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13. ABSTRACT

An analytical study of the two-dimensional viscous, incompressible steady flow over an airfoil of arbitrary shape was made. Theodorsen's method was used to analyze the potential flow around the airfoil, providing edge velocities for the boundary layer equations, which were then solved by the Karman-Pohlhausen method. The resulting boundary layer displacement thickness was then added to the original airfoil shape to obtain a better potential flow solution. Iteration was continued in this manner until the desired accuracy was obtained. A computer program was written to effect this airfoil analysis technique. Potential flow surface velocity distribution, angle of attack at zero lift, and wall shearing stress were shown to agree well with results of other investigators.

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AN ANALYSIS OF THE FLOW FIELD AROUND A 2-D BODY OF ARBITRARY SHAPE

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

by

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List of Symbols

For Theodorsen's Method

Symbol	Description
a	Dimensional proportionality constant (length)
A_n, B_n, C_n	Fourier coefficients
i	$\sqrt{-1}$
m	Intermediate quantity in Joukowski transformation, defined on page 9
R	Radius vector in exact circle plane
u	Velocity in x-direction
V	Potential flow surface velocity
V_∞	Freestream velocity
v	Velocity in y-direction
w	Complex potential function
x	Cartesian coordinate in freestream flow direction
y	Cartesian coordinate perpendicular to freestream flow direction
z	Complex position vector in exact circle plane
z'	Complex position vector in pseudo-circle plane
α	Angle of attack
β	Angle of attack at zero lift
Γ	Circulation
ε	Conformal angular distortion function
ζ	Complex position vector in airfoil plane
θ	Angular measure in exact circle plane
λ	Dummy variable of integration corresponding to θ

Symbol	Description
ρ	Radius vector in pseudo-circle plane
Φ	Potential function
ϕ	Magnification exponent in exact circle plane
Ψ	Stream function
ψ	Magnification exponent in pseudo-circle plane, as a function of ω
$\bar{\psi}$	ψ, as a function of θ
ω	Angular measure in pseudo-circle plane

For Karman-Pohlhausen Method

Symbol	Description
a,b,c,d	Coefficients in assumed velocity profile
F, f_1, f_2, Z, K	Boundary layer parameters
U	Boundary layer edge velocity
x	Streamwise coordinate
y	Cross-stream coordinate
δ	Boundary layer thickness
δ_1	Boundary layer displacement thickness
δ_2	Boundary layer momentum thickness
η	Cross-stream coordinate referenced to boundary layer thickness, y/δ
Λ	Shape factor
μ	Absolute viscosity
ν	Dynamic viscosity
ρ	Fluid density
τ_w	Shear stress at airfoil surface
o	Subscript indicating stagnation point

Symbol	Description
1	Subscript indicating first point downstream of stagnation point
n	Subscript indicating general point

AN ANALYSIS OF THE FLOW FIELD AROUND A 2-D BODY OF ARBITRARY SHAPE

I. Introduction

Purpose

The purpose of this study was to determine the velocity field and, therefore, the pressure distribution about a body of arbitrary shape in a two-dimensional, steady, incompressible, constant viscosity flow, utilizing a combination of Theodorsen's method and the Karman-Pohlhausen method. A computer program to implement this combined method has been written, and results from this program are described herein.

Some practical and theoretical considerations leading to this combined method are presented in the remainder of this section.

Background

The theoretical analysis or design of an airfoil shape is based primarily upon a knowledge of the velocity field in the neighborhood of the body. Since the time of Prandtl, aerodynamicists have realized that the flow over a body could be divided, for purposes of analysis, into two regions. Observations indicate that viscous forces predominate over inertia forces in a very thin layer near the surface of the body, and this region has come to be called the boundary layer. Outside this region, the flow may, with little error, be considered inviscid. Since inviscid flow with a uniform parallel onset velocity may be described in terms of a velocity potential, the inviscid region is also called the region of potential flow. If the velocity field can be determined in the inviscid flow, then the velocity at the edge of the boundary layer will be known and may be used as a boundary condition for solution of the boundary layer equations. Each of these regions will now be considered.

Region of Inviscid Flow

For thin streamlined airfoils at low angles of attack, small perturbation theory permits a number of simplifying assumptions in the equations of motion for inviscid flow. However, no such simplified potential theory was available for airfoils of arbitrary shape until 1933, when Theodorsen's method (Ref 2) was published.

Theodorsen made use of the fact that any closed curve, such as an airfoil, defined in the complex plane, may be transformed mathematically into a circle. This transformation may be further required to be conformal; that is, to preserve the local angular relationship between lines passing through each point. Conformality, therefore, ensures that the angle of attack of the airflow is unaffected by the transformation. The problem then becomes that of analyzing the two-dimensional flow over a circle. Since this is one of the very few cases for which exact solutions to the equations of motion are known, the velocity at each point on the circle may be obtained immediately. Then, through an inverse transformation, the potential flow velocity at the corresponding point on the surface of the airfoil may also be determined. This velocity may be used as an approximation, (since boundary layer thickness has not yet been accounted for) to the velocity at the edge of the boundary layer.

Straightforward though this method appears, its use before the advent of high-speed computers was limited in practice by the enormity of the task of solving the necessary integral equations iteratively to the desired accuracy. The transformation is very time-consuming and tedious when performed by hand, providing numerous opportunities to make minor mathematical errors which may invalidate subsequent work. Since these are precisely the disadvantages in computation that a digital computer is designed to overcome, Theodorsen's method lends itself well to formulation for machine use.

The rate at which the iterative solution converges is largely dependent upon how nearly the original airfoil approximates a circle. Therefore, Theodorsen recommends the use of an intermediate analytical transformation (the Joukowski transformation) which converts the airfoil shape into a pseudo-circle conformally, and, thereby, reduces the computation time required.

Boundary Layer Region

The boundary layer equations may be derived from the Navier-Stokes equations for viscous fluid motion under the assumptions of steady, incompressible, constant viscosity flow at large Reynold's numbers. The Karman-Pohlhausen method (Ref 3) of solving the boundary layer equations is based on the further assumption that the velocity profile may be adequately represented as a fourth-degree polynomial satisfying appropriate boundary conditions, including the edge velocity distribution previously found from Theodorsen's method.

Combined Flow Fields

The boundary layer flow determined from the Karman-Pohlhausen method is only a first approximation, because a body with a boundary layer appears to the potential flow to be thicker by the amount of the displacement thickness. Therefore, the displacement thickness of the boundary layer must be computed for the approximate edge velocity; then this thickness must be added to the actual thickness of the body; and the inviscid velocity field for the resulting shape determined by Theodorsen's method. Thus, the velocity field solutions are continually refined until the change in displacement thickness from one iteration to the next is negligible. At this point, the problem of finding the velocity field may be considered solved. With the velocity field known, other aerodynamic parameters such as lift, drag, circulation, and separation point may also be computed, using standard methods.

II. Theodorsen's Method

Method of Presentation

Theodorsen's method of irrotational flow analysis depends upon several important results of potential theory and the theory of conformal transformations of complex functions. These results will be briefly summarized before a detailed presentation of the Joukowski and Theodorsen transformations is given. Finally, the process for obtaining surface velocities by an inverse transformation will be presented.

Review of Potential Theory

The flow outside the boundary layer is assumed to be steady, irrotational, and incompressible, permitting its description in terms of a velocity potential ϕ or a stream function Ψ, defined so that:

$$
\begin{aligned}
u &= \frac{\partial \phi}{\partial x} = \frac{\partial \Psi}{\partial y} &\quad& \text{velocity in x-direction} \\
v &= \frac{\partial \phi}{\partial y} = -\frac{\partial \Psi}{\partial x} &\quad& \text{velocity in y-direction}
\end{aligned}
\tag{1}
$$

The lines of constant Ψ (streamlines) and lines of constant ϕ (equipotential lines) are orthogonal families of curves.

Any functions of ϕ and Ψ which satisfy Laplace's equation, $V^2\phi = V^2\Psi = 0$, represent possible types of fluid motion. The boundary conditions of impermeability of the airfoil surface and uniform flow at infinity define a unique solution.

Conformal Transformation

If $w(\zeta) = \Phi(x,y) + i\Psi(x,y)$ is an analytic function of the complex variable $\zeta = x + iy$, and the components Φ and Ψ satisfy Laplace's equation, then certain useful relations exist between the ζ-plane and the w-plane. Specifically, any simple curve $f(\zeta)$, such as an airfoil surface, maps into a curve $f(w)$ in the w-plane in such a manner that the angles between pairs of lines passing through any point in the ζ-plane remain unchanged at the image point in the w-plane, although local rotations and magnifications may occur. Therefore, the streamlines and equipotential lines of the ζ-plane remain orthogonal in the plane of transformation. An exception occurs at points where $\frac{dw}{d\zeta} = 0$. These points correspond to flow stagnation points in the exact circle plane, and also in the airfoil plane, unless the trailing edge is cusped, in which case the airfoil has no trailing edge stagnation point.

Riemann has shown that the transformed curve may be specified without negating the conformal characteristic of the transformation. In particular, the interior of any simply-connected region can be mapped inside a circle, with the curve enclosing the region mapping onto the circumference of the circle. This transformation is unique when the origins and orientations of the coordinate systems in the two planes are specified.

Joukowski Transformation

The transformation $\zeta = z' + a^2/z'$, where the ζ-plane is the plane of the airfoil ($\zeta = x + iy$) is the Joukowski transformation. If the airfoil is elliptical or one of the special class of airfoils known as Joukowski airfoils, then a circle results in the z'-plane. If not, then a curve (pseudo-circle in Fig. 1) closely approximating a circle results for conventional airfoil shapes. While this transformation is not essential to Theodorsen's method, it improves the rate of convergence of the subsequent transformation. The constant "a" is included to preserve dimensions and has the dimension of length. The curve in the z'-plane can also be represented as $z' = ae^{\psi+i\omega}$ in polar coordinates, where $\rho = ae^{\psi}$ is the radius vector and the angular coordinate is ω. Substituting the polar form of z' into the transformation equation yields

$$\zeta = 2a \cosh (\psi + i\omega) \qquad (2)$$

or

$$\zeta = 2a \cosh \psi \cos \omega + 2ia \sinh \psi \sin \omega \qquad (3)$$

Noting that $\zeta = x + iy$, and equating real and imaginary parts, the airfoil coordinates may be found explicitly in terms of the transformation coordinates

$$x = 2a \cosh \psi \cos \omega$$

$$y = 2a \sinh \psi \sin \omega \qquad (4)$$

The inverse relations between coordinates will now be determined. Solving Eqs (4) for cosh ψ and sinh ψ and substituting into the identity $\cosh^2 \psi - \sinh^2 \psi = 1$ gives

$$\left(\frac{x}{2a \cos \omega}\right)^2 - \left(\frac{y}{2a \sin \omega}\right)^2 = 1 \qquad (5)$$

which can be solved for ω

$$\omega = \sin^{-1} \sqrt{1/2[m + \sqrt{m^2 + (y/a)^2}]} \qquad (6)$$

where

$$m = 1 - \left(\frac{x^2 + y^2}{4a^2}\right)$$

Likewise, Eqs (4) may be solved for cos ω and sin ω and substituted into the identity $\cos^2 \omega + \sin^2 \omega = 1$, giving

$$\left(\frac{x}{2a \cosh \psi}\right)^2 + \left(\frac{y}{2a \sinh \psi}\right)^2 = 1 \qquad (7)$$

so that,

$$\psi = \sinh^{-1} \sqrt{1/2[-m + \sqrt{m^2 + (y/a)^2}]} \qquad (8)$$

Theodorsen Transformation

The Theodorsen transformation $z' = z\, e^{\sum_{n=0}^{\infty} \frac{C_n}{z^n}}$ maps the pseudo-circle in the z'-plane into an exact circle (Fig. 1) in the z-plane. The coefficients C_n are complex numbers of the form $A_n + iB_n$. The condition that the flow at infinity

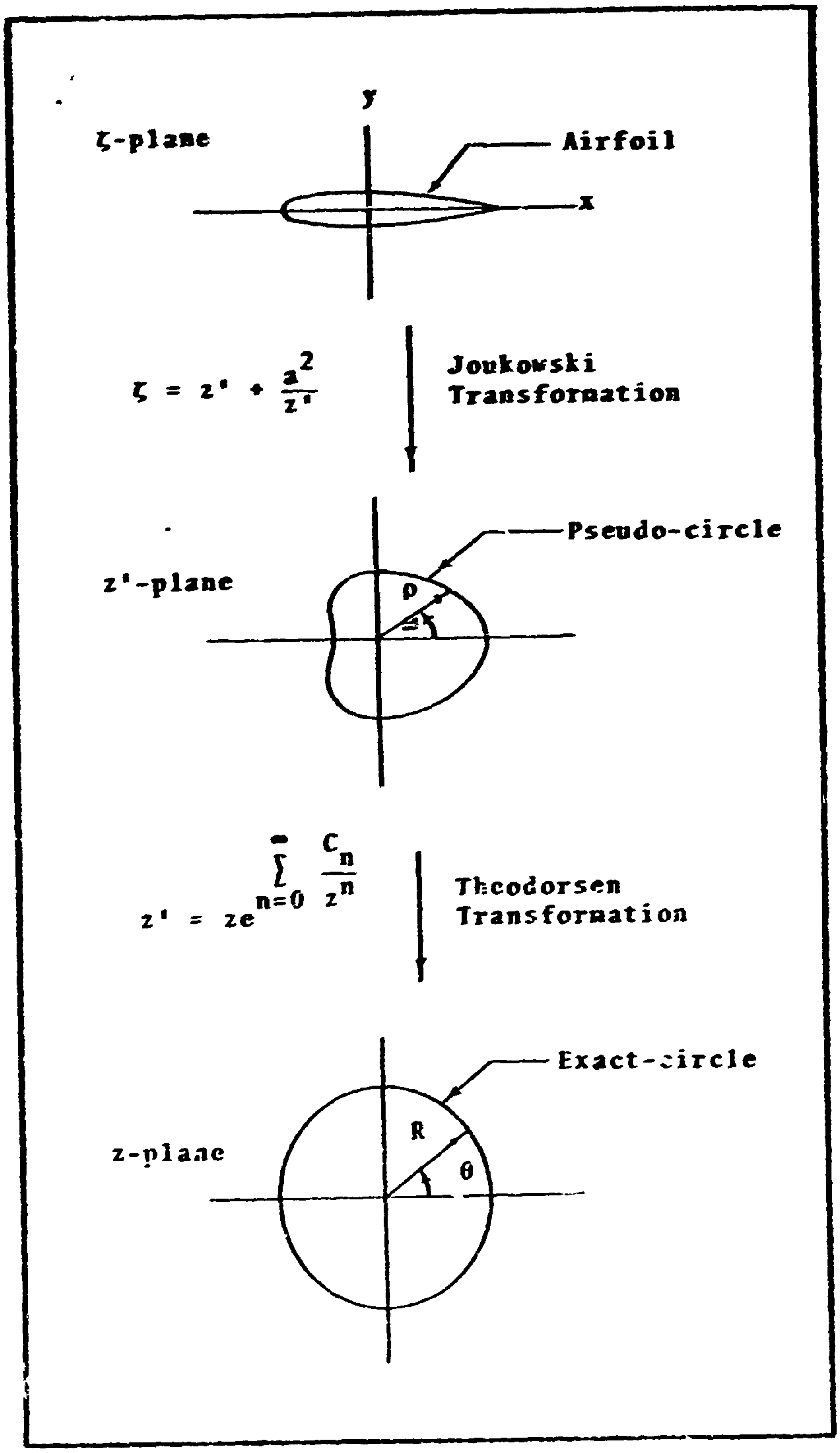

Fig. 1. Transformation Planes and Notation.

must be identical in airfoil and exact circle planes requires that $A_0 = B_0 = 0$. The transformation then becomes

$$z' = z\, e^{\sum_{n=1}^{\infty} \frac{C_n}{z^n}} = z\, e^{\sum_{n=1}^{\infty} \frac{A_n + iB_n}{z^n}} \qquad (9)$$

Suitable means must now be found for determining the constants A_n and B_n.

