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METHODOLOGIES FOR EVALUATING THE VULNERABILITY OF NATIONAL SYSTEMS. VOLUME I. METHODOLOGIES AND EXAMPLES

James T. McGill, et al

Institute for Defense Analyses

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Volume I: METHODOLOGIES AND EXAMPLES

James T. McGill, *Project Leader*
Jerome Bracken
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INSTITUTE FOR DEFENSE ANALYSES
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13. ABSTRACT

This study documents methodologies that can be used to evaluate some vulnerabilities of the U.S. to strategic nuclear attack. The collection of component methodologies is called MEVUNS, Methodologies for Evaluating the Vulnerability of National Systems.

The study comprises three volumes. The first contains methodologies and examples of their use. The second contains results of a sensitivity analysis performed with the methodologies. The third is a user's guide for MEVUNS computer programs.

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for
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IDA
INSTITUTE FOR DEFENSE ANALYSES
PROGRAM ANALYSIS DIVISION
400 Army-Navy Drive, Arlington, Virginia 22202

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II

FOREWORD

This study reports research sponsored by the Defense Civil Preparedness Agency (DCPA) under Contract Number DAHC 20 70 C 2087, Task 4114B, Evaluation of National Total Civil Defense Systems. The goal of this study is to integrate into an operable system those methodologies developed under the sponsorship of the Systems Evaluation Division of DCPA so that evaluations may be made of the effectiveness of civil defense systems.

Authors of the study are:

Dr. Jerome Bracken
Mrs. Carolyn D. Davis
Ms. Lynn P. Dolins
Dr. James T. McGill
Mr. Hugh M. Pitcher
Miss Sheryll A. Pratt
Dr. Leo A. Schmidt, Jr.
Miss Jacqueline E. Thompson

L. Schmidt developed the active defense models and the attack generator. Modifications to the damage assessment routine, known as ANCET, were the responsibility of C. Davis and J. McGill. H. Pitcher was responsible for the final development and documentation of the economic model. L. Dolins assumed responsibility for the development and documentation of the data for the economic model. J. Bracken and J. McGill developed the integrating model. Major responsibility for compiling and modifying the data bases was assumed by C. Davis, S. Pratt, and J. Thompson.

Edward S. Pearsall and L. Dolins of IDA developed the initial conceptual framework for the economic model. The first version of the computer program was developed by Dr. Pearsall.

Aiding Ms. Dolins in the development of the economic data bases were K. Glass, J. Severo, and L. Ziglar of the IDA Computer Group.

R. E. Kutscher, D. Eldridge, and W. Karr of the Bureau of Labor Statistics and J. Rodgers of Jack Faucett Associates provided key data.

Mr. Neal FitzSimons, Director, Systems Evaluation Division, and Mr. Donald Hudson, the contracting office's technical representative, both of the Defense Civil Preparedness Agency, deserve special mention for their direction and contributions.

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SUMMARY

This report documents methodologies which can be used in evaluating some of the vulnerabilities of the continental United States to a strategic nuclear attack. In the compilation and development of the methodologies, particular attention was given to those that show the sensitivity of the post-attack state of the nation to major active and passive defense measures. Most analyses of defense measures have used population casualties as a basis for comparison. The present methodology also estimates casualties, but, in addition, includes models for estimating the economic impact of a nuclear attack. With the economic models, the post-attack recovery period is considered so that estimates of GNP, GNP per capita, and other economic measures can be obtained. In addition, a methodology for integrating costs, the effectiveness of defense systems, and survivability requirements is developed. The collection of component methodologies is referred to in the report as MEVUNS, Methodologies for Evaluating the Vulnerability of National Systems.

I

INTRODUCTION

The Defense Civil Preparedness Agency Work Statement (T.O. 4114B) for IDA for fiscal year 1972 calls for a study of methodologies useful in the evaluation of the national civil defense system. The results of the present study are in response to the following task statement:

> "This study will determine methodological means of relating ballistic missile defense and other defense postures to the vulnerability of national systems. The methodologies will be responsive to changes in assumptions about the threat, the missile defense, pre-attack population posture and economic structure, and post-attack national economic policies. The performance of the post-attack economic system, for example, will depend upon surviving capital, surviving labor, and the capability to integrate these two factors to redevelop a functioning national economy.
>
> "Available quantitative methodologies which can be used for evaluating the effects of defense alternatives on national system performance will be compiled and examined. This effort will include methodologies for:
>
> (a) targeting a ballistic missile attack on counterforce and countervalue nodes;
>
> (b) specifying the interactions of a ballistic missile attack with ballistic missile defenses and other defense measures;
>
> (c) assessing damages, both in terms of the social and economic systems; and
>
> (d) predicting long-run post-attack economic redevelopment.

These methodologies will be evaluated with respect to their scope, data requirements, ease of implementation, and overall usefulness in providing relationships between missile defense postures and national post-attack viability. Where necessary and feasible, consistent with study time and fundings, new methodologies will be developed. Recommendations for further development or refinement will be given.

"A selected set of these methodologies will be implemented on the IDA computer. Documentation of these computer programs will be provided.

"The computer programs will be fully exercised with reasonable data so as to provide a tested and integrated methodology. Examples of the types of sensitivity analyses that can be performed will be given."

Consistent with the formal task description, several guidelines were established for the study. First, it was recognized that a considerable amount of previous work had been done in evaluating the effects of nuclear attack. It was thus desirable, and feasible, to draw heavily on established methodologies. Second, complete documentation of the methodologies was to be provided, so that the computer programs could be used by a wider audience of analysts than was previously possible. Third, demonstration of the capabilities of the methodologies was to be accomplished by exhibiting the results of sensitivity analyses. Finally, an evaluation of the strengths and weaknesses of the methodologies, singly and together, was to be presented.

The report comprises three volumes. The first is Methodologies and Examples, which presents an overview of all the methodologies, describing how they can be used singly or in tandem, plus a description of each of the major component models and data bases. When issued, the second volume will contain the results of sensitivity analyses performed with the methodologies. The third volume, User's Guide for MEVUNS Computer Programs, with restricted distribution, is intended for the analyst who needs a detailed understanding of the computer programs for the methodologies. Descriptions of inputs, types of outputs, and the program structure will be given.

II

OVERVIEW OF MEVUNS

A. INTRODUCTION

The MEVUNS study provides the development and documentation of methodologies, rather than their exercise in substantive analyses. The primary objective has been to develop an integrated means of evaluating the effects of a nuclear attack, which can then be used in future analyses of substantive strategic defense planning issues.

The integrated methodology links several component models. Each of these has utility in its own right for certain types of analyses. The fully-linked stream of MEVUNS models, however, gives the analyst an expanded capability for performing a broad spectrum of studies relatively easily and quickly.

The integrated methodology consists of five main components: a population and industrial data base with a high degree of geographic resolution, a terminal and area defense model, along with attack-generation model for targeting population and/or economic sectors, damage assessment models for both population and industry, an economic recovery model, and a mathematical programming model. These are fully linked. However, options are provided so that a user need not employ the full stream of models. In addition, a mathematical programming model is given for integrating the cost and effectiveness of defense measures with survivability requirements for population and industry. This model can use the results of the other models.

The methodolcgies are national in scope. While their primary use is in the evaluation of the effects of a nationwide attack, certain components can be used to assess the effects in a local geographic area.

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A brief description of each of the component methodologies is given in this chapter, followed by a description of the links and interrelationships among the components. More detailed specification of the individual components can be found in the following chapters. Major inputs and outputs for each of the models are summarized in the appropriate chapters. This chapter concludes with an overall evaluation of the strengths and weaknesses of the methodologies. Some substantive analyses that could make use of MEVUNS are also given.

Figure 1 provides a schematic of the components and how they are linked. The boxes within heavy lines indicate the scope of each of the five major components. The boxes with dotted edges indicate non-MEVUNS inputs that can be used in the models.

B. POPULATION AND ECONOMIC DATA BASES

The data base for the population distribution is the geographic nodal network (GEONN).[1] The data in GEONN are aggregations of US population into a total of 3434 urban clusters and 3041 rural clusters. Each of these 6475 clusters is described by its population, a population distribution (either elliptical normal or uniform), its geographical location, parameters relating to its area, and several qualitative attributes, such as OBE area and size class. Population figures are based on a projected 1975 continental US population of nearly 224 million; of this total, nearly 165 million are included in the urban clusters.

To evaluate the effects on the economy of a nuclear attack, geographic-specific descriptors of the distribution of industry are needed. The data base used in MEVUNS was derived from the 1964 Census of Agriculture, County Business Patterns, 1964, the 1963

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1. Petersen, D.L. and L.A. Schmidt, Jr., Arrangements of U.S. Population by Urban and Rural Geometric Clusters, IDA Paper P-706 (Arlington, Va., September 1970).

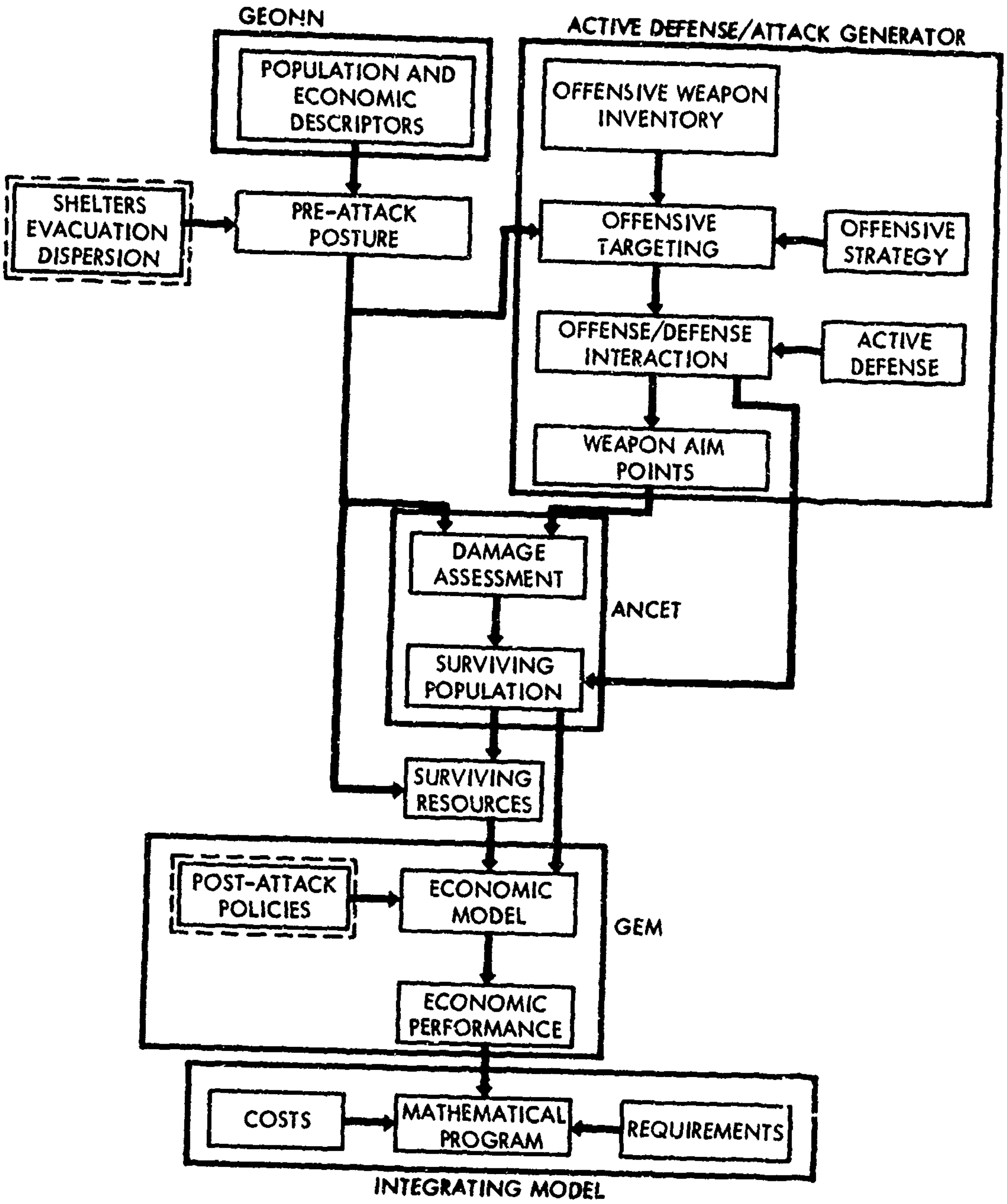

FIGURE 1. Schematic of MEVUNS

Census of Mineral Industries, and the 1963 Census of Manufactures[2] and is extrapolated to 1975. There are 82 economic sectors corresponding to those used in economic input/output analyses. For each county in the United States, the economic value added, the gross output, and the number of establishments are given for each economic sector.

The GEONN has a level of geographic resolution higher than that of the economic data base. Each of the 6475 population nodes is contained in a county,[3] and there is more than one node in some counties. Damage assessment is computed on a node-by-node basis using the GEONN. A method for aggregating the nodal results to a county is used to estimate economic damage. This procedure is described in Section E.

If there were adequate warning preceding a nuclear attack, the distribution of population and industrial capital stock might change. In particular, people might be evacuated from the urban areas, and some of the capital stock might be dispersed. Thus, the distributions of people and industry could be significantly different at the time of attack than those given in the data bases. While MEVUNS does not explicitly include the evacuation and dispersion processes, the results of such movement can be incorporated into the methodology by appropriately modifying the population and economic distributions.

In addition to the geographic distribution of population and industry, vulnerability to the effects of a nuclear attack is also described. Shelters are not explicitly represented in MEVUNS, but their effect can be modeled by changing vulnerability parameters. Fallout shelters provide protection against radiological fallout. Protection factors (PFs) for fallout are accommodated in the damage

2. Jack Faucett Associates, Inc., 1963 Output Measures for Input-Output Sectors by County (Silver Spring, Md., December 1968).

3. There are a few exceptions (of no consequence here) to this general rule. See Petersen and Schmidt, *op. cit*.

assessment model. These factors can be varied from population area to population area, allowing an assessment of the effect of differential sheltering of, say, urban nodes versus rural nodes. The effect of blast shelters is to increase the hardness of population against blast effects. The mean lethal overpressure of each population node can be specified in the damage assessment model, thus modeling the effect of blast shelters. Industrial hardness to blast effects can also be modeled in a similar manner.

C. ACTIVE DEFENSE

A simple active defense model, consisting of both terminal and area components, is used to degrade the number of warheads in an attack that reach their specified targets. The model is optional. If used, the size and location of the defense must be specified.

The terminal defense is described by the number of terminal interceptors at a node. Such interceptors are assumed to be perfect; thus, in order to inflict damage on a given node, there must be more attacking warheads than interceptors at that node.

The area defense consists of a specified number of nonoverlapping areas, each with a given number of area interceptors. The area covered must be specified. The model calculates those nodes in the GEONN that are covered. It is assumed that warheads entering the area are randomly engaged by the area interceptors, thus allowing leakage through the area defense. The amount of leakage depends on the size of the attack relative to the number of area interceptors.

D. ATTACK GENERATION

An attack-generation procedure determines the ground zeroes of the warheads in an attack. Damage assessment is then made for the weapon laydown. Descriptors of the threat must be specified. These include the number of warheads and certain characteristics of each warhead.

An attack may be given in one of two modes; it may be geographic specific on nonspecific. In the former case, the desired ground zero (DGZ) for each warhead is specified by a latitude and longitude. In the latter case, DGZs are not given by the user, but rather a targeting strategy is specified. Based on this strategy, DGZs are developed by an attack-generation procedure that maximizes value destroyed.

For a geographic-specific attack, each warhead is described in terms of its CEP, yield, height of burst (air or surface), detonation time, and fission/fusion ratio. These parameters can then be used to assess blast and fallout effects against population and estimate blast effects against industry. In this case, the attack-generation procedure is circumvented. For a geographic nonspecific attack, all warheads are assumed to have the same characteristics. The attack generator then develops DGZs for each weapon in the attack.

A targeting or attack strategy may concentrate on population, on an individual economic sector, on groups of economic sectors, on the whole economy, or on a weighted mix of population and economy. The attack generator uses the population and economic data bases to determine those nodes in the GEONN which are most attractive for attack and assigns weapons to nodes in order to maximize value destroyed. The maximization procedure is based on blast effects only and uses the square root damage law as an approximation for damage incurred. After assigning weapons to nodes, DGZs are calculated for the weapons targeted. With an active defense the user may specify a defense-avoidance attack (thereby not targeting any nodes covered by the defense) or an attack of the defense.

As mentioned, the attack-generation program uses the square-root damage law to obtain an estimate of mortalities from blast effects. This assessment may be used in lieu of the results of the more detailed damage assessment procedure, described below.

E. DAMAGE ASSESSMENT

The population damage assessment procedure estimates the number of fatalities and injuries to the population in nodes of the GEONN.[4] Blast effects, fallout effects, and combined effects are estimated. An analytical procedure is used in calculating blast effects.[5] Fallout effects are estimated by one of two standard fallout models.[6] Output data are available at several different levels of aggregation, including node-by-node, urban totals and rural totals, and nationwide totals.

Since the economic data are not at the same geographical resolution as the population, the procedure for estimating damage to economic sectors extrapolates population blast effects fatalities to industrial destruction. The result of the calculations is an estimate of the nationwide percent of capital stock that is destroyed in each of the 82 economic sectors.

F. ECONOMIC RECOVERY

While population casualties are a useful measure of the results of a nuclear attack, they do not provide information about the post-attack integrity of major institutions. The economic recovery model is designed to estimate the capability of the post-attack economy, thus incorporating economic institutions in the assessment of national vulnerability.

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4. The model used for these calculations is ANCET. The latest reference is: Woodside, Mary B., ANCET Improvements, Final Report, Vol. I, Research Triangle Institute (Research Triangle Park, N.C., November 1968).

5. Hunter, J.J., An Analytical Technique for Urban Casualty Estimation From Multiple Nuclear Weapons, Operations Research, Vol. 15, 1967, pp. 1096-1108.

6. Pugh, G.E. and R.J. Galiano, An Analytic Model of Close-In Deposition of Fallout for Use in Operational-Type Studies, WSEG Research Memorandum No. 10 (1 October 1961); and Polan, M., An Analysis of the Fallout Prediction Models, Volume I--Analysis, Comparison, and Classification of Models, USNRDL-TRC-68 (12 December 1966).

The economic model embodies the structure of the pre-attack macro-economy. It has demand and supply mechanisms linked by prices. It is nationwide in scope, resolving the economy to 82 economic sectors.[7] The damage assessment models provide (1) the population surviving (from which the size of the post-attack labor force can be calculated) and (2) the surviving capital stock in each of the economic sectors.

With these surviving resources, the economic model calculates the time-phased recovery of the post-attack economy. Such measures as total GNP, GNP per capita, capital stocks, relative prices, investment, and other demands are derived.

An important feature of the model is an indication of "bottleneck" sectors--that is, those sectors whose capital stocks have been sufficiently degraded to affect the workings of the remainder of the economy. Effects of "surgical" attacks on selected industries can thus be highlighted.

The model is sensitive to several different policy-related inputs. Recovery rates for achieving post-attack levels of capital construction, inventory accumulation, government expenditure, and exports are examples. The size of the labor force is another. Also, the demand structure may be altered to reflect rationing policies within certain sectors.

G. COST, EFFECTIVENESS, AND REQUIREMENTS INTEGRATION

A methodology for integrating the cost and effectiveness of defense measures with given survivability requirements is presented. The methodology computes a least-cost mix among a selected set of defensive systems that is capable of meeting a multi-dimensional set of specified requirements.

---

7. There are 87 sectors in the Office of Business Economics model used to collect the data for the economic model. Data for the first 79 of these sectors are generated on a county by county basis. Sectors 80-82 are considered productive sectors without a geographic distribution. Sectors 83-87 exist only for accounting, purposes. Depending on the situation the model is referred to as a 79, 82 or 87 sector model.

The types of requirements that can be accommodated are restricted only by the outputs that can be obtained with the methodologies described above. These include percent of surviving population (national and/or regional), surviving capital stock (by sector), level of output in given industries, GNP per capita (one, two, and five years post-attack, say), and so on.

The integrating methodology may also be used to consider simultaneously several different scenarios--Soviet or Chinese threat, amount of warning, size of post-attack military force, and so on. In addition, the methodology will compute optimal targeting by the offense.

The damage assessment procedures and the economic recovery model provide means of estimating the outcome of a nuclear attack. The effectiveness of defense systems can be generated by using these results. Outcomes are computed for different levels of defense to allow the implicit calculation of their marginal effects. Cost functions for the defense systems are supplied to the integrating model.

H. OPTIONAL USES OF THE METHODOLOGIES

As the MEVUNS structure is modular, certain components, or sets of components, may be used in isolation. Figure 2 illustrates the options available to the user. Possible entry points in the stream of models are indicated. For all entry points, the GEONN and/or the economic data bases are necessary. The user may exit at any point in the stream.

If the type of attack consisting of a specified set of DGZs for an inventory of weapons is used, these laydown points can reflect warhead reliability, active defense, and other such factors which degrade the number of impacting warheads. In this case, the active defense model and the attack generation model are not used. The second way to present an attack is as a total inventory of warheads to be targeted on the nation (more precisely, on some specified set of nodes in the GEONN). The attack-generation model is then used to allocate the

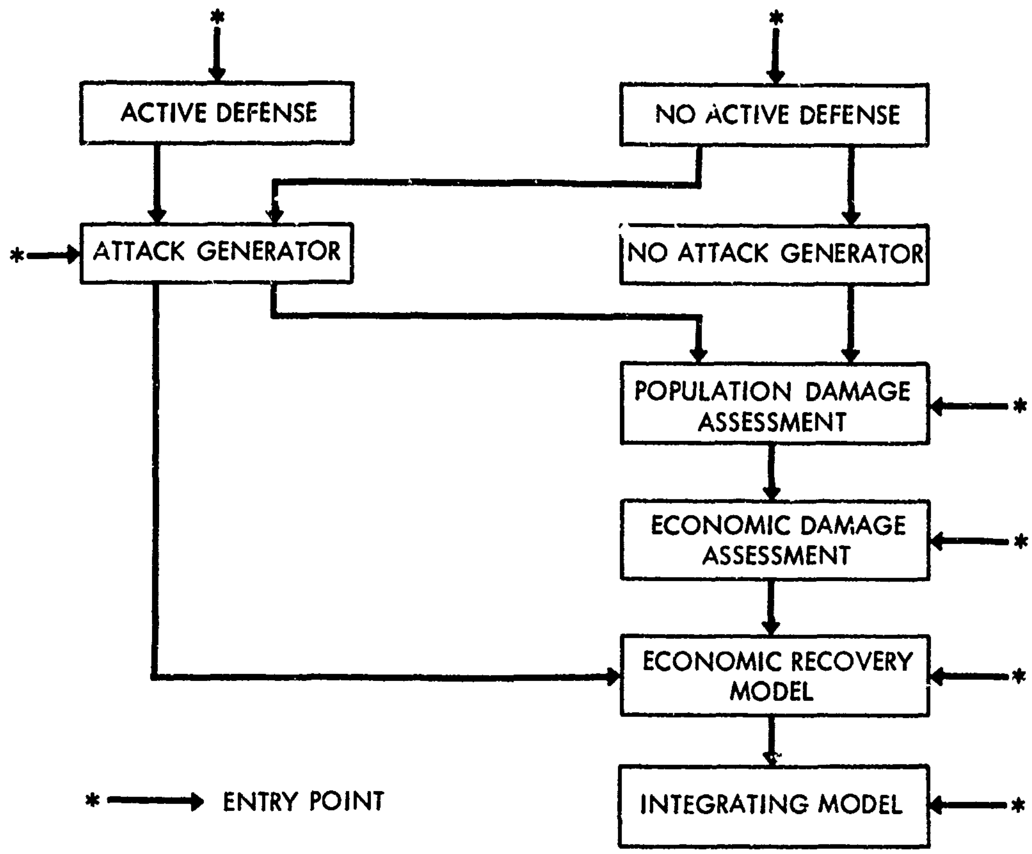

FIGURE 2. User Options for MEVUNS

inventory of weapons to specific nodes. In the presence of an active defense, the model will degrade the number of warheads arriving at the targets, dependent upon the size and location of the active defenses.

For the first type of attack, the population damage assessment model must be employed. For the second type of attack, there is an option of using instead the attack-generation model estimates of casualties from the blast effects. The results of either of these population assessments may be used in the economic damage assessment model.

I. OVERALL EVALUATION OF THE METHODOLOGIES

1. Introduction

The results of the calculations of MEVUNS should not be interpreted as being predictors of the outcome of any particular nuclear attack. There are far too many uncertainties associated with the pre-attack, attack, and post-attack processes to predict such outcomes in absolute terms. Many assumptions have been made concerning numerical values of the inputs and the damage and recovery processes themselves. In addition, all of the models are deterministic. There are no random numbers used in any of the calculations.

The lack of predictive capability notwithstanding, the methodologies should prove useful and credible for comparative analyses. For instance, the relative magnitudes of lives saved or post-attack per capita GNP can be assessed for a varying array of shelter postures. For such comparative analyses, the methodologies were designed to be flexible and to allow changes to be made easily in the major inputs to the models.

Examples of the types of analyses for which the methodologies could be employed are:

1. Evaluating the effectiveness of a rural sheltering program or a combined rural sheltering and urban evacuation program.
2. Evaluating the effectiveness of an urban blast shelter program.
3. Determining the vulnerability of various industries to different levels of attack.
4. Determining which industries are most vulnerable to varying levels of attack.
5. Finding those industries which could be surgically attacked to "bottleneck" the whole economy.
6. Assessing the effects of post-nuclear attack hostilities in terms of the capability to recover economically.
7. Generating cost-effective mixes of defensive systems for various scenarios and assumptions about the threat, warning time, and technology of an active defense.

2. Strengths

The computer programs described in this report are programmed, implemented, documented, and available for use. The documentation permits decisionmakers and analysts to understand and use them. They are flexible, with a wide array of optional inputs and outputs. The computer programs are relatively fast running.[8]

The scope of the models comprehends the primary active and passive defense systems and measures. An important contribution is the capability to assess the effects of damage to industrial capital stock. The integrating model provides a means of simultaneously considering costs, effectiveness, and requirements of defense systems.

The models have a high degree of geographic resolution. Thus, effects of specific attacks can be assessed. When appropriate local civil defense models are developed, they can be linked to MEVUNS through the GEONN.

3. Limitations

The industrial data are aggregated to the county level, since locations of firms within counties are not available. For this reason, industrial damage is assessed as an extrapolation of urban population fatalities. If a more detailed industrial data base were available, an analysis of damage to plants and machinery should be conducted to estimate industrial damage functions.

The active defense model is a rough representation of a ballistic missile defense. If appropriate, a more detailed model of active defense could be accommodated. The terminal defense consists of perfectly reliable interceptors, and the area defense randomly destroys incoming warheads.

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8. Computation times for each model are discussed in Volume II.

The attack-generation procedure does not necessarily allocate warheads to cities in an optimal fashion. One reason is that it is not clear what measure of effectiveness should be used. The present allocation procedure is based on the square root damage law. Counterforce targets are not presently included in the attack generator.

The damage assessment routine assumes bivariate normal population distributions and cumulative normal casualty functions. Furthermore, only the effects of a limited number of weapons can be estimated for each node.

The economic model assumes that institutional structures remain intact after an attack. That is, the financial system, government, and social frameworks remain unscathed. In addition, the economic model assumes that goods and labor can flow without hindrance from region to region.

There is no explicit representation of local civil defense measures. A highly effective local program would change the casualty estimates in the present model.

Each of these limitations is discussed in more detail in the following chapters.

III

GEOGRAPHIC DISTRIBUTIONS OF POPULATION AND ECONOMIC DATA

A. POPULATION DATA

The population data base used in the MEVUNS study is called the Geographic Nodal Network (GEONN). It was developed at IDA[1] from population data prepared by the Bureau of Census.[2] Some 47,000 Standard Location Areas (SLAs),[3] are described by a code number,[4] latitude, longitude, and 1960 census population. These data were then extrapolated by the Bureau of Census to 1975 population.

The Census data distinguish between urban and rural SLAs, but provide no means of aggregating population into urban clusters other than by political boundaries. The development of the GEONN was motivated by the need to describe population clusters by a few parameters.

The GEONN has population distribution parameters for each county, for the rural area in each county, and for each urban cluster in the

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1. Petersen and Schmidt, _op. cit._

2. U.S. Bureau of Census, _National Location Code_, FG D 3.1/4 (1962).

3. A Standard Location Area is similar to a census tract; for most urban areas the two are the same; in rural areas several census tracts have usually been combined.

4. The Region, State, Area, County (RSAC) code consists of eight alphanumeric symbols. The first symbol identifies one of eight regions in the nation, the first two symbols combined define the state, the third symbol identifies an area within the state, and the first four symbols combined identify the county. The last four symbols give a numbering of the SLAs in the county. No specific order is maintained.

