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RECENT DEVELOPMENTS IN FLIGHT FLUTTER
TESTING IN THE UNITED STATES

E. F. Baird, et al

Advisory Group for Aerospace Research
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on

Recent Developments in Flight Flutter Testing in the United States

by

E.F. Baird and W.B. Clark

Supplement to the

**MANUAL ON AEROELASTICITY
VOLUME IV CHAPTER 10**

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RECENT DEVELOPMENTS IN FLIGHT FLUTTER TESTING IN THE
UNITED STATES

by

E.F.Baird and W.B.Clark

Grumman Aerospace Corporation
Bethpage, New York, USA

Supplement to the
MANUAL ON AEROELASTICITY
VOLUME IV
CHAPTER 10

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PREFACE

The in-flight vibration test, carried out under conditions very close to those of the future operation of the aircraft, is the most conclusive test to ensure the safety of a new type of aircraft, since it does not involve any of the theoretical assumptions on which predictive calculations are based. While flight flutter tests are delicate, its application is essential if theoretical calculations have revealed a tendency to flutter in certain areas of the aircraft, even if a considerable safety margin is predicted.

Such tests can be carried out according to various processes which are selected in relation to the size of the aircraft and the accuracy aimed at. In an early article of the Manual on Aeroelasticity (Volume IV, Chapter 10, 1961), the methods used in this area about twelve years ago are described by Messrs M.O.W. Wolfe and W.T. Kirby. In a more recent supplement (Revision 1969), French developments relative to the various means of excitation and the various methods for utilizing the data collected were reported by Mr Piazzoli.

This is a new addition in which the authors review the testing methods used in the United States. This document will undoubtedly prove extremely useful to engineers and specialists desirous of being informed of the present state-of-the-art in this particular field and may even provide guidelines for the selections which they will be called upon to make in the future.

R. MAZET

Editor of the Manual on Aeroelasticity

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RECENT DEVELOPMENTS IN FLIGHT FLUTTER TESTING IN THE UNITED STATES

1. INTRODUCTION.

The critical development schedules and high performance requirements of recently developed military and commercial aircraft in the United States have demanded the use of new techniques for the rapid and accurate determination of flutter characteristics and the establishment of safe flight envelopes. Although significant advances have been made in theoretical flutter analysis methods and in wind tunnel testing techniques over the last 10 years, it has been only in the recent past that flight flutter testing methods have shown significant advances in the state of the art. These advances are principally the result of more sophisticated analysis methods which are now possible because large high speed computers have been dedicated to the flight test operation to reduce the overall flight demonstration time required. Since consideration of flutter is a pacing item for expanding the speed envelope for any new airplane, considerable effort has been spent to reduce the number of flights required to obtain flutter data.

There are several different flight flutter testing techniques currently in use or being developed in the United States. Most of the methods employ some form of sinusoidal excitation, using either aerodynamic vanes, internal mass shakers or power control systems. The more promising methods all seem to employ fast sweep rates. The old reliable shake and stop method still appears to be in use, as does the pilot impulse technique; there has also been at least one recent use of ballistic impulses. The use of random excitation techniques does not seem to have gained much popularity in the U.S. as yet; however, several of the analysis schemes appear to have the capability of handling the response to random inputs.

With regard to data reduction, the Kennedy-Pancu method, Reference 1, or modifications of it, still appears to have merit and is being used by several companies. Techniques based on the fast Fourier transform, Reference 2, have also gained popularity for flight flutter testing.

This paper presents a few comments on some flight flutter testing procedures in use or under development, as reported by some of the major U.S. aerospace companies. No attempt is made to provide a comprehensive review of flight flutter testing philosophy since a great many papers have already been written on the subject, e.g., References 3 through 7. The main part of the paper is devoted to the Grumman model-matching approach.

NOTATION

b_0, b_s, c_0, c_s	Coefficients used in the representation of the unsteady generalized aerodynamic force
$f(t)$	Generalized force due to a vane shaker or mass shaker, etc.
$f_h(t)$	Approximate generalized force obtained by applying a data hold to a sampled force signal
ϕ_v	Displacement at the shaker location in the mode which is responding
m	Generalized mass at zero airspeed based on a unit modal displacement at the location of the response transducer
n	Integer used to identify a particular sampled value
s	Laplace transform variable
t	Time
v	Flight speed
z	Z transform variable
γ	Coefficient used in the exponential approximation to the indicial aerodynamic force
$\delta(t)$	Dirac delta function
$\delta_T(t)$	Sampling function
λ_1, λ_2	Complex conjugate roots which determine the oscillatory motion characteristic of an aeroelastic mode
$\lambda_3 = \lambda_r$	Real root which determines the exponential motion characteristic of an aeroelastic mode
τ	Sampling interval
ω_n	Resonant frequency at zero airspeed
$\xi(t)$	Generalized coordinate
$\bar{f}(s)$	Laplace transform of $f(t)$ $\mathcal{L}\{f(t)\} = \bar{f}(s)$
$F(z)$	Z transform of $f(n\tau)$ $\mathcal{Z}\{f(n\tau)\} = F(z)$

2. A SUMMARY OF FLIGHT FLUTTER TESTING METHODS

The following summary describes the most recent methods used by some of the major United States aerospace companies.

McDonnell Aircraft Company

Excitation is accomplished through the stabilator and aileron actuators by means of an electrical signal which feeds directly into the servo of the power cylinders. The exciter can be set either on automatic linear frequency sweep or be manually controlled by the pilot. In the latter case, constant frequency shutoff operation of the exciter may be employed to obtain decay records. A fully automated analysis for determining frequency and damping from a transfer function has been developed based on a modification to the Kennedy-Pancu method. These data are stored in a computer as a function of altitude and Mach number. Tracking and curve fitting of frequency and damping as a function of dynamic pressure are employed to identify the critical modes. At each speed after the first two, the flutter margin, Reference 3, is calculated at constant altitude or constant Mach number as desired. A cathode ray display of any of the derived quantities is available on line in essentially real time. The accuracy of the determined frequency, damping, and flutter margin has been checked using theoretical and wind tunnel model data.

Douglas Aircraft Company

The DC-10 airplane has aerodynamic vanes installed on all surfaces for flutter excitation. The response transducer signals are digitized and stored on magnetic tapes in the flight test aircraft, and are also telemetered to the ground facility. The ground facility provides instant replay of the last 45 seconds of data on live analog display and also provides for post flight digital analysis. One notable method of post flight analysis involves a least squares fit of the aircraft transient response to pilot inputs. The analysis follows the technique given in Reference 9 except that a two step iteration procedure is used. The zero offset amplitude, and phase angle are found in the first step by a direct least squares analysis; a Taylor expansion and least squares fit is used to find the damping and frequencies in the second step. This two step procedure is repeated several times, converging on all terms to completely define the amplitude, phase angle, frequency, and damping of each mode in the response. The operator has a choice of searching for one, two, three or four degrees of freedom.

It has been found that reruns with minor changes in the start and stop times give repetitive results for "true data" but results derived from noise are different for each modified run. The computer is programmed to ignore any amplitudes that do not exceed a certain percentage of the maximum, thus preventing it from trying to converge on very low level signals.

Lockheed-California Company

The Lockheed technique utilizes the response to variable frequency sinusoidal excitations to evaluate the dynamic aeroelastic stability (modal damping) of the aircraft structure. The method is based upon the theoretical response of a single-degree-of-freedom spring-mass-damper system when excited by a sinusoidal force of constant amplitude, but of linearly increasing or decreasing period. The method was first developed by Dr. E. A. Bartsch of Lockheed-California Company in conjunction with flutter tests on the F-104 fighter and the Electra transport aircraft beginning about 1957. The distinguishing feature of the method is its reliance on the frequency sweep-rate effect. The sweep effect produces a shift in the response frequency at resonance which increases with increasing sweep rate and with decreasing damping. By evaluating the shift in resonant frequency between an up-sweep and a down-sweep, relative to the average response frequency, a very sensitive measure of damping is obtained for all critical modes.

In practice, the computerized version of the analysis technique uses flight recorded structural responses to symmetric and antisymmetric variable frequency excitation forces. These forces are generated aerodynamically on the L-1011 wing by wing tip vanes and on the horizontal tail by two tail tip vanes. The analysis data consist of time series measurements of the appropriate pair of vane excitation forces together with 10 to 20 wing or tail response measurements. The program tabulates the time series of the excitation forces and the responses, as well as the output/input ratios and the instantaneous frequency sweep rate. Plots of these quantities versus time are provided together with plots of the output/input ratios versus frequency. The peak response frequencies, frequency shifts, and amplitudes are then evaluated to determine the frequency and damping of the critical aeroelastic modes for all flight conditions of interest.

General Dynamics Corporation

Flight flutter testing follows the practice of detecting major response frequencies and obtaining the damping of these modes at successive speed increments. The variation of damping with speed is monitored via telemetry to allow safe expansion of the allowable flight envelope.

Excitation is provided by oscillating the control surfaces if the frequency response of the actuating system is satisfactory over the frequency range of interest. Otherwise, excitation is supplied by either aerodynamic tabs or oscillating mass shakers. The tabs and shakers are driven hydraulically with provisions for both amplitude and frequency control. Automatic frequency sweep capability with sweep rate proportional to frequency is utilized. A typical time for frequency sweep from 0 to 40 Hz is one minute.

Accelerometers, strain gages, and position pickups are used to measure the airframe response to excitation. The output of selected pickups is plotted on x-y plots during the frequency sweeps. Major resonances observed on these plots are noted and corresponding damping values obtained from shake-and-stop tests.

The Boeing Company

The following paragraphs describe the method which Boeing proposed for use on its SST airplane.

Aerodynamic vanes located at the wing tips of the SST were selected for in-flight excitation of wing modes which were of primary concern from a flutter standpoint. The wing tip aerodynamic vane was chosen because it provides adequate force for the least weight. The choice was based on the results of analytical studies in the subcritical speed range indicating the wing tip to be an effective location from which to excite the wing flutter mode. Maximum frequency for the aerodynamic vane shaker was to be approximately 20 Hz.

Response of the airplane to a rapid sinusoidal sweep was to be measured by accelerometers installed in the wing and body. The outputs were to be recorded on magnetic tapes and digitized before undergoing a fast Fourier analysis. The output was to appear as a frequency spectrum plot, for each sweep and transducer, obtained from the ratio of Fourier transforms of output to input. These data allow calculation of the modal damping for each sweep by the Kennedy-Pancu method.

3. THE MODEL MATCHING METHOD

In 1968, Grumman decided to purchase a large computer facility for use exclusively by the flight test organization. The purpose of this investment was to effect a major reduction in the time taken to flight test a new airplane. The reduction was to be accomplished by shortening all phases of the test program; one of the most significant of these was the time spent at each test point during the high speed build-up to acquire flutter data.

When the decision was made to use a computer, we had the opportunity to develop a new technique that could provide frequency and damping data in a relatively short time. A review of Grumman methods in use at that time, Reference 10, showed that a dedicated computer could readily reduce data reduction time; however, data acquisition time could not be significantly altered. A technique which reduces data analysis time and is compatible with relatively fast data acquisition is described in Reference 11 and became the basis of the current Grumman procedure. The initial implementation of this technique was accomplished by Astrodata, Inc. under the direction of R. G. Madsen.

The technique can best be described as "model matching". An analysis model of known functional form represented by a difference equation is programmed in the computer; constants are derived which cause the model response to be identical, within some prescribed degree of accuracy, to the aircraft response to an input force; these constants are then used to determine the damping and frequency of the mode(s). The matching is done for the aircraft response data over a limited frequency range. Each significant frequency band of data is analyzed separately and a different model match is computed.

While different types of force inputs can be used with this method, it is necessary that the force be such that it causes the modes in question to be excited to reasonable signal/noise ratios and that it be related to some transducer signal by a transfer function of known form. In order to keep the time for data acquisition to a minimum, a force is chosen which has a frequency content consistent with the modes of interest and has a duration of application only long enough to give the modes time to respond to practical levels. Grumman has chosen a relatively fast frequency sweep for its method. Sweeps from 5 to 50 Hz in 15 seconds are typical. This rate is fast enough to get the test data quickly but slow enough to insure that the analog records still contain the familiar resonance peaks of a steady state response plot. This latter feature is desirable to permit the test engineers to gain additional understanding of the response characteristics by studying the analog traces.

The derivation of the difference equations for the analysis model will now be presented. This will be followed by a description of the associated computer program and the methods employed to gain confidence in the approach. Finally, some results from a recent flight test program will be shown.

3.1 Derivation of the Difference Equation

For simplicity, the derivation of the required difference equation will be given for a single mode model. The comparable derivation for higher order models would only require more elaborate algebraic procedures.

We begin with the generalized equation of motion for the mode.

$$\ddot{\xi} + \omega_n^2 \xi + \frac{1}{m} \gamma \dot{\xi} + \frac{1}{m} \gamma^2 \xi + \frac{1}{m} \frac{\partial}{\partial t} \int_0^1 \gamma b_0 \xi(t_1) + \gamma^2 c_0 \xi(t_1) e^{-\gamma(t-t_1)} dt_1 = \frac{\phi_v}{m} f(t) \quad (1)$$

The first two terms on the left are inertial and structural; the last three are aerodynamic. This form of the unsteady aerodynamics has been chosen because it is applicable to nonharmonic motion. A single exponential term has been used in the approximation of the indicial aerodynamic response for simplicity. A description of the derivation of equations of this type for a general system of modes is given in Reference 12.

Equation (1) describes the continuous response of the mode starting from rest. To obtain the transfer function between input force and acceleration response, the Laplace transform is taken

$$s^2 \bar{\xi}(s) = \frac{1}{m} \frac{(s+\gamma)s^2}{(s-\lambda_1)(s-\lambda_2)(s-\lambda_3)} \phi_v \bar{f}(s) \quad (2)$$

where λ_1 and λ_2 are the complex conjugate roots associated with the frequency and damping and λ_3 is a real root associated with the indicial aerodynamics. It can be shown that the λ 's are roots of the cubic polynomial

$$s^3 + \left(\gamma + \frac{1}{m} \gamma b_0 + \frac{1}{m} \gamma b_0 \right) s^2 + \left(\frac{\gamma}{m} \gamma b_0 + \frac{1}{m} \gamma^2 c_0 + \frac{1}{m} \gamma^2 c_0 \right) s + \left(\gamma \omega_n^2 + \frac{\gamma}{m} \gamma^2 c_0 \right) = 0 \quad (3)$$

The objective of the model-matching program is to solve for the roots of Equation (2) using a difference equation which relates sampled values of acceleration and force. The sampled values are obtained by feeding analog response signals, telemetered from the aircraft, to an analog to digital converter in the ground station and then passing this digital data directly to the computer.

The difference equation is derived by using the theory of sampled-data systems, Reference 13. The functional representation of the sampling process is

$$y_f(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

The sampled force is then simply $f(t) \delta_T(t)$.
The Laplace transform of this sampled force is

$$\sum_{n=0}^{\infty} f(n\tau) e^{-n\tau s}$$

Now, a hold or data reconstruction function is required to convert this sampled function back into one which is essentially the same as the original continuous function $f(t)$. Since the conversion is not exact, we will call the reconstructed signal $f_h(t)$. The hold function used in the Grumman model is called a polygonal hold; its transfer function is given by:

$$\frac{e^{\tau s} (1 - e^{-\tau s})^2}{\tau s^2}$$

We illustrate the overall process in Figure 1(A). Using Z transforms, the desired difference equation can be derived for the sampled-data model shown in this figure. Letting $z = e^{\tau s}$,

$$\sum_{n=0}^{\infty} f(n\tau) z^{-n} = F(z)$$

and

$$\sum_{n=0}^{\infty} \ddot{x}(n\tau) z^{-n} = \ddot{X}(z)$$

it follows that

$$F(z) T_z(z) = \ddot{X}(z) \quad (4)$$

where

$$T_z(z) = \frac{e^{\tau s} (1 - e^{-\tau s})^2}{\tau s^2} \frac{(s+\gamma) s^2 \frac{\phi_v}{m}}{(s-\lambda_1)(s-\lambda_2)(s-\lambda_3)}$$

If the indicated Z transform is carried out, we have

$$T_z(z) = \frac{a_4 + (a_5 - 2a_4) z^{-1} + (a_5 - 2a_4) z^{-2} + a_5 z^{-3}}{1 - a_1 z^{-1} - a_2 z^{-2} - a_3 z^{-3}}$$

where

$$a_1 = e^{\lambda_1 \tau} + e^{\lambda_2 \tau} + e^{\lambda_3 \tau}$$

$$a_2 = -e^{(\lambda_1 + \lambda_2)\tau} - e^{(\lambda_1 + \lambda_3)\tau} - e^{(\lambda_2 + \lambda_3)\tau}$$

$$a_3 = e^{(\lambda_1 + \lambda_2 + \lambda_3)\tau}$$

$$a_4 = \frac{\phi_v}{\tau m} \frac{1}{(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)(\lambda_2 - \lambda_3)} \left[e^{\lambda_1 \tau} (\lambda_1 + \gamma)(\lambda_2 - \lambda_3) + e^{\lambda_2 \tau} (\lambda_2 + \gamma)(\lambda_3 - \lambda_1) + e^{\lambda_3 \tau} (\lambda_3 + \gamma)(\lambda_1 - \lambda_2) \right]$$

$$a_5 = \frac{\phi_v}{\tau m} \frac{1}{(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)(\lambda_2 - \lambda_3)} \left[e^{(\lambda_1 + \lambda_2)\tau} (\lambda_3 + \gamma)(\lambda_1 - \lambda_2) + e^{(\lambda_1 + \lambda_3)\tau} (\lambda_2 + \gamma)(\lambda_3 - \lambda_1) + e^{(\lambda_2 + \lambda_3)\tau} (\lambda_1 + \gamma)(\lambda_2 - \lambda_3) \right]$$

By taking the inverse Z transform of Equation (4), the difference equation, in the sampled time domain, relating past and present values of the system input and acceleration response is obtained:

$$a_4 f(n\tau) + (a_5 - 2a_4) f(n-1)\tau + (a_5 - 2a_4) f(n-2)\tau + a_5 f(n-3)\tau = \ddot{x}(n\tau) - a_1 \ddot{x}(n-1)\tau - a_2 \ddot{x}(n-2)\tau - a_3 \ddot{x}(n-3)\tau, n = 3, 4, 5, \dots$$

This equation can be written in matrix form as follows:

$$\begin{bmatrix} \ddot{x}(2\tau) & \ddot{x}(\tau) & \ddot{x}(0) & [f(3\tau) - 2f(2\tau) + f(\tau)] & [f(2\tau) - 2f(\tau) + f(0)] \\ \ddot{x}(3\tau) & \ddot{x}(2\tau) & \ddot{x}(\tau) & [f(4\tau) - 2f(3\tau) + f(2\tau)] & [f(3\tau) - 2f(2\tