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**A NOMOGRAPHIC PROCEDURE TO CALCULATE
GAS FLOW PARAMETERS**

by
M. Rhoden

April 1972

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AD 745898



U.S. ARMY MISSILE COMMAND

Redstone Arsenal, Alabama

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ABSTRACT

The purpose of this report is to present graphical solutions to the equations for isentropic flow of ideal gases at subsonic, sonic, and supersonic velocities. Knowing only the weight flow, molecular weight, pressure, temperature, k values, and the exit pressures (vacuum), the reader will be able to design a flow system by specifying pipe sizes, venturi throats, and exit diameters without the use of computers.

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1. Introduction

In laser work optical energy is extracted from thermally excited species of fast flowing hot gases. For this extraction a number of ambient temperature gases must be moved in ducts, the mass flow must be controlled by venturis, and they must be combined to burn at low pressure and high velocity in the cavity.

Design calculations must be made for every line and every venturi. Despite the relative simplicity of the calculations, they are time consuming. A nomographic procedure enables engineers to make fast calculations without use of a computer. The graphs in the report are designed for commonly used variables such as temperatures in degrees Fahrenheit.

2. Discussion

It is assumed that the flow in the ducts is adiabatic. The basic equations to describe the flow are taken from "Compressible Fluid Flow," Volume I, by A. H. Shapiro, and are as follows.

$$\frac{W}{A \sqrt{\text{mol wt}}} = \sqrt{\frac{k g}{R}} \frac{P_0}{\sqrt{T_0}} \frac{M}{\left[1 + \frac{k-1}{2} M^2\right]^{\frac{k+1}{2(k-1)}}} \quad (1)$$

(p 84, Eq. 4.16)

$$\frac{P_0}{P} = \left(1 + \frac{k-1}{2} M^2\right)^{\frac{k}{k-1}} \quad (\text{p 83, Eq. 4.14b}) \quad (2)$$

$$C_0^2 = k g R T_0 \left(\frac{1}{\text{mol wt}}\right) \quad (\text{p 79, Eq. 4.5b}) \quad (3)$$

$$C^{*2} = g \frac{2k}{k+1} R T_0 \left(\frac{1}{\text{mol wt}}\right) \quad (\text{p 80, Eq. 4.5c}) \quad (4)$$

$$V = M^* C^* \quad (\text{p 81}) \quad (5)$$

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2 \quad , \quad (p \ 80, \text{Eq. 4.7}) \quad (6)$$

where

W = weight flow (g/sec)

Mol wt = g/mol

D = diameter (in.)

A = flow area (in.²)

P₀ = pressure at no flow conditions (note that if the pressure is reduced by a pressure reducer it is still P₀) (mm Hg)

P = pressure at flow conditions (mm Hg)

T₀ = stagnation temperature (°F)

k = specific heat ratio (C_p/C_v)

R = gas constant

M = Mach number

g = gravitational constant (ft/sec²)

C₀ = velocity of sound in gas at temperature (°F)

C* = velocity of gas when flowing at the speed of sound, M=1.0 (ft/sec)

V = velocity (ft/sec).

Nomographs were developed for these six equations. Graph 1 solves the factor $(1 + (k-1) M^2/2)$ common to Equations (1), (2), and (7). Graph 2 solves Equation (1) and Graph 3 solves Equation (2). The velocities of sound, C₀, and the velocity of gases flowing, C*, Equations (3), (4), and (5) are solved by Graph 4. Equation (6) is solved by Graph 6.

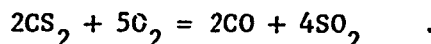
The graphical solutions of these equations (presented in the report) are accurate but inaccuracies using a straight edge, marking the reference lines and going to the next line, will degrade them so that the

solutions will be only within ± 10 percent. The advantage of this method of calculation is that it can be done without the use of mathematics and that it is almost impossible to make a mistake by a factor of 10 or 100 which is easy to do on a slide rule. A further advantage is that trends can very easily be observed by varying certain key inputs within limits.

The equations are correct for ideal gases only, but they will give acceptable results for gases at low pressure and at temperatures high above the critical temperature. They are also good for mixtures of gases (such as air) if the average specific heat ratios k , and the average molecular weights are used.

If burning takes place, there is a different set of gases before and after the reaction. The temperature will change due to the heat of reaction, the number of moles will not be the same, and the value of the specific heat ratio will also change.

For example the burning of carbon disulfide with oxygen to carbon monoxide and sulphur dioxide uses 7 moles of reactants to produce 6 moles of products as shown:



The k values for the chemicals are as follows:

$$\text{CS}_2 - 1.21$$

$$\text{CO} - 1.40$$

$$\text{O}_2 - 1.40$$

$$\text{SO}_2 - 1.29$$

The molecular weight of the products is $7/6$ times that of the reactants.

Furthermore the heat ratios change with temperature and pressure and most experimental data are only for ambient temperatures. The International Critical Tables, Volume 5, pp. 80-82, show for example, that for oxygen at 1 atmosphere the k value increases from 1.40 to 1.45 as the temperature drops from 15°C to -181°C . Using Graph 3 to calculate the pressure at Mach 1, the error introduced would be approximately 1.5 percent. However, if experimental data for the reaction products at elevated temperatures are available the equations may be solved for the flow problems.

3. Description of the Nomographs

The variables in Graph 1 are k and M . The solution is the factor $(1 + (k - 1) M^2)$ common to Graphs 2, 3, and 6.

Graph 2 is the solution of Equation (1). Straightforward use solves for the flow area but any other variable may be found as shown in the following example. For flow velocities slower than Mach 0.4, it is not necessary to solve for the factor in Graph 1. The calibration on the lower part of line 2 is sufficient for the solution but accuracy is impaired. Graph 3 solves Equation (2); Graph 4 solves Equations (3), (4), and (5); and Graph 6 solves Equation (6).

Graph 2 can be used straightforward if the unknown is the area. If the unknown is any other variable, the procedure is the same until the unknown is reached then the solution is exactly backwards starting with the area. For low velocities (less than Mach 0.4), line 2 is calibrated in Mach numbers and this should be the starting point rather than line A.

Graph 3 solves for the flowing pressure and is more accurate at velocities greater than Mach 1. Note that at the intersection of line B and line 1 the total pressure and the flowing pressure are equal. If the pressure range is not on this graph (like 3100 mm Hg) use 31 and multiply the answer by a factor of 100 as well (read $16.4 \times 100 = 1640$ mm Hg for air at Mach 1.0).

Graph 4 solves for the gas velocity at Mach 1 (in ft/sec) and then by straight multiplication for any other velocity. The conversion chart shows the distance traveled in mm/ μ sec.

Graph 5 shows a comparison of inlet and exit diameters to the throat diameters for k values ranging from 1.1 to 1.66. It is a solution of Equation 1.

Graph 6 gives the temperature at the flow velocity in conjunction with Graph 1.

The development of the graphs using the formulas indicated was straightforward. The graphs were prepared using the technique outlined in "Easy Way to Make Nomographs," Chemical Engineering, November 1952.

Instead of calibrating the scales with their actual values, input values such as k were used. For example in Graph 1 the specific heat ratio scale is calibrated from $k = 1.1$ to $k = 1.65$. The actual value at the point 1.1 is 0.05 which is $(1.1 - 1)/2$ and at $k = 1.6$ it is 0.3.

The slanting line in Graphs 2 and 3 occurs because of raising the result from Graph 1 to a power. Again the k scale (slanting line) is calibrated in terms of k rather than $(k + 1)/2(k - 1)$ as in Graph 2, or $k/(k - 1)$ as in Graph 3.

The temperature scales in Graphs 2 and 4 are calibrated in °F. The actual value at 0°F is 460 (°R) and at 2000°F it is 2460 (°R). The length of 1 log scale is from 100°R (-360°F) to 1000°R (540°F).

4. Method of Solution

Sample solutions using each and every graph will show how to solve problems.

Air was used in the problem because the reference book furnishes short-cut formulas for solving the equations. A numerical solution follows the graphical one so that the reader may compare results. It should be noted, however, that the graphs will solve problems for any gas while the short-cut formulas are valid only for air.

EXAMPLE:

The following data are given:

Working fluid	Air	Air
Molecular weight	28.9 g/mol	28.9 lb/lb mol
Weight flow (W)	45.4 g/sec	0.1 lb/sec
Stagnation temperature	1140°F	1600°R
Stagnation pressure	3100 mm Hg	60 psia
Specific heat ratio (k)	1.4	1.4
Universal gas constant (R)		1544
Gravitational constant (g)		32.2 ft/sec ²

The following are required:

- The throat diameter of a critical flow venturi
- The pressure at the throat
- The temperature at the throat
- The velocity of sound at the stagnation conditions

- e) The actual velocity of the gas flowing through the throat
- f) The velocity of the gas in a 1-inch diameter (ID) line.

a. Solution

1) At a sonic throat the velocity is always Mach 1.0. On Graph 1 connect $k = 1.4$ with $M = 1.0$ and read 1.2. On Graph 2 connect 1.2 on line A with 1.4 on line B and mark line 1. Connect the point on line 1 with the Mach number 1.0 on line A (note that line A is used twice, and for different values each time) and mark line 2. Connect with the pressure of 3100 mm Hg on line C and mark line 3. Connect with the weight flow of 45.4 g/sec on line D and mark line 4. Connect with the specific heat ratio $k = 1.4$ on line E and mark line 5. Connect with the temperature of 1140°F on line F and mark line 6. Connect with the molecular weight on line G and read the flow area as 0.13 square inch. The horizontal line below gives the throat diameter as 0.41 inch.

2) For the pressure at the throat, connect on Graph 1 $k = 1.4$ with $M = 1.0$ and read 1.2. On Graph 3, connect 1.2 on line A with 1.4 on line B and mark line 1. Connect with line marked "Total Pressure" (3100 mm Hg) and read 1640 mm Hg on the line marked "Pressure."

3) For the temperature at the throat, use again the factor 1.2 from Graph 1. On Graph 6, connect this number with 1140°F on the line marked "Stagnation Temperature" and read 890°F on the line marked "Flowing Temperature."

4) The velocity of sound is calculated by Equation (3) and is the velocity of the propagation of sound in a medium at rest. On Graph 4 the left-hand scale of line A (k to calculate C_0) is used, connect 1.4 with 28.9 on line B and mark line C. Connect with 1140°F on line D and read 1950 ft/sec on line E.

5) The actual velocity of the gas flowing through the throat is given by the same graph (Graph 4) but using the right-hand scale on line A (k to calculate C^*). Connect 1.4 on line A (right-hand scale) with 28.9 on line B and mark line C. Connect with 1140°F on line D and read 1750 ft/sec velocity on line E.

6) The velocity of the gas in a 1-inch ID line is subsonic because the throat diameter (for sonic flow) is 0.41 inch. Because the flow diameter and thereby the area are given and the velocity of the gas is unknown, Graph 2 must be solved backwards. On the horizontal line 1-inch diameter corresponds to 0.785-inch² area. Connect 0.785 on line marked "Flow Area" with 28.9 on line G and mark line 6. Connect with 1140°F on line F and mark line 5. Continue until line 3 is marked. Connect with 3100 mm Hg pressure on line C and read Mach 0.09 on line 2. The value of C^* (1750 ft/sec) should be multiplied by 0.09 to give a velocity of 157 ft/sec on the 1-inch line.

b. Numerical Solution

1) For the throat diameter, Flieguers formula (p 85 - 4.18) is used with the English units (such as $W = 0.1$ lb/sec) called for in the text.

$$A^* = \frac{W \sqrt{T_0}}{P_0 \times 0.532} = \frac{0.1 \text{ lb/sec} \sqrt{1600^\circ\text{R}}}{60 \text{ psia} (0.532)} = 0.125 \text{ in.}^2$$

$$A = \frac{\pi D^2}{4} ; \quad D^2 = \frac{0.125 \times 4}{\pi} = 0.1585$$

$$D = 0.4 \text{ in.}$$

2) For the pressure of the throat, the formula $P^*/P_0 = 0.5283$ can be used in mm Hg absolute because P^*/P_0 is a ratio
 $P^* = 3100 \times 0.5283 = 1635 \text{ mm Hg.}$

3) The temperature at the throat is again in ratio form $T^*/T_0 = 0.8333$, but absolute units such as Rankin or Kelvin must be used.

$$T^* = T(0.8333) = (1600)(0.8333) = 1335^\circ\text{R}$$

$$T^* = 1335 - 460 = 875^\circ\text{F} \quad .$$

4) The velocity of sound is given by

$$C_0^2 = k g R T_0 \frac{1}{(\text{mol wt})}$$

$$C_0 = \left(\frac{1.4 \times 32.2 \times 1544 \times 1600}{28.9} \right)^{\frac{1}{2}} = 1963 \text{ ft/sec} \quad .$$

5) The actual velocity of the gas flowing through the throat is given by

$$C^*^2 = \frac{2k}{k+1} g R T_0 \frac{1}{(\text{mol wt})}$$

$$C^* = \left[\left(\frac{2.8}{2.4} \right) (32.2) (1544) (1600) \frac{1}{(28.9)} \right]^{\frac{1}{2}} = 1792 \text{ ft/sec} .$$

6) The velocity of the gas in a 1-inch ID pipe must be solved by trial and error

$$\frac{W \sqrt{T_0 R}}{A_x P_0 \sqrt{(\text{mol wt}) k g}} = \frac{M}{\left(1 + \frac{k-1}{2} M^2 \right)^{\frac{k+1}{2(k-1)}}}$$

may be reduced to

$$\frac{W \sqrt{T_0 R}}{A_x P_0 \sqrt{(\text{mol wt}) k g}} = \frac{M}{(1 + 0.2 M^2)^3}$$

because

$$\frac{k-1}{2} = \frac{1.4-1}{2} = 0.2$$

and

$$\frac{k+1}{2(k-1)} = \frac{2.4}{2(.4)} = 3.0 .$$

This may be rewritten to

$$A = \frac{W \sqrt{T_0 R}}{P_0 \sqrt{(\text{mol wt}) k g}} \frac{(1 + 0.2 M^2)^3}{M} = \left(\frac{0.1}{60} \right) \sqrt{\frac{(1600)(1544)}{(28.9)(1.4)(32.2)}} \times \frac{(1 + 0.2 M^2)^3}{M}$$

$$A = 0.0726 \frac{(1 + 0.2 M^2)^3}{M} = 0.785$$

Since the graphical solution gave $M = 0.09$, this will be the assumed starting point.

<u>M</u>	<u>0.2 M²</u>	<u>(1 + 0.2 M²)³</u>	<u>$\frac{(1 + 0.2 M^2)^3}{M}$</u>	<u>$0.0726 \frac{(1 + 0.2 M^2)^3}{M}$</u>
0.09	0.00162	1.00486	11.1	0.805
0.095	0.00182	1.00547	10.55	0.77
0.100	0.002	1.006	10.06	0.73

Since the actual flow area is 0.785 in.², the Mach number must be between 0.09 and 0.095.

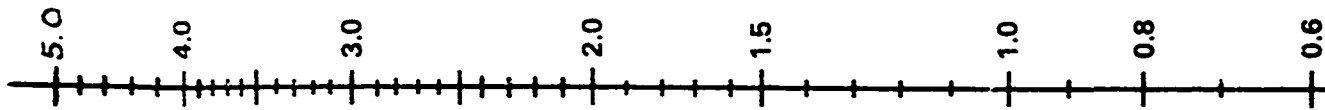
Assuming $M = 0.093$,

$$A = 0.0726 \times \frac{[1 + (0.2)(0.093)^2]^3}{(0.093)} = 0.785$$

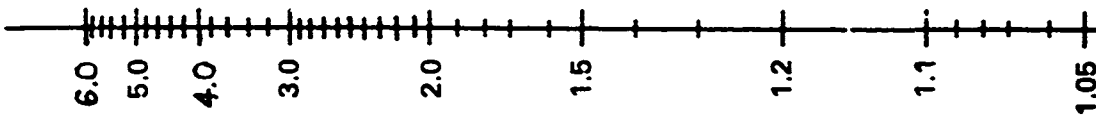
The velocity then is $M^* C^* = 0.093 \times 1792 = 167$ ft/sec.

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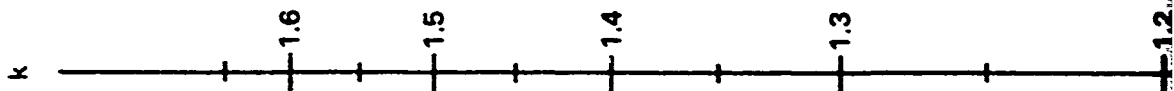
MACH NO.
M