The equation for the exact circle may be expressed as $z = ae^{\phi+i\theta}$ in polar coordinates where ϕ is a constant and $R = ae^{\phi}$ is the radius vector and the angular coordinate is θ. Eliminating the constant "a" between the polar definitions of z and z' yields

$$z' = z\, e^{(\psi-\phi) + i(\omega-\theta)} \qquad (10)$$

Now, combining with the transformation equation

$$(\psi-\phi) + i(\omega-\theta) = \sum_{n=1}^{\infty} \frac{A_n + iB_n}{z^n} \qquad (11)$$

Note that

$$z^n = (ae^{\phi+i\theta})^n = [R(\cos\theta + i\sin\theta)]^n = \frac{R^n}{\cos n\theta - i\sin n\theta} \qquad (12)$$

Substituting into Eq (11)

$$(\psi-\phi) + i(\omega-\theta) = \sum_{n=1}^{\infty} \frac{A_n + iB_n}{R^n} (\cos n\theta - i \sin n\theta) \qquad (13)$$

expanding

$$(\psi-\phi) + i(\omega-\theta) = \sum_{n=1}^{\infty} \left[\frac{A_n}{R^n} \cos n\theta + \frac{B_n}{R^n} \sin n\theta \right] +$$

$$+ \, i \sum_{n=1}^{\infty} \left[\frac{B_n}{R^n} \cos n\theta - \frac{A_n}{R^n} \sin n\theta \right] \qquad (14)$$

and equating real and imaginary parts of Eq (14) establishes the following conjugate Fourier expansions:

$$\psi - \phi = \sum_{n=1}^{\infty} \left[\frac{A_n}{R^n} \cos n\theta + \frac{B_n}{R^n} \sin n\theta \right] \qquad (15)$$

$$\omega - \theta = \sum_{n=1}^{\infty} \left[\frac{B_n}{R^n} \cos n\theta - \frac{A_n}{R^n} \sin n\theta \right] \qquad (16)$$

The Fourier coefficients $\frac{A_n}{R^n}$, $\frac{B_n}{R^n}$, and ϕ (the constant term) of Eq (15) are given by

$$\frac{A_n}{R^n} = \frac{1}{\pi} \int_0^{2\pi} \bar{\psi}(\lambda) \cos n\lambda \, d\lambda \qquad (17)$$

$$\frac{B_n}{R^n} = \frac{1}{\pi} \int_0^{2\pi} \bar{\psi}(\lambda) \sin n\lambda \, d\lambda \qquad (18)$$

$$\phi = \frac{1}{2\pi} \int_0^{2\pi} \bar{\psi}(\lambda) \, d\lambda \qquad (19)$$

where λ is an angular measure corresponding to θ and $\bar{\psi}(\lambda) = \psi[\omega(\lambda)]$, since ψ is known only as a function of ω. Therefore, a relation between ψ and θ (or λ) must be found. For any particular value of θ, the coefficients given in Eqs (17) and (18) may be substituted into Eq (16), thereby eliminating these coefficients from that relation

$$\omega - \theta = \frac{1}{\pi} \sum_{n=1}^{\infty} \left[\cos n\theta \int_0^{2\pi} \bar{\psi}(\lambda) \sin n\lambda \, d\lambda - \sin n\theta \int_0^{2\pi} \bar{\psi}(\lambda) \cos n\lambda \, d\lambda \right] \tag{20}$$

which reduces to

$$\omega - \theta = \frac{1}{\pi} \sum_{n=1}^{\infty} \left[\int_0^{2\pi} \bar{\psi}(\lambda) \sin n(\lambda-\theta) d\lambda \right] = \frac{1}{\pi} \int_0^{2\pi} \bar{\psi}(\lambda) \left[\sum_{n=1}^{\infty} \sin n(\lambda-\theta) \quad d\lambda \right] \tag{21}$$

But,

$$\sum_{n=1}^{N} \sin n(\lambda-\theta) = \frac{1}{2} \cot \frac{\lambda-\theta}{2} - \frac{\cos \left[(2N+1) \frac{\lambda-\theta}{2} \right]}{2 \sin \frac{\lambda-\theta}{2}} \tag{22}$$

so that

$$\omega - \theta = \lim_{N\to\infty} \left[\frac{1}{2\pi} \int_0^{2\pi} \bar{\psi}(\lambda) \cot \frac{\lambda-\theta}{2} \, d\lambda - \right.$$

$$\left. \frac{1}{2\pi} \int_0^{2\pi} \bar{\psi}(\lambda) \frac{\cos\left[(2N+1)\frac{\lambda-\theta}{2}\right]}{2 \sin \frac{\lambda-\theta}{2}} \, d\lambda \right] \qquad (24)$$

Since the second term is identically zero and the first one is unaffected in the limit,

$$\omega - \theta = \frac{1}{2\pi} \int_0^{2\pi} \bar{\psi}(\lambda) \cot \frac{\lambda-\theta}{2} \, d\lambda \qquad (25)$$

The difference, $(\omega-\theta)$ or $(\omega-\lambda)$, between angular coordinates of a point and its image in the pseudo-circle and exact circle planes is called the conformal angular distortion function, and is denoted by ε, so that

$$\varepsilon(\theta) = \frac{1}{2\pi} \int_0^{2\pi} \psi[\varepsilon(\lambda) + \lambda] \cot \frac{\lambda-\theta}{2} \, d\lambda \qquad (26)$$

Equation (26) is of fundamental importance in the practical application of Theodorsen's method, as it may be solved iteratively for $\varepsilon(\theta)$, from which $\omega(\theta)$ is obtained permitting the solution of Eq (19) for the constant ϕ. Note that the relationship between polar coordinates in the pseudo-circle and exact circle planes has been obtained without having to solve for the Fourier coefficients directly.

Surface Velocity

It is shown in potential flow theory that the complex velocity potential for a circle in the z-plane immersed in steady, x-directed flow is

$$w(z) = \Phi + i\Psi = -V_\infty \left(z + \frac{R^2}{z}\right) - \frac{i\Gamma}{2\pi} \ln \frac{z}{R} \qquad (27)$$

where V_∞ is the freestream velocity, $R = ae^\phi$ is the radius of the circle, and Γ is the circulation. The fluid velocity in the circle plane is obtained by differentiating w(z)

$$V(z) = \frac{dw}{dz} = u + iv = -V_\infty \left(1 - \frac{R^2}{z^2}\right) - \frac{i\Gamma}{2\pi z} \qquad (28)$$

This velocity will be zero (u = v = 0) at stagnation points. Therefore, the locations (z_0) of these points is given by the value for z when $\frac{dw}{dz} = 0$. That is,

$$z_0 = \frac{-i\Gamma \pm \sqrt{16\pi^2 R^2 V_\infty{}^2 - \Gamma^2}}{4\pi V_\infty} \qquad (29)$$

A non-zero angle of attack α may be accounted for by replacing z by $ze^{i\alpha}$ in the real part of Eq (27). This has the effect of rotating the flow field about the circle by the angle α.

$$w(z) = -V_\infty \left(ze^{i\alpha} + \frac{R^2}{z} e^{-i\alpha}\right) - \frac{i\Gamma}{2\pi} \ln z \qquad (30)$$

and

$$V(z) = -V_\infty e^{i\alpha}\left(1 - \frac{R^2}{z^2} e^{-2i\alpha}\right) - \frac{i\Gamma}{2\pi z} \qquad (31)$$

The well-known Kutta-Joukowski condition requires that the trailing edge ($\omega = \pi$) on an airfoil which does not have a cusped trailing edge be a stagnation point in order that infinite velocities and pressure gradients may be avoided. The circulation required to meet this condition may be found by setting $z_o = Re^{i(\pi+\beta)} = -Re^{i\beta}$ (where β is the negative of the value of ε at $\omega = \pi$, and is called the angle of zero lift for the airfoil) and $V(z) = 0$ in Eq (31), and solving for the corresponding Γ:

$$\Gamma = 4\pi RV_\infty \sin(\alpha+\beta) \qquad (32)$$

Then

$$V(z) = \frac{dw}{dz} = -V_\infty e^{i\alpha}\left(1 - \frac{R^2}{z^2} e^{-2i\alpha}\right) - i\,\frac{4\pi RV_\infty \sin(\alpha+\beta)}{2\pi z} \qquad (33)$$

The velocity, $v(z)$, in the plane of the circle must now be converted to the velocity, $v(\zeta)$, at the surface of the airfoil through the use of transformation derivatives:

$$v(\zeta) = \left|\frac{dw}{d\zeta}\right| = \left|\frac{dw}{dz}\right| \left|\frac{dz}{dz'}\right| \left|\frac{dz'}{d\zeta}\right| \qquad (34)$$

and these derivatives must now be determined. Recall that $z = Re^{i\theta}$ on the surface of the circle. Substituting this value for z in Eq (33) gives

$$\frac{dw}{dz} = -V_\infty e^{i\alpha}[1 - e^{-2i(\alpha+\theta)}] - 2iV_\infty e^{-i\theta} \sin (\alpha+\beta) \tag{35}$$

which reduces to

$$\frac{dw}{dz} = -2iV_\infty e^{-i\theta}[\sin (\alpha+\theta) + \sin (\alpha+\beta)] \tag{36}$$

the absolute value of which is

$$\left|\frac{dw}{dz}\right| = 2V_\infty[\sin (\alpha+\theta) + \sin (\alpha+\beta)] \tag{37}$$

From the definitions of z, z', and ε

$$\frac{z'}{z} = \frac{ze^{\psi+i\omega}}{ae^{\phi+i\theta}} = e^{(\psi-\theta)+i\varepsilon} \tag{38}$$

Differentiating and noting that ϕ is a constant

$$\frac{dz'}{dz} = e^{(\psi-\phi)+i\varepsilon} + ze^{(\psi-\phi)+i\varepsilon} \frac{d}{dz} [(\psi-\phi)+i\varepsilon] =$$

$$\frac{z'}{z} [1 + z \frac{d}{dz} (\psi+i\varepsilon)] \tag{39}$$

But

$$\frac{1}{z} dz = \frac{1}{ae^{\phi+i\theta}} d(ae^{\phi+i\theta}) = id\theta \tag{40}$$

Substitution into Eq (39) gives

$$\frac{dz'}{dz} = \frac{z'}{z}\left[1 + \frac{d(\psi+i\varepsilon)}{i\,d\theta}\right] = \frac{z'}{z}\left(1 - i\,\frac{d\psi}{d\theta} + \frac{d\varepsilon}{d\theta}\right) \tag{41}$$

Further algebraic manipulation on this equation gives

$$\frac{dz'}{dz} = \frac{z'}{z}\left[\frac{(d\theta/d\omega) + (d\varepsilon/d\omega) - i\,(d\psi/d\omega)}{(d\theta/d\omega)}\right] \tag{42}$$

Applying the definition $\varepsilon = \omega - \theta$, and reducing

$$\frac{dz'}{dz} = \frac{z'}{z}\left[\frac{1 - i\,(d\psi/d\omega)}{1 - (d\varepsilon/d\omega)}\right] \tag{43}$$

And finding the absolute value

$$\left|\frac{dz'}{dz}\right| = e^{\psi-\phi}\left[\frac{\sqrt{1 + (d\psi/d\omega)^2}}{1 - (d\varepsilon/d\omega)}\right] \tag{44}$$

Inverting

$$\left|\frac{dz}{dz'}\right| = e^{\phi-\psi}\left[\frac{1 - (d\varepsilon/d\omega)}{\sqrt{1 + (d\psi/d\omega)^2}}\right] \tag{45}$$

which is the desired derivative. Substituting the definition $z' = ae^{\psi+i\omega}$ into the Joukowski transformation

$$\zeta = ae^{\psi+i\omega} + ae^{-\psi-i\omega} = 2a\cosh(\psi+i\omega) \tag{46}$$

Differentiating

$$\frac{d\zeta}{dz'} = 2\sinh(\psi+i\omega)e^{-\psi-i\omega} \tag{47}$$

Expanding

$$\frac{d\zeta}{dz'} = 2e^{-\psi-i\omega}\,(\sinh\psi\,\cosh i\omega + \cosh\psi\,\sinh i\omega) \qquad (48)$$

Noting that $\cosh i\omega = \cos\omega$ and $\sinh i\omega = i\sin\omega$, this becomes

$$\frac{d\zeta}{dz'} = 2e^{-\psi-i\omega}\,(\sinh\psi\,\cos\omega + i\cosh\psi\,\sin\omega) \qquad (49)$$

Then

$$\left|\frac{d\zeta}{dz'}\right|^2 = 4e^{-2\psi}\,(\sinh^2\psi\,\cos^2\omega + \cosh^2\psi\,\sin^2\omega) \qquad (50)$$

which reduces to

$$\left|\frac{d\zeta}{dz'}\right|^2 = 4e^{-2\psi}\,(\sinh^2\psi + \sin^2\omega) \qquad (51)$$

Taking the square root and inverting gives the final derivative

$$\left|\frac{dz'}{d\zeta}\right| = \frac{e^{\psi}}{2\sqrt{\sinh^2\psi + \sin^2\omega}} \qquad (52)$$

The derivatives given in Eqs (37), (45), and (52) will now be substituted into Eq (34) to find the airfoil surface velocity

$$V(\zeta) = \frac{V_\infty e^{\phi}[\sin(\alpha+\theta) + \sin(\alpha+\beta)][1 - (d\varepsilon/d\omega)]}{\sqrt{(\sinh^2\psi + \sin^2\omega)[1 + (d\psi/d\omega)^2]}} \qquad (53)$$

This is the velocity which will then be used as an approximation for the edge velocity in the analysis of the boundary layer.

III. Karman-Pohlhausen Method

Origin

General motion of a viscous fluid is described by the Navier-Stokes equations, which were derived from first principles. Schlichting (Ref 3) has shown that, if Reynolds number is large, viscosity is constant, and flow is steady and incompressible, then the equations of motion for flow over a surface may be approximated by the momentum integral equation:

$$U^2 \frac{d\delta_2}{dx} + (2\delta_2 + \delta_1)\ U \frac{dU}{dx} = \frac{\tau_w}{\rho} \tag{54}$$

where the boundary layer displacement and momentum thicknesses are, respectively,

$$\delta_1 = \int_o^\infty \left(1 - \frac{u}{U}\right) dy \tag{55}$$

$$\delta_2 = \int_o^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \tag{56}$$

and where x and y are, respectively, the streamwise and cross-stream coordinates of a general point in the flow (Fig. 2), with the corresponding velocities given by u and v. Note that the boundary layer edge velocity U is a parameter common to both the potential flow and boundary layer regions.

The following analysis is due originally to Pohlhausen, but is presented in the form developed by Holstein and

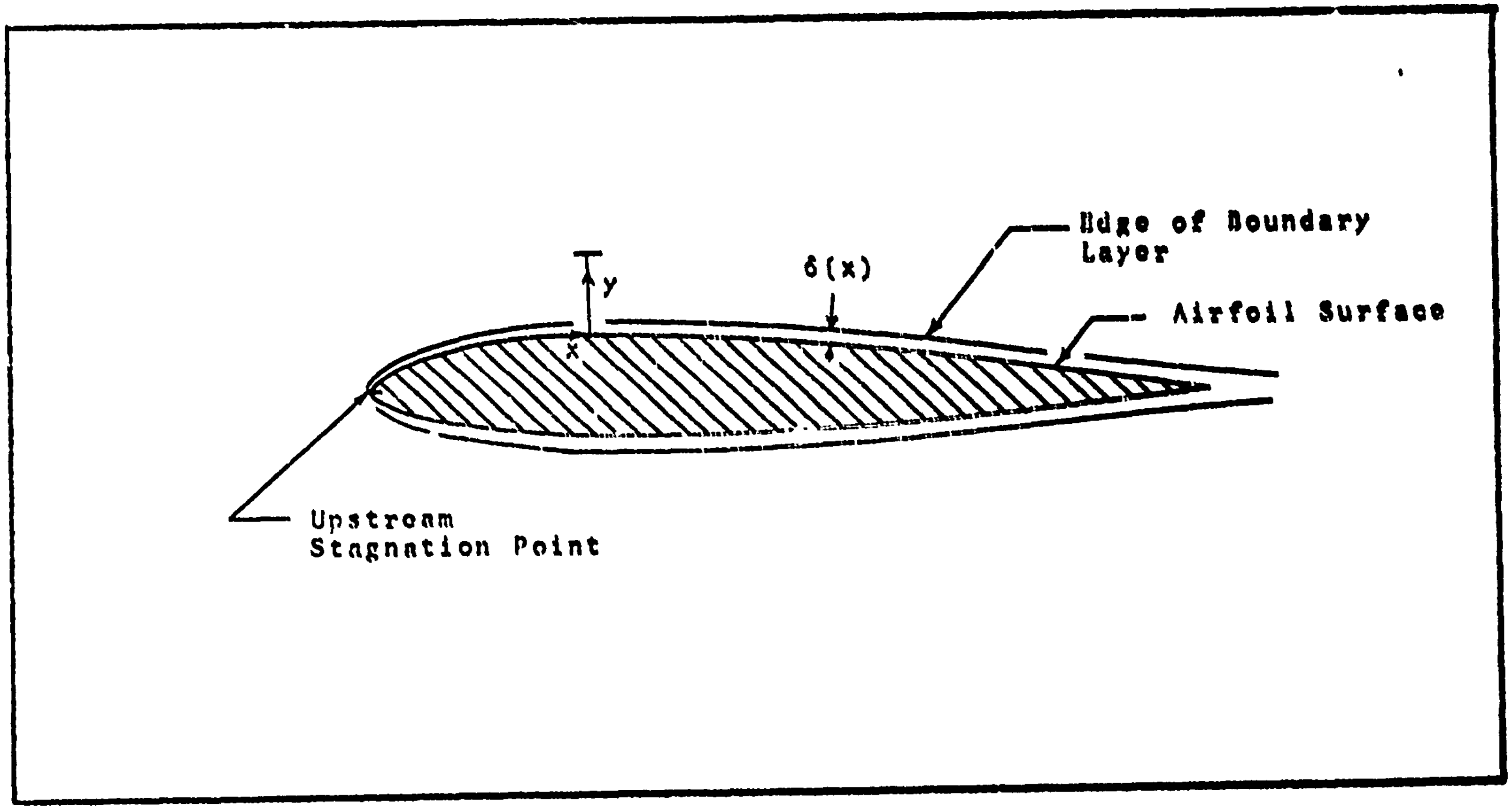

Fig. 2. Boundary Layer Coordinate System.

Bohlen, and described by Schlichting.

Velocity Profile

A velocity profile of the following form is now assumed:

$$\frac{u}{U} = a\eta + b\eta^2 + c\eta^3 + d\eta^4 \tag{57}$$

where $\eta = y/\delta$ and a, b, c, and d are coefficients to be determined.

Observations indicate that the following boundary conditions should be applied to the solution of Eq (54):

$$u = 0 \qquad \text{at } y = 0 \tag{58}$$

$$\nu \frac{\partial^2 u}{\partial y^2} = -U \frac{dU}{dx} \qquad \text{at } y = 0 \tag{59}$$

$$u = U \qquad \text{at } y = \delta \tag{60}$$

$$\frac{\partial u}{\partial y} = 0 \qquad \text{at } y = \delta \tag{61}$$

$$\frac{\partial^2 u}{\partial y^2} = 0 \qquad \text{at } y = \delta \tag{62}$$

where δ, called the boundary layer thickness, is the cross-stream distance from the surface at which the boundary layer meets the potential flow region.