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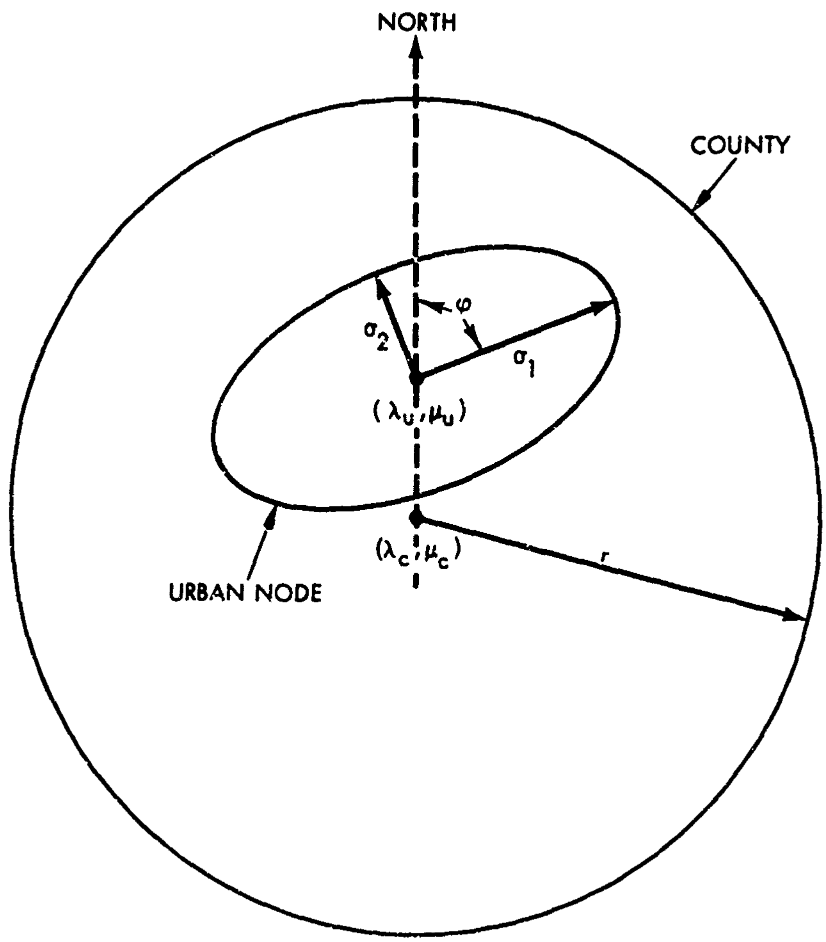
county. Thus, counties consist of one rural node and varying numbers of urban nodes. A distinguishing feature of this description of population is that the urban clusters are defined independently of political subdivisions within the county. For each node, the population, the latitude and longitude of the centroid of the population, and the standard deviations of the population distribution about the centroid are given. Two standard deviations are presented, along the semi-major and semi-minor axes of the ellipse which best approximates the actual population distribution. The angle of the principal axis from the north also is presented. Figure 3 illustrates the parameters used to describe population distributions in the MEVUNS data base. All rural nodes are taken to be circular with a uniform density. Certain small adjustments to the county structure were made (for example, cities over a certain size in the state of Virginia are defined as separate counties in the Census data but are handled differently in the GEONN).

A total of 3434 individual urban nodes in 3041 counties were obtained. The total population is 223,727,140, of which 164,740,567 is urban. Population clusters crossing county boundaries, called Multinodal Complexes, have also been constructed. There are only 124 such Complexes, containing 312 urban nodes. These tend to be larger cities, so that the population in these Multinodal Complexes totals 93,424,379 or about 57 percent of the urban population.

Note that this data base differs from that of damage assessment routines using Standard Metropolitan Statistical Areas (SMSAs) as the base. Not only is the population distribution represented differently, but the total urban population at risk from blast effects is different. Such differences can make comparisons of alternative damage calculation models difficult.

A data base developed from the 1970 census could give a more accurate representation of 1970-1980 population. Such a data base would contain not only a better estimate of population and its distribution, but also a better definition of urban and rural



A = AREA OF COUNTY

σ_1 = STANDARD DEVIATION OF POPULATION ALONG MAJOR AXIS

σ_2 = STANDARD DEVIATION OF POPULATION ALONG MINOR AXIS

φ = ANGLE OF INCLINATION OF MAJOR AXIS FROM NORTH

r = RADIUS OF COUNTY

$= \sqrt{A/\pi}$

(λ_u, μ_u) = LATITUDE AND LONGITUDE OF URBAN NODE CENTER

(λ_c, μ_c) = LATITUDE AND LONGITUDE OF COUNTY CENTER

P_c = TOTAL COUNTY POPULATION

P_u = URBAN POPULATION

ρ_c = DENSITY OF POPULATION IN NON-URBAN AREA OF COUNTY

$= (P_c - P_u) / (A - \sum_{\text{URBAN NODES}} \pi \sigma_1 \sigma_2)$

FIGURE 3. Population Distribution Within a County

areas. Moreover, several new features, for example, distribution of people by skill classes, could be included in a new GEONN which could prove useful for performing damage assessment calculations.

B. ECONOMIC DATA

Only the Total Value Added (TVA) by input-output sector for each county is used in the damage assessment routines. Other economic data are used in the economic recovery model and are discussed in Chapter VIII.

A 1963 data base was used as the base year for the 1975 TVA projection. The output data were compiled by county for the first 79 input-output sectors.[5] The county output measures were based on aggregations at the regional, state, and national levels. The county output was constructed by first developing area measures of economic activity closely related to output. These measures consisted of data on (1) the value of agricultural product sales; (2) the value of mining and manufacturing shipments; (3) industry payroll statistics; (4) industry employment statistics; and (5) miscellaneous data, such as selected population statistics and government revenues and expenditures. National, state, and metropolitan groupings of these statistics were used to calculate relationships between output and the proxy measures. The relationships were then applied to the county proxy data to construct county output measures.

Both the 1960 and the projected 1975 population by county are used in the projections of TVA. The TVA was projected to 1975 by county and by sector, using the following formula:

$$V_{ij}^{75} = \left[\hat{V}_j^{75} \Big/ \sum_{k=1}^{m} \frac{P_k^{75}}{P_k^{60}} V_{kj}^{63} \right] \frac{P_i^{75}}{P_i^{60}} V_{ij}^{63} \quad , \qquad \text{for all } i \text{ and } j \ ,$$

5. Jack Faucett Associates, Inc., op. cit.

where

$i,k = 1, \ldots, m$, indices for counties,

$j = 1, \ldots, 79$, an index of industrial sectors,

V_{ij}^{75} = projected 1975 TVA in county i for industry j,

P_i^{60} = 1960 population of county i,

V_{ij}^{63} = 1963 output in county i for industry j,

P_i^{75} = projected 1975 population of county i,

$\hat{V}_j^{75}$ = projected national TVA in 1975 for industry j.[6]

Thus, it is assumed that the growth of a sector's TVA in a county is proportional to the county population growth, P_i^{75}/P_i^{60}, modified by the ratio of the Bureau of Labor Statistics (BLS) estimate of total 1975 sectoral TVA to an estimate of 1975 total sectoral TVA based upon total population growth:

$$\hat{V}_j^{75} \Bigg/ \sum_{k=1}^{m} \frac{P_k^{75}}{P_k^{60}} V_{kj}^{63} .$$

6. The source of projected 1975 total national sector TVA was a Department of Labor projection to 1970. Annual average growth rates between 1962 and 1970 were used to extrapolate the data to 1975. Bureau of Labor Statistics, U.S. Department of Labor, Projections 1970, Interindustry Relationships, Potential Demand, Employment, Bulletin No. 1536, U.S. Government Printing Office (Washington, D.C., 1966).

IV

PASSIVE DEFENSES

A. FALLOUT PROTECTION

The main population damage assessment model (ANCET, Analytical Casualty Estimation Technique) requires a specification of fallout protection on a node-by-node basis. Thus it is possible to do extensive sensitivity analyses on varying fallout shelter programs.

The fallout models in ANCET compute the total unshielded radiological dose received by sectors in a GEONN node. The effects of fallout protection are estimated by modifying the unshielded dose. The shielded dose for an individual in a sector is calculated by dividing the unshielded dose by a protection factor (PF). That is, if UD is the unshielded dose and SD the shielded dose, then SD = UD/PF. A distribution of PFs within a node is allowed. Let f_j be the fraction of the total population, P, in a sector which is protected to the level PF_j. Then f_jP is the number of people exposed to a shielded dose of UD/PF_j.

The damage assessment model allows up to 15 different PF categories in each city. Thus, fallout shelter postures are modeled by specifying the values of f_j, the fraction of the population in a shelter which provides a protection factor of PF_j, for $j = 1, 2, \ldots, 15$. For any particular run of ANCET, up to six different shelter postures may be evaluated. The incremental computer running time for the evaluation of an additional shelter posture is very small relative to the total time for damage assessment.

B. BLAST PROTECTION

The ANCET model allows for the implicit consideration of blast shelters. The blast-effects calculations in ANCET are based on casualty functions relating the probability of fatality (or injury) to distance from the weapon ground zero. This probability for any distance will depend upon the hardness of the population. In particular, it will be a function of the mean lethal overpressure (MLOP).

Probability curves for different values of MLOP have been constructed for surface bursts and for air bursts. The air burst relationships are based on the height of burst which optimizes the radius of the 10 psi contour. The basic relationships between pressure and distance are given in Table 1.[1] For a given MLOP, the distance at which the probability of fatality is .5 can be determined from the table.

A set of six different curves relating probability to distance has been constructed for use in ANCET. Each of the curves intersects at the distance dictated by a given MLOP. Figure 4 illustrates the curves for a surface burst with an MLOP of six, intersecting at a distance of 2.56 miles. The choice of the curve to be used in the calculation of blast effects is left to the user to reflect his best information regarding its shape. Some sensitivity analysis on the choice is given in Volume II of the report.

Differing types of blast shelter protection can be accommodated in ANCET. Casualty functions have been constructed for MLOPs of 2, 3, 6, 10, 15, 20, 30, 50, and 90. In addition, the Defense Civil Preparedness Agency (DCPA)[2] has developed casualty curves for the 13 different shelter types shown in Table 2. Thus, the user has 22 sets of six casualty curves available for blast effects assessment. The MLOP is assumed to be the same for all the population

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1. The figures in the table are derived from S. Glasstone (ed.), The Effects of Nuclear Weapons, United States Department of Defense, April, 1962, p. 139. Actual calculations were made by the DCPA.

2. N. FitzSimons and G. Sisson have reported these curves in a DCPA internal communication, dated May, 1971.

Table 1

PRESSURE-DISTANCE RELATIONSHIPS, ONE-MEGATON WEAPONS

psi	Surface Burst (Nautical Miles)	Air Burst[a] (Nautical Miles)
1	7.197	11.174
2	4.735	7.197
3	3.61	5.5
4	3.087	4.451
5	2.8	3.95
6	2.557	3.598
7	2.32	3.35
8	2.140	3.125
10	1.932	2.756
12	1.80	2.5
15	1.544	2.201
20	1.354	1.061
25	1.226	.781
30	1.108	.502
50	.866	
72	.763	
90	.683	
100	.633	
200	.477	

a. Optimum Height of Burst (HOB) at 10 psi

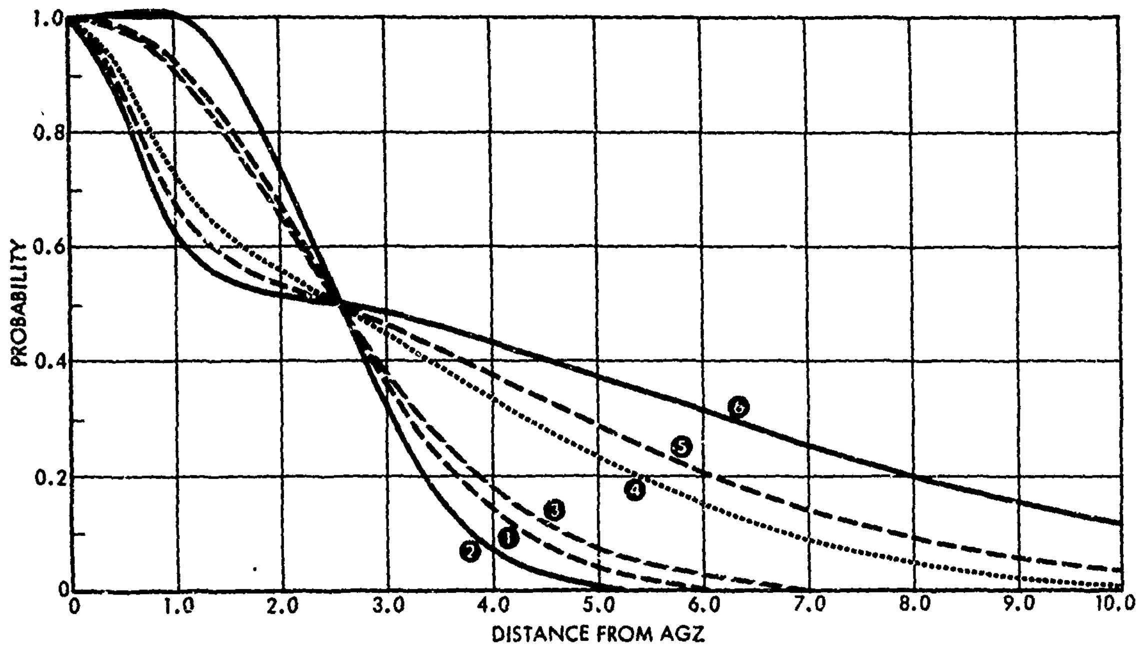

FIGURE 4. ANCET Blast Casualty Functions

in a node, although it may vary from node to node. It would be preferable to allow for differential MLOPs with a node, but this modification would entail a major change in the damage-assessment methodology.

There is a high degree of uncertainty in the blast casualty functions. They are based on data obtained from the Japanese explosions and subsequent testing. Although the relationship of MLOP to range from ground zero is relatively well understood, the probability-of-casualty curves (as a function of the distance from ground zero) are tenuous.

Table 2

BLAST FATALITY OVERPRESSURES

No.	Shelter Description	Necessary Overpressure (psi) for Given Percent of Fatalities				
		99	90	50	10	1
1	Outside-exposed to thermal pulse	5	4	3	2	1
2	Outside-shielded from thermal pulse or, One- and two-family residence: above ground	8	7	5	3	2
3	One- and two-family residence: basement	18	13	10	7	3
4	NFSS buildings (weak-walled: above ground)	7	5	4	3	2
5	NFSS buildings (strong-walled: above ground)	10	8	7	6	4
6	NFSS buildings (massive-basement)	20	15	12	9	5
7	NFSS buildings (flat plate-basement)	10	7	5	3	2
8	Slanted basement designed for 15 psi (open)*	29	27	25	23	21
9	Basement designed for 25 psi (closed)**	92	83	72	60	50
10	Basement designed for 50 psi (closed)**	130	112	90	63	50
11	Single-purpose fallout shelter (r/c arch)	45	45	45	45	45
12	Single-purpose fallout shelter (r/c box)	32	30	20	12	10
13	Single-purpose 30 psi shelter (r/c arch)	95	85	70	48	42

*1600-person basement.

**School basement.

Note: All weapons are presumed to be one megaton.

C. EVACUATION

The process of evacuating people from urban areas is not explicitly considered in MEVUNS. However, sensitivity analyses on the effectiveness of an evacuation program can be accomplished. Each of the nodes in the population data base has some pre-warning population, say p_i, for node i. An evacuation changes the population in some or all of the nodes. Thus the evacuation process has the effect of changing p_i to some p_i', the post-warning population in node i.

If an evacuation model is developed for relating warning time, capacity of transportation links, and other factors affecting the movement process, then it can be linked with the GEONN data base and the damage assessment routines in MEVUNS, since a change in the population of a node is easily accomplished by input. An explicit linking of an evacuation model to MEVUNS is desirable. This added feature would provide an expanded capability for analyzing relevent civil defense measures.

V

ACTIVE DEFENSES

A. INTRODUCTION

There are a number of levels of detail at which the capabilities of a ballistic missile defense (BMD) may be represented. Simple representations of both terminal and area defenses are used for the MEVUNS active defense model. This was done to (1) allow a larger variety of defense systems and threats to be represented,(2) simplify the attack-generation problem and make it easier to see the relations between defense assumptions and attack patterns, and (3) minimize the number of parametric variations that must be considered in comparing a range of possible attack and defense options. The model selected is one consistent with the level of detail in the other components of MEVUNS. A translation from physical parameters, such as radar capabilities, interceptor reliabilities, interceptor flyout times, suite of penetration aides employed by the attacker, and so on, is needed to generate the input to the MEVUNS active defense models. The active defense parameters are listed in Table 3.

B. TERMINAL DEFENSE

1. Terminal Defense Model

The terminal defense is represented by a basic "price" model. Each terminally defended city has a specified number of "reliable interceptors". Each reliable interceptor can intercept one incoming warhead. It is assumed that no damage is inflicted on the city until all interceptors are exhausted, after which all incoming warheads penetrate to the city. In the simplest representation of such a model, the no-defense curve of value destroyed (as a function

Table 3

ACTIVE DEFENSE PARAMETERS

Terminal Defense
1. Number of sites (maximum of 400)
2. Number of perfectly reliable interceptors (for each site)
3. Node identification number (for each site)
Area Defense
1. Number of islands (maximum of 40)
2. Number of perfectly reliable interceptors (for each island)
3. Latitude and longitude of center of island (for each island)
4. Radius of coverage in nautical miles (for each island)

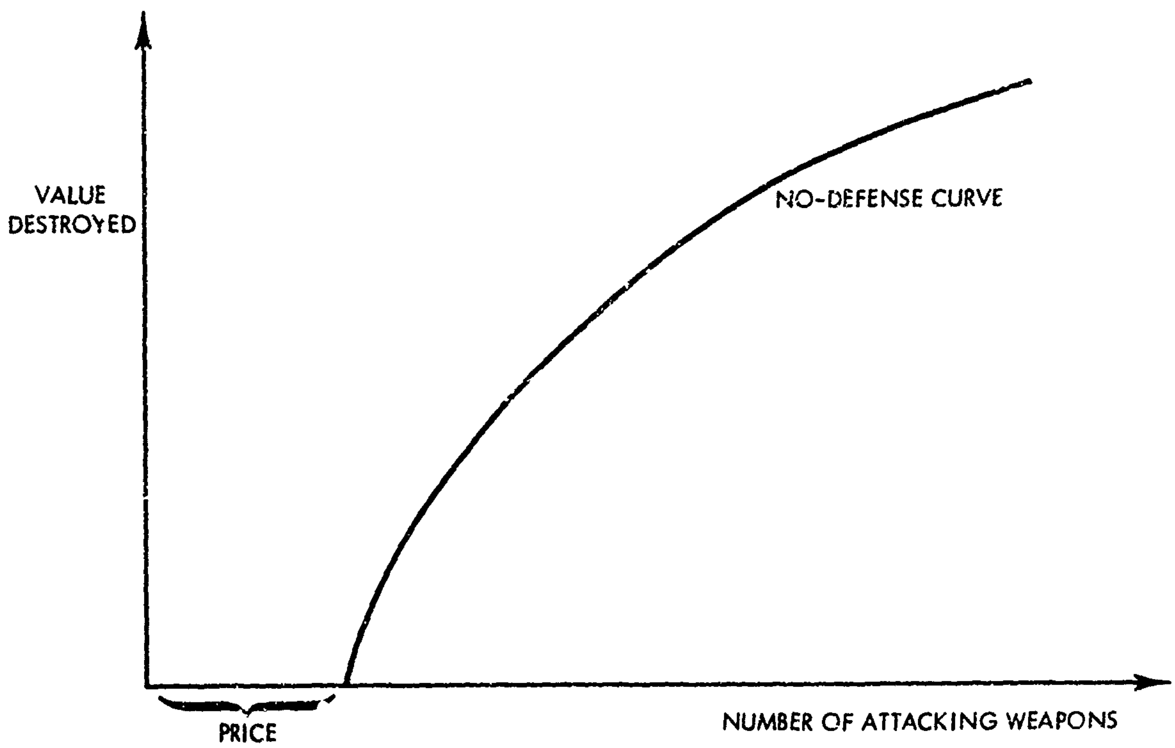

FIGURE 5. Terminally-Defended City Damage Functions

of the number of targeted warheads) is shifted to the right by the number of reliable interceptors (see Figure 5). Such a representation requires only one parameter to characterize the terminal defense, namely the price or number of reliable interceptors. The relation between the number of reliable interceptors and actual interceptors depends upon many factors, including tactics of both the offense and defense, as well as physical parameters. A commentary on the general applicability of this model and upon means of translating BMD parameters to a price is given in Subsection 2 below.

The terminal defense model, as presently implemented, can cover only one city at a time. No city in the population data base extends into more than a single county. This means that the coverage of any terminal defense battery is restricted to only the part of the metropolitan area which lies in a single county. In some cases, e.g., Los Angeles, such a restriction may not be serious, but in others the county structure causes artificial boundaries. For example, New York City itself is represented by five counties in the data base--Manhattan, Kings, Queens, Bronx, and Staten Island. While the basic population data base is structured so that complete population aggregations are identified, a significant amount of additional programming would be needed to allow these population clusters to become the object protected by a terminal defense battery.

2. General Applicability and Interpretation

Because of the presence of stochastic effects, the damage curve, as a function of number of weapons aimed, represents an expected value. In general, it is not true that simply combining expected values of separate processes yields a correct result, since the expected value of a function of a stochastic variable is not necessarily equal to the function of the expected value of that variable. If the terminal defense and damage process were modeled in detail, then a Monte Carlo simulation, or similar procedure, could be carried out to obtain the distribution of the results.

From this distribution the expected value could be calculated. The expected value model used here is justified as a simplification arising from the lack of an explicit detailed model.

A simulation yielding the probability distribution of damage could be developed. As an example of such a model suppose that, for a large attack, one interceptor is allocated to each incoming warhead until the interceptors are all used. If the interceptors, each with reliability ρ , are allocated to n incoming warheads, then the number of attackers destroyed by the terminal defense is binomally distributed with mean $n\rho$.[1] If it is further assumed that the square root damage law perfectly represents the damage as a function of number of arriving warheads, the appropriate summations could be performed to calculate the resulting expected damage curve. Such a curve would be different in shape than the square root damage law used in the terminal defense calculations, but would approximate it in regions where the rate of change in the slope of the square root damage law is small.

Even for this simple model, a more accurate calculation would involve combining the distribution of warheads penetrating the terminal defense with the probability distribution of damage from the penetrators. Since the latter distribution cannot be analytically expressed,[2] a numerical integration would be needed to find the overall distribution of destruction.

The defense model described in Subsection 1 represents one of a number of possible defense models. That model would represent an accurate portrayal of the defense if (1) none of the attacking weapons were aimed at the defensive battery itself; (2) if all weapons were aimed at equal-value portions of the target; (3) if all objects assessed as warheads by the radar were, in fact, live

1. This is true if the number of warheads is at least as large as the number of interceptors and if the interceptors engage warheads independent of each other.

2. This is due to the irregular distribution of target value.

warheads of equal yield; (4) if the defense knew the attack size would equal or exceed the number of interceptors; and (5) if the success of each interceptor against an incoming warhead is independent of the allocation of all the other interceptors.

Regarding the first assumption, if some weapons are aimed at the defense battery itself, then for an optimal defense these weapons would be attacked with more interceptors, giving a higher probability of kill of these particular warheads. If the defense knew the number of warheads the attacker aimed at the battery and also knew the interceptor reliability, then the optimal allocation of interceptors to self-defense and to defense of the target area could be determined. In practice, neither of these factors are known *a priori* (testing of interceptors cannot be done with live nuclear warheads). Thus, the optimal self-defense allocation is a problem of a nontrivial sort.

Concerning the second assumption, the defense should attempt to preferentially intercept those weapons aimed at the high-value portions of the target, if possible. If the defense battery can estimate the impact point of arriving warheads, additional interceptors should be allocated to protect the high-value portions of the targets. Moreover, the defense can save interceptors by not allocating them to protect areas which have already been subjected to severe weapon effects. The additional allocation of resources for protecting high-value areas would depend upon the total number of attacking warheads and whether the attacker had shifted his attack to compensate for a possible preferential terminal defense. It also should reflect the fact that the defense is probably more capable for some portions of the defense footprint.

Concerning the third assumption, if an attacker has decoys, the defense would have to assess the likelihood of an incoming object being a live warhead and allocate resources on that basis. As a longer time in the atmosphere assists in unmasking decoys, the decoy discrimination problem tends to imply a later commitment time for interceptors (which couples with the preferential defense question). The probability of unmasking a decoy depends upon the direction of approach, as well as the radar capability. In an intense battle situation radar visibility will be lowered by previous warhead detonations, possible enemy electronic countermeasures, and radar traffic-handling capability, the decoy unmasking at some times will present different problems than at others, thus tending to make the optimal allocation of interceptors a function of time.

If the fourth assumption were violated and the defense overestimated the size of an attack, then the defense could end up saving interceptors for warheads which never presented themselves. The knowledge the defense may have concerning an attack depends in part upon its capability for early observation of enemy missile trajectories, as well as the attacker tactics in spacing incoming missiles.

Finally, concerning the assumption of independence, degradations of the radar environment due to nuclear weapon detonations force an optimal defense to look ahead in time to insure that unacceptable radar blinding of later warheads is not caused by the engagement of earlier ones. Moreover, limitations on radar capability may force decisions between allocating radar effort to searching for new objects, discrimination of penetration aids, and tracking and intercepting of particular objects. Thus, the doctrine for allocating radar effort would of necessity have to consider several objects at a time.

Due to the complexities of a real terminal defense system, it was felt that an attempt to use distributional rather than expected value calculations in MEVUNS would only obscure the primary effects of a terminal defense. Thus, the characterization of the terminal defense in terms of the expected number of warheads intercepted is judged to be adequate in the absence of a very detailed analysis.

Other types of basic models might be hypothesized. They could include multiple allocation of interceptors on incoming objects, preferential defense capability, defense suppression attacks, saturation attacks, etc. As in the case described above, various factors would be operating to tend to degrade the validity of the assumptions needed for these other types of models.

With more complex models it becomes necessary to describe the characteristics of the attack and the defense in more detail, requiring a more specific definition of defense system parameters, attacking weapon parameters, and tactics on the part of both the offense and the defense. Such studies are more appropriately carried out with computer programs specifically designed to simulate the battle situation in considerable detail. The results of such efforts can be reflected in the MEVUNS terminal defense model through the price parameter.

There would be little difficulty in modifying the terminal defense model to use a damage curve more appropriately representing an active defense, if a detailed analysis of a particular system produced such a curve. Including a preferential terminal defense capability in the model would represent a somewhat more difficult undertaking. The basic description of the preferential defense capability of specific hardware configurations which could be related to the price would have to be developed. Also, a means of expressing the attacker reaction to the preferential defense capability would have to be derived. Similar problems arise for area ballistic missile defense, as discussed below. The assumption of normally distributed population in cities may be inadequate to represent properly the benefits possible from preferential terminal defense. If this is the case, an even more extensive change in methodology is needed.

C. AREA DEFENSE

1. General Discussion

There is a profusion of models describing the capabilities of an area defense system. These are not reviewed in detail here, but some indication of the types of such models is given to place the selection of the MEVUNS model in context.

A single defensive battery in an area ballistic missile defense can cover a number of targets. The coverage footprints of these batteries may, or may not, overlap and may, or may not, cover the entire country. There is a coupling between targets within a footprint in that the probability of intercepting a warhead aimed at one target is not independent of the number of warheads aimed at other targets in the footprint.

From a modeling viewpoint, a basic distinction between terminal and area defense is the knowledge of the opposing side's strategy. Against terminal defenses it is usually assumed that the attacker knows the number of interceptors protecting each target. For area defense this assumption is not valid; the defender can, to some degree, allocate his interceptors to defend selected targets, the choice of which is not known to the attacker.

A basic variable then in modeling area defense is the degree to which the defender can preferentially defend targets within a footprint. If the defender is allowed perfect knowledge of the attack within an island and also has the capability to allocate his interceptors against any incoming warhead in the island, relatively good results are achieved by the defense. As a general rule, the defender would completely defend certain high-value targets with area interceptors, and not defend other targets. This tactic is preferred, because once the defender begins to defend a target, he obtains increasing marginal returns for each additional warhead destroyed, and should continue to allocate interceptors until all enemy warheads aimed at that target are destroyed.

Because of the all-or-nothing character of preferential area defense, the problem of leakage through the defense is (in contrast to terminal defense) quite significant. If interceptors are not assumed perfectly reliable, then the defense should allocate interceptors to insure that an unreliable interceptor is not responsible for the large increment of destruction which occurs when the first warhead hits the target. If the defense has adequate time, the most efficient means of allocation would be a shoot-look-shoot mode; that is, an interceptor would be launched at an incoming object, followed by a second interceptor only if the first interceptor is not credited with a kill of the incoming warhead. Lacking such a shoot-look-shoot capability, the less efficient allocation doctrine of simultaneously launching several interceptors might be employed.

It is characteristic of area defense that different targets within a battery footprint have different shoot-look-shoot capabilities. Targets near the forward edge of a footprint require that interceptors be launched as soon as a valid track is established and hence have no shoot-look-shoot capability. Targets near the rear edge of a footprint have more time between the track establishment and interceptor launch. Thus, the choice of those targets which are defended would in part depend upon their geographical location.

The establishment of a shoot-look-shoot capability for each target requires a rather detailed analysis of the location of the area defense radars and interceptor farms in relation to the target, as well as assumptions about enemy launch locations and missile trajectories.[3] Some restrictions, such as radar horizons, are primarily independent of the system, but others such as radar power, target cross section, interceptor fly out times and distances, etc., are system dependent. A detailed active defense model must incorporate a number of specifics for the particular system being studied. Even if the system components are defined, the area defense shoot-look-shoot capabilities are quite dependent on siting requirements and on the degree of cooperation between different batteries. The impossibility of locating radars or interceptor

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3. Primarily whether all minimum energy trajectories are assumed, or whether lofting is permitted.

farms in the ocean, or in Canada, renders area defense of some locations inherently more difficult than others.

The tactics of the attacker can significantly affect the area defense interceptor allocation process. The attacker can deny the defense complete knowledge of his attack by making his attack in waves, with the spacing between waves large enough so that the defense must commit interceptors to a first wave before the next can be seen. In such a case the problem becomes a game (in the game theoretic sense) where each side must allocate with incomplete knowledge of the actions of the other. In this situation, the solution has the characteristic of both the attacker and the defender tending to adopt all-or-nothing tactics: the attacker either attacks a target heavily or not at all, and the defender either defends heavily or not at all. The size of targets attacked and defended and the strength of the attack are parts of the problem which can be addressed analytically, while the actual selection of targets must be done in a different manner.

