-DIFFERENCE MODEL OF MOUNT M3

feet. Points 7 and 10 are the intersection points between centerlines of the traverse bar and left and right rear legs, respectively. Point 4, the so-called leg confluence point, is actually the intersection of the rear leg centerlines with the centerline of the tapered vertical pin of the pintle. The centerline of the front leg does not actually pass through point 4, as can be seen in Figure 1, but this slight discrepancy is disregarded in the formation of the mathematical model because it would unnecessarily burden the computer programming.

The logic of positioning the rest of nodal points is based on the requirement that any finite element must be uniform throughout its entire length in both mass density per lineal inch and in area properties. From this it follows that any discontinuities of leg cross-section must be located at nodal points. Thus, for instance, points 2, 15 and 18 may be assigned to the locations where the outer tube of the respective leg terminates; points 3, 6 and 9 may be assigned to locations where the telescoped inner tube of the leg terminates inside its respective outer tube. More about this will be said in Section 2 and Appendix A.

In Figure 2 the circles around nodal points 1, 4, 7, 10, 16 and 19 indicate the location of concentrated masses since in the finite-element approach all material that is not distributed along the finite elements must be lumped and applied at the nodal points. This fact was taken into consideration when segmenting the tripod -- certain nodal points were positioned at the approximate C.G.-locations of the lumped masses. Thus the lumped masses at points 1, 16 and 19 represent the tripod feet; the mass at point 4 consists of the pintle and the tripod head assembly, plus part of the machine gun; the mass at point 7 is the fixed sleeve; and the mass at point 10 consists of the sliding sleeve and its stop.

All nineteen elements of the tripod model are assumed to be linearly elastic and possess no structural damping (which, by the way, is quite negligible compared to that of the soil). The ends of elements that form junction points were treated as being constrained as follows:

1) Elements (4) and (7) are hinged so that they can rotate freely about an axis perpendicular to the plane of the rear legs. A moment connection, however, exists about the Z-axis.

2) At points 7 and 10 elements (10) and (13) are hinged about axes perpendicular to the plane of rear legs. However, moments about the other two axes can be transmitted between the traverse bar and the rear legs.

3) At points 1, 16 and 19 elements (1), (16) and (19) are constrained by ground reaction components as shown in Figure 3. At these three points all six degrees of freedom are possible.

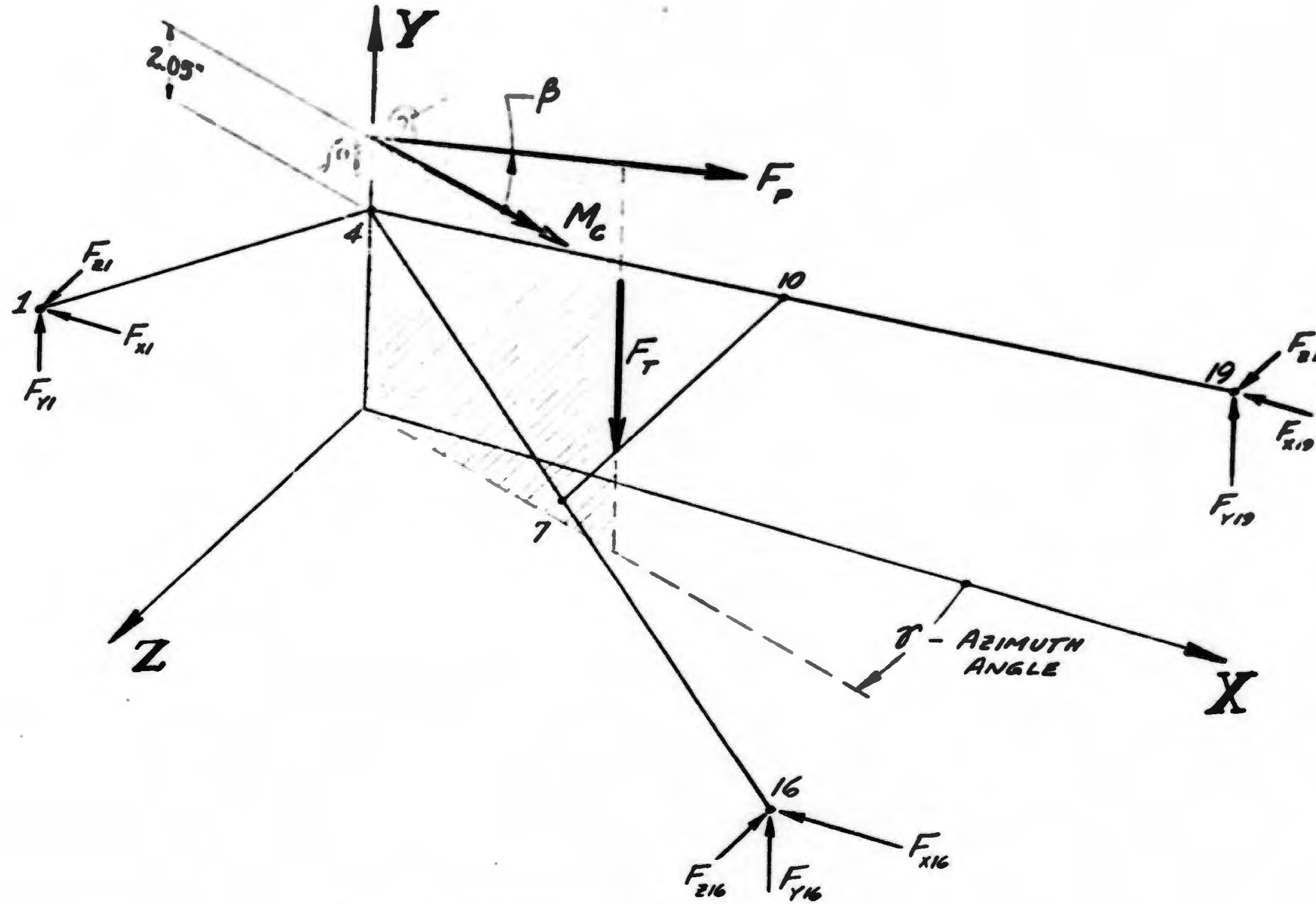


Figure 3: FREE-BODY DIAGRAM OF TRIPOD MOUNT M3

Figure 3 also indicates the external forces applied to the tripod during the firing of the machine gun. These are the traverse-bar force, the pintle force, and a moment applied about an axis passing through the horizontal pintle pin and at right angles to it. More detail on this subject is presented in Section 4.

2.2 Soil Model

Mathematical modeling of soil to simulate the dynamic response of a (for all practical purposes) semi-infinite mass of particulate matter under a rapidly fluctuating point load is a very difficult problem for which no satisfactory solution has been found in the literature. Although constitutive equations for various types of soils under dynamic loading do exist, generally they are rather complicated and do not lend themselves easily to application to problems of practical interest Specifically, there are currently no analytical methods nor empirical formulas that could be employed in estimating the contact forces and moments between the ground and a tripod foot such that the following would be fully accounted for:

a) dynamic loading of a periodic nature,
b) a complicated geometric shape of the buried part of the foot (i.e the spade plus the web), and
c) plastic yielding of the soil.

True, there are methods in existence for estimating the load-bearing capacity of building foundations on both cohesive and cohesionless soils, but these methods have severe limitations when it comes to applying them to the problem at hand because:

a) Foundation-engineering formulas are good only for statically-applied loads.

b) Foundations are usually treated as infinitely-long bodies so that the problem becomes two-dimensional. Current methods cannot be used to treat a three-dimensional partly-buried body such as a tripod foot.

c) Methods and formulas have been developed for the specific purpose of estimating the so-called failure load at which the foundation will be on the verge of sinking or tilting. What happens when such a failure load is exceeded, i.e., what is the subsequent force-displacement-time history for an overloaded foundation, is a subject that does not interest a foundation designer. Consequently, force-displacement-time relationships have never been obtained for sinking foundations or for other buried or partly-buried bodies penetrating the soil. And it is precisely these relationships for the post-yield regime of soils that is needed for estimation of the soil-tripod interaction forces.

In view of the above deficiencies in the current state-of-the-art of soil mechanics, it was decided to incorporate into the computer program the next best thing available -- a set of expressions taken from Ref. 1. These expressions have been derived for calculation of vibration amplitudes of heavy machine foundations subjected to vibratory force excitation. These expressions are for equivalent mass, spring constant and dashpot constant of a lumped-parameter system in which the mass is that of the foundation plus a certain portion of soil next to it; and where the spring and dashpot constants represent the linear response of a semi-infinite expanse of soil to displacement and velocity of the foundation. The derivations are based on the assumptions of linear elasticity and linear damping, and of soil's homogeneity with respect to all its properties. Thus we see that the expressions for spring and dashpot constants, k and c, are limited to the linear elastic range of soil's behavior, and therefore cannot possibly be used to predict any residual displacements of the tripod feet in case the foot-soil contact forces exceed the soil's bearing capacity. The fact that these expressions were used in the programming means simply this: no matter how soft the soil, no matter what the ammunition used and no matter how long the firing burst, the computer output will indicate no residual displacements at any of the feet.

The formulas for k and c that are listed in Ref. 1 come in four sets, each set for a distinct mode of excitation, as follows:

1) vertical,
2) horizontal,
3) rocking (about an axis lying in the plane of contact between the foundation and the soil), and
4) torsional (about a vertical axis).

Modes (1) and (3) above induce primarily compressive soil stresses at the soil-foundation interface, whereas modes (2) and (4) induce shear stresses at the same interface.

Expressions for all four of the above-mentioned excitation modes are listed in Section 4 for the case of circular foundations. This geometric shape is the best available approximation to the actual configuration of the tripod feet, which in addition to the circular planview also have vertical projections (= spades) whose effect on the effective mass and spring rate of the soil cannot at present be analytically assessed.

2.3 Scope and Limitations

The tripod-soil model and the resultant computer program accounts for the following:

1) Variation in section properties of the tripod legs along their length (i.e., it recognizes the fact that the inner tube and the outer tube portions thereof have different cross-sections).

2) Telescoping of the legs (the analytic treatment of the telescoped lengths is further discussed in Section 4).

3) Degree of leg extension (i.e., fully extended or fully retracted legs, plus three intermediate discrete positions).

4) Type of soil underneath the tripod legs.

5) Elevation and azimuth angles of the gun barrel.

6) Lifting of the front leg off the ground during a portion of each firing cycle.

7) The interaction between the tripod and the weapon is simulated to a limited extent by adding the gun's mass to the lumped masses of the tripod at two locations (such as points 4 and 12, for instance, which corresponds to the case of a zero azimuth angle).

Following is a list of the most important deviations from the actual physical system that was mathematically modeled. Without these simplifying assumptions the modeling would have been hardly possible within the scope of this program.

1) Backlash in all hinged connections was assumed to be zero.

2) The flexural properties of the tripod head were taken to be equal to those of the leg elements adjacent to it -- as shown in Figure 2 the section properties of elements (3), (4) and (7) are those of the respective outer tubes of the legs, while the tripod head itself is reduced to a lumped mass.

3) The traverse bar is imagined to span the distance between centerlines of the rear legs -- which means that the traverse bar was taken to be somewhat longer than it actually is, and that the slight torsional moment in the rear legs due to the attachment of traverse bar is neglected.

4) The beam-column interaction was neglected in the rear legs where it would be most significant: rough estimates indicate that the peak compressive loads in each rear leg will amount to less than 4% of the Euler column buckling load. From this it can be concluded that the column loads in the legs will have virtually no effect on leg deflections due to beam action.

5) Azimuth angle in the input cannot be specified at any arbitrary value. As dictated by the finite-element method, forces external to the structure must be applied at nodal points only. Since the traverse bar is divided into four equal-length elements, there are only three nodal points internal to the traverse bar, and they are at fixed locations -- which means that the traverse-bar reaction can be applied at only these three points on the bar. Thus the only azimuth angles available for computer input are 0° and ± 14°55'.

6) The effect of gravity forces on the motions of and stresses in the tripod are neglected since the dynamic forces due to gun barrel recoil predominate, being an order of magnitude higher than gravity forces. For this reason there will be no difference in computer output whether the tripod is emplaced on a slope or on level ground.

7) The calculated tripod foot displacements will be periodic and, at the end of the firing burst, will show no residual values -- i.e., there will be no indication of sinkage into the ground and/or rearward ploughing. This is because of the nature of the soil model used, remembering that this model postulates linear elastic behavior of soil, with no restrictions on peak values of applied forces.

SECTION 3

OUTPUT AND INPUT

This section spells out the type of output generated by the computer program, and specifies the input required in order to run the program.

3.1 Output Generated

The output generated falls into two categories -- geometrical variables relating to the stability of gun during firing, and the peak dynamic values of stress in the tripod legs.

The geometric output variables are defined by Figure 4; although the program at every time step generates 414 bits of information associated with the 19 nodal points, printing out such a mass of data would serve no useful purpose. Only the following geometrical variables are considered to be of practical importance in assessing the operation of the tripod-weapon system, and are therefore printed out.

1) Time from initiation of the explosion of the propellant charge in the first round of a particular firing burst (time is the one and only independent variable).

2) Displacements and velocities (at every time step), in the three global coordinate directions, of the midpoint of pintle pin and each of the three tripod feet (i.e. x_i, y_i, z_i and $\dot{x}_i$, $\dot{y}_i$, $\dot{z}_i$ for i = 4, 1, 16 and 19, where i denotes the index of the nodal point).

3) Change in elevation angle, ϕ, as measured from the initial position of the gun barrel before firing has started. This variable is probably the best indicator of the effect of mount resilience on gun stability.

The rest of the output data (comprising three columns on the output sheet), also printed out at every time step, relates to the dynamic stresses in the tripod elements. Listed in the three right-hand-most columns, these data appear in the following order:

1) Element number. This number indicates the particular finite element in which the peak stress was determined. The elements subjected to stress analysis in this program are the four center-most elements of the left rear leg, the four center-most elements of the right rear leg, and all four elements of the traverse bar. The front leg is not stress-investigated at all, as it is much less severely loaded than the rest of the tripod structure.

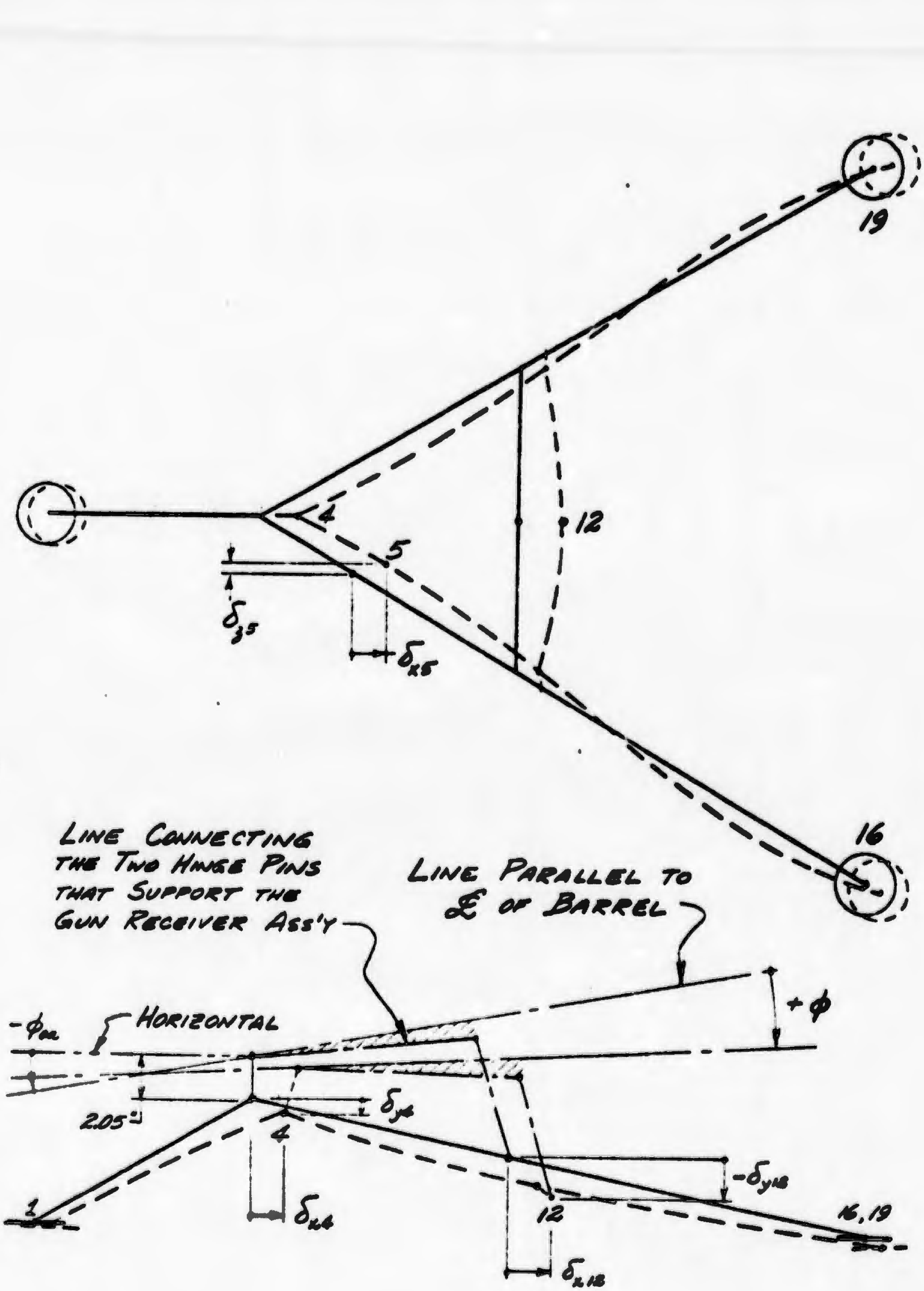


Figure 4: OUTPUT VARIABLES DEFINED

2) Peak combined stress. Printed out opposite each element number is the peak combined stress in that element This value represents the maximum compressive stress due to flexure with the axial compressive stress superposed.

3) Location of peak stress. This value is the distance from one end of the element at which the peak stress value (appearing in the preceding column) occurs in this particular element. The distance is always measured in the positive Z-direction on the traverse bar, in the general positive X-direction along both rear legs.

3.2 Input Required

This section lists and describes all of the inputs necessary for operating the computer program. Some of these input variables are defined in Figure 5; all of the variables are defined verbally in the same order as they are entered on the input cards.

1) Included angle from front to rear legs, α(ALF) Angle α is defined in Figure 5, and always remains constant at 137° 20' -- it is a tripod constant.

2) Front leg angle of kink, γ(GAM). Positive direction of γ is shown in Figure 5.

3) Fixed front-leg length from pintle. This is another tripod constant, always set equal to 6.01". It denotes the distance between the leg confluence point and the front leg hinge.

4) Time increment, Δt(DTAU). This is the independent variable used in obtaining the output; a value of .001 sec is recommended.

5) Element number, (LL), identifies the particular finite element, (1) through (19).

6) Length, (SEG). This is the length of the particular finite element (LL). The various lengths, with the exception of those for elements of the traverse bar, are variable as they depend on the degree of leg extension.

7) Nodal points at which concentrated masses are located: 1, 4, 7, 10, 16 and 19.

8) Concentrated mass, M (AM). These are the values of the concentrated mass, in lbs, associated with each nodal point in Item (7) above.

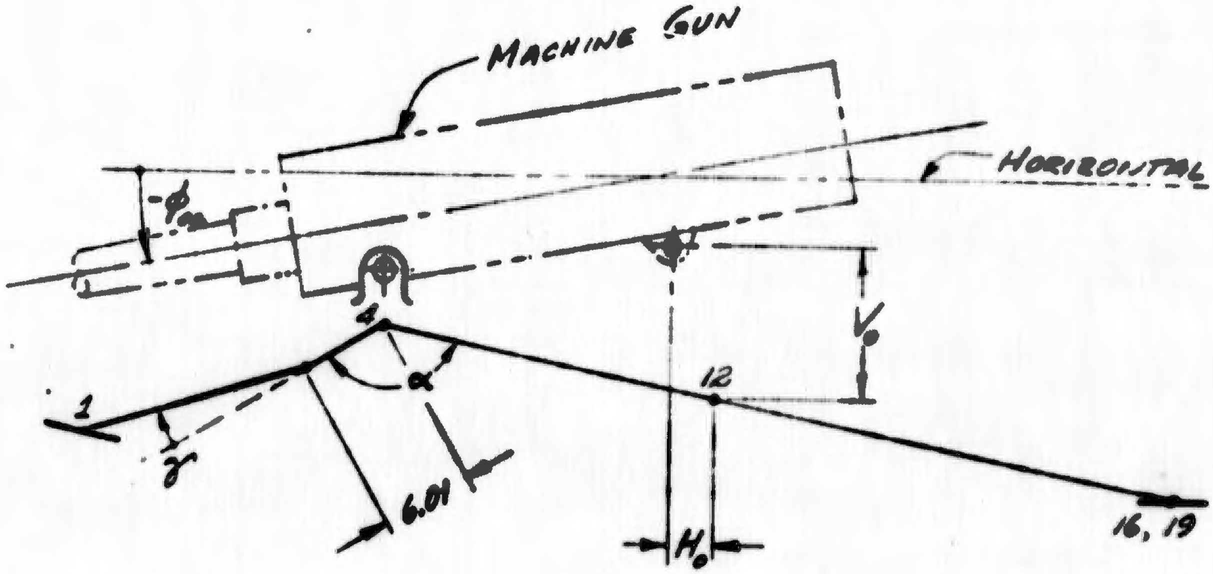


Figure 5: INPUT VARIABLES DEFINED

9) Mass moments of inertia, JX, JY, JZ (AJ). These are the mass moments of inertia of the concentrated masses, taken about the three orthogonal axes in the global system whose origin is located at each particular nodal point listed in Item (7). Thus, for instance, for the rear left foot: JX is the moment about a diametral axis lying in the upper surface of the foot disk; JZ -- about a similar diametral axis at right angles; and JY is the polar moment of inertia about a vertical axis passing through the center of the foot disk.

10) Additional concentrated mass due to gun. This item represents the introduction of the gun mass into the mass matrix of the tripod, and it consists of two numerical constants. One constant, WTP = 72.1 lbs, is the portion of the gun's weight that is statically supported by the tripod at nodal point 4. The other constant, WTT = 9.2 lbs, is the portion of the gun's weight supported by the elevating mechanism. The particular nodal point (12, 13 or 11) at which this latter mass is lumped is determined by the input value of azimuth angle.

The following five items (11 through 15) constitute the only information that is required for identification of the soil, and for the subsequent evaluation of the soil's spring rate and damping constant.

11) Radius of foot disk r_o (RO). This always equals 2.30 inches, since it represents an invariable dimension of the tripod foot.

12) Soil density ρ, (RHO), in (lbs/in^3).

13) Poisson's Ratio ν, (ANU) of the soil.

14) Void ratio e, (E) of the soil.

15) Soil type, IG. For angular sand or cohesive soils the indicator IG must be taken as 1; for round-grained sands, IG = 2.

16) Initial gun elevation angle, ϕ_{oa} (ELEV). As shown in Figure 5, ϕ_{oa} is the angle between the centerline of the barrel and the XZ-plane, and it constitutes the reference from which one of the output variables -- the change in elevation angle, ϕ, -- is measured. ϕ_{oa}-values may range continuously from +5.6° to -14°.

17) Initial gun azimuth angle (AZIM). The values entered here must be consistent with the nodal-point locations mentioned in Item (10): the only admissible azimuth angle values are those which correspond to the elevating mechanism's position at nodal points 11, 12 or 13.

18) Elevation mechanism setting, V_o. This is the vertical distance between the centerline of the traverse bar and the center of pin that joins the elevation mechanism to the gun receiver (see Figure 5).

The next five items, (19) through (23), have to do with specifying the forcing function.

19) Time from initiation of the first round fired in a burst. The values entered here (up to 50 entries are admissible) are the discrete time instants at which the components of the forcing function (Items 19 through 22) must be known.

20) Pintle force, F_P is the instantaneous value of the dynamic interaction force between the gun receiver and the pintle pin, located 2.05" above nodal point 4 (see Figure 3). It is entered as a discrete-point function, as are all of the forcing function components.

21) Traverse bar force, F_T is the instantaneous value of the dynamic interaction force between the gun receiver and the elevating mechanism. Its direction is always assumed to be vertical. Also note that, as shown in Figure 3, F_P and F_T always lie in the same vertical plane.

22) Pintle moment, M_C is the reaction couple due to feeding of the ammunition belt in the gun.

23) Angle of action for pintle force, β, is the instantaneous value of angle between the pintle force vector and the XZ-plane (see Figures 3 and A.7).

In addition to all of the above-enumerated external inputs that have to be entered onto data cards, the program embodies a considerable number of internal inputs in the form of permanently stored constants. This internal input consists of the following:

1) Young's Modulus for steel (= 29 x 10^6 psi)
2) Shear modulus for steel (= 11 x 10^6 psi)
3) Weight density of steel (= .285 lbs/in^3)
4) Section properties for every one of the 19 finite-difference elements, namely:
   a) cross-sectional area,
   b) area moment of inertia,
   c) torsional constant, and
   d) outside radius (i.e., one-half of the OD of the leg or the traverse bar).

Specific details on input preparation are presented in Appendix A — such as the required input format, how to specify the element lengths once the degree of leg extension is chosen, etc.

SECTION 4

EQUATIONS OF MOTION

A basic feature of the finite-element technique is the fact that the equations of motion for the individual finite elements must be assembled in matrix form before subjecting them to simultaneous solution for accelerations at the nodal points. This section is primarily concerned with listing, without derivations, of expressions that are the building blocks of matrices that constitute the equations of motion for all possible degrees of freedom of each element.

4.1 Equations of Motion for Tripod Elements

In the most general case, the equations of motion for a finite element of prismatic shape may be written in matrix notation as follows (Ref. 2). The below expression is based on a treatment of the finite element from the beam-vibrations rather than the wave-propagation standpoint; Eq. (1) is essentially a statement of forced vibration of a beam wherein the rotary inertia and transverse shear are neglected.

$$\underline{M}_{\textcircled{1}}\, \ddot{\underline{q}}_{\textcircled{1}} + \underline{C}_{\textcircled{1}}\, \dot{\underline{q}}_{\textcircled{1}} + \underline{K}_{\textcircled{1}}\, \underline{q}_{\textcircled{1}} = \underline{Q}_{\textcircled{1}}(t) \qquad (1)$$

Here the subscript $\textcircled{1}$ refers to End 1 of the element, so that a formally identical equation may be written for End 2 of the same element.

In Eq. (1) $\underline{M}$, $\underline{C}$ and $\underline{K}$ stand for square matrices, known as mass, damping and stiffness matrices; and $\ddot{\underline{q}}$, $\dot{\underline{q}}$ and $\underline{q}$ are the column matrices for generalized accelerations, velocities and displacements. And $\underline{Q}$ is the column matrix for generalized forces applied at End 1 of the element.

The terms of the mass and stiffness matrices are listed in the following two subsections (4.1.1 and 4.1.2). The damping matrices for all tripod elements are assumed to be identically equal to zero, since -- as already mentioned -- structural damping of steel was considered negligible as compared to soil damping.

4.1.1 Elements of Mass Matrix

The expressions appearing in this subsection were obtained from Ref. 2; a sample derivation of one of them is presented in Appendix D.

Consider a prismatic element (typically, a segment of a uniform beam) subjected to all manner of forces and moments at both ends, and accelerating as a result of the action of all these applied forces and moments (Figure 6).

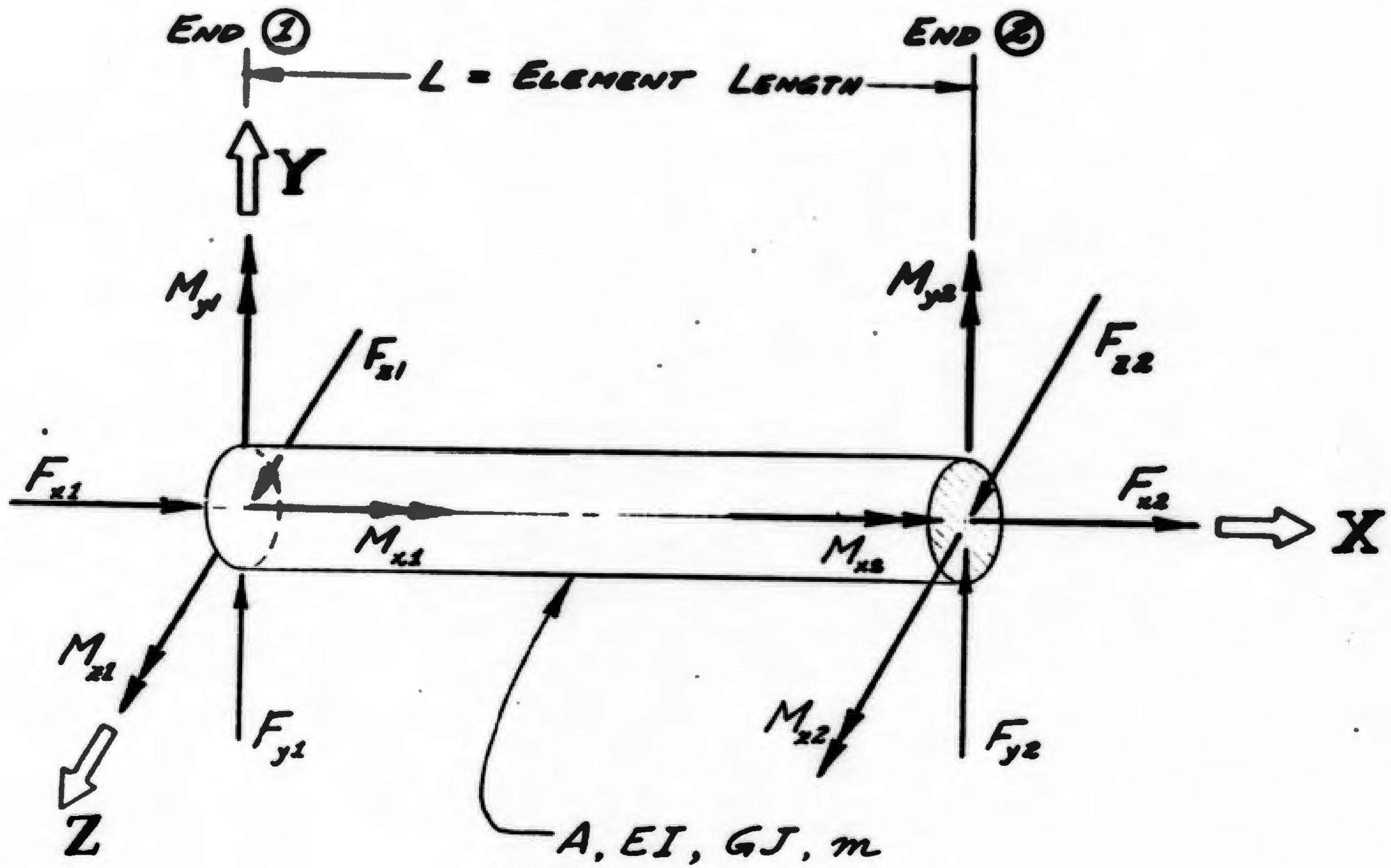


Figure 6: FREE-BODY DIAGRAM OF A FINITE ELEMENT

The directions of forces and moments as shown in Figure 6 are all positive; positive moments are defined by the right-hand rule throughout all subsequent analysis.

Now, each one of the twelve generalized forces acting on the ends of the finite element consists of two components: one of the components, denoted by superscript m, equilibrates the inertial forces within the element; the other component, denoted by superscript k, equilibrates the elastic forces within the element that are caused by differences in displacements at the two ends of the element. Thus:

$$F_{x1} = F^m_{x1} + F^k_{x1}$$

$$M_{x1} = M^m_{x1} + M^k_{x1} \qquad (2)$$

This subsection is concerned with listing of all the inertial components of F_i's and M_i's associated with both ends of the finite element. These components are seen to depend only on the lineal density m (mass per inch of length), the length of element L, and the acceleration in a particular direction as measured at the end of the element. Figures 7 and 8 are supplied as an aid in identifying the particular mode of inertial loading with the force-acceleration expression associated with it. The sign convention for forces, moments, displacements and their derivatives, as well as the labeling of the two ends, in Figures 7 and 8 is identical with that of Figure 6.

Figure 7a: End reactions due to axial acceleration at End 1, $\ddot{\delta}_{x1}$ (all other acceleration components are zero; End 2 thus may be considered either as stationary, or moving with constant velocity):

$$F^m_{x1} = \frac{1}{3} mL \ddot{\delta}_{x1}$$

$$F^m_{x2} = \frac{1}{6} mL \ddot{\delta}_{x1} \qquad (3)$$

Figure 7b: End reactions due to axial acceleration at End 2, $\ddot{\delta}_{x2}$ (again, all other $\ddot{\delta}_{i1}$'s and $\ddot{\delta}_{i2}$'s are zero):

$$F^m_{x1} = \frac{1}{6} mL \ddot{\delta}_{x2}$$

$$F^m_{x2} = \frac{1}{3} mL \ddot{\delta}_{x2} \qquad (4)$$

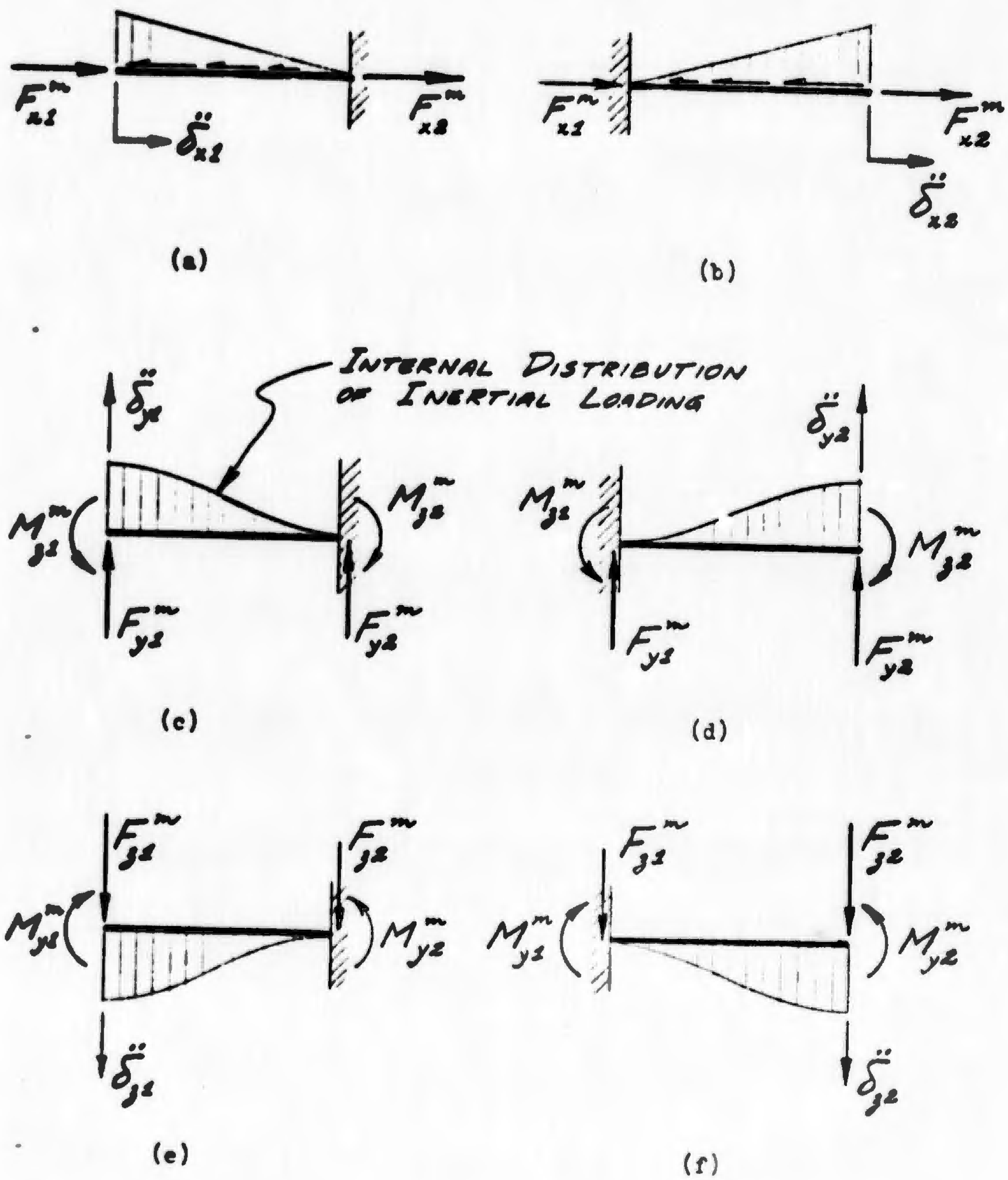


Figure 7: INERTIAL FORCE COMPONENTS DUE TO LINEAR ACCELERATIONS AT THE ENDS OF A FINITE ELEMENT

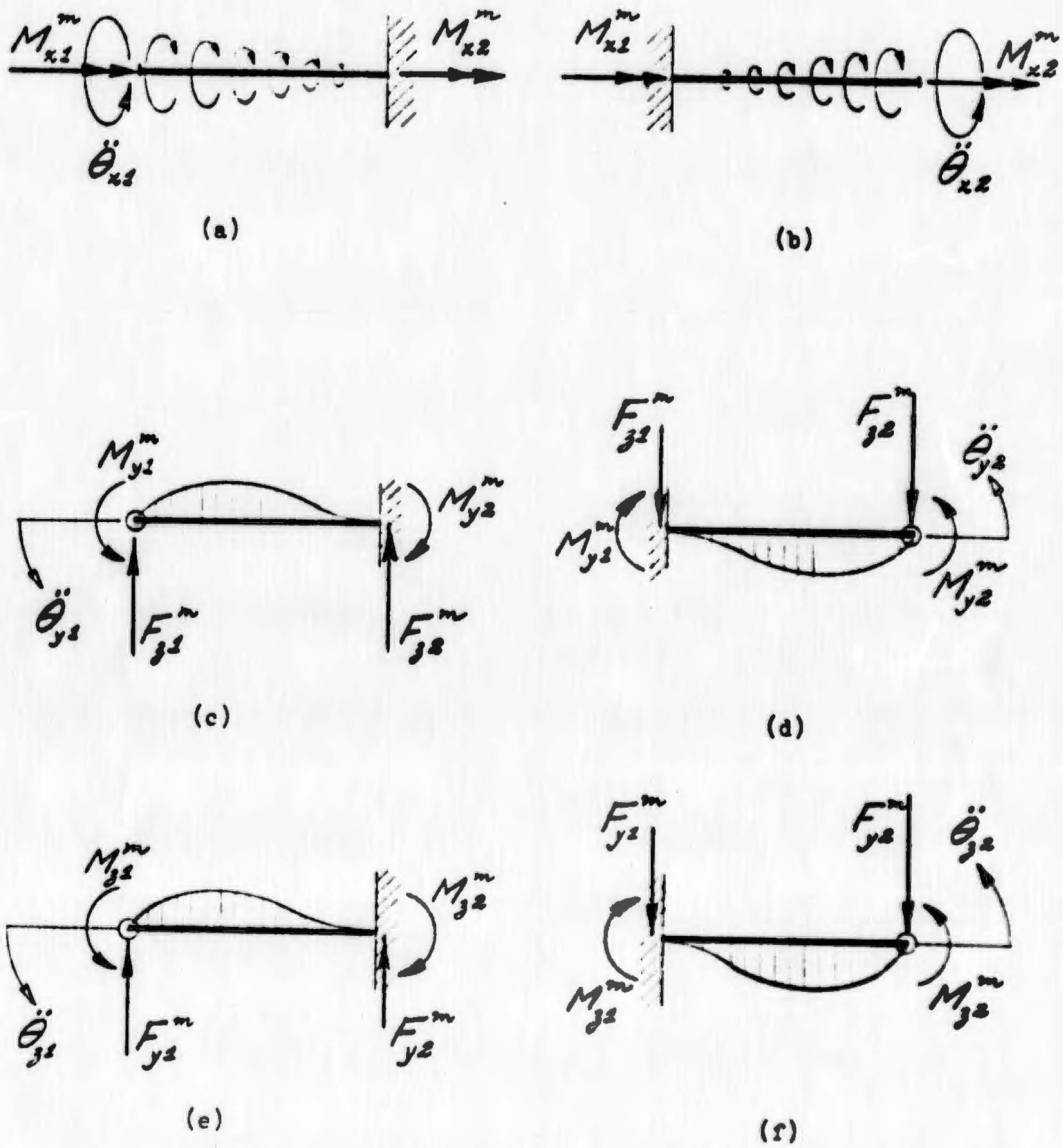


Figure 8: INERTIAL FORCE COMPONENTS DUE TO ANGULAR ACCELERATIONS AT THE ENDS OF A FINITE ELEMENT

Figure 7c: End reactions due to transverse acceleration at End 1, $\ddot{\delta}_{y1}$:

$$F^{m}_{y1} = \frac{13}{35} mL \, \ddot{\delta}_{y1}$$

$$F^{m}_{y2} = \frac{9}{70} mL \, \ddot{\delta}_{y1}$$

$$M^{m}_{z1} = \frac{11}{210} mL^{2} \, \ddot{\delta}_{y1} \qquad (5)$$

$$M^{m}_{z2} = -\frac{13}{420} mL^{2} \, \ddot{\delta}_{y1}$$

Figure 7d: End reactions due to transverse acceleration at End 2, $\ddot{\delta}_{y2}$:

$$F^{m}_{y1} = \frac{9}{70} mL \, \ddot{\delta}_{y2}$$

$$F^{m}_{y2} = \frac{13}{35} mL \, \ddot{\delta}_{y2}$$

$$M^{m}_{z1} = \frac{13}{420} mL^{2} \, \ddot{\delta}_{y2} \qquad (6)$$

$$M^{m}_{z2} = -\frac{11}{210} mL^{2} \, \ddot{\delta}_{y2}$$

Figure 7e: End reactions due to transverse acceleration at End 1, $\ddot{\delta}_{z1}$:

$$F^{m}_{z1} = \frac{13}{35} mL \, \ddot{\delta}_{z1}$$

$$F^{m}_{z2} = \frac{9}{70} mL \, \ddot{\delta}_{z1}$$

$$M^{m}_{y1} = -\frac{11}{210} mL^{2} \, \ddot{\delta}_{z1} \qquad (7)$$

$$M^{m}_{y2} = \frac{13}{420} mL^{2} \, \ddot{\delta}_{z1}$$

$\ddot{\delta}_{z2}$: **Figure 7f**: End reactions due to transverse acceleration at End 2,

$$F^m_{z1} = \frac{9}{70} mL \, \ddot{\delta}_{z2}$$

$$F^m_{z2} = \frac{13}{35} mL \, \ddot{\delta}_{z2}$$

$$M^m_{y1} = -\frac{13}{420} mL^2 \, \ddot{\delta}_{z2} \qquad (8)$$

$$M^m_{y2} = \frac{11}{210} mL^2 \, \ddot{\delta}_{z2}$$

$\ddot{\theta}_{x1}$: **Figure 8a**: End reactions due to a torsional acceleration at End 1,

$$M^m_{x1} = \frac{1}{3} m \left(\frac{JL}{A}\right) \ddot{\theta}_{x1}$$

$$M^m_{x2} = \frac{1}{6} m \left(\frac{JL}{A}\right) \ddot{\theta}_{x1} \qquad (9)$$

$\ddot{\theta}_{x2}$: **Figure 8b**: End reactions due to a torsional acceleration at End 2,

$$M^m_{x1} = \frac{1}{6} m \left(\frac{JL}{A}\right) \ddot{\theta}_{x2}$$

$$M^m_{x2} = \frac{1}{3} m \left(\frac{JL}{A}\right) \ddot{\theta}_{x2} \qquad (10)$$

Figure 8c: End reactions due to an angular acceleration about Y-axis, $\ddot{\theta}_{y1}$ (at End 1):

$$F^m_{z1} = -\frac{11}{210} mL^2 \, \ddot{\theta}_{y1}$$

$$F^m_{z2} = -\frac{13}{420} mL^2 \, \ddot{\theta}_{y1}$$

$$M^m_{y1} = \frac{1}{105} mL^3 \, \ddot{\theta}_{y1} \qquad (11)$$

$$M^m_{y2} = -\frac{1}{140} mL^3 \, \ddot{\theta}_{y1}$$

Figure 8d: End reactions due to an angular acceleration about Y-axis, $\ddot{\theta}_{y2}$ (at End 2):

$$F^m_{z1} = \frac{13}{420} mL^2 \ddot{\theta}_{y2}$$

$$F^m_{z2} = \frac{11}{210} mL^2 \ddot{\theta}_{y2}$$

$$M^m_{y1} = -\frac{1}{140} mL^3 \ddot{\theta}_{y2} \qquad (12)$$

$$M^m_{y2} = \frac{1}{105} mL^3 \ddot{\theta}_{y2}$$

Figure 8e: End reactions due to an angular acceleration about Z-axis, $\ddot{\theta}_{z1}$ (at End 1):

$$F^m_{y1} = \frac{11}{210} mL^2 \ddot{\theta}_{z1}$$

$$F^m_{y2} = \frac{13}{420} mL^2 \ddot{\theta}_{z1}$$

$$M^m_{z1} = \frac{1}{105} mL^3 \ddot{\theta}_{z1} \qquad (13)$$

$$M^m_{z2} = -\frac{1}{140} mL^3 \ddot{\theta}_{z1}$$

Figure 8f: End reactions due to an angular acceleration about Z-axis, $\ddot{\theta}_{z2}$ (at End 2):

$$F^m_{y1} = -\frac{13}{420} mL^2 \ddot{\theta}_{z2}$$

$$F^m_{y2} = -\frac{11}{210} mL^2 \ddot{\theta}_{z2}$$

$$M^m_{z1} = -\frac{1}{140} mL^3 \ddot{\theta}_{z2} \qquad (14)$$

$$M^m_{z2} = \frac{1}{105} mL^3 \ddot{\theta}_{z2}$$

4.1.2 Elements of Stiffness Matrix

Expressions appearing in this subsection were derived through the transposition of some standard strength-of-materials equations that are readily available in a multitude of sources.

The expressions subsequently listed are for the elastic force components acting at both ends of the finite element (the F^k_{ij}'s and M^k_{ij}'s of Eq. (2)). These force components are to be interpreted as the end reactions resulting from differences in displacements and rotations between one end of the element and the other. Hence they are functions of only the element length and section properties, and the impressed displacements or rotations at either end of the element. Figures 9 and 10 are supplied for the purpose of identifying and defining a particular mode of elastically straining the element with the force-displacement expression associated with it. The sign convention in these figures is again identical with that of Figure 6.