$1 + \frac{k-1}{2} M^2$



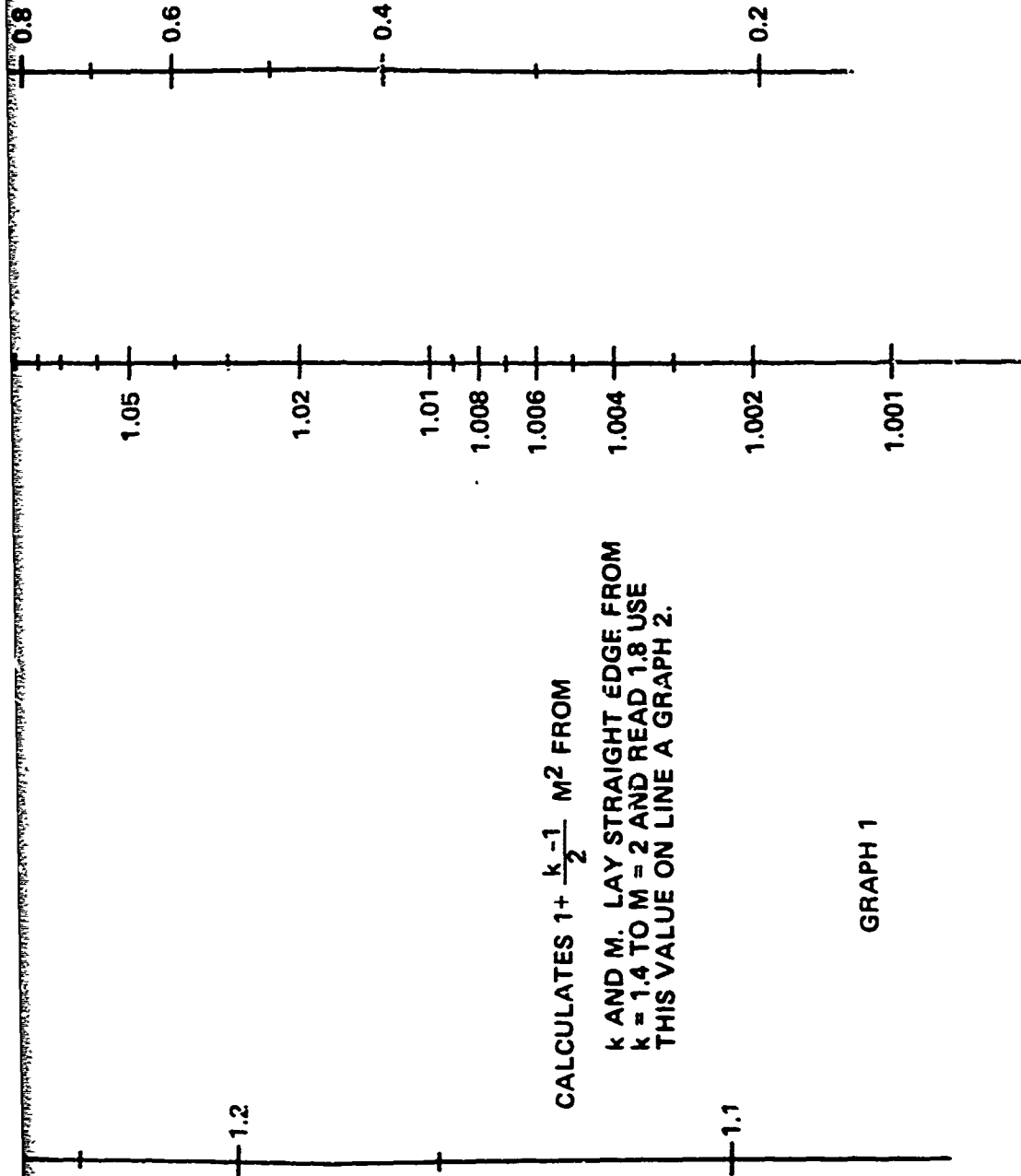
SPECIFIC HEAT RATIO
k



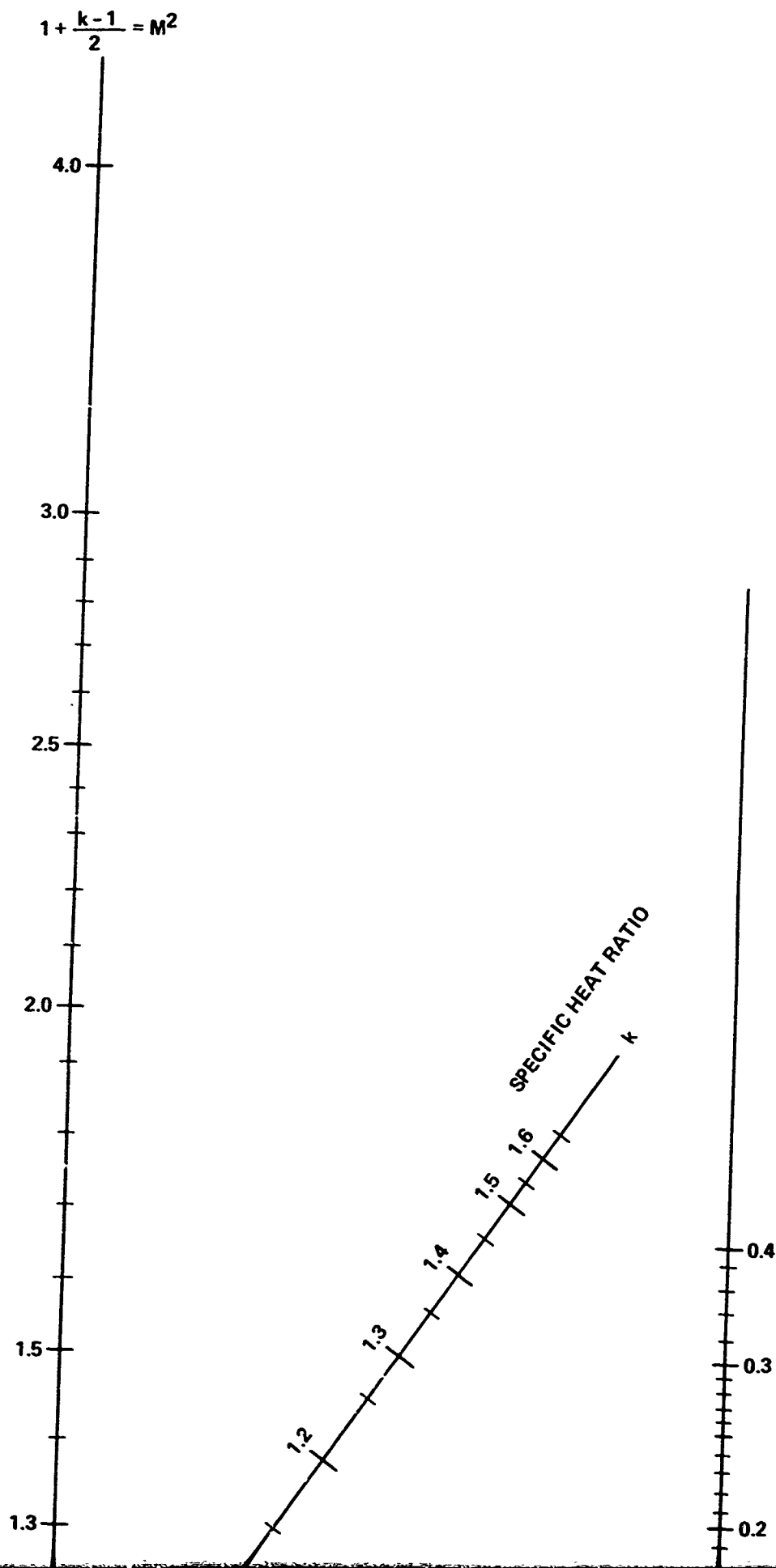
CALCULATES $1 + \frac{k-1}{2} M^2$ FROM
 k AND M. LAY STRAIGHT EDGE FROM
 k = 1.4 TO M = 2 AND READ 1.8 USE
 THIS VALUE ON LINE A GRAPH 2.

GRAPH 1

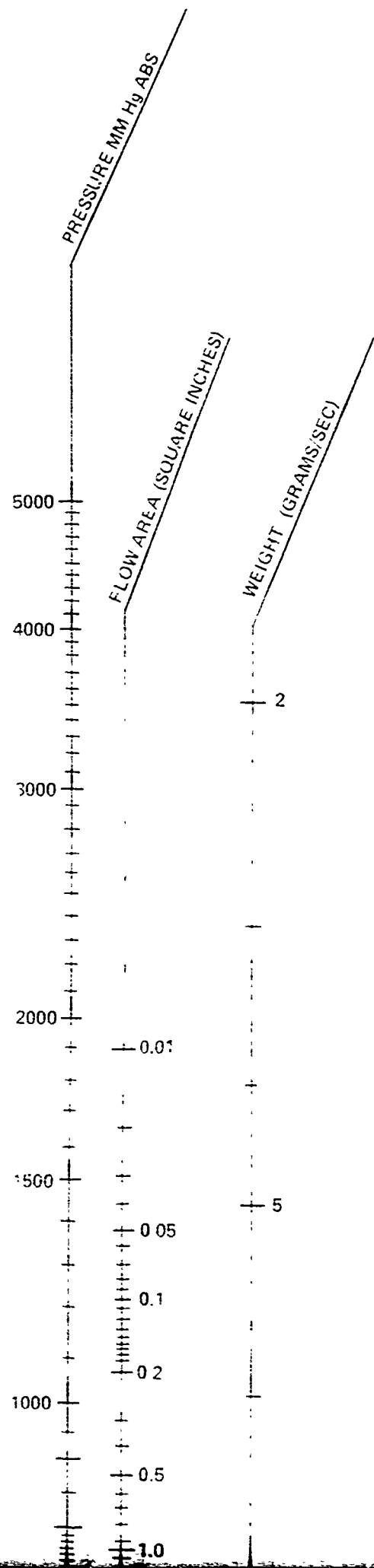
USE STRAIGHT EDGE BETWEEN k AND M AND READ SOLUTION
 OF EQUATION ON CENTER LINE



A



B



B

W/SEC

SPECIFIC HEAT RATION (K)