The attacker may also employ various kinds of penetration aids. Since the area defense interceptors must be launched while the penetrating objects are still exoatmospheric, atmospheric discrimination is impossible.[4] Thus, the penetration aids which might be employed by the attacker are different for area defense than for terminal defense. In general, lighter penetration aids can be used, but the defender has a longer time to observe incoming objects and decide whether a particular object is on a sufficiently threatening trajectory.

A serious problem in area defense is the obscuration of the radar due to nuclear weapon effects. Because the effects of an exoatmospheric explosion covers a larger volume, and recombination

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4. An interceptor, of course, might be maneuvered after launch. Depending on the degree of such maneuver capability and the attacker tactics, various degrees of atmospheric filtering could be achieved.

of ionized gases is slower at rarefied densities, and area defense radars tend to operate at lower frequencies, the blackout problem tends to be more severe for area defense than for terminal defense. This obscuration of vision tends to lessen the possibilities of preferential defense unless the attack is widely spaced.

To some extent, area defense and terminal defense are complementary. The large kill radii which area defense weapons might extract from exoatmospheric detonations tend to make the attacker spread his attack to avoid having more than one object destroyed at a time. However, penetration of terminal defense may be more effective if an attack is closely spaced. Thus, an attacker is placed in a quandary if it is necessary to penetrate this defense in depth. Whether the attacker is rash or conservative in attempting to resolve this problem can strongly influence the nature of the attack. In a rash attack, the attacker would go for a larger expected payoff with an appreciable chance of much smaller payoffs, whereas in a conservative attack, an attacker would go for a smaller expected payoff, with a much smaller chance of very low payoffs.

An attacker may attempt to negate preferential defense capabilities by first attacking the defense site, forcing the defender to defend this site, and then after all the defense interceptors are exhausted, attacking the remainder of the target system. Such a tactic is influenced by whether the defender defends the area defense site with a terminal defense battery. If so, then the complications of the preceding paragraph are encountered in attempting to model the results of such an effort. In any case, if the attacker spreads an attack to exhaust area defense interceptors, an appreciable time might be required before penetration can be guaranteed. The attacker may not wish to expend the necessary time before doing anything besides attacking the defense site.

The simplest type of model allows for pure preferential defense of the entire country with perfect interceptors. Next in complexity are those models which divide the country into nonoverlapping preferential defense islands and allow pure preferential defense in each island. Here the attacker must choose which islands to attack. Finally, pure preferential defense with overlapping islands can be analyzed. In this case there is a coupling between islands which complicates the methodology considerably.

The above sequence can also include interceptors with a non-reprogrammable reliability of less than one. In this case, the number of interceptors allocated per warhead may be preassigned, or it may be subject to optimization. The optimization can assume various degrees of shoot-look-shoot capability.

Several models have investigated the degradation of preferential defense capability when the attacker attacks in waves. These are complications added to the well-studied situation where it is assumed that neither side has knowledge of the opponent's strategy, and the situation is analyzed as a game.

At the other extreme from pure preferential defense are models which assume the attacker can attack the defense directly. The attacker must pay a price to overcome the area defense of an island, but then is allowed to attack the rest of the island as if no area defense is present. If the islands are overlapping, techniques of mathematical programming such as "branch and bound" are needed to select the islands to attack.

2. Area Defense Model

As an alternative to the two-sided optimization approach, a simplifying assumption is often made that the defense can allocate area defense interceptors at random against an attack which has been optimized by the attacker in each island as if there were no area defense. This approach is adopted in the MEVUNS area defense model and also in the attack generator. The defense islands are assumed to be nonoverlapping. The attacker is permitted the option of not attacking an island if he can achieve a better return by using his warheads elsewhere. The assumption of random interceptor allocation allows the area defense to be treated as a degradation of the attacker missile reliability, where the degradation depends upon the number of missiles attacking each island. If this number (for a particular island) is given by n_a, and the number of the actual intercepts in an island is given by d_a, then the probability of a warhead penetrating is given by

$$p = (1 - \frac{1}{n_a})^{d_a} .$$

This model yields a large degradation in attacker capability when only a few warheads attack an island, but only a small degradation for large attacks.

An advantage of this model is that it allows an expected value calculation to represent the physical interactions fairly well. The pure preferential defense and the two-sided optimization approaches both yield all-or-nothing type strategies. There, an expected destruction of a target of 50 percent usually means there is a 50 percent chance of the target being unattacked and a 50 percent chance of the target being completely destroyed. Hence, the distribution is severely bimodal. Under the assumptions adopted here, however, a 50 percent expected damage means the most likely amount of damage is 50 percent, with only a small chance of no damage or complete destruction.

This model could be readily extended by allowing the attacker to react to the defense by reallocating his attack to account for lower overall reliability of the attacking weapons. In fact various degrees of reaction could be achieved by simply allowing a percentage degradation of attacking missile reliability. Several different physical interpretations of this parameter might be made, such as ignorance on the part of the attacker of defense capabilities, or degraded preferential capabilities by the blinding of radars.

A more desirable, but methodologically more difficult, extension is to allow varying degrees of preferential defense capability. Here some work is needed to find appropriate parameters to describe this capability which not only are simple, but which are amenable to physical interpretation. Examples of types of parameters are degradation of interceptor reliability, limits on the numbers of interceptors which could be allocated to a single target, and use of a fraction of the desired interceptor allocation. It is not clear, moreover, whether expected value calculations can be used, or whether more complex methods will be needed to represent the stochastic variations appropriately.

One tactic, which can readily be analyzed in cases where area defense does not cover the entire country, is to force the attacker to bypass the defense completely. Such a tactic might represent small attack situations where the attacker cannot suffer the degradation necessary to penetrate the area defense.

It is also possible to allow the attacker to attack as if the defense is not present, and then account for the defense action by degrading the no-defense attack afterwards. This is most readily done by simply eliminating some of the attacker weapons from the attack against an undefended target. While such calculations are readily performed, the rational for selecting the weapons to be eliminated, especially in an expected value calculation, is often difficult to relate to any physical model.

The model implemented in MEVUNS assumes a fixed defense deployment. Another direction for extension is to vary the defense deployments. In order to do so (in addition to including the appropriate defense optimization algorithms), it is probably desirable to include a more specific representation of a particular system to allow a better means of controlling the variation of the description of the defensive capability.

VI

ATTACK GENERATOR

A. INTRODUCTION

As a part of the MEVUNS study a computer program was developed to generate nationwide attacks for optimizing the destruction from the blast effects of nuclear weapons. This attack generator has the capability to structure attacks against economic resources, as well as against population. In order to assess the influence of a ballistic missile defense, the active defense model described in Chapter V is linked to the attack generator. The program is written to provide attacks which can be used as input to the ANCET damage assessment model, described in Chapter VII.

The attack generator calculates the effects of nuclear weapons upon target areas by the "square root damage law". The target descriptors required for the square root damage law are a target value and a measure of the size of the target. The standard deviation of the population distribution is the measure of target size used here. The required economic data consist of total value added (TVA) for each of 79 economic sectors for each county of the United States. It is assumed that the economic capacity for each sector in a county is collocated with the urban population and is equally vulnerable. These assumptions were not made to simplify the computer programming, but rather because of the lack of appropriate data.

The methodology employed is basically a one-sided Lagrange multiplier optimization of damage. A "cell" structure of the target system is assumed; that is, the damage on one target is independent of that on others, since only blast effects are considered.

The input data for the attack generator are summarized in Table 4.

Table 4

ATTACK GENERATOR INPUTS

A. Weapon Descriptors

1. Number of weapons
2. Yield
3. CEP
4. Overall reliability
5. Population MLOP
6. Height of burst (air or surface)

B. Population Descriptors for Counties and Cities (from GEONN)

1. Identification number
2. Name
3. Population
4. Geographic parameters (See Figure 3)

C. Economic Descriptors

1. Total value added (TVA)
2. TVA for each economic sector

D. Active Defense Descriptors (See Table 3)

E. Targeting Descriptors

1. Terminal defense avoidance (yes or no)
2. Area defense avoidance (yes or no)
3. Relative weight of population with respect to total TVA
4. Weight on individual economic sectors

B. TARGETING OBJECTIVES

To allow for various targeting strategies, a weighted payoff function for each node in the GEONN is calculated. The attack generator uses a vector of weights, $w = (w_0, w_1, \ldots, w_{79})$, where w_0 is the population weight and $w_1, w_2, \ldots, w_{79}$ are the weights for the economic sectors. Thus, an attack to optimize fatalities would have $w_0 = 1$ and $w_j = 0$ for $j = 1, 2, \ldots, 79$. To attack a particular economic sector, the user would specify all the weights to be zero except the weight corresponding to the chosen sector.

The user of the attack generator actually supplies a value of β , defined to be the weight of population relative to total economic weight. Thus, $\beta = 1$ implies equal importance for population and economic targets. A value of $\beta = 0$ instructs the attack generator to ignore population in the targeting. The relative weights for the economic sectors, w_j, $j = 1, 2, \ldots, 79$, are also supplied by the user. The weight w_0 is calculated by the computer program: $w_0 = \beta\, T/V$, where T is the total urban population and V is the total TVA.

It is assumed that each of the 79 economic sectors in a county has the same vulnerability as the urban population in that county. For some sectors, such an assumption is clearly false (for example, agriculture) while for others it may be reasonably correct. In most cases this assumption leads to an overestimate of damage to industry, since plants and equipment tend to be less vulnerable than people.

For each county being attacked, a weighted payoff function is used for optimizing the attack. Let P_i be the population in urban node i in a county with n urban nodes. Then $P = \sum_{i=1}^{n} P_i$ is the total urban population in the county. Let v_j be the TVA in economic sector j in the county. The payoff function for the county as a whole is then

$$w_0 P + \sum_{j=1}^{79} w_j v_j \quad .$$

The payoff function for urban node i in the county is

$$\frac{P_i}{P}\left[w_0 P + \sum_{j=1}^{79} w_j v_j\right] = w_0 P_i + \frac{P_i}{P}\sum_{j=1}^{79} w_j v_j \quad .$$

If f_i denotes the fraction of damage in urban node i, then $f_i P_i$ is the estimated number of blast fatalities in node i, and $\frac{f_i P_i}{P} v_j$ is the estimated amount of value added in sector j that is destroyed in node i.

Thus, the targeting mechanism does reflect county-by-county variations of economic capability. The output includes the nation-wide fraction of the TVA destroyed in each economic sector which can then be used as input to the economic model. Conversely, the economic model can provide indicators of the criticality of various sectors which can guide the selection of weighting factors for generating attacks designed to maximize economic scarcities.

Different values of industrial vulnerability, expressed by mean lethal overpressures, could be readily introduced into the model and would probably increase the validity of the calculations. As an example of the use of these data, the sector weighting factors might be modified to represent a desire to attack vulnerable sectors preferentially. If these vulnerabilities represent the estimated over-pressures needed to achieve 50 percent destruction of the physical facilities, a me sure of the loss of immediate production capacity, or possibly of inventories of finished goods, might be obtained. The economic model, as discussed in Chapter VIII, does not model immediate post-attack economic conditions. Between the attack time and the time when the economic model is applicable, a considerable amount of rehabilitation of damaged facilities could be accomplished. This fact could be reflected by raising the mean lethal overpressures.

The economic data on number of establishments in each county could be used to estimate the degree of collocation with urban population. For example, if a sector has a few large establishments located outside of cities, then an attack on this sector could be better described by allocating one weapon to each establishment and assuming that the sector is destroyed. In this case, the damage calculations in the attack generator should be modified to allow for this different method of destruction.

The present model does not include counterforce or other types of military targets. These types of targets could be added to the data base, allowing user-specified values to be estimated, and calculations to be made to maximize total value destroyed.

Several other types of relationships might be approached through variations of targeting objectives. For example, regional economic relations might be studied by defining economic regions and structuring attacks specifically against such regions to maximize disruption within the region. Economic sectors that do not lead to particularly serious bottlenecks on a nationwide basis might lead to serious complications within a region.

The question of the social vulnerability of the nation has been raised; however, no serious attempts to analyze the possible seriousness of social disruption have been made on a nationwide basis. Economic data, or data available on a county basis from the census such as numbers of households, age distribution, income characterization, etc., might provide the grist for an analysis which attempts to obtain a better understanding of societal vulnerability than now exists.

C. URBAN DAMAGE CALCULATIONS

The "square root damage law" can be derived by assuming:[1]

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1. See Galiano, Robert J. and Hugh Everett, III, Defense Models IV, Family of Damage Functions for Multiple-Weapon Attacks, Lambda Corp., Paper 6 (Arlington, Va., March 1967).

(a) Weapons are infinitesimal in size and can be represented by a weapon density, ω .

(b) The fraction of damage at any location is given by $1 - e^{-k\omega}$, where k is a scaling constant.

(c) The target value is circular normal in distribution.

(d) Weapons are optimally targeted to maximize value destroyed.

From these assumptions the square root damage law can be derived which gives the fraction of survivors, S , as

$$S = e^{-\sqrt{x}} (1 + \sqrt{x})$$

where x = KN with K a constant depending on the city, and N the number of weapons attacking the city.

Define β by

$$\beta = \frac{\pi R_L^2 \rho_d}{\bar{\pi} \sigma^2} ,$$

with R_L being the lethal radius of the weapons employed, ρ_d the non-reprogrammable reliability, and σ the standard deviation of the value distribution in the target area.

A study of the applicability of the square root damage law to a target area was done by comparing the square root damage law results with results of a weapon-by-weapon computer optimized laydown. Excellent agreement was found between the two methods.[2] The value of K is taken to be $\alpha\beta$, where α has been calculated for some cities by scaling the square root damage law to obtain

2. Schmidt, L.A., Jr., A Sensitivity Analysis of Urban Blast Fatality Calculations, IDA Paper P-762 (Arlington, Va., January 1971).

good agreement with the computer-optimized laydowns. A value of α of about 2 was found, varying by about 25 percent from target to target. The city size, the weapon lethal radius, the slope of the probability curve of kill as a function of distance, and the weapon delivery probability also induce variations in α, ranging from about 1.5 to about 3.5. The square root law appeared to fit the computer optimization results much better than would be expected from the assumptions in the theoretical derivation. The goodness of these fits, the simplifications of the analysis resulting from its use, and the wide prior usage of this formula all dictated its adoption here.

The major target characteristic that affects the value of α is target size. The square root damage law is not affected if the target value distribution is assumed to be elliptical normal rather than circular normal in shape. To better represent differences between targets, more descriptors (e.g., specific fits of the square root damage law to individual cities) are needed. Since these are not available in the present data base, only an average value of α is used in the attack generator. The user has the option of specifying $\alpha = 2.0$ or letting the program calculate α, taking account of weapon CEP and delivery probability.

It should be mentioned here that the damage assessment routine, ANCET (described in Chapter VII), uses the same target data base as the attack generator. Because ANCET also suffers from the inability to account properly for the population distribution in cities, errors of about 25 percent from city to city might be expected.

Other damage assessment systems use methods different from those outlined here to determine the value of K in the square root law. None of these methods are sufficiently sensitive to reflect the parametric variations that appear to dominate the city-to-city differences. Moreover, these other calculations of K are based upon different data bases than the one used here. For these reasons, the overall damage calculations produced here may well be different from those produced by other damage assessment schemes. An appreciable effort would be required to achieve adequate calibration between methodologies.[3] If comparability between different systems is desired, or a method of properly reflecting changes in a sensitivity analysis is needed, such efforts could be undertaken. If, however, the intent of such calculations is to predict the outcome of nuclear war, the unknowns of the threat or physical effects probably outweigh errors in the damage assessment methodology.

It is possible to apply the square root damage law assumptions to calculate injuries due to blast effects, as well as total casualties. Suppose that a constant K_c is used in the square root damage law corresponding to a large weapon radius which gives the area within which someone is either killed or injured. If it is assumed that the targeting doctrine, that is, to maximize fatalities, is unchanged, the resulting injuries are given as

$$I = e^{-\sqrt{x}} \frac{e^{-\alpha \sqrt{x}} - 1}{\alpha} + \sqrt{x} \quad ,$$

where

$$\alpha = \frac{K_c}{K} - 1 \quad .$$

This formula may be used to yield estimates of blast injuries to supplement the fatality calculations.

Well over a thousand nodes are defined in the data base by a single, isolated Standard Location Area (SLA). It is assumed that the SLA is sufficiently small so that a single weapon aimed at such a node will destroy it completely. This assumption considerably decreases the calculation time needed to estimate damage.

For other small nodes defined by more than one SLA, a single weapon aimed at the center would destroy most of the node. If R_L

3. Ibid.

is the weapon's lethal radius, and σ_c the standard deviation of the node, the fraction killed is

$$\frac{1}{\left(\frac{1.386\sigma_c^2}{R_L^2} + 1\right)} \quad .$$

If the surviving value is found to be smaller than the least payoff acceptable for a weapon, the formula is used to calculate the expected payoff instead of the square root damage law. This not only simplifies the calculations but provides a more accurate estimate than the square root damage law for only one weapon.

Large metropolitan clusters of population often spread across several county borders. By considering the population county-by-county, instead of simultaneously for an entire area, an error is committed. To investigate the possible size of this error, a severe exemplar case, the Washington metropolitan area, was chosen and analyzed with the square root damage law.[4] About 50 percent more weapons are needed than when the area is split into counties. This indicates that the county division of population tends to overestimate fatalities. While computing fatalities for an entire metropolitan area and then splitting results into counties is conceptually simple, the computer manipulations are somewhat involved and have not been implemented.

D. DESCRIPTION OF TARGETING ALGORITHMS

1. Introduction

This section describes the procedures used in generating a weapon laydown. The user supplies the size of the attack, descriptors of the weapons in the attack, the weights for population and economic sectors, and the characteristics of the active defense, if any. The

4. Schmidt, *op. cit.*

attack generator then gives an assignment of weapons to nodes in the GEONN and also estimates population and economic damage. Several output options are available.

2. <u>No-Active Defense Optimization</u>

Define

x_i = number of warheads targeted on node i ,
$i = 1, 2, \ldots, n$,
$f_i(x_i)$ = value destroyed in node i by x_i warheads,
m = total number of warheads to be targeted,
r = overall reliability of a warhead.

The damage function, $f_i(\cdot)$, is constructed for each node in the data base, taking cognizance of the weights on population and economic value added. Let V_i denote the value of node i .

If the node has only a single census tract, it is assumed that if one weapon arrives, all of the node value is destroyed, so that the expected value destroyed is the node value multiplied by the weapon's delivery probability. Since a number of nodes in the data base consist of only a single census tract, this provision can appreciably decrease computer running time. The damage function for this type of node is

$$f_i(x_i) = \begin{cases} 0 , & \text{if } x_i = 0 , \\ rV_i , & \text{if } x_i = 1, 2. \end{cases}$$

For nodes constructed from more than one census tract, a test is first made to determine whether the node is small enough so that a single weapon on its center can destroy most of the value. If so, the damage function is

$$f_i(x_i) = \begin{cases} 0 , & \text{if } x_i = 0 , \\ rg_iV_i , & \text{if } x_i = 1, 2, \ldots , \end{cases}$$

where g_i is the fraction of the node destroyed by one weapon.

For larger nodes, the expected value destroyed is calculated by using the square root damage law and V_i. In particular,

$$f_i(x_i) = V_i \left[1 - (1 + \sqrt{k_i x_i}\ ) e^{-\sqrt{k_i x_i}} \right] ,$$

where k_i is a vulnerability parameter defined in Section C.

The targeting problem is to choose an integer-valued vector $x = (x_1, x_2, \ldots, x_n)$ to

$$\text{maximize} \sum_{i=1}^{n} f_i(x_i)$$

subject to

$$\sum_{i=1}^{n} x_i \leq m \quad .$$

Since the functions $f_i(\cdot)$ are concave, nondecreasing, and pass through the origin, a straightforward multiplier method may be used to determine x .[5]

Tables giving the marginal return for weapons are constructed for each node. These tables are truncated by using a specified minimal marginal return. For example, with an attack against population only, a specified marginal return of 60,000 fatalities per weapon will limit the total number of one-megaton weapons on the United States to about 400. A return of 10,000 fatalities per weapon implies a maximum of about 3400 one-megaton weapons. The number of weapons implied by this marginal return must be at least as large as the desired number of weapons in an attack. To conserve

5. See McGill, J.T., Solution of Singly-Constrained Concave Allocation Problems, IDA Paper P-619 (Arlington, Va., January 1970) for discussion and proof of the details of the solution procedure.

computer running time, however, the marginal return value should be judiciously selected. The maximum number of weapons which can be handled in an attack scenario is limited by computer storage requirements. Currently, the limit is 4000 weapons.

The weapons are ordered in terms of decreasing payoff

$$P_i(x_i) \equiv f_i(x_i) - f_i(x_i - 1) \quad .$$

An optimal attack of m weapons is obtained by simply selecting the m weapons whose payoffs are highest.

3. Terminal Defense Optimization

If a terminal defense is specified, the procedures described below are added to those previously given. Each terminal defense battery is characterized by the identification number of the node it defends and its number of perfectly reliable interceptors, denoted by t_i for node i. Those nodes having a terminal defense are assigned a price. The price is the number of perfectly reliable interceptors divided by the reliability of an attacking warhead, t_i/r. The damage function for this case is $g_i(x_i)$, where

$$g_i(x_i) = \begin{cases} 0 & , \quad \text{if } x_i < t_i/r \quad , \\ f_i(x_i - t_i/r) & , \quad \text{if } x_i \geq t_i/r \quad . \end{cases}$$

Figure 6 provides an illustration of $g_i(x_i)$. As discussed and heuristically justified in Section B.2, this damage function is a function of an expected value and is not the expected value of damage.

The algorithm used to calculate an optimal attack against nodes, some of which are terminally defended, begins (as in the no-defense case) by constructing a weapons list arranged in decreasing order of payoff. For undefended nodes, the list is made as before. For nodes which are terminally defended, the number of weapons which maximizes average return is computed. Let s_i be the smallest such number and

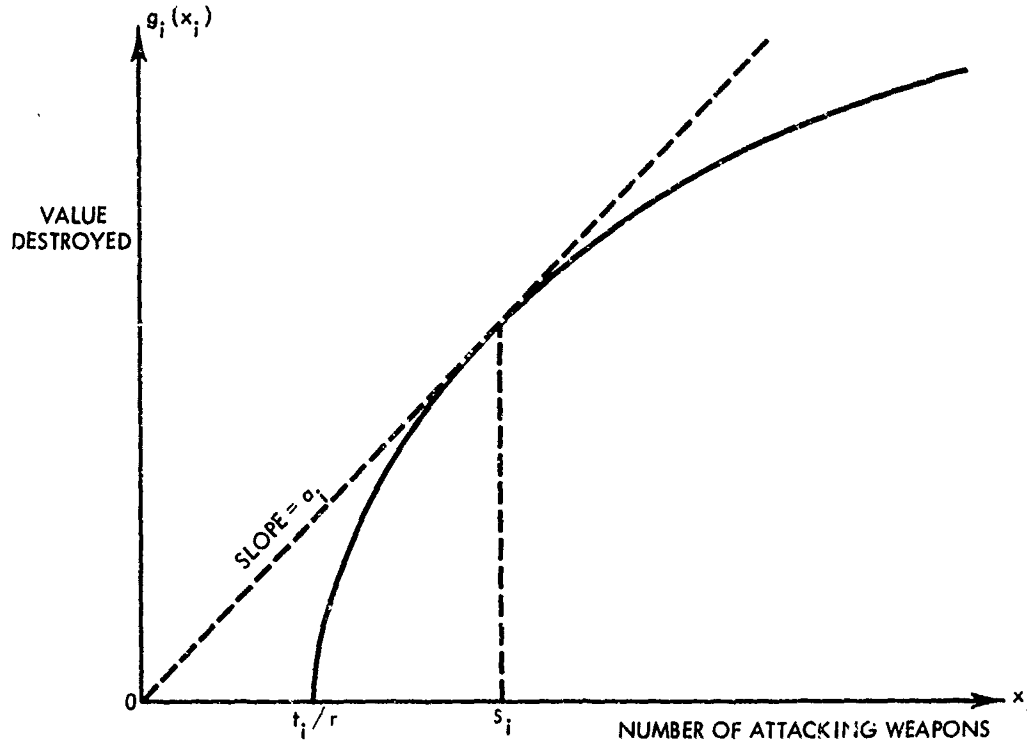

FIGURE 6. Illustration of the Function g_i

a_i the corresponding average return for node i . Then

$$a_i = \frac{g_i(s_i)}{s_i} \geq \frac{g_i(x_i)}{x_i} \quad , \quad \text{for } x_i = 1, 2, \ldots \quad .$$

The list of marginal returns is then constructed for terminally defended nodes as follows. For the first s_i weapons on the node, the weapon payoff is taken as a_i. For $x_i > s_i$, $P_i(x_i) = f_i(x_i - t_i/r) - f_i(x_i - 1 - t_i/r)$. The list is reordered by descending payoff. Since for terminally defended items the first s_i weapons have the same payoff, they will enter the list as a group. The first m weapons from the list comprise the attack. If the attack is such that the last weapon selected from the list is in a group of weapons which had been inserted at the maximal average payoff level, the attack is not optimal and the output listing so indicates.[6] In this case, the resultant total value destroyed is an upper bound on the optimal solution. A lower bound can be obtained by evaluating the damage function at the number of weapons allocated. For large attacks relative to total number of interceptors, the ratio of these two bounds approaches 1 .

4. Area Defense Optimization

The following data are input for each area defense site: the site latitude and longitude, the number of interceptors at each site, and descriptors used to calculate the area defense site footprint. A calculation is made to determine which counties an area defense site protects, i.e., the size of the area defense island. Provision is made to allow various types of area defense footprint calculations but at present only one type is implemented, namely, nonoverlapping circles of a radius which is specified for each site. In this calculation if a county is covered by more than one site, it is allocated to that site nearest to the county.

Let

d_j = number of warheads which can be destroyed by area defense interceptors in island j , $j = 1, 2, \ldots, \ell$,

y_{ij} = number of warheads targeted on node i in island j , $i = 1, 2, \ldots, r_j$,

z_j = number of warheads targeted at island j

$$= \sum_{i=1}^{n_j} y_{ij} \ ,$$

$p_j(z_j)$ = probability that any warhead targeted in island j penetrates the defense when z_j warheads are targeted against the island,

6. If weapon m had been entered at its marginal payoff, the attack is optimal. This fact follows from the main theorem in Everett, H., Generalized Lagrange Multiplier Method for Solving Problems of Optimum Allocation of Resources, Operations Research, Vol. 11, 1963, pp. 399-417.

$g_{ij}(\cdot)$ = damage function for node i in island j .

The penetration probability is a function of both the offense and defense, and is the same for each node within an island, and for each warhead entering the defense. That is, it is assumed that area defense interceptors are randomly allocated against interceptors. See Section C.1 for a discussion of the selection of this particular model. In particular, we shall use

$$p_j(z_j) = \left(1 - \frac{1}{z_j}\right)^{d_j} .$$

The targeting problem against an area defense is represented as one of finding nonnegative integer values of y_{ij} to

$$\text{maximize} \sum_{j=1}^{\ell} \sum_{i=1}^{n_j} p_j\left(\sum_{i=1}^{n_j} y_{ij}\right) g_{ij}(y_{ij})$$

subject to

$$\sum_{j=1}^{\ell} \sum_{i=1}^{n_j} y_{ij} \leq m .$$

The problem is solved in two parts. First, for each value of j find y_{ij} to

$$\text{maximize} \sum_{i=1}^{n_j} p_j(z_j) g_{ij}(y_{ij})$$

subject to

$$\sum_{i=1}^{n_j} y_{ij} \leq z_j$$

for given values of z_j. This problem is analogous to the one considered in the terminal defense case where the county is replaced by those nodes within an island. The function $g_{ij}(\cdot)$ depends implicitly upon the weapon reliability. For this problem, the weapon reliability is $p_j(\cdot)r$. Since, however, $g_{ij}(\cdot)$ is often only a weak function of $p_j(\cdot)r$, an approximation is made whereby only r is used for the weapon reliability.

For each z_j, suppose $h_j(z_j)$ is the value of the objective function for the solution of the above problem. The second step is to solve the problem:

$$\text{maximize} \sum_{j=1}^{\ell} h_j(z_j)$$

subject to

$$\sum_{j=1}^{\ell} z_j \leq m \quad .$$

This problem must be solved by different methods, since $h_j(\cdot)$ is not necessarily concave. Call $a_j(z_j)$ the average return ($= h_j(z_j)/z_j$) in island j when expending z_j weapons. Let s_j be the value of z_j which maximizes a_j, i.e., $a_j(s_j) \geq a_j(z_j)$, $z_j = 1, 2, \ldots$.

We call $h_j(z_j)$ and S-shaped function if h_j has increasing average returns for $z_j \leq s_j$ and is concave for $z_j \geq s_j$; in other words if:

(1) $$a_j(z_j) \leq a_j(z_j + x_j) \quad ,$$

for $z_j = 1, 2, \ldots, s_j$ and $x_j = 1, 2, \ldots, s_j - z_j$, and

(2) $$h_j(z_j + 2) - 2h_j(z_j + 1) + h_j(z_j) \leq 0$$

where $z_j = s_j, s_j + 1, \ldots$.

The method implemented for solving this latter problem is exact for S-shaped functions, and, heuristically, appears almost exact for almost-S-shaped functions. If the functions $g_{ij}(\cdot)$ within an island are the same, then $h_j(\cdot)$ is S-shaped. If a few of the $g_{ij}(\cdot)$ decrease rapidly, and the rest decrease slowly, $h_j(\cdot)$ may not be S-shaped. It is expected that in most real cases $h_j(\cdot)$ is S-shaped, and a check has been put in the program to indicate if the $h_j(\cdot)$ are not. Further additions to the algorithm are needed to handle serious non-S-shaped functions.