Figure 9a: End reactions due to axial displacement, δ_{x1}, only:

$$F^k_{x1} = \left(\frac{EA}{L}\right) \delta_{x1}$$

$$F^k_{x2} = -\left(\frac{EA}{L}\right) \delta_{x1} \qquad (15)$$

Figure 9b: End reactions due to axial displacement, δ_{x2}, only:

$$F^k_{x1} = -\left(\frac{EA}{L}\right) \delta_{x2}$$

$$F^k_{x2} = \left(\frac{EA}{L}\right) \delta_{x2} \qquad (16)$$

Figure 9c: End reactions due to transverse displacement δ_{y1}, while end rotations θ_{z1} and θ_{z2} are constrained to remain zero:

$$F^k_{y1} = \left(\frac{12EI}{L^3}\right) \delta_{y1}$$

$$F^k_{y2} = -\left(\frac{12EI}{L^3}\right) \delta_{y1}$$

$$M^k_{z1} = \left(\frac{6EI}{L^2}\right) \delta_{y1} \qquad (17)$$

$$M^k_{z2} = \left(\frac{6EI}{L^2}\right) \delta_{y1}$$

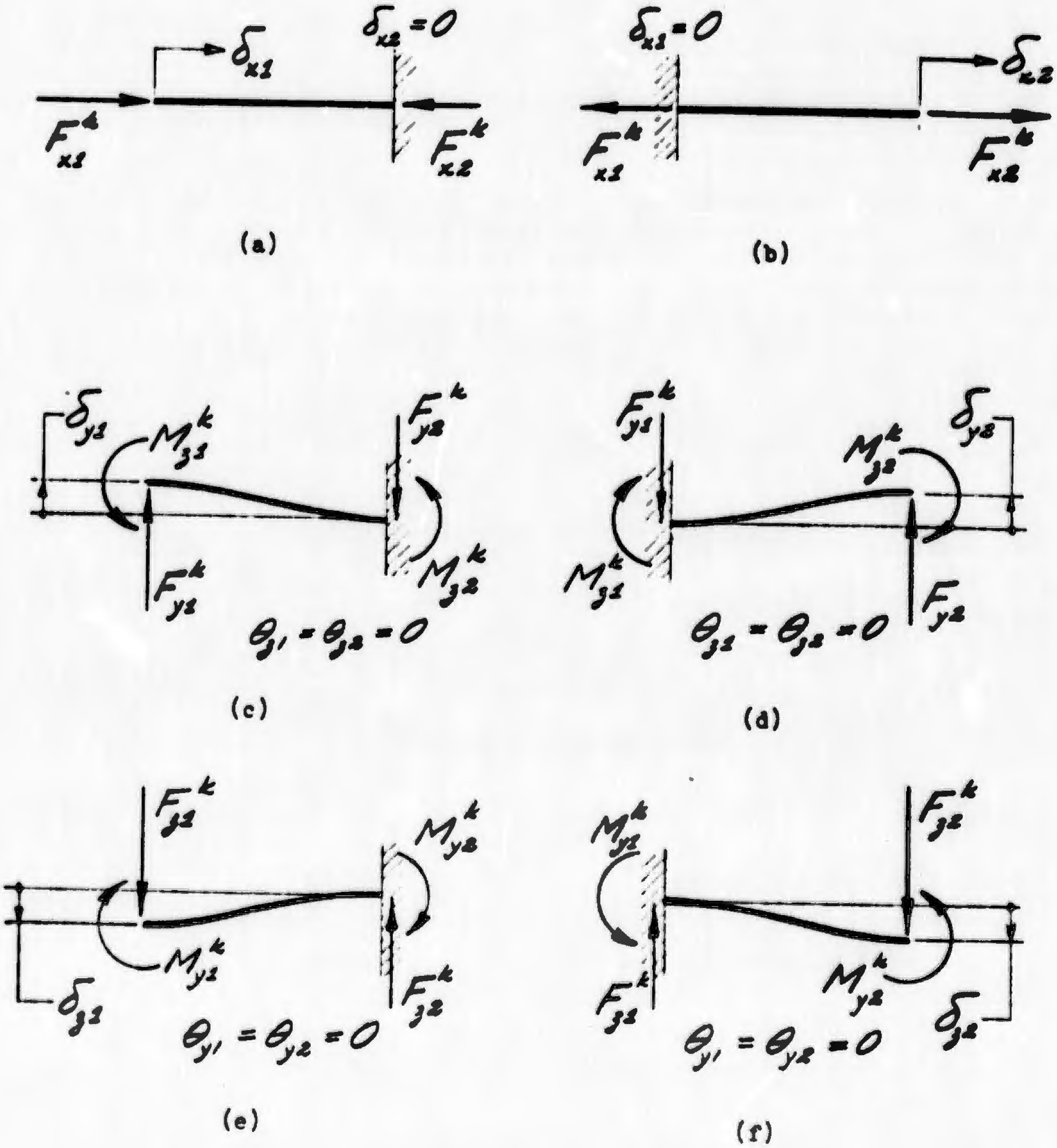


Figure 9: ELASTIC FORCE COMPONENTS DUE TO LINEAR DISPLACEMENTS AT THE ENDS OF A FINITE ELEMENT

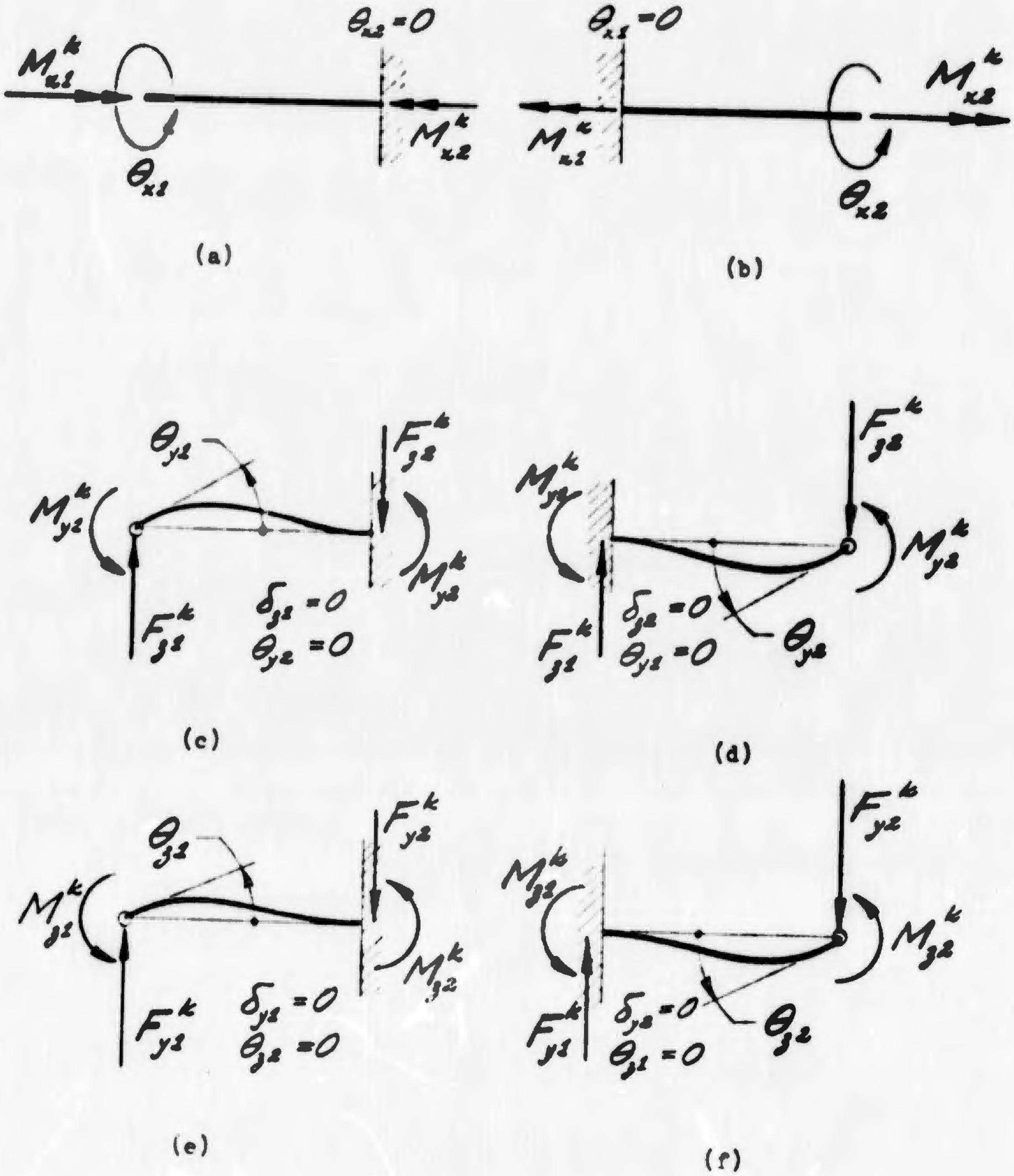


Figure 10: ELASTIC FORCE COMPONENTS DUE TO ANGULAR DISPLACEMENTS AT THE ENDS OF A FINITE ELEMENT

Figure 9d: End reactions due to transverse displacement δ_{y2}, while end rotations θ_{z1} and θ_{z2} are constrained to remain zero:

$$
\begin{aligned}
F^k_{y1} &= -\left(\frac{12EI}{L^3}\right) \delta_{y2} \\
F^k_{y2} &= \left(\frac{12EI}{L^3}\right) \delta_{y2} \\
M^k_{z1} &= -\left(\frac{6EI}{L^2}\right) \delta_{y2} \\
M^k_{z2} &= -\left(\frac{6EI}{L^2}\right) \delta_{y2}
\end{aligned}
\qquad (18)
$$

Figure 9e: End reactions due to transverse displacement, δ_{z1}, while end rotations θ_{y1} and θ_{y2} are constrained to remain zero:

$$
\begin{aligned}
F^k_{z1} &= \left(\frac{12EI}{L^3}\right) \delta_{z1} \\
F^k_{z2} &= -\left(\frac{12EI}{L^3}\right) \delta_{z1} \\
M^k_{y1} &= -\left(\frac{6EI}{L^2}\right) \delta_{z1} \\
M^k_{y2} &= -\left(\frac{6EI}{L^2}\right) \delta_{z1}
\end{aligned}
\qquad (19)
$$

Figure 9f: End reaction due to transverse displacement δ_{z2}, while end rotations θ_{y1} and θ_{y2} are constrained to remain zero:

$$
\begin{aligned}
F^k_{z1} &= -\left(\frac{12EI}{L^3}\right) \delta_{z2} \\
F^k_{z2} &= \left(\frac{12EI}{L^3}\right) \delta_{z2} \\
M^k_{y1} &= \left(\frac{6EI}{L^2}\right) \delta_{z2} \\
M^k_{y2} &= \left(\frac{6EI}{L^2}\right) \delta_{z2}
\end{aligned}
\qquad (20)
$$

Figure 10a: End reactions due to angular displacement, θ_{x1}, about the torsional axis:

$$M^k_{x1} = (\frac{GJ}{L})\,\theta_{x1}$$

$$M^k_{x2} = -(\frac{GJ}{L})\,\theta_{x1} \qquad (21)$$

Figure 10b: End reactions due to angular displacement, θ_{x2}, about the torsional axis:

$$M^k_{x1} = -(\frac{GJ}{L})\,\theta_{x2}$$

$$M^k_{x2} = (\frac{GJ}{L})\,\theta_{x2} \qquad (22)$$

Figure 10c: End reactions due to end rotation θ_{y1}, while end displacements δ_{z1} and δ_{z2} are constrained to remain zero:

$$F^k_{z1} = -(\frac{6EI}{L^2})\,\theta_{y1}$$

$$F^k_{z2} = (\frac{6EI}{L^2})\,\theta_{y1}$$

$$M^k_{y1} = (\frac{4EI}{L})\,\theta_{y1} \qquad (23)$$

$$M^k_{y2} = (\frac{2EI}{L})\,\theta_{y1}$$

Figure 10d: End reactions due to end rotation θ_{y2}, while end displacements δ_{z1} and δ_{z2} are constrained to remain zero:

$$F^k_{z1} = -(\frac{6EI}{L^2})\,\theta_{y2}$$

$$F^k_{z2} = (\frac{6EI}{L^2})\,\theta_{y2}$$

$$M^k_{y1} = (\frac{2EI}{L})\,\theta_{y2} \qquad (24)$$

$$M^k_{y2} = (\frac{4EI}{L})\,\theta_{y2}$$

Figure 10e: End reactions due to end rotation θ_{z1}, while end displacements δ_{y1} and δ_{y2} are constrained to remain zero:

$$F^k_{y1} = (\frac{6EI}{L^2})\ \theta_{z1}$$

$$F^k_{y2} = -(\frac{6EI}{L^2})\ \theta_{z1}$$

$$M^k_{z1} = (\frac{4EI}{L})\ \theta_{z1} \qquad (25)$$

$$M^k_{z2} = (\frac{2EI}{L})\ \theta_{z1}$$

Figure 10f: End reactions due to end rotation θ_{z2}, while end displacements δ_{y1} and δ_{y2} are constrained to remain zero:

$$F^k_{y1} = (\frac{6EI}{L^2})\ \theta_{z2}$$

$$F^k_{y2} = -(\frac{6EI}{L^2})\ \theta_{z2}$$

$$M^k_{z1} = (\frac{2EI}{L^2})\ \theta_{z2} \qquad (26)$$

$$M^k_{z2} = (\frac{4EI}{L^2})\ \theta_{z2}$$

4.2 Treatment of Telescoped Portions of Legs

As shown in Figure 1, each tripod leg may be thought of as being divided into three zones such that within each zone the sectional properties remain constant. Thus in Zones 1 and 6 the section properties (A, EI, GJ, m) are those of the inner tube (i.e., the lower portion of legs); in Zones 3 and 4 the section properties are those of the outer tube (i.e., the upper portion of legs). Zones 2 and 5 are characterized by the fact that here the inner tubes are telescoped within their respective outer tubes. Question arises -- what are the effective values of section properties in those two zones? The following discussion briefly states the assumptions made in the attempt to answer this question, and the conclusions reached.

Since the upper and lower portions of the legs are manufactured with a diametral clearance (when telescoped) ranging from .006 to .012 inch, we can expect the inner tube to touch the outer tube at two points only, as shown in Figure 11: at the termination of the outer tube where it is squeezed to a smaller diameter by the clamp, and at the termination of the inner tube. Figure 11a shows the general arrangement, whereas Figure 11b shows the actual pinned-and-clamped support at the end of the outer tube idealized as a circumferential knife-edge for the purposes of analysis. The derivation of the expression for the equivalent moment of inertia then proceeded from the following two assumptions:

a) Static loading of the telescoped tube group by the force system shown in Figure 11b, and
b) Simply supported ends of inner tube.

The equivalency criterion invoked is (Figure 11b):

$$\rho_{actual} = \rho_{equivalent}$$

which means that the sum of end rotations of the "equivalent beam" (moment of inertia = I_e) should be equal to the sum of end rotations of the outer tube in the actual tripod, given an identical loading in both cases.

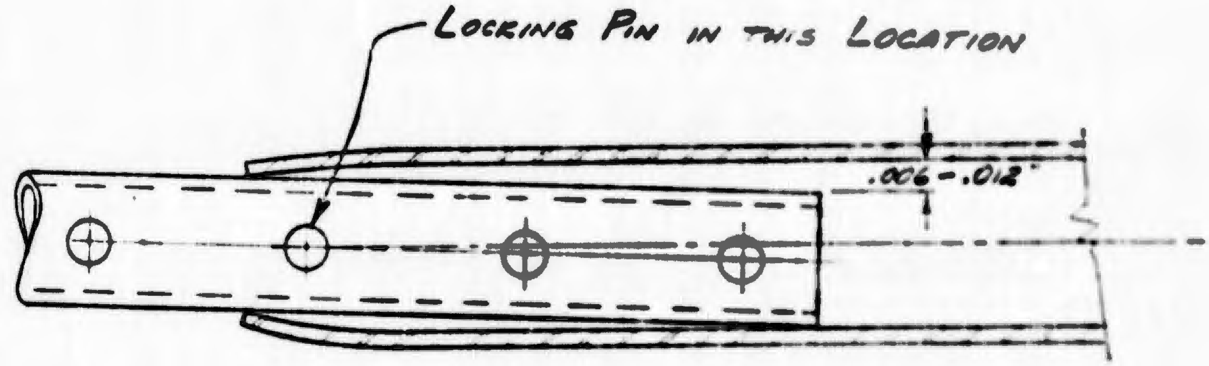
The derivation of the expression for I_e is found in Appendix D; only the end result is given below:

$$I_e = \frac{3}{2}\left[\frac{2M_L + V_L L_T}{\left(1 + \frac{I_3}{I_1}\right) M_L + V_L L_T}\right] I_3 \qquad (27)$$

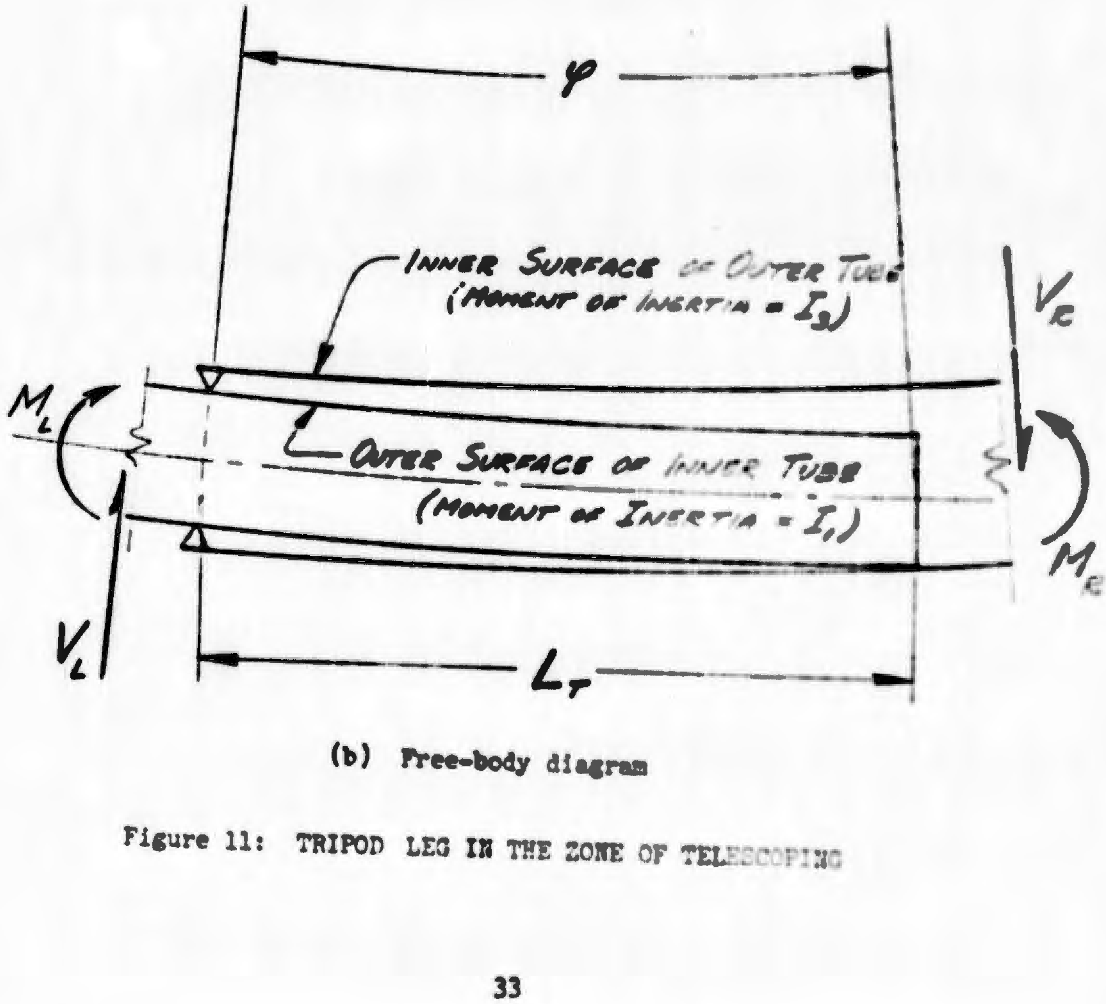
where all the symbols are defined in Figure 11. From this expression it appears that I_e is a function of not only geometry, but also of applied loads. However, we are fortunate in that the tripod was designed with almost equal moments of inertia for inner and outer tubes (I_1 = .131 in^4; I_3 = .124 in^4), because when $I_1 = I_3$, then I_e ceases to depend on loading:

$$I_e = 1.5\, I_3$$

for the case of $I_1 = I_3$, regardless of the length of telescoping, L_T, and regardless of moments and shears applied. Therefore, for the sake of convenience in computer programming, the slight inequality between I_1 and I_3, and between I_4 and I_6, was overlooked. Thus the following two values of I_e were used in the program whenever the finite element under consideration consists of telescoped tubes:



(a) General arrangement



(b) Free-body diagram

Figure 11: TRIPOD LEG IN THE ZONE OF TELESCOPING

For rear legs:

$$I_e = 1.5\left(\frac{I_1 + I_3}{2}\right) = .191 \text{ in}^4$$

For the front leg:

$$I_e = 1.5\left(\frac{I_4 + I_6}{2}\right) = .199 \text{ in}^4$$

Here, as a compromise to the inequality of I's, the average value of I's for inner and outer tube was taken in lieu of I for the outer tube as prescribed by Eq. (27).

Since any compressive or tensile loads transmitted along the legs do not load (in tension or compression) that part of the inner tube which lies between the locking pin and the free end of the inner tube, it follows that:

$A_e = A_3$ (for rear legs)

$A_e = A_4$ (for the front leg)

In other words, the "equivalent" (i.e., load-carrying) cross-sectional area in the zone of telescoping may be taken as equal to the area of the outer tube only.

The above comment also applies to the torsional constant J if one neglects the friction between the inner and the outer tube:

$J_e = J_3$ (for rear legs)

$J_e = J_4$ (for the front leg)

The equivalent mass per unit length, m, to be used in expressions for the inertial-force components (see Section 4.1.1) was taken as the sum of m's for the inner and outer tube:

$m_e = m_1 + m_3$ (for rear legs)

$m_e = m_4 + m_6$ (for the front leg)

4.3 Equations of Motion for Soil

The so-called equations of motion for soil are, in the strictest sense, expressions for evaluating the X, Y and Z components of the contact force between the ground and a tripod foot, as well as for evaluating the components of resisting couples exerted on the foot as it executes angular displacements. These equations (taken from Ref. 1) are based on a lumped-parameter treatment of a vibrating system consisting of a rigid machine foundation resting on top of the earth's surface. Each component of the contact force or moment (denoted by subscript i) consists of two subcomponents, as follows:

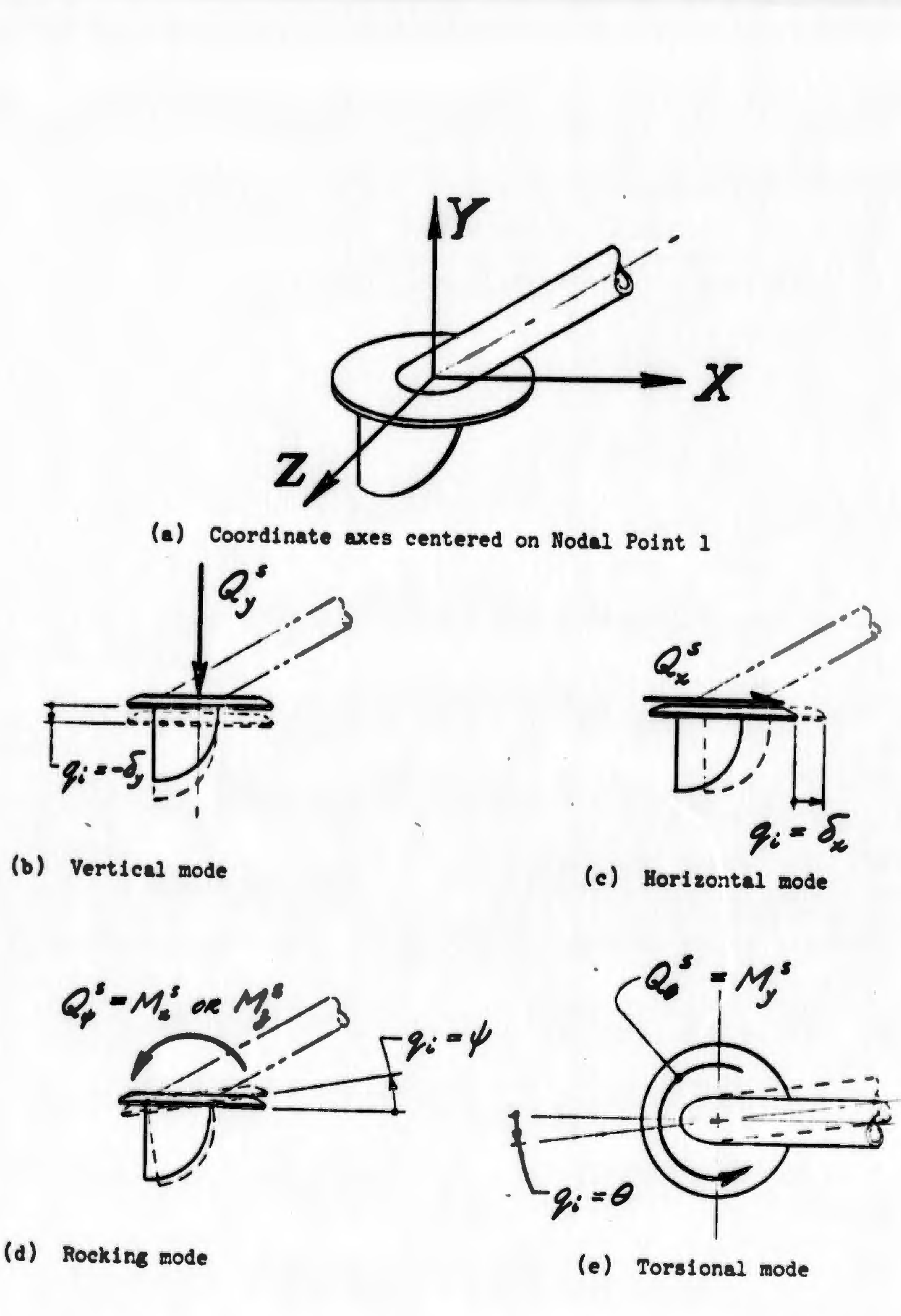
$$Q^s_{ij} = k_{ij} q_{ij} + c_{ij} \dot{q}_{ij} \qquad (28)$$

Here j = 1, 16, 19 refers to the nodal points that identify the location of feet. The first component in Eq. (28) is the spring force, whereas the second one is the damping force. Thus the spring constant k_{ij} represents a linear relationship between the applied load and displacement of the tripod foot (in the source, Ref. 1, it is the vibrating foundation), and this implies a linear stress-strain relation for the soil. Likewise, the damping constant c_{ij} represents a linear relationship between the force and the velocity of the foot in the direction i. The series of expressions quoted below and used in the computer program are for k_{ij}'s and c_{ij}'s, derived for a rigid foundation with a circular plan view that rests on the surface of the elastic half-space. In all these expressions the only parameter of the foot (originally, of the foundation) that must be known is the radius of the foot disk, r_o; the other needed information is soil parameters such as its shear modulus G, its Poisson's Ratio ν, density ρ and void ratio e.

The listed expressions refer to some particular mode of excitation of a tripod foot as shown in Figure 12. Although the figure shows the front foot, the equations are just as applicable to the rear feet since they take no account of the presence of spade -- that is, no matter what the spade configuration (even if the foot has no spade at all), the interaction force as predicted by Eq. (28) will be the same.* Since there are only four distinct excitation (or displacement) modes possible but each foot possesses six degrees of freedom, obviously some of the listed expressions can be associated with more than one coordinate of motion (i.e., degree of freedom), as pointed out below.

---

*) Herein lies one of the principal weaknesses of the equations taken from Ref. 1 -- namely, their inability to account for geometric intricacies beyond those of a simple, flat-bottomed circular footing. The other principal weakness is, as mentioned before, the fact that these equations cover only the linear elastic regime -- which means that prediction of permanent displacements of the soil due to post-yield loads is totally impossible.



(a) Coordinate axes centered on Nodal Point 1

(b) Vertical mode

(c) Horizontal mode

(d) Rocking mode

(e) Torsional mode

Figure 12: COMPONENTS OF FORCE EXCITATION TO WHICH A TRIPOD FOOT MAY BE SUBJECTED

Vertical Mode (Figure 12b)

This is an up-and-down bounce of the foot, producing compressive soil stresses at the interface. It is always associated with the Y-axis in the global coordinate system of the tripod.

$$k_Y = \frac{4G\, r_o}{1 - \nu}$$

$$c_Y = \frac{3.4\, r_o^2}{1 - \nu} \sqrt{G\rho} \qquad (29)$$

Horizontal Mode (Figure 12c)

This is a back-and-forth motion, producing shear stresses in the soil at the interface. It can be associated with either the X or the Z-axis in the global coordinate system.

$$k_X = k_Z = \frac{32\,(1 - \nu)}{7 - 8\,\nu}\; G\, r_o$$

$$c_X = c_Z = .576\; \frac{32\,(1 - \nu)}{7 - 8\,\nu} \sqrt{G\rho} \qquad (30)$$

Rocking Mode (Figure 12d)

This is an angular displacement about any diametral axis of the foot disk producing compressive soil stresses at the interface. Rocking can be associated with both the X and the Z-axes.

$$k_\psi = \frac{8G\, r_o^3}{3(1-\nu)}$$

$$c_\psi = \frac{0.80}{1-\nu} \left[\frac{r_o^4}{1 + \frac{3(1-\nu)}{8} \left(\frac{I_\psi}{\rho r_o^5} \right)} \right] \sqrt{G\rho} \qquad (31)$$

Here I_ψ is the mass moment of inertia of the foot about a particular diametral axis (i.e., about X or Z).

Torsional Mode (Figure 12e)

This is an angular displacement about the polar axis of the foot disk, producing shear stresses in soil at the interface. This displacement takes place in the XZ-plane.

$$k_\theta = \frac{16}{3} G r_o^3$$

$$c_\theta = \frac{2.31 r_o^2}{1 + \left(\frac{2I_\theta}{\rho r_o^5}\right)} \sqrt{\frac{GI_\theta}{r_o}} \qquad (32)$$

Here I_θ is the mass moment of inertia of the foot about its polar, or Y, axis.

The soil's shear modulus G in all the above equations must be evaluated from either one or the other of the following two empirical expressions.

For angular sands and cohesive soils, use

$$G = \frac{1230 (2.97 - e)^2}{1 + e} \sqrt{\bar{\sigma}_o} \qquad (33a)$$

For round-grained sands, use

$$G = \frac{2630 (2.17 - e)^2}{1 + e} \sqrt{\bar{\sigma}_o} \qquad (33b)$$

Here e is the void ratio of the soil in its natural state at the surface; and $\bar{\sigma}_o$ is the so-called effective octahedral normal stress in soil directly under the foot disk. If the Poisson's ratio of soil, ν, is known, then this stress may be estimated from the following expression:

$$\bar{\sigma}_o = \frac{1}{3} \left(\frac{1 + \nu}{1 - \nu} \right) \sigma_y \qquad (34)$$

where σ_y is the nominal vertical (bearing) stress at the foot-soil interface. In the computer program the values used for σ_y were estimated from the static-equilibrium reactions at the three feet due to the dead weight of the tripod plus the machine gun. These vertical reactions range -- depending on the degree of leg extension -- from 27 to 29 lbs at each rear foot, and from 69.5 to 73.4 lbs at the front foot. Dividing the above values by the area of horizontal contact between the foot and the ground (area = πr_o^2 = 16.9 in^2), we obtain the following average values of σ_y that are built into the computer program as internal, invariable inputs:

Each rear foot: σ_y = 1.775 psi
Front foot: σ_y = 4.142 psi

4.4 Forcing Function

What is meant here by forcing function is the sum total of forces and moments that are applied to the tripod by the machine gun while the latter is firing. Since the machine gun is supported by the tripod at only two points, the number of these generalized interaction forces is only three -- two forces and one moment, as shown in Figure 3. One of these forces, F_T, is the elevation mechanism reaction; its direction is practically constant, no matter what the gun barrel elevation, and for all practical purposes it can be taken as vertical. The other force, F_P, is the pintle-pin reaction; the orientation of this force vector can be expected to vary during the firing cycle, so that the angle β is regarded as time-dependent. As shown in Figure 3, note that both of these two forces lie in a vertical plane (cross-hatched in the figure) that passes through the centerline of the gun barrel.

The moment M_C is applied through the pintle; its vector is always parallel to the centerline of the gun barrel. The source of this moment is the inertia forces due to feeding of the ammunition belt into the gun, whereas the source of F_P and F_T is the accelerations of bolt and barrel masses undergoing recoil and counter-recoil motions.

The forcing function is obviously time-dependent and periodic -- one period being the cycle time for one round fired. Furthermore, each one of its three components can be expected to have a very irregular shape when plotted vs. time. For this reason it was decided not to attempt to represent the forcing function by analytic curves -- a more expedient method is to specify it as four tabular functions. Presently the computer program is written so that the total number of data cards specifying the forcing function must be 50. This means that each one of the four tabular functions must be known at 50 discrete time instants during a firing cycle.* The time increments, Δt_i, here need not be equal, as shown in Figure 13; where the rate of change of the function is more abrupt, Δt_i may be chosen smaller than in areas where the function changes

*) If the computer run is intended for, say, a 3-round burst, then the forcing function in each firing cycle must be specified by 17 discrete values.

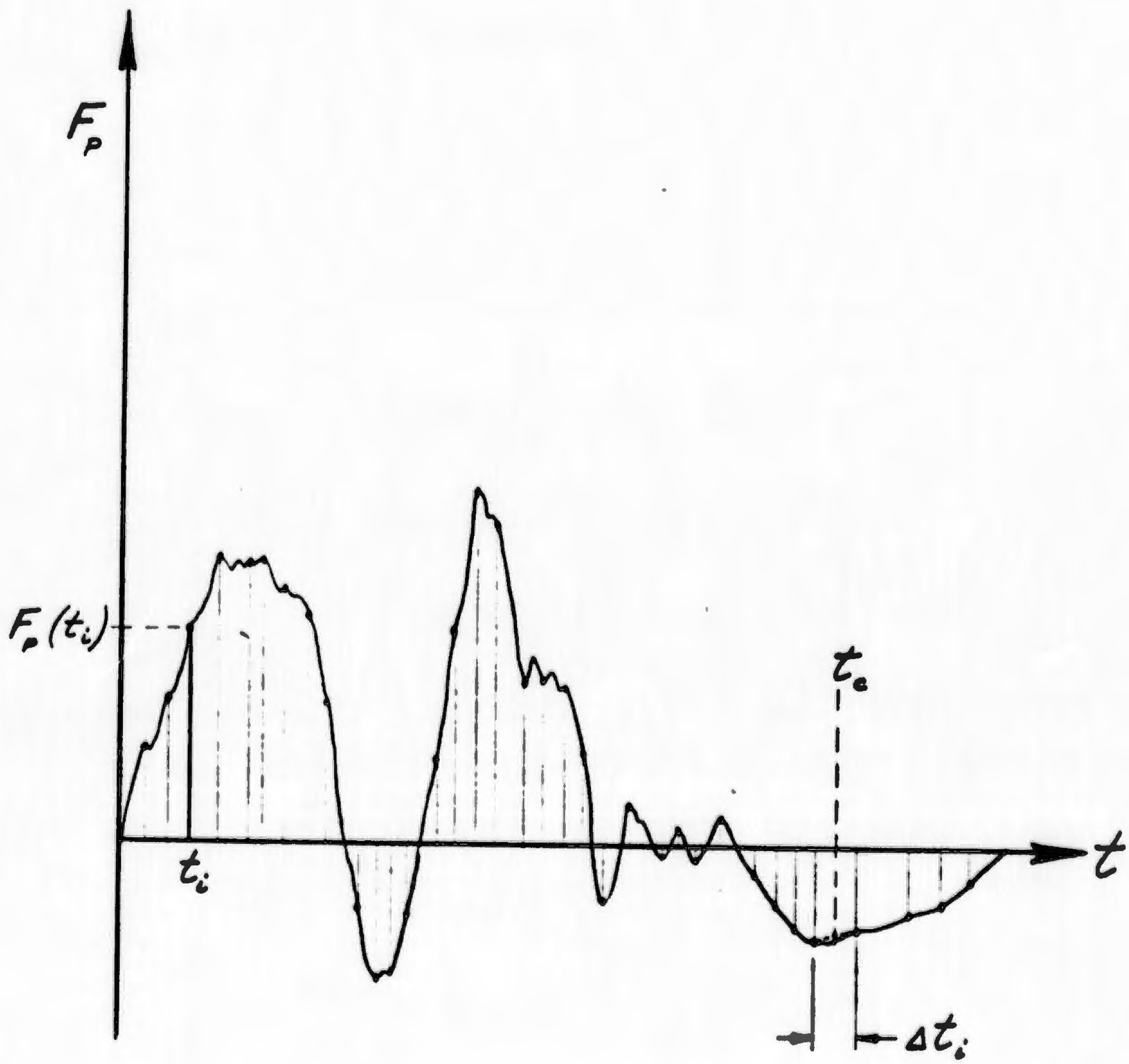


Figure 13: A COMPONENT OF THE FORCING FUNCTION REPRESENTED AS A TABULAR FUNCTION

slowly. It must be emphasized that the F_P-curve in Figure 13 is strictly illustrative -- its shape does not purport to represent an actual pintle-pin loading.

The four tabular functions that comprise the forcing function have already been identified in Section 3.2, but are repeated below in functional notation to indicate that they are specified at discrete time instants t_i ($i = 1, \ldots 50$).

1) $F_P(t_i)$ -- magnitude of the pintle-pin force,

2) $F_T(t_i)$ -- magnitude of the traverse-bar force (= force transmitted from the gun through the elevation mechanism to the traverse-bar),

3) $M_C(t_i)$ -- magnitude of the pintle moment,

4) $\beta(t_i)$ -- direction of the $F_P(t_i)$-vector.

The sign convention for positive directions of all four above variables is as shown in Figure 3.

The computer program embodies an interpolation subroutine to generate values of the forcing function components whenever the time instant for which the output is being calculated, t_c, does not coincide with the forcing function specification time, t_i. Such a case is illustrated in Figure 13 where the instant t (dashed vertical line) falls between two t_i's (solid lines).

The problem of generating the forcing function is discussed in considerable detail in Appendix A: there a procedure is outlined for conversion of the USAWECOM Machine Gun M2 computer program's output into values of F_P, F_T, M_C and β. The above machine gun program is based on a one-dimensional gun model; consequently, the gun-tripod interaction forces that form part of the program's output are parallel to the gun-barrel centerline. One of these forces is due to the motion of the bolt, the other one due to the motion of the barrel group. The lines of action of these two forces are a certain fixed distance apart and parallel. From the knowledge of this distance as well as some other dimensional data of the gun receiver, plus the printed-out values of the two interaction forces, it is possible to calculate the forcing function values for the subject computer program.

SECTION 5

METHOD OF SOLUTION

5.1 Mass and Stiffness Matrices for a Finite Element

The elements appearing in the element mass and stiffness matrices were presented in Section 4. Before proceeding in this section, it will be useful to present these matrices in assembled form, together with the element displacement and force vectors.

Let us first define the 1 x 12 displacement and force vectors.

Displacement Vector

$$\underline{q} = \begin{Bmatrix} \delta_{x1} \\ \delta_{y1} \\ \delta_{z1} \\ \theta_{x1} \\ \theta_{y1} \\ \theta_{z1} \\ \delta_{x2} \\ \delta_{y2} \\ \delta_{z2} \\ \theta_{x2} \\ \theta_{y2} \\ \theta_{z2} \end{Bmatrix} \qquad (35)$$

Force Vector

$$\underline{Q} = \begin{Bmatrix} F_{x1} \\ F_{y1} \\ F_{z1} \\ M_{x1} \\ M_{y1} \\ M_{z1} \\ F_{x2} \\ F_{y2} \\ F_{z2} \\ M_{x2} \\ M_{y2} \\ M_{z2} \end{Bmatrix} \tag{36}$$

Using equations (3) through (26), the 12 x 12 mass and stiffness matrices may now be written.

Mass Matrix

$$\underline{W} = \begin{Bmatrix} 2\alpha & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 & 0 & \delta & 0 & \lambda & 0 & 0 & 0 & -\mu \\ 0 & 0 & \beta & 0 & -\delta & 0 & 0 & 0 & \lambda & 0 & \mu & 0 \\ 0 & 0 & 0 & 2\gamma & 0 & 0 & 0 & 0 & 0 & \gamma & 0 & 0 \\ 0 & 0 & -\delta & 0 & \rho & 0 & 0 & 0 & -\mu & 0 & -\tau & 0 \\ 0 & \delta & 0 & 0 & 0 & \rho & 0 & \mu & 0 & 0 & 0 & -\tau \\ \alpha & 0 & 0 & 0 & 0 & 0 & 2\alpha & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & \mu & 0 & \beta & 0 & 0 & 0 & -\delta \\ 0 & 0 & \lambda & 0 & -\mu & 0 & 0 & 0 & \beta & 0 & \delta & 0 \\ 0 & 0 & 0 & \gamma & 0 & 0 & 0 & 0 & 0 & 2\gamma & 0 & 0 \\ 0 & 0 & \mu & 0 & -\tau & 0 & 0 & 0 & \delta & 0 & \rho & 0 \\ 0 & -\mu & 0 & 0 & 0 & -\tau & 0 & -\delta & 0 & 0 & 0 & \rho \end{Bmatrix} \tag{37}$$

where

$$\alpha = mL/6$$

$$\beta = 13mL/35$$

$$\gamma = jL/6$$

$$\delta = 11mL^2/210$$

$$\lambda = 9mL/70 \qquad (38)$$

$$\mu = 13mL^2/420$$

$$\rho = mL^3/105$$

$$\tau = mL^3/140$$

<u>Stiffness Matrix</u>

$$\underline{K} = \begin{Bmatrix} \alpha^* & 0 & 0 & 0 & 0 & 0 & -\alpha^* & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta^* & 0 & 0 & 0 & \delta^* & 0 & -\beta^* & 0 & 0 & 0 & \delta^* \\ 0 & 0 & \beta^* & 0 & -\delta^* & 0 & 0 & 0 & -\beta^* & 0 & -\delta^* & 0 \\ 0 & 0 & 0 & \gamma^* & 0 & 0 & 0 & 0 & 0 & -\gamma^* & 0 & 0 \\ 0 & 0 & -\delta^* & 0 & 2\lambda^* & 0 & 0 & 0 & \delta^* & 0 & \lambda^* & 0 \\ 0 & \delta^* & 0 & 0 & 0 & 2\lambda^* & 0 & -\delta^* & 0 & 0 & 0 & \lambda^* \\ -\alpha^* & 0 & 0 & 0 & 0 & 0 & \alpha^* & 0 & 0 & 0 & 0 & 0 \\ 0 & -\beta^* & 0 & 0 & 0 & -\delta^* & 0 & \beta^* & 0 & 0 & 0 & -\delta^* \\ 0 & 0 & -\beta^* & 0 & \delta^* & 0 & 0 & 0 & \beta^* & 0 & \delta^* & 0 \\ 0 & 0 & 0 & -\gamma^* & 0 & 0 & 0 & 0 & 0 & \gamma^* & 0 & 0 \\ 0 & 0 & -\delta^* & 0 & \lambda^* & 0 & 0 & 0 & \delta^* & 0 & 2\lambda^* & 0 \\ 0 & \delta^* & 0 & 0 & 0 & \lambda^* & 0 & -\delta^* & 0 & 0 & 0 & 2\lambda^* \end{Bmatrix} \qquad (39)$$

where

$$\alpha^* = AE/L$$

$$\beta^* = 12EI/L^3$$

$$\gamma^* = GJ/L \qquad (40)$$

$$\delta^* = 6EI/L^2$$

$$\lambda^* = 2EI/L$$

5.2 Transformation to Global Coordinates

In the solution of the tripod motions, it has proved convenient to refer all displacements, velocities, accelerations, forces and moments to a "global" system of coordinates, X-Y-Z, as shown in Figure 14. However, the element mass and stiffness matrices, presented in Section 4, have been referred to systems of local coordinates pertinent to the particular elements. These matrices must now be transformed to the global system before they are assembled into overall tripod system matrices.

Referring again to Figure 14, the local coordinate systems taken for each of the four legs are shown. In each case, the pertinent local x-axis is directed along the axis of the leg. For the front leg, leg 1, the local y-axis lies in the X-Y plane and the local z-axis is parallel to the Z-axis. The local y-axes for the two rear legs, legs 2 and 3, are normal to the plane of those legs. The local z-axes for legs 2 and 3, determined by orthogonality to the x- and y-axes, lie in the X-Z plane. For leg 4, the traverse bar, the local x-axis is parallel and opposite to the Z-axis, the local z-axis lies in the plane of legs 2 and 3, and the local y-axis is normal to the plane of legs 2 and 3.

The only configurational variables, necessary for specifying the transformations, are the angles θ_1, θ_2, and θ_3, shown in Figure 14. For convenience, another angle, θ_4, is also employed, but it is dependent upon θ_2 and θ_3.

$$\tan \theta_4 = \tan \theta_2 / \cos \theta_3 \qquad (41)$$

Consider first the transformation of a 1 x 3 vector quantity from global to local coordinates, through use of a transformation matrix, $\underline{T}$:

$$\underline{V}^{\ell} = \underline{T}\, \underline{V}^{g} \qquad (42)$$

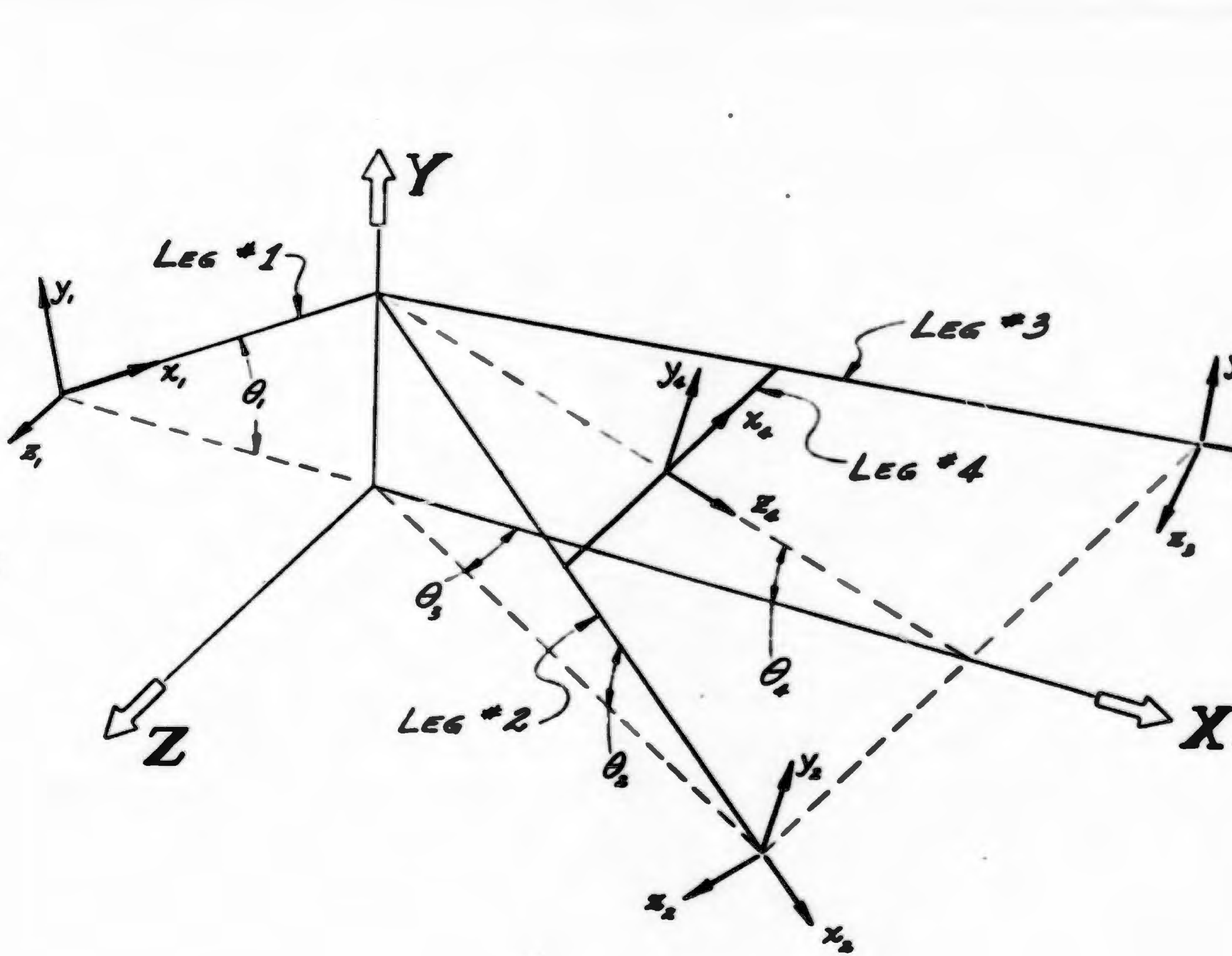


Figure 14: LOCAL COORDINATE AXES

The transformation matrices are easily derived, and are as follows:

Leg 1:

$$\dot{T}_1 = \begin{Bmatrix} \cos\theta_1 & \sin\theta_1 & 0 \\ -\sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{Bmatrix} \qquad (43a)$$

Leg 2:

$$\dot{T}_2 = \begin{Bmatrix} \cos\theta_2\cos\theta_3 & -\sin\theta_2 & \cos\theta_2\sin\theta_3 \\ \sin\theta_2\cos\theta_3 & \cos\theta_2 & \sin\theta_2\sin\theta_3 \\ -\sin\theta_3 & 0 & \cos\theta_3 \end{Bmatrix} \qquad (43b)$$

Leg 3:

$$\dot{T}_3 = \begin{Bmatrix} \cos\theta_2\cos\theta_3 & -\sin\theta_2 & -\cos\theta_2\sin\theta_3 \\ \sin\theta_2\cos\theta_3 & \cos\theta_2 & -\sin\theta_2\sin\theta_3 \\ \sin\theta_3 & 0 & \cos\theta_3 \end{Bmatrix} \qquad (43c)$$

Leg 4:

$$\dot{T}_4 = \begin{Bmatrix} 0 & 0 & -1 \\ \sin\theta_4 & \cos\theta_4 & 0 \\ \cos\theta_4 & -\sin\theta_4 & 0 \end{Bmatrix} \qquad (43d)$$

Now, the transformation of a 3-dimensional matrix quantity may be written

$$U^{\ell} = \dot{T}\, U^{g}\, \dot{T}^{T} \qquad (44)$$

where $\dot{T}^{T}$ is the transpose of $\dot{T}$, and $\dot{T}$ is the same matrix as employed in transforming the vector.

