AD-751 178

Seismic Design of Building Structures

New Mexico University

prepared for

Army Construction Engineering Laboratory

JULY 1972

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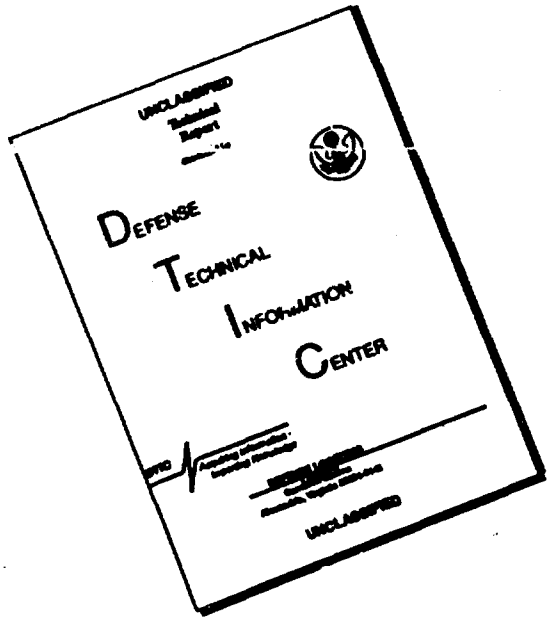

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TECHNICAL REPORT S-10
July 1972

C E R L

SEISMIC DESIGN OF BUILDING STRUCTURES

by

J. T. P. Yao
C. Omid'varan
A. Gürpinar
C. L. Hulsbos

CONSTRUCTION ENGINEERING RESEARCH LABORATORY
Champaign, Illinois 61820

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DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Department of the Army CONSTRUCTION ENGINEERING RESEARCH LABORATORY P.O. Box 4005 Champaign, Illinois 61820	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

SEISMIC DESIGN OF BUILDING STRUCTURES

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Final Report

5. AUTHOR(S) (First name, middle initial, last name)

J.T.P. Yao, C. Omid'varan, A. Gürpinar, C.L. Hulsbos

6. REPORT DATE July 1972	7a. TOTAL NO. OF PAGES 150	7b. NO. OF REFS 289
8a. CONTRACT OR GRANT NO. DACA 23-70-C-0062 b. PROJECT NO. 4DM78012AOK1 c. Task -03 Work Unit -007 d.	9a. ORIGINATOR'S REPORT NUMBER(S) TECHNICAL REPORT S-10 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) *Obtain AD# from block 1 address	

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES * Copies of this report available from: National Technical Information Service Springfield, Virginia 22151	12. SPONSORING MILITARY ACTIVITY Department of the Army

13. ABSTRACT

The purpose of this project was to formulate possible improvements in the design codes concerning building structures which are subjected to earthquake loads. In addition, suggestions were made concerning ways to incorporate the concepts and methods of discrete mechanics, statistical analysis, as well as earthquake behavior of concrete and metal structures into the seismic design code.

Available literature on existing design codes is summarized herein. Design philosophies and methodologies in earthquake engineering were also studied. A "design-tree" technique was then adapted to present the recommended improvements in the context of existing specifications. Moreover, approaches to certain expected problems for the implementation of these improvements were outlined along with a set of recommendations. Furthermore, literature reviews and some original contributions in discrete mechanics, statistical methods, and dynamic behavior of structures are given in three appendices as backgroung information.

14. KEY WORDS

Seismic Design Earthquake resistant structures: concrete construction
steel construction

DD FORM 1 NOV 65 1473 REPLACES DD FORM 1473, 1 JAN 64, WHICH IS OBSOLETE FOR ARMY USE.

UNCLASSIFIED
Security Classification

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by

J. T. P. Yao
C. Omid'varan
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research performed at

University of New Mexico
CONTRACT No.: DACA 23-70-C-0062

July 1972

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Champaign, Illinois 61820

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FOREWORD

This investigation was performed in the Department of the Civil Engineering at the University of New Mexico for the U.S. Army Construction Engineering Research Laboratory (CERL) under Contract No.: DACA 23-70-C-0062. This research was performed under Project 4DM78012AOK1, "Engineering Criteria for Design and Construction," Task 03, "Systems Criteria for Environmental Isolation and Control," Work Unit 007, "Earthquake Effects." Technical Monitor was Dr. James Prendergast.

This research was accomplished May - November 1970 by Dr. J. T. P. Yao, who prepared Appendices B and D and was responsible for coordinating the research effort; Dr. C. Omid'varan, who prepared Appendix A; Mr. A. Gürpinar, who prepared Part II; and Dr. C. L. Hulsbos who prepared Appendix C.

Dr. Walter E. Fisher, Chief of Buildings Branch, CERL, initiated the project. The advice and guidance of both Drs. Fisher and Prendergast throughout the duration of this project were appreciated. Mr. J. J. Healy, Chief of Construction Systems Laboratory, CERL, also gave his support and encouragement to these investigators. Col. E. S. Townsley and Dr. L. R. Shaffer were Director and Deputy Director, respectively, of CERL.

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SEISMIC DESIGN OF BUILDING STRUCTURES

PART I: INTRODUCTION

General

It is a well known fact that, throughout man's history, human life and property have been lost during strong-motion earthquakes. As examples, (a) property damage from the 1964 Alaska Earthquake was on the order of $300 million, (b) almost 11,000 persons lost their lives during the 1968 Iran Earthquake (1, 2)*, and (c) over 40,000 persons died during the recent 1970 Peru Earthquake. Detailed descriptions of the damage from the Alaska Earthquake as well as other recent earthquakes can be found elsewhere (3-7). While it is not possible at present to prevent the occurrence of strong-motion earthquakes, continous efforts have been made to improve the design of structures in order to minimize the earthquake damage of civilian as well as military structures.

Existing specifications concerning earthquake loads such as the Uniform Building Code (8), Recommended Lateral Force Requirements (9), and Seismic Design for Buildings (10), provide a set of equivalent static lateral loads. Although these codes are based on the dynamic analysis of earthquakes (11), Blume (12) has shown that structural deformations induced by loads based on code requirements may be considerably less than those computed for the same structure subjected to the excitation of a recorded strong-motion earthquake. As Blume pointed out, the safety of the structure against collapse is greater than that indicated by the linear analysis because a considerable amount of damping is introduced by the failure of "architectural clothing" as well as the elasto-plastic behavior of the frame.

Veletsos and Newmark (13) studied an elasto-plastic single-degree-of-freedom system subjected to earthquake motions corresponding to the 1940 El Centro and the 1933 Vernon earthquakes. Penzien (14) investigated the response of an elasto-plastic system to the ground motion of the 1940 El Centro earthquake, which confirmed the thought that the maximum displacements are less in elasto-plastic systems than those in corresponding elastic systems. Since then, nonlinear and yielding seismic structures have been studied by Jennings (15), Goel and Berg (16), and Penzien

*Parenthetic numerals indicate references.

and Liu (17). The results of these studies seem to confirm a generally accepted design philosophy which aims at an elastic response in the case of small (and frequent) earthquakes and to permit inelastic response in the case of strong-motion (and infrequent) earthquakes. However, it is well known that plastic deformations can cause cumulative damage, which could lead to low-cycle fatigue failure of structures (18).

Due to the random characteristics of earthquakes, several probabilistic models have been proposed to represent earthquake ground motions. Housner (19) represented the earthquake as a series of impulses that were random in time. Thomson (20) showed that it is reasonable to use white noise to describe earthquake motions. Bycroft (21) then proposed a specific white noise with a constant spectral density and a fixed duration for the representation of a standard strong-motion earthquake. Bogdanoff, Goldberg, and Bernard (22) suggested a nonstationary random process consisting of a sum of damped sinusoids to represent earthquake accelerations. In a subsequent paper, this model was modified by Goldberg, Bogdanoff, and Sharpe (23). Lin (24) proposed a nonstationary random process resulting from a filtered shot noise, which has become a popular model in the study of seismic structures to date.

Stationary random processes for earthquake simulation have been developed by Housner and Jennings (25) using a digital computer, and by Ward (26) using an analog computer. Earthquake ground motion is generally regarded as a non-stationary random process. Digital computer simulation of nonstationary earthquake motion has been performed by Ar n and Ang (27), Jennings, Housner, and Tsai (28), Shinozuka and Sato (29), Rascon and Cornell (30), Levy (31), Hou (32), and Iyengar and Iyengar (33). The simulation proposed by Rascon and Cornell (30) attempts to account for most of the pertinent physical characteristics of earthquakes. The results of Jennings, Housner, and Tsai (28) as well as Iyengar and Iyengar (33) contain parameters which account for different earthquake types. Recently, Wirsching and Yao (34) completed a Monte Carlo study of bilinear seismic structures using analog computation.

The distribution of earthquake occurrences in time and space has been considered by Rosenblueth (35), Benjamin (36), Cornell (37), and Shinozuka (38). Benjamin (36) used Bayes theorem and computed the posterior probabilities of the earthquake occurrences. Cornell (37) showed that the earthquake intensity is a function of distance from a fault. Recently Wirsching and Yao discussed the probability distribu-

tion of structural response to simulated earthquakes (39).

The traditional techniques for dynamic analysis of highly redundant framed structures can be referred to as "piece by piece" methods of analysis, which call for tedious and independent dynamic analysis of each structure in the sense that any variation in the framework properties and applicable parameters requires the development of a new solution. Recent developments in the field of mechanics makes it feasible to develop explicit functional solutions and formulas for dynamic characteristics of frames, applicable to prescribed patterns and formations with arbitrary number of bays and levels. The classical literature concerning functional field solution to structures is due to Bleich and Melan (40). The development and application of the technique to dynamic analysis of framed structures is due to Wah (41), Leimback and McDonald (42), Ellington and McCallion (43), and Dean and his colleagues (44). The technique employs the calculus of finite differences and discrete mechanic concepts to derive the difference-differential equations describing the dynamic behavior of framed structures. It results in functional solutions which are exact within the scope of linear beam theory and yet describes the dynamic behavior of various types of framed structures.

Although the existing seismic design codes are based somewhat on a dynamic analysis, the use of a set of "equivalent" static loads does not result in a solution which fully describes the dynamic behavior of structures under earthquake loading conditions. The practice of employing an "equivalent static loading" is known as a compromise solution with traceable deficiencies. It was recommended in lieu of the complexity of a dynamic analysis and limitations of the state of the art. Moreover, the statistical characteristics of earthquakes were totally disregarded. Results of many investigations concerning the dynamic behavior of framed structures are now available. These and field solution techniques are promising as a supplemental to statistical methods in serving to provide a rational basis for development of new and comprehensive codes and specifications.

Objective and Scope

The objective of this research program was to formulate possible improvements in the code specifications for the seismic design of buildings. Available literature on existing design codes was reviewed and summarized in PART II. In PART III, design philosophies and methodologies in earthquake engineering were studied, and the recommended improvements are presented with the use of a "design-tree" technique.

Moreover, approaches to some problems resulting from implementation of these improvements are outlined. In PART IV, conclusions and recommendations are made. An introduction, as well as some original solutions, to discrete mechanics is included in Appendix A; a literature review of statistical methods in earthquake engineering is given in Appendix B; the seismic behavior of concrete and metal structures is summarized in Appendix C; and an illustration for the direct approach portion of the design tree is given in Appendix D.

PART II: EXISTING SEISMIC DESIGN CODES

General

Until the 1940's, the forces induced by earthquake accelerations were treated as static laterial forces in building codes all over the world. The equivalent static force was simply calculated by assuming a rigid-body structure. During these past three decades, engineers have adopted more rational and refined methods to calculate these forces.

In the United States, the first attempt to better understand the dynamic properties of building structures was made after the Long Beach, California, earthquake of March 10, 1933, when the United States Coast and Geodetic Survey measured the natural periods of vibration of 212 buildings in two major directions (45). In fact, these data have remained the empirical basis for computing natural periods of building structures to date (46). With the development of high speed computers, a new era began for the design and construction of buildings. Closely connected with this development was the study of dynamic response of building structures to earthquake loads. Any techniques which could be adapted to computer methods have been emphasized greatly. Of these, matrix methods are the most popular. Either a stiffness or a flexibility matrix is constructed for the total structure and the dynamic responses of the structure are thus calculated. One of the more serious limitations of the application of these methods in earthquake engineering was the fact that they are applicable only to linear elastic systems. Therefore, the results were astonishing at first. The responses calculated analytically by using these methods were much larger than those recommended by the building codes (12, 4). However, later it was understood that the difference was due to the nonlinear and inelastic behavior of structures during strong-motion earthquakes.

Earthquakes Regulations are given in Sec. 2314 of the Uniform Building Code (8). These regulations are taken from the recommendations of the Seismology Committee of the Structural Engineering Association of California which dates back to 1957 (9). Considering the amount of research, and the volume of literature which has been published on the subject of earthquake engineering since that time, it is necessary to look at these recommendations more closely and compare them with some of the other building codes around the world. In the following, recommendations of the Uniform Building Code concerning different forces and requirements will be reviewed, critiqued, and compared with recommendations of

other existing building codes.

Equivalent Lateral Forces

Recommendations for computing the equivalent lateral force, V, are a very important part of the earthquake design regulation, because the seismic coefficient relates the weight of the building to the equivalent lateral force. In all the building codes which are reviewed herein, the relationship between the weight of the building and the total lateral force can be characterized by a simple formula as follows:

$$V = SW \tag{1}$$

where

V = Total lateral force

W = Weight of the building

S = Seismic coefficient

However, the seismic coefficient S contains different factors in various building codes.

1. Uniform Building Code (8)

In the Uniform Building Code, the relationship is given by,

$$V = ZKCW \tag{2}$$

hence we have

$$S = ZKC \tag{3}$$

where

C = the dynamic factor which depends on the period of the building.

K = a factor which varies for different types of buildings and indicates their ductility.

Z = a factor which depends on the earthquake zoning of the locations of the building.

With these factors in mind, it is appropriate to divide this section on "Equivalent Lateral Force" into four parts. The first three will discuss the factors C, K, and Z separately; the fourth will be devoted to the distribution of this lateral force along the height of the building.

1.1 The factor C depends on the natural period of vibration of the building, T, with the following relationship:

$$C = \frac{0.05}{\sqrt[3]{T}} \tag{4}$$

with the restriction that,

$$C \leq 0.10 \tag{5}$$

Therefore the computation of the natural period of vibration of the building requires attention; hence the discussion will start with an analysis of T.

In the absence of technical data and rigorous calculation the Uniform Building Code recommends the following formulas for T:

$$T = 0.1N, \quad \text{for ductile frames} \tag{6}$$

$$T = \frac{0.05h_n}{\sqrt{D}}, \quad \text{otherwise} \tag{7}$$

where

h_n = height of the building (in feet)

D = dimension of the building in a direction parallel to applied forces (in feet)

N = number of stories

T = period of vibration (in sec.)

Following the Long Beach, California, earthquake of March 10, 1933, the United States Coast and Geodetic Survey measured the period of 212 (pre-1940) buildings (45), and obtained Equations 6 and 7 on this basis. However, the form of the formulas has some theoretical justification as well (46). Equation 6 is valid only for a building in which the lateral resisting system consists of a moment resisting space frame, which resists 100% of the required lateral forces and the frame is not inclosed nor adjoined by more rigid elements, which tend to prevent the frame from resisting lateral forces. Equation 6 does not involve the depth D nor the breadth B, because (a) the stiffness of each floor is a function of the number of columns on that floor which in turn is proportional to the floor area BD, (b) the mass of each story is also proportional to the floor area, and (c) the two effects are said to cancel each other in the calculation of the natural period T (46).

For a shear-wall type structure with equal story heights and stiffnesses, the Rayleigh method of computing the natural period yields:

$$T = 2\pi\sqrt{\frac{M}{k}}\ (0.63N) \tag{8}$$

where

M = mass of the floor

k = story stiffness

It is to be noted that the story stiffness is a function of the product of wall stiffness per unit length and the wall length.

Therefore, Equation 8 can also be written in the form:

$$T = 2\pi\sqrt{\frac{M}{k_w}}\ \frac{(0.45N)}{\sqrt{D}} \tag{9}$$

where

k_w = wall stiffness per unit length.

Consequently, Equation 7 can be a reasonable formula if the ratio M/k_w is a constant for all shear wall buildings. Some other suggestions for calculating the natural period are summarized below (46).

Assume that the first mode of vibration controls the motion and that the displacement of the building in this mode is given by:

$$y = Cf(x)\ \sin\frac{2\Pi t}{T} \tag{10}$$

where,

C = a constant

$f(x)$ = mode shape

Using Rayleigh's method and taking the static deflection curve as the mode shape, we obtain,

$$T = \left(2\Pi\ \frac{\Sigma y_n^2}{g\Sigma y_n}\right)^{1/2} \tag{11}$$

where

y_n = deflection produced by a 1g lateral load.

For shear-wall type buildings, the stiffness and mass are assumed to be:

$$k = 2D\ k_w \tag{12}$$

$$M = BDm_f + 2hm_w\ (B + D) \tag{13}$$

where,

m_w = average mass of wall per unit area

m_f = mass per unit floor area

h = story height

This leads to an equation of the type:

$$T = C_1 N \sqrt{B} = C_1' H \sqrt{B} \tag{14}$$

where, C_1 = a constant

C_1' = a constant

H = total height.

For shear-beam buildings, the mass is assumed to be:

$$M = m_f BD \tag{15}$$

For the stiffness of space frame buildings, three assumptions can be made. The story stiffness can be assumed to be proportional to the weight above the story. This leads to:

$$k_n = k_N (N-n+1) \tag{16}$$

and

$$K_n = k_n BD \tag{17}$$

where

K_n = effective stiffness of the nth story in a building of N stories

k_n = stiffness per unit area of the nth story

k_N = stiffness per unit area of the Nth story

Using the relationship for the displacement of nth floor subjected to a unit load and Equations 15, 16, and 17, we have,

$$T = 2\pi\sqrt{\frac{m_f}{k_N}}\,(0.8\sqrt{N}) = C_2\sqrt{N} = C_2' H \tag{18}$$

If, however, the assumption is that the stiffness, k, of the story is the same and is proportional to the total number of stories, i.e.,

$$k = k_n BDN \tag{19}$$

Again we have,

$$T = 2\pi\sqrt{\frac{m_f}{k_N}}\,(0.63\sqrt{N}) = C_3\sqrt{N} = C_3'\sqrt{H}. \tag{20}$$

The third assumption is that the story stiffnesses are the same and independent of the total number of stories.

Then,

$$k = k_N \, BD \tag{21}$$

and

$$T = C_4 N = C_4' H \tag{22}$$

Considering the Uniform Building Code, the assumptions underlying Equations 6 and 7 can now be better understood. From Equation 9, it can be seen that Equation 7 would give reasonable results only if the mass to wall stiffness per unit length ratio of all shear wall buildings were constant. Equation 6 assumes the relationship expressed by Equation 21. Both of these assumptions are very restrictive. In fact, for shear-wall type buildings, Equation 7 as given in the Uniform Building Code gives the poorest fit to actual data comparing with Equations 18, 20, and 22 (46). It is probably too much to expect from a simple relationship to give reasonable results for a variety of buildings. Therefore, recommending different empirical formulas for different assumptions about the mass and stiffness distribution of the building should lead to more reasonable results.

A plausible approach to this problem has been presented by Salvadori and Heer (48), who proposed to calculate the natural period by combining the effects of shearing deformation, bending deformation, rocking motion as well as foundation translational motion. For the Alexander Building in San Francisco, their calculations for the natural period of vibration of the building had errors of only between 4.5% - 8.7%. By using extreme values of all parameters for normal buildings, the authors plotted two curves which enveloped all the possible periods for these structures. This method could be adapted to give a range of value for T rather than a single value, and thus improve the validity of empirical formulas.

Since the response is given as an explicit function of the period of the structure, Equation 4 suggests the use of a response spectrum technique in its derivation. Therefore the following background information on response spectrum technique is presented (49).

For a multi-degree-of-freedom system and using the elastic modal superposition, we have the following governing equation:

$$Y_n + 2_n \lambda_n \omega_n Y_n + \omega_n^2 Y_n = \frac{P_n^*}{M_n^*} \tag{23}$$

in which,

Y_n = amplitude of mode "n"

λ_n = damping ratio of mode "n"

ω_n = natural frequency of mode "n"

also,

$$M_n^* = \sum_{i=1}^{N} \phi_{in}^2 M_i \tag{24}$$

$$P_n^* = \sum_{i=1}^{N} \phi_{in} P_i \tag{25}$$

where,

ϕ_{in} = n^{th} mode shape.

M_i = i^{th} mass

P_i = i^{th} force

In the earthquake response problem, let

$$P_n^* = L_n \ddot{V}_g(t) \tag{26}$$

where,

$\ddot{V}_g(t)$ = ground acceleration,

and

$$L_n = \sum_{i=1}^{N} M_i \phi_{in} \tag{27}$$

The solution for each mode can be written in the form of the Duhamel integral, or for earthquake response,

$$Y_n(t) = \frac{L_n}{M_n^* \omega_n} V_n(t) \tag{28}$$

where,

$$V_n(t) \equiv \int_0^t \ddot{V}_g(\tau) e^{-\lambda\omega(t-\tau)} \sin\omega(t-\tau) d\tau \tag{29}$$

We have the following relationships and definitions:

$$\text{"displacement"} \quad v(t) = \frac{1}{\omega} V(t) \tag{30}$$

"base shear," $Q(t) = Kv(t) = M\omega^2 v(t) = M\omega V(t)$ (31)

"spectral density," $S = V_{max}$ (32)

"spectral displacement," $S_d = \frac{S}{\omega}$ (33)

"spectral acceleration," $S_a = \omega S$ (34)

The effective acceleration is given by:

$$\ddot{Y}_n(t) = \omega_n^2 Y_n(t) = \frac{L_n}{M_n^*} \omega_n V_n(t) \tag{35}$$

Thus the force at level "i" is

$$F_{in}(t) = M_i \ddot{V}_{in}(t) = M_i \phi_{in} \ddot{Y}_n (t) = M_i \phi_{in} \frac{L_n}{M_n^*} \omega_n V_n(t) \tag{36}$$

Then, the base shear becomes,

$$Q_n = \sum_{i=1}^{N} F_{in} = \sum_{i=1}^{N} M_i \phi_{in} \cdot \frac{L_n}{M_n^*} \cdot \omega_n V_n(t) = \frac{L_n^2}{M_n^*} \cdot \omega_n V_n(t) \tag{37}$$

The maximum base shear is obtained using response spectrum superposition:

$$Q_{n_{max}} = \frac{L_n^2}{M_n^*} \omega_n S_{vn} = \frac{L_n^2}{M_n^*} S_{an} \tag{38}$$

Since the maxima of each mode do not occur simultaneously, the following approximation is made to find the total maximum.

$$Q_{max} \cong \sqrt{Q_1^2 + Q_2^2 + \ldots + Q_N^2} \tag{39}$$

The above procedure is based on modal superposition and the effects of all modes of vibration are taken into account. However, the Uniform Building Code formula for the coefficient C is based on a more approximate method which only considers the first mode of vibration. It also assumes that the first vibration mode shape is a straight line from the base to the top. For this method:

$$L = \sum_{i=1}^{N} M_i \phi_i \tag{40}$$

$$M^* = \sum_{i=1}^{N} M_i \phi_i^2 \tag{41}$$

which results in:

$$Q_{max} = \frac{L^2}{M^*} \omega S_v \tag{42}$$

There is little apparent resemblance between Equation 42 and the Uniform Building Code formula except for the dependence on the natural period (or frequency) of the building. This is a result of empirical considerations in the derivation of Equation 4. Another difference is that Equation 42 is a result of purely elastic analysis; therefore it gives much higher values than that specified in the Uniform Building Code. A good discussion on how to incorporate ductility and therefore the energy absorption capacity of buildings into response spectrum techniques can be found elsewhere (50). It is to be noted that the inelastic action which is taken into account in the coefficient C is further modified by another factor K depending on the type of structure according to its ductility.

1.2 The factor K represents the effect of the type or arrangement of the resisting elements of the structure, and is an indication of the structure's overall ductility. In the Uniform Building Code, K varies from 0.67 for moment resisting ductile space frames to 1.33 for box systems and 3.00 for elevated tanks. In a dual bracing system where both shear walls and a moment resisting space frame (to take at least 25% of the total lateral force) are designed to carry the lateral loads, the factor K is taken to be 0.80. For all other buildings with normal ductility, the factor K is considered as being unity.

The difference between the elastic analysis and the Uniform Building Code recommendations is due to the effects of inelastic action, which the code has taken into account. Equations 4 and 5 already include a "ductility factor" which reduces the maximum response of the structure. This "ductility factor" is chosen for a normal building, where the lateral deflection is partly shear deflection and partly flexural. It is seen that for this type of building the "ductility factor" which is already incorporated in C is untouched (i.e. K = 1.00). However, when other types of resisting arrangements are used, then this "ductility factor" is modified by increasing or decreasing K. As an example, for a ductile moment resisting space frame, the ductility factor increases; therefore the K factor reduces (to reduce the base shear) to 0.67. For specially designed dual bracing systems in which the space frame can take 25% of the lateral load, K is equal to 0.80. For more rigid structures such as box systems, the ductility factor decreases and K increases to a value of 1.33.

When the structure in question is expected to act as statically determinate (e.g., elevated tanks), inelastic action results in collapse; therefore for these types of structures, K assumes its highest value of 3.00.

1.3 The numerical coefficients for the factor Z depend upon the seismic zone map of the United States as given in the Uniform Building Code. For locations in Zone No. 1 'Z' shall be equal to one-fourth. For locations in Zone No. 2 'Z' shall be equal to one-half. For locations in Zone No. 3 'Z' shall be equal to one.

Seismic zoning or regionalization of the United States is a very difficult task and is usually based on two kinds of observations. The first one is a statistical approach where the history of earthquakes in a given location acts as a guideline to estimate the intensity and the frequency of future earthquakes. The best source of data which is available concerning seismic activity in the United States is the observation of the frequency of small shocks for a given location. Unfortunately, a speculative correlation between the number of small shocks observed in a location and the maximum expected intensity of a future earthquake can be misleading (51). Because the interest of structural engineers lies with the maximum expected intensity, the use of small shock data in earthquake engineering is rather questionable. The second source of observation, on which the seismic regionalization can be based, is geological. Usually, both of these observations have been evaluated for seismic regionalization.

In the United States seismic regionalization was attempted in 1950 and 1951, when the "Seismic Probability Map of the United States" was prepared by Roberts and Ulrich (52, 53). Richter indicated, however, that this "was not strictly a regionalization map, since it was directed to estimate risk rather than maximum intensity" (51). Again quoting from Richter, "The Seismic Probability Map was officially retired in 1952, as 'subject to misinterpretation and too general to satisfy the requirements of many users.' This action was not taken in consequence of scientific criticism, but as a result of pressure from a business group interested in lower ranking in their community." It is interesting to note that this map is still being published, with a few revisions, as a part of the Uniform Building Code.

1.4 The Uniform Building Code recommendations for the distribution of the total lateral force V are as follows:

$$F_t = 0.004\ V \left(\frac{h_n}{D_s}\right)^2 \tag{43}$$

with the following restrictions,

$$F_t \leq 0.15V \tag{44}$$

and

$$F_t = 0, \text{ for } \frac{h_n}{D_s} \leq 3 \tag{45}$$

$$F_x = \frac{(V-F_t)W_x h_x}{\sum_{i=1}^{n} W_i h_i} \tag{46}$$

where,

F_t = force at the top level

D_s = the plan dimension of the vertical, lateral force resisting system in feet

F_x = force at level "x"

W_i, W_x = weight at levels "i" and "x", respectively

h_i, h_x = height at levels "i" and "x", respectively

Equations 43 through 46 are not applicable for one- or two-story buildings, for which the distribution is assumed to be uniform.

