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13 ABSTRACT

A computational procedure is presented to explore and formulate the quantitative relations between the preference distribution of a population for menu items and a corresponding set of optimum choicegroups on a selective menu schedule. The investigation is limited to the decision rules applicable for the determination of a single choicegroup, and it is carried out in two stages. First, it is shown that for any preference matrix a unique process of choicegroup augmentation exists with the properties that the augmented choicegroup will provide a maximum increase in population preferences along with the probability estimates of selections. Second, it is shown that if the preference matrix is updated in the function of time and predicted selections, a multistage process of choicegroup generation can determine a sequence of choicegroups with optimal properties concerning the size or preference contribution involved. A computer program which operates on these principles is attached, and is being utilized for column generating functions of constrained optimization models of selective menu scheduling.

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I INTRODUCTION

Menus are shopping lists of ready to eat products, called menu items, which are displayed or considered to be available for the consumer at food service establishments. From an economic point of view, these products fall naturally into well defined categories according to the existence of complementary or substitution effects in their use. The categories between which complementary effect is normally assumed are the courses on the menu. This means that most people prefer a meal containing a complementary combination of menu items such as appetizers, entries, vegetables, deserts, beverages, etc. If this combination is fixed by the management, the menu is called nonselective, and it leaves the trivial choice for the consumer of either eating the predetermined set of items, or only a subset of the items. That is, there is no substitution effect to speak of. On the other hand, if the menu lists more than one item in a course category, these items are considered to be substitutes in use, and a selection is allowed from them, mostly on a mutually exclusive basis. Such menus are called selective menus, and the set of items associated with one given course is referred to as a choicegroup. In these terms, any selective menu can be regarded as a list of more or less non-overlapping choicegroups.

Since the practical size of any choicegroup on a menu is much less than the number of menu items eligible for a particular course, food service management repeatedly faces a decision problem: which items and how many of them should be included in a choicegroup. Judged from the varying sizes of choicegroups seen on the menus of basically similar food service organizations and the various expert opinions on the subject [4] there seems to be no common policy or rule of thumb in use, and definitely no sign of theoretical work which is considered acceptable or followed by food service managers. Although the problem is closely related to the process known as product diversification [3] in the economic literature, the corresponding mathematical relations are not applicable to the specific short run character of selectivity on the menus.

Decisions concerning choicegroups are repeatedly made in the process of scheduling a menu, but the essential elements of the decision problem are the same for each choicegroup. For this reason, the present study focuses on the limited problem of analysing the conditions under which an internally consistent decision rule can be found for determining the set membership and size of a <u>single choicegroup</u> - disregarding for the time being other choicegroups of the menu.

The point of departure for the analysis is the realization that every selection from a choicegroup is incidental with revealing a particular consumer's preference for an item in the choicegroup relative to the others. The selections in general, therefore, are the manifestations of preferences existing in the population for menu items. An earlier study of the authors' [2] and others [5] have shown that the preferences for menu items are measurable quantities, although the exact relation between an individual's preference rating and his behavior at the choicepoint are not yet fully understood. Nevertheless, for the purpose of this study it will be assumed that the quantitative values of the preferences of individuals at the time of selection are known deterministically. This assumption serves only expository purposes by creating manageable conditions for the formulation of a mathematical model which will define optimum decision rules. It is hoped that by understanding and analyzing an abstraction of reality, progress can be continued toward more realistic applications of the principles determined by this study.

In part II of the report the hypothetical conditions concerning population preferences for a set of menu items are formulated in terms of a preference matrix. It is shown that within this structure the concept of the most preferred choicegroup and the population preference increment due to increased choicegroup size can be uniquely determined. An essential by-product of this formulation is full information on the relative proportions of item selections from choicegroups.

Part III presents a review of the assumed time dependent behavior of preferences studied by the authors earlier, and describes the process of updating the preference matrix of a population in the function of individual selections from a choicegroup.

Part IV deals with the process of finding the most preferred choicegroups for a sequence of meals or days with the choicegroup size limited to a predetermined value or to a predetermined level of population preference, but with no other constraints such as cost or nutrition considered.

The study of the choicegroup generation principles formulated under Parts II and IV has been accomplished by a computer program written for this purpose and attached in the Appendix.

II THE MATHEMATICAL BACKGROUND OF CHOICEGROUP AUGMENTATION

Consider the (m x n) matrix H of preference ratings for a class of n menu items belonging to the same course as rated by m individuals. The general element h_{ij} of H is the preference rating of the i-th individual on a like-dislike scale for menu item j . It is known that the matrix H is not defined without consideration for the history of exposure of the items to the population. Only $h_{ij}(t)$ is defined well enough for mathematical treatment, and techniques for its estimation have been developed [2]. First assume that the $h_{ij}(t)$ values are all known for any arbitrary t value, and thus the time effect can be taken out of consideration. Table 1 illustrates such a matrix H for m = 15 and n = 10.

Let h_j be the j-th column vector of H , i.e., the preference ratings of m individuals for item j . Then the number h(j) and $\overline{h}(j)$ are defined as follows:

$$h(j) = \sum_{i=1}^{m} (h_{ij} \mid h_{ij} \geq t_i) \quad (1)$$

$$\overline{h}(j) = \frac{h(j)}{m}$$

h(j) is the total population preference for item j , $\overline{h}(j)$ is the average population preference for item j , with t_i being the threshold level of preference below which "skipping" takes place. If j^* is the one single item most preferred by the population,

$$h(j^*) = \max_j h(j)$$

Although $h(j^*)$ is a maximum, this does not imply that item j^* is the most preferred item among all the n items for all individuals in the population. In other words, it <u>does not imply</u> that for all i (individuals), $h_{ij^*} \geq h_{ij}$ for any j . This phenomenon is well known in the food service business, and leads intuitively to the policy of offering selective menus.

Any choicegroup of size k constitutes k items that can be offered from any course on a selective menu. Let Ω_k be the set of all choicegroups of size k taken from n items, and let s_k be a general element of Ω_k defined by

$$s_k = \{j_1, j_2, \cdots, j_k\}$$

Just as h_j was defined as the j-th column of the preference matrix H, with h_{ij} being the preference of the i-th individual for the j-th item, a column vector, h_{s_k} can now be defined with h_{is_k} being the preference of the i-th individual for choicegroup s_k. The i-th element of h_{s_k} is defined as

$$h_{is_k} = \max_{j \epsilon s_k} \{h_{ij}\} \tag{2}$$

The important underlying assumption in (2) is that the preference of a person for a choicegroup is equal to the person's preference for the menu item he prefers most in the choicegroup. The total population preference with choicegroup s_k on the menu schedule is defined as

$$h(s_k) = \sum_{i=1}^{m} [h_{is_k} \mid h_{is_k} \geq t_i] \tag{3}$$

Note that we differentiate between $h(s_k)$ and h_{s_k}. The latter is a column vector of preferences of individuals for choicegroup s_k whereas the former is the total preference of the population for choicegroup s_k. Thus h_{s_k} is a vector, while $h(s_k)$ is a scaler.

The number of persons out of m who will select item j_ℓ of the choicegroup s_k is given by

$$m_{j_\ell} = \sum_{i=1}^{m} [1 | h_{ij_\ell} = h_{is_k}, h_{ij_\ell} \geq t_i] \tag{4}$$

The number m_{j_ℓ} is therefore the number of persons whose preferences for item j_ℓ exceed the preferences for all other items in choicegroup s_k. Thus the preference matrix has information not only about what the preference for a choicegroup is, but also about what proportion of a population will select a particular item if it is offered together with other specified menu items in a choicegroup. This, therefore, gives the sales estimates for the items in a given choicegroup.

The choicegroup s_k^* for which the population preference is a maximum is such that

$$h(s_k^*) = \max_{s_k \varepsilon \Omega_k} h(s_k) \tag{5}$$

It is thus possible to determine which choicegroup of size k is most preferred by evaluating all the $\frac{n!}{(n-k)!k!}$ combinations of n items taken k at a time, and applying (3) in the summations.

An example is given to illustrate the inherent simplicity of the mathematics of preferences as given by the above formula for m=5 and n=3. (Also see [1].)

$$\text{Let } H = \begin{bmatrix} 8 & 7 & 8 \\ 4 & 7 & 6 \\ 3 & 8 & 7 \\ 9 & 7 & 7 \\ 7 & 4 & 6 \end{bmatrix}$$

Let $t_i = 4$, $\quad i = 1, 2, \cdots, 5$

By (1), $h(1) = 28$, $h(2) = 33$, $h(3) = 34$

The single item with the highest preference total is $j^* = 3$.

By (2) and (3)

$$h_{12} = \begin{bmatrix} 8 \\ 7 \\ 8 \\ 9 \\ 7 \end{bmatrix} \quad h(1,2) = 39 \qquad h_{13} = \begin{bmatrix} 8 \\ 6 \\ 7 \\ 9 \\ 7 \end{bmatrix} \quad h(1,3) = 37$$

$$h_{23} = \begin{bmatrix} 8 \\ 7 \\ 8 \\ 7 \\ 6 \end{bmatrix} \quad h(2,3) = 35 \qquad h_{123} = \begin{bmatrix} 8 \\ 7 \\ 8 \\ 9 \\ 7 \end{bmatrix} = h_{12}$$

Hence the most preferred choicegroup of size 2 should contain items 1 and 2.

By (4), $m_1 = 3$ and $m_2 = 2$ if items 1 and 2 are offered in a choicegroup. Item number 3, although the most preferred single item, is not present in the most preferred choicegroup of size 2, In fact, $h_{123} = h_{12}$, which means that item 3 does not contribute to the preference of the choicegroup, and is dominated by the column vectors corresponding to the preferences for items 1 and 2.

Suppose it is possible to describe the values of preferences of the j-th item over the population by a probability density function $f_j(x)$ and its associated distribution function $F_j(x)$. This implies that the probability that the preference of the i-th individual for the j-th item lies between any arbitrary limits a and b, is given by the following:

$$P[a \leq h_{ij} \leq b] = \int_a^b f_j(x)\,dx$$

and

$$F_j(x) = P[h_{ij} \leq x] = \int_{-\infty}^{x} f_j(z)\,dz$$

The number $h(s_k)$ is the total preference derived from choicegroup s_k by the population, and let $\bar{h}(s_k)$ be the average preference derived per person from the population. Since the density function of the distribution of preferences for all the items is assumed to be known, it is now possible to find an expression for the expected value of $\bar{h}(s_k)$ in terms of arbitrary density and distribution functions.