Transformation from local to global coordinates may also be written:

$$V^{g} = \dot{T}^{T}\, V^{\ell} \qquad (45)$$

$$U^{g} = \dot{T}^{T}\, U^{\ell}\, \dot{T} \qquad (46)$$

The actual vectors and matrices to be transformed in the analysis -- namely, $\underline{q}$, $\underline{Q}$, $\underline{K}$, $\underline{W}$, $\underline{C}$ -- are 12-dimensional. However, they are composed of 3-dimensional parts -- i.e., the displacement vector, $\underline{q}$, is composed of four 3-dimensional vectors, stacked one upon the other. As such, suitable 12 x 12 transformation matrices may be easily formed from the 3 x 3 matrices, as follows.

$$\underline{T} = \left[\begin{array}{cc|cc} \tilde{\underline{T}} & \underline{O}_3 & \multicolumn{2}{c}{\underline{O}_6} \\ \underline{O}_3 & \tilde{\underline{T}} & & \\ \hline \multicolumn{2}{c|}{\underline{O}_6} & \tilde{\underline{T}} & \underline{O}_3 \\ & & \underline{O}_3 & \tilde{\underline{T}} \end{array}\right] \qquad (47)$$

where $\underline{T}$ is the 12 x 12 transformation matrix, $\tilde{\underline{T}}$ is the 3 x 3 transformation matrix, $\underline{O}_3$ is a 3 x 3 null matrix and $\underline{O}_6$ is a 6 x 6 null matrix.

Transformations of $\underline{q}$, $\underline{Q}$, $\underline{W}$, $\underline{C}$ and $\underline{K}$ may now be written. As an example, for an element on leg 1:

$$\begin{aligned} \underline{q}^{g} &= \underline{T}_1^T \underline{q}^{\ell} \\ \underline{Q}^{g} &= \underline{T}_1^T \underline{Q}^{\ell} \\ \underline{W}^{g} &= \underline{T}_1^T \underline{W}^{\ell} \underline{T}_1 \\ \underline{C}^{g} &= \underline{T}_1^T \underline{C}^{\ell} \underline{T}_1 \\ \underline{K}^{g} & \quad \underline{T}_1^T \underline{K}^{\ell} \underline{T}_1 \end{aligned} \qquad (48)$$

5.3 <u>Construction of Overall System Mass and Stiffness Matrices</u>

Solution for tripod motions follows from a number of simultaneous equations. These equations are mostly force balance equations -- three force equations and three moment equations at each nodal point of the tripod. In addition, there are 24 equations expressing required displacement compatibility at the leg confluence point, and at both ends of the traverse bar.

The equations of motion for the overall system may be expressed as follows.

$$\underline{M}^* \ddot{\underline{q}}^* + \underline{C}^* \dot{\underline{q}}^* + \underline{K}^* \underline{q}^* = \underline{Q}^* \qquad (49)$$

In our system there are 19 finite elements and 23 nodal points: 3 points are at what is labeled as Nodal Point 4, and 2 points are at each end of the traverse bar. The displacement vector, $\underline{q}^*$, then has 3 displacements and 3 rotations for each nodal point -- a total of 138 elements. The force vector, $\underline{Q}^*$, has 3 forces and 3 moments at each of 19 nodal points (only one at the leg confluence point and one at each end of the traverse bar), plus 24 extra zeros to accommodate the necessary compatibility equations -- a total of 138 elements. $\underline{M}^*$, $\underline{C}^*$ and $\underline{K}^*$, of course, are 138 x 138 matrices.

5.3.1 Notation

In the following, certain notational conventions will prove convenient.

Referring to Figure 2, the finite leg elements have been numbered from 1 to 19 and nodal points have been numbered from 1 to 19. Four extra nodal points are introduced with respect to the displacements, to account for multiple-valuedness of displacements at the leg confluence point and the ends of the traverse bar -- i.e., the rotations at the leg confluence point of the end of element (3) are not identical to those of the ends of elements (4) and (7).

Displacements

At each nodal point, i, the 1 x 6 vector of displacements will be denoted by $\underline{q}_i$. At the leg confluence point -- Nodal Point 4 -- the displacements of the end of leg 1 will be denoted by $\underline{q}_{4①}$, the displacements at the end of leg 2 will be denoted $\underline{q}_{4②}$, and the displacements at the end of leg 3 will be denoted $\underline{q}_{4③}$. Similarly at nodal point 7, the displacements of leg 2 will be denoted by $\underline{q}_{7②}$ and the displacements of leg 4 by $\underline{q}_{7④}$. Also at nodal point 10, we have $\underline{q}_{10③}$ and $\underline{q}_{10④}$.

Each of the 1 x 6 displacement vectors is referred to the global coordinate system and arranged as follows:

$$\begin{Bmatrix} \delta_x \\ \delta_y \\ \delta_z \\ \theta_x \\ \theta_y \\ \theta_z \end{Bmatrix}$$

Forces

The 1 x 6 vector of forces and moments at each nodal point, i, will be denoted by $\underline{Q}_i$. This vector will include only externally applied forces and moments, thus requiring no multiple vectors at the pintle and traverse bar ends. Each of these force vectors shall be referred to global coordinates and arranged as

$$\begin{Bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{Bmatrix}$$

Element Mass and Stiffness Matrices

The 12 x 12 mass and stiffness matrices, presented in equations (37) and (39), are defined for each finite element. For the present discussion, we will deal with these matrices after they have been transformed to global coordinates through the appropriate transformations. Consider, for instance, the mass matrix for element i. It will be broken up into four 6 x 6 matrices as indicated in the following.

$$\underline{W}_i = \left\{\begin{array}{c|c} \underline{W}_{iI} & \underline{W}_{iII} \\ \hline \underline{W}_{iIII} & \underline{W}_{iIV} \end{array}\right\} \qquad (50)$$

Similarly, the stiffness matrix is decomposed into four 6 x 6 "quadrant" matrices for each element, i:

$$\underline{K}_i = \left\{\begin{array}{c|c} \underline{K}_{iI} & \underline{K}_{iII} \\ \hline \underline{K}_{iIII} & \underline{K}_{iIV} \end{array}\right\} \qquad (51)$$

5.3.2 Formation of $\underline{q}^*$ and $\underline{Q}^*$ Vectors

The overall displacement vector $\underline{q}^*$ has 138 elements, which have been divided into 23 groups of 6 elements apiece. Vector $\underline{q}^*$ is formed by stacking the 23 groups, one upon the other. The following order has been chosen, starting from the top:

$\underline{q}_1, \underline{q}_2, \underline{q}_3, \underline{q}_4①, \underline{q}_4②, \underline{q}_5, \underline{q}_6, \underline{q}_7②, \underline{q}_4③, \underline{q}_8, \underline{q}_9,$
$\underline{q}_{10}③, \underline{q}_7④, \underline{q}_{11}, \underline{q}_{12}, \underline{q}_{13}, \underline{q}_{10}④, \underline{q}_{14}, \underline{q}_{15}, \underline{q}_{16},$
$\underline{q}_{17}, \underline{q}_{18}, \underline{q}_{19}.$

Similarly, the $\underline{Q}^*$ vector has 138 elements which have been divided in 19 groups of 6 elements, together with 4 additional groups of 6 zeros (to accommodate the compatibility equations). Ordering, from the top, is as follows:

$\underline{Q}_1, \underline{Q}_2, \underline{Q}_3, \underline{0}_6, \underline{0}_6, \underline{Q}_5, \underline{Q}_6, \underline{Q}_7, \underline{Q}_4, \underline{Q}_8, \underline{Q}_9,$
$\underline{Q}_{10}, \underline{0}_6, \underline{Q}_{11}, \underline{Q}_{12}, \underline{Q}_{13}, \underline{0}_6, \underline{Q}_{14}, \underline{Q}_{15}, \underline{Q}_{16},$
$\underline{Q}_{17}, \underline{Q}_{18}, \underline{Q}_{19}.$

where $\underline{0}_6$ is a group of six zeros.

5.3.3 Formation of $\underline{W}^*$ and $\underline{K}^*$ Matrices

The $\underline{W}^*$ and $\underline{K}^*$ matrices are 138 x 138; they may be further divided into 529 cells, each cell being a 6 x 6 matrix. First, the various 6 x 6 "quadrant" matrices of $\underline{K}_1$ and $\underline{W}_1$ are entered into these cells as indicated in Table I.

TABLE I

Cell (row, column)	$\underline{W}^*$	$\underline{K}^*$
1, 1	$\underline{W}_{1I}$	$\underline{K}_{1I}$
1, 2	$\underline{W}_{1II}$	$\underline{K}_{1II}$
2, 1	$\underline{W}_{1III}$	$\underline{K}_{1III}$
2, 2	$\underline{W}_{1IV} + \underline{W}_{2I}$	$\underline{K}_{1IV} + \underline{K}_{2I}$
2, 3	$\underline{W}_{2II}$	$\underline{K}_{2II}$

TABLE I (Continued)

Cell (row, column)	$\underline{W}^*$	$\underline{K}^*$
3, 2	$\underline{W}_{2III}$	$\underline{K}_{2III}$
3, 3	$\underline{W}_{2IV} + \underline{W}_{3I}$	$\underline{K}_{2IV} + \underline{K}_{3I}$
3, 4	$\underline{W}_{3II}$	$\underline{K}_{3II}$
6, 5	$\underline{W}_{4III}$	$\underline{K}_{4III}$
6, 6	$\underline{W}_{4IV} + \underline{W}_{5I}$	$\underline{K}_{4IV} + \underline{K}_{5I}$
6, 7	$\underline{W}_{5II}$	$\underline{K}_{5II}$
7, 6	$\underline{W}_{5III}$	$\underline{K}_{5III}$
7, 7	$\underline{W}_{5IV} + \underline{W}_{6I}$	$\underline{K}_{5IV} + \underline{K}_{6I}$
7, 8	$\underline{W}_{6II}$	$\underline{K}_{6II}$
8, 7	$\underline{W}_{6III}$	$\underline{K}_{6III}$
8, 8	$\underline{W}_{6IV} + \underline{W}_{14I}$	$\underline{K}_{6IV} + \underline{K}_{14I}$
8, 13	$\underline{W}_{10I}$	$\underline{K}_{10I}$
8, 14	$\underline{W}_{10II}$	$\underline{K}_{10II}$
8, 18	$\underline{W}_{14II}$	$\underline{K}_{14II}$
9, 3	$\underline{W}_{3III}$	$\underline{K}_{3III}$
9, 4	$\underline{W}_{3IV}$	$\underline{K}_{3IV}$
9, 5	$\underline{W}_{4I}$	$\underline{K}_{4I}$
9, 6	$\underline{W}_{4II}$	$\underline{K}_{4II}$
9, 9	$\underline{W}_{7I}$	$\underline{K}_{7I}$
9, 10	$\underline{W}_{7II}$	$\underline{K}_{7II}$
10, 9	$\underline{W}_{7III}$	$\underline{K}_{7III}$
10, 10	$\underline{W}_{7IV} + \underline{W}_{8I}$	$\underline{K}_{7IV} + \underline{K}_{8I}$

TABLE I (Continued)

Cell (row, column)	$\underline{W}^*$	$\underline{K}^*$
10, 11	$\underline{W}_{8II}$	$\underline{K}_{8II}$
11, 10	$\underline{W}_{8III}$	$\underline{K}_{8III}$
11, 11	$\underline{W}_{8IV} + \underline{W}_{9I}$	$\underline{K}_{8IV} + \underline{K}_{9I}$
11, 12	$\underline{W}_{9II}$	$\underline{K}_{9II}$
12, 11	$\underline{W}_{9III}$	$\underline{K}_{9III}$
12, 12	$\underline{W}_{9IV} + \underline{W}_{17I}$	$\underline{K}_{9IV} + \underline{K}_{17I}$
12, 16	$\underline{W}_{13III}$	$\underline{K}_{13III}$
12, 17	$\underline{W}_{13IV}$	$\underline{K}_{13IV}$
12, 21	$\underline{W}_{17II}$	$\underline{K}_{17II}$
14, 13	$\underline{W}_{10III}$	$\underline{K}_{10III}$
14, 14	$\underline{W}_{10IV} + \underline{W}_{11I}$	$\underline{K}_{10IV} + \underline{K}_{11I}$
14, 15	$\underline{W}_{11II}$	$\underline{K}_{11II}$
15, 14	$\underline{W}_{11III}$	$\underline{K}_{11III}$
15, 15	$\underline{W}_{11IV} + \underline{W}_{12I}$	$\underline{K}_{11IV} + \underline{K}_{12I}$
15, 16	$\underline{W}_{12II}$	$\underline{K}_{12II}$
16, 15	$\underline{W}_{12III}$	$\underline{K}_{12III}$
16, 16	$\underline{W}_{12IV} + \underline{W}_{13I}$	$\underline{K}_{12IV} + \underline{K}_{13I}$
16, 17	$\underline{W}_{13II}$	$\underline{K}_{13II}$
18, 8	$\underline{W}_{14III}$	$\underline{K}_{14III}$
18, 18	$\underline{W}_{14IV} + \underline{W}_{15I}$	$\underline{K}_{14IV} + \underline{K}_{15I}$
18, 19	$\underline{W}_{15II}$	$\underline{K}_{15II}$
19, 18	$\underline{W}_{15III}$	$\underline{K}_{15III}$

TABLE I (Continued)

Cell (row, column)	$\underline{W}^*$	$\underline{K}^*$
19, 19	$\underline{W}_{15IV} + \underline{W}_{16I}$	$\underline{K}_{15IV} + \underline{K}_{16I}$
19, 20	$\underline{W}_{16II}$	$\underline{K}_{16II}$
20, 19	$\underline{W}_{16III}$	$\underline{K}_{16III}$
20, 20	$\underline{W}_{16IV}$	$\underline{K}_{16IV}$
21, 12	$\underline{W}_{17III}$	$\underline{K}_{17III}$
21, 21	$\underline{W}_{17IV} + \underline{W}_{18I}$	$\underline{K}_{17IV} + \underline{K}_{18I}$
21, 22	$\underline{W}_{18II}$	$\underline{K}_{18II}$
22, 21	$\underline{W}_{18III}$	$\underline{K}_{18III}$
22, 22	$\underline{W}_{18IV} + \underline{W}_{19I}$	$\underline{K}_{18IV} + \underline{K}_{19I}$
22, 23	$\underline{W}_{19II}$	$\underline{K}_{19II}$
23, 22	$\underline{W}_{19III}$	$\underline{K}_{19III}$
23, 23	$\underline{W}_{19IV}$	$\underline{K}_{19IV}$

The next items to be entered into the $\underline{W}^*$ matrix will be the compatibility conditions at the leg confluence point and at the ends of the traverse bar.

<u>Leg Confluence Point</u>

At Nodal Point 4 the translational displacements referred to legs 1, 2 and 3 must be identical, as must the rotational displacements about the Z-axis. However, legs 2 and 3 are free to rotate about local axes y_2 and y_3, respectively (see Figure 14), while their respective rotations about local axes, x_2 and x_3, respectively, are constrained to be equal to those of leg 1. The free rotation conditions imply moment conditions on legs 2 and 3, which can in turn be expressed in terms of the displacements in $\underline{q}_4^{(2)}$, $\underline{q}_4^{(3)}$, $\underline{q}_5$ and $\underline{q}_6$. After some simplification, two sets of six compatibility equations may be written (referred to global coordinates):

<u>**Set A:**</u>

$$(\delta_x)_4 \textcircled{3} = (\delta_x)_4 \textcircled{1}$$

$$(\delta_y)_4 \textcircled{3} = (\delta_y)_4 \textcircled{1}$$

$$(\delta_z)_4 \textcircled{3} = (\delta_z)_4 \textcircled{1}$$

$$
\begin{aligned}
(\theta_x)_4 \textcircled{3} = \; & [(\delta_x)_4 \textcircled{1} \left(\frac{3}{2L} \sin\theta_3 \sin\theta_4\right) + (\delta_z)_4 \textcircled{1} \left(\frac{3}{2L} \cos\theta_3 \sin\theta_4\right) \\
& + (\theta_x)_4 \textcircled{1} (\cos\theta_2 \cos\theta_4) - (\theta_y)_4 \textcircled{1} (\cos\theta_2 \sin\theta_4) \\
& + (\theta_z)_4 \textcircled{1} (\sin\theta_2 \sin\theta_3 \sin\theta_4) - (\delta_x)_8 \left(\frac{3}{2L} \sin\theta_3 \sin\theta_4\right) \\
& - (\delta_z)_8 \left(\frac{3}{2L} \cos\theta_3 \sin\theta_4\right) - (\theta_x)_8 \left(\frac{1}{2} \sin\theta_2 \cos\theta_3 \sin\theta_4\right) \\
& - (\theta_y)_8 \left(\frac{1}{2} \cos\theta_2 \sin\theta_4\right) + (\theta_z)_8 \left(\frac{1}{2} \sin\theta_2 \sin\theta_3 \sin\theta_4\right)] \\
& / [\cos\theta_2 \cos\theta_4 + \sin\theta_2 \cos\theta_3 \sin\theta_4] \qquad (52)
\end{aligned}
$$

$$
\begin{aligned}
(\theta_y)_4 \textcircled{3} = \; & [(\delta_x)_4 \textcircled{1} \left(\frac{3}{2L} \sin\theta_3 \cos\theta_4\right) + (\delta_z)_4 \textcircled{1} \left(\frac{3}{2L} \cos\theta_3 \cos\theta_4\right) \\
& - (\theta_x)_4 \textcircled{1} (\sin\theta_2 \cos\theta_3 \cos\theta_4) + (\theta_y)_4 \textcircled{1} (\sin\theta_2 \cos\theta_3 \sin\theta_4) \\
& + (\theta_z)_4 \textcircled{1} (\sin\theta_2 \cos\theta_3 \cos\theta_4) - (\delta_x)_8 \left(\frac{3}{2L} \sin\theta_3 \cos\theta_4\right) \\
& - (\delta_z)_8 \left(\frac{3}{2L} \cos\theta_3 \cos\theta_4\right) - (\theta_x)_8 \left(\frac{1}{2} \sin\theta_2 \cos\theta_3 \cos\theta_4\right) \\
& - (\theta_y)_8 \left(\frac{1}{2} \cos\theta_2 \cos\theta_4\right) + (\theta_z)_8 \left(\frac{1}{2} \sin\theta_2 \sin\theta_3 \cos\theta_4\right)] \\
& / [\cos\theta_2 \cos\theta_4 + \sin\theta_2 \cos\theta_3 \sin\theta_4]
\end{aligned}
$$

$$(\theta_z)_4 \textcircled{3} = (\theta_z)_4 \textcircled{1}$$

where L refers to element 7.

Set B:

$$(\delta_x)_4 ② = (\delta_x)_4 ①$$

$$(\delta_y)_4 ② = (\delta_y)_4 ①$$

$$(\delta_z)_4 ② = (\delta_z)_4 ①$$

$$(\theta_x)_4 ② = [-(\delta_x)_4 ① (\tfrac{3}{2L} \sin\theta_3 \sin\theta_4) + (\delta_z)_4 ① (\tfrac{3}{2L} \cos\theta_3 \sin\theta_4)$$
$$+ (\theta_x)_4 ① (\cos\theta_2 \cos\theta_4) - (\theta_y)_4 ① (\cos\theta_2 \sin\theta_4)$$
$$- (\theta_z)_4 ① (\sin\theta_2 \sin\theta_3 \sin\theta_4) + (\delta_x)_5 (\tfrac{3}{2L} \sin\theta_3 \sin\theta_4)$$
$$- (\delta_z)_5 (\tfrac{3}{2L} \cos\theta_3 \sin\theta_4) - (\theta_x)_5 (\tfrac{1}{2} \sin\theta_2 \cos\theta_3 \sin\theta_4)$$
$$- (\theta_y)_5 (\tfrac{1}{2} \cos\theta_2 \sin\theta_4) - (\theta_z)_5 (\tfrac{1}{2} \sin\theta_2 \sin\theta_3 \sin\theta_4)]$$
$$/ [\cos\theta_2 \cos\theta_4 + \sin\theta_2 \cos\theta_3 \sin\theta_4] \qquad (53)$$

$$(\theta_y)_4 ② = [-(\delta_x)_4 ① (\tfrac{3}{2L} \sin\theta_3 \cos\theta_4) + (\delta_z)_4 ① (\tfrac{3}{2L} \cos\theta_3 \cos\theta_4)$$
$$- (\theta_x)_4 ① (\sin\theta_2 \cos\theta_3 \cos\theta_4) + (\theta_y)_4 ① (\sin\theta_2 \cos\theta_3 \sin\theta_4)$$
$$- (\theta_z)_4 ① (\sin\theta_2 \cos\theta_3 \cos\theta_4) + (\delta_x)_5 (\tfrac{3}{2L} \sin\theta_3 \cos\theta_4)$$
$$- (\delta_z)_5 (\tfrac{3}{2L} \cos\theta_3 \cos\theta_4) - (\theta_x)_5 (\tfrac{1}{2} \sin\theta_2 \cos\theta_3 \cos\theta_4)$$
$$- (\theta_y)_5 (\tfrac{1}{2} \cos\theta_2 \cos\theta_4) - (\theta_z)_5 (\tfrac{1}{2} \sin\theta_2 \sin\theta_3 \cos\theta_4)]$$
$$/ [\cos\theta_2 \cos\theta_4 + \sin\theta_2 \cos\theta_3 \sin\theta_4]$$

$$(\theta_z)_4 ② = (\theta_z)_4 ①$$

where L refers to element 4.

Ends of Traverse Bar

At each end of the traverse bar (leg 4), the translational displacements of the traverse bar are identical to those of the leg to which it is joined -- i.e., leg 2 at one end, leg 3 at the other. Similarly the rotations about the Z-axis must be equal, as must the rotations about the x_4-axis (see Figure 14). On the other hand, the ends of the traverse bar are free to rotate about the y_4-axis. A set of six compatibility equations may be derived at each end, in much the same way as those at the pintle were derived.

Set C:

$$(\delta_x)_{7 \textcircled{4}} = (\delta_x)_{7 \textcircled{2}}$$

$$(\delta_y)_{7 \textcircled{4}} = (\delta_y)_{7 \textcircled{2}}$$

$$(\delta_z)_{7 \textcircled{4}} = (\delta_z)_{7 \textcircled{2}}$$

$$\begin{aligned}(\theta_x)_{7 \textcircled{4}} = {} & (\delta_x)_{7 \textcircled{2}} \left(\frac{3}{2L} \sin \theta_4 \cos \theta_4\right) - (\delta_y)_{7 \textcircled{2}} \left(\frac{3}{2L} \sin^2 \theta_4\right) \\ & + (\theta_x)_{7 \textcircled{2}} (\cos^2 \theta_4) - (\theta_y)_{7 \textcircled{2}} (\sin \theta_4 \cos \theta_4) \\ & - (\delta_x)_{11} \left(\frac{3}{2L} \sin \theta_4 \cos \theta_4\right) + (\delta_y)_{11} \left(\frac{3}{2L} \sin^2 \theta_4\right) \\ & - (\theta_x)_{11} \left(\frac{1}{2} \sin^2 \theta_4\right) - (\theta_y)_{11} \left(\frac{1}{2} \sin \theta_4 \cos \theta_4\right) \end{aligned} \qquad (54)$$

$$\begin{aligned}(\theta_y)_{7 \textcircled{4}} = {} & (\delta_x)_{7 \textcircled{2}} \left(\frac{3}{2L} \cos^2 \theta_4\right) - (\delta_y)_{7 \textcircled{2}} \left(\frac{3}{2L} \sin \theta_4 \cos \theta_4\right) \\ & - (\theta_x)_{7 \textcircled{2}} (\sin \theta_4 \cos \theta_4) + (\theta_y)_{7 \textcircled{2}} (\sin^2 \theta_4) \\ & - (\delta_x)_{11} \left(\frac{3}{2L} \cos^2 \theta_4\right) + (\delta_y)_{11} \left(\frac{3}{2L} \sin \theta_4 \cos \theta_4\right) \\ & - (\theta_x)_{11} \left(\frac{1}{2} \sin \theta_4 \cos \theta_4\right) - (\theta_y)_{11} \left(\frac{1}{2} \cos^2 \theta_4\right) \end{aligned}$$

$$(\theta_z)_{7 \textcircled{4}} = (\theta_z)_{7 \textcircled{2}}$$

where L refers to element 10.

Set D:

$$(\delta_x)_{10\textcircled{4}} = (\delta_y)_{10\textcircled{3}}$$

$$(\delta_y)_{10\textcircled{4}} = (\delta_y)_{10\textcircled{3}}$$

$$(\delta_z)_{10\textcircled{4}} = (\delta_z)_{10\textcircled{3}}$$

$$\begin{aligned}
(\theta_x)_{10\textcircled{4}} = & - (\delta_x)_{10\textcircled{3}}\left(\frac{3}{2L} \sin \theta_4 \cos \theta_4\right) + (\delta_y)_{10\textcircled{3}}\left(\frac{3}{2L} \sin^2 \theta_4\right) \\
& + (\theta_x)_{10\textcircled{3}}(\cos^2 \theta_4) - (\theta_y)_{10\textcircled{3}}(\sin \theta_4 \cos \theta_4) \\
& + (\delta_x)_{13} \left(\frac{3}{2L} \sin \theta_4 \cos \theta_4\right) - (\delta_y)_{13} \left(\frac{3}{2L} \sin^2 \theta_4\right) \\
& - (\theta_x)_{13} \left(\frac{1}{2} \sin^2 \theta_4\right) - (\theta_y)_{13} \left(\frac{1}{2} \sin \theta_4 \cos \theta_4\right) \qquad (55) \\
(\theta_y)_{10\textcircled{4}} = & -(\delta_x)_{10\textcircled{3}}\left(\frac{3}{2L} \cos^2 \theta_4\right) + (\delta_y)_{10\textcircled{3}}\left(\frac{3}{2L} \sin \theta_4 \cos \theta_4\right) \\
& - (\theta_x)_{10\textcircled{3}}(\sin \theta_4 \cos \theta_4) + (\theta_y)_{10\textcircled{3}}(\sin^2 \theta_4) \\
& + (\delta_x)_{13} \left(\frac{3}{2L} \cos^2 \theta_4\right) - (\delta_y)_{13} \left(\frac{3}{2L} \sin \theta_4 \cos \theta_4\right) \\
& - (\theta_x)_{13} \left(\frac{1}{2} \sin \theta_4 \cos \theta_4\right) - (\theta_y)_{13} \left(\frac{1}{2} \cos^2 \theta_4\right)
\end{aligned}$$

$$(\theta_x)_{10\textcircled{4}} = (\theta_z)_{10\textcircled{3}}$$

where L refers to element 13.

It is computationally more convenient when the compatibility equations can be expressed in terms of the second time derivatives of the translations and rotations. This differentiation is performed immediately by simply replacing all translations and rotations by their second time derivatives. The resulting equations then give rise to the following 6 x 6 matrices which must be entered into the $\underline{W}^*$ matrix.

$$\mathcal{A}_1 = \begin{Bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{Bmatrix} \tag{56}$$

$$\mathcal{A}_2 = \begin{Bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{3}{2L} S_3S_4/\bar{\alpha} & 0 & \frac{3}{2L} C_3S_4/\bar{\alpha} & C_2C_4/\bar{\alpha} & -C_2S_4/\bar{\alpha} & S_2S_3S_4/\bar{\alpha} \\ \frac{3}{2L} S_3C_4/\bar{\alpha} & 0 & \frac{3}{2L} C_3C_4/\bar{\alpha} & -S_2C_3C_4/\bar{\alpha} & S_2C_3S_4/\bar{\alpha} & S_2C_3C_4/\bar{\alpha} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{Bmatrix} \tag{57}$$

where

$$\begin{aligned} C_2 &= \cos\theta_2 & S_2 &= \sin\theta_2 \\ C_3 &= \cos\theta_3 & S_3 &= \sin\theta_3 \\ C_4 &= \cos\theta_4 & S_4 &= \sin\theta_4 \end{aligned}$$
$$\bar{\alpha} = C_2C_4 + S_2C_3S_4 \tag{58}$$

$$\mathcal{A}_3 = \begin{Bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{3}{2L} S_3S_4/\bar{\alpha} & 0 & -\frac{3}{2L} C_3S_4/\bar{\alpha} & -\frac{1}{2} S_2C_3S_4/\bar{\alpha} & -\frac{1}{2}C_2S_4/\bar{\alpha} & \frac{1}{2}S_2S_3S_4/\bar{\alpha} \\ -\frac{3}{2L} S_3C_4/\bar{\alpha} & 0 & -\frac{3}{2L} C_3C_4/\bar{\alpha} & -\frac{1}{2} S_2C_3C_4/\bar{\alpha} & -\frac{1}{2}C_2C_4/\bar{\alpha} & \frac{1}{2}S_2S_3C_4/\bar{\alpha} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{Bmatrix} \tag{59}$$

$$\Phi_4 = \Phi_1 \tag{60}$$

$$\Phi_5 = \begin{Bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -\frac{3}{2L}S_3S_4/\bar{\alpha} & 0 & \frac{3}{2L}C_3S_4/\bar{\alpha} & C_2C_4/\bar{\alpha} & -C_2S_4/\bar{\alpha} & -S_2S_3S_4/\bar{\alpha} \\ -\frac{3}{2L}S_3C_4/\bar{\alpha} & 0 & \frac{3}{2L}C_3C_4/\bar{\alpha} & -S_2C_3C_4/\bar{\alpha} & S_2C_3S_4/\bar{\alpha} & -S_2C_3C_4/\bar{\alpha} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{Bmatrix} \tag{61}$$

$$\Phi_6 = \begin{Bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{3}{2L}S_3S_4/\bar{\alpha} & 0 & -\frac{3}{2L}C_3S_4/\bar{\alpha} & -\frac{1}{2}S_2C_3S_4/\bar{\alpha} & -\frac{1}{2}C_2S_4/\bar{\alpha} & -\frac{1}{2}S_2S_3S_4/\bar{\alpha} \\ \frac{3}{2L}S_3C_4/\bar{\alpha} & 0 & -\frac{3}{2L}C_3C_4/\bar{\alpha} & -\frac{1}{2}S_2C_3C_4/\bar{\alpha} & -\frac{1}{2}C_2C_4/\bar{\alpha} & -\frac{1}{2}S_2S_3C_4/\bar{\alpha} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{Bmatrix} \tag{62}$$

$$\Phi_7 = \Phi_1 \tag{63}$$

$$\Phi_8 = \begin{Bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{3}{2L}S_4C_4 & -\frac{3}{2L}S_4^2 & 0 & C_4^2 & -S_4C_4 & 0 \\ \frac{3}{2L}C_4^2 & -\frac{3}{2L}S_4C_4 & 0 & -S_4C_4 & S_4^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{Bmatrix} \tag{64}$$

$$\underline{a}_9 = \left\{ \begin{array}{cccccc} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{3}{2L} s_4 c_4 & \frac{3}{2L} s_4^2 & 0 & -\frac{1}{2} s_4^2 & -\frac{1}{2} s_4 c_4 & 0 \\ -\frac{3}{2L} c_4^2 & \frac{3}{2L} s_4 c_4 & 0 & -\frac{1}{2} s_4 c_4 & -\frac{1}{2} c_4^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right\} \quad (65)$$

$$\underline{a}_{10} = \underline{a}_1 \quad (66)$$

$$\underline{a}_{11} = \left\{ \begin{array}{cccccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -\frac{3}{2L} s_4 c_4 & \frac{3}{2L} s_4^2 & 0 & c_4^2 & -s_4 c_4 & 0 \\ -\frac{3}{2L} c_4^2 & \frac{3}{2L} s_4 c_4 & 0 & -s_4 c_4 & s_4^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right\} \quad (67)$$

$$\underline{a}_{12} = \left\{ \begin{array}{cccccc} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{3}{2L} s_4 c_4 & -\frac{3}{2L} s_4^2 & 0 & -\frac{1}{2} s_4^2 & -\frac{1}{2} s_4 c_4 & 0 \\ \frac{3}{2L} c_4^2 & -\frac{3}{2L} s_4 c_4 & 0 & -\frac{1}{2} s_4 c_4 & -\frac{1}{2} c_4^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right\} \quad (68)$$

It is to be noted here that coefficients used in these matrices shall refer to L values of particular elements as follows:

$\underline{a}_2$, $\underline{a}_3$	element 7
$\underline{a}_5$, $\underline{a}_6$	element 4
$\underline{a}_8$, $\underline{a}_9$	element 10
$\underline{a}_{11}$, $\underline{a}_{12}$	element 13

Placement of $\underline{a}_1$ through $\underline{a}_{12}$ in $\underline{W}^*$ shall be as indicated in Table 2.

TABLE 2

Cell (row, column)	Matrix
4, 4	$\underline{a}_2$
4, 9	$\underline{a}_1$
4, 10	$\underline{a}_3$
5, 4	$\underline{a}_5$
5, 5	$\underline{a}_4$
5, 6	$\underline{a}_6$
13, 8	$\underline{a}_8$
13, 13	$\underline{a}_7$
13, 14	$\underline{a}_9$
17, 12	$\underline{a}_{11}$
17, 16	$\underline{a}_{12}$
17, 17	$\underline{a}_{10}$

Soil stiffness was considered in Section 4, and must now be entered into the K^* matrix From equations (29) through (32), a 6 x 6 soil-stiffness matrix, $\underline{a}$, is constructed:

$$\underline{a} = \begin{Bmatrix} K_x & 0 & 0 & 0 & 0 & 0 \\ 0 & K_y & 0 & 0 & 0 & 0 \\ 0 & 0 & K_z & 0 & 0 & 0 \\ 0 & 0 & 0 & K_\psi & 0 & 0 \\ 0 & 0 & 0 & 0 & K_\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & K_\psi \end{Bmatrix} \qquad (69)$$

$\underline{a}$ is then added into the $\underline{K}^*$ matrix at the following cell locations:

<u>Cell (row, column)</u>

1, 1

20, 20

23, 23

Finally, several 6 x 6 matrices are added into the $\underline{M}^*$ matrix to account for lumped masses in the tripod (i.e., those lumped masses which do not contribute to the tripod stiffness) and the gun mass. Particular tripod masses to be considered here are located at the pintle, the ends of the traverse bar, and the tripod feet (ends of legs 1, 2, 3). The gun mass was taken to be distributed between the pintle and a point along the traverse bar, the location of this point being that where the elevation mechanism is in contact with the traverse bar.

Each of the lumped tripod masses were expressed as a 6 x 6 matrix $\underline{i}$ of the form

$$\underline{i} = \begin{Bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & J_X & 0 & 0 \\ 0 & 0 & 0 & 0 & J_Y & 0 \\ 0 & 0 & 0 & 0 & 0 & J_Z \end{Bmatrix} \qquad (70)$$

where m is the lumped mass, and J_x, J_y and J_z are mass moments of inertia of the lumped mass, referred to global coordinates. Cross moments of inertia were neglected. These 6 x 6 $\underline{L}$ matrices were added into the $\underline{W}^*$ matrix at the following 6 cells.

Cell (row, column)	$\underline{L}$ based upon lumped mass	
1, 1	foot on leg 1	($\underline{L}_1$)
8, 8	mass at node 7	($\underline{L}_7$)
9, 4	pintle	($\underline{L}_4$)
12, 12	mass at node 10	($\underline{L}_{10}$)
20, 20	foot on leg 2	($\underline{L}_{16}$)
23, 23	foot on leg 3	($\underline{L}_{19}$)

The machine gun mass, distributed between the pintle and a point on the traverse bar, was assumed to have translational inertia only, so that the necessary 6 x 6 matrices were of the form

$$\underline{L} = \begin{Bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{Bmatrix} \qquad (71)$$

where m is the mass taken at the particular location. Distribution between the two locations was determined by static equilibrium reactions of the gun. The $\underline{L}$ matrix corresponding to gun mass taken at the pintle, $\underline{L}_4$, was added into the $\underline{W}^*$ matrix at cell

9, 4

The $\underline{L}$ matrix arising from gun mass taken at the traverse bar, $\underline{L}_n$, was added into the $\underline{W}^*$ matrix at the cell corresponding to the node at which the elevating mechanism contacts the traverse bar:

14, 14 if contact is at node 11

15, 15 if contact is at node 12

16, 16 if contact is at node 13

5.3.4 **Presentation of $\underline{M}^*$, $\underline{K}^*$, $\underline{Q}^*$ and $\underline{q}^*$**

The developed form of the $\underline{M}^*$ and $\underline{K}^*$ matrices is shown in Figures 15 and 16, respectively. These figures show the 138 x 138 matrices, and divided into 529 6 x 6 cells, and the 6 x 6 matrices entered into each cell are indicated there. In the $\underline{M}^*$ matrix, the entries of $\underline{L}$ are shown in parentheses, since only one cell will contain that matrix, as described in 5.3.3.

Prior to computation, however, a modification was made to the $\underline{M}^*$ and $\underline{K}^*$ matrices, to help reduce computational error. In the ninth row of cells, the cells in columns 5 and 9 were eliminated through use of equation sets (52) and (53), respectively. This elimination of course yielded additions to columns 4, 6 and 10 of that row.

Similarly, equation set (54) was used to eliminate column 13 in row 8, yielding additions to columns 8 and 14 of that row. Also, equation set (55) was applied to row 12 to eliminate column 17, with additions going to columns 12 and 16.

The form of vectors $\underline{Q}^*$ and $\underline{q}^*$, divided into 1 x 6 cells is shown in Figure 17, following from the discussion in 5.3.2.

5.4 The Damping Matrix

As noted in Section 4, structural damping is neglected in our analysis, so that the only damping considered is that arising at the positions of support on the soil. The $\underline{C}^*$ matrix, mentioned in Section 5.3, represents the damping to the system. This matrix, like $\underline{M}^*$ and $\underline{K}^*$, is 138 x 138 and may be divided into 529 6 x 6 cells. The only cells which are not identically zero are those at the feet of legs 1, 2 and 3.

Damping constants at the soil-foot connection were presented in Section 4 -- equations (29) through (32). At each foot, then, the 6 x 6 damping matrix is written (referred to global coordinates)

$$\underline{c} = \begin{Bmatrix} c_x & 0 & 0 & 0 & 0 & 0 \\ 0 & c_y & 0 & 0 & 0 & 0 \\ 0 & 0 & c_z & 0 & 0 & 0 \\ 0 & 0 & 0 & c_\psi & 0 & 0 \\ 0 & 0 & 0 & 0 & c_\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & c_\psi \end{Bmatrix} \qquad (72)$$

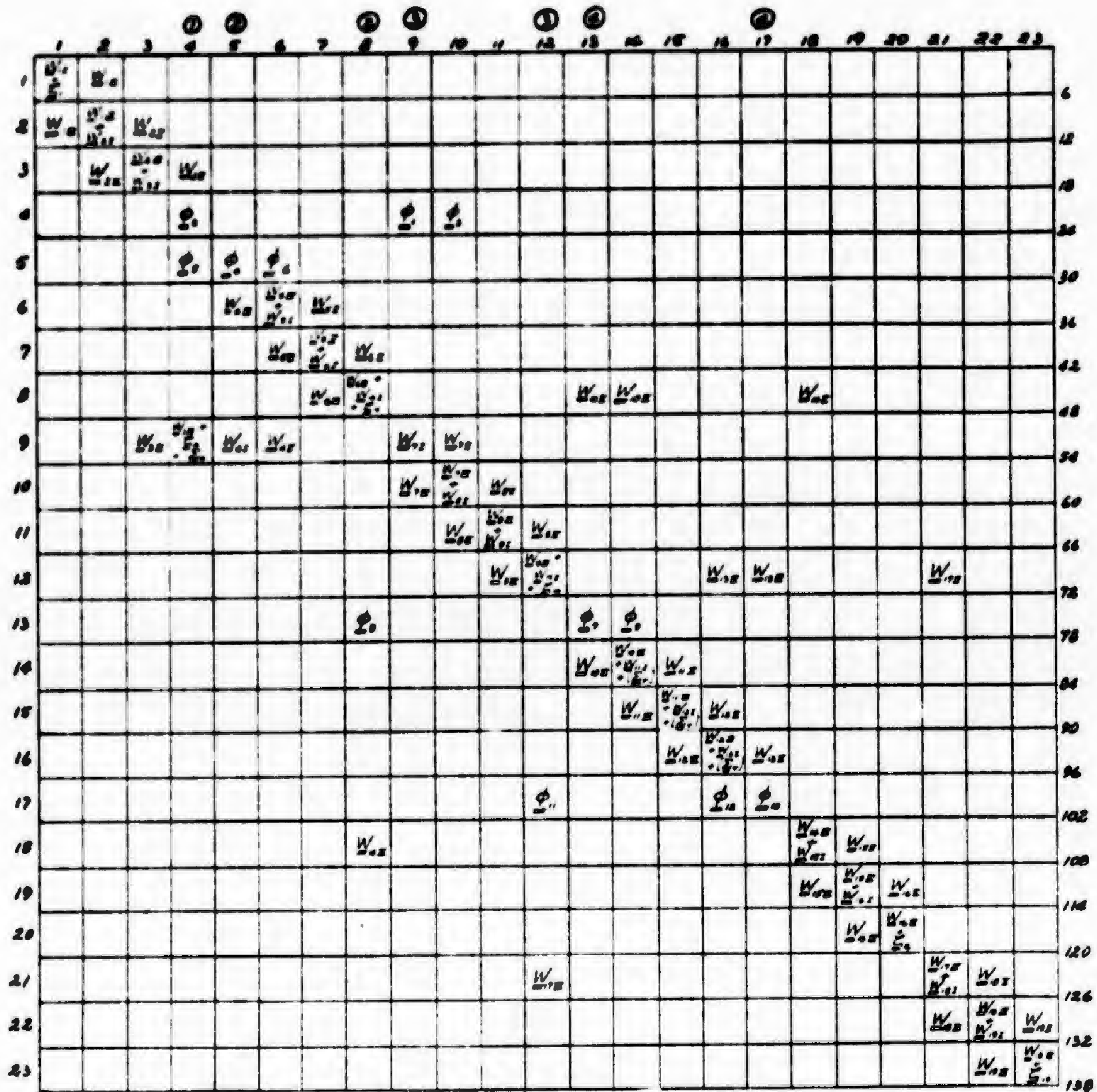


Figure 15: MASS MATRIX OF THE TRIPOD MOUNT

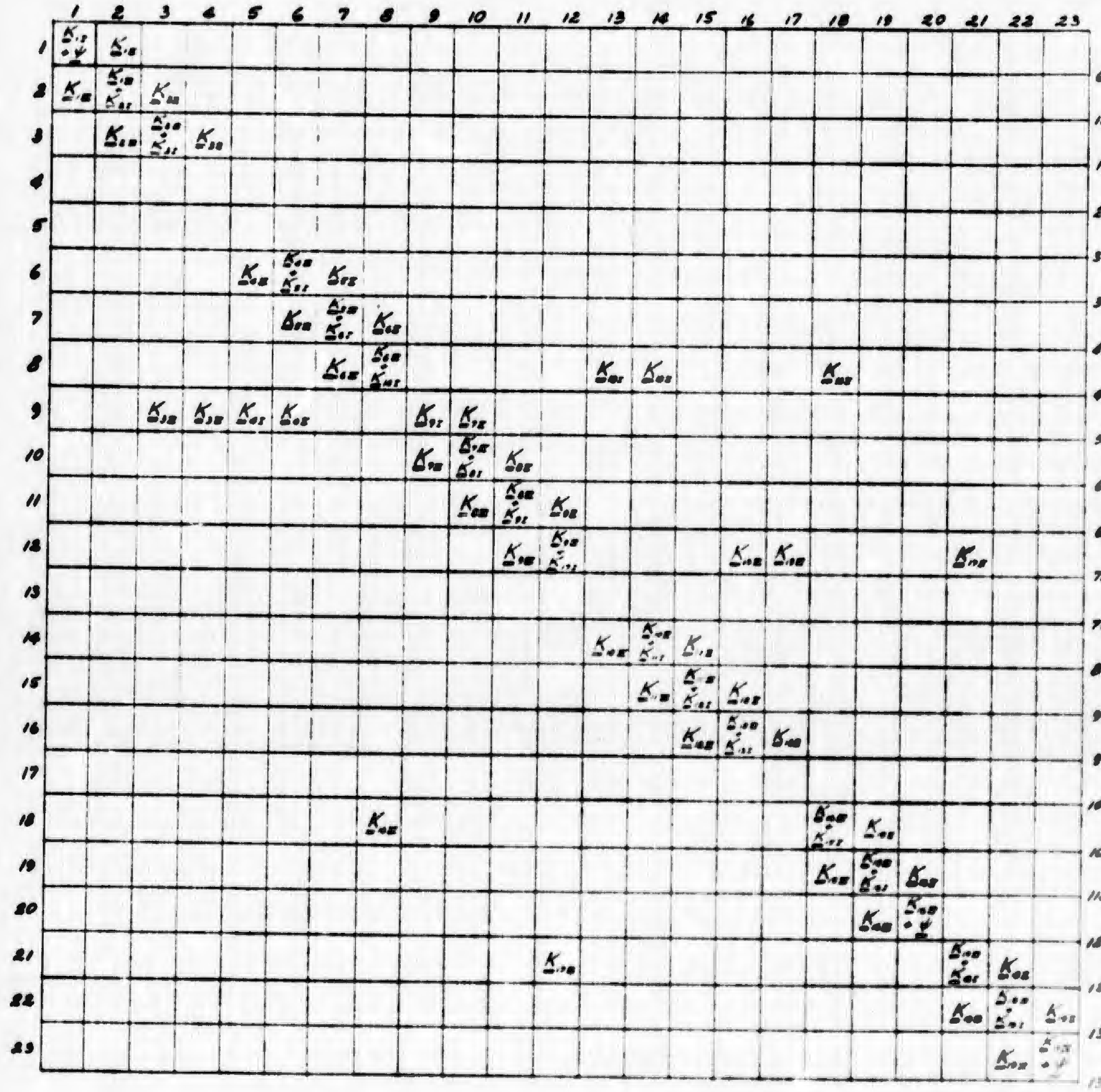


Figure 16: STIFFNESS MATRIX OF THE TRIPOD MOUNT

$\underline{q}_1$	$\underline{Q}_1$
$\underline{q}_2$	$\underline{Q}_2$
$\underline{q}_3$	$\underline{Q}_3$
$\underline{q}_4$ (1)	$\underline{0}_6$
$\underline{q}_4$ (2)	$\underline{0}_6$
$\underline{q}_5$	$\underline{Q}_5$
$\underline{q}_6$	$\underline{Q}_6$
$\underline{q}_7$ (2)	$\underline{Q}_7$
$\underline{q}_4$ (3)	$\underline{Q}_4$
$\underline{q}_8$	$\underline{Q}_8$
$\underline{q}_9$	$\underline{Q}_9$
$\underline{q}_{10}$ (3)	$\underline{Q}_{10}$
$\underline{q}_7$ (4)	$\underline{0}_6$
$\underline{q}_{11}$	$\underline{Q}_{11}$
$\underline{q}_{12}$	$\underline{Q}_{12}$
$\underline{q}_{13}$	$\underline{Q}_{13}$
$\underline{q}_{10}$ (4)	$\underline{0}_6$
$\underline{q}_{14}$	$\underline{Q}_{14}$
$\underline{q}_{15}$	$\underline{Q}_{15}$
$\underline{q}_{16}$	$\underline{Q}_{16}$
$\underline{q}_{17}$	$\underline{Q}_{17}$
$\underline{q}_{18}$	$\underline{Q}_{18}$
$\underline{q}_{19}$	$\underline{Q}_{19}$

Figure 17: DISPLACEMENT VECTOR $\underline{q}^*$ AND FORCING-FUNCTION VECTOR $\underline{Q}^*$

This matrix $\underline{c}$ is entered into the $\underline{C}^*$ matrix at the following cells:

Cell (row, column)
1, 1
20, 20
23, 23

The resulting $\underline{C}^*$ matrix is shown in Figure 18.