It can be seen from these above formulas that with the exception of the top story where the force is assumed to be a little greater, the distribution is linearly increasing from bottom to top when the weight and story heights are the same.

As it was discussed in item 1.1, only the first mode of vibration was considered in the determination of the factor C. The inverted triangular distribution is a simple extension of the first mode assumption. However, assuming the shape of the first mode to be a straight line is overly restrictive. Even though the formulas are to be used for any type of structural system, the straight line assumption can be considered to be good only for a combination of shear wall and space frame building. For all other types of structural systems, the first mode shape would definitely be represented by a curve. Furthermore, the first mode approximation holds only for relatively short buildings. For tall buildings the effects of the second and even the third mode of vibration should be included. The question of accuracy of the first mode approximation was considered by

Clough in 1962 (54), when the errors generated by the first mode approximation were computed for four- to twenty-story buildings subjected to 1940 El Centro, 1949 Olympia, and 1934 El Centro Earthquakes. The following conclusion were reached:

(a) The errors given by all approximations increase with the number of stories because the higher mode contributions become more significant as the period of vibration increases.

(b) First mode approximation shows the greatest discrepancies in total shear envelopes; because the higher modes are most effective in developing shears. First mode approximation does not even represent the shape of the envelope.

2. French Aseismic Code of 1964 (55)

The seismic coefficients (for masses along the height of the building) are given in the form of a product of four factors which are discussed as follows:

2.1 The intensity coefficient, α, is somewhat analogous to the factor Z of the Uniform Building Code as discussed in item 1.3. However, the intensity coefficient, α, is more closely related to the maximum response because it is a function of the Modified Mercalli Intensity scale. The reference intensity has been chosen as VIII, so that for the design intensity VIII, α is taken to be unity. It is assumed that α doubles whenever the intensity becomes one degree higher. This assumption is generally considered to be on the conservative side.

2.2 The response coefficient β depends on the natural period of vibration and the damping of the building, and is expressed by the following three empirical formulas depending on the assumed amount of damping:

(a) For common buildings and houses ("normal" damping)

$$\beta(T) = \frac{0.065}{\sqrt[3]{T}} \tag{47}$$

and

$$0.05 \leq \beta(T) \leq 0.10$$

(b) Buildings with large spans and lower density of partitions, or with partitions tied only in places ("medium" damping)

$$\beta(T) = \frac{0.085}{\sqrt[3]{T}} \tag{48}$$

and

$$0.065 \leq \beta(T) \leq 0.13$$

(c) Structures reduced to a skeleton with no external friction (low damping)

$$\beta(T) = \frac{0.095}{\sqrt[4]{T^3}}$$

and

$$0.06 \leq \beta(T) \leq 0.20 \tag{49}$$

The curves for Equations 47 and 48 are not exactly response spectra. Alterations have been made in order to take into consideration higher modes of vibration as well as possible plastic actions. For Equation 49, the curve is much closer to the true spectrum, so that it can be applied to flexible and slender structures, which require more explicit consideration of higher modes of vibration.

Obviously, $\beta(T)$ is a counterpart of C of the Uniform Building Code discussed in item 1.1. It can be seen that the French Aseismic Code gives comparatively more conservative values to $\beta(T)$ than the Uniform Building Code does to C. In fact, even Equation 48, which is for the highest damping, is still more conservative (by 30%) than the single formula given by the Uniform Building Code. An extreme difference of 100% is obtained in the case of a slender structure with low daming, since the upper limit of $\beta(T)$ is 0.20 for Equation 49 as opposed to 0.1 for C.

The use of $\beta(T)$ seems to be more versatile and adaptable than the coefficient C in the Uniform Building Code. The qualitative consideration of damping is also another parameter recognized by the French Aseismic Code. The effect of damping is recognized by the Uniform Building Code, but it is not explicitly included.

2.3 The foundation coefficient reflects the effect of (a) the soil conditions of the immediate location and (b) the characteristics of the building foundation. It varies from 0.8 for deep foundations on firm rock to 1.3 for piling in moist ground. Response curves corresponding to periods lower than 0.5 sec. are reduced when the soil of the foundation is a part of a very large geological formation of soft ground. It is to be noted that this coefficient is completely ignored by the present Uniform Building Code.

2.4 The distribution coefficient γ is the coefficient of the term $\Gamma(T)/g$ in Equation 50. It depends only upon the characteristics of the structure and describes the variation of the response with respect to the level considered.

For a system with concentrated masses M(Z) and distributed masses m(z), the seismic coefficient $\sigma(h)$ similar to the factor S defined by Equation 1 applicable to the masses situated at a height h above the base of the structure is given as:

$$\sigma(h) = \gamma \frac{\Gamma(T)}{g} = \frac{\Sigma M(Z)X(Z) + \int m(z)X(z)dz}{\Sigma M(Z)X^2(Z) + \int m(z)X^2(z)dz} X(h) \frac{\Gamma(T)}{g} \tag{50}$$

in which,

Z, z = ordinates of concentrated masses M and distributed masses m, respectively

X(h) = a function giving the shape of the fundamental mode

g = acceleration of gravity

$\Gamma(T)$ = the maximum response (acceleration) of a single mass oscillator of the same period T and damping as the fundamental mode.

This coefficient, γ, shows a basic difference between the French Aseismic Co and the Uniform Building Code. In the Uniform Building Code there is no "distribution coefficient" and the distribution of the lateral force is predetermined by an inverted triangular shape regardless of the height or the type of structure. The distribution coefficient γ in the French Code, however, varies from floor to floor not depending on a predetermined function but on the mass at each story and the shape of the mode (not necessarily linear) under consideration. In this respect the French Aseismic Code gives more freedom to the design engineer to use his judgment about the shape of the mode. In addition, the French Aseismic Code also requires the consideration of the first three modes of vibration for slender structures with low damping. This also seem to be a step in the right direction if the discussions of item 1.4 are considered.

3. Standards and Regulations for Buildings in Seismic Regions of the USSR 1957 (56)

The "Standards and Regulations for Buildings in Seismic Regions" is a long and involved document and the entire document

will not be discussed here. Only exerpts from sections I and III will be discussed. However, the titles of all sections are cited as follows for additional information:

I. Earthquake Zone of an Area or a Building Site and Design Rating of Buildings and Structures

II. Planning Cities and Towns

III. Seismic Forces for Residential, Civic, Industrial, and Farm Buildings and Structures

IV. Industrial and Civic Buildings and Structures

V. Water Works and Sewerage

VI. Highways and Railways

VII. Hydraulic Structures

VIII. Rural Structures

IX. Field Work and Control of Seismic Requirements

Seismic regionalization on the basis of maximum expected intensity began in the USSR as early as 1933 (51). In 1947, Gorshkov came up with the first seismic map of the USSR (57). Both statistical and geological data were used in the preparation of the map, but the map was criticized by Gubin on geological grounds (58). However, the map was adopted as it was with only minor alterations (59).

The seismic regions are established according to the GEOFIAN scale which is similar to the Modified Mercalli scale. The different degrees of intensity are defined according to the amplitude of a standard pendulum. Therefore, the divisions between different intensities in the GEOFIAN scale are more definite than those in the Modified Mercalli scale.

Both the Modified Mercalli scale and the GEOFIAN scale contain twelve divisions and they both descend from the Mercalli-Concani scale. The GEOFIAN scale can be summarized in Table 1.

In the USSR regulations, buildings are classified according to their use and importance as well as their location with respect to the seismic regionalization. Buildings are classified according to their use into four groups. As examples, Group I includes very large and important buildings, large radio stations, etc. For this group the design intensity is taken to be one degree higher than that indicated on the seismic regionalization map. Group IV consists of temporary light buildings. For this group the design intensity is given to be six regardless of the location.

TABLE 1

GEOFIAN Scale for Earthquakes (56)

Division	Qualitative Description of Earthquake	Instrumental Amplitude Range	Instrumental Amplitude Mean
6	Strong	1.1 to 2.0 mm	1.6 mm
7	Very strong	2.1 to 4.0 mm	3.0 mm
8	Destructive	4.1 to 8.0 mm	6.0 mm
9	Devastating	8.1 to 16.0 mm	12.0 mm
10	Annihilating	16.1 to 32.0 mm	24.0 mm
11	Catastrophic	> 32.0 mm	---
12	Greatly Catastrophic	---	---

The design seismic force at any point with a mass Q_k is given by using the following formula:

$$S_x = Q_x k_c \beta \zeta_x \tag{51}$$

in which,

Q_x = dead load plus 0.8 times the live load at point x (except for warehouses, etc., where the live load is not reduced).

k_c = seismic coefficient. For design rating 7, $k_c = 1/40$; for 8, $k_c = 1/20$; and for 9 $k_c = 1/10$.

β = dynamic coefficient.

$$\beta = \frac{0.9}{T} \tag{52}$$

and $0.6 < \beta < 3.0$

ζ_x = a coefficient which is a function of the deformation curve resulting from free vibration, and of the position of the load Q_x within the structure.

$$\zeta_x = \frac{q(x_k) \sum_{j=1}^{n} Q_j q(x_j)}{\sum_{j=1}^{n} Q_j q^2(x_j)} \tag{53}$$

where $q(x_k)$ and $q(x_j)$ represent deflections at levels k and j, respectively.

For normal buildings, only the first mode of vibration need be considered. For slender and flexible structures, the first three modes must be considered, and β is multiplied by 1.6.

The similarities between the above regulations and the French Aseismic Code are quite obvious. Therefore, comparisons and discussions which were carried out in section 2 will not be repeated here.

One of the strongest points of the USSR Regulations is the attention given to seismic regionalization. The use of the more exact GEOFIAN intensity scale and the combined statistical and geological data have resulted in a more scientifically sound seismic regionalization.

Another new item in the USSR Regulations is the incorporating of the use or the importance of the building into the seismic coefficient k_c. It seems to be desirable to reduce the risk where the building is massive and important and its collapse may involve a great consequence in the loss of life and money.

4. Aseismic Provisions for the Federal District, Mexico (60)

According to the subsoil characteristics, the Federal District is divided in two zones in accordance with the chapter on foundations. In addition, buildings are classified into three groups, A, B, and C, depending on their use and importance. Group A includes government municipal and public buildings, hospitals, museums, schools, stadiums, etc. Group B consists of private habitation and public places where conglomerations of people are uncommon (office and industrial buildings, restaurants, gas stations, etc.). Isolated and unoccupied buildings are in Group C. Another classification is made according to the structural characteristics. Three types are thus classified. Type 1 calls for framed structures in which the frame is designed to take 50% of the shear. Type 2 consists of structures where the frame is designed to take 25% of the shear. Type 3 includes water tanks, chimney stacks, one column structures, etc.

The base shear coefficients are tabulated for group B buildings. According to the zone and the type of structure, this coefficient varies between 0.15 and 0.04. These tabulated values are to be multiplied by 1.3 for group A buildings. On the other hand, Group C buildings (mostly unoccupied, isolated buildings) do not require aseismic design.

Three different types of analyses are suggested: simplified, static and dynamic. The selection of the method depends on the problem at hand and is described in detail in the code. The simplified method includes buildings with height to length ratio of not more than 1.5 plus other requirements. The static analysis also assumes the inverted triangular shear distribution along the height of the building.

For dynamic analysis a coefficient "a" is defined as follows:

(a) In the zone of low compressibility,

for $T \leq 0.5$, $a = 1.0$ (54)

for $T > 0.5$, $a = \frac{0.5}{T}$ (55)

(b) In the zone of high compressibility,

for $T \leq 1.0$, $a = 0.5(1+T)$ (56)

for $2.5 \geq T \geq 1.0$, $a = 1.0$ (57)

for $T \geq 2.5$, $a = \frac{2.5}{T}$ (58)

For the purpose of computation, it is assumed that the ground suddenly undergoes a constant acceleration, which is equal to (ag) times the base shear coefficient. It is to be noted that the dynamic analysis requires the use of the root-mean-square of the responses corresponding to each mode.

One of the new items in this code which has not been covered so far in the codes already mentioned is the suggested three methods of analysis for different conditions. This permits the mode to be simple for simple cases and elaborate and not too simplistic for buildings which require special attention. Another new point is the superposition of the modes of vibration of the structure by the root-mean-square method. This criterion is based on probability analyses (61).

5. Earthquake Resistant Design Provisions of Other Countries

In the preceding sections 2 through 4, earthquake regulations of France, the USSR and Mexico were discussed and compared with the recommendations of the Uniform Building Code. In this section the provisions of the building codes of four more countries (Romania, Japan, Turkey, Israel) will be mentioned. However, a lengthy discussion or a comparison is not undertaken because the recommendations of these codes are qualitatively covered in the three codes which have been

mentioned already. The main reason to choose the codes discussed in sections 2 through 4 as a basis for comparison with the Uniform Building Code was due to their uniqueness as well as their accessibility.

The following will then be a summary of the earthquake regulations of the aforementioned countries concerning the total lateral force and its distribution.

5.1 Romanian Regulations for Aseismic Design* (62)

The shear force S is the product of the total weight Q and the seismicity coefficient c. This seismic coefficient is a product of five parameters.

$$c = k_s \cdot \eta \cdot \beta \cdot \delta \cdot \psi \tag{59}$$

with the restriction,

$$c \leq 0.02 \tag{60}$$

where,

k_s = indicates the seismic intensity based on the Modified Mercalli scale

η = depends on the soil resistance

β = a dynamic coefficient indicating the response spectrum (acceleration)

$$\beta = \frac{0.9}{T} \tag{61}$$

δ = a correction coefficient compensating for the single degree of freedom assumption and given by,

$$\delta = \frac{[\sum_1^h Q_k q_k]^2}{\sum_1^n Q_k \sum_1^h Q_k^2} \tag{62}$$

where,

n = number of stories

h = story under consideration

q_k = the deflection of the multi-degree of freedom system for the considered mode of vibration

Q_k = the mass of the considered point of deflection

*Note: These regulations were established taking into consideration the different codes used in other countries, especially the USSR and USA.

ψ = a measure of damping

The distribution of this shear force is given by:

$$S_k = S \frac{Q_k q_k}{\sum^{n} Q_k q_k} \tag{63}$$

where q_k is the deflection of the point k under the Q_k loadings. For simplification, q_k can be taken as a parabola.

5.2 Some Recommendations from the Japanese Building Code (63)

The seismic force is calculated by multiplying the sum of dead and live loads with the coefficient of horizontal force, which is not an explicit function of period T. It is taken to be 0.2 for buildings up to 16 meters, then 0.01 is added to the coefficient for each additional 4 meters above 16 meters. For wooden buildings on soft soil, water tanks and projected chimneys, this coefficient is increased to 0.3.

A reduction coefficient which can be used to reduce the coefficient of horizontal force is designated for combination of type of construction and the soil conditions.

Table 2

Reduction Coefficients in Japanese Code (63)

Type of Const. / Kind of Ground	Wood	Steel	R/C or Composite
Kind I*	0.6	0.6	0.8
Kind II**	0.8	0.8	0.9

* Kind I: Ground consisting of rock, hard sandy gravel, etc. classified as Terhary or older strata over a considerable area around the structure.

**Kind II: Ground consisting of sandy gravel, sandy hard clay, loam, etc. classified as diluvial, or gravelly alluvium, about 5 meters or more in thickness, over a considerable area around the structure.

Another table is provided to reduce the coefficient further either by 0.8 or by 0.9 depending on the location. This reduction is a result of seismic regionalization.

5.3 Regulations for Construction in Regions of Hazard in Turkey (64)

The total seismic coefficient C is made up of four different coefficients.

$$C = C_o a \beta \psi \tag{64}$$

C_o = coefficient of seismic regionalization and can take on three different values

a = a coefficient of foundation soil conditions

β = coefficient of building importance

ψ = dynamic coefficient

$$\text{for } T \leq 0.5, \quad \psi = 1.0 \tag{65}$$

$$\text{for } T > 0.5, \quad \psi = \frac{0.5}{T} \tag{66}$$

and

$$\psi \leq 0.3 \tag{67}$$

The story weights are calculated according to the formula:

$$W_i = G_i + nP_i$$

W_i = total weight of story "i"

G_i = dead load of story "i"

P_i = live load of story "i"

n = is taken to be 1 for public buildings and 0.5 for others.

The total shear is to be distributed according to the following formula:

$$F_i = F \frac{W_i h_i}{\sum_1^n W_i h_i} \tag{68}$$

F_i = force at story "i"

h_i = height of story "i"

5.4 Israel Standard Code Seismic Loads on Buildings (Proposed) (65)

The total horizontal force H is calculated by:

$$H = C(G + \alpha P) \tag{69}$$

where,

C = depends on the seismic region (there are two regions), type of building and foundation soil condition (six combinations are considered)

G = dead load

P = live load

α = varies from 0 to 1 depending on the importance of the structure

Overturning Moment

The following are the recommendations of the 1967 Uniform Building Code (without revision) concerning the computation of the overturning moments. For the base of the structure the overturning moment M is given by:

$$M = J(F_t h_n + \sum_{i=1}^{n} F_i h_i) \tag{70}$$

$$J = \frac{0.5}{\sqrt[3]{T^2}} \tag{71}$$

and

$$0.3 \leq J \leq 1.00 \tag{72}$$

The overturning moment at level "x":

$$M_x = J_x[F_t(h_n - h_x) + \sum_{i=x}^{n} F_i(h_i - h_x)] \tag{73}$$

$$J_x = J + (1-J)\left(\frac{h_x}{h_n}\right)^3 \tag{74}$$

where all the symbols have been defined in section 1 of this Part.

It can be seen that the overturning moments are calculated statically from the story shear forces reduced by the factor J_x. The reduction factors J_x and J for the overturning moment at the base decrease as the period of vibration

of the building increases (or as the building gets taller and more slender). Assuming that T = 0.1N, where N is the number of stories of the building, we get the following approximate relationships: J = 1.0 for a three-story building, J = 0.5 for a ten-story building and J = 0.33 for an eighteen story building. Justifications for this reduction for more slender buildings are qualitative in the following manner (47, 50). For low buildings (i.e., short T) the assumption that the structure acts as a cantilever is a good one. But the application of this concept to tall buildings is overconservative, because maximum total moments on a section through the building and maximum shears on such sections do not occur for the same combinations of modal components. The formula for the distribution of overturning moment provides simply for a straight line variation of moment from M at the base to zero at the top, even though this is not consistent with the static moments computed from the lateral force distribution. However, it is reasonable for slender structures (50).

Another point made is that no tall building has been observed to simply "topple over" during an earthquake, whereas for shorter buildings this is not an uncommon occurrence. However, this argument cannot be considered valid because the number of observations on tall building behavior during earthquakes is still limited. In addition, the fact that no tall building was observed to have "toppled over" does not mean that the overturning moments do not excite a different form of failure.

The following comments against the code formulas were made by Derrick in his commentary to Reference 47. "In the SEAOC proposal the reduced values F_x are used to compute M, the overturning moment. The true value of M is the sum of the moments of the equivalent inertia forces capable of producing the maximum dynamic distortion at maximum response. The results of both F_x and F_{xn} act at approximately 2/3 of the height of the structure and from the ratio V_n/V the F_xh_x is $(V_n/V)(F_{xn}h_x)$, where F_{xn} denotes the distributed shear of nth mode on x^{th} story. Hence to evaluate the maximum overturning moment from F_x, there should be a correction factor increasing the F_xh_x by (V_n/V). The SEAOC proposal provides a correction factor, but it decreases the summation by $J = 0.5/\sqrt[3]{T^2}$ with limits 1.0 to 0.33." Derrick further comments that the factor J moves the position of ΣF_x to a point lower than what the triangular distribution assumption would predict. As an example, the triangular distribution assumption would place the resultant at 2H/3. However, with the use of J-factor, the resultant is placed at 2H/9. Derrick claimed that this difference was not adequately ex-

plained.

In the above discussion the subscript "n" refers to the dynamic analysis response and therefore V_n/V is the ratio of the shear value obtained from a dynamic analysis to the recommended SEAOC value.

In an analytical study, Bustamante calculated the ratios of dynamic to static overturning moments (66). Both direct addition and the root mean square values of the spectral superposition were computed. These ratios were then compared with the J factor of the Uniform Building Code. It was concluded that there could be large differences between the computed ratios and the reduction factor of the SEAOC code. The errors involved could be as large as 100% even for moderate periods of less than 0.35 seconds. However, all errors were found to be on the safe side. The considerations were formulated by assuming that there were no errors in the estimation of T. As far as the distribution was concerned, the author concluded that "the ratio of static to dynamic moments increases pronouncedly towards the top. The tendency is opposite the one concerning shears, for which the larger factors of safety were at the bottom." This study by Bustamante seems to confirm the qualitative arguments of Clough (47) and Blume, Newmark and Corning (50).

The foreign building codes which were cited previously do not provide any regulations with regard to the overturning moment. Therefore it is assumed that the overturning moments are simply calculated from the story shears for which the provisions of these codes are extensive. One exception is the "Aseismic Provisions for the Federal District of Mexico" (60). But even in this code the provisions for overturning moment are restricted to the "Static Analysis," and are as follows: "For design purposes, overturning moments may be reduced, but for each frame or group of resisting elements it shall not be taken smaller than the design shear at the elevation considered times its distance to the center of gravity of all the corresponding masses above."

Torsion

In the Uniform Building Code, the static method is used to calculate torsional moments. Torsional analysis is required for all buildings (symmetrical buildings included) and the minimum design eccentricity is to be taken as 5% of the maximum dimension of the building.

Considering the importance of the problem, the Uniform Building Code treats it very lightly. Four references will

be cited below about the problem of torsion and some of their important observations and conclusions will be quoted. One area of unanimous agreement is the importance and the potential danger of the problem.

The following observations were made by Housner and Outinen (67):

> "It has been shown that an unsymmetrical building undergoing torsional accelerations can be expected to sustain stresses in the more flexible wall that are higher than those predicted by the customary static method of analysis. If the objective of the design is to keep the maximum stress within the usual allowable limits, a correction should be made to the results given by the usual static method of analysis. On the other hand, if the objective of the design is to provide a certain ultimate strength, the relative rigidity of the wall is not so important a factor as is the ability of the wall to absorb energy."

Similar and additional conclusions are also reached by Bustamante and Rosenblueth (68).

> "(a) Dynamic eccentricity may exceed statically computed values. The excess is particularly important when the polar moments of inertia are close to their critical values; these are attained within the range of usual characteristics of buildings.
>
> Dynamic eccentricity is defined as follows:

$$e_d = \left(\frac{M_1^2 + M_2^2}{V_2^2 + V_2^2}\right)^{1/2} \tag{75}$$

where,

M_i = torsional moment in the ith mode

V_i = shear force in the ith mode of a statically eccentric structure.

> "(b) A rough estimate of torsional dynamic effects in multistory buildings can be obtained from the response of a single story structure with similar characteristics.
>
> "(c) Excessive dynamic eccentricity due to closeness to the critical polar moments of inertia can be greatly reduced by changing the critical values.

This may be accomplished by increasing the stiffness of perimetral frames."

From these observations we can conclude that the "core" type buildings where the resisting elements are not at the perimeter of the structure are more susceptible to failure due to torsion. This effect should be incorporated in the recommendations of building codes.

Shiga (69) concurs with Bustamante and Rosenblueth in the equivalent single story concept and further adds that "It should be noted that when the torsional rigidity is less than the lateral rigidity the fundamental vibration is rotational and torsion is serious; when the building is not rigid against the ground motion, the torsion becomes more serious and vibration pattern becomes more complicated." In addition Skinner, Skilton and Laws add that close torsional and translational periods are dangerous (70).

In nearly all the foreign codes cited before, torsion is passed rather lightly and in none is a dynamic torsion analysis suggested. In the Aseismic Provisions for the Federal District of Mexico (60), the torsional eccentricity is to be calculated for all stories and the design eccentricity is to be 1.5 times the computed value plus or minus 0.05 times the maximum plan dimension. The 1.5 factor is based on Housner's suggestions for single story buildings (67).

Allowable Stresses

The ACI and AISC codes provide for a 33-1/3% increase in the allowable stress of concrete for dynamic loads (wind and earthquake) in Article 1004 (71). This increase in the allowable stresses (both concrete and steel) has also been adopted by the Uniform Building Code. Also for the ultimate strength design of ductile reinforced concrete moment resisting space frames in seismic zones 2 and 3 the ultimate capacity U is given by:

$$U = 1.40\ (D + L + E) \tag{76}$$

or

$$U = 0.90\ D + 1.25\ E \tag{77}$$

whichever is greater. Where,

D = dead load

L = live load

E = earthquake load

U = ultimate capacity

The building codes all around the world have recognized the increase in the allowable stresses of different materials under temporary loads such as wind and earthquakes. However, the magnitude of the increase varies greatly.

In the USSR a service factor of 1.4 is given for steel and wooden structures, 1.2 for reinforced concrete and 1.0 for prestressed concretes. A service factor of 1.4 is equivalent to increasing the design strength of the member by 40% (from translator's note in Reference 56).

In Turkey the increase in the allowable stresses depends on the foundation soil conditions. The increase is taken to be 50% for good conditions and 30% for fair foundation soil conditions. If the conditions are poor, no increase is permitted (64). The Japanese Building Code allows an increase in the allowable stresses of 50% for reinforcing steel and 100% for concrete in reinforced concrete (63). Also, increases of 50% for structural and reinforcing steel and timber, and 33% for concrete are recognized by the Mexican provisions (60).

An interesting treatment of this problem is in the French Aseismic Code. There is an option of choosing one of the two methods suggested. The so-called "ordinary method" increases the allowable stresses for steel, reinforced concrete and prestressed concrete by 50%. The second method is the "state limit" method. State limit is defined as the state beyond which the construction can no longer remain appropriate for normal use. This corresponds to the elastic limit for steel structures and to the yielding of the steel. The seismic forces corresponding to the nominal design intensity I_N are considered as being normal loads, and those corresponding to $I_N + 1/2$ as being exceptional ones. In addition, for various types of loads, different "ponderation coefficients" are assigned to the capacity of the structure for inelastic action. This way, a more uniform design can be obtained (55).

In all cases, the increases in the allowable stresses apply only to the combination of gravity and lateral loads. Also, they are given for working-stress designs. In fact, this is the basic reason for the inconsistencies in the increase permitted for the allowable stresses by the above cited codes. Blume, Newmark and Corning explain these differences as follows (50):

> "It can be shown that the reserve capacity of structural members to resist severe earthquake loading will depend upon the additional stress requirement to cause yielding and failure. Thus

a bracing member proportioned only for earthquake stress (no dead or live load participation) will tend to have less reserve strength and energy capacity than a member the size of which is determined by dead and live load requirements as well as earthquake. The result is an inconsistency in the strength provided for different members in the same structure, when a working stress or even a modified working stress design procedure is used. A design procedure based on ultimate strength or energy capacity would avoid these inconsistencies."

Other Considerations

1. Lateral Force on Parts of Buildings and Appendages

The Uniform Building Code formula for the "Lateral force on parts or portions of buildings and other structures" which is designated by F_p is as follows:

$$F_p = ZC_pW_p \tag{78}$$

where

W_p = the weight of the portion under consideration

C_p = a coefficient given by Table 23-D of the Uniform Building Code and which varies from 1.00 for "cantilever parapet and other cantilever walls (except retaining walls)" and "exterior and interior ornamentations and appendages" to 0.10 for "tank plus effective mass of its contents, when resting on the ground."

This formula is analogous to the base shear formula, but the coefficient C_p is always equal to or greater than C. Since there is no dependence of C_p on the period of vibration Equation 78 is a result of an equivalent static analysis.

The problem of appendages on buildings is often a serious one. The attached portions are subject to the acceleration of the building which in general is much larger than ground acceleration. Normally all such appendages have different periods of vibration; therefore a rigorous analysis is generally very difficult. Another factor is that usually these appendages are statically determinate and therefore cannot develop much ductility, which is desirable in buildings designed to resist earthquakes. This is why a factor which is equivalent to K in Equation 2 has been omitted from Equation 78.

The following conclusions are drawn about appendage behavior during earthquakes by Penzien and Chopra (72).

(a) The two-degree-of-freedom method of analysis quite accurately predicts the maximum dynamic response of an appendage attached to the top of a multi-story building even when its period of vibration coincides with the fundamental period of the building; thus, the availability of two-degree-of-freedom response spectra makes this method of analysis practical.

(b) The single-degree-of-freedom method of analysis becomes considerably in error when the period of the appendage is near the period of one of the lower building modes; therefore, this method of analysis should not be used in such cases.

(c) To greatly reduce the seismic forces in an appendage, it should be designed so that its period of vibration differs considerably from the first mode of vibration of the building and also does not coincide with other lower building modes.

(d) The seismic forces developed in an appendage, even when designed in accordance with the recommendations of (c) above, are larger than code values; therefore, the desirable effects of inelastic deformations must be considered as a standard practice in the design of buildings.

With regard to this point there is not much that the other international codes would contribute since their provisions are generally in the same form as that of the Uniform Building Code. The Turkish Code gives a coefficient as a function of, in fact three times, the coefficient C for the whole building (64). The code of Israel specifies a coefficient C_p which can be as much as twenty times the coefficient C of the whole building, although the relationship is not explicit (65). The USSR provisions specify the product $\beta\eta$ (see section 3 on page 18 for definitions) to be at least five for portions of buildings and attached structures.

2. Effect of Vertical Seismic Force

In most building codes, including the Uniform Building Code, the effects of the vertical seismic force are totally neglected. Generally the reasons given for ignoring the vertical acceleration are: (a) the large safety factor (1.65) which the building is designed for in the vertical

direction, (b) the general acceptance of the fact that the vertical acceleration is considerably less than the horizontal accelerations, and (c) rareness of failure due to vertical acceleration from observations at the field. All these reasons, however, are delusive and at best simplistic. It is true that most building structures are very stiff in the vertical direction and for this reason the dynamic amplification in the vertical direction is almost negligible. Therefore, the axial stresses in the vertical members usually are not critical owing to the factor of safety. However, the main problem is not one of stresses but one of dynamic instability. The general notion that the vertical acceleration is considerably less than the horizontal acceleration is also false. Obviously the average intensity of the vertical and horizontal components of an earthquake depends upon the relationship between the epicentral distance to the seismograph and the focal depth. If these two distances were equal, the average intensities of the vertical and horizontal components would also be equal. Therefore the problem is magnified especially for deep focus earthquakes. However, this relationship can only be assumed for the average intensities. For the extreme values of acceleration such simple relations do not hold, since the vertical and horizontal accelerations are actually components of a random process. From the data that he takes from Housner (73), Despeyroux points to this fact in the following way (55):

> "At a distance of 48 km. from the epicentre of the El Centro earthquake of 1940, the focal depth being 24 km., the maximum vertical acceleration was 60% of the maximum horizontal one; and for the Taft earthquake of 1952, it was also 60% at a distance of 64 km. for the same focal depth of 24 km."

Therefore, if the failure criterion is taken to be the maximum value and not the cumulative effect, then even for shallow earthquakes and considerable epicentral distances the vertical acceleration cannot be neglected in comparison to the horizontal component. In fact, for a stability analysis the maximum value would be the failure criterion. As for the lack of evidence that failure does not occur due to vertical acceleration, the following argument can be presented. First, the amount of data that we have about the failures during earthquakes is not enough to exclude any mode of failure. Second, the evidence of the Good Friday earthquake of Alaska as well as that of Venezuela are sufficient to conclude exactly the opposite (74). However, even if there were no evidence whatsoever about stability failure due to vertical earthquake acceleration, this problem still cannot be discarded. The problem is a coupled one

between flexural lateral deflection and axial vibration. Therefore the transfer of energy from one mode to the other is quite probable. In fact, it has been shown that the energy of one mode can almost be totally transferred into the other one (75). Therefore, a seemingly lateral failure can actually be induced by vertical forces. This is nothing new, and it is a natural outcome of any coupled system.

As was mentioned earlier, most codes do not include any provisions for vertical forces. Two exceptions are the Romanian and the French Aseismic Codes. The recommendations of the Romanian regulations for aseismic design read as follows (62): "Besides horizontal forces, supplementary vertical forces are also taken into account by increasing the vertical forces from 25 to 100 per cent according to the degree of intensity." Also the recommendations of the French Aseismic Code are given as (55):

> "Vertical and horizontal seismic forces are considered to act simultaneously. As the maximum horizontal and vertical responses are not likely to occur at the same time, the vertical component taken into account is only a part of the maximum response. However, for structural elements in which lateral forces are of no concern, like vaultties and cantilevers, higher values might be imposed, with the idea of penalizing projected facades and so on."

PART III: POSSIBLE IMPROVEMENTS IN CODE FORMAT

General

The Uniform Building Code (8) specifies equivalent static loads for design. Although these static loads are based on dynamical considerations, discrepancy in their accuracy is noted in many areas. For example, Kanai and Yoshizawa (76) have shown that the equation given in the building code for the natural period of vibration of buildings is not in agreement with experimental results. Similar conclusions have been drawn by Arias and Husid (77). Blume (12) has shown that the cumulative displacement due to dynamic loads could critically exceed static load deflections specified by codes. A review of existing codes as given in Part II indicates that the Uniform Building Code takes into account the dynamic effects in a different manner from French and other codes. At the present time, there is a considerable amount of research work being done in earthquake engineering (78). Nevertheless, the current research programs are primarily concerned with specific problems. It seems to be desirable to review the philosophies behind these design codes.

In Part III, some design philosophies including the concept of structural control are examined in the context of earthquake engineering. Design methodologies are also reviewed with emphasis on the design tree and the fully systematic method, which are later applied to the formulation of a new code format. Moreover, the expected problems in the implementation of the proposed code format are listed along with an outline of necessary procedures for their solutions.

Some Design Philosophies and Methodologies

According to Asimow (79), there are the following three parts in a philosophy of engineering design:

1. a set of consistent principles and their logical derivations,
2. an operational discipline leading to action, and
3. a feedback mechanism measuring the advantages, detecting weakness, and pointing out the direction for improvement.

The principles and their logical derivations being used in support of existing design codes are primarily deterministic in nature. Whenever it is possible, dynamic effects are represented by "equivalent static forces." Moreover, the seismic structures are designed to remain elastic or nearly so under the influence of moderate and frequent earthquakes, but to allow local yielding without collapsing under severe and infrequent earthquakes (50, 80). However, the effect of repeated plastic deformation is known to be cumulative (81, 82). Recently, it was found to be possible to have low-cycle fatigue failure of structures under earthquake loads (18).

The relations between pure research, applied research, and practical applications are given by the Committee on Earthquake Engineering Research as shown in Figure 1 (2). Because of differences in objectives, the pure research in seismology does not contribute directly to engineering design under earthquake conditions. As an example, the structural engineer is interested to know the structural deformations caused by strong-motion earthquakes, which cannot be measured with overly sensitive seismographs utilized by seismologists. In general, however, research results in various related specialties such as seismology, geology, and geophysics are helpful to earthquake engineering in terms of basic information such as (a) the locations, magnitudes, and frequencies of occurrence of earthquakes, (b) the distinction between active fault zones and potentially active faults, (c) the locations and amounts of surface fault movements, and (d) the nature of crustal deformation (2). The applied research as shown in Figure 1 may be described as a feedback mechanism, which serves to point out the direction for improvement of the practical application. A design code may be considered as a component of practical applications as shown in Figure 1.

Several attempts have been made to "control" the structural response recently. These include (a) the application of "static control" by Zuk (83), (b) "active systems" by Wright (84) and Nordell (85), and (c) "adaptive systems" by Yao (86). As an example of the adaptive system, the bilinear system was studied is some detail (34, 87, 88). Along the same line of thought, a statistical study of passive motion-reducing devices in earthquake engineering was completed very recently (89). While these studies indicate possible improvements in the seismic structural design, the available results are not yet directly applicable.

Generally speaking, specifications refer to (a) functional requirements and (b) physical limitations in the engineering design (90). This information can be provided by

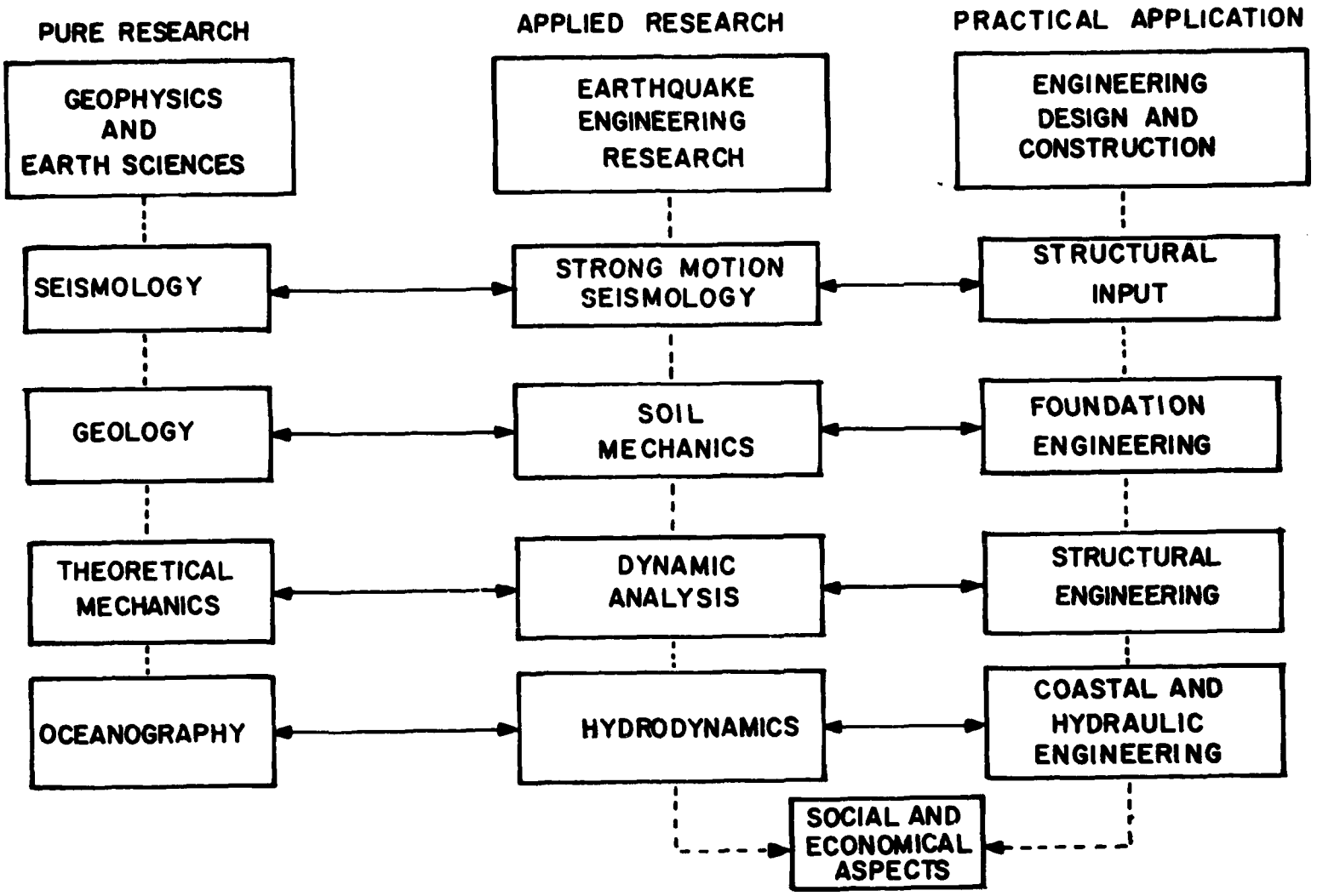


Figure 1. Relations Between Pure Research, Applied Research, and Practical Applications (2).

clients, authorities or agencies concerned with public safety, and engineering organizations. In earthquake engineering, design specifications have been proposed by professional engineering organizations such as the Structural Engineers Association of California (47) and adopted as a part of the Uniform Building Code (8). The guidance for the design of seismic structures is also given in the form of manuals as the one prescribed by the Departments of the Army, the Navy and the Air Force in 1966 (10).

A comprehensive summary of design methodologies is given by Eder (91). These include (a) expenses, (b) modification and running redesign, (c) checklists, (d) design trees, (e) the fully systematic method, and (f) the system search methods. In this section, the design trees and the fully systematic methods are examined in some detail, because they are used in the development and presentation of the new code format.

In engineering design (92), the cyclic design sequence of (a) analysis of the problem, (b) proposing solutions, (c) delineating these solutions, and (d) modifying them can be plotted as a design tree as shown in Figure 2, where a problem is denoted by a vertical line and a solution is denoted by a slanting line. The tree is started with a statement of the primary problem, which is referred to as the zeroth order, Q_0. The alternative solutions, first order, are represented by slanting lines A_1, A_2, etc. For each alternative solution, such as A_2, there may be several problems to be solved, i.e., Q_{21}, Q_{22}, etc. Any one or more of the alternative solutions are sufficient to solve the problem. On the other hand, it is necessary to solve all problems before a lower order alternative solution is obtained. As an example, the bold lines in Figure 2 indicate one possible set of solutions.

In the fully systematic method, a list of factors relevant to the problem or its solution factors must be decided in the beginning. It is important to include all ideas without criticism and to allow duplications. Each factor, described by a single statement, should be numbered in the order of appearance. Furthermore, any factor that contains a possible solution of a sub-problem should begin a new category. Eventually, all factors must be classified into a set of categories. The influence of one category upon another can be studied with the use of an interaction matrix, which can be transformed into interaction nets. The resulting interactions can now be transformed into performance specifications, which are used to define performance (e.g., "a column must support the floor load", not "a column must be

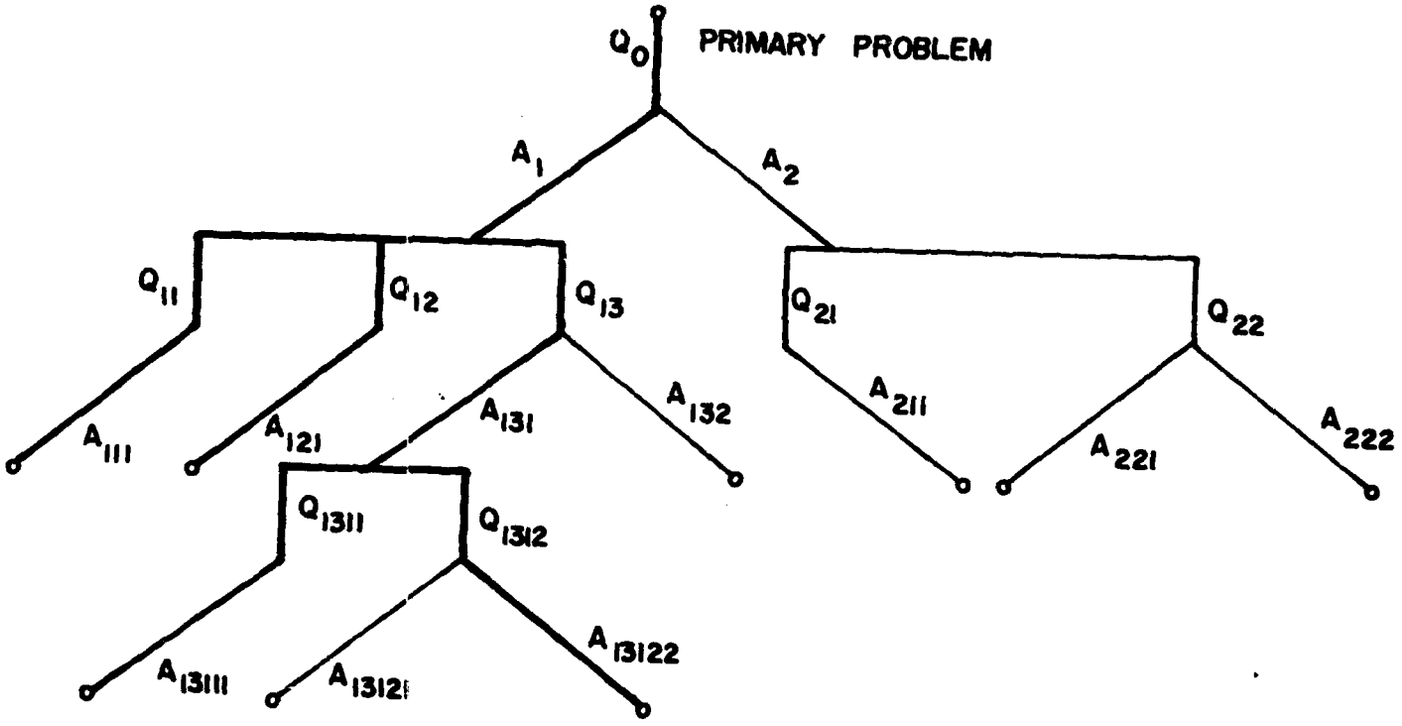


Figure 2. Design Tree

of a certain size"). The objective of the performance specifications is to completely separate the problem. Therefore, these specifications must not make reference to shape, material, or design. Each performance specification can now be considered to conceive the idea of solutions and to list them in a morphological chart (93). The method of the design tree is used in the process of obtaining an improved seismic design code in this report.

Direct Approach

The "primary question," Q_0, in the present research program is "what constitutes a seismic design code?" There are two first-order alternative answers, namely A_1: the "equivalent static forces" approach which summarizes all of the major existing codes as shown in Figure 3, and A_2: a direct approach which is shown in Figure 4. All detailed "answers," A's, in both Figures 3 and 4, are tabulated in Table 3 immediately following these two figures. The equivalent static forces approach was discussed in detail in Part II. Therefore, only the "direct approach" is elaborated and discussed herein.

To date, the existing seismic design codes specify a set of equivalent static forces for use by structural engineers in their designs. This approach has the advantage of being easy to use, because the engineer is spared the need to understand structural dynamics. On the other hand, the simple formulae given in the code cannot possibly cover all possible situations accurately and precisely. With (a) more sophisticated education of young engineers, and (b) widespread use of tools such as the electronic digital computer in recent years, it is believed that the present generation of engineers can be trusted with more analysis on their own than ever before. The "direct approach" as suggested is also a more flexible one, which is expected to yield safer as well as more economical structural designs.

As shown in Figure 4, there can be three second-order questions in the direct approach A_2, namely, Q_{21}: excitation, Q_{22}: structural model and/or analysis, and Q_{23}: response limitations for functioning and safety. There can be two alternative third-order answers to each of these second-order questions. For Q_{21}, the excitation can be specified by A_{211}: a single sample function (deterministic), or by A_{212}: a family of sample functions (probabilistic). For Q_{22}, the code can give A_{221}: the methods of computing parameters for the mathematical model of the structure, or A_{222}: the model as well as computed responses using discrete mechanics. The alternate approach A_{222} can be

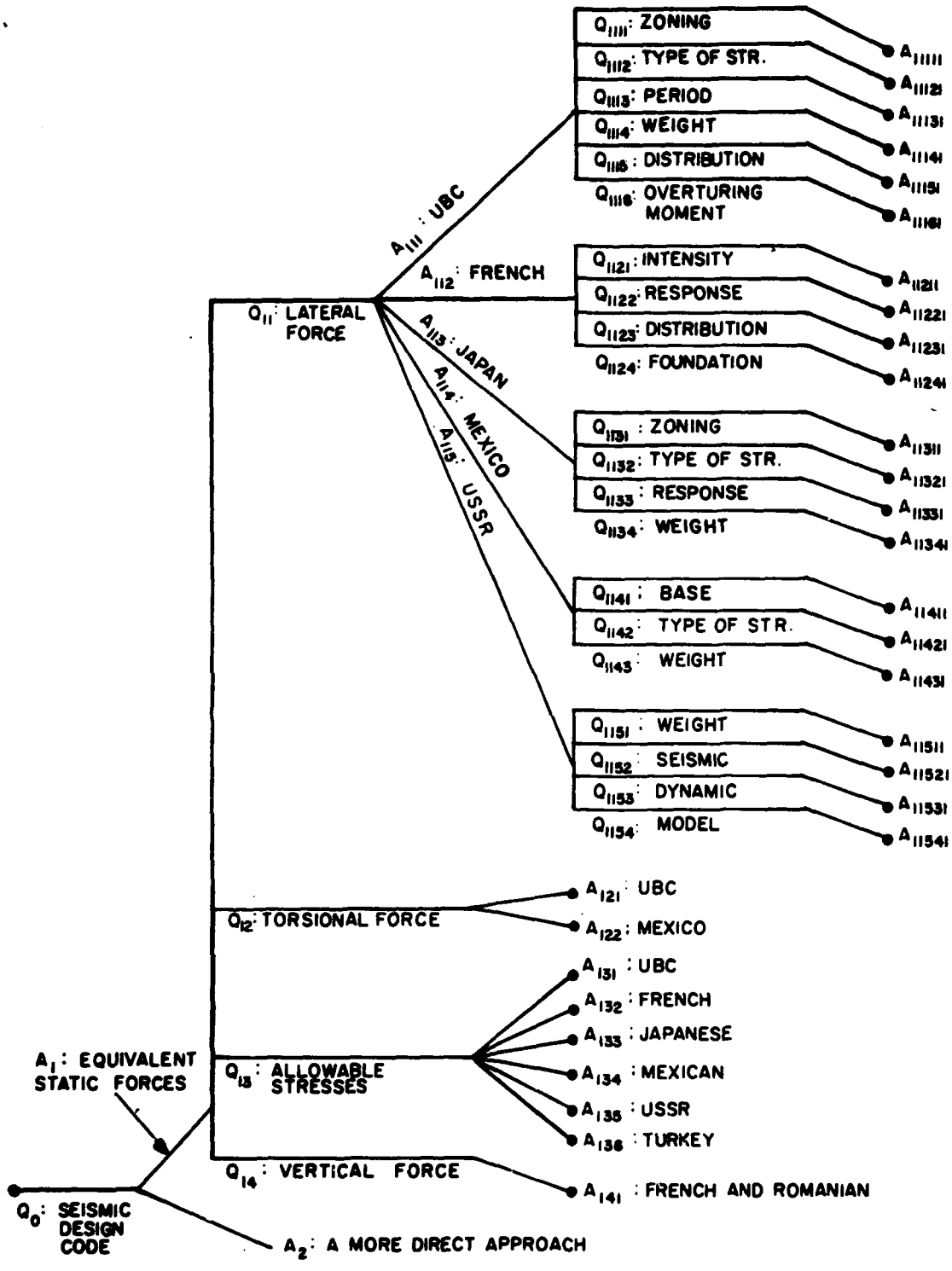


Figure 3. "Design Tree" for Existing Seismic Design Codes

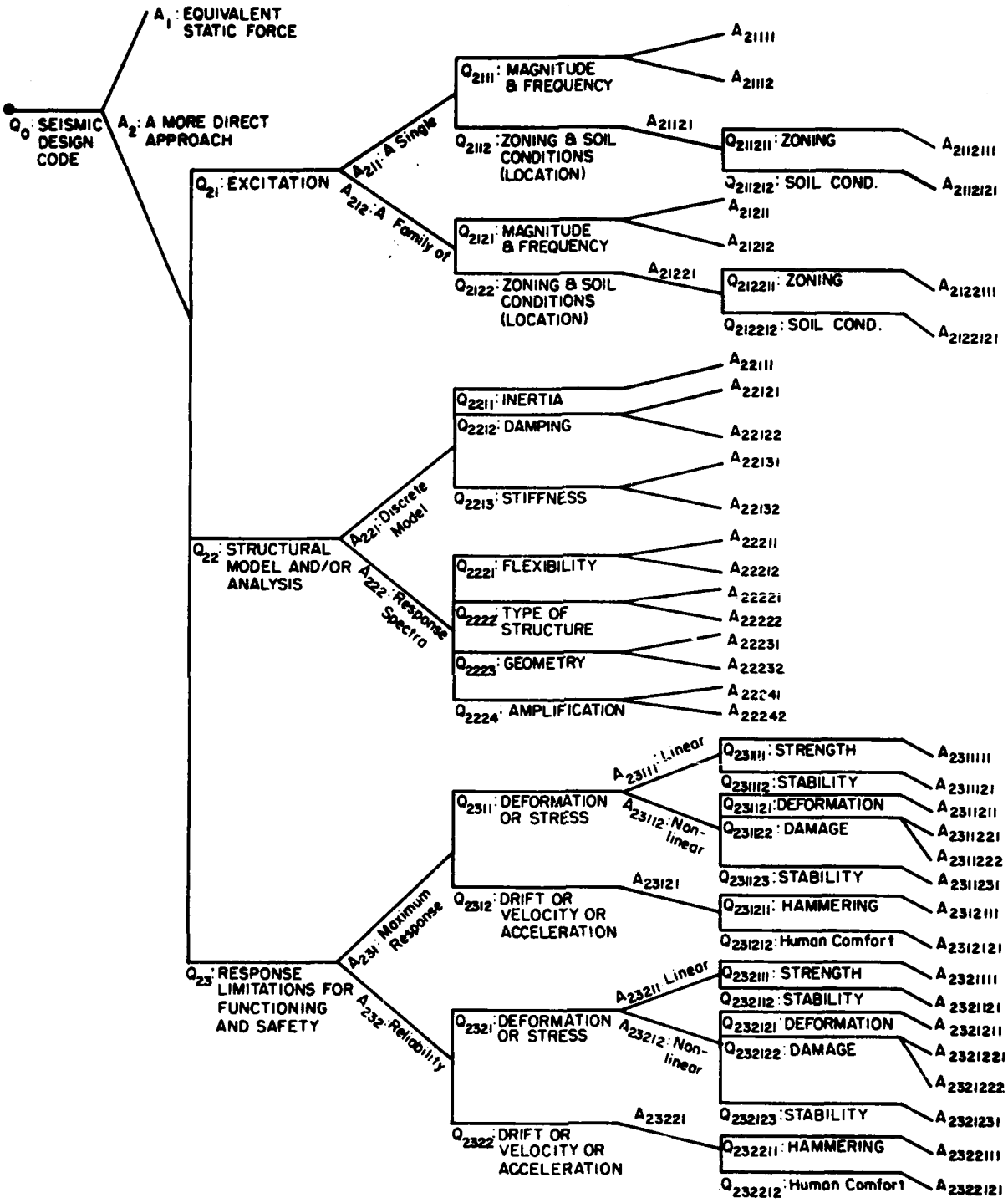


Figure 4. "Design Tree" for a Direct Approach

Table 3

"Answers" in Design Trees as Shown in Figures 3 and 4

A_{11111}:

Zone No.	0	1	2	3
Z	0	.25	.50	1.0

A_{11121}: K = .67 for Ductile Frames
1.33 for Box Systems (e.g., shear walls)
3.0 for Elevated Tanks

A_{11131}: $C = \frac{.05}{\sqrt[3]{T}} \leq 0.1; \quad T = \begin{cases} 0.1\ N, & \text{for Ductile Frames} \\ \frac{.05}{\sqrt{D}}\ h, & \text{otherwise} \end{cases}$

A_{11141}: W = total weight

A_{11151}: $F_f = 0.004\ V \left(\frac{h_n}{D_s}\right)^2 \leq .15\ V$, top of structure

$$F_x = \frac{(V-F_t)\ w_x h_x}{\sum_{i=1}^{n} w_i h_i}, \text{ otherwise}$$

A_{11161}: $M = J(F_t h_N + \sum_{i=1}^{n} F_i h_i), \quad J = \frac{.5}{\sqrt[3]{T^2}} \leq 1.0$

$$M_x = J_x[F_t(h_N - h_x) + \sum_{i=x}^{N} F_i(h_i - h_x)],$$

$$J = J + (1-J)\left(\frac{h_x}{h_N}\right)^3, \text{ for other floors}$$

A_{11211}:

MMI	VIII	IX	X	XI	XII
	1	2	4	8	16

A_{11221}: $.05 \leq \beta = \frac{.065}{\sqrt[3]{T}} \leq .10$; Common Buildings (normal damping)

Table 3 (continued)

$.065 \leq \beta = \frac{.085}{\sqrt[3]{T}} \leq .13$; Large Span & Little Partitions (medium damping)

$.06 \leq \beta = \frac{.095}{\sqrt[4]{T^3}} \leq .20$; Skeleton, no external friction (low damping)

A_{11231}:

$$\sigma_x = \frac{\sum_{i=1}^{x} M(z_i)q(z_i) + \int_0^{h_x} m(z)q^2(z)dz}{\sum_{i=1}^{x} M(z_i)q^2(z_i) + \int_0^{h_x} m(z)q^2(z)dz} q(h_N)\frac{\Gamma(T)}{g}$$

$q(h)$ = shape fcn. of the fundamental mode,

$\Gamma(T)$ = max. response (acceleration of a SDF system with the same period T and damping as the fundamental mode*

*The first three modes are considered for slender structures with low damping.

A_{11241}: f = 0.8 for deep foundation of firm rock

= 1.3 for piling in moist ground

A_{11311}: z = 0.8 or 0.9 depending on seismic zone

A_{11321}: s = 0.6 for steel frame or wooden structure on firm ground,

= 0.9 for semi-firm ground and reinforced concrete structure,

= 1.0 for soft ground

A_{11331}: c = 0.2 for structures up to 16 m., and add 0.01 for each 4 m. in addition to 16 m.

A_{11341}: W = total weight

A_{11411}: Type 1 = structures having, at right angles with the direction being considered, two or more elements capable of resisting shear, and whose deformation under lateral loads is essentially due to flexure in members.

Table 3 (continued)

Type 2 = Structures whose deformations under lateral loads are due to shearing stresses or axial forces in members.

Type 3 = Elevated tanks, chimney stacks, etc.

Type \ Zone	1	2
1	.06	.04
2	.08	.08
3	.15	.10

A_{11421}: k = 1.3 for Group A, government, municipal and public buildings

= 1.0 for Group B, private housing

0 for Group C, isolated and unimportant buildings

A_{11431}: W = total weight

A_{11511}: Z_q = D.L. + 0.8 L.L. (for warehouses, use full L.L.)

A_{11521}:

GEOFIAN Scale	7	8	9
α	1/40	1/20	1/10

4 Types of Bldgs:

I: Massive bldgs. of great importance, increase rating by 1

II: Public Bldgs, industrial bldgs. of primary importance, same rating

III: Industrial bldgs. of secondary importance, one-story houses, same ratings for "6", "7", reduce "8" and "9" by 1.

IV: Use rating "6" for all farm bldgs., barns, sheds, etc.

A_{11531}: $0.6 < \delta = \frac{0.9}{T} < 3.0$, and use 1.6 δ for slender and flexible structures

Table 3 (continued)

A_{11541}: $$\zeta_x = \frac{q(x_k) \sum_{i=1}^{N} Q_i q(x_i)}{\sum_{i=1}^{N} Q_i q^2(x_i)}$$

use 1st mode for normal buildings, and use the first 3 modes for slender and flexible bldgs.

A_{121}: $T_N = S_N(1.5\ e_N + 0.5\ B_n)$

shear eccentricity max. dimension

A_{122}: $e_D = 1.5\ e_c \pm 0.05\ D$

A_{131}: May be increased by 33%

A_{132}: May be increased by 50%

A_{133}: May be increased by 50% for reinforcing steel, and 100% for concrete in reinforced concrete

A_{134}: May be increased by 50% for steel, and 33% for concrete

A_{135}: May be increased by 40% for steel, 20% for reinforced concrete, and 0% for prestressed concrete

A_{136}: May be increased by 50% for good soil conditions, 30% for fair soil conditions, and 0% for bad soil conditions

A_{141}: French and Romanian codes mentioned it. However, it is not clear as to how to do it in designs

A_{21111}: Take a typical set of earthquake records, e.g., 1940 El Centro

A_{21112}: Find the least favorable excitation, e.g., Drenick

$A_{21112111}$: Zoning (deterministic)

Table 3 (continued)

$A_{21112121}$: Soil Conditions (deterministic)

A_{21211}: Take a set of artificial functions, e.g., Shinozuka and Sato, Amin and Ang, etc.

A_{21212}: Take several sets of artificially generated functions, e.g., Jennings, Housner and Tsai

$A_{2122111}$: Zoning (statistical)

$A_{2122121}$: Soil Conditions (statistical)

A_{22111}: Analytical

A_{22121}: Analytical

A_{22122}: Experimental

A_{22131}: Analytical

A_{22211}, A_{22221}, A_{22231}, A_{22241}: Linear-Discrete Mechanics*

*discrete mechanics, based on the calculus of finite differences, can be used to obtain field or functional solutions for systems, which are more accurately represented by a lattice or a network of elements.

A_{22212}, A_{22222}, A_{22232}, A_{22242}: Nonlinear analysis

$A_{2311111}$: $|S_{max}| \leq s_y$

$A_{2311121}$: Stable

$A_{2311211}$: $|y_{max}| \leq y_{limit}$

$A_{2311221}$: $\sum D_i \leq 1$

$A_{2311222}$: Energy Absorbed $\leq E_{crit}$.

Table 3 (continued)

$A_{2311231}$:	Stable
$A_{2312111}$:	Hammering Prevented
$A_{2312121}$:	Comfortable
$A_{2321111}$:	$P(\lvert s_{max} \rvert \leq s_y) \leq p_1$
$A_{2321121}$:	$P(\text{Unstable}) \leq p_2$
$A_{2321211}$:	$P(\lvert y_{max} \rvert \geq y_{limit}) \leq p_3$
$A_{2321221}$:	$P(\sum D_i \geq 1) \leq p_4$
$A_{2321222}$:	$P(E_{ab} \geq E_{cr.}) \leq p_5$
$A_{2321231}$:	$P(\text{Unstable}) \leq p_6$
$A_{2322111}$:	$P(\text{Hammering}) \leq p_7$
$A_{2322121}$:	$P(\text{Discomfort}) \leq p_8$
	p_i's should be functions of type of structure, function (importance) of structure, consequence of failure, material properties, etc.

considered as an intermediate code between the existing ones and that given by A_{221}. For Q_{23}, the response limitations for functioning and safety can be specified by A_{231}: a single value (deterministic) or by a probability (probabilistic). The fourth-order questions related to these third-order approaches are discussed in the next section, which also includes fifth-order answers.

Implementation Problems and Possible Approaches

1. Excitation (Q_{21})

To specify the excitation for the dynamic analysis of structures, time record(s) of the gound motion is.(are) needed. One possible answer, which has been used by many consulting engineers, is A_{21111}: use a typical set of strong-motion earthquake records such as those of the famous 1940 El Centro earthquake. An alternate approach, A_{21112}, is to make use of a "least favorable excitation." Drenick (94) showed that, for a class of excitation x(t) with a finite total energy M^2, (a) $x(t) = (M/N)h(-t)$ causes the least favorable response of a linear structure with the impulse response function h(t), and (b) the maximum energy response is MN, where

$$N^2 = \int_{-\infty}^{\infty} h^2(t)\,dt$$

More recently, Shinozuka (94) showed how to estimate the least favorable structural response to probabilistic and earthquake-like excitation. With digital or analog computers, it is also possible to apply Monte Carlo methods using artificially generated earthquake-like motions (27-34). All these time functions should be modified acording to the location of the structure, which in turn can be expressed as a function of the zoning and local soil conditions as shown in Figure 4.

2. Structural Model and/or Analysis (Q_{22})

To analyse a structural system subjected to dynamic loads, it is necessary to have a mathematical model. With the recent advances in digital computation, the discrete model is a possible and useful one (A_{221} in Figure 4). Frequently, the structure can be represented by a set of ordinary differential equations with parameters representing the inertia (translational and/or rotational), the damping (linear or nonlinear), and stiffness (linear or nonlinear). Although the study of structural dynamics has reached a mature stage with certain given models such as the linear shear-type building frames, theoretical and experimental work seems to be needed in relating these parameters to real structures. For example, though many studies have been made on material and structural damping (96, 97), research and development work is needed to specify damping coefficients

for various types of structures and partitions. Moreover, the effect of cracking or cumulative damage due to previous earthquakes on these parameters is not clearly understood at the present time.

An intermediate seismic design code might be the specification of response spectra (A_{222} in Figure 4).

The dynamic analysis of multi-story building structures subjected to earthquake excitations is a complicated problem in the field of mechanics. Redundancy of such structures, random nature of loads, difficulties in determining contribution to damping by architectural clothing, behavior of foundation under shakedown, questions related to dynamic stability of structure, significance of nonlinear analysis accounting for secondary stresses, and complexity of elastic-plastic behavior of frames are among problems which contribute to the intricacy of the subject area.

A reasonable objective for the codes and specifications in line with economic factors appears to be that seismic design codes should provide guidelines for design of structures:

(a) to withstand frequent earthquakes without significant damage to structure or facings,

(b) to avoid total collapse in major earthquakes,

(c) to have sufficient stiffness to exclude sensible vibration when subjected to wind load in normal conditions.

Subject to a few restrictions, it is feasible to predict the dynamic behavior of almost any structure by the use of high speed digital computers, as well as mathematical manipulation. The alternative approach A_{222} is the development of working formulas which are simple enough to be included in the codes for use by practicing engineers and yet describe the dynamic behavior of a broader category of structures. A more detailed outline of the problem areas will include the following:

(a) Formulas or charts for response spectrum of rigid and braced multi degree-of-freedom frames subject to decided critical earthquake excitation to include (i) displacement response spectrum in shear, torsion, or bending, (ii) shear response spectrum, (iii) bending response spectrum, (iv) torsion response spectrum, (v) determination of influence on response spectrum (shearing forces, bending. torsion, and displacements) accounting for the

interaction of vertical earthquake excitation with the horizontal accelerations, (vi) determination of influence on response spectra (displacements, shearing forces, bending, and torsion) in a non-linear analysis accounting for secondary stresses and the interaction between lateral and vertical forces known as the P-Δ effect, (vii) dynamic responses of elastic-plastic frames, (viii) determination of time-dependent excitation most critical for any particular structure, and (ix) formulation of dynamic stability of structures under shakedown.

(b) Energy absorption capacity of structures:

(i) formulas for natural frequencies of vibration of rigid and braced structures. Structures with orderly pattern and boundary conditions as well as stepped-up structures and structures with appendages need to be considered. (ii) Influence charts for the effects of variations in mass distribution, ratio of stiffnesses, shapes, damping factors, sizes, etc., on energy absorption capacity of structures. (iii) Optimum distribution of ductility to aid maximum energy absorption capacity.

(c) Problems related to foundations and structural materials and their properties:

(i) necessary modifications in response spectra due to nature of subfoundation soil layers, (ii) liquefaction of sand and silt under intense vertical acceleration, (iii) the uneven foundation properties and their effects, (iv) damping factors most closely descriptive of different construction, (v) dynamic behavior of reinforced concrete members, joints, precast or prestressed members, (vi) dynamic characteristics of multistory reinforced concrete buildings.

Besides those approximate methods which fall under the classification of static equivalent methods, two methods of solution are available. Clough (98) suggested matrix formulation of each independent structure and use of generally applicable but somewhat lengthy computer programs. Newmark (80) formulated the response spectra for single degree-of-freedom system. Then, a multi degree-of-freedom system is treated by transformation to an equivalent single degree-of-freedom system. At the present time, this technique is being pursued at the University of Illinois (80).

Research results under the present project have indicated the suitability of employing an alternate approach to formulate the response spectra of multi degree-of-freedom systems. The proposed method of solution bypasses much of the difficulty inherent in the earlier lines of thinking by assuming a discrete model representation of highly redundant structures and using the calculus of finite differences and the concepts of discrete mechanics to derive a solution. These concepts are illustrated in Appendix A, where a single bay multi-story frame is analysed. Closed functional solutions are found for natural frequencies of vibration and for lateral force distribution. Furthermore, the formulation of the dynamic response in combined torsion and biaxial shear is given.

Earthquakes produce torsional oscillation and stress in structures due to nonsymmetric mass and stiffness properties of buildings. There is, however, a lack of rational solutions for dynamic behavior of structures in torsion due to complexity of the problem. The problem of dynamic behavior in torsion and biaxial shear has received little attention even though the interaction of biaxial shear displacement with torsion is apparent. A general formulation of the dynamic response of structures in torsion and biaxial shear is given in Appendix A. This formulation as well as other research work based on discrete mechanics performed under this research program serve to illustrate the potential of the techniques of discrete mechanics.

3. Response Limitations (Q_{23})

For the consideration of functioning and safety of structures, limitations can be specified in terms of either maximum response (A_{231}) or reliability or probability of failure (A_{232}). These specifications can be made with the use of statistical methods and on the basis of seismic behavior of structures. A review of statistical methods in earthquake engineering is given in Appendix B. Moreover, the seismic behavior of concrete and metal structures is summarized and discussed in Appendix C.

When the structure behaves in a linear manner, the deformation is proportional to either stress or strain. Therefore, strength limits can be specified. On the other hand, deformation limits must be given in the case of inelastic structural behavior. Because the damage due to plastic deformation is cumulative, it is also desirable to restrict the allowable damage in building structures.

The structural stability under the vertical as well as horizontal ground motions is a complicated problem. There appears to be little work done in this regard at the present time. The hammering effect of closely-spaced buildings against one another should be prevented. Moreover, it is desirable to minimize the discomfort of human inhabitants during earthquakes.

In both the deterministic specification A_{231} and the probabilistic approach A_{232}, it is desirable to make statistical analysis of available data. In addition, the decision analysis can be a useful tool in the choice of these limitations (99).

In Appendix B, following an introduction to basic definitions, topics covered include statistics of strong-motion earthquakes, random response of seismic structures, structural safety in earthquake engineering, and decision analysis. In Appendix C, a review is made of the seismic behavior of various types of structures during recent earthquakes. In Appendix D, a simple design problem is given as an illustration of the "direct approach" portion of the design tree.

PART IV: CONCLUSIONS AND RECOMMENDATIONS

Conclusions

A survey of existing and available seismic design codes was made. The "design tree" technique was used to summarize these existing specifications as well as to present possible improvements. Moreover, some original work has been performed applying the discrete mechanics to structural dynamics as given in Appendix A. Literature reviews of statistical methods in earthquake engineering and seismic behavior of concrete and metal structures are given in Appendices B and C, respectively.

A "direct approach" to the seismic design code is proposed herein. With this direct approach, the code would specify (a) the excitation, (b) the structural model, and (c) the response limitations. It is believed that engineers using this type of code will be able to design building structures in a more precise, realistic, and economic manner. However, it might be desirable to perform this type of analysis and design enough times to determine if simpler analysis would provide the required information at cost and time savings.

Recommendations

The following is a list of problems, which are delineated for possible further studies by CERL personnel or interested persons elsewhere.

1. Specification of Ground Motions - the concept of the least favorable excitation originally developed by Drenick (94) and recently modified by Shinozuka (95) seems to be very promising. Nevertheless, further work is required before the present highly mathematical forms can be incorporated into any design code. In case it is desirable to specify several artificially generated records for statistical analysis, development work is also required for code adoption of existing techniques. Moreover, studies should be made to develop methods with which these design ground-motions can be modified with respect to the location and soil conditions at the site of the structure.

2. Computation of Parameters for Given Structures - Mathematical models should be as representative of the actual structure as possible. In the iterative design

process, the engineer should be given a set of formulae for the computation of structural parameters such as damping coefficients. Personnel of a research facility such as CERL are in a unique position of conducting analytical as well as experimental investigations, which could result in successfully relating abstract and mathematical models to real structures.

3. Response Spectra - As an intermediate step, formulae and/or charts can be developed for response spectra of multistory frame structures subjected to various earthquake loads. These solutions can be obtained as functions of number of floors, ratio of stiffnesses, differences in natural frequencies of vibration, mass, etc. Criteria for the intensity and distribution of loads to be suggested in the codes must be provided based on the formulation of the response spectra. Displacement response spectrum in shear, torsion, bending and their combination; as well as shearing force response spectrum, bending response spectrum, torsion response spectrum need to be formulated and the recommended load distribution for each action needs to be defined. The examples as given in Appendix A (some are original contributions resulting from this research program) illustrate the potential usefulness of discrete mechanics in structural dynamics.

4. Dynamic Stability of Seismic Structures - Although methods for three-dimensional dynamic analysis of seismic structures are available, the stability of seismic structures has not been studied extensively. To minimize the chance of structural failure due to instability under earthquake loads, it is necessary to develop new knowledge in this regard.

5. Inelastic Behavior of Structures - Because the principle of superposition is not applicable to inelastic structural analysis, most available solutions are results of numerical analyses. Perhaps, improvements can be made in these numerical methods as well as problem-oriented computer languages.

6. Low-Cycle Failure of Seismic Structures - Experimental results are available for concrete members (100) as well as steel members (101) subjected to repeated applications of prescribed large deformations. The fatigue behavior of structures under random loads needs further study (102). Moreover, there seems to be a need for experimental studies on the fatigue behavior of structures under earthquake-like random loads,

though some data are available in aerospace engineering fields for coupon specimens (103).

7. Reliability Considerations - Structural reliability has become a major field of study in structural engineering since Freudenthal's pioneering works (104, 105) some twenty-five years ago. In the determination of acceptable probabilities of failure, the extended reliability concept of Ang and Amin (106, 107) seems to provide a reasonable basis for further development, which incorporates subjective decision-making into objective probabilistic analyses. In this regard, the decision analysis such as the applications of Benjamin (108, 109) can be useful in considering the economic consequences of failure.

8. Importance of Structures - It seems to be desirable for the design engineer to make use of a quantitative assessment of the importance of structures. Obviously, the safety of a school is more important than that of a warehouse. Perhaps, the acceptable probabilities of failure can be made as functions of these "measures of importance" for various structures.

9. Soil-Structure Interaction - The structural response is certainly dependent on the foundation and surrounding soils. This problem should also be studied further.

10. Interaction of Structural and Non-structural Components - Although it is known that non-structural components affect the overall structural response to earthquake loads, these do not appear to exist in any concise solution to this problem. It seems to be desirable to study interaction of structural and non-structural components both analytically and experimentally at CERL.

These recommendations for further study are not meant to be exhaustive. Nevertheless, they seem to be the significant ones in relation to the development of a new seismic code, with which safer and more economical building structures can be designed.

APPENDIX A

REVIEW OF DISCRETE MECHANICS AND SOME ORIGINAL SOLUTIONS

General

The "discrete mechanics" or "discrete field mechanics" can be used to obtain field or functional solutions for structural systems, which can be represented with a lattice or a network of elements. In discrete mechanics, the mathematical model is a set of difference equations or recurrence relations among lattice nodes. These equations are then solved in closed-form with the use of the calculus of finite differences (110), which is concerned with functions of integer variables. The calculus of finite differences is the suitable mathematical tool in discrete mechanics in the same manner as the differential calculus is effective in continuum mechanics.

From the viewpoint of continuum mechanics, the infinitesimal elements of an elastic body are used to express compatibility and dynamic conditions, which along with the material properties lead to a set of differential equations. On the other hand, in discrete mechanics, a finite number of elastic segments are examined to satisfy the compatibility and dynamic requirements at the joints, which yield a set of difference equations or difference-differential equations (111-113). Functional solutions to these equations are field solutions, which describe the dynamic behavior of the nodal points in the structure.

For further references, Dean (114) gave a comprehensive outline of basic concepts in discrete mechanics. Leimback and McDonald (42) applied the technique to investigate steady state vibration of certain frames. Wah (41) formulated natural frequencies of vibration of grids vibrating out of plane using the technique. Manvi, Duvall and Lowell (112) formulated the wave propagation in lattices. Lin et al. (115) has adopted a technique resembling discrete approach to formulate ribbed panel vibration problem.

Dean and Ugarte (44) applied this technique to the analysis of building frames. Omid'varan (116) investigated behavior of slender high-rise frames using finite difference approach.

The formulation of governing difference-differential equations of latticed frames based on energy approach was given by Omid'varan (117). Omid'varan (118) and Guthowski

(119) and Dean and Ugarte (44) have all investigated the stability of frameworks. Space lattices and structural nets were treated by Dean (120, 121) and Dean and Ugarte (122-124). The basis for investigation of behavior of composite discrete and continuous elastic media is given by Dean, Omid'varan and Gangaro (125, 126). Additional application of the technique which more represents the historic development of the technique is found in references (127-129).

In the following, examples are given to illustrate the application of the calculus of finite differences in discrete mechanics. In addition, original solutions to several problems in structural dynamics are given.

Example 1 - Continuous Beams*

A closed functional solution was derived for deformations and stresses at the supports of a beam continuous over N panels herein. The moments at the supports represent a finite number of ordinates of a continuous function, namely the moment in the continuous beam. As stated earlier, discrete mechanics can be employed to develop closed functional solutions to a finite number of ordinates of a continuous function. Given a continuous beam loaded in an arbitrary manner as shown in Figure 5, an equivalent system as far as joint deformation and support moment is concerned carries concentrated moments m_r at the supports.

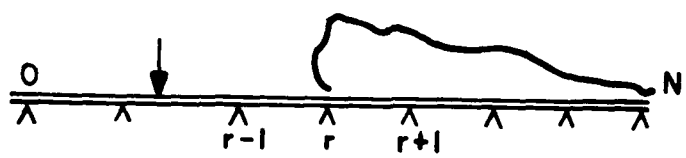


Figure 5. Continuous Beam

$$m_r = M_F(r) + M_F'(r-1) \tag{79}$$

where m_r denotes the sum of fixed end moments at support "r" due to loading of the panel preceding and the panel succeed-

*Originally presented by D. L. Dean.

ing joint "r". M_F (r) and M'_F(r-1) denote fixed end moments at the support "r." The free body diagram of the joint is shown in Figure 6.

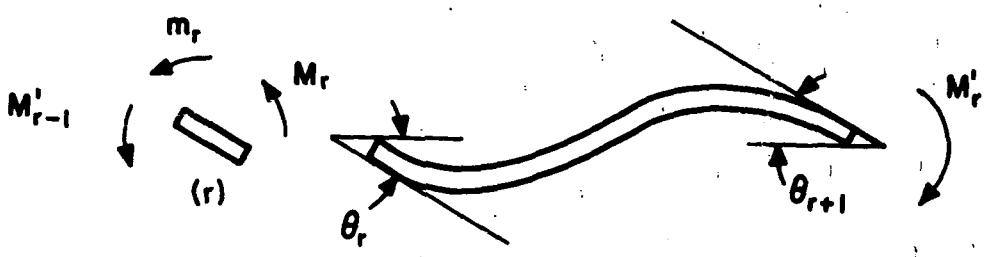


Figure 6. The Free Body Diagram of Joint Loaded Equivalent System

Considering the joint free body diagram at the support as shown in Figure 6, we have

$$M_r + M'_{r-1} + m_r = 0 \tag{80}$$

The panel force-deformations are:

$$\begin{bmatrix} M_r \\ \\ M'_r \end{bmatrix} = 2k \begin{bmatrix} (E+2) \\ \\ (2E+1) \end{bmatrix} \Theta_r \tag{81}$$

In the above presentation k denotes the panel flexural stiffness and E denotes the Bool's difference operator. Operators such as E, delta Δ, Nebla ∇, Devla $\Delta\nabla$, and multa $\boxed{-}$ are among the most frequent difference operators in the calculus of finite differences and are defined as follows:

$$E(\Theta_r) \equiv \Theta_{r+1}, \text{ or } E^{-1}(\Theta_r) \equiv \Theta_{r-1} \tag{82}$$

$$\Delta(\cdot) \equiv (E-1)(\cdot); \text{ e.g., } (\Delta\Theta_r = \Theta_{r+1} - \Theta_r) \tag{83}$$

$$\nabla(\cdot) \equiv (1-E^{-1})(\cdot) \tag{84}$$

$$\Delta\nabla(\cdot) \equiv (E-2+E^{-1})(\cdot) \tag{85}$$

$$\square(\cdot) \equiv \frac{1}{2}(E-E^{-1})(\cdot) \tag{86}$$

Introduce Equation 81 into Equation 80 to obtain the governing difference equation for Θ_r. The result for the case when panel stiffness properties are identical (i.e., for k = constant and for arbitrary loading) is given by

$$(\Delta\nabla + 6)\Theta_r = \frac{-1}{2k} m_r \tag{87}$$

This second order difference equation is the counterpart of the following differential equation in differential calculus

$$(D^2 + 6)\ Y(\underline{X}) = \frac{-1}{2k} m(x) \tag{88}$$

where $D^2 = d^2/dx^2$. The solution to Equation 87 based on calculus of finite differences will contain a homogeneous and a particular solution

$$\Theta_r = (-1)^r(C_1 \sinh ar + C_2 \cosh ar) + \Theta_p(r) \tag{89}$$

where

$$a = \cosh^{-1}(2) \tag{90}$$

The particular solution $\Theta_p(r)$ may be evaluated for any loading condition. The most general loading case is that of a unit impulse moment at r=b. All other solutions can be obtained by superposition of this unit impulse solution.

$$\Theta_p(r) = \frac{(-1)^{r-b}}{2k \sinh a} \sinh a(r-b)\ H(r-b) \tag{91}$$

where H(r-b) denotes discrete step function such that,

$$H(r-b) = \begin{cases} o, & \text{for } r<b \\ 1, & \text{for } r\geq b \end{cases} \tag{92}$$

For fixed end conditions $\Theta_o = \Theta_N = 0$, these constants are given by

$$C_2 = -\Theta_p(0), \quad C_1 = \frac{1}{\sinh aN}\left[(-1)^{1+N}\ \Theta_p(N) + \Theta_p(r) \cosh aN\right] \tag{93}$$

Given the closed functional solution to Θ_r, Equation 81 can be used to express M_r as follows:

$$M_r = 2k(\Theta_r + 2\Theta_{r+1}) \tag{94}$$

Results of this example showed that discrete mechanics represents an extension of finite element method. In the finite element method, the properties of individual segments are employed to express joint equilibrium (or dynamic requirements) and compatibility requirements. The result is a set of linear equations which are solved numerically. In discrete mechanics the segment properties are employed to derive a difference equation describing the behavior of nodal points at the joints between the segments. A closed solution is derived based on a solution to difference equations based on calculus of finite differences.

Example 2 - A Truss

A cross-braced pin connected truss as shown in Figure 7 is a simple example of a discrete elastic body. Field solutions to stresses are given to illustrate our point of view. We shall formulate the stresses in the chord segment and the bracing identified in a sequence as the chord by bracing "r". These are denoted by N_r and G_r respectively as shown in Figure 8. In this manner, N_r and G_r are functions of an integer variable "r".

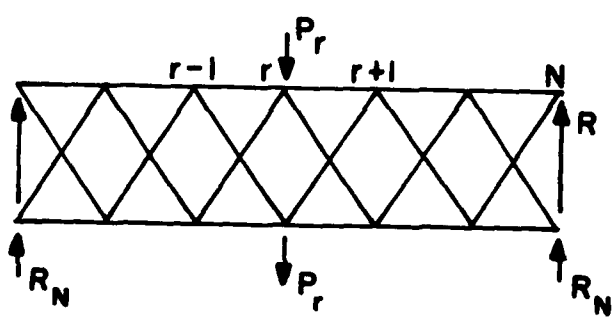


Figure 7. Cross-Braced Truss

Consideration of the equilibrium requirements of the joint "r" as shown in Figure 8 of a symmetrically loaded truss will give:

$$(G_r + G_{r-1}) \cos\alpha + N_r - N_{r-1} = 0 \tag{95}$$

$$(G_r - G_{r-1}) \sin\alpha + P_r = 0 \tag{96}$$

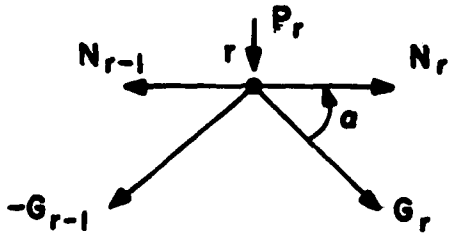


Figure 8. Free Body Diagram of Joint "r"

Equations 95 and 96 represent a set of two first-order difference equations, which will be solved for G_r and N_r. For constant α Equation 96 yields,

$$\Delta G_r = \frac{-P_{r+1}}{\sin} ; \tag{97}$$

where Δ denotes the first forward difference operator, i.e., $\Delta G_r \equiv G_{r+1} - G_r$.

The solution will include a homogeneous solution which in this case is a constant and particular solution $G_r(r)$ which depends on loading term P_r:

$$G_r = C_1 + G_p(r) \tag{98}$$

For uniform loading $P_r = P$ and

$$G_p(r) = \frac{-P_r}{\sin\alpha} \tag{99}$$

The constant C_1 is determined in view of the end condition at $r = 0$.

$$R_o = NP = G_o \sin\alpha, \text{ or}$$

$$G_o = \frac{PN}{\sin\alpha} \tag{100}$$

From Equation 100, we obtain

$$C_1 = \frac{PN}{\sin\alpha} \tag{101}$$

Thus, a functional field solution to the stress in bracing "r" is given by

$$G_r = \frac{P(N-r)}{\sin\alpha} \tag{102}$$

To obtain a closed solution to stresses in the chords, Equations 95 and 102 will be considered From Equation 95,

$$\Delta N_r = -\cos\ (G_r + G_{r+1}) \tag{103}$$

In view of Equation 102,

$$\Delta N_r = -P\ \cot\alpha\ (2N-2r-1) \tag{104}$$

The solution to Equations 104 will include a summation constant C_2 and a particular solution, as follows,

$$N_r = C_2 + P\ \cot\alpha(r^2-2Nr) \tag{105}$$

Using the condition $N_o = -PN\ \cot\alpha$,

$$C_2 = -PN\ \cot\alpha \tag{106}$$

As a result, field solution to stresses in the chords is found to be:

$$N_r = P\ \cot\alpha(-N+r^2-2Nr) \tag{107}$$

Since the truss considered in this example is a statically determinate structure, the stiffness properties of the individual segments did not have to be considered in formulation of the stresses. The stiffness properties become necessary should we choose to formulate a field solution to truss deformations.

The following example will illustrate the application of discrete mechanics in the dynamic analysis of a statically indeterminate system.

Example 3 - Steady State Vibration of a Lumped Beam*

The frequencies of vibration of a system of N concentrated equal masses connected by massless elastic bars of equal lengths as shown in Figure 9 will be formulated to illustrate the dynamic application of discrete mechanics. A multi-story shear-frame in vibration remotely resembles a lumped cantilever beam.

The free body diagram of the mass M at the node point "r" is shown in Figure 10. The notation employed to designate the moment and shear in the elastic segment preceding the rth mass is shown in Figure 11.

*Prepared by C. Omid'varan in collabroation with C. Bacchus.

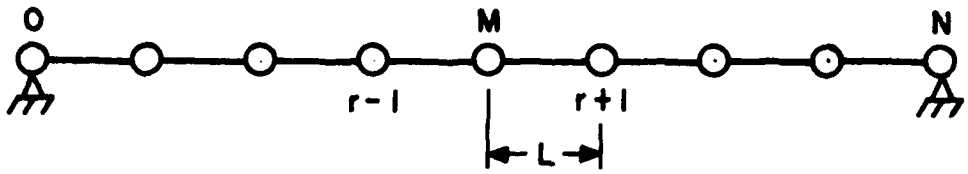


Figure 9. Lumped Mass Beam

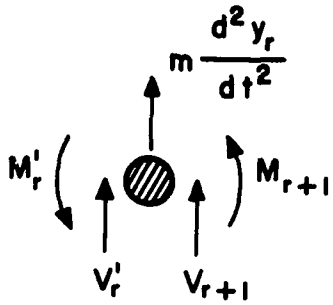


Figure 10. The Free Body Diagram of Mass r

Figure 11. The Free Body Diagram of Panel r

The lateral displacement and rotation of mass r are denoted by $Y_r(t)$ and $\Theta_r(t)$ respectively, and $Y_r(t)$ and $\Theta_r(t)$ are discrete functions of r and continuous functions of t, the time element.

Neglecting rotational inertia, the dynamic consideration of mass r as shown in Figure 10 will require that:

$$M'_r + M_{r+1} = 0 \tag{108}$$

$$V'_r - V_{r+1} = m \frac{d^2Y_r(t)}{dt^2} \tag{109}$$

The stiffness properties of elastic bars connecting the masses are given by:

$$M_r = k[4\Theta_{r-1} + 2\Theta_r - \frac{6}{L}(Y_r - Y_{r-1})] \tag{110}$$

$$M'_r = k[2\Theta_{r-1} + 4\Theta_r - \frac{6}{L}(Y_r - Y_{r-1})] \tag{111}$$

$$V_r = V'_r = \frac{1}{L}(M_r + M'_r) \tag{112}$$

where $k \equiv EI/L$.

Eliminate the joint rotations from Equations 110 and 111 for a difference equation relating M_r, M'_r, and Y_r. The result may be introduced into Equation 108 to obtain:

$$(\Delta\nabla_r + 6)M'_r - \frac{6k}{L} \Delta\nabla_r Y_r = 0 \tag{113}$$

The difference operation $\Delta\nabla_r$ (Debla), is an operator concerning typical three-node relations.

$$\Delta\nabla_r Y_r \equiv Y_{r-1} - 2Y_r + Y_{r+1} \tag{114}$$

$$\Delta\nabla_r M'_r \equiv M'_{r-1} - 2M'_r + M'_{r+1} \tag{115}$$

Equation 113 represents a generalized three-moment equation for non-zero displacement at the node points.

In view of Equation 108, Equation 112 will give,

$$V_r = V'_r = \frac{1}{L}(M'_r - M'_{r-1}) \tag{116}$$

Introduce Equation 116 into Equation 109, to obtain,

$$\Delta\nabla_r M'_r + mL \frac{d^2Y_r}{dt^2} = 0 \tag{117}$$

This is a second order difference-differential equation. For a steady state harmonic motion with angular frequency ω, we have

$$\frac{d^2Y_r}{dt^2} = -\omega^2 Y_r \tag{118}$$

As a result of Equation 118, Equation 117 can be transformed into the following difference equation:

$$\Delta\nabla_r M'_r - mL\omega^2 Y_r = 0 \tag{119}$$

The solution to a set of two difference equations as given by Equations 113 and 118 for M'_r and Y_r represents a closed functional solution to the problem. In a matrix form, these are

$$\begin{bmatrix} (\Delta\nabla_r + 6) & -\frac{6k}{L}\Delta\nabla_r \\ \Delta\nabla_r & -mL\omega^2 \end{bmatrix} \begin{bmatrix} M'_r \\ Y_r \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{120}$$

For simple support conditions at r=0 and r=N, both M'_r and Y_r must vanish at the boundaries.

The following finite series will satisfy the required end conditions naturally.

$$m'_r = \sum_{i=0}^{N} A_i \sin\frac{i\pi r}{N} \tag{121}$$

$$Y_r = \sum_{i=0}^{N} \beta_i \sin\frac{i\pi r}{N} \tag{122}$$

Introduce Equations 121 and 122 into Equation 120, we obtain

$$\begin{bmatrix} 2(2+\cos\beta_i) & -12\frac{k}{L}(\cos\beta_i - 1) \\ 2(\cos\beta_i - 1) & -mL\omega^2 \end{bmatrix} \begin{bmatrix} A_i \\ \beta_i \end{bmatrix} = 0 \tag{123}$$

where

$$\beta_i = \frac{i\pi}{N} \tag{125}$$

For a non-trivial solution the determinate of coefficients must be identical with zero, and we obtain,

$$\omega^2 = \frac{12k}{mL^2} \frac{(\cos\beta_i - 1)^2}{\cos\beta_i + 2}, \quad i=1,\ldots N \tag{125}$$

The N different angular frequencies of vibration of a lumped beam are given by Equation 125. The angular frequencies are found to approach those of continuous beams rapidly as the number of lumped masses replacing a continuous beam increase. For 10 equal masses $\omega = 9.87$ in contrast with π^2 for a continuous beam given that $k/mL^2 = 1$.

Vibration of Multi-Story Frames

A single-bay multi-level frame with uniform properties as shown in Figure 12 is used to illustrate the derivation of a closed functional solution to dynamic properties and response of structures. This work illustrates the effectiveness of discrete mechanics in resolving several problems in the implementation of a new code.

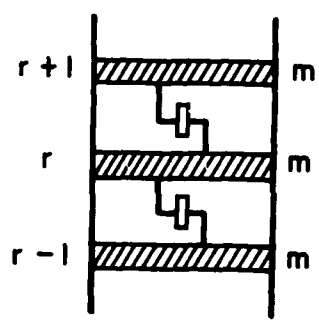


Figure 12. Discrete Model of a Multi-Story Frame

The masses are assumed to be identified and denoted with a sequence of numbers ... r-1, r, r+1 .... The forces acting on the mass identified as mass "r" in Figure 12 include the inertia force F_I, the damping force F_D, the elastic force F_S, and the forcing function P(t).

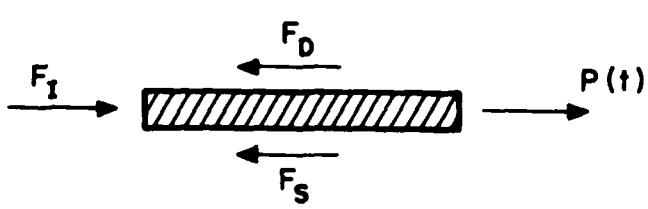


Figure 13. Free Body of Mass "r"

Considering the free body of mass r, we have

$$-F_I + F_D + F_S = P(t) \tag{126}$$

Denoting the displacement of this mass by $V_r(t)$, the inertia force is given by

$$F_I = -m\ddot{V}_r(t) \tag{127}$$

The elastic force on mass "r" will depend on displacement of mass (r-1) preceding and mass (r+1) following this mass shown in Figure 14.

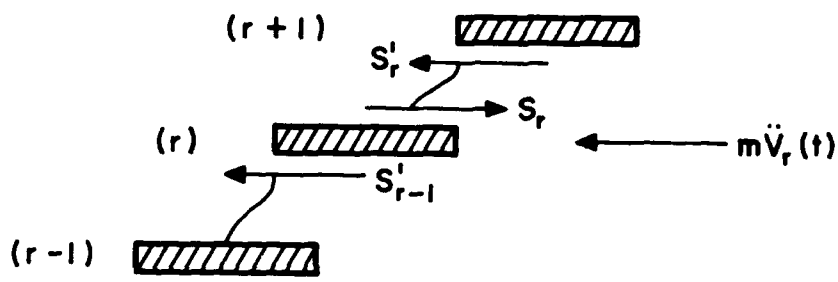


Figure 14. Shearing Forces on Mass r

Let k denote the frame shearing stiffness relating the shearing force and relative end displacements of any two successive levels.

$$S_r + S'_r = k(V_{r+1} - V_r) \tag{128}$$

Then the elastic force is given by

$$F_S = S'_{r-1} - S_r = k[V_r(t) - V_{r-1}(t) - k[V_{r+1}(t) - V_r(t)] \tag{129}$$

Thus

$$F_S = -k\Delta\nabla_r V_r(t) \tag{130}$$

where $\Delta\nabla_r$, Debla, denotes the second central difference operator, such that

$$\Delta\nabla_r V_r(t) \equiv [V_{r+1}(t) - 2V_r(t) + V_{r-1}(t)] \tag{131}$$

The damping force, F_D, will in a similar manner depend on damping coefficient C and second central difference of velocities at three nodes

$$F_D = -C\Delta\nabla_r \dot{V}_r(t) \tag{132}$$

Introducing Equations 127, 130, and 132 into Equation 126, we obtain:

$$m\ddot{V}_r(t) - k\Delta\nabla_r V_r(t) - C\Delta\nabla_r \dot{V}_r(t) = P(t) \tag{133}$$

The second-order difference-differential equation as given by Equation 133 represents the mathematical model for the frame considered. It is a differential equation with respect to continuous element t, and a difference equation with respect to integer variable r.

First the steady state harmonic motion will be investigated. A solution will be given for an undamped structure ($C = 0$) subject to earthquake excitation ($P(t) = 0$). Then Equation 133 will reduce to

$$m\ddot{V}_r(t) - k\Delta\nabla_r V_r(t) = 0 \tag{134}$$

In view of harmonic nature of structure in the absence of damping we may assume that,

$$V_r(t) = F_r \sin\omega t \tag{135}$$

where ω denotes the angular frequency of vibration. Substituting Equation 135 into Equation 134, we obtain,

$$(\Delta\nabla_r - 2\beta)F_r = 0; \quad \beta = -\frac{m\omega^2}{2k} \tag{136}$$

The solution to Equation 136 is found to be:

$$F_r = A \sin\gamma r + B \cos\gamma r, \quad \text{for } (-2 < \beta < 0) \tag{137}$$

$$F_r = (-1)^r [A_1 \cosh\eta r + A_2 \sinh\eta r], \text{ for } (\beta < -2) \tag{138}$$

where

$$\gamma = \cos^{-1}(1+\beta), \quad \eta = \cosh^{-1}(-1-\beta) \tag{139}$$

The summation constants A, B, A_1 and A_2 will first be determined. Then Equation 135 will be used to express functional solution to displacement $V_r(t)$. For the purpose of illustration, the solution corresponding to $(-2 < \beta < 0)$ only is presented herein.

For a steady-state vibration of a "N" story frame fixed at $r = 0$, the end conditions are assumed to be:

$$V_o(t) = 0 \tag{140}$$

$$\dot{V}_o(t) = 0 \tag{141}$$

$$\ddot{V}_o(t) = 0 \tag{142}$$

$$[V_{r+1}(t) - V_r(t)]_{r=N} = 0 \tag{143}$$

Equation 143 implies that the shear at r = N vanishes. Equation 140 will given B = 0. Then Equations 141 and 142 are also satisfied. Equation 143 will give:

$$A[\sin\gamma \cos\gamma N + (\cos\gamma - 1) \sin\gamma N] = 0 \tag{144}$$

For a nontrivial solution ($A \neq 0$) we must have

$$\sin\gamma \cos\gamma N + (\cos\gamma - 1) \sin\gamma N = 0 \tag{145}$$

Equation 145 is a function of three parameters, N, k/M, and ω, angular frequency. A computer program can be employed to plot ω versus k/M for different choices of N. Such a graphical solution is rationally based and reflects considerable improvement over existing solutions to frame natural frequencies.

A functional solution to the shear force at differential levels of a multi-story frame due to a unit impulse acceleration at the base can also be formulated. The response due to any particular earthquake acceleration is the result of integration of response due to unit impulse load.

The boundary conditions for a unit impulse acceleration acting at the base of a multi-story frame are assumed to be:

$$V_r(o) = 0 \tag{146}$$

$$\dot{V}_o(o) = 1 \tag{147}$$

$$[V_{r+1}(t) - V_r(t)]_{r=N} = 0 \tag{148}$$

Equation 146 is naturally satisfied in view of Equation 135. Equations 147 and 148 yield:

$$B = \frac{1}{\omega} \tag{149}$$

$$A = \frac{[\sin\gamma \sin\gamma N - \beta \cos\gamma N]}{[\sin\gamma \cos\gamma N - \beta \sin\gamma N]\omega} \tag{150}$$

Given summation constants A and B, a closed functional solution to displacement response $V_r(t)$ is obtained, and we will be in a position to formulate shear response spectrum. The shearing force $Q_r(t)$ is equal to the total sum of inertia forces acting on the masses above the rth level.

$$Q_r(t) = - \sum_{i=r+1}^{N} m\omega^2 V_i(t) \tag{151}$$

In view of Equation 135, Equation 151 will give,

$$Q_r(t) = -m\omega^2 \sin t \sum_{i=r+1}^{N} F_i \tag{152}$$

Calculus of finite difference (110) provides means of evaluations of finite sums of discrete functions in the manner differential calculus helps with integrals of continuous functions. These will be employed to simplify Equation 152.

$$Q_r(t) \; -m\omega^2 \sin\omega t \; [\Delta_i^{-1} F_i]_{i=r+1}^{N+1} \tag{153}$$

which will give:

$$Q_r(t) = \frac{-m\omega^2 \sin\omega t}{2 \sin\frac{\gamma}{2}} [-A \cos\gamma \; (i-1/2)$$

$$+ \beta \sin\gamma \; (1-1/2)]_{i=r+1}^{N+1} \tag{154}$$

or

$$Q_r(t) = \frac{-m\omega^2 \sin\omega t}{4 \sin\frac{\gamma}{2}} [A \cos\gamma (N+1/2) - A \cos\gamma \; (r+1/2)$$

$$+ \beta \sin\gamma \; (N+1/2) - \beta \sin\gamma \; (r+1/2)] \tag{155}$$

Equation 155 represents a solution to the shear force at any level as a function of time due to a unit impulse acceleration at the base. In addition, it can be used for design purposes, taking into account factors which are neglected at the present time.

Torsion and Biaxial Shear

Inspection of buildings subjected to earthquakes has indicated the presence of torsional oscillations and torsional damage in these structures. Most structures are not symmetric in mass and/or stiffness. Further, shearing stiffness properties of structures are different along principal stiffness axes in normal cases. The nonsymmetrical features will affect the induced stresses subjected to a dynamic excitation in an arbitrary direction. Biaxial shear displacement as well as torsion will result in structures.

In the case of one degree of freedom systems it is possible to assume an equivalent static force acts through the center of mass of structure and determines the forces based on elastic analysis. However, for multi-degree of freedom systems, it is difficult to conceive of a static method of

analysis and there is a dearth of rational solutions for torsional oscillation of multi-story frames, and little is known regarding combined torsion and biaxial shear behavior of high rise structures. Consciousness of their importance has, nevertheless, led to adoption of provisions in most codes and specifications. Housner and Outinen (67) have investigated the dynamic behavior of single-story structures in torsion. Bustamante and Rosenblueth (68) have made a general examination of code provisions and available literature. Shiga (69) has presented a manner of formulation of dynamic behavior of multi-story frames in torsion in cases when the centers of mass and rigidity coincide. However, the method is not easily applicable since it requires predetermination of rigidity ratios in shear and torsion of each level in relation to all other levels.

The presentation here illustrates a method of formulation of dynamic properties and response in torsion and biaxial shear. The illustration assumes identical mass and geometry for all levels.

The displacements of level "r" are designated by u_r, v_r, and θ_r. u_r and v_r denote displacement parallel with x and y, the principal elastic axis. θ_r denotes the angle of distortion and for convenience will be transformed into a parameter w_r:

$$\frac{w_r}{R^2} = \theta_r \tag{156}$$

R = radius of gyration of mass m of each level.

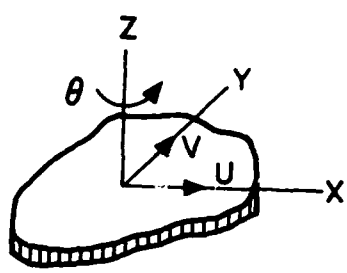


Figure 15. Principal Elastic Axis

First influence coefficients will be defined for displacement and force interaction between any two successive levels. k_y^x will be used to designate the force in x direction of any one floor due to a unit displacement in y direction for a rotation corresponding to unit w of following

floor. k_x^z will denote torque in z-direction for a rotation corresponding to unit w of following panel.

$$k_y^x = k \begin{matrix} \text{reactive force direction} \\ \text{unit displacement direction} \end{matrix} \tag{157}$$

Given , the ratio of rigidity against shear in y direction to that in x direction; we have

$$k_y^y = \alpha k_x^x \tag{158}$$

In view of the choice of x and y as principal elastic axes we have

$$k_y^x = k_x^y = 0 \tag{159}$$

Equation 159 will serve as the basis for choice of principal axes x and y. β, γ, η, e_1, e_2 will be used to designate other influence coefficient ratios as defined by the following relations:

$$k_x^z = e_1 k_z^z \tag{160}$$

$$k_y^z = e_2 k_z^z \tag{161}$$

$$k_z^z = \beta k_x^x \tag{162}$$

$$k_z^x = \gamma k_x^x \tag{163}$$

$$k_z^y = \eta k_y^y \tag{164}$$

In the absence of damping, inertia and elastic forces in x, y and z, directions will be active. The inertia forces are given by:

$$I_x = -m\ddot{u}_r \tag{165}$$

$$I_y = -m\ddot{v}_r \tag{166}$$

$$I_z = -m\ddot{w}_r \tag{167}$$

The shearing elastic forces acting on panel r are a function of displacements of the mass preceding the mass succeeding this panel. V_x, the shearing force in x direction, is given by Figure 16.

$$V_x = S_r - S'_{r-1} \tag{168}$$

In view of influence coefficients defined earlier, we find

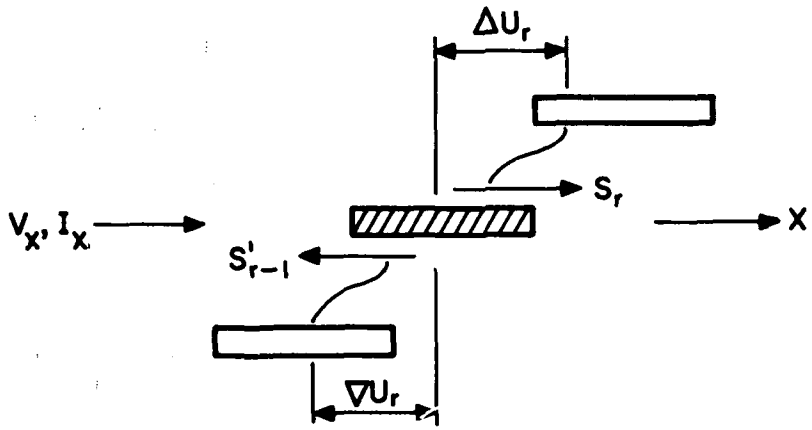


Figure 16. Shearing Forces in x Direction

$$S_r = k_x^x \, \Delta u_r + k_y^x \, \Delta v_r + k_z^x \, \Delta w_r \tag{169}$$

$$S_{r-1} = k_x^x \, \nabla u_r + k_y^x \, \nabla v_r + k_z^x \, \nabla w_r \tag{170}$$

Where Δ and ∇ denote the first forward and backward difference operators

$$\Delta u_r \equiv u_{r+1} - u_r \qquad \nabla u_r \equiv u_r - u_{r-1} \tag{171}$$

Considering Equation 159, and introducing Equation 169 and 170 into Equation 168, we obtain

$$V_x = k_x^x \, [\Delta\nabla \, u_r + \gamma\Delta\nabla w_r] \tag{172}$$

The shearing force V_y in the y-direction is found in a similar manner to be:

$$V_y = \alpha k_x^x \, [\Delta\nabla v_r + \eta\Delta\nabla w_r] \tag{173}$$

The torque T_z is found to be:

$$T_z = \beta k_x^x \, [\Delta\nabla w_r + e_2\Delta\nabla v_r + e_1\Delta\nabla u_r] \tag{174}$$

It will be assumed that the principal mass and elastic axes are parallel as shown in Figure 17.

Denoting the eccentricities between center of mass c and elastic center G by a_1 and a_2, consideration of the free body diagram of mass r as shown in Figure 17 will require that:

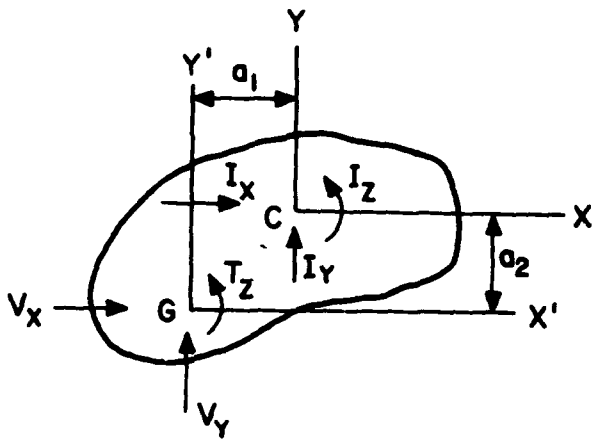


Figure 17. Principal Directions and Forces

$$I_x + V_x = 0 \tag{175}$$

$$I_y + V_y = 0 \tag{176}$$

$$I_z + T_z + V_x a_2 - V_y a_1 = 0 \tag{177}$$

where a_1 and a_2 are positive as shown in Figure 16. In view of Equations 165-167 and 172-174, we obtain

$$-M\ddot{u}_r + \Delta\nabla k_x^x \, [u_r + \quad w_r] = 0 \tag{178}$$

$$-M\ddot{v}_r + \Delta\nabla\alpha k_x^x \, [v_r + \quad w_r] = 0 \tag{179}$$

$$-M\ddot{w}_r + \Delta\nabla\beta k_x^x \, [A_1 w_r + A_2 u_r + A_3 v_r] = 0 \tag{180}$$

wherein,

$$M = \frac{m}{k_x^x}, \quad A_1 = \beta + a_2\gamma - a_1\alpha\eta, \quad A_2 = e_1\beta + e_2,$$

$$A_3 = \beta e_2 - a_1\alpha \tag{181}$$

Equations 178-180 represent a set of three difference-differential equations for u_r, v_r, and w_r. The mathematical model for torsion combined with biaxial shear of a shear frame is a second order differential equation with respect to time element and second order difference equation with respect to displacement elements.

In view of the fact that damping is assumed to be absent, we have,

$$u_r = U_r q(t), \quad v_r = V_r q(t), \quad w_r = W_r q(t) \tag{182}$$

where

$$q(t) = A \cos\omega t + B \sin\omega t \tag{183}$$

Considering Equations 182 and 183, we find

$$\ddot{u}_r = -\omega^2 u_r, \quad \ddot{v}_r = -\omega^2 v_r, \quad \ddot{w}_r = -\omega^2 w_r \tag{184}$$

Equation 184 will serve to separate difference and differential mathematical models. In view of Equation 184, Equations 178-180 will give:

$$\begin{bmatrix} (\Delta\nabla + M\omega^2) & 0 & \gamma\Delta\nabla & u_r \\ 0 & (\alpha\Delta\nabla + M\omega^2) & \alpha\eta\Delta\Sigma & v_r \\ A_2\Delta\nabla & A_3 & A_1\Delta\nabla + M\omega^2 & w_r \end{bmatrix} = [0] \tag{185}$$

A solution to vector difference Equation 185 to meet physical boundary conditions of multi-story frame is feasible by the aid of the stress function approach. For fixed end conditions at r=0 and r=N, a solution is easily feasible and will be given to conclude this illustration. Fixed end conditions can be met naturally through assumption of the following finite series representation for displacements:

$$\begin{bmatrix} u_r \\ v_r \\ w_r \end{bmatrix} = \sum_{i=1}^{N} \begin{bmatrix} A_i \\ B_i \\ C_i \end{bmatrix} \sin b_i r, \quad b_k = \frac{i\pi}{N} \tag{186}$$

Introduce Equation 183 into Equation 182, where we obtain the following:

$$\begin{bmatrix} (2\cos b_i - 2) + M\omega^2 & 0 & 2\gamma(\cos b_i - 1) \\ 0 & 2\alpha(\cos b_i - 1) + M\omega^2 & 2\alpha\eta(\cos b_i - 1) \\ 2A_2(\cos b_i - 1) & 2A_3(\cos b_i - 1) & 2A_1(\cos b_1 - 1) + M\omega^2 \end{bmatrix} = [0]$$

$$i=1,\ldots,N) \tag{187}$$

The N different frequencies of vibration of frame (i=1 to N) in torsion and biaxial shear are contained in the solution to the linear equation derived from the expansion of the determinate in Equation 187.

The strength of the technique in discrete mechanics illustrated in these examples will be more fully appreciated

considering the fact that torsional vibration of frameworks is recognized as one of the more complex problems in earthquate engineering. Scarcity of literature regarding the subject is an evidence of this point of view. This illustration has been concerned with more than torsional vibration in formulating the case of torsion combined with biaxial shear displacement.

APPENDIX B

STATISTICAL METHODS

General

There exist uncertainties in the prediction of the occurrence of future earthquakes as well as the estimation of the resistance of structural materials. Therefore, it is desirable to apply statistical methods to problems of earthquake engineering.

The objective of this appendix is to review and summarize available literature in this regard. The terminology of statistics is introduced in this section as well as wherever it is necessary. The statistical characteristics of strong-motion earthquakes and their computer simulations are then discussed. The random response of seismic structures is reviewed. Finally, the general problem of structural safety in earthquake engineering is described and discussed, along with a brief introduction to decision analysis.

Although the magnitude of an earthquake can be represented with the use of the Richter scale and the intensity can be described with the use of the Modified Mercalli scale, the most basic and useful data of earthquake engineering are the recordings of ground accelerations during strong-motion earthquakes (130). The record of an individual earthquake, say $x(\omega_1,t)$, can be considered as a sample function of a random process $X(\omega_1,t)$, the definition of which along with some other terminologies are introduced herein (131).

A random process (or stochastic process or random function) is a family of random variables indexed by an argument ω in a sample space Ω and a parameter t in an index set T, i.e., $X(t) \equiv \{X(\omega\ ,t) : \omega\varepsilon\Omega,\ t\varepsilon T\}$. In earthquake engineering applications, the parameter t denotes time. For a fixed value ω in Ω, say ω_1, $X(\omega_1,\cdot) = X(\omega_1)$ is a function of t representing a possible observation of the random process. The function $X(\omega_1)$ is called a realization or a sample function of the process. For a fixed value of t, say t_1, $X(t_1,\cdot) = X(t_1)$ is a random variable, which is

a function on the sample space Ω such that a specific value is assigned to each and every outcome ω. The probability that a random variable is less than or equal to a specific value x is called the distribution function, i.e.,

$$F_{X(t_1)}(x) \equiv P(X(t_1) \leq x) \tag{188}$$

Equation 188 is called the first-order distribution of the random process X(t) with its corresponding density function as follows:

$$p_{X(t_1)}(x) = \frac{\partial F_{X(t_1)}(x)}{\partial x} \tag{189}$$

Given two times instances t_1 and t_2, there are two random variables $X(t_1)$ and $X(t_2)$. The second-order distribution and density functions are respectively as follows:

$$F_{X(t_1)X(t_2)}(x_1,x_2) \equiv P(X(t_1) \leq x_1,\ X(t_2) \leq x_2) \tag{190}$$

$$p_{X(t_1)X(t_2)}(x_1,x_2) = \frac{\partial^2}{\partial x_1 \partial x_2} F_{X(t_1)X(t_2)}(x_1,x_2) \tag{191}$$

The description of a random process can also be made by the following moment functions:

$$\mu_X(t_1) = E[X(t_1)] = \int_{-\infty}^{\infty} x\, p_{X(t_1)}(x)\, dx \tag{192}$$

$$\begin{aligned}\phi_{XX}(t_1,t_2) &= E[X(t_1)X(t_2)] \\ &= \int_{-\infty}^{\infty}\int x_1 x_2\, p_{X(t_1)X(t_2)}(x_1,x_2)\, dx_1 dx_2\end{aligned} \tag{193}$$

where the first moment $\mu_X(t_1)$ and second moment $\phi_{XX}(t_1,t_2)$ are called the mean-value function and the autocorrelation function, respectively. The averaging operation, E, denotes expectation. Another important second-order moment is the autocovariance function defined as follows:

$$\begin{aligned}\Gamma_{XX}(t_1,t_2) &= E([X(t_1) - \mu_X(t_1)]\ [X(t_2) - \mu_X(t_2)]) \\ &= \phi_{XX}(t_1,t_2) - \mu_X(t_1)\ \mu_X(t_2)\end{aligned} \tag{194}$$

If for a random process X(t), for all values of n,

$$p_{X(t_1)X(t_2)\ \ldots\ X(t_n)}\ (x_1, x_2, \ldots, x_n)$$
$$= p_{X(t_1+a)\ X(t_2+a)\ \ldots\ X(t_n+a)}\ (x_1, x_2, \ldots, x_n) \tag{195}$$

the process is called completely stationary or strongly stationary or stationary in the strict sense. If Equation 195 is applicable for n=1 and n=2, the process is said to be weakly stationary or stationary in the wide sense. Then the expected value is a constant and the auto correlation is denoted by $R_{XX}(\tau)$ which is a function of $\tau = t_2 - t_1$. Although seismic ground motions are nonstationary, they are sometimes represented with stationary random processes for the sake of simplicity, as it will be discussed later.

For a weakly stationary process X(t), a frequency decomposition of the autocorrelation function $R_{XX}(\tau)$ can be made as follows:

$$R_{XX}(\tau) = \int_{\infty}^{\infty} \Phi_{XX}(\omega)\ \exp(i\omega\tau)\ d\omega \tag{196}$$

$$\Phi_{XX}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{XX}(\tau)\ \exp(-i\omega\tau)\ d\tau \tag{197}$$

If X(t) is a real-valued process, $\Phi_{XX}(\omega)$ is a non-negative and even function of ω. Equations 196 and 197 are usually called the Wiener-Khintchine relations. Note that,

$$R_{XX}(0) = \int_{-\infty}^{\infty} \Phi_{XX}(\omega)\ d\omega = E[X^2(t)] \tag{198}$$

Equation 198 can be interpreted to mean that $\Phi_{XX}(\omega)$ describes the density of the total mean-square value over the frequency domain. Therefore, the function $\Phi_{XX}(\omega)$ is called mean-square spectral density. It is also called the power spectrum because $E[X^2(t)]$ is often a measure of average energy.

Statistics of Strong-Motion Earthquakes

Housner (19) represented the earthquake as a series of impulses which are random in time. The ground acceleration X(t) is given by

$$X(t) = \sum_j V_j \, \delta(t - jt_0 - t_j) \tag{199}$$

where V_j is a random variable representing the magnitude of the jth impulse, t_0 is an average period computed by dividing the total time by the total number of pulses, $jt_0 + t_j$ is the time elapsed between the beginning time and the arrival of the jth pulse, and $\delta(t)$ is the unit impulse function. The random variable V_j is specified with given mean $E[V]$ and mean-square $E[V^2]$. Later, Thomson (20) showed that the power spectrum of a random process as given by Equation 199 and with $E[V] = 0$ is as follows:

$$\Phi(\omega) = \frac{1}{\pi t_0} \left| g(\omega) \right|^2 E[V^2] \tag{200}$$

where $g(\omega)$ is the Fourier transform of the pulse shape. When the pulse duration is short in comparison to the period of vibration, $|g(\omega)|$ approaches unity irrespective of the pulse shapes. Consequently, the power spectrum was found to be proportional to the weighted sum of the mean-square values, which is the same as the mean-square value of all the pulses. In other words, the power spectrum in this case becomes a constant regardless of the pulse shape and time spacing.

Goodman, Rosenblueth, and Newmark (61) used a velocity pulse model, in which the pulses are either uniformly or randomly occurring in time. Later Rosenblueth (132), and Rosenblueth and Bustamante (133) used an acceleration process which was generated from closely spaced velocity steps of random sign and magnitude with a constant intensity per unit of time.

When the power spectrum of a random process is found to be constant, the process is called "white noise" because the energy content of the process is considered to be uniformly distributed over the entire frequency range. Bycroft (21) proposed such a white noise with a constant spectral density of 0.75 ft^2/sec^4cps and a duration of 30 seconds for the representation of typical strong-motion earthquakes. The basis of comparison for this study is the response spectrum obtained by Housner (134). Ward (26) adopted a similar white noise process for analog simulations with some consideration to emphasize power at certain frequencies.

Because observations showed that certain frequencies predominate the ground motion for given soil conditions, Kanai (135) obtained a response process of the soil overburden to a white noise excitation at bedrock. Tajimi (136) suggested determining spectral density from spliced and repeated sections of the actual earthquake records. Caughey and Stumpf (137) used this type of model and found root-mean-square response to the 1940 El Centro earthquake.

Bogdanoff, Goldberg, and Bernard (22) studied the use of a nonstationary random process in representing earthquake accelerations as follows:

$$X(t) = \sum_{j=1}^{n} t\, a_j(\exp[-\alpha_j t]) \cos(\beta_j t + \gamma_j),\ t \geq 0 \qquad (201)$$

where a_j, α_j, and β_j are given sets of positive numbers with $\beta_1 < \beta_2 < \ldots < \beta_n$, and γ_j are n independent random variables uniformly distributed over an interval $[0,2\pi]$. The mean value is zero and its variance function has a shape as expected from an earthquake record. Later, Goldberg, Bogdanoff, and Sharpe (23) used the same process except making β_j also independent random variables, each uniformly distributed over the interval [6,46]. This modification causes little effect in the appearance of sample functions, but there seems to be considerable difference in their response spectra.

Lin (24) proposed the use of a nonstationary random process, which is obtained by passing a nonstationary shot noise through a linear filter. The resulting process is also called a filtered Poisson process (138,139). A shot noise S(t) refers to a series of random pulses, the arrivals of which are defined by a Poisson process N(t), i.e.,

$$S(t) = \sum_{k=1}^{N(t)} Y_k \cdot \delta(t - \tau_k) \qquad (202)$$

where Y_k are mutually independent random variables and $\delta(\cdot)$ denotes the unit impulse function. Furthermore,

$$E[S(t)] = 0 \qquad (203)$$

$$E[S(t_1)S(t_2)] = I(t_1)\delta(t_2 - t_1) \qquad (204)$$

In addition to generating an earthquake-like process, the linear filter can also be used in matching the frequency content of the process to that of recorded ground motions.

Because it is frequently difficult to obtain analytical solutions for structural problems in earthquake engineering, statistical answers can be obtained by the simulation of these problems with the use of digital or analog computers. Simulation techniques for earthquake motions as stationary processes were developed by Housner and Jennings (25) using a digital computer and by Ward (26) using an analog computer. Nonstationary earthquake motions have been simulated with the use of digital computers by Shinozuka and Sato (29), Amin and Ang (27), Jennings, Housner and Tsai (28), Hou (32), Rascon and Cornell (30), Iyengar and Iyengar (33), and Levy (31). As it is summarized by Levy (31), the typical operation of the digital simulation is (a) to generate an equally spaced time sequence of uncorrelated random numbers by drawing numbers following a normal distribution with zero mean, (b) to multiply these numbers by a time-dependent multiplier, and (c) to filter these numbers with a given impulse response function to obtain the sample function of the earthquake-like acceleration.

In earthquake engineering, passive analog computers have been used by various investigators including Alford, Housner and Martel (140), Naka,Kato, and Yuasa (141), and Murphy, Bycroft, and Harrison (142). Bycroft (21) used an active analog system with amplifiers in studying the response of linear as well as elasto-plastic single-degree-of-freedom systems to white noise excitation. Other investigators using the analog computer in earthquake simulations include Merchant and Hudson (143), Cherry and Brady (144), Brady (145), Ward (26), and Wirsching and Yao (146). A comprehensive treatment of random-process simulation in general can be found in Korn (147).

Random Response of Seismic Structures

Until approximately 10 years ago, the principal applications of random vibration studies have been to missiles, satellites, and other space vehicles. Recently, the random vibration of seismic structures has been emphasized. Irrespective of the type of structures studied, the ultimate objective of the study of random vibration is to ensure the maximum reliability of the structure. Some aspects of structural reliability will be discussed in the next section. In this section, the fundamentals of random vibration will be reviewed along with its application in earthquake engineering.

When the excitation can be represented by a random process, the problem of computing certain statistics of the

response belongs to the study of random vibration. During the past decade, many text or reference books concerning this subject area have been published (131, 148-153).

A linear system can be characterized by its response to (a) the sinusoidal forcing function, and (b) the impulsive forcing function. As an example, consider a mechanical system with the following equation of motion:

$$\ddot{x} + 2\zeta\omega_o\dot{x} + \omega_o^2 x = \frac{f(t)}{m} \tag{205}$$

where

$$\omega_o^2 = \frac{k}{m}$$

$$\zeta = \frac{c}{2\omega_o m}$$

m = mass

x = displacement

c = damping constant

k = spring constant, and

$f(t)$ = forcing function.

When the forcing function is sinusoidal, i.e.,

$$f(t) = Ae^{i\omega t},$$

let the particular solution be

$$x_p(t) = Ke^{i\omega t} \tag{206}$$

Substituting Equation 206 into 205, we obtain,

$$K = \frac{A}{m(\omega_o^2 - \omega^2 + 2i\zeta\omega_o\omega)} = \frac{A}{Z(\omega)} = AH(\omega) \tag{207}$$

where $Z(\omega)$ is called the "mechanical impedance" of the system, and $H(\omega)$ is called the "frequency response function" or the "transfer function" of the system.

When the forcing function is impulsive, i.e.,

$$f(t) = C\,\delta(t),$$

the particular solution becomes,

$$x_p(t) = C\,h(t) = \frac{C}{i\sqrt{1-\zeta^2}\,\omega_o m} \exp[(-\zeta + i\sqrt{1-\zeta^2}\,\omega_o)t],$$

$$\text{for } t \geq 0 \qquad (208)$$

The function h(t) in Equation 208 is called the "impulse response function."

It is to be noted here that the response of a stable linear system to an arbitrary excitation does not depend a great deal on the initial conditions after a fairly long time. This is known as the steady-state condition, during which the following relations exist with the use of the principle of superposition:

$$x_s(t) = \int_{-\infty}^{\infty} H(\omega)\,\bar{f}(\omega)\,\exp(i\omega t)\,d$$

$$= \int_{0}^{t} f(\tau)\,h(t-\tau)\,d\tau \qquad (209)$$

where

$$f(\omega) = \frac{1}{2\pi}\int_{0}^{\infty} f(t)\,\exp(i\omega t)dt, \text{ and} \qquad (210)$$

$$f(t) = \int_{-\infty}^{\infty} \bar{f}(\omega)\,\exp(i\omega t)d\omega = \int_{0}^{\infty} f(\tau)h(t-\tau)d\tau \qquad (211)$$

The integral $\int_0^t f(\tau)h(t-\tau)d\tau$ is called the "Duhamel integral." From Equation 209, the following convolution theorem of Fourier transformation can be obtained:

$$h(u) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)\,\exp(i\omega u)d\omega, \qquad (212)$$

and

$$H(\omega) = \int_{-\infty}^{\infty} h(t)\,\exp(-i\omega t)dt \qquad (213)$$

When the excitation is represented by a random process F(t), the response also becomes a random process X(t). Correspondingly, Equation 205 becomes

$$\ddot{X}(t) + 2\zeta\omega_o\dot{X}(t) + \omega_o^2 X(t) = \frac{1}{m} F(t) \tag{214}$$

assuming that (a) the derivatives of X(t) exist in mean-square sense (131) and (b) the random excitation begins at t = 0.

Because the principle of superposition holds for a linear system, we have

$$X(t) = \int_o^t F(\tau)\ h(t - \tau)d\tau \tag{215}$$

Also because integration is a linear operator, we obtain,

$$E[X(t)] = \int_o^t E[F(\tau)]\ h(t - \tau)d\tau \tag{216}$$

and

$$\phi_{XX}(t_1,t_2) = \int_o^{t_1} \int_o^{t_2} \phi_{FF}(\tau_1,\tau_2)h(t_1-\tau_1)h(t_2-\tau_2)d\tau_1 d\tau_2 \tag{217}$$

If F(t) is stationary in the wide sense, i.e.,

$$E[F(t)] = \mu_F = \text{constant} \tag{218}$$

and

$$E[F(t)F(t + \tau)] = R_{FF}(\tau) \tag{219}$$

it can be shown that

$$E[X(t)] = \mu_X = \text{constant} \tag{220}$$

and

$$E[X(t)X(t + \tau)] = R_{XX}(\tau) \tag{221}$$

Equations 220 and 221 indicate that the response of a linear system to weakly stationary excitations is also weakly stationary. Furthermore, the spectral density of the response process is equal to the product of the spectral density of the forcing process and the "transmittancy function" or the "system function" $|H(\omega)|^2$, i.e.,

$$\Phi_{XX}(\omega) = |H(\omega)|^2\ \Phi_{FF}(\omega) \tag{222}$$

For the given system subjected to a white noise excitation with $\Phi_{FF}(\omega) = W_o$, the mean-square response is found to be

as follows:

$$E[X^2(t)] = R_{XX}(o) = W_o \int_{-\infty}^{\infty} |H(\omega)|^2 d\omega = \frac{\pi W_o}{2\zeta\omega_o^3 m^2} \qquad (223)$$

More advanced topics in random vibration are summarized and discussed by Lin (131).

Rosenblueth and Bustamante (133) found a distribution function for the structural response to an idealized earthquake excitation, which was represented by a series of impulses random in time. Bogdanoff, Goldberg, and Schiff (154) studied the effect of ground transmission time on the response of long structures. Shinozuka, Hakuno, and Itagaki (155) evaluated the response of a multi-story frame structure to nonstationary random excitation. A general dynamic analysis for both stationary and nonstationary response of linear structures was proposed by Shinozuka and Yang (156). A numerical method for the analysis of complex structures subjected to nonstationary random excitation was also made available by Yang and Shinozuka (157). Recently, the evolutionary power of random processes and its application to earthquake engineering was presented by Shinozuka (158) and Brant and Shinozuka (159). The application of random vibration theory to earthquake engineering problems was also contributed by Bogdanoff, Goldberg, et al. (22,23) and Penzien and Liu (17). Moreover, the problem of random vibration of nonlinear systems has been studied by many investigators (160-167).

Structural Safety in Earthquake Engineering

The basic problem of structural safety can be described in the following manner. Consider a structural component. Usually, the resistance is considered as a function of the applied force, and failure is assumed to occur whenever the force, $S(t)$, exceeds the resistance $R(t)$. In general, both force and resistance are random in nature. Therefore, the "life" of this structural component can be represented by random variable T. The reliability function of this structural component, $L_T(t)$, is defined as the probability of the event that its useful life will be at least t, i.e.,

$$L_T(t) = P(T > t)$$

$$= P(R(\tau) > S(\tau), \quad 0 \leq \tau \leq t) \qquad (224)$$

Because of wear, cumulative damage, and increasing chance of encountering larger loads with increasing time, the relia-

bility of a structure is usually a monotonically decreasing function.

By definition, the distribution function for the life, T is then,

$$F_T(t) = P(T \leq t) = 1 - L_T(t) \tag{225}$$

For a special case where $R(t) = r$ = constant, and $S(t)$ is a known random function, the problem of finding $F_T(t)$ belongs to the "first passage" (or "barrier" or "crossing") problem, which will be discussed later. Another special case is that when $R(t) = R$, $S(t) = S$, both R and S are random variables with known distributions. Freudenthal first proposed the basic concepts of structural safety more than twenty years ago (104). Later, Asplund (168), Brown (169), and Freudenthal and Shinozuka (170) derived the expression for the probability of failure. Shinozuka (171) then found the probability of survival under n applications of load. A comprehensive review of the classical studies of structural reliability was made by Freudenthal, Garrelts, and Shinozuka in 1964 (105). In 1967, Cornell (172) gave reliability bounds for cases where the loads and the resistance are not statistically independent. Amin and Ang (173) established a monotonic property of the hazard function of structures, which is subjected to a sequence of random loads. Recent studies of structural reliability include the use of an uncertainty factor (106,107), redundant systems (174,175), statically indeterminate structures (176,177), and plastic collapse of structural frames (178).

In addition to its use as a common basis for comparing the relative safety among alternative designs, the probability of failure can be used as a constraint in the optimum design of structures. Hilton and Feign (179) used a Lagrange multiplier technique to find the minimum weight of a simple structure with statistically distributed force and resistance. This optimization technique was later refined by Kalaba (180). In 1964, the simplex linear programming method was used to solve the optimization problem following the development of a linear relationship between the weight and the probability of failure (181). Then Moses and Kinser (182) developed computational techniques for the determination of failure probabilities of elastic frames and trusses under multiple loading conditions, which were used in the optimum structural design. More recently, the effect of proof-load testing to reliability-based structural optimization was studied by Shinozuka, et al. (183,184).

In earthquake engineering, structural failures can re-

sult from excessive deformation, dynamic instability, or fatigue damage. Collapse of yielding structures during earthquakes was studied by Jennings and Husid (185). The failure probability of seismic structures was treated as a barrier problem by Shinozuka, et al. (155, 186-193) and others (22, 23,194,195).

The deterministic problem of dynamic stability has been studied by many investigators (196-202). When the axial excitation of the column is represented by a stationary Gaussian process, various stability conditions were obtained by Ariaratnam (203), Kozin (204), and Caughey and Gray (205). Later, Caughey and Dickerson (206) studied the stability of systems subjected to a narrow-band random excitation. Results from a recent study showed that, for a concentrically loaded column without sidesway, dynamic instability due to the vertical component of the earthquake excitation is not likely to be a problem (207). However, significant lateral motion of the structure occurs and thus may cause the columns to become unstable during an actual earthquake.

To improve the design of earthquake-resistant structures, it is desirable to know more about the failure mechanisms of these structures. Although it is possible to have low-cycle fatigue failure in seismic structures (18,208), such effects have not been studied statistically. To date, the studies of random fatigue consist of cases with stationary loading processes, which were summarized recently by Tang and Yao (102). Miles (209) was the first to compute the mean value of cumulative fatigue damage for a narrow-band random process. An approximation was suggested by Powell (210) for the case where the stress did not follow a narrow-band process. The variance of the cumulative damage resulting from narrow-band random stresses were computed by Crandall, Mark and Khabbaz (211), Bendat (212), and Shinozuka (213). Parzen (214) and Freudenthal and Shinozuka (215) applied the renewal theory to the study of random fatigue. Sweet and Kozin (216) incorporated physical properties into the renewal model, which was compared to experimental results. Rice (217) developed an approximate density function of the rise and fall statistics, which was used for the prediction of fatigue crack propagation by Rice, Beer, and Paris (218). A comprehensive review of experimental studies in the field of random fatigue was made by Swanson (219).

Decision Analysis in Earthquake Engineering

In engineering problems, the engineer must decide on

the action to take among a space of possible acts $A = \{a\}$. Also, it is usually assumed that there exists a space of possible "states of nature (the way things really are)" $\Theta = \{\theta\}$. In order to obtain information about the state of nature, engineers can conduct certain experiments from a family of possible experiments $E = \{e\}$. The family of experiments E is said to include the "dummy" experiment which refers to a decision-making without experimentation. The collection of all possible outcomes for various experiments is denoted by $\Omega = \{\omega\}$. For each combination $(a,\theta,e,\omega) \varepsilon A X \Theta X E X \Omega$, it is assumed that the engineer can assign a consequence $c \varepsilon C$.

As an example, the action to be decided upon might be whether to accept or reject a lot of reinforcing steel bars. The possible states of nature might be (a) that all bars meet the specifications, (b) that majority meet the specification, (c) that only few bars meet the specification, and (d) none meets the specification. To obtain some information concerning the state of nature, the engineer can perform certain experiments on a selected number of these bars. The result of these experiments can be either satisfactory or unsatisfactory. For each possible combination of (a,θ,e,ω), the consequence can range from excessive cost to defective structure as an end product. The decision analysis can be used to guide a decision maker such as the engineer in his choosing an action under uncertainty.

A comprehensive introduction to decision analysis with a minimum of mathematical demands is given by H. Raiffa (220). Other introductory references include G. Hadley (221), H. Raiffa and R. Schlaifer (222), Tribus (99), and H. Chernoff and L. E. Moses (223). Recently, many applications of the decision theory to structural engineering have been proposed by several authors (108,109, 224-228). Also, the application of decision analysis and probabilistic concepts to design codes was explored by several investigators (229-232).

In 1763, Reverend Thomas Bayes suggested the use of Bayes' Theorem in combining the probability based on hunches with that based on relative frequencies. Generally, the Bayesian, or subjective, approach to probabilistic methods incorporates or introduces intuitive judgements and feelings into the formal analysis of a decision problem. The resulting choice of an action is said to be consistent with the decision maker's preference for various consequences as well as his judgement about the uncertainties involved in the problem.

To begin the decision analysis, it is desirable to construct a "decision-flow diagram" or "tree." The engineer has a choice at the very beginning: to be involved or not to be involved (e.g., he can quit the job in case he chooses not to be involved). If he decides to be involved in the decision-making process, he must decide then whether to obtain more information through experimentation $(e_0, e_1, \ldots)$. For each of these experiments, there are possible outcomes $(\omega_1, \omega_2, \ldots)$, which depend on chance. At this point, the engineer can choose to take the actions $(a_1, a_2, \ldots)$. Then, by chance, the state of nature may be $(\theta_1, \theta_2, \ldots)$. A typical decision flow diagram is given in Figure 18. Following Raiffa (220), a "decision fork" is denoted by a small square and a "chance fork" by a small circle. Note that there exists a "consequence" c for each possible "path" in the tree.

At each chance fork, it is important to know the probability that any one of the alternative branches will be taken by chance. In other words, we need to know the probabilities of experimental results, i.e., $P(\omega_i)$, and the conditional probabilities of states of nature given that a given experimental outcome has occurred, i.e., $P(\theta_j|\omega_i)$. To compute these probabilities $P(\theta_j|\omega_i)$, (from quantities $P(\theta_j)$, $P(\omega_i|\theta_j)$, the following Bayes' Theorem can be used.

$$P(\theta_j|\omega_i) = \frac{P(\omega_i|\theta_j)P(\theta_j)}{\sum_j P(\omega_i|\theta_j)P(\theta_j)} \tag{226}$$

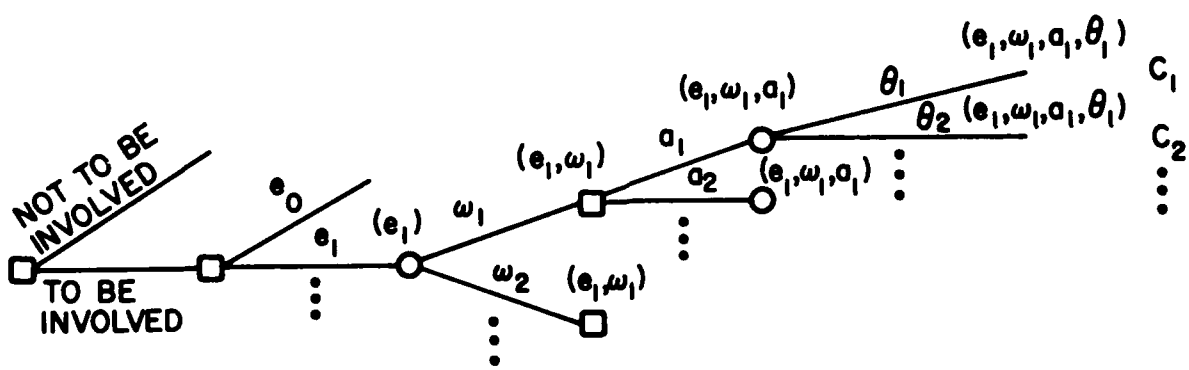


Figure 18. Typical Decision-Flow Diagram

For quantitatively representing consequences, it is first necessary to define a preference relation " " over the set of consequences C = c such that (a) if $c_i, c_j \varepsilon C$, then either $c_i \succ c_j$, $c_j \succ c_i$, or both and (b) if $c_i \succ c_j$, and $c_j \succ c_k$, then $c_i \succ c_k$. The decision maker can then express his preference for consequences by a real-valued "utility function" $u(\cdot)$ such that

$$c_i \succ c_j \text{ if and only if } u(c_i) > u(c_j)$$

The choice of experiment e* and action a* should be made such that

$$E_{\Theta\Omega}[u(e^*,\omega,a^*,\theta)] = \max_{\substack{a\varepsilon A \\ e\varepsilon E}} E_{\Theta\Omega}[u(e,\omega,a,\theta)] \tag{227}$$

APPENDIX C

SEISMIC BEHAVIOR OF CONCRETE AND METAL STRUCTURES

General

The behavior of steel, reinforced concrete, prestressed concrete, and composite structures has been tested by the recent strong motion earthquakes of Skopje, Yugoslavia, in 1963; Niigata, Japan, in 1964; Alaska, U.S.A., in 1964; and Caracas, Venezuela, in 1967. Especially the earthquakes of Alaska and Caracas have revealed valuable information regarding the behavior of different materials as well as the overall behavior of multistory buildings. Recently it has been customary for teams of structural engineeers to go to the site of an earthquake immediately after its occurrence and to study the various forms of failure and behavior of different materials. The reports of these engineers are therefore very useful for locating areas of further research.

First of all, these damage reports of the four aforementioned earthquakes are examined to help locate the problem areas. Then the research which has been done concerning these problem areas will be cited and discussed.

Damage Reports from Niigata Earthquake

1. Steel Building Structures (Minou Makina, Building Research Institute) (233)

Most of the steel structures in the city took the earthquake quite well. Where there were fissures and depressions and if the subsoil conditions were bad there was considerable damage; this was the case for the Niigata Timber Manufacturing Center. There were occasional failures at beam-column connections (Plywood Factory in Niigata, Pipeline Station for Petroleum Gas in Tsukiji Village), gusset plates (Agricultural Warehouse of Niigata Economic Union in Niigata), and fracture of tensile members (a factory building in Nikaho Town). None of these buildings was multistory and poorness of subsoil conditions was shown to be the primary reason for most failures. A boiler house in Niigata which is a steel gabled frame house (35 m. high) and which sits on a mat foundation with 8 m. concrete piles was structurally undamaged even in a destructive area of subsoil.

2. R/C Constructions (Koichiro Ogura, Meiji University) (234)

Out of a total number of 340 normal reinforced concrete

buildings, 236 had some damage and 104 had no damage. Again out of 77 box-type reinforced concrete buildings, only 17 experienced some damage while 60 experienced none.

Table 4 summarizes the influence of settlement or slant on the amount of cracking of concrete in all the damaged reinforced concrete structures.

Table 4

Effect of Settlement on Structural Damage (234)

	Notation	Settlement and Slant	Cracks on the Body	Number of Buildings	
Damage	A	Building had shown sinking, unequal settlement, or slant due to a displacement or deformation of the ground.	Big cracks occurred on the body over the whole building.	30	353
	B		Small cracks occurred, or deflection at all the expansion joints occurred.	46	
	C		Cracks scarcely occurred or no cracks were found.	140	
	D	Building had shown no sinking, unequal settlement or slant.	Big cracks occurred (likely due to vibration).	12	
	E		Small cracks occurred, cause of which could not be seen.	25	
No damage	F	No or slight settlement or slant.*	No cracks or slight cracks.	164	

*Slight slant means the slope is less than one-hundredth.

3. Reinforced Concrete Structures, Syuzo Takada (Chiba University) (235)

From Table 4 the following information is obtained. Nine such (reinforced concrete) buildings were observed and four different modes of failure were considered: (a) damage to body, (b) damage to finish, (c) overturning, and (d) unequal sinking. These buildings are of moderate height (6-11 stories) and none of them experienced any "damage to body." Only one was observed to have "damage to finish." However, three of them experienced both "overturning" and "unequal sinking" (same three).

4. Building Damage and Soil Condition, Yorihito Ohsaki (Building Research Institute) (236)

After a big fire in 1955 in Niigata City, reinforced concrete construction expanded greatly and at the time of the 1964 earthquake there were a total of 1,530 reinforced concrete structures. Of these, 340 were more or less damaged by the earthquake. The following is an excerpt from the report of Y. Ohsaki:

"In every case of earthquakes in the past, wooden buildings used to suffer heavy damage and reinforced concrete structures remained safe. However, during the earthquake of Niigata last year, the situation was completely reversed, a fact which may be regarded as the most characterstic feature of this earthquake. Considering this situation, it seems appropriate to focus attention mainly upon the damage to reinforced concrete buildings."

The Venezuela Earthquake of 1967 (Magnitude 6.5 on Richter Scale)

Reinforced concrete construction is by far the most widespread type of construction in Venezuela. In the Los Palos Grandes district where four buildings collapsed, all the multistory construction is reinforced concrete with the exception of the two 30-story steel frame towers of the Simon Bolivar Center. Therefore the discussion will be focused on the performance of reinforced concrete structures in Caracas during the 1967 earthquake.

The concrete used in construction was of very good quality and varies in strength from 2400 psi to 4300 psi. The ordinary reinforcing steel has a minimum yield strength of 34000 psi which corresponds to a working strength of about 17000 psi. A type of high strength reinforcing steel (Heliacero) which is commonly used has a minimum yield strength of 56000 psi and a working strength of about 28000 psi. This higher strength is obtained by cold working the

steel and alters the stress-strain diagram by eliminating the flat plateau between the yield point and the start of the strain hardening region.

The typical construction of multistory buildings in Caracas consists of reinforced concrete frames with hollow tiles as fillers. Most of the design practices would also correspond to the provisions of the U.S. building codes. One exception may be the absence of top reinforcing in the center of joists and slab steel parallel to the joist. This would be considered as normal in areas where the seismic risk is very small but not in relatively severe seismic areas such as California in the United States. One major problem which was completely overlooked in design was the interaction of the reinforced concrete frame with the filler wall.

Of the 157 multistory (from 4-22 stories, most between 10-20 stories) buildings in Los Palos Grandes district of Caracas, Venezuela, four of them (10-12 stories) completely collasped, 31 suffered major and 15 suffered minor damages (after subjective evaluation of Hanson and Degenkolb).

The absence of the top reinforcing steel in the joists may have been the reason for the failure of three of the collapsed buildings. This, coupled with deviations from original plans and calculations led to the collapse of four buildings.

Causes for major structural damage can be classified in three groups. The first one is due to the unfavorable interaction between the reinforced concrete frame and the filler hollow tile. This localized the damage to the "soft" stories where no filler was used (especially first stories which were used for parking). However, the frames in these "soft" stories were not designed to carry the larger forces and therefore failed. The second type of failure was due to the unexpectedly high overturning moments. These moments caused compression failures in the columns and in some cases ruptured the less ductile Heliacero reinforcing steel in tension. Last, but not least, is the failure due to poor detailing of connections. This is one area which is very important and common to both reinforced concrete and steel structures in earthquake resistant design.

The above discussion was based on a report by Hanson and Degenkolb for the American Iron and Steel Institute (237). The PCA report (238) also finds the Caracas codes and design procedures comparable to U.S. practices for a-seismic design. It is concluded that "...most of the dis-

tress resulted not from inadequate codes, but from a lack of understanding of the dynamic behavior of structures."

Some statistics are given about the number of "unsafe" and collapsed buildings. "Out of 7000 buildings higher than four stories, approximately 180 buildings were damaged and listed as temporarily unsafe. Of these 180, approximately 40 had structural distress. Five out of the 1000 buildings higher than 10 stories collapsed."

As the AISI report also points out, most of the high-rise buildings in Caracas were reinforced concrete construction. One of the interesting and plausible reasons of failure of these reinforced concrete frame buildings is discussed in the PCA report. "High-rise buildings in Caracas are typically reinforced concrete frame construction without shear walls. Although the buildings were designed as frames, the interior partitions and exterior walls are constructed with brittle clay tile of low strength but high rigidity. These walls give the buildings the stiffness and response of shear wall buildings and influenced the distribution of earthquake forces within the structure."

This adverse effect of frame-partition interaction was also pointed out in the AISI report and shows how different structures can behave under dynamic loading.

Skopje Earthquake of 1963 (239)

According to the data gathered by the Statistical Institute of the Socialist Republic of Macedonia the damage inflicted upon the housing due to the earthquake can be summarized in Table 5:

Table 5

Earthquake Damages in Skopje (239)

Degree of Damage	Flats Damaged (%)	Living Space Affected (%)	Population Affected (%)
Collapsed	8.5	7.05	8.5
Heavily damaged (most buildings have to be demolished)	33.6	29.9	36.4
Moderately damaged	36.3	39.9	30.6
Slightly damaged	19.0	19.8	20.3
Undamaged	2.6	3.4	4.2

Ambraseys (240) divided the buildings in Skopje into four categories: (a) old adobe construction with or without timber bracing; (b) load-bearing brick wall construction supporting reinforced concrete, or wood floors supported partly by masonry walls and partly by reinforced concrete columns and beams; (c) reinforced concrete skeleton buildings with and without concrete shear walls; (d) there was also one roof structure of prestressed concrete and a few isolated cases where use of prefabricated, prestressed or ordinary elements were made.

The new steel mill in the outskirts of Skopje was the only steel frame construction.

The adobe and the mixed construction performed badly as expected. The reinforced concrete skeleton buildings were far better and only two collapsed. The taller buildings (up to 15 stories) in this category performed much better since they were designed for wind forces. The prestressed construction totally collapsed when the supporting columns failed. The steel mill which was still under construction had only minor damage. None of the buildings was designed for earthquake forces.

Alaskan Earthquake of March 27, 1964 (241)

The damage caused during this earthquake is of special importance because the many buildings affected by the earthquake were designed according to the earthquake provisions of the Uniform Building Code. The damage was caused either by the seismic sea wave (tsunami) effects, or by earth vibration effects. Most of the buildings of structural interest were in the Anchorage area, which experienced primarily the earth vibration effects and therefore this aspect will be considered here. Another big cause of damage was the landslides which took place because of the earthquake; a detailed study of the soil conditions leading to these landslides can be found in the report made by the Shannon and Wilson firm (242). This earthquake also demonstrated the correlation between the epicentral distance to the structures mostly affected by the earthquake. The epicenter of the Alaskan earthquake was about 75 miles from Anchorage and therefore the dominant periods were quite long (more than 0.5 sec.). For this reason, taller and more flexible structures attracted more force, and this fact should be kept in mind when the damage reports are studied. The following is a description of the damage on the metal and concrete structures as quoted by Steinbrugge (241).

"All metal buildings performed excellently as may be typified by the usual absence of damage to all steel gasoline service stations.

"Some damage, however, was noted in all metal structures having large elevated masses. Equipment shifted, one bin fell, X-bracing broke, metal skin buckled, and steel columns twisted at the Chugach steam power generating plant located in the Ship Creek section of Anchorage; differential settlements indicated that foundation problems probably accentuated this damage. Nearby, the Elmendorf Air Force Base had its principal damage restricted to the steel connections from a large elevated pit to the main structure. Overall damage to both power plants was estimated to be slight, and overall equipment damage probably could also be classified as being slight. A third power plant, owned by the City of Anchorage and also located in the Ship Creek area, had apparently negligible damage.

"Hollow concrete block was a common construction material for small mercantile structures as well as for industrial structures. Roof and supported floors were usually wood. When small in area, when one story high and when not located in the land movement areas, such damage as sometimes occurred was usually no more than slight to moderate. Some parapets fell, unanchored roofs punched out sections of the hollow concrete block walls, but collapse was uncommon. Wire webbing for reinforcement was usually laid in selected horizontal joints of the hollow concrete block. The placement of vertical steel was inconsistent. This reinforcement, in general, appeared to be of a size and amount that would not be considered to be fully adequate in other sections of the United States which are considered seismically very active. Workmanship was often poor where collapsed walls were noted; the concrete grout did not fill all of the cells in many observed instances.

"Poured-in-place reinforced concrete wall construction performed well for small buildings. Usually the roof and supported floor materials were wood for the small buildings, although not always. Instances of metal deck and metal open web joist roof and floor systems were found. The performance of these structures was generally good and somewhat better than similar size structures of different masonry materials when not located in the land movement areas."

The five-story Penney Building which was entirely of reinforced concrete (both poured-in-place and precast) collapsed. Poor detailing especially of the poured-in-place to precast connections was a major cause for the collapse.

Also, large torsional forces were developed in the upper stories because second floor and above was structurally a U-shaped building.

The Four Seasons Apartment Building was six stories and was only structurally completed (it had not yet been finished). It had prestressed concrete (in both directions) slabs for floors which were supported by steel columns. It also had two cores of poured-in-place reinforced concrete. These two cores failed and the building collapsed. Faulty splicing and shear connections, and the lack of grouting of the prestressing cables, were pointed out to be the major causes of failure. The building was very closely located to the main graben of the "L" Street landslide.

Hill Building which was eight stories had a central core of poured-in-place reinforced concrete, a steel frame, and at the upper stories poured-in-place reinforced concrete one-way slabs. The major damage was concentrated at the central core. Laboratory tests for the concrete indicated excessive organic material which had resulted in extremely low strength concrete.

Steinbrugge (241) completes his report with the following conclusion: "Precast reinforced concrete performed rather poorly in too many instances. However, failures were almost inevitably associated with the connections to the precast elements. It would appear obvious that the material was usually not at fault, rather a lack of sound engineering judgment was the principal cause. There was no evidence to indicate that one construction material was superior to another when given comparable design and construction attention.

After presenting these damage reports of the four earthquakes, we shall now turn to the research done in the areas of steel, reinforced concrete, prestressed concrete, and composite structures under strong motion earthquakes. A comparison of the performance of different materials is unwarranted. Steinbrugge's quotation in the last paragraph quite accurately sums up the situation. However, one point should be made regarding concrete structures. Concrete is a newcomer to the field of earthquake engineering. "Ductile concrete" is a very recent concept (50). Therefore the majority of the literature concerns reinforced or prestressed concrete behavior under earthquake loading.

One of the biggest problems common to both steel and concrete structures concerns detailing. Beam-to-column joints are one of the major problem areas. The following

will comprise a detailed discussion of connections, both concrete and steel, and also a representative paper will be presented for each material emphasizing both the merits and the shortcomings. Other research will be cited briefly.

Reinforced Concrete

Splicing is one of the important details of beam-column connections. At the University of Illinois a series of ten tests were carried out on lapped splices in beam-column connections (243). The main objective was to study the effects of splice length, splice location, and stirrup spacing on the behavior of tension splices in column specimens. The following is an excerpt from the conclusions of the investigators (243):

"1. For equal splice lengths, splicing away from the point of maximum moment increases both the ultimate deflection and the steel stress at ultimate.

"2. Placing the through bar on the inside of the cross section resulted in a lower load and deflection at ultimate.

"3. A closer spacing of stirrups over the splice length resulted in greater ductility and also larger maximum steel stresses for the same concrete stress.

"4. The ultimate deflection of all the spliced specimens, except C-10, was less than half that of a similar unspliced column."

Another valuable study of reinforced concrete beam-column joints was carried out by Hanson and Connor (244). This study specifically concerned the seismic resistance of the joints. The authors take the recommendations of Blume, Newmark, and Corning (50) as a basis for their investigations and summarize them as follows:

"Crucial recommendations for earthquake resistant design are the following:

"1. Sufficient transverse and shear reinforcement to provide a shear strength greater than the flexural strength.

"2. Limitations on the amount of tensile reinforcement or required use of compression reinforcement, to ensure ductility and energy absorbing capacity.

"3. Confinement of the concrete by hoops or spirals of critical sections such as beam column connections, to increase the ductility of columns under combined axial load and bending.

"4. Special attention to details, such as splices in reinforcement and exclusion of planes of weakness that would

result from bending or terminating all bars at the same sections."

With these recommendations in mind, the investigators devised a test to determine joint reinforcement required to ensure maintaining ultimate capacity for cast-in-place beams and columns subjected to multiple reversals of loading of major earthquake magnitude. An exterior beam-column connection was selected for study since this is the most critical in a multistory structure.

The following conclusions were drawn by the authors:

"The series of tests demonstrate that properly designed and detailed cast-in-place reinforced concrete frames can resist moderate earthquakes without damage and severe earthquakes without loss of strength. Adequate energy absorption is provided by ductility of reinforcing steel. Joints connecting beams and columns need special attention in design:

"1. Hoops are required for unconfined (isolated) beam-column joints. A design procedure for hoops based on supplying adequate confinements and shear resistance will provide safe designs.

"2. Corner joints with beams on only two column faces should be designed as unconfined joints requiring hoops until tests are made for this case.

"3. Hoops are not required for exterior joints confined on at least three sides by beams or spandrels of approximately equal depth and meeting ACI 318-63 requirements for concrete strength needed to transfer the column load through the joint.

"4. The cumulative ductility of a test specimen provided a measure of the ability of a structure to withstand seismic deformation. Well detailed joints sustained high values of cumulative ductility while they maintained their strength. Omission of important hoop reinforcement reduced amount of cumulative ductility which could be sustained.

"These details of joint reinforcement are not limited to seismic design. Good design practice requires that joints be designed as strong as the adjoining members. These tests indicate that for the rare case of an isolated joint, a design for the usual forces of wind, dead, and live load will require hoops."

More specialized literature includes the earthquake effects on reinforced concrete chimneys (245,246), walls, (247), shells and plates (248), frames (249), and buildings

(250-252). The dynamic characteristics of plain concrete (253,254), and the effect of reinforcement on the ductility of concrete members (255), are also relevant topics to earthquake engineering.

Since the topics of dynamic torsion and dynamic stability have not been explored extensively, static torsion (256,257) and axial loads (258) must serve as guidelines for the time being.

Prestressed Concrete

Before the Alaskan earthquake of 1964, prestressed concrete structures had not been tested by a strong motion earthquake. Unfortunately, during this earthquake some prestressed concrete structures performed very badly. It was later discovered that the failures were mainly due to faulty aseismic design and foundation failure, rather than the inadequacy of the material. Nevertheless, these failures coupled with the comparative lack of knowledge about the seismic behavior of prestressed concrete, made many engineers uneasy about this material for use in earthquake resistant structures. Most of the aseismic design codes around the world do not make explicit reference to prestressed concrete. An exception for this is the French code entitled "Règles Paraseismiques P.S. 64," which explicitly admits prestressed concrete without providing any special limitation. There are misconceptions and conflicting ideas about the seismic performance of prestressed concrete, and extensive experimental data is urgently needed on this subject.

In a paper by Despeyroux, some of the questions which most frequently arise regarding the seismic behavior of prestressed concrete, are discussed (259).

On the subject of resistance to reversed loadings, Despeyroux points out that as long as the reversed loading does not exceed the yield stresses, fatigue is not a problem. For reinforced concrete elements the resistance to reversed loading remains unchanged up to 80% of the ultimate strength (260,261). Therefore, he argues, there is no reason not to expect the same performance from prestressed members. However, this theory has not yet been confirmed by experimental results.

The second question pertains to the excessive flexibility of prestressed concrete members compared to members of the same bearing capacity made with other materials. This, in turn, leads to damage in brittle non-structural elements. Despeyroux first points out the advantages of flexibility in aseismic design. The major advantage is, of course, the

fact that flexible structures in general, attract less earthquake forces than more rigid structures. Then Despeyroux recommends using stricter code provisions against drift and hammering to prevent excessive unwanted deformation.

The third question concerns the ductility and energy-absorbing capacity. Using the experimental results of Caulfield and Patton (260) on prestressed concrete, and his own theoretical moment-curvature relationship for reinforced concrete, the author concludes that both the energy-absorbing capacities (areas under moment-curvature curves), and the ductilities (ratios of the ultimate deflection to yield deflection) of prestressed and reinforced concrete are of the same order of magnitude. Below is the graph from which he draws his conclusions.

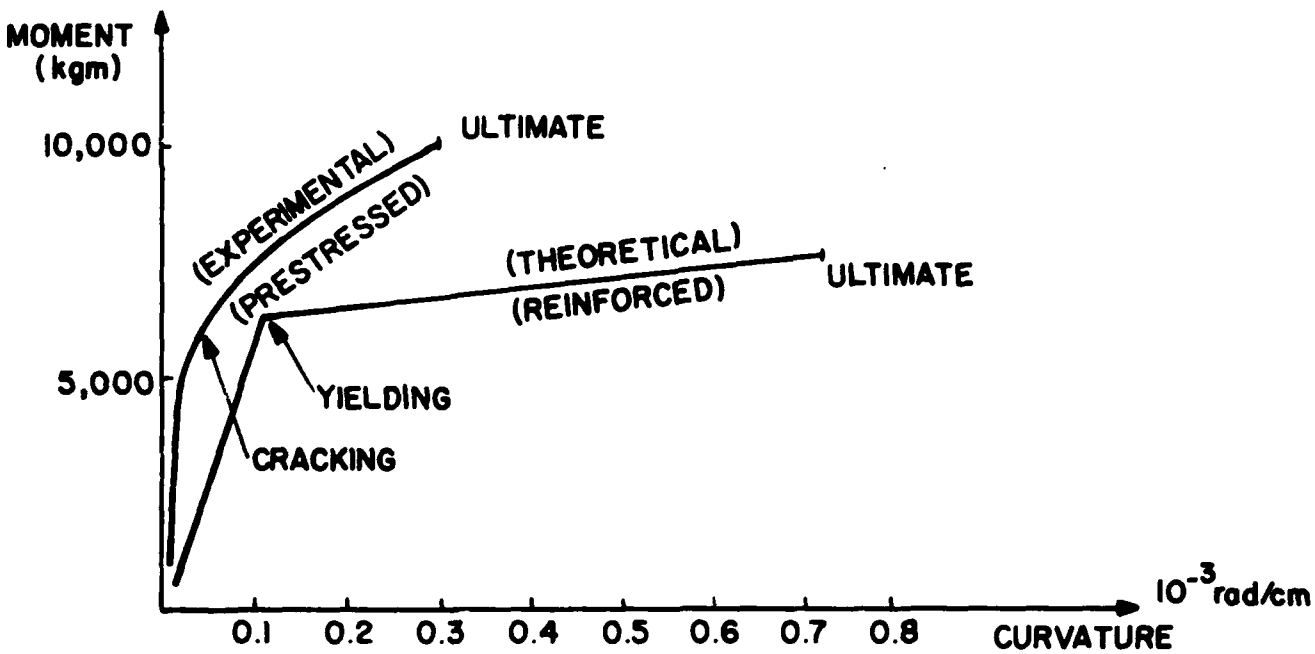


Figure 19. Moment vs Curvature

This conclusion of Despeyroux is also experimentally confirmed by Guyon (261) and Sutherland (262).

As for the joints of prestressed concrete members, Despeyroux cautions the design engineer to use his good judgment. He says that these problems are well known and refers to some literature on the subject (263).

In a similarly written paper Lin (264) concurs with just about every point that Despeyroux makes. Other useful material on the subject includes some experimental model studies by Nakano (265) and determination of dynamic response of prestressed concrete beams by Hamilton (266).

Steel

Most engineers view steel as the most suitable and reliable material for multistory earthquake resistant buildings. Both the strength and the ductility of steel structures tend to be greater than similar buildings of different material. It is also true that during strong-motion earthquakes, steel buildings suffered relatively little damage. However, one very important point in steel design as well as design of buildings with any other material, is the connections. Low-cycle fatigue and brittle failure of beam-column joints of steel structures may have occurred. For this reason an experimental work by Bouwkamp (267) on girder-to-column steel connections will be discussed below.

This experiment was a detailed study on four large full-size girder-to-column connections. Each connection was composed of a riveted, 36-inch by 36-inch built-up column with bolted T-sections and welded plates which acted as moment resistant supports for 42-inch-deep welded girders. The material used in the design of girders and columns was a combination of A7 and A373 steel. The two-way framing system without any interior columns made the length of the girders 93 feet. There were two objectives of this project and they are summarized as follows in the report:

"The first objective was to determine the stiffness of the T-Sections which accomodate the flanges of the 93-foot long floor girders. This information was important in order to evaluate the end-restraining effect (rotation) of the large span girders and to increase if necessary the rigidity of these connections in order to reduce the floor deflections. Furthermore, an acceptable rigidity would favorably contribute to the overall lateral stiffness of these buildings. The second objective was to determine the yield and ultimate strength of the 1-1/2-inch and 2-inch thick welded plates between the column flanges and web. These plates are 30 inches wide and H. S. bolted to the flanges of the 43-inch deep girders spanning the short distance between the columns in the exterior frames. Since no previous informa-

tion was available about the strength of these welded plates as affected by residual stresses due to welding, two geometrically identical specimens of the types shown in Figures 2 and 3 were to be investigated, one non-stress relieved and one stress relieved. Should the tests indicate the need for stress relieving, all girder to column connections were to be treated to relieve the residual stresses."

The following was the test program:

"1. Specimen 1 (Type I, third floor connection, non-stress relieved),

2. Specimen 2 (Type I, third floor connection, stress relieved),

3. Specimen 3 (Type II, fourteenth floor connection, non-stress relieved),

4. Specimen 4 (Type II, fourteenth floor connection, stress relieved)."

The yield and ultimate loads for the welded plates were as follows:

"Specimen 1. non-stress relieved; welded plate 30x2 in^2.

Py=1800 k, fy=30 ksi; P_{ult}=2805 k, f_{ult}=47 ksi

Specimen 2. stress relieved; welded plate 30x2 in^2.

Py=2200 k, fy=37 ksi; P_{ult}=2750 k, f_{ult}=46 ksi

Specimen 3. non-stress relieved, welded plate 30x1-1/2 in^2.

Py=1400 k, fy=31 ksi; P_{ult}=2200 k, f_{ult}=49 ksi

Specimen 4. stress relieved; welded plate 30x1-1/2 in^2.

After load of 1600 k was reached end weld was cut

P_{ult}=1344 k, $f_{shear\ ult}$=44 ksi

"The results for Py indicated that the stress distribution in the moment plates was improved by stress relieving. Similar conclusions were derived from the stress distribution along critical sections close to the welds. These welds indicated that the distribution of the load over the several welds was better for the two stress relieved connections than for the non-stress relieved specimens."

The prototype for this experiment was the Health Sciences Instruction and Research Buildings for the University of California, San Francisco Medical Center, as

designed by Ried and Tarics, Architects and Engineers, San Francisco. This complex consists of two buildings: the Main East Building which is 15 stories high and the West Building which is 16 stories. There is also a connecting corridor between the two buildings, two mechanical service towers, and an elevator tower. The Main East Building has been the site for some more dynamic testing done by the University of California, Berkeley (268). These involve frequency response, time response, mode shapes, energy transfer, and damping characteristics.

Some of the important conclusions are listed below:

1. "The steel frame of the service tower had very little effect on the dynamic behavior of the East Building in the summer of 1964, and the East Building was tested virtually in isolation. However, when the steel frame had been encased in concrete, the service tower did affect the behavior of the East Building; in fact, both buildings then formed a new structural system. This interconnected system had much larger damping capacity then any of the isolated buildings."

2. "The increase of damping after the summer of 1964 was caused mainly by the connections between buildings."

3. "A standard open frame model was found to reproduce accurately the dynamical behavior of the East Building. This model was subjected to the ground acceleration record of the El Centro earthquake. It was found that the amount of damping in the model played an important role in determining the extent to which yielding occurred."

Damping of this building is discussed in more detail in reference (269).

Behavior of welded connections (270) and inelastic response of steel structures (271-275) also have application in the field of earthquake engineering.

More experimental research is necessary to determine the properties and the behavior of materials. The parameter that is used in the equations of motion, and the idealization that is made in the analysis, depend on the material properties to a great extent. Also as was mentioned earlier, the behavior of details and connections of structural elements made from different materials, is very important.

The dynamic effect can be divided into two effects as applied to materials. The first effect is that of the

strain rate $\dot{\varepsilon}$. As the strain rate increases, the yield stress and the modulus of elasticity (very slightly) increase. However, this is much more pronounced in blast type loadings and the increases can be ignored for earthquake considerations. The second one is the effect of reversed action or what may be called the Bauschinger effect. This results in deterioration or decrease in both the yield stress and the modulus of elasticity. This is a problem in earthquake-type loading and should be investigated further. Some literature on the subject (276-280) is presently available.

Idealization of Mechanical Behavior

No matter what material is to be used, there must be an idealization of the material, the cross section, the member, and the structure. Nonlinear and inelastic idealizations proved to be the most suitable for strong motion earthquake-resistant structural analysis. Therefore these topics have received considerable attention in recent years (13,14,281-288). It is well known that the damage due to repeated deformations is cumulative and could lead to low-cycle fatigue failures (81,82). Preliminary results indicate that it is possible to have low-cycle fatigue failures of seismic structures (18).

Below is a summary of different idealizations and their deviations from the "true behavior" (289).

The following relationships must be studied:

σ vs. ε for the material	(stress vs. strain)
M vs. ϕ for the cross section	(moment vs. curvature)
M vs. θ for the member	(moment vs. rotation)
H vs. δ for the structure	(load vs. deflection)

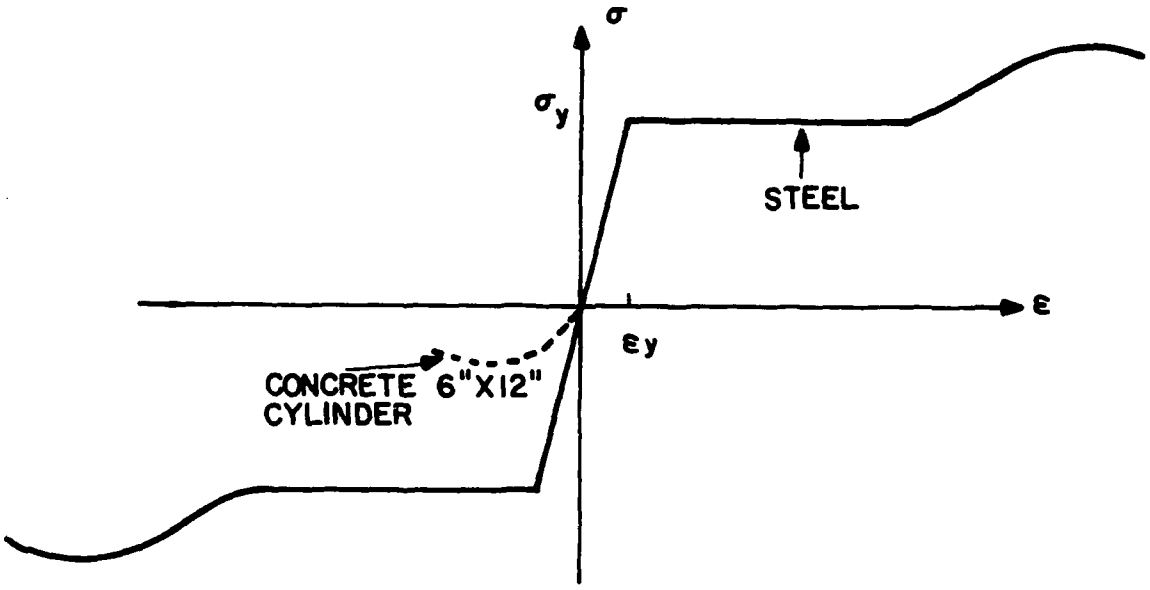


Figure 20. Stress-Strain Relations

There are two problems concerning the stress-strain relationship.

A. Does the strain rate $\dot{\varepsilon}$ have any effect on σ vs. ε? This effect is usually considerable in blast loading, but for earthquakes the increase in σ_y, E and σ_{ult} is negligible.

B. What is the effect of the reversed action (the Baushinger effect)? This may be a problem in earthquake type loading.

1. M vs. ϕ of a cross section

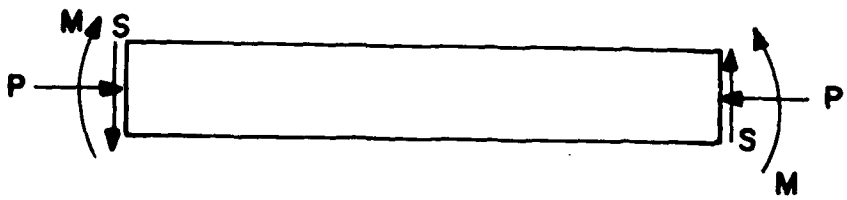
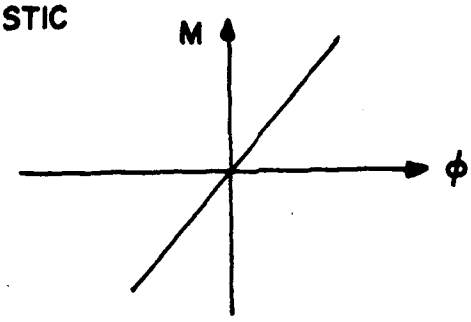
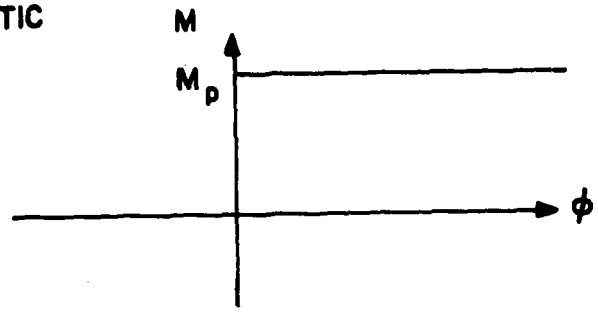


Figure 21. Moment vs. Rotations

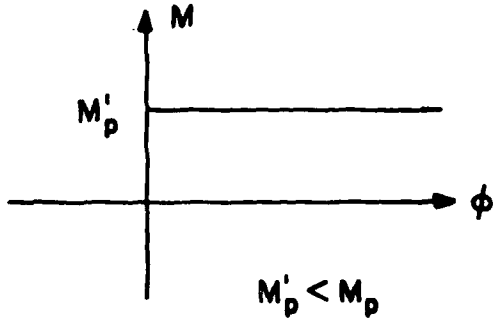
(a) LINEAR ELASTIC



(b) RIGID PLASTIC



(c) RIGID PLASTIC (INCLUDING EFFECT OF P AND S)



(d) LINEAR ELASTIC-PERFECTLY PLASTIC

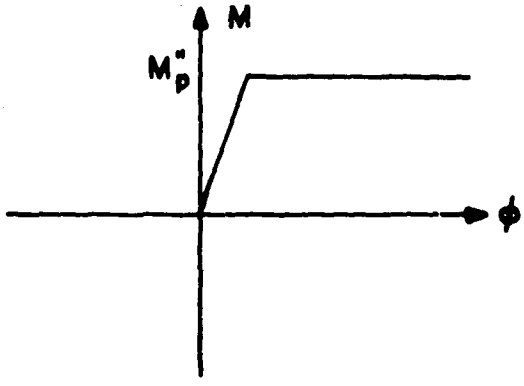
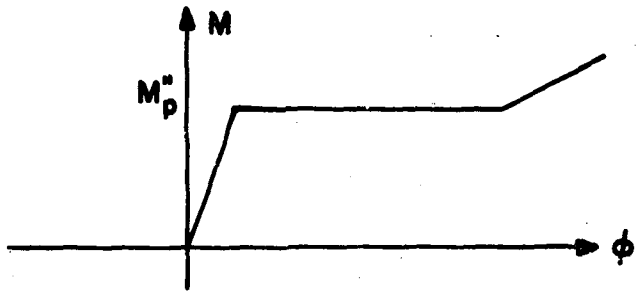
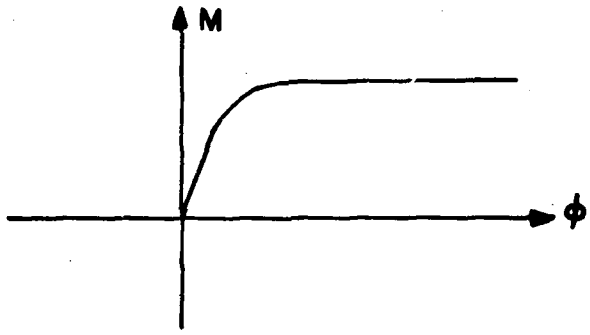


Figure 21 (continued)
Moment vs. Rotations

(e) LINEAR ELASTIC-PERFECTLY PLASTIC (WITH STRAIN HARDENING)



(f) RESIDUAL STRESSES INCLUDED



(g) (f) + STRAIN HARDENING

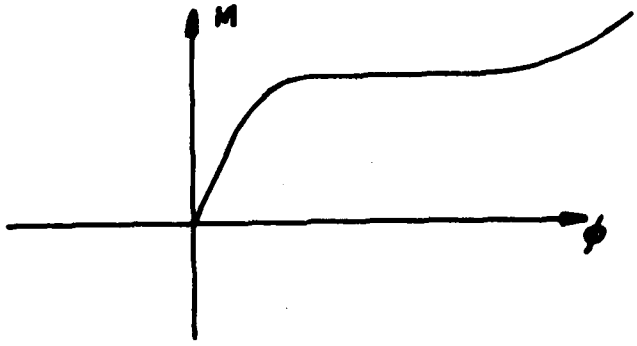


Figure 21 (continued)
Moment vs. Rotations

2. M vs. θ for the member

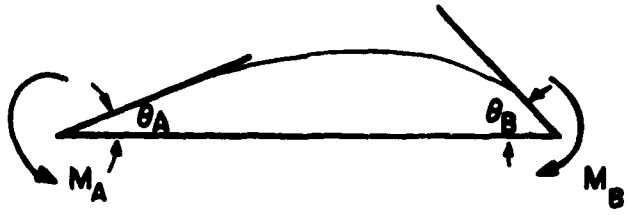


Figure 22. Free-Body Diagram of a Beam

(a) First order theory (neglect change in geometry).

(b) Second order theory beam-column problem.

3. H vs. δ for the structure

(a) First order theory

(b) Second order theory--P-Δ effects

All the combinations of assumptions under 1, 2, and 3 are plotted on Figure 23 (289), which also includes the so-called "true behavior." This diagram dramatically illustrates the differences in the load-deflection relations of unbraced frames under various idealizations.

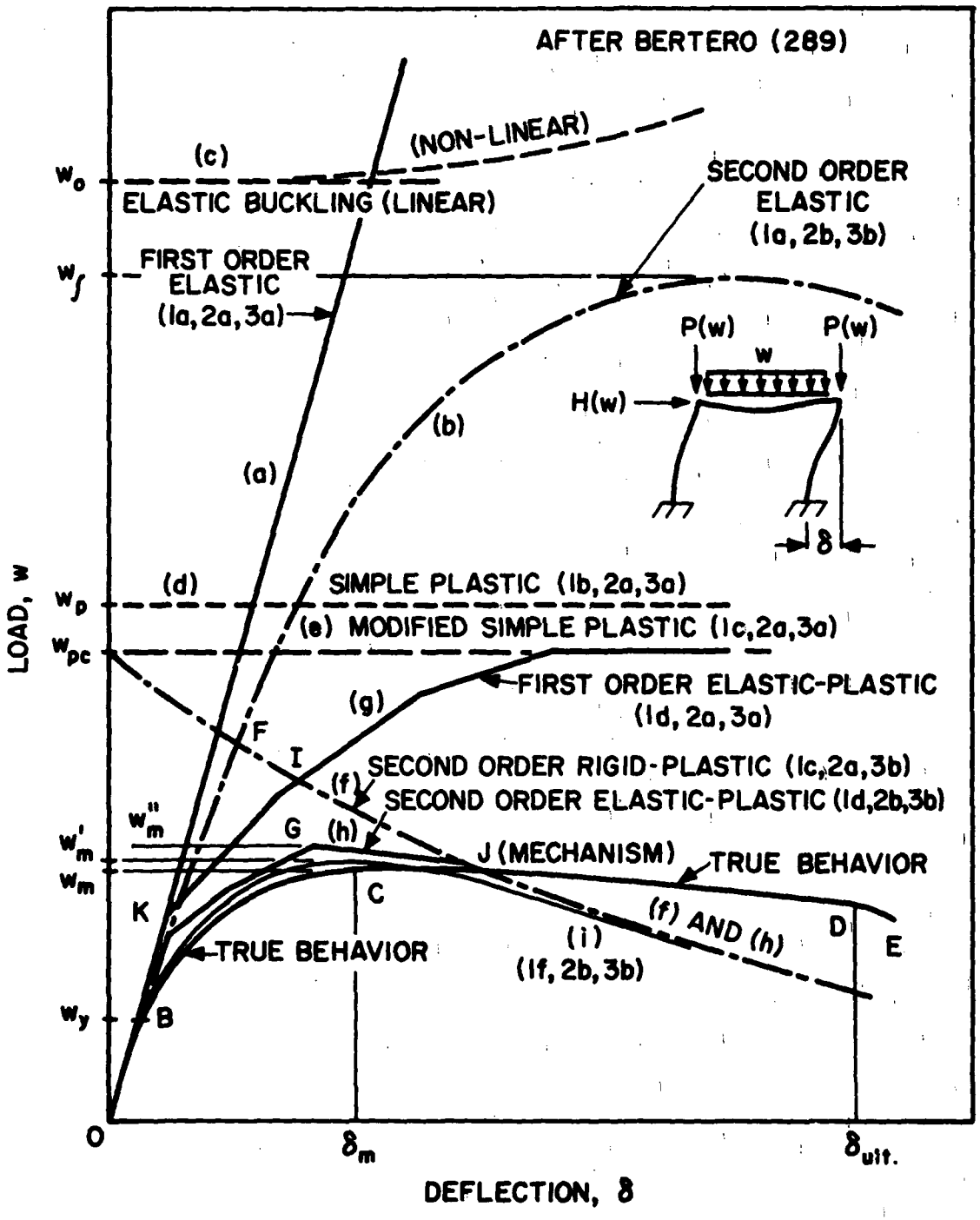


Figure 23. Behavior of Unbraced Frames

APPENDIX D

AN ILLUSTRATION OF THE DIRECT APPROACH

Until an extensive research program is carried out, it is difficult to find a numerical example demonstrating completely the direct approach in the seismic design of building structures. Nevertheless, it is possible to illustrate the concepts involved herein with a simple example, which is described in this appendix.

Consider the seismic design of a single-degree-of-freedom bilinear hardening structural frame. The seismic behavior of such a structure was studied by Yeh and Yao (87), who found that the maximum displacement response of the bilinear structure to earthquake excitation can be less than the maximum displacement of a linear system with either a soft or a hard stiffness. Moreover, this bilinear hardening system compares favorably with nonlinear systems undergoing plastic deformations because, with a proper design, the members remain elastic to avoid possible low-cycle fatigue damage in the structure(18).

For this example, the direct approach portion of the design tree can be constructed as shown in Figure 24. The alternative solution A_{21111} to the excitation question Q_{2111} is chosen to be the 1940 El Centro earthquake record. The digitized plot of this acceleration time-history is shown along with a response function in Figure 25. In a similar manner, the second alternative solution A_{21112} is chosen to be the 1952 Taft earthquake record.

The alternative solution A_{221} to the model question Q_{22} is chosen to be a discrete model as shown in Figure 26. The diagonal bracings are purposely made slack initially. As the deformation reaches a predetermined value Δ, one of these cables will become tight and begin to contribute to the frame stiffness. This structural frame can be represented by a mechanical system with bilinear stiffness as shown in Figure 27. The equation of motion for this system is given as follows:

$$m\ddot{u} + c\dot{u} + k_1 u + \mathrm{sgn}(u)(k_2-k_1)(|u|-\Delta)H(|u|-\Delta) = f(t) \qquad (228)$$

where m is the mass of the system

c is a damping constant

k_1 and k_2 are spring constants $(k_2>k_1)$

f(t) is the forcing function

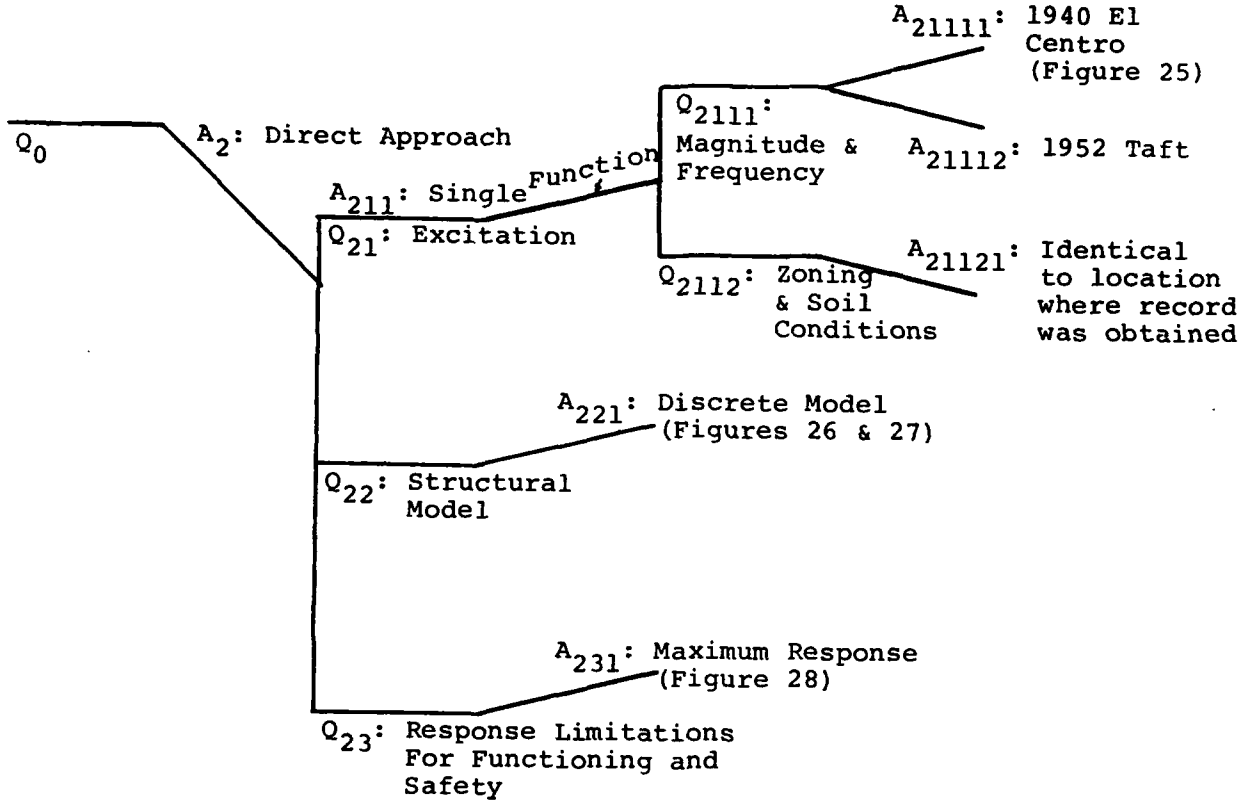


Figure 24. Application of Design Tree in the Design of a Bilinear Structural Frame

Following Yeh and Yao (87)

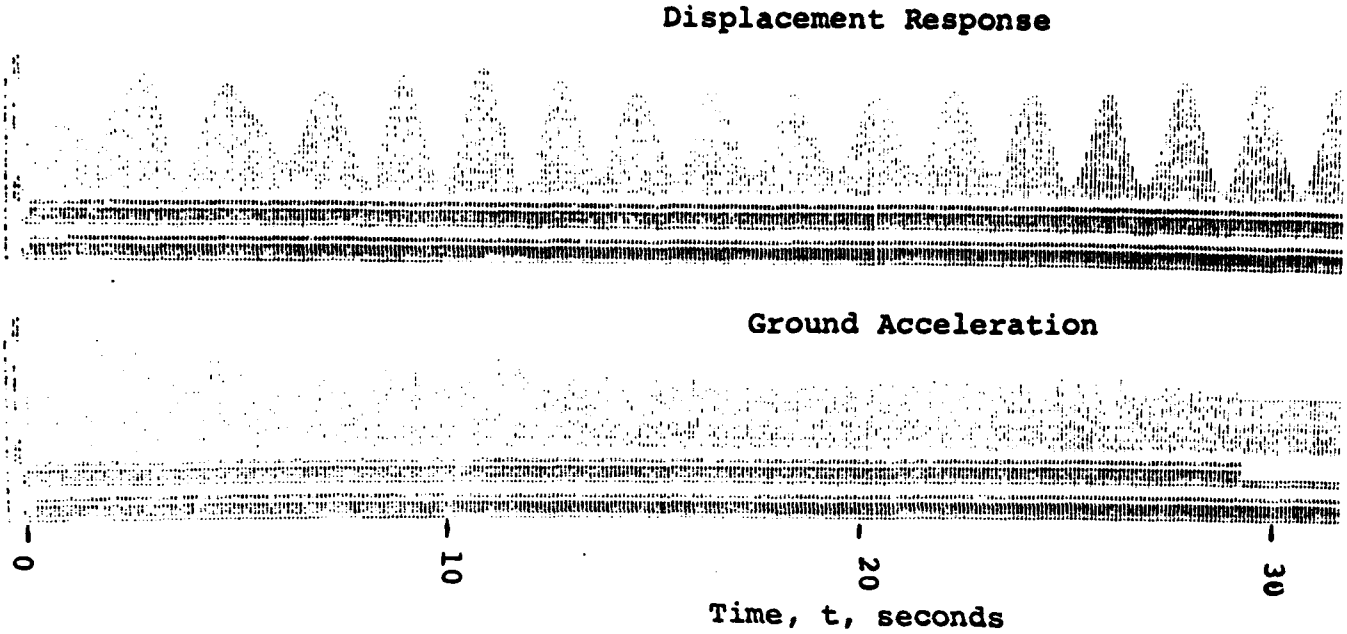


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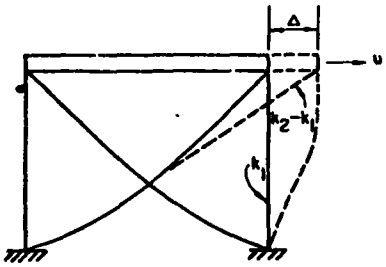
Figure 25. Acceleration Record of 1940 El Centro Earthquake and a Typical Structural Response



$$\sqrt{\frac{k_1}{m}} = 3.23 \text{ rad/sec.}$$

$$\sqrt{\frac{k_2}{m}} = 3.76 \text{ rad/sec.}$$

Figure 26. One-Degree-of-Freedom Bilinear Structural System

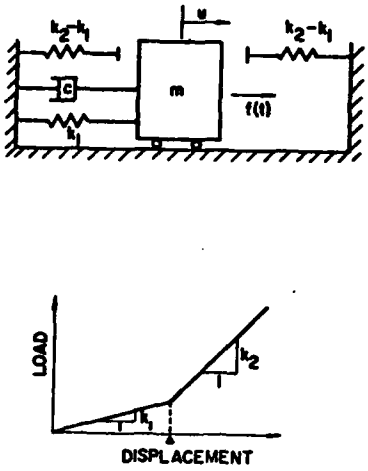


Figure 27. One-Degree-of-Freedom Bilinear Mechanical System

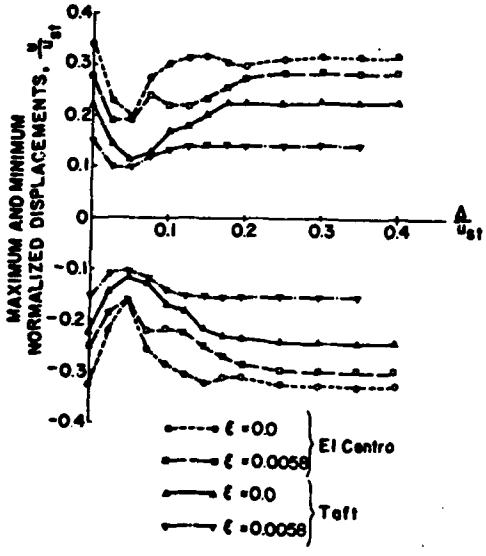


Figure 28. Responses of Single-Degree-of Freedom System to El Centro and Taft Earthquakes

u is a variable denoting the displacement response

Δ is the distance at which the displacement-force relationship is piecewise continuous.

$$\mathrm{sgn}(u)=\begin{cases}1, & \text{if } u>0\\ 0, & \text{if } u=0\\ -1, & \text{if } u<0\end{cases}$$

H(x) is the Heaviside step function

For convenience the following expressions are also introduced,

$$u_{st} = \frac{mg}{k.}$$

g is a gravitational constant

$$\varepsilon = \frac{C}{2\ mk.}$$

For this particular case, $\sqrt{\frac{k_1}{m}}$ and $\sqrt{\frac{k_2}{m}}$ were arbitrarily chosen to be 3.23 rad/sec and 3.76 rad/sec, respectively, and ε, a dimensionless damping factor, was assumed to be 0.0 and 0.0058.

Suppose that we are interested in selecting a design with the best value of Δ/u_{st} on the basis of maximum response A_{231}. Using the Continuous System Modeling Program with an IBM 360/40 digital computer, the relationship between the maximum response and the nondimensionalized Δ was found as shown in Figure 28. On this basis, the design engineer could choose a value of Δ/u_{st} of 0.05, which results in a maximum displacement of 0.2 of the corresponding static displacement u_{st} for zero damping and the 1940 El Centro earthquake excitation.

This simple example is given herein to illustrate the concepts involved in the direct approach method, particularly the use of the design-tree in seismic design. As it is pointed out in the text of this report, the seismic design of building structures is a complex problem. For example, the proper choice of damping coefficients for various types of buildings is to be determined. Consequently, more research is required to make the direct approach really practical in seismic design.

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