The probability that for any individual, item j_1 has a preference between x and $x+dx$, and is more preferred than all other items in the choicegroup s_k is given by

$$P[h_{ij_1} \varepsilon (x,x+dx),\ h_{ij_2} < x,\ h_{ij_3} < x,\ \cdots,\ h_{ij_k} < x]$$

$$= f_{j_1}(x)dx \cdot F_{j_2}(x) \cdot F_{j_3}(x) \cdots F_{j_k}(x)$$

where $x \geq t$, the threshold preference level.

Putting

$$G_{j_r}(x) = \prod_{\ell \neq r} F_{j_\ell}(x)$$

the above expression becomes

$$f_{j_1}(x) \cdot G_{j_1}(x) \cdot dx$$

The probability of selection of item j_1 if choicegroup s_k is offered is therefore

$$\int_t^{\infty} f_{j_1}(x) \cdot G_{j_1}(x) \cdot dx$$

In general, the probability of selection of item j_r in choicegroup s_k is

$$\int_t^{\infty} f_{j_r}(x) \cdot G_{j_r}(x) \cdot dx \qquad (6)$$

This probability also gives the fraction of the population which can be expected to select item j_r if choicegroup s_k is offered. The skipping probability with a threshold preference level t is thus given by

$$\sum_{j_r \varepsilon s_k} \int_{-\infty}^{t} f_{j_r}(x) \cdot G_{j_r}(x)dx$$

It is instructive to show that the skipping probability, together with the selection probabilities as given by (6) for all the items, does indeed sum to 1.

$$\text{To show} \quad I \equiv \sum_{j_r \varepsilon s_k} \int_{-\infty}^{\infty} f_{j_r}(x) \cdot G_{j_r}(x)dx = 1$$

The first term of the summation in the above expression is

$$I_1 \equiv \int_{-\infty}^{\infty} f_{j_1}(x) \cdot G_{j_1}(x)dx \qquad (7)$$

Differentiating

$$G_{j_1}(x) = F_{j_2}(x) \cdot F_{j_3}(x) \cdots F_{j_k}(x)$$

$$G'_{j_1}(x) = \sum_{\substack{j_\ell \varepsilon s_k \\ j_\ell \neq j_1}} f_{j_\ell}(x) \cdot \prod_{\substack{j_\eta \neq j_\ell \\ j_\eta \neq j_1}} F_{j_\eta}(x)$$

Integrating (7) by parts results in

$$G_{j_1}(x) \cdot F_{j_1}(x) \Bigg|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} F_{j_1}(x) \sum_{\substack{j_\ell \varepsilon s_k \\ j_\ell \neq j_1}} f_{j_\ell}(x) \cdot \prod_{\substack{j_\eta \neq j_\ell \\ j_\eta \neq j_1}} F_{j_\eta}(x)$$

which is equivalent to

$$I_1 = 1 - \sum_{\substack{j_\ell \varepsilon s_k \\ j_\ell \neq j_1}} \int_{-\infty}^{\infty} f_{j_\ell}(x) \cdot G_{j_\ell}(x) \cdot dx \qquad (8)$$

However

$$I_1 + \sum_{\substack{j_\ell \varepsilon s_k \\ j_\ell \neq j_1}} \int_{-\infty}^{\infty} f_{j_\ell}(x) \cdot G_{j_\ell}(x) \cdot dx = I$$

Therefore, by (8) $I = 1$. This completes the proof.

The sales estimate for item j_r for choicegroup s_k on the schedule is given by

$$m \int_t^{\infty} f_{j_r}(x) \cdot G_{j_r}(x) \cdot dx$$

where m is the number of persons in the population. But to be able to make the primary decision about which choicegroup is to be on the schedule, it is necessary to know the average preference that each person may be expected to derive from each choicegroup. This expected value $E[\overline{h}(s_k)]$ is given by

$$E[\overline{h}(s_k)] = \sum_{j_r \varepsilon s_k} \int_t^{\infty} x \cdot f_{j_r}(x) \cdot G_{j_r}(x) \cdot dx \qquad (9)$$

The choicegroup of size k which is most preferred is s_k^*, and has the property that $E[\overline{h}(s_k^*)] = \max\limits_{s_k \varepsilon \Omega_k} E[\overline{h}(s_k)]$.

If there are n possible items from which a choicegroup of size k must be selected, the maximum achievable preference, (or the total preference of the population if all n items are offered) is given by $h(s_n)$ in the case where the matrix H is explicitly known, and by $E[h(s_n)]$ in the case where only the density functions of the column of preferences in the H matrix are explicitly defined. The average maximum achievable preference is then $\bar{h}(s_n)$ and $E[\bar{h}(s_n)]$ respectively.

For any choicegroup size k under consideration, it will be convenient to use as a reference point the case where all the n items are offered, and so one can look at the fraction $E[\bar{h}(s_k^*)]/E[\bar{h}(s_n)]$ (or just $\bar{h}(s_k^*)/\bar{h}(s_n)$ if the matrix H is explicitly known) or its associated percentage, which expresses the percentage of the maximum preference actually achieved.

If $f_j(x)$ is known for $j=1, \cdots, n$, application of (9) to all the $n!/[(n-k)!\ k!]$ different choicegroups yields the choicegroup of size k which is most preferred. $E[\bar{h}(s_k^*)]/E[\bar{h}(s_n)]$ then gives the percentage of the maximum preference achieved.

As an illustrative example, consider the case where

$$f_1(x) = f_2(x) = \cdots\cdot = f_n(x) = f(x)$$

This assumption implies that the density function and the expected preference from all choicegroups of equal size is the same. In spite of its lack of realism, this simplifying assumption affords the analytical simplicity to illustrate the effect of increasing choicegroup size.

Relation (9) now reduces to

$$E[\bar{h}(s_k^*)] = E[\bar{h}(s_k)] = k \int_t^\infty x \cdot f(x) \cdot [F(x)]^{k-1} dx \tag{10}$$

Relation (10) can be used to determine $E[\bar{h}(s_k^*)]$ for any choicegroup size k if f(x) is known. The integral in (10) may not always yield to analytical attack for all f(x), but its numerical evaluation is always possible, as the examples below indicate.

(i) Suppose, for example, that f(x) follows the uniform distribution as defined by

$$f(x) = \begin{cases} \frac{1}{b} \text{ for } 0 < x < b \\ 0 \text{ Otherwise} \end{cases}$$

$$F(x) = \int_0^x \frac{dz}{b} = \frac{x}{b}$$

If the threshold preference t=0, applying (10)

$$E[\bar{h}(s_k^*)] = k \int_0^b x \cdot \frac{1}{b} \cdot (\frac{x}{b})^{k-1} dx$$

$$= \frac{k}{b^k} \int_0^b x^k dx$$

$$= \frac{k}{b^k} [x^{k+1}/(k+1)]_0^b$$

$$= \frac{bk}{k+1}$$

$$\text{So } E[\bar{h}(s_n)] = \frac{bn}{n+1}$$

Therefore the fraction of the maximum preference that can be expected to be achieved with choicegroup size k is

$$(\frac{bk}{k+1}) / (\frac{bn}{n+1}) = \frac{k}{(k+1)} \cdot \frac{(n+1)}{n}$$

This formula provides the first analytical insight with regard to the expected benefits of a population due to choicegroup augmentation. By increasing the size of the choicegroup k the percentage of the maximum preference achievable by the population will increase proportionally with $k/(k+1)$, i.e., at a decreasing rate. Figure 1 shows the monotonically increasing step-function associated with this process in case of uniformly distributed preferences.

(ii) Let $f(x)$ follow the exponential distribution as defined by

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{for } x > 0 \\ 0 & \text{Otherwise} \end{cases}$$

$$F(x) = \int_0^x \lambda e^{-\lambda z}\, dz = (1-e^{-\lambda x})$$

Computations similar to the case of the uniform distribution yield

$$E[\bar{h}(s_k^*)] = \frac{k}{\lambda} \sum_{i=1}^{\infty} \frac{1}{i(k+i)}$$

Figure 1 shows the stepfunction of the percentage of the maximum preference achieved by augmenting the choicegroup in the case of exponentially distributed preferences. It is noticeable that the preference effect of choicegroup augmentation is significantly less than before.

(iii) If it is assumed that $f(x)$ is normally distributed, analytical expression for (10) is no longer available, but the percentage of the maximum achievable preference of the population still can be estimated by simulation techniques. The result of the simulation is shown on Figure 1, and compares well with the analytically defined values.

In each of the above described cases identical distribution of preferences was assumed over a population of indefinite size to facilitate analytically well defined conclusions concerning the effects of choicegroup augmentation. The results displayed in Figure 1 exhibit the rapidly diminishing utility of adding additional items to the choicegroup, irrespective of the assumed probability density function of $f(x)$.

This effect became even more prevalent when the choicegroup augmentation process was applied to a (15x10) matrix of preferences of 15 individuals for 10 dessert items as shown in Table 1. The matrix entries were computed from estimated parameters of realistic preference-time functions observed by experiments [2] and the data were initialized so they reflect past histories of selections as it would occur in reality. The maximum achievable preference in this case is the sum of the row maximums. Offering one choice alone (item 1) as Figure 1 shows, would realize only 71.69% of the maximum. The best choicegroup of two items (1 and 8) would increase the percentage by 13.09%, and the addition of a third item would contribute only 6.78%. Moreover, 8 out of 10 items are sufficient to achieve the maximum. The conclusion for the realistic case is that convergence to the maximum achievable preference is much faster than in all the cases with assumed identical density functions of the preferences. The discrepancy can be easily explained by the arbitrary assumption made in the simulated cases that all items are, on the average, equally preferred. In reality, it is expected that there will be a wide variation, even in the average preference of the population for different items. As a consequence, the most preferred item will contribute most heavily to the maximum achievable preference, and the contributions will diminish more rapidly as we keep adding less and less preferred items.

In order to test the validity of this explanation, uniformly distributed preferences were simulated with uniformly distributed means producing a mixture of preferences with nonidentical means. Figure 1 shows that it is very likely that the heterogeneity of the population preferences is causing the sharp initial increase in the achievable maximum population preference. The points corresponding to the mixed uniform preferences are indeed fairly close to the points obtained from realistic data.