1.7

1.6

1.5

MOLECULAR WEIGHT

80

70

60

55

50

40

35

30

25

20

15

10

9

8

7

6

5

4

3

2

TEMPERATURE °F

-100

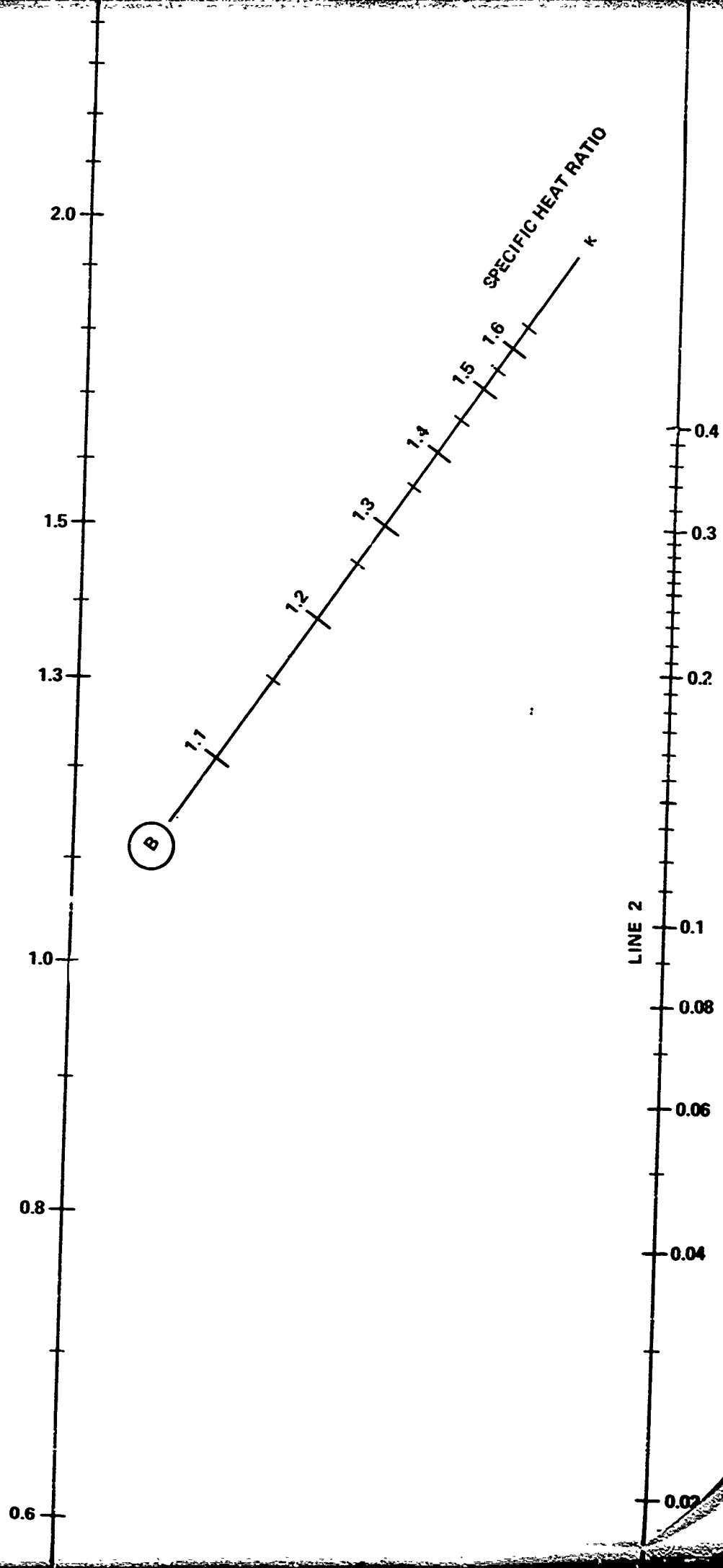
0

100

300

500

22



77-1

0.4
0.3
0.2
0.1
0.08
0.06
0.04

LINE 1

LINE 3

LINE 4

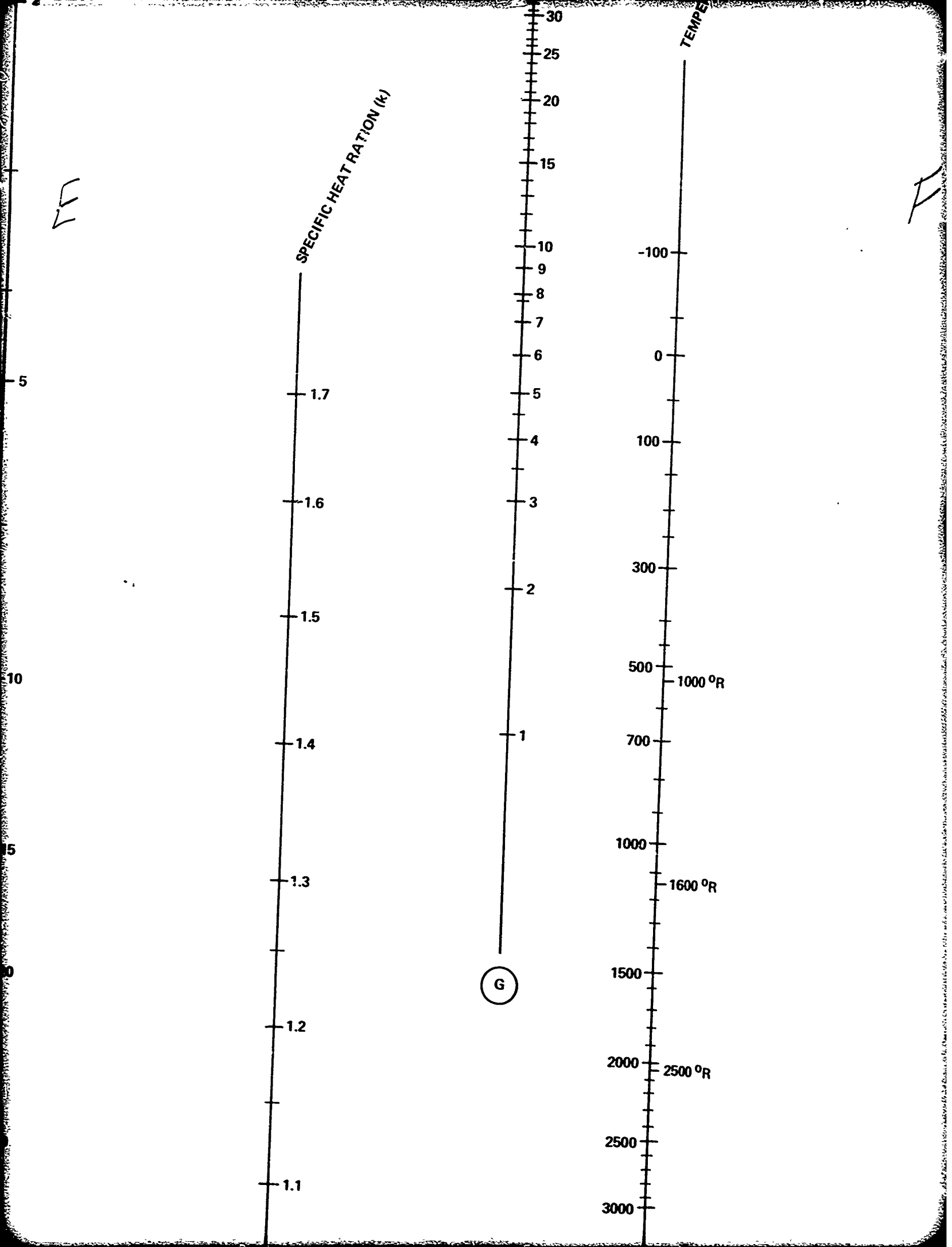
LINE 5

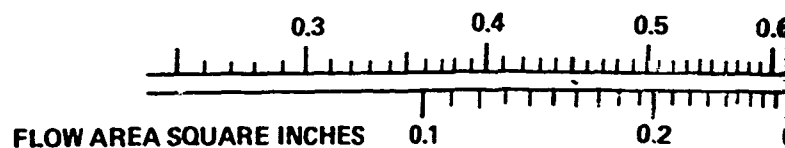
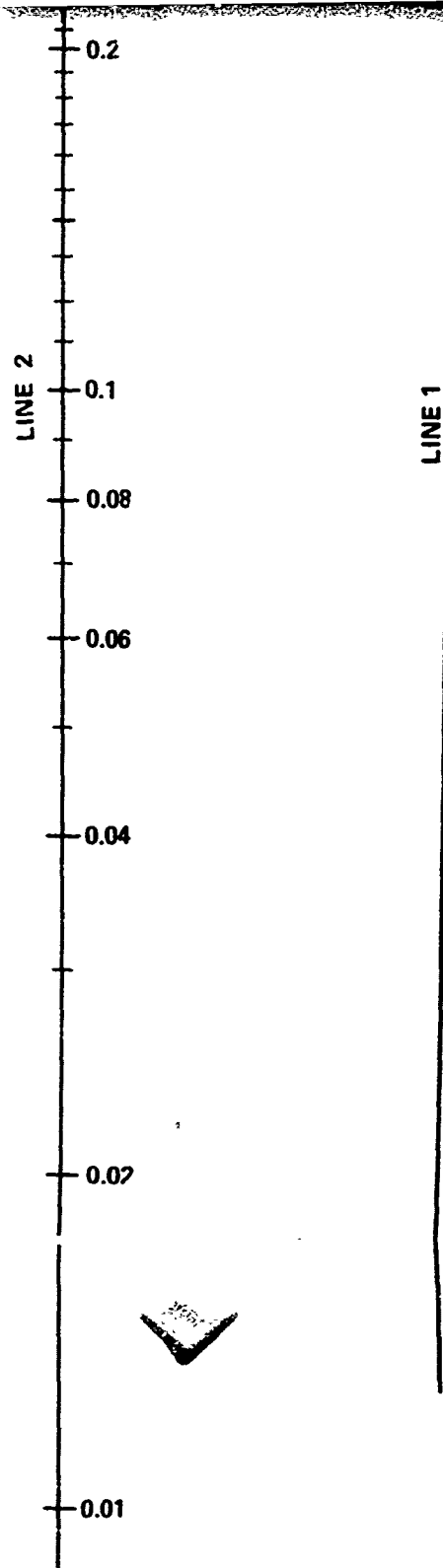
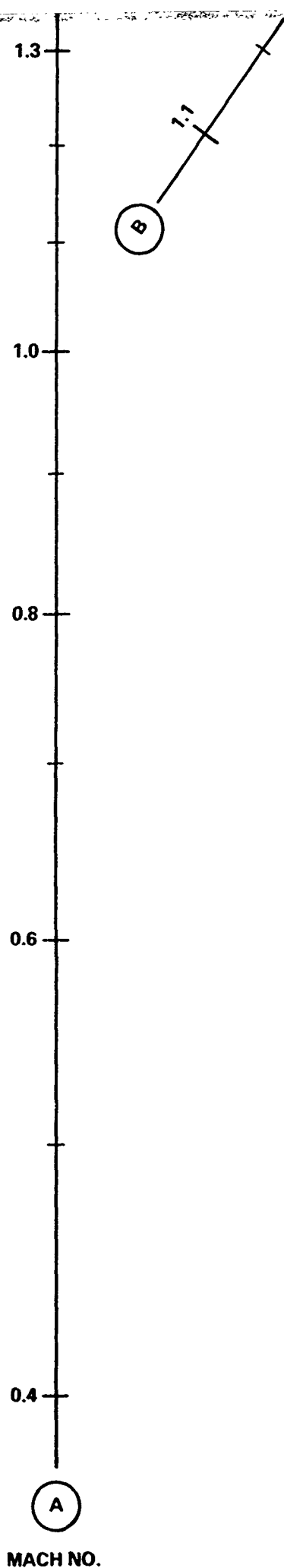
LINE 6

2000
1500
1000
500
400
300
200

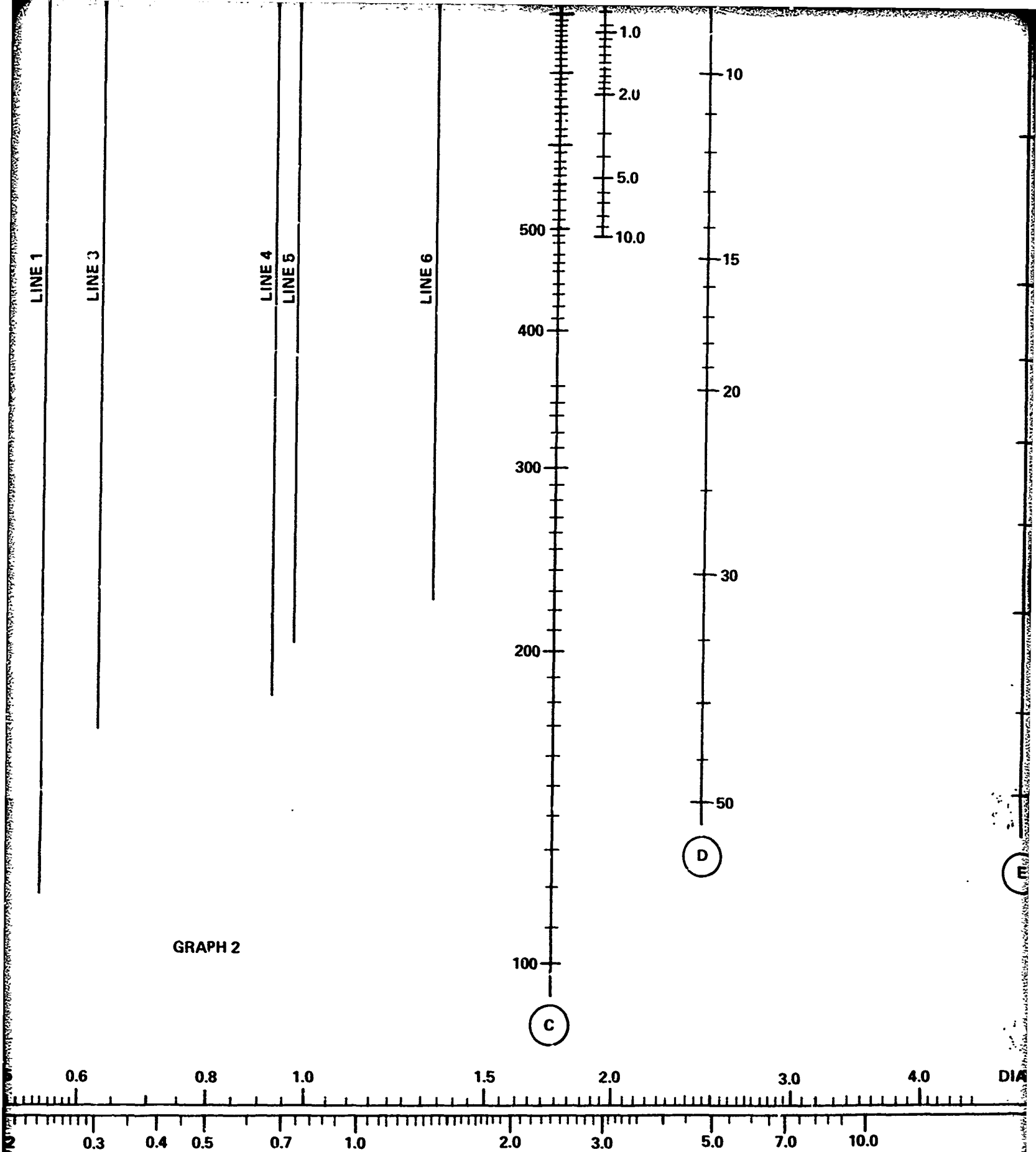
0.01
0.05
0.1
0.2
0.5
1.0
2.0
5.0
10.0

5
10
15
20
30

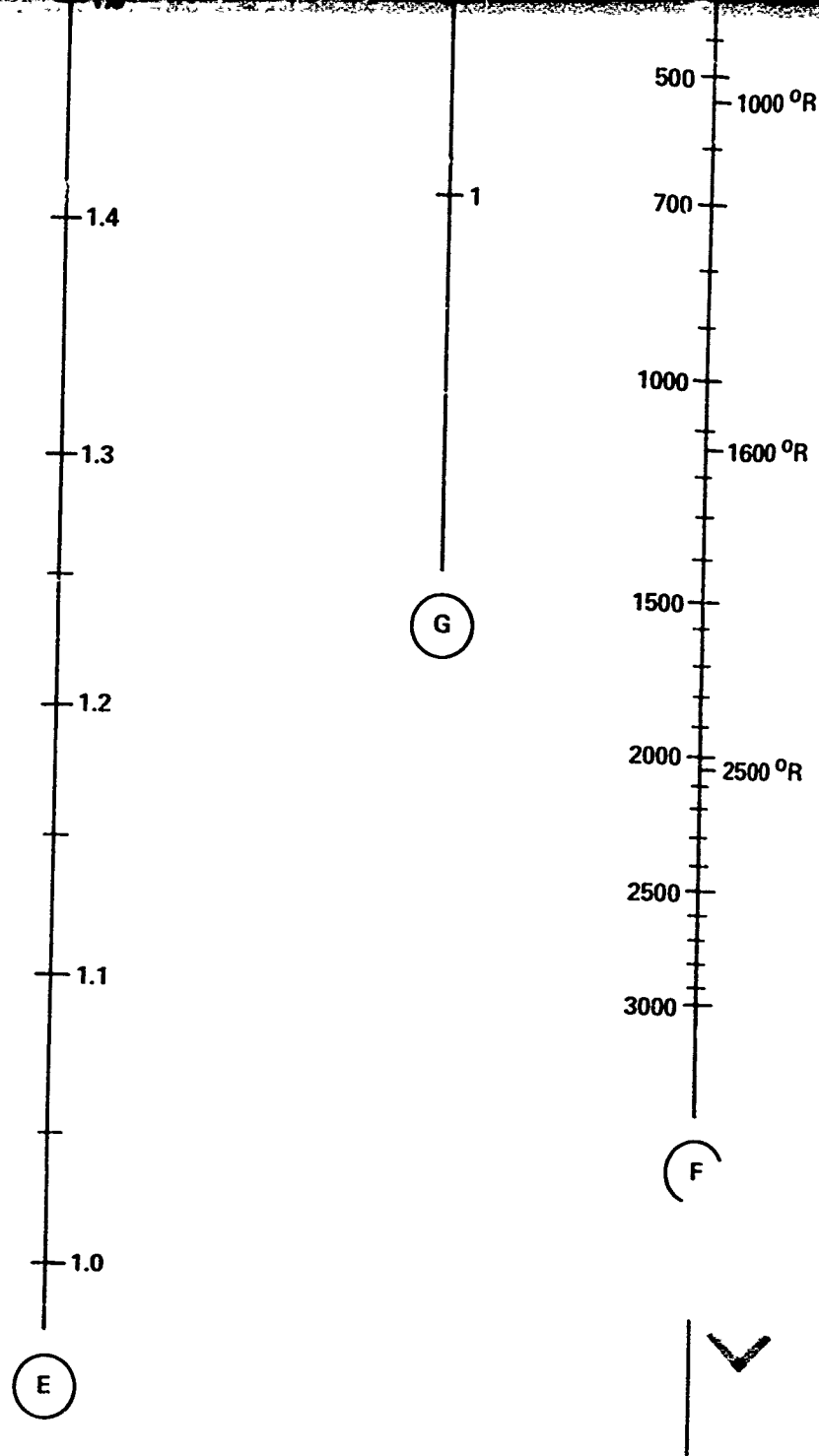




USE RESULT FROM GRAPH 1 ON LINE A
ON LINE 1; CONNECT WITH MACH NO. ON
3-D LINE 4-E LINE 5-F LINE 6-G-F



ON LINE A, CONNECT WITH B AND MARK
EACH NO. ON LINE A, MARK LINE 2→C→LINE
LINE 6→G→REND FLOW AREA

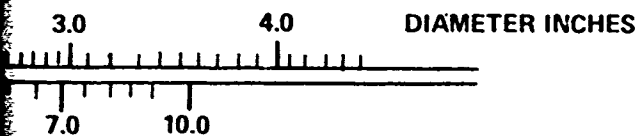


THIS GRAPH SOLVES THE EQUATION

$$\frac{W}{\sqrt{\text{MOL WT}} \times A} = \frac{P_0 \times M \times \sqrt{k}}{\sqrt{R \times \left[1 + \frac{k-1}{2} M^2\right]^{\frac{k+1}{k-1}} \times T_0}}$$

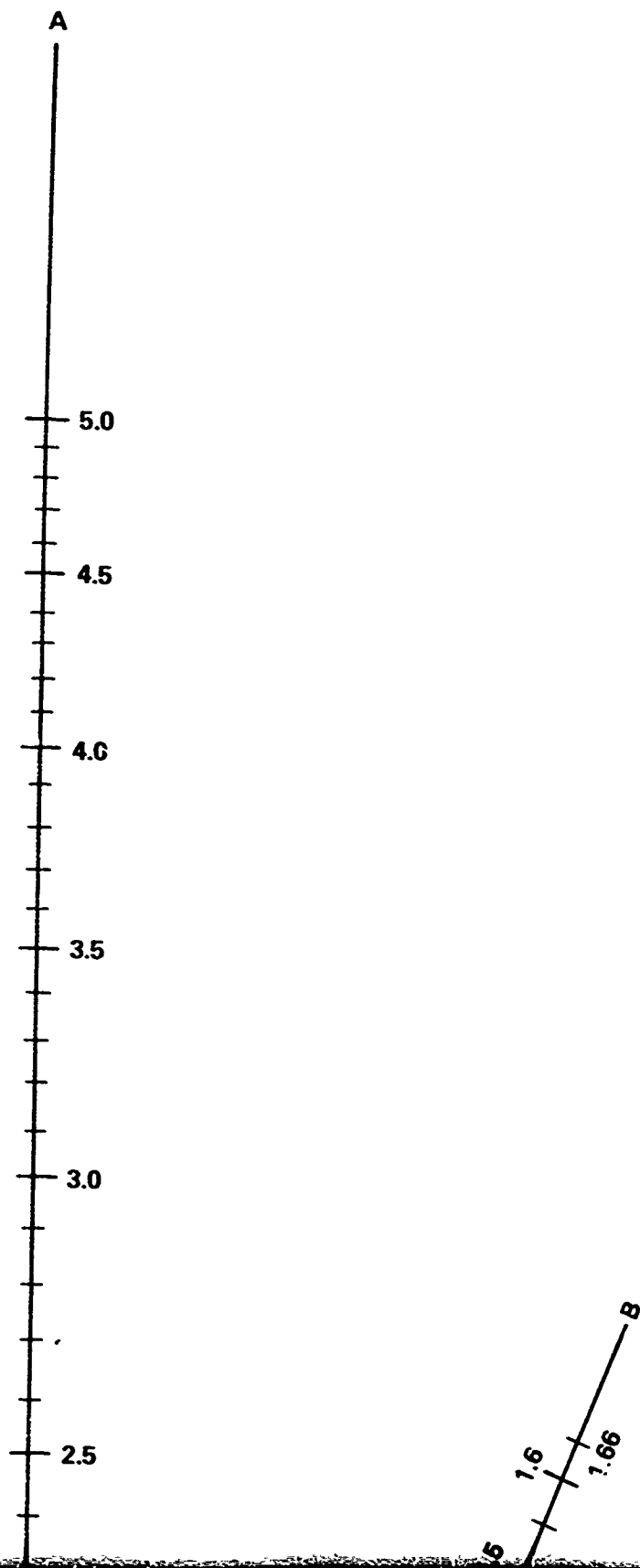
FROM
"COMPRESSIBLE FLOW" SHAPIRO VOL 1 p.84

10,000 °R

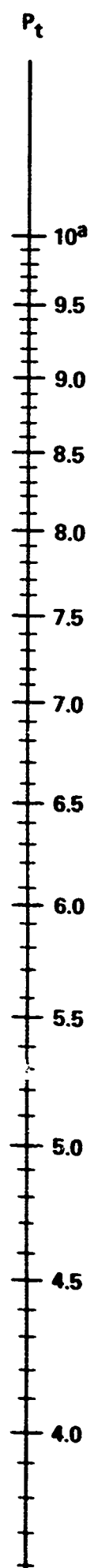
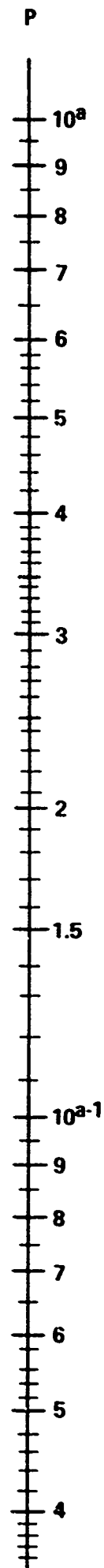