Note that the first boundary condition, Eq (58), is identically satisfied by the assumed solution by virtue of the omission of a constant term in the polynomial. There remain, therefore, four boundary conditions to determine a, b, c, and d. In terms of the assumed velocity profile, the second through fifth boundary conditions become, respectively,

$$U\nu\left[\frac{\partial^2}{\partial\eta^2}\left(\frac{u}{U}\right)\left(\frac{\partial\eta}{\partial y}\right)^2\right] = -U\,\frac{dU}{dx} \qquad \text{at } \eta = 0 \qquad (63)$$

$$\frac{u}{U} = 1 \qquad \text{at } \eta = 1 \qquad (64)$$

$$U\left[\frac{\partial}{\partial\eta}\left(\frac{u}{U}\right)\left(\frac{\partial\eta}{\partial y}\right)\right] = 0 \qquad \text{at } \eta = 1 \qquad (65)$$

$$U\left[\frac{\partial^2}{\partial\eta^2}\left(\frac{u}{U}\right)\left(\frac{\partial\eta}{\partial y}\right)^2\right] = 0 \qquad \text{at } \eta = 1 \qquad (66)$$

Application of these conditions to the assumed solution and defining a shape factor

$$\Lambda = \frac{\delta^2}{\nu}\,\frac{dU}{dx} \qquad (67)$$

leads to the following expression for the velocity profile:

$$\frac{u}{U} = (2\eta - 2\eta^3 + \eta^4) + \frac{\Lambda}{6}(\eta - 3\eta^2 + 3\eta^3 - \eta^4) \qquad (68)$$

By applying the physical constraints that $\frac{\partial u}{\partial y} = 0$ at $y = 0$ at the separation point and that $\frac{u}{U} \leq 1$ within the boundary layer, the upper and lower physical limits on Λ are readily determined. These have been found to be +12 and -12, respectively.

Other Parameters

The displacement thickness, momentum thickness, and shear stress will now be found in terms of Λ.

Substituting the above velocity profile and the definition for η into the definition for displacement thickness given by Eq (55) gives

$$\frac{\delta_1}{\delta} = \int_0^1 \Big[1 - (2\eta - 2\eta^3 + \eta^4) - \frac{\Lambda}{6}(\eta - 3\eta^2 + 3\eta^3 - \eta^4)\Big]d\eta \qquad (69)$$

And, finally,

$$\frac{\delta_1}{\delta} = \frac{3}{10} - \frac{\Lambda}{120} \qquad (70)$$

Likewise,

$$\frac{\delta_2}{\delta} = \frac{37}{315} - \frac{\Lambda}{945} - \frac{\Lambda^2}{9072} \qquad (71)$$

Now, the shear stress, by definition, is

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} \qquad (72)$$

Evaluating the derivative and multiplying both sides by $\frac{\delta}{\mu U}$, gives

$$\frac{\tau_w \delta}{\mu U} = 2 + \frac{\Lambda}{6} \qquad (73)$$

Simplification of Momentum Integral Equation

The momentum integral equation can now be put into a form from which practical results may be obtained.

Multiplying both sides of Eq (54) by $\frac{\delta_2}{\nu U}$, and reducing, leads to

$$\frac{\tau_w \delta_2}{\mu U} = \frac{U}{\nu} \delta_2 \frac{d\delta_2}{dx} + \left(2 + \frac{\delta_1}{\delta_2}\right) \frac{{\delta_2}^2}{\nu} \frac{dU}{dx} \qquad (74)$$

Equations (67), (68), (70), (71), and (74), together with the known edge velocity distribution, form a solvable system. However, elimination of dependence of the other variables on δ is desirable, since the boundary layer thickness is physically meaningless. Furthermore, the $\frac{d\delta_2}{dx}$ quantity in Eq (74) leads to a requirement for $\frac{d^2U}{dx^2}$ at every point on the surface. By suitable mathematical manipulation, this can be reduced to a requirement for $\frac{d^2U}{dx^2}$ at the upstream stagnation point only. The following analysis proceeds, therefore, toward these objectives.

Introduction of several new parameters will permit simplification of the form of Eq (74). Define

$$K = \frac{{\delta_2}^2}{\nu} \frac{dU}{dx} \qquad (75)$$

Then,

$$K = \left(\frac{\delta_2}{\delta}\right)^2 \frac{\delta^2}{\nu} \frac{dU}{dx} = \Lambda \left(\frac{\delta_2}{\delta}\right)^2 = \Lambda \left(\frac{37}{315} - \frac{\Lambda}{945} - \frac{\Lambda^2}{9072}\right)^2 \qquad (76)$$

Let

$$Z = \frac{\delta_2^{\,2}}{\nu} \tag{77}$$

So that

$$\frac{dz}{dx} = \frac{2}{\nu}\,\delta_2\,\frac{d\delta_2}{dx} \tag{78}$$

Also, from Eqs (75) and (77)

$$K = Z\,\frac{dU}{dx} \tag{79}$$

The ratio of displacement to momentum boundary layer thicknesses may be defined

$$f_1 = \frac{\delta_1}{\delta_2} \tag{80}$$

Or, using Eqs (70) and (71)

$$f_1 = \frac{\frac{\delta_1}{\delta}}{\frac{\delta_2}{\delta}} = \frac{\frac{3}{10} - \frac{\Lambda}{120}}{\frac{37}{315} - \frac{\Lambda}{945} - \frac{\Lambda^2}{9072}} \tag{81}$$

And, finally, the definition

$$f_2 = \frac{\tau_w \delta_2}{\mu U} \tag{82}$$

gives, using Eqs (71) and (73),

$$f_2 = \frac{\tau_w \delta}{\mu U}\left(\frac{\delta_2}{\delta}\right) = \left(2 + \frac{\Lambda}{6}\right)\left(\frac{37}{315} - \frac{\Lambda}{945} - \frac{\Lambda^2}{9072}\right) \tag{83}$$

Now, if these definitions are substituted into Eq (74), the result is

$$f_2 = \frac{U}{2}\,\frac{dZ}{dx} + (2 + f_1)K \tag{84}$$

which may be rearranged to give

$$U\,\frac{dZ}{dx} = 2f_2 - (4 + 2f_1)K \tag{85}$$

The right side of Eq (85) is a function of Λ alone, as shown by Eqs (76), (81), and (83). Therefore, if a new parameter F is defined such that

$$F = 2f_2 - (4 + 2f_1)K \tag{86}$$

Then, it may be shown that

$$F = 2\left(\frac{37}{315} - \frac{\Lambda}{945} - \frac{\Lambda^2}{9072}\right)$$
$$\left[2 - \frac{116}{315}\Lambda + \left(\frac{2}{945} + \frac{1}{120}\right)\Lambda^2 + \frac{2}{9072}\Lambda^3\right] \tag{87}$$

And Eq (85), the equation of motion, becomes

$$\frac{dZ}{dx} = \frac{F}{U} \tag{88}$$

Conditions at Stagnation Point

Enough relations now exist to permit a stepwise solution for all the boundary layer parameters, once a starting condition is known. Starting conditions, therefore, will now be determined at the upstream stagnation point.

Since the flow is brought to rest at the stagnation point, $U_0 = 0$ (where the "o" subscript means that an associated variable is to be evaluated at stagnation point conditions). Then, from Eq (88), F_0 must also be zero if $\left(\frac{dZ}{dx}\right)\Big|_0$ is to exist. Therefore, Λ_0 must be a root of Eq (87) when $F = 0$. The roots of this equation are -72.2, -37.8, 7.052, 17.9, and 28.2. Because of the physical limitations on the range of Λ, it may be seen that

$$\Lambda_0 = 7.052 \qquad (89)$$

Then from Eq (76),

$$K_0 = .0770 \qquad (90)$$

And, from Eq (79),

$$Z_0 = \frac{.0770}{\left(\frac{dU}{dx}\right)\Big|_0} \qquad (91)$$

which determines Z_0, since the edge velocity distribution is known from the potential flow analysis.

Now, $\left(\frac{dZ}{dx}\right)\Big|_0$ may be found by applying L'Hospital's Rule to Eq (88)

$$\left(\frac{dZ}{dx}\right)\bigg|_o = \lim_{x\to 0}\left[\frac{(dF/dx)}{(dU/dx)}\right] = \lim_{x\to 0}\left[\frac{(dF/dK)\,(dK/dx)}{(dU/dx)}\right] \tag{92}$$

But, from Eq (79),

$$\frac{dK}{dx} = \frac{dZ}{dx}\,\frac{dU}{dx} + Z\,\frac{d^2U}{dx^2} \tag{93}$$

Substituting this expression into Eq (92) and passing to the limit gives

$$\left(\frac{dZ}{dx}\right)\bigg|_o = Z_o\left[\frac{\left(\frac{dF}{dK}\right)\Big|_o}{1 - \left(\frac{dF}{dK}\right)\Big|_o}\right]\left[\frac{\left(\frac{d^2U}{dx^2}\right)\Big|_o}{\left(\frac{dU}{dx}\right)\Big|_o}\right] \tag{94}$$

But, using Eqs (76) and (87),

$$\left(\frac{dF}{dK}\right)\bigg|_o = \frac{\left(\frac{dF}{d\Lambda}\right)\Big|_o}{\left(\frac{dK}{d\Lambda}\right)\Big|_o} = -5.57 \tag{95}$$

Substituting the values for Z_o and $\left(\frac{dF}{dK}\right)\Big|_o$ given by Eqs (91) and (95), respectively, into Eq (94) provides an expression for $\left(\frac{dZ}{dx}\right)\Big|_o$ in terms of the edge velocity distribution:

$$\left(\frac{dZ}{dx}\right)\bigg|_o = -.0652\,\frac{\left(\frac{d^2U}{dx^2}\right)\Big|_o}{\left(\frac{dU}{dx}\right)\Big|_o} \tag{96}$$

Using the Method

With starting conditions known at the stagnation point, the value for Z at a distance Δx downstream is

$$Z_1 = Z_o + \left(\frac{dZ}{dx}\right)\Big|_o \Delta x \tag{97}$$

Then, from Eq (79),

$$K_1 = Z_1 \left(\frac{dU}{dx}\right)\Big|_1 \tag{98}$$

The corresponding value of Λ, from Eq (76) may now be used to find F_1 from Eq (87). Using Eq (88),

$$\left(\frac{dZ}{dx}\right)\Big|_1 = \frac{F_1}{U_1} \tag{99}$$

so that the stepping process may be continued indefinitely, the value of Z at point "n" being

$$Z_n = Z_{n-1} + \left(\frac{dZ}{dx}\right)\Big|_{n-1} \Delta x \tag{100}$$

Knowledge of Λ_n permits determination of f_{1_n}, f_{2_n}, and the velocity profile $\left(\frac{u}{U}\right)\Big|_n$, using Eqs (45), (83), and (68), respectively. Substituting Z_n into Eq (77) yields δ_{2_n}, which in turn produces τ_{w_n} from Eq (82), and, the ultimate objective, δ_{1_n} from Eq (80). This value of δ_{1_n} must be added to the original airfoil thickness to form a better approximation of the potential flow airfoil shape to be analyzed again by Theodorsen's method.

Limitation

The Karman-Pohlhausen method of boundary layer analysis does not properly describe the flow downstream of a separation point (the point at which the shear stress at the wall vanishes).

IV. Computer Study

Purpose of Computer Study

A Fortran Extended program for the CDC-6600 computer was written as part of this study and is listed in Appendix A. It implements the theory discussed in the preceding chapters and lays a foundation for more sophisticated airfoil analysis programs. Flow parameters from this program may be compared with those from simplified airfoil theories in order to determine the validity of thin airfoil and inviscid assumptions. Some pertinent facts about the program will be discussed in this chapter.

Program Composition

The computer program is composed of a main program and several subprograms and functions. The main program is called MAGIC and provides the means for reading input data, controlling data flow and sequencing among subprograms, and testing for completion of the iteration on displacement thickness. One principal subprogram is THEO, which applies Theodorsen's method to the airfoil; the other is BOUND, which solves the boundary layer equations using the Karman-Pohlhausen method. Other subprograms and their purposes include: FNEVAL, which defines integrands; SIMPS, which performs integrations using Simpson's Rule; MTXEQ, which solves systems of algebraic equations; PLSQ, a polynomial least-square curve-fitting routine; ATKN, an interpolating function; several subroutines for the CALCOMP plotter; and numerous standard functions from the system library.

Input

Program MAGIC requires an alphanumeric description of the airfoil (e.g., NACA 1408), the number of data points, the Cartesian coordinates of each point, a plotting index, freestream velocity, viscosity, density, and the angle of attack on the chord line. The airfoil chord line is assumed to lie on the x-axis with the trailing edge in the positive x-direction and the midchord point at the origin. Points defining the airfoil surface must begin at the trailing edge and be numbered in the counterclockwise direction. All dimensions must be referenced to chord length. The plotting index causes the airfoil and its transformations in subprogram THEO: (1) to be plotted on the same axes, (2) not to be plotted, or (3) to be plotted on separate axes, according as its value is -1, 0, or +1, respectively. The angle of attack must be given in degrees.

Additionally, accuracy requirements on various computations may be reset by modifications to the program.

Output

Program MAGIC computes and prints Cartesian coordinates in all three planes, polar coordinates in the pseudo-circle and exact circle planes, potential flow surface velocity, coefficients for fifth-order polynomials for $\bar{\psi}(\theta)$ and $\psi(\omega)$, radius of exact circle, zero-lift angle of attack, circulation, section lift, pressure coefficients, shear stress at surface, momentum thickness, displacement thickness, shape factor, and several important quantities used in computing these. The airfoil surface in all three planes is plotted (provided that the plotting index is non-zero), as are pressure coefficient, shear stress, and displacement thickness as functions of chordwise station. Samples of output format are shown in Appendix A.

Limitations on Program

Although subprogram THEO gives correct results for non-zero angles of attack, subprogram BOUND does not, since the shift of upstream stagnation point with angle of attack has not been accounted for. Therefore, accurate results for the complete program may be expected only for zero angle of attack.

Values of shear stress and displacement thickness generated by subprogram BOUND downstream of the separation point become very large. These values are printed as output, but are actually limited to a reasonable maximum value before being plotted or transmitted to subprogram THEO for another iteration. In particular, the displacement thickness at the trailing edge is set to zero in order that the new airfoil shape considered by subprogram THEO will be a closed curve.

V. Results

Conformal Transformations

Transformations were accomplished by subprogram THEO on several airfoils, including NACA 1408 and NACA 4424. The output plot of the transformation on the NACA 4424 airfoil is presented in Fig. 3. Success of the transformation on these airfoils was measured by the accuracy of the output angle of zero lift, β. The values for these angles as given by Abbott and von Doenhoff (Ref 1) are presented for comparison in Table 1. Comparison data was not available for the other airfoils used.

Table 1

Comparison of Zero Lift Angles of Attack

Airfoil	(Program MAGIC)	(Ref 5)
NACA 1408	.95°	.95°
NACA 4424	3.58°	3.50°

Potential Flow Surface Velocities

Surface velocities generated by subprogram THEO were also compared with those given by Abbott and von Doenhoff. A typical comparison is shown in Table 2. The velocities agree within 2% in most cases. Exceptions occur at points where the radius of curvature is relatively small; that is, near the leading edge. This probably reflects poor accuracies in numerical derivatives in that area.

Table 2

Comparison of Potential Flow Velocities for NACA 1408 Airfoil

Upper Surface			Lower Surface		
Chordwise Station (percent chord)	V/V_∞ (Program MAGIC)	V/V_∞ (Ref 1)	Chordwise Station (Percent chord)	V/V_∞ (Program MAGIC)	V/V_∞ (Ref 1)
0	0	0	0	0	0
1.189	.780	1.061	1.311	.829	1.039
2.413	1.067	1.121	2.582	1.079	1.089
4.896	1.135	1.151	5.104	1.121	1.105
7.386	1.142	1.161	7.614	1.112	1.105
9.883	1.145	1.161	10.117	1.102	1.099
14.889	1.153	1.164	15.111	1.096	1.092
19.904	1.152	1.162	20.096	1.085	1.082
24.926	1.148	1.157	25.074	1.075	1.071
29.950	1.139	1.149	30.050	1.065	1.063
40.000	1.126	1.131	40.000	1.054	1.047
50.020	1.107	1.110	49.980	1.038	1.034
60.034	1.087	1.089	59.966	1.023	1.019
70.041	1.067	1.070	69.959	1.012	1.008
80.039	1.042	1.043	79.961	.998	.991
90.027	1.014	1.003	89.973	.982	.965
95.016	.966	.983	94.984	.953	.955
100.000	0	0	100.000	0	0

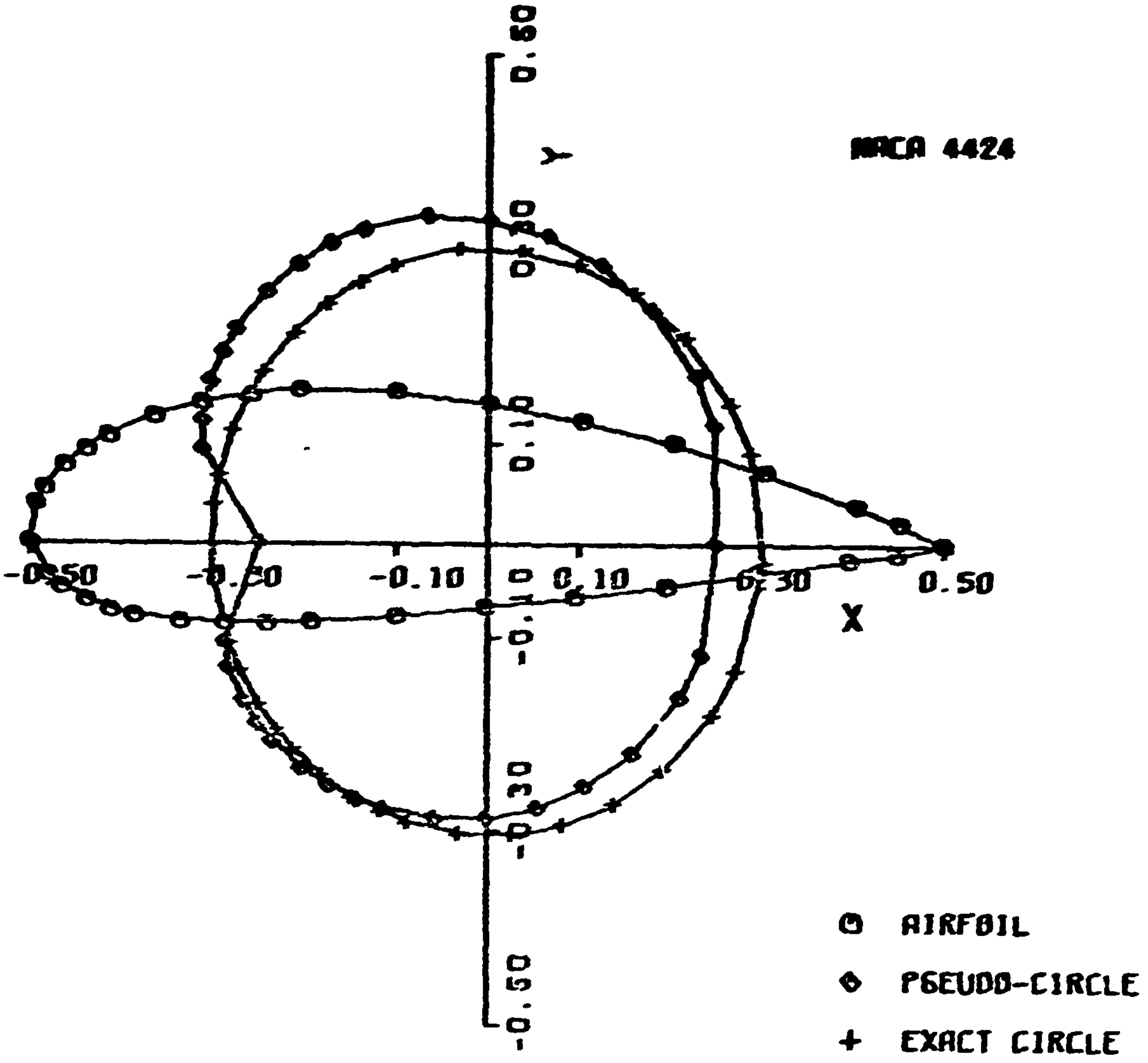

Fig. 3. Transformation of NACA 4424 Airfoil.

