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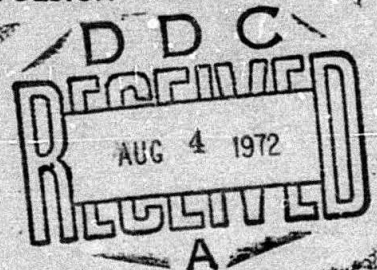
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**THERMONUCLEAR MICRO-BOMB ROCKET PROPULSION**

February 1972

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**III**

### Abstract

An advanced nuclear rocket propulsion system based on a chain of exploding thermonuclear micro-bombs ignited in front of a concave magnetic reflector open at one side was proposed previously<sup>(1,2,3,4)</sup>. It is shown that for such a propulsion system to work with ideally high efficiency the following conditions have to be satisfied:

1. The thermonuclear material for the micro-bombs has to be a stoichiometric mixture of  $\text{He}^3$  and D in solid or liquid form.
2. The ignition of the micro-bombs has to be accomplished by an intense relativistic electron beam.

Requirement 1) results from the property of the  $\text{He}^3$ -D thermonuclear reaction to lead to charged fusion products only, in contrast to the T-D reaction in which 80% of the released energy is set free in neutrons. Requirement 2) results from the much higher ignition temperature of the  $\text{He}^3$ -D thermonuclear reaction as compared to the T-D reaction, to be accompanied by a substantial increase in the ranges of the charged fusion products. This would normally lead to energy inputs for ignition of the  $\text{He}^3$ -D reaction many orders of

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magnitude larger than those for the T-D reaction, therefore excluding laser systems for ignition. Intense relativistic electron beams, however, can bypass this difficulty by virtue of their very strong self-magnetic fields. This strong self-magnetic beam field leads to three effects reducing the minimum energy input required for ignition substantially: first, it will confine the charged fusion products to within a sufficiently small volume; second, it will reduce

the electronic heat conduction losses within the plasma of the micro-bomb; and third, it will delay the thermal expansion losses of the micro-bomb plasma by magnetic confinement.

The electron beam method for thermonuclear micro-bomb ignition has the further distinct advantage over laser ignition systems that intense relativistic electron beams can be produced with much higher efficiency than laser beams, which is of special importance for any propulsion system. It is shown furthermore, that intense relativistic electron beams can be produced even more efficiently by a novel inductive energy storage system which is distinguished by its low weight, another property of great importance for its application to propulsion.

The proposed advanced propulsion system is a high specific impulse high thrust system which will make possible flights to Mars in less than a week, to the planet Jupiter in less than a month and to the planet Pluto at the edge of our solar system within a year. These short interplanetary transit times would make it the ideal propulsion system for future manned missions within the solar system.

## 1. Introduction

One of the principal problems towards manned exploration of the solar system is the development of a high specific impulse high thrust propulsion system. It is recognized that only with a propulsion system, producing both, sufficient thrust and a high specific impulse, will it be possible to reduce the transit times for manned interplanetary missions to comfortable levels. It is furthermore obvious that such a system has to make use of nuclear energy.

Nuclear propulsion systems of proven feasibility are all utilizing the release of nuclear energy by the fission process.

In the NERVA system hydrogen is heated up to several thousand degrees Centigrade by a solid core nuclear reactor. Although such a system can produce substantial thrust, it is limited in its specific impulse arising from material problems of the reactor core setting an upper limit for the technically attainable reactor temperature. As a result, the maximum specific impulse which can be reached with such a system is approximately twice as large as compared with advanced chemical rockets. This increase in specific impulse is indeed a great progress over chemical propulsion and can be compared to the development in aircraft propulsion by going from propeller to jet driven systems. Raising the specific impulse by a factor of two, for example, would greatly reduce the cost for lunar missions and could be thus of great importance in the future establishment of permanent lunar bases. An increase in a factor two over chemical propulsion in the attainable specific impulse, however, is

insufficient to reduce the transit times within the solar system to levels desirable for manned missions and will still lead to trips lasting of the order years even for the nearest planets like Mars and Venus.

The other nuclear rocket propulsion systems of proven feasibility are without exception based on the use of electric power generated by a space nuclear power plant to drive either a beam of ions or a plasma by a variety of means of accelerating a jet, either by thermal heating, or magnetohydrodynamically, or electrostatically. All these propulsion systems are limited due to the maximum attainable power per unit mass of any space nuclear power plant primarily as a result of the large radiator mass. In spite of having a high specific impulse such systems can only produce moderate thrusts. Although the high specific impulse would make the attainment of high vehicle velocities possible, required for short transit times for interplanetary missions, the low thrust leads to acceleration times (to attain these velocities), which are of the order of years, and which in effect again result in interplanetary transit times of years.

In summary, one can therefore safely say that the state of the art does not permit interplanetary missions in reasonable short times as they are already possible for trips to the moon.

It is widely hoped that a way out of this dilemma will arrive with the development of the gas core reactor rocket engine. But even here, quite apart from the two principal problems of fuel containment and heat transfer, the expected gain in specific impulse is in no way so dramatic as one would expect it to be the

case for the employment of nuclear energy. It should be conceded, however, that a breakthrough for such a concept would be of the greatest importance in the field of propulsion. Until now and in spite of intensive studies such a breakthrough has not been achieved.

There are two other proposed advanced nuclear propulsion systems frequently mentioned in the literature, the Orion bomb propulsion concept and thermonuclear propulsion.

In the Orion concept it is proposed to propel the spacecraft by a sequence of exploding atomic bombs. Such a system promises both large thrust and large specific impulse. It is obvious, however, that the technical problems associated with such a system are considerable, quite apart from the political considerations in conjunction with the test ban treaty. Furthermore, such a system seems to be only prudent for very large spacecraft and payloads.

The other mentioned propulsion system, that is the thermonuclear propulsion system, is not clearly defined since it depends upon the still unsolved problem of controlled thermonuclear fusion. When mentioned in the literature, however, it is normally understood that such a system depends upon some kind of magnetic plasma confinement device whereby the "magnetic bottle" has an opening for the hot thermonuclear plasma to enter an exhaust nozzle. The principal problem of such a system is the weight factor. Even if the magnetic confinement problem can be eventually solved, it is widely agreed that such a system must have an enormous size and consequently a weight of many thousands of tons, making it again only interesting for very large spacecraft.

## 2. Thermonuclear micro-bomb propulsion

A distinct advantage of the Orion propulsion concept is that it is a pulsed system. It is well known that material can withstand much higher heat fluxes if exposed to them only for a short period of time. This is one of the reasons for the good performance of the internal combustion engine, which is a chemically pulsed system. The principal disadvantage of the Orion propulsion concept is that it depends on a sequence of exploding fission bombs. Fission bombs have always a critical mass making only sufficiently large nuclear fission explosions economical.

In contrast to fission bombs the situation is quite different for thermonuclear bombs. As long as a thermonuclear bomb relies on a fission bomb as trigger, it again has a lower explosion size limit, dictated by the minimum explosive power of the fission trigger. However, if a fissionless trigger mechanism can be found, which like an exploding fission bomb can produce the required high temperatures for thermonuclear ignition, but unlike a fission bomb within a much smaller volume and thus with a much smaller energy release, thermonuclear micro-explosions could be set off by igniting pellets of thermonuclear explosives a few mm across. The energy release in such micro-explosions could be conceivable equivalent to the explosive power of one ton of TNT, explaining the proper word thermonuclear micro-bomb, since a thermonuclear bomb is usually associated with an energy release in the megaton range.

Provided a fissionless trigger mechanism can be found, a chain of such thermonuclear micro-explosions could be used for rocket

propulsion by letting the explosions take place in a concave reflector open at one side.

The use of thermonuclear micro-bombs instead of fission bombs as in the Orion concept has two distinct advantages: first, it works with much smaller explosions per pulse and which are much easier to control; second, each thermonuclear micro-explosion results in the formation of an optically transparent plasma ball which can be effectively reflected by a strong magnetic field. This magnetic field can be created by a proper arrangement of superconducting high magnetic field coils.

Furthermore, in comparing such a thermonuclear micro-bomb rocket propulsion system with the conventional thermonuclear systems based on the magnetic confinement of a plasma, a third important advantage does emerge. Suppose that each micro-bomb releases an energy  $\epsilon_{out}$ . There is then a relation in between the atomic number density  $N$  in the micro-bomb plasma, the inertial confinement time  $\tau$  of the micro-explosion and the plasma volume  $V$  given by

$$\epsilon_{out} = \text{const. } N^2 \tau V \quad (2.1)$$

The Lawson criterion for a positive energy balance requires furthermore that

$$N\tau \geq a \quad (2.2)$$

where, for example,  $a \approx 10^{14} \text{ sec cm}^{-3}$ , (approximately valid for both the T-D and  $\text{He}^3\text{-D}$  reaction). Combining (2.1) and (2.2) results in

$$\epsilon_{\text{out}} = \text{const. } NV, \quad (2.3)$$

and thus for a required value  $\epsilon_{\text{out}}$

$$V \propto N^{-1} \quad (2.4)$$

From eq. (2.4) follows that the plasma volume is inversely proportional to the plasma density. The plasma volume, however, strongly influences the size of the system and hence its weight, which for purpose of propulsion has to be chosen as small as possible. In order to minimize  $V$ , it is therefore desirable to choose  $N$  as large as possible, preferably by choosing the solid or liquid state for the thermonuclear material, which is the case realized in the micro-bomb. From eq. (2.4) follows furthermore, that the total mass of the plasma  $M \propto NV = \text{const.}$  The micro-bomb system is therefore the most compact system under all systems of equal plasma mass.

The principle of the propulsion system is drawn in Fig. 1. A sequence of micro-bombs are injected into the focus of the magnetic reflector and are ignited by the trigger apparatus. By means of M. H. D. energy conversion part of the released energy is fed back into the trigger apparatus to be recharged to trigger the following micro-explosion.

The principal problem of the micro-bomb concept is a fissionless trigger device. There are at least three known possible means for the ignition of a thermonuclear micro-explosion:

- 1) Ignition by the impact of a fast moving projectile.<sup>(5,6)</sup>
- 2) Ignition by an intense relativistic electron beam.<sup>(1,2)</sup>
- 3) Ignition by a pulsed laser beam.<sup>(7)</sup>

All three methods have in common the conversion of kinetic energy into heat by impact upon a thermonuclear target.

The idea of thermonuclear micro-bomb rocket propulsion was proposed by the author several years ago<sup>(8)</sup>; first, in connection with the possibility to perform the ignition by projectile impact and, second, with the prospect of ignition by an intense relativistic electron beam.<sup>(1,2)</sup>

Although very elegant in principle, the ignition by projectile impact is confronted with the yet unsolved problem of accelerating projectile of the required dimension to the needed velocity of  $10^8$  cm/sec required for thermonuclear ignition by an accelerator of reasonable dimensions. The third method of micro-bomb ignition, first proposed by Basov and Krokhin, is confronted with the problem of low laser efficiency. In contrast to laser beams, intense relativistic electron beams of several megajoule energy output in  $10^{-8}$  seconds can be produced with high efficiency. The second method of micro-bomb ignition on first sight therefore seems to be the most promising of all known methods.

Calculations for the T-D thermonuclear reaction indicate a trigger energy of at least  $10^6$  Joule<sup>(1,2)</sup>. An energy of this amount is difficult to produce by lasers in contrast to relativistic electron beams.

As the following arguments show, there is, however, a more important reason why for a high efficiency rocket propulsion system lasers are even at a much greater disadvantage in comparison to electron beams. In order to get a propulsion system of high efficiency the T-D thermonuclear reaction is unsuitable because 80% of the energy released in that reaction goes into kinetic energy of neutrons. These energetic neutrons cannot be reflected by the magnetic field and to a substantial part would be absorbed in the system structure which would be heated up. The situation is different for the  $\text{He}^3\text{-D}$  thermonuclear reaction where all the energy is released as kinetic energy of charged fusion products which can be deflected by a magnetic field. The  $\text{He}^3\text{-D}$  thermonuclear reaction, however, has a much higher ignition temperature than the T-D reaction, resulting in much higher energy inputs for ignition. It was therefore suggested previously<sup>(3,4)</sup>, that one might first trigger a smaller T-D explosion which then sets off a larger  $\text{He}^3\text{-D}$  reaction. In this mode of operation the T-D would only act as an ignition cap. More detailed studies, however, revealed that even then a large amount of undesirable energetic neutrons are produced by the T-D reaction which again would reduce the overall system efficiency. These studies suggest strongly that in order to have an efficient rocket propulsion system the exclusive use of the  $\text{He}^3\text{-D}$  reaction is highly desirable.

The higher ignition temperature of the  $\text{He}^3\text{-D}$  reaction has two important consequences. First, because of the higher ignition temperature more energy has to be deposited per unit volume of the  $\text{He}^3\text{-D}$  thermonuclear material. Second, the stopping ranges of

the charged fusion products depend on the plasma temperature  $T$  as  $T^{3/2}$ , resulting in much larger ranges of the charged fusion products for the  $\text{He}^3$ -D thermonuclear reaction as compared to the T-D reaction. This is very important since the minimum micro-bomb size and thus plasma volume to be heated up to thermonuclear temperatures in order to achieve detonation is determined by the ranges of the fusion products which have to dissipate their kinetic energy within the reacting volume, a condition necessary for detonation. This greatly increased plasma volume in combination with the increased thermonuclear ignition temperature would raise the ignition energy of a  $\text{He}^3$ -D micro-bomb beyond a technically feasible value.

The problem resulting from the increased range of the charged fusion products can be, however, greatly reduced by placing the  $\text{He}^3$ -D thermonuclear material inside a strong magnetic field. If this magnetic field is strong enough then the micro-bomb size is determined by the Larmor-radius of the charged fusion products rather than by the stopping range. In this case the strong magnetic field confines the charged fusion products within the thermonuclear reaction zone.

Calculations presented below show that this would require magnetic fields in excess of  $10^6$  gauss. Fields of this strength can be produced in a pulsed operation employing implosion type techniques. Such techniques, however, are highly unsuitable for a repetitive light weight propulsion system as it is envisioned here. It is for this reason in conjunction with the requirement to use the  $\text{He}^3$ -D thermonuclear reaction that laser ignition systems are excluded for micro-bomb rocket propulsion.

The situation is, however, quite different for intense relativistic electron beams. The reason for this is that in contrast to laser beams, intense relativistic electron beams not only can be produced with high efficiency and large total energy output, but also can carry and concentrate a strong self-magnetic field of many megagauss within the plasma target volume just during the right moment when the beam deposits its energy into the thermonuclear material. This strong self-magnetic beam field to be entrapped within the thermonuclear material will lead to three important effects: 1) it will quench the stopping range of the charged fusion products to within the reacting region which is required for the onset of a thermonuclear detonation, 2) reduce the electronic heat conduction losses, and 3) delay the thermal expansion of the thermonuclear plasma by magnetic confinement. The self-magnetic field of the beam can assume many megagauss within the thermonuclear target and the beam therefore forms a very strong, small "magnetic bottle". As a consequence of this strong self-magnetic beam field, much smaller energy inputs for igniting a thermonuclear reaction seem to be sufficient than previously considered omitting this effect. This reduction in the required trigger energy is, of course, of great importance for a mobile propulsion system since the system weight is strongly determined by the size of the trigger apparatus, the weight of which grows in proportion with the trigger energy.

Calculations taking the self-magnetic beam field into account suggest minimum energy inputs for ignition as low as  $10^4$  Joule in case of the T-D reaction.<sup>(9)</sup> In case of the  $\text{He}^3$ -D reaction, the

minimum energies are, of course, larger and of the order  $10^6$  Joule  
as will be shown below.

### 3. General description of the propulsion system in which a chain of thermonuclear micro-bombs is ignited by an intense relativistic electron beam

The overall functioning of the proposed propulsion system is outlined in the flow diagram Fig. 2. The micro-bombs, together with the propellant hydrogen to be added to the exhaust jet, are stored in S and injected into the focus of the magnetic mirror reflector. After being ignited by an electron beam from the electron beam generator E, the exploding micro-bombs are reflected by the magnetic field of M and are ejected. Part of the energy of the micro-bomb explosion is converted magnetohydrodynamically by the loop L. The electromagnetic energy drawn from the magnetohydrodynamic loop L is used to recharge the trigger apparatus for the subsequent beam shot. The trigger apparatus consists of the energy storage system and the electron beam generator.

As will be seen below, there are two ways to store the energy in the trigger apparatus. In the first way it is stored capacitively in a system known as a Marx generator. In this case the relatively high voltage from the magnetohydrodynamic loop L can be used directly to recharge the capacitor bank of the Marx generator. In the second way, which is the inductive analog to the capacitive Marx generator, a system of magnetic coils is magnetized by a strong current. In this case, the pulse coming from the magnetohydrodynamic loop has to be first transformed down to a lower voltage by a pulse transformer PT shown in Fig. 2. The energy is then stored intermediately in a unipolar machine U (see Fig. 2) from where a strong current is drawn to magnetize the coils of the inductive Marx generator.

The inductive Marx generator, like any other inductive energy storage system, has the advantage of a much smaller volume and hence weight as a capacitive system of the same total energy storage, a factor of great importance for propulsion because of its weight-saving property.

For the initial micro-bomb explosion the trigger energy has to be taken from a small nuclear reactor which may slowly store the required energy for the first electron beam into the trigger apparatus. The reactor has to be used also to initially energize the magnetic field coils of M and also to liquify the helium required to keep the coils of M superconducting. In addition to the reactor a radiator is required to dispose of the several sources of waste heat within the system. The largest source of waste heat is to be expected from the mirror reflector due to the absorption of Bremsstrahlung and of neutrons from undesired D-D reactions of the  $\text{He}^3$ -D thermonuclear micro-bomb material. Depending on the amount of hydrogen added to the exhaust jet, the specific impulse can reach values of the order  $10^5$  sec corresponding to an exhaust velocity of  $10^8$  cm/sec. If smaller exhaust velocities are sufficient or being desired this can be accomplished by adding hydrogen propellant to the exhaust jet. The same hydrogen, prior to its injection into the exhaust jet, can also be used to cool certain system components which otherwise would have to be cooled solely by the radiator.

The principle of the electron beam generator is shown in Fig. 3. Its main component consists of a coaxial strip line seen from the side. The upper and lower plate of the line are circular

disks with a proper dielectric substance in between, for example, mylar. The center of the two disks have a circular hole. The hole of the upper disk is closed by a concave field emission cathode. The upper disk is connected onto the field emission electrode by a triggered spark gap switch. After the upper disk is being charged up to a high negative potential, by either a capacitive or inductive high voltage Marx generator, the spark gap switch is closed and an electron beam is emitted from the field emission electrode. The beam is propagating down a discharge tube which is filled with a tenuous plasma, the density of which is properly chosen with an electron density less than the electron density in the beam. The reason for letting the electron beam propagate inside a tenuous plasma is that only in this case can it carry a strong self-magnetic field <sup>(10)</sup>. The discharge tube is permeated by an axial magnetic field, produced by external superconducting field coils. The axial field helps guiding and stabilizing the beam. The reason for selecting superconducting field coils to produce the guiding field is the required field strength of the order 100 kilogauss, which can be most economically produced in this way. The axial magnetic field forms at the end of the discharge tube a magnetic mirror into which the thermonuclear micro-bomb is placed. The purpose of the magnetic mirror is to concentrate the electron beam onto the thermonuclear material of the micro-bomb.

In the electrostatic Marx generator a series of capacitors is charged up in parallel to a high voltage and discharged in series over spark gap switches raising the voltage in proportion to the number of capacitors switched in series. Similarly in an inductive

Marx generator a system of inductances is magnetized in series by a large current and discharged in parallel raising the resulting current in proportion to the number of coils.

The principle of the electrostatic capacitive Marx generator is shown in Fig. 4. A charging voltage  $V$  drawn from the magneto-hydrodynamic loop charges in parallel a bank of  $N$  capacitors. After the voltage across one spark gap reaches a critical value it closes within a time of the order  $10^{-9}$  sec followed by a rapid closing of the remaining spark gaps, the voltages across of which are raised in proportion to the number of spark gaps already closed. As a result, the bank of  $N$  capacitors is switched from parallel into series such that their voltages add up to a resultant voltage  $NV$ . This resultant high voltage pulse is then applied to the coaxial strip line capacitor of the electron beam generator.

In the inductive Marx generator, explained in Fig. 5, a charging current  $I$  drawn from the unipolar machine first magnetizes  $N$  coils in series. After the inductive charging by the current  $I$  is completed the switches  $S$  are opened rapidly. As a result a surge voltage will occur at the spark gap switches closing them and thus switching the system of coils from series into parallel such that the currents carried by the  $N$  coils will add up to a resultant current  $NI$ . This resultant high current pulse is then transmitted to the strip line of the electron beam generator.

The most difficult problem in an inductive energy storage device is the need for the rapid switching-off a large current. In view of the much higher energy concentration of magnetically stored energy the feasibility of such a switch-off technique will

also be of great importance to other space oriented applications. A more detailed discussion of the inductive energy storage system together with a possible solution to the switch-off problem is given in the appendix.

The concentration of the electron beam onto the thermonuclear target is shown in Fig. 6. The target is placed in the center of a magnetic mirror and surrounded by a heavy Z material tamp. The externally applied magnetic mirror field focusses the electron beam onto the target. The large azimuthal self-magnetic beam field penetrates into the initially non-conducting target. Estimated values of the penetration depth are of the order of a centimeter. It turns out that the beam dissipation length within the target is expected to be of the same order of magnitude. After the beam has penetrated into the target, the target material will be heated to thermonuclear temperatures. The strong self-magnetic beam field confines the beam within the target to a very narrow channel further increasing the magnetic field within the beam. The large beam field gives rise to the three important effects mentioned before and which are expected to reduce substantially the trigger energy for the thermonuclear micro-explosion. The reacting region is not spherical, as in schemes based on target bombardment by laser beams, but rather cylindrical forming a relatively long cylinder of very small cross section. This optimal shape for the reacting region is, of course, a consequence of the symmetry associated with the strong self-magnetic beam field and has no counterpart in the laser method of thermonuclear micro-bomb ignition.

If the high  $Z$  material tamp is made of some reasonably good conducting material, which is the case for any metallic tamp, the Foucault currents induced in the tamp by the electron beam within the target plasma will help to stabilize the high current discharge.

Since the beam is projected into a tenuous plasma, in order to carry a large self-magnetic field, the tenuous background plasma may not have a sufficient electron density to carry the electron return current, going radially from the target towards the reflector wall which is the return conductor. In case the tenuous background-plasma is unable to carry the return current radially away from the target, the target would be charged up electrostatically. This, however, can be easily avoided by externally filling the space within the reflector, but not the space within the electron beam carrying tube, with a puff of gas just prior to the beam bombardment of the target. In this case the plasma in the tube carrying the electron beam will remain tenuous and the beam will still be able to carry a large self-magnetic field. In the reflector space, however, the plasma density will be sufficiently high to ensure a strong return current of electrons to flow radially towards the wall of the reflector.

Finally, we have to make a remark with regard to the system component of the magnetic mirror reflector. In order to protect the superconducting d.c. field coils from a rapidly induced current resulting from the magnetohydrodynamic interaction with the expanding fire ball of the thermonuclear micro-explosion, the coils have to be shielded against these currents by a layer of a

good metallic conductor, for example copper. The expanding fire ball, in pushing the magnetic field aside will then induce currents in this shield only. The magnetic field resulting from these induced currents will compensate the changed magnetic field inside the reflector space and screen the superconducting coils. The induced current  $\underline{j}$  in the shield together with the field  $\underline{H}$  of the superconducting field coils will result in an electromagnetic body force  $\frac{1}{c} \underline{j} \times \underline{H}$  driving the spacecraft. The magnetic field acts furthermore as a cushion by softening the impact of the detonation shock wave, which is a desirable side effect of the micro-bomb propulsion system.

#### 4. The Beam Model

The electron beam is projected into a tenuous hydrogen plasma prior to hitting the thermonuclear target forming the micro-bomb. We therefore consider an intense electron beam propagating through a tenuous plasma. We define as  $n_e$  the electron number density of the beam and as  $n_e^{(0)} = n_i^{(0)}$  the electron and ion number densities of the hydrogen plasma into which the beam is projected.

The electric and magnetic field  $\underline{E}$  and  $\underline{H}$  of the beam are then determined by Maxwell's equation ( $\rho$  electric charge density and  $\underline{j}$  electric current density in electrostatic c.g.s. units)

$$\text{div } \underline{E} = 4\pi\rho \quad (4.1)$$

$$\frac{4\pi}{c} \underline{j} = \text{curl } \underline{H} \quad (4.2)$$

where

$$\rho = -e (n_e - n_i^{(0)}) \quad (4.3)$$

$$\begin{aligned} \underline{j} &= -e (n_e \underline{v} - n_e^{(0)} \underline{v}_e) \\ &= -e (n_e \underline{v} - n_i^{(0)} \underline{v}_e) \end{aligned} \quad (4.4)$$

In eq. (4.4)  $\underline{v}$  is the electron drift velocity and  $\underline{v}_e$  the electron velocity of the induced upstreaming current in the background plasma. It should be mentioned that the induced upstreaming current will be

locally separated from the original current due to the charge separation within the background plasma resulting in the net charge density given by eq. (4.3). The average over the current density, however, is unaffected by this local charge separation and so is the magnetic field to be computed from eq. (4.4). We define a degree of electric charge neutralization  $\phi_e$  by

$$n_e - n_i^{(0)} = n_e (1 - \phi_e) \quad (4.5)$$

and similar of electric current neutralization  $\phi_m$  by

$$n_e \underline{v} - n_i^{(0)} \underline{v}_e = n_e (1 - \phi_m) \underline{v} \quad (4.6)$$

It is here assumed that the beam is only slightly neutralized both electrically and magnetically by a background plasma. This means that  $\phi_e \geq 1$  and  $\phi_m \geq 1$ . With these definitions we obtain for the electric and magnetic fields in cylindrical coordinates at the beam radius  $r$  under the assumption of uniform charge and current distribution

$$E_r = - 2\pi r n_e (1 - \phi_e) \quad (4.7)$$

$$H_\phi = - 2\pi r n_e (1 - \phi_m) \frac{v}{c} \quad (4.8)$$

Beam equilibrium requires that

$$E_r = \frac{v}{c} H_\phi \quad (4.9)$$

hence

$$\left(\frac{v}{c}\right)^2 = \frac{1-\phi_e}{1-\phi_m} \quad (4.10)$$

Since the l.h.s. of eq. (4.10) is smaller than one it follows that

$\phi_m < \phi_e$ . Eq. (4.10) permits to rewrite eq. (4.8):

$$H_\phi = -2\pi r n_e \sqrt{(1-\phi_e)(1-\phi_m)} \quad (4.11)$$

Equation (4.10) for the beam equilibrium is valid only for vanishing transverse beam temperature an assumption made for the sake of simplicity.

From the definition of  $\phi_e$  in eq. (4.5) it follows that

$$\phi_e = \frac{n_1^{(o)}}{n_e} \quad (4.12)$$

and similar from eq. (4.6)

$$\phi_m = \frac{v_e}{v} \frac{n_1^{(o)}}{n_e} = \frac{v_e}{v} \phi_e \quad (4.13)$$

If  $n_1^{(0)} = n_e$  one has  $\phi_e = \phi_m = 1$  and both electric and magnetic field of the beam will vanish. At the other hand if  $n_1^{(0)} \ll n_e$  it follows that  $\phi_e \ll 1$  and  $\phi_m \ll 1$ . Under these conditions the beam can carry very large selfelectric and selfmagnetic fields approaching the values

$$E_r = - 2\pi r n_e \quad (4.14)$$

$$H_\phi = - 2\pi r n_e \frac{v}{c} \quad (4.15)$$

The magnetic field given by eq. (4.15) can be of course also written in the form

$$\begin{aligned} H_\phi &= + \frac{2I}{rc} \\ &= + \frac{0.2I}{r} \end{aligned} \quad (4.16)$$

where  $I = - \pi r^2 n_e v$  is the total beam current, and the latter being valid if  $I$  is expressed in Ampere. If, for example,  $I = 10^6$  amp and  $r = 1$  cm, it follows that  $H_\phi = 2 \times 10^5$  gauss. The electric field in volt/cm is obtained by multiplying this number with the factor  $300 \frac{v}{c}$  resulting in  $E_r = 6 \times 10^8 \frac{v}{c}$  volt/cm.

The beam radius in the tenuous plasma is determined by the transverse beam temperature for a specified degree of beam neutralization. Since the degree of beam neutralization can be varied by properly choosing the density of the background plasma, any desired beam radius

is possible. The beam may develop instabilities unless an axial magnetic field  $H_z$  is applied along its direction of propagation which is of the same order as the beam magnetic field ( $H_z \sim H$ ). For beam fields of the order of  $2 \times 10^5$  gauss the axial field can be produced by superconducting magnetic field coils. The beam magnetic and electric field may also help to enhance stability by induced mirror forces in the wall of the discharge tube.

### 5. The He<sup>3</sup>-D Plasma Parameters

It is assumed that the thermonuclear material is composed of a stoichiometric mixture of liquid He<sup>3</sup> and D.

The atomic number density of liquid D is given by  $5 \times 10^{22} \text{ cm}^{-3}$ . With the density of liquid He<sup>3</sup> which is  $0.095 \text{ g/cm}^3$  one computes for the atomic number density  $1.89 \times 10^{22} \text{ cm}^{-3}$ . From these values one then computes as a stoichiometric mixture the equal atomic number densities  $N_1 = N_2 = N$  for He<sup>3</sup> and D to be  $N_1 = N_2 = N = 1.37 \times 10^{22} \text{ cm}^{-3}$ .

The total number of ions  $N_i$  in a fully ionized He<sup>3</sup>-D plasma is equal to  $N_i = 2N$  and the number of electrons  $N_e$  is equal to  $N_e = 3N$ , since for each He<sup>3</sup> ion one must count two electrons. The total number of particles is  $N_t = 5N$ . The plasma pressure is thus given by ( $k$  Boltzmann constant,  $T$  temperature °K)

$$p = 5NkT \quad (5.1)$$

The electron plasma frequency is given by ( $e$  and  $m$  electronic charge and mass)

$$\begin{aligned} \omega_p &= (12\pi Ne^2/m)^{1/2} \\ &= 1.14 \times 10^{16} \text{ sec}^{-1}. \end{aligned} \quad (5.3)$$

The kinetic plasma energy  $E$  per unit volume is given by

$$E = (15/2)NkT = (3/2) p \quad (5.2)$$

The density of the liquid  $\text{He}^3\text{-D}$  mixture and thus the plasma density is computed to be  $\rho = 0.115 \text{ g/cm}^3$ .

## 6. The penetration of the beam magnetic field into the thermonuclear target

Prior to the bombardment by the electron beam, the  $\text{He}^3\text{-D}$  target is in a liquid or solid state and has a low electrical conductivity. After the beam hits the target the temperature of the  $\text{He}^3\text{-D}$  material will rise, first by classical collision effects and later, after reaching the plasma state, in addition by collective effects due to beam-plasma instabilities. Because of the initial low value of the target conductivity the selfmagnetic beam field can penetrate into the target. The actual process of initial target heating is rather complex. For example, the sudden rise of the strong beam electric field in front of the beam should result in dielectric breakdown and streamer development within the cold target material. For the present study we omit this and other effects and will only make a rough estimate to support qualitatively our claim that the beam magnetic field will penetrate sufficiently deep into the target material.

We will assume that the initial target conductivity  $\sigma_0$  is of the order  $\sigma_0 \approx 10^9 \text{ sec}^{-1}$  which is a typical value for an organic semiconductor. It will be furthermore assumed that the conductivity remains constant until the target temperature  $T_0$  has reached the value  $T_0 \approx 10^4 \text{ }^\circ\text{K} \approx 1 \text{ ev}$ . For temperatures above  $T_0 \approx 10^4 \text{ }^\circ\text{K}$  it is assumed that the conductivity rises by many orders of magnitude due to the transformation of the target into a plasma. The penetration depth for the magnetic field of the electron beam can then be estimated by the skin depth  $\delta$  for the initial value of the conductivity and for the time  $\tau_D$  required to heat the target above the temperature

$$T_0 = 1 \text{ ev}$$

$$\delta = c (\tau_D / 4\pi\sigma_0)^{1/2} \quad (6.1)$$

The time  $\tau_D$  in eq. (6.1) is then the diffusion time for the magnetic field. The ignition of the thermonuclear target is achieved at a temperature of  $T_1 = 10^9 \text{ K} = 100 \text{ keV} = 10^5 T_0$ . The required pulse length of the electron beam is of the order  $10 \text{ ns} = 10^{-8} \text{ sec}$ . Under the assumption that the target is heated uniformly in time to the final temperature  $T = T_1 = 100 \text{ keV}$  the fraction of the time for the beam to heat the target to  $T = T_0 = 1 \text{ ev}$  is thus  $10^{-5} \times 10^{-8} = 10^{-13} \text{ sec}$ . It is this time in which the beam has to penetrate into the target and which has to be put in eq. (6.1) equal to  $\tau_D$ , resulting in  $\delta = 0.1 \text{ cm}$ . The actual penetration depth is likely to be larger since the heating of the target is non-uniform in time. The reason for this is that prior to the target ionization the heating is caused by single particle interactions which are much less efficient than the collective interactions becoming operational after the target has been transformed into a plasma. It therefore seems more likely that the time for preheating up to  $T_0 = 1 \text{ ev}$  will be 100 times larger that is  $10^{-10} \text{ sec}$ . If this is the case the penetration depth will be 10 times larger. It therefore seems not unreasonable to assume that the beam magnetic field can penetrate into the target material by the order of a cm. After the magnetic field has penetrated into the target and the target has been transformed into a plasma, the magnetic field will be henceforth frozen into the plasma. This fact is of great importance as we will see below.

Similarly, the beam electric field will penetrate into the target as long as the target conductivity is low, but after the target has been transformed into a plasma this electric field inside the plasma will decay within a time of  $\omega_p^{-1} \approx 10^{-16}$  sec. In contrast to the magnetic field, the electric field will not be frozen into the target after the target has been transformed into a plasma.

## 7. The interaction of the electron beam with the plasma target

After the target has been transformed into a plasma, single collisional interactions of the beam electrons with the target electrons or ions become less significant with increasing plasma temperature. As a result of this, it would be difficult to heat the target to thermonuclear temperatures on the basis of collisional beam dissipation alone. However, for intense electron beams interacting with a plasma target, the beam energy can be dissipated rapidly by collective beam plasma interactions primarily due to the electrostatic two-stream instability.

The growth rate for this collective interaction in the linearized beam plasma interaction theory and if collisions are neglected is given by<sup>(11)</sup>

$$\sigma = 0.55 (n_2/n_1)^{1/3} \omega_p \gamma^{-1/3}, \quad (7.1)$$

where

$$\gamma \equiv (1-\beta^2)^{-1/2}$$

and where  $n_1$  and  $n_2$  are the electron number densities in the target and the beam. The stopping range of an electron beam in the linearized approximation is then given by

$$\begin{aligned} \lambda_D &= (v/\sigma) \ln (F_0/F_{th}) \\ &= (c/\sigma) \ln (F_0/F_{th}) \end{aligned} \quad (7.2)$$

The approximation in which  $v$  is replaced by  $c$  is valid for electron energies above 100 keV. In eq. (7.2)  $E_0$  is the initial electron energy the  $E_{th}$  the thermal electron energy at a given target temperature. Expressing the electron number density in the beam by the electric current density  $j$  one has

$$n_2 = 2.1 \times 10^8 j, \quad (7.3)$$

where  $j$  is expressed in A/cm<sup>2</sup>. For a beam current  $I$  [A] and a beam radius  $r_0$  [cm] one has  $j = I/\pi r_0^2$  hence

$$n_2 = 6.7 \times 10^7 I/r_0^2, \quad (7.4)$$

and thus from eq. (7.2)

$$\lambda_D = 0.9 r_0^{2/3} (\gamma/I)^{1/3} \log (E_0/E_{th}). \quad (7.5)$$

If, for example,  $r_0 = 10^{-1}$  cm,  $I = 10^6$  Ampere,  $E_0 = 1$  MeV,  $E_{th} = 100$  keV one then has  $\lambda_D \approx 2 \times 10^{-3}$  cm. From eq. (7.5) it follows that the stopping range decreases with increasing beam current, a fortunate behavior since large beam currents are required for our purpose anyway. The actual stopping length is larger due to a slowing down of the wave-growth by nonlinear effects.

Computer experiments<sup>(12)</sup> taking the nonlinear growth rate of the two-stream instability into account indicate that the actual growth rate can be  $10^3$  times smaller than the ones computed from eq. (7.1) and hence the stopping distances larger by a factor  $10^3$ .

Although the special assumptions under which these results were obtained do not apply to our case, it should be stated that other authors in unpublished communications and under different assumptions (J. M. Dawson, A. T. Lin and J. E. Rowe) have arrived at similar conclusions.

In the absence of any other data, we will therefore assume that the results of these computer experiments also apply to our situation and henceforth take as the beam stopping length a value which is  $10^3$  times larger than the one computed from eq. (7.5) with the result

$$\lambda_D = 10^3 r_0^{2/3} (\gamma/I)^{1/3} \log (E_0/E_{th}) \quad . \quad (7.6)$$

For the given numerical example above this would result in  $\lambda_D = 1$  cm.

8. The ignition of thermonuclear reactions by the magnetically focussed electron beam within the target plasma

In order to ignite a small thermonuclear explosion the kinetic energy of the charged fusion products has to be dissipated within the reacting volume. If the stopping range of the fusion products is  $\lambda$  then the probability  $P$  for the fusion products to be stopped within a volume  $V$  is given by

$$P = \frac{1}{\lambda V} \int \int \frac{e^{-|r_1 - r_2|/\lambda}}{4\pi|r_1 - r_2|^2} dr_1 dr_2 \quad (8.1)$$

If the volume is cylindrical in shape with a radius  $r_0$ , as it is the case for an electron beam heating up to a thermonuclear temperatures a plasma cylinder of radius  $r_0$ , and if  $x \equiv r_0/\lambda$  one obtains from eq. (8.1)

$$P(x) = 1 - 2x \int_x^\infty I_1(x) K_1(x) dx / x^2 \quad (8.2)$$

In eq. (8.2)  $I_1$  and  $K_1$  are modified Bessel functions conventionally defined. A useful approximation to eq. (8.2) is given by

$$P(x) \approx 1 - \frac{0.183}{x^3} [2.56x^2 - 1/2 + (1/2 + 1.6x) e^{-3.2x}] \quad (8.3)$$

$P(x)$  approaches 1 for  $x = 1$ , resp.  $r_0 = \lambda$ , implying that the ignition of a small thermonuclear explosion will require that  $r_0 \approx \lambda$ . There are two charged fusion products of the  $\text{He}^3\text{-D}$  thermonuclear reaction, an  $\alpha$  particle and a proton with kinetic energies of

$$\alpha - \text{particle: } \epsilon_1 = 3.66 \text{ MeV} = 5.86 \times 10^{-6} \text{ erg}$$

$$\text{proton: } \epsilon_2 = 14.65 \text{ MeV} = 2.34 \times 10^{-5} \text{ erg}$$

with velocities of

$$\alpha - \text{particle: } V_1 = 1.3 \times 10^9 \text{ cm/sec,}$$

$$\text{proton: } V_2 = 5.1 \times 10^9 \text{ cm/sec.}$$

with these values one can compute the stopping ranges  $\lambda_1$  and  $\lambda_2$  of the charged fusion products in a plasma of temperature  $T$  (see for example reference 13). The result of these calculations is ( $T$  in keV):

$\alpha - \text{particle}$

$$\lambda_1 = 3.96 \times 10^{-2} T^{3/2} \quad (8.4)$$

proton:

$$\lambda_2 = 1.53 \times 10^{-1} T^{3/2} \quad (8.5)$$

If for example  $T = 100$  keV one has  $\lambda_1 = 28$  cm and  $\lambda_2 = 153$  cm. For  $T = 30$  keV one would have instead  $\lambda_1 = 5.4$  cm,  $\lambda_2 = 29$  cm. Since the ignition temperature for the  $\text{He}^3\text{-D}$  reaction is above 30 keV, the ranges of the charged fusion products are thus at least several cm. In order to ignite a thermonuclear detonation the radius  $r$  of the plasma sphere to be heated up to the ignition temperature  $T$  must be of the order of the fusion product range. In this case the probability  $P(x)$  is close to one and the kinetic energy of the fusion products is dissipated within the reacting plasma sphere. Assuming  $r \approx \lambda$  and  $\lambda = 3$  cm, furthermore  $T = 100$  keV one thus would obtain for the required minimum energy input required for ignition,  $E_{in} = (15/2)NkT(4\pi/3)r^3 \approx 2 \times 10^{18}$  erg  $= 2 \times 10^{11}$  Joule  $\sim 40$  tons TNT. This value for the minimum energy input is so large that for practical reasons it can neither be produced by a laser or electron beam.

The energy input for ignition, however, can be substantially suppressed by applying a strong magnetic field to the thermonuclear target material. The strength of the magnetic field is determined from the condition  $r \approx r_L$  where  $r_L$  is the Larmor-radius of the charged fusion product. If this condition is satisfied the fusion products will dissipate their energy within a distance  $\approx r_L$  from their origin rather than within the distance of the stopping length in the absence of a magnetic field. One computes for the Larmor-radius of the fusion products the following values

$$\alpha \text{ - particle: } r_L^{(1)} = 2.78 \times 10^5 / H \quad (8.6)$$

$$\text{proton: } r_L^{(2)} = 5.56 \times 10^5 / H \quad (8.7)$$

In (8.6) and (8.7)  $r_L$  is given in cm and the magnetic field  $H$  in gauss. For a thermonuclear target size  $r \leq 0.1$  cm it thus follows that magnetic fields are required in excess of  $5 \times 10^6$  gauss. The energy required for ignition would be then reduced to a value of  $E_{in} = 10^{14}$  erg =  $10^7$  Joule.

In the electron beam method of bombarding a thermonuclear target, the required strong magnetic fields have not to be produced separately but are readily available due to the strong electric currents of these beams and are expected to reach values far in excess of the value given above, thus reducing the minimum energy input substantially below the amount which was estimated for a field of  $5 \times 10^6$  gauss. In the laser method, however, the magnetic field has to be produced independently by some external means such as imploding foils. Although magnetic fields of several megagauss can be produced this way, the method seems highly impractical for a rapidly pulsating energy-producing system. Furthermore, since the attainable magnetic fields are here limited to a few megagauss, the required laser energies of  $10^7$  Joule are far beyond present day technology.

To incorporate the concept of a magnetically reduced fusion-product-range into the expression for the probability  $P(x)$  given above one can make the reasonably good approximation

$$\lambda \approx 2r_L \quad , \quad (8.8)$$

since the dissipation in the presence of a strong magnetic field takes place over a length which in fact is equal to twice the value of the Larmor-radius.

After the beam and its strong self-magnetic field have penetrated the target and raised it to thermonuclear temperatures, it shall be assumed that the pressure of the self-magnetic beam field is balanced by the thermodynamic plasma pressure. This particular assumption seems somewhat ambiguous. We justify it on the ground that it is energetically favored. The situation is then similar as for the ordinary pinch discharge with the important distinction that here the current is drawn from an externally applied beam of electrons and that furthermore the boundary of the discharge is not a vacuum but a mantle of cooler plasma. At the beam radius it is thus assumed that the usual pinch condition

$$\frac{H^2}{8\pi} = p = 5NkT \quad (8.9)$$

holds. If the beam heats the  $\text{He}^3\text{-D}$  target up to thermonuclear temperatures,  $T$  is of the order of  $10^9^\circ\text{K}$ . Expressing  $T$  in units of  $\text{keV}$  and putting  $N = 1.37 \times 10^{22} \text{cm}^{-3}$  one obtains from eq. (8.9)

$$H = 1.66 \times 10^7 T^{1/2} \quad (\text{gauss}) \quad (8.10)$$

For  $T = 100 \text{ keV}$  one would obtain  $H = 1.6 \times 10^8 \text{ gauss}$ . As already pointed out, this strong self-magnetic field of the beam will serve three important purposes: First, it will quench the range of the charged fusion products within the reacting volume thus facilitating the ignition of a small thermonuclear reaction; second, it will reduce the heat conduction losses radially; and third, it will reduce the radial expansion losses by magnetic confinement. It is, however,

not known how much it will affect the stability of the plasma column. But since the discharge column is surrounded by a dense plasma, which enhances the stability of the discharge column, it is hoped that the characteristic times for instability are at least as long and in the order of magnitude as the discharge times so that the beam plasma instabilities will be relatively unimportant.

By combining eq. (8.10) with the expression for the magnetic field at the beam radius  $r_0$ ,  $H = 0.2 I/r_0$ , one obtains an expression for the beam radius within the thermonuclear material

$$r_0 = 1.2 \times 10^{-8} I/T^{1/2} \quad (\text{cm}) \quad (8.11)$$

Assuming, for example, that  $T = 100$  keV,  $I = 10^6$  Amp one obtains  $r_0 = 1.2 \times 10^{-3}$  cm.

The energy production of the  $\text{He}^3\text{-D}$  thermonuclear reaction can be expressed as follows ( $T$  in keV)

$$\epsilon_f = 4.33 \times 10^{-12} N^2 T^{-2/3} \exp(-32.6 T^{-1/3}) \cdot \epsilon \left[ \text{erg/cm}^3 \text{sec} \right] \quad (8.12)$$

where  $\epsilon = \epsilon_1 + \epsilon_2$  is the total energy delivered by the two fusion products each of which delivers the energy  $\epsilon_1$  resp.  $\epsilon_2$ .

If the probability for these two fusion products, i.e. the  $\alpha$  particle and the proton, to dissipate their kinetic energy within a cylinder of radius  $r_0$  be  $P_1$  and  $P_2$ , one then has clearly

$$P_1 = P(x_1) \quad , \quad (8.13)$$

$$P_2 = P(x_2) \quad , \quad (8.14)$$

where

$$x_1 = r_0/\lambda_1, \quad (8.15)$$

and

$$x_2 = r_0/\lambda_2. \quad (8.16)$$

Defining  $x = x_1$  and  $x_2 = ax_1 = ax$ , where  $a = \lambda_1/\lambda_2$ , one has

$$P_1 = P(x), \quad (8.17)$$

and

$$P_2 = P(ax). \quad (8.18)$$

Furthermore, since the fusion product ranges are determined by the magnetic field one has

$$\lambda_1 = 2r_L^{(1)} = 5.56 \times 10^5/H, \quad (8.19)$$

hence

$$x = r_0/\lambda_1 = 1.8 \times 10^{-6} H r_0, \quad (8.20)$$

and

$$a = \lambda_1/\lambda_2 = 0.5 \quad . \quad (8.21)$$

Since the magnetic deflection of the charged particle is determined by the maximum magnetic field within the discharge channel we can with good approximation insert into eq. (8.20) the maximum value of  $H$  given by eq. (8.10) resulting in

$$x = 3.0 \times 10^1 r_o T^{1/2} \quad . \quad (8.22)$$

The thermonuclear energy being dissipated within the beam volume per unit beam length in the discharge channel is then given by

$$E_f = \frac{\epsilon_f}{\epsilon} (\epsilon_1 P_1 + \epsilon_2 P_2) \pi r_o^2 \quad , \quad (8.23)$$

or by introducing eq. (8.12), (8.17), (8.18) and the values for  $\epsilon_1$  and  $\epsilon_2$  results in

$$E_f = 2.54 \times 10^{-17} N^2 T^{-2/3} \exp(-32.6 T^{-1/3})$$

$$\left( P(x) + 4.0 P(0.5x) \right) \pi r_o^2 \text{ [erg/cm sec]} \quad . \quad (8.24)$$

The energy losses result from Bremsstrahlung and heat conduction. The Bremsstrahlungs-losses can be calculated<sup>(14)</sup> by using the plasma parameters for the He<sup>3</sup>-D plasma given above. The Bremsstrahlungs-losses per unit beam length are then given by

$$E_r = 8.02 \times 10^{-23} N^2 T^{1/2} \pi r_o^2 \text{ [erg/cm sec]} \quad . \quad (8.25)$$

The heat conduction losses from a cylindrical discharge column in the presence of a strong perpendicular magnetic field have been calculated for a hydrogen-plasma<sup>(15)</sup>. A corresponding calculation for a He<sup>3</sup>-D plasma has not been published in the literature. In the absence of such a calculation and also since the heat conduction losses in the presence of a strong magnetic field are not the predominant loss mechanism we will assume that the same expression for these losses as in the case of a hydrogen-plasma also holds for a He<sup>3</sup>-D plasma, at least approximately. These losses per unit beam length are then given by

$$E_c = 2 \times 10^9 (p/H)^{1.4} \text{ [erg/cm sec]} \quad (8.26)$$

In eq. (8.26)  $p = 5NkT$  is the plasma pressure. With the help of eq. (8.9) one has

$$\frac{p}{H} = \left( \frac{5NkT}{8\pi} \right)^{1/2} \quad (8.27)$$

or for  $N = 1.37 \times 10^{22} \text{ cm}^{-3}$  and  $T$  given in keV

$$\frac{p}{H} = 2.08 \times 10^6 T^{1/2} \quad (8.28)$$

and hence

$$E_c = 1.9 \times 10^{18} T^{0.7} \text{ [erg/cm sec]} \quad (8.29)$$

The ignition condition for a small thermonuclear explosion is now obtained by the energy balance of the energy gain supplied by the fusion products to the discharge channel against the losses by Bremsstrahlung and heat conduction. One thus has

$$E_f = E_r + E_c, \quad (8.30)$$

or after inserting from eq. (8.24), (8.25) and (8.29) there results

$$\begin{aligned} T^{-2/3} \exp(-32.6 T^{-1/3}) (P(x) + 4.0 P(0.5x)) \\ = 3.16 \times 10^{-6} T^{1/2} + 1.27 \times 10^{-10} T^{0.7} r_0^{-2} \end{aligned} \quad (8.31)$$

For sufficiently large values of  $r_0$  eq. (8.31) has two roots  $T = T(r_0)$ , the lower of which defines the ignition temperature. When  $r_0 \rightarrow \infty$  then also  $x \rightarrow \infty$  and  $P \rightarrow 1$ . In this case eq. (8.31) assumes the simplified form

$$1.58 \times 10^6 T^{-7/6} \exp(-32.6 T^{-1/3}) = 1 \quad (8.32)$$

Eq. (8.32) results from the energy balance of all the energy supplied by the fusion products against the losses by Bremsstrahlung. Eq. (8.32) is therefore obtained by equating (8.24) with (8.25). This also shows that eq. (8.32) is valid for all densities  $N$ . Eq. (8.32) has a lower root at  $T = T(\infty) = 32.6$  keV, valid for an infinitely extended thermonuclear plasma. The ignition temperatures for a finite plasma radius obtained from eq. (8.31) are necessarily larger.

The input energy per unit beam length required for ignition is given by

$$\begin{aligned} E_0(r_0) &= (15/2) NkT\pi r_0^2 \\ &= 5.17 \times 10^{14} r_0^2 T \end{aligned} \quad (8.33)$$

The computed value of the ignition temperature then also determines the value of the magnetic field given by eq. (8.10) and the required beam current

$$\begin{aligned} I &= 5Hr_0 \\ &= 0.835 \times 10^8 T^{1/2} r_0 \end{aligned} \quad (8.34)$$

Computed values for the energy input per unit beam length together with the corresponding ignition temperature have been plotted in Fig. 7a.

The energy for ignition has to be supplied to the target material in a time which is shorter than the radiation loss time given by

$$\tau_R = 1.1 \times 10^{-8} T^{1/2} \quad [\text{sec}] \quad (8.35)$$

For  $T = 100$  keV,  $\tau_R \approx 10^{-7}$  sec. Since the computed ignition temperatures are of the same order it follows that the ignition energy has to be supplied in a time less than 100 nsec.

The remarkable result displayed in Fig. 7a are the rather small energy input values required for the ignition of the  $\text{He}^3\text{-D}$  thermonuclear micro-explosion, which would be impossible without the effect of the large beam magnetic field. Since the laser method of target ignition has to work without a self-magnetic beam field the advantage of the electron beam method over the laser method has thus been proved to be very distinct.

It has to be emphasized that the fusion reaction will proceed into a thermonuclear deflagration only as long as the electron beam is switched on, requiring thus in general more energy than the value given by eq. (8.33). A simple estimate for the increased amount of input energy required for such a thermonuclear deflagration to develop can be obtained easily. From Fig. 7a, it can be seen that the energy input will assume a minimum at a beam radius of  $r_0 \approx 5 \times 10^{-3}$  cm. The stopping length  $\lambda_D$  of the beam was estimated above to be of the order one cm. It thus follows an initial energy input for ignition of  $E_{\text{ign}} = E_0 \lambda_D = 1.7 \times 10^{12}$  erg =  $1.7 \times 10^5$  Joule. We will demand that the deflagration sets free an energy which is  $10^4$  times larger i.e.  $1.7 \times 10^{16}$  erg which has the equivalent of the explosive power of  $\sim 0.3$  ton of TNT. This means that the thermonuclear deflagration propagating radially has to reach a radius 100 times larger than the initial radius of  $r_0 = 5 \times 10^{-3}$  cm, that is it has to reach the radius  $r_1 = 0.5$  cm. In this case the reacting volume has increased  $10^4$  fold and so has the energy output. The ratio of the Larmor-radius of the fusion products to the radius of the reacting cylinder  $r$  is a constant since  $r_L \propto 1/H$  and  $H \propto I/r$ . The quenching of the fusion products

to the reacting region is only affected by the ratio  $r/r_L \approx 1$  and thus independent of  $r$  itself. The probabilities  $P(x)$  will be therefore constant during the radial development of the deflagration as long as the electron beam current is being kept constant.

The time for the deflagration to propagate to the radius  $r_1 = 10^2 r_0$  can be estimated by the Lawson criterion, which for the  $\text{He}^3\text{-D}$  reaction and under the assumption of  $T = 130$  keV (at the minimum of  $E_0$  in Fig. 7a and  $N$  as defined above) is given by

$$N\tau_L \geq 1.9 \times 10^{14} \text{ sec cm}^{-3}. \quad (8.36)$$

For  $N = 1.37 \times 10^{22} \text{ cm}^{-3}$  one has  $r_L = 1.4 \times 10^{-8} \text{ sec}$ . The time to increase the energy by a factor  $10^4$  is then given by  $t = (4/\log 2)r_L = 13.4\tau_L = 1.85 \times 10^{-7} \text{ sec}$ . The beam would thus have to be sustained for about 200 nsec.

From Fig. 7a follows that the energy input assumes a minimum at a radius for which the ignition temperature is  $T = 130$  keV. The required magnetic field corresponding to this temperature is obtained from eq. (8.10) with the result  $H = 1.9 \times 10^8$  gauss. The beam current is then given by eq. (8.34) and is  $I = 4.75 \times 10^6$  amp. The beam radius according to eq. (8.11) is  $r_0 = 5 \times 10^{-3} \text{ cm}$  which is consistent with the value at which  $E_0$  is a minimum. Let us assume that the ignition energy is deposited by the beam in the time  $\tau_{in} = 30 \text{ ns} < \tau_R$ . With  $E_{ign} = 1.7 \times 10^5$  joule,  $I$  the beam current in Ampere and  $V$  beam voltage in Volt, we have

$$E_{ign} = IV\tau_{in} \quad (8.37)$$

From eq. (8.37) one obtains for the required beam voltage  $V = 1.2 \times 10^6$  volt.

To support the thermonuclear deflagration would require that the electron beam is sustained for about  $2 \times 10^{-7}$  sec, which would require an energy of  $\approx 10^6$  joule.

We would like to discuss briefly how our results are changed if for the thermonuclear material a gas target is used instead of a solid or liquid. The gas target has a number of advantages over a solid target. One such advantage is that the gas could be injected into the target area by opening a valve letting the gas flow into the reaction chamber prior to the bombardment by the electron beam. This mode of operation may be more expedient since it would not require a large storage of solid targets. It was suggested to the author by Dr. von Braun that an even more expedient way may be the rapid pulsed injection of the thermonuclear material as liquid jet into the reaction chamber. In such a case the jet would evaporate into a dense gas target prior to reaching the center of the reaction chamber. A second advantage of going to a gas target is in the increase of the radiation loss time which is proportional to  $N^{-1/2}$ , implying that an electron beam of less power and longer pulse length can be used. If, for example, the target density is reduced by a factor 100 this would mean that the pulse length could be increased from  $3 \times 10^{-8}$  sec to  $3 \times 10^{-7}$  sec.

If the atomic number density in the gas target is  $n$  we put

$$N = \alpha n_0, \quad \alpha \leq 1 \quad (8.38)$$

where  $N_0 = 1.37 \times 10^{22} \text{ cm}^{-3}$  is the number density at liquid resp. solid target densities. With this definition eq. (8.31) has to be modified as follows

$$\begin{aligned} & T^{-2/3} \exp(-32.6 T^{-1/3}) (P(x) + 4.0 P(0.5x)) \\ & = 3.16 \times 10^{-6} T^{1/2} + 1.27 \times 10^{-10} T^{0.7} \alpha^{-1.3} r_0^{-2} \end{aligned} \quad (8.39)$$

where

$$x = 3.0 \times 10^1 r_0 \alpha^{1/2} T^{1/2}, \quad (8.40)$$

$$E_0 = 5.17 \times 10^{14} \alpha r_0^2 T, \quad (8.41)$$

$$H = 1.66 \times 10^7 \alpha^{1/2} T^{1/2}, \quad (8.42)$$

$$r_0 = 1.2 \times 10^{-8} I / \alpha^{1/2} T^{1/2}, \quad (8.43)$$

$$\tau_L > 1.4 \times 10^{-8} \alpha^{-1}, \quad (8.44)$$

$$\tau_R = 1.1 \times 10^{-8} \alpha^{-1} T^{1/2}, \quad (8.45)$$

$$\lambda_D = 10^3 r_0^{2/3} (\gamma/I)^{1/3} \log(E_0/E_{th}) \alpha^{-1/3}. \quad (8.46)$$

Fig. 7b and 7c display computed values of  $E_0$  and  $T$  for  $\alpha = 0.1$  and  $\alpha = 0.01$ .

Let us discuss how our results are affected by going to  $\alpha = 0.01$ . First, the minimum energy input there is at  $r_0 = 0.09$  cm with  $E_0 = 5.2 \times 10^{12}$  erg/cm and  $T = 125$  keV. From eq. (8.42) then follows  $H = 1.86 \times 10^7$  gauss and furthermore from eq. (8.34)  $I = 8.4 \times 10^6$  Ampere. From eq. (8.43) follows  $r_0 = 0.09$  cm which again is consistent with the value  $r_0$  for the minimum energy input. The Lawson time is  $\tau_L = 1.4 \times 10^{-6}$  sec and the radiation loss time  $\tau_R = 10^{-5}$  sec. The beam stopping length is increased by the factor  $\alpha^{-1/3} = 4.6$ . The energy input for ignition would be thus  $E_{ign} = E_0 \lambda_D = 2.4 \times 10^{13}$  erg =  $2.4 \times 10^6$  joule. With  $\tau_{in} = 10^{-6}$  sec  $< \tau_R$  it follows that  $V \approx 300$  kvolt.

A system with a gas target given by  $\alpha = 0.01$  resembles more the conditions which are prevalent in the plasma focus. The initiation of a thermonuclear deflagration is here probably more difficult to achieve and we will omit a discussion of it.

### 9. The interaction of the expanding fire ball with the superconducting magnetic reflector

From the requirement that the expanding fire ball be reflected by the magnetic field, a condition for the dimension of the magnetic mirror for a given magnetic field strength can be derived. If the reflector has a hemispherical shape, roughly half of the energy of the exploding fire ball will have to be absorbed by the magnetic field. In this way, the exploding fire ball produces a cavity in the magnetic field of the hemisphere corresponding to a magnetic energy of

$$E_H = \frac{2\pi}{3} R^3 \frac{H^2}{8\pi}, \quad (9.1)$$

where  $R$  is the radius of the hemisphere. This magnetic energy has to be equated to the explosion energy of the micro-bomb in order to reflect the fire ball. If this energy is given by  $E_B$  one has from eq. (9.1)

$$R = (12E_B)^{1/3} H^{-2/3}. \quad (9.2)$$

Let us assume that  $E_B = 10^{16}$  erg  $\approx$  100 kg TNT,  $H = 10^5$  gauss (which is attainable with superconductors), one thus would have  $R \approx 230$  cm, representing a very reasonable dimension.

The formation of the magnetic cavity is accompanied by induced eddy currents in the external field coils. Since high field superconductors do not operate at rapidly varying magnetic fields, they have to be protected by a shield of an ordinary conductor placed on

the inner side of the concave magnetic mirror reflector. The skin depth  $\delta$  in this conductor is given by

$$\delta = c(\tau/4\pi\sigma)^{1/2}, \quad (9.3)$$

where  $\tau$  is the time during which the plasma ball expands to the radius  $R$  and  $\sigma$  is the electrical conductivity of the shielding material. Since  $\tau = R/v_0$  where  $v$  is the expansion velocity of the fire ball, we have

$$\delta = c(R/4\pi\sigma v)^{1/2}. \quad (9.4)$$

If one puts, for example,  $R \approx 200$  cm,  $\sigma \approx 10^{18}$  sec<sup>-1</sup>,  $v \approx 10^7$  cm/sec one then obtains  $\delta \approx 4 \times 10^{-2}$  cm, which is very small. Thus an effective shielding could be obtained even with materials of relatively low electrical conductivity, such as steel or graphite.

The currents induced in the shield will interact with the magnetic field to produce an electromagnetic body force  $(1/c)\underline{j} \times \underline{H}$  ( $\underline{j}$  being the current density), which serves to propel the spacecraft, which is rigidly connected to the mirror. Except for regions near the mirror-point, there will be no ablation resulting from direct contact with the fire ball.

From Ampere's law it follows that the total current in the field coils is of the order of  $I \approx HR \approx 10^7$  Amp. The maximum current density in superconductors of the second kind is  $j_{\max} \approx 10^4$  A/cm<sup>2</sup>. If the coils extend radially by the amount  $\Delta R$ , the surface area through which the current can flow is of the order

$R\Delta R$ , hence

$$j_{\max} R\Delta R = I = HR$$

Therefore we have

$$\Delta R = H/j_{\max}$$

$$\approx 10 \text{ cm}$$

The mass of the reflector is of the order  $M \approx 2\pi\rho_m R^2 \Delta R$ , with  $\rho_m \approx 4\text{gm/cm}^3$  the overall density of the reflector material. For  $R \approx 200 \text{ cm}$ , one has  $M \approx 10^7 \text{ gm} \approx 10 \text{ tons}$ .

Finally, the shape of the magnetic mirror must be optimized. If the magnetic mirror is shaped more like the pusher plate in the Orion propulsion concept, i.e., less concave than is indicated, for example, in Fig. 3, the heating of the mirror will be reduced at the expense of rocket efficiency. This results from the increased transverse component of the jet velocity. Here again, this important trade-off effect has to be studied with regard to its optimal conditions.

11. The recharging of the trigger apparatus by magnetohydrodynamic energy conversion drawn from the expanding fire ball.

The magnetic coupling with external conductors can be used to draw energy from the thermonuclear explosion by magnetohydrodynamic power conversion, for recharging the trigger apparatus. For this we consider a coaxial conducting loop with an axial thickness  $D$  placed a distance  $\rho$  from the axis of rotation of the mirror. (See Fig. 3.) The induced voltage in a single loop will be (electrostatic units)

$$\begin{aligned} V &= \frac{1}{c} \pi \rho^2 \frac{\partial H}{\partial t} \\ &= \frac{1}{c} \frac{H \pi \rho^2}{\tau} \\ &\approx \pi \rho H v_0 / c \end{aligned} \quad (11.1)$$

where  $\tau \approx \rho/v$  is the characteristic time for the change of the magnetic field. Assuming  $\pi \rho \approx 100$  cm,  $H = 10^5$  gauss,  $v/c = 3 \times 10^{-4}$ , one has  $V = 3 \times 10^3$  esu  $\approx 10^6$  volts. In order to estimate the required thickness  $D$ , we request that an energy equal to  $E_0 = 10^6$  joules be drawn from the explosion required to trigger the next micro-explosion. The magnetic energy in a disk with radius  $\rho$  and thickness  $D$  is given by

$$\begin{aligned} E_M &= \pi \rho^2 D \frac{H^2}{8\pi} \\ &= \rho^2 D H^2 / 8 \end{aligned} \quad (11.2)$$

Putting  $E_M = E_0$ , we have

$$D = 8E_0/\rho^2 H^2 \quad (11.3)$$

For  $E_0 = 10^{13}$  ergs,  $\rho = 30$  cm,  $H = 10^5$  gauss, we have  $D = 1$  cm.

The high voltage pulse is accompanied by a high current. The following condition must hold in between the energy of the pulse

$E_0$  (Joule) the voltage  $V$ , the current  $I$  and the pulse time  $\tau = \rho/v$ :  $E_0 = IV\rho/v$ . With the above given values ( $E_0 = 10^6$  Joule,  $V = 10^6$  volt,  $\rho = 30$  cm  $v = 10^7$  cm/sec) one has  $I = 3 \times 10^5$  amp.

In case of a capacitive energy storage the voltage is high enough to directly charge the capacitor bank. In case of the inductive energy storage the energy from the high voltage pulse is first stored onto a unipolar machine. Because of the high voltage of  $10^6$  volt drawn from the magnetohydrodynamic loop, a pulse transformer reducing the voltage has to be put in between the M.H.D. loop and the unipolar machine as shown in Fig. 2.

The unipolar machine is essentially an electromagnetic flywheel and the energy stored in it is equal to the mechanical rotational energy of the rotor. If  $v_{\max}$  is the maximum radial velocity of the flywheel and  $M$  its mass, then the stored energy is given by

$$E_s = \frac{M}{4} v_{\max}^2 \quad (11.4)$$

Assume  $v_{\max} = 10^4$  cm/sec and  $E_s = E_0 = 10^6$  Joule =  $10^{13}$  erg one has  $M = 0.4$  tons. In the unipolar machine a strong axial magnetic field is applied to

the rotor resulting in a radial electric field  $E_r = (v/c) H$ .

Putting  $v = \omega r$ , where  $\omega$  is the angular velocity of the rotor, one obtains for the voltage  $V$  (in electrostatic cgs units),

$$V = \int_0^R E_r dr = \frac{H\omega}{c} \int_0^R r dr$$

$$= \frac{1}{2} \frac{H v_{\max}}{c} R \quad (11.5)$$

Assume a rotor with a specific gravity of  $3 \text{ gm/cm}^3$  and a diameter equal to its length, it then follows that  $R = 28 \text{ cm}$ . Assume furthermore that  $H = 10^5$  gauss one then has  $V = 140 \text{ volt}$ .

In order to provide the trigger energy for the first micro-bomb explosion a small nuclear reactor of 100kW power would be sufficient, which could charge the trigger apparatus in about 10 seconds. A reactor employing thermionic energy conversion would be especially suitable for this purpose. Since thermionic energy conversion produces a low d.c. voltage, an alternator and transformer would be needed in the capacitive energy storage system. In the inductive storage system the d.c. current drawn from the thermionic converter could be directly fed into the unipolar machine. The small reactor power plant can be also used to provide the energy to initially magnetize the superconducting coils and to operate the refrigerator for the liquid helium cryostat.

## 12. The mass of the trigger apparatus

If a capacitor bank is used for the trigger apparatus it will constitute the largest single mass component of the propulsion system. Ordinary capacitor banks have an energy storage capacity of  $10^{-2}$  joule/cm<sup>3</sup>. For the required energy storage of  $10^6$  joules, this would require a capacitor volume of  $10^9$  cm<sup>3</sup>. For an assumed density of 2 gm/cm<sup>3</sup> for the overall capacitor bank material, this would amount to a total weight of 200 tons.

An energy density of  $10^{-2}$  joule/cm<sup>3</sup> corresponds to an electric field of  $E = 5 \times 10^5$  volts/cm, assuming a dielectric material with  $\epsilon = 1$ . The conventional capacitor design uses certain oil papers for which  $\epsilon = 1$ . For conventional, ground based capacitor banks, weight considerations are of minor importance. This, however, is not true for space flight requirements which make a light weight capacitor bank highly important. Fortunately, there are materials with large dielectric constants available, i.e., the ferroelectrics. The most outstanding representative of this group of materials is Bariumtitanate ( $\text{BaTiO}_3$ ), with a dielectric constant  $\epsilon_1 = \epsilon_2 = 5 \times 10^3$  along two crystal axes. The dielectric constant along the third axis is only  $\epsilon_3 = 160$ , so a single crystal layer would be required for capacitor design. A further, more serious problem is the reduction in the maximum attainable breakdown voltage of  $10^4$  volt/cm and hence the reduction in the attainable energy density,  $\epsilon E^2/8\pi = 0.02\text{j/cm}^3$ . This is about the same as for conventional capacitors. It seems likely that the lower breakdown voltage results from inhomogeneities in the Bariumtitanate single crystals, combined with the small value for the dielectric constant along the third

principal crystal axis. It seems possible that this problem can eventually be overcome by technological progress in controlling the growth of single Bariumtitanate crystals. In such a case the attainable energy density could be increased by a factor of  $5 \times 10^3$  and the weight of the capacitor bank reduced by the same amount, down to one ton. A much less ambitious goal would still be very satisfactory. Conjecturing that the breakdown voltage problem will be less severe for an isotropic dielectric tensor, the use of the following four substances, available as ceramics, seems to be highly interesting:

- 1) Cadmium niobate ( $\text{Cd}_2\text{Nb}_2\text{O}_7$ ),  $\epsilon \sim 310$
- 2) Lead metaniobate ( $\text{Pb}(\text{NbO}_3)_2$ ),  $\epsilon \sim 280$
- 3) Lead metatantalate ( $\text{Pb}(\text{TaO}_3)_2$ ),  $\epsilon \sim 1200$
- 4) Manganese metatantalate ( $\text{MnTaO}_3$ ),  $\epsilon \sim 750$

Let us assume a value of  $\epsilon = 500$  with a breakdown voltage of  $E = 10^5$  volts/cm; one then obtains for the energy density  $\epsilon E^2/8\pi = 0.2 \text{ joule/cm}^3$ , which is an improvement by a factor 20 over the conventional value of  $10^{-2} \text{ joule/cm}^3$ . The weight of the capacitor bank would be reduced by this factor from 200 tons down to 10 tons.

If instead of a capacitive, an inductive Marx generator is employed, a much more compact energy storage system can be built. If the electric and magnetic field are both measured in electrostatic units then the electric and magnetic energy densities are given by  $E^2/8\pi$ , resp.  $H^2/8\pi$ , where  $E$  and  $H$  are the electric and magnetic field strengths in electrostatic c.g.s. units. If, for example, the magnetic energy is stored in a field of  $H = 2 \times 10^4$  gauss then to achieve the same energy density with an electric field would require  $E = 2 \times 10^6$

volt/cm, a very large electric field, not yet attainable in capacitors. The energy density for the value of  $H = 2 \times 10^4$  gauss is  $H^2/8\pi = 1.6 \times 10^7$  erg/cm<sup>3</sup> = 1.7 joule/cm<sup>3</sup>. If the energy is stored in magnetic coils the coil mass to store the energy can be made rather small since the center of each solenoid is hollow. An average mass density per coil volume of 0.5 g/cm<sup>3</sup> seems to be thus quite reasonable. To store  $10^6$  joule with an energy density of 1.7 joule/cm<sup>3</sup> would require a volume of  $6 \times 10^5$  cm<sup>3</sup> = 0.6 m<sup>3</sup>, which with the assumed density 0.5 g/cm<sup>3</sup> would then result in a mass of 0.3 tons. The actual weight will be larger because the inductive energy storage system needs the additional components of the pulse transformer and unipolar machine. The weight of the unipolar machine was estimated to be of the order 0.4 tons. The weight of the pulse transformer may be of the order one ton such that the total weight would not exceed the amount of 2 tons, but which is still better by a factor 5 than the best which can be expected from ferroelectric capacitors.

### 13. Fuel propellant optimization and system weight analysis

For electric propulsion systems it has been shown by Stuhlinger<sup>(16)</sup> that there exists an optimal exhaust velocity which depends on the specific power of the energy production system and on the mission, the latter defined in terms of a certain terminal velocity and the propulsion time. For the optimal exhaust velocity, the mass ratio  $M_0/M_L$  of the initial to the final (load) vehicle mass will be a minimum.

In the case of a micro-bomb rocket propulsion system, it is not so important to minimize the mass ratio  $M_0/M_L$  as it is to minimize the amount of consumed thermonuclear fuel, a condition dictated by its high cost. The question to be answered is, therefore, the following: which fuel-propellant ratio to be ejected in the exhaust will minimize the total amount of required fuel mass per payload mass. This minimum will require a higher mass ratio than the minimum possible mass ratio. This, however, is of minor importance because of the high fuel cost.

In order to derive the condition for optimization, we start from the basic rocket equation

$$\begin{aligned} \exp(u/v) &= M_0/M_L \\ &= (M_f + M_h + M_L)/M_L \end{aligned} \quad , \quad (13.1)$$

where  $u$  is the terminal rocket vehicle velocity,  $v$  the exhaust velocity,  $M_f$ ,  $M_h$  and  $M_L$  the masses of the fuel, the propellant, and the load. The load consists of 1) the ignition system mass  $M_T$ ,

which includes the trigger apparatus, the magnetic mirror, and the power reactor with the radiator: and 2) the payload mass  $M_{PL}$ , hence

$$M_L = M_T + M_{PL} \quad (13.2)$$

At a temperature  $T$ , the thermonuclear plasma will have a characteristic exhaust velocity  $v_0$  determined by  $v_0 = (3kT/M)^{1/2}$  ( $k$  is Boltzmann's constant,  $M$  is the average ion mass of thermonuclear plasma). For thermonuclear temperatures one has  $v_0 = 10^8$  cm/sec. If the thermonuclear plasma is mixed with a propellant, as for example hydrogen, the resulting exhaust velocity  $v$  is determined by conservation of energy,

$$v = (M_f / (M_f + M_h))^{1/2} v_0 \quad (13.3)$$

and hence

$$M_h / M_f = (v_0 / v)^2 - 1 \quad (13.4)$$

Substituting (13.4) into (13.1) leads to

$$\exp(u/v) = 1 + (v_0 / v)^2 (M_f / M_L) \quad (13.5)$$

from which one obtains

$$M_f / M_L = (v / v_0)^2 (\exp(u/v) - 1). \quad (13.6)$$

Eq. (13.6) gives the ratio of the fuel to load mass as a function of the exhaust velocity for a given terminal velocity  $u$ . The characteristic velocity  $v_0$  is to be considered a fixed parameter.

Putting

$$u/v = x$$

$$M_f/M_L = (u/v_0)^2 y \quad , \quad (13.7)$$

one obtains from (13.6)

$$y = x^{-2}(\exp(x) - 1) \quad . \quad (13.8)$$

The minimum of this equation is determined by the solution of

$$\exp(-x) = 1 - x/2 \quad , \quad (13.9)$$

with  $x \approx 1.6$  and  $y \approx 1.56$ . We therefore have under optimal conditions

$$M_f/M_L = 1.56 (u/v_0)^2 \quad . \quad (13.10)$$

Furthermore, because of (13.4),

$$M_h/M_f = 2.55 (v_0/u)^2 - 1 \quad , \quad (13.11)$$

and, because of (13.1) and (13.9),

$$M_o = e^{1.6} M_L = 5.0 M_L \quad , \quad (13.12)$$

and

$$M_f + M_h = 4.0 M_L \quad . \quad (13.13)$$

The optimal exhaust velocity, according to (13.7) and (13.9), is given by

$$v = u/1.6 = 0.62u \quad . \quad (13.14)$$

Another important quantity, the ratio of thrust to initial weight, can be obtained from these results under optimal conditions. This ratio obeys the relation

$$F/M_o g = 2 P/vM_o g \quad , \quad (13.15)$$

where  $F$  is the thrust,  $g$  is the acceleration due to gravity at sea level,  $P$  is the power of the rocket engine in ergs/sec, and  $v$  is the exhaust velocity in cm/sec. In Eq. (13.14) we used the relation between thrust and power given by

$$F = 2 P/v \quad , \quad (13.16)$$

Inserting into (13.15) the optimal values for  $M_o$  and  $v$  given by (13.14), we have

$$F/M_0g = 6.6 \times 10^{-4} P/uM_L \quad (13.17)$$

The thrust to final weight ratio is given by

$$F/M_Lg = 5.0 F/M_0g = 3.3 \times 10^{-3} P/uM_L \quad (13.18)$$

and the average is given by

$$\overline{F/Mg} = 2.0 \times 10^{-3} P/uM_L \quad (13.19)$$

Assume, for example, the following parameters:  $M_{PL} = 100$  tons,  $M_T = 5$  tons,  $u = 10^7$  cm/sec, and  $v_0 = 10^8$  cm/sec. We thus have  $v = 6.2 \times 10^6$  cm/sec,  $M_f/M_L = 1.56 \times 10^{-2}$ ,  $M_h/M_f = 2.54 \times 10^2$ , hence  $M_f = 1.64$  tons,  $M_h = 4.17 \times 10^2$  tons,  $M_0 = 525$  tons. For the ratio of thrust to initial weight we have (putting  $M_L = 1.05 \times 10^8$  gm)  $F/M_0g = 6.3 \times 10^{-19}P$ . Let us assume that we have 30 explosions per second, each explosion equivalent to 0.2 ton of TNT, i.e.,  $10^{16}$  ergs, so that  $P = 3 \times 10^{17}$  ergs/sec. We then have  $F/M_0g = 0.19$ . The ratio of thrust to final weight is given by  $F/M_Lg = 0.95$  and the average of the ratio is given by  $\overline{F/Mg} = 0.6$ . This ratio is larger by a factor of  $10^3$  than for other contemplated advanced propulsion systems.

The maximum exhaust velocity attainable with the thermonuclear fuel plasma alone is of the order  $v_0 = 10^8$  cm/sec. Let us, therefore, put  $v = v_0 = 10^8$  cm/sec. We then have an optimal vehicle velocity of  $u = 1.6 \times 10^8$  cm/sec. In this case, we have furthermore  $M_f/M_L = 4.0$ ,  $M_h/M_f = 0$ , hence  $M_f = 420$  tons,  $M_h = 0$ ,  $M_0 = 525$  tons. The

amount of the expensive  $\text{He}^3\text{-D}$  fuel has increased from the 1.64 tons for the 100 km/sec mission up to 420 tons for the 1600 km/sec mission. To achieve a high vehicle velocity becomes, therefore, increasingly expensive and, according to Eq. (10.10), increases as  $u^2$ . The ratio of thrust to initial weight in this case is given by  $F/M_0g = 3.9 \times 10^{-20}P$ , which, for  $P = 3 \times 10^{17}$  ergs/sec, results in  $F/M_0g = 1.2 \times 10^{-3}$ . Furthermore,  $F/M_Lg = 6 \times 10^{-3}$  and  $F/Mg = 3.6 \times 10^{-3}$ . It follows that even for this extreme case the thrust-to-weight ratio is still better by one order of magnitude than in any other known system.

Finally, we shall calculate under optimal conditions the time required for the mission to reach a vehicle velocity  $u$ . The acceleration of the rocket vehicle with variable velocity  $u$  follows from Eq. (13.16)

$$\frac{du}{dt} = \frac{F}{M} = \frac{2P}{Mv} \quad (13.20)$$

From the basic rocket equation we have

$$M = M(t) = M_0 \exp(-u/v) \quad (13.21)$$

hence

$$\exp(-u/v) \frac{du}{dt} = \frac{2P}{M_0v} \quad (13.22)$$

Integration of this from  $u = 0$  to  $u$  and from  $t = 0$  to the mission time  $t = t_0$  yields

$$t_0 = \frac{M_0 v^2}{2P} \left( 1 - \exp(-u/v) \right) \quad (13.23)$$

Inserting the optimal values, we have

$$t_0 = 0.8 M_L u^2 P \quad (13.24)$$

For  $M_L = 1.05 \times 10^8$  gm,  $u = 10^7$  cm/sec, and  $P = 3 \times 10^{17}$  ergs/sec, one has  $t_0 = 2.8 \times 10^4$  sec  $\approx$  8 hours. For  $u = 1.6 \times 10^8$  cm/sec, one would have  $t_0 = 7 \times 10^6$  sec  $\approx$  81 days. In Table I is compiled a list of the various quantities of interest for a number of flight parameters ranging from a fuel vehicle velocity of 50 km/sec up to 1600 km/sec.

It is clear that with such a propulsion system, a manned trip to Mars could be made in a week. With a vehicle velocity of  $3 \times 10^7$  cm/sec, a trip to Saturn could be made in one to two months.

Table I

| $u[\text{km/sec}]$ | $v[\text{km/sec}]$ | $M_f(\text{tons})$ | $M_h(\text{tons})$ | $\overline{F/Mg}$ | $t_o[\text{days}]$ |
|--------------------|--------------------|--------------------|--------------------|-------------------|--------------------|
| 50                 | 31                 | 0.41               | 420                | 1.1               | 0.07               |
| 100                | 62                 | 1.64               | 417                | 0.6               | 0.3                |
| 300                | 188                | 14.7               | 404                | 0.19              | 2.7                |
| 1600               | 1000               | 420                | 0                  | 0.0036            | 81                 |

$M_{PL} = 100 \text{ to}, M_t = 5 \text{ to}, M_L = 105 \text{ to}$

$M_O = 524 \text{ to}, P = 3 \times 10^{17} \text{ erg/sec} = 15 \text{ ton TNT/sec}$

## APPENDIX

Inductive Marx Generator

The principle of the inductive Marx generator is being shown in Fig. 5. As already pointed out, in contrast to the capacitive Marx generator where a bank of  $N$  capacitors is charged in parallel to the voltage  $V$  and discharged in series raising the voltage to the value  $NV$ , in the inductive Marx generator a bank of  $N$  coils is magnetized in series by a current  $I$  and discharged in parallel, raising the resulting current to the value  $NI$ .

In the inductive Marx generator the switch-off problem is facilitated by the fact that if the total required current to be delivered to the target is  $I_t$  then  $I_t = NI$  and only the current  $I = I_t/N$  has to be interrupted in each of  $N$  switches  $S$  as shown in Fig. 5. If, for example, the total required current is  $I_t = 4 \times 10^6$  amp then a total of  $N = 400$  coils with 400 switches would require that in each switch a current of only  $10^4$  amp would have to be interrupted.

We will derive certain relations for such an inductive Marx generator. In deriving these relations, we use here the more convenient MKS units. We define as  $\epsilon$  the energy in joule stored in one coil of a bank consisting of  $N$  coils. If the total energy in the bank is  $E$  then

$$E = N\epsilon$$

(A1)

Furthermore, if  $L$  is the inductance of one coil in henry then

$$\epsilon = LI^2/2 \quad (A2)$$

and if  $\tau$  is the required discharge time the induced voltage in volts is given by

$$V = L \frac{dI}{dt}$$

$$= LI/\tau \quad (A3)$$

From eq. (A1), (A2) and (A3) together with the relation

$$I_t = NI \quad (A4)$$

one obtains the following relations

$$L = 2EN/I_t^2 \quad (A5)$$

$$V\tau = 2E/I_t \quad (A6)$$

If, for example,  $E = 10^6$  joule and  $I_t = 4.7 \times 10^6$  amp it follows that  $L = 0.89 \times 10^{-7}$  N henry and  $V\tau = 0.42$  volt sec. With  $\tau = 3.5 \times 10^{-7}$  sec it would follow  $V = 1.2 \times 10^6$  volt. These numbers are in good agreement with our requirements.

We are directing now our attention specifically to a single coil of our bank and express here for better understanding everything in electrostatic c.g.s. units, with the exception of the current  $I$  which is expressed in Ampere. The inductance of a coil of length  $l$ , radius  $r$ , and  $n$  number of turns is then given by

$$L = 4\pi^2 n^2 r^2 / l \quad (A7)$$

The capacitance of a coil can be approximately determined by considering the coil as a conducting cylinder of radius  $r$  and length  $l$ , hence

$$C = l/2 \ln(l/r) \quad (A8)$$

The discharge time which is one quarter of the Thomson time is given in electrostatic units by

$$\begin{aligned} \tau_o &= (LC)^{1/2}/2c \\ &= \frac{\pi^2 n r}{c [\ln(l/r)^2]^{1/2}} = \frac{\pi n r}{c} \Lambda \end{aligned} \quad (A9)$$

$$\Lambda \equiv \frac{\pi}{[\ln(l/r)^2]^{1/2}}$$

Introducing the length  $\lambda = 2\pi n r$  of the coil wire, we may also write

$$\tau_o = \frac{\lambda}{2c} \Lambda \quad (A10)$$

In eq. (A10) the factor  $\lambda/2c$  is equal to the time needed for an electromagnetic wave to travel along the wire of the coil from the

center to one of the terminals. The remaining factor in eq. (A10) is of the order one and does not depend very sensitively on the particular coil geometry. If, for example,  $l = 46$  cm,  $r = 6.6$  cm, then  $\Lambda \approx 2.0$ . If the coil has a current of  $I$  (amp) it will produce a magnetic field  $H$  (gauss) which is given by

$$H = 0.4\pi n I / l \quad (A11)$$

If we request that  $I = 10^4$  amp, and  $H = 10^4$  gauss then for  $l = 46$  cm eq. (A11) yields  $n \approx 37$  turns. With this value of  $n$  one obtains then from eq. (A9)  $\tau_0 = 2.6 \times 10^{-8}$  sec. The actual discharge time  $\tau$  for the coil can be made longer but never shorter than  $\tau_0$ , hence

$$\tau \geq \tau_0 \quad (A12)$$

How short  $\tau$  can be actually made depends on how fast the coil can be switched off. In our case, for example, the required switch-off times have to be of the order of  $\tau = 3.5 \times 10^{-7}$  sec, which is more than 10 times longer than the time  $\tau_0$ .

The energy  $\epsilon$  in erg stored in one coil is computed from multiplying the magnetic energy density  $H^2/8\pi$  with coil volume  $\pi r^2 l$

$$\begin{aligned} \epsilon &= \pi r^2 l \frac{H^2}{8\pi} \\ &= 0.2 n^2 r^2 I^2 / l \end{aligned} \quad (A13)$$

For our example ( $n = 37$ ,  $r = 6.6$  cm,  $l = 46$  cm,  $I = 10^4$  amp) one has  $\epsilon \approx 2.6 \times 10^{10}$  erg = 26kjoule. The example was chosen in such a way as to have in the inductive Marx generator with  $N = 400$  coils, a total current of  $I_t \approx 4 \times 10^6$  amp and a stored energy of  $E \approx 10^6$  joule, which is in good agreement with our requirements.

The 400 coils of the inductive Marx generator would occupy a volume of only  $2.5\text{m}^3$ , which again underlines the efficiency of the magnetic energy storage. By increasing the number of turns in each coil by a factor 3 the magnetic field would go up to  $3 \times 10^4$  gauss and the energy density would rise by a factor of about 10. Under othersise unchanged parameters the total energy output of the inductive Marx generator would then increase tenfold.

The functioning of the inductive Marx generator depends critically on the ability to switch off large currents of the order of  $10^4$  Ampere in a time as short as  $10^{-7}$  sec. The switching-off of large currents in a short period of time has been early recognized as the principal problem in any inductive energy storage system. In contrast to the opening of a switch the closing of a switch can be accomplished very rapidly, in a matter of a few nanoseconds by a spark gap switch. This is the reason why capacitively stored energy can be very rapidly released.

A well known system by which large currents can be switched off in a period of several 100 nanoseconds makes use of the rapid change in the conductivity of an exploding wire. The exploding wire technique, however, has the disadvantage that it depends on a fuse, the exploding wire, which has to be replaced for each subsequent discharge.

It was pointed out above that in the inductive Marx generator, consisting of a bank of  $N$  coils, and requiring  $N$  switches as shown in Fig. 5, the switching problem is facilitated by the subdivision of the total current to be delivered to the load in proportion to the number of coils in the storage bank; and it was specifically pointed out that if, for example, as in our case, the required current to the load is  $4 \times 10^6$  amp and the energy is stored in 400 coils, each switch has only to interrupt a current of  $4 \times 10^6 / 400 = 10^4$  amp.

We will here describe a switching technique, which seems to promise to switch off currents of this magnitude in times of the order of nanoseconds. The principle of the idea is explained in Fig. 8, showing a cylindrical vacuum tube in a cross section along its axis. The tube is inside a magnetic field coil producing a strong axial magnetic field. The tube has a field emission cathode and a collector anode. Around the center of the tube is an auxiliary ring electrode, which is brought against the field emission cathode onto a large positive potential of several million volts. This can be done, for example, by connecting the ring electrode with a Van De Graaff or with a Marx Blumlein line high voltage generator. This high voltage source is represented in Fig. 8 by the auxiliary capacitor  $C$ . After a high voltage is applied to the ring electrode a field emission current will start to develop. Normally, this field emission current would be discharged onto the ring electrode. If, however, a strong axial magnetic field is applied as shown in Fig. 8, the current can be prevented from flowing to the ring electrode if the following condition holds

$$E_r \ll H_z$$

(A12)

where  $E_r$  is the radial electric field component and  $H_z$  the axial magnetic field strength applied externally, the electric field by the ring electrode and the magnetic field by the field coil, both measured in electrostatic units.

If condition (A12) is satisfied the electron current will be confined radially. The electrons after being accelerated in the first half part of the tube towards its center at the position of the ring electrode are decelerated in the second half part of the tube and reach the collector anode with zero energy if the potential difference in between the cathode and anode is zero and if the processes of emission, acceleration and collection are without losses. However, this idealized condition is not expected to hold rigouously and a small potential difference in between the cathode and the anode has to be sustained in order to make up for these losses.

After the ring electrode has been connected to the high voltage source a current will flow from the cathode to the anode. In order to interrupt this current a triggered spark gap switch  $s$  at the auxiliary capacitor  $C$  connected to the ring electrode has to be closed. This can be done in a time as short as  $10^{-9}$  sec. The voltage at the ring electrode will then drop in a time given by the  $1/4$  Thomson time (in electrostatic c.g.s. units)

$$= \frac{\pi}{2c} (LC)^{1/2}$$

$$= D/c$$

(A13)

where  $L$  and  $C$  are the inductance and capacitance of the capacitor  $C$

and the ring electrode.  $D$  is the linear system of this system of conductors. If, for example,  $\tau = 3 \times 10^{-7}$  sec it follows that  $D = 10^4$  cm = 100 meters, a value which can be easily realized by a Van De Graaff or Marx generator. After the voltage at the ring electrode has dropped, in the time given by eq. (A13) the field emission current will stop instantaneously and the electron current is suddenly interrupted. The same would be the case if the current from the cathode is produced by other processes, for example, by thermionic emission. In this case, after the voltage at the ring electrode has dropped off, the uncompensated electric space charge field would also immediately stop the current.

In Fig. 8 the negative terminal of the high voltage source is grounded against the cathode and anode over large resistors  $R$  in order to prevent the current from bypassing the vacuum tube.

The rapid switching off the electron current is possible because of the very small inertia of the electronic charge carriers. Since in our case the electrons will be accelerated between the field emission electrode and the center of the tube to relativistic energies the acceleration time is given by

$$t_a \approx \frac{mc}{eE_z}, \quad (A14)$$

where  $c$  and  $m$  are the electronic charge and mass and  $E_z$  the axial electric field component in electrostatic units. If the tube has a length  $2l$  and a radius  $R$  which is equal to the radius of the ring electrode and if a voltage  $V$  is applied to the ring electrode, the electric field  $E_z$  due to this applied voltage  $V$  near the fi

emission electrode is given by (in electrostatic units, V in volt)

$$E_z = V / 300(\ell^2 + R^2) \quad , \quad (A15)$$

hence from (A14)

$$t_a = 1.7 \times 10^{-5} (\ell^2 + R^2) / \ell V \quad . \quad (A16)$$

If, for example,  $V = 10^6$  volt and  $\ell = R = 10$  cm it follows  $t_a = 3.4 \times 10^{-10}$  sec, and for all practical purposes  $\tau \gg t_a$ .

The condition (A12) for confined space charge flow (Brillouin flow) is necessary but not sufficient and has to be supplemented by a number of other restraints.

First, condition (A12) does not include the self-electric and self-magnetic field of the beam current in the tube. But since the electrons attain relativistic energies in most parts of the tube, along their trajectory, except near the cathode and collector anode, the azimuthal beam magnetic field  $H_\phi^B$  is almost completely compensated by the radial beam electric field  $E_r^B$ . Therefore, in all practical cases the externally applied axial magnetic field can be made sufficiently strong to over-compensate the repulsive forces of the beam self-electric field over the self-magnetic field. Since the electron energies near the emitter and collector electrodes are small, the externally applied magnetic field near the electrodes is not assisted by the self-magnetic beam field. To compensate the self-electric beam field one is therefore on the safe side if one simply requests that the externally applied magnetic field must be everywhere

sufficiently strong to over-compensate both the radial electric field component resulting from the high voltage externally applied to the ring electrode and the radially oriented self-electric field of the beam.

Second, in order to avoid the diocotron instability the centrifugal force on the electrons resulting from the  $\underline{E} \times \underline{H}$  drift velocity must be small compared to the Lorentz force.

Third, for a space charge flow to be possible the axial electric field component of the externally applied electric field has to be strong enough to compensate the axial component of the space charge field resulting from the electron current.

We will now quantitatively formulate all the additional conditions as listed.

1) The radial self-electric field  $E_r$  of the current having a radius  $r < R$  is given by (in electrostatic c.g.s. units)

$$E_r^B = - 2\pi n e r \quad , \quad (A17)$$

where  $n$  is the electron number density in the beam current. Since the electron current obeys the relation ( $v$  electron drift velocity)

$$I = - \pi r^2 n e v \quad (A18)$$

one has from (A17) and (A18)

$$\begin{aligned} E_r^B &= \frac{2I}{v r} \\ &= \frac{c}{v} \frac{0.2I}{r} \quad , \quad (A19) \end{aligned}$$

the latter being valid if  $I$  is expressed in amp. The azimuthal magnetic beam field at the other hand is given by

$$H_{\phi}^B = \frac{0.2I}{r} \quad (A20)$$

so that

$$H_{\phi}^B = \frac{v}{c} E_r^B$$

respectively

$$H_{\phi}^B < E_r^B, \quad \text{for } v < c \quad (A21)$$

For  $v \leq c$  one has

$$E_r^B \geq 0.2I/r \quad (A22)$$

Take for example  $r = 10$  cm,  $I = 10^4$  amp it follows  $E_r^B = 2 \times 10^2$  esu =  $6 \times 10^4$  volt/cm.

The electric field at the emitter or collector electrode due to the externally applied voltage  $V$  is given by ( $E_r$  in electrostatic units,  $V$  in volt)

$$E_r = VR/300(l^2 + R^2) \quad (A23)$$

Put, for example,  $R = l = 10$  cm and  $V = 10^6$  volt one then has  $E_r = 1.7 \times 10^2$  esu =  $5 \times 10^4$  volt/cm.

In the center of the tube the radial component of the electric field is given by

$$E_r \approx \frac{1}{300} \frac{V}{R} \quad (A24)$$

For the given example this would yield  $E_r = 3.3 \times 10^2 \text{ esu} = 10^5 \text{ volt/cm}$ . The total electric field near the emitter or collector electrode is thus the sum of the values given by eq. (A22) and (A23). Near the center of the tube the self-electric field is almost compensated by the self-magnetic beam field so that the radial electric field strength is there determined by eq. (A24) alone. The condition (A12) for confined space charge flow near the emitter or collector electrode has to be therefore refined as follows

$$\frac{1}{300} \frac{RV}{l^2 + R^2} + \frac{0.2I}{r} \ll H_z \quad (A25)$$

and in the center of the tube as follows

$$\frac{1}{300} \frac{V}{R} \ll H_z \quad (A26)$$

2) In order to avoid the diocotron instability the following relation must hold

$$\frac{mv_D^2}{r} \ll \frac{ev_D H_z}{c} \quad (A27)$$

where

$$v_D = cE_r/H_z \quad (A28)$$

is the azimuthal electron drift velocity and where it was already assumed that  $E_r \ll H_z$ . From eq. (A27) and (A28) one obtains

$$H_z^2 \gg \frac{mc^2}{e} \frac{E_r}{r} \quad (A29)$$

$E_r$  in eq. (A29) must, of course, include both the contribution resulting from the externally applied voltage as well as the beam field. The maximum contribution to  $E_r$  resulting from the externally applied field is given by eq. (A24) and the maximum of the self-electric field of the beam is given by eq. (A22), hence by substituting in eq. (A29) the sum of both contributions and by putting  $v \approx c$  results in

$$\begin{aligned} H_z^2 &\gg \frac{mc^2}{e} \frac{1}{r} \left[ \frac{V}{300R} + \frac{0.2I}{r} \right] \\ &= 1.7 \times 10^3 \frac{1}{r} \left[ \frac{V}{300R} + \frac{0.2I}{r} \right] \end{aligned} \quad (A30)$$

3) In order to sustain a space charge flow axially an axial electric field  $E_z$  has to be applied. In our case  $E_z$  is given by eq. (A15). The current  $I$  which can be sustained under the application of an external voltage and an electrode separation  $l$  is given by the space charge flow equation ( $I$  in amp,  $V$  in volt)

$$I = 7.3 \times 10^{-6} \left[ \ell^2 / (\ell^2 + R^2) \right]^{3/2} (r/\ell)^2 V^{3/2} \quad (A31)$$

Or if a certain current  $I$  is required one can solve eq. (A31) for  $V$  giving

$$V = 2.7 \times 10^3 (\ell/r)^{4/3} \left[ (\ell^2 + R^2)/\ell^2 \right] I^{2/3} \quad (A32)$$

Take, for example,  $I = 10^4$  amp  $\ell = r = 10$  cm it thus follows  $V = 2.4 \times 10^6$  volt.

We now check if the different conditions expressed by the inequalities (A25), (A26) and (A30) are satisfied, for the following final set of parameters,  $V = 2.4 \times 10^6$  volt,  $I = 10^4$  amp,  $\ell = R \geq r = 10$  cm. We obtain from (A25):  $H_z \gg 600$  gauss, from (A26):  $H_z \gg 800$  gauss and from (A30):  $H_z \gg 400$  gauss. From these values it thus follows that a magnetic field of  $H_z = 10^4$  gauss would suffice. Such a field can be produced without any difficulty.

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## Figure Captions

- Fig. 1 Basic flow diagram of thermonuclear micro-bomb propulsion engine.
- Fig. 2 Flow diagram for propulsion system in which a chain of thermonuclear micro-bombs is ignited by an intense electron beam. P = payload, S storage for micro-bombs and hydrogen, E electron beam generator, M magnetic mirror reflector, M.B. micro-bomb in focus of mirror, L magnetohydrodynamic loop, PT pulse transformer, U unipolar machine, N.R. nuclear reactor, Rad radiator: 1 flow of micro-bombs from S to focus of M, 2 electron beam, 3 electric energy flow from M.H.D. loop L to unipolar machine U, 4 energy flow from U to electron beam generator E, 5 energy flow from nuclear reactor N.R. to unipolar machine, flow of energy from N.R. to magnetic field coils of M: 7, 8, and 9 flow of waste heat to radiator Rad.
- Fig. 3 The principle of the thermonuclear micro-bomb rocket engine.
- Fig. 4 Capacitive high voltage Marx generator.
- Fig. 5 Inductive high voltage Marx generator.
- Fig. 6 Position of thermonuclear target within magnetic mirror and target bombardment by electron beam.
- Fig. 7a, b, c Ignition temperature and energy input for  $\alpha = 1.0$ , 0.1, and 0.01.
- Fig. 8 The switch for rapidly interrupting a large current. C is a high voltage capacitor, s is an auxiliary triggered spark gap switch, R are large resistors, (I is directed along the electron current).

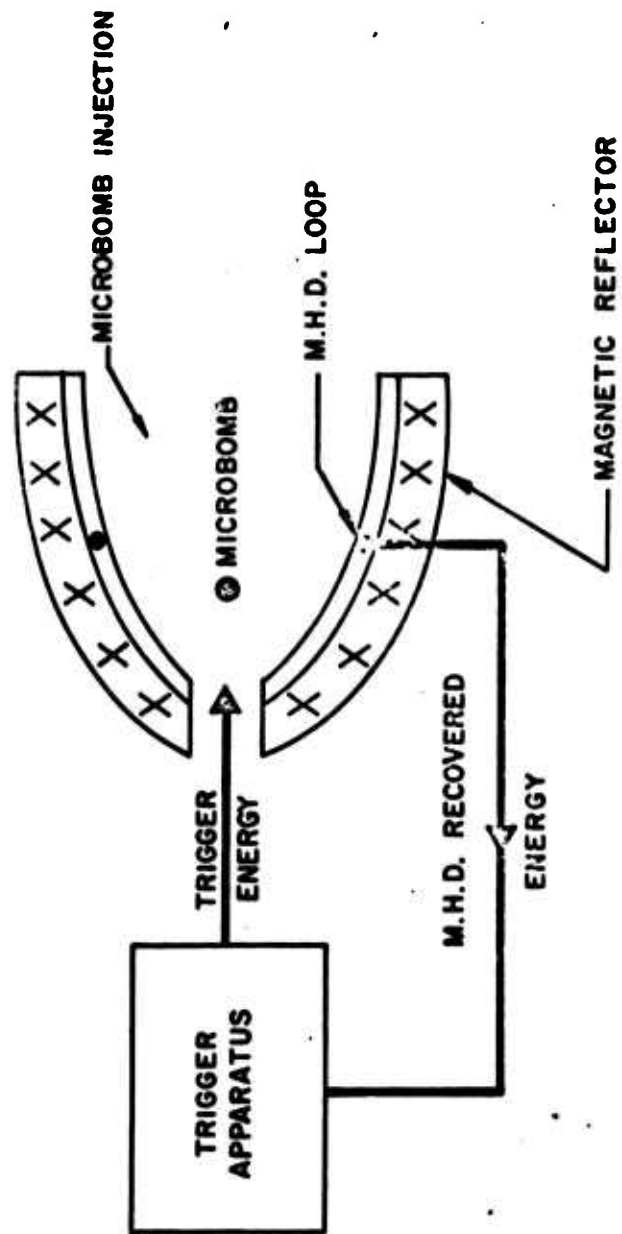


Fig. 1

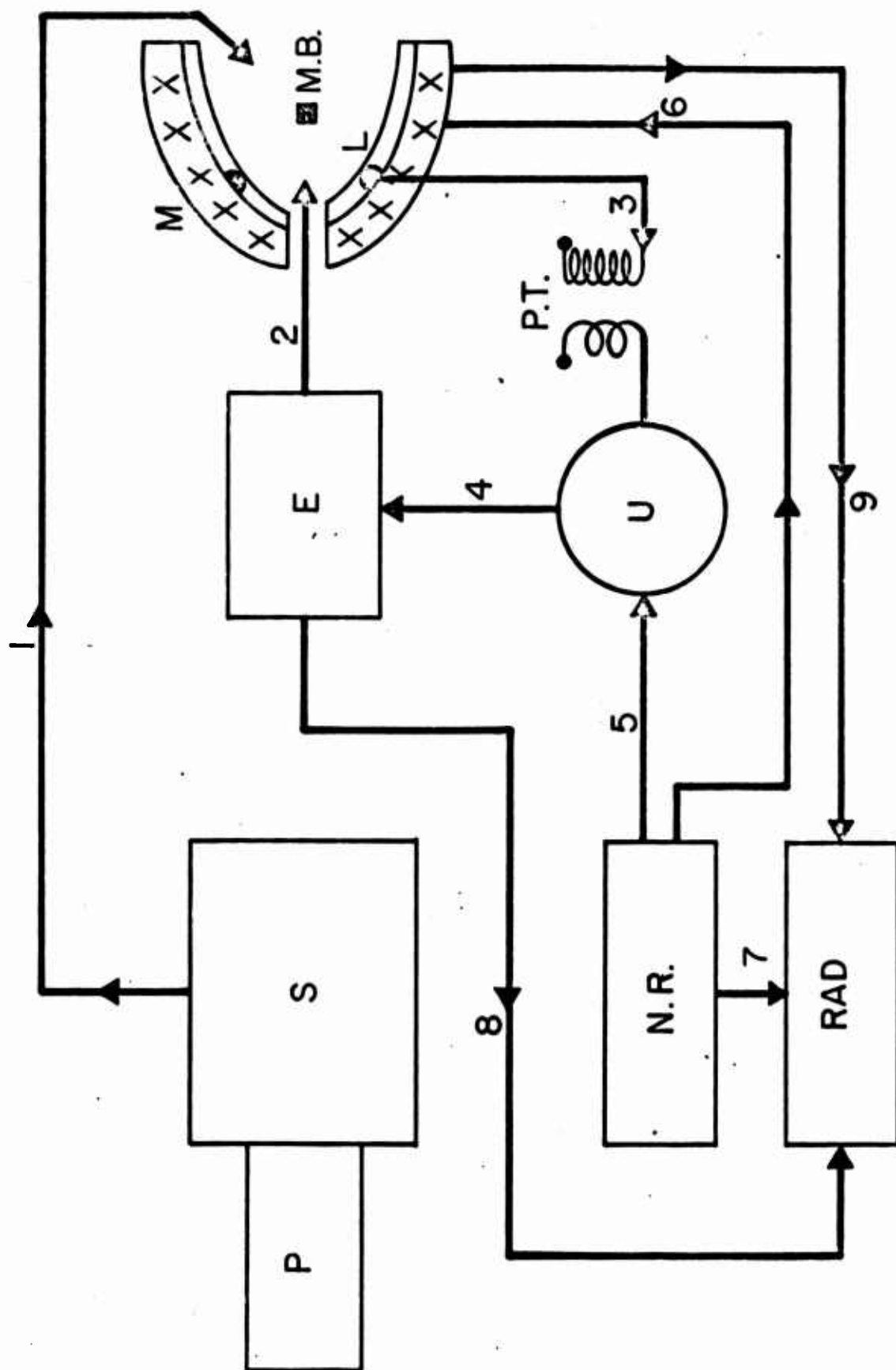


Fig. 2

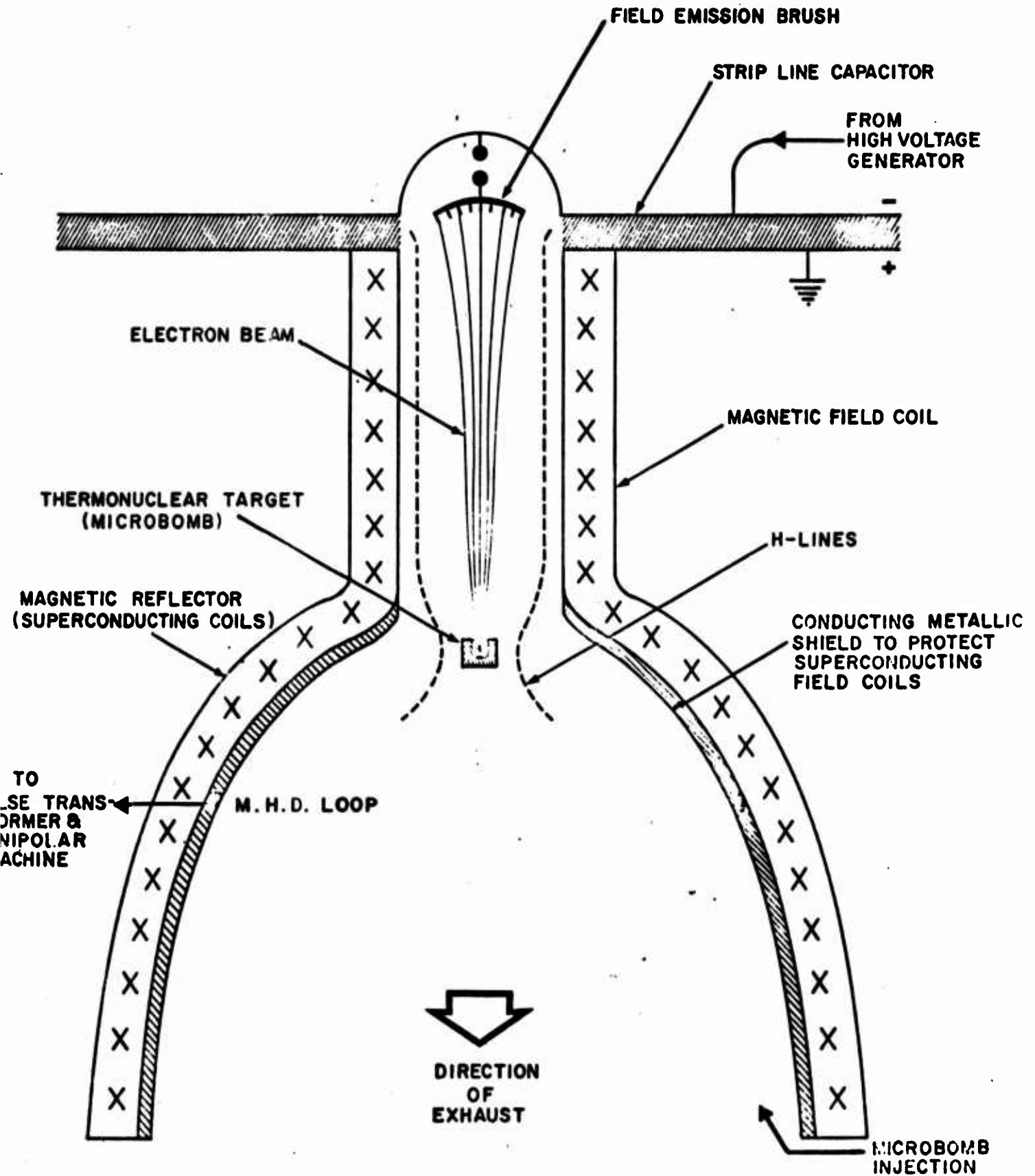


Fig. 3

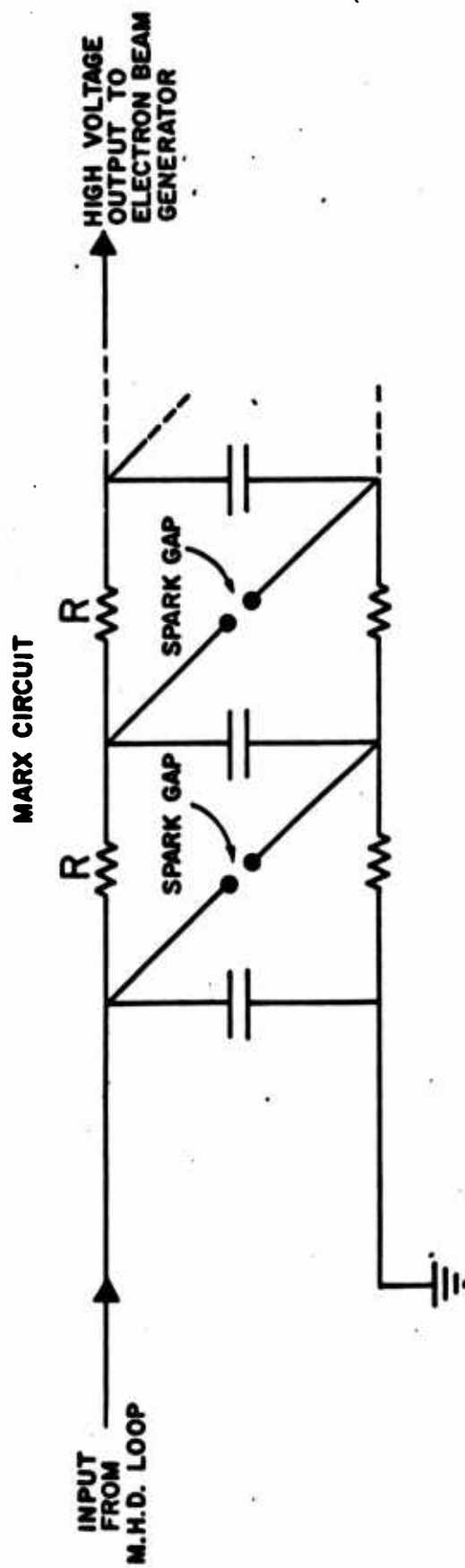


Fig. 4

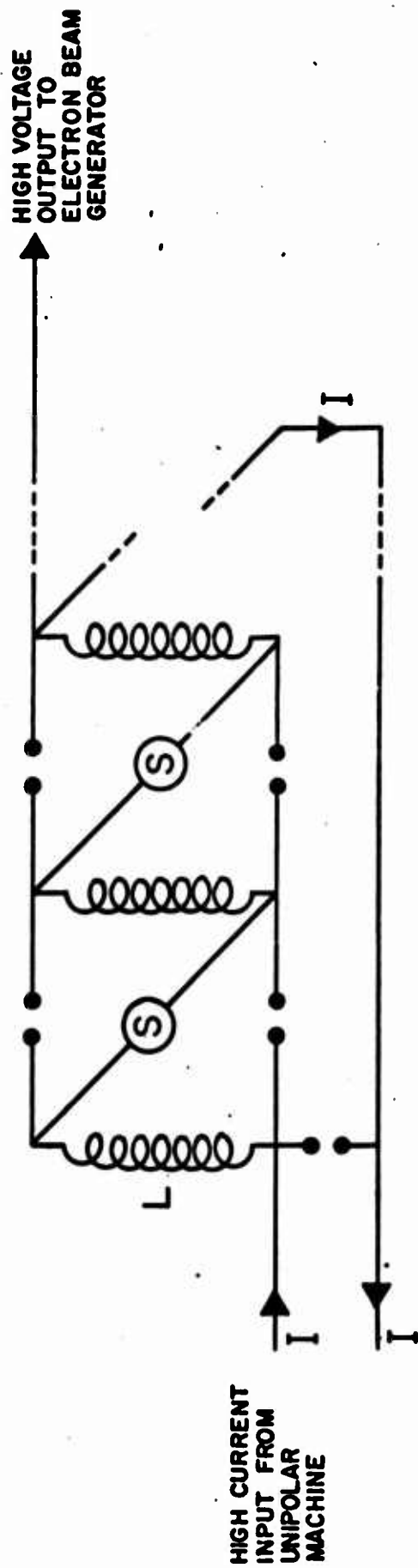


Fig. 5

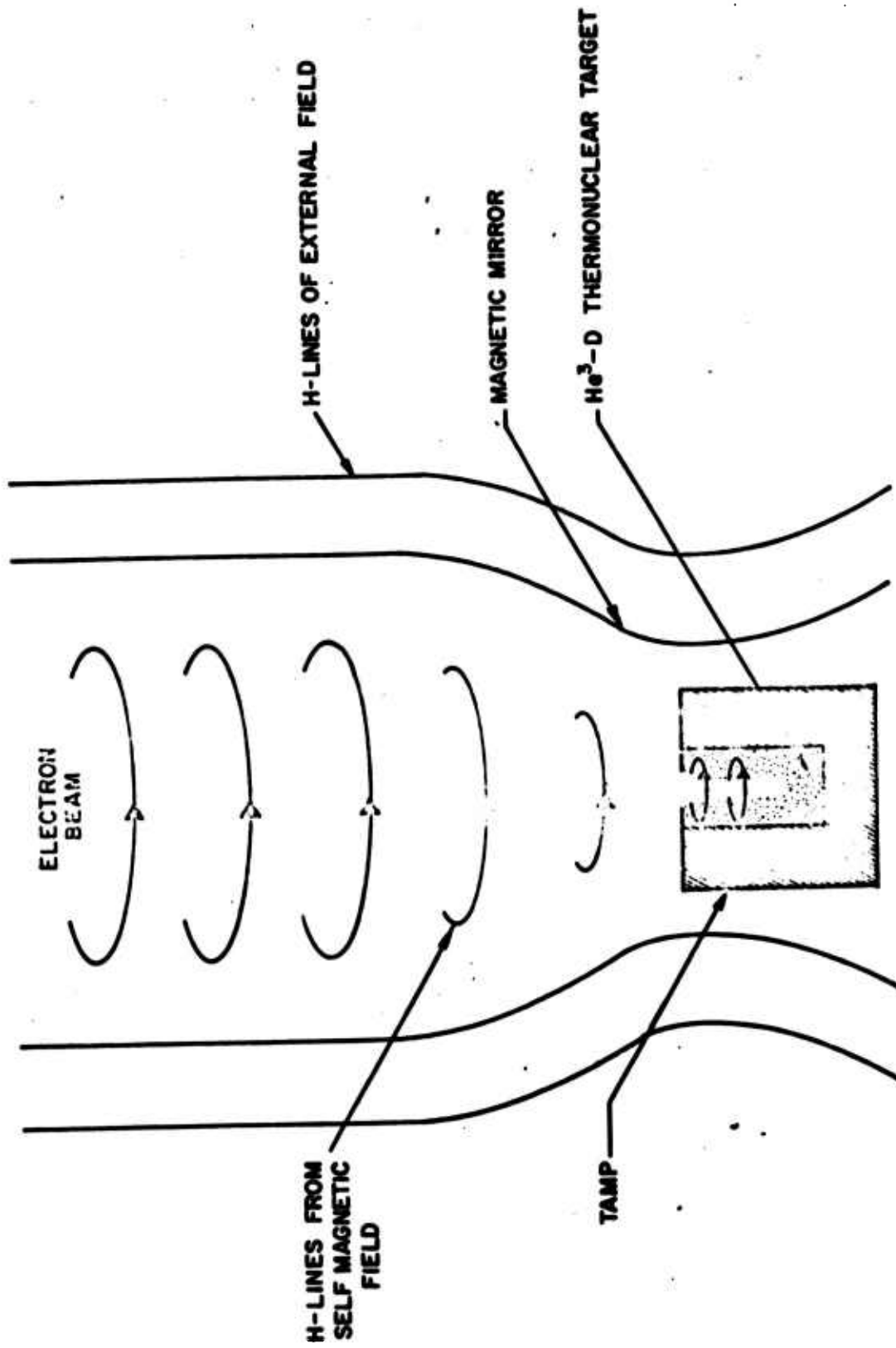


Fig. 6

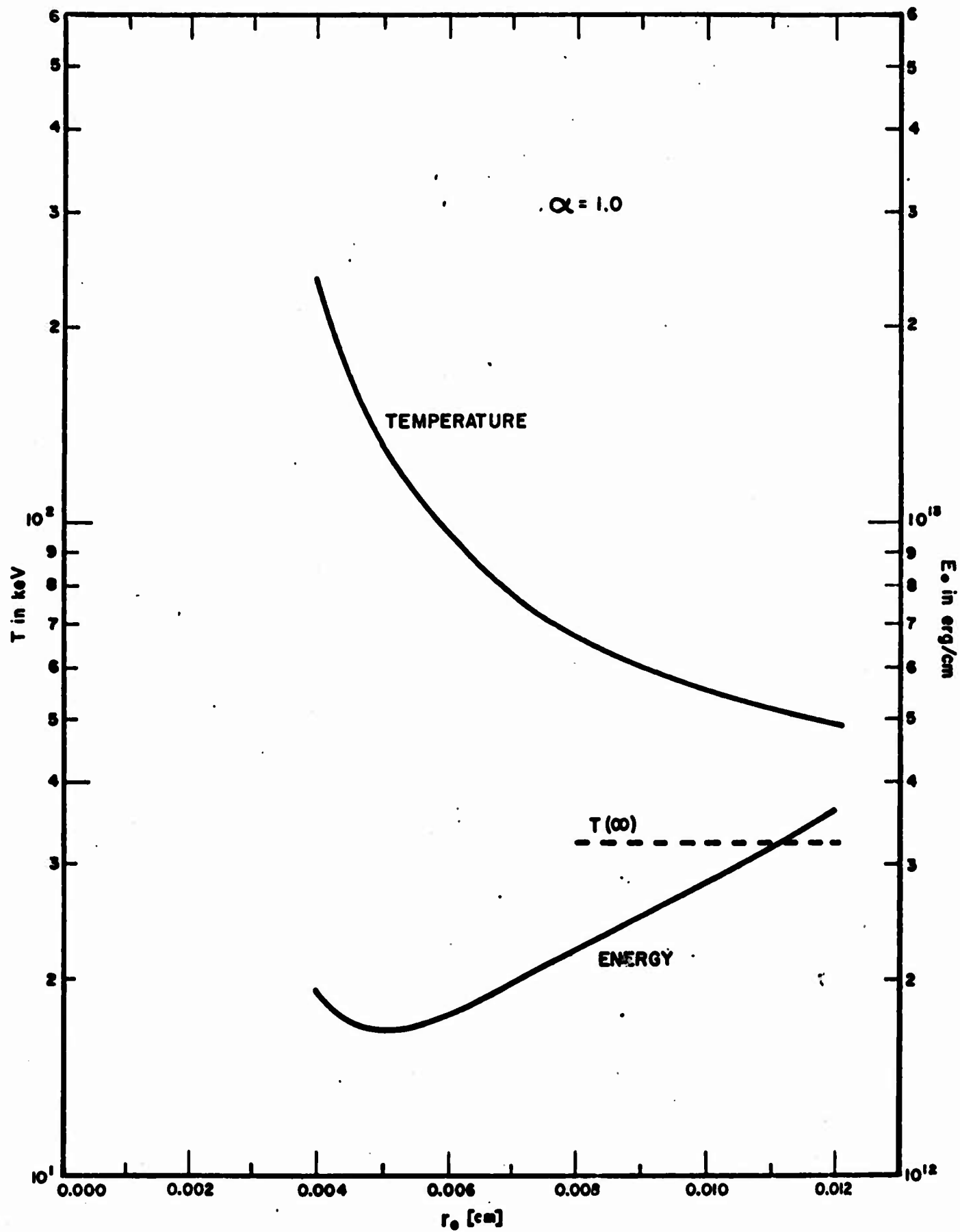


Fig. 7a

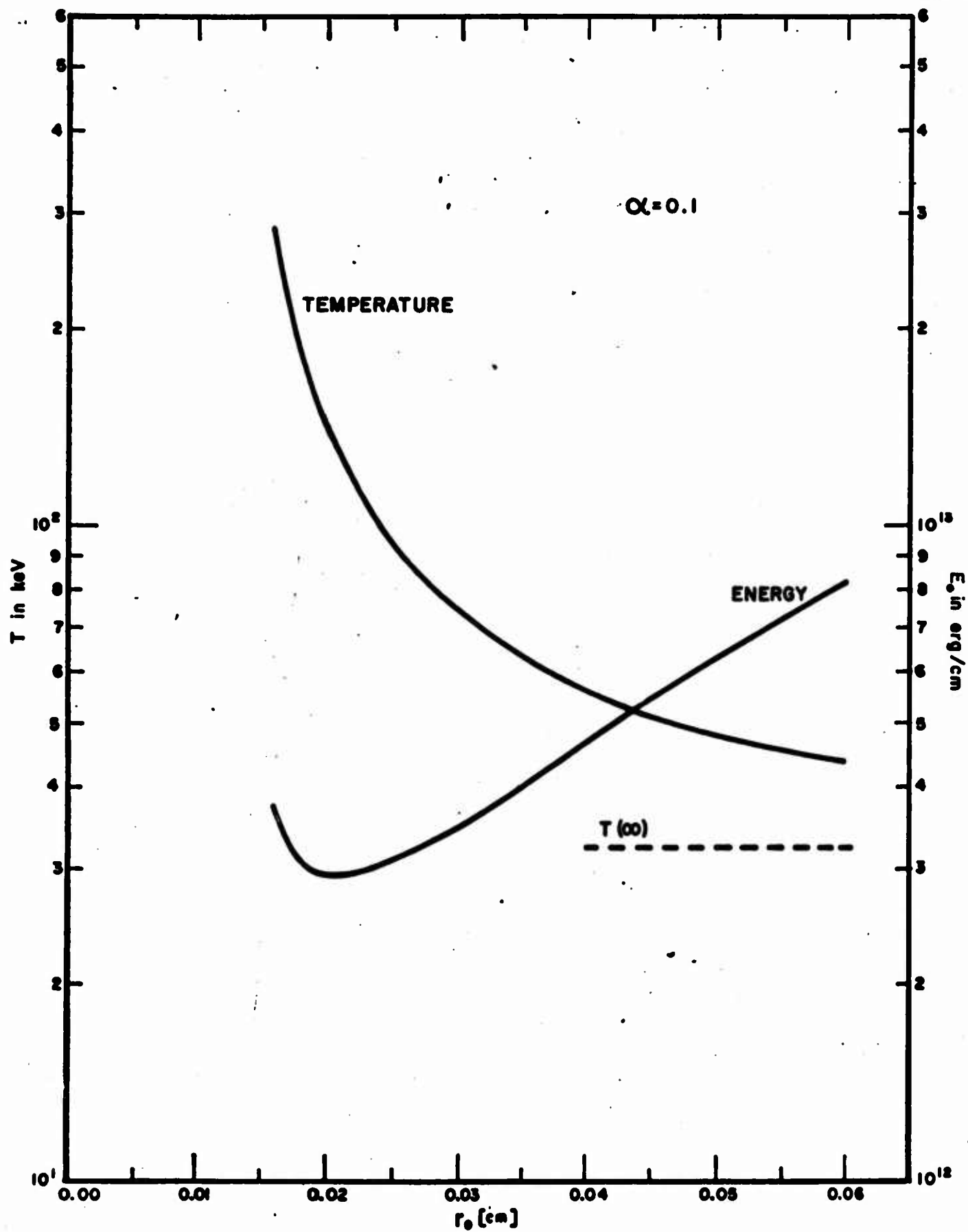


Fig. 7b

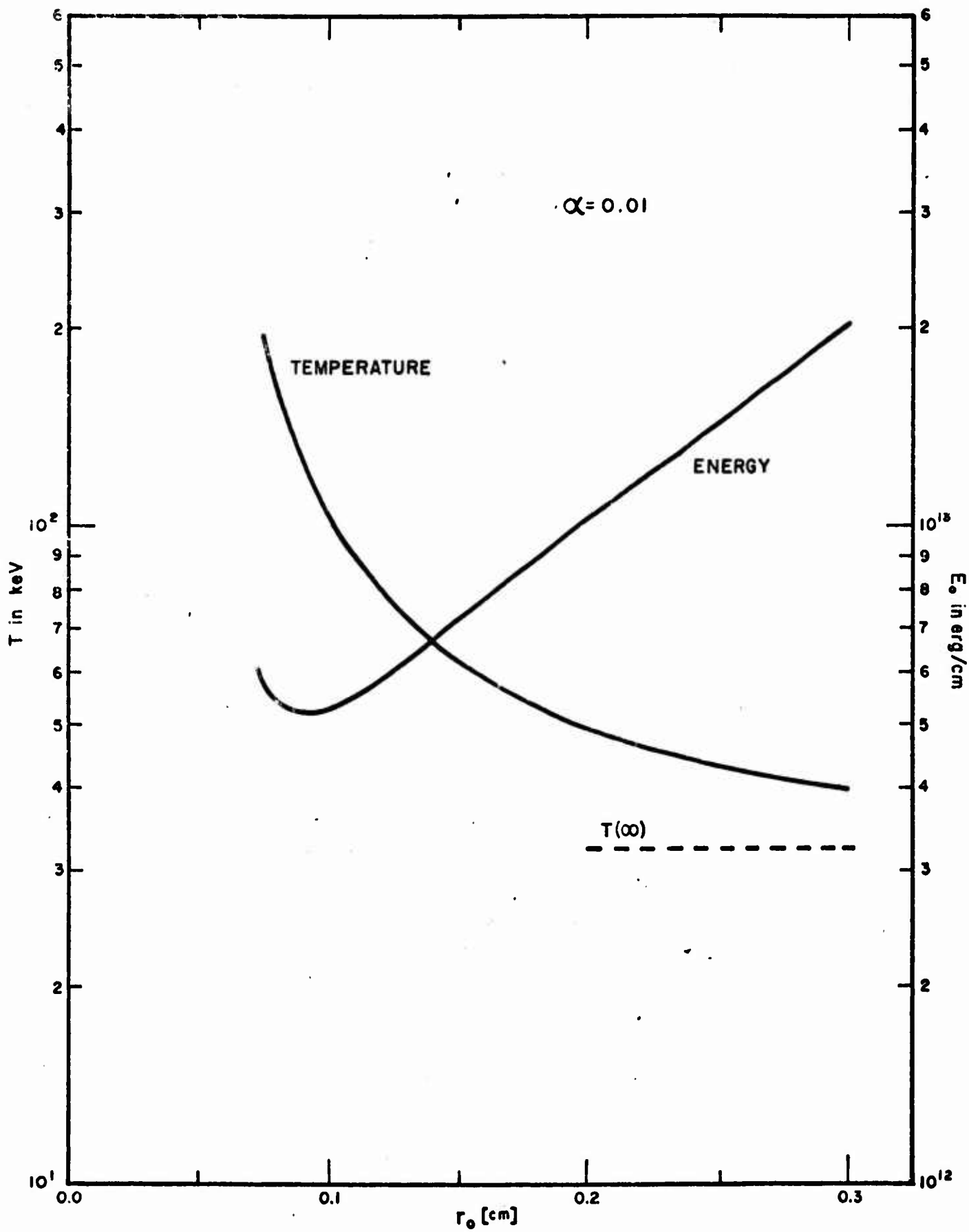


Fig. 7c

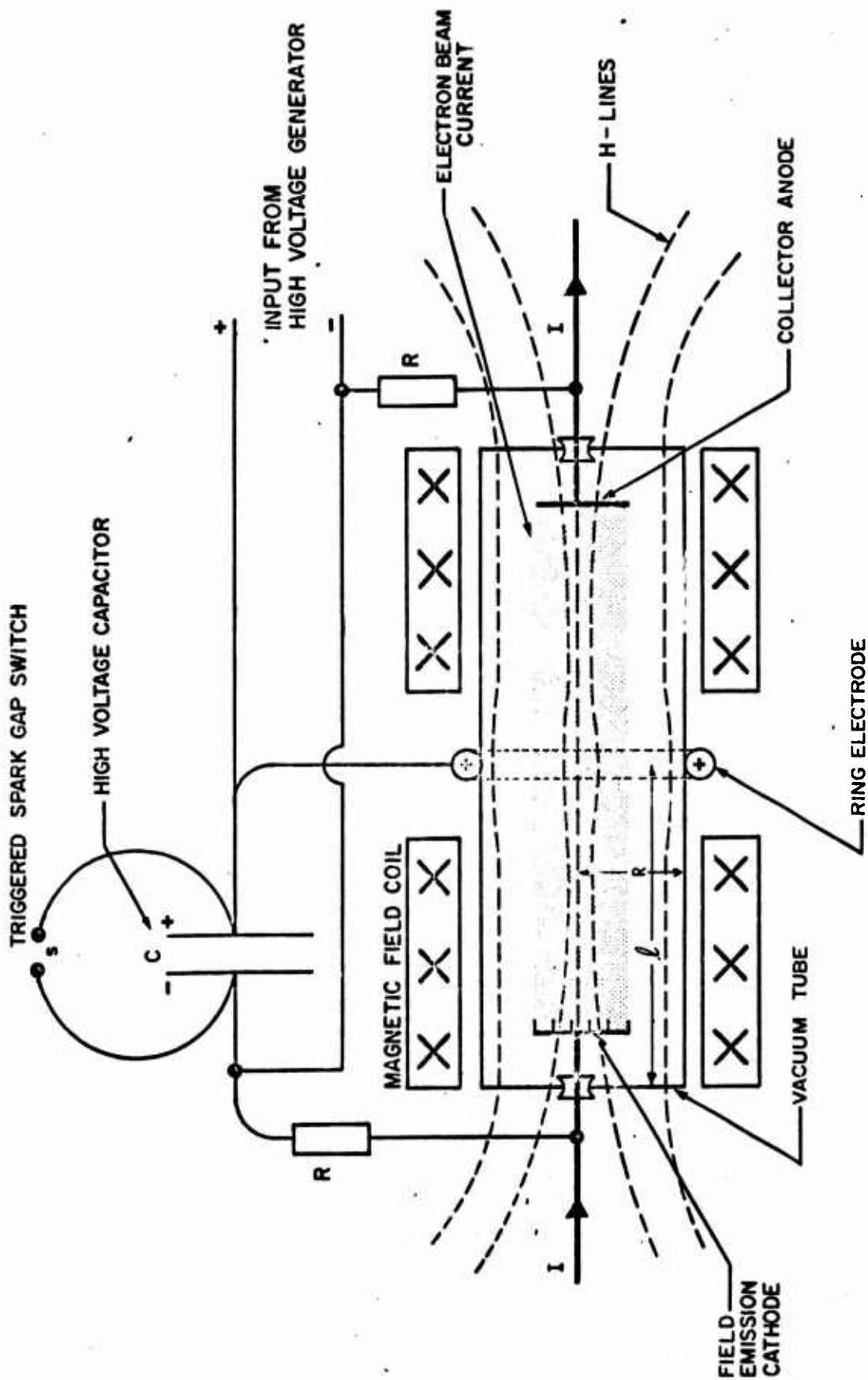


Fig. 8