1

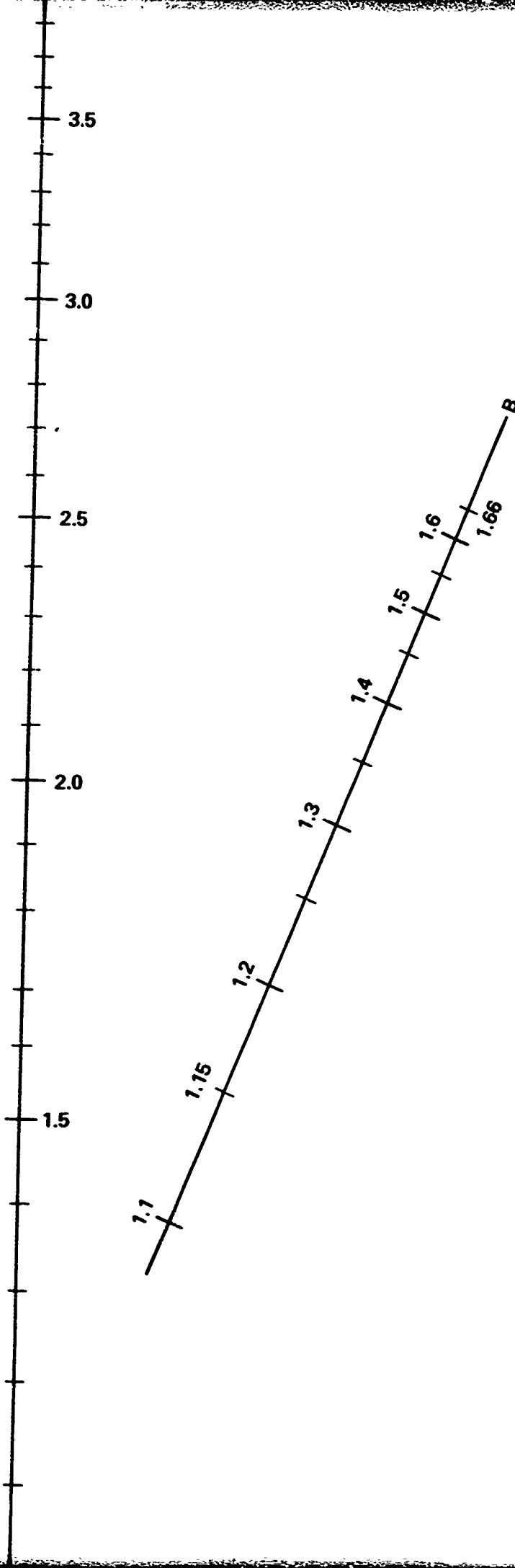
A



B



C



10

1.5

10^{a-1}

9

8

7

6

5

4

3

2

1.5

10^{a-2}

9

8

7

6

5

4

6.0

5.5

5.0

4.5

4.0

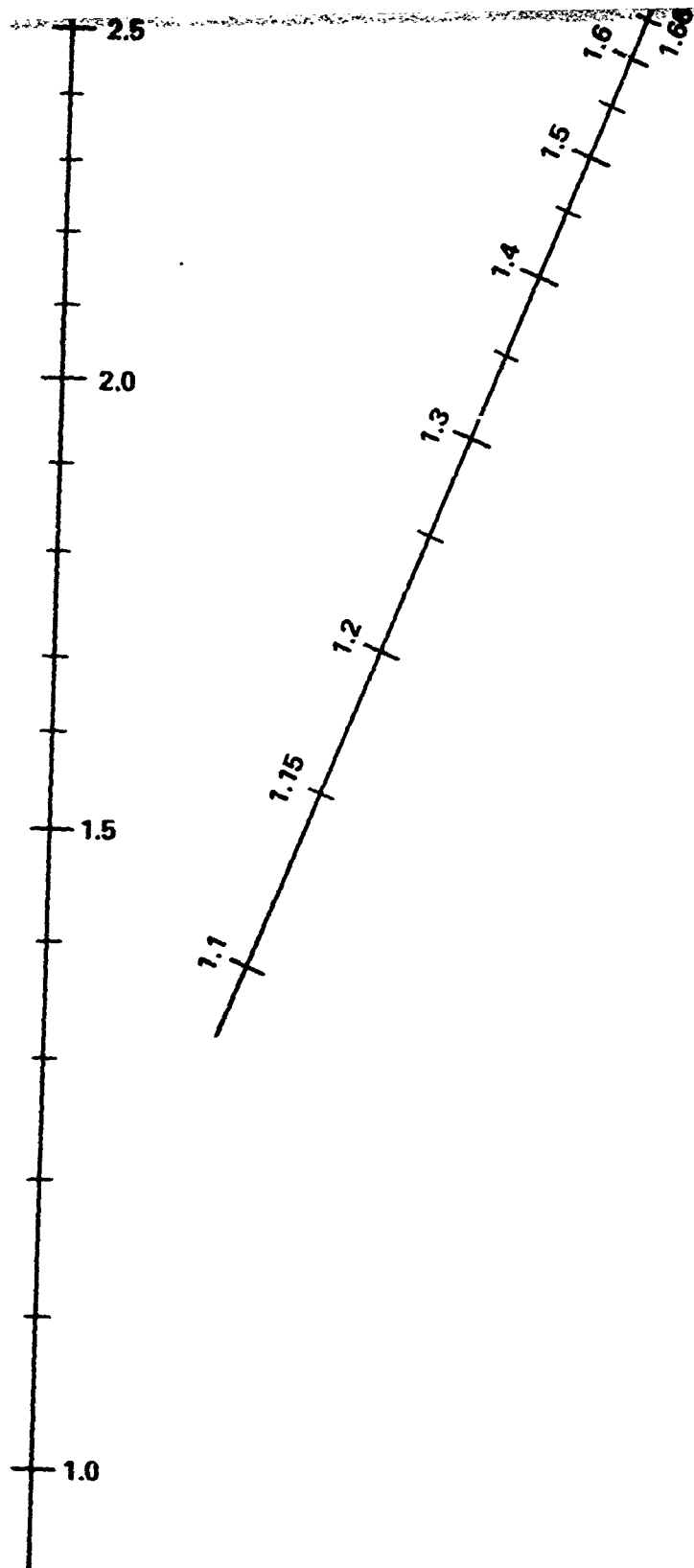
3.5

3.0

2.5

2.0



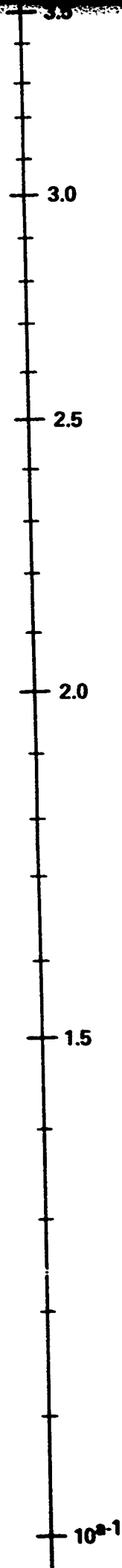
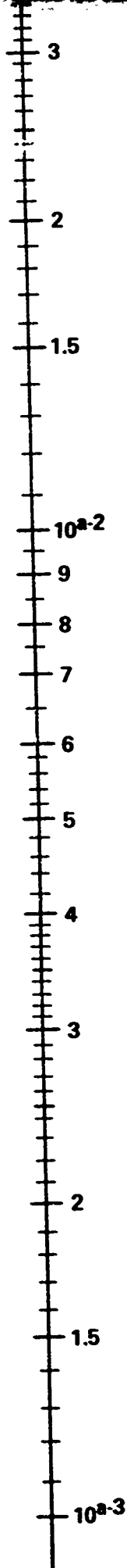


GRAPH 3

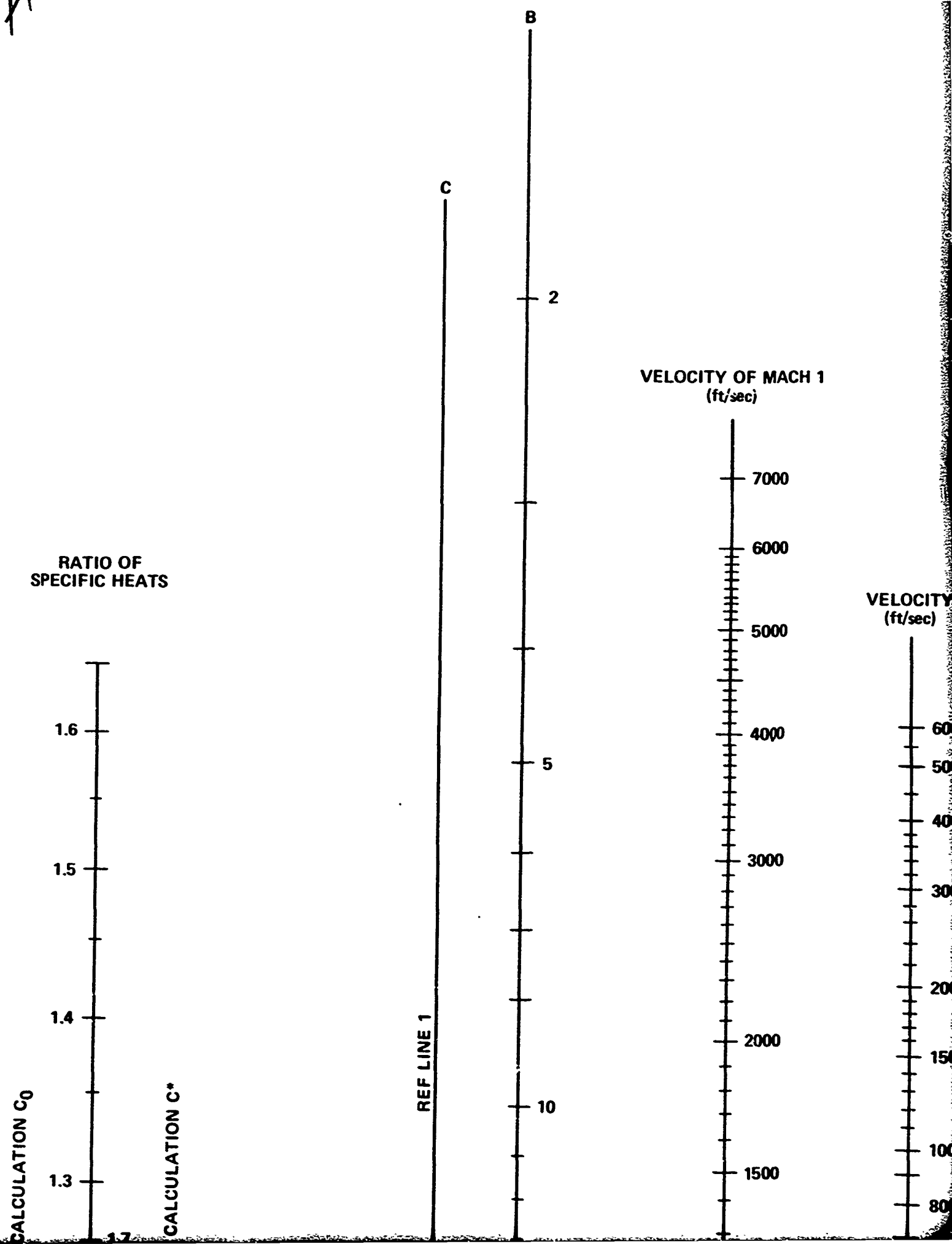
MARK VALUE OF GRAPH 1 ON LINE A. CONNECT K VALUE ON LINE B AND MARK ON LINE 1. CONNECT THIS POINT WITH VALUE [ABS PRESSURE] ON LINE P_t AND READ "P".