5.5 Forward Solution in Time

The equations of motion of the system have been written as

$$\underline{M}^* \ddot{\underline{q}}^* + \underline{C}^* \dot{\underline{q}}^* + \underline{K}^* \underline{q}^* = \underline{Q}^* \qquad (73)$$

For solution of tripod motions, a set of zero initial conditions is taken:

$$\begin{aligned} \underline{q}^* (0) &= \underline{0} \\ \dot{\underline{q}}^* (0) &= \underline{0} \end{aligned} \qquad (74)$$

The initial accelerations may now be calculated from solution of

$$\underline{M}^* \ddot{\underline{q}}^* (0) = \underline{Q}^* (0) \qquad (75)$$

Forward time integration may now be effected. Let $\underline{q}^*$, $\dot{\underline{q}}^*$ and $\ddot{\underline{q}}^*$ be known at t. Using approximate forward integration techniques (based on the assumption that accelerations vary linearly with time between successive time steps):

$$\dot{\underline{q}}^* (t + \Delta t) = \dot{\underline{q}}^* (t) + \frac{\Delta t}{2} [\ddot{\underline{q}}^* (t + \Delta t) + \ddot{\underline{q}}^* (t)] \qquad (76)$$

$$\underline{q}^*(t + \Delta t) = \underline{q}^*(t) + \Delta t\, \dot{\underline{q}}^*(t) + \frac{(\Delta t)^2}{3} \ddot{\underline{q}}^*(t) + \frac{(\Delta t)^2}{6} \ddot{\underline{q}}^*(t + \Delta t) \qquad (77)$$

Substituting into (73) and rearranging:

$$[\underline{M}^* + \frac{\Delta t}{2} \underline{C}^* + \frac{(\Delta t)^2}{6} \underline{K}^*]\, \ddot{\underline{q}}^* (t + \Delta t) = \underline{Q}^*(t + \Delta t)$$

$$- \underline{C}^* [\dot{\underline{q}}^*(t) + \frac{\Delta t}{2} \ddot{\underline{q}}^*(t)] - \underline{K}^* [\underline{q}^*(t) + \Delta t\, \dot{\underline{q}}^*(t) + \frac{(\Delta t)^2}{3} \ddot{\underline{q}}^*(t)] \qquad (78)$$

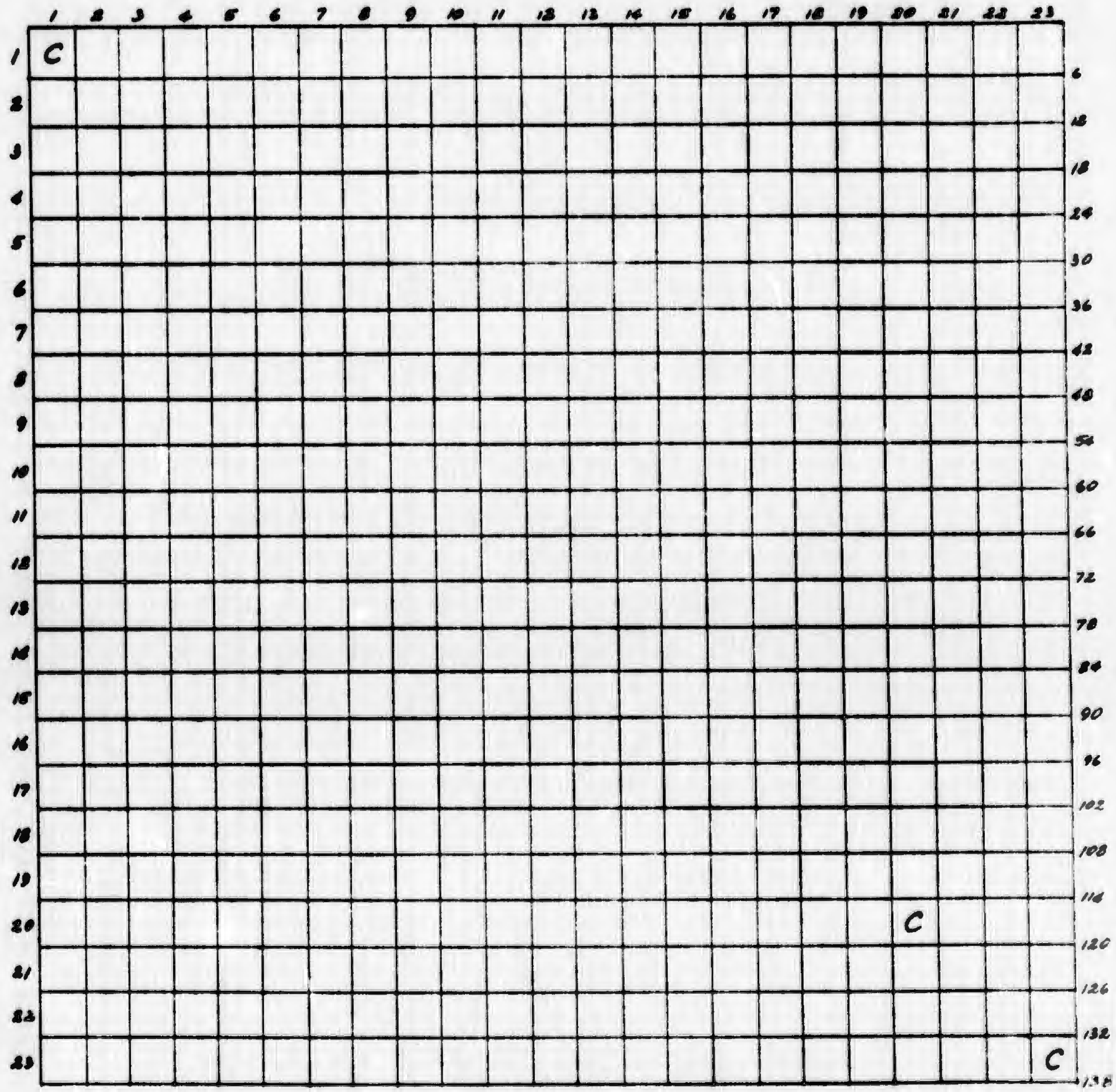


Figure 18: DAMPING MATRIX OF THE TRIPOD MOUNT

Defining

$$\underline{\Omega} = \underline{M}^* + \frac{\Delta t}{2}\underline{C}^* + \frac{(\Delta t)^2}{6}\underline{K}^*$$

$$\underline{A}(t) = \dot{\underline{q}}^*(t) + \frac{\Delta t}{2}\ddot{\underline{q}}^*(t) \qquad (79)$$

$$\underline{B}(t) = \underline{q}^*(t) + \Delta t\,\dot{\underline{q}}^*(t) + \frac{(\Delta t)^2}{3}\ddot{\underline{q}}^*(t)$$

(78) may be written,

$$\underline{\Omega}\,\ddot{\underline{q}}^*(t + \Delta t) = \underline{Q}^*(t + \Delta t) - \underline{C}^*\underline{A}(t) - \underline{K}^*\underline{B}(t) \qquad (80)$$

Solution of equation (80) allows determination of $\ddot{\underline{q}}^*(t + \Delta t)$. Then $\dot{\underline{q}}^*(t + \Delta t)$ and $\underline{q}^*(t + \Delta t)$ may be calculated from:

$$\dot{\underline{q}}^*(t + \Delta t) = \underline{A}(t) + \frac{\Delta t}{2}\ddot{\underline{q}}^*(t + \Delta t)$$

$$(81)$$

$$\underline{q}^*(t + \Delta t) = \underline{B}(t) + \frac{(\Delta t)^2}{6}\ddot{\underline{q}}^*(t + \Delta t)$$

The next time step may then be initiated.

In general, the integration from time t_n to time t_{n+1} may be written,

$$\underline{\Omega}\,\ddot{\underline{q}}^*(t_{n+1}) = \underline{Q}^*(t_{n+1}) - \underline{C}^*\underline{A}(t_n) - \underline{K}^*\underline{B}(t_n)$$

$$\dot{\underline{q}}^*(t_{n+1}) = \underline{A}(t_n) + \frac{\Delta t}{2}\ddot{\underline{q}}^*(t_{n+1}) \qquad (82)$$

$$\underline{q}^*(t_{n+1}) = \underline{B}(t_n) + \frac{(\Delta t)^2}{6}\ddot{\underline{q}}^*(t_{n+1})$$

where

$$\underline{A}(t_n) = \dot{\underline{q}}^*(t_n) + \frac{\Delta t}{2}\ddot{\underline{q}}^*(t_n)$$

$$\underline{B}(t_n) = \underline{q}^*(t_n) + \Delta t\,\dot{\underline{q}}^*(t_n) + \frac{(\Delta t)^2}{3}\ddot{\underline{q}}^*(t_n) \qquad (83)$$

$$\underline{\Omega} = \underline{M}^* + \frac{\Delta t}{2}\underline{C}^* + \frac{(\Delta t)^2}{6}\underline{K}^*$$

$$\Delta t = t_{n+1} - t_n$$

5.6 Gauss-Seidel Method of Solution

Once the equations of motion for the finite elements have been assembled into matrices, the next problem then is to find the solution to a system of 138 simultaneous linear equations. The two types of equations to be solved are:

(1) For the initial acceleration field -

$$\underline{M}^* \ddot{\underline{q}}^*_o = \underline{Q}^*_o - \underline{C}^* \dot{\underline{q}}^*_o - \underline{K}^* \underline{q}^*_o$$

(2) For subsequent acceleration field -

$$\underline{\Omega} \ddot{\underline{q}}^*_n = \underline{Q}^*_n - \underline{C}^* \underline{A} - \underline{K}^* \underline{B}$$

where $\underline{\Omega} = \underline{M}^* + \frac{\Delta t}{2} \underline{C}^* + \frac{(\Delta t)^2}{6} \underline{K}^*$

and $\underline{A} = \dot{\underline{q}}^*_{n-1} + \frac{\Delta t}{2} \ddot{\underline{q}}^*_{n-1}$

$$\underline{B} = \underline{q}^* + \Delta t \, \dot{\underline{q}}^*_{n-1} + \frac{(\Delta t)^2}{3} \ddot{\underline{q}}^*_{n-1}$$

Both of these equations may be reduced to the form:

$$\underline{\Lambda} \underline{x} = \underline{\Psi}$$

each individual equation being:

$$\sum_{j=1}^{m} \lambda_{ij} x_j = \psi_i \qquad \begin{matrix} i = 1, 2, \ldots 138 \\ m = 138 \end{matrix}$$

Elimination methods and matrix inversion methods for solving simultaneous linear equations yield sufficiently accurate solutions for as many as 15 to 20 equations, depending upon the precision of digital computer arithmetic operations and round-off errors. However, for large numbers of equations these methods are unsuitable without considerable error correction. The Gauss-Seidel iteration method has been used successfully for large numbers of equations but has the basic disadvantage of not always converging to a solution or sometimes converging very slowly. Fortunately, for our case, the method does converge to a solution after approximately 13 iterations.

The critical step toward obtaining convergence is the starting trial solution. It was found that specifying the order of solution, beginning with equations for which $\psi_i \neq 0$ and working away from these unknowns, successful convergence was achieved.

Initially let $x_i^{[o]} = 0$ for all i. Then for n = 0, m = 138, compute:

$$x_i^{[n+1]} = \frac{1}{\lambda_{ii}} \left[\psi_i - \sum_{k=1}^{i-1} \lambda_{ik} x_k^{[n+1]} - \sum_{k=i+1}^{m} \lambda_{ik} x_k^{[n]} \right]$$

$i = 1, 2, \ldots 138$

These will produce a set of starting solutions. For all subsequent iterations the unknowns were computed by:

$$x_i^{[n+1]} = \frac{\omega}{\lambda_{ii}} \left[\psi_i - \sum_{k=1}^{i-1} \lambda_{ik} x_k^{[n+1]} - \sum_{k=i+1}^{m} \lambda_{ik} x_k^{[n]} \right] + (1-\omega) x_i^{[n]}$$

$i = 1, 2, \ldots 138$

where ω is the relaxation coefficient. After several trial runs it was determined that an $\omega = 1.057$ produced a 27% reduction in iterations required for convergence.

Convergence is relative and must be detected computationally by an acceptable region of convergence. So that defining:

$$\delta = \left| \frac{x_i^{[n+1]} - x_i^{[n]}}{x^{[n+1]}} \right|$$

it is possible to prescribe an ε such that

$$\delta \leq \varepsilon$$

is acceptable as a solution. Since the unknowns x_i may vary in value by several orders of magnitude, it is difficult to prescribe a universal value of ε. For the present, a value of $\varepsilon = 0.8$ is prescribed but should be investigated further. As a rule, ε should be as large as possible provided that no "significant" change in the results is observed. Making ε too small will require a large number iterations and is wasteful of computer time. Thus, the region of convergence must be tailored to the individual situation.

5.7 Generation of Desired Output Variables

As just discussed in the preceding sections, the immediate product of the Gauss-Seidel iterative solution is nineteen sets of acceleration values, one set for each nodal point. These accelerations are then integrated twice to yield values of displacements. Such data are, however, of little use and must be further processed to result in computer output that can be more readily interpreted. As already covered in Section 3.1, probably the most meaningful output variables are those describing the motions executed by the gun barrel, and values of peak stresses in the tripod elements. The computer program embodies a number of subroutines that digest the raw output data (displacements, velocities, accelerations of nodal points) in such a manner as to produce the requisite output; what follows is the enumeration of the governing equations that were employed in these subroutines.

5.7.1 Change in Elevation Angle and Pintle Motion

The variables that best describe the firing stability of a gun-tripod system are the time histories of pintle displacements and velocities, and of the change in elevation angle.

Figure 19 shows the side view of the tripod in its undeflected (solid lines) and deflected (dotted lines) states. From this figure it is obvious that the displacement and velocity components of point P on the pintle pin (located at its midspan) can be calculated from the following simple expressions:

$$\left.\begin{aligned} x_P &= \delta_{x4} - 2.05\ \theta_{z4} \\ y_P &= \delta_{y4} \\ z_P &= \delta_{z4} + 2.05\ \theta_{x4} \end{aligned}\right\} \quad \text{in inches}$$

$$\left.\begin{aligned} \dot{x}_P &= \dot{\delta}_{x4} - 2.05\ \dot{\theta}_{z4} \\ \dot{y}_P &= \dot{\delta}_{y4} \\ \dot{z}_P &= \dot{\delta}_{z4} + 2.05\ \dot{\theta}_{x4} \end{aligned}\right\} \quad \text{in (in/sec)}$$

where 2.05" is the distance between the centerline of the pintle point and Nodal Point 4.

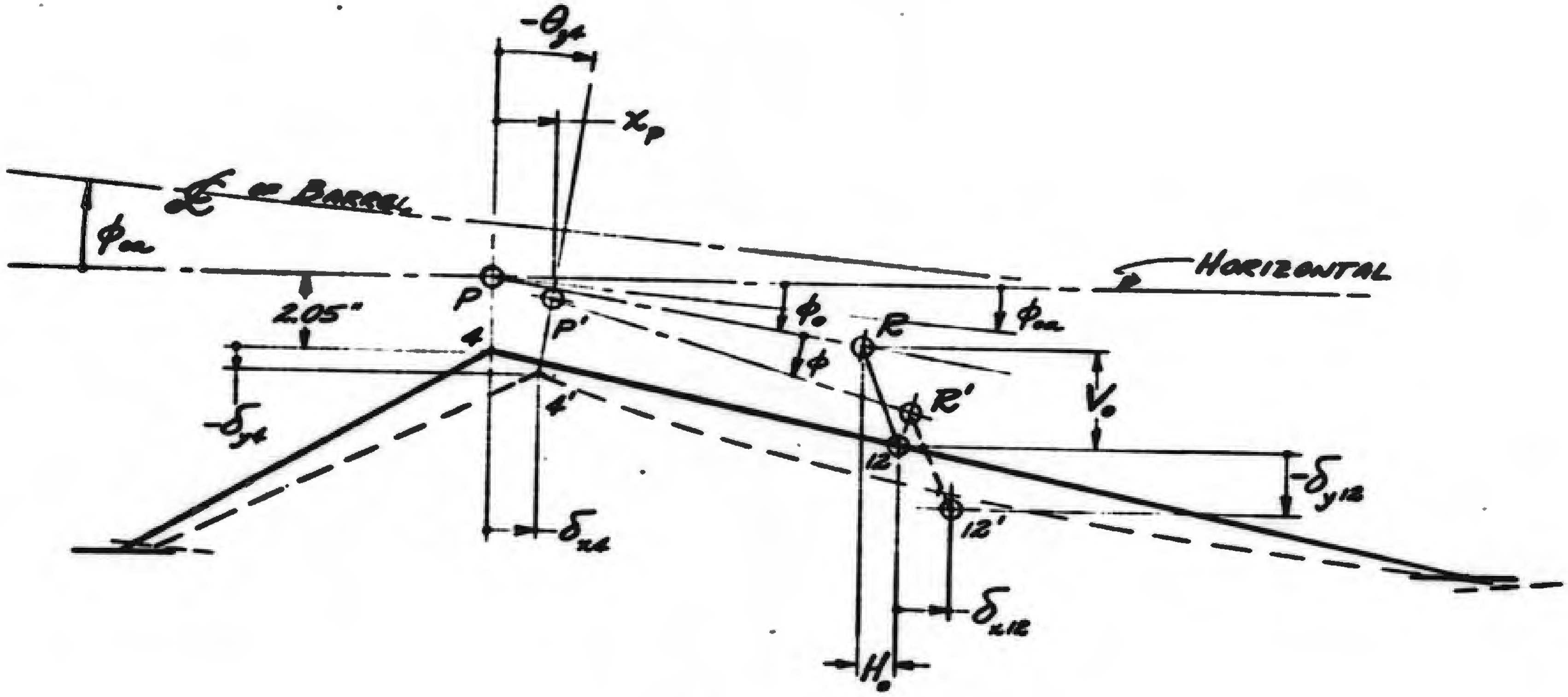


Figure 19: PINTLE DISPLACEMENT x_p AND CHANGE IN ANGLE OF ELEVATION, ϕ

Another subroutine deals with changes in the barrel elevation angle, ϕ (Figure 19), which is calculated by the following formula whose derivation is a rather trivial exercise in plane geometry:

$$\phi = \frac{H_o \Delta_x - V_o \Delta_y}{D_1 \left[(V_o + \Delta_y) \cos \phi_o - (H_o + \Delta_x) \sin \phi_o \right]} \tag{85}$$

Here H_o and V_o are distances related to the setting of the elevating mechanism -- by decreasing V_o the initial elevation angle is increased, while H_o remains essentially constant (the computer uses a fixed value of 2.22 inches for H_o). A value of 13.50" is used for D_1 which is the fixed distance between the pintle pin P and the pin R that connects the gun receiver to the elevation mechanism. The angle ϕ_o is the initial value of angle between line PR and the horizontal, and it bears the following relationship to the initial barrel elevation angle ϕ_{oa}:

$$\phi_o = \phi_{oa} + 4°40'$$

And finally, the two time-dependent variables in Eq. (85) are defined as follows:

$$\Delta_x \equiv \delta_{x12} - \delta_{x4}$$

$$\Delta_y \equiv \delta_{y4} - \delta_{y12}$$

The above definitions of Δ_x and Δ_y hold for the case of a zero azimuth angle; for the cases when the azimuth angle is nonzero, δ_{x12} and δ_{y12} must be replaced by δ_{x13} and δ_{y13}, respectively.

5.7.2 Peak Combined Stresses

By comparison with the subroutines for calculating pintle motions and barrel tilting, the subroutine for estimating the combined stresses in the tripod is a much more complicated affair. It involves a rather lengthly DO-loop that is invoked 120 times at every time step of the main program: there are twelve finite elements designated for stress analysis, and in each element the stresses are evaluated at ten cross-sectional locations. The above DO-loop is outlined below as a series of seven major computational steps.

Step 1: Transform the last calculated values of displacements and accelerations from the global to the local coordinate system. The transformation is performed only for the following nodal points: 5, 6, 15, 18, 7, 14; 8, 9, 10, 17; 11, 12, 13. These nodal points are the end points defining the elements that are to be stress-analyzed.

Now, the program zeros in on one particular element and performs the following six steps. Starting out with Element (5), it labels Nodal Point 5 as End ①, and Point 6 as End ② of this element. In all of the subsequent expressions the subscripts 1 and 2 of the δ's, θ's and their derivatives refer to either End ① or End ② of the particular element under consideration. For instance, for Element 5 $\delta_{x1} \equiv \delta_{x5}$, $\delta_{x2} \equiv \delta_{x6}$, etc.

Step 2: For the particular element of length L and flexural rigidity EI, calculate the following parameters:

$$\beta = \frac{12EI}{L^3}$$

$$\delta \equiv \frac{6EI}{L^2}$$

$$\lambda \equiv \frac{2EI}{L}$$

Step 3: Calculate the values of bending moments and transverse forces acting on End ① of the finite element.

Bending moments induced by unequal displacements and rotations at the two ends:

$$M^k_{y1} = \delta(-\delta_{z1} + \delta_{z2}) + \lambda(2\theta_{y1} + \theta_{y2})$$

$$M^k_{z1} = \delta(\delta_{y1} - \delta_{y2}) + \lambda(2\theta_{z1} + \theta_{z2})$$

Transverse forces induced by unequal displacements and rotations at the two ends:

$$F^k_{y1} = \beta(\delta_{y1} - \delta_{y2}) + \delta(\theta_{z1} + \theta_{z2})$$

$$F^k_{z1} = \beta(\delta_{z1} - \delta_{z2}) - \delta(\theta_{y1} + \theta_{y2})$$

Bending moments induced by inertial forces within the element:

$$M^m_{y1} = -mL^2 \left(\frac{11}{210}\ddot{\delta}_{z1} + \frac{13}{420}\ddot{\delta}_{z2}\right) + mL^3 \left(\frac{1}{105}\ddot{\theta}_{y1} - \frac{1}{140}\ddot{\theta}_{y2}\right)$$

$$M^m_{z1} = mL^2 \left(\frac{11}{210}\ddot{\delta}_{y1} + \frac{13}{420}\ddot{\delta}_{y2}\right) + mL^3 \left(\frac{1}{105}\ddot{\theta}_{z1} - \frac{1}{140}\ddot{\theta}_{z2}\right)$$

Transverse forces induced by inertial forces within the element:

$$F^m_{y1} = mL \left(\frac{13}{35}\ddot{\delta}_{y1} + \frac{9}{70}\ddot{\delta}_{y2}\right) + mL^2 \left(\frac{11}{210}\ddot{\theta}_{z1} - \frac{13}{420}\ddot{\theta}_{z2}\right)$$

$$F^m_{z1} = mL \left(\frac{13}{35}\ddot{\delta}_{z1} + \frac{9}{70}\ddot{\delta}_{z2}\right) + mL^2 \left(\frac{11}{210}\ddot{\theta}_{y1} - \frac{13}{420}\ddot{\theta}_{y2}\right)$$

<u>Step 4</u>: Calculate the total bending moments in two orthogonal planes at a particular value of ξ, where ξ is the distance (measured along the element) from End ① to the particular cross-section at which the stresses are being evaluated. Since each element is stress-analyzed at ten equidistant cross-sections, values of ξ will range from 0 (at End ①) to L (at End ②).

Total bending moment in the local XY-plane:

$$M_{xy}(\xi) = (M^k_{z1} + M^m_{z1}) - (F^k_{y1} + F^m_{y1})\xi$$

$$+ m\,\xi^2 \left[\frac{1}{10}\left(\frac{\xi}{L}\right)^3 - \frac{1}{4}\left(\frac{\xi}{L}\right)^2 + \frac{1}{2}\right] \ddot{\delta}_{y1}$$

$$- m\,\xi^2 \left[\frac{1}{10}\left(\frac{\xi}{L}\right)^3 - \frac{1}{4}\left(\frac{\xi}{L}\right)^2\right] \ddot{\delta}_{y2}$$

$$+ m\,\xi^3 \left[\frac{1}{20}\left(\frac{\xi}{L}\right)^2 - \frac{1}{6}\left(\frac{\xi}{L}\right) + \frac{1}{6}\right] \ddot{\theta}_{z1}$$

$$+ m\,\xi^3 \left[\frac{1}{20}\left(\frac{\xi}{L}\right)^2 - \frac{1}{12}\left(\frac{\xi}{L}\right)\right] \ddot{\theta}_{z2}$$

Total bending moment in the local XZ-plane:

$$
\begin{aligned}
M_{xz}(\xi) = {} & (M^{k}_{y1} + M^{m}_{y1}) + (F^{k}_{z1} + F^{m}_{z1})\xi \\
& - m\,\xi^2 \left[\frac{1}{10}\left(\frac{\xi}{L}\right)^3 - \frac{1}{4}\left(\frac{\xi}{L}\right)^2 + \frac{1}{2}\right] \ddot{\delta}_{z1} \\
& + m\,\xi^2 \left[\frac{1}{10}\left(\frac{\xi}{L}\right)^3 - \frac{1}{4}\left(\frac{\xi}{L}\right)^2\right] \ddot{\delta}_{z2} \\
& + m\,\xi^3 \left[\frac{1}{20}\left(\frac{\xi}{L}\right)^2 - \frac{1}{6}\left(\frac{\xi}{L}\right) + \frac{1}{6}\right] \ddot{\theta}_{y1} \\
& + m\,\xi^3 \left[\frac{1}{20}\left(\frac{\xi}{L}\right)^2 - \frac{1}{12}\left(\frac{\xi}{L}\right)\right] \ddot{\theta}_{y2}
\end{aligned}
$$

__Step 5__: Calculate the maximum bending stress at the particular location ξ:

$$
\sigma_b(\xi) = \left(\frac{c}{I}\right) \sqrt{[M_{xy}(\xi)]^2 + [M_{xz}(\xi)]^2}
$$

__Step 6__: Calculate the axial compressive stress at location ξ:

$$
\sigma_c(\xi) = \frac{E}{L}(\delta_{x1} - \delta_{x2}) + \frac{mL}{A}\left\{ \left[\frac{1}{3} - \left(\frac{\xi}{L}\right) + \frac{1}{2}\left(\frac{\xi}{L}\right)^2\right] \ddot{\delta}_{x1} + \left[\frac{1}{6} - \frac{1}{2}\left(\frac{\xi}{L}\right)^2\right] \ddot{\delta}_{x2} \right\}
$$

__Step 7__: Combine the above stress components to obtain the maximum compressive stress at location ξ. This is the nominal stress at extreme fibers of the tubular element, and it does not reflect the effect of holes or other stress raisers that might happen to be in the vicinity of location ξ.

$$
\sigma(\xi) = \sigma_c(\xi) + \sigma_b(\xi)
$$

After completing the above seven steps, the program stores the value of $\sigma(\xi)$, indexes the value of ξ and repeats the calculations. Then, when the full set of ten $\sigma(\xi)$-values is obtained, the program searches for the highest value among the ten stored values of $\sigma(\xi)$, and having found it, prints out the following:

1) element number
2) maximum value of combined stress, $\sigma(\xi_m)$
3) value of ξ_m (in inches).

After such a printout is completed, the program indexes to the next specified finite element, executes the ten DO-loops, and prints out the above three output items. This is repeated until all twelve elements to be stress-analyzed are covered; only then does the program index the time variables and proceed with the calculations for the next time increment.

SECTION 6

CONCLUDING REMARKS

6.1 Discussion of Computed Results

The output data given in Appendix C is the result of a partial run using the USAWECOM forcing function received October 12, 1970. The output for the forcing function received was first reduced to approximately 150 data entries to the auxiliary program SJLFF. In performing these data selections, the USAWECOM output was scanned for all maxima, minima and abrupt changes of sign in the Bolt Force (F_1), Barrel Force (F_2) and receiver acceleration (a_R). After processing the selected data, the same procedure was followed in selecting all maxima, minima and change-of-sign data for the resulting Pintle Force and Traverse Bar Force. This resulted in the selection of 37 sets of data for the Forcing Function as shown in Appendix C. It may be noted here that several extreme spikes, having orders-of-magnitude larger values and very short duration, were ignored. To include these values in the Forcing Function would require prohibitively small time increments.

From a plot of the forcing function, it became apparent that a time increment of 0.001 second would generally include every peak value of the Pintle and Traverse Bar Forces. A symmetric case for loading was chosen, i.e., with the initial azimuth angle given as zero. Furthermore, initial elevation angle was chosen as zero, and the tripod was taken as having all three legs extended by two hole stops. Soil properties data were selected, and concentrated masses and their mass moments of inertia were computed to supply the necessary input data for the demonstration run. The data deck was assembled as outlined in Appendix A and the program was executed at the GARD Computer Center using the IBM 1130.

The program executed successfully for the demonstration run. However, after several time increments, several problems arose which made the results unacceptable. Since the tripod was being loaded symmetrically, the displacements must be symmetrical also. As seen from the output, the Y-displacements for nodal points 16 and 19 beginning with time 0.003 are no longer equal. Furthermore, a mounting error for a non-zero Z-displacement of the front leg (nodal point 1) becomes evident.

The source of these problems was traced to computer precision coupled with Gauss-Seidel solution accuracy. As discussed in Section 5.6, the solution is deemed acceptable when all 138 unknowns being sought fall within a region of convergence. If the region of convergence is narrowed down in an attempt to increase accuracy, the computational precision creeps in and does not allow convergence. To force convergence of all values, particularly those unknowns which are very small, a crude region of convergence was specified. With a crude region of convergence, control is lost over the more meaningful unknowns and exhibits itself with the non-symmetric results observed in the output. It was impossible to increase precision of the calculations with the IBM 1130 computer. Since computer capacity and particularly the deck file are very near their limitations. Consequently, further refinements are not feasible with the present computer system.

The computer program as presented herein is fully operational in that no errors in logic and computational procedures are evident. However, further refinements must be made, notably the modifications and execution of the program with higher precision and lower regions of convergence. This can only be accomplished with a larger computer facility for which suitable program changes would be made to make the program compatible with the new system.

6.2 Conclusions

The tripod mount for the 0.50 caliber machine gun has been shown to lend itself to analysis by the finite-element approach. The analysis has been successfully programmed for solution on the digital computer. The computer program is operational with no apparent errors in the logic and computational procedures. However, to produce acceptable results, further modifications and refinements of the computer program must be made which would increase computational precision, thereby allowing greater accuracy.

6.3 Recommendations

In order to bring the program to a fruitful conclusion, the following course of action is suggested.

1. The computer program should be exercised on a computer with larger capacity than that of IBM 1130. Only then could the necessary computational precision be achieved without which meaningful output is not possible.

2. An improved soil model should be incorporated, supplanting the linearly-elastic, non-yielding model that is presently built into the computer program.

3. Validation of the computer output is also desirable: after incorporation of an improved soil model and successful achievement of the required convergence, the computed values of tripod motions and stresses should be compared to corresponding values experimentally obtained from an actual firing test.

LIST OF REFERENCES

1. "Vibration of Soils and Foundations," by F. E. Richart, Jr., J. R. Hall, Jr., R. D. Woods; Prentice-Hall, 1970.

2. "Consistent Mass Matrix for Distributed System," by J. S. Archer; Proc. ASCE 89 ST4 P161, 1963.

3. "Formulas for Stress and Strain," R. J. Roark, 1954.

NOMENCLATURE

A	cross-sectional area of a finite element, (in^2)
$\underline{C}^*$	damping matrix (Figure 18)
E	Young's modulus, (lbs/in^2)
e	void ratio of soil, (dimensionless)
F_{ij}	($i=x,y,z$; $j=1,2,...23$) force components of the $\underline{Q}_i$-vector, (lbs)
F_P	pintle force, (lbs)
F_T	traverse-bar force, (lbs)
G	shear modulus, (lbs/in^2)
I	area moment of inertia of a finite element, (in^4)
J	torsional constant of a finite element, (in^4)
J_i	mass moments of inertia of a lumped mass about global coordinate axes $i = X,Y,Z$ ($slug\text{-}in^2$)
$\underline{K}^*$	stiffness matrix (figure 16)
L	length of the finite element, (in)
M_{ij}	moment components of the $\underline{Q}_i$-vector, (in-lbs)
M_c	pintle moment, (in-lbs)
m	mass per lineal inch of finite element, (slug/in); lumped mass in the mass matrix $\underline{M}$, (slugs)
$\underline{Q}_i$	a 1 x 6 vector of forces and moments that are externally applied to nodal point i
$\underline{Q}$	a matrix, defined by Equation (79)
$\underline{q}_i$	a 1 x 6 vector of displacements at nodal point i
r_o	radius of foot disc, (in)

$\tilde{T}_i$ transformation matrix (global to local coordinates) for leg i (i = 1,2,3,4)

t time, (sec)

$\underline{V}$ an arbitrary 1 x 3 vector

V_c elevation mechanism setting, (in) (Figure 5)

$\underline{W}^*$ mass matrix (Figure 15)

X,Y,Z global coordinates

x,y,z local coordinates

α = mL/6; angle between front leg and the plane of rear legs (Figure 5)

β = 13mL/35; angle of action for pintle force F_p

γ = jL/6; kink angle of front leg (Figure 5)

δ = $11mL^2/210$

δ_{ij} displacement of nodal point j in direction i(i = x,y,z)

$\underline{\Delta}$ a 6 x 6 matrix, accouting for the lumped masses of tripod (a component of the $\underline{W}^*$)

θ_n(n=1,2,3,4) angles between tripod elements (Figure 14)

θ_{ij} angular displacement at nodal point j, about coordinate axis i (i = X,Y,Z)

λ = 9mL/70

μ = $13mL^2/420$

ν Poisson's ratio of soil

ξ position variable inside a particular finite element, (in)

ρ = $mL^3/105$; soil density, (lbs/in^3)

$\underline{\Sigma}$		a 6 x 6 matrix, accounting for the mass of the machine gun (a component of $\underline{W}^*$)
σ		stress, (psi)
τ	=	$mL^3/140$
α^*	=	AE/L
β^*	=	$12EI/L^3$
γ^*	=	GJ/L
δ^*	=	$6EI/L^2$
λ^*	=	$2EI/L$
ϕ		change in gun-barrel elevation angle during firing
ϕ_o		angle defined in Figure 19
ϕ_{oa}		initial value of the gun-barrel elevation angle (Figure 19)
$\underline{\Phi}_n (n=1,...12)$		displacement compatibility matrices, defined by Equations (56) through (68)
$\underline{\Psi}$		a 6 x 6 matrix, accounting for soil stiffness (a component of $\underline{K}^*$)

APPENDIX A

DESCRIPTION OF COMPUTER PROGRAM

APPENDIX A

DESCRIPTION OF COMPUTER PROGRAM

A.1 Computer Program Operation

A.1.1 Overall Program Description

A digital computer program was written to compute the transient response of the Tripod Mount M3 encompassing the analysis contained in this report. The program was written in FORTRAN IV for the IBM 1130 Disk Operating System and trial-executed at the GARD computer facilities. The program is divided into 5 linked main programs with 13 supporting subroutines. An auxiliary program was also written to generate the forcing function data cards required by the main program which utilizes output from USAWECOM's machine gun program. A listing of the entire program deck is given in Appendix B.

The five linked main programs are:

(1) Program SJLMS: Mass and Stiffness Matrix Writer
(2) Program SJLMI: Mass and Stiffness Matrix Modifier
(3) Program SJLFQ: Forcing Function Reader and (Q)-Matrix Writer
(4) Program SJLIN: Initial Acceleration Calculator
(5) Program SJLTM: Transient Displacement, Velocity, and Acceleration Calculator.

A brief description of each of these main programs follows. It may be noted here that since the program is operational on the IBM 1130 system, detailed output beyond what is normally required is possible through the use of sense switches. Normal operation, however, is with all switches off.

A.1.2 Program SJLMS (Link 1)

There are massive amounts of data contained in the matrices to be manipulated, so it was decided to generate and store these matrices as disk files. These files are accessible to the computer upon execution but never remain entirely in core. Each major matrix is stored as 138 records; each record being 276 words long. Since standard precision is used throughout, each real number generated requires 2 words.

The first linked program (SJLMS) begins by clearing the files 1 and 2 which are designated as the Mass and Stiffness Matrices, respectively. Once these files are cleared, the caption card (CAP) which identifies the case being analyzed is read. This caption card is common to all linked programs and is printed on all output pages from this and all succeeding programs. The tripod geometrical data is read into the computer next. The program has stored properties of the tripod, but if any

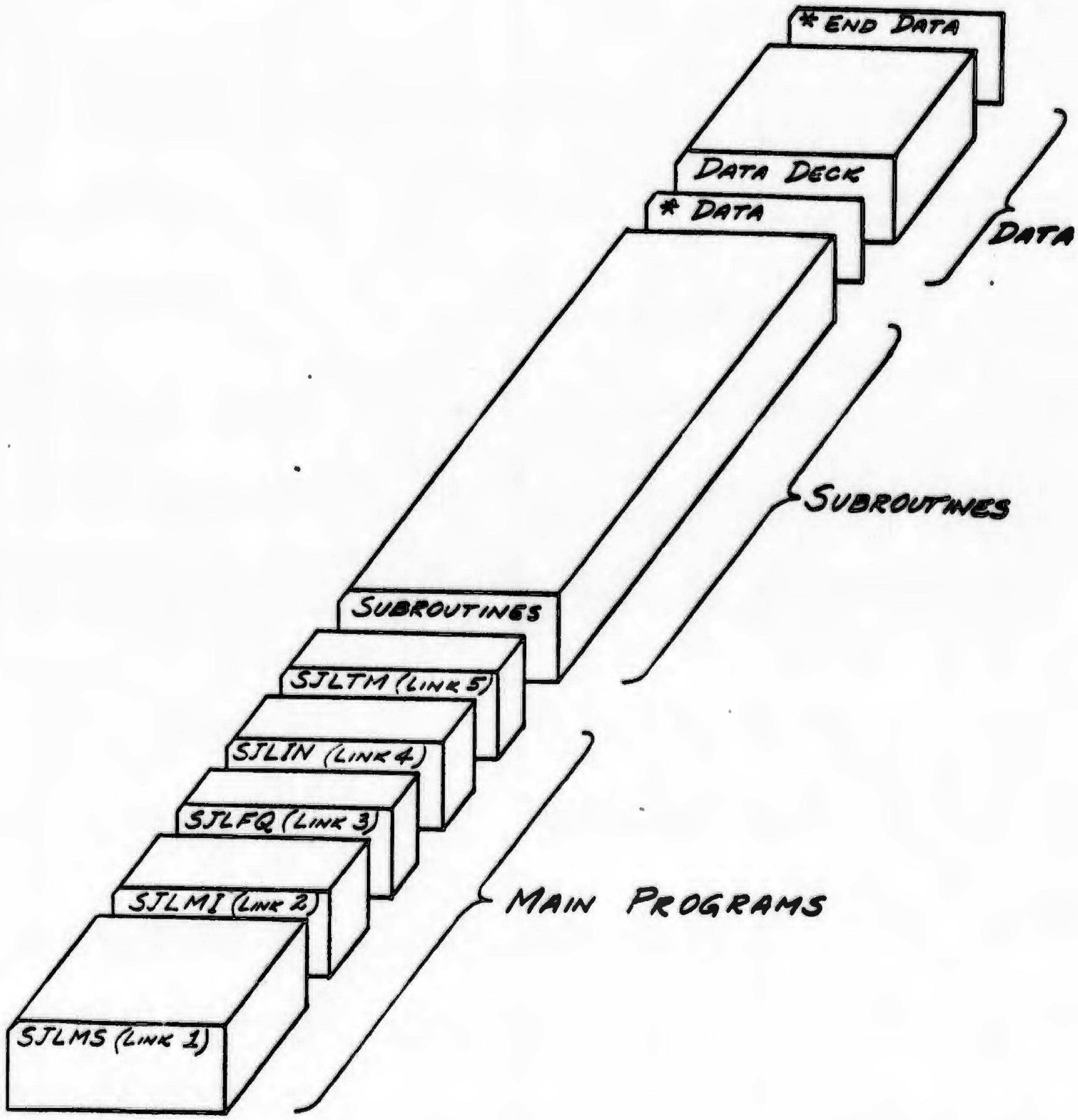


Figure A.1: PROGRAM DECK

of these properties are to be changed for a particular case being analyzed, provisions have been made to accept corrections of data which differ from those shown in Table A-I. The first page of normal output is then produced which recapitulates the input and/or stored tripod properties.

The computations proceed to the geometry for the system which is used in the development of the Transformation Matrix (SJLTI). Proceeding now for each successive element, the terms for the Mass Matrix are computed and transformed into global coordinates (SJLMX) and sorted (SJLFY) into the Mass Matrix file for all nineteen tripod elemental lengths. The boundary conditions are computed and written (SJLBD) on file to complete the Mass Matrix file for all the distributed masses. The calculations then proceed in similar manner to compute, transform, sort and write the Stiffness Matrix file. At the conclusion, the option is made to print the Mass and Stiffness Matrix (sense switch 1 on) or for normal operation links control to the next program (sense switch 1 off).

A.1.3 Program SJLMI (Link 2)

The purpose of the second linked program (SJLMI) is to modify the Mass and Stiffness Matrix files for concentrated masses and soil stiffness. The program calls for input data of mass and mass moments of inertia to be read in for the ordered nodal points 1, 4, 7, 10, 16 and 19 (see Figure 2), followed by the initial gun elevation angle, azimuth angle and the value of V_o (see Section 3.2). The program then computes a geometrically compatible azimuth angle to provide confluence with traverse bar nodal points 11, 12 or 13. In this sense the azimuth angle (AZIM), read in as input, is used so that if:

If AZIM < 0 azimuth is computed through nodal point 13
If AZIM = 0 azimuth is computed through nodal point 12
If AZIM > 0 azimuth is computed through nodal point 11

The second page of normal output recapitulates the concentrated masses and mass moments of inertia Then the gun mass components are read as input to be lumped at the pintle and traverse bar (see Section 3.2), and are printed as output. This completes the mass modifications data required so that now these terms are inserted into the mass matrix file in their proper place.

The soil properties are then read as input and recapitulated as output. From the soil properties the soil characteristics are calculated (see Section 4.3). After distributing the terms of the soil characteristics in proper order, the procedure is repeated, this time inserting the soil characteristics into the Stiffness Matrix file in their proper place.

TABLE A-I

TRIPOD PROPERTIES

(LL) ELEMENT NUMBER	(A) AREA (IN^2)	(W) WEIGHT PER UNIT LENGTH (LBS/IN)	(I) MOMENT OF INERTIA (IN^4)	(J) POLAR MOMENT OF INERTIA (IN^4)	(C) OUTER RADIUS (IN)
1	0.633	0.180	0.1310	0.2620	0.717
2	0.455	0.310	0.1990	0.2700	0.817
3	0.455	0.130	0.1350	0.2700	0.817
4	0.420	0.120	0.1240	0.2480	0.810
5	0.420	0.120	0.1240	0.2480	0.810
6	0.420	0.120	0.1240	0.2480	0.810
7	0.420	0.120	0.1240	0.2480	0.810
8	0.420	0.120	0.1240	0.2480	0.810
9	0.420	0.120	0.1240	0.2480	0.810
10	0.490	0.140	0.0429	0.0858	0.502
11	0.490	0.140	0.0429	0.0858	0.502
12	0.490	0.140	0.0429	0.0858	0.502
13	0.490	0.140	0.0429	0.0858	0.502
14	0.420	0.120	0.1240	0.2480	0.810
15	0.420	0.300	0.1920	0.2480	0.810
16	0.633	0.180	0.1310	0.2620	0.717
17	0.420	0.120	0.1240	0.2480	0.810
18	0.420	0.300	0.1920	0.2480	0.810
19	0.633	0.180	0.1310	0.2620	0.717

During the course of the calculations, values to be used in subsequent programs are saved and stored in COMMON. Likewise, the unmodified Stiffness and Mass Matrix files are saved as files 5 and 6, respectively.

A.1.4 Program SJLFQ (Link 3)

The third linked program (SJLFQ) comprises the last preparatory program of the system. Its purpose is to read in the forcing function and to form the (Q) matrix. The first input data required is the time increment (DTAU) which will be used to step the transient calculations. Then the forcing function deck is read. The program provides for up to 50 forcing function data cards. If less than 50 is being used, the signal for this situation is to make the last data card negative in the first data field. Once all the forcing function data is read, the entire forcing function table is recapitulated as output.

The calculations now proceed to forming the (Q) matrix. The Mass matrix is read from file 1, the Stiffness matrix is read from file 2, and the ground damping is computed and held in COMMON as C. Each term of the (Q)-matrix is computed and file 3 is created. In view of the boundary conditions, certain terms of the (Q)-matrix can be eliminated and expressed in terms of other variables. Thus, (Q)-matrix modifications are performed (SJLQB) on file 3, leaving it prepared for all legs under the influence of soil damping and stiffness. Normal operation (sense switch 6 OFF) would cause control to be given to the next linked program; however, for more detailed output, the entire (Q)-matrix file may be output (sense switch 6 ON).

A.1.5 Program SJLIN (Link 4)

The purpose of the third linked program (SJLIN) is to compute the initial acceleration field. For normal operation (sense switch 0, OFF) it is assumed that the initial displacements and velocities are zero. However, the option has been provided to initialize on a given set of displacements and velocities for a given initial time, TAU. If this option is desired (sense switch 0, ON) a set of data cards providing this information is required.

Given the initial conditions, the computations proceed to find the value of the forcing function (SJLQQ) at the initial time (TAU). Next, the initial acceleration field is solved (see Section 5.6) by the Gauss-Seidel method (SJLGS). If more detailed output is required (sense switch 8, ON), the initial displacements, velocities, and accelerations are output. For normal operation, however, (sense switch 8, OFF), the calculations proceed toward the computations of pintle pin displacements, gun orientation and maximum combined stresses in selected elements of the tripod (SJLDG). Once this is completed, the results are output. Control is then transferred to the next linked program.

A.1.6 Program SJLTM (Link 5)

The fifth and last linked program (SJLTM) is the culmination of the computations It provides for the calculation of the transient displacements, velocities and acceleration at each nodal point in the tripod system. The last bit of input data (TMAX) is read, namely, the maximum time towards which the computations are to proceed. The calculations begin by stepping time to the first time increment. The value of the forcing function (SJLQQ) is determined and the (A) and (B) vectors are formed (see Section 5.6). Next the right-hand sides of simultaneous equations (see Eq. 82) are formed. The acceleration field is then solved (SJLGS) for the first time increment; followed by the corresponding displacement and velocity field. A check of the computed displacements is made to see if any of the three legs are lifted out of the ground. If a leg lifting is detected, and the solution was for conditions other than for those detected, leg lifting modifications are made (SJLLL) and a new solution is sought for the correct conditions. If no leg lifting is detected and the solution was for these conditions, the solution is accepted and the computations proceed.

For a valid solution of the transient response the pintle displacement, gun orientation and maximum combined stresses in selected elements of the tripod are computed (SJLDG). These values are then output (SJLOU). A check is made to see if the maximum time (TMAX) is exceeded. If not, the time is incremented and the entire procedure is repeated. If TMAX is exceeded, the program EXIT's.

Normal output (sense switch 8, OFF) would consist of the transient response giving the time, displacement and velocity of the pintle and tripod feet, gun elevation and azimuth angle changes, and maximum combined stresses in selected elements of the tripod legs. However, for more detailed output, it is possible (sense switch 8, ON) to output the displacements, velocity, and acceleration for each nodal point

A.2 Computer Program Input Data

A.2.1 Discussion of Input Data

A typical data deck will contain a minimum of 66 data cards. More cards may be required:

1) If tripod properties differ from those shown in Table A-I correction cards will be required for each appropriate correction.

2) If initial displacements and velocities are to be prescribed other than at rest at time zero, the displacement and velocity fields must be given.

The following discussion may be used as a guide for preparing the data deck. The data deck described is the one used for the demonstration computer run made at the GARD computer center. The output from this run will be discussed in Section A.3.

A.2.2 Identification Caption Data

The first card to be prepared is the identification caption card. Any legal alphameric character may be used to identify the case being analyzed in the computer run. In the example shown in Figure A-2 the caption shows that:

ALL THREE LEGS ARE EXTENDED BY TWO HOLE STOPS.

The identification follows since the segment leg lengths are taken from Table A-IV. The user may also identify the conditions for which the USAWECOM's computer program output was obtained on this same caption card. In which case it is suggested that the card be prepared with this additional information, the only restriction being that no more than 80 legal alphameric characters are used.

A.2.3 Tripod Element Length Data

Cards 2, 3, 4 and 5 are associated with the element lengths of the tripod and its geometric and physical properties. (See Figure A-3.) As discussed in Section A.1.2, the program has built-in physical properties of the tripod materials. The segment lengths possible for the tripod are given in Tables A-II through A-VI. The preparation of cards 2 and 3 is fairly straightforward since all that is required is to utilize the segment lengths given in the tables. The first four values of card 4 are the remaining segment lengths. However, the fifth value is the included angle from the front to the back legs, and the sixth value is the angle of kink for the front leg. Card 5 must always be included in the data deck. If tripod properties corrections are necessary (as for example, in the case taken from Table A-II), the correction cards are placed between card 4 and card 5. In this way the built-in tripod properties are amended to conform with the required corrections.

A.2.4 Concentrated Mass and Mass Moments of Inertia Data

Concentrated masses and their corresponding mass moments of inertia relative to the global coordinate system comprise the next set of input data cards (see Figure A-4). These cards must be ordered so that:

Card 6 corresponds to nodal point 1
Card 7 corresponds to nodal point 4
Card 8 corresponds to nodal point 7
Card 9 corresponds to nodal point 10
Card 10 corresponds to nodal point 16
Card 11 corresponds to nodal point 19

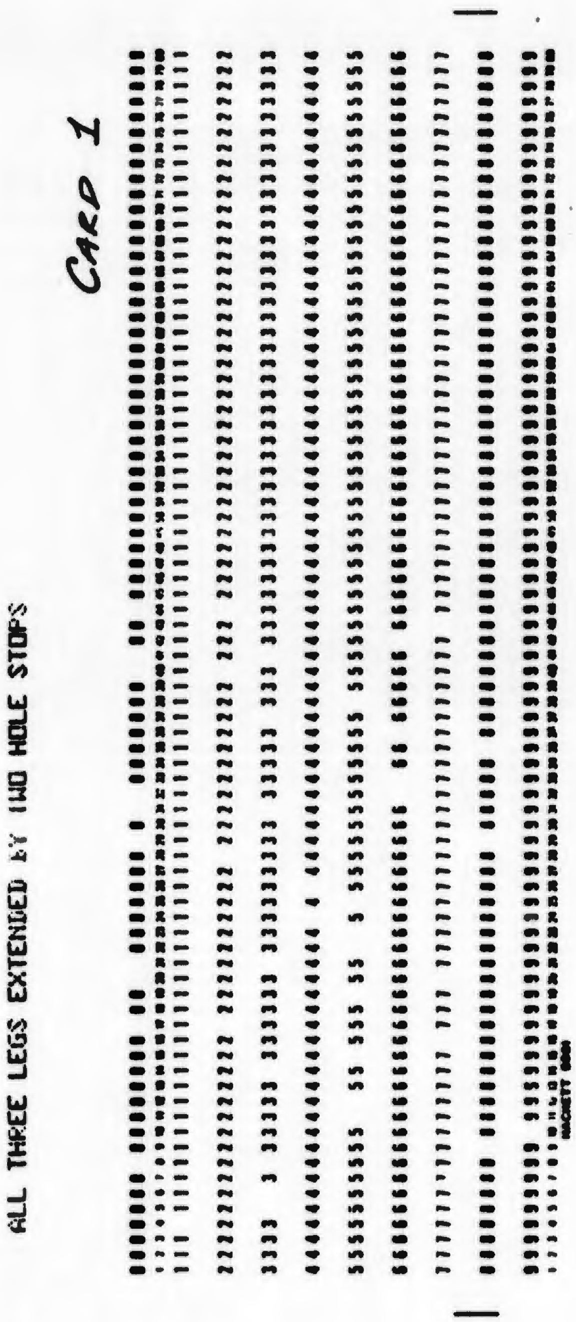


Figure A.2: TYPICAL IDENTIFICATION CAPTION CARD.