Call ℓ dimensional vector $z^*(A) = (z_1^*(A), z_2^*(A), \ldots, z_\ell^*(A))$ a solution to the problem:

$$\text{maximize} \sum_{j=1}^{\ell} h_j(z_j)$$

subject to

$$\sum_{j=1}^{\ell} z_j \leq A$$

and let

$$H(A) = \sum_{j=1}^{\ell} h_j(z_j^*) \quad .$$

Call $\{\overline{A}\}$ the set of all values of A for which the solution to the above problem can be obtained by the method of Lagrange multipliers.[7] If $m \in \{\overline{A}\}$ then the Lagrange solution is optimal for $A = m$. The set $\{\overline{A}\}$ is readily constructed by computing marginal payoffs $P(z_j) = h_j(z_j) - h_j(z_j - 1)$, ordering these payoffs, successively increasing values of the Lagrange multiplier λ, and dropping weapons from the list when the marginal payoff becomes less than λ. If at any time z_j becomes less than s_j for any island, then

7. See Everett, op. cit. In Everett's terminology, this is the set of all Lagrange-accessible points.

z_j is set = 0. Thus this solution will have no values of z_j such that $0 < z_j < s_j$. This sudden decrease in resources used causes certain resource levels not to be included in the method of solution.

Suppose $m \notin \bar{A}$. Then call A_L the largest element of $\bar{A}$ less than m, and let b assume values 0, 1, 2, ..., $m - A_L$. Call all islands where $z_j^*(m - b)$ islands of Class I, and the others of Class II. Call p(b) the folution of the problem:

$$\text{maximize} \sum_{j \epsilon II} h_j(z_j)$$

subject to

$$\sum_{j \epsilon II} z_j \leq b \quad .$$

Find b* that maximizes

$$[p(b) + H(m - b)] \quad .$$

Since p(b) and H(m - b) are both optimal solutions to their respective problems, and since the overall problem is separable, p(b*) + H(m - b*) is an optimal solution to the overall problem.

It remains to describe the procedure for finding p(b). This procedure relies on the fact that since all $h_j(z_j)$ are assumed S-shaped, every member of Class II has increasing average returns as z_j increases to s_j. Only values of $z_j \leq s_j$ are of interest here, since values of $z_j > s_j$ are in the concave region of $h_j(z_j)$ and thus are Lagrange accessible so they would be included in Class I solutions. To start the procedure set a variable c = 0, b = 1, Class II′ = Class II, Class II′′ = null set, and c = 0. Class II′ will contain those islands in Class II which have fewer than s_j weapons allocated, whereas Class II′′ will have at least s_j weapons.

Set d = b - c. Find j* to

$$\underset{j \epsilon II}{\text{maximize}} \; a_j(d) = \frac{h_j(d)}{d} \quad ,$$

and put j* in Class II′. The function p(b) is given by

$$h_j(s_j) = h_{j*}(d) \quad , \qquad j \in II' \quad .$$

If at any time $z_{j*} = s_{j*}$, then the area defense island denoted by j* is transferred from Class II′ to Class II′′ , and c is replaced by $c + s_{j*}$. Moreover, if $a_{j*}(z_j)$ is greater than any $a_j(s_j)$ for j in Class II′′ , these elements are moved back to Class II′ and c is replaced by $c - s_j$.

To obtain p(b) for all values of b , b is incremented by one, a new value of d is calculated, and the process is repeated.[8]

E. PREPARATION OF ANCET DAMAGE ASSESSMENT INPUT

The output of the attack generator is a listing of the nodes in the GEONN, accompanied by the number of weapons which are targeted on each node. To use the ANCET damage assessment routine, desired ground zeroes (DGZs) must be specified for each weapon. A procedure for accomplishing this is described below. The ANCET calculations limit the number of weapons contributing blast effects to five per node. The attack generator may yield targeting patterns with more than five weapons per node.

If there are more than five weapons on a node, say n , then a routine is used to convert them to five weapons with equivalent total lethal area. As stated in Section C, the fraction of survivors in a node is estimated by the square root damage law to be

$$(1 + \sqrt{kn}\,)\, e^{-\sqrt{kn}} \quad ,$$

where n is the number of weapons and where

---

8. The proof of the optimality of this algorithm is contained in a forthcoming paper: Schmidt, L.A., Jr., An Optimal Algorithm for a Class of Separable Non-Convex Programs, IDA Paper P-869 (Arlington, Va.), draft.

$$k = \alpha \frac{R_L^2 \rho_d}{\sigma^2} \quad .$$

The parameter, R_L, the weapon lethal radius for each of the n weapons, reflects the yield of the individual weapons. Fewer weapons with a larger lethal radius, R_L' yield the same fraction of survivors.

In particular, if $n > 5$, then the following holds:

$$\frac{\alpha R_L^2 \rho_d}{\sigma^2} n = \frac{\alpha (R_L')^2 \rho_d}{\sigma^2} 5 \quad .$$

Solving for R_L' gives

$$R_L' = R_L \sqrt{\frac{n}{5}} \quad .$$

Using the yield-to-radius scaling law, the individual yield of the five new weapons, Y', as a function of the yield of the n original weapons, Y, is

$$Y' = Y \left(\frac{R_L'}{R_L} \right)^3 \quad .$$

This method for conversion to five weapons yields[9] indicates that five weapons of a given yield with a total lethal area equal to that of a number of smaller weapons can result in errors of 25 percent. The variation is in the direction of making the larger weapons less efficient. In heavy attacks on a city, the error in number of survivors will be less.

9. Schmidt, *op. cit.*

The procedure for generating DGZs for five or fewer weapons is now described. A brief summary of the salient features of ANCET is given.

The nodes considered in ANCET have population distributions of three types: (1) elliptical normal, (2) uniform on a circle, and (3) uniform on a ring. The elliptical normal node can be completely characterized by its center point (given by a latitude and longitude), the standard deviations along its semi-major and semi-minor axis, and its population. For the analysis below it is convenient to define a coordinate axis such that the center of the node is at (0,0) and the semi-major axis corresponds with the x-axis. Define

σ_1 = standard deviation along the semi-major axis,

σ_2 = standard deviation along the semi-minor axis,

P = constant related to population.

A weapon is characterized by its CEP as well as its DGZ. It is assumed that all weapons have the same CEP, denoted r. For computing casualties, ANCET uses a casualty function of the form

$$a_1 e^{-c_1 R^2} + a_2 e^{-c_2 R^2} ,$$

where R is the distance from the actual ground zero, and where a_1, c_1, a_2, and c_2 are parameters with $a_1 + a_2 = 1$. For the calculations given below, the casualty function is approximated by the normal curve e^{-cR^2}, where c is related to a_1, c_1, a_2, and c_2 by $c = a_1 c_1 + a_2 c_2$. This approximation seems to be sufficient in that the casualty functions now used in ANCET have a shape close to the normal curve. In addition to the above notation also define $M = 1 + 2cr^2$ and $L = c/M$.

Hunter[10] gives the basic analytic formulas for the calculation of casualties. His results are used in the derivations given below. Let A, appropriately subscripted, denote the expected number of casualties. For instance, A_2 denotes these effects for weapon 2, A_{13} for weapons 1 and 3 jointly, and so on.

Since ANCET can calculate the blast effects on a node for five or fewer weapons, DGZs are derived for each of five cases--one through five weapons on a node. The DGZs given below do not necessarily maximize expected casualties, but do give lower bounds on their optimal laydown. Figure 7 shows the pattern assumed for the laydowns in each case.

One Weapon. In this case the optimal DGZ is at the center of the ellipse, (0,0).

Two Weapons. Locate both weapons on the semi-major axis equidistant from the center of the ellipse. The weapon DGZs are (-x,0) and (x,0) with x to be found. The expected number of casualties is given by $A_1 + A_2 - A_{12}$. Define

$$\alpha_i = \frac{1}{2\sigma_i^2} + L$$

$$\beta_i = \frac{1}{2\sigma_i^2} + 2L \quad .$$

Then

$$A_1 = A_2 = \frac{P\pi}{M(\alpha_1\alpha_2)^{1/2}} \exp\left\{- \frac{Lx^2}{2\alpha_1\sigma_1^2}\right\}$$

$$A_{12} = \frac{P\pi}{M^2(\beta_1\beta_2)^{1/2}} \exp\left\{- 2Lx^2\right\} \quad .$$

10. Hunter, *op. cit.*

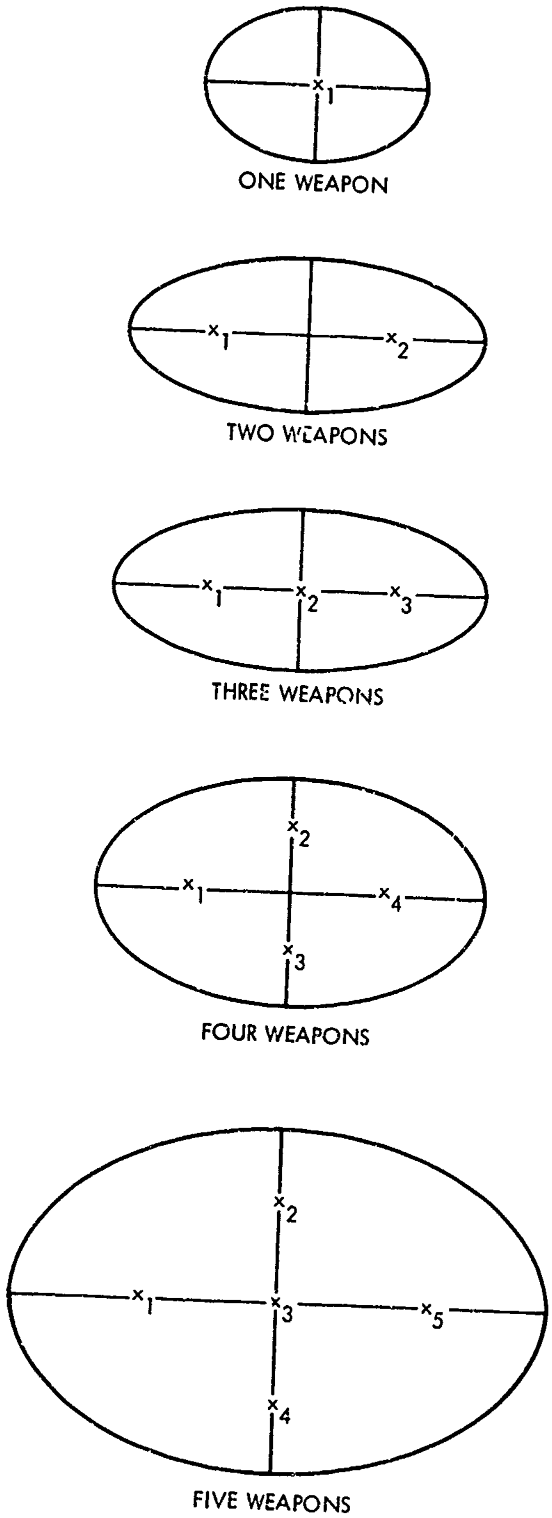

FIGURE 7. Weapon Laydown Patterns

Use of differential calculus gives x^* as the value of x maximizing $A_1 + A_2 - A_{12}$ where

$$x^* = \left\{ \frac{2\alpha_1\sigma_1^2}{L(1 + 4L\sigma_1^2)} \ln \left[\frac{2\alpha_1\sigma_1^2}{M} \left(\frac{\alpha_1\alpha_2}{\beta_1\beta_2} \right)^{1/2} \right] \right\}^{1/2} .$$

If the argument of the natural logarithm is less than one, then $x^* = 0$.

Three Weapons. All three weapons are located on the semi-major axis, one at the center and the remaining two equidistant from the center. To derive a closed-form analytical expression, only the joint effects from adjacent weapons are considered. The impact of this assumption is to underestimate the casualties that would be obtained from a consideration of the joint effects of all three weapons. The pertinent DGZs for weapons 2 and 3 are (0,0) and (x,0), with x to be determined. The expected number of casualties is $A_2 + A_3 - A_{23}$, where

$$A_2 = \frac{P\pi}{M(\alpha_1\alpha_2)^{1/2}}$$

$$A_3 = \frac{P\pi}{M(\alpha_1\alpha_2)^{1/2}} \exp \left\{ - \frac{Lx^2}{2\alpha_1\sigma_1^2} \right\}$$

$$A_{23} = \frac{P\pi}{M^2(\beta_1\beta_2)^{1/2}} \exp \left\{ - L \frac{\alpha_1}{\beta_1} x^2 \right\} .$$

Differential calculus yields x^* as a maximum of $A_2 + A_3 - A_{23}$ where

$$x^* = \left\{\frac{\alpha_1\beta_1}{L^3} \ln\left[\frac{2\alpha_1^2\sigma_1^2}{M\beta_1}\left(\frac{\alpha_1\alpha_2}{\beta_1\beta_2}\right)^{1/2}\right]\right\}^{1/2} .$$

The DGZ for weapon 1 is $(-x^*,0)$, If the argument of the natural logarithm is less than one, then $x^* = 0$.

Four Weapons. The joint effects of weapons 1 and 4 and, separately, weapons 2 and 3 are considered, ignoring the other weapon interactions. Under these conditions, the calculations made in the two-weapon case can be used. The resultant DGZs are given in Table 5.

Five Weapons. The pairwise joint effects of weapons 1, 2, 4, and 5 on weapon 3 are considered, all other joint effects being ignored. The calculations used in the three-weapon case are used to give the laydowns in Table 5.

Table 5 displays the laydowns for each of the five cases, where, for i = 1, 2,

$$s_i = \left\{\frac{2\alpha_i\sigma_i^2}{L(1 + 4L\sigma_i^2)} \ln\left[\frac{2\alpha_i\sigma_i^2}{M}\left(\frac{\alpha_1\alpha_2}{\beta_1\beta_2}\right)^{1/2}\right]\right\}^{1/2} ,$$

and

$$t_i = \left\{\frac{\alpha_i\beta_i}{L^3} \ln\left[\frac{2\alpha_i^2\sigma_i^2}{M\beta_i}\left(\frac{\alpha_1\alpha_2}{\beta_1\beta_2}\right)^{1/2}\right]\right\}^{1/2} .$$

For uniform population distributions the DGZs for five or fewer weapons are computed as described in the following discussion.

Again let the population center be at (0,0), and let (x_i,y_i) denote the coordinates of weapon i. Also let the outer radius of the circle be R and the inner radius (for a ring) be r.

Table 5

WEAPON DGZs

Weapon Number	x-coordinate	y-coordinate
One Weapon		
1	0	0
Two Weapons		
1	$-s_1$	0
2	s_1	0
Three Weapons		
1	$-t_1$	0
2	0	0
3	t_1	0
Four Weapons		
1	$-s_1$	0
2	0	s_2
3	0	$-s_2$
4	s_1	0
Five Weapons		
1	$-t_1$	0
2	0	t_2
3	0	0
4	0	$-t_2$
5	t_1	0

For the circular distribution r = 0. See Figure 8 for the geometrical interpretation. If there is a single weapon on a circular node, it should be placed at (0,0). Otherwise, for n weapons the coordinates are taken to be

$$(x_i, y_i) = \left(\frac{R + r}{2} \sin\left[\frac{(i - 1)}{n}\right], \frac{R + r}{2} \cos\left[\frac{(i - 1)}{n}\right]\right) .$$

This procedure will not necessarily maximize casualties, but does provide a reasonable targeting pattern.

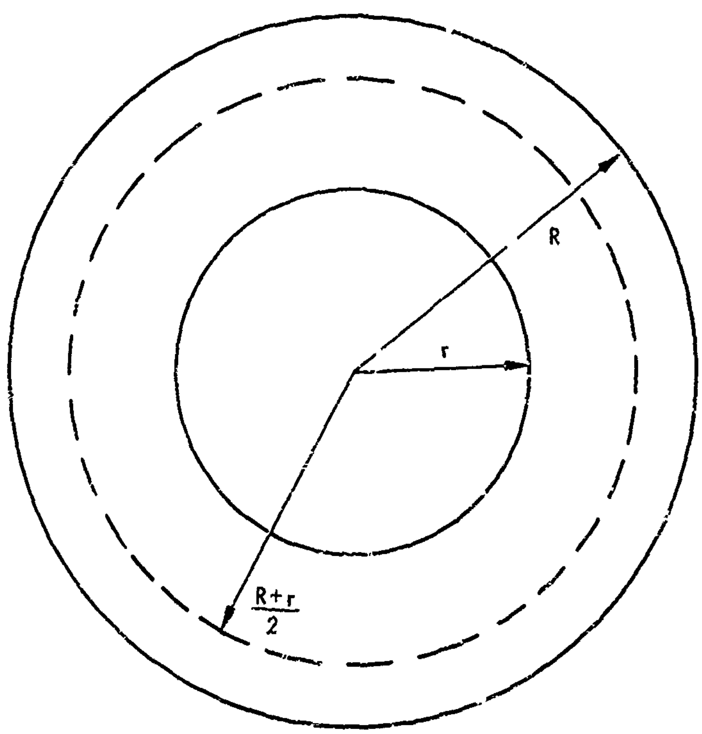

FIGURE 8. Circular Distribution

VII

DAMAGE ASSESSMENT

A. INTRODUCTION

The damage assessment procedures described in this chapter are based on the geographical location of population, industry, and the desired ground zeroes of the weapons in an attack. In contrast to the damage assessment method used in the attack generator, distances from the ground zeroes of the weapons to elements of the population are explicitly considered. This added detail is possible because the population distribution can be used to derive an analytical expression for expected population casualties. In addition, injuries, as well as fatalities, are estimated and fallout effects are considered.

The means of assessing nuclear effects against population is ANCET (Analytical Nuclear Casualty Estimation Technique). The ANCET computer program was developed at the Research Triangle Institute and has been extensively documented.[1] Modifications to the program have been made in the MEVUNS Study. These changes are reported in Volume II.

ANCET is an expected value model; it does not generate random numbers, but rather uses assumptions about the probability distributions of some of the uncertain parameters. This feature, plus its specially built input processor, allows extensive sensitivity analyses to be made.

There are two main components of the population damage assessment calculations: blast effects and fallout effects. Common to both the blast and fallout effects calculations are a population data base and an attack specification. The blast-effects model uses a

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1. Woodside, Mary B., ANCET Improvements, op. cit.

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set of parameterized casualty functions, and the fallout model requires a set of fallout protection parameters.

The population data base for ANCET has a high degree of geographic resolution. In particular, it requires a specification of population centers by latitude and longitude in the geographic area to be considered. The distribution of population within a given center must also be specified. The population data base used in MEVUNS describes these distributions as being elliptical normal, circular uniform, or uniform in a ring.[2] All urban nodes (from the GEONN) are approximated by an elliptical normal distribution. This assumption implies that the distribution on any vertical plane cutting the city is normal. Rural nodes are treated as having uniform distributions. If there are no urban nodes contained in a county, the rural node for that county is treated as a circle. If there are urban nodes, the rural population is uniformly spread over a circular ring, the center of which has an area equal to the area of all urban nodes within the county.

The description of the threat is also geographic specific. The desired ground zero (DGZ) for each weapon is required. Other parameters relating to its direct and fallout effects are also necessary.

The ANCET input processor (AIP)[3] takes the geographic location of the weapon DGZs and develops a weapons table for each population center. This table can include up to five weapons which contribute blast effects on the center and up to 25 weapons contributing fallout effects. The table is constructed by searching through the list of weapons in an attack and matching the weapons to the center. A weighting procedure is used to keep the number of weapons in the table within the stated bounds.[4]

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2. ANCET allows other distributions to be used. Since the MEVUNS data base does not have the information necessary for such additional distributions, the ANCET computer program has not been tested for any distribution other than the three named above.

3. Thornton, R.H., ANCET Input Processor, Final Report, Volume II, Research Triangle Institute (Research Triangle Park, N.C., October 1967).

4. Details of this procedure can be found in Thornton, ibid.

The AIP then gives ANCET the weapons list for each population center. ANCET computes the blast and fallout effects center by center and cumulates these across centers for summary statistics. Thus, procedures for assessing the effects of weapons on one city are the basis for the national totals.

Table 6 summarizes the inputs necessary for ANCET.

Table 6

ANCET INPUTS

A. Attack Inputs (for each weapon)
   1. Latitude and longitude of designated ground zero
   2. CEP
   3. Yield
   4. Height of burst (air or surface)
   5. Time of detonation
   6. Fission-fusion ratio
   7. Wind direction
   8. Wind velocity

B. Direct Effects Casualty Functions

C. Fallout Effects Parameters (for each node)
   1. Time at which fallout dose calculations stop
   2. Time of fallout cloud formation
   3. Radiation decay exponent
   4. Fallout cloud parameters
   5. Terrain attenuation factor
   6. Crosswind shear
   7. Lethal dose parameters
   8. Casualty dose parameters

D. Fallout Protection Parameters (for each node)

E. Population Descriptors for nodes (from GEONN, see Figure 3)

B. BLAST EFFECTS

Multiple weapon effects on a city are established by appropriately aggregating single weapon effects. The aggregation accounts for the overlapping of weapons so that the casualty estimate does not double-count fatalities or injuries. It is assumed that the effects of two or more weapons are not synergistic; that is, a survivor from one weapon's effect is assessed for casualty from a second weapon in a manner no different from the assessment from the first weapon. The following description of the blast effects estimation technique focuses first on the effect of a single weapon.

Casualties are derived from knowledge of three probability distributions:

1. Population distribution for the city,
2. Weapon CEP,
3. Casualty probabilities for a single weapon.

The analytical nature and consequently, the speed of computation in ANCET is due to the assumptions made about the form of these three distributions.

The population distribution provides a two-dimensional description of the population density. Both elliptical normal and uniform distributions are considered in ANCET. In the former case, an exact analytical expression for expected casualties can be derived. In the latter case an approximation is used.

The weapon CEP is assumed to be circular normal. The CEP value is the radius of the circle about the desired ground zero (DGZ) which would contain 50 percent of the actual ground zeroes (AGZs).

Finally, the casualty probability distribution is assumed to be a functional sum of two normal distributions. It gives the probability that an individual is a casualty as a function of his distance from the AGZ of a weapon. Several different casualty categories may be assessed. Each category necessitates a separate calculation. Figure 9 illustrates the shape of the casualty function. The analytical expression for the probability of casualty, P(R), as a

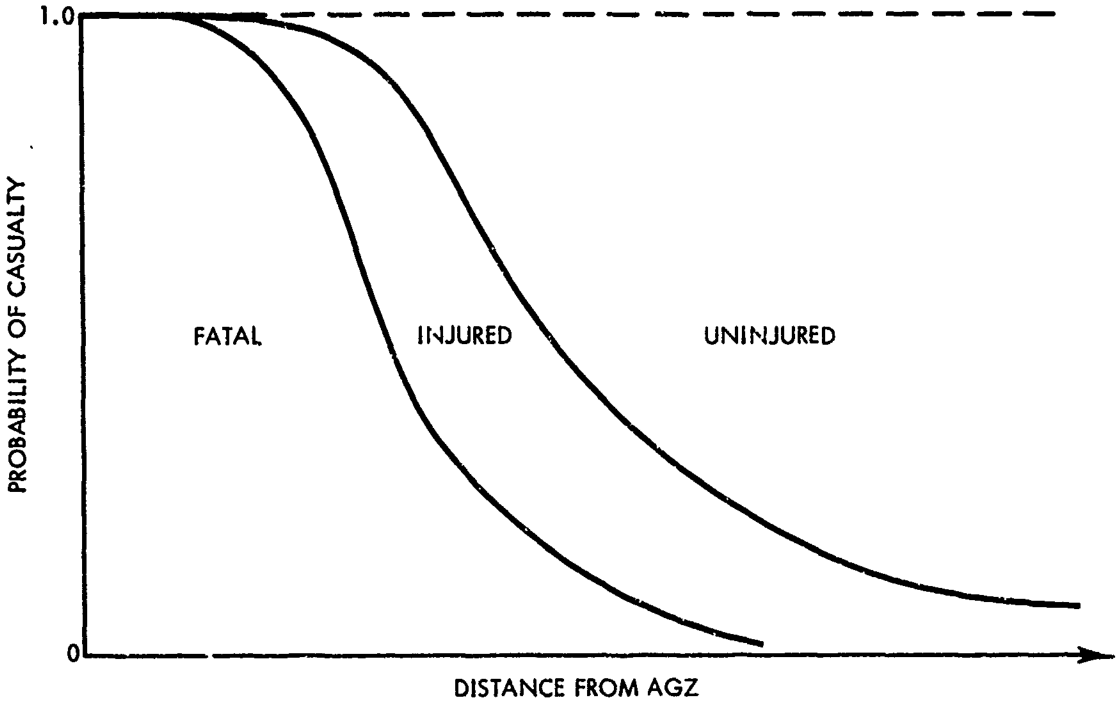

FIGURE 9. Casualty Probability Distribution

function of distance from the AGZ R, is

$$P(R) = a_1 e^{-c_1 R^2} + a_2 e^{-c_2 R^2} ,$$

where the parameters a_1, c_1, a_2, c_2 may vary with the casualty category, weapon yield, the height of burst, MLOP of population, and so on. It is required that $a_1 + a_2 \leq 1$, $0 \leq c_1$ and $0 \leq c_2$. The parameters must be estimated from a given casualty curve. Numerical values for the parameters used in MEVUNS are given in Volume II.

For a single weapon the three distributions are combined with conditional probability calculations to estimate the expected number of casualties at each point in the plane. The schematic in Figure 10 indicates the calculation for a point (x,y) from a single

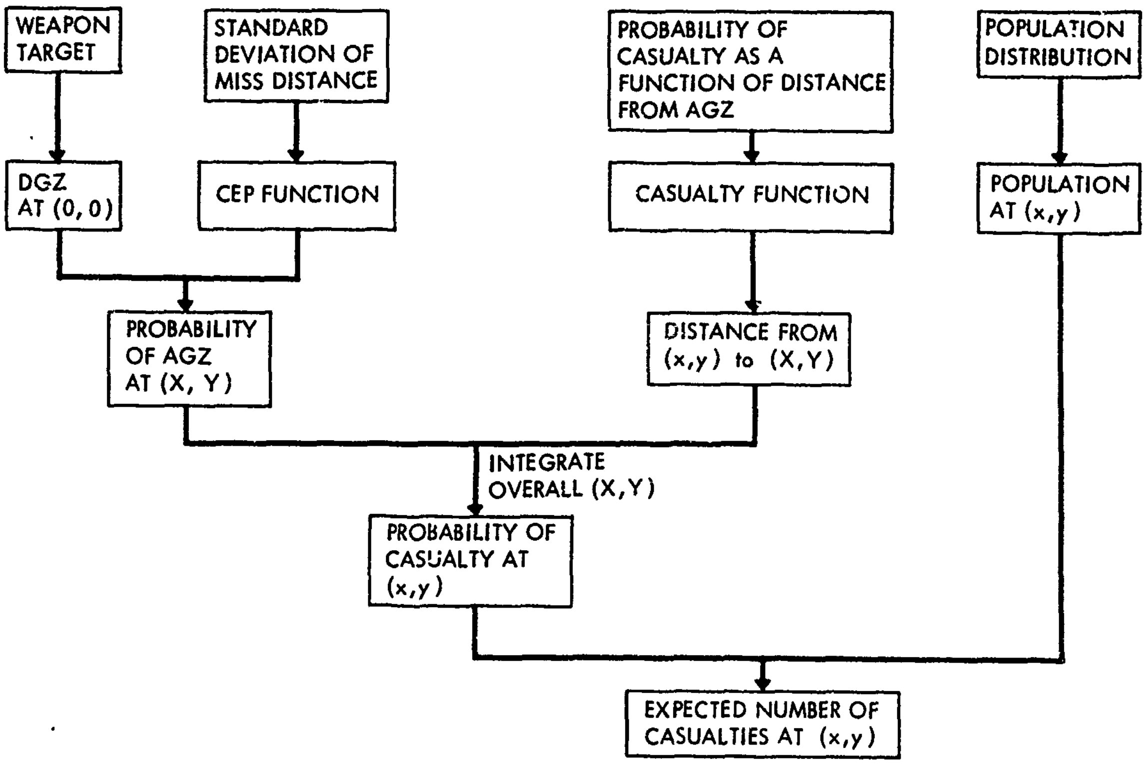

FIGURE 10: Casualty Estimation Procedure for a Single Weapon

weapon with DGZ of (0,0). Subsequent integration over the plane yields the total expected casualties for the center.[5]

Multiple weapons effects on a city are accounted for in the same way as a single weapon's effects. For two weapons, say, the joint probability of a casualty at (x,y) is derived. Then the total expected number of casualties from the two weapons, say c_{12}, is computed. Finally, the overlap of effects is taken into account to yield the casualty figures for the city: $c_1 + c_2 - c_{12}$, where $c_i (i = 1, 2)$ is the expected number of casualties from one weapon.

5. See Hunter, J.J., op. cit.

C. FALLOUT EFFECTS

The fallout model in ANCET calculates the total unshielded radiological dose from one or more weapons. Two alternative methods are provided in ANCET: the WSEG-10[6] model and the National Academy of Sciences' modified WSEG-10 model.[7] The user can specify his choice.

The calculation of total unshielded dose is made for a specific geographic point. The dosage depends upon the location of the AGZs of those weapons contributing fallout, their height of burst, their fission/fusion ratio, their cloud formation, and the wind effects. Because of the relatively more complicated phenomenology of fallout, the distribution of population is not used explicitly in an analytical expression for calculating casualties. Rather, dosage is computed for a maximum of five separate geographical locations (called sectors) in a population node.[8] Fallout effects are assessed separately for each sector and are cumulated for total city effects. Fallout protection is assumed to be the same within each sector of the city.

The shielded dose (SD) in a sector is computed from the unshielded dose (UD) by dividing by a protection factor (PF). That is, SD = UD/PF. The shielded dose is used to calculate the fatalities and injuries to the population. In particular, the probability distribution of fatalities is assumed to be normal in the shielded dose. The mean and standard deviation of this distribution are inputs to ANCET--one pair for fatalities and another pair for nonfatal injuries.