It is noticeable that under any assumption the achievable population preference is monotonically increasing as the choicegroup is augmented. This is to say that in terms of the previously adopted notations, the property $h(s_k^*) \leq h(s_{k+1}^*)$ holds as the set s_k is augmented by one item, resulting in set s_{k+1}. Hence the notation:

$$\Delta h_{k+1} = h(s_{k+1}^*) - h(s_k^*)$$

will express the increment in preferences due to the addition of the k+1-st item to the choicegroup. It is instructive to reconstruct this Δh_{k+1} value directly from the elements of the preference matrix H. Suppose k=1 and thus k+1=2, i.e., a nonselective choicegroup is augmented to a selective one according to (5). Let S denote the set of m individuals. If $\{j_1\}=s_1^*$ and $\{j_1,j_2\}=s_2^*$, then S can be partitioned into two subsets such that

$S = S_1 \cup S_2$ where

$$S_1 = \{i \varepsilon S \mid h_{ij_1} \geq h_{ij_2}\}$$

$$S_2 = \{i \varepsilon S \mid h_{ij_1} < h_{ij_2}\}$$

Consequently S_2 is the set of individuals who will be benefited by the second choice. One can also write that

$$h(s_2^*) = \sum_{i \varepsilon S_1} h_{ij_1} + \sum_{i \varepsilon S_2} h_{ij_2}$$

and thus the increment of preferences due to the j_2 item is

$$\Delta h_2 = h(s_2^*) - h(s_1^*) = \sum_{i \varepsilon S_2} (h_{ij_2} - h_{ij_1})$$

which means that the preference gain is generated only over a subset of the population. This result can be generalized to the case where the k+1-st item is added to k existing ones where $k > 1$. In this case S can be partitioned into k disjoint subsets according to the number of individuals who prefer any one of the k elements of s_k^* over the others. Consequently S_ℓ is defined as

$$S_\ell = \{i \varepsilon S \mid h_{ij_\ell} \geq h_{ij_\rho}\} \; ; \; \rho \neq \ell, \; j_\rho \varepsilon s_k^*, \; j_\ell \varepsilon s_k^*$$

and

$$\bigcup_{\ell=1}^{k} S_\ell = S$$

Then each S_ℓ can be partitioned in turn into S_{ℓ_k} and $S_{\ell_{k+1}}$ when the k+1-th item is included in s^*_{k+1} as follows:

$$S_{\ell_{k+1}} = \{i\varepsilon S_\ell \mid h_{i,k+1} > h_{ij_\ell}\}$$

and

$$S_{\ell_k} = S_\ell - S_{\ell_{k+1}}$$

Consequently the general expression of preference increment due to the new k+1-st item in the choicegroup is

$$\Delta h_{k+1} = \sum_{\ell=1}^{k} \sum_{i\varepsilon S_{\ell_{k+1}}} (h_{i,k+1} - h_{ij_\ell})$$

It should be noted that the quantity in the parenthesis is always non-negative by virtue of the definition of the $S_{\ell_{k+1}}$ sets, and if Δh_{k+1} is zero increasing the choicegroup size by k+1 is unwarranted. This effect includes the side benefit of eliminating skipping - or lost demand - by increasing the choicegroup.

The increase of population preference due to choicegroup augmentation is a measure of the benefits of selective menus which, however, cannot be realized without incurring some cost due to the increased number of items to produce. The production cost of a menu item can be viewed as made up of two parts: the set up cost, or fixed cost of putting the item on the menu, and the variable cost, which is proportional to the number of items sold or selected. Let $n_{\ell_{k+1}}$ be the cardinality of $S_{\ell_{k+1}}$ and consider S unchanged while s^*_k is augmented to s^*_{k+1}

The incremental cost Δc_{k+1} is then:

$$\Delta c_{k+1} = a_{k+1} + n_{k+1}c_{k+1} - \sum_{\ell=1}^{k} n_{\ell_{k+1}} c_{j_\ell}$$

where a_{k+1} is the fixed cost of item k+1 and c_{k+1} is the variable (unit) cost of the new item, while n_{k+1} is the cardinality of the union of the $S_{\ell_{k+1}}$ sets, i.e., the total number of item (k+1) demanded.

The negative terms in the expression indicate the reduction in the total cost due to a reduction in the demand of the other items. Since S is assumed to be unchanged, the quantity of k items will be decremented proportionally with their variable cost. The addition of the k+1-st item, if it gives a strictly positive value for Δh_{k+1} can never reduce the demand of any of the original k items to zero. Suppose this were possible. Let $j_r \varepsilon s_k^*$ be one of the items (or the only one) whose demand is driven to zero. Let a choicegroup s_k' consist of all the k+1 items of s_{k+1}^* except item j_r. Then, as item j_r does not contribute to the preference of the choicegroup, $h(s_k') = h(s_{k+1}^*)$. But $h(s_{k+1}^*) > h(s_k^*)$, and hence $h(s_k') > h(s_k^*)$. This contradicts the fact that s_k^* is the most preferred choicegroup of size k .

The development of expressions for Δh_{k+1} and Δc_{k+1} thus far has involved the important assumption that $j \varepsilon s_k^* \Rightarrow j \varepsilon s_{k+1}^*$, i.e. that the best choicegroup of size k+1 includes all items from the best choicegroup of size k . This need not be true for an arbitrary matrix H . Consider two preference matrices H_1 and H_2 of size 2x3, defined by

$$H_1 \equiv \begin{bmatrix} 4 & 8 & 2 \\ 7 & 6 & 5 \end{bmatrix} \qquad H_2 \equiv \begin{bmatrix} 2 & 7 & 6 \\ 7 & 3 & 6 \end{bmatrix}$$

With a threshold preference level of zero, the matrix H_1 yields $s_1^* = \{2\}$

and $s_2^* = \{1,2\}$, and therefore $s_1^* \subset s_2^*$. But for the matrix H_2, $s_1^* = \{3\}$ and $s_2^* = \{1,2\}$, and $s_1^* \not\subset s_2^*$.

Consider a general case in which a choicegroup s_k of size k is augmented by adding a set of r items, s_r, none of which are in s_k. The set S_ℓ of persons who will prefer item j_ℓ to all others in choicegroup s_k is defined as

$$S_\ell = \{i \varepsilon S \mid h_{ij_\ell} \geq h_{ij_\rho}\};\ \rho \neq \ell,\ j_\ell \varepsilon s_k,\ j_\rho \varepsilon s_k$$

It may be noted that no condition is laid down at this stage about s_k being the best choicegroup of size k . If h_{s_r} is constructed according to (2), and if S_{ℓ_j} is the subset of individuals from S_ℓ who are benefited by the introduction of item j of the set s_r, then

$$S_{\ell_j} = \{i \varepsilon S_\ell \mid h_{is_r} > h_{ij_\ell}\} \text{ for } j \varepsilon s_r$$

and $\tilde{S}_\ell$, the set of persons who prefer item j_ℓ in s_k in spite of the additional choice is given by

$$\tilde{S}_\ell = S_\ell - \sum_{j \varepsilon s_r} S_{\ell_j} \qquad \text{for } j_\ell \varepsilon s_k$$

The set of individuals $\tilde{S}_j$ who are benefited by the introduction of the item j of set s_r is given by

$$\tilde{S}_j = \sum_{\ell=1}^{k} S_{\ell_j} \qquad \text{for } j \varepsilon s_r$$

It is conceivable that a certain number of the sets S_ℓ, will be empty This implies that the augmentation of s_k by s_r has resulted in the demand for some existing items going to zero. Also, $\tilde{S}_j$, $j\varepsilon s_r$, may be zero, implying that there is no demand for some of the items from s_r. Let ρ be the number of sets from $\{\tilde{S}_\ell \mid j_\ell \varepsilon s_k\}$ which are empty, and let ν be the number of sets from $\{\tilde{S}_j \mid j\varepsilon s_r\}$ which are empty. As all items whose demand is zero do not contribute to the preference of the choicegroup, they can be dropped from the choicegroup. So the effective choicegroup size by uniting s_k and s_r into one choicegroup is $k+r-\rho-\nu$.

If $\tilde{r} = r - \rho - \nu$, the general expression for the preference increment due to the addition of the set s_r is then given by

$$\Delta h_{k+\tilde{r}} = \sum_{\ell=1}^{k} \sum_{\mu\varepsilon s_r} \sum_{i\varepsilon S_{\ell_\mu}} (h_{is_r} - h_{ij_\ell})$$

The incremental cost is then given by

$$\Delta c_{k+\tilde{r}} = \sum_{\substack{\mu\varepsilon s_r \\ \tilde{S}_\mu \neq 0}} (a_\mu + n_\mu c_\mu) - \sum_{\mu\varepsilon s_k} \Delta n_\mu c_\mu - \sum_{\substack{\mu\varepsilon s_k \\ \tilde{S}_\mu = 0}} a_\mu$$

where a_μ is the fixed cost of including the μ-th item on the schedule, c_μ is the variable per unit cost of item μ, n_μ is the cardinality of S_μ and Δn_μ is the cardinality of $(S_\mu - \tilde{S}_\mu)$ for $\mu\varepsilon s_k$. The first negative term indicates the reduction in total cost due to a reduction in demand, and the second negative term indicates the fixed cost savings because of demand being reduced to zero.

If the choicegroup of size k being augmented is s_k^*, a necessary condition for s_{k+1}^* to result is that $\tilde{r}=1$, and <u>it is not required that r be 1.</u>

In conclusion, the computational rules of the cost benefit analysis of choicegroup augmentation is fully provided. The values of $\Delta h_{k+\tilde{r}}$ above, with the values $\Delta c_{k+\tilde{r}}$ at every step of augmentation are uniquely determined with the above formulas. The computation and comparison of these values relative to a given preference matrix and cost structure will enable food service management to evaluate and balance the marginal benefit and cost of changing the choicegroup size. One of the options of the Choicegroup Generator Demonstration Program (CGDP) listed in the Appendix is to compute the maximum achievable preferences for different choicegroup sizes relative to a given preference matrix H , and hence to calculate the increased preference achieved by choicegroup augmentation. A by-product of these computations is the sales estimates for items in any choicegroup, and the change in sales estimates due to choicegroup augmentation. The points of the curve in Figure 1 for the realistic case were derived from the CGDP relative to the preference matrix of Table 1.