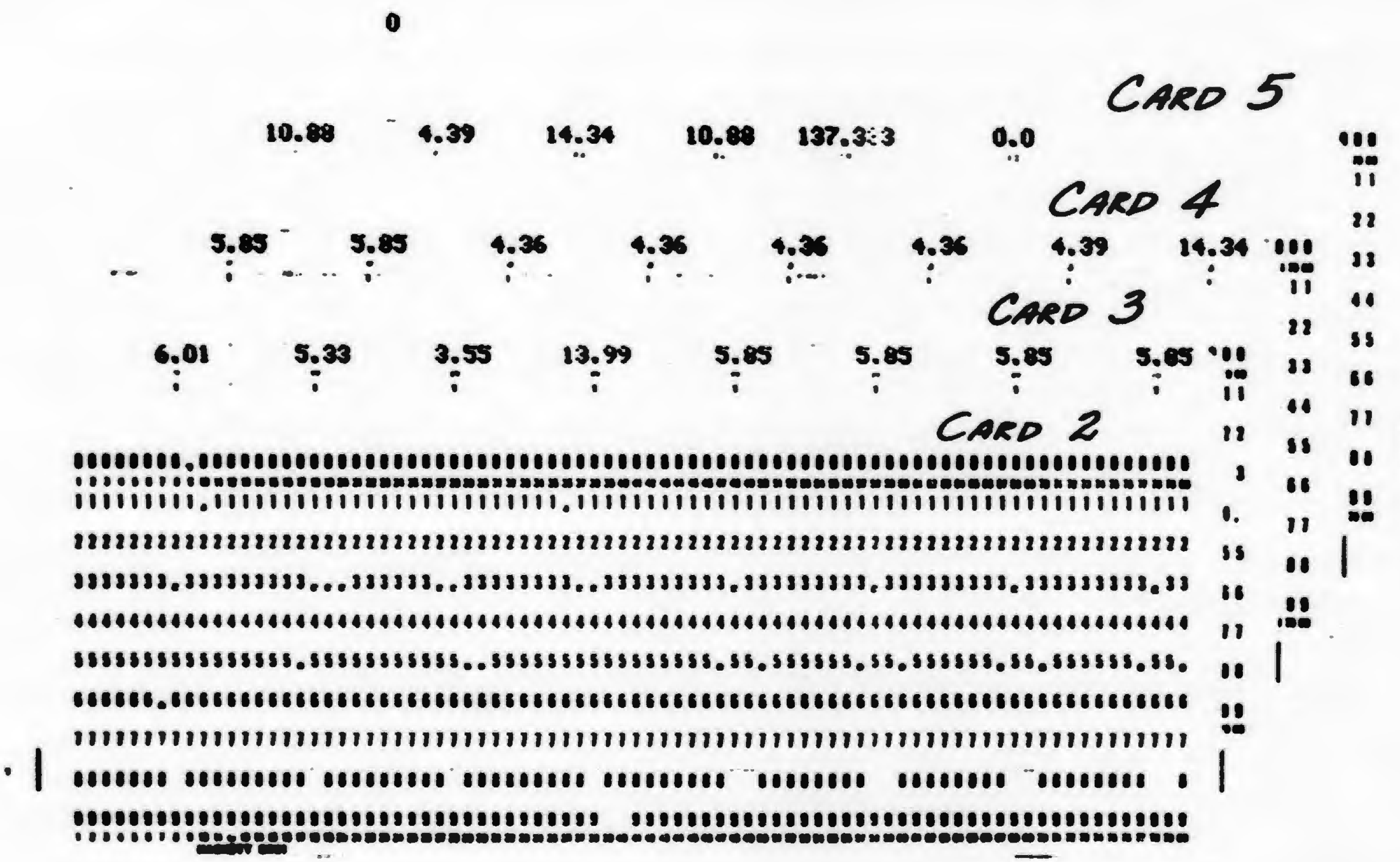


Figure A.3: TYPICAL TRIPOD ELEMENT LENGTH CARDS

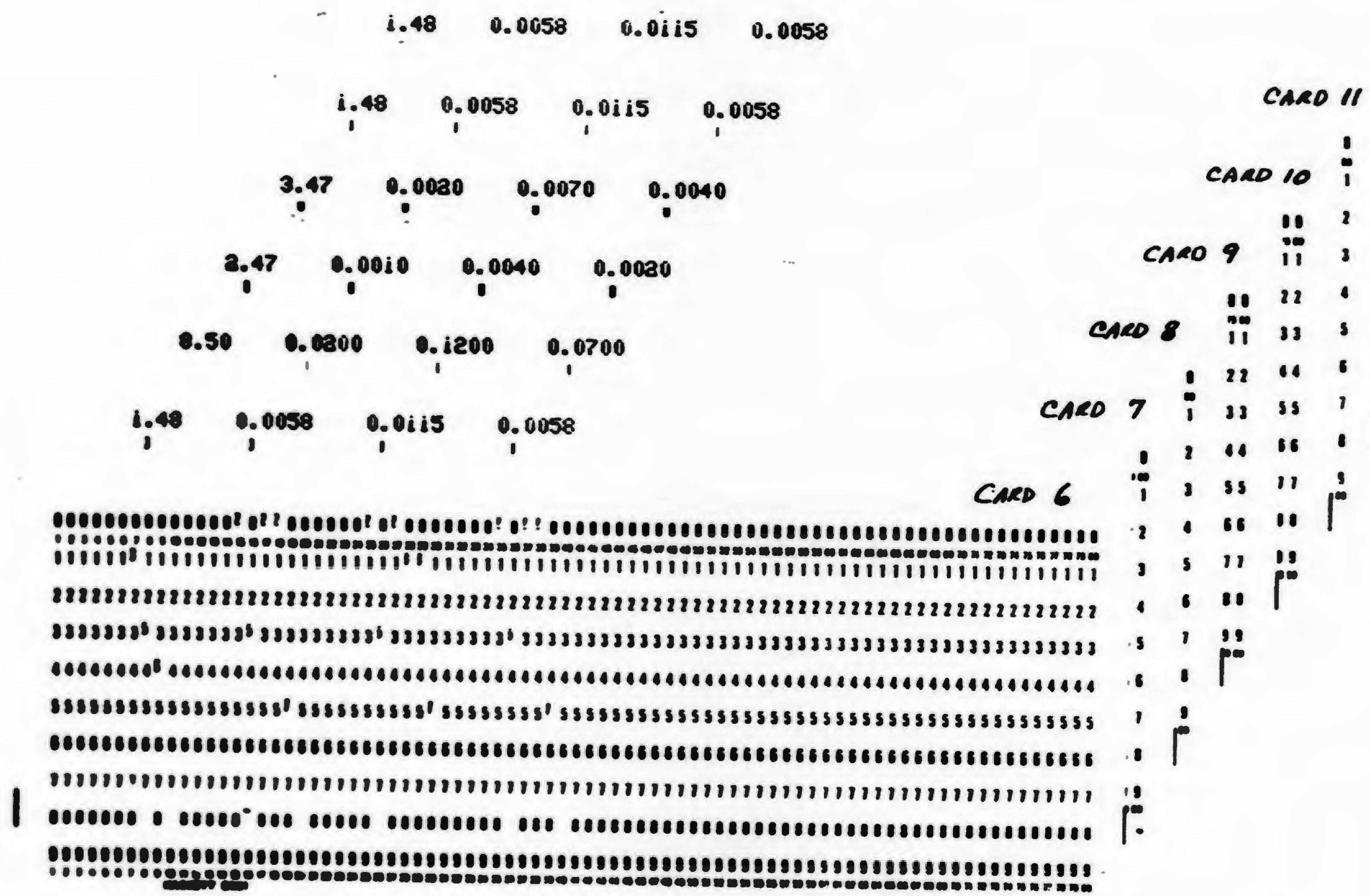


Figure A.4: TYPICAL CONCENTRATED MASS AND MASS MOMENTS OF INERTIA CARDS

TABLE A-II

TRIPOD SEGMENT LENGTHS

ALL THREE LEGS FULLY RETRACTED

LENGTHS REQUIRED

AL = 6.010 in.

ELEMENT NUMBER	SEGMENT LENGTH (in)
1	2.33
2	7.41
3	10.99
4	7.02
5	7.02
6	3.51
7	7.02
8	7.02
9	3.51
10	4.36
11	4.36
12	4.36
13	4.36
14	9.37
15	9.36
16	2.98
17	9.37
18	9.36
19	2.98

TRIPOD PROPERTIES CORRECTIONS

ELEMENT NUMBER	AREA (in^2)	WEIGHT PER UNIT LENGTH (lbs/in)	MOMENT OF INERTIA (in^4)	POLAR MOMENT OF INERTIA (in^4)	OUTER RADIUS (in)
6, 9, 14, 17	.420	.300	.192	.248	.810

TABLE A-III

<u>TRIPOD SEGMENT LENGTHS</u>

ALL THREE LEGS EXTENDED BY ONE HOLE STOP

LENGTHS REQUIRED

AL = 6.010 in.

ELEMENT NUMBER	SEGMENT LENGTH (in)
1	3.83
2	5.05
3	12.49
4	5.85
5	5.85
6	5.85
7	[illegible].85
8	5.85
9	5.85
10	4.36
11	4.36
12	4.36
13	4.36
14	9.37
15	9.36
16	6.93
17	9.37
18	9.36
19	6.93

TRIPOD PROPERTIES CORRECTIONS

ELEMENT NUMBER	AREA (in^2)	WEIGHT PER UNIT LENGTH (lbs/in)	MOMENT OF INERTIA (in^4)	POLAR MOMENT OF INERTIA (in^4)	OUTER RADIUS (in)
14, 17	.420	.300	.192	.248	.810

TABLE A-IV

TRIPOD SEGMENT LENGTHS

ALL THREE LEGS EXTENDED BY TWO HOLE STOPS

LENGTHS REQUIRED

AL = 6.010 in.

ELEMENT NUMBER	SEGMENT LENGTH (in)
1	5.33
2	3.55
3	13.99
4	5.85
5	5.85
6	5.85
7	5.85
8	5.85
9	5.85
10	4.36
11	4.36
12	4.36
13	4.36
14	4.39
15	14.34
16	10.88
17	4.39
18	14.34
19	10.88

TRIPOD PROPERTIES CORRECTIONS

NONE

TABLE A-V

TRIPOD SEGMENT LENGTHS

ALL THREE LEGS EXTENDED BY THREE HOLE STOPS

LENGTHS REQUIRED

AL = 6.010 in.

ELEMENT NUMBER	SEGMENT LENGTH (in)
1	6.83
2	2.05
3	15.49
4	5.85
5	5.85
6	5.85
7	5.85
8	5.85
9	5.85
10	4.36
11	4.36
12	4.36
13	4.36
14	8.34
15	10.39
16	14.83
17	8.34
18	10.39
19	14.83

TRIPOD PROPERTIES CORRECTIONS

NONE

TABLE A-VI

TRIPOD SEGMENT LENGTHS

ALL THREE LEGS FULLY EXTENDED

LENGTHS REQUIRED

AL = 6.010 in.

ELEMENT NUMBER	SEGMENT LENGTH (in)
1	8.33
2	8.77
3	8.77
4	5.85
5	5.85
6	5.85
7	5.85
8	5.85
9	5.85
10	4.36
11	4.36
12	4.36
13	4.36
14	12.29
15	12.61
16	12.61
17	12.29
18	12.61
19	12.61

TRIPOD PROPERTIES CORRECTIONS

ELEMENT NUMBER	AREA (in^2)	WEIGHT PER UNIT LENGTH (lbs/in)	MOMENT OF INERTIA (in^4)	POLAR MOMENT OF INERTIA (in^4)	OUTER RADIUS (in)
2, 3	.455	.130	.135	.270	.817
15,16; 18,19	.633	.180	.131	.262	.717

A.2.5 Gun Orientation and Lumped Gun Mass Data (Figure A.5)

Card 12 represents the initial gun orientation. The first word is the initial gun elevation angle; the second word is the initial gun azimuth angle; and the last word is the value of the constant V (see Section 3.2). It is not important that the value of the azimuth angle be given very accurately. The program utilizes only the sign of this value by applying it to a computed geometrically compatible value of azimuth passing through one of the traverse bar nodal points.

Card 13 represents the lumped masses due to the gun considered at the pintle and the traverse bar, respectively. These lumped masses are determined from static equilibrium of the gun mass with the reactions at the pintle and traverse bar (see section 3.2).

A.2.6 Soil Properties Data (Figure A.6)

Card 14 supplies the information for the soil properties necessary to compute the soil damping and stiffness (Figure A-6). The first word is the radius of the tripod foot disk; the second word is the soil density; and the third word is the Poisson ratio. Typical values of the Poisson ratio are:

$.25 \leq \nu \leq .35$ for cohesionless soils
$.35 \leq \nu \leq .45$ for cohesive soils.

The fourth word is the void ratio which may vary extensively. For example, the values for the void ratio can be:

e = 0.51 for uniform sand, dense
e = 0.85 for uniform sand, loose
e = 0.25 for glacial till, very mixed-grained
e = 3.0 for soft very organic clay.

The last value represents the soil type index, IG. The only possible indices are:

IG = 1 signifies Angular Sands or Cohesive Soils
IG = 2 signifies Round-grained Sands.

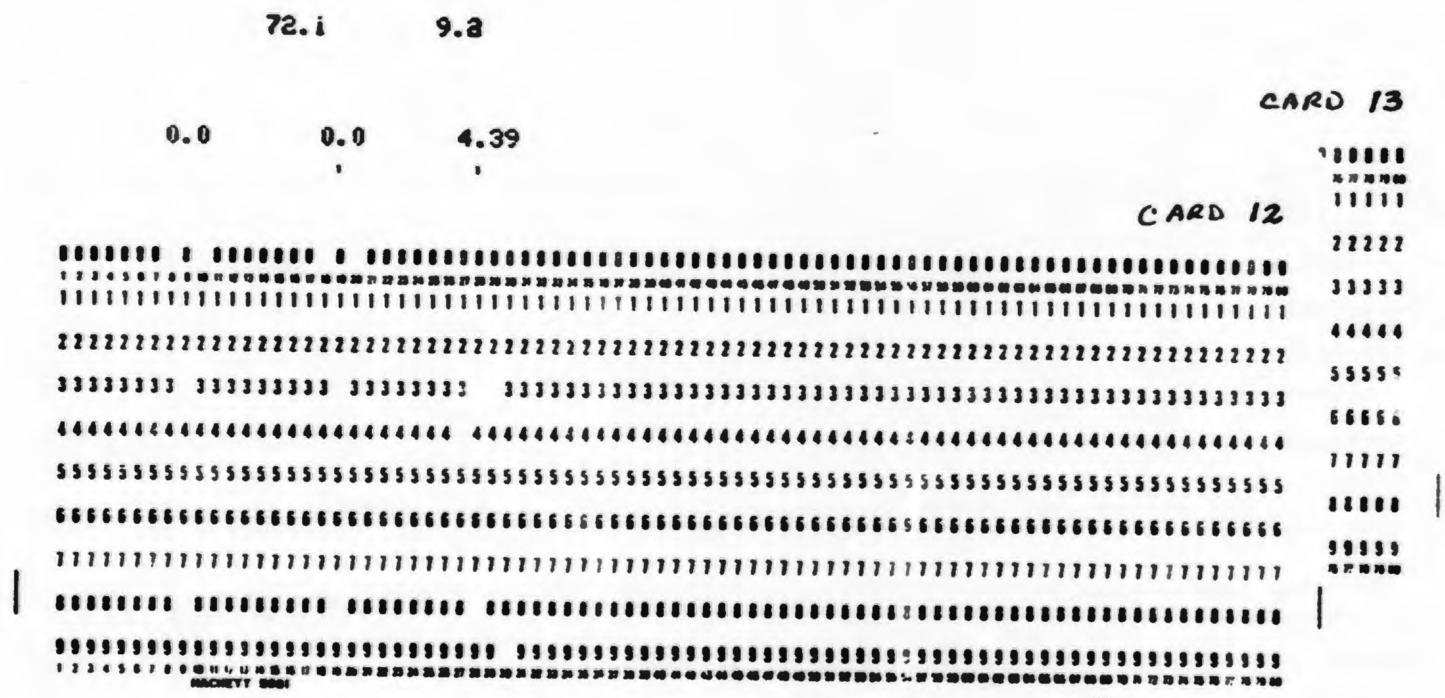


Figure A.5: TYPICAL INITIAL GUN ORIENTATION AND LUMPED GUN MASS CARDS

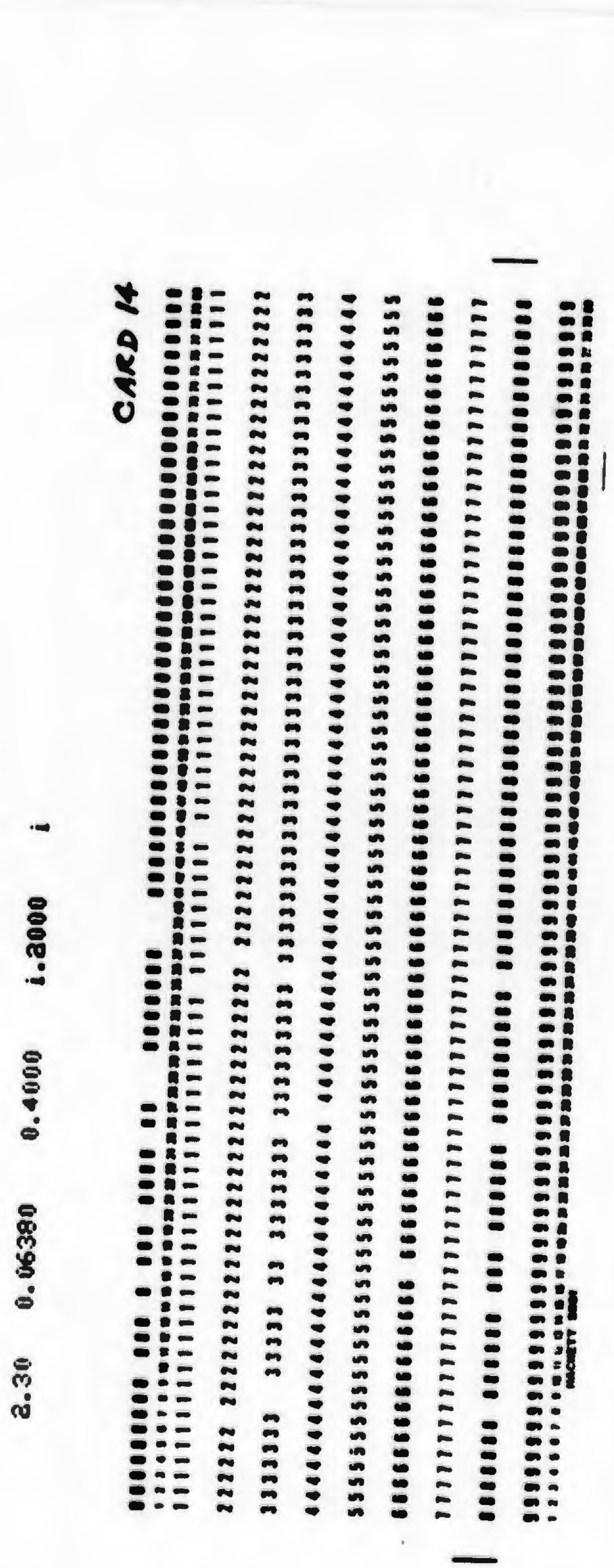


Figure A.6: TYPICAL SOIL PROPERTIES CARD

A.2.7 Preparation of the Forcing Function Data

The input format requires that the forcing function consist of 50 sets of four numbers, where each set specifies the four components of the forcing function at a given time instant during the firing of the gun. The data needed for the preparation of these four components (F_P, F_T, M_C and β) comes from two sources:

1) Computer output of the USAWECOM's machine-gun program, and

2) Certain data relating to the geometry of the gun-tripod system, as follows:

   a) D_1 = 13.50": fixed distance between the points at which the receiver is supported by the tripod.

   b) 15.67" and 5.49: fixed dimensions, shown in Figure A.7.

   c) Barrel elevation angle, ϕ_{oa}. This is one of the variable inputs, already entered on Data Card No. 12.

   d) M_R = 1.406 slugs: mass of the gun receiver.

The output variables from the gun program that must be used are the following (see Figure A.7):

1) Bolt force F_1. This is the sum of forces exerted on the gun receiver by the driving spring, the back-plate buffer spring, and any damping devices associated with the bolt.

2) Barrel force F_2. This is the force that the oil buffer exerts on the gun receiver.

3) Receiver acceleration, a_R. Its positive direction is as shown in Figure A.7.

From the preceding geometrical data, compute the orientation angle of the traverse bar, θ:

$$\theta = \arctan\left[\frac{5.49 - 13.50 \sin\phi_o}{15.67 - 13.50 \cos\phi_o}\right]$$

where

$$\phi_o = \phi_{oa} + 4^\circ 40'$$

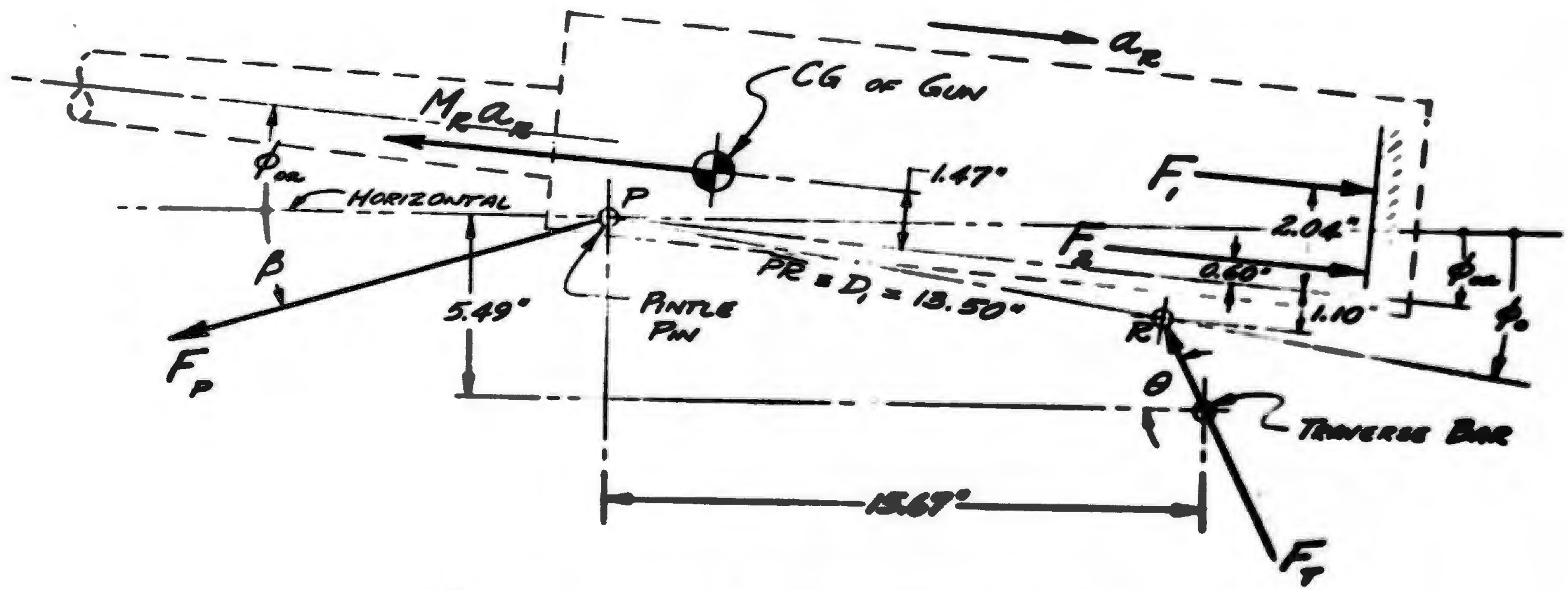


Figure A.7: FREE-BODY DIAGRAM OF THE GUN RECEIVER

With the above-calculated value of θ, one may proceed to compute the forcing function. The equations used for this purpose are derived from the free-body diagram of the gun receiver shown in Figure A.7. The procedure is as follows.

<u>Step 1</u>: Plot the three output functions F_1, F_2 and a_R vs. time, all to a common time scale.

<u>Step 2</u>: Divide the time scale of the above plot into 49 intervals for the purpose of obtaining 50 discrete values of each output function. The time increments resulting from this division need not be equal to each other -- the determination of increment size is left to the discretion of the preparer of input data. Only one rule is suggested: the steeper the F_2-curve, the finer the time divisions in that region.

<u>Step 3</u>: From these three plotted curves, read and record the values of F_1, F_2 and a_R for each time instant t_i (i = 0, 1, 2, ...50). These 50 sets of discrete functional values are now substituted as many times in the equations of Steps 4, 5 and 6.

<u>Step 4</u>: Calculate the 50 discrete values of the traverse-bar force, F_T, from the following expression:

$$F_T = \frac{2.04\,F_1 + 0.60\,F_2 - 1.47\,M_R a_R}{13.50 \sin(\theta - \phi_0)}$$

where M_R = .1172 (lb-in^{-1}-sec^2) must be used if a_R is expressed in (in-sec^2). Naturally, the values of F_1, F_2 and a_R must all be taken at the same time instant t_i.

<u>Step 5</u>: Calculate the 50 discrete values of the pintle-pin force, F_P, from the following expression that employs the previously calculated values of F_T:

$$F_P = \sqrt{\left(\Sigma F \cos\phi_{oa} - F_T \cos\theta\right)^2 + \left(F_T \sin\theta - \Sigma F \sin\phi_{oa}\right)^2}$$

where

$$\Sigma F \equiv F_1 + F_2 - M_R a_R$$

Step 6: Calculate the 50 discrete values of the angle-of-action of the pintle-pin force, β:

$$\beta = \arc \tan \left[\frac{F_T \sin\theta - \Sigma F \sin\phi_{aa}}{\Sigma F \cos\phi_{aa} - F_T \cos\theta} \right]$$

Step 7: Set the pintle moment M_C to zero for all t_i, since the USAWECOM gun program in its present form does not output this variable.

A typical forcing function deck is shown in part in Figure A-8. Card 15 is the time increment (DTAU). Cards 16-65 are the forcing function values containing the time, pintle force, traverse bar force, pintle moment, and pintle force action angle. Card 66 is the maximum time for the cut-off of the transient solution.

The program requires that forcing function cards be less than or equal to 50 in ascending order for time. Should less than 50 cards be used, the last card must have the time (first word of the last card) given as negative. This signals the program that this is the last card and it is treated accordingly.

A.2.8 Forcing Function Data Generator

To facilitate the translation of forcing function information from USAWECOM's computer output to the requirements of GARD's input, an auxiliary program (SJLFF) was written. This program requires that the data outlined in Section A.2.7 be taken and put on input cards for the auxiliary program. All the required calculations are made by the auxiliary program and a set of forcing function cards are output which may then be used in the main program.

The first required data card is the initial elevation angle:

ELEV (FIO.3) (Degrees)

Then a set of data cards are required, each containing:

Time Col. 1-10 (sec) (F10.6)
Bolt Force Col. 11-20 (lbs) (E15 5)
Barrel Force Col. 21-30 (lbs) (E15.4)
Receiver Acceleration Col. 31-40 (in/sec^2) (E15.4)
Pintle Moment Col 41-50 (in-lbs) (E15 3)

```
0.03
-0.067800   -0.18976E 02   0.18948E 02   0.00000E 00   -0.65187E 02   CARD 66
0.067700    0.21679E 03    0.32278E 02   0.00000E 00    0.77791E 01
0.064100   -0.25954E 03   -0.30429E 02   0.00000E 00    0.61182E 01
0.060300    0.22319E 03    0.32341E 02   0.00000E 00    0.75694E 01
0.001600    0.21997E 03   -0.57011E 02   0.00000E 00   -0.13627E 02
0.000001    0.14585E 02    0.26860E 01   0.00000E 00    0.96375E 01
0.000000    0.00000E 00    0.00000E 00   0.00000E 00    0.00000E 00   CARD 16
0.001                                                                 CARD 15
```

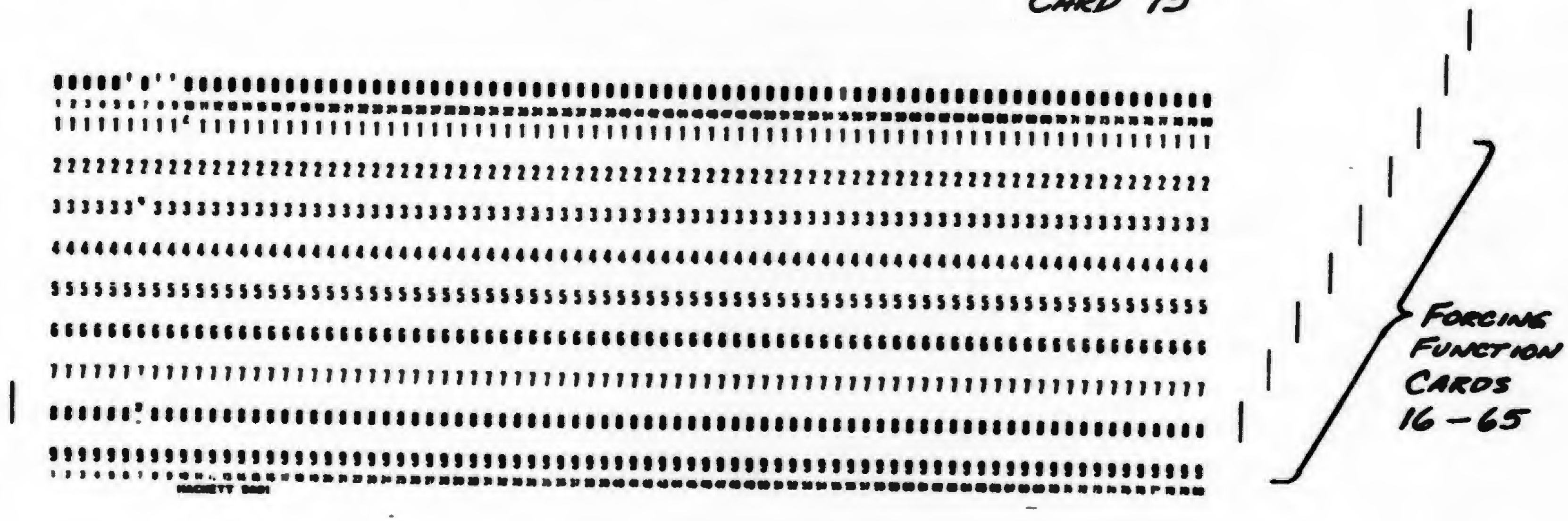


Figure A.8: TYPICAL FORCING FUNCTION CARDS

Again, the auxiliary program requires that these data be less than or equal to 50 cards. If less than 50 cards, the last card must have the time (first word last card) given as negative. The printed output from the auxiliary program is given in Figures A-9 and A-10.

A.2.9 Initial Conditions Data

The computer program normally assumes that the calculations begin at rest for time zero. However, the option is available to input initial conditions, thereby providing a degree of flexibility to the program. To take advantage of this option, a group of 47 data cards are required. Card A contains the initial time (TAU) for the option. Cards B_1-B_{23} contain the displacements and rotations for time TAU, and cards C_1-C_{23} contain the velocities and angular velocities for time TAU. These data must be ordered:

$$\delta_x, \delta_y, \delta_z, \theta_x, \theta_y, \theta_z$$

on each card and the cards must be in sequential order according to the displacement vector given in Sections 5.1 and 5.3.2.

A.2.10 Summary of Input Data

To summarize and simplify input data preparation, the charts presented on the following pages have been prepared. The charts show input data card sequence, format, and data requirements. In short, all that is necessary to make up the input data deck is specified. (See pages A-32 to A-36.)

A.3 Computer Program Output Data

The first three pages of output data of the computer program are a recapitulation of the data used in program execution. The first page (Figure A-11) shows the tripod properties. Element numbers correspond to the segmented tripod sections previously defined. The length of each element follows from input data. The remaining physical properties are those used by the program after all corrections, if any, are made by the input.

The second page (Figure A-12) recapitulates the concentrated mass, mass moments of inertia, gun lumped masses, and soil properties as read in by input data.

The third page (Figure A-13) shows the forcing function data. These data are exactly as read in by the input data deck which may have been generated by the auxiliary program (SJLFF).

FORCING FUNCTION INPUT DATA -

INITIAL GUN ELEVATION ANGLE = 0.000 DEG.

TIME (SEC)	BOLT FORCE (LBS)	BARREL FORCE (LBS)	RECEIVER ACCEL. (IN/SEC2)	PINTLE MOMENT (IN-LBS)
0.000000	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00
0.000001	0.1550E 02	0.0000E 00	0.0000E 00	0.0000E 00
0.001600	0.1605E 02	0.1103E 04	0.7927E 04	0.0000E 00
0.003000	0.1707E 02	0.1290E 04	0.1001E 03	0.0000E 00
0.004189	0.1801E 02	0.1221E 04	-0.1995E 05	0.0000E 00
0.007000	0.2198E 02	0.2836E 04	-0.6781E 04	0.0000E 00
0.007900	0.2371E 02	0.8603E 04	0.4642E 05	0.0000E 00
0.009600	0.2680E 02	0.0000E 00	-0.5318E 05	0.0000E 00
0.012700	0.3313E 02	-0.9[illegible]39E 03	0.3734E 05	0.0000E 00
0.014300	0.3616E 02	-0.4751E 04	0.1581E 05	0.0000E 00
0.015900	0.3897E 02	-0.9967E 03	0.2388E 05	0.0000E 00
0.016900	0.4063E 02	0.0000E 00	-0.5601E 03	0.0000E 00
0.017159	0.1872E 05	0.0000E 00	0.1616E 06	0.0000E 00
0.017890	0.4106E 02	0.0000E 00	-0.6122E 05	0.0000E 00
0.019200	0.4023E 02	0.0000E 00	-0.1023E 06	0.0000E 00
0.020900	0.3968E 02	0.1149E 05	0.5975E 05	0.0000E 00
0.023300	0.3858E 02	-0.7210E 04	-0.5908E 05	0.0000E 00
0.025500	0.3763E 02	0.0000E 00	0.1000E 06	0.0000E 00
0.028400	0.3549E 02	-0.1065E 03	-0.1689E 05	0.0000E 00
0.029000	0.3505E 02	0.0000E 00	-0.2973E 05	0.0000E 00
0.030100	0.3428E 02	0.8516E 04	0.2605E 05	0.0000E 00
0.032000	0.3289E 02	-0.2295E 03	-0.8251E 05	0.0000E 00
0.033500	0.3226E 02	0.1921E 04	0.4627E 04	0.0000E 00
0.035500	0.3131E 02	0.0000E 00	0.6157E 05	0.0000E 00
0.036900	0.3025E 02	-0.9753E 04	-0.4035E 05	0.0000E 00
0.038300	0.2929E 02	0.0000E 00	0.4728E 05	0.0000E 00
0.040100	0.2774E 02	0.2667E 04	0.1798E 05	0.0000E 00
0.041800	0.2615E 02	0.1618E 04	-0.5982E 05	0.0000E 00
0.045300	0.2395E 02	0.0000E 00	0.3450E 04	0.0000E 00
0.046100	0.2347E 02	0.0000E 00	0.2837E 05	0.0000E 00
0.048600	0.2168E 02	-0.2712E 04	0.4410E 05	0.0000E 00
0.048700	0.2161E 02	0.6384E 01	0.1053E 04	0.0000E 00
0.054500	0.1841E 02	-0.1868E 02	-0.1495E 03	0.0000E 00
0.060300	0.1717E 02	-0.2973E 02	-0.2110E 04	0.0000E 00
0.064100	0.1636E 02	-0.3503E 02	0.2151E 04	0.0000E 00
0.067700	0.1553E 02	-0.4090E 02	-0.2164E 04	0.0000E 00
-0.067800	0.0000E 00	-0.2565E 03	-0.2188E 04	0.0000E 00

Figure A.9: Printout of Forcing Function Input Data

FORCING FUNCTION OUTPUT DATA -

TIME (SEC)	PINTLE FORCE (LBS)	TAVERSE BAR FORCE (LBS)	PINTLE MOMENT (IN-LBS)	PINTLE FORCE ACTION ANGLE (DEG)
0.000000	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00
0.000001	0.1458E 02	0.2686E 01	0.0000E 00	0.9637E 01
0.001600	0.2199E 03	-0.5701E 02	0.0000E 00	-0.1362E 02
0.003000	0.1268E 04	0.6724E 02	0.0000E 00	0.2761E 01
0.004189	0.3443E 04	0.3571E 03	0.0000E 00	0.5413E 01
0.007000	0.3556E 04	0.2475E 03	0.0000E 00	0.3628E 01
0.007900	0.3292E 04	-0.2367E 03	0.0000E 00	-0.3748E 01
0.009600	0.5975E 04	0.7829E 03	0.0000E 00	0.6840E 01
0.012700	-0.5045E 04	-0.5875E 03	0.0000E 00	0.6077E 01
0.014300	-0.6387E 04	-0.4672E 03	0.0000E 00	0.3813E 01
0.015900	-0.3610E 04	-0.3935E 03	0.0000E 00	0.5687E 01
0.016900	0.1008E 03	0.1523E 02	0.0000E 00	0.7893E 01
0.017159	-0.9904E 03	0.8791E 03	0.0000E 00	-0.5379E 02
0.017890	0.6888E 04	0.9030E 03	0.0000E 00	0.6844E 01
0.019200	0.1148E 05	0.1504E 04	0.0000E 00	0.6838E 01
0.020900	0.4629E 04	-0.2848E 03	0.0000E 00	-0.3206E 01
0.023300	-0.6472E 03	0.5038E 03	0.0000E 00	-0.4504E 02
0.025500	-0.1115E 05	-0.1456E 04	0.0000E 00	0.6820E 01
0.028400	0.1819E 04	0.2479E 03	0.0000E 00	0.7116E 01
0.029000	0.3359E 04	0.4411E 03	0.0000E 00	0.6856E 01
0.030100	0.5472E 04	0.5874E 02	0.0000E 00	0.5590E 00
0.032000	0.9038E 04	0.1201E 04	0.0000E 00	0.6941E 01
0.033500	0.1396E 04	0.3578E 02	0.0000E 00	0.1334E 01
0.035500	-0.6859E 04	-0.8956E 03	0.0000E 00	0.6817E 01
0.036900	-0.5035E 04	0.9867E 02	0.0000E 00	-0.1020E 01
0.038300	-0.5262E 04	-0.6868E 03	0.0000E 00	0.6814E 01
0.040100	0.6481E 03	-0.1223E 03	0.0000E 00	-0.9885E 01
0.041800	0.8299E 04	0.9624E 03	0.0000E 00	0.6[illegible]1E 01
0.045300	-0.3635E 03	-0.4634E 02	0.0000E 00	0.6654E 01
0.046100	-0.3152E 04	-0.4111E 03	0.0000E 00	0.6809E 01
0.048600	-0.7566E 04	-0.7796E 03	0.0000E 00	0.5376E 01
0.048700	-0.9127E 02	-0.1134E 02	0.0000E 00	0.6485E 01
0.054500	0.1592E 02	0.4427E 01	0.0000E 00	0.1463E 02
0.060300	0.2231E 03	0.3234E 02	0.0000E 00	0.7569E 01
0.064100	-0.2595E 03	-0.3042E 02	0.0000E 00	0.6118E 01
0.067700	0.2167E 03	0.3227E 02	0.0000E 00	0.7779E 01
-0.067800	-0.1897E 02	0.1894E 02	0.0000E 00	-0.6518E 02

Figure A.10 Printout of Forcing Function Output Data

TRIPOD MOUNT M 3 FOR 0.50 CALIBER MACHINE GUN
CASE ANALYZED- ALL THREE LEGS EXTENDED BY TWO HOLE STOPS

TRIPOD PROPERTIES -

INCLUDED ANGLE FROM FRONT TO BACK LEGS = 137.333 DEG.

FRONT LEG ANGLE OF KINK = 0.000 DEG.

FIXED FRONT LEG LENGTH FROM PINTLE = 6.010 IN.

(LL) ELEMENT NUMBER	(L) LENGTH (IN)	(A) AREA (IN2)	(W) WEIGHT PER UNIT LENGTH (LBS/IN)	(I) MOMENT OF INERTIA (I.4)	(J) POLAR MOMENT OF INERTIA (IN4)	(C) OUTER RADIUS (IN)
1	5.330	0.633	0.180	0.1310	0.2620	0.717
2	3.550	0.455	0.310	0.1990	0.2700	0.817
3	13.990	0.455	0.130	0.1350	0.2700	0.817
4	5.850	0.420	0.120	0.1240	0.2480	0.810
5	5.850	0.420	0.120	0.1240	0.2480	0.810
6	5.850	0.420	0.120	0.1240	0.2480	0.810
7	5.850	0.420	0.120	0.1240	0.2480	0.810
8	5.850	0.420	0.120	0.1240	0.2480	0.810
9	5.850	0.420	0.120	0.1240	0.2480	0.810
10	4.360	0.490	0.140	0.0429	0.0858	0.502
11	4.360	0.490	0.140	0.0429	0.0858	0.502
12	4.360	0.490	0.140	0.0429	0.0858	0.502
13	4.360	0.490	0.140	0.0429	0.0858	0.502
14	4.390	0.420	0.120	0.1240	0.2480	0.810
15	14.340	0.420	0.300	0.1920	0.2480	0.810
16	10.880	0.633	0.180	0.1310	0.2620	0.717
17	4.390	0.420	0.120	0.1240	0.2480	0.810
18	14.340	0.420	0.300	0.1920	0.2480	0.810
19	10.880	0.633	0.180	0.1310	0.2620	0.717

Figure A.11: Tripod Properties Output

TRIPOD MOUNT M 3 FOR 0.50 CALIBER MACHINE GUN
CASE ANALYZED- ALL THREE LEGS EXTENDED BY TWO HOLE STOPS

CONCENTRATED MASSES AND MASS MOMENTS OF INERTIA -

NODAL POINT	CONC. MASS M (LBS)	MASS MOMENTS OF INERTIA (IN - LBS - SEC 2) JX	JY	JZ
1	1.480	0.0058	0.0115	0.0058
4	8.500	0.0200	0.1200	0.0700
7	2.470	0.0010	0.0040	0.0020
10	3.470	0.0020	0.0070	0.0040
16	1.480	0.0058	0.0115	0.0058
19	1.480	0.0058	0.0115	0.0058

ADDITIONAL CONCENTRATED MASS DUE TO GUN -

72.100 LBS ADDED AT NODAL POINT 4

9.200 LBS ADDED AT NODAL POINT 12

SOIL PROPERTIES -

RADIUS OF FOOT DISK = 2.300 IN.

SOIL DENSITY = 0.0638 LBS/IN3

POISSON RATIO = 0.40

VOID RATIO = 1.20

SOIL TYPE = ANGULAR SANDS OR COHESIVE SOILS

Figure A.12: Recapitulated Concentrated Masses, Mass Moments of Inertia and Soil Properties

FORCING FUNCTION - TRIPOD MOUNT M 3 FOR 0.50 CALIBER MACHINE GUN
CASE ANALYZED- ALL THREE LEGS EXTENDED BY TWO HOLE STOPS

INITIAL GUN ELEVATION ANGLE = 0.000 DEG.
INITIAL GUN AZIMUTH ANGLE = 0.000 DEG.

TIME (SEC)	PINTLE FORCE (LBS)	TRAVERSE BAR FORCE (LBS)	PINTLE MOMENT (IN-LBS)	PINTLE FORCE ACTION ANGLE (DEG)
0.000000	0.0000E 00	0.0000E 00	0.0000E 00	0.0000E 00
0.000001	0.1458E 02	0.2686E 01	0.0000E 00	0.9637E 01
0.001600	0.2199E 03	-0.5701E 02	0.0000E 00	-0.1362E 02
0.003000	0.1268E 04	0.6724E 02	0.0000E 00	0.2761E 01
0.004169	0.3443E 04	0.3573E 03	0.0000E 00	0.5413E 01
0.007000	0.3556E 04	0.2475E 03	0.0000E 00	0.3628E 01
0.007900	0.3292E 04	-0.2367E 03	0.0000E 00	-0.3748E 01
0.009600	0.5975E 04	0.7829E 03	0.0000E 00	0.6840E 01
0.012700	-0.5045E 04	-0.5875E 03	0.0000E 00	0.6077E 01
0.014300	-0.6387E 04	-0.4672E 03	0.0000E 00	0.3813E 01
0.015900	-0.3610E 04	-0.3935E 03	0.0000E 00	0.5687E 01
0.016700	0.1008E 03	0.1523E 02	0.0000E 00	0.7893E 01
0.017159	-0.9904E 03	0.8791E 03	0.0000E 00	-0.5379E 02
0.017890	0.6888E 04	0.9030E 03	0.0000E 00	0.6844E 01
0.019200	0.1148E 05	0.1504E 04	0.0000E 00	0.6838E 01
0.020900	0.4623E 04	-0.2845E 03	0.0000E 00	-0.3206E 01
0.023300	-0.6472E 03	0.5338E 03	0.0000E 00	-0.4504E 02
0.025500	-0.1115E 05	-0.1456E 04	0.0000E 00	0.6820E 01
0.028400	0.1819E 04	0.2479E 03	0.0000E 00	0.7116E 01
0.029000	0.3359E 04	0.4411E 03	0.0000E 00	0.6856E 01
0.030100	0.5472E 04	0.5874E 02	0.0000E 00	0.5590E 00
0.032000	0.9038E 04	0.1201E 04	0.0000E 00	0.6941E 01
0.033500	0.1376E 04	0.3578E 02	0.0000E 00	0.1334E 01
0.035500	-0.6859E 04	-0.8956E 03	0.0000E 00	0.6817E 01
0.036900	-0.5035E 04	0.9867E 02	0.0000E 00	-0.1020E 01
0.038300	-0.5262E 04	-0.6868E 03	0.0000E 00	0.6814E 01
0.040100	0.6481E 03	-0.1223E 03	0.0000E 00	-0.9885E 01
0.041800	0.8299E 04	0.9624E 03	0.0000E 00	0.6051E 01
0.045300	-0.3635E 03	-0.4634E 02	0.0000E 00	0.6654E 01
0.046100	-0.3152E 04	-0.4111E 03	0.0000E 00	0.6809E 01
0.046600	-0.7566E 04	-0.7798E 03	0.0000E 00	0.5376E 01
0.048700	-0.9127E 02	-0.1134E 02	0.0000E 00	0.6485E 01
0.054500	0.1592E 02	0.4427E 01	0.0000E 00	0.1563E 02
0.060300	0.2231E 03	0.3234E 02	0.0000E 00	0.7569E 01
0.064100	-0.2595E 03	-0.3042E 02	0.0000E 00	0.6118E 01
0.067700	0.2167E 03	0.3227E 02	0.0000E 00	0.7779E 01
0.067800	-0.1897E 02	0.1894E 02	0.0000E 00	-0.6518E 02

Figure A.13: Forcing Function Output

Beginning with the fourth page of the output, the transient response of the tripod system is given. The displacements and velocities for the pintle (nodal point 4) and each of the leg nodal points in contact with the ground are given (nodal points 1, 16, 19). The change in elevation angle is given as well as the azimuth angle of the gun barrel. At the present time, however, the azimuth angle is not being computed so that the azimuth angle does not change from the given initial value computed from the input.

The peak combined stresses for selected elements in the rear legs and traverse bar are given after each time increment. These stresses are computed after the solution for all nodal points is obtained.

The transient response calculation continues up to the maximum time TMAX as prescribed by input data card no. 66. At which time the program calls EXIT. Program running time is approximately 20 minutes per time increment on the IBM 1130. Based on a rule of thumb of a 10:1 ratio, this should correspond to approximately 2 minutes per time increment for the IBM 360-65 system.

WORD	1
COLUMNS	1-80
FORMAT	20A4
CARD 1	Identification Caption Card
SYMBOL	CAP(I), I = 1,20

WORD	1	2	3	4	5	6	7	8
COLUMNS	1-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80
FORMAT	8F10.3							
CARD 2	Fixed Front Leg Length (in)	Length of Element 1 (in)	Length of Element 2 (in)	Length of Element 3 (in)	Length of Element 4 (in)	Length of Element 5 (in)	Length of Element 6 (in)	Length of Element 7 (in)
SYMBOL	AL	SEG(1)	SEG(2)	SEG(3)	SEG(4)	SEG(5)	SEG(6)	SEG(7)

WORD	1	2	3	4	5	6	7	8
COLUMNS	1-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80
FORMAT	8F10.3							
CARD 3	Length of Element 8 (in)	Length of Element 9 (in)	Length of Element 10 (in)	Length of Element 11 (in)	Length of Element 12 (in)	Length of Element 13 (in)	Length of Element 14 (in)	Length of Element 15 (in)
SYMBOL	SEG(8)	SEG(9)	SEG(10)	SEG(11)	SEG(12)	SEG(13)	SEG(14)	SEG(15)

WORD	1	2	3	4	5	6
COLUMNS	1-10	11-20	21-30	31-40	41-50	51-60
FORMAT	6F10.3					
CARD 4	Length of Element 16 (in)	Length of Element 17 (in)	Length of Element 18 (in)	Length of Element 19 (in)	Angle from Front to Back Leg (DEG)	Angle of Front Leg Kink (DEG)
SYMBOL	SEG(16)	SEG(17)	SEG(18)	SEG(19)	ALF	GAM

WORD	1	2	3	4	5	6
COLUMNS	1-10	11-20	21-30	31-40	41-50	51-60
FORMAT	I10, 5F10.3					
CARD 5	Element Number	Area (in^2)	Weight per Unit Length (lbs/in)	Moment of Inertia (in^4)	Polar Moment of Inertia (in^4)	Outer Radius (in)
	0	SIGNIFIES NO MORE TRIPOD PROPERTIES CORRECTIONS				
SYMBOL	LL	PROP(LL,1)	PROP(LL,2)	PROP(LL,3)	PROP(LL,4)	PROP(LL,5)

NOTE: If corrections are made, a Zero card must appear after the last correction card.

WORD	1	2	3	4
COLUMNS	1-10	11-20	21-30	31-40
FORMAT	4F10.3			
CARD 6-11	Concentrated Mass (lbs)	Mass Moments of Inertia About X Axis	About Y Axis ($in-lbs-sec^2$)	About Z Axis
SYMBOL	AM(I)	AJ(I,1)	AJ(I,2)	AJ(I,3) I = 1,5

NOTE: These data are for nodal points 1, 4, 7, 10, 16, 19, respectively.

WORD	1	2	3
COLUMNS	1-10	11-20	21-30
FORMAT	3F10.3		
CARD 12	Initial Gun Elevation Angle (DEG)	Initial Gun Azimuth Angle (DEG)	Geometric Distance (See Figure) (in)
SYMBOL	ELEV	AZIM	VO

WORD	1	2
COLUMNS	1-10	11-20
FORMAT	2F10.3	
CARD 13	Weight of Gun Lumped at Pintle (lbs)	Weight of Gun Lumped at Traverse (lbs)
SYMBOL	WTP	WTT

WORD	1	2	3	4	5
COLUMNS	1-10	11-20	21-30	31-40	45
FORMAT	4F10.3, I5				
CARD 14	Soil Properties Card Radius of Foot Disk (in)	Soil Density (lbs/in3)	Poisson Ratio of Soil	Void Ratio of Soil	Soil Type Index
SYMBOL	RO	RHØ	ANU	E	IG

NOTE: IG = 1 Signifies angular sands or cohesive soils
IG = 2 Signifies round-grained sands

WORD	1
COLUMNS	1-10
FORMAT	F10.3
CARD 15	Time Increment (sec)
SYMBOL	DTAU

WORD	1	2	3	4	5
COLUMNS	1-10	11-20	21-30	31-40	41-50
FORMAT	5F10.3				
CARD 16-65	Forcing Function Cards Time (sec)	Pintle Force (lbs)	Traverse Bar Force (lbs)	Pintle Moment (in-lbs)	Pintle Force Action Angle (DEG)
SYMBOL	F(I,1)	F(I,2)	F(I,3)	F(I,4)	F(I,5)

NOTE: If less than 50 forcing function cards are used make last F(I,1) negative.

WORD	1
COLUMNS	1-10
FORMAT	F10.3
CARD 66	Maximum Time (sec)
SYMBOL	TMAX

OPTIONAL CARDS: If initial conditions are given place cards A, $B_1 \ldots B_{23}$, $C_1 \ldots C_{23}$ before card 66.

WORD	1
COLUMNS	1-10
FORMAT	F10.3
CARD A	Initial Time (sec)
SYMBOL	TAU

WORD	1	2	3	4	5	6
COLUMNS	1-13	14-26	27-39	40-52	53-65	66-78
FORMAT	6E13.4					
CARD $B_1 - B_{23}$	Displacements of Nodal Point I			Rotations of Nodal Point I		
	δ_x (in)	δ_y (in)	δ_z (in)	θ_x (radians)	θ_y (radians)	θ_z (radians)
SYMBOL	D(J1)	D(J1+1)	D(J1+2)	D(J1+3)	D(J1+4)	D(J1+5)

NOTE: (J1 = 6I - 5), I = 1,23
There are 23 cards in this set.

WORD	1	2	3	4	5	6
COLUMNS	1-13	14-26	27-39	40-52	53-65	66-78
FORMAT	6E13.4					
CARD $C_1 - C_{23}$	Velocities of Nodal Point I			Angular Velocities of Nodal Point I		
	$\dot{\delta}_x$ (in/sec)	$\dot{\delta}_y$ (in/sec)	$\dot{\delta}_z$ (in/sec)	$\dot{\theta}_x$ (radians/sec)	$\dot{\theta}_y$ (radians/sec)	$\dot{\theta}_z$ (radians/sec)
SYMBOL	DOT(J1)	DOT(J1+1)	DOT(J1+2)	DOT(J1+3)	DOT(J1+4)	DOT(J1+5)

NOTE: (J1=6I-5),I=1,23
There are 23 cards in this set.

APPENDIX B

FORTRAN LISTING OF COMPUTER PROGRAM

PROGRAM SJLMS (LINK 1)

```
C
C
C     A DIGITAL COMPUTER PROGRAM TO COMPUTE THE TRANSIENT RESPONSE OF
C     THE TRIPOD MOUNT M 3 FOR THE 0.50 CALIBER MACHINE GUN
C     UNDER CONTRACT NO.  DAAF03-70-C-0032
C         U.S. ARMY WEAPONS COMMAND,  ROCK ISLAND, ILLINOIS
C       BY GENERAL AMERICAN REASEARCH DIVISION
C         GENERAL AMERICAN TRANSPORTATION CORPORATION
C         7449 N. NATCHEZ AVE.    NILES, ILLINOIS  60648
C
C     PROGRAMMED IN  FORTRAN IV  FOR THE I B M  1130 D O S
C        BY   S.J. LIS,  RESEARCH ENGINEER
C             R. VAITYS,  PROJECT ENGINEER
C             A.N. HENZI,    RESEARCH ENGINEER
C
C     INPUT DATA
C          CAP  -ALPHAMERIC CAPTION INFORMATION
C          AL   -FIXED FRONT LEG LENGTH FROM PINTLE, IN.
C          SEG  -SEGMENT LENGTHS FOR EACH OF 19 ELEMENTS,  IN.
C          ALF  -INCLUDED ANGLE FROM FRONT TO BACK LEGS,  DEG.
C          GAM  -FRONT LEG ANGLE OF KINK,  DEG.
C          LL   -ELEMENT INDEX
C          PROP -A (19X5) ARRAY OF TRIPOD MATERIAL PROPERTIES
C                PROP(LL,1)  AREA, IN2
C                PROP(LL,2)  WEIGHT PER UNIT LENGTH,  LBS/IN
C                PROP(LL,3)  MOMENT OF INERTIA,  IN4
C                PROP(LL,4)  POLAR MOMENT OF INERTIA,  IN4
C                PROP(LL,5)  OUTER RADIUS, IN.
C          ELEV -INITIAL GUN ELEVATION ANGLE, DEG.
C          AZIM -INITIAL GUN AZIMUTH ANGLE,  DEG.
C          AM   -AN ARRAY OF CONCENTRATED MASSES,  LBS.
C          AJ   -AN ARRAY OF MASS MOMENTS OF INERTIA,  IN-LBS-SEC2
C          WTP  -ADDITIONAL CONC. MASS OF GUN LUMPED AT PINTLE,  LBS
C          WTT  -ADDITIONAL CONC. MASS OF GUN LUMPED AT TRAVERSE BAR
C          RO   -RADIUS OF FOOT DISK,  IN.
C          RHO  -SOIL DENSITY,  LBS/IN3
C          ANU  -SOIL POISSON RATIO
C          E    -SOIL VOID RATIO
C          IG   -SOIL TYPE INDEX
C          TAU  -TIME,  SEC.
C          DTAU -TIME INCREMENT, SEC.
C          F    -AN ARRAY OF THE FORCING FUNCTION
C                F(I,1)  TIME, SEC.
C                F(I,2)  PINTLE FORCE COMPONENT,  LBS.
C                F(I,3)  TRAVERSE BAR FORCE COMPONENT,  LBS.
C                F(I,4)  PINTLE MOMENT,  IN-LBS
C                F(I,5)  ANGLE OF ACTION FOR PINTLE FORCE,  DEG.
C
C     DATA SWITCHES
C
C          NORMAL OPERATION IS WITH ALL SWITCHES OFF
C
C          SW 0 ON, INITIAL DISPL. AND VELOCITIES ARE INPUT
C          SW 1 ON, MASS AND STFNS MATRICES ARE OUTPUT
C          SW 2 ON, MASS AND STFNS SUB-MATRICES ARE OUTPUT
C          SW 3 ON, BOUNDARY CONDITIONS ARE OUTPUT
C          SW 4 ON, CONC. MASS AND STFNS SUB-MATRICES ARE OUTPUT
C          SW 5 ON, SOIL DAMPING MATRIX IS OUTPUT
C          SW 6 ON, (Q)  MATRIX IS OUTPUT
C          SW 7 ON, EACH ITERATION OF SOLN. FOR ACCEL. IS OUTPUT
C          SW 8 ON, DISPL.,VEL.,ACCEL., OF EACH NODAL POINT IS OUTPUT
C
C
```

```
C         PROGRAM   SJLMS     (LINK 1)
C          MASS AND STIFFNESS MATRIX WRITER
C
C          SUPPORTING SUBROUTINES
C              SJLTI    TRANSFORMATION MATRIX WRITER
C              SJLMX    MATRIX ELEMENT WRITER
C                   MTRA     MATRIX TRANSPOSE  (IBM SSP)
C                   GMPRD    GENERAL MATRIX PRODUCT  (IBM SSP)
C              SJLFY    FILE SORTER
C              SJLBD    BOUNDARY CONDITIONS WRITER
C
C          REQ'D INPUT DATA
C              CAP    (20A4)
C              AL,(SEG(I),I=1,7)    (8F10.3)
C              (SEG(I),I=8,15)    (8F10.3)
C              (SEG(I),I=16,19),ALF,GAM    (6F10.3)
C              I, (ALL(J),J=1,5)    (I10,5F10.3)
C                NOTE-  I=0   SIGNIFIES NO FURTHER CORRECTIONS OF PROP
C                       I=LL  SIGNIFIES PROP CORRECTIONS BEING MADE
C                         I=0 IS REQ'D AFTER LAST CORRECTION CARD
C
C          NORMAL OUTPUT
C              ALF    INCLUDED ANGLE FROM FRONT TO BACK LEGS
C              GAM    FRONT LEG ANGLE OF KINK
C              AL     FIXED FRONT LEG LENGTH FROM PINTLE
C              LL,L,((PROP(I,J),J=1,5),I=1,19)    TRIPOD PROPERTIES
C
      DEFINE FILE 1(138,276,U,N), 2(138,276,U,N)
      DIMENSION PROP1(19),PROP2(19),PROP3(19),PROP4(19),PROP5(19)
      DIMENSION ALL(6),AMTX(6,6,4),FYLE(138),TI(3,3),PROP(19,5)
      COMMON N
      COMMON THET1,THET2,THET3,THET4,PRO (19,5),SEG(19),CAP(20)
      EQUIVALENCE (PROP(1,1),PROP1(1)), (PROP(1,2),PROP2(1))
      EQUIVALENCE (PROP(1,3),PROP3(1)), (PROP(1,4),PROP4(1))
      EQUIVALENCE (PROP(1,5),PROP5(1))
      DATA PROP1/0.633,2*0.455,6*0.420,4*0.490,2*0.420,0.633,2*0.420,
     1 0.633/
      DATA PROP2/0.180,0.310,0.130,6*0.120,4*0.140,0.120,0.300,0.180,
     1 0.120,0.300,0.180/
      DATA PROP3/0.131,0.199,0.135,6*0.124,4*0.0429,0.124,0.192,0.131,
     1 0.124,0.192,0.131/
      DATA PROP4/0.262,2*0.270,6*0.248,4*0.0858,2*0.248,0.262,2*0.248,
     1 0.262/
      DATA PROP5/0.717,2*0.817,6*0.810,4*0.502,2*0.810,0.717,2*0.810,
     10.717/
   10 FORMAT(20A4)
   11 FORMAT(3(8F10.3,/))
   12 FORMAT(1H1,'PRINT OUT OF MASS MATRIX')
   13 FORMAT(1H1,'PRINT OUT OF STIFFNESS MATRIX')
   14 FORMAT(1H0,2I5)
   15 FORMAT(1H ,I10,6E18.4)
   16 FORMAT(1H0,20X,I4,F14.3,F10.3,F14.3,2F15.4,F14.3)
   17 FORMAT(I10,5F10.3)
   18 FORMAT(1H1,20X,'TRIPOD MOUNT M 3 FOR 0.50 CALIBER MACHINE GUN',/
     121X,'CASE ANALYZED- ',20A4,//,1X,'TRIPOD PROPERTIES -')
   19 FORMAT(1H ,20X,'INCLUDED ANGLE FROM FRONT TO BACK LEGS =',F10.3,
     12X,'DEG.')
   20 FORMAT(1H0,35X,'FRONT LEG ANGLE OF KINK =',F10.3,2X,'DEG.')
   21 FORMAT(1H0,24X,'FIXED FRONT LEG LENGTH FROM PINTLE =',F10.3,2X,
     1'IN.')
   22 FORMAT(1H0,22X,'(LL)',8X,'(L)',7X,'(A)',10X,'(W)',12X,'(I)',13X,
     1'(J)',11X,'(C)',/,21X,'ELEMENT',5X,'LENGTH',5X,'AREA',6X,'WEIGHT',
     2' PER',6X,'MOMENT OF',5X,'POLAR MOMENT',6X,'OUTER',/,22X,'NUMBER',
     326X,'UNIT LENGTH',6X,'INERTIA',7X,'OF INERTIA',6X,'RADIUS',/,34X,
     4'(IN)',5X,'(IN2)',8X,'(LBS/IN)',8X,'(IN4)',11X,'(IN4)',9X,
     5'(IN)',/,19X,90('-'))
C
```

```
C        IN=2  SIGNIFIES LOGICAL UNIT 2 (CARD) INPUT
C        MO=5  SIGNIFIES LOGICAL UNIT 5 (PRINTER) OUTPUT
C
      IN=2
      MO=5
C
C        CLEAR FILES
C
      DO 100 I=1,138
  100 FYLE(I)=0.0
      DO 110 KASE=1,2
      N=1
      DO 110 I=1,138
  110 WRITE(KASE'N) FYLE
C
C        READ TRIPOD GEOMETRICAL DATA
C
      E=29.0E+06
      G=11.0E+06
      GVTY=386.4
      DNSTY=0.285/GVTY
      READ(IN,10 )  CAP
      READ(IN,11 )  AL,SEG,ALF,GAM
  120 READ(IN,17 ) I,(ALL(J), J=1,5)
      IF(I)   130,  150,  130
  130 DO 140 J=1,5
  140 PROP(I,J)= ALL(J)
      GO TO   120
C
C        WRITE OUT TRIPOD PROPERTIES
C
  150 WRITE(MO,18 )  CAP
      WRITE(MO,19 ) ALF
      WRITE(MO,20 ) GAM
      WRITE(MO,21 )  AL
      WRITE(MO,22 )
      DO 160 I=1,19
      DO 160 J=1,5
  160 PRO (I,J)=PROP(I,J)
      DO 170 I=1,19
  170 WRITE(MO,16 )  I,SEG(I),(PROP(I,J), J=1,5)
      ALF=0.01745*ALF
      GAM=0.01745*GAM
C
C        COMPUTE SECTION LENGTHS BY SUMMATION OF ELEMENTS
C
      IJ=0
      DO 210 J=1,6
      IMAX=3
      ALL(J)=0.0
      IF(J-4)   190,  180,  190
  180 IMAX=4
  190 DO 200 I=1,IMAX
      II=IJ + I
  200 ALL(J)=ALL(J) + SEG(II)
  210 IJ=IJ + IMAX
C
C        COMPUTE GEOMETRY OF SYSTEM
C
      PI=3.14159
      S=0.0
      DO 220 J=2,4
  220 S=S + ALL(J)
      S=0.5 * S
      SML=(S-ALL(2)) *(S- ALL(3))
      PHI=2.0*ATAN(SQRT(SML/(ALL(2)*ALL(3)-SML)))
      B5=(ALL(2)+ALL(5))*COS(PHI/2.0)
      GAMO=ALL(1)*COS(GAM) - B5 * COS(ALF) + AL
      GAMO=GAMO/(ALL(1)*SIN(GAM) + B5*SIN(ALF) )
      GAMO=ATAN(GAMO)
```

```
      THET1=PI/2.0 - GAM - GAMO
      THETO=PI/2.0 - GAMO
      ALO=ALL(1)*SIN(THET1)/SIN(THETO)
      B6=B5*SIN(ALF-GAMO)
      B0=(AL+ALO)*SIN(GAMO)
      H=SQRT(B5**2-B6**2)
      THET4=ATAN(H/B6)
      B2=SQRT((ALL(2)+ALL(5))**2 - H**2)
      THET2=ATAN(H/B2)
      B3=ALL(4)*(ALL(2) + ALL(5))/ALL(2)
      THET3=    ATAN(B3/(2.0*B6))
C
C        KASE= 1   SIGNIFIES MASS MATRIX
C        KASE= 2   SIGNIFIES STIFFNESS MATRIX
C
C        KODE= 1   ELEMENTS WHICH REQUIRE TRANSFORMATION   1
C        KODE= 2   ELEMENTS WHICH REQUIRE TRANSFORMATION   2
C        KODE= 3   ELEMENTS WHICH REQUIRE TRANSFORMATION   3
C        KODE= 4   ELEMENTS WHICH REQUIRE TRANSFORMATION   4
C        KODE= 5   ELEMENTS WHICH REQUIRE TRANSFORMATION   2
C        KODE= 6   ELEMENTS WHICH REQUIRE TRANSFORMATION   3
C
      N=1
      KASE=1
      LUMP=0
  230 DO 510 L=1,22
      IF(L-4)    240,   510,   250
  240 LL=L
      KODE=1
      GO TO   440
  250 IF(L-6)    510,   260,   260
  260 IF(L-8)    290,   280,   270
  270 IF(L-9)    280,   400,   310
  280 KODE=3
      GO TO   300
  290 KODE=2
  300 LL=L-1
      GO TO   440
  310 IF(L-12)    320,   410,   330
  320 LL=L-2
      KODE=3
      GO TO   440
  330 IF(L-14)    510,   350,   340
  340 IF(L-17)    350,   420,   370
  350 KODE=4
  360 LL=L-3
      GO TO   440
  370 IF(L-20)    380,   430,   390
  380 KODE=5
      GO TO   360
  390 KODE=6
      GO TO   360
  400 LL=4
      KODE=2
      LUMP=1
      GO TO   440
  410 LL=10
      KODE=4
      LUMP=2
      GO TO   440
  420 LL=14
      KODE=5
      LUMP=3
      GO TO   440
  430 LL=17
      KODE=6
```

```
      LUMP=6
  440 DO 450 I=1,6
      DO 450 J=1,6
      DO 450 K=1,6
  450 AMTX(I,J,K)=0.0
C
C          SETUP TRANSFORMATION MATRIX
C
      CALL SJLTI(KODE,TI)
  460 IF(KASE - 1)   470,  470,  480
C
C          COMPUTE MASS MATRIX TERMS
C
  470 ALPHA=PROP(LL,2) * SEG(LL) / (6.0*GVTY)
      BETA=13.0 * PROP(LL,2) * SEG(LL) / (35.0 * GVTY)
      GAMMA=DNSTY * PROP(LL,4) * SEG(LL) / 6.0
      DELTA=11.0 * PROP(LL,2) * SEG(LL)**2 / (210.0 * GVTY)
      AMBDA=9.0 * PROP(LL,2) * SEG(LL) / (70.0 * GVTY)
      AMU=13.0 * PROP(LL,2) * SEG(LL)**2 / (420.0 * GVTY)
      RHO=PROP(LL,2) * SEG(LL)**3 / (105.0 * GVTY)
      TAU=PROP(LL,2) * SEG(LL)**3 / (140.0 * GVTY)
      GO TO   490
C
C          COMPUTE STIFFNESS MATRIX TERMS
C
  480 ALPHA=PROP(LL,1)* E / SEG(LL)
      BETA=12.0 * E * PROP(LL,3)/ SEG(LL)**3
      GAMMA=G * PROP(LL,4)/ SEG(LL)
      DELTA=6.0 * E * PROP(LL,3)/ SEG(LL)**2
      AMBDA=2.0 * E * PROP(LL,3)/ SEG(LL)
C
C          COMPUTE ELEMENT MATRICES AND SORT INTO FILES
C
  490 CALL SJLMX(KASE,ALPHA,BETA,GAMMA,DELTA,AMBDA,AMU,RHO,TAU,AMTX,TI,
     1            L)
      CALL SJLFY(L,LUMP,KASE,AMTX,FYLE)
      IF(LUMP)    510,  510,  500
  500 LUMP=0
  510 CONTINUE
      IF(KASE - 1)   520,  520,  540
C
C         COMPUTE BOUNDARY CONDITIONS
C
  520 C4=COS(THET4)
      S4=SIN(THET4)
      DO 530 KODE=1,6
      II=3*(KODE-1) + 4
      ALPHA=SIN(THET2)*COS(THET3)
      BETA=3.0/(2.0*SEG(II))
      GAMMA=COS(THET2)*COS(THET4) + ALPHA*SIN(THET4)
      CALL SJLBD(KODE,C4,S4,ALPHA,BETA,GAMMA,AMTX,FYLE)
  530 CONTINUE
      KASE=2
      GO TO   230
C
C          SW 1  ON  WRITE OUT STORED MATRICES
C          SW 1 OFF  LINK TO SJLMI
C
  540 CALL DATSW(1,ISW)
      GO TO (550,660),ISW
  550 KASE=1
      WRITE(NO,12 )
  560 N=1
      LINE=0
      DO 640 I=1,138
      II=(I-1)/6 + 1
      READ(KASE'N) FYLE
      WRITE(NO,14 )  II,I
      J1=1
      J2=6
```

```
      DO 600 K=1,23
      DO 570 KJ=J1,J2
      IF(FYLE(KJ))   580,  570,  580
  570 CONTINUE
      GO TO   590
  580 WRITE(MO,15 )  K,(FYLE(J), J=J1,J2)
      LINE =  LINE  + 1
  590 J1=J1+6
  600 J2=J2 + 6
      IF(LINE-48)   640,  610,  610
  610 LINE=0
      IF(KASE - 1)   620,  620,  630
  620 WRITE(MO,12 )
      GO TO   640
  630 WRITE(MO,13 )
  640 CONTINUE
      IF(KASE-1)   650,  650,  660
  650 WRITE(MO,13 )
      KASE = 2
      GO TO   560
  660 CALL LINK(SJLMI)
      END
```

PROGRAM SJLMI (LINK 2)

```
C
C
C          PROGRAM  SJLMI      (LINK 2)
C          MASS AND STIFFNESS MATRIX MODIFIER
C
C          SUPPORTING SUBROUTINES
C              NONE
C
C          REQ'D INPUT DATA
C              (AM(I),(AJ(I,J),J=1,3)),I=1,6)    (4F10.3)
C              ELEV,AZIM,VO    (3F10.3)
C              WTP,WTT     (2F10.3)
C              RO,RHO,ANU,E,IG.   (4F10.3,I5)
C
C          NORMAL OUTPUT
C              NP,AM(I),(AJ(I,J),J=1,3)    CONC. MASSES AND MASS MOMENTS
C              WTP,WTT    ADDITIONAL CONC. MASS DUE TO GUN
C              RO,RHO,ANU,E,IG      SOIL PROPERTIES
C
C
      DEFINE FILE 1(138,276,U,N), 2(138,276,U,N), 3(138,276,U,N)
      DEFINE FILE 4(19044,2,U,N4),5(138,276,U,N),6(138,276,U,N)
      DIMENSION AM(6),FYLE(138),AMM(6,6),NP(6)
      COMMON N
      COMMON THET1,THET2,THET3,THET4,PROP(19,5),SEG(19),CAP(20)
      COMMON AJ(6,3),G1,G2,ANU,RO,RHO
      COMMON C(6,3),F(50,5),DTAU,BETA,AZIM,IM,VO,PHIO
      COMMON N4,AAM(3,4)
      DATA NP/1,4,7,10,16,19/
   10 FORMAT(1H1,20X,'TRIPOD MOUNT M 3 FOR 0.50 CALIBER MACHINE GUN',/,
     1 21X,'CASE ANALYZED- ',20A4)
   11 FORMAT(4F10.3,I5)
   12 FORMAT(1H0,'CONCENTRATED MASSES AND MASS MOMENTS OF INERTIA -',//,
     121X,'NODAL',5X,'CONC.',9X,'MASS MOMENTS OF INERTIA',/,21X,'POINT',
     25X,'MASS',13X,'(IN - LBS - SEC 2)',/,33X,'M',11X,'JX',8X,'JY',8X,
     3'JZ',/,31X,'(LBS)',/,21X,50('-'))
```

```
   13 FORMAT(1H0,20X,I3,F12.3,3X,3F10.4)
   14 FORMAT(1H0,'ADDITIONAL CONCENTRATED MASS DUE TO GUN -',
     12(//,20X,F8.3,2X,'LBS ADDED AT NODAL POINT',I4))
   15 FORMAT(1H0,'SOIL PROPERTIES -',/,21X,'RADIUS OF FOOT DISK =',F9.3,
     1' IN.',//,28X,'SOIL DENSITY =',F9.4,' LBS/IN3',//,27X,
     2'POISSON RATIO =',F6.2,//,30X,'VOID RATIO =',F8.2,//,31X,
     3'SOIL TYPE =')
   16 FORMAT(1H+,44X,'ANGULAR SANDS OR COHESIVE SOILS')
   17 FORMAT(1H+,44X,'ROUND-GRAINED SANDS')
   18 FORMAT(1H ,'FORCING FUNCTION -',/,21X,'INITIAL GUN ELEVATION',
     1' ANGLE  =',F10.3,' DEG.',/,23X,'INITIAL GUN AZIMUTH ANGLE  =',
     2 F10.3,' DEG.')
      IN=2
      MO=5
      GVTY=386.4
C
C        READ MASS AND MOMENTS OF INERTIA
C
C        AM(1), (AJ(1,J),J=1,3)  CONC MASS,(JX,JY,JZ)  AT NODAL POINT 1
C        AM(2), (AJ(2,J),J=1,3)  CONC MASS,(JX,JY,JZ)  AT NODAL PT  4
C        AM(3), (AJ(3,J),J=1,3)  CONC MASS,(JX,JY,JZ)  AT NODAL PT  7
C        AM(4), (AJ(4,J),J=1,3)  CONC MASS,(JX,JY,JZ)  AT NODAL PT  10
C        AM(5), (AJ(5,J),J=1,3)  CONC MASS,(JX,JY,JZ)  AT NODAL PT  16
C        AM(6), (AJ(6,J),J=1,3)  CONC MASS,(JX,JY,JZ)  AT NODAL PT  19
C
      DO 100 I=1,6
      READ(IN,11 )  AM(I), (AJ(I,J), J=1,3)
      DO 100 J=1,6
  100 AMM(I,J)=0.0
      READ(IN,11 ) ELEV,AZIM,VO
      PHIO=0.01745*(ELEV+4.666667)
C
C        CALC GEOMETRICALLY COMPATABLE AZIMUTH ANGLE
C        DETERMINE TAVERSE BAR NODAL LOAD POINT
C
      IF(AZIM)   130,  120,  110
  110 NPT=11
      NNPT=79
      ALEG=SEG(4) + SEG(5) + SEG(6)
      TRAV=SEG(10)+ SEG(11)
      SEGL=SEG(11)
      GO TO   140
  120 NPT=12
      NNPT=85
      AZIM=0.0
      GO TO   150
  130 NPT=13
      NNPT=91
      ALEG=SEG(7) + SEG(8) + SEG(9)
      TRAV=SEG(12) + SEG(13)
      SEGL=SEG(12)
  140 H=COS(THETA) * SQRT(ALEG**2 - TRAV**2)
      AZ=ATAN(SEGL/H)
      AZIM=SIGN(AZ,AZIM)
  150 WRITE(MO,10 )  CAP
C
C        WRITE OUT CONC. MASS AND MASS MOMENTS OF INERTIA
C
      WRITE(MO,12 )
      DO 160 I=1,6
      WRITE(MO,13 ) NP(I),AM(I),(AJ(I,J), J=1,3)
  160 AM(I)=AM(I)/GVTY
C
C        READ GUN MASS COMPONENTS LUMPED AT PINTLE AND TRAVERSE BAR
C
      READ(IN,11 )  WTP,WTT
      WRITE(MO,14 ) WTP,NP(2),WTT,NPT
      AM(2)= AM(2) + WTP/GVTY
```

```
C
C
C        SAVE ORIGINAL M MATRIX AS FILE  6
C
      N=1
      DO 170 I=1,138
      READ(1'N) FYLE
      N=N-1
  170 WRITE(6'N) FYLE
C
C
C        KASE=1    MASS MATRIX FILE BEING MODIFIED
C        KASE=2    STIFFNESS MATRIX BEING MODIFIED
C
      JMAX=6
      DO 560 KASE=1,2
      DO 440 K=1,JMAX
      DO 210 J=1,6
      IF(J-3)   180,  180,  200
  180 IF(KASE - 1)   190,  190,  210
  190 AMM(J,J)=AM(K)
      GO TO   210
  200 AMM(J,J)=AJ(K,J-3)
  210 CONTINUE
      IF(KASE-1)   220,  220,  230
  220 KK=K
      GO TO   270
  230 KK=1 + (K/2)*(K+2)
      IF(K-1)   240,  240,  250
  240 G=G1
      GO TO   260
  250 G=G2
  260 AMM(1,1)=(32.0*(1.0-ANU)/(7.0-8.0*ANU))*G*RO
      AMM(2,2)=4.0*G*RO/(1.0-ANU)
      AMM(3,3)=AMM(1,1)
C
C        SAVE ROWS 4,5,6  OF  (AJ)
C                    4(   (KX1)   (KY1)    (KZ1)    )
C        (AJ)=       5(   (KX16)  (KY16)   (KZ16)   )
C                    6(   (KX19)  (KY19)   (KZ19)   )
C
      AJ(K+3,1)=AMM(1,1)
      AJ(K+3,2)=AMM(2,2)
      AJ(K+3,3)=AMM(3,3)
  270 GO TO (280,290,300,310,320,330),KK
  280 N=1
      JJ=N-1
      GO TO   340
  290 N=43
      JJ=18
      GO TO   340
  300 N=43
      JJ=N-1
      GO TO   340
  310 N=67
      JJ=N-1
      GO TO   340
  320 N=115
      JJ=N-1
      GO TO   340
  330 N=133
      JJ=N-1
  340 DO 380 I=1,6
      READ(KASE'N) FYLE
      J1=JJ + I
      FYLE(J1)=FYLE(J1) + AMM(I,I)
      N=N - 1
      WRITE(KASE'N) FYLE
      IF(KASE - 1)   350,  350,  380
  350 IF(K - 2)   380,  370,  360
  360 IF(K - 4)   370,  370,  380
```

```
  370 N=N - 1
      READ(6'N) FYLE
      FYLE(J1)=FYLE(J1) + AMM(I,I)
      N=N - 1
      WRITE(6'N) FYLE
  380 CONTINUE
C
      CALL DATSW(4,ISW)
C        SW 4 ON  WRITE OUT MASS AND/OR STFNS MATRIX MODIFICATIONS
C        SW 4 OFF CONTINUE WITH MATRIX MODIFICATIONS
C
      GO TO (390,400),ISW
  390 WRITE(MO,13 ) K,((AMM(II,JJ), JJ=1,6),II=1,6)
  400 IF(KASE - 1)   410,  410,  440
C
C        SAVE MASSES AND MOMENTS AS (AAM)
C                  (    M1    JX1    JY1    JZ1    )
C        (AAM)=    (    M16   JX16   JY16   JZ16   )
C                  (    M19   JX19   JY19   JZ19   )
C
  410 IF(K-1)    440,  430,  420
  420 IF(K-5)    440,  430,  430
  430 KK=1 + K/3
      AAM(KK,1)=AM(K)
      AAM(KK,2)=AJ(K,1)
      AAM(KK,3)=AJ(K,2)
      AAM(KK,4)=AJ(K,3)
  440 CONTINUE
      IF(KASE-1)    450,  450,  570
C
C        READ SOIL PROPERTIES DATA
C        RO         RADIUS
C        RHO        DENSITY  (LB/IN3)
C        ANU        POISONS RATIO
C        E          SOIL VOID RATIO
C        IG         SOIL TYPE INDEX
C             IG=1  ANGULAR SANDS AND COHESIVE SOILS
C             IG=2  ROUND GRAINED SANDS
C
C
  450 READ(IN,11 )  RO,RHO,ANU,E,IG
C
C        WRITE OUT SOIL PROPERTIES
C
      WRITE(MO,15 ) RO,RHO,ANU,E
      GO TO (460,470),IG
  460 WRITE(MO,16 )
      GO TO   480
  470 WRITE(MO,17 )
  480 RHO=RHO/GVTY
C
C        CONCENTRATED MASS ON TRAVERSE BAR NODAL POINT
C
      N=NNPT
      NNPT1=NNPT + 2
      DO 490 J=NNPT,NNPT1
      READ(6'N) FYLE
      FYLE(J)=FYLE(J) + WTT/GVTY
      N=N-1
      WRITE(6'N) FYLE
      N=N -1
      WRITE(1'N) FYLE
  490 CONTINUE
C
C        SAVE ORIGINAL K MATRIX AS FILE  5
C
      N=1
      DO 500 I=1,138
      READ(2'N) FYLE
      N=N-1
```

```
  500 WRITE(5'N) FVLE
C
C        COMPUTE SOIL CHARACTERISTICS
C
      SIG=(1.0+ANU)/(3.0*(1.0-ANU))
      SIG1=4.142*SIG
      SIG2=1.779*SIG
      GO TO (510,520),IG
  510 G=1230.*(2.97-E)**2/(1.0+E)
      GO TO  530
  520 G=2630.*(2.17-E)**2/(1.0+E)
  530 G1=G*SQRT(SIG1)
      G2=G*SQRT(SIG2)
      JMAX=3
C
C        SAVE ROWS 1,2,3  OF  (AJ)
C                     1(   (KTX1)   (KTY1)   (KTZ1)   )
C        (AJ)=        2(   (KTX16)  (KTY16)  (KTZ16)  )
C                     3(   (KTX19)  (KTY19)  (KTZ19)  )
C
      DO 560 I=1,3
      IF(I-1)   540,  540,  550
  540 AJ(I,1)=8.0*G1*R0**3/(3.0*(1.0-ANU))
      AJ(I,2)=16.0*G1*R0**3/3.0
      AJ(I,3)=AJ(I,1)
      GO TO   560
  550 AJ(I,1)=8.0*G2*R0**3/(3.0*(1.0-ANU))
      AJ(I,2)=16.0*G2*R0**3/3.0
      AJ(I,3)=AJ(I,1)
  560 CONTINUE
  570 AZ=AZIM/0.01745
      WRITE(MU,10 ) CAP
      WRITE(MU,18 )  ELEV,AZ
      CALL LINK(SJLFQ)
      END
```


PROGRAM SJLFQ (LINK 3)

```
C
C
C        PROGRAM  SJLFQ      (LINK 3)
C        FORCING FUNCTION READER AND (Q)  MATRIX WRITER
C
C        SUPPORTING SUBROUTINES
C            SJLQB    (Q)  MTRX  MOD  FOR BDRY CONDS
C
C        REQ'D INPUT DATA
C            DTAU    (F10.3)
C            (F(I,J),J=1,5)      (F10.6,4E15.5)
C               NOTE- FORCING FUNCTION DATA MUST BE
C                     LESS THAN OR EQUAL TO 50  DATA CARDS
C
C        NORMAL OUTPUT
C            ELEV   INITIAL GUN ELEVATION ANGLE
C            AZIM   INITIAL GUN AZIMUTH ANGLE
C            TAU,(F(I,J),J=1,5)   FORCING FUNCTION TABULATION
C
      DEFINE FILE 1(138,276,U,N), 2(138,276,U,N), 3(138,276,U,N)
      DEFINE FILE 4(19044,2,U,N4),5(138,276,U,N),6(138,276,U,N)
      DIMENSION FVLE(138),X(138)
      COMMON [illegible]
```

```
      COMMON THET1,THET2,THET3,THET4,PROP(19,5),SEG(19),CAP(20)
      COMMON AJ(6,3),G1,G2,ANU,RO,RHO
      COMMON C(6,3),F(50,5),DTAU,BETA,AZIM,IM,VO,PHIO
      COMMON A4,AAM(3,4)
   10 FORMAT(F10.6,4E15.5)
   11 FORMAT(4F10.3,I5)
   12 FORMAT(1H0,'DAMPING MATRIX')
   13 FORMAT(1H ,I2,6E15.4)
   14 FORMAT(1H1,'PRINT OUT OF  (Q)  MATRIX')
   15 FORMAT(1H0,2I5)
   16 FORMAT(1H ,I10,6E18.4)
   17 FORMAT(1H0,21X,'TIME',7X,'PINTLE',7X,'TRAVERSE',8X,'PINTLE',6X,
     1'PINTLE FORCE',/,34X,'FORCE',7X,'BAR FORCE',7X,'MOMENT',6X,
     2'ACTION ANGLE',/,21X,'(SEC)',8X,'(LBS)',9X,'(LBS)',8X,'(IN-LBS)',
     37X,'(DEG)',/,19X,68('-'))
   18 FORMAT(1H ,18X,F9.6,E13.4,E15.4,2E14.4)
      IN=2
      MO=5
C
C        READ FORCING FUNCTION DATA
C              FORCING FUNCTION DATA MUST BE LESS OR EQUAL TO 50
C              IF LESS THAN 50,  MAKE LAST  F(I,1)  NEGATIVE
C
  100 READ(IN,11 ) DTAU
      WRITE(MO,17 )
      DO 110 I=1,50
      READ(IN,10 ) (F(I,J), J=1,5)
      IF(F(I,1))   120,  110,  110
  110 WRITE(MO,18 )  (F(I,J), J=1,5)
      IM=50
      GO TO   130
  120 F(I,1)=-F(I,1)
      WRITE(MO,18 ) (F(I,J), J=1,5)
      IM=I
  130 N=1
C
C        MAKE UP  (Q)  MATRIX   STORE AS FILE  3
C
      DO 260 K=1,23
      II=6*(K-1)
      DO 260 I=1,6
      II=II + 1
      READ(2'N)  X
      N=N-1
      READ(1'N) FYLE
      N=N-1
      FIND(3'N)
      IF(K-1)   180,  180,  140
  140 IF(K-20)   160,  190,  150
  150 IF(K-23)   160,  200,  200
C
C        NO DAMPING TERMS
C        (Q) = (M)  +  (DTAU**2)/6  (K)
C
  160 DO 170 J=1,138
  170 X(J)=FYLE(J) + (DTAU**2/6.0) * X(J)
      GO TO   250
C
C        CALCULATE DAMPING TERMS,  STORE AS (C)
C        (Q) = (M) + DTAU/2  (C)  +  (DTAU**2)/6  (K)
C
  180 KK=1
      G=G1
      GO TO   210
  190 KK=2
      G=G2
      GO TO   210
```

```
  200 KK=3
      G=G2
  210 C(1,KK)=0.576*(32.*(1.-ANU)*RO**2/(7.-8.*ANU))*SQRT(G*RHO)
      C(2,KK)=(3.4*RO**2/(1.-ANU))*SQRT(G*RHO)
      C(3,KK)=C(1,KK)
      C(4,KK)=RO**4/(1.0+3.0*(1.0-ANU)*AAM(KK,2)/(8.0*RHO*RO**5))
      C(4,KK)=C(4,KK)*(0.8/(1.-ANU))*SQRT(G*RHO)
      C(5,KK)=(2.31*RO**2/(1.+2.*AAM(KK,3)/(RHO*RO**5)))
      C(5,KK)=C(5,KK)*SQRT(G*AAM(KK,3)/RO)
      C(6,KK)=RO**4/(1.0+3.0*(1.0-ANU)*AAM(KK,4)/(8.0*RHO*RO**5))
      C(6,KK)=C(6,KK)*(0.8/(1.-ANU))*SQRT(G*RHO)
      DO 240 J=1,138
      IF(II-J)   230,  220,  230
  220 X(J)=FYLE(J) + (DTAU/2.0) * C(I,KK) + (DTAU**2/6.0) * X(J)
      GO TO   240
  230 X(J)=FYLE(J) + (DTAU**2/6.0) * X(J)
  240 CONTINUE
  250 WRITE(3'N) X
  260 CONTINUE
C
C         MAKE  (D)  MATRIX MODIFICATIONS
C
      IFYLE=3
      CALL SJLQB(FYLE,IFYLE)
C
      CALL DATSW(5,ISW)
      GO TO (270,290),ISW
C        SW 5  ON  PRINT OUT DAMPING MATRIX
C        SW 5 OFF  PROCEED
C
  270 WRITE(MO,12 )
      DO 280 I=1,3
  280 WRITE(MO,13 ) I,(C(J,I), J=1,6)
C
C         MODIFIED M MATRIX STORED IN FILE 1
C         MODIFIED K MATRIX STORED IN FILE 2
C         MODIFIED D MATRIX STORED IN FILE 3
C         ORIGINAL K MATRIX STORED IN FILE 5
C         ORIGINAL M MATRIX STORED IN FILE 6
C
C
  290 CALL DATSW(6,ISW)
      GO TO (300,350),ISW
C        SW 6  ON  PRINT OUT  (D)  MATRIX
C        SW 6 OFF  LINK TO SJLTM
C
  300 WRITE(MO,14 )
      N=1
      DO 340 I=1,138
      II=(I-1)/6 + 1
      READ(3'N)  FYLE
      WRITE(MO,15 )  II,I
      J1=1
      J2=6
      DO 340 K=1,23
      DO 310 KJ=J1,J2
      IF(FYLE(KJ))    320,  310,  320
  310 CONTINUE
      GO TO   330
  320 WRITE(MO,16 ) K, (FYLE(J), J=J1,J2)
  330 J1=J1+6
      J2=J2+6
  340 CONTINUE
  350 CALL LINK(SJLIN)
      END
```

```
C
C
C     PROGRAM  SJLIN      (LINK 4)
C     INITIAL ACCELERATION CALCULATOR
C
C     SUPPORTING SUBROUTINES
C         SJLMP   MTRX PRODUCT OF FILE MTRX BY COLUMN MTRX
C         SJLC3   FORCING FUNCTION INTERPOLATOR
C         SJLGS   GAUSS-SEIDEL SOLN FOR SIMULTANEOUS EQNS
C             SJLOO   ORDER OF SOLN FOR GAUSS-SEIDEL
C         SJLDO   FINITE DISPL, GUN ORIENTATION, AND STRESSES
C             SJLTI   TRANSFORMATION MTRX WRITER
C             GMPRD   GENERAL MTRX PRODUCT  (IBM SSP)
C
C     REQ'D INPUT DATA
C         SW 0  OFF   NO INPUT DATA REQ'D
C                     INITIAL DISPL AND VLCTY ARE ZERO
C         SW 0   ON   INITIAL DISPL, VLCTY, AND ACCEL ARE INPUT
C             TAU   (E13.4)
C             ((D(J),J=J1,J2),I=1,23)   (6E13.4)
C             ((DDT(J),J=J1,J2),I=1,23)   (6E13.4)
C             ((DDDT(J),J=J1,J2), I=1,23)  (6E13.4)
C                 WHERE      J1=6 I  - 5
C                            J2=J1  +  5
C
C     NORMAL OUTPUT
C         TRANSIENT RESPONSE,ELEV,AZIM,AND STRESSES
C         FOR INITIAL TIME INCREMENT
C
      DEFINE FILE 1(130,276,U,N), 2(130,276,U,N),3(130,276,U,N)
      DEFINE FILE 4(19044,2,U,N4),5(130,276,U,N),6(130,276,U,N)
      DIMENSION W(130)
      DIMENSION A(130),B(130)
      DIMENSION ID(12,3),ID1(12),ID2(12),ID3(12),SIGMA(12,2)
      COMMON N
      COMMON THET1,THET2,THET3,THET4,PROP(19,5),SEG(19),CAP(20)
      COMMON AJ(6,3),G1,G2,ANU,RO,RHO
      COMMON C(6,3),F(50,5),DTAU,BETA,AZIM,IM,VO,PHIO
      COMMON N4,AAN(3,4)
      COMMON D(130),DDT(130),DDDT(130),TAUU
      EQUIVALENCE((ID(1,1),ID1(1)),(ID(1,2),ID2(1)),(ID(1,3),ID3(1)))
      DATA ID1/5,6,14,15,8,9,17,18,10,11,12,13/
      DATA ID2/1,2,3,12,4,5,6,14,8,9,10,11/
      DATA ID3/2,3,12,13,5,6,14,15,7,8,9,10/
   10 FORMAT(1H0,'TAU=',F7.3,' SEC.')
   11 FORMAT(1H ,6E15.4)
   12 FORMAT(6E13.4)
   13 FORMAT(1H1,20X,'TRIPOD MOUNT M 3 FOR 0.50 CALIBER MACHINE GUN',/
     1 21X,'CASE ANALYZED- ',20A4,//,1X,'TRANSIENT RESPONSE -')
   14 FORMAT(1H0,2X,'TIME',3X,'NODAL',7X,'DISPLACEMENTS AND VELOCITIES',
     1 8X,'ELEVATION',3X,'AZIMUTH',3X,'ELEMENT',3X,'PEAK COMBINED',3X,
     2 'OCCURRING AT',/,2X,'(SEC.)  POINT',12X,'(IN)',10X,'(IN/SEC)',11X,
     3 'ANGLE',6X,'ANGLE',5X,'NUMBER',7X,'STRESS',8X,'LOCATION',/,23X,
     4 'X',19X,'Y',13X,'Z',3X,2(5X,'(DEG)'),19X,'(PSI)',6X,'(IN.FROM END)
     5',/,1X,120('-'))
   15 FORMAT(1H ,F7.5,I5,3E14.3,F11.5,F10.3,I8,E19.5,F12.3)
   16 FORMAT(1H ,7X,I5,3E14.3,21X,I8,E19.5,F12.3)
   17 FORMAT(1H ,12X,3E14.3,21X,I8,E19.5,F12.3)
   18 FORMAT(1H ,75X,I8,E19.5,F12.3)
      IN=2
      MO=5
      TAU=0.0
      KUU=T=0
      WRITE(MO,13 ) CAP
      WRITE(MO,14 )
```

```
      DO 100 I=1,138
      D(I)=0.0
      DOT(I)=0.0
  100 DDOT(I)=0.0
      CALL DATSW(0,ISW)
      GO TO (110,150),ISW
C
C        SW  0  ON   READ INITIAL  (D) , (DOT), AND (DDOT)  DATA
C        SW  0  OFF  (D) = (DOT) = 0.0
C
  110 READ(IN,12 )  TAU
      TAUO=TAU
      DO 120 I=1,23
      J1=6*I - 5
      J2=J1 + 5
      READ(IN,12 )  (D(J),  J=J1,J2)
      READ(IN,12 )   (DOT(J),   J=J1,J2)
  120 READ(IN,12 )   (DDOT(J),   J=J1,J2)
      CALL SLITE(0)
      IF(DDOT(1))  165,135,165
C
C        FORM  (C) (DOT)  STORE TEMPORARILY IN (B)
C
  135 IFYLE=4
      CALL SJLMP(IFYLE,DOT,B,KC)
C
C        FORM  (K) (D)   STORE TEMPORARILY IN  (A)
C
      IFYLE=2
      CALL SJLMP(IFYLE,D,A,KC)
      CALL SJLQQ(Q,TAU,F,BETA,AZIM,IM)
C
C        FORM  (Q) -(K) (D)  -(C) (DOT)   STORE AS (Q)
C
      DO 140 I=1,138
  140 Q(I)=Q(I) - A(I) - B(I)
      GO TO   160
  150 CALL SJLQQ(Q,TAU,F,BETA,AZIM,IM)
C
C        SOLVE FOR INITIAL  (DDOT)
C
  160 IFYLE=1
      KODE=0
      CALL SLITE(1)
      CALL SJLGS(IFYLE,DDOT,Q)
C
  165 CALL DATSW(8,ISW)
      GO TO (170,190),ISW
C        SW 8  ON  WRITE OUT (D),  (DOT),  AND  (DDOT)
C        SW 8  OFF  PROCEED WITH CALCULATIONS
C
  170 DO 180 K=1,23
      I1=6*K - 5
      I2=I1 + 5
      WRITE(MO,11 ) (D(II),  II=I1,I2)
      WRITE(MO,11 ) (DOT(II), II=I1,I2)
      WRITE(MO,11 )  (DDOT(II), II=I1,I2)
      WRITE(MO,11 )
  180 CONTINUE
  190 CALL SJLDG(D,DOT,DDOT,ID,XP,YP,ZP,DXP,DYP,DZP,SIGMA,ELEV)
      IF(KOUNT - 3)   210,  200,  200
  200 KOUNT=0
      WRITE(MO,13 )  CAP
      WRITE(MO,14 )
  210 KOUNT=KOUNT + 1
      DO 280 I=1,10,3
      IF(I-1)   220,  220,  230
  220 NP=4
```

```
      AZ=AZIM/0.01745
      WRITE(MO,15 )  TAU,NP,XP,YP,ZP,ELEV,AZ  ,ID(I,1),
     1(SIGMA(I,J), J=1,2)
      WRITE(MO,17 ) DXP,DYP,DZP,ID(I+1,1),(SIGMA(I+1,J),J=1,2)
      GO TO   280
  230 II=(I-1)/3
      GO TO (240,250,260),II
  240 NP=1
      J1=1
      J2=3
      GO TO   270
  250 NP=16
      J1=115
      J2=117
      GO TO   270
  260 NP=19
      J1=133
      J2=135
  270 WRITE(MO,16 ) NP,(D(J),J=J1,J2),ID(I,1),(SIGMA(I,J), J=1,2)
      WRITE(MO,17 )  (DOT(J),J=J1,J2),ID(I+1,1),(SIGMA(I+1,J),J=1,2)
  280 WRITE(MO,18 )  ID(I+2,1),(SIGMA(I+2,J),J=1,2)
      CALL LINK(SJLTM)
      END
```


PROGRAM SJLTM (LINK 5)

```
C
C       PROGRAM  SJLTM      (LINK 5)
C       TRANSIENT DISPL, VELOCITY, AND ACCEL  CALCULATOR
C
C       SUPPORTING SUBROUTINES
C           SJLMP   MTRX PRODUCT OF FILE MTRX BY COLUMN MTRX
C           SJLQQ   FORCING FUNCTION INTERPOLATOR
C           SJLGS   GAUSS-SEIDEL SOLN FOR SIMULTANEOUS EQNS
C               SJLOD   ORDER OF SOLN FOR GAUSS-SEIDEL
C           SJLLL   MATRICES MODIFIER FOR LEG LIFTING
C               SJLKQ   K AND Q  MODIFICATIONS FOR LEG LIFTING
C                   SJLLB   (Q) MTRX MOD  FOR BRDRY CONDS
C           SJLDG   PINTLE DISPL,GUN ORIENTATION, AND STRESSES
C           SJLOU   OUTPUT GENERATOR AND WRITER
C
C       REQ'D INPUT DATA
C           TMAX   MAXIMUM TIME  (SEC)  (F10.3)
C
C       NORMAL OUTPUT
C           TRANSIENT RESPONSES,ELEV,AZIM,AND STRESSES
C           FOR SUCCESSIVE TIME INCREMENTS
C
      DEFINE FILE 1(138,276,U,N), 2(138,276,U,N),3(138,276,U,N)
      DEFINE FILE 4(19044,2,U,N4),5(138,276,U,N)
      DEFINE FILE 7(138,276,U,N), 8(138,276,U,N)
      DIMENSION Q(138)
      DIMENSION A(138),B(138),Y(138)
      DIMENSION ID(12,3),ID1(12),ID2(12),ID3(12),SIGMA(12,2)
      COMMON N
      COMMON THET1,THET2,THET3,THET4,PROP(19,5),SEG(19),CAP(20)
      COMMON AJ(6,3),G1,G2,ANU,RO,RHO
      COMMON C(6,3),F(50,5),DTAU,BETA,AZIM,IM,VO,PHIO
      COMMON N4,AAM(3,4)
      COMMON D(138),DOT(138),DDOT(138),TAUU
      EQUIVALENCE(ID(1,1),ID1(1)),(ID(1,2),ID2(1)),(ID(1,3),ID3(1))
```

```
      DATA ID1/5,6,14,15,8,9,17,18,10,11,12,13/
      DATA ID2/1,2,3,12,4,5,6,14,8,9,10,11/
      DATA ID3/2,3,12,13,5,6,14,15,7,8,9,10/
   10 FORMAT(1H0,'TAU=',F7.3,' SEC.')
   11 FORMAT(1H ,6E15.4)
   12 FORMAT(F10.3)
   13 FORMAT(6E13.4)
      TAU=TAUU
      IF(DDOT(1))  90, 80, 90
   80 CALL SLITE(1)
      TAU=0.0
   90 IN=2
      MO=5
      KC=0
      KKC=0
      LL=1
      KKKC=0
      READ(IN,12 ) TMAX
      KOUNT=1
      KODE=0
  100 TAU=TAU + DTAU
      CALL SJLQQ(Q,TAU,F,BETA,AZIM,IM)
C
C        FORM (A)  AND  (B)   MATRICES
C
      DO 110 I=1,138
      A(I)=DOT(I) + (DTAU/2.0) * DDOT(I)
  110 B(I)=D(I) + DTAU*DOT(I) + (DTAU**2/3.0) * DDOT(I)
C
C        FORM  (C) (A)   STORE AS (Y)
C
  120 IFYLE=4
      CALL SJLMP(IFYLE,A,Y,KC)
C
C        MAKE   (Q) = (Q) - (C) (A)
C
      DO 130 I=1,138
  130 Q(I)=Q(I) - Y(I)
C
C        FORM (K) (B)   STORE AS (Y)
C
C
C        LL=1   NO LEGS ARE LIFTED
C        LL=2   AT LEAST ONE LEG IS LIFTED
C
      GO TO (140,150),LL
  140 IFYLE=2
      GO TO   160
  150 IFYLE=7
  160 CALL SJLMP(IFYLE,B,Y,KC)
C
C        MAKE  (Q) = (Q) - (C) (A) - (K) (B)
C
      DO 170 I=1,138
  170 Q(I)=Q(I) - Y(I)
C
C        CALCULATE  (DDOT)  FROM  MODIFIED (K) AND (M)
C
      GO TO (180,190),LL
  180 IFYLE=3
      GO TO   200
  190 IFYLE=8
  200 CALL SJLGS(IFYLE,DDOT,Q)
      CALL DATSW(8,ISW)
      GO TO (210,220),ISW
  210 WRITE(MO,10 ) TAU
C
C        CALCULATE  (DOT)  AND  (D)
C
```

```
  220 DO 230 I=1,138
      DOT(I)=A(I) + (DTAU/2.0) * DDOT(I)
  230 D(I)=B(I) + (DTAU**2/6.0)*DDOT(I)
      KC=0
      IF(D(2))   250,  250,  240
  240 KC=KC + 1
  250 IF(D(116))   270,  270,  260
  260 KC=KC + 3
  270 IF(D(134))   290,  290,  280
  280 KC=KC + 5
  290 IF(KC)   300,  300,  320
  300 IF(KODE)   350,  350,  310
  310 LL=1
      KODE=0
      CALL SJLDD(D,TAU,F,BETA,AZIM,IM)
      GO TO   120
  320 IF(LL-2)   340,  330,  330
  330 IF(KKKC-KC)   340,  350,  340
  340 CALL SJLLL(KODE,KC,KKC,KKKC,TAU,U,D,DOT,LL,D2,D116,D134)
      GO TO   120
C
  350 CALL DATSW(8,ISW)
      GO TO (360,380),ISW
C         SW 8  ON  WRITE OUT (D),  (DOT),  AND  (DDOT)
C         SW 8  OFF  PROCEED WITH CALCULATIONS
C
  360 READ(IN, 12)  DUMMY
      DO 370  K=1,23
      I1=6*K - 5
      I2=I1 + 5
      WRITE(MO,11 ) (D(II),  II=I1,I2)
      WRITE(IN, 13)  (D(II), II=I1,I2)
      WRITE(MO,11 ) (DOT(II), II=I1,I2)
      WRITE(IN, 13)  (DOT(II),  II=I1,I2)
      WRITE(MO,11 )  (DDOT(II), II=I1,I2)
      WRITE(IN,13 )   (DDOT(II),  II=I1,I2)
      WRITE(MO,11 )
  370 CONTINUE
      WRITE(IN, 12)  TAU
  380 CALL SJLOG(D,DOT,DDOT,ID,XP,YP,ZP,DXP,DYP,DZP,SIGMA,ELEV)
      CALL SJLOU(KOUNT,ID,SIGMA,ELEV,TAU,XP,YP,ZP,DXP,DYP,DZP)
      IF(TAU - TMAX)   100,  100,  390
  390 CALL EXIT
      END
```

SUBROUTINE SJLBD   BOUNDARY CONDITIONS WRITER

```
      SUBROUTINE SJLBD(KODE,C4,S4,ALPHA,BETA,GAMMA,AMTX,FYLE)
      DIMENSION AMTX(6,6,4), FYLE(138)
      COMMON N
      COMMON THET1,THET2,THET3,THET4,PROP(19,5),SEG(19),CAP(20)
   10 FORMAT(1H1)
   11 FORMAT(1H0,'BOUNDARY CONDITIONS MATRIX  KODE=', I4)
   12 FORMAT(1H ,6(6E12.3,/,1X),/)
      MO=5
      S2=SIN(THET2)
      C2=COS(THET2)
      S3=SIN(THET3)
      C3=COS(THET3)
C
C         CLEAR  AMTX  AS WORKING ARRAY
C
```

```
      DO 100 I=1,6
      DO 100 J=1,6
      DO 100 K=1,4
  100 AMTX(I,J,K)=0.0
C
      GO TO (110,120,130,140),KODE
C        KODE=1   CALC BDRY CONDS LINE 5
C        KODE=2   CALC BDRY CONDS LINE 4
C        KODE=3   CALC BDRY CONDS LINE 13
C        KODE=4   CALC BDRY CONDS LINE 17
C
C
C        SETUP BDRY COND FOR LINE 5
C
  110 N=25
      J1=1
      J2=18
      K1=1
      K2=3
      JJ1=37
      JJ2=138
      JJJ1=0
      JJJ2=0
      SIG=1.0
      GO TO   150
C
C        SETUP BDRY COND FOR LINE 4
C
  120 N=19
      J1=1
      J2=18
      K1=1
      K2=1
      JJ1=25
      JJ2=48
      KK1=2
      KK2=3
      JJJ1=61
      JJJ2=138
      SIG=-1.0
      GO TO   150
C
C        SETUP BDRY COND FOR LINE 13
C
  130 N=73
      J1=1
      J2=42
      K1=1
      K2=1
      JJ1=49
      JJ2=72
      KK1=2
      KK2=3
      JJJ1=85
      JJJ2=138
      SIG=1.0
      K=2
      KK=3
      GO TO   310
C
C        SETUP BDRY COND FOR LINE 17
C
  140 N=97
      J1=1
      J2=66
      K1=1
      K2=1
      JJ1=73
      JJ2=90
      KK1=2
```

```
      KK2=3
      JJJ1=103
      JJJ2=138
      SIG=-1.0
      K=3
      KK=2
      GO TO   310
C
C        COMPUTE TERMS FOR BDRY CONDS LINES 4 AND 5
C
  150 DO 200 I=1,6
      IF(I-4)    170,  180,  160
  160 IF(I-5)    170,  190,  170
  170 AMTX(I,I,1)=1.0
      GO TO   200
  180 AMTX(I,I,1)=C2*C4/GAMMA
      AMTX(I,I-1,1)=BETA*C3*S4/GAMMA
      AMTX(I,I-3,1)=-SIG*BETA*S3*S4/GAMMA
      AMTX(I,I+1,1)=-C2*S4/GAMMA
      AMTX(I,I+2,1)=-SIG*S2*S3*S4/GAMMA
      AMTX(I,I,3)=-ALPHA*S4/(2.0*GAMMA)
      AMTX(I,I-1,3)=-AMTX(I,I-1,1)
      AMTX(I,I-3,3)=-AMTX(I,I-3,1)
      AMTX(I,I+1,3)=AMTX(I,I+1,1)/2.0
      AMTX(I,I+2,3)=AMTX(I,I+2,1)/2.0
      GO TO   200
  190 AMTX(I,I,1)=ALPHA*S4/GAMMA
      AMTX(I,I-1,1)=-ALPHA*C4/GAMMA
      AMTX(I,I-2,1)=BETA*C3*C4/GAMMA
      AMTX(I,I-4,1)=-SIG*BETA*S3*C4/GAMMA
      AMTX(I,I+1,1)=-SIG*ALPHA*C4/GAMMA
      AMTX(I,I,3)=-C2*C4/(2.0*GAMMA)
      AMTX(I,I-1,3)=AMTX(I,I-1,1)/2.0
      AMTX(I,I-2,3)=-AMTX(I,I-2,1)
      AMTX(I,I-4,3)=-AMTX(I,I-4,1)
      AMTX(I,I+1,3)=-SIG*S2*S3*C4/(2.0*GAMMA)
  200 AMTX(I,I,2)=-1.0
  210 DO 230 I=1,6
      DO 230 J=1,6
      DO 230 K=1,3
      IF(AMTX(I,J,K))   220,  230,  220
  220 TX=EXP(ALOG(AMTX(I,J,K)))
      AMTX(I,J,K)=SIGN(TX,AMTX(I,J,K))
  230 CONTINUE
C
      CALL DATSW(3,ISW)
C        SW 3  ON  WRITE OUT BOUNDARY CONDITIONS
C        SW 3 OFF  CONTINUE CALC OF BOUNDARY CONDITIONS
C
      GO TO (240,270),ISW
  240 IF(KODE-1)    250,  250,  260
  250 WRITE(MO,10 )
  260 WRITE(MO,11 ) KODE
      WRITE(MO,12 ) (((AMTX(I,J,K),J=1,6),I=1,6),K=1,3)
  270 DO 300 I=1,6
      IF(JJJ1)    280,  280,  290
  280 WRITE(1'N) (FYLE(J),J=J1,J2),((AMTX(I,J,K),J=1,6),K=K1,K2),
     1(FYLE(J),J=JJ1,JJ2)
      GO TO   300
  290 WRITE(1'N) (FYLE(J),J=J1,J2),((AMTX(I,J,K),J=1,6),K=K1,K2),
     1(FYLE(J),J=JJ1,JJ2),((AMTX(I,J,K),J=1,6),K=KK1,KK2) ,
     2(FYLE(J),J=JJJ1,JJJ2)
  300 CONTINUE
      RETURN
C
C        COMPUTE TERMS FOR BDRY CONDS LINES  13 AND 17
C
  310 DO 360 I=1,6
      IF(I-4)    330,  340,  320
```

```
320 IF(I-5)   330,  350,  330
330 AMTX(I,I,1)=1.0
    GO TO   360
340 AMTX(I,I,1)=C4**2
    AMTX(I,I-2,1)=-SIG*BETA*S4**2
    AMTX(I,I-3,1)=SIG*BETA*S4*C4
    AMTX(I,I+1,1)=-S4*C4
    AMTX(I,I,KK)=-S4**2/2.0
    AMTX(I,I-2,KK)=-AMTX(I,I-2,1)
    AMTX(I,I-3,KK)=-AMTX(I,I-3,1)
    AMTX(I,I+1,KK)=AMTX(I,I+1,1)/2.0
    GO TO   360
350 AMTX(I,I,1)=S4**2
    AMTX(I,I-1,1)=-S4*C4
    AMTX(I,I-3,1)=-SIG*BETA*S4*C4
    AMTX(I,I-4,1)=SIG*BETA*C4**2
    AMTX(I,I,KK)=-C4**2/2.0
    AMTX(I,I-1,KK)=AMTX(I,I-1,1)/2.0
    AMTX(I,I-3,KK)=-AMTX(I,I-3,1)
    AMTX(I,I-4,KK)=-AMTX(I,I-4,1)
360 AMTX(I,I,K)=-1.0
    GO TO   210
    END
```

SUBROUTINE SJLOG   PINTLE DISPL, GUN ORIENTATION, AND STRESSES

```
      SUBROUTINE SJLOG(D,DOT,DDOT,ID,XP,YP,ZP,DXP,DYP,DZP,SIGMA,ELEV)
      DIMENSION D(138),DDOT(138),TI(3,3),DLOC(15,6,2),X(6),Y(6),Z(6)
      DIMENSION T(6,6),ID(12,3),SIGMA(12,2),DOT(138)
      COMMON N
      COMMON THET1,THET2,THET3,THET4,PROP(19,5),SEG(19),CAP(20)
      COMMON AJ(6,3),G1,G2,ANU,RO,RHO
      COMMON C(6,3),F(50,5),DTAU,BETA,AZIM,IM,VO,PHIO
      COMMON N4,AAM(3,4)
C
C        COMPUTE THE PINTLE DISPLACEMENTS AND GUN ELEV AND AZIM
C
      XP=D(19) - 2.05*D(24)
      YP=D(20)
      ZP=D(21)  +  2.05*D(22).
      DXP=DOT(19) - 2.05*DOT(24)
      DYP=DOT(20)
      DZP=DOT(21)  +  2.05*DOT(22)
      D1=13.50
      HU=2.22
      AJJ=1.0
      IF(AZIM)  70,80,70
 70   AJJ=-6.0*SIGN(AJJ,AZIM)
      GO TO  90
 80   AJJ=0.0
 90   JJ=AJJ
      JJ=85  +  JJ
      DX=D(JJ)  -  D(19)
      DY=D(20)  -  D(JJ+1)
      ELEV=(HU*DX+VO*DY)/(D1*((VO+DY)*COS(PHIO)-(HU+DX)*SIN(PHIO)))
      AZIM=AZIM
C
C        TRANSFORM SELECTED DISPLS. AND ACCELS.  TO LOCAL COORD
C
      DO 260 I=1,15
      IF(I-3)   100,  100,  110
```

```
100 I1=1
    I2=30
    KODE=2
    GO TO   150
110 IF(I-14)   120,  140,  140
120 I1=4
    I2=56
    KODE=2*(I/12)  +  3*(6/I)
    IF(KODE)   150,  130,  150
130 KODE=4
    GO TO   150
140 I1=14
    I2=120
    KODE=3
150 JJ=6*(I-I1)  +  I2
    DO 160 J=1,6
    JJ=JJ + 1
    X(J)=D(JJ)
160 Y(J)=DDOT(JJ)
    DO 180 J=1,6
    IF(X(J))    200,  170,  200
170 IF(Y(J))    200,  180,  200
180 CONTINUE
    DO 190 K=1,2
    DO 190 J=1,6
190 DLOC(I,J,K)=0.0
    GO TO   260
200 CALL SJLTI(KODE,TI)
    DO 210 J=1,6
    DO 210 K=1,6
210 T(J,K)=0.0
    DO 220 L=1,4,3
    K=L-1
    DO 220 II=1,3
    DO 220 JJ=1,3
    IK=II + K
    JK=JJ + K
220 T(IK,JK)=TI(II,JJ)
    N=6
    M=6
    L=1
    K=1
    CALL GMPRD(T,X,Z,N,M,L)
230 DO 240 J=1,6
240 DLOC(I,J,K)=Z(J)
    IF(K-2)   250,  260,  260
250 K=2
    CALL GMPRD(T,Y,Z,N,M,L)
    GO TO   230
260 CONTINUE
C
C        CALC STRESSES IN EACH SELECTED ELEMENT AND DETERMINE MAX STRESS
C
    E=29.0E+06
    GVTY=386.4
    DO 290 I=1,12
    LL=ID(I,1)
    XI=0.0
    DXI=0.1*SEG(LL)
    SIGM=0.0
    XII=0.0
    DO 280 IX=1,11
    I1=ID(I,2)
    I2=ID(I,3)
    BET=12.0*E*PROP(LL,3)/SEG(LL)**3
    DELTA=6.0*E*PROP(LL,3)/SEG(LL)**2
    AMBDA=2.0*E*PROP(LL,3)/SEG(LL)
    AM=PROP(LL,2)/GVTY
    AL=SEG(LL)
```

```
      AMZ1K=DELTA*(DLOC(I1,2,1)-DLOC(I2,2,1))
      AMZ1K=AMZ1K + AMBDA *(2.0*DLOC(I1,6,1) + DLOC(I2,6,1))
      AMZ1M=(AM*AL**2/420.)*(22.*DLOC(I1,2,2)+13.*DLOC(I2,2,2))
      AMZ1M=AMZ1M + AM*AL**3*(DLOC(I1,6,2)/105.-DLOC(I2,6,2)/140.)
      FY1K=BET*(DLOC(I1,2,1)-DLOC(I2,2,1))+DELTA* DLOC(I1,6,1)
      FY1K=FY1K + DELTA*DLOC(I2,6,1)
      FY1M=(AM*AL/70.)*(26.*DLOC(I1,2,2)+9.*DLOC(I2,2,2))
      FY1M=FY1M+(AM*AL**2/420.)*(22.*DLOC(I1,6,2)-13.*DLOC(I2,6,2))
      AMY1K=DELTA*(-DLOC(I1,3,1)+DLOC(I2,3,1))
      AMY1K=AMY1K + AMBDA*(2.0*DLOC(I1,5,1)+DLOC(I2,5,1))
      AMY1M=(-AM*AL**2/420.)*(22.*DLOC(I1,3,2)+13.*DLOC(I2,3,2))
      AMY1M=AMY1M +AM*AL**3*(DLOC(I1,5,2)/105.-DLOC(I2,5,2)/140.)
      FZ1K= BET *(DLOC(I1,3,1)-DLOC(I2,3,1))-DELTA*DLOC(I1,5,1)
      FZ1K=FZ1K - DELTA*DLOC(I2,5,1)
      FZ1M=(AM*AL/70.)*(26.*DLOC(I1,3,2)+9.*DLOC(I2,3,2))
      FZ1M=FZ1M + (AM*AL**2/420.)*(22.*DLOC(I1,5,2)-13.*DLOC(I2,5,2))
      Q1=AMZ1K + AMZ1M - XI*(FY1K+FY1M)
      Q2=0.1*(XI/AL)**3-0.25*(XI/AL)**2 + 0.5
      Q3=Q2 - 0.5
      Q2=AM*XI**2*(Q2*DLOC(I1,2,2)-Q3*DLOC(I2,2,2))
      Q3=(XI/AL)**2/20.0-(XI/AL)/6.0 + 1.0/6.0
      Q4=(XI/AL)**2/20.0 - (XI/AL)/12.0
      Q3=AM*XI**3*(Q3*DLOC(I1,6,2) + Q4*DLOC(I2,6,2))
      AMXY=Q1 + Q2 + Q3
      Q1=AMY1K + AMY1M + XI*(FZ1K+FZ1M)
      Q2=0.1*(XI/AL)**3-0.25*(XI/AL)**2 + 0.5
      Q3=Q2 - 0.5
      Q2=AM*XI**2*(Q3*DLOC(I2,3,2)- Q2*DLOC(I1,3,2))
      Q3=(XI/AL)**2/20.0 -(XI/AL)/6.0  + 1.0/6.0
      Q4=(XI/AL)**2/20.0 - (XI/AL)/12.0
      Q3=AM*XI**3*(Q3*DLOC(I1,5,2) + Q4*DLOC(I2,5,2))
      AMXZ=Q1 + Q2 + Q3
C        ALL TERMS FOR BENDING AND COMPRESSIVE STRESSES ARE COMPUTED
      SIGB=(PROP(LL,5)/PROP(LL,3))*SQRT(AMXY**2+AMXZ**2)
      Q1=(L/AL)*(DLOC(I1,1,1)-DLOC(I2,1,1))
      Q2=1./3. - (XI/AL)  + 0.5*(XI/AL)**2
      Q3=1./6. - 0.5*(XI/AL)**2
      SIGC=Q1 +  (AM*AL/PROP(LL,1))*(Q2*DLOC(I1,1,2)+Q3*DLOC(I2,1,2))
      SIG=SIGB + SIGC
      IF(ABS(SIG) - ABS(SIGM))   280,  280,  270
  270 SIGM=SIG
      XII=XI
  280 XI=XI + DXI
      SIGMA(I,1)=SIGM
      SIGMA(I,2)=XII + 0.00001
  290 CONTINUE
      RETURN
      END
```


<u>SUBROUTINE SJLGS   GAUSS-SEIDEL SOLN FOR SIMULTANEOUS EQNS</u>

```
      SUBROUTINE SJLGS(IFYLE,DDOT,Q)
      DIMENSION FYLE(138),Q(138),DDOT(138),X(138,2)
      DIMENSION IDD(23,2)
      COMMON N
      COMMON THET1,THET2,THET3,THET4,PROP(19,5),SEG(19),CAP(20)
      COMMON AJ(6,3),G1,G2,ANU,RO,RHO
      COMMON C(6,3),F(50,5),DTAU,BETA,AZIM,IM,VO,PHIO
      COMMON N4,AAM(3,4)
   10 FORMAT(1H0,'IT=',I5)
   11 FORMAT(1H ,2I5,6E15.4)
   12 FORMAT(1H0,'MORE THAN 50 ITERATIONS REQ''D,  JOB TERMINATED')
```

```
      MO=5
      IT=0
      OMEGA=1.057
      EPS=0.8
      CALL SJLOO(IOD)
      CALL SLITET(1,ISW)
      GO TO (120,100),ISW
  100 DO 110 I=1,138
      X(I,1)=DUOT(I)
  110 X(I,2)=X(I,1)
      GO TO   170
  120 DO 130 I=1,138
      X(I,1)=0.0
  130 X(I,2)=0.0
      DO 160 I=1,23
      N=6*IOD(I,1) - 5
      II=N
      DO 160 J=1,6
      JJ=6*(IOD(I,2)-1) + J
      READ(IFYLE'N) FYLE
      X(JJ,1)=W(II)/FYLE(JJ)
      II=II+1
      DO 150 K=1,138
      IF(K-JJ)   140,  150,  140
  140 X(JJ,1)=X(JJ,1) - FYLE(K)*X(K,1)/FYLE(JJ)
  150 CONTINUE
  160 X(JJ,2)=X(JJ,1)
  170 CALL DATSW(7,ISW)
      GO TO (180,220),ISW
  180 WRITE(MO,10 )IT
      DO 210 K=1,23
      J1=6*K - 5
      J2=J1 + 5
      DO 190 J=J1,J2
      IF(X(J,1))   200,  190,  200
  190 CONTINUE
      GO TO   210
  200 WRITE(MO,11 )K,J1,(X(J,1), J=J1,J2)
  210 CONTINUE
  220 IT=IT + 1
  230 DO 260 I=1,23
      N=6*IOD(I,1) - 5
      II=N
      DO 260 J=1,6
      JJ=6*(IOD(I,2) - 1) + J
      READ(IFYLE'N) FYLE
      X(JJ,2)=W(II)/FYLE(JJ)
      II=II+1
      DO 250 K=1,138
      IF(K-JJ)   240,  250,  240
  240 X(JJ,2)=X(JJ,2) - FYLE(K)*X(K,2)/FYLE(JJ)
  250 CONTINUE
  260 X(JJ,2)=OMEGA*X(JJ,2) + (1.0-OMEGA)*X(JJ,1)
  270 CALL DATSW(7,ISW)
      GO TO (280,320),ISW
  280 WRITE(MO,10 )IT
      DO 310 K=1,23
      J1=6*K - 5
      J2=J1 + 5
      DO 290 J=J1,J2
      IF(X(J,2))   300,  290,  300
  290 CONTINUE
      GO TO   310
  300 WRITE(MO,11 )K,J1,(X(J,2),J=J1,J2)
  310 CONTINUE
  320 DO 390 I=1,138
      DIFF=X(I,2) - X(I,1)
      IF(ABS( DIFF) - 1.0E-01)  325,325,340
  325 IF(ABS(X(I,1)) - 1.0E-02)  330,330,340
```

```
330 X(I,2)=0.0
    GO TO   390
340 IF(X(I,2))    360,  350,  360
350 IF(X(I,1))    370,  390,  370
360 DIFF=DIFF/X(I,2)
    GO TO   380
370 DIFF=DIFF/X(I,1)
380 IF(ABS(DIFF) - EPS)   390,  390,  400
390 CONTINUE
    GO TO   420
400 DO 410 I=1,138
410 X(I,1)=X(I,2)
    IF(IT-50)  220,415,415
415 WRITE(MO, 12)
    CALL EXIT
420 DO 430 I=1,138
430 DDOT(I)=X(I,2)
    RETURN
    END
```


SUBROUTINE SJLKQ   K AND Q MODIFICATIONS FOR LEG LIFTING

```
      SUBROUTINE SJLKQ(KC)
      DIMENSION FYLEK(138),FYLEM(138)
      COMMON N
      COMMON THET1,THET2,THET3,THET4,PROP(19,5),SEG(19),CAP(20)
      COMMON AJ(6,3),G1,G2,ANU,RO,RHO
      COMMON C(6,3),F(50,5),DTAU,BETA,AZIM,IM,VO,PHIO
      COMMON N4,AAM(3,4)
C
C        KC=1  SIGNIFIES NODAL POINT 1 LIFTED
C        KC=3  SIGNIFIES NODAL POINT 16 LIFTED
C        KC=4  SIGNIFIES NODAL POINTS 1 + 16 LIFTED
C        KC=5  SIGNIFIES NODAL POINT 19 LIFTED
C        KC=6  SIGNIFIES NODAL POINTS 1 + 19 LIFTED
C        KC=8  SIGNIFIES NODAL POINTS 16 + 19 LIFTED
C        KC=9  SIGNIFIES NODAL POINTS 1 + 16 + 19 LIFTED
C
      KODE=0
  100 J2=3
      I2=24
      GO TO (110,170,120,140,130,150,170,160,170),KC
  110 I1=20
      I2=23
      J1=2
      J2=3
      GO TO   180
  120 I1=1
      I2=23
      J1=1
      J2=3
      GO TO   180
  130 I1=1
      I2=20
      J1=1
      J2=2
      GO TO   180
  140 I1=23
      J1=3
      GO TO   180
  150 I1=20
      J1=2
      GO TO   180
```

```
  160 I1=1
      J1=1
      GO TO   180
  170 I1=24
  180 IF(KODE)   190,  190,  280
C
C          FORM (K)  MATRIX FOR LEGS IN GROUND
C
  190 N=1
      DO 270 I=1,23
      II=6*I - 5
      DO 270 J=1,6
      READ(5'N) FYLEK
      IJ=II + J - 1
      IF(I - I1)   260,  200,  210
  200 IJJ=J1
      GO TO   230
  210 IF(I - I2)   260,  220,  260
  220 IJJ=J2
  230 IF(J-3)   240,  240,  250
  240 FYLEK(IJ)=FYLEK(IJ) + AJ(IJJ+3,J)
      GO TO   260
  250 FYLEK(IJ)=FYLEK(IJ) + AJ(IJJ,J-3)
  260 N=N - 1
C
C          FILE 7  CONTAINS PARTLY MODIFIED (K) MATRIX
C
      WRITE(7'N) FYLEK
  270 CONTINUE
      KODE=1
      GO TO   100
C
C          FORM (Q) MATRIX FROM (K) AND (M) AND C
C          STORE MODIFIED (Q)  AS FILE 8
C
  280 N=1
      DO 390 I=1,23
      II=6*I - 5
      JJ=II
      DO 390 J=1,6
      JJ=JJ + 1
      READ(7'N) FYLEK
      N=N - 1
      READ(1'N) FYLEM
      IJ=II + J - 1
      IF(I - I1)   360,  290,  300
  290 IJJ=J1
      GO TO   320
  300 IF(I - I2)   360,  310,  360
  310 IJJ=J2
  320 DO 350 K=1,138
      IF(K - JJ)   340,  330,  340
  330 FYLEM(K)=FYLEM(K) + (DTAU/2.0) * C(J,IJJ) + (DTAU**2/6.0)*FYLEK(K)
      GO TO   350
  340 FYLEM(K)=FYLEM(K) + (DTAU**2/6.0) * FYLEK(K)
  350 CONTINUE
      GO TO   380
  360 DO 370 K=1,138
  370 FYLEM(K)=FYLEM(K) + (DTAU**2/6.0) * FYLEK(K)
  380 N=N - 1
      WRITE(8'N) FYLEM
  390 CONTINUE
      IFYLE=8
      CALL SOLUB(FYLEM,IFYLE)
      RETURN
      END
```

SUBROUTINE SJLLL   MATRICES MODIFIER FOR LEG LIFTING

```
      SUBROUTINE SJLLL(KODE,KC,KKC,KKKC,TAU,G,D,DOT,LL,D2,D116,D134)
      DIMENSION D(138),DOT(138),G(138)
      COMMON N
      COMMON THET1,THET2,THET3,THET4,PROP(19,5),SEG(19),CAP(20)
      COMMON AJ(6,3),G1,G2,ANU,RO,RHO
      COMMON C(6,3),F(50,5),DTAU,BETA,AZIM,IM,VO,PHIO
      COMMON V4,AAA(3,4)
      IF(KODE)   270,  100,  350
  100 CALL SJLGO(C,TAU,F,BETA,AZIM,IM)
      I3=139
      D2=D(2)
      D116=D(116)
      D134=D(134)
      GO TO (110,170,120,130,140,150,170,160,170),KC
  110 I1=2
      I2=139
      GO TO   180
  120 I1=116
      I2=139
      GO TO   180
  130 I1=2
      I2=116
      GO TO   180
  140 I1=134
      I2=139
      GO TO   180
  150 I1=2
      I2=134
      GO TO   180
  160 I1=116
      I2=134
      GO TO   180
  170 I1=2
      I2=116
      I3=134
  180 DO 260 I=1,138
      IF(I-I1)    190,  210,  190
  190 IF(I-I2)    200,  210,  200
  200 IF(I-I3)    260,  210,  260
  210 IF(I-116)    220,  230,  240
  220 II=4
      GO TO   250
  230 II=5
      GO TO   250
  240 II=6
  250 G(I)=-AJ(II,2)*D(I) - C(2,II-3)*DOT(I)
  260 CONTINUE
      KKC=KC
      KODE=-1
      LL=1
      RETURN
  270 EPS=0.8
      DIFF1=ABS(D2-D(2))
      DIFF2=ABS(D116-D(116))
      DIFF3=ABS(D134-D(134))
      IF(D2)   290,  280,  290
  280 IF(D(2))    300,  310,  300
  290 DIFF1=DIFF1/ABS(D2)
      GO TO   320
  300 DIFF1=DIFF1/ABS(D(2))
      GO TO   320
  310 DIFF1=0.0
  320 IF(D116)    340,  330,  340
  330 IF(D(116))    350,  360,  350
  340 DIFF2=DIFF2/ABS(D116)
      GO TO   370
```

```
350 DIFF2=DIFF2/ABS(D(116))
    GO TO   370
360 DIFF2=0.0
370 IF(D134)    390,  380,  390
380 IF(D(134))    400,  410,  400
390 DIFF3=DIFF3/ABS(D134)
    GO TO   420
400 DIFF3=DIFF3/ABS(D(134))
    GO TO   420
410 DIFF3=0.0
420 GO TO (430,520,440,450,470,480,520,500,520),KC
430 IF(DIFF1-EPS)    550,  550,  100
440 IF(DIFF2-EPS)    550,  550,  100
450 IF(DIFF1-EPS)    460,  460,  100
460 IF(DIFF2-EPS)    550,  550,  100
470 IF(DIFF3-EPS)    550,  550,  100
480 IF(DIFF1-EPS)    490,  490,  100
490 IF(DIFF3-EPS)    550,  550,  100
500 IF(DIFF2-EPS)    510,  510,  100
510 IF(DIFF3-EPS)    550,  550,  100
520 IF(DIFF1-EPS)    530,  530,  100
530 IF(DIFF2-EPS)    540,  540,  100
540 IF(DIFF3-EPS)    550,  550,  100
550 KODE=1
    LL=2
    CALL SJLQQ(Q,TAU,F,BETA,AZIM,IM)
    IF(KKKC-KC)   560,  570,  560
560 CALL SJLKQ(KC)
    KKKC=KC
570 RETURN
    END
```


SUBROUTINE SJLMP   MATRIX PRODUCT OF FILE MATRIX BY COLUMN MATRIX

```
      SUBROUTINE SJLMP(IFYLE,B,R,KC)
      DIMENSION B(1),R(1),FYLE(138)
      COMMON NN
      COMMON THET1,THET2,THET3,THET4,PROP(19,5),SEG(19),CAP(20)
      COMMON AJ(6,3),G1,G2,ANU,RO,RHO
      COMMON C(6,3),F(50,5),DTAU,BETA,AZIM,IM,VO,PHIO
      COMMON N4,AAM(3,4)
      N=138
      M=138
      L=1
C
C        IFYLE      FILE NUMBER OF FIRST INPUT MATRIX  (ON FILE)
C        B          NAME OF SECOND INPUT MATRIX  (COLUMN)
C        R          NAME OF OUTPUT MATRIX  (COLUMN)
C        N          NUMBER OF ROWS IN FILE MATRIX AND ROWS OF R
C        M          NUMBER OF COLUMNS IN FILE MATRIX AND ROWS OF B
C        L          NUMBER OF COLUMNS IN B AND R
C        KC         GROUND DAMPING OPTION INDEX
C                   KC=0      ALL LEGS BEING DAMPED
C                   KC=1      16 AND 19 BEING DAMPED
C                   KC=3      1 AND 19  BEING DAMPED
C                   KC=4             19  BEING DAMPED
C                   KC=5      1 AND 16  BEING DAMPED
C                   KC=6             16  BEING DAMPED
C                   KC=8              1  BEING DAMPED
C                   KC=9      NO LEGS    BEING DAMPED
C
```

```
      IF(IFYLE - 4)   290,  100,  290
C
C
C        CLEAR FILE  4
C
  100 DO 110 I=1,138
  110 FYLE(I)=0.0
      N4=1
      DO 120 I=1,138
  120 WRITE(4'N4) FYLE
C
C        SPECIAL CASE WHEN  IFYLE=4  (DAMPING MATRIX)
C
      IF(KC)   130,  130,  140
  130 KC=2
  140 IC=1
  150 GO TO (160,220,170,180,190,200,320,210,350),KC
  160 GO TO (250,270,320,320),IC
  170 GO TO (230,270,320,320),IC
  180 GO TO (270,320,320,320),IC
  190 GO TO (230,250,320,320),IC
  200 GO TO (250,320,320,320),IC
  210 GO TO (230,320,320,320),IC
  220 GO TO (230,250,270,320),IC
  230 DO 240 I=1,6
      N4=138*(I-1) + 1
  240 WRITE(4'N4) C(I,1)
      IC=IC + 1
      GO TO   150
  250 II=0
      DO 260 I=1,6
      N4=15846 + 138*(I-1) + 1
      II=II + 1
  260 WRITE(4'N4) C(II,2)
      IC=IC + 1
      GO TO   150
  270 II=0
      DO 280 I=1,6
      N4=18348 + 138*(I-1) + 1
      II=II + 1
  280 WRITE(4'N4) C(II,3)
      IC=IC + 1
      GO TO   150
C
C        GENERAL CASE  IFYLE= 1, 2, 3, 5, OR 6
C
  290 IR=1
      NN=1
      DO 310 J=1,N
      R(IR)=0.0
      READ(IFYLE'NN) FYLE
      DO 300 I=1,M
  300 R(IR)=R(IR)  +  FYLE(I) * B(I)
  310 IR=IR + 1
      RETURN
  320 IR=1
      N4=1
      DO 340 J=1,N
      R(IR)=0.0
      READ(4'N4) FYLE
      DO 330 I=1,M
  330 R(IR)=R(IR) + FYLE(I) * B(I)
  340 IR=IR + 1
      RETURN
  350 DO 360 I=1,138
  360 R(I)=0.0
      RETURN
      END
```

```
      SUBROUTINE SJLMX(KASE,ALPHA,BETA,GAMMA,DELTA,AMBDA,AMU,RHO,TAU,
     1 AMTX,TI,LL)
      DIMENSION AMTX(6,6,4),TI(3,3),T(12,12),Y(12,12),Z(12,12),X(12,12)
   10 FORMAT(1H0,'MASS MATRIX')
   11 FORMAT(1H0,'STIFFNESS MATRIX')
   12 FORMAT(1H ,12E10.3)
   13 FORMAT(1H0)
      MU=5
C
C        CLEAR X,Y,Z, AND  T  ARRAYS
C
      DO 100 I=1,12
      DO 100 J=1,12
      X(I,J)=0.0
      Y(I,J)=0.0
      Z(I,J)=0.0
  100 T(I,J)=0.0
C
C        FILL T  WITH  TI  ALONG  DIAGONAL
C
      DO 110 L=1,10,3
      K=L-1
      DO 110 I=1,3
      DO 110 J=1,3
      IK=I + K
      JK=J + K
  110 T(IK,JK)=TI(I,J)
C
      GO TO (120,150),KASE
C        KASE=1     FILL AMTX WITH MASS MATRIX TERMS
C        KASE=2     FILL AMTX WITH STIFFNESS MATRIX TERMS
C
  120 SIG=1.0
      DO 130 K=1,4,3
      AMTX(1,1,K)=2.0*ALPHA
      AMTX(2,2,K)=BETA
      AMTX(3,3,K)=BETA
      AMTX(4,4,K)=2.0*GAMMA
      AMTX(5,5,K)=RHO
      AMTX(6,6,K)=RHO
      AMTX(6,2,K)=DELTA*SIG
      AMTX(5,3,K)=-DELTA*SIG
      AMTX(3,5,K)=-DELTA*SIG
      AMTX(2,6,K)=DELTA*SIG
  130 SIG=-1.0
      SIG=-1.0
      DO 140 K=2,3
      AMTX(1,1,K)=ALPHA
      AMTX(2,2,K)=AMBDA
      AMTX(3,3,K)=AMBDA
      AMTX(4,4,K)=GAMMA
      AMTX(5,5,K)=-TAU
      AMTX(6,6,K)=-TAU
      AMTX(6,2,K)=-AMU*SIG
      AMTX(5,3,K)=AMU*SIG
      AMTX(3,5,K)=-AMU*SIG
      AMTX(2,6,K)=AMU*SIG
  140 SIG=1.0
      GO TO  180
  150 SIG=1.0
      DO 160 K=1,4,3
      AMTX(1,1,K)=ALPHA
      AMTX(2,2,K)=BETA
      AMTX(3,3,K)=BETA
      AMTX(4,4,K)=GAMMA
      AMTX(5,5,K)=2.0*AMBDA
      AMTX(6,6,K)=2.0*AMBDA
```

```
      AMTX(6,2,K)=DELTA*SIG
      AMTX(5,3,K)=-DELTA*SIG
      AMTX(3,5,K)=-DELTA*SIG
      AMTX(2,6,K)=DELTA*SIG   .
  160 SIG=-1.0
      SIG=1.0
      DO 170 K=2,3
      AMTX(1,1,K)=-ALPHA
      AMTX(2,2,K)=-BETA
      AMTX(3,3,K)=-BETA
      AMTX(4,4,K)=-GAMMA
      AMTX(5,5,K)=AMBDA
      AMTX(6,6,K)=AMBDA
      AMTX(6,2,K)=-DELTA*SIG
      AMTX(5,3,K)=DELTA*SIG
      AMTX(3,5,K)=-DELTA*SIG
      AMTX(2,6,K)=DELTA*SIG
  170 SIG=-1.0
C
C          FILL  X  WITH   AMTX   TERMS
C
  180 KI=0
      KJ=0
      DO 230 L=1,4
      DO 190 I=1,6
      DO 190 J=1,6
      IK=I + KI
      JK=J + KJ
  190 X(IK,JK)=AMTX(I,J,L)
      GO TO (200,210,220,230),L
  200 KJ=6
      GO TO   230
  210 KI=6
      KJ=0
      GO TO   230
  220 KJ=6
  230 CONTINUE
C
C          PERFORM  (T TRANSPOSE)  (X)  (T)
C
      N=12
      M=12
      L=12
      CALL MTRA(T,Z,N,M,0)
      CALL GMPRD(Z,X,Y,N,M,L)
      CALL GMPRD(Y,T,Z,N,M,L)
      DO 250 I=1,12
      DO 250 J=1,12
      IF(Z(I,J))   240,  250,  240
  240  ZZ    =EXP(ALOG(Z(I,J)))
      Z(I,J)=SIGN(ZZ,Z(I,J))
  250 CONTINUE
C
C          SELECT TRANSFORMATION FOR ELEMENT  LL
C
      IF(LL-3)   260,  260,  270
  260 ITR=1
      GO TO   350
  270 IF(LL-8)   280,  310,  290
  280 ITR=2
      GO TO   350
  290 IF(LL-9)   280,  280,  300
  300 IF(LL-11)   310,  310,  320
  310 ITR=3
      GO TO   350
  320 IF(LL-16)   330,  330,  340
  330 ITR=4
      GO TO   350
  340 IF(LL-19)   280,  280,  310
```

```
  350 GO TO (360,460,460,520),ITR
C              ITR=1  SIGNIFIES TRANSFORMATION  1
C              ITR=2  SIGNIFIES TRANSFORMATION  2
C              ITR=3  SIGNIFIES TRANSFORMATION  3
C              ITR=4  SIGNIFIES TRANSFORMATION  4
C
C
C         MAKE ZERO CELLS FOR TRANSFORMATION  1  ELEMENTS
C
  360 KI=0
      DO 450 L=1,2
      DO 440 I=1,6
      IK=I + KI
      IF(I-2)   380,  380,  370
  370 IF(I-6)   410,  380,  380
  380 KJ=0
      DO 400 K=1,2
      DO 390 J=3,5
      JK=J + KJ
  390 Z(IK,JK)=0.0
  400 KJ=6
      GO TO   440
  410 KJ=0
      DO 430 K=1,2
      DO 420 J=1,2
      JK=J + KJ
  420 Z(IK,JK)=0.0
      Z(IK,KJ+6)=0.0
  430 KJ=6
  440 CONTINUE
  450 KI=6
      GO TO   610
C
C         MAKE ZERO CELLS FOR TRANSFORMATION 2 AND 3  ELEMENTS
C
  460 KI=0
      DO 510 I=1,2
      KJ=0
      DO 500 K=1,2
      DO 490 J=1,6
      IK=KI + J
      IF(J-3)    470,  470,  480
  470 JK=KJ + J + 3
      GO TO   490
  480 JK=KJ + J -3
  490 Z(IK,JK)=0.0
  500 KJ=6
  510 KI=6
      GO TO   610
C
C         MAKE ZERO CELLS FOR TRANSFORMATION  4  ELEMENTS
C
  520 KI=0
      DO 600 L=1,2
      DO 590 I=1,6
      IK=I + KI
      KJ=0
      DO 580 K=1,2
      DO 570 J=1,6
      JK=KJ + J
      IF(I-J)     530,  570,  530
  530 IF(J-3)     540,  560,  540
  540 IF(J-6)     550,  560,  550
  550 JJ=6-J
      IF(I-JJ)    560,  570,  560
  560 Z(IK,JK)=0.0
  570 CONTINUE
  580 KJ=6
  590 CONTINUE
```

```
  600 KI=6
C
  610 CALL DATSW(2,ISW)
      GO TO (620,680),ISW
C         SW 2  ON   WRITE OUT SUB MATRIX BEFORE AND AFTER TRANSFORMATION
C         SW 2 OFF   TRANSFER  X  INTO  AMTX  AND RETURN
C
  620 IF(KASE-1)   630,  630,  640
  630 WRITE(MO,10 )
      GO TO   650
  640 WRITE(MO,11 )
  650 DO 660 I=1,12
  660 WRITE(MO,12 ) (X(I,J), J=1,12)
      WRITE(MO,13 )
      DO 670 I=1,12
  670 WRITE(MO,12 ) (Z(I,J), J=1,12)
  680 KI=0
      KJ=0
      DO 730 L=1,4
      DO 690 I=1,6
      DO 690 J=1,6
      IK=I + KI
      JK=J + KJ
  690 AMTX(I,J,L)=Z(IK,JK)
      GO TO (700,710,720,730),L
  700 KJ=6
      GO TO   730
  710 KI=6
      KJ=0
      GO TO   730
  720 KJ=6
  730 CONTINUE
      RETURN
      END
```