To illustrate, let μ and σ be the mean and standard deviation for the fatality distribution, and let $\Phi(\cdot)$ be the standardized

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6. Pugh, E.G. and R.J. Galiano, *op. cit.*

7. Polan, M., *op. cit.*

8. Cruze, A.M., D.B. Wilkerson, and M. B. Woodside, The ANCET Computer Program, Final Report, Volume III, Research Triangle Institute (Research Triangle Park, N.C., 15 March 1967). The means of splitting a city into sectors and determining the one geographic location for effects calculations is described on pages 103-105.

normal cumulative distribution function. Then the probability that an individual is a fatality from radiological exposure, at a protection level of PF, is

$$\Phi\left(\frac{UD/PF - \mu}{\sigma}\right) \quad .$$

The population of a city is allowed to be differentially protected from fallout. If f_j is the fraction of the population protected to the level PF_j, then the expected number of fatalities in a city with a population of P is

$$P \sum_j f_j \, \Phi\left(\frac{UD/PF_j - \mu}{\sigma}\right) \quad .$$

The same type of calculation is made for nonfatal injuries with different values for μ and σ.

D. COMBINED EFFECTS

Total fatalities and nonfatal injuries are computed for each population center. The calculations assume that the blast effects and the fallout effects are independent. Let d_1, d_2, and d_3 be the number of people killed, injured, and uninjured, respectively, by blast effects for the population center. Also, define f_1, f_2, and f_3 to be the fraction of people in each of these catagories due to fallout effects only. Then, the combined totals, c_1, c_2, and c_3 are:

$$c_1 = d_1 + f_1 (d_2 + d_3)$$
$$c_2 = d_2 (f_2 + f_3) + d_3 f_2$$
$$c_3 = d_3 f_3 \quad .$$

One of the modifications made to ANCET during the study was a correction in the combined effects calculations. Previously, that part of the population injured by blast effects and uninjured by fallout was being cumulated in the combined uninjured totals. A modification in the output of combined effects was also incorporated. Distinction is now made between that part of the population which is injured by both blast and fallout effects and that injured by only one of the effects. Thus, subject to availability of the appropriate data, the synergistic effects of injuries from both blast effects and fallout effects can be treated outside of the model.

E. ECONOMIC DAMAGE

The economic recovery model requires estimates of damage to the capital stock in each of the 79 economic input-output sectors. It would be desirable to assess economic damage in terms of the actual location and distribution of capital stocks in each economic sector in the same way that damage is assessed against population. However, the available industrial data base contains only the value added for each of the 79 economic sectors by US county. Since the geographic area of most US counties is larger than the lethal radius of the nuclear weapons, direct assessment on industries cannot presently be accomplished. Thus, a means of extrapolating from population damage is used.

The assumptions implicit in the procedure are:

1. Economic value added is proportional to capital stock. Thus, a 50 percent post-attack reduction in value added in a sector is assumed to imply a 50 percent destruction of capital stock in that sector.

2. The present reduction in value added for a sector in a given county is the same as the percent of the urban population in that county killed by blast effects. If industry is collocated with the urban population, capital stock damage should be proportional to urban population effects.[9] The calculation for industrial damage can be based on an MLOP (mean lethal overpressure) different from

9. N. FitzSimons of the DCPA analyzed the collocation of people and industry and found that, indeed (with a few exceptions), they are collocated.

that for population. Thus, for instance, industrial capital stock might have a MLOP of 10, while population has a lower average MLOP.

3. Each of the sector's capital stocks are equally "hard"; that is, the MLOP is the same across sectors. This assumption can be circumvented by using different population MLOPs for the industrial calculation. However, this process would require (1) knowledge of MLOPs by sector, and (2) an increase in computation time.

Since there may be more than one urban population node in a county, the percent of total blast fatalities in the county must be calculated. This percent is then applied on a sector-by-sector basis to obtain the value added that has been destroyed.

Let n be the number of counties in the United States, and let

i index counties ($i = 1, 2, \ldots, n$),

j index urban notes within the county ($j = 1, 2, \ldots, \ell_i$),

k index economic sectors ($k = 1, 2, \ldots, 79$).

Further, define

P_{ij} = population in node j contained in county i,

f_{ij} = blast fatalities in node j contained in county i,

v_{ik} = economic value added in sector k in count i,

T_k = nationwide total of economic value added destroyed in sector k.

Then

$$T_k = \sum_{i=1}^{n} v_{ik} \left(\sum_{j=1}^{\ell_i} f_{ij} \Bigg/ \sum_{j=1}^{\ell_i} P_{ij} \right)$$

for $k = 1, 2, \ldots, 79$.

VIII

ECONOMIC MODEL

A. INTRODUCTION

The General Economic Model (GEM) is designed to exhibit the sensitivity of the economy to nuclear attack, by exhibiting aggregate levels of economic activity, identifying industries in which bottlenecks will occur in a post-attack environment, and estimating the rate at which the economy might be expected to recover from an attack.

The motivation for the model is to expand the criteria usually used for evaluating civil defense programs (those based on the amount of population damage) to include post-attack economic conditions. Post-attack standard of living, as reflected in GNP (Gross National Product) per capita, can be considered as one measure of overall economic performance. Factors relevant to GNP per capita are the surviving labor force, the existence of specific kinds of shortages, the proper functioning of the distribution networks, the proper functioning of governmental agencies, and the availability of sufficient amounts of materials that must be imported. It is not possible within the context of GEM to handle all of these problems. Specifically, GEM assumes that the networks and the governmental agencies are capable of functioning in a reasonable fashion and that necessary imports are available.

In order to look at the behavior of the economy in a post-attack environment, it is necessary to make some assumptions about how consumers and producers will behave. In the absence of any better information, the model assumes that behavior patterns will be the same as they were in the pre-attack environment. In extreme ranges of behavior, it has been necessary to modify this assumption. These modifications will be explained in the body of the paper.

A further assumption has been made in the damage assessment procedure. Specifically, if a certain percentage of the productive capacity in an industry is destroyed, it is assumed this distribution will not have disruptive effects on the organizational efficiency of that industry. Thus, it is possible to use the same production functions coefficients in the post-attack period as in the pre-attack period. The overall implication of the assumptions made in GEM is that a nuclear attack will remove some of the capacity in each sector and part of the population. The attack is assumed to have no further effect on the economy.

The model consists of two parts. The first is the supply sector. Using a combination of an input-output matrix and constant elasticity of substitution production functions, the supply side determines labor allocation, feasibility of producing final demands, factor prices, and prices of final products. The second part is a series of equations that determine final demand by consumers, demand for investment goods, demand for inventories, federal government expenditures, state and local government expenditures, and export demand. An equilibrium mechanism equates total final demand with supply and adjusts the overall average wage rate so that the available labor supply is utilized. The linkage between periods is provided by adjusting capital stocks and updating the values of lagged variables. Except for these changes, the structure of the model is invariant over time.

The discussion of the model begins with a description of its broad properties. Next, there is a detailed discussion of the various components of the model. Finally, there is a description of the data sources for the model and of the imputation techniques that were used when data were not available.

B. OVERVIEW

The GEM is a general equilibrium model which considers only real, as opposed to monetary, phenomena. The basic structure of the model consists of 87 industries as given in Table 7. The first 82 have both a supply and a demand equation. The remaining 5 sectors are

Table 7

ECONOMIC SECTORS

1 Livestock and Livestock Products
2 Other Agricultural Products
3 Forestry and Fishery Products
4 Agriculture, Forestry, and Fishery Service
5 Iron and Ferroalloy Ores Mining
6 Nonferrous Metal Ores Mining
7 Coal Mining
8 Crude Petroleum and Natural Gas
9 Stone and Clay Mining and Quarrying
10 Chemical and Fertilizer Mineral Mining
11 New Construction
12 Maintenance and Repair Construction
13 Ordnance and Accessories
14 Food and Kindred Products
15 Tobacco Manufactures
16 Broad and Narrow Fabrics - Yarn Mills
17 Miscellaneous Textile Goods and Floor Covering
18 Apparel
19 Miscellaneous Fabricated Textile Products
20 Lumber and Wood Products, Except Containers
21 Wooden Containers
22 Household Furniture
23 Other Furniture and Fixtures
24 Paper and Allied Products, Except Containers
25 Paperboard Containers and Boxes
26 Printing and Publishing
27 Chemicals and Selected Chemical Products
28 Plastics and Synthetic Materials
29 Drugs, Cleaning, and Toilet Preparations
30 Paints and Allied Products
31 Petroleum Refining and Related Industries
32 Rubber and Miscellaneous Plastics Products
33 Leather Tanning and Industrial Leather
34 Footwear and Other Leather Products
35 Glass and Glass Products
36 Stone and Clay Products
37 Primary Iron and Steel Manufacturing
38 Primary Nonferrous Metals Manufactures
39 Metal Containers
40 Heating, Plumbing, Fabricated Structural Metal
41 Screw Machine Products
42 Other Fabricated Metal Products
43 Engines and Turbines
44 Farm Machinery and Equipment
45 Construction, Mining, Oil Field Machinery
46 Materials Handling Machinery and Equipment
47 Metalworking Machinery and Equipment
48 Special Industry Machinery and Equipment
49 General Industrial Machinery and Equipment
50 Machine Shop Products
51 Office, Computing, and Accounting Machinery
52 Service Industry Machines
53 Electric Transmission and Distribution Equipment
54 Household Appliances
55 Electric Lighting and Wiring Equipment
56 Radio, Telephone, and Communications Equipment
57 Electronic Components and Accessories
58 Miscellaneous Electrical Machinery, Equipment
59 Motor Vehicles and Equipment
60 Aircraft and Parts
61 Other Transportation Equipment
62 Professional, Scientific Instruments
63 Optical, Ophthalmic, Photographic Equipment
64 Miscellaneous Manufacturing
65 Transportation and Warehousing
66 Communications, Except Radio and T.V.
67 Radio and T.V. Broadcasting
68 Electric, Gas, Water, and Sanitary Service
69 Wholesale and Retail Trade
70 Finance and Insurance
71 Real Estate and Rental
72 Hotels, Personal, and Repair Service, Ex.Auto.
73 Business Services
74 Research and Development
75 Automobile Repair and Services
76 Amusements
77 Medical, Educational Service, Nonprofit Org.
78 Federal Government Enterprises
79 State and Local Government Enterprises
80 Gross Imports of Goods and Services
81 Business Travel, Entertainment, Gifts
82 Office Supplies
83 Scrap, Used, and Secondhand Goods
84 Government Industry
85 Rest-of-the-World Industry
86 Household Industry
87 Inventory Valuation Adjustment

discussed in Section H.12. The demand for the output of each industry is, in general, determined by the price of the commodity, the prices of other commodities, and past consumption patterns. The supply in each industry is determined by the available capital stock in the industry, the availability of the necessary inputs from other industries, and the total available stock of labor.

In addition to markets for commodities, there are markets for labor and capital. The aggregate supply of labor is fixed and a price of labor is found which will just exhaust the labor supply. Relative wages across industries are held fixed during this adjustment process. Only the average wage rate is varied. The source of funds is assumed to be unlimited in the capital market. The optimum stock of capital is determined for the prevailing price. Then the demand for capital goods is set equal to a fraction of the difference between desired and actual capital stocks. Finally, these demands are broken down into the amounts each industry must supply to meet investment demand. After a solution for the various markets is found, the capital stocks are revised to reflect depreciation and investment, and the values of lagged variables used in various behavioral equations are updated. This procedure is repeated for each period.

In the period immediately before the attack, the model assumes that equilibrium occurs in the labor and capital markets as well as in the product markets. In the post-attack period, equilibrium will occur in the product market if remaining capital stocks are sufficiently large. If the capital stocks are not large enough, equilibrium will not occur in the product market. In general, however, in the post-attack period, equilibrium will not occur in either the labor or capital markets, i.e., neither labor nor capital will receive its respective marginal products.

Figure 11 is a flow chart for the model. The beginning steps assess the effects of an attack on the economy and initialize the many parameters used in the model. Then the status of the economy in the pre-attack period is computed. The values determined here

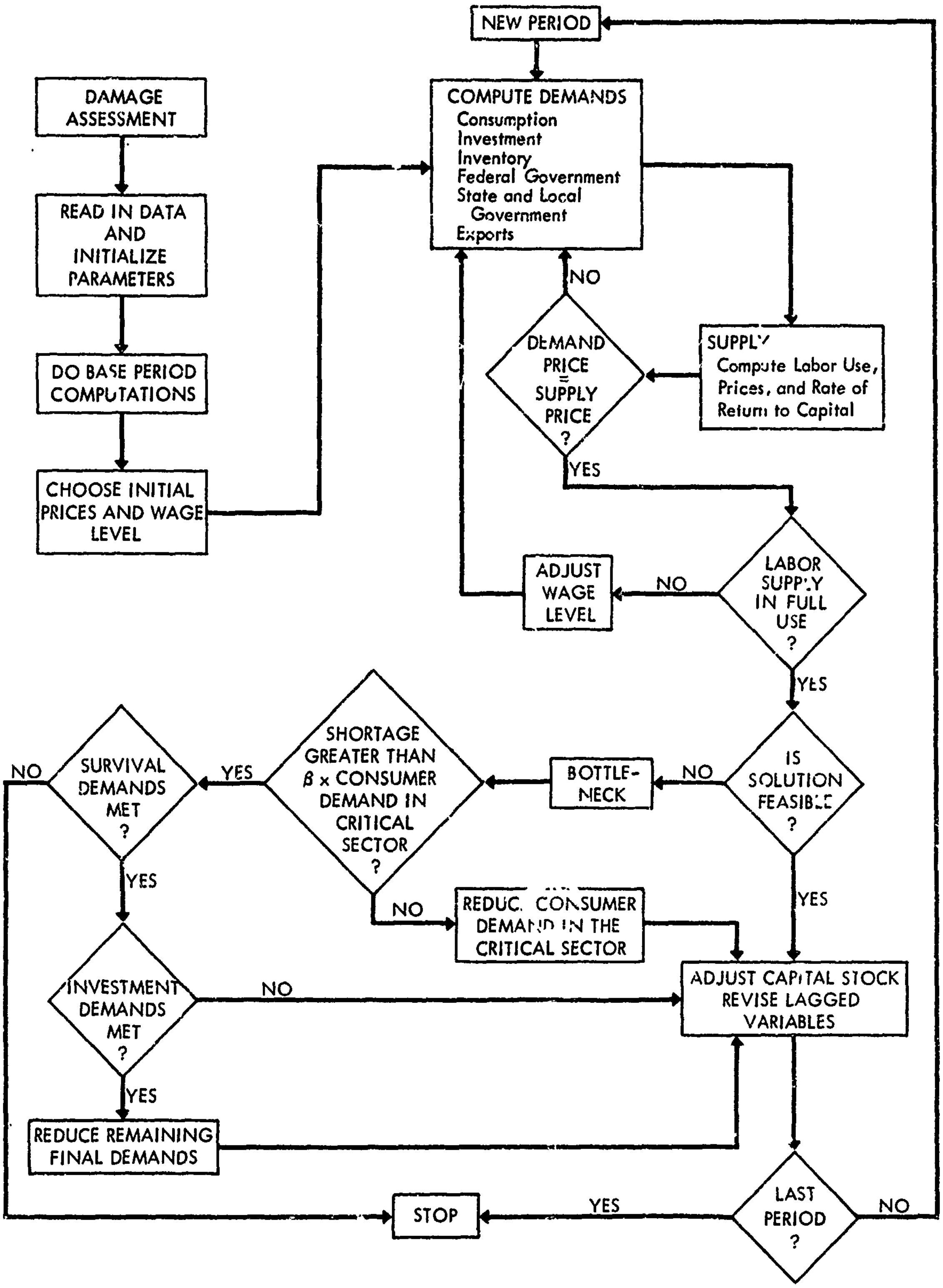

FIGURE 11. GEM Flow Chart

are used to initialize prices and wages in the solution procedure in the immediate post-attack period. These prices and wages are used to determine initial demands. As Figure 11 indicates, these demands are broken down into six components. A brief explanation of how each demand is computed is given below.

Consumption demands are determined by a set of equations taken from work done by Houthakker and Taylor.[1] This work assumes that, for durable goods, consumers have in mind a certain desired stock of goods and for nondurable goods they have formed consumption habits that remain stable. On this basis, a behavioral equation is derived that takes into account current price of the commodity, current level of aggregate consumption, past price of the commodity, past level of consumption of the commodity, and past level of aggregate consumption.[2]

Aggregate investment is determined by a rather complex procedure in the model, but in essence, it can be described in a fairly straightforward way. Given current demand levels, and assuming that these demand levels will continue unchanged, information about the current cost of capital and about how long it takes to complete a capital investment, individual firms within an industry can determine what their desired levels of capital stock will be at a date sufficiently far into the future that a decision to invest now can change the capital stock available at that future date. A firm adjusts its capital stock by a fraction of the difference between actual and desired capital stock. The actual value of this fraction for each sector can be specified by the user. The resulting expenditures

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1. See Houthakker, H.S., and Lester D. Taylor, Consumer Demand in the United States, Analysis and Projections, Harvard University Press (Cambridge, Mass., 1966). For a precise description of the equations used in this model, see Dolins, Lynn P., An Interindustry Projection to 1985 of Consumer Demand and Stocks of Consumer Durables, IDA Paper P-578 (Arlington, Va., 1969).

2. For a precise derivation of the equation, see Houthakker and Taylor, op. cit., pages 5-29. A somewhat expanded explanation is given in Section D below.

for capital goods are assumed to be spread evenly over the time it takes to complete the capital project. Summing up current expenditures for all the projects currently underway within an industry gives total current investment expenditures within that industry. Breaking these expenditures down into the industries from which the capital goods come will yield the total investment demand faced by each industry.

Inventory demand is computed as a user-determined percentage of the difference between desired and actual inventories. Desired inventories are determined by the square root law, which states that desired inventories are proportional to the square root of total real output divided by the cost of capital. The constant of proportionality can be interpreted as the cost of ordering inventories.[3]

Federal government demand, state and local government demands, and export demand are all determined by essentially the same mechanism. It is assumed that desired expenditure rates are those existing in the pre-attack period. Current expenditures are then determined to be last period's expenditures plus a user-controlled percentage of the difference between desired expenditures and last period's expenditures. Government demands are determined on a per-capita basis, and export demands on an aggregate basis.

Once demands have been determined, they are summed to get the total final demand that each industry must supply. The supply model then uses an input-output model to determine the aggregate level of output required of each industry. Assuming that value added is a fixed proportion of real output, the model determines how much value added each industry must contribute. Value added is assumed to be related to inputs of labor and capital by a constant-elasticity-of-substitution production function.

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3. For a derivation of this law, see Baumol, Economic Theory and Operations Analysis, Prentice Hall (Englewood Cliffs, N.J., 1965), pp. 5-10. For an explanation of how the rule is applied in this model, see Subsection D.3.

Capital is assumed to be industry specific and, therefore, fixed in the short run. Thus, given a certain level of value added, the necessary amount of labor can be determined

The marginal product of labor can be computed once the amount of labor used is known. By comparing marginal product of labor with the wage rate in the industry, the extent of the disequilibrium in the labor market can be determined. The markup in the industry is adjusted upward if labor is used too intensively and downward if labor is not used intensively enough. Thus, a new set of prices is computed. The solution mechanism in the model alternates between the demand and the supply model until prices have stabilized.

Once prices have stabilized, the model computes total labor usage. If usage is greater than supply, the price of labor is raised, and equilibrium prices in the product market are again determined. If labor usage is less than the labor supply, the price of labor is lowered and a new equilibrium is found. This process continues until an average price for labor is found which just uses up the labor supply. At this point, the model checks to see whether capacity in any industry has been exceeded. If capacity has been exceeded, a bottleneck has occurred. If capacity has not been exceeded, the model updates the capital stock and the lagged variables and proceeds to the next period.

In the event that capacity is exceeded, a check is made to see if the shortfall in capacity is less than a fixed percentage of consumer demand. If the shortfall is less than this amount, consumed final demand in the affected sector is cut to the point where capacity is not exceeded and the model continues as usual. If the shortfall is more than this amount, then a bottleneck has occurred.

The model checks to be sure that sufficient capacity exists to meet survival demands. If sufficient productive capacity remains to produce investment goods for the bottleneck industry, investment goods receive the next priority. Remaining capacity in the bottleneck industry is divided on a pro-rata basis between the remaining final demands. Again, capital stocks are updated. Treatment of the lagged variables differs in this case from the no-bottleneck case. Further discussion is presented in Section F.

C. THE SUPPLY STRUCTURE

The first part of the model to be discussed in detail is the supply structure. This part of the economy will undergo the greatest change in the event of a nuclear attack. Since it is impossible to predict how technology might change in the event of an attack, the technical relations are left unchanged. The entire impact of the attack is on the quantities of productive factors that will be available to producers.

The supply structure uses a combination of an input-output model and production functions to determine labor used, output prices, and factor prices in each of the 82 productive sectors. First, the input-output structure is presented,[4] and the production functions are discussed. Finally, the way prices are determined is discussed.

1. Input-Output Submodel

The input-output (I-O) relations provide each sector with a list of input materials necessary to produce the output of that sector. To these inputs, varying combinations of labor and capital can be added to produce the final output. The difference between the cost of the inputs and the price of output is called value added. Labor and capital are used to produce value added. Thus, the I-O structure can be viewed as determining required inputs and the production function as determining the necessary amount of labor and capital.

The I-O relationships are represented by a matrix of coefficients where column i of the matrix gives the amount of input from each sector needed to produce one unit of output in sector i. Call the matrix of coefficients A where a_{ij} represents the amount

4. For a basic description of this model see Goldman, Morris R., Martin L. Marimont, and Beatrice N. Vacarra, "The Interindustry Structure of the United States", Survey of Current Business, November 1964, pp. 10-29.

of output from industry i required to produce one unit of output in industry j. Let $y' = (y_1, \ldots, y_n)$ be a vector, where y_i is total final demand faced by industry i. Let $x' = (x_1, \ldots, x_n)$ be a vector where x_i is gross output produced by industry i.

Total output in sector i is

$$x_i = y_i + \sum_{j=1}^{n} a_{ji} x_j$$

or

$$x = y + A'x \quad .$$

Solving for x, the level of gross output, yields $x = (I-A')^{-1} y$. Let $(I-A')^{-1}$ have elements α_{ij}. Then $x_i = \sum_j \alpha_{ij} y_j$. Thus it is possible to determine total output levels necessary tc support a given set of final demands.

There is an assumption implicit in this framework that the ratio of any particular input to output is constant for all levels of output. Alternatively, no substitution between inputs is possible. This is more rigid than would be expected in reality. In a nuclear attack there would probably be some substitution, but it is difficult to tell just how this would be accomplished. Therefore, we assume that no such changes would occur. The data for the matrix A are described in Section H.2.

2. Production Function Submodel

The quantity $A'x$ represents the amount of intermediate inputs necessary to produce the output level y. The question is how these inputs can be turned into the finished product. GEM uses production functions to define the relation between value added and the amounts of labor and capital used. Value added is defined as the difference between the price of a commodity and the cost of intermediate goods necessary to produce it. Leaving until later the question of how the price of goods is determined, examine the cost of intermediate goods. Define p_i as the price of output of sector i. The total cost of intermediate goods in sector i is

$$\sum_j a_{ji} p_j \quad .$$

Then total value added per unit of output is

$$p_i - \sum_j a_{ji} p_j \quad .$$

Dividing by p_i yields

$$u_i = 1 - \sum_j a_{ji} p_j / p_i \quad ,$$

which is the value added per dollar of output in sector i. The values of $u_1, \ldots, u_n$ for the base period are input data and are discussed in Section H.3. The vector $V' = (v_1, \ldots, v_n)$ represents the total amount of value added, where

$$v_i = u_i x_i \quad .$$

The production function gives the set of technologically efficient combinations of labor and capital which can be used to process the inputs to produce the required level of value added. There are three forms of production functions used in this model. The first is the constant elasticity of substitution (CES) production function. This has the form

$$V_i = H_i \left(\delta_i L_i^{-\rho_i} + (1 - \delta_i) K_i^{-\rho_i} \right)^{-\lambda_i/\rho_i} \quad , \tag{1}$$

where

$$H_i = \zeta e^{\gamma_i t} N_i^{1-\lambda_i} \quad ,$$

where

T represents the time from 1963 to the year of the attack,

γ_i represents the rate of neutral technical change,

N_i represents the number of establishments in the industry,

λ_i represents the returns to scale parameter,

ζ_i represents the value of the efficiency parameter in 1963,

L_i represents the amount of labor used in industry i,

K_i represents the amount of capital used in industry i,

δ_i is a parameter representing the labor intensity of the production process,

and

$\rho_i = \frac{1}{\sigma_i} - 1$, where σ_i represents the elasticity of substitution.

The second form is the Cobb-Douglas production function. In this form

$$V_i = H_i L^{\lambda_i \delta_i} K_i^{\lambda_i (1-\delta_i)} , \qquad (2)$$

where the variables and parameters have the same interpretation as above. The Cobb-Douglas production function is the limiting form of the CES production function as σ_i approaches 1.

The third form is

$$V_i = H_i (\Psi_i N_i + \omega_i L_i)^{\lambda_i} . \qquad (3)$$

Except for Ψ_i and ω_i the variables and parameters have the same interpretation as above. The parameters Ψ_i and ω_i determine the relative importance of the number of establishments and the amount of labor in determining output. This equation is used only for sectors 80, 81, and 82. These are not productive sectors in the sense that the other sectors are, because 80, 81, and 82 are used only for accounting purposes. Therefore this rather artificial form of the production function is used. Parameters of the production functions are discussed in Section H.4.

The production functions are used in the model to determine the amount of labor needed to produce given demand levels. In any given time period, the capital stock, K_i, is fixed. Output levels can only be varied by changing the amount of labor employed. Given a demand level, the producer can decide how much labor is needed.

Because of some of the properties of the production functions used in this model, it is necessary to place a minimum and maximum on the value of the labor/capital ratio. For the CES production function, which includes the Cobb-Douglas production function as a special case, there can be situations when the capital stock is too small to allow a given level of output to be attained with any amount of labor, or so small the output can be attained only by using a very large amount of labor. The allocation of this much labor is unreasonable. Therefore, a maximum value is placed on the labor/capital ratio. An economic rationale for an upper limit of this type is discussed in Section F.

Similarly, if the capital stock is very large relative to demand, very small amounts of labor will be allocated. Again, this is unreasonable. The solution is to place a minimum on the value the labor/capital ratio can take. Economically, this can be treated as a requirement that some of the capital stock remain idle in such circumstances.

3. Pricing Mechanism

Once the level of labor usage has been determined the marginal product of labor can be computed from the production function. This is a very important part of the price adjustment mechanism used in this model. This mechanism adjusts value added in the sector in proportion to the disequilibrium between the price of labor and the marginal product of labor. In the short run, the sector faces a fixed price for labor and a fixed demand. Sector prices are set so that:

$$p_i = \sum_j a_{ji}p_j + u_i w_i/(\partial V_i/\partial L_i) \quad . \tag{4}$$

The term $\sum_j a_{ji}p_j$ represents the cost of inputs in the industry, u_i represents base period value added, w_i gives the price of labor, and $\partial V_i/\partial L_i$ represents the marginal product of labor. Relative wages are fixed in the model. Only the average wage level is allowed to change. The relative wage data are discussed in Section H.5.

If the value of the ratio of wages to marginal output exceeds one, the optimum output level in this industry has been exceeded and prices should be raised. If the ratio is less than one, prices should fall. Prices can be directly computed in the I-O model from (4). They are

$$p_i = \sum_j \alpha_{ij}u_j w_j/(\partial V_j/\partial L_j) \quad , \tag{5}$$

where (α_{ij}) are the elements of the matrix $(I-A')^{-1}$.

The specific forms for the derivative $\partial V_i/\partial L_i$ for the production functions used in this model are now given. For the CES production function,

$$\frac{\partial V_i}{\partial L_i} = \lambda_i \delta_i H_i \left(\delta_i L_i^{-\rho_i} + (1 - \delta_i)K_i^{-\rho_i}\right)^{-\lambda_i/\rho_i - 1} L^{-\rho_i - 1} \quad . \tag{6}$$

For the Cobb-Douglas production function,

$$\frac{\partial V_i}{\partial L_i} = \lambda_i \delta_i \frac{V_i}{L_i} \quad . \tag{7}$$

For the third production function,

$$\frac{\partial V_i}{\partial L_i} = \lambda_i \omega_i H_i (\Psi_i N_i + \omega_i L_i)^{\lambda_i - 1} \quad . \tag{8}$$

The other price which remains to be determined in the model is the rental rate of capital. It is defined as the price of using a capital good for one year, and is determined by assuming that the same relative disequilibrium exists in the capital market as exists in the labor market in each sector. Let R_i be the rental rate of capital in sector i. Then

$$R_i = (w_i/(\partial V_i/\partial L_i))\partial V_i \partial K_i \quad . \qquad (9)$$

Thus the rental rate of capital exceeds the marginal product of capital if the price of labor exceeds the marginal product of labor.

The specific forms for the derivative of output with respect to capital follow. For the CES production function,

$$\frac{\partial V_i}{\partial K_i} = \lambda_i(1 - \delta_i)H_i\left(\delta_i L_i^{-\rho_i} + (1 - \delta_i)K_i^{-\rho_i}\right)^{\lambda_i/\rho_i - 1} K_i^{-\rho_i - 1} \quad . \qquad (10)$$

For the Cobb-Douglas production function,

$$\frac{\partial V_i}{\partial K_i} = \lambda_i(1 - \delta_i)\frac{V_i}{K_i} \quad . \qquad (11)$$

Finally, for the third production function, the marginal product of capital can be estimated by

$$\frac{\lambda_i V_i - \frac{\partial V_i}{\partial L_i} L_i}{K_i} = \frac{\lambda_i \Psi_i N_i H_i (\Psi_i N_i + \omega_i L_i)^{\lambda_i - 1}}{K_i} \quad . \qquad (12)$$

D. THE DEMAND STRUCTURE

The second part of the model is the demand structure. The model portrays the economy as having 82 productive sectors. These sectors are the ones used in the 82-sector input-output model published by the Office of Business Economics (OBE). Since this is the basic structure of the model, it is necessary to present the demands for output on a sector basis. The sections below indicate how this is done for each of the six categories comprising final demands.