III THE PREFERENCE TIME FUNCTION AND THE UPDATING PROCESS

As has been asserted earlier, the preference of an individual for a familiar menu item depends on the history of exposure of the item to to individual. Experimental data from a recent study [2] support this hypothesis and provide a functional relation which makes possible the estimation of these preferences at any time. A brief review of the results of the above mentioned study, pertinent to this report, is included here.

First, assume that the preference of an individual for an item is related only to the time the item was consumed last; the effect of previous exposures being ignored. If $f_{ij}(t)$ is the preference of the i-th person for the j-th item at an elapsed time t from last consumption of the item,

$$f_{ij}(t) = a_{ij} - b_{ij}e^{-c_{ij}t} \tag{11}$$

where a_{ij}, b_{ij} and c_{ij} are parameters which depend on the individual and the item, and are identifiable from questionnaires. The above formulation implies that the time since last consumption determines the preference. In doing so, the formulation ignores the effect of the history prior to the time at last consumption. Now let t_{ij} be the time elapsed since last consumption of menu item j by person i and let h'_{ij} be the person's preference for the item at last selection. If $h_{ij}(t)$ is the preference of the person for the item at absolute time t, then the effect of history of eating can be included in the recursive relation;

$$h_{ij}(t) = f_{ij}(t_{ij}) - e^{-r_{ij}t_{ij}}(a_{ij} - h'_{ij}) \qquad (12)$$

where $r > 0$ is a parameter also identifiable from questionnaires and $f_{ij}(t_{ij})$ is defined by (11). Methods for the routine estimation of the parameters of the preference time function have been developed. To be able to determine $h_{ij}(t)$ from (11) and (12), the parameters a_{ij}, b_{ij}, c_{ij}, r_{ij} must be known, and so must the time elapsed since last consumption t_{ij}, and h'_{ij} the preference at last consumption. Each of the parameters of the preference-time function can be conceived of as embodying some characteristic of the function. The parameter a_{ij} is the preference of the individual for the item if he has not consumed it for a very long time. It is also the asymptotic maximum which cannot be exceeded by h(t). The intuitively appealing premise for the existence of a_{ij} is that a person would desire a familiar item most if he were not exposed to it for the longest time. The parameter b_{ij} is the decrease in preference ensuing immediately after consuming an item, because of the consumption of the item. So if an item is consumed after a very long time, the preference for it is a_{ij}. As soon as it is consumed, the preference drops to $(a_{ij} - b_{ij})$. As we get further away in time from the consumption of an item, the immediate satiation effect b_{ij} wears off. The rate of decay of satiation, or looking at it from another viewpoint, the rate of buildup of desire for the item, is controlled by the parameter c_{ij}. The larger the value of c_{ij}, the faster the effect of satiation wears off. The parameter c_{ij} can be expected to be large for staples like bread. Thus the effect of satiation is embodied in the term $b_{ij}\exp(-c_{ij}t_{ij})$. When $t_{ij} = 0$ (the item has just been consumed), the term is equal to b_{ij}; as t_{ij} grows, $b_{ij}\exp(-c_{ij}t_{ij})$ declines. Finally, the parameter r_{ij} determines how much the effect of history prior to last consumption of an item affects the current preference for the item. The larger the value of r_{ij}, the less the previous history affects current preference for an item.

The use of (11) and (12) to determine the preference at any time can best be illustrated by an example.

Let

$$
\begin{aligned}
a_{ij} &= 100.00 \\
b_{ij} &= 40.00 \\
c_{ij} &= 0.05 \\
r_{ij} &= 0.40 \\
t_{ij} &= 9 \quad \text{on day 1} \\
h'_{ij} &= 67.00 \quad \text{on day 1}
\end{aligned}
\tag{13}
$$

Suppose the item is not consumed at time $t = 1, 2, \cdots, 8$. The preference at $t = 9$ is given by

$$h(9) = 100 - 40e^{-0.05(17)} - e^{-0.40(17)}(100-67) = 82.87$$

If the item is offered on the 9th day, it may still not be consumed. If it is not consumed, h'_{ij} and the reference point for t_{ij} do not change, and the preferences can be determined for subsequent days just as h(9). On the other hand, if it is consumed, t_{ij}, the time at last consumption, is measured from $t = 9$, and $h'_{ij} = 67.00$ must now be replaced by $h'_{ij} = 82.87$.

Figure 2 illustrates the preference change over a 30 day period for an item characterized by (13). As the sharp preference drops in the figure indicate, the item is consumed on day 9, 16, 19, and 28.

Relation (11) and (12) thus provide a means for the day to day updating of the preferences of all individuals for all the items, if the initial conditions and the parameters of the preference-time functions are known.

Table 1 shows a matrix of preferences for an arbitrary day. Relative to this preference matrix, repeated application of (2) and (3) to all possible choicegroups of size 2 indicated that items 1 and 8 constitute the best choicegroup of size 2. An asterisk after an element h_{ij} of Table 1 indicates that item j is in the best choicegroup of size 2, and the i-th person will select item j. The symbol "+" indicates an item offered but not selected. Table 2 shows how the preferences look on a subsequent day. Upward and downward arrows in columns corresponding to items 1 and 8 indicate the increase and decrease of preferences after offering items 1 and 8 on the previous day. For each location in the H matrix bearing an asterisk in Table 1, the preferences have dropped to the levels indicated in Table 2. For each location in the H matrix of Table 1 bearing the symbol "+" (item offered but not selected) or having no symbol at all (item not offered), the preferences have increased to the levels indicated in Table 2. The preference-time function parameters used for obtaining these preferences were computed from responses to questionnaires and by the method outlined in [2]. The initial conditions were arbitrarily selected.

IV MULTISTAGE SCHEDULING OF OPTIMUM CHOICEGROUPS

The mechanism of continuously updating the elements of the preference matrix H from day to day can now be used in conjunction with the method of evaluating all $n!/[(n-k)!\ k!]$ choicegroups of size k to find and schedule, i.e., generate, the optimum choicegroups of menu items from day to day.

Let $H(t)$ be the preference matrix for the t-th day. Clearly, $H(t+1)$ and $H(t)$ are not independent. In fact, $H(t+1)$ is obtained from $H(t)$ in a manner depending on which items are offered on the t-th day, and which items are selected from those offered. A change of a single item on any day can affect the preferences for all choicegroups on all subsequent days. Even for a small number of items and a small choicegroup size, the number of distinct schedules becomes astronomical. For example, with 10 items and a choicegroup size of 2, the number of distinct choicegroups possible on any day is $10!/(8!2!) = 45$. The number of distinct schedules for a 7-day cycle is $(45)^7 = 373669453125$. This clearly points to the need for a highly selective technique to explore the maze of possible solutions.

A logical stage of a multi-day schedule is a day itself, and this suggests a technique to circumvent the problem of a combinatorially burgeoning solution space. The problem of selecting the optimum choicegroups can be tackled sequentially from day to day, without consideration of the following days, hence the name "multistage scheduling." It may seem that the practical effect of this simplification is a loss of guaranteed optimality.

This is true if the required schedule is for a finite period of p days, and p is relatively small. Otherwise the multistage schedule will likely arrive at a steady state - some time after initialization - exhibiting an internally defined period p ; thus in this case the period is no longer a constraint on the schedule, and it does not affect the optimality. The problem here is, however, that this internal period may be too long for practical considerations. This problem area is currently the subject of further investigation.

The most trivial case of multistage scheduling is a sequence of nonselective, i.e., unit size, choicegroups. This schedule contains the sequence of the most preferred menu items as defined by the successive updating of the H(t) matrices. Computational experience and Figure 1 suggest that the percentage of achievable preference of the population will fluctuate around 70% for such nonselective schedules. If the sequence was generated from n menu items, the steady state period, i.e., a nonrepeating subsequence of the items will usually exceed the value of n by several times.

At this point food service management may consider a selective menu for improving acceptability. There are two ways to proceed. Management can decide on a fixed choicegroup size k in which case the multistage process will find the optimum choicegroup s_k^* from each updated preference matrix H(t) and the optimum schedule will contain a sequence of s_k^* choicegroups. The principles laid out in section II will assure that on any day the percentage of the achievable preference is at maximum, and choicegroups of size k in the steady state will therefore achieve higher population preference than choicegroups of size (k-1). Consequently, the findings of choicegroup augmentation can be directly applied to the multistage schedule.

The other possible policy for management is to prescribe the percentage of the achievable preference for the population, and leave it to the choicegroup generating process to determine the necessary size k for every day or meal of the schedule. This policy will tend to maintain a uniformly acceptable menu schedule over time, but will require flexibility in adjusting the choicegroup size.

For the purposes of studying these policies, the principles of choicegroup generation have been incorporated in an experimental demonstration program CGDP (Choicegroup Generator Demonstration Program). A program listing is given in the Appendix. The listing corresponds to a Fortran IV version of the CGDP currently operating in an on-line time-sharing environment on the UMASS (Unlimited Machine Access from Scattered Sites) system at the University of Massachusetts. A sample run, including user-computer dialogue during program execution, follows the program listing in the Appendix.

Summary of computational procedures for algorithmic steps

(i) Computing elements of matrix H for current day t: when entering any day, for each person i, and each menu item j, there are available the parameters a_{ij}, b_{ij}, c_{ij} and r_{ij}, the days since last selection of item j by person i, t_{ij}, and the preference at last selection, h'_{ij}. By applying (11) and (12), each element $h_{ij}(t)$ of the preference matrix $H(t)$ is computed as follows:

If $\quad f_{ij}(t) = a_{ij} - b_{ij}e^{-c_{ij}t_{ij}}$

then $\quad h_{ij}(t) = f_{ij}(t) - e^{-c_{ij}t_{ij}}(a_{ij} - h'_{ij})$

(ii) Calculating maximum achievable preference (pmax):

$$\text{pmax} = \sum_{i=1}^{m} [\max_{1\le j\le n} \{h_{ij}\} \mid \max_{1\le j\le n} \{h_{ij}\} \ge \text{thresh}]$$

where thresh = threshold preference level below which no item is selected.

(iii) Generating best choicegroup of size k :

Generate all possible choicegroups of size k , and s_k^* is the best choicegroup if $h(s_k^*) = \max_{s_k \in \Omega_k} \{h(s_k)\}$

where, if $s_k = j, j, \cdots, j_k$, by relation (2) and (3)

$$h(s_k) = \sum_{i=1}^{m} [\max_{1\le \ell\le k} \{h_{ij_\ell}\} \mid \max_{1\le \ell\le k}\{h_{ij_\ell}\} \ge \text{thresh}]$$

The percentage of the maximum achievable preference actually achieved $= 100\, h(s_k^*)/\text{pmax}$

(iv) Calculating of sales extimates of menu items:

Let the best choicegroup of size k be

$$s_k^* = \{j_1^*, j_2^*, \cdots, j_k^*\}$$

If m_{j^*} is the sales estimate of item j ,

$$m_{j_\ell^*} = \sum_{i=1}^{m} [1 \mid h_{ij_\ell^*} = \max\{h_{ij_1^*}, h_{ij_2^*}, \cdots, h_{ij_k^*}\},\ h_{ij_\ell^*} \ge \text{thresh}]$$

(v) Updating t_{ij} and h'_{ij}:

If $h_{ij_\ell^*} = \max\{h_{ij_1^*}, h_{ij_2^*}, \cdots, h_{ij_k^*}\}$, $h_{ij_\ell^*} \geq$ thresh

then set $t_{ij_\ell^*} = 0$ and $h'_{ij_\ell^*} = h_{ij_\ell^*}$

For every i and j, set $t_{ij} = t_{ij} + 1$

Figure 3 illustrates a sample output from a CGDP run with the choicegroup size fixed at 3. A schedule for only 2 days is presented. As can be expected, none of the menu items scheduled on the first day appear on the subsequent day.* Figure 4 shows an output with the percentage of the maximum achievable preference that must be achieved fixed at 85%. For the first day, the menu items are thus exactly the same as those on the first day in Figure 3. But on the second day only two items are able to achieve 85% of the maximum achievable preference.

The above described method basically formulates a multistage, unconstrained optimization process for choicegroup generation in the sense that for an indefinite period, p, the optimum sequence of choicegroups is determined parametrically for any desired choicegroup size or population preference level. Relative to this indication of optimality, conventional menu schedules appear to have two principal flaws. One is the tendency to pair up equally liked or disliked items in the same choicegroup. Such policy is not corroborated by the results shown of Figures 3 and 4, and it is likely to lead to suboptimal schedules from the point of view of acceptability.

---

* It is a coincidence that the percentages of the maximum achievable preferences for the two days are so close to one another.

The other is the tendency to rotate short menu cycles when the computational evidence indicates that the hypothetical distribution of population preferences would favor periods much longer than presently used.

In conclusion, the two immediate potential uses of the results of this study deserve mention. One is the educational, training and demonstration aspect of operating the CGDP program as it is documented. The other is the extension of the approach to constrained optimization problems, i.e., realistic selective menu scheduling problems, where the population preference is to be optimized subject to food cost, capacity and nutritional constraints. Essential elements of the CGDP program are constructed to serve as column generators for the pivoting rules of a stochastic programming model of menu scheduling, which is under development by the authors. In this approach the concept of choicegroups can be extended to the more realistic concept of choicegroups of pairwise combinations of items as is suggested in [1].

**********day number 1

******preference matrix

	*****menu items*****									
persons	1	2	3	4	5	6	7	8	9	10
1	43.1*	45.3	43.1	47.7	36.0	47.0	48.1	40.0†	41.4	49.3
2	1.9†	0	-4.2	4.7	0	6.9	14.7	19.6*	6.0	3.7
3	7.6*	5.2	11.1	0	1.9	0	7.9	-7.7†	6.8	8.9
4	52.3*	13.8	15.0	58.8	24.6	0	27.7	29.7†	32.0	33.7
5	13.2*	1.6	12.9	10.5	[illegible]	5.2	9.0	10.0†	9.9	10.3
6	22.8†	48.7	14.3	24.6	20.8	19.3	7.6	23.7*	26.4	9.4
7	56.8†	51.7	25.8	18.7	0	56.1	84.7	93.9*	65.7	86.3
8	24.4*	18.6	9.7	27.2	0	16.6	2.7	-10.3†	0	21.6
9	5.2†	19.7	27.9	30.9	18.4	21.6	44.3	40.0*	37.6	37.2
10	19.0*	12.7	0	14.7	12.5	13.5	11.8	4.3†	10.7	6.4
11	26.9*	15.4	9.1	27.3	16.3	27.0	22.6	15.7†	21.9	29.2
12	15.0†	22.9	0	17.2	10.5	12.6	8.7	17.9*	9.5	23.4
13	9.3*	0	2.1	15.0	0	20.3	8.4	7.6†	5.1	10.5
14	9.7*	11.2	21.2	0	0	7.4	10.2	8.6†	17.4	18.2
15	19.5*	9.8	19.4	8.8	11.0	0	10.0	8.0†	6.7	5.3

Table 1.

A preference matrix H for m=15 and n=10. The elements of the matrix were generated from actual preference-time functions evaluated on an arbitrary day. With respect to a choicegroup of size 2 comprising items 1 and 8, for each individual, a "*" indicates the more preferred item between the two, and a "†" indicates the less preferred of items 1 and 8.

**********day number 2

******preference matrix

	*****menu items*****									
persons	1	2	3	4	5	6	7	8	9	10
1	19.3↓	45.8	44.8	48.0	37.1	48.3	48.8	42.9↑	47.1	49.9
2	2.0↑	0	-2.4	4.8	0	8.7	16.2	18.4↓	6.2	4.5
3	3.1↓	5.5	11.1	0	2.8	0	9.2	-6.5↑	7.6	11.9
4	33.0↓	17.7	21.7	59.8	24.7	0	28.0	49.2↑	34.3	35.7
5	-5.9↓	6.3	13.1	10.5	2.1	5.2	9.3	10.1↑	10.0	19.4
6	23.0↑	49.6	14.4	24.7	20.9	19.5	15.0	6.5↓	28.1	12.5
7	57.3↑	52.4	26.0	18.9	0	59.1	89.4	82.4↓	66.1	20.8
8	-2.1↓	19.9	13.5	28.1	0	19.6	11.3	-9.5↑	0	23.4
9	5.2↑	19.8	31.1	30.0	18.4	24.3	50.6	25.5↓	37.9	38.5
10	-.9↓	15.5	0	15.4	14.3	14.9	12.2	8.0↑	11.3	9.9
11	11.0↓	15.5	9.1	27.4	16.4	28.3	23.3	20.8↑	21.9	31.8
12	15.4↑	23.5	0	17.3	10.5	12.9	3.3	17.1↓	9.9	24.0
13	5.5↓	0	2.1	15.4	0	22.6	8.7	9.7↑	9.1	11.5
14	5.0↓	11.2	11.2	0	0	7.8	11.0	10.6↑	17.6	19.3
15	17.8↓	10.1	20.7	8.9	11.3	0	10.0	10.2↑	6.8	5.4

Table 2.

The updated preference matrix H on the day subsequent to that depicted by Table 1, when items 1 and 8 were offered on the day corresponding to Table 1. Upward and downward arrows after preferences indicate the rise and drop of preferences with respect to Table 1 for offered items selected and not selected respectively.

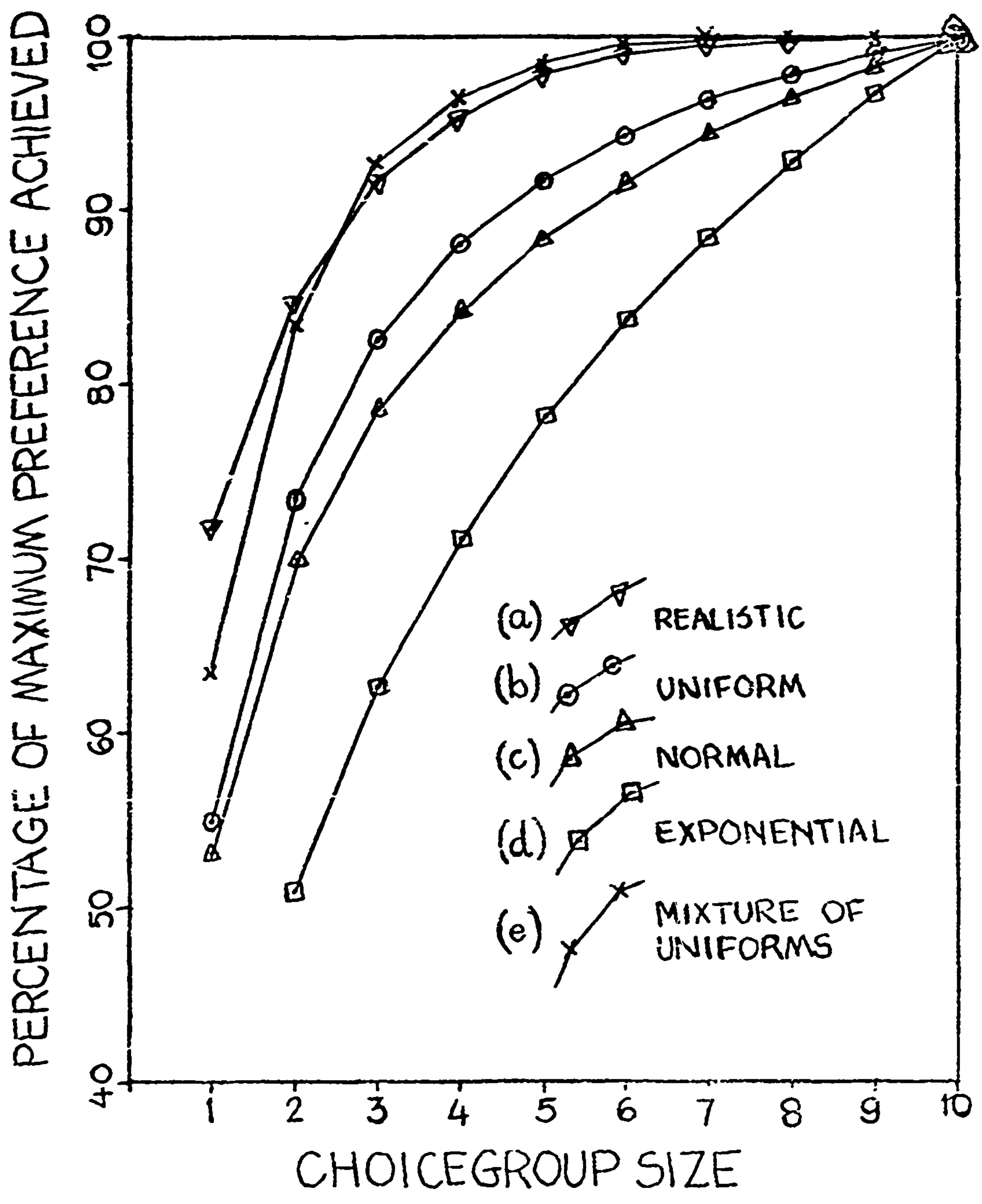

Figure 1. The percentage of maximum preference achieved as a function of choicegroup size with $n=10$ for preferences from (a) realistic data (b) uniform distribution in the interval (0,100) (c) normal distribution with mean 50 and standard deviation $50/\sqrt{3}$ (d) exponential distribution with mean 50, and (e) an arbitrary mixture of uniform distributions.

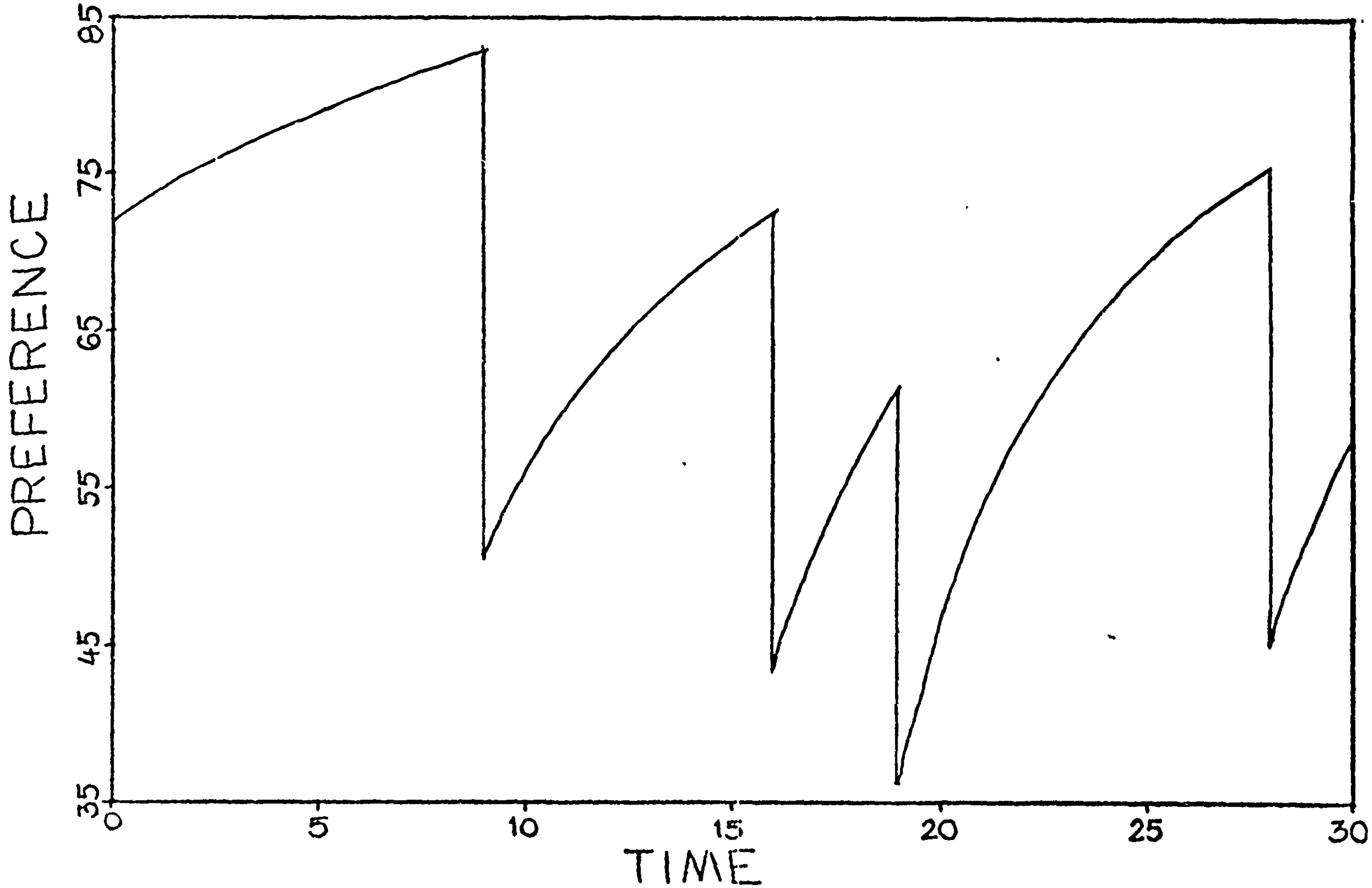

Figure 2. The change of preference over time when the item is consumed on days 9, 16, 19 and 28, the item having been consumed 9 days prior to day 1. The parameters of the preference-time function used for this plot are: a = 100, b = 40, c = 0.05, r = 0.4.

```
**********day number    1

maximum achievable preference =  495.65

size of choice group =   3

total preference achieved =   453.84

percentage of maximum =    91.56 percent

item num.   number of persons   proportion(%)

      1              8               53.33
      2              4               26.67
      8              3               20.00
  omitted            0                  00

**********day number    2

maximum achievable preference =  501.47

size of choice group =   3

total preference achieved =   458.53

percentage of maximum =    91.44 percent

item num.   number of persons   proportion(%)

      4              6               40.00
      7              3               20.00
     10              6               40.00
  omitted            0                   0
```

Figure 3. Sample output from a CGDP run with the choicegroup size fixed at 3 for 2 days. The preference matrix for the first day is depicted on Table 1.

```
**********day number    1

maximum achievable preference =  495.65

size of choice group =    3

total preference achieved =    453.84

percentage of maximum =     91.56 percent

item num.   number of persons   proportion(%)

      1              8               53.33
      2              4               26.67
      8              3               20.00
  omitted            0                   0

**********day number    2

maximum achievable preference =  501.47

size of choice group =    2

total preference achieved =    433.85

percentage of maximum =     86.51 percent

item num.   number of persons   proportion(%)

      4              8               53.33
     10              7               46.67
  omitted            0                   0
```

Figure 4. Sample output from a CGDP run with the percentage of the maximum preference to be achieved fixed at 85% for 2 days. The preference matrix for the first day is depicted on Table 1.

APPENDIX

```
1 program cgdp
2*      choice group generator demonstration program (cgdn)
3*      developed by  J.L. Balintfy  and  P. Sinha under  o.n.r.
4*      contract nr-047-100 at the univ.of mass. feb.  1972
5*
6*      this experimental program determines the most preferred
7*      choice group of menu items relative to the preference
8*      ratings of a population.
9*
10*     *****************************************************
11*     instructions for use:
12*
13*     problem size values and other options will be requested
14*     by the program during execution.  answer all questions
15*     by a "yes" or a "no" except where otherwise specified.
16*
17*     hypothetical preference-time function parameters for
18*     the data can be generated in the program itself, or the
19*     parameters can be externally supplied by appending them
20*     to the end of the program, i.e. from line 5000 onwards,
21*     or placing them in a binary file.  the unit number where
22*     the data is located will be requested by the program.
23*
24*     if preference-time function parameters for all pairwise
25*     combinations of the "m" individulas and the  "n" items
26*     are externally supplied, they must be in the following
27*     order  (format free):
28*     (no statement no.s required if data is in a binary file)
29*
30*     5000 a(1,1),b(1,1),c(1,1),r(1,1),time(1,1),hlast(1,1)
31*     5005 a(1,2),b(1,2),c(1,2),r(1,2),time(1,2),hlast(1,2)
32*     ....   ..     ..     ..     ..     ..        ..
33*     .... a(1,n),b(1,n),c(1,n),r(1,n),time(1,n),hlast(1,n)
34*     .... a(2,1),b(2,1),c(2,1),r(2,1),time(2,1),hlast(2,1)
35*     ....   ..     ..     ..     ..     ..        ..
36*     .... a(2,n),b(2,n),c(2,n),r(2,n),time(2,n),hlast(2,n)
37*     ....   ..     ..     ..     ..     ..        ..
38*     ....   ..     ..     ..     ..     ..        ..
39*     ....   ..     ..     ..     ..     ..        ..
40*     ....   ..     ..     ..     ..     ..        ..
42*     .... a(m,n),b(m,n),c(m,n),r(m,n),time(m,n),hlast(m,n)
43*
44*     where a(i,j),b(i,j),c(i,j) and r(i,j) are the parameters
45*           of the preference time function of the ith person
46*           for the jth item.
47*           time(i,j) is the number of days since the ith
48*           person selected the ith item and hlast(i,j) is
49*           the preference of the ith person for the jth
50*           item at last selection of the item.
51*
52*     the options and requests during program execution include:
53*
54*     1)  printout of preference matrix
55*     2)  threshold preference level, i.e. preference below
56*         which no menu item is selected.
57*     3)  number of persons and number of menu items
58*     4)  generation of fictitious data by program or num
59*         of data by user by appending to the end of the p  ram
60*         or by placing in a numbered binary file.
61*     5)  numbers for internal generation of data from uniform,
62*         normal or truncated normal distributions.
63*     6)  ....fixed preference matrix,varying choice group size
64*               to observe effect of increasing choice group size.
65*         or..multiday schedule with fixed choice group size
66*         or..multiday schedule with fixed percentage of max.
67*               preference which must be achieved.
68*     *****************************************************
69*
70 common idays,kmin,kmax,m,n,k,range,ex,stdx,tl,tu,in,ln,jj(10),
71ah(50,25),time(50,25),a(50,25),c(50,25),h(50,25),r(50,25),extra(10),
72ahlast(50,25),isol(10),thresh,num(10),nmax,pref,p(10),pere
75 do 78 j=1,5
76 read 82,(extra(i),i=1,8)
78 print 82,(extra(i),i=1,8)
79 print 81
80 input,ians
81 format(//" do you want to print the preference matrix ?")
82 format(5x,8a8)
85 kp=0 $ if(ians .eq. 3hyes)kp=1
10f print 107
10' format("type in threshold preference level below which no
10ua */* item selected (floating point)")
109 input,thresh
110 print 111
111 format(" type in the number of persons - not more than 50 and"/
112a " the number of menu items - not more than 25(integer)")
115 input,m,n
120 if(m .lt. 51)go to 140
130 print 131 $ go to 110
131 format("not more than 50 persons - please")
140 if(n .lt. 26)go to 160
150 print 151 $ go to 110
151 format("not more than 25 items - please")
160 continue
170 print 175
175 format(" type in unit number(integer) where data is located"/
176a " type zero if data is to be generated by the program")
177 extra(10)=0.0
180 input,ians
185 if(ians .eq. 0)go to 220
190 read(ians)(((a(i,j),b(i,j),c(i,j),r(i,j)
192a ,time(i,j),hlast(i,j)),j=1,n),i=1,m)
```

```
193 extra(10)=1.
196 do 200 i=1,15
197 do 200 j=1,10
198 h(i,j)=a(i,j)-h(i,j)*exp(-time(i,j)*e(i,j))-exp(-r(i,j)*time(i,j))
198a*(a(i,j)-hlast(i,j))
200 continue
210 go to 370
220 print 221
221 format(*type 1 if numbers to be generated from normal distr.*/
222a *type 2 if numbers to be generated from uniform distr.*)
230 input,ians
240 if(ians .eq. 1)go to 280
250 print 251
251 format(*type in lower limit,upper limit for uniform distr.(real)*)
255 input,r1,r2
256 range=r2-r1
260 call uniform(r1)
270 go to 370
280 print 281
281 format(*type mean,standard dev. for normal distr.(real)*)
290 input,ex,stdx
300 print 301
301 format(*type 1 if you want truncated normal,2 otherwise*)
310 input, ians
320 if(ians .eq.1)go to 340
330 tl=-1000  $ tu=1000 $go to 360
340 print 341
341 format(*type in lower limit,upper limit for truncation(real)*)
350 input,tl,tu
360 call normal
370 print 371
371 format(*type 1 if you want to see the effect of increasing the*/
372a *choice group size with a fixed preference matrix*/
373a *type 2 if you want a multi-day schedule with a fixed size*/
374a *of choice group for each day*/
375a *type 3 if you want to fix the percentage of the maximum*/
376a *preference which must be achieved for a multi-day schedule*/
377a *(options 2 and 3 use the day to day updating feature)*)
380 input,ians
390 if(ians-2)400,480,540
400 print 401
401 format(*type in maximum choice group size(integer)-not more than 8*)
410 input,kmax
420 jp=1 $ kmin=1 $ idays=1
421 extra(3)=0.
430 if(kmax .le. 8)go to 470
440 print 450
450 format(* choice group maximum not more than 8 - please*)
460 go to 400
470 go to 800
480 print 481
481 format(*type choice group size and number of days(integer)*/
482a *for which schedule required*)
490 input,kmax,idays
500 kmin=kmax
501 extra(3)=00.
502 jp=0
510 if(kmax.le. 8)go to 535
520 print 450
530 go to 480
535 if(extra(10) .gt. .5)go to 809.
536 go to 698
540 kmin=1 $ kmax=8
545 print 550
550 format(*specify number of days(integer) and percentage of
555a maximum*/*(real) to be achieved*/)
560 input,idays,extra(3)
561 jp=0
590 if(extra(10) .gt. .5)go to 800
698 do 730 i=1,m
699 do 730 j=1,n
702 a(i,j)=(1.2+.8*ranf(-1))*h(i,j)
703 x=ranf(-1)=1.
704 b(i,j)=a(i,j)*.5*x+a(i,j)
705 y=ranf(-1)
706 if(y .lt. .01 .or. y .gt. .1)go to 705
707 e(i,j)=y
708 y=ranf(-1)
709 if(y .lt. .1)go to 708
710 r(i,j)=y
715 y=ranf(-1)*20.
716 if(y .lt. 1.01)go to 715
717 iy=y $ y=iy
718 time(i,j)=y
719 y=ranf(-1)
720 if(y .lt. .2 .or. y .gt. 1.)go to 719
721 hlast(i,j)=a(i,j)*y
725 h(i,j)=a(i,j)-h(i,j)*exp(-time(i,j)*e(i,j))-exp(-r(i,j)*time(i,j))
726a*(a(i,j)-hlast(i,j))
730 continue
800 do 910 iiii=1,idays
805 print 815,iiii
810 if(kp .gt. 0)call prints
815 format(//20h**********day number,i3)
825 call fpmax
835 do 905 k=kmin,kmax
840 call irun(kk)
845 call gpsize
850 perc=pref*100./pmax
855 if(perc-extra(3))905,860,860
860 call compute(pp)
865 call irun(kkk)
870 atime=(float(kkk-kk)/1000.)
875 call printl(pp)
880 if(jp .ne. 1)call update
895 if(extra(3) .gt. 0.)go to 910
```

```
905 cont.r..
910 continue
915 print 920
920 format(* type 1 to continue,0 to terminate*)
925 input,ians $ if(ians .eq. 1)go to 79
930 end
1000 subroutine print2
1002 common idays,kmin,kmax,m,n,k,range,ex,stdx,tl,tu,jp,lp,jj(10),
1003ah(50,25),time(50,25),a(50,25),c(50,25),b(50,25),r(50,25),extra(10),
1004ahlast(50,25),isol(10),thresh,num(10),pmax,pref,p(10),pere
1005 print 1006
1006 format(/23h******preference matrix)
1010 jf=0
1020 if(jf .eq. n)return
1025 js=jf+1
1030 format(1x,i2,2x,10(1x,f5.1))
1040 jf=js+9
1050 if(jf .gt. n)jf=n
1060 print 1070,(i,i=js,jf)
1070 format(/25x,20h*****menu items*****//,4x,10(4x,i2))
1080 print 1090
1090 format(*persons*)
1220 do 1230 i=1,m
1230 print 1030,i,(h(i,j),j=js,jf)
1240 go to 1020
1250 end
2000 subroutine normal
2002 common idays,kmin,kmax,m,n,k,range,ex,stdx,tl,tu,jp,lp,jj(10),
2003ah(50,25),time(50,25),a(50,25),c(50,25),b(50,25),r(50,25),extra(10),
2004ahlast(50,25),isol(10),thresh,num(10),pmax,pref,p(10),pere
2010 do 2120 i=1,m
2010 do 2120 j=1,n
2030 sum=0.
2060 do 2080 ii=1,12
2070 r=ranf(-1)
2080 sum=sum+r
2090 x=stdx*(sum-6.)+ex
2100 if(x .lt. tl)go to 2050
2110 if(x .gt. tu)go to 2050
2120 h(i,j)=x
2130 return
2140 end
2200 subroutine uniform(rl)
2202 common idays,kmin,kmax,m,n,k,range,ex,stdx,tl,tu,jp,lp,jj(10),
2203ah(50,25),time(50,25),a(50,25),c(50,25),b(50,25),r(50,25),extra(10),
2204ahlast(50,25),isol(10),thresh,num(10),pmax,pref,p(10),pere
2220 do 2240 i=1,m
2225 do 2240 j=1,n
2230 x=ranf(-1)
2240 h(i,j)=rl+range*x
2250 return
2260 end
```

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```
2400 subroutine compute(pp)
2402 common idays,kmin,kmax,m,n,k,range,ex,stdx,tl,tu,jp,lp,jj(10),
2403ah(50,25),time(50,25),a(50,25),e(50,25),b(50,25),r(50,25),extra(10),
2404ahlast(50,25),isol(10),thresh,num(10),pmax,pref,p(10),pere
2410 pp=100.
2430 do 2440 i=1,10
2440 num(i)=0
2460 do 2650 i=1,m
2470 icount=1 $amax=-100
2480 do 2550 l=1,k
2490 ii=isol(l)
2500 if(h(i,ii)-amax)2550,2510,2540
2510 icount=icount+1
2520 jj(icount)=ii
2530 go to 2550
2540 icount=1 $jj(1)=ii
2545 amax=h(i,ii)
2550 continue
2555 if(amax .lt. thresh)go to 2650
2560 idum=1
2570 if(icount .eq. 1)go to 2600
2580 idum=ibyte(47,ranf(1),0,0)
2590 idum=mod(idum,icount)+1
2600 time(i,jj(idum))=0.
2605 hlast(i,jj(idum))=h(i,jj(idum))
2610 do 2640 ii=1,k
2620 if(isol(ii) .ne. jj(idum))go to 2640
2630 num(ii)=num(ii)+1
2640 continue
2650 continue
2670 do 2710 i=1,k
2680 aa=num(i)
2690 bb=m
2700 p(i)=aa*100./bb
2710 pp=pp-p(i)
2730 return
2730 end
2800 subroutine update
2802 common idays,kmin,kmax,m,n,k,range,ex,stdx,tl,tu,jp,lp,jj(10),
2803ah(50,25),time(50,25),a(50,25),e(50,25),b(50,25),r(50,25),extra(10),
2804ahlast(50,25),isol(10),thresh,num(10),pmax,pref,p(10),pere
2810 do 2850 i=1,m
2820 do 2850 j=1,n
2830 time(i,j)=time(i,j)+1.
2840 h(i,j)=a(i,j)-b(i,j)*exp(-time(i,j)*e(i,j))-exp(-r(i,j)*time(i,j))
2841a*(a(i,j)-hlast(i,j))
2850 continue
2910 return
2920 end
2950 subroutine print1(pp)
3002 common idays,kmin,kmax,m,n,k,range,ex,stdx,tl,tu,jp,lp,jj(10),
3003ah(50,25),time(50,25),a(50,25),e(50,25),b(50,25),r(50,25),extra(10),
3004ahlast(50,25),isol(10),thresh,num(10),pmax,pref,p(10),pere
```

```
3010 nump=m
3050 print 3060,pmax
3060 format(/*maximum achievable preference = *f7.2)
3110 print 3120,k
3120 format(/*size of choice group = *,i3)
3130 print 3140,pref,perc
3140 format(/*total preference achieved = *f8.2,/
3145a /*percentage of maximum = *,f8.2,* percent *)
3150 print 3160
3160 format(/13h item num.    ,17hnumber of persons,
3170a 19h   proportion(%)    ,/)
3180 do 3200 j=1,k
3190 format(5x,i3,5x,7x,i3,13x,f7.2)
3195 nump=nump-num(j)
3200 print 3190,isol(j),num(j),p(j)
3210 print 3220,nump,pp
3220 format(13h   omitted    ,7x,i3,13x,f7.2)
3230 return
3240 end
3900 subroutine fpmax
3902 common idays,kmin,kmax,m,n,k,range,ex,stdx,tl,tu,jp,lp,jj(10),
3903ah(50,25),time(50,25),a(50,25),c(50,25),h(50,25),r(50,25),extra(10),
3904ahlast(50,25),isol(10),thresh,num(10),pmax,pref,p(10),perc
3930 pmax=0
3910 do 4000 i=1,m
3950 amax=-1000
3960 do 3900 j=1,n
3970 if(h(i,j) .gt. amax)amax=h(i,j)
3980 continue
3990 pmax=pmax+amax
4000 continue
4010 return
4020 end
4100 subroutine gpsize
4102 common idays,kmin,kmax,m,n,k,range,ex,stdx,tl,tu,jp,lp,jj(10),
4103ah(50,25),time(50,25),a(50,25),c(50,25),h(50,25),r(50,25),extra(10),
4104ahlast(50,25),isol(10),thresh,num(10),pmax,pref,p(10),perc
4124 dimension nn(10),hh(75,10)
4125 do 4129 i=1,m
4126 do 4129 j=1,n
4127 if(h(i,j) .lt. thresh)h(i,j)=0.
4129 continue
4130 pref=-1000
4140 do 4560 n1=1,n-k+1
4142 do 4143 i=1,m
4143 hh(i,1)=h(i,n1)
4150 nn(1)=n1
4160 if(k .eq. 1)go to 4170
4170 do 4550 n2=n1+1,n-k+2
4172 do 4176 i=1,m
4173 if(hh(i,1)-h(i,n2))4174,4175,4175
4174 hh(i,2)=h(i,n2) $ go to 4176
4175 hh(i,2)=hh(i,1)
```



```
4176 continue
4180 nn(2)=n2
4190 if(k .eq. 2)go to 4370
4200 do 4540 n3=n2+1,n-k+3
4202 do 4206 i=1,m
4203 if(hh(i,2)-h(i,n3))4204,4205,4205
4204 hh(i,3)=h(i,n3) $ go to 4206
4205 hh(i,3)=hh(i,2)
4206 continue
4210 nn(3)=n3
4220 if(k .eq. 3)go to 4370
4230 do 4530 n4=n3+1,n-k+4
4232 do 4236 i=1,m
4233 if(hh(i,3)-h(i,n4))4234,4235,4235
4234 hh(i,4)=h(i,n4) $ go to 4236
4235 hh(i,4)=hh(i,3)
4236 continue
4240 nn(4)=n4
4250 if(k .eq. 4)go to 4370
4260 do 4520 n5=n4+1,n-k+5
4262 do 4266 i=1,m
4263 if(hh(i,4)-h(i,n5))4264,4265,4265
4264 hh(i,5)=h(i,n5) $ go to 4266
4265 hh(i,5)=hh(i,4)
4266 continue
4270 nn(5)=n5
4280 if(k .eq. 5)go to 4370
4290 do 4510 n6=n5+1,n-k+6
4292 do 4296 i=1,m
4293 if(hh(i,5)-h(i,n6))4294,4295,4295
4294 hh(i,6)=h(i,n6) $ go to 4296
4295 hh(i,6)=hh(i,5)
4296 continue
4300 nn(6)=n6
4310 if(k .eq. 6)go to 4370
4320 do 4500 n7=n6+1,n-k+7
4322 do 4326 i=1,m
4323 if(hh(i,6)-h(i,n7))4324,4325,4325
4324 hh(i,7)=h(i,n7) $ go to 4326
4325 hh(i,7)=hh(i,6)
4326 continue
4330 nn(7)=n7
4340 if(k .eq. 7)go to 4370
4350 do 4490 n8=n7+1,n-k+8
4352 do 4356 i=1,m
4353 if(hh(i,7)-h(i,n8))4354,4355,4355
4354 hh(i,8)=h(i,n8) $ go to 4356
4355 hh(i,8)=hh(i,7)
4356 continue
4360 nn(8)=n8
4370 apref=0.
4380 do 4390 i=1,m
4390 apref=apref+hh(i,k)
```

```
4400 if(apref .le. pref)go to 4480
4410 pref=apref
4460 do 4470 ii=1,k
4470 ipol(ii)=nn(ii)
4480 go to(4560,4550,4540,4530,4520,4510,4500,4490)k
4490 continue
4500 continue
4510 continue
4520 continue
4530 continue
4540 continue
4550 continue
4560 continue
4570 return
4580 end
4590 ordprog
4600 ************************************************************
4610  choice group generator demonstration program  (cgdp)
4620  if you are totally unfamiliar with this program, stop it
4630  and list 1,69 for instructions......good luck.........
4640 ************************************************************
```



```
run cgdp

16k
     ************************************************************
      choice group generator demonstration program  (cgdp)
      if you are totally unfamiliar with this program, stop it
      and list 1,69 for instructions......good luck.........
     ************************************************************

do you want to print the preference matrix ?
?yes

type in threshold preference level below which no
item selected (floating point)
?3.0

type in the number of persons - not more than 50 and
the number of menu items - not more than 25(integer)
?20 10

type in unit number(integer) where data is located
type zero if data is to be generated by the program
?0

type 1 if numbers to be generated from normal distr.
type 2 if numbers to be generated from uniform distr.
?1

type mean,standard dev. for normal distr.(real)
?.5. 5.

type 1 if you want truncated normal,2 otherwise
?1

type in lower limit,upper limit for truncation(real)
?0.0 30.0

type 1 if you want to see the effect of increasing the
choice group size with a fixed preference matrix
type 2 if you want a multi-day schedule with a fixed size
of choice group for each day
type 3 if you want to fix the percentage of the maximum
preference which must be achieved for a multi-day schedule
(options 2 and 3 use the day to day updating feature)
?3

specify number of days(integer) and percentage of maximum
(real) to be achieved

?2 85.0
```

**********day number 1

******preference matrix

*****menu items*****

persons	1	2	3	4	5	6	7	8	9	10
1	.3	6.2	4.4	20.0	7.7	15.5	10.3	24.3	5.8	8.8
2	.9	7.7	9.0	13.9	9.2	4.0	20.7	6.5	11.0	9.1
3	6.6	13.3	19.3	-.3	11.6	5.2	25.5	13.7	16.7	20.6
4	3.1	17.4	13.3	12.1	10.9	8.5	13.9	11.2	7.8	23.7
5	20.7	-2.6	4.5	15.9	18.8	13.1	12.7	15.1	2.1	8.0
6	13.5	33.4	-3.8	20.3	1.4	8.0	6.1	19.2	8.0	5.8
7	20.9	20.0	15.1	6.7	22.4	7.3	-3.1	21.7	8.6	3.7
8	3.5	15.8	2.7	13.3	22.3	12.4	21.5	18.2	1.3	6.4
9	14.4	23.7	11.1	4.2	13.4	15.8	10.1	19.6	10.8	8.1
10	19.2	21.2	3.6	21.7	21.3	23.2	9.6	16.8	22.2	.3
11	7.0	7.3	7.8	6.7	11.4	13.2	11.1	27.4	6.1	12.3
12	[illegible]	[illegible]	7.4	14.4	3.3	-2.6	17.3	10.9	7.0	10.6
13	2.4	14.3	10.1	12.8	5.1	2.1	11.8	5.9	3.6	4.4
14	6.3	2.5	6.9	8.4	14.3	18.9	10.8	20.9	7.8	-1.4
15	18.9	13.8	15.5	17.2	7.0	15.9	17.5	12.3	13.6	4.9
16	9.9	12.6	9.9	18.7	21.1	9.9	10.9	12.6	13.3	12.2
17	9.4	16.3	.6	7.5	3.5	7.2	4.8	11.6	1.6	5.4
18	-12.3	9.4	16.0	4.6	17.1	18.6	5.6	14.6	3.9	4.2
19	14.6	.0	13.4	10.7	16.5	5.0	20.1	6.1	20.6	13.5
20	11.6	6.8	12.3	19.3	8.0	8.5	17.8	12.7	10.0	15.1

maximum achievable preference = 437.00

size of choice group = 3

total preference achieved = 401.81

percentage of maximum = 91.78 percent

item num.	number of persons	proportion(%)
2	6	30.00
7	6	30.00
8	8	40.00
omitted	0	0

**********day number 2

******preference matrix

*****menu items*****

persons	1	2	3	4	5	6	7	8	9	10
1	1.3	6.4	7.8	30.1	8.3	15.9	10.0	9.5	6.0	9.4
2	1.4	9.6	10.1	14.6	9.5	6.6	-3.2	6.9	12.1	9.1
3	6.0	14.1	20.1	.9	12.0	8.4	1.6	14.3	17.2	21.1
4	4.1	6.9	18.8	14.0	12.0	9.4	14.7	12.8	8.7	25.4
5	21.1	-.7	7.2	16.9	19.5	13.3	13.2	-.5	4.1	8.2
6	13.7	12.0	.4	20.0	3.9	8.3	7.2	20.9	8.5	5.9
7	20.3	21.5	15.3	7.0	23.1	7.5	1.6	6.2	8.7	4.8
8	4.8	16.5	6.0	13.6	23.0	13.3	5.8	18.0	3.5	6.9
9	14.6	10.2	11.2	5.6	14.0	16.6	19.4	20.4	11.1	9.2
10	10.6	3.9	6.8	22.7	22.1	13.6	10.5	17.1	22.6	1.4
11	8.7	8.5	8.4	7.1	12.4	14.3	11.3	8.0	6.9	13.0
12	6.6	4.1	8.7	14.7	3.8	.8	17.6	-.2	9.1	11.6
13	4.0	-7.3	11.8	13.1	5.4	2.4	12.1	6.2	3.8	4.7
14	7.8	6.3	8.1	8.8	17.1	19.9	19.3	5.7	8.1	3.5
15	10.3	13.9	16.4	17.8	8.1	16.2	7.1	13.3	14.2	5.2
16	20.3	12.8	10.1	19.0	21.7	10.0	12.3	1.1	14.3	12.4
17	10.2	4.8	2.6	8.0	6.0	8.1	5.2	11.9	3.7	8.0
18	-4.4	10.0	16.4	6.1	17.5	19.1	5.8	2.0	4.7	4.4
19	15.2	2.0	14.6	11.0	17.1	8.6	6.2	6.9	21.4	16.2
20	12.6	8.3	12.9	19.6	8.2	0.8	6.1	13.3	11.8	15.6

maximum achievable preference = 400.93

size of choice group = 2

total preference achieved = 352.75

percentage of maximum = 87.98 percent

item num.	number of persons	proportion(%)
4	10	50.00
5	10	50.00
omitted	0	0

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