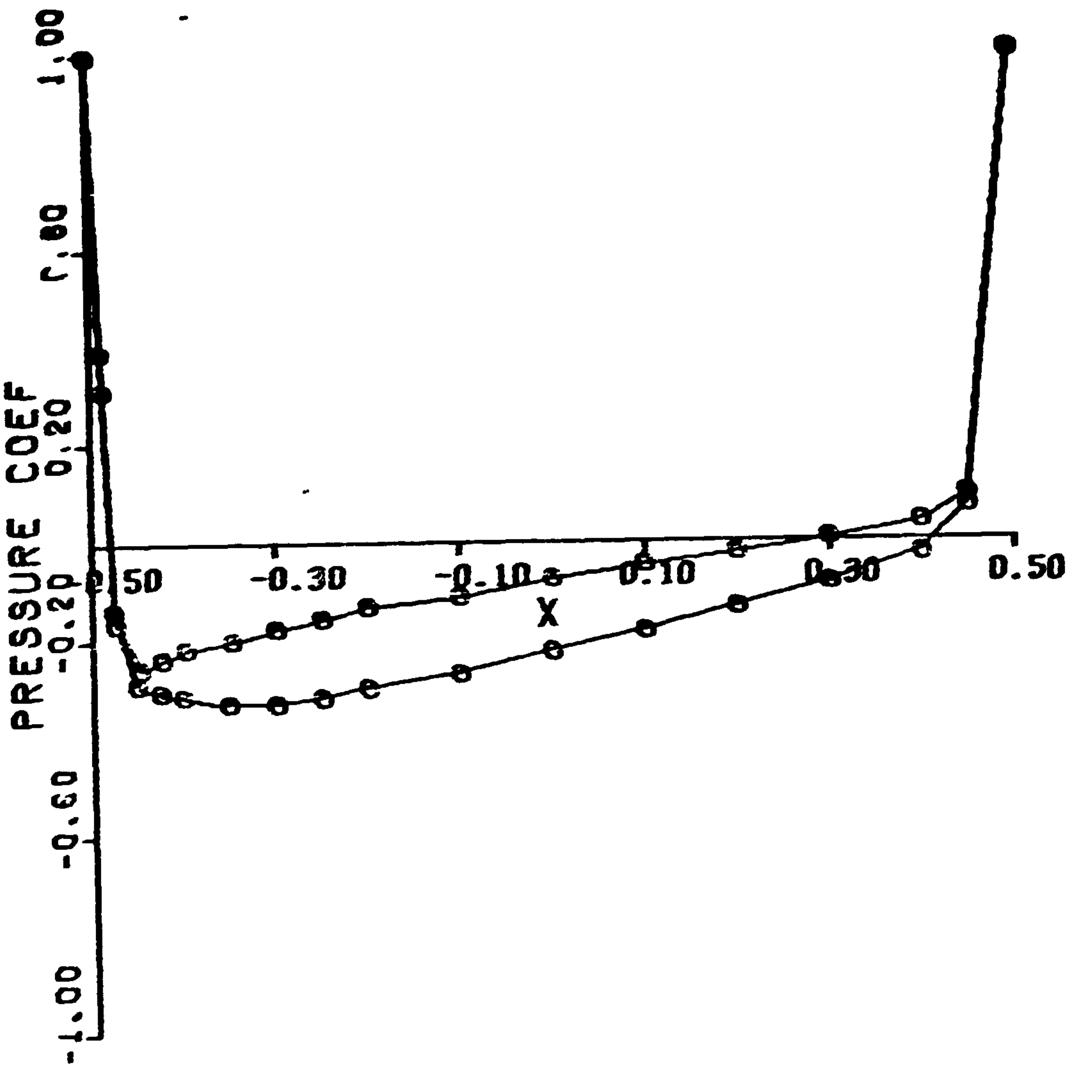

Fig. 4. Pressure Distribution on NACA 1408 Airfoil.

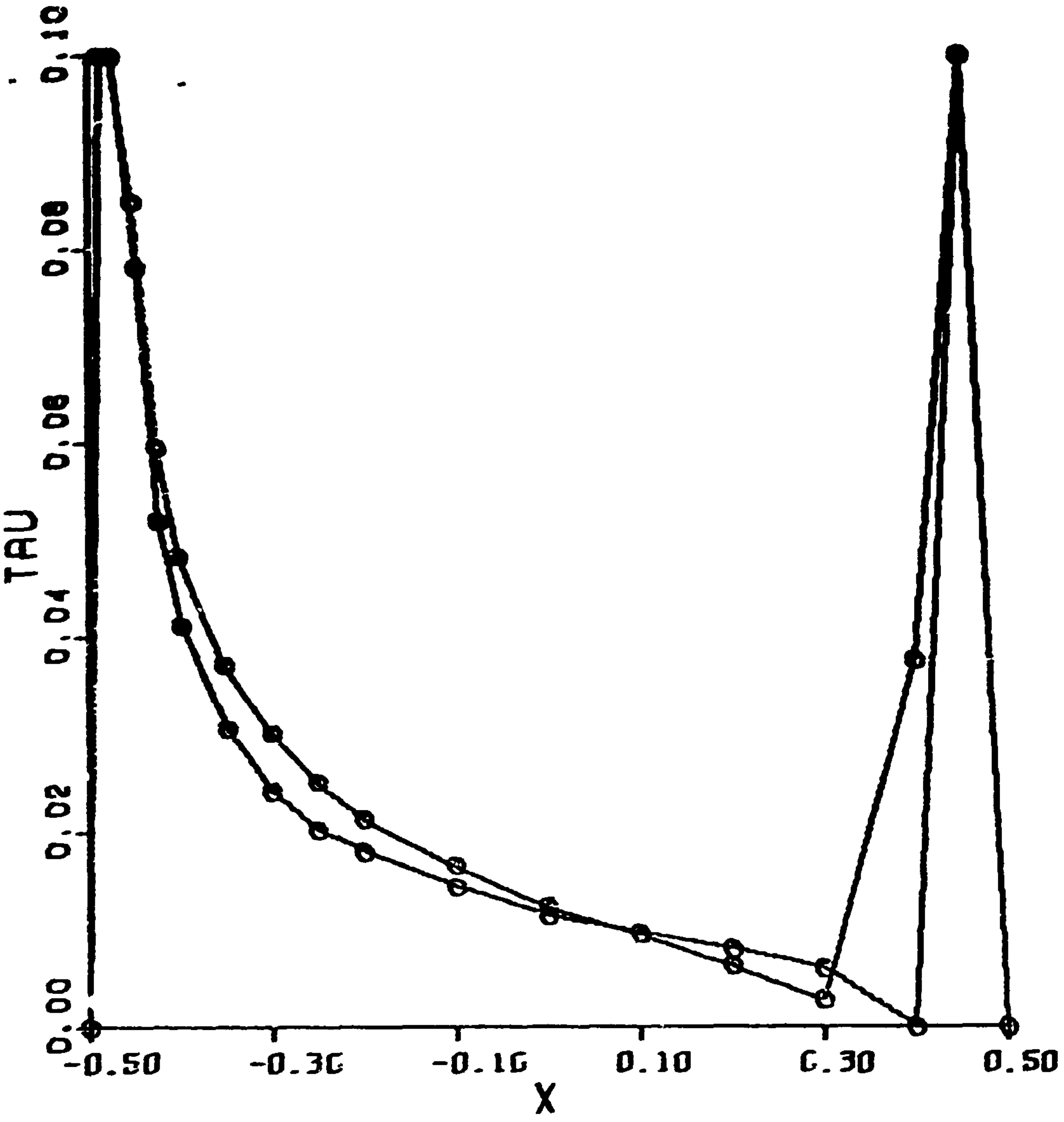

Fig. 5. Shear Stress on NACA 1408 Airfoil.

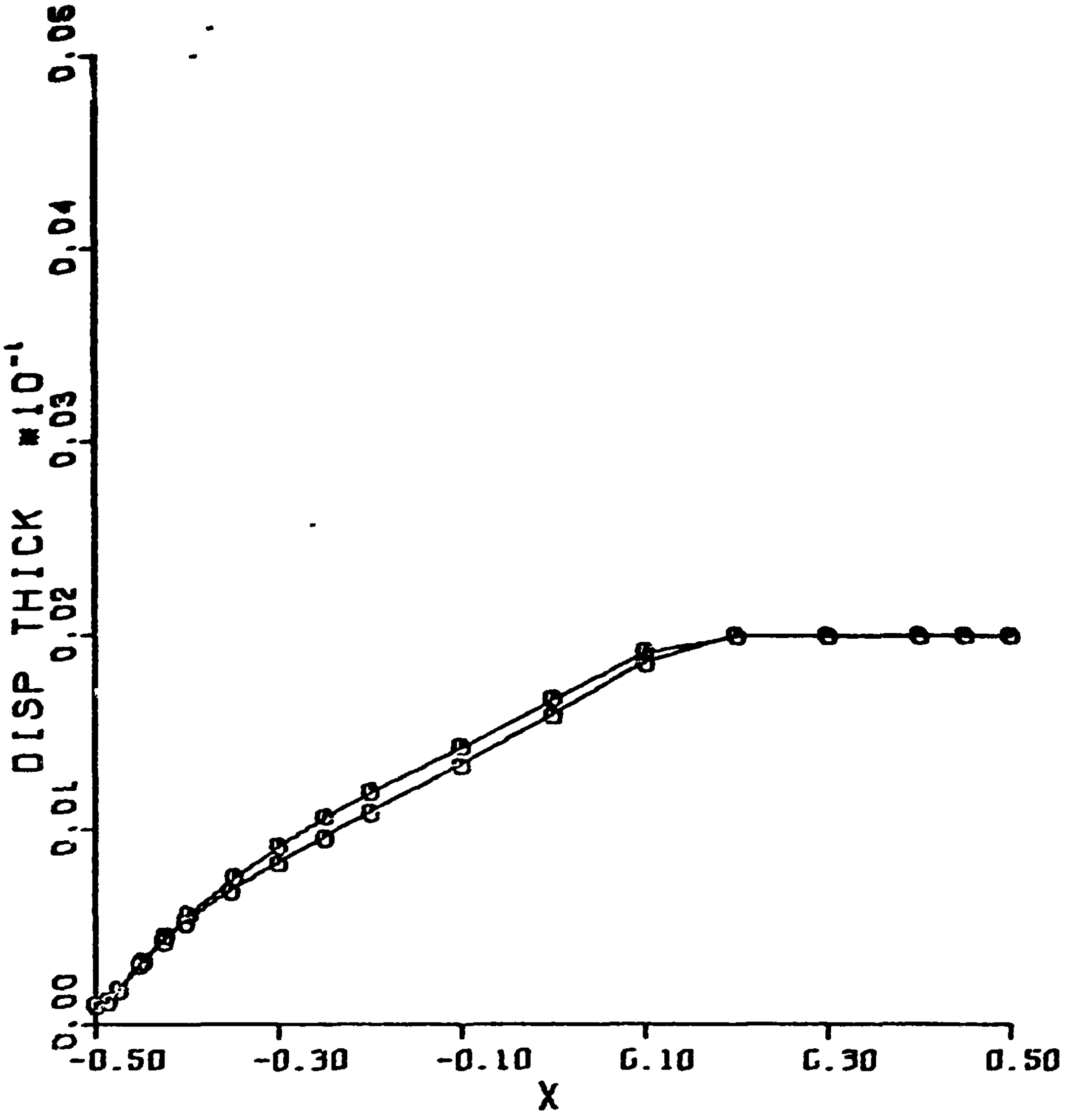

Fig. 6. Boundary Layer Displacement Thickness on NACA 1408 Airfoil.

Boundary Layer Parameters

Pressure coefficients, shear stress, and boundary layer displacement thickness distributions are qualitatively correct (Figs. 4, 5, and 6) upstream of the separation point. As a quantitative check, the separation point for uniform flow around a circle was computed. Subprogram BOUND found this point to be at 105° from the upstream stagnation point. This agrees well with values from 104.5° to 108.8° determined by various investigators, as cited by Schlichting.

Running Time and Core Memory Required

Computer time required for program MAGIC is a function of airfoil shape, number of points, and accuracy required. The NACA 1408 airfoil, with 34 points defined, required approximately 50 seconds of CDC-6600 central processor time and two iterations to obtain a displacement thickness accuracy of .1% of chord length.

Approximately 40,000 octal words of core memory were required.

VI. Conclusions and Recommendations

Conclusions

Based upon the results shown in the preceding chapter, the following conclusions were drawn from this study:

1. Theodorsen's method computes values of zero lift angle of attack and potential flow surface velocities very accurately.

2. The Karman-Pohlhausen method of solving the boundary layer equations yields good results upstream of the separation point. No useful information is obtained from these equations downstream of the separation point. Knowledge of the shape factor permits determination of the coefficients in the assumed velocity profile at any point along the surface.

3. Program MAGIC is a practical means of analyzing incompressible flow about an airfoil, without resort to small perturbation or inviscid assumptions. Core memory and central processor requirements are very reasonable for an analysis of this magnitude. Input data requirements are minimal.

Recommendations

The following recommendations are made for future studies in extension of this one:

1. Program BOUND could be modified to permit boundary layer computations when angle of attack is not zero.

2. Integration of pressure distribution and shear stress would provide drag, moment, and center of pressure information.

3. According to Giesing (Ref 2), Theodorsen's method can be extended to linear arrangements of identical airfoils. Program MAGIC, so extended, could be of considerable value in cascade theory.

4. Theodorsen states that accounting for leading edge and trailing edge radii in locating the coordinate system origin improves the rate of convergence of the iteration on ε. Inclusion of this correction might result in some saving in computer time.

5. Numerical derivatives of a higher order of accuracy near the leading edge would probably improve velocities computed in that region.

6. The inverse problem (see Ref 4) could be programmed. That is, given a desired pressure distribution, the required airfoil shape could be determined.

7. A more graphic means of input and output could be obtained by adapting the program to the cathode ray tube display terminal.

Bibliography

1. Abbott, Ira H. and Albert E. von Doenhoff. Theory of Wing Sections. New York: McGraw-Hill Book Co., 1949.

2. Giesing, Joseph P. Potential Flow About Two-Dimensional Airfoils. Report No. LB31946. McDonnell-Douglas Aircraft Company, 1 December 1965 (Revised 10 May 1968).

3. Schlichting, H. Boundary Layer Theory (Sixth Edition). New York: McGraw-Hill Book Co., 1968.

4. Theodorsen, T. and I. E. Garrick. General Potential Theory of Arbitrary Wing Sections. NACA Report No. 452. Washington: National Advisory Committee for Aeronautics, 1933.

Appendix A

Computer Program and Sample Output Format

```
      PROGRAM MAGIC(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,PLOT)
C
C MAGIC IS A GENERALIZED PROGRAM FOR ANALYSIS OF STEADY,VISCOUS,INCOMPRESSIBLE,
C TWO-DIMENSIONAL FLOW AROUND AN AIRFOIL OF ARBITRARY SHAPE.  IT UTILIZES
C THEODORSENS METHOD AND THE KARMAN-POHLHAUSEN METHOD.  OUTPUTS ARE DESCRIBED
C IN SUBROUTINES THEO AND BOUND.
C
C MAGIC REQUIRES THE FOLLOWING INPUT(AND FORMAT)-
C
C     DESIGNATION OF AIRFOIL(A10)
C     NUMBER OF POINTS(I3)
C     X AND Y FOR EACH POINT(2F10.5)
C     PLOTTING INDEX(I2)
C     ANGLE OF ATTACK(F10.5)
C     DENSITY,ABSOLUTE VISCOSITY, FREESTREAM VELOCITY(3F10.5)
C
C THE NUMBER OF POINTS IS LIMITED TO 47.  SEE SUBROUTINE THEO FOR ADDITIONAL
C INPUT DATA RESTRICTIONS.
C
C MAGIC REQUIRES APPROXIMATELY 45,000 OCTAL WORDS OF CORE MEMORY.
C RUNNING TIME IS A FUNCTION OF AIRFOIL SHAPE, NUMBER OF POINTS, AND ACCURACY
C REQUIRED.
C
C DEFINITION OF PRINCIPAL OUTPUT VARIABLES-
C
C
C ARRAY C GIVES HIGH-TO-LOW ORDER COEFFICIENTS OF PSI(OMEGA) POLYNOMIAL.
C
C ARRAY D GIVES HIGH-TO-LOW ORDER COEFFICIENTS OF PSI(THETA) POLYNOMIAL.
C
C DEL1 IS DISPLACEMENT THICKNESS.
C
C DEL2 IS MOMENTUM THICKNESS.
C
C TAU IS SHEAR STRESS AT THE AIRFOIL SURFACE.
C
C LAM IS THE SHAPE FACTOR.
C
C (X,Y), (XX,YY), AND (XXX,YYY) ARE CARTESIAN COORDINATES IN THE THREE
C TRANSFORMATION PLANES.
C
C (RHO,OMEGA) AND (R,THETA) ARE POLAR COORDINATES IN THE PSEUDO-CIRCLE
C AND EXACT CIRCLE PLANES.
C
C PSI IS THE MAGNIFICATION EXPONENT IN THE PSEUDO-CIRCLE PLANE.
C
C PHI IS THE CONSTANT MAGNIFICATION EXPONENT IN THE EXACT CIRCLE PLANE.
C
C VTV IS THE SURFACE VELOCITY/FREESTREAM VELOCITY RATIO.
C
C CP IS THE PRESSURE COEFFICIENT.
C
C EPS IS (OMEGA-THETA).
C
C ALPHA IS THE ANGLE OF ATTACK.
```

```
C
C ZETA IS THE ZERO LIFT ANGLE OF ATTACK.
C
C
C
C
      DIMENSION XSAVE(5 )
      COMMON/BLOCK3/XIN(50),YIN(50),N,CPLY(50),MU,DNSTY,VFREER
      COMMON/BLOCK4/X(50),Y(50),IPLOT,ALPHA,LA,NN,ALPHAC,DNSTY,VFREE
      REAL MU
      ITER=0
      TEST=.001
      PI=3.1415926535
C READ AND WRITE INPUT DATA
      READ(5,940) DA
  940 FORMAT(A10)
      WRITE(6,941) DA
  941 FORMAT(1H ,*AIRFOIL IS*,2X,A10)
      READ(5,935) M
  935 FORMAT(I3)
      WRITE(6,936) M
  936 FORMAT(1H ,I3,* POINTS ARE DEFINED*)
      WRITE(6,926)
  926 FORMAT(1H ,*INPUT DATA*)
      WRITE(6,927)
  927 FORMAT(1H ,7X,1HI,7X,1HX,8X,1HY)
      N=M+1
      M=N
      DO 100 I=1,M
      READ(5,930) XIN(I),YIN(I)
  930 FORMAT(2F10.5)
  100 WRITE(6,931) I,XIN(I),YIN(I)
  931 FORMAT(1H ,I8,F13,7X,F8.5)
      WRITE(6,950)N
  950 FORMAT(1H ,*POINTS*,I3,* IS DEFINED INTERNALLY TO BE EQUAL TO POINT
     1 1 FOR CLOSURE.*)
      READ(5,937)IPLOT
  937 FORMAT(I2)
      IF(IPLOT) 170,171,172
  170 WRITE(6,920)
  920 FORMAT(1H ,*AIRFOIL, PSEUDO-CIRCLE, AND EXACT CIRCLE WILL BE PLOTT
     1ED ON SAME AXES.*)
      GO TO 173
  172 WRITE(6,921)
  921 FORMAT(1H ,*AIRFOIL, PSEUDO-CIRCLE, AND EXACT CIRCLE WILL BE PLOTT
     1ED ON SEPARATE AXES.*)
      GO TO 173
  171 WRITE(6,922)
  922 FORMAT(1H ,*PLOT WILL BE OMITTED.*)
  173 CONTINUE
      READ(5,950)ALPHAD
  950 FORMAT(F10.5)
      ALPHA=(ALPHAD*PI)/180.
      READ(5,932) DNSTY,MU,VFREE
  932 FORMAT(3F15.5)
```

```
      DNSTYP=DNSTY
      VFREEP=VFREE
      WRITE(6,901) DNSTY,MU,VFREE
  901 FORMAT(1H ,*DENSITY=*,F12.5,5X,*VISCOSITY=*,E12.5,5X,*FREESTREAM V
     1ELOCITY=*,F12.5)
      REY=(DNSTY*VFREE)/MU
      WRITE(6,802) REY
  802 FORMAT(1H ,*REYNOLDS NUMBER=*,F12.5,3X,*BASED ON CHORD LENGTH.*)
      XIN(N)=XIN(1)
      YIN(N)=YIN(1)
      DO 6 I=1,N
      X(I)=XIN(I)
      Y(I)=YIN(I)
    6 YSAVE(I)=Y(I)
    1 CALL THRS
      CALL BOUND
      DO 7 I=1,N
      DELI(I)=SIGN(DELI(I),YIN(I))
      Y(I)=YIN(I)+DELI(I)
      IF(ABS(Y(I)-YSAVE(I)).GE.TEST) ITER =1
    7 CONTINUE
      IF(ITER.EQ.0) GO TO 4
      DO 3 I=1,N
    3 YSAVE(I)=Y(I)
      Y(1)=0.
      Y(N)=0.
      ITER=0
      GO TO 1
    4 WRITE(6,5)
    5 FORMAT(1H ,*ITERATION IS COMPLETE.*)
      END
```

```
      SUBROUTINE THEO
C
C THEO TRANSFORMS AN ARBITRARY AIRFOIL SHAPE FIRST INTO A PSEUDO-CIRCLE,
C THEN INTO AN EXACT CIRCLE. THE AIRFOIL CHORD LINE IS ASSUMED TO LIE ON
C THE X-AXIS WITH THE TRAILING EDGE IN THE +X DIRECTION AND THE MIDCHORD
C POINT AT THE ORIGIN. POINTS DEFINING THE AIRFOIL SURFACE MUST BEGIN AT
C THE TRAILING EDGE AND CONTINUE IN THE COUNTERCLOCKWISE DIRECTION. ALL
C DIMENSIONS MUST BE REFERENCED TO A UNIT CHORD LENGTH.
C
C THEO REQUIRES THE FOLLOWING INPUT
C
C      DESIGNATION OF AIRFOIL
C      NUMBER OF POINTS
C      X AND Y FOR EACH POINT
C      PLOTTING INDEX
C           -1 FOR PLOTS ON SAME AXES
C            0 FOR PLOTS OMITTED
C           +1 FOR PLOTS ON SEPARATE AXES
C      ANGLE OF ATTACK
C      DENSITY
C      FREESTREAM VELOCITY
C
C THEO OUTPUTS THE COORDINATES IN ALL THREE PLANES, POLAR COORDINATES IN
C THE PSEUDO-CIRCLE AND EXACT CIRCLE PLANES, AND VARIOUS QUANTITIES USED
C TO COMPUTE THESE COORDINATES, AS WELL AS SURFACE/FREE-STREAM VELOCITY
C RATIO, PRESSURE COEFFICIENT, CIRCULATION, AND LIFT.
C
C THE METHOD OF TRANSFORMATION IS THEODORSEN'S METHOD.
C
C SUBROUTINE THEO USES SUBROUTINES PLTVAL, TIMES, PLTS, AND XTKFG AND
C FUNCTION ATAN, IN ADDITION TO LIBRARY SUBROUTINES AND FUNCTIONS.
C
      DIMENSION XX(50),YY(50),PHI(50),PSI(50),VVV(50)
      DIMENSION [illegible]
      DIMENSION [illegible]
      COMMON/BLOCK1/THETA,PI,TWT,C(50),D(50),THETA(50),PSI(50),DPT(50)
      COMMON/BLOCK2/NAME,A(50),N,EPS(50)
      COMMON/BLOCK3/VXV(50)
      COMMON/BLOCK4/X(50),Y(50),IPLOT,ALPHA,RHO,V,ALPHAC,EPS,[illegible]
      REAL LIFT
      PI=3.14159265
      [illegible]
      DELTA=.1
      A=.5
      N=NN
      M=N-1
C COMPUTE OMEGA
      DO 311 I=1,N
      PHI=1.+(X(I)*X(I)+Y(I)*Y(I))/(4.*[illegible])
      ARG=SQRT(ABS(.5*(-[illegible]+SQRT(ABS(PHI*PHI-(Y(I)/.5)**2))))))
      OMEGA(I)=ASIN(ARG)
      IF (X(I)) 301,301,302
  310 IF (Y(I)) 302,303,303
  302 OMEGA(I)=OMEGA(I)+PI
      GO TO 310
```

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```
      237 OMEGA(I)=PI-OMEGA(I)
          GO TO 210
      231 IF (Y(I)) 236,210,210
      236 OMEGA(I)=2.*PI-OMEGA(I)
      210 CONTINUE
          OMEGA(N)=OMEGA(1)+2.*PI
    C COMPUTE PSI
          ARGR=SQRT(SQRT(1.+[illegible](Y(I)/A)**2)))))
          PSI(I)=ALOG(ARGR+SQRT(ARGR*ARGR-1.))
    C COMPUTE RHO
          RHO(I)=EXP(PSI(I))
    C COMPUTE XX AND YY
          XX(I)=RHO(I)*COS(OMEGA(I))
      171 YY(I)=RHO(I)*SIN(OMEGA(I))
    C CURVE FIT PSI(OMEGA)
          CALL FLSQ(OMEGA,PSI,N,M,C,B,EMAX,RMS,EMPS)
          [illegible]
    C COMPUTE THETA
          DO 207 I=1,N
          THETA(I)=OMEGA(I)
          EPS(I)=0.
      207 EEPS(I)=0.
          J=0
      211 INC=0
      212 DO 270 I=1,N
          EEPS=0.
          EPS=0.
          THETA(I)=OMEGA(I)-EPS(I)
          THETA=THETA(I)
          IF((THETA(I)[illegible]) GO TO 251
          IF(THETA(I)[illegible]) 250,250,251
      250 [illegible]
          [illegible]
          CALL [illegible],ALOWER,JTOLB)
          GO TO 252
      251 [illegible]
          [illegible]
          CALL [illegible],FLARMA,JTOLA)
          [illegible]
          CALL [illegible],FLADWN,JTOLB)
          EPS(I)=[illegible]
      252 IF(ABS(EPS(I)-EEPS(I)).LE.[illegible]) GO TO 270
          INC=1
      270 CONTINUE
          J=J+1
          DO 515 I=1,N
      515 EEPS(I)=EPS(I)
          IF(J.GT.15) GO TO 520
          IF(INC.NE.0) GO TO 211
          GO TO 600
      520 WRITE(6,521)
      521 FORMAT(1H ,*J.GT.15*)
      600 CONTINUE
    C CURVE FIT PSI(THETA)
          CALL FLSQ(THETA,PSI,N,M,C,B,EMAX1,RMS1,EMPS1)
```

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```
SUBROUTINE  [illegible]      TRACE                    [illegible]      PAGE      3
      [illegible]
C COMPUTE [illegible]
      CALL [illegible]
C COMPUTE [illegible]
      [illegible]
      DO [illegible]
C COMPUTE [illegible]
      [illegible]
  1-3 [illegible]
C COMPUTE [illegible]
      [illegible]
      [illegible]
      [illegible]
      [illegible]
      [illegible]
      [illegible]
      [illegible]
      [illegible]
   [illegible]
      [illegible]
      DO [illegible]
      [illegible]
      [illegible]
      [illegible]
  [illegible]
      [illegible]
      [illegible]
      [illegible]
      [illegible]
      [illegible]
      [illegible]
  [illegible]
      [illegible]
  [illegible]
      [illegible]
      [illegible]
C [illegible]
      [illegible]
  [illegible]
     [illegible]
      DO [illegible]
  [illegible]
     [illegible]
  [illegible]
      [illegible]
  [illegible]
      [illegible]
  [illegible]
  [illegible]
      [illegible]
  [illegible]
      DO [illegible]
  [illegible]
  [illegible]
      [illegible]
```

```
  910 FORMAT(1H0,6HTHETA=,E11.4,5X,3HR0=,E11.4,5X,6HALPHA=,E11.4,1X,3HDEG,
     1 5X,5HDELTA=,E11.4,1X,3HDEG)
      WRITE(6,916) CIRC,LIF
  916 FORMAT(1H0,*CIRCULATION=*,E11.4,5X,*LIFT=*,E11.4)
C PLOT COORDINATES
      X(N+1)=0.
      Y(N+1)=0.
      XX(N+1)=0.
      YY(N+1)=0.
      YYY(N+1)=0.
      YYY(N+1)=0.
      X(N+2)=.2
      Y(N+2)=.2
      XX(N+2)=.2
      YY(N+2)=.2
      YYY(N+2)=.2
      YYY(N+2)=.2
      IF(IPLOT) 6 [illegible]
  600 CALL PLOT(10.,-12.,-3)
      CALL PLOT(4.,6.5,-3)
      CALL AXIS(-3.,0.,3.,22H                     X,=23,0.,0.,
     1-.5,.2)
      CALL AXIS(3.,-3.,5.,22H                     Y,=30,0.,90.,
     1-.5,.2)
      CALL LINE(X,Y,N,1,1,1)
      CALL LINE(XX,YY,N,1,1,0)
      CALL LINE(XXX,YYY,N,1,1,2)
      CALL SYMBOL(2.,5.,.1,[illegible])
      CALL SYMBOL(4.,-1.5,.1,[illegible])
      CALL SYMBOL(2.,-2.6,.1,7HAIRFOIL,0.,7)
      CALL SYMBOL(2.,-2.7,.1,[illegible])
      CALL SYMBOL(2.,-2.,.1,[illegible])
      CALL SYMBOL(2.,-2.75,.1,[illegible])
      CALL SYMBOL(2.,-2.,.1,[illegible])
      CALL PLOT(10.,-12.,-3)
      GO TO 631
  601 CALL PLOT(10.,-10.,-3)
      CALL PLOT(4.,5.5,-3)
      CALL AXIS(-2.,0.,3.,22H                     X,=23,0.,0.,
     1-.5,.2)
      CALL AXIS(3.,-2.,5.,22H                     Y,=30,0.,90.,
     1-.5,.2)
      CALL LINE(X,Y,N,1,1,1)
      CALL SYMBOL(2.,2.,.1,[illegible])
      CALL SYMBOL(2.,-2.,.1,7HAIRFOIL,0.,7)
      CALL PLOT(11.,0.,-3)
      CALL AXIS(0.,-2.,5.,22H                     Y,=30,0.,90.,
     1-.5,.2)
      CALL AXIS(-2.,5.,3.,22H                     X,=20,0.,0.,
     1-.5,.2)
      CALL LINE(XX,YY,N,1,1,1)
      CALL SYMBOL(2.,-2.,.1,[illegible],0.,12)
      CALL PLOT(11.,0.,-3)
      CALL AXIS(-2.,5.,3.,22H                     X,=20,0.,0.,
     1-.5,.2)
```

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```
      SUBROUTINE  THEO       TRACE                   CDC 6600 FTN V3.0-P310 OPT=1  11/04/71  14.29.55.      PAGE      9
                 CALL AXIS(0.,-2.5,25H                                Y,-30,5.,90.,
                1-.5,.2)
                 CALL LINE(XXX,YYY,N,1,1,1)
                 CALL SYMBOL(2.,-2.,.1,12HEXACT CIRCLE,0.,12)
 225             CALL PLOT(10.,-10.,-3)
             401 CONTINUE
           C PLOT PRESSURE COEFFICIENT
                 X(N+1)=-.5
                 X(N+2)=.2
 230             CP(N+1)=-1.0
                 CP(N+2)=.4
                 CALL PLOT(10.,-10.,-3)
                 CALL PLOT(1.,1.,-3)
                 CALL AXIS(0.,0.,13HPRESSURE COEF,13,5.,90.0,-1.0,.4)
 235             CALL AXIS(0.,2.5,1HX,-1,5.,0.0,-.5,.2)
                 CALL LINE(X,CP,N,1,1,1)
                 END
```

```
SUBROUTINE  FNEVAL    TRACE                      CDC 6600 FTN V3.0-P316 OPT=1  11/04/71  16.29.55.      PAGE     1
                    SUBROUTINE FNEVAL(X,FX)
              C
              C FNEVAL DEFINES THE INTEGRANDS NEEDED IN COMPUTING PHI AND EPS, AND REQUIRES
              C FUNCTION ATKN, AS WELL AS LIBRARY FUNCTIONS.
              C
                    COMMON/BLOCK1/THET,PI,INT,C(6),D(6),THETA(50),EPS(50),PSI(50)
                    COMMON/BLOCK1/OMEGA(50),N,EPSO(50)
                    IF(INT)11,10
              C
              C FORM PHI INTEGRAND
              C
                 11 FX=(D(1)*X**5+D(2)*X**4+D(3)*X**3+D(4)*X**2+D(5)*X+D(6))/(2.*PI)
                    GO TO 12
              C
              C FORM EPS INTEGRAND
              C
                 10 Z=ATKN(THETA,EPSO,N,3,X)*X
                    CT=COS(.5*(X-THET))/SIN(.5*(X-THET))
                    W=ATKN(OMEGA,PSI,N,3,X)
                    FX=(W*CT)/(2.*PI)
                 12 RETURN
                    END
```

```
      SUBROUTINE BOUND
C
C BOUND COMPUTES BOUNDARY LAYER PARAMETERS FOR AN ARBITRARY AIRFOIL
C SHAPE IMMERSED IN LAMINAR INCOMPRESSIBLE FLOW.  THE KARMAN-POHLHAUSEN
C METHOD IS USED.
C
C BOUND REQUIRES THE FOLLOWING INPUT
C
C     NUMBER OF POINTS
C     X AND Y FOR EACH POINT
C     TANGENTIAL/FREE-STREAM VELOCITY RATIO FOR EACH POINT
C     VISCOSITY
C     DENSITY
C     FREESTREAM VELOCITY
C
C BOUND OUTPUTS SHEAR STRESS AT SURFACE AND MOMENTUM AND DISPLACEMENT
C BOUNDARY LAYER THICKNESSES, AS WELL AS VARIOUS QUANTITIES USED IN
C THESE COMPUTATIONS.
C
C BOUND USES FUNCTION ATKN IN ADDITION TO LIBRARY SUBROUTINES AND FUNCTIONS.
C
      DIMENSION Z(50),DZET(50),K(34),F(34),F1(34),F2(34),LAMBDA(34),
     1KK(50),FF(50),FF1(50),FF2(50),LAM(50),DEL2(50),TAU(50),
     2X(50),Y(50),RAD(50),ZETA(50),ANG(50),S(50),DVTV(50)
      COMMON/BLOCK2/VTV(50)
      COMMON/BLOCK1/XIN(50),YIN(50),N,DELL1(50),MU,DNSTY1,VFREES
      REAL NU,MU,K,KK,LAMBDA,LAM
      DATA LAMBDA/12.,11.,10.,9.,8.,7.8,7.6,7.4,7.2,7.052,7.,6.8,6.6,6.4,
     16.2,6.,5.,4.,3.,2.,1.,0.,-1.,-2.,-3.,-4.,-5.,-6.,-7.,-8.,-9.,
     2-10.,-11.,-12./
      DATA K/.0948,.0941,.0919,.0882,.0831,.0819,.0807,.0794,.0781,
     1.0772,.0767,.0752,.0737,.0721,.0706,.0689,.0599,.0497,.0385,.0264,
     2.0135,0.,-.0140,-.0284,-.0429,-.0575,-.0720,-.0862,-.0999,-.1130,
     3-.1254,-.1369,-.1474,-.1567/
      DATA F/-.0948,-.0912,-.0840,-.0598,-.0335,-.0271,-.0203,
     1-.0132,-.0061,0.,.0021,.0107,.0186,.0274,.0362,.0459,.0579,
     2.1579,.2365,.3034,.3820,.4609,.5413,.6230,.7060,.7898,.8720,
     3.9577,1.0481,1.1387,1.4167,1.5299,1.6857,1.7261/
      DATA F1/2.250,2.253,2.260,2.273,2.289,2.293,2.297,2.301,2.305,
     12.308,2.309,2.314,2.319,2.323,2.328,2.333,2.361,2.392,2.427,2.466,
     22.508,2.554,2.614,2.647,2.716,2.779,2.847,2.921,2.999,3.085,3.176,
     33.276,3.383,3.500/
      DATA F2/.356,.355,.351,.347,.340,.338,.337,.335,.333,.332,.331,
     1.330,.329,.326,.324,.321,.317,.297,.283,.269,.252,.235,.217,.199,
     2.179,.160,.140,.120,.100,.079,.059,.039,.019,0./
      DELLIM=.002
      TAULIM=.1
      DO 1550 I=1,N
      VTV(I)=VTV(I)*VFREES
      X(I)=XIN(I)
 1550 Y(I)=YIN(I)
      NU=MU/DNSTY1
C
C COMPUTE POLAR COORDINATES OF EACH POINT AND FIND LEADING EDGE
C
```

```
      DO 1001 I=1,N
      RAD(I)=SQRT(X(I)**2+Y(I)**2)
      ZETA(I)=ASIN(Y(I)/RAD(I))
      IF(Y(I).EQ.-.5) LE=I
 1001 CONTINUE
C
C COMPUTE ARC LENGTHS
C
      ARC(1)=0.
      DO 1002 I=2,N
 1002 ARC(I)=SQRT(ABS(RAD(I)**2+RAD(I-1)**2-2.*RAD(I)*RAD(I-1)*COS(ZETA(
     1I)-ZETA(I-1))))
C
C COMPUTE STREAMWISE COORDINATE OF EACH POINT
C
      LEM1=LE-1
      S(LE)=0.
      I=LEM1
 1003 S(I)=S(I+1)+ARC(I+1)
      IF(I.EQ.1) GO TO 1010
      I=I-1
      GO TO 1003
 1010 CONTINUE
      LEP1=LE+1
      DO 1004 I=LEP1,N
 1004 S(I)=S(I-1)+ARC(I)
C
C COMPUTE VELOCITY DERIVATIVES ON UPPER SURFACE
C
      DVTV(LE)=(-3.*VTV(LE)+4.*VTV(LE-1)-VTV(LE-2))/S(LE-2)
      DVTV(1)=(VTV(3)-4.*VTV(2)+3.*VTV(1))/(S(1)-S(3))
      I=LEM1
 1035 DVTV(I)=(-VTV(I+1)+VTV(I-1))/(S(I-1)-S(I+1))
      IF(I.EQ.2) GO TO 1011
      I=I-1
      GO TO 1035
 1011 CONTINUE
      D2VTV=(-3.*DVTV(LE)+4.*DVTV(LE-1)-DVTV(LE-2))/S(LE-2)
C
C COMPUTE BOUNDARY LAYER PARAMETERS ON UPPER SURFACE
C
      Z(LE)=.0770/DVTV(LE)
      DZDS(LE)=(-.0652*D2VTV)/(DVTV(LE))**2
      I=LEM1
 1012 Z(I)=Z(I+1)+DZDS(I+1)*(S(I)-S(I+1))
      KK(I)=Z(I)*DVTV(I)
      FF(I)=ATKN(K,F,34,4,KK(I))
      DZDS(I)=FF(I)/VTV(I)
      FF1(I)=ATKN(K,F1,34,4,KK(I))
      FF2(I)=ATKN(K,F2,34,4,KK(I))
 1036 LAM(I)=ATKN(K,LAMBDA,34,4,KK(I))
      IF(I.EQ.1) GO TO 1013
      I=I-1
      GO TO 1012
 1013 CONTINUE
```

```
      KK(LE)=.0779
      FF(LE)=0.
      FF1(LE)=2.308
      FF2(LE)=.332
      LAM(LE)=7.052
      I=LE
 1014 DEL2(I)=SQRT(NU*Z(I))
      DEL1(I)=FF1(I)*DEL2(I)
 1037 TAU(I)=(FF2(I)*MU*VTV(I))/DEL2(I)
      IF(I.EQ.1) GO TO 1015
      I=I-1
      GO TO 1014
 1015 CONTINUE
C
C WRITE ANSWERS FOR UPPER SURFACE
C
      WRITE(6,1924)
 1924 FORMAT(1H1,*UPPER SURFACE*)
      WRITE(6,1920)
 1920 FORMAT(1H ,2H PT,5X,1HZ,9X,4HDZDS,8X,2HKK,9X,2HFF,8X,3HFF1,8X,
     13HFF2,8X,3HLAM,8X,4HDEL1,7X,4HDEL2,7X,3HTAU)
      DO 1928 I=1,LE
 1928 WRITE(6,1921) I,Z(I),DZDS(I),KK(I),FF(I),FF1(I),FF2(I),LAM(I),
     1DEL1(I),DEL2(I),TAU(I)
 1921 FORMAT(1H ,I3,10E11.4)
 1926 CONTINUE
C
C COMPUTE VELOCITY DERIVATIVES ON LOWER SURFACE
C
      DVTV(LE)=(-3.*VTV(LE)+4.*VTV(LE+1)-VTV(LE+2))/S(LE+2)
      DVTV(N)=(VTV(N-2)-4.*VTV(N-1)+3.*VTV(N))/(S(I+1)-S(I-1))
      NM1=N-1
      DO 1043 I=LEP1,NM1
 1043 DVTV(I)=(-VTV(I-1)+VTV(I+1))/(S(I+1)-S(I-1))
      D2VTV=(-3.*DVTV(LE)+4.*DVTV(LE+1)-DVTV(LE+2))/S(LE+2)
C
C COMPUTE BOUNDARY LAYER PARAMETERS ON LOWER SURFACE
C
      Z(LE)=.0779/DVTV(LE)
      DZDS(LE)=(-.0779*D2VTV)/(DVTV(LE))**2
      DO 1041 I=LEP1,N
      Z(I)=Z(I-1)+DZDS(I-1)*(S(I)-S(I-1))
      KK(I)=Z(I)*DVTV(I)
      FF(I)=ATKN(K,F,74,4,KK(I))
      DZDS(I)=FF(I)/VTV(I)
      FF1(I)=ATKN(K,F1,74,4,KK(I))
      FF2(I)=ATKN(K,F2,74,4,KK(I))
      LAM(I)=ATKN(K,LAMBDA,74,4,KK(I))
      DEL2(I)=SQRT(NU*Z(I))
      DEL1(I)=FF1(I)*DEL2(I)
      TAU(I)=(FF2(I)*MU*VTV(I))/DEL2(I)
 1041 CONTINUE
C
C WRITE ANSWERS FOR LOWER SURFACE
C
```

```
      WRITE(6,1922)
 1922 FORMAT(1H0,*LOWER SURFACE*)
      WRITE(6,1920)
      DO 1029 I=LF,N
 1029 WRITE(6,1921) I,Z(I),DZDS(I),XX(I),FF(I),FF1(I),FFR(I),LAP(I),
     1DEL1(I),DEL2(I),TAU(I)
      DO 770 I=1,N
      IF(DEL1(I).GE.DELLIM) DEL1(I)=DELLIM
      IF(TAU(I).GE.TAULIM) TAU(I)=TAULIM
      IF(TAU(I).LE.0.) TAU(I)=0.
  770 CONTINUE
C
C PLOT SHEAR STRESS
C
      X(N+1)=-.5
      X(N+2)=.2
      TAU(N+1)=0.
      TAU(N+2)=.02
      CALL PLOT(10.,-10.,-3)
      CALL PLOT(1.,1.,-3)
      CALL AXIS(0.,0.,3HTAU,3,5.,90.0,0.,.02)
      CALL AXIS(0.,0.,1HX,-1,5.,0.0,-.5,.2)
      CALL LINE(X,TAU,N,1,1,1)
C
C PLOT DISPLACEMENT THICKNESS
C
      X(N+1)=-.5
      X(N+2)=.2
      DEL1(N+1)=0.
      DEL1(N+2)=.001
      CALL PLOT(10.,-10.,-3)
      CALL PLOT(1.,1.,-3)
      CALL AXIS(0.,0.,10HDISP THICK,10,5.,90.0,0.,.001)
      CALL AXIS(0.,0.,1HX,-1,5.,0.0,-.5,.2)
      CALL LINE(X,DEL1,N,1,1,1)
      CALL PLOT(0.,0.,-3)
 1927 CONTINUE
      END
```

```
      SUBROUTINE MTXEQ(A,X,B,N,K)                                       MTXEQ001
C                                                                       MTXEQ002
C     MATRIX EQUATION SOLVER     (7094 FORTRAN IV)                      MTXEQ003
C                                                                       MTXEQ004
C     USAGE...                                                          MTXEQ005
C                                                                       MTXEQ006
C     TO SOLVE THE LINEAR SYSTEM     AX=B                               MTXEQ007
C                                                                       MTXEQ008
C     CALL MTXEQ(A,X,B,N,K)                                             MTXEQ009
C                                                                       MTXEQ010
C        WHERE A MUST BE DIMENSIONED N X N                              MTXEQ011
C              X MUST BE DIMENSIONED N X K                              MTXEQ012
C              B MUST BE DIMENSIONED N X K                              MTXEQ013
C              N IS THE NO. OF EQUATIONS (ROWS IN A,X,B)                MTXEQ014
C              K IS THE NO. OF SOLUTION VECTORS (COLS. IN X,B)          MTXEQ015
C                                                                       MTXEQ016
C     564 CELLS OF BLANK COMMON ARE USED.                               MTXEQ017
C                                                                       MTXEQ018
C     NOTE...   TO CHANGE DIMENSIONS OF ARRAYS C AND PIV, ALSO          MTXEQ019
C            CHANGE VALUES OF NMAX AND NKMAX IN DATA STATEMENT.         MTXEQ020
C                                                                       MTXEQ021
C                                                                       MTXEQ022
      DIMENSION   A(N,N), B(N,K), X(N,K)                                MTXEQ023
      COMMON   AIPP, I, IERCM, IP1, IPIV, IIC,                          MTXEQ024
     1         J, KP, L, NP, NP1, NPJ, NPK, PM                          MTXEQ025
      COMMON     PIV(25), C(24,26)                                      MTXEQ026
      DATA     NMAX, NKMAX/ 24, 26/                                     MTXEQ027
C                                                                       MTXEQ028
C     GET ARGUMENTS N AND K                                             MTXEQ029
C                                                                       MTXEQ030
      NP=N                                                              MTXEQ031
      KP=K                                                              MTXEQ032
C                                                                       MTXEQ033
C     TEST N AND K FOR CORRECT RANGE                                    MTXEQ034
C                                                                       MTXEQ035
      IF(NP.LE.0.OR.NP.GT.NMAX) GO TO 150                               MTXEQ036
      IF(KP.LE.0.OR.(NP+KP).GT.NKMAX) GO TO 190                         MTXEQ037
C                                                                       MTXEQ038
C     MOVE ARRAYS A(I,J) AND B(I,J) INTO C(I,J)                         MTXEQ039
C                                                                       MTXEQ040
      DO 10 J=1,NP                                                      MTXEQ041
      DO 10 I=1,NP                                                      MTXEQ042
   10 C(I,J)=A(I,J)                                                     MTXEQ043
      DO 20 J=1,KP                                                      MTXEQ044
      NPJ=NP+J                                                          MTXEQ045
      DO 20 I=1,NP                                                      MTXEQ046
   20 C(I,NPJ)=B(I,J)                                                   MTXEQ047
C                                                                       MTXEQ048
C     SET TO PERFORM N ELIMINATION SWEEPS (I=1,N)                       MTXEQ049
C                                                                       MTXEQ050
      NP1=NP+1                                                          MTXEQ051
      NPK=NP+KP                                                         MTXEQ052
      DO 120 I=1,NP                                                     MTXEQ053
      IP1=I+1                                                           MTXEQ054
C                                                                       MTXEQ055
```

```
C     SEARCH FOR NEXT PIVOT ROW (I-TH PIVOT IS IN COL. I)
C
      ATPE=0.
      DO 40 J=I,NR
      IF (ABS(C(J,I))-ATPE) 40,35,35
   35 ATPE=ABS(C(J,I))
      IPIV=J
   40 CONTINUE
C
C     OPERATE ON THE PIVOT ROW
C
      IF (ATPE) 210,210,50
   50 DO 60 J=IP1,NCK
   60 PIV(J)=C(IPIV,J)/C(IPIV,I)
C
C     PERFORM ELIMINATIONS BELOW THE DIAGONAL (COL. I)
C
      IFROW=NR
      ITO=NR
   70 IF (IFROW-IPIV) 8 ,110,80
   80 RM=C(IFROW,I)
      DO 90 J=IP1,NCK
   90 C(ITO,J)=C(IFROW,J)-RM*PIV(J)
      ITO=ITO-1
  100 IFROW=IFROW-1
      IF (IFROW-I) 110,70,70
C
C     PUT THE I-TH PIVOT ROW IN THE VACATED ROW I
C
  110 DO 120 J=IP1,NCK
  120 C(I,J)=PIV(J)
C
C     NOW DO THE BACK SOLUTION
C
      I=NR
  130 IP1=I
      I=I-1
      IF (I) 160,160,140
  140 DO 150 J=NR1,NCK
      DO 150 L=IP1,NR
  150 C(I,J)=C(I,J)-C(I,L)*C(L,J)
      GO TO 130
C
C     MOVE THE SOLUTION TO ARRAY X(I,J)
C
  160 DO 170 J=1,MR
      NRJ=NR+J
      DO 170 I=1,NR
  170 X(I,J)=C(I,NRJ)
  180 RETURN
C
  190 PRINT 1,20, NR, MR
      CALL SYSTEM (71,1L )
      STOP
  210 PRINT 1001
```

```
SUBROUTINE  MTXEQ     TRACE                       CDC 6600 FTN V3.0-261A OPT=1  11/04/71  14.29.55.      PAGE      3
                 CALL SYSTEM (203,1L )                                                    MTXEQ111
                 STOP                                                                     MTXEQ112
            1000 FORMAT(3H N=I12,5H   M=I12,35H ARE INCORRECT FOR SUBROUTINE MTXEQ)MTXEQ113
            1001 FORMAT (37HCOEFF(A)=0 IN CALL TO SUBROUTINE MTXEQ)                       MTXEQ114
                 END                                                                      MTXEQ115
```

```
      FUNCTION ATKN(X,Y,N,K,XI)                                         ATKN0001
C                                                                       ATKN0002
C     ATKN     AITKEN INTERPOLATING FUNCTION                            ATKN0003
C                                                                       ATKN0004
C     USAGE...                                                          ATKN0005
C                                                                       ATKN0006
C     Z=ATKN(X,Y,N,K,XI)                                                ATKN0007
C                                                                       ATKN0008
C        WHERE...                                                       ATKN0009
C                                                                       ATKN0010
C        X - TABLE OF INDEPENDENT VARIABLE VALUES.                      ATKN0011
C            (MAY BE ASCENDING OR DESCENDING).                          ATKN0012
C        Y - TABLE OF DEPENDENT VARIABLE VALUES.                        ATKN0013
C        N - NO. OF POINTS IN TABLES X AND Y.                           ATKN0014
C        K - DEGREE OF INTERPOLATION DESIRED.                           ATKN0015
C        XI- X-VALUE FOR WHICH INTERPOLATION IS DESIRED.                ATKN0016
C                                                                       ATKN0017
C        THE INTERPOLATED VALUE IS RETURNED AS THE FUNCTION VALUE.      ATKN0018
C                                                                       ATKN0019
C        NO CELLS OF BLANK COMMON ARE USED.                             ATKN0020
C                                                                       ATKN0021
      DIMENSION   X(N), Y(N)                                            ATKN0022
      INTEGER     LI, KI, NI, LL, LU                                    ATKN0023
      COMMON      XX(17), YY(17)                                        ATKN0024
      DATA        KMAX/ 12/                                             ATKN0025
C                                                                       ATKN0026
      IF ( K .GT. KMAX .OR. K .LT. 1 )      GO TO 310                   ATKN0027
C                                                                       ATKN0028
      KIP=K+1                                                           ATKN0029
      IF (X(N)-X(1)) 110,10,10                                          ATKN0030
   10 IF (XI-X(1)) 20,20,30                                             ATKN0031
   20 LL=1                                                              ATKN0032
      GO TO 200                                                         ATKN0033
   30 IF (X(N)-XI) 40,40,50                                             ATKN0034
   40 LL=N-K                                                            ATKN0035
      GO TO 200                                                         ATKN0036
   50 LL=1                                                              ATKN0037
      LU=N                                                              ATKN0038
   60 IF (LU-LL-1) 180,180,70                                           ATKN0039
   70 LI=(LL+LU)/2                                                      ATKN0040
      IF (X(LI)-XI) 80,90,90                                            ATKN0041
   80 LL=LI                                                             ATKN0042
      GO TO 60                                                          ATKN0043
   90 LU=LI                                                             ATKN0044
      GO TO 60                                                          ATKN0045
  110 IF (XI-X(1)) 120,20,20                                            ATKN0046
  120 IF (X(N)-XI) 130,40,40                                            ATKN0047
  130 LL=1                                                              ATKN0048
      LU=N                                                              ATKN0049
  140 IF (LU-LL-1) 180,180,150                                          ATKN0050
  150 LI=(LL+LU)/2                                                      ATKN0051
      IF (X(LI)-XI) 160,170,170                                         ATKN0052
  160 LU=LI                                                             ATKN0053
      GO TO 140                                                         ATKN0054
  170 LL=LI                                                             ATKN0055
```

```
FUNCTION    ATKN      TRACE                     CDC 6600 FTN V3.0-261A OPT=1  11/04/71  14.29.55.      PAGE     2

            GO TO 140                                                      ATKN0056
        180 LL=LM-(K1+1)/2                                                 ATKN0057
            IF (LL) 20,210,190                                             ATKN0058
        190 IF (LL+K1-M) 200,200,40                                        ATKN0059
  5     200 DO 30  I=1,K1                                                  ATKN0060
            I1=LL+I                                                        ATKN0061
            XX(I)=X(I1)-X0                                                 ATKN0062
         30 YY(I)=Y(I1)                                                    ATKN0063
            DO 50  I=1,K                                                   ATKN0064
 50         DO 50  J=I,K                                                   ATKN0065
         50 YY(J+1)=(1./(XX(J+1)-XX(I)))*(YY(I)*XX(J+1)-YY(J+1)*XX(I))     ATKN0066
            ATKN=YY(K1)                                                    ATKN0067
            RETURN                                                         ATKN0068
                                                                           ATKN0069
              [illegible]                                                  ATKN0070
         40 [illegible]       =,I12,22H   IS INCORRECT FOR FUNCTION ATKN)  ATKN0071
            [illegible]       ,0)                                          ATKN0072
```

```
      SUBROUTINE PLSQ(X,Y,N,K,C,LIST,EMAX,ERMS,EMEQ)                      PLSC0001
C                                                                         PLSC0002
C     PLSQ     POLYNOMIAL LEAST SQUARE CURVE FIT                          PLSC0003
C                                                                         PLSC0004
C                                                                         PLSC0005
C          PLSQ WILL FIT A GIVEN SET OF DATA TO A                         PLSC0006
C     POLYNOMIAL OF DEGREE K OF THE FORM...                               PLSC0007
C                                                                         PLSC0008
C          Y=C(K+1)+C(K)*X+C(K-1)*X**2+...+C(2)*X**(K-1)+C(1)*X**K        PLSC0009
C                                                                         PLSC0010
C     PLSQ THEN COMPUTES THE MAXIMUM ERROR AND ROOT                       PLSC0011
C     MEAN SQUARE ERROR OBTAINED BY USING THE C                           PLSC0012
C     COEFFICIENTS TO RE-COMPUTE Y FROM X                                 PLSC0013
C                                                                         PLSC0014
C     USAGE...                                                            PLSC0015
C                                                                         PLSC0016
C     DIMENSION X(N), Y(N), C(L)                                          PLSC0017
C          WHERE L IS K+1                                                 PLSC0018
C     CALL PLSQ(X,Y,N,K,C,LIST,EMAX,ERMS,EMEQ)                            PLSC0019
C                                                                         PLSC0020
C        WHERE,                                                           PLSC0021
C                                                                         PLSC0022
C        X  IS THE ARRAY OF N INDEPENDENT VARIABLES                       PLSC0023
C                                                                         PLSC0024
C        Y  IS THE ARRAY OF N DEPENDENT VARIABLES                         PLSC0025
C                                                                         PLSC0026
C        N  IS THE NUMBER OF INDEPENDENT(DEPENDENT)                       PLSC0027
C           VARIABLES                                                     PLSC0028
C                                                                         PLSC0029
C        K  IS THE DEGREE OF THE LEAST SQUARES POLYNOMIAL                 PLSC0030
C                                                                         PLSC0031
C        C  IS THE ARRAY OF THE COEFFICIENTS,HIGH ORDER                   PLSC0032
C           TO LOW ORDER, OF THE LEAST SQUARES POLYNOMIAL                 PLSC0033
C                                                                         PLSC0034
C        LIST  =0   SUPPRESSES THE ERROR ANALYSIS OUTPUT                  PLSC0035
C              =1   GIVES THE ERROR ANALYSIS OUTPUT                       PLSC0036
C                                                                         PLSC0037
C        EMAX  IS THE MAXIMUM ABSOLUTE ERROR OBTAINED                     PLSC0038
C              BY USING THE COMPUTED C COEFFICIENTS TO                    PLSC0039
C              APPROXIMATE THE DEPENDENT VARIABLE                         PLSC0040
C                                                                         PLSC0041
C        ERMS  IS THE ROOT MEAN SQUARE ERROR OBTAINED                     PLSC0042
C              BY USING THE COMPUTED C COEFFICIENTS TO                    PLSC0043
C              APPROXIMATE THE DEPENDENT VARIABLE                         PLSC0044
C                                                                         PLSC0045
C        EMEQ  IS THE MAXIMUM DEVIATION FROM UNITY                        PLSC0046
C              IN THE LINEAR SYSTEM CHECK SOLUTION                        PLSC0047
C                                                                         PLSC0048
C                                                                         PLSC0049
C     PLSQ CALLS SUBROUTINES EXEM AND MTXEQ                               PLSC0050
C                                                                         PLSC0051
C     PLSQ USES 1339 CELLS OF BLANK COMMON                                PLSC0052
C                                                                         PLSC0053
C                                                                         PLSC0054
      COMMON    MTXEQT(664), CF, DIF, I, J, JC, JK,                       PLSC0055
```

```
     *          L, LL, LU, M, SUM, XI, XM(576),                         PLSQ0056
     *          XMAX, XMIN, XP, YC, YM(48)                              PLSQ0057
      DIMENSION    X(N), Y(N), C(24),                                   PLSQ0058
     *             XDP(48), XYDP(24)                                    PLSQ0059
      EQUIVALENCE    (MTXEQT(1),XDP(1)), (MTXEQT(97),XYDP(1))           PLSQ0060
      DATA    KMAX/ 23/                                                 PLSQ0061
C                                                                       PLSQ0062
C     CHECK K AND N FOR PROPER RANGE                                    PLSQ0063
C                                                                       PLSQ0064
      IF (K .GT. KMAX .OR. N .LE. K .OR. K .LE. 0)    GO TO 200         PLSQ0065
      L=K+1                                                             PLSQ0066
C                                                                       PLSQ0067
C     FIND MINIMUM AND MAXIMUM VALUES FOR X                             PLSQ0068
C                                                                       PLSQ0069
      XMIN=X(1)                                                         PLSQ0070
      XMAX=X(1)                                                         PLSQ0071
      DO 10 I=2,N                                                       PLSQ0072
      XMIN = AMIN1(XMIN,X(I))                                           PLSQ0073
   10 XMAX = AMAX1(XMAX,X(I))                                           PLSQ0074
C                                                                       PLSQ0075
C     ZERO DOUBLE PRECISION ARRAYS FOR SUMMING                          PLSQ0076
C                                                                       PLSQ0077
      M=2*K+1                                                           PLSQ0078
      DO 20 I=1,M                                                       PLSQ0079
   20 XDP(I) = 0.0                                                      PLSQ0080
      DO 25 I=1,L                                                       PLSQ0081
   25 XYDP(I) = 0.0                                                     PLSQ0082
C                                                                       PLSQ0083
C     TRANSFORM RANGE OF X TO (-1,+1) AND                               PLSQ0084
C     COMPUTE SUMS OF POWERS OF X AND SUMS                              PLSQ0085
C     OF Y TIMES POWERS OF X                                            PLSQ0086
C                                                                       PLSQ0087
      C1 = 2.0 / (XMAX - XMIN)                                          PLSQ0088
      C2 = (XMAX + XMIN) / (XMAX - XMIN)                                PLSQ0089
      LL=K+2                                                            PLSQ0090
      LU=2*K+1                                                          PLSQ0091
      DO 40 I=1,N                                                       PLSQ0092
      XP = 1.0                                                          PLSQ0093
      XI = C1 * X(I) - C2                                               PLSQ0094
      DO 30 J=1,L                                                       PLSQ0095
      XDP(J)=XDP(J)+XP                                                  PLSQ0096
      XYDP(J)=XYDP(J)+XP*Y(I)                                           PLSQ0097
   30 XP=XP*XI                                                          PLSQ0098
      DO 40 J=LL,LU                                                     PLSQ0099
      XDP(J)=XDP(J)+XP                                                  PLSQ0100
   40 XP=XP*XI                                                          PLSQ0101
C                                                                       PLSQ0102
C     STORE ABOVE COMPUTED SUMS IN ARRAY XM                             PLSQ0103
C     AND COMPUTE ROW SUMS FOR CHECK SOLUTION                           PLSQ0104
C                                                                       PLSQ0105
      DO 50 I=1,L                                                       PLSQ0106
      LL=I+L                                                            PLSQ0107
      YM(LL)=0.0                                                        PLSQ0108
      LU=(I-1)*L                                                        PLSQ0109
      JK=I-1                                                            PLSQ0110
```

```
              DO 50 J=1,L                                                     PLSC0111
              JK=JK+1                                                         PLSC0112
              JC=LU+J                                                         PLSC0113
              XM(JC)=XDP(JK)                                                  PLSC0114
   50         YM(LL)=YM(LL)+XM(JC)                                            PLSC0115
              DO 60 I=1,L                                                     PLSC0116
   60         YM(I)=XYDP(I)                                                   PLSC0117
        C                                                                     PLSC0118
        C     SOLVE THE SYSTEM XM*C=YM                                        PLSC0119
        C                                                                     PLSC0120
              CALL MTXEQ(XM,YM,YM,L,2)                                        PLSC0121
        C                                                                     PLSC0122
        C     REORDER AND MOVE SOLUTION TO C AND FIND                         PLSC0123
        C     MAXIMUM ERROR IN CHECK SOLUTION                                 PLSC0124
        C                                                                     PLSC0125
              EMEQ=0.0                                                        PLSC0126
              DO 70 I=1,L                                                     PLSC0127
              JK=K-I+2                                                        PLSC0128
              C(JK)=YM(I)                                                     PLSC0129
              JC=I+L                                                          PLSC0130
   70         EMEQ=AMAX1(EM[illegible]M(JC)-1.0))                             PLSC0131
        C                                                                     PLSC0132
        C     ADJUST COEFFICIENTS FOR ORIGINAL RANGE                          PLSC0133
        C     OF X                                                            PLSC0134
        C                                                                     PLSC0135
              DO 80 I=1,K                                                     PLSC0136
              DO 80 J=1,I                                                     PLSC0137
   80         C(J) = C(J) * C1                                                PLSC0138
              C1 = (XMAX + YMIN) / 2.0                                        PLSC0139
              DO 90 I=1,K                                                     PLSC0140
              M=L-I+1                                                         PLSC0141
              DO 90 J=2,M                                                     PLSC0142
   90         C(J) = -C1 * C(J-1) + C(J)                                      PLSC0143
        C                                                                     PLSC0144
        C     INITIATE PRINT OF ERROR ANALYSIS IF LIST .NE. 0                 PLSC0145
        C                                                                     PLSC0146
              IF (LIST.EQ.1) PRINT 1001                                       PLSC0147
        C                                                                     PLSC0148
        C     COMPUTE MAXIMUM AND ROOT MEAN SQUARE ERRORS                     PLSC0149
        C     AND OUTPUT ERROR ANALYSIS IF LIST .NE. 0                        PLSC0150
        C                                                                     PLSC0151
              EMAX=0.0                                                        PLSC0152
              SUM=0.0                                                         PLSC0153
              DO 120 I=1,N                                                    PLSC0154
              YC=C(1)                                                         PLSC0155
              DO 100 J=1,K                                                    PLSC0156
  100         YC=YC*X(I)+C(J+1)                                               PLSC0157
              DIF=YC-Y(I)                                                     PLSC0158
              IF (LIST.EQ.0) GO TO 120                                        PLSC0159
              IF (I .GT. L)    GO TO 110                                      PLSC0160
              PRINT 1002, I, X(I), Y(I), YC, DIF, C(I)                        PLSC0161
              GO TO 120                                                       PLSC0162
  110         PRINT 1002, I, X(I), Y(I), YC, DIF                              PLSC0163
  120         EMAX=AMAX1(EMAX,ABS(DIF))                                       PLSC0164
  130         SUM=SUM+DIF**2                                                  PLSC0165
```

```
SUBROUTINE  PLSQ      TRACE                    CDC 6600 FTN V3.0-261A OPT=1  11/04/71  14.29.55.      PAGE     4
                ERMS=SQRT(SUM/FLOAT(N))                                                 PLSC0166
                IF (LIST.EQ.1) PRINT 1003, EMAX, ERMS, EMEQ                             PLSC0167
                RETURN                                                                  PLSC0168
          C                                                                             PLSC0169
          C     GIVE ERROR MESSAGE AND RETURN TO                                        PLSC0170
          C     SYSTEM VIA EXEM                                                         PLSC0171
          C                                                                             PLSC0172
            200 PRINT 1000, N, K                                                        PLSC0173
                CALL SYSTEM (200,1L )                                                   PLSC0174
                RETURN                                                                  PLSC0175
           1000 FORMAT (3H0N=,I12,3H K=,I12,29HINCORRECT FOR SUBROUTINE PLSQ)           PLSC0176
           1001 FORMAT (1H1,20X,32HPLSQ     POLYNOMIAL LEAST SQUARE                     PLSC0177
               *24HCURVE FIT ERROR ANALYSIS//                                           PLSC0178
               *6H0     I,11X,9HY - GIVEN,11X,9HY - GIVEN,11X,                          PLSC0179
               *10HY - FITTED,12X,5HERROR,16X,4HB(I)//)                                 PLSC0180
           1002 FORMAT (1X,I5,8X,5(1PE14.6,6X))                                         PLSC0181
           1003 FORMAT (1H0,9X,5HEMAX=,1PE15.6,9X,5HERMS=,E15.6,                        PLSC0182
               *9X,5HEMEQ=,E15.6)                                                       PLSC0183
                END
```

```
SUBROUTINE  SIMPS     TRACE                  CDC 6600 FTN V3.0-261A OPT=1  11/04/71  14.29.55.      PAGE    1

      SUBROUTINE SIMPS(A,B,TOL,J,JMIN,ANSW,ALARM,JTOL)                    SIMPS001
CSIMPS INTEGRATION SUBROUTINE                                             SIMPS002
C        CALLING SEQUENCE                                                 SIMPS003
C        CALL SIMPS(A,B,TOL,JMIN,ANSW,ALARM,JTOLN)                        SIMPS004
C         A=LOWER LIMIT                                                   SIMPS005
C         B=UPPER LIMIT                                                   SIMPS006
C         TOL=10**-S MEANS S SIGNIF. DIGITS                               SIMPS007
C         J=MAX NO. OF ITERATIONS ALLOWED                                 SIMPS008
C         JMIN=MIN NO. OF ITERATIONS                                      SIMPS009
C         ANSW=VALUE OF INTEGRAL                                          SIMPS010
C         ALARM=0. INDICATES CONVERGENCE                                  SIMPS011
C              =1. OTHERWISE                                              SIMPS012
C         JTOL=NO. OF ITERATIONS REQUIRED FOR CONVERGENCE                 SIMPS013
C     SIMPS REQUIRES WRITING OF SUBROUTINE FNEVAL TO EVALUATE INTEGRAND SIMPS014
 1    FORMAT(///10X,31HINTEGRAL DID NOT CONVERGE AFTER,                   SIMPS015
     12X,I5,2X,10HITERATIONS///)                                          SIMPS016
      IF(A-B) 15,10,15                                                    SIMPS017
 10   D=0.0                                                               SIMPS018
      GO TO 75                                                            SIMPS019
 15   ALARM=0.0                                                           SIMPS020
      JTOL=1                                                              SIMPS021
      DH=B-A                                                              SIMPS022
      CALL FNEVAL(A,FA)                                                   SIMPS023
      CALL FNEVAL(B,FB)                                                   SIMPS024
C     FNEVAL EVALUATES INTEGRAND AT SPECIFIC POINTS                       SIMPS025
      DK=FA+FB                                                            SIMPS026
      DL=DK                                                               SIMPS027
 20   DM=DK                                                               SIMPS028
      S=0.0                                                               SIMPS029
      D=DH/2.0                                                            SIMPS030
      AO=A+D                                                              SIMPS031
 30   X=AO                                                                SIMPS032
      CALL FNEVAL(X,FY)                                                   SIMPS033
      S=S+FY                                                              SIMPS034
      AO=AO+DH                                                            SIMPS035
      IF(AO-B)30,40,40                                                    SIMPS036
 40   DK=DL+4.0*S                                                         SIMPS037
      DL=DL+2.0*S                                                         SIMPS038
      DH=D                                                                SIMPS039
      DMX2=2.0*DM                                                         SIMPS040
      DMAX=AMAX1(DK,DMX2)                                                 SIMPS041
      IF(ABS(DK-DMX2)-ABS(DMAX)*TOL) 70,70,50                             SIMPS042
 50   JTOL=JTOL+1                                                         SIMPS043
      IF(JTOL-J)60,65,65                                                  SIMPS044
 60   GO TO 20                                                            SIMPS045
   65 PRINT 1,J                                                           SIMPS046
      ALARM=1.0                                                           SIMPS047
      RETURN                                                              SIMPS048
 70   IF(JTOL-JMIN)50,75,75                                               SIMPS049
 75   ANSW=DK*D/3.0                                                       SIMPS050
      RETURN                                                              SIMPS051
       END                                                                SIMPS052
```

```
AIRFOIL IS   NACA [illegible]
 34 POINTS ARE [illegible]
THEY ARE

  PT        X         Y
   1    .[illegible] [illegible]
   2    .[illegible] .[illegible]
   3    .[illegible] .[illegible]
   4    .[illegible] .[illegible]
   5    .[illegible] .[illegible]
   6    .[illegible] .[illegible]
   7    .[illegible] .[illegible]
   8   -.[illegible] .[illegible]
   9   -.[illegible] .[illegible]
  10   -.25074   .[illegible]
  11   -.[illegible] .[illegible]
  12   -.[illegible]111 .[illegible]
  13   -.4[illegible] .[illegible]
  14   -.[illegible] .[illegible]
  15   -.[illegible] .[illegible]
  16   -.[illegible] .[illegible]
  17   -.[illegible] .[illegible]
  18   -.[illegible] [illegible]
  19   -.[illegible] -.[illegible]
  20   -.[illegible] -.[illegible]
  21   -.[illegible] -.[illegible]
  22   -.[illegible] -.[illegible]
  23   -.[illegible] -.[illegible]
  24   -.[illegible] -.[illegible]
  25   -.[illegible] -.[illegible]
  26   -.[illegible] -.[illegible]
  27   -.[illegible] -.[illegible]
  28   -.[illegible] -.[illegible]
  29   -.[illegible] -.[illegible]
  30    .[illegible] -.[illegible]
  31    .[illegible] -.[illegible]
  32    .[illegible] -.[illegible]
  33    .[illegible] -.[illegible]
  34    .[illegible] -.[illegible]

POINT 35 IS DEFINED INTERNALLY TO BE EQUAL TO POINT 1 FOR CLOSURE.
AIRFOIL, PSEUDO-CIRCLE, AND EXACT CIRCLE WILL BE PLOTTED ON SAME AXES.
DENSITY= .[illegible]E-02     VISCOSITY= .[illegible] E-06     FREESTREAM VELOCITY= .10000E+03
REYNOLDS NUMBER= .[illegible]   BASED ON CHORD LENGTH.
```

```
PT    X          Y          XX         YY         OMEGA      RHO        PSI        XXX        YYY        THETA      VTY
 1  .5830E+00 0.         .2500E+00 0.         0.         .2500E+00 0.         .2669E+00 -.4387E-02 -.1644E-01 0.
 2  .4593E+00  .6893E-02  .2327E+00  .1121E+00  .4513E+00  .2581E+00  .3203E-01  .2449E+00  .1064E+00  .4100E+00  .9662E+00
 3  .4033E+00  .1771E-01  .2065E+00  .1569E+00  .6429E+00  .2659E+00  .4234E-01  .2205E+00  .1534E+00  .5986E+00  .1014E+01
 4  .3094E+00  .2755E-01  .1509E+00  .2119E+00  .9276E+00  .2645E+00  .5759E-01  .1701E+00  .2057E+00  .8799E+00  .1042E+01
 5  .2034E+00  .3157E-01  .1072E+00  .2457E+00  .1159E+01  .2689E+00  .6962E-01  .1181E+00  .2292E+00  .1112E+01  .1067E+01
 6  .1033E+00  .3931E-01  .5414E-01  .2650E+00  .1369E+01  .2709E+00  .8316E-01  .6499E-01  .2589E+00  .1325E+01  .1087E+01
 7  .2033E-02  .4527E-01  .1503E-03  .2775E+00  .1572E+01  .2735E+00  .8992E-01  .1089E-01  .2667E+00  .1533E+01  .1107E+01
 8 -.1033E+00  .4790E-01 -.4404E-01  .2701E+00  .1771E+01  .2761E+00  .9921E-01 -.4416E-01  .2622E+00  .1737E+01  .1126E+01
 9 -.2073E+00  .4939E-01 -.1113E+00  .2757E+00  .1991E+01  .2784E+00  .1075E+00 -.1002E+00  .2474E+00  .1956E+01  .1139E+01
10 -.2617E+00  .4419E-01 -.1382E+00  .2497E+00  .2097E+01  .2753E+00  .1109E+00 -.1284E+00  .2340E+00  .2073E+01  .1148E+01
11 -.3013E+00  .4574E-01 -.1679E+00  .2264E+00  .2212E+01  .2822E+00  .1139E+00 -.1568E+00  .2160E+00  .2199E+01  .1152E+01
12 -.3511E+00  .4171E-01 -.1959E+00  .2117E+00  .2347E+01  .2809E+00  .1147E+00 -.1852E+00  .1922E+00  .2338E+01  .1153E+01
13 -.4032E+00  .3589E-01 -.2240E+00  .1633E+00  .2493E+01  .2812E+00  .1175E+00 -.2135E+00  .1602E+00  .2498E+01  .1145E+01
14 -.4261E+00  .3174E-01 -.2390E+00  .1494E+00  .2585E+01  .2812E+00  .1176E+00 -.2275E+00  .1355E+00  .2592E+01  .1142E+01
15 -.4513E+00  .2607E-01 -.2514E+00  .1246E+00  .2691E+01  .2813E+00  .1169E+00 -.2415E+00  .1136E+00  .2702E+01  .1135E+01
16 -.4754E+00  .1967E-01 -.2640E+00  .9197E-01  .2813E+01  .2832E+00  .1141E+00 -.2553E+00  .7793E-01  .2845E+01  .1067E+01
17 -.4891E+00  .1324E-01 -.2757E+00  .5777E-01  .2894E+01  .2769E+00  .1354E+00 -.2617E+00  .5247E-01  .2944E+01  .7804E+00
18 -.5033E+00 0.         -.2851E+00  .2341E-01  .3147E+01  .2500E+00 0.         -.2669E+00 -.6098E-02  .3164E+01 0.
19 -.4937E+00 -.1360E-01 -.2673E+00 -.7287E-01  .3309E+01  .2755E+00  .9723E-01 -.2589E+00 -.6450E-01  .3387E+01  .8293E+00
20 -.4743E+00 -.1677E-01 -.2627E+00 -.9111E-01  .3474E+01  .2752E+00  .9793E-01 -.2515E+00 -.8995E-01  .3487E+01  .1079E+01
21 -.4493E+00 -.2174E-01 -.2467E+00 -.1237E+00  .3616E+01  .2749E+00  .9566E-01 -.2365E+00 -.1236E+00  .3623E+01  .1121E+01
22 -.4209E+00 -.2459E-01 -.2312E+00 -.1459E+00  .3747E+01  .2743E+00  .9159E-01 -.2222E+00 -.1491E+00  .3730E+01  .1112E+01
23 -.3089E+00 -.2809E-01 -.2177E+00 -.1651E+00  .3794E+01  .2721E+00  .8823E-01 -.2077E+00 -.1676E+00  .3821E+01  .1102E+01
24 -.3643E+00 -.3159E-01 -.1997E+00 -.1951E+00  .3947E+01  .2714E+00  .8211E-01 -.1795E+00 -.1975E+00  .3975E+01  .1056E+01
25 -.2993E+00 -.3174E-01 -.1600E+00 -.2106E+00  .4037E+01  .2659E+00  .7651E-01 -.1517E+00 -.2156E+00  .4108E+01  .1035E+01
26 -.2497E+00 -.2260E-01 -.1375E+00 -.2239E+00  .4197E+01  .2655E+00  .7142E-01 -.1243E+00 -.2362E+00  .4228E+01  .1075E+01
27 -.1995E+00 -.1639E-01 -.1085E+00 -.2485E+00  .4353E+01  .2627E+00  .6673E-01 -.9475E-01 -.2455E+00  .4349E+01  .1065E+01
28 -.1013E+00 -.7690E-02 -.5203E-01 -.2604E+00  .4511E+01  .2651E+00  .5957E-01 -.4363E-01 -.2673E+00  .4549E+01  .1054E+01
29 -.2013E-02 -.2759E-02 -.1171E-03 -.2673E+00  .4712E+01  .2631E+00  .5110E-01  .9610E-02 -.2667E+00  .4748E+01  .1038E+01
30  .9967E-01 -.2151E-01  .5213E-01 -.2564E+00  .4917E+01  .2613E+00  .4753E-01  .6180E-01 -.2599E+00  .4946E+01  .1023E+01
31  .1983E+00 -.1860E-01  .1077E+00 -.2377E+00  .5123E+01  .2554E+00  .3652E-01  .1143E+00 -.2412E+00  .5155E+01  .1012E+01
32  .2995E+00 -.1187E-01  .1547E+00 -.2161E+00  .5300E+01  .2576E+00  .2979E-01  .1651E+00 -.2097E+00  .5379E+01  .9978E+00
33  .3997E+00 -.6601E-02  .2069E+00 -.1577E+00  .5635E+01  .2555E+00  .2153E-01  .2159E+00 -.1565E+00  .5655E+01  .9820E+00
34  .4409E+00 -.3768E-02  .2294E+00 -.1111E+00  .5821E+01  .2544E+00  .1731E-01  .2412E+00 -.1143E+00  .5941E+01  .9529E+00
35  .5830E+00 0.          .2500E+00 -.4409E-17  .6283E+01  .2500E+00 0.          .2669E+00 -.4408E-02  .6267E+01 0.

PT       CP         CPE
 1     .1[illegible]  .1880E-01
 2     .3042E-01  .4142E-01
 3    -.2046E-01  .4421E-01
 4    -.3160E+00  .4772E-01
 5    -.5190E+00  .4709E-01
 6    -.1923E+00  .4461E-01
 7    -.2960E+00  .4142E-01
 8    -.3672E+00  .3422E-01
 9    -.3672E+00  .2931E-01
10    -.3977E+00  .1976E-01
11    -.3367E+00  .1137E-01
12    -.3294E+00  .3742E-02
13    -.3110E+00 -.4703E-02
14    -.3049E+00 -.1139E-01
15    -.2499E+00 -.2015E-01
16    -.1770E+00 -.3009E-01
17     .7010E+00 -.4571E-01
18     .1000E+01 -.2912E-01
19     .3172E+00  .3409E-02
20    -.1037E+00 -.4649E-02
21    -.2577E+00 -.1609E-01
22    -.2776E+00 -.2967E-01
23    -.3156E+00 -.3667E-01
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 24      -.2011E+00 -.3140E-01
 25      -.1759E+00 -.3416E-01
 26      -.1564E+00 -.3576E-01
 27      -.1333E+00 -.3666E-01
 28      -.1102E+00 -.3688E-01
 29      -.7799E-01 -.3647E-01
 30      -.4650E-01 -.3347E-01
 31      -.2395E-01 -.2976E-01
 32        .4478E-02 -.2461E-01
 33        .3571E-01 -.1677E-01
 34        .9197E-01 -.9709E-02
 35        .1000E+01  .1658E-01

     C          Γ
 -.2203E-03 -.2376E-03
  .3559E-02  .3274E-02
 -.1855E-01 -.1573E-01
  .2102E-01  .1497E-01
  .5986E-01  .5538E-01
 -.5056E-03  .7661E-03

PHI=  .6570E-01     E=  .2150E+00     ALPHA= 1.        DEG     BETA=  .9505E+00 DEG

CIRCULATION=  .5864E+01     LIFT=  .1737E+01
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UPPER SURFACE

PT	X	AMP	KV	F2	FF1	FF2	LAM	CFL1	DFL2	TAU
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.

LOWER SURFACE

PT	X	AMP	KV	F2	FF1	FF2	LAM	CFL1	DFL1	TAU
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.

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PT  X  Y  VX  VY  OMEGA  THO  PSI  XXX  YYY  THETA  VTV
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24     -.[illegible]  -.[illegible]
25     -.18000E+00    -.[illegible]E-01
26     -.[illegible]E+00  -.34462E-01
27     -.11482E+00    -.[illegible]E-01
28     -.11080E+00    -.[illegible]E-01
29     -.94432E-01    -.[illegible]E-01
30     -.59987E-01    -.[illegible]E-01
31     -.34807E-01    -.27197E-01
32     -.70226E-02    -.21[illegible]E-01
33      .13762E-01    -.13[illegible]E-01
34      .67257E-01    -.111[illegible]E-02
35      .10001E+01     .1[illegible]E-01

     B           C
-.21364E-03 -.19901E-03
 .33114E-02  .70746E-02
-.10[illegible]E-01 -.14220E-01
 .12094E-01  .7[illegible]E-02
 .40946E-01  .73032E-01
 .17714E-02  .37[illegible]E-02

PHI= .64979E-01     XR .25278E+00     ALPHA= 9.          DEG     BETA= .90986E+00 DEG

CIRCULATION= .57198E-01     LIFT= .17459E-01
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-75-

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UPPER SURFACE
 PT      Y         [illegible]     KX          FF          FF1         FF2         LAM         DEL1        DEL2        TAU
  1  [illegible]   [illegible] [illegible] [illegible] .2883E+37   .1759E+38  -.9592E+37   .6652E+37   .1307E+01  0.
  2  [illegible]   [illegible] [illegible] [illegible] .2975E+09   .1001E+08  -.9934E+09   .7671E+08   .1234E-02   .5273E+06
  3  [illegible]   [illegible] [illegible] [illegible] [illegible] .6911E+00  -.1045E+03   .2441E-01   .9496E-03   .1947E-01
  4  [illegible]   [illegible] [illegible] [illegible] [illegible] .2644E-01  -.7647E+01   .2579E-02   .8446E-03   .3942E-02
  5  [illegible]   [illegible] [illegible] [illegible] [illegible] .1710E+00  -.5324E+01   .2147E-02   .7512E-03   .6968E-02
  6  [illegible]   [illegible] [illegible] [illegible] [illegible] .1829E+00  -.2843E+01   .1842E-02   .6680E-03   .9871E-02
  7  [illegible]   [illegible] [illegible] [illegible] [illegible] .1937E+00  -.4063E+00   .1577E-02   .5828E-03   .1282E-01
  8  [illegible]   [illegible] [illegible] [illegible] [illegible] .2033E+00  -.1872E+01   .1224E-02   .5021E-03   .1678E-01
  9  [illegible]   [illegible] [illegible] [illegible] [illegible] .2148E+00  -.1152E+01   .1037E-02   .4197E-03   .2161E-01
 10  [illegible]   [illegible] [illegible] [illegible] [illegible] .2214E+00  -.7614E+00   .9893E-03   .3701E-03   .2526E-01
 11  [illegible]   [illegible] [illegible] [illegible] [illegible] .2314E+00  -.2016E+00   .9267E-03   .3224E-03   .3040E-01
 12  [illegible]   [illegible] [illegible] [illegible] [illegible] .2356E+00   .2643E+00   .5905E-03   .2717E-03   .3737E-01
 13  [illegible]   [illegible] [illegible] [illegible] [illegible] .2407E+00   .7315E+00   .5311E-03   .2092E-03   .4542E-01
 14  [illegible]   [illegible] [illegible] [illegible] [illegible] .2417E+00   .3882E+00   .4204E-03   .1692E-03   .5971E-01
 15  [illegible]   [illegible] [illegible] [illegible] [illegible] .2642E+00   .1167E+01   .3160E-03   .1264E-03   .8375E-01
 16  [illegible]   [illegible] [illegible] [illegible] [illegible] .2747E+00   .2441E+01   .1884E-03   .7695E-04   .1395E+00
 17  [illegible]   [illegible] [illegible] [illegible] [illegible] .2927E+00   .5518E+01   .1490E-03   .6390E-04   .1447E+00
 18  [illegible]   [illegible] [illegible] 0.          [illegible] .3228E+00   .7082E+01   .1260E-03   .5460E-04   .6709E-01

LOWER SURFACE
 PT      Y         [illegible]     KX          FF          FF1         FF2         LAM         DEL1        DEL2        TAU
 18  [illegible]   [illegible] [illegible] 0.          [illegible] .3323E+00   .7057E+01   .1260E-03   .5460E-04   .6709E-01
 19  [illegible]   [illegible] [illegible] [illegible] [illegible] .3115E+00   .5154E+01   .1451E-03   .6158E-04   .1590E+00
 20  [illegible]   [illegible] [illegible] [illegible] [illegible] .2872E+00   .1949E+01   .1906E-03   .7721E-04   .1773E+00
 21  [illegible]   [illegible] [illegible] [illegible] [illegible] .2442E+00   .5422E+00   .3317E-03   .1312E-03   .7469E-01
 22  [illegible]   [illegible] [illegible] [illegible] [illegible] .2272E+00  -.4460E+00   .4605E-03   .1798E-03   .5201E-01
 23  [illegible]   [illegible] [illegible] [illegible] [illegible] .2269E+00  -.4562E+00   .5750E-03   .2232E-03   .4120E-01
 24  [illegible]   [illegible] [illegible] [illegible] [illegible] .2229E+00  -.6798E+00   .7576E-03   .2927E-03   .3075E-01
 25  [illegible]   [illegible] [illegible] [illegible] [illegible] .2139E+00  -.1217E+01   .9165E-03   .3509E-03   .2437E-01
 26  [illegible]   [illegible] [illegible] [illegible] [illegible] .2069E+00  -.1605E+01   .1065E-02   .4093E-03   .2023E-01
 27  [illegible]   [illegible] [illegible] [illegible] [illegible] .2187E+00  -.1573E+01   .1199E-02   .4586E-03   .1793E-01
 28  [illegible]   [illegible] [illegible] [illegible] [illegible] .2081E+00  -.1726E+01   .1437E-02   .5444E-03   .1461E-01
 29  [illegible]   [illegible] [illegible] [illegible] [illegible] .1516E+00  -.1772E+01   .1656E-02   .6232E-03   .1203E-01
 30  [illegible]   [illegible] [illegible] [illegible] [illegible] .1847E+00  -.5706E+00   .1884E-02   .6991E-03   .1001E-01
 31  [illegible]   [illegible] [illegible] [illegible] [illegible] .1706E+00  -.1113E+01   .2125E-02   .7741E-03   .8247E-02
 32  [illegible]   [illegible] [illegible] [illegible] [illegible] .1632E+00  -.2047E+01   .2354E-02   .8501E-03   .7093E-02
 33  [illegible]   [illegible] [illegible] [illegible] [illegible] .3154E-01  -.1027E+02   .3054E-02   .9243E-03   .1327E-02
 34  [illegible]   [illegible] [illegible] [illegible] [illegible] .2809E+07  -.1445E+09   .4268E+05   .9877E-03   .9305E+05
 35  [illegible]   [illegible] [illegible] [illegible] [illegible] .9789E+34  -.4729E+36   .1274E+38   .8826E+03  0.

ITERATION IS COMPLETE.
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Vita

Ellie B. Underwood, Jr., was born on 27 January 1939 in Beeville, Texas. He was graduated in June 1956 from Ballinger High School, Ballinger, Texas. In June 1961, he was awarded a Bachelor of Science degree in Aerospace Engineering from the University of Texas, Austin, Texas, and was commissioned in the United States Air Force at the same time. In June 1960, he was assigned to the Air Force Institute of Technology, Wright-Patterson Air Force Base, Ohio.

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