1. Demand for Consumption Goods

The demand for consumption goods is determined on the basis of personal consumption expenditure (PCE) categories.[5] The crucial determinants of consumer demand are past consumer demand, current consumer income, past consumer income, current product prices, and past product prices.

Let $c' = (c_1, \ldots, c_n)$ represent current consumption, with c_i representing per capita personal consumption expenditure (PCE), category i. Define c_{-1i} as PCE in category i in the previous period. Denote current PCE category prices by $s' = (s_1, \ldots, s_n)$, where these prices are expressed in percentage terms relative to the base year 1958. Let s_{-1i} represent PCE prices lagged one year. In some PCE categories, other arguments are necessary to explain the behavior of demand. These are represented by O_i. The relation which is used to determine PCE for category i is:

$$c_i = \beta_{0i} + \beta_{1i}c_{-1i} + \beta_{2i}s_i + \beta_{3i}s_{-1i} + \beta_{4i}\left(\sum_j c_j\right) + \beta_{5i}\left(\sum_j c_{-1j}\right) + \beta_{6i}O_i \quad . \qquad (13)$$

The coefficients $\beta_{0i}, \ldots, \beta_{6i}$ are determined by regression analysis. A discussion of the source of these coefficients and of the behavioral model that underlies (13) is contained in Section H.6.

The equation can be rearranged so that current PCE in category i appears only on the left. Doing so yields

5. For an explanation of personal consumption expenditure categories and their use in this model, see Section H. The derivation of parameter values is explained in Dolins, L.D., op. cit., p. 41 ff.

$$c_i = \beta_{Ci} + \beta_{1i}c_{-1i} + \beta_{2i}s_i + \beta_{3i}s_{-1i} + \beta_{5i}\left(\sum_j c_{-1j}\right) + \beta_{6i}O_i$$

$$+ \frac{\beta_{4i}}{1 - \sum_j \beta_{4j}} \sum_j \Big(\beta_{0j} + \beta_{ij}c_{-1j} + \beta_{2j}s_j + \beta_{3j}s_{-1j}$$

$$+ \beta_{5j}\left(\sum_k c_{-1k}\right) + \beta_{6j}O_j\Big) \quad .$$

If the model is to be stable, it is required that $\sum_j \beta_{4j} < 1$. The coefficients currently used by GEM satisfy this constraint.

These PCEs must be converted to I-O sector expenditures. This is done by using a matrix $B = (b_{ij})$, where the coefficient b_{ij} gives the fraction of the goods in PCE category j that are produced in I-O sector i. The source for these coefficients is given in Section H.6.

Letting $CONS_i$ be consumer demand in I-O sector i on a per-capita basis, then

$$CONS_i = \sum_j b_{ij}c_j \quad .$$

The same matrix of coefficients can be used to relate PCE and I-O prices. Let p_i be the price in I-O sector i expressed as a fraction of the price in the base year 1958. Then,

$$s_i = 100 \sum_j b_{ji}p_j \quad .$$

2. Demands Generated by Purchases of Investment Goods

Investment demand is determined by a model which assumes that sector behavior can be described by a profit-maximizing decision-maker who faces a lag between the time an investment decision is made and the time in which the investment becomes a productive part of the industry's capital stock. The decisionmaker is assumed to base his decisions on the difference between the anticipated capital stock, at the earliest date in the future at which a change in the capital stock can be made, and the desired capital stock at that date. The investment decision is made on a continuous basis and the expenditures on a particular investment good are assumed to be at a uniform rate between the date of the decision and date at which the investment comes into use. The submodel incorporating these elements produces the demand for investment goods by industry. These demands are then broken down into the outputs from each industry which are necessary to supply the investment good. These outputs constitute fixed capital investment demand.

The discussion of the investment submodel begins with the method for determining the desired stock of capital at a future date. From there it continues with a discussion of how the continuous investment decisions of the firm can be converted to the discrete intervals used in the model. Next it looks at how the continuous stream of expenditures on any given project is allocated to time periods. Finally, the way in which the expenditures are converted into required industry outputs is discussed.

Suppose that the rate of return on capital is expected to remain constant over time. Let R_i be the rate of return for sector i. Suppose also that capital depreciates at a constant percentage rate over time. Denote this rate by d_i for sector i. Assume no depreciation occurs in the first year. Then the present value of one dollar's worth of investment in section i, z_i, is given by

$$z_i = R_i \sum_{t=0}^{\infty} \frac{(1 - d_i)^t}{(1 + r_i)^{t+1}} ,$$

where r_i is the opportunity cost, expressed in the form of a rate of return, associated with investment in sector i. This can be simplified to

$$z_i = \frac{R_i}{r_i + d_i} . \qquad (14)$$

The data for r_i and d_i are discussed in Sections H.7 and H.8.

Substituting (9) in (14) yields

$$z_i = \left(\frac{w_i}{r_i + d_i}\right)\left(\frac{\partial V_i}{\partial K_i}\right)\Big/\left(\frac{\partial V_i}{\partial L_i}\right) . \qquad (15)$$

If the specific partial derivatives from expressions (6)-(8) and (10)-(12) are substituted in equation (15), and the resulting equations are solved for K_i, the following are obtained:

a) CES production function

$$K_i = \left[\frac{w_i(1 - \delta_i)}{z_i(r_i + d_i)\delta_i}\right]^{\sigma_i} L_i \quad , \qquad (16)$$

where

$$\sigma_i = 1/(1 + \rho_i) \quad ;$$

b) Cobb-Douglas production function

$$K_i = \frac{w_i(1 - \delta_i)}{z_i(r_i + d_i)\delta_i} L_i \quad ; \qquad (17)$$

c) Third production function

$$K_i = \frac{w_i}{z_i(r_i + d_i)} \frac{\alpha_i N_i}{\beta_i} \quad . \qquad (18)$$

Note that the equations as treated so far have no empirical content. To achieve that, it will be necessary to fix the value z_i. To do this, assume that perfect competition exists in the capital goods market. Then the price of an investment good is given as the sum of the prices of its components. Define e_{ij} as the amount of final demand generated in I-O sector i for one dollar's worth of investment expenditure by sector j. These data are discussed in Section H.9. Then $z_i = \sum_j e_{ji} p_i$.[6] Thus, z_i is the supply price of capital goods in industry i. Substituting z_i in (16)-(18) will yield a value for the desired capital stock.

Given a value for the desired stock of capital, the decision-maker determines the current rate of investment by using the equation:

$$I(t) = \gamma[K^*(t - m) - K(t)] + dK(t) \quad , \qquad (19)$$

where

γ is an adjustment rate,
K^* represents the desired stock of capital,
K represents the actual stock of capital, and
d the rate of depreciation.

For convenience, the sector subscript i has been suppressed. The index m represents the time between the decision to invest and the availability of the investment good for production purposes. The data for m are discussed in Section H.10.

The relation given by equation (19) is based on two assumptions. The first is that desired capital stock at date t is the same as desired capital stock at date t + m. The second is that the desired rate of capital accumulation is the same at date t as it is at date t + m. These assumptions allow the derivation of (19) from the basic behavioral equation

$$I(t) = \gamma[K^*(t + m) - K(t + m)] + dK(t + m) \quad .$$

This model suggests that the principal determinants of investment are desired capital stock at date t + m and depreciation at date t + m.

The relation given by (19) is assumed to hold for all t . It is necessary to convert it to an expression for discrete time.

6. The precise equation is identical in concept to the one used to determine I-O prices. See Section C.3 for an explanation.

Essentially, the procedure is to integrate over time and substitute approximations for the values of the integrals.[7]

Let τ denote the length of a period and t_o some point in time. Then the amount of investment, capital stock service, and desired capital stock service in one time period are, respectively,

$$\bar{I}_{t_o} = \int_{t_o}^{t_o+\tau} I(t)dt \quad ,$$

$$\bar{K}_{t_o} = \int_{t_o}^{t_o+\tau} K(t)dt \quad ,$$

$$\bar{K}^*_{t_o} = \int_{t_o}^{t_o+\tau} K^*(t)dt \quad .$$

Thus,

$$\bar{I}_{t_o} = \gamma[\bar{K}^*_{t_o-m} - \bar{K}_{t_o}] + d\bar{K}_{t_o} \quad . \tag{20}$$

Approximating the capital stock service in the period by the initial amount in the period plus the average amount invested in the period yields

$$\bar{K}_{t_o} \approx K(t_o) + \bar{I}_{t_o}/2 \quad . \tag{21}$$

Substituting (21) in (20) and solving gives

$$\bar{I}_{t_o} = \frac{1}{\left(1 + \frac{\gamma - d}{2}\right)}\left[\gamma\bar{K}^*_{t_o-m} + (d - \gamma)K(t_o)\right]$$

for all t_o.

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7. This procedure is similar to that used by Houthakker and Taylor to derive the estimating equation they used in their consumption-function work. See Houthakker and Taylor, op. cit., pp. 11-21.

Since m is not necessarily an integer, we need to approximate the value of $\bar{K}^*_{t-m}$ at noninteger values. Let n be the largest integer in m. Then, assuming $\bar{K}^*_t$ changes in a linear fashion over the interval $n - 1$ to n, we have

$$\bar{K}^*_{-m} = (m - n)K^*_{-n-1} + [1 - (m - n)]K^*_{-n} \quad . \qquad (22)$$

Substituting (22) in (21) yields

$$\bar{I}_{t_o} = \left\{\frac{1}{1 + \frac{\gamma}{2} - \frac{d}{2}}\right\}\left\{\gamma\left[(m - n)K^*_{-n-1} + (1 - (m - n))K^*_{-n}\right] + (d - \gamma)K_0\right\} \quad .$$

Under the assumptions which have been made, all the values in the above equation are known. Thus, desired accruals to the capital stock are known.

Next it is necessary to determine desired accruals to the capital stock over the next $n + 1$ periods. Accruals to the capital stock are now denoted I_t. We can write

$$I_t = \mu_0 K^*_{t-n-1} + \mu_1 K^*_{t-n} + \mu_2 K_t \text{ , where}$$

$$\mu_0 = \frac{\gamma(m - n)}{1 + \gamma/2 - d/2} \quad ,$$

$$\mu_1 = \frac{\gamma[1 - (m - n)]}{1 + \gamma/2 - d/2} \quad ,$$

and

$$\mu_2 = \frac{d - \gamma}{1 + \gamma/2 - d/2} \quad ,$$

and

$$I_{t+1} = \mu_0 K^*_{t-n} + \mu_1 K^*_{t-n+1} + \mu_2 K_{t+1}$$

and so on until

$$I_{t+n} = \mu_0 K^*_{t-1} + \mu_1 K^*_t + \mu_2 K_{t+n}$$

and

$$I_{t+n+1} = \mu_0 K^*_t + \mu_1 K^*_{t+1} + \mu_2 K_{t+n+1} \quad .$$

The value for K^*_{t+1} is determined by assuming $K^*_{t+1} = K^*_t$.

Having determined desired accruals to the capital stock over n+1 future time periods, it is possible to determine the current investment expenditures necessary to attain these desired accruals. Assume from the time of inception until a period m time units later investment expenditures are made at a uniform rate. Next, for all projects in progress at the beginning of the current period, determine the level of expenditures in the current period. The difficult projects to evaluate in terms of expenditure will be those that either begin or end within the current period. Projects that terminate during the interval are examined first.

The exact time at which a project is finished is not known. However, the total value of projects finished during the period is known. The assumption is made that the rate of project completion is uniform during the period. For projects being completed during the current period, expenditures will be at the rate I_t/m at the beginning of the period and at the rate 0 at the end of the period. Thus, the overall rate of expenditures for projects being completed during this period is $I_t/2m$. For projects not being completed in the period (t, t+1), the rate of expenditure will be I_t/m. Projects beginning with the period (t, t+1) fall into two categories and relate to accruals expected to occur both in period t+n and in period t+n+1. Some projects scheduled for completion in period t+n will have begun before the current period. Expenditures on these projects will total $\frac{(m - n)}{m} I_{t+n}$. Projects begun during the current period and scheduled to end in period t+n have expenditures totaling

$\frac{(n+1-m)(1+m-n)}{2m} I_{t+n}$. Finally, projects begun in the current period n+1 will have total expenditures $I_{t+n+1} \frac{(m-n)^2}{2m}$. Summing up these terms yields total expenditures in period 0 , denoted IEXP. This is given by

$$IEXP_o = \frac{I_o}{2m} + \sum_{i=1}^{n-1} \frac{I_i}{m} + I_n \left[\frac{m-n}{m} + \frac{(n+1-m)(1+m-n)}{2m}\right]$$

$$+ I_{n+1} \frac{(m-n)^2}{2m} .$$

Once expenditures by I-O sector on investment goods have been determined, it is possible to determine the final demand, by I-O sector, that is due to investment goods. As before, e_{ij} gives the amount of final demand arising in sector i for one dollar's worth of investment expenditure by sector j. Let $ICAP_i$ be the final demand in I-O sector i arising due to investment expenditure, then

$$ICAP_i = \sum_j e_{ij} (IEXP_o)_j .$$

3. Inventory Demand

The third component of final demand is that due to investment or disinvestment in inventories. This quantity is found by comparing desired and actual inventory accumulations. In particular, the rate of inventory investment in sector i, INV_i, is given by

$$INV_i = \eta(Q_i^* - Q_i) ,$$

where Q_i^* is desired inventory level and Q_i is the actual level. The parameter η is an adjustment parameter giving the fraction of the difference between actual and desired inventories to be made up.

Desired holdings of inventories are assumed to be determined by the square root law:

$$Q_i^* = \sqrt{\frac{k_i x_i}{2r_i p_i}} \quad , \qquad (23)$$

where k_i is inventory reorder costs, x_i is gross output in the current period for industry i, r_i is the rate of return to capital, and p_i is the price of output.[8] The data used to compute k_i are discussed in Section H.11.

Again, as in the investment model, the instantaneous model given above must be converted to discrete time periods. The procedure is similar to that used in the investment model. Upon integrating (23), approximating inventory service in a period, and substituting,

$$INV_i = \frac{\eta}{1+\eta/2}(Q_i^* - Q_i) \quad .$$

4. Federal Government Expenditure

The federal government is assumed to want to maintain the same per-capita expenditure after the attack as it had in the pre-attack economy. Actual federal government demand is determined by applying the following formula:

$$FEDG_i = FEDG_{-1,i} + \varkappa(FEDG_i^* - FEDG_{-1,i}) \quad ,$$

where $FEDG_i$ represents actual expenditures on a per-capita basis and $(FEDG^*)_i$ is desired expenditures on a per-capita basis. $FEDG_{-1,i}$ represents per-capita expenditures from the prior period. The parameter $\varkappa$ represents the rate at which actual expenditures will be adjusted to desired expenditures. Total federal government demand is given by $P \cdot FEDG_i$ where P is the population. It would be possible to modify this demand sector to reflect a specific government program if the demands arising from the program can be distributed among the productive sectors in the model.

8. See Baumol, op. cit.

5. State and Local Government Expenditures

State and local governments are also assumed to want to continue the same level of expenditures on a per-capita basis that they had prior to the attack. Let this be $SALG_i^*$. Then actual per-capita expenditures are determined by

$$SALG_i = SALG_{-1,i} + \pi (SALG_i^* - SALG_{-1,i}) \quad ,$$

where $SALG_i$ is actual expenditures in the current period, $SALG_{-1,i}$ is actual expenditures last period, and π is the rate of adjustment. Total state and local government expenditures are then given by $P \cdot SALG_i$.

6. Export Demand

Desired exports are assumed to equal the pre-attack exports. Actual exports are then determined by a mechanism similar to that used in the government sector. Specifically, define $EXPT_i$ as actual exports in sector i, $EXPT_{-1,i}$ as previous period exports, $EXPT_i^*$ as desired exports, and θ as the rate of adjustment of actual to desired exports.

Then

$$EXPT_i = EXPT_{-1,i} + \theta(EXPT_i^* - EXPT_{-1,i}) \quad .$$

7. Total Demand

Total final demand is then defined as the sum of consumption, investment, inventory, government, and exports. Thus

$$y_i = P \cdot CONS_i + ICAP_i + INV_i + P \cdot FEDG_i + P \cdot SALG_i + EXPT_i \quad ,$$

where y_i is final demand in sector i and P is population.

The final demands generated by investment, inventory, federal government, state and local government, and exports depend upon lagged adjustment parameters. The values of these parameters determine the effects of supply shortages in the indicated final demand components on the level of final demand in those components, allowing for differential consideration of the various sectors. Thus, different recovery policies can be modeled.

For instance, if a critical shortage occurs in industry 10, the following values might be used: investment rate = 1.0, inventory rate = 0.0, and export rate 0.0. These values would force investment in capital stock, thus increasing productive capacity. The values assigned to the two government rates in this case would depend upon how critical this particular sector is in allowing completion of government projects.

E. THE SOLUTION MECHANISM

The solution mechanism consists of two parts. The first part takes a given average wage rate, then finds a set of prices and quantities that will satisfy the demand and supply equations. The second part adjusts the wage rate until a specified amount of labor is in use. The property of the solution from an economic point of view is that product markets have cleared, i.e., the quantity supplied equals the amount demanded. However, the markets for the factors of production will not, in general, be in equilibrium. In the labor market, by virtue of the requirement that a specified amount of labor be used, it will almost always be the case that a particular industry would like to use either more or less labor than it is using. If the industry is using as much labor as it desires, the marginal product of labor would equal the wage rate.[9] The market for capital goods is also not in equilibrium due to the fact that there is a lag in the adjustment of the capital stock to market conditions. Further, the initial solution reached by the model is

9. This statement is an oversimplification. In fact, the conditions necessary for this result are not present in the model. However, by specifying that overall labor usage is predetermined, the model prevents the firm from deciding how much labor it would like to employ. This is sufficient to indicate that this market is not in equilibrium.

not guaranteed to be feasible. There is no guarantee that sufficient capital stocks and labor are present to produce the quantities required by the solution. However, the bottleneck procedure, detailed in Section F, will reduce demands to feasible levels in the event of insufficient capacity.

1. Solution Mechanism for Prices and Quantities

Given the demand and supply equations, it is necessary to find a set of prices and final demands which satisfy both sets of equations. From a purely theoretical point of view, it would be desirable to establish both that a solution exists and that the solution is unique. Empirically, the existence of a solution has been established. However, a mathematical characterization of the solution and a demonstration that the solution is unique have not been achieved, although preliminary investigation indicates that the desired characteristics are, in fact, present.

The solution technique used in the computer program embodying the model is iterative. The price of labor is fixed and prices from the last period, called p_{i0}, are used as a starting point. These prices are given to the demand sector to determine a set of demands, x . These demands are given to the supply sector to determine a new set of prices, p_{i1}. The function is computed:

$$ERR = \sum_{i=1}^{82} (p_{i0} - p_{i1})^2 \quad .$$

If this is less than some specified value, ϵ , then p_{i1} is considered to be the solution set of prices. The set of demand x_{i1} is considered to be the solution set of demands. If ERR > ϵ, a new starting price vector is determined by the formula

$$p^*_{i0} = (p_{i0} + p_{i1})/2 \quad .$$

Convergence to a solution is not always achieved using this algorithm.

The significant errors which occur because of this failure appear to be those relating to price levels. Difficulties in finding a solution appear when prices change a great deal in response to a small change in output. This will normally be the case in only a few of the sectors.[10] The available labor supply is the most important determinant of level of GNP. Thus, failure to achieve convergence will have only limited impact on the overall accuracy of the results.

2. Solution Mechanism for Average Wage Level

The wage rate is determined by adjusting a fixed relative wage structure up and down until total labor usage equals the available labor force. If it is assumed that relative wages are fixed and that labor is a homogeneous good, it is necessary to have only one labor market, rather than one labor market for each I-O sector. The actual procedure for finding the required average wage rate begins with a trial average wage rate. This determines the specific wage rate in each industry. Through (5), prices are determined. Prices will determine total final demand in each sector. Total final demands will determine aggregate output levels in each sector. As described in the previous section, iteration will continue between prices and quantities until equilibrium occurs in the product markets. The total amount of labor required can then be computed. If labor required exceeds the available labor force, the average wage rate will be raised. If total labor in use is less than the labor force, the average rate will be met. Adjustment of the average wage rate and computation of the resulting labor use will continue until total labor in use equals the labor force.

The available labor force, $\bar{L}$, is determined by taking a proportion, ℓ, of the uninjured population, P_u. Thus, $\bar{L} = \ell P_u$. Total population, P, is the sum of uninjured plus injured, P_i.

10. Particulary troublesome industries are 15, 47, and 49. Most other industries will converge to their solution value within two or three iterations.

Thus, $P = P_u + P_i$. The uninjured population is assumed to grow at the rate of g per year. Population injured in an attack is assumed to have a first year recovery rate of υ. Injured who do not recover in the first year after an attack are assumed to be permanently disabled. The injured group is assumed to have a death rate of ζ. Thus, population in the second year after an attack would be

$$P_2 = (1 + g)\, P_{u,1} + \upsilon P_{i,1}(1 - \zeta) + (1 - \upsilon)P_{i,1}(1 - \zeta)$$

$$P_{u,2} = (1 + g)\, P_{u,1} + \upsilon P_{i,1}(1 - \zeta)$$

$$P_{i,2} = (1 - \upsilon)\, P_{i,1}(1 - \zeta) \quad .$$

The rationale for using all of the available labor force is that the proper criterion in a post-attack environment is maximum possible output rather than some measure of output which might reflect less than complete usage of a scarce resource. The average price of labor then can be considered to be a measure of how scarce labor is relative to capital, given the current level of demand. A low average price for labor indicates that labor is relatively abundant. A high price would indicate that labor is relatively scarce. Since labor is allocated to each sector based on demand, the model cannot exhibit relative labor scarcities among sectors. However, GEM can exhibit relative capital scarcities since the capital stock in each sector is assumed to be specific to that sector. In those sectors where capital is scarce, the marginal product of capital will exceed the rate of return. Where capital is relatively abundant, the reverse will hold true.

F. BOTTLENECK PROCEDURE

If capacity in an industry must be exceeded for the solution demands to be satisfied, a bottleneck has occurred. In the event of a bottleneck, the price mechanism used in the model will not perform the function of allocating the scarce product among competing demands. This happens for two reasons. First, when the maximum capacity is reached, price will no longer rise. When maximum capacity is reached, the model will not allow any more labor to be allocated to the industry in question even though, because of the properties of the production functions used in the model, the marginal product of labor is still positive. This limit on the amount of labor an industry can use implies there is an upper limit on the price of the industry's product. Thus, the price mechanism no longer works. Secondly, a number of the components of demand are not sensitive to price. It is possible that these demands could exceed available capacity. These two factors would have to be corrected if the price mechanism were to work in the presence of inadequate capacity.

An alternative way to correct this problem in the model is to impose some type of rationing scheme. This is the solution used. There are definite real-world situations in which rationing schemes are used. A post-nuclear attack situation is likely to be such a situation. The particular rationing scheme chosen is arbitrary. In the event that a bottleneck occurs, solution prices are used. However, quantities supplied are adjusted so that capacity in the bottleneck industry is not exceeded. Capacity is determined by first observing the labor-capital ratio in the base year period. The output associated with this labor-capital ratio and the capital stock available to the industry in the current period is assumed to be the normal operating level in the industry. This operating level is multiplied by an emergency capacity ratio.[11] The resulting figure is treated as maximum capacity in the industry.

The use of available capacity in the event that available capacity is inadequate depends on the extent of the shortage. If the shortage is only slight, then consumer demands in the affected sector are cut sufficiently to allow available demands to be supplied.

11. The emergency capacity ratios used were taken from Bickley, L.J., J.F. Crane, and E.S. Pearsall, Estimates of the Potential of the U.S. Economy Following a Strategic Attack in 1975, IDA Study S-305 (Arlington, Va., 1967), pp. 26-27.

The model then continues in its usual fashion. If the shortage is more severe, final demands in all sectors are cut. In this case, available capacity is allocated first to survival demands, then to investment industries with insufficient capacity to meet solution demands, and finally to remaining demands.

Minimum survival demands are determined by using a vector giving minimum outputs necessary for an individual to survive and then multiplying by the surviving population level to get aggregate output levels necessary for survival.[12] If the minimum consumption vector cannot be met out of current production, the economy stops and a statement as to the infeasibility is printed out. This does not necessarily indicate the collapse of the economy, but rather that some substantially less interdependent form of organization would occur. Modeling this form is beyond the current scope of the work. Of necessity, any such model would be much more conjectural than the current one.

If minimum consumption can be met out of production, then

$$x_i^* = x_i^m + \left(\frac{x_i^s - x_i^m}{x_i^d} \right) \left(x_i^{\cdot} - x_i^m \right),$$

where

x_j^s is maximum supply in sector i,

x_i^m is the level of output of this sector needed to meet minimum survival and investment demands

x_i^d is total demand for output of sector i,

x_i^* is actual output in sector i, and

x_i is desired output in sector i.

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12. The individual demand vector is taken from Bickley, et al., op. cit., pp. 33-44.

The economy will then proceed to the next period. As soon as there are no sector scarcities, the economy will return to the normal mode of production.

G. TIME STRUCTURE OF THE MODEL

One of the primary goals of the model is to trace out the recovery process that might occur after a nuclear attack. The model accomplishes this by examining the behavior of the economy over a four-year period. The validity of the time path traced out by the model depends on the accuracy of the intertemporal relations incorporated in the behavioral relationship in the model. The time links in the model occur on both the demand and supply sides.

On the supply side, the time links are incorporated in the supply of labor and of capital. The supply of labor is primarily dependent on the passage of time, since there is no model that relates economic performance to the labor force participation rate. The capital stock is incremented each year by the amount of investment which occurs less the amount of depreciation which has occurred. In addition, a certain amount of neutral technical change is assumed to occur in some industries. Thus, the efficiency of inputs may increase over time.

On the demand side of the model, the time structure is somewhat more complex. In the case of consumer demand, the passage from period to period is reflected in changes in the values of past consumption and prices in the sector, and past total consumption. These past values represent the influence of habits, or the size of stocks held by consumers, as is appropriate, for the commodity in question. In the case of investment demand, the linkage depends upon past values of desired capital and the length of time it takes to complete an investment. In the case of inventories, the principal component of interest in total inventories is the previous period. The remaining three sectors of final demand have a time path that is independent of the performance of the economy when a bottleneck occurs.

Lagged values of demand and total PCE in the PCE equation are used to represent habit formation by consumers. In the event of a bottleneck, consumption is held to artificially low levels that are not a valid representation of consumer expectations. Therefore, in the event of a bottleneck, the lagged values are not updated. Since a bottleneck would quite typically be expected to occur in the immediate post-attack period, this has the effect of carrying pre-attack habits over into the post-attack era. This is only one of many possible assumptions that could be made. Its chief merit is that it is consistent with the other assumptions made in the model. It is not necessary to adjust any of the lagged variables in any of the other demand sectors. In each of these sectors, the failure to obtain last period's demand will be reflected in current demand.

H. DATA BASE

1. Introduction

The general economic model requires an extensive data base for its operation. Some of the data can be obtained directly from the literature and some must be imputed from the values of other data. This section describes data which can either be obtained directly from the literature or by imputation, using standard procedures. Those data whose imputation require more elaborate procedures are discussed in Section I. In this section the required data are discussed in the order in which they were previously introduced.

2. Direct Input-Output Coefficients

Direct input-output coefficients are required for 1972. These coefficients reflect the purchases required from other producing industries per dollar of output. The relationships between producing and consuming industries change over time as the result of changes in technology, product mix, and price competition.

There is no projection of these coefficients available for 1975. However, the Bureau of Labor Statistics has projected a set of direct coefficients for 1970 and one for 1980. Rather than use one or the other for 1975, a fit was made between 1970 and 1980 to obtain annual rates of change and projections for each year, between 1970 and 1975.

Several problems occurred in attempting to find these growth rates. Initial projections were made with a constant percentage rate of growth. As shown in Figure 12, the growth rate has a logarithmic shape (A), but for some sectors the rate was such that by 1985 some of the coefficients were greater than one, a theoretically unacceptable condition in an input-output model. Therefore, projections were made using the formula

$$a_{ij}^{t} = a_{ij}^{80} + a_{ij}^{t-1} (1 - r^{10-t}) ,$$

where r is the annual rate of change between 1970 and 1980. This has the effect of concentrating most of the change in the coefficients in the earlier part of the period. The resulting rate of change yields a curve shape like A^* rather than A .

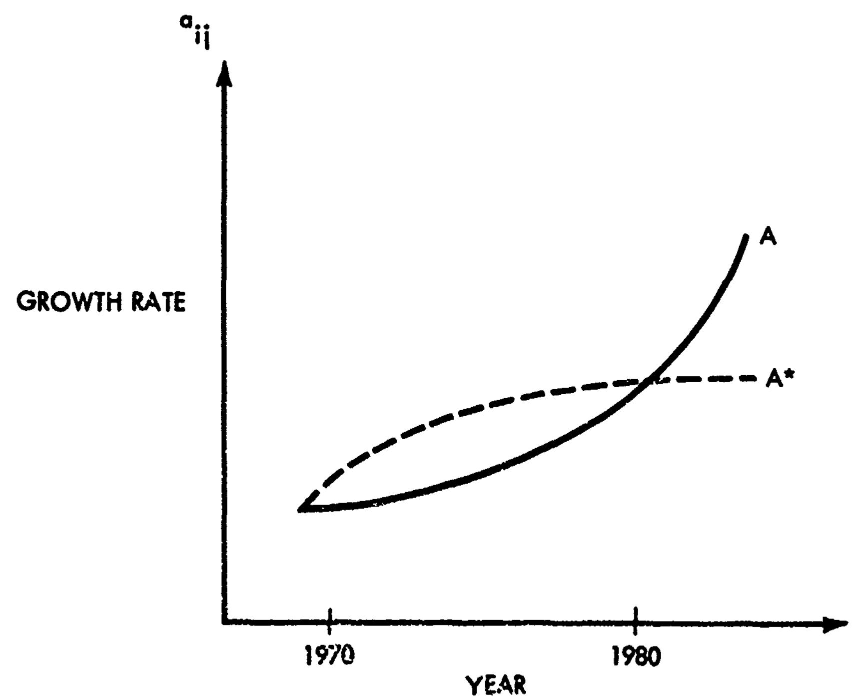

FIGURE 12. Projection Trends for Direct Coefficients

In addition, because of sectoral definition changes in sectors 74 and 78, the 1980 sectoral definitions were chosen and new coefficients for the 1970 matrix were created. This was accomplished by using a 1965 to 1980 Bureau of Labor Statistics index of the coefficient change based on 1980 sectoral definitions.[13] These rates were used to project these sectors to 1985, to project any sector that was zero in 1970 and positive in 1980, and to project a few cells with unstable coefficients.[14]

3. Value-Added Coefficients by Input-Output Sector

A projection of value-added coefficients must be made before the input-out direct coefficients projections can be completed. Value added is the value of the output added by the production process. Summing down the column of an input-output direct coefficient matrix, the sum of the producing sector coefficients was projected separately from the matrix. The growth rate of the sum was calculated from the 1970 and 1980 values. These growth rates were also calculated as shown in Figure 16.

With this distinct value-added coefficient, the column sums of the direct coefficients plus value added were calculated. If they were greater than one, the proportion of error was applied to each coefficient to force the sum to one.

4. Production Function Parameters

The parameters of the production function used in the model were obtained in part from regression estimates and in part from estimating procedures internal to GEM. The procedures for the

13. U.S. Department of Labor, Bureau of Labor Statistics, "Patterns of U.S. Economics Growth", Bulletin 1672, U.S. Government Printing Office, Washington, D.C., hereafter referred to as Projections 80, p. 30.

14. These were 28th row by 34th column
57th row by 47th column
57th row by 49th column
57th row by 61st column
63rd row by 82nd column.

parameters estimated within GEM are described in Subsection I.1. The parameters taken from regression estimates are now described.

The basic production function is given by

$$V = \zeta_i \, e^{\gamma_i t} \, N_i^{1-\lambda_i} \left[\delta_i L_i^{-\rho_i} + (1 - \delta_i) K_i^{-\rho_i} \right]^{-\lambda_i/\rho_i} \quad . \qquad (24)$$

The parameters γ_i, λ_i, and ρ_i are determined by regression estimates. ζ_i and δ_i are determined internally. For input-output sectors 13-64, the values of γ_i, λ_i, and ρ_i are determined by a combination of cross-section and time-series estimates.[15] For those sectors in which estimated returns to scale were less than one, returns to scale were set equal to one because profit-maximizing decisionmakers would not build plants larger than the point at which diminishing returns would be incurred. Thus, if decreasing returns are estimated, it will be due to observational errors or other statistical problems.

For industries 1-12 and 65-82, data were not available to estimate the parameters γ, λ, and ρ. In these sectors γ has been set to zero, λ to one, and ρ to one. The errors are probably more important for sectors 1-12, the agricultural sectors, than they are for sectors 65-82, since these are primarily service sectors.

In some industries the value of ρ was not statistically different than zero. Equation (24) is not defined for ρ equal zero. In this case, the Cobb-Douglas production function is used.

The projected total number of firms, by sector and employment class, was derived by projecting 1963 data with an average annual rate of growth based upon 1958 and 1963 data.[16]

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15. For a complete description of the procedure used, see Grimm, Bruce T., Estimation of CES Production Functions for US Manufacturing by Input-Output Sector, IDA Paper P-525 (Arlington, Va., July 1969).

16. U.S. Bureau of the Census, Census of Manufactures, op. cit.

The number of firms, by county and sector, was projected to 1975, using the same approach as used to project value added:

$$N_{ijk}^{75} = \left[\hat{N}_{jk}^{75} \Bigg/ \sum_{\ell=1}^{m} \frac{P_\ell^{75}}{P_\ell^{60}} N_{\ell jk}^{63} \right] \frac{P_i^{75}}{P_i^{60}} N_{ijk}^{63} \quad , \qquad \text{for all i, j, and k ,}$$

where

k = employment class ($k = 1, 4$),

N_{ijk}^{75} = projected number of firms in county i , of sector j , in class k ,

$\hat{N}_{jk}^{75}$ = projected total number of firms in sector j , in employment class k ,

N_{ijk}^{63} = 1963 number of firms in county i , of sector j , in class k .

The internal procedure for estimating the remaining parameters of the production functions requires information on the ratio of labor income to value added. The labor income figure is projected by means of separate projections of wages and employment. The wage data are described in Subsection 5. The employment projections are developed by using an estimate of gross output and projections of the ratio of the number of employees to gross output. These data were acquired directly from BLS. Data for 1969 and 1980 were used to compute an annual rate of change, by sector, which was used as a constant proportion to project employment-output ratios for the years of the data base.

5. Wages by Input-Output Sector

The data on wages were acquired from Jack Faucett Associates.[17] Data on the value of payrolls and the number of employers, by

17. Jack Faucett Associates of Maryland released the wage data. Faucett is engaged in research on the input-output structure of the US economy for the Defense Civil Preparedness Agency.

input-output sector, were presented for 1958 and 1963 in current dollars. The 1963 data were deflated to 1958 dollars with the Consumer Price Index. Average wages for each sector were calculated by dividing the value of payrolls by the number of employees. Faucett did not include data for sectors 80, 81, 82, 83, 85, and 87. The mean of the average wage for the remaining 81 sectors was used for these sectors. Using a constant annual percentage rate of growth fit between 1958 and 1963, average wages were projected to 1985.

6. Final Demand by Input-Output Sector

GEM requires a projection of the 1975 components of final demand by input-output sector. The total output is consumed either by intermediate use in the production of goods or by final consumption. The components of final demand are personal consumption expenditures (PCE), gross private fixed capital formation, net inventory change, and net exports, federal government expenditures, state and local government expenditures. The sum of all of these components is the Gross National Product (GNP). The source for projections of these data is the Bureau of Labor Statistics (BLS).[18]

For the purposes of the model, BLS projections of GNP and of the input-output structure of the economy in 1980 were used. The GNP projection reflects the assumption of a reduction of present level of defense expenditures due to a resolution of the Viet Nam war and a four percent unemployment rate. The projection reflects a combination of past trends modified to take account of anticipated developments. PCE and state and local government expenditures, as proportions of GNP, are higher than in the past. Federal government purchases show a sizable drop as a proportion of GNP, particularly when compared with current levels, which include a large amount of Viet Nam-related expenditures. The drop occurs only in defense-related expenditures. The proportion of nondefense expenditures relative to GNP is assumed to increase.

18. Projections 80, op. cit.

a. Personal Consumption Expenditures (PCE)

(1) Derivation of PCE equation. The behavioral model from which the equations are derived is of the following form:

$$c_j = b_0 + b_1\Omega_j + b_2 T + b_3 s_j \quad , \qquad (25)$$

where Ω_j represents the stock of the durable commodity or habitual comsumption levels in the case of a nondurable commodity and $T = \sum_i c_i$.

The individual's current demand for a good is not only a function of his current income, but of his stock of that good and of its price. In the case of a durable commodity, b_1 will be negative because the more of the good he possesses the less he will currently want. However, b_1 will be positive for habit-forming commodities of which the consumer normally does not hold large inventories. Consider tobacco, a habit-forming commodity, for which the present consumption is positively influenced by past consumption. This psychological state variable is difficult to measure for habit-forming commodities. In addition, for durable goods the depreciation rate for the stocks, δ, is difficult to approximate accurately. Therefore, through algebraic manipulation and differentiation with respect to time, Houthakker and Taylor eliminated an explicit reference to the variable Ω_i from the equation.[19]

This yields the regression equation

$$c_j = a_0 + a_1 c_{-1,j} + a_2 \Delta T + a_3 T_{-1} + a_4 \Delta s_j + a_5 s_{-1,j} \quad . \qquad (26)$$

The structural parameters of (25) are related to the regression parameters of (26) as follows:

$$b_0 = \frac{2a_0(a_2 - 1/2a_3)}{a_3(a_1 + 1)}$$

19. Houthakker and Taylor, op. cit., pp. 9-12.

$$b_1 = \frac{2(a_1 - 1)}{a_1 + 1} + \frac{a_3}{a_2 - 1/2a_3}$$

$$b_2 = \frac{2(a_2 - 1/2a_3)}{a_1 + 1}$$

$$b_3 = \frac{2(a_4 - 1/2a_5)}{a_1 + 1} \quad .$$

In some of the habit-forming commodities analyzed by Houthakker and Taylor, the basic behavioral model implied implausibly high rates of depreciation which indicated that the basic equation might not hold in these cases. An alternative model conceived by A.R. Bergstrom from the London School of Economics was used. Instead of the assumption of stock adjustment in the Houthakker-Taylor model, the dynamics of the Bergstrom model assume that consumers try to bring actual consumption in line with some desired level, which is a function of total PCE. The structural form of the Bergstrom model consists of

$$c_j = \theta(\hat{c}_j - c_j)$$

$$\hat{c}_j = \xi + \mu T \quad ,$$

where c_j is the rate of change of consumption over time and $\hat{c}_j$ is the desired level of consumption.[20] The final estimating equation is:

$$c_j = a_0 + a_1 c_{-1,j} + a_2(T + T_{-1})$$

20. The Bergstrom model was used to estimate demand for 14 of the 82 PCE categories. A static model ($q_t = C_1 x_t = C_2 P_t$) was used to estimate demand for category 5.2 (Kitchen and Other Household Appliances) and for category 7.1 (Brokerage Charges and Investment Counseling). No equation was specified for category 2.4 (Standard Clothing Issued to Military Personnel) because of its peculiar policy-dependent character. The remaining PCE category demands were estimated with the Houthakker-Taylor behavioral dynamic model.

where

$$\theta = \frac{2(1 - a_1)}{1 + a_1} \quad ,$$

$$\xi = \frac{a_0}{1 - a_1} \quad ,$$

$$\mu = \frac{2a_2}{1 - a_1} \quad .$$

These estimating equations do a credible job of explaining consumer demand between 1930 and 1965. The question here is: How credible will their prediction of post-attack consumer demand be? The parameters of these equations have captured the tastes and behavior of consumers during prosperous as well as depressed times. Thus, they should reliably predict consumer response to great changes in income and price, as would occur during the economic dislocations of the post-attack period.

(2) Equations. The projection of PCE by input-output sector was accomplished by using the consumer demand equations described above. The coefficients for these equations were estimated by Houthakker and Taylor using a data base for the years 1929 to 1964.[21] Houthakker and Taylor estimated two sets of equations, one based on a 1929 to 1961 data base and the other on 1929 to 1964 data. The equations were of the following form:

$$c_j = a_0 + a_1 c_{-1,j} + a_2 \sum_i c_i + a_3 \sum_i c_{-1,i} + a_4 s_j + a_5 s_{-1,j} + a_6 0$$

where

c_j = current consumption in category j ,

21. Houthakker and Taylor, op. cit., and the second edition of the same title published in 1970.

$c_{-1,j}$ = per capita consumption in category j, lagged one year,

$\sum_i c_i$ = total PCE per capita,

$\sum_i c_{-1,i}$ = total PCE per capita, lagged one year,

s_j = relative price in category j,

$s_{-1,j}$ = relative price, lagged one year in category j, and

O = variables specific to certain equations such as the percent of total population 18 years or older, the number of shares sold on the New York Stock Exchange per capita, and farm income.

The choice between the two available sets of coefficients was made on the basis of forecasting efficiency by using a set of PCE prices derived from input-output prices.[22]

Projecting the input-output sector demands required a data base for projecting the PCE demands and a matrix for converting the results from personal consumption expenditure categories to input-output sectors.

(3) Data base for PCE projections. A data base is required to project PCE from 1970 to 1985. PCE by input-output sector for 1969 was made available by BLS. The data required to solve the demand equations for PCE projection from 1970 to 1985 were:

i. PCE consumption in year t - 1 (C_{-1j}) was initially the 1969 PCE, by I-O sector, acquired from BLS. This was converted to PCE category by using the inverse of the PCE-I-O conversion matrix. This matrix is described below. For the years following 1970, the previous year's projection was used.

ii. The variable $\sum_i c_i$ or total PCE was needed for the years 1969 to 1985. This was calculated by projecting from

22. Dolins, L.D., op. cit., p. 21. The BLS projections are based upon the 1929-1964 data base.

1969 using a constant annual percentage rate of growth based upon 1969 and the BLS estimate for 1980. The sum of the individual equations was constrained to equal this estimate of total PCE. This was accomplished by adjusting the arithmetic average of PCE prices up or down until the individual components of demand summed to total demand.[23]

iii. Prices by PCE sector were estimated through an application of the inverse of the PCE-I-O conversion coefficient matrix to prices by input-output sector. Using the value-added figures which are described in Subsection 2 and the direct coefficients which are described in Subsection 1, the input-output prices were estimated annually as follows:

$$p = (I - A')^{-1} v \quad .$$

Here p is a vector containing prices, A is a matrix containing direct coefficients, and v is a vector containing value added.

iv. The equations required per-capita data. Thus, a projection of total population between 1965 and 1985 was required. The Bureau of the Census was consulted about which series of projections best fit the latest developments in population growth. Series E was chosen. The point estimates around which the annual projections were made are:

1970	203.185 million (as of 1 April)
1975	214.735
1980	225.510
1986	236.918

This is not the same series used in the projections of the other components of the GNP data (namely, Series C). The discrepancy is not large, however.

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23. See Dolins, L.D., op. cit., pp. 17-18, for a more complete discussion of the procedure.

v. The percent of the population greater than or equal to 18 years of age was projected with Series E estimates.

vi. The projection of the percent of farmers in the population was based upon a constant percentage growth rate taken from actual experience between 1960 and 1968.

vii. The number of stocks sold on the New York Stock Exchange per capita was projected to grow at a rate of 2.9 percent per annum.

viii. Disposable farm income per farm capita was projected at a rate of 6 percent to 1970 and at a slower rate of 2.75 percent to 1985. These rates were determined by Houthakker and Taylor in the creation of their data base.

(4) PCE to I-O conversion matrix. Even though the ultimate concern here are projections of consumer demand distributed according to input-output sector, the initial projections were made with equations classified according to the type of product consumed rather than the producing industry. When attempting to predict consumer behavior, it is more logical to use a set of consumer equations classified as the consumer would tend to think about his purchases, i.e., by type of product, not by producing industry. The equations were classified by the 82 Personal Consumption Expenditure categories published by the U.S. Department of Commerce.[24]

Once consumer demand is estimated for each of these categories, it can be converted into the 87 producing industry sectors of the input-output model published by the Office of Business Economics of the Department of Commerce.[25] The coefficients used to convert consumer demand of a PCE category into its input-output sector components

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24. See Table 2.5 in the July National Income Accounts issue of the U.S. Department of Commerce's Survey of Current Business.

25. National Economics Division Staff, Office of Business Economics, Department of Commerce, "The Transactions Table of the 1958 Input-Output Study and Revised Direct and Total Requirements Data", Survey of Current Business, Vol. 45, No. 9 (September 1965), pp. 33-49, 56. Sectors 83-87 are discussed in Subsection 12.

were based upon a Department of Commerce table detailing the industrial composition of 1958 consumer expenditures.[26] The coefficients are calculated in the following manner:

$$b_{ij} = \frac{c_{ij}}{\sum_{i=1}^{n} c_{ij}} \quad (i = 1, \ldots, m)(j = 1, \ldots, n) \quad ,$$

where c_{ij} is the amount of the j^{th} PCE category coming from the i^{th} input-output sector and $c_j = \sum_i c_{ij}$ is the total expenditure for that PCE category.

These coefficients are used to distribute the projected values of the PCE categories among the input-output sectors as follows:

$$CONS_i = \sum_{j=1}^{m} b_{ij} c_j \quad (i = 1, \ldots, n) \quad .$$

The personal consumption expenditure allocated to the i^{th} input-output sector ($CONS_i$) is equal to the sum of the portions of all the PCE categories composed of goods from that input-output sector.

The table from which these coefficients were derived was based on the 1958 input-output table. Similar figures, based upon the 1963 input-output table, are not available. However, BLS made available an updated version of the original data, taking into account some of the changes in the industrial composition of consumer demand. As an example, the consumer demand in PCE category 1.1 (Food Purchased for Off-Premises Comsumption) will not reflect more purchases from the packaged food industries and fewer direct purchases from the agricultural sector than the 1958 table indicates. It is from this BLS version that the matrix of coefficients for conversion from PCE categories to input-output sectors was calculated.

26. Simon, Nancy W., "Personal Consumption Expenditures in the 1958 Input-Output Study", Survey of Current Business (October 1965), pp. 7-20. A more recent table is not yet available.

b. Other Components of Final Demand. Given the values of the components of final demand in 1969 and their projections for 1980, projections for the intermediate years were made, assuming a constant percentage rate of growth between 1969 and 1985[27]. This method was used to ascertain the annual values between 1969 and 1985 by input-output sector for gross private fixed capital formation, net inventory change,[28] net exports, federal government expenditures, and state and local government expenditures.

7. Rates of Return in Capital Assets by Input-Output Sector

The investment calculation requires rates of return to capital. The source of these data is a study conducted by George J. Stigler for the National Bureau of Economic Research.[29] Stigler calculates the rate of return on capital assets using the sum of the following components in the denominator:

(1) depreciated machinery and equipment
(2) depreciated buildings
(3) land
(4) inventories
(5) other working capital, i.e., cash, accounts receivable, government securities, other assets.

The net earnings figure in the numerator consists of the sum of business receipts, rents, and royalties depleted by deductions such as losses on non-capital assets.

Stigler's calculation of rate of return was presented by three-digit Standard Industrial Classification (SIC) Code for the years

27. The Bureau of Labor Statistics was the source of the 1969 distribution of final demand by input-output sector. The data-base tape contains annual projections from 1969 through 1985.

28. Projections 80 provides projections of Gross Private Fixed Capital Formation and Gross Private Domestic Investment (GPDI). The Net Inventory figures are a residual of the subtraction of Capital Formation from GPDI.

29. Stigler, George J., Capital and Rates of Return in Manufacturing Industries, National Bureau of Economic Research, Princeton University Press (Princeton, N.J., 1963), pp. 220-226.

1957 and 1958.[30] The data used were averages of these two years compiled into input-output sectors. The scheme used to take three-digit SIC-coded data and put them into the input-output sector scheme of classification was published by the Office of Business Economics.[31]

8. Lifetime of Capital Assets by Input-Output Sector

Depreciation rates for equipment in manufacturing industries were calculated on the basis of the length of life, in years.[32] These data were given by two-digit major SIC codes that were transferred to input-output sector by the method discussed above. The average life span of manufacturing equipment was used for the non-manufacturing input-output sectors. The depreciation rate was calculated on a constant percentage basis, constrained so that 50 percent of the capital stock remained halfway through the life span:

$$d = 1 - (.5)^{\frac{2}{L}} ,$$

where d is the depreciation rate and L the life span of a capital asset.

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30. The Standard Industrial Classification scheme was developed for use in the classification of establishments by type of activity in which it is engaged. Establishments are classified by industry on a two-, three-, or four-digit basis, according to the degree of detail in formation. Thus, we have major Group 25--Furniture and Fixtures; Group 251--Household Furniture; and Industry No. 2511--Wood Household Furniture, Except Upholstered. See Bureau of the Budget, U.S. Technical Committee on Industrial Classification, Standard Industrial Classification Manual, 1967, U.S. Government Printing Office, Washington, D.C. 1967.

31. Office of Business Economics, The Transactions Table, loc. cit., p. 33.

32. Cramer, Dobrovolsky, and Borenstein, Capital in Manufacturing, p. 223, cited in Stigler, op. cit., p. 121.

9. <u>Capital-Flow Matrix</u>

A capital-flow matrix was created so that, given expenditures for investment goods by each input-output sector, it would be possible to calculate the output by input-output sector required to supply that demand. The data source for this matrix was 1958 capital flows as measured by producers' value.[33] The figures for the row totals of the original data matrix were compiled to correspond with the 1958 input-output matrix. The data were adjusted to assure that the sums across the row and down the column equalled the row and column totals, which was not the case with the original data.

10. <u>Construction Time of New Plants by Input-Output Sector</u>

The data on new plant construction time were developed by the National Planning Association (NPA).[34] NPA presents estimates, in months, of the new plant construction time, including design and procurement. The data for large plants were chosen for the data base. The size of a large plant varies by the type of industry involved, e.g., in food-processing industries a large plant has more than 50 employees, whereas in computer hardware manufacturing industries a large plant has more than 1000 employees. These data were presented on the basis of four-digit SIC codes that were then aggregated to input-output sector by using the OBE table discussed above.[35]

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33. U.S. Department of Labor, Bureau of Labor Statistics, "Capital Flow Matrix, 1958", Bulletin No. 1601, U.S. Government Printing Office, October 1968. There is as yet no capital-flow data for any later date.

34. Economic Programming Center, <u>Capacity Expansion Planning Factors, Manufacturing Industries</u>. National Planning Association (Washington, D.C., 1966).

35. Office of Business Economics, <u>op. cit.</u>

11. Rates of Change of Gross Output by Input-Output Sector

The calculation of inventory reorder costs requires average annual rates of change of gross output by input-output sector.[36] The rates used were those for the period 1965-1980, published by BLS in Projections 80.[37]

12. Special Sectors 83-87

In addition to the 82 sectors in the input-output matrix, five other sectors are included in the complete model. Only sectors 84 and 86 have any employment attributed to them. Based on employment data,[38] sector 84 is assumed to grow at an annual rate of 3.5 percent, and sector 86 at an annual rate of 0.5 percent.

The employment figures for sector 84 also include employment from sectors 78 and 79. Therefore, in computing total employment, sectors 78 and 79 must be subtracted from sector 84.

Output from sectors 83-87 is assumed to grow at the rates given in Projections 80.[39] The inventory valuation adjustment is projected at zero. Using 1965 figures as a base and the growth rates for both labor and employment, coefficients giving labor per unit of output and per-capita output are found for each sector. Final demand is then determined, using per-capita output ratios and surviving population. For the two sectors which use labor, labor demand is computed using the labor-per-unit-of-output coefficients. These figures are then used in computing total GNP and total labor usage.

I. PARAMETER INITIALIZATION

Data sources for a number of the variables used in this model were described in the preceding section. Because it was not possible

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36. See Section I.2

37. Bureau of Labor Statistics, op. cit., p. 97.

38. Projections 80, op. cit., p. 99.

39. Projections 80, op. cit., p. 99. Rates from the 3.0 percent base model are used.

to find empirical sources for all the parameters used in the model, estimates of parameter values were made when sources could not be found. Estimates were based on available data and whatever assumptions were necessary. Included in the list of estimated parameters or data points are the parameters ζ and δ from the production function, the minimum and maximum capital labor ratios allowed, the size of the capital stock, the cost-of-reordering-inventories parameter, and the lagged adjustment rates in the various demand sectors.

1. Production Function Parameters

For the purposes of this section the production function will be written as

$$V_i = H_i\left(\beta_i L_i^{-\rho_i} + \alpha_i K_i^{-\rho_i}\right)^{-\lambda_i/\rho_i} \tag{27}$$

for the CES case,

$$V_i = H_i\left(\alpha_i L_i^{\beta_i} K_i^{1-\beta_i}\right)^{\lambda_i}$$

for the Cobb-Douglas case, and

$$V_i = H_i(\alpha_i N_i + \beta_i L_i)^{\lambda_i}$$

for the third case, where H_i now has the form

$$e^{\gamma_i t} N_i^{1-\lambda_i} .$$

The parameters ζ_i and δ_i in equations (1), (2), and (3) are related to α_i and β_i as follows:

In the CES case,

$$\delta_i = \frac{\beta_i}{\alpha_i + \beta_i} ,$$

and $$\zeta_i = (\alpha_i + \beta_i)^{-\lambda_i/\rho_i} .$$

In the Cobb-Douglas case,

$$\zeta_i = \alpha_i^{\lambda_i} ,$$

and $$\delta_i = \beta_i .$$

In the third case,

$$\alpha_i = \Psi_i ,$$

$$\beta_i = \omega_i ,$$

$$\delta_i = 1 .$$

The problem is to determine the values for α_i and β_i and the ratio between capital and output, Ψ_i, in the base year. These parameters are determined by assuming that capital and labor are both paid their marginal product.[40] Thus, in the CES case,

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40. Economic theory requires the presence of a Lagrange multiplier in Eq. (28). This multiplier represents the value to the firm of an additional unit of output. Deleting it implicitly assumes that the industry's demand curve is perfectly elastic and that product price is one in the base year. Neither of these conditions is true in the model. In addition, if λ_i is greater than one, then factor payments will exceed the net income of the industry. It is preferable to have econometric estimates of the parameters. Lacking these, it is desirable to have a logically consistent method for estimating the parameters. However, given the limitations of available data, neither of these approaches is available. The errors in the model results attributable to poor parameter estimates are difficult to assess precisely. Since the parameters involved are rather

$$w_i = H_i\left(\beta_i L_i^{-\rho_i} + \alpha_i K_i^{-\rho_i}\right)^{-\lambda_i/\rho_i - 1} \lambda_i \beta_i L_i^{-\rho_i - 1} \quad . \tag{28}$$

Simplifying and solving for β yields

$$\beta_i = \frac{w_i}{\lambda_i H_i}\left(\frac{H_i}{V_i}\right)^{1+\rho_i/\lambda_i} L_i^{1+\rho_i} \quad .$$

For the Cobb-Douglas case,

$$w_i = H_i\left(\alpha_i L_i^{\beta_i} K_i^{1-\beta_i}\right)^{\lambda_i - 1} \lambda_i \alpha_i \beta_i L_i^{\beta_i - 1} K_i^{1-\beta_i} \quad .$$

Simplying and solving for β_i yields

$$\beta_i = \frac{w_i L_i}{\lambda_i V_i} \quad .$$

In the third case,

$$w_i = H_i \lambda_i \beta_i \left(\alpha_i N_i + \beta_i L_i\right)^{\lambda_i - 1} \quad .$$

Solving for β_i yields

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crucial in determining the behavior of any given sector, these sector results must be somewhat suspect. However, the effect of changes in available resources should be reflected fairly accurately. Further, the effect of these errors should be much smaller on the whole economy since it is the aggregate levels of resources rather than their distribution which determines aggregate performance of the economy.

$$\beta_i = \frac{w_i}{\lambda_i V_i}\left(\frac{V_i}{H_i}\right)^{1/\lambda_i} .$$

To find α_i, assume $K_i = V_i$. This assumption is made only for the base year. Substituting V_i for K_i in (27) gives

$$V_i = H_i\left(\alpha_i V_i^{-\rho_i} + \beta_i L_i^{-\rho_i}\right)^{-\lambda_i/\rho_i} .$$

Simplifying and solving for α_i gives

$$\alpha_i = \beta_i\left(\frac{V_i}{L_i}\right)^{\rho_i}\left(\frac{\lambda_i V_i}{w_i L_i}\right) - 1 .$$

In the Cobb-Douglas case,

$$V_i = H_i\left(\alpha_i L_i^{\beta_i} V_i^{1-\beta_i}\right)^{\lambda_i}$$

after the substitution $K_i = V_i$ is made. Sovling for α_i then yields

$$\alpha_i = \frac{\left(\frac{V_i}{H_i}\right)^{1/\lambda_i}\left(\frac{V_i}{L_i}\right)^{\beta_i}}{V_i} .$$

In the third case, no substitution is necessary. Thus,

$$V_i = H_i\left[\alpha_i N_i^o + \beta_i L_i^o\right]^{\lambda} ,$$

and

$$\alpha_i = \frac{\left(\frac{V_i}{H_i}\right)^{1/\lambda_i} - \beta_i L_i}{N_i} \quad .$$

Once the parameters α_i and β_i are determined, it is necessary to determine the relationship between capital stock and output in the base period. To determine Ψ_i, assume that capital is paid its marginal product. If V_i is used to measure the capital stock, the marginal product of capital will not equal its price. Let c_i equal the ratio of the marginal product of capital to the price of capital when capital is measured in value-added terms. For the Cobb-Douglas case, $\Psi_i = c_i$ and for the CES case, $\Psi_i = c_i^{-\sigma_i}$, In the I-O case,

$$\Psi_i = \frac{\lambda_i \alpha_i N_i H_i (\alpha_i N_i + \beta_i L_i)^{\lambda_i - 1}}{K_i (r_i + d_i) \sum_{j=1}^{82} b_{ji}} \quad .$$

Then, to get an initial estimate of the capital stock, value added in the base year is multiplied by Ψ_i. In subsequent years, the value of the capital stock is obtained by updating the initial value to take account of new investment and depreciation.

Given the values of these parameters, it is then necessary to determine the minimum and maximum labor/capital ratios to be allowed. The maximum labor/capital ratio poses no problem in the current version of the model. High labor/capital ratios imply that the price of labor is too high and prices should be raised. This occurs automatically in the course of the model. Therefore, the upper limit is set arbitrarily at $100(L/K)^o$ where $(L/K)^o$ represents the base year labor/capital ratio. This situation may appear to allow too much labor to be used with a given capital stock. However, in the bottleneck portion of the model, maximum permissible capital/labor ratios are computed which reflect estimates of maximum capacity. The reason for not applying these limits in the main part of the model is that they interfere with the operation of the price mechanism. The minimum labor/capital ratio is set at $.67(L/K)^o$. As labor usage falls below this level there will be a decline in the amount of capital stock in use.

2. Inventory Reorder Costs

In the inventory demand model, it is necessary to have an estimate of k_i, the inventory-reorder-cost parameter. Desired inventories are determined by

$$Q_i^{*o} = \sqrt{\frac{k_i x_i}{2 r_i p_i}} \ .$$

Assume that, in the base year, firms have succeeded in changing inventory by exactly the desired amount. Thus,

$$\left(\frac{dQ_i^*}{dt}\right)^o = \sqrt{\frac{k_i}{2 r_i p_i^o}} \left\{\left(\frac{dx_i}{dt}\right)^o \Big/ x_i^o\right\} \frac{\sqrt{x_i^o}}{2} \ . \qquad (29)$$

Let U_i be actual inventory change in the base year. Let $(dx_i/dt)^o/x_i^o = g_i$ where g_i is the rate of growth of output in the base year. Substituting in (29) and solving for k_i yields

$$k_i = \frac{8 r_i \rho_i^o}{x_i^o} \left(\frac{U_i^o}{g_i^o}\right)^2 \ .$$

IX

INTEGRATING MODEL

A. INTRODUCTION

The models presented in this chapter provide a means of integrating cost and effectiveness of strategic defensive forces to satisfy planning objectives. They generate least-cost mixes of forces to meet specified objectives. There may be more than one objective, and objectives may be specified in terms of more than one measure of effectiveness. For instance, objectives can be stated in terms of surviving population, surviving economic capacity, and other such measures. Specifications may be handled for one or more scenarios, concerning warning time and whether the threat is Soviet or Chinese, for instance.

The following generic example will serve to illustrate the methodology. Let x denote a vector of strategic defensive forces and let y denote the threat. The function $c(x)$ represents the cost of providing defensive forces x. For a nuclear attack, let $O(x,y)$ denote the outcome. For instance, $O(x,y)$ might be the number of survivors in an attack y with a defense x. Suppose that r survivors of the attack are specified. A mathematical programming model representing the problem is to choose x (for a given threat y) to

$$\text{minimize } c(x)$$

subject to

$$O(x,y) \geq r \quad .$$

Now expand the formulation to include four scenarios: Soviet attack with warning (SW), Soviet attack with no warning (SN), Chinese attack with warning (CW), and Chinese attack without warning (CN). The outcome of an attack will probably be different in each case. Let y^S and y^C denote the Soviet and Chinese threats, respectively, and the effectiveness functions and requirements be superscripted to represent the scenarios. The mathematical program is to choose x to

$$\text{minimize } c(x)$$

subject

$$O^{SW}(x,y^S) \geq r^{SW}$$
$$O^{SN}(x,y^S) \geq r^{SN}$$
$$O^{CW}(x,y^C) \geq r^{CW}$$
$$O^{CN}(x,y^C) \geq r^{CN} \quad .$$

The solution to such a mathematical program will provide a minimum-cost array of defensive forces x to meet the specified requirement on survivors for any of the four scenarios.

The formulations of the problems given above assume a given enemy targeting of his forces. To reflect adequately enemy capabilities, the methodology should consider the optimal targeting of his weapons. Let the generic set Y denote the available inventory of weapons and y their targeting. Then the mathematical program is to choose x to

$$\text{minimize } c(x)$$

subject to

$$\min_{y^{SW} \in Y^{SW}} O^{SW}(x,y^{SW}) \geq r^{SW}$$

$$\min_{y^{SW} \in Y^{SN}} O^{SN}(x,y^{SN}) \geq r^{SN}$$

$$\min_{y^{CW} \in Y^{CW}} O^{CW}(x,y^{CW}) \geq r^{CW}$$

$$\min_{y^{CN} \in Y^{CN}} O^{CN}(x,y^{CN}) \geq r^{CN} \quad .$$

Recent theoretical results have been developed with regard to the capability of nonlinear programming algorithms to find optimal solutions for this type of mathematical program.[1] Models of a number of military problems have been formulated.[2] Computational procedures for solving mathematical programs with nonlinear programs in the constraints also have been developed.[3]

To use this type of methodology, the following must be accomplished:

- Quantitative specification of the defensive systems,
- Quantitative specification of the offensive threat,
- Determination of a cost function for the defensive systems,
- Development of a means for obtaining measures of the outcome of an attack. The outcome descriptors should be multi-dimensional).

The defensive system specification could include both active and passive components. The active defense can be specified by the number of interceptors of various types and the number of radars. The passive components could include fallout protection [measured by the average protection factor (PF)], blast protection [measured by

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1. Bracken, J. and J.T. McGill, Mathematical Programs With Optimization Problems in the Constraints, IDA Paper P-725 (Arlington, Va., May 1971).

2. Bracken J., and J.T. McGill, Mathematical Programs With Optimization Problems in the Constraints: Applications to Defense Analyses, IDA Paper P-784 (Arlington, Va., July 1971), draft.

3. Bracken, J. and J. T. McGill, Computer Program for Solving Mathematical Programs With Nonlinear Programs in the Constraints, IDA Paper P-801 (Arlington, Va., March, 1972).

the mean lethal overpressure (MLOP)], and evacuation capability (measured perhaps by the number of people that can be accommodated in rural reception centers and by the ability of people to reach the centers).

The threat specification could include the number and size of the warheads targeted.

Cost functions, which include investment and operating costs, could be used. These functions would have to be compatible with the measures used to quantify the defensive system.

The outcome of an attack can be modeled quantitatively in two ways. Analytic expressions for the outcome may be postulated, based on knowledge of qualitative relationships. For instance, the square root damage law is often used to estimate blast effects on population. Alternatively, detailed damage assessment models, such as ANCET and GEM, can be used to generate a set of outcomes for various offense and defense levels. Curve-fitting techniques can then be used to fit analytic functions to this set of data. Such an approach to obtaining effectiveness functions is used in general purpose forces planning methodologies.[4]

The first model presented in this chapter determines passive defenses meeting both population-survival and industrial-survival requirements. The attacker can use his weapons optimally against population or industry, whichever he chooses; the defender's objectives are satisfied in the face of these optimal attacker allocations. The least-cost mix of evacuation capability, hardness of population, and dispersion and hardness of industry, by location, is determined.

The second model presented in this chapter determines both active and passive defenses. Post-attack requirements, by sector, are specified. Destruction of both the capital stock and labor supply is considered. Active defenses and passive defenses are

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4. Bracken, et al., Methodologies for General Purpose Forces Planning, WSEG Report 165 (April 1971) (SECRET NOFORN).

included for the defender. After observing the defensive resources, the attacker optimally allocates a given arsenal of offensive weapons, by type, to location, to minimize post-attack production in whichever economic sector he chooses. In the face of the possible attacks, the defender specifies the post-attack capabilities by sector.

A computer program is available for the second model, to be used with the master computer program given in IDA P-801.[5] Results from sample selection of parameters are presented in Part II of this volume.

In both models the outcome functions for the attack are given by analytic expressions which seem to be reasonable. Alternatively, extensive runs of ANCET/GEM could be used to generate more realistic outcome functions.

B. PASSIVE DEFENSE MODEL WITH POPULATION AND INDUSTRIAL SURVIVAL REQUIREMENTS

The problem is to provide minimum-cost passive strategic defensive systems for Side 1 to guarantee specified levels of surviving population and industry, after absorbing an attack on population or an attack on industrial capacity by Side 2 with known strategic offensive forces. Side 1 deploys strategic defensive systems to locations. Side 2 observes the defenses of Side 1 and allocates offensive weapons to either population destruction or industrial destruction. Side 1 must attain at least a specified level of population survival and industrial survival, regardless of the attack chosen by Side 2. Both Side 1 and Side 2 know the damage functions for population and industrial capacity by location.

In particular, let Side 1 provide hardening and/or evacuation capabilities for population and hardening and/or dispersion capabilities for industry. The passive defense measures are to be supplied

5. Bracken and McGill, Computer Programs for Solving Mathematical Programs, op. cit.

at minimum cost to protect against attacks on either population or industry.

First consider the problem of protecting the population. Let $i = 1, \ldots, p$ denote location and $j = 1, \ldots, q$ denote offensive weapon type. Define

y_{ij} = number of offensive weapons of type j targeted to location i ,

Y_j = number of offensive weapons of type j available,

z_{i1} = hardness of population at location i ,

z_{i2} = evacuation capability of population from location i ,

α_{ij} = scaling factor for population damage by offensive weapon of type j hitting location i ,

β_{i1} = population hardness parameter at location i ,

β_{i2} = population evacuation parameter at location i ,

P_i = unevacuated population at location i .

The number of survivors at location i for offensive allocations $y_{ij}(j = 1, \ldots, q)$ will be taken to be

$$P_i \left\{ 1 = \exp \left(- \beta_{i1}z_{i1} - \beta_{i2}z_{i2} \right) \left[1 - \exp \left(- \sum_{j=1}^{q} \alpha_{ij}y_{ij} \right) \right] \right\} .$$

The expression $\exp(- \beta_{i1}z_{i1} - \beta_{i2}z_{i2})$ gives the fraction of the population that is susceptible to attack. Thus, the effect of increasing z_{i1} and/or z_{i2} is to remove a portion of the population from the attack base. If $z_{i2} = 0$, then none of the population is evacuated. The variable z_{i1} may be bounded from below, say by $\overline{z}_{i1}$, to represent the natural hardness of the population. An increase in z_{i1} may be interpreted as an increase in the MLOP of the population. The parameters β_{i1} and β_{i2} provide a means of scaling the relative contributions of evacuation and hardness of the population.

The expression $1 - \exp\left(- \sum_{j=1}^{q} \alpha_{ij}y_{ij}\right)$ yields the fraction of the susceptible population which is destroyed by the attack

$y_{ij}(j = 1, \ldots, q)$. The sum $\sum_{j=1}^{q} \alpha_{ij} y_{ij}$ provides a measure of the joint effects of various types of weapons. The parameters α_{ij} scale weapons of different yields to an equivalent number of a standard weapon.

A cost function for blast shelters is

$$c(z_{i1} - \bar{z}_{i1})^d \quad ,$$

where $c > 0$, $z_{i1} \geq \bar{z}_{i1}$, and $d \geq 1$. For illustrative purposes, assume that the cost of evacuation capability is linear, namely $e_i z_{i2}$.

The attack and protection of industry can similarly be considered. Let the undispersed industry at location i be I_i, where $i = 1, \ldots, p$. Let the superscript 1 denote population and the superscript 2 denote industry.

Let the surviving population and industry requirements be r^1 and r^2. The overall problem of providing defensive forces at minimum cost is to choose $z_{i1}^1(i = 1, \ldots, p)$; $z_{i2}^1(i = 1, \ldots, p)$, $z_{i1}^2(i = 1, \ldots, p)$, $z_{i2}^2(i = 1, \ldots, p)$; $y_{ij}^1(i = 1, \ldots, p; j = 1, \ldots, q)$, $y_{ij}^2(i = 1, \ldots, p; j = 1, \ldots, q)$ to

$$\text{minimize} \sum_{i=1}^{p} c^1 \left(z_{i1}^1 - \bar{z}_{i1}^1\right)^{d^1} + \sum_{i=1}^{p} e_i^1 z_{i2}^1$$

$$+ \sum_{i=1}^{p} c^2 \left(z_{i1}^2 - \bar{z}_{i1}^2\right)^{d^2} + \sum_{i=1}^{p} e_i^2 z_{i2}^2$$

subject to

$$\left[\begin{array}{c} \underset{y_{ij}^1}{\text{minimum}} \sum_{i=1}^{p} P_i \left\{1 - \exp\left(- \beta_{i1}^1 z_{i1}^1 - \beta_{i2}^1 z_{i2}^1\right)\left[1 - \exp\left(- \sum_{j=1}^{q} \alpha_{ij}^1 y_{ij}^1\right)\right]\right\} \\ \sum_{i=1}^{p} y_{ij}^1 \leq Y_j \ , \quad j = 1, \ldots, q \end{array}\right] \geq r^1 \ ,$$

$$\left[\begin{array}{c}\underset{y_{ij}^2}{\text{minimum}} \sum_{i=1}^{p} I_i\left\{1 - \exp\left(- \beta_{i1}^2 z_{i1}^2 - \beta_{i2}^2 z_{i2}^2\right)\left[1 - \exp\left(- \sum_{u=1}^{q} \alpha_{ij}^2 y_{ij}^2\right)\right]\right\} \\ \sum_{i=1}^{p} y_{ij}^2 \leq Y_j \ , \quad j = 1, \ldots, q \end{array}\right] \geq r^2 \ .$$

$$z_{i1}^1 \geq \bar{z}_{i1}^1 \ , \quad i = 1, \ldots, p$$

$$z_{i1}^2 \geq \bar{z}_{i1}^2 \ , \quad i = 1, \ldots, p \ .$$

It should be noted that an alternative strategic defense model is given in Section IV of P-784.[6] The model given here, and in the following section, is based on Section V of P-784.

C. ACTIVE/PASSIVE DEFENSE MODEL WITH POST ATTACK ECONOMIC REQUIREMENTS

The problem is to provide minimum-cost active and passive defenses which satisfy post-attack economic objectives. The attacker is assumed to know the active and passive defenses at the time he targets his weapons. He also knows the post-attack economic functions, including effects of the defense and of offensive weapons, and is able to target a fixed quantity of attacking weapons optimally against any chosen economic sector. There are several economic sectors, several locations, several types of defenses, and several types of offensive weapons.

Let the indexes be $i = 1, \ldots, m$ on economic sectors, $j = 1, \ldots, n$ on locations, $k = 1, \ldots, p$ on defensive resources, and $\ell = 1, \ldots, q$ on offensive resources.

Define

x_{jk} = number of defensive resources of type k assigned to location j ,

$v_{j\ell}^i$ = number of offensive resources of type ℓ targeted on

6. Bracken and McGill, Mathematical Programs with Optimization Problems, op. cit.

location j when the attack is on economic sector i ,

V_ℓ = inventory of offensive weapons of type ℓ .

The post-attack production function in the sector i is represented by

$$P_i\left(x_{jk}, v^i_{j\ell}\right) = \sum_{j=1}^{n} H_{ij}\left[1 - e^{-\sum_{k=1}^{p} a^H_{ijk} x_{jk}\left(1 - e^{-\sum_{\ell=1}^{q} b^H_{ij\ell} v^i_{j\ell}}\right)}\right]$$

$$\times \quad K_{ij}^{\alpha_{ij}}\left[1 - e^{-\sum_{k=1}^{p} a^K_{ijk} x_{jk}\left(1 - e^{-\sum_{\ell=1}^{q} b^K_{ij\ell} v^i_{j\ell}}\right)}\right]^{\alpha_{ij}}$$

$$\times \quad L_{ij}^{\beta_{ij}}\left[1 - e^{-\sum_{k=1}^{p} a^L_{ijk} x_{jk}\left(1 - e^{-\sum_{\ell=1}^{p} b^L_{ij\ell} v^i_{j\ell}}\right)}\right]^{\beta_{ij}}$$

The coefficient H_{ij} represents efficiency of the economy in sector i in location j . The coefficients K_{ij} and α_{ij} reflect the contributions of capital, and the coefficients L_{ij} and β_{ij} reflect the contributions of labor. The exponents α_{ij} and β_{ij}, are chosen to reflect diminishing marginal productivity in each economic sector and location.

The expressions

$$\left[1 - e^{-\sum_{k=1}^{p} a_{ijk} x_{jk}\left(1 - e^{-\sum_{\ell=1}^{q} b_{ij\ell} v^i_{j\ell}}\right)}\right]$$

modify the efficiency, capital and labor terms. The a_{ijk} term reflects the effectiveness of defensive weapon type k in protecting economic sector i in location j , and the $b_{ij\ell}$ term reflects the effectiveness of offensive weapon type ℓ in destroying economic sector i in location j. The superscripts H, K, and L on a_{ijk} and $b_{ij\ell}$ differentiate among efficiency, capital, and labor. It should be noted that the defensive resources x_{jk} protect all three economic factors by location, and the offensive resources $v^i_{j\ell}$ destroy all three factors, which would be true if efficiency, capital, and labor are collocated.

Similar to the passive defense model given previously, the post-attack production function allows for targets to be essentially removed from susceptibility to attack (or, equivalently, made relatively more difficult to destroy) by adding defensive resources. This removal process has diminishing marginal productivity. The function allows for surviving targets to be destroyed by attacking weapons, with the destruction also having diminishing marginal productivity.

The cost of defensive resources is taken to be $\sum_{k=1}^{p} \sum_{j=1}^{n} c_{jk}x_{jk}$, where c_{jk} is the unit cost of defensive resource k in location j.

The requirements for surviving post-attack economic capacity are given by $r^i(k = 1, \ldots, m)$.

The overall model is to choose $x_{jk}(j = 1, \ldots, n; k = 1, \ldots, p)$ and $v^i_{j\ell}(i = 1, \ldots, m; j = 1, \ldots, n; \ell = 1, \ldots, q)$ to

$$\text{minimize} \sum_{k=1}^{p} \sum_{j=1}^{n} c_{jk}x_{jk}$$

subject to

$$
\left[\begin{array}{l}
\underset{v^i_{j\ell}}{\text{minimum}} \; \sum_{j=1}^{n} H_{ij}\left[1 - e^{-\sum_{k=1}^{p} a^H_{ijk} x_{jk}\left(1 - e^{-\sum_{\ell=1}^{q} b^H_{ij\ell} v^i_{j\ell}}\right)}\right] \\
\quad \times \; K_{ij}^{\alpha_{ij}}\left[1 - e^{-\sum_{k=1}^{p} a^K_{ijk} x_{jk}\left(1 - e^{-\sum_{\ell=1}^{q} b^H_{ij\ell} v^i_{j\ell}}\right)}\right]^{\alpha_{ij}} \\
\quad \times \; L_{ij}^{\beta_{ij}}\left[1 - e^{-\sum_{k=1}^{p} a^L_{ijk} x_{jk}\left(1 - e^{-\sum_{\ell=1}^{q} b^L_{ij\ell} v^i_{j\ell}}\right)}\right]^{\beta_{ij}} \\
\quad \sum_{j=1}^{n} v^i_{j\ell} \leq V_\ell \;, \qquad \ell = 1, \ldots, q
\end{array}\right] \geq r^i \;, \quad i = 1, \ldots, m.
$$

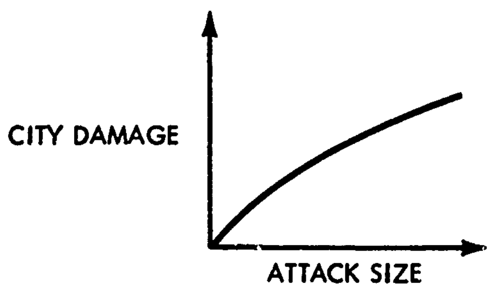
A computer program has been written, and an example problem formulated and solved, for the following dimensions

		Computer Program Dimensions	Example Problem Dimensions
Sectors	$(i = 1, \ldots, m)$	$m = 5$	$m = 2$
Locations	$(j = 1, \ldots, n)$	$n = 10$	$n = 3$
Defenses	$(k = 1, \ldots, q)$	$p = 3$	$p = 2$
Offenses	$(\ell = 1, \ldots, q)$	$q = 2$	$q = 2$

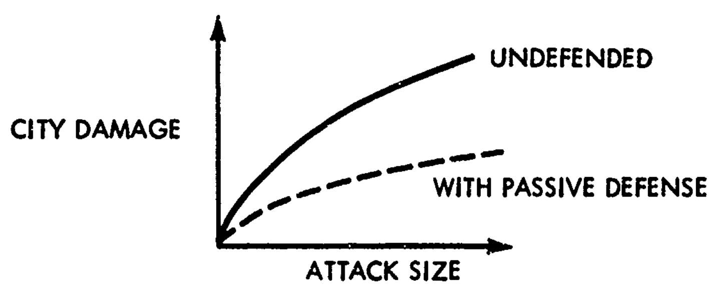
D. EFFECTIVENESS FUNCTIONS IN INTEGRATING MODELS

The treatment of effectiveness functions for offensive forces and for active and passive defenses in integrating models is not discussed.

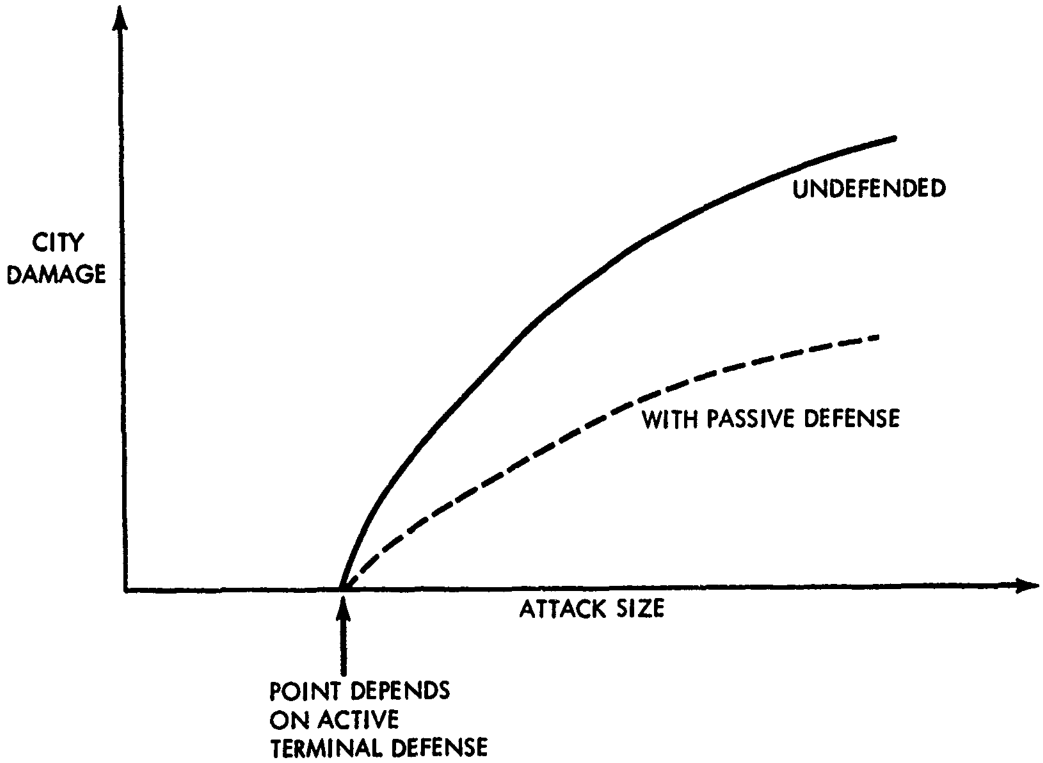
If the model is at the level of detail of individual cities and there are no terminal active or passive defenses, city damage as a function of attack size has the following shape:



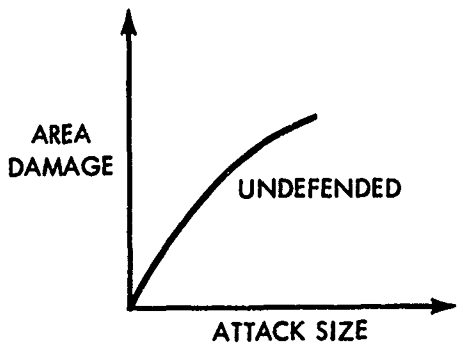
A passive defense tends to move the function downward, as follows:

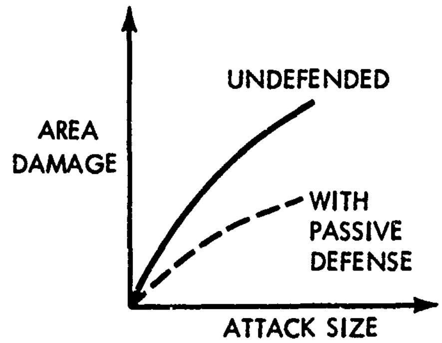


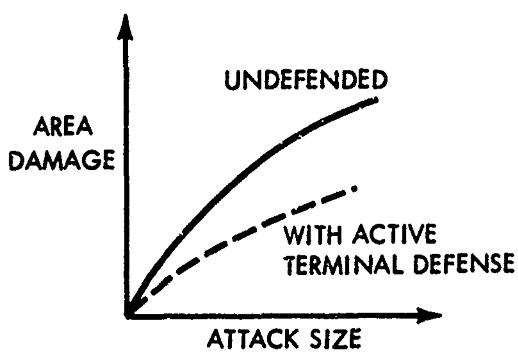
If a subtractive terminal defense is present, then damage as a function of attack size has the following shape:

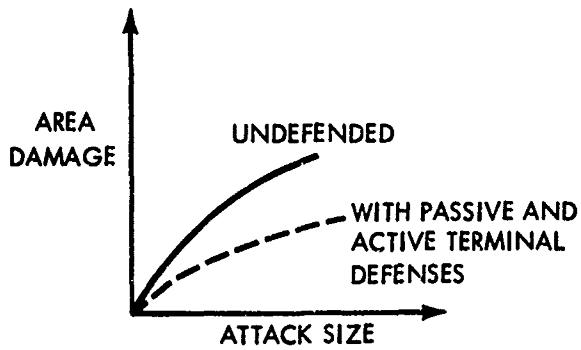


If the model is at the level of an area composed of a number of cities and there is no active area defense, damage as a function of attack size should have one of the following shapes:



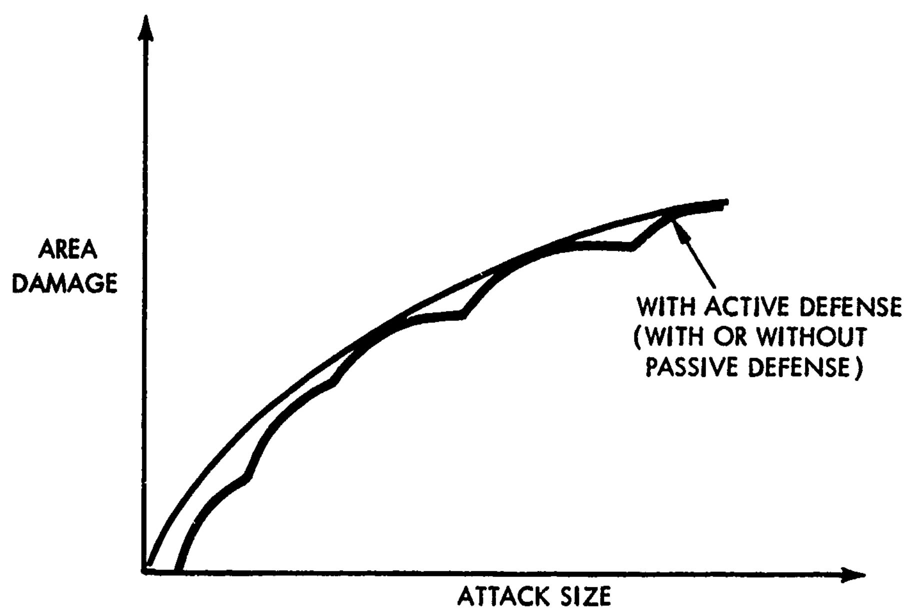






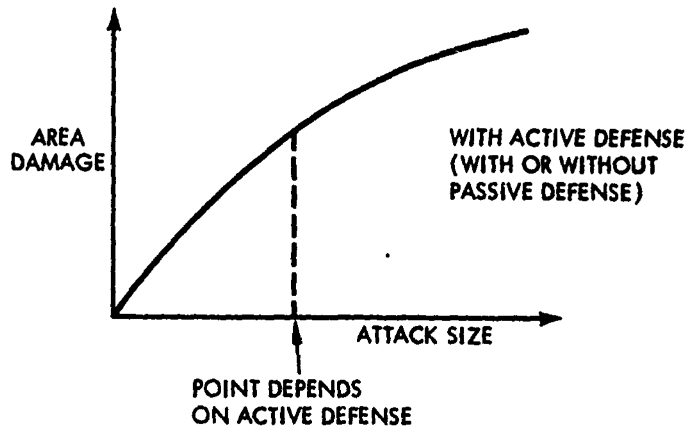
The treatment of effectiveness functions for offense forces in the integrating model essentially requires that the functions pass through the origin. Modifications can be made, but the assumption is very useful and important.

The third and fourth curves require some justification. For a large area, the attacker would attack defended cities in priority of payoff and achieve the following segmented damage curve, which can be approximated by a concave function.

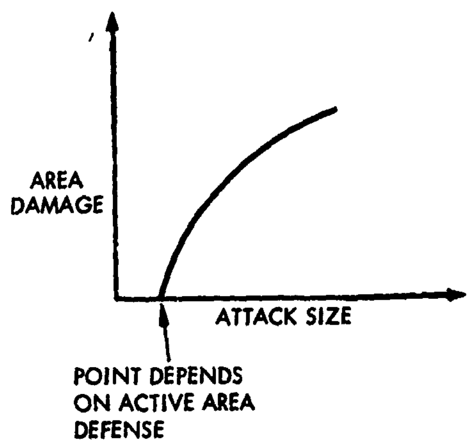


Also, with an optimally allocated active terminal defense, the marginal return of each attacking weapon is equal for attacks up to a certain size. For larger attacks, the marginal return is decreasing.[7] The damage curve in this case is as follows:

7. This argument is confirmed by L. Schmidt in an unpublished paper.



Continuing with the model at the level of an area comprised of a number of cities, if a subtractive active area defense is present, the damage function is as follows:



If there are numerous cities and subtractive active terminal defenses, attack optimization requires combinatorial treatment, and the offense optimization in the inside problem must be handled with a special algorithm. The procedures of P-801 are not sufficient.[8]

With several areas protected by subtractive active area defense, the combinatorial problems can be handled by considering combinations of areas to be attacked.

8. Bracken and McGill, Computer Programs for Solving Mathematical Programs, op. cit.

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