66

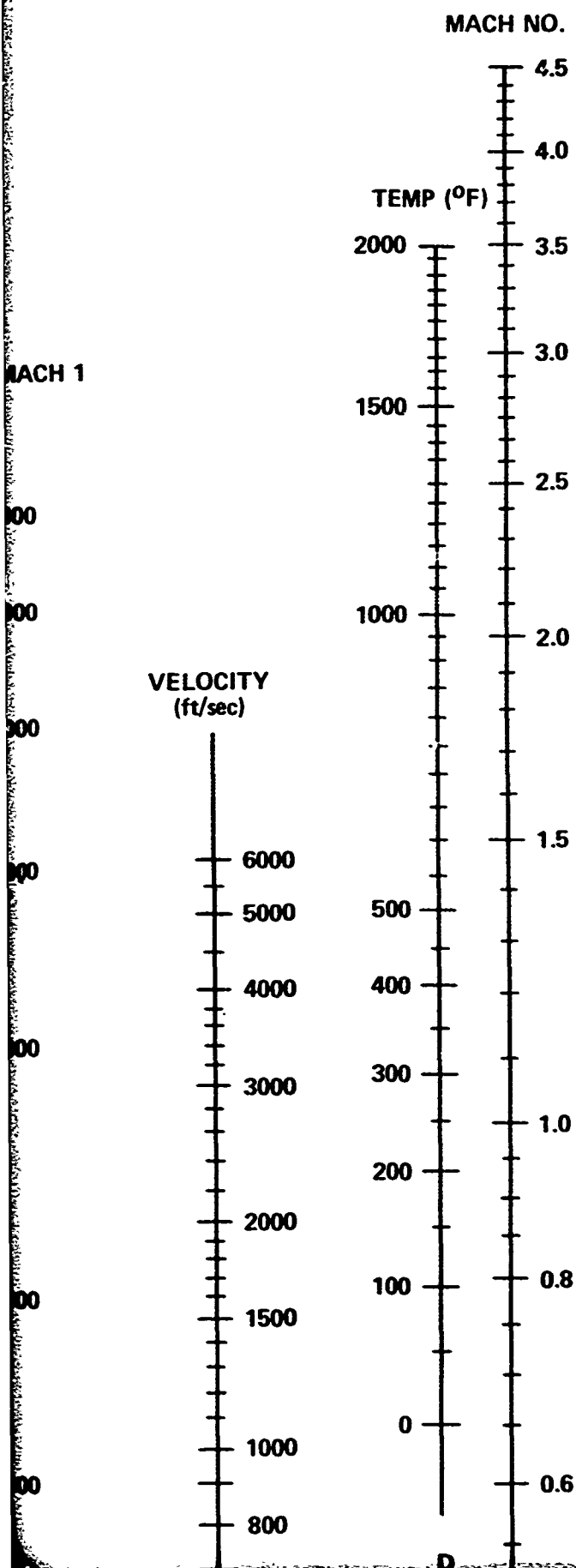
100



A



B

MM/MICRO
SEC ft/

2.0

1.5

1.0

0.9

0.8

0.7

0.6

0.5

0.4

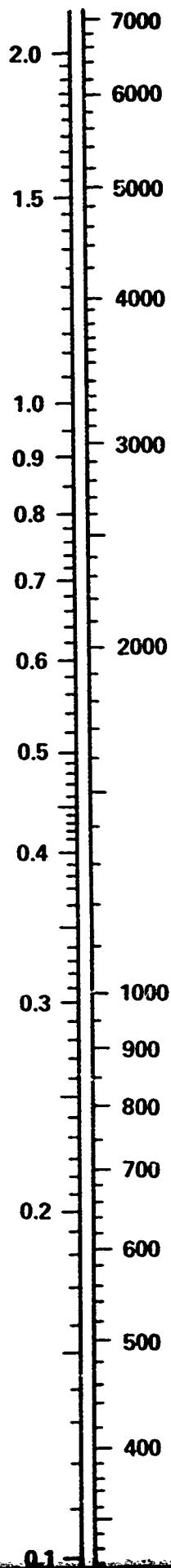
0.3

0.2

0.1

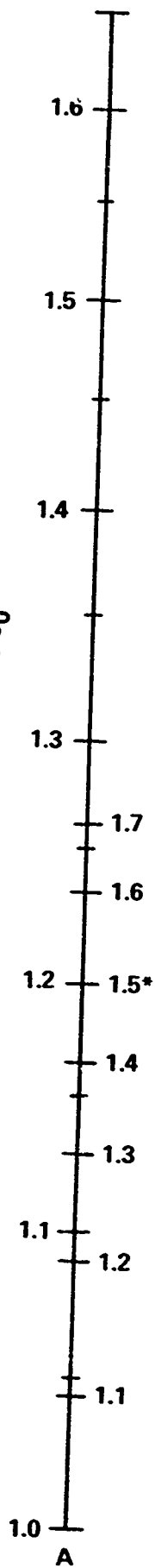
B

MM/MICRO
SEC ft/sec



C

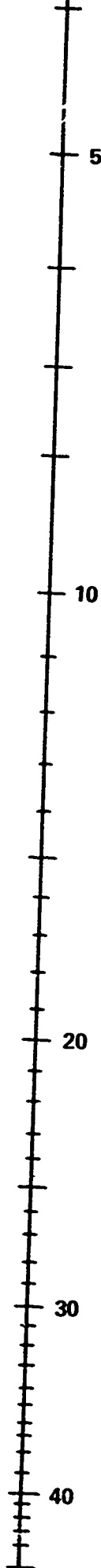
k TO CALCULATION C_0



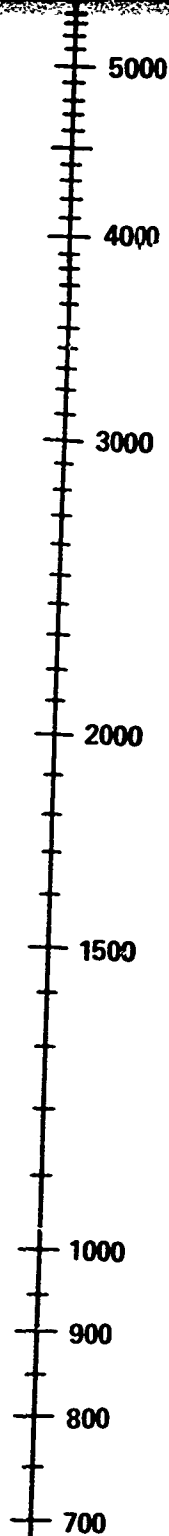
k TO CALCULATION C^*

REF LINE 1

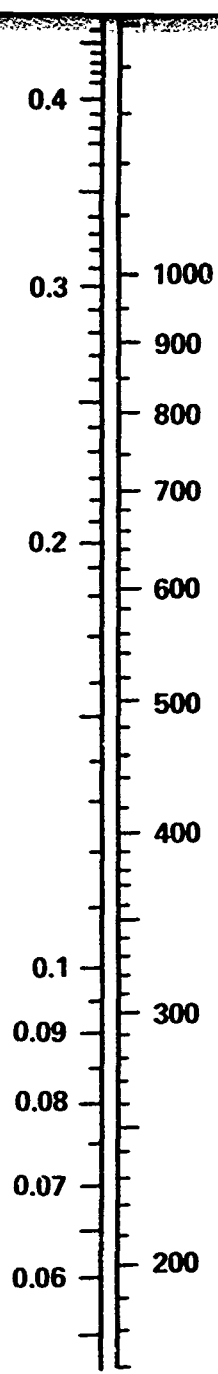
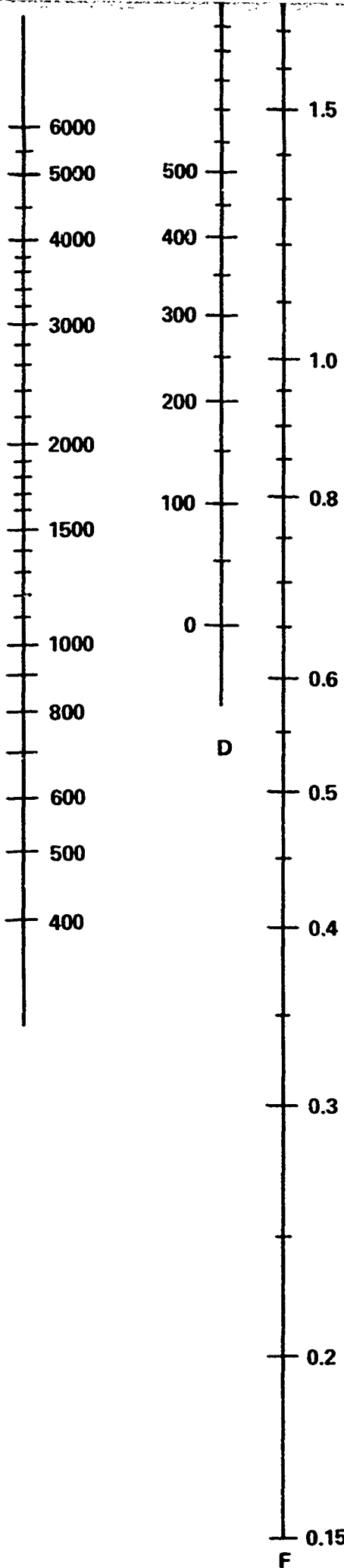
MOL WEIGHT



E



GRAPH 4



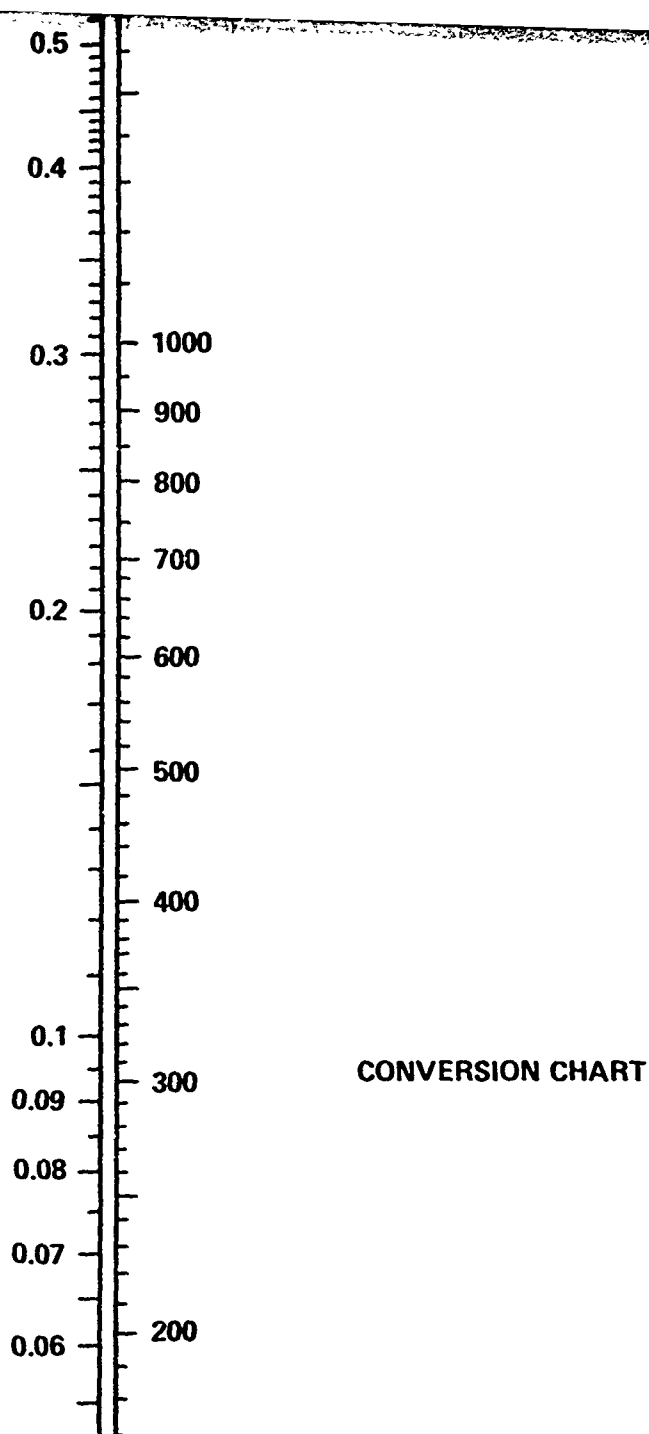
THIS GRAPH SOLVES THE EQUATION AND

$$V_0 = C_0 \times M \text{ WHERE } C_0^2 = k g R T_0 / \text{MOL WEIGHT}$$

$$V^* = C^* \times M \text{ WHERE } C^{*2} = k g R T^* / \text{MOL WEIGHT}$$

CONNECT LINE A WITH B, MARK C (REF LINE 1). GO TO D, READ AND MARK E. THEN GO TO F AND READ VEL

6



THIS GRAPH SOLVES THE EQUATION AND

$$V_0 = C_0 \times M \text{ WHERE } C_0^2 = k g R T_0 / \text{MOL WEIGHT}$$

$$V^* = C^* \times M \text{ WHERE } C^{*2} = k g R T^* / \text{MOL WEIGHT} = \frac{2k}{k+1} g R T_0 / \text{MOLES}$$

CONNECT LINE A WITH B, MARK C (REF LINE 1). GO TO
D, READ AND MARK E. THEN GO TO F AND READ VELOCITY.

7.0

6.0

D/D*

RATIO OF EXIT DIAMETER TO THROAT DIAMETER VS MACH NUMBER

$$\frac{D}{D^*} = \frac{1}{\sqrt{M}} \left[\left(\frac{2}{K+1} \right) \left(1 + \frac{K-1}{2} M^2 \right) \right]^{\frac{K+1}{4(K-1)}}$$

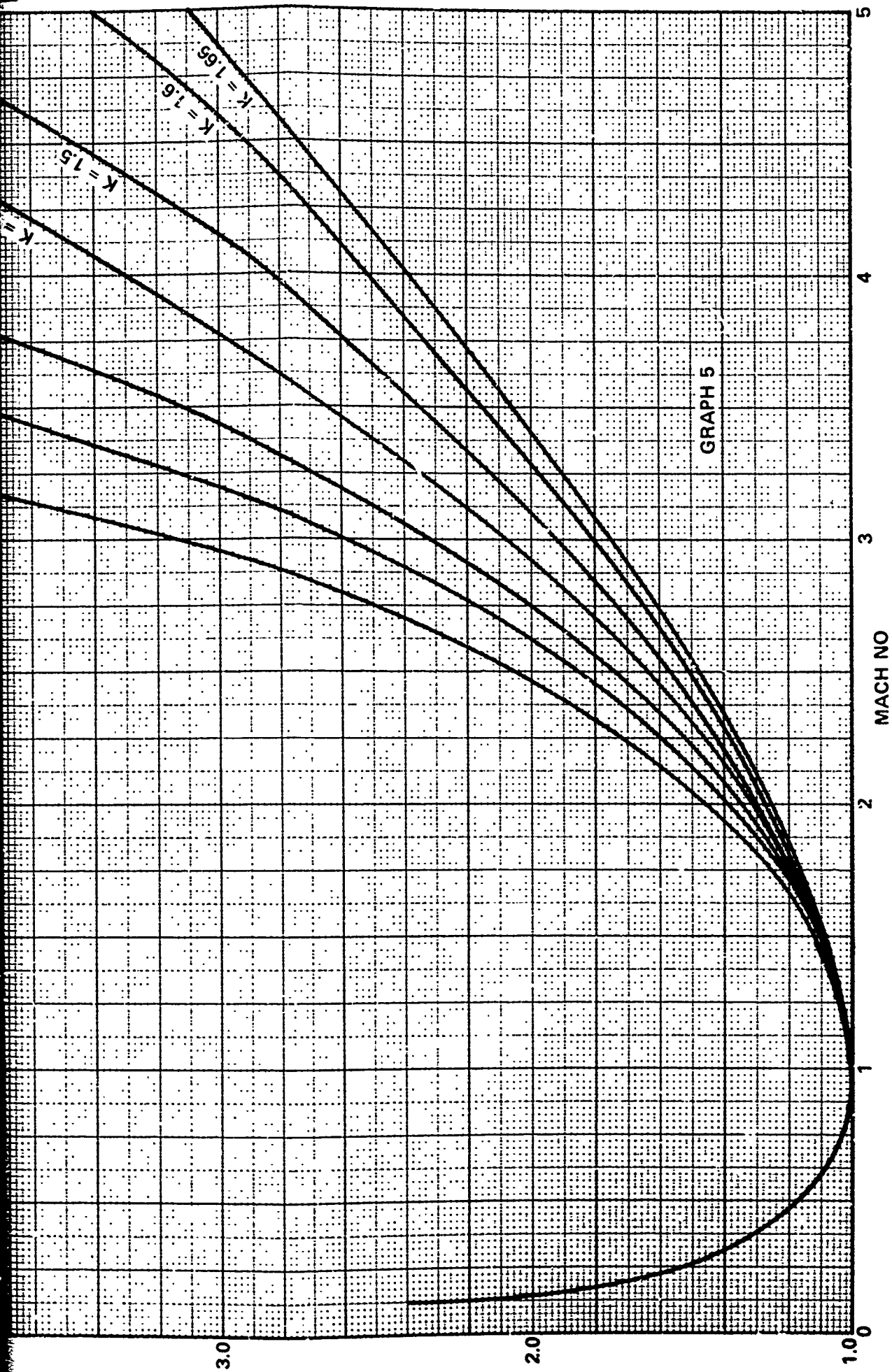
COM. FLUID FLOW Vol. p. 86 (Eq. 4.19)

K=1.1

K=1.2

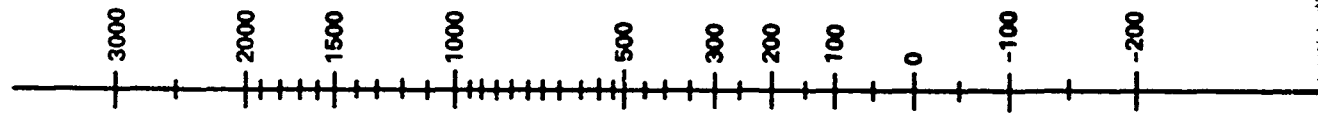
K=1.3

K=1.4

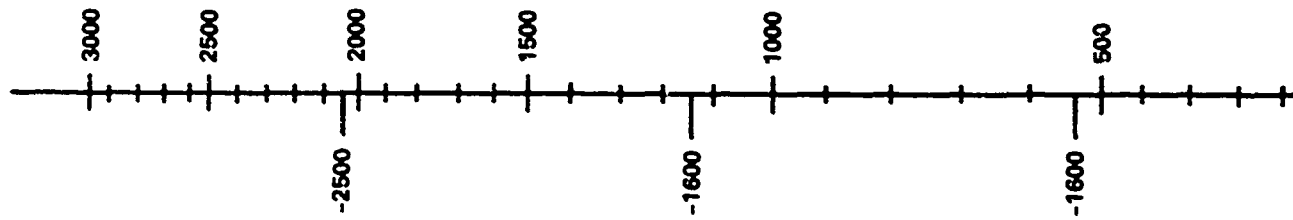


~~XXXX~~

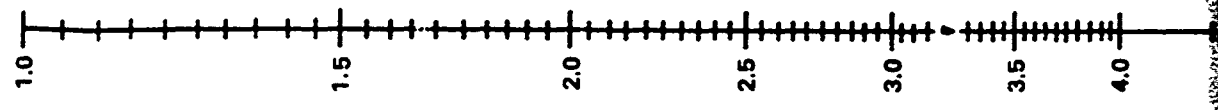
FLOWING
TEMPERATURE
(°F)

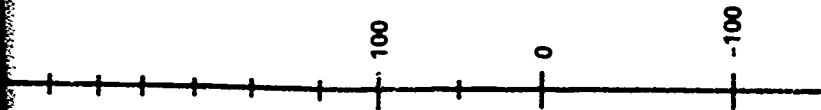


STAGNATION
TEMP (°F)



FACTOR
GRAPH 1

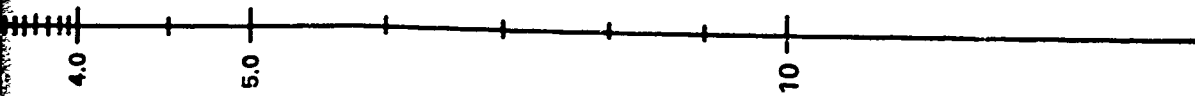




GRAPH 6

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2$$

(EQUATION 4.14a) SHAPIRO p.83



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