SUBROUTINE SJLOD   ORDER OF SOLN FOR GAUSS-SEIDEL

```
      SUBROUTINE SJLOD(IOD)
      DIMENSION IOD(23,2)
      I=1
      IOD(I,1)=9
      IOD(I,2)=4
      I=I+1
      DO 100 K=1,2
  100 IOD(I,K)=15
      DO 140 J=1,5
      I=I+1
      JJ=6-J
      IF(J-2)   110,  130,  110
  110 DO 120 K=1,2
  120 IOD(I,K)=JJ
      GO TO   140
  130 IOD(I,1)=4
      IOD(I,2)=9
  140 CONTINUE
      J1=5
      DO 250 J=1,17
      I=I+1
      IF(J-4)   200,  210,  150
  150 IF(J-8)   200,  220,  160
  160 IF(J-10)   200,  170,  180
  170 I=I-1
      GO TO   250
  180 IF(J-12)   200,  190,  190
  190 J1=6
  200 JJ=J + J1
      GO TO   230
  210 JJ=13
      GO TO   230
  220 JJ=17
  230 DO 240 K=1,2
  240 IOD(I,K)=JJ
  250 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SJLOU(KOUNT,ID,SIGMA,ELEV,TAU,XP,YP,ZP,DXP,DYP,DZP)
      DIMENSION ID(12,3),SIGMA(12,2)
      COMMON N
      COMMON THET1,THET2,THET3,THET4,PROP(19,5),SEG(19),CAP(20)
      COMMON AJ(6,3),G1,G2,ANU,RO,RHO
      COMMON C(6,3),F(50,5),DTAU,BETA,AZIM,IM,VO,PHIO
      COMMON N4,AAM(3,4)
      COMMON D(138),DOT(138),DDOT(138)
   10 FORMAT(1H1,20X,'TRIPOD MOUNT M 3 FOR 0.50 CALIBER MACHINE GUN',/
     1 21X,'CASE ANALYZED- ',20A4,//,1X,'TRANSIENT RESPONSE -')
   11 FORMAT(1H0,2X,'TIME',3X,'NODAL',7X,'DISPLACEMENTS AND VELOCITIES',
     118X,'ELEVATION',3X,'AZIMUTH',3X,'ELEMENT',3X,'PEAK COMBINED',3X,
     2'OCCURRING AT',/,2X,'(SEC.)  POINT',12X,'(IN)',10X,'(IN/SEC)',11X,
     3'ANGLE',6X,'ANGLE',5X,'NUMBER',7X,'STRESS',8X,'LOCATION',/,23X,
     4'X',13X,'Y',13X,'Z',3X,2(5X,'(DEG)'),19X,'(PSI)',6X,'(IN.FROM END)
     5',/,1X,120('-'))
   12 FORMAT(1H0,F7.5,I5,3E14.3,F11.5,F10.3,I8,E19.5,F12.3)
   13 FORMAT(1H ,7X,I5,3E14.3,21X,I8,E19.5,F12.3)
   14 FORMAT(1H ,12X,3E14.3,21X,I8,E19.5,F12.3)
   15 FORMAT(1H ,75X,I8,E19.5,F12.3)
      MO=5
      IF(KOUNT - 3)   110,  100,  100
  100 KOUNT=0
      WRITE(MO,10 )  CAP
      WRITE(MO,11 )
  110 KOUNT=KOUNT + 1
      DO 180 I=1,10,3
      IF(I-1)   120,  120,  130
  120 NP=4
      ELV=ELEV/0.01745
      AZ=AZIM/0.01745
      WRITE(MO,12 )TAU,NP,XP,YP,ZP,ELV,AZ,ID(I,1),(SIGMA(I,J),J=1,2)
      WRITE(MO,14 ) DXP,DYP,DZP,ID(I+1,1),(SIGMA(I+1,J),J=1,2)
      GO TO   180
  130 II=(I-1)/3
      GO TO (140,150,160),II
  140 NP=1
      J1=1
      J2=3
      GO TO   170
  150 NP=16
      J1=115
      J2=117
      GO TO   170
  160 NP=19
      J1=133
      J2=135
  170 WRITE(MO,13 ) NP,(D(J),J=J1,J2),ID(I,1),(SIGMA(I,J), J=1,2)
      WRITE(MO,14 )  (DOT(J),J=J1,J2),ID(I+1,1),(SIGMA(I+1,J),J=1,2)
  180 WRITE(MO,15 )  ID(I+2,1),(SIGMA(I+2,J),J=1,2)
      RETURN
      END
```

```
      SUBROUTINE SJLTI(KODE,TI)
      DIMENSION TI(3,3)
      COMMON N
      COMMON THET1,THET2,THET3,THET4,PROP(19,5),SEG(19),CAP(20)
      DO 100 II=1,3
      DO 100 JJ=1,3
  100 TI(II,JJ)=0.0
      GO TO (110,120,130,140,120,130),KODE
C
C          COMPUTE SUBMATRIX FOR TRANSFORMATION  1
C
  110 TI(1,1)=COS(THET1)
      TI(2,2)=COS(THET1)
      TI(2,1)=-SIN(THET1)
      TI(1,2)=SIN(THET1)
      TI(3,3)=1.0
      GO TO   150
C
C          COMPUTE SUBMATRIX FOR TRANSFORMATION  2
C
  120 TI(1,1)=COS(THET2)*COS(THET3)
      TI(1,2)=-SIN(THET2)
      TI(1,3)=COS(THET2)*SIN(THET3)
      TI(2,1)=SIN(THET2)*COS(THET3)
      TI(2,2)=COS(THET2)
      TI(2,3)=SIN(THET2)*SIN(THET3)
      TI(3,1)=-SIN(THET3)
      TI(3,3)=COS(THET3)
      GO TO   150
C
C          COMPUTE SUBMATRIX FOR TRANSFORMATION  3
C
  130 TI(1,1)=COS(THET2)*COS(THET3)
      TI(1,2)=-SIN(THET2)
      TI(1,3)=-COS(THET2)*SIN(THET3)
      TI(2,1)=SIN(THET2)*COS(THET3)
      TI(2,2)=COS(THET2)
      TI(2,3)=-SIN(THET2)*SIN(THET3)
      TI(3,1)=SIN(THET3)
      TI(3,3)=COS(THET3)
      GO TO   150
C
C          COMPUTE SUBMATRIX FOR TRANSFORMATION  4
C
  140 TI(1,3)=-1.0
      TI(2,1)=SIN(THET4)
      TI(2,2)=COS(THET4)
      TI(3,1)=COS(THET4)
      TI(3,2)=-SIN(THET4)
  150 RETURN
      END
```

SUBROUTINE SJLOB (Q) MATRIX MODIFIER FOR BOUNDARY CONDITIONS

```
      SUBROUTINE SJLOB(FYLE,M)
      DIMENSION FYLE(130),QL(12,130)
      COMMON N
      N=19
      DO 100 I=1,12
  100 READ(M'N) (QL(I,J),J=1,130)
C
C        ELIMINATE THE -1.0 COEFF FROM BDRY COND EQNS LINES 4 AND 5
C
      I1=1
      I2=6
      J1=48
      DO 120 K=1,2
      DO 110 I=I1,I2
      DO 110 J=1,6
      JJ=J + J1
  110 QL(I,JJ)=0.0
      J1=24
      I1=7
      I2=12
  120 CONTINUE
C
C        CONVERT LINE 9.  ELIMINATING COLS 5 AND 9
C
      N=49
      DO 170 I=1,6
      READ(M'N) FYLE
      J1=1
      J2=6
      JJ1=48
      DO 160 L=1,2
      DO 150 J=J1,J2
      JJ=J + JJ1
      COEF=FYLE(JJ)
      FYLE(JJ)=0.0
      DO 150 K=1,130
      CQ=COEF*QL(J,K)
      IF(CQ)   130,  150,  130
  130 CCQ=EXP(ALOG(CQ))
      FYLE(K)=FYLE(K)  +  SIGN(CCQ,CQ)
      IF(ABS(FYLE(K)/CCQ) - 1.0E-04)   140,  140,  150
  140 FYLE(K)=0.0
  150 CONTINUE
      J1=7
      J2=12
      JJ1=18
  160 CONTINUE
      N=N-1
      WRITE(M'N) FYLE
  170 CONTINUE
      N=73
      J1=72
      N1=43
      DO 230 L=1,2
C
C     L=1  ELIMINATE THE -1.0 COEFF FROM BDRY EQNS LINE 13
C     L=2  ELIMINATE THE -1.0 COEFF FROM BDRY EQNS LINE 17
C
      DO 180 I=1,6
  180 READ(M'N) (QL(I,J),J=1,130)
      DO 190 I=1,6
      DO 190 J=1,6
      JJ=J + J1
  190 QL(I,JJ)=0.0
C
C     L=1  CONVERT LINE 8, ELIMINATING COL 13
C     L=2  CONVERT LINE 12, ELIMINATING COL 17
C
```

```
      N=N1
      DO 220 I=1,6
      READ(M'N) FYLE
      DO 210 J=1,6
      JJ=J + J1
      COEF=FYLE(JJ)
      FYLE(JJ)=0.0
      DO 200 K=1,138
  200 FYLE(K)=FYLE(K) + COEF*GL(J,K)
  210 CONTINUE
      N=N-1
      WRITE(M'N) FYLE
  220 CONTINUE
      N=97
      J1=46
      N1=67
  230 CONTINUE
      RETURN
      END
```

SUBROUTINE SJLOU   FORCING FUNCTION INTERPOLATOR

```
      SUBROUTINE SJLOU(G,TAU,F,BETA,AZIM,IM)
      DIMENSION G(138),F(50,5)
      DO 100 I=1,IM
      IF(TAU - F(I,1))   110,  110,  100
  100 CONTINUE
      I=IM
  110 IF(I-1)   130,  130,  120
  120 F1=(TAU - F(I-1,1))/(F(I,1) - F(I-1,1))
      FP=F1*(F(I,2)- F(I-1,2)) + F(I-1,2)
      FT=F1 * (F(I,3) - F(I-1,3)) + F(I-1,3)
      AMX=F1*(F(I,4)-F(I-1,4)) + F(I-1,4)
      BETA=(F1*(F(I,5)-F(I-1,5))+F(I-1,5))*0.01745
      GO TO  140
  130 FP=F(1,2)
      FT=F(1,3)
      AMX=F(1,4)
      BETA=0.01745*F(1,5)
  140 FPX=FP * COS(BETA) * COS(AZIM)
      FPY=-FP * SIN(BETA)
      FPZ=FP * COS(BETA) * SIN(AZIM)
      DO 150 I=1,138
  150 G(I)=0.0
      G(49)=FPX
      G(50)=FPY
      G(51)=FPZ
      G(52)=AMX
      G(54)=-FPX * 2.05
      G(86)=-FT
      RETURN
      END
```

```
C
C          PROGRAM SJLFF        (AUXILIARY)
C          FORCING FUNCTION DATA GENERATOR
C
C          SUPPORTING SUBROUTINES
C              NONE
C
C          REQ'D INPUT DATA
C              ELEV      (F10.3)    INITIAL GUN ELEVATION ANGLE
C              TAU,F1,F2,AR,AMC      (F10.6,E15.5,2E15.4,E15.3)
C                   TAU    TIME  (SEC)
C                   F1     BOLT FORCE   (LBS)
C                   F2     BARREL FORCE   (LBS)
C                   AR     RECEIVER ACCELERATION    (IN/SEC2)
C                   AMC    PINTLE MOMENT   (IN-LBS)
C              NOTE-  THESE DATA CARDS MUST BE LESS THAN OR EQUAL TO 50
C                       IF LESS THAN 50, MAKE LAST TAU NEGATIVE
C
C          PRINTED OUTPUT
C              ELEV   INITIAL GUN ELEVATION ANGLE
C              TAU,F1,F2,AR,AMC   RECAPITULATION OF INPUT DATA
C
C          PUNCHED AND PRINTED OUTPUT
C              (F(I,J), J=1,5)    FORCING FUNCTION DATA
C
C
C
      DIMENSION F(50,5)
   10 FORMAT(1H1,'FORCING FUNCTION INPUT DATA -')
   11 FORMAT(F10.6,E15.5,2E15.4,E15.3)
   12 FORMAT(1H0,20X,'INITIAL GUN ELEVATION ANGLE =',F10.3,' DEG.',/21X
     1,'TIME',7X,'BOLT',9X,'BARREL',8X,'RECEIVER',8X,'PINTLE',/,34X,
     22('FORCE',9X),'ACCEL.',9X,'MOMENT',/,21X,'(SEC)',8X,'(LBS)',9X,
     3'(LBS)',8X,'(IN/SEC2)',7X,'(IN-LBS)',/,19X,68('-'))
   13 FORMAT(1H ,18X,F9.6,E13.4,E15.4,2E14.4)
   14 FORMAT(1H1,'FORCING FUNCTION OUTPUT DATA -',/21X,'TIME',8X,
     1'PINTLE',8X,'TAVERSE',8X,'PINTLE',6X,'PINTLE FORCE',/,34X,'FORCE',
     27X,'BAR FORCE',7X,'MOMENT',6X,'ACTION ANGLE',/,21X,'(SEC)',8X,
     3'(LBS)',7X,'(LBS)',8X,'(IN-LBS)',9X,'(DEG)',/,19X,68('-'))
   15 FORMAT(F10.6,4E15.5)
      IN=2
      IP=2
      MO=5
      WRITE(MO,10 )
      D1=13.50
      READ(IN,11 )  ELEV
      WRITE(MO,12 )  ELEV
      AMR=0.1172
      PHI0=0.01745*(ELEV + 4.66666667)
      ELEV=0.01745*ELEV
      THETA=ATAN((5.49-D1*SIN(PHI0))/(15.67-D1*COS(PHI0)))
      DO 110 I=1,50
      READ(IN,11 )  TAU,F1,F2,AR,AMC
      WRITE(MO,13 )  TAU,F1,F2,AR,AMC
      FT=(2.04*F1+0.60*F2-1.47*AMR*AR)/(D1*SIN(THETA-PHI0) )
      SUMF=F1 + F2 - AMR*AR
```

```
      BETA=FT*SIN(THETA) - SUMF*SIN(ELEV)
      BETA=ATAN(BETA/(SUMF*COS(ELEV)-FT*COS(THETA)))
      FP=(SUMF*COS(ELEV)-FT*COS(THETA))**2
      FP=SQRT(FP+(FT*SIN(THETA)-SUMF*SIN(ELEV))**2)
      FP=SIGN(FP,SUMF)
      ANX=ANG
      BETA=BETA/0.01745
      F(I,1)=TAU
      F(I,2)=FP
      F(I,3)=FT
      F(I,4)=ANX
      F(I,5)=BETA
      IF(TAU)   100,  110,  110
  100 IM=I
      GO TO   120
  110 CONTINUE
      IM=50
  120 READ(IN,11 )  DUMMY
C        OUTPUT DATA
      WRITE(NO,14 )
      DO 130 I=1,IM
      WRITE(NO,13 )  (F(I,J), J=1,5)
      WRITE(IP,15 )  (F(I,J), J=1,5)
  130 CONTINUE
      CALL EXIT
      END
```

SUBROUTINE MTRA (IBM SSP)

```
      SUBROUTINE MTRA(A,R,N,M,MS)
      DIMENSION A(1),R(1)
C        IF MS IS 1 OR 2, COPY A
      IF(MS) 10,20,10
   10 CALL MCPY(A,R,N,M,MS)
      RETURN
C        TRANSPOSE GENERAL MATRIX
   20 IR=0
      DO 30 I=1,N
      IJ=I-N
      DO 30 J=1,M
      IJ=IJ+N
      IR=IR+1
   30 R(IR)=A(IJ)
      RETURN
      END
```


SUBROUTINE GMPRD (IBM SSP)

```
      SUBROUTINE GMPRD(A,B,R,N,M,L)
      DIMENSION A(1),B(1),R(1)
      IR=0
      IK=-M
      DO 10 K=1,L
      IK=IK+M
      DO 10 J=1,N
      IR=IR+1
      JI=J-N
      IB=IK
      R(IR)=0
      DO 10 I=1,M
      JI=JI+N
      IB=IB+1
   10 R(IR)=R(IR)+A(JI)*B(IB)
      RETURN
      END
```

SUBROUTINE SJLFY FILE SORTER

```
      SUBROUTINE SJLFY(L,LUMP,KASE,AMTX,FYLE)
      DIMENSION AMTX(6,6,4),FYLE(138),Z(138)
      COMMON N
      IF(LUMP)   100,  100,  270
  100 IF(L-8)   110,  120,  110
  110 N=6*L - 5
      GO TO   130
  120 N=49
  130 J1=N- 1
      JJ1=J1 + 6
      J2=N + 12
      DO 170 I=1,6
      READ(KASE'N) Z
      DO 140 J=1,6
      JJ=J + J1
  140 Z(JJ)=Z(JJ) + AMTX(I,J,1)
      N=N - 1
      IF(L-22)   150,  160,  160
  150 WRITE(KASE'N)(Z(J),J=1,JJ1),(AMTX(I,J,2),J=1,6),(FYLE(J),J=J2,138)
      GO TO   170
  160 WRITE(KASE'N)(Z(J),J=1,JJ1),(AMTX(I,J,2),J=1,6)
  170 CONTINUE
      IF(L-3)   190,  240,  180
  180 IF(L-16)   190,  250,  210
  190 IF(L-1)   200,  200,  210
  200 WRITE(KASE'N)(((AMTX(I,J,K),J=1,6),K=3,4),(FYLE(J),J=J2,138),
     1I=1,6)
      GO TO   410
  210 IF(L-22)   220,  230,  230
  220 WRITE(KASE'N)((FYLE(J),J=1,J1),((AMTX(I,J,K),J=1,6),K=3,4),
     1(FYLE(J),J=J2,138),I=1,6)
      GO TO   410
  230 WRITE(KASE'N)((FYLE(J),J=1,J1),((AMTX(I,J,K),J=1,6),K=3,4),I=1,6)
      GO TO   410
  240 N=49
      GO TO   210
  250 N=67
      DO 260 I=1,6
      READ(KASE'N) Z
      N=N - 1
      WRITE(KASE'N) (Z(J),J=1,J1),((AMTX(I,J,K),J=1,6),K=3,4),
     1(FYLE(J),J=J2,138)
  260 CONTINUE
      GO TO   410
  270 GO TO (280,290,340,350),LUMP
  280 N=49
      NN=31
      JJ1=24
      J1=NN-1
      J2=37
      JJ2=31
      GO TO   300
  290 N=43
      NN=79
      JJ1=72
      J1=NN-1
      J2=85
      JJ2=79
```

```
300 DO 310 I=1,6
    READ(KASE'N) Z
    N=N - 1
    WRITE(KASE'N)(Z(J),J=1,JJ1),((AMTX(I,J,K),J=1,6),K=1,2),
   1(Z(J),J=J2,138)
310 CONTINUE
    N=N4
    DO 330 I=1,6
    READ(KASE'N) Z
    DO 320 J=1,6
    JJ=J + J1
320 Z(JJ)=Z(JJ) + AMTX(I,J,4)
    N=N - 1
    WRITE(KASE'N)(Z(J),J=1,JJ1),(AMTX(I,J,3),J=1,6),(Z(J),J=JJ2,138)
330 CONTINUE
    GO TO   410
340 N=43
    NN=103
    JJ1=102
    J1=42
    J2=109
    JJ2=49
    GO TO   360
350 N=67
    NN=121
    JJ1=120
    J1=66
    J2=127
    JJ2=73
360 DO 380 I=1,6
    READ(KASE'N) Z
    DO 370 J=1,6
    JJ=J + J1
370 Z(JJ)=Z(JJ) + AMTX(I,J,1)
    N=N - 1
380 WRITE(KASE'N)(Z(J),J=1,JJ1),(AMTX(I,J,2),J=1,6),(Z(J),J=J2,138)
    N=N4
    DO 400 I=1,6
    READ(KASE'N) Z
    DO 390 J=1,6
    JJ=J + JJ1
390 Z(JJ)=Z(JJ) + AMTX(I,J,4)
    N=N - 1
400 WRITE(KASE'N)(Z(J),J=1,J1),(AMTX(I,J,3),J=1,6),(Z(J),J=JJ2,138)
410 RETURN
    END
```

APPENDIX C

SAMPLE OUTPUT OF COMPUTER PROGRAM

TRIPOD MOUNT M 3 FOR 0.50 CALIBER MACHINE GUN
CASE ANALYZED-   ALL THREE LEGS EXTENDED BY TWO HOLE STOPS

TRANSIENT RESPONSE -

TIME (SEC.)	NODAL POINT	DISPLACEMENTS AND VELOCITIES (IN) (IN/SEC) X	Y	Z	ELEVATION ANGLE (DEG)	AZIMUTH ANGLE (DEG)	ELEMENT NUMBER	PEAK COMBINED STRESS (PSI)	OCCURRING AT LOCATION (IN.FROM END)
0.00000	4	0.000E 00	0.000E 00	0.000E 00	0.00000	0.000	5	0.000000E 00	0.000
		0.000E 00	0.000E 00	0.000E 00			6	0.000000E 00	0.000
							14	0.000000E 00	0.000
	1	0.000E 00	0.000E 00	0.000E 00			15	0.000000E 00	0.000
		0.000E 00	0.000E 00	0.000E 00			8	0.000000E 00	0.000
							9	0.000000E 00	0.000
	16	0.000E 00	0.000E 00	0.000E 00			17	0.000000E 00	0.000
		0.000E 00	0.000E 00	0.000E 00			18	0.000000E 00	0.000
							10	0.000000E 00	0.000
	19	0.000E 00	0.000E 00	0.000E 00			11	0.000000E 00	0.000
		0.000E 00	0.000E 00	0.000E 00			12	0.000000E 00	0.000
							13	0.000000E 00	0.000
0.00100	4	0.263E-03	0.205E-04	0.920E-08	-0.00084	0.000	5	0.263216E 03	0.000
		0.784E 00	0.615E-01	0.276E-04			6	0.123722E 03	5.850
							14	0.652228E 02	0.000
	1	0.487E-04	-0.590E-05	0.000E 00			15	0.241548E 02	0.000
		0.146E 00	-0.177E-01	0.000E 00			8	0.263108E 03	0.000
							9	0.123628E 03	5.850
	16	0.165E-04	-0.332E-05	0.337E-05			17	0.645798E 02	0.000
		0.496E-01	-0.998E-02	0.101E-01			18	0.239108E 02	0.000
							10	0.869368E 03	0.000
	19	0.164E-04	-0.330E-05	-0.336E-05			11	0.754968E 03	4.360
		0.493E-01	-0.991E-02	-0.101E-01			12	0.757916E 03	0.000
							13	0.870226E 03	4.360
0.00200	4	0.165E-02	0.223E-03	0.935E-07	-0.00473	0.000	5	0.140216E 04	0.000
		0.179E 01	0.423E 00	0.170E-03			6	0.777736E 03	5.850
							14	0.528658E 03	0.000
	1	0.562E-03	-0.231E-04	-0.133E-07			15	0.242726E 03	0.000
		0.110E 01	0.142E-02	-0.401E-04			8	0.140598E 04	0.000
							9	0.786538E 03	5.850
	16	0.228E-03	-0.502E-04	0.693E-04			17	0.526058E 03	0.000
		0.466E 00	-0.110E 00	0.167E 00			18	0.240678E 03	0.000
							10	0.509358E 04	0.000
	19	0.226E-03	-0.498E-04	-0.689E-04			11	0.433318E 04	4.360
		0.483E 00	-0.110E 00	-0.166E 00			12	0.434518E 04	0.000
							13	0.588128E 04	4.360

TRIPOD MOUNT M 3 FOR 0.50 CALIBER MACHINE GUN
CASE ANALYZED- ALL THREE LEGS EXTENDED BY TWO HOLE STOPS

TRANSIENT RESPONSE -

TIME (SEC.)	NODAL POINT	DISPLACEMENTS AND VELOCITIES (IN) (IN/SEC) X	Y	Z	ELEVATION ANGLE (DEG)	AZIMUTH ANGLE (DEG)	ELEMENT NUMBER	PEAK COMBINED STRESS (PSI)	OCCURRING AT LOCATION (IN.FROM END)
0.00300	4	0.532E-02	0.871E-03	0.491E-05	-0.00614	0.000	5	0.34162E 04	0.000
		0.723E 01	0.796E 00	0.140E-01			6	0.13916E 04	5.650
	1	0.277E-02	0.172E-03	-0.284E-06			14	0.10137E 04	0.000
		0.361E 01	0.547E 00	-0.693E-03			15	0.69191E 03	0.000
							8	0.34455E 04	0.000
	16	0.135E-02	-0.469E-03	0.560E-03			9	0.14532E 04	5.850
		0.201E 01	-0.945E 00	0.991E 00			17	0.10759E 04	0.000
							18	0.72485E 03	0.000
	19	0.134E-02	-0.725E-04	-0.703E-03			10	0.16579E 05	4.360
		0.194E 01	0.242E 00	-0.142E 01			11	0.88952E 04	4.360
							12	0.87169E 04	0.000
							13	0.16383E 05	0.000
0.00400	4	0.161E-01	0.190E-02	0.453E-04	0.00833	0.000	5	0.76762E 04	0.000
		0.127E 02	0.142E 01	0.795E-01			6	0.22054E 04	5.650
	1	0.839E-02	0.152E-02	-0.244E-05			14	0.15938E 04	4.390
		0.795E 01	0.246E 01	-0.447E-02			15	0.13041E 04	7.170
							8	0.76087E 04	0.000
	16	0.456E-02	-0.153E-02	0.260E-02			9	0.24586E 04	5.850
		0.444E 01	-0.570E 00	0.346E 01			17	0.17189E 04	0.000
							18	0.12458E 04	5.736
	19	0.471E-02	-0.362E-03	-0.271E-02			10	0.44517E 05	4.360
		0.506E 01	-0.179E 01	-0.206E 01			11	0.18235E 05	4.360
							12	0.17977E 05	0.000
							13	0.43676E 05	0.000
0.00500	4	0.329E-01	0.385E-02	0.173E-03	0.06169	0.000	5	0.14704E 05	0.000
		0.247E 02	0.245E 01	0.174E 00			6	0.82083E 04	0.000
	1	0.196E-01	0.660E-02	-0.186E-04			14	0.41600E 04	0.000
		0.152E 02	0.891E 01	-0.366E-01			15	0.32092E 04	0.000
							8	0.14725E 05	0.000
	16	0.121E-01	-0.246E-02	0.711E-02			9	0.82933E 04	0.000
		0.126E 02	-0.275E 01	0.479E 01			17	0.42053E 04	0.000
							18	0.32679E 04	0.000
	19	0.122E-01	-0.635E-03	-0.737E-02			10	0.11113E 06	4.360
		0.106E 02	0.525E 01	-0.103E 02			11	0.50120E 05	4.360
							12	0.51178E 05	0.000
							13	0.11046E 06	0.000

APPENDIX D

SAMPLE DERIVATIONS OF SOME EQUATIONS

APPENDIX D

SAMPLE DERIVATIONS OF SOME EQUATIONS

D.1 Sample Derivation of Finite-Element Relationships

In this section, derivations will be given for equations (15), (3), (17), and (5), quoted in Section 4.1.

D.1.1 Derivation of (15)

Consider a bar of cross-sectional area A, length L, and elastic modulus E. Let the right end of the bar be constrained from moving, while the left end is subjected to a displacement δ_{x1} along the axis of the bar, as shown in Figure D.1. A single applied axial force, F^k_{x1}, is required to cause this displacement and a single axial reaction force F^k_{x2} arises at the constrained end — superscript k has been added here.

This is a simple one-dimensional problem. The axial strain is uniform along the bar,

$$\epsilon_x = -\frac{\delta_{x1}}{L}$$

and the axial stress is therefore uniform along the bar, given by

$$\sigma_x = E\epsilon_x = -\frac{E\delta_{x1}}{L}$$

The required applied force is now written as

$$F^k_{x1} = -\sigma_x A = \left(\frac{EA}{L}\right)\delta_{x1}$$

For equilibrium, it follows that

$$F^k_{x2} = -F^k_{x1}$$

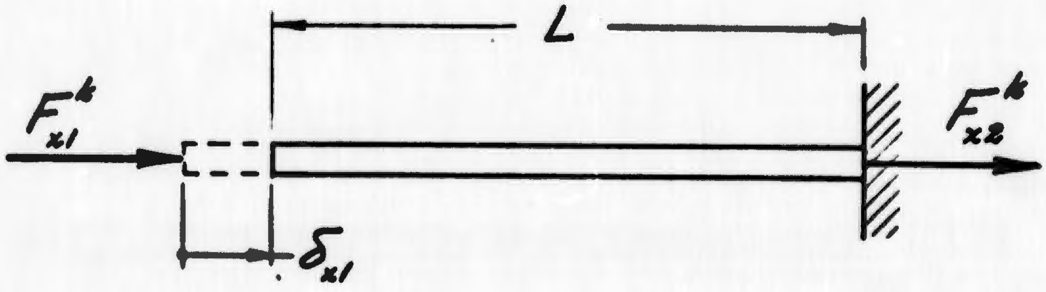


Figure D.1: AXIAL DISPLACEMENT OF BAR

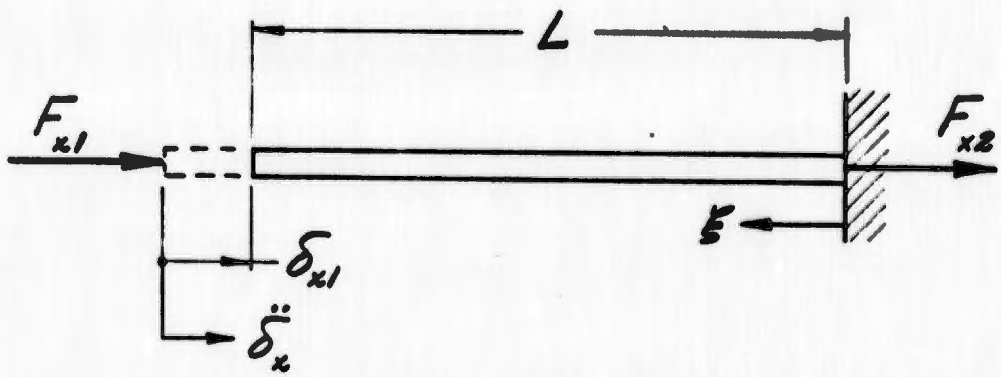


Figure D.2: AXIALLY ACCELERATING BAR

Then we have

$$F_{x1}^{k} = \left(\frac{EA}{L}\right)\delta_{x1}$$
$$F_{x2}^{k} = -\left(\frac{EA}{L}\right)\delta_{x1} \qquad (D.1)$$

D.1.2 <u>Derivation of (3)</u>

Consider now the rod subjected to an axial acceleration $\ddot{\delta}_{x1}$, referring to Figure D.2. The applied force and reaction required to maintain such an acceleration are

$$F_{x1} = F_{x1}^{k} + F_{x1}^{m}$$
$$F_{x2} = F_{x2}^{k} + F_{x2}^{m} \qquad (D.2)$$

where F_{x1}^{m} is the force required to maintain the necessary acceleration of the bar mass.

The acceleration may be given as a function of distance, ξ, from the constrained end:

$$\ddot{\delta}_x(\xi) = \left(\frac{\xi}{L}\right)\ddot{\delta}_{x1} \qquad (D.3)$$

An equation of motion may now be written:

$$EA\frac{\partial^2}{\partial\xi^2}(\delta_x) = m\,\ddot{\delta}_x \qquad (D.4)$$

where m is the mass of rod per unit length. Substituting (D.3) into (D.4) and integrating twice with respect to ξ yields

$$\delta_x = \frac{m}{6EAL}\ddot{\delta}_{x1}\xi^3 + C\xi + D \qquad (D.5)$$

where C and D are integration constants. The boundary condition

$$\delta_x(0) = 0 \qquad (D.6)$$

implies that D is zero, and the condition

$$\delta_x(L) = \delta_{x1} \tag{D.7}$$

implies

$$C = \frac{\delta_{x1}}{L} - \frac{mL}{6EA}\ddot{\delta}_{x1} \tag{D.8}$$

so that

$$\delta_x = \frac{m}{6EAL}\ddot{\delta}_{x1}\left(\xi^3 - \xi L^2\right) + \frac{1}{L}\delta_{x1}\,\xi \tag{D.9}$$

Now the axial stress may be calculated :

$$\sigma = -E\frac{\partial}{\partial \xi}(\delta_x)$$

$$= -\frac{m}{6AL}\left(3\xi^2 - L^2\right)\ddot{\delta}_{x1} - \left(\frac{E}{L}\right)\delta_{x1} \tag{D.10}$$

and, now

$$F_{x1} = -A\sigma(L)$$

$$= \left(\frac{mL}{3}\right)\ddot{\delta}_{x1} + \left(\frac{EA}{L}\right)\delta_{x1}$$

$$F_{x2} = A\sigma(0)$$

$$= \left(\frac{mL}{6}\right)\ddot{\delta}_{x1} - \left(\frac{EA}{L}\right)\delta_{x1}$$

Restating,

$$F_{x1} = \left(\frac{mL}{3}\right)\ddot{\delta}_{x1} + \left(\frac{EA}{L}\right)\delta_{x1}$$

$$F_{x2} = \left(\frac{mL}{6}\right)\ddot{\delta}_{x1} - \left(\frac{EA}{L}\right)\delta_{x1} \tag{D.11}$$

Using (D.2) and (D.1), we have finally

$$F_{x1}^{m} = \frac{mL}{3}\ddot{\delta}_{x1}$$

$$F_{x2}^{m} = \frac{mL}{6}\ddot{\delta}_{x1} \qquad \text{(D.12)}$$

D.1.3 Derivation of (17)

Consider now a bar constrained to be motionless on the right end, but subjected to a lateral translation, in the y direction, with no rotation, at the left end (see Figure D.3). A differential equation for the deflection shape is

$$\frac{d^4}{d\xi^4}(\delta_y) = 0 \qquad \text{(D.13)}$$

Integrating four times, we obtain

$$\delta_y(\xi) = A\xi^3 + B\xi^2 + C\xi + D$$

The boundary conditions may be written

$$\delta_y(0) = \delta_{y1}$$

$$\delta_y(L) = 0$$

$$\frac{d}{d\xi}(\delta_y)\Big|_{\xi=0} = 0 \qquad \text{(D.14)}$$

$$\frac{d}{d\xi}(\delta_y)\Big|_{\xi=L} = 0$$

with the result that

$$\delta_y(\xi) = \frac{\delta_{y1}}{L^3}\left(2\xi^3 - 3\xi^2 L + L^3\right) \qquad \text{(D.15)}$$

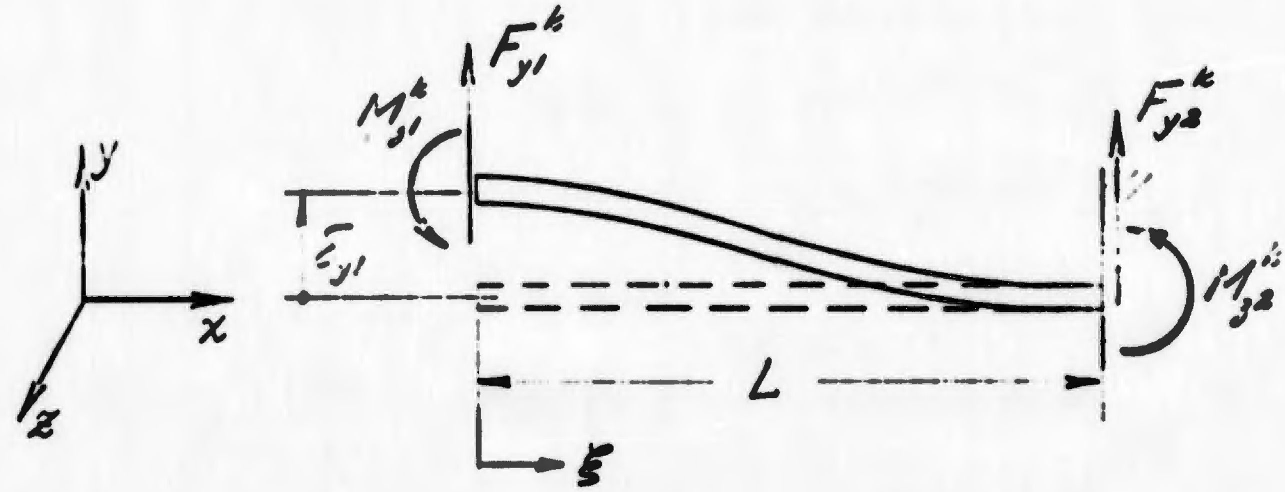


Figure D.3: LATERAL DISPLACEMENT OF BAR

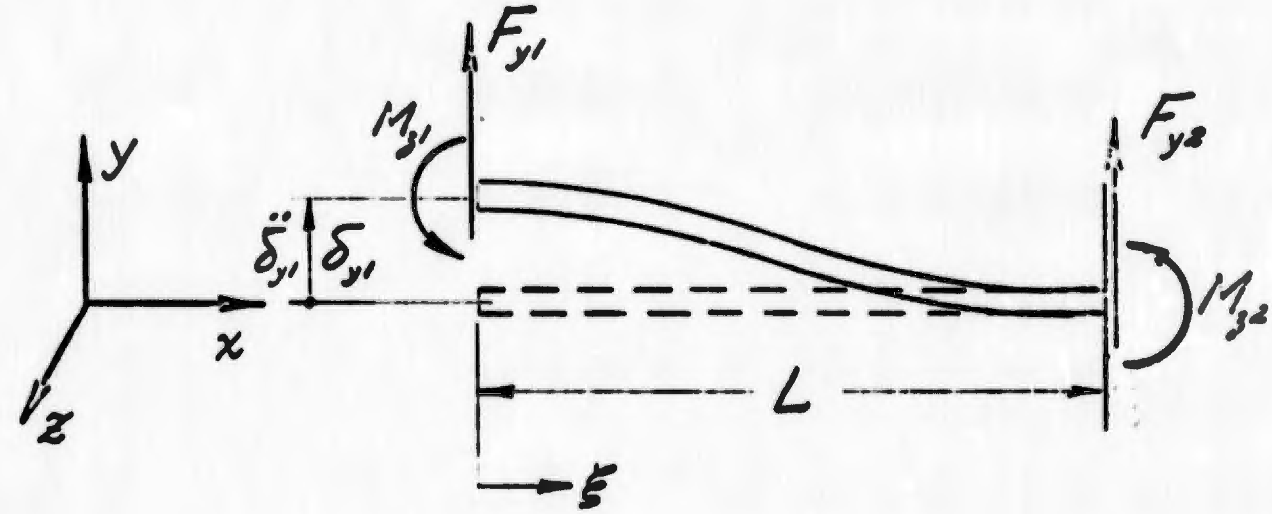


Figure D.4: LATERALLY ACCELERATING BAR

Now, F^k_{y1}, F^k_{y2}, M^k_{z1}, and M^k_{z2} may be evaluated:

$$F^k_{y1} = EI \frac{d^3}{d\xi^3}(\delta_y)\Big|_{\xi=0}$$

$$F^k_{y2} = -EI \frac{d^3}{d\xi^3}(\delta_y)\Big|_{\xi=L}$$

$$M^k_{z1} = -EI \frac{d^2}{d\xi^2}(\delta_y)\Big|_{\xi=0}$$

$$M^k_{z2} = EI \frac{d^2}{d\xi^2}(\delta_y)\Big|_{\xi=L} \qquad \text{(D.16)}$$

where I is the centroidal area moment of inertia of the beam cross-section about the z-axis. One obtains

$$F^k_{y1} = \frac{12EI}{L^3}\delta_{y1}$$

$$F^k_{y2} = -\frac{12EI}{L^3}\delta_{y1}$$

$$M^k_{z1} = \frac{6EI}{L^2}\delta_{y1}$$

$$M^k_{z2} = \frac{6EI}{L^2}\delta_{y1} \qquad \text{(D.17)}$$

D.1.4 Derivation of (5)

The rod shall now be considered as accelerating, as shown in Figure D.4. The forces and moments are written :

$$
\begin{aligned}
F_{y1} &= F_{y1}^{k} + F_{y1}^{m} \\
F_{y2} &= F_{y2}^{k} + F_{y2}^{m} \\
M_{z1} &= M_{z1}^{k} + M_{z2}^{k} \\
M_{z2} &= M_{z2}^{k} + M_{z2}^{m}
\end{aligned}
\tag{D.18}
$$

At any position along the bar, the lateral acceleration is given by

$$\ddot{\delta}_y(\xi) = \ddot{\delta}_{y1}\left(2\xi^3 - 3\xi^2 L + L^3\right)/L^3 \tag{D.19}$$

where $\ddot{\delta}_{y1}$ is the lateral acceleration at the left end.

The equation of motion, neglecting transverse shear and rotary inertia, is given by

$$EI\frac{\partial^4}{\partial\xi^4}(\delta_y) = -m\,\ddot{\delta}_y \tag{D.20}$$

Inserting (D.18) and integrating,

$$
\begin{aligned}
EI\,\delta_y(\xi) = -\frac{m}{L^3}\ddot{\delta}_{y1}\Big(&\frac{1}{420}\xi^7 - \frac{1}{120}\xi^6 L \\
&+ \frac{1}{24}\xi^4 L^3 + B\xi^3 + C\xi^2\Big) + D\xi + E
\end{aligned}
\tag{D.21}
$$

The boundary conditions (D.14) are again employed, yielding :

$$
\begin{aligned}
EI\,\delta_y(\xi) = -\frac{m}{L^3}\ddot{\delta}_{y1}\Big(&\frac{1}{420}\xi^7 - \frac{1}{120}\xi^6 L + \frac{1}{24}\xi^4 L^3 \\
&- \frac{13}{210}\xi^3 L^4 + \frac{11}{420}\xi^2 L^5\Big) + \frac{EI}{L^3}\delta_{y1}\left(2\xi^3 - 3\xi^2 L + L^3\right)
\end{aligned}
\tag{D.22}
$$

Now

$$F_{y1} = EI \frac{d^3}{d\xi^3} (\delta_y) \Big|_{\xi=0}$$

$$F_{y2} = -EI \frac{d^3}{d\xi^3} (\delta_y) \Big|_{\xi=L}$$

$$M_{z1} = -EI \frac{d^2}{d\xi^2} (\delta_y) \Big|_{\xi=0} \qquad \text{(D.23)}$$

$$M_{z2} = EI \frac{d^2}{d\xi^2} (\delta_y) \Big|_{\xi=L}$$

yielding

$$F_{y1} = \frac{13mL}{35} \ddot{\delta}_{y1} + \frac{12EI}{L^3} \delta_{y1}$$

$$F_{y2} = \frac{9mL}{70} \ddot{\delta}_{y1} - \frac{12EI}{L^3} \delta_{y1}$$

$$M_{z1} = \frac{11mL^2}{210} \ddot{\delta}_{y1} + \frac{6EI}{L^2} \delta_{y1} \qquad \text{(D.24)}$$

$$M_{z2} = -\frac{13mL^2}{420} \ddot{\delta}_{y1} + \frac{6EI}{L^2} \delta_{y1}$$

Using (D.18) and (D.17), we have

$$F_{y1}^m = \frac{13mL}{35} \ddot{\delta}_{y1}$$

$$F_{y2}^m = \frac{9mL}{70} \ddot{\delta}_{y1}$$

$$M_{z1}^m = \frac{11mL^2}{210} \ddot{\delta}_{y1} \qquad \text{(D.25)}$$

$$M_{z2}^m = -\frac{13mL^2}{420} \ddot{\delta}_{y1}$$

D.2 Equivalent Moment of Inertia for the Telescoped Portions of Legs

For problem definition, refer to Figure 11 and Section 4.2 -- presented herein is only the derivation of the required formula which is built up of simple expressions for beam deflection, such as found in Ref. 3.

Referring to Figure D.5a, the sum of end rotations, φ, in the actual case of telescoped tubes is given by:

$$\varphi = \theta_L + \theta_R = \left(\frac{M_L}{3EI_1} + \frac{M_R}{3EI_3}\right) L_T \tag{D.26}$$

Here θ_L is the end rotation of the inner tube, loaded by an end couple M_L and simply supported at both ends. Likewise, θ_R is the end rotation of the outer tube, loaded by an end couple M_R, and also simply supported at both ends. But from equilibrium conditions we have

$$M_R = M_L + V_L L_T$$

which, when substituted in Eq. (D.26), yields the following:

$$\varphi = \frac{L_T}{3E}\left[\left(\frac{1}{I_1} + \frac{1}{I_3}\right) M_L + \frac{L_T}{I_3} V_L\right] \tag{D.27}$$

Shown in Figure D.5b is an "equivalent beam" with a moment of inertia I_e, loaded in an identical manner as the actual telescoped tubes. To determine I_e we stipulate that the sum of end rotations for this equivalent beam be equal to the sum of end rotations for the telescoped tubes, i.e.:

$$\varphi_e = \varphi \tag{D.28}$$

End rotation at the left end of the equivalent beam:

$$\theta_L' = \frac{M_L L_T}{3EI_e} + \frac{M_R L_T}{6EI_e}$$

End rotation at the right end of the equivalent beam:

$$\theta_R' = \frac{M_R L_T}{3EI_e} + \frac{M_L L_T}{6EI_e}$$

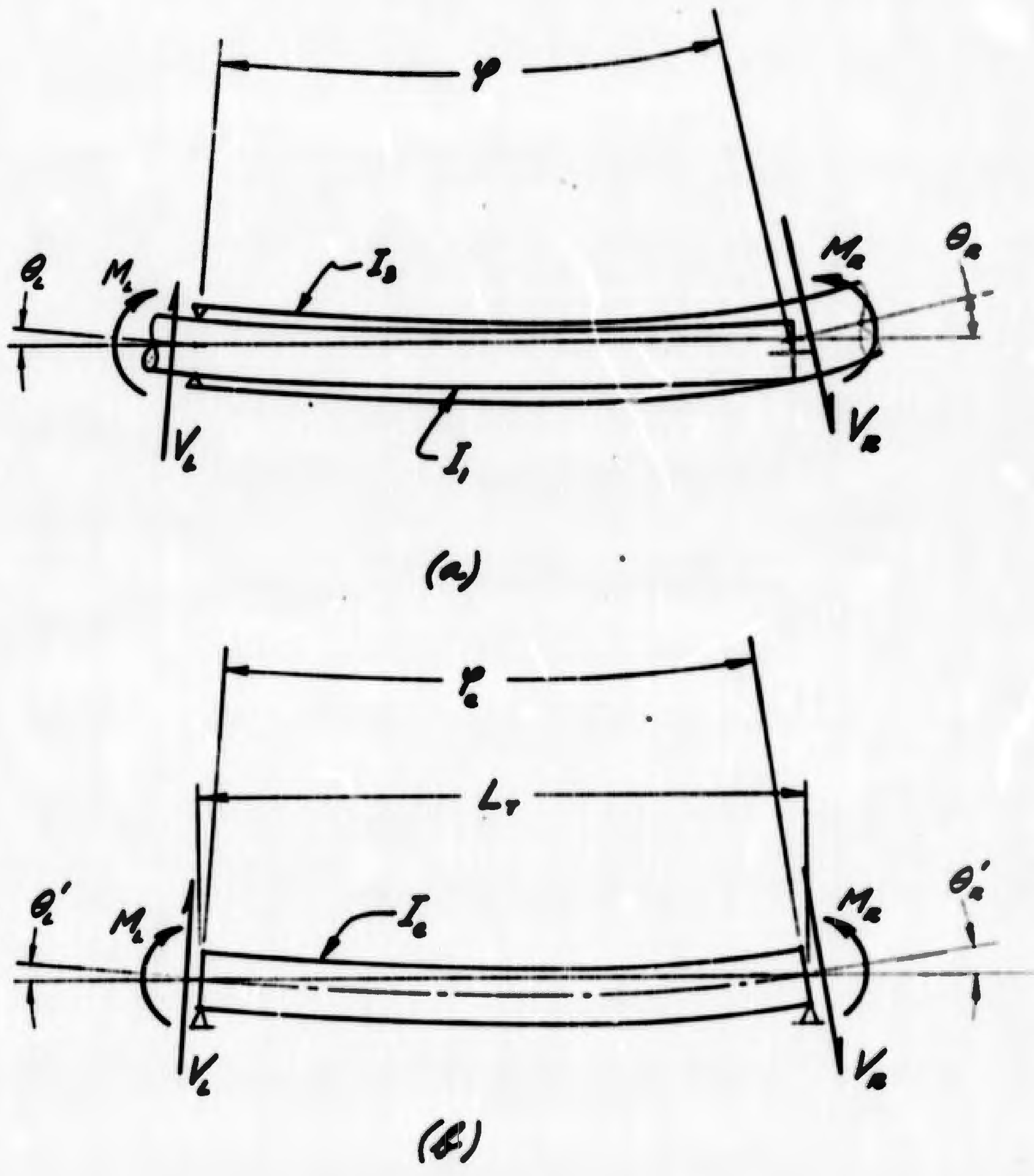


Figure D.5: BENDING DEFLECTION OF TELESCOPED TUBES (a), AND OF AN EQUIVALENT BEAM (b).

Adding the preceding two expressions we get:

$$\varphi_e = \theta_L' + \theta_R' = \frac{L_T}{2EI_e}\left(M_L + M_R\right) \qquad \text{(D.29)}$$

Invoking the equivalency condition, Eq. (D.28), and solving the resulting equality for I_e, we get the following expression:

$$I_e = \frac{3}{2}\left[\frac{2M_L + V_L L_T}{\left(1 + \frac{I_3}{I_1}\right)M_L + V_L L_T}\right] I_3 \qquad \text{(D.30)}$$