AD-753 264

ACOUSTIC RADIATION FROM A FREE FLOODED PIEZOELECTRIC RING TRANSDUCER OF RECTANGULAR CROSS SECTION

John L. Butler

Parke Mathematical Laboratories, Incorporated

Prepared for:

Navy Underwater Sound Laboratory

December 1966

DISTRIBUTED BY:

NTIS

National Technical Information Service
U. S. DEPARTMENT OF COMMERCE
5285 Port Royal Road, Springfield Va. 22151

Best
Available
Copy

Parke
[illegible]
Cop. #6

Technical Memorandum No. 3

Contract N00140 66 C 0172

December 1966

• • a/ • applied • mathematics • •

Acoustic Radiation From a Free Flooded Piezoelectric Ring Transducer of Rectangular Cross Section

by

John L. Butler

AD-753 264

for

U.S. Navy Underwater Sound Laboratory

PARKE MATHEMATICAL LABORATORIES, Inc.
One River Road • Carlisle, Massachusetts

• applied • mathematics • applied • mathematics •

Parke
15787
Cop. # 6

Technical Memorandum No. 3

Contract N00140 66 C 0172

December 1966

applied mathematics

Acoustic Radiation from a Free Flooded Piezoelectric Ring Transducer of Rectangular Cross Section

by

John L. Butler

for

U.S. Navy Underwater Sound Laboratory

PARKE MATHEMATICAL LABORATORIES, Inc.
One River Road • Carlisle, Massachusetts

Table of Contents

I. Introduction

The acoustic radiation from a free-flooded ring transducer of rectangular cross section depends not only on the oscillation of the mean diameter, but on the pulsation of the ring wall and height. On the expansion of the mean diameter a positive pressure is produced on the outer surface while a negative pressure is produced on the inner surface. These two pressures tend to cancel one another. Because of this, the pressure arising from other portions of the ring can not, in general, be neglected. The object of this report is to present a simple model which easily accommodates these modes of vibration which are coupled through the piezoelectric state equations. A further object is to incorporate the piezoelectric coupling into the torus model developed by Parke and Sherman[1,2].

The new model presented will be referred to as the ring model. This model is derived in an elementary fashion by replacing the separate vibrating surfaces of a ring-transducer of rectangular cross section by separate thin rings with source strengths equal to the strengths of the respective vibrating surfaces. The total field is then obtained by superposing the field from each ring. Although the model is approximate, there is the advantage that the motion of the separate surfaces may be directly related to the piezoelectric equations which describe the motion of the actual physical model.

A comparison of the far-field expressions for the ring and torus models is made and it is seen that the expressions are quite similar, but not, indeed, identical. A comparison is also made between the results calculated from the ring model, and actual measurements made by McMahon[3] on a piezoelectric free flooded transducer. It is found that this model yields a favorable description of the far field pattern not only at ring resonance but at cavity resonance.

II. Development of the Ring Model

In this section we shall derive the equation for the far-field pressure of the ring model. A vibrating ring-transducer of small cross section will be approximated by four rings two separated by the wall thickness b and two separated by the height h. The strength of these rings will be set equal to the strengths of the corresponding surfaces and the piezoelectric equation will be used to couple the motions of the surfaces to the motion of the mean radius a. It will be assumed that the presence of one ring does not affect the field of the other, and that the total field is simply due to the superposition of the fields of all the individual rings.

Now the far-field pressure response $p(r,\theta)$ for a thin ring of radius ρ vibrating with a uniform sinusoidal pulsation of the cross section may be shown to be;

$$p(r,\theta) = \frac{i\rho_0 c k}{4\pi}\,\frac{e^{-ikr}}{r}\,Q(t)\,J_0(k\rho\sin\theta) \qquad (1)$$

where ρc is the characteristic impedance of the medium, k is the wave number of the medium, J_0 is the zeroth order Bessel function, r is the distance to the far field point from the center of the ring and θ is the angle between r and the axis of the ring. The "source strength" $Q(\rho)$ is the product of the normal velocity and the surface area of the ring. We have written Q as $Q(\rho)$ to emphasize that it is a function of the radius.

We will now superpose solutions of the above form to obtain a new model for the transducer. Let the actual physical model be as shown in Fig. 1. The ring representation is shown below in Fig. 2. The rings i, t, o, b will be used to represent the motions of the inside, top, outside, and bottom of the actual physical model. The strengths of the rings will be taken to be equal to the strengths of the corresponding surface strengths of the actual physical model. Thus

$$Q_i\,(a - b/2) = v_i\, 2\pi\, (a - b/2)\, h \tag{2a}$$

$$Q_t\,(a) = v_t\, 2\pi\, a b \tag{2b}$$

$$Q_o\,(a + b/2) = v_o\, 2\pi\, (a + b/2)\, h \tag{2c}$$

$$Q_b\,(a) = v_b\, 2\pi\, a b \tag{2d}$$

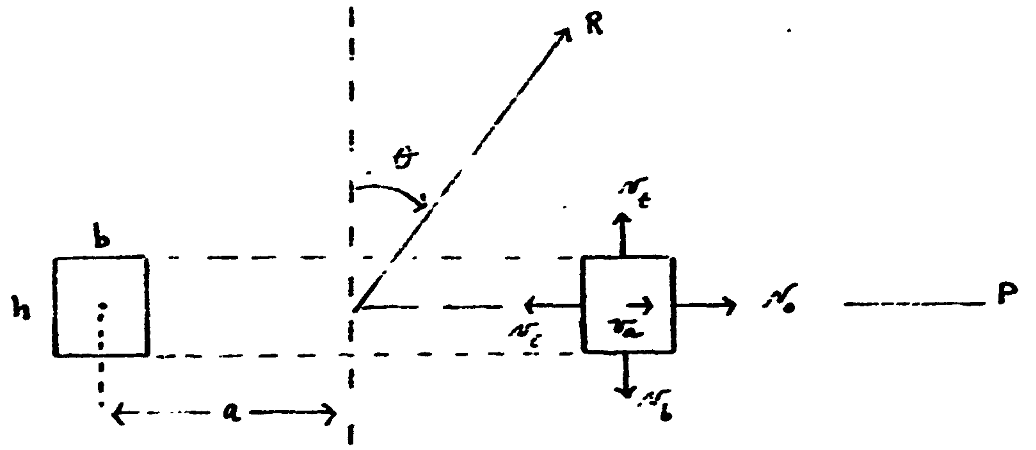

Figure 1. Cross-Section of Actual Physical Model

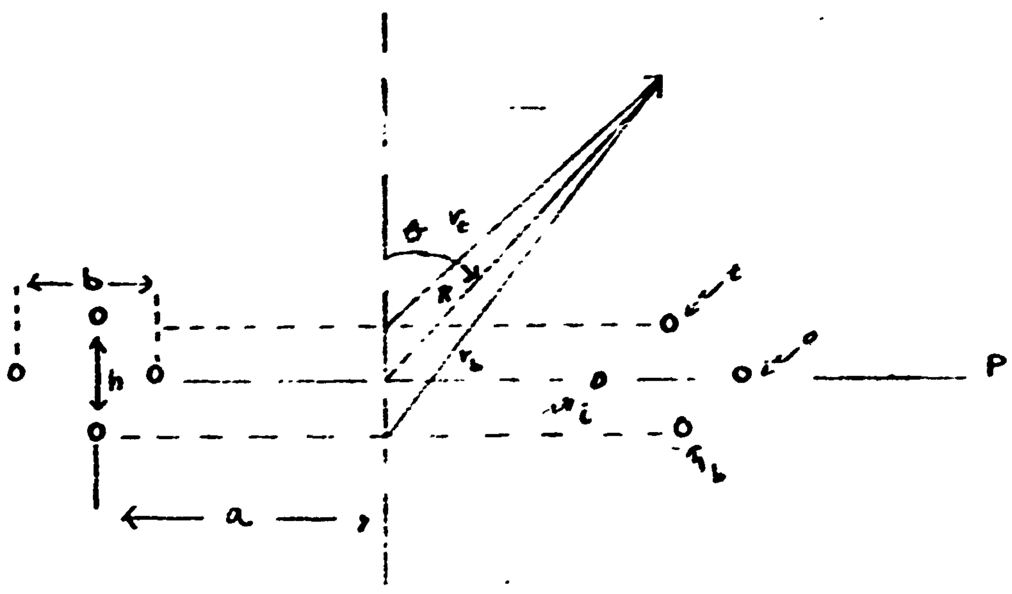

Figure 2. Cross Section of Ring Model

Now the rings i and o are centered in the same corresponding plane P as the actual physical model; thus, for these rings $r = R$. Since the pressure $p(r, \theta)$ is to be evaluated in the far field, r_t, R, and r_b are nearly parallel; and hence,

$$r_t \approx R - \frac{h}{2} \cos \theta \tag{3a}$$

$$r_b \approx R + \frac{h}{2} \cos \theta \tag{3b}$$

where the above expressions are to be used in the phase, and $r_t \approx r_b \approx R$ is to be used in the amplitude of Eqs. (2b) and (2d).

Hence for the i, t, o and b rings we have the respective pressures

$$p_i = \frac{i \rho_0 c k}{2} \frac{e^{-ikR}}{R} v_i h (a - b/2) J_0 \{k(a - b/2) \sin \theta\} \tag{4a}$$

$$p_t = \frac{i \rho_0 c k}{2} \frac{e^{-ikR}}{R} v_t b a J_0 \{ka \sin \theta\} e^{i \frac{kh}{2} \cos \theta} \tag{4b}$$

$$p_o = \frac{i \rho_0 c k}{2} \frac{e^{-ikR}}{R} v_o h (a + b/2) J_0 \{k(a + b/2) \sin \theta\} \tag{4c}$$

$$p_b = \frac{i \rho_0 c k}{2} \frac{e^{-ikR}}{R} v_b b a J_0 \{ka \sin \theta\} e^{-i \frac{kh}{2} \cos \theta} \tag{4d}$$

and the total pressure p_T is the sum.

The summation of the above equations can be reduced to a simpler form by considering the velocity distributions v_i, v_t, v_o and v_b. We note first that because of symmetry, the top and bottom velocities will be equal in magnitude and phase. With the notation v_H for these velocities (v_t and v_b) the total pressure contribution from the top and bottom vibrations is

$$p_t + p_b = i\rho_0 c k \frac{e^{-ikR}}{R}\, b a\, v_H\, J_0(ka \sin\theta)\cos\left(\frac{kh}{2}\cos\theta\right) \tag{5}$$

The net velocity on the outside surface v_o and the net velocity on the inside surface v_i may be decomposed in the following manner. On expansion of the mean circumference, the mean radius a moves outward with velocity v_a (See Fig. 1). In addition to this motion there is the coupled vibration $v_H = v_t = v_b$, as noted, and the expansion and contraction of the thickness b of the ring. Calling this motion v_B we then may write

$$v_o = v_B + v_a \tag{6a}$$

$$v_i = v_B - v_a \tag{6b}$$

where the direction of v_B is outward from the cross section. With this notation the total pressure contribution from the inside and outside surfaces is

$$p_i + p_o = i\rho_0 c k \frac{e^{-ikR}}{R}\, v_B \frac{h}{2}\left\{(a+b/2)J_0[k(a+b/2)\sin\theta] + (a-b/2)J_0[k(a-b/2)\sin\theta]\right\}$$

$$+ i\rho_0 c k \frac{e^{-ikR}}{R}\, v_a \frac{h}{2}\left\{(a+b/2)J_0[k(a+b/2)\sin\theta] - (a-b/2)J_0[k(a-b/2)\sin\theta]\right\}. \tag{7}$$

The first term is due to the pulsation of the wall of the ring, while the second term is due to the oscillation of the wall. The Bessel functions may be expanded by the addition formula[4]

$$J_0\left[k\left(a \pm \tfrac{b}{2}\right)\sin\theta\right] = \sum_{n=0}^{\infty} \epsilon_n J_n(ka\sin\theta)\, J_n\left(\pm \frac{kb}{2}\sin\theta\right) \qquad (8)$$

$$\epsilon_0 = 1 \quad , \qquad \epsilon_{n\neq 0} = 2$$

and, thus for $b \ll a$ and $kb \ll 1$ we have

$$p_i + p_o \simeq i\rho_0 c k \frac{e^{-ikR}}{R} v_B \pi a\, J_0[ka\sin\theta]$$

$$+ i\rho_0 c k \frac{e^{-ikR}}{R} v_a \frac{\pi b}{2}\left\{J_0(ka\sin\theta) - ka\sin\theta\, J_1(ka\sin\theta)\right\}. \qquad (9)$$

Collecting terms, we then have the total pressure $p_T = (p_t + p_b) + (p_i + p_o)$. That is,

$$p_T = i\, v_a \rho_0 c k \frac{bh}{2} \frac{e^{-ikR}}{R} \left\{A(\theta) J_0(ka\sin\theta) - ka\sin\theta\, J_1(ka\sin\theta)\right\} \qquad (10a)$$

where

$$A(\theta) = 1 + \frac{v_B}{v_a}\frac{2a}{b} + \frac{v_H}{v_a}\frac{2a}{h}\cos\left(\frac{kh}{2}\cos\theta\right). \qquad (10b)$$

In this report we shall confine our attention to the far-field angular response rather than an absolute determination of the pressure level. The quantity of interest will then be the far field directivity function R_n normalized at $\theta = 90^{\circ}$; that is

$$R_n = p_T(\theta) / p_T(\theta = 90^{\circ}) . \tag{11}$$

Our analysis has led us to the point where we must now evaluate the velocity ratios v_B / v_a and v_H / v_a for a complete determination of the function $A(\theta)$.

III. Piezoelectric Coupling Between the Modes of Vibration

In this section we shall determine the relations between the expansion of the circumference, the thickness, and the height of the ring. From this analysis the velocity ratio will be determined for the complete descriptions of the directivity function R_n. Since a description of the pertinent modes of vibration has been given by R. S. Woollett[5], we need only relate his results to our model.

Before considering piezoelectric effects, let us suppose the ring to be a simple mechanical vibrator isotropic in the circumferential, thickness, and height directions. With Δ denoting the total extension in length the velocity ratios may be related to Poissonb ratio σ in the following fashion. With Poisson's ratio defined as

$$\sigma = - \frac{\Delta h / h}{\Delta(2\pi a)/2\pi a} = - \frac{\Delta b / b}{\Delta a / a} \tag{12}$$

we then have

$$v_H / v_a = \Delta h / 2\Delta a = - \frac{b}{2a} \sigma \tag{13a}$$

$$v_B / v_a = \Delta b / 2\Delta a = - \frac{h}{2a} \sigma . \tag{13b}$$

Thus, for this case we have from Eq. (10b) that

$$A(\theta) = \left[1 - \sigma - \sigma \cos\left(\frac{kh}{2} \cos\theta \right) \right]. \tag{14}$$

For $\frac{kh}{2}$ extremely small Eq. (10a) may then be written as

$$p_T = i\, v_a\, \rho_0 c\, k\, b\, \frac{h}{2}\, \frac{e^{-ikR}}{R} \left\{ (1 - 2\sigma) J_0(ka \sin\theta) - ka \sin\theta\, J_1(ka \sin\theta) \right\} \tag{15}$$

It is seen that, for this approximation, the directivity function is independent of the height h and the thickness b .

In the more general case of an anisotropic piezoelectric ring the coupling to the thickness mode may be different than the coupling to the height mode and will depend on the elastic moduli and the piezoelectric d constants. Defining σ_B and σ_H as piezoelectric Poisson's constants,

$$\sigma_B \equiv -\frac{\Delta b}{b}\,\frac{a}{\Delta a} \qquad \sigma_H \equiv -\frac{\Delta h}{h}\,\frac{a}{\Delta a} \tag{16}$$

and noting that

$$v_B / v_a = \Delta b / 2\Delta a \;, \qquad v_H / v_a = \Delta h / 2\Delta a \tag{17}$$

it may be seen that

$$A(\theta) = 1 - \sigma_B - \sigma_H \cos\left(\frac{kh}{2} \cos\theta \right). \tag{18}$$

Thus with a description of σ_B and σ_H the far-field pressure would be determined by Eqs. (10a) and (18).

From the equations developed in R. S. Woollett's notes the relations for σ_B and σ_M may be written for the cases of a circumferential field, radial field, and axial field (field directed between the top and bottom surfaces). These results are summarized in Table I. For convenient reference we have listed in Table II the material constants for Clevite Ceramic B, PZT-4, and PZT-5. The approximate values for Nickel are also given.

In the development of the equations displayed in Table I it was assumed that the medium offered no reaction and that the finite "Q" was due to the internal losses in the ring (and mounting). These equations then would be most applicable for the ring operating in air. For the cases where the radiation loading is not negligible (as in underwater operations) we shall assume the equations to be approximately true and interpret f_r to be the mean resonant frequency (where applicable) and interpret "Q" to be the measured value from a frequency response curve.

The Piezoelectric equations given in the previous section may also be incorporated into the basic far field radiation equation for the torus developed by Sherman and Parke[1,2]. This equation may be written in terms of the present notations as

$$p_T = i U_1 \rho_0 c k R_1 \pi R_1 \frac{e^{-ikr}}{R} \left\{ B J_0(ka \sin\theta) - ka \sin\theta \, J_1(ka \sin\theta) \right\} \tag{19a}$$

where $$B = \frac{(ka)^2}{3} - \frac{1}{2} - \sigma . \tag{19b}$$

In the development of the above equations σ is the Poisson's ratio for a long thin isotropic circular rod. Following the same analysis but

Table I Equations for σ_B and σ_H

Radial Field (ceramic ring):

$$\sigma_B = -d_{33}/d_{31} + \gamma (f/f_r)^2 \left(d_{33}/d_{31} - s_{13}^E / s_{11}^E \right)$$

$$\sigma_H = -1 + \gamma (f/f_r)^2 \left(1 - s_{12}^E / s_{11}^E \right) \qquad \text{(Ia)}$$

Axial Field* (ceramic ring):

$$\sigma_B = -1 + \gamma (f/f_r)^2 \left(1 - s_{12}^E / s_{11}^E \right)$$

$$\sigma_H = -d_{33}/d_{31} + \gamma (f/f_r)^2 \left(d_{33}/d_{31} - s_{13}^E / s_{11}^E \right) \qquad \text{(Ib)}$$

Circumferential Field (segmented ceramic ring):

$$\sigma_B = \sigma_H = -d_{31}/d_{33} + \gamma (f/f_r)^2 \left(d_{31}/d_{33} - s_{13}^E / s_{33}^E \right) \qquad \text{(Ic)}$$

Circumferential Field (magnetostrictive scroll):

$$\sigma_B = \sigma_H = -d_{31}/d_{33} + \gamma (f/f_r)^2 \left(d_{31}/d_{33} - s_{13}^H / s_{33}^H \right) \qquad \text{(Id)}$$

where f_r is the resonant frequency and $\gamma = 1 - \frac{j}{Q}$

*This case was added for completeness.

Table II Material Constants

Material	d_{31}/d_{33}	s_{12}^E/s_{11}^E	s_{13}^E/s_{11}^E	s_{13}^E/s_{33}^E
Ceramic B	-.392	-.302	-.314	-.297
PZT-4	-.425	-.329	-.431	-.342
PZT-5A	-.457	-.350	-.440	-.384
Nickel*	≈ -.5	---	---	≈ -.3

*These are the approximate values suggested by Woollett and are for the case of a constant magnetic field H rather than a constant electric field $\mathcal{E}$.

considering a long thin anisotropic rod of rectangular cross section we find, through comparison, that

$$2\sigma = -\frac{\Delta b}{b}\,\frac{a}{\Delta a} - \frac{\Delta h}{h}\,\frac{a}{\Delta h} = \sigma_B + \sigma_H \,. \qquad (20)$$

Thus, the equations in Table I may be incorporated into the torus model equation through the use of Eq. (20).

If we compare the equations for the torus model [Eqs. (19a), (19b), (20)] with the equations for the ring model [Eqs. (10a) and (10b)] it may be seen that there is a close similarity. The basic difference seems to be between the fractions $A(\theta)$ and B. The function $A(\theta)$ shows, in a simple fashion, that the far-field pressure depends on the height. For the case where kh is extremely small both B and $A(\theta)$ are equal for

$$(ka)^2/3 = 3/2 - (\sigma_B + \sigma_H)/2 \,.$$

With $(\sigma_B + \sigma_H)/2 \approx 1/3$ both A and $\bar{B}$ would be equal for $ka = 1.87$ which is close to the value for resonant operation of most ceramic ring transducers. Thus, we would expect that both models would give nearly the same results for resonance operations.

IV. Comparison With Measurements

A comparison of the theoretical and measured far-field response for the transducer described by McMahon[3] has been made. A comparison at ring resonance and cavity resonance was previously made using the torus model with an assumed simple mechanical coupling between the oscillation of the mean circumference and the poisition of the ring cross section. The theoretical model we shall use is the ring model discussed in Sections II and III of this report.

The transducer described by McMahon was poled radially. Although it was claimed to be a PZT-4 material, it was felt that the measured Young's modulus indicated a more compliant material such as PZT-5A. Accordingly, we used the PZT-5A constants in the equation for the far-field pattern given by Eqs. (11) and (16a) along with Eqs. (16) and (1a).

The two unknowns in Eq. (1a) are ζ^* and f_r. For the purposes of this initial comparison we assumed $\zeta^* \gg 1$; and therefore, let ρ^* equal to unity in Eq. (1a). The value of f_r was then chosen to be equal to the average of the ring resonant frequency 26 Kc and the cavity resonant frequency 18 Kc as measured by McMahon.

In Figs. 3 and 4 the experimental and theoretical curves may be compared at ring resonance 26 Kc and cavity resonance 18 Kc respectively. Here we chose f_r to be 22 Kc and $\rho^* = 1$ as discussed. It is seen that the agreement is quite good not only at ring resonance but at cavity resonance.

In order to evaluate the effectiveness of the piezoelectric coupling, a calculation was made at cavity resonance using the ring model with only the mechanical coupling as described by Eq. (15). The value of σ used was 0.367 as determined from Table II. As seen in Fig. 5, this simpler mechanized model does not compare as favorably as the model with the piezoelectric coupling.

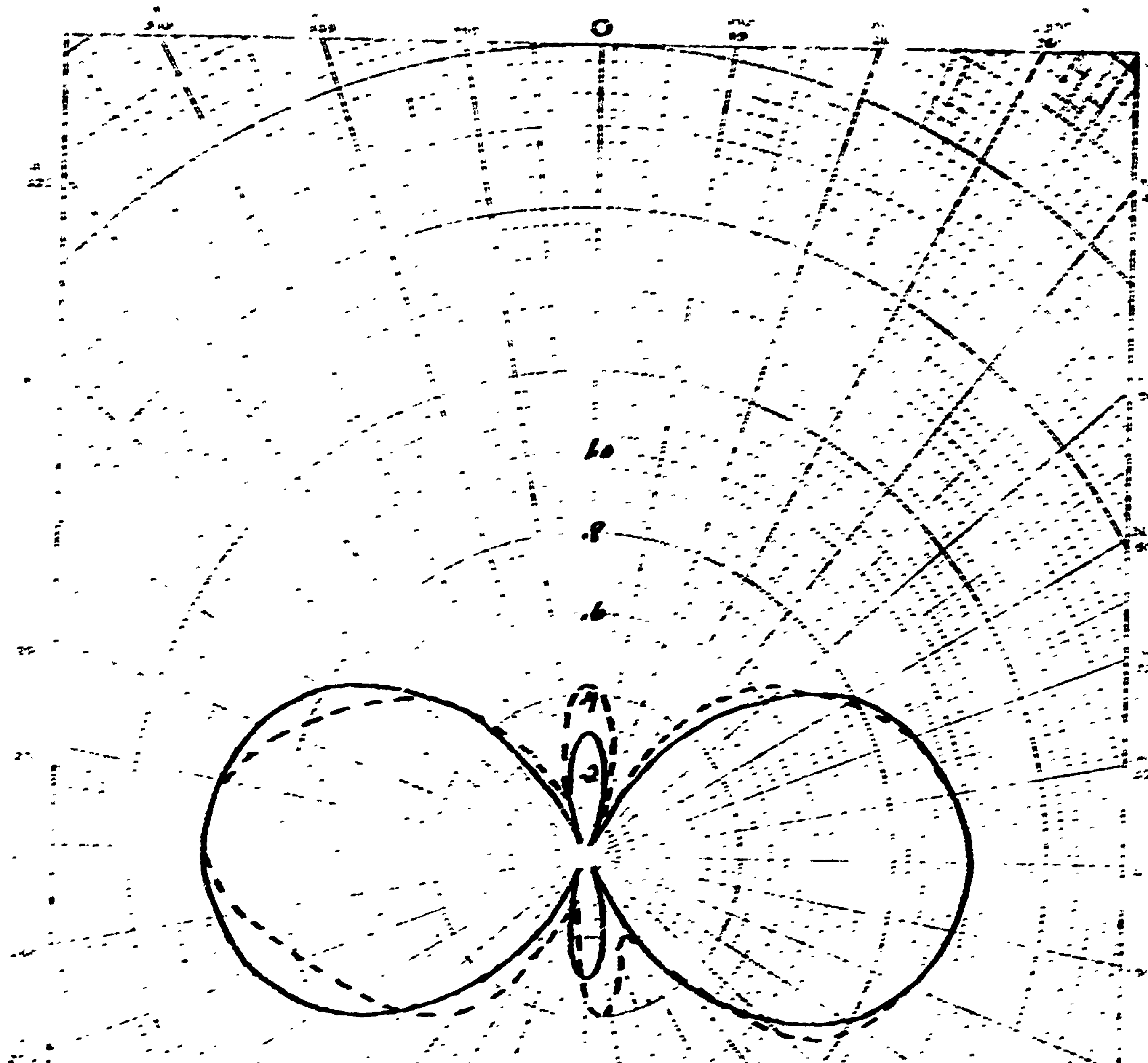

Fig. 3 Pressure Amplitude at Ring Resonance (26 kc)

- - - - Experimental

——— Ring Theory, with Piezo-coupling

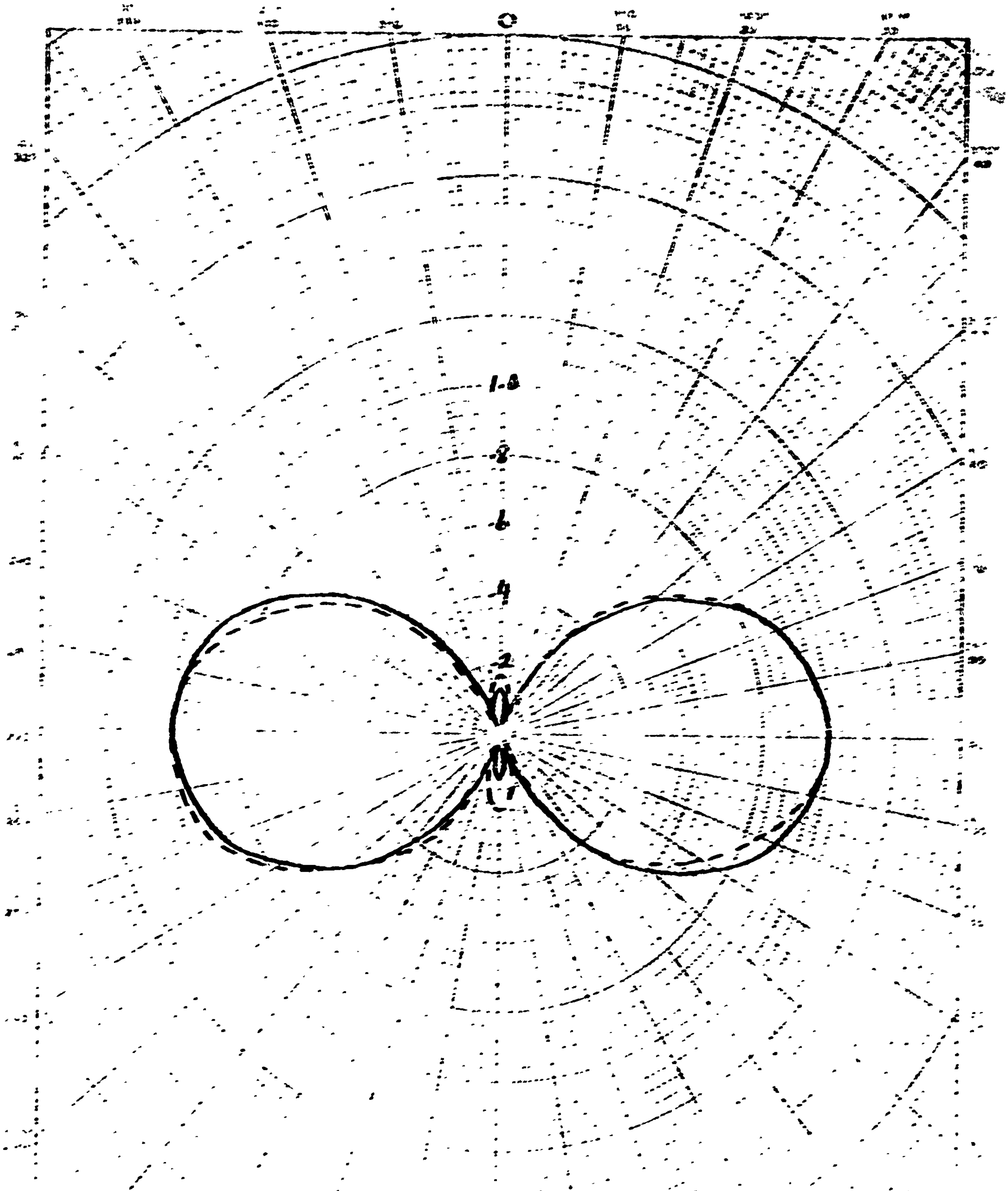

Fig. 4 Pressure Amplitude at Cavity Re-ance (18kc)

— — — Experimental

——— Ring Theory with Piezo-coupling

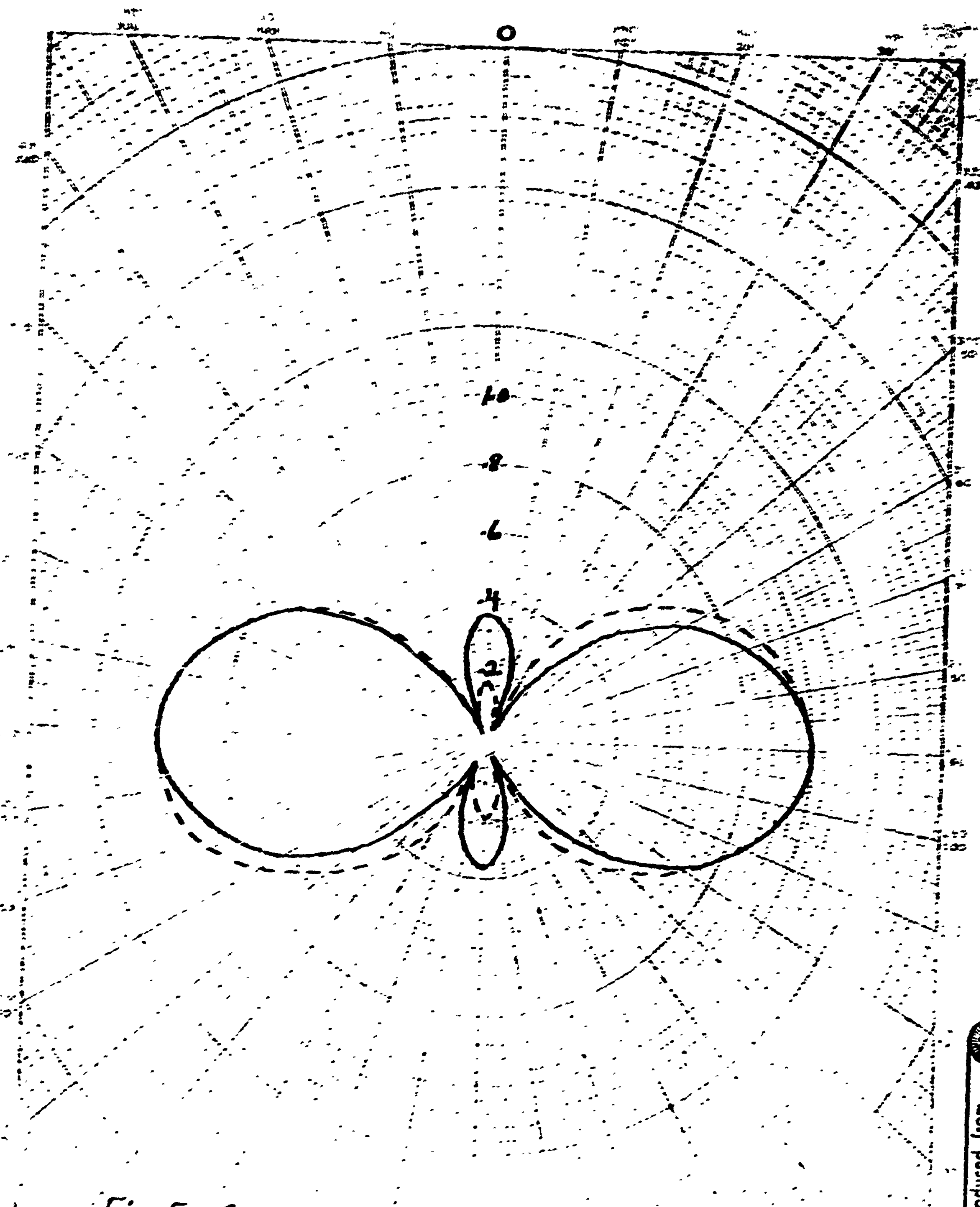

Fig. 5 Pressure Amplitude at Cavity Resonance (18kc)

— — — Experimental

——— Ring Theory with Mechanical Coupling

V. Summary and Discussion

In this report we have presented a new theoretical model for the acoustic radiation from a free flooded piezoelectric ring transducer. Although approximate, the model does yield a simple and fairly accurate description of the far-field directivity function. A comparison with McMahon's data showed that, at least in this case, the inclusion of the piezoelectric coupling effects yielded a better agreement at cavity resonance than with the inclusion of only mechanical coupling effects. The agreement at cavity resonance is also better than previously obtained with the torus model[1,2].

Clearly, the theory has its short-comings, and a limited range of applicability. It was assumed that the vibrating surface may be represented by simple vibrating rings. Thus, we did not take into account the fact that the radiation from inner and outer surfaces is, in general, dependent on the height and wall thickness. It would seem, therefore, that the theory presented would be most applicable to short, thin-walled rings. Another deficiency is involved in the coupled piezoelectric equations in the sense that in their present form assumed values of "Q" and resonance must be used.

In the future, we hope to compare the theory with more experimental data before adding refinements. One way will be to make a comparison with measurements on different free-flooded ring transducers; that is, transducers excited in different ways, and transducers of the same diameter and wall thickness but of varying height. This comparison should yield a good indication of the limitations of the present simple theory. Another test will be to compare the experimental and theoretical far-field pressure frequency response. The present theory can be easily adapted to yield such a prediction by relating the velocity σ_a to the input voltage.

It appears that the most difficult region to obtain a good prediction of the far-field pressure will be in the vicinity of the axis of the cylinder. In this region the oscillations of the mean circumference contributes little to the pressure, and consequently, the radiation from the height and wall thickness will be comparatively significant. Thus, here the relative magnitudes of the vibrations will be important. The manner of excitation (direction of remanent polarization) along with the values of the physical constants will indeed influence the radiation in this region.

Although we have recently acquired some new* data on free-flooded electrostrictive rings, no data in addition to that previously reported on free-flooded magnetostrictive scrolls seems to be available. We would like to suggest that measurements be made on a scroll with a square cross-section that is considerably less than one-fourth of a wavelength. A complete set of data including impedance loops, calibrated frequency responses (at $\theta = 0^{\circ}$ and 90°), and directivity patterns would be most helpful. Possibly a small - high frequency model would lend itself most easily to a precisely controlled laboratory set up and measurement.

*Including considerable unpublished data kindly given to us by Harris A.S.W., Westwood, Mass.

References

1. C.H. Sherman and H.G. Parke, Parke Mathematical Laboratories, Inc. Scientific Report No. 2, Contract NONR 1140(70X64)T5[illegible]B December 1964

2. C.H. Sherman and H.G. Parke, J. Acoust. Soc. Am. 38, 715-722 (1965)

3. G.W. McMahon, J. Acoust. Soc. Am. 36, 5[illegible]-533 (1964)

4. W. Magnus and F. Oberhettinger, "Special Functions of Mathematical Physics" Chelsea Publishing Co., New York, 1949, p.21

5. R.S. Woollett, "Vibrations of a Thin Short Toroid of Rectangular Cross-Section". Unpublished notes, August 15, 1966

6. C.H. Sherman and B.T. Howard, Parke Mathematical Laboratories, Inc. Technical Memorandum No. 1, Contract N00140 66C 0172 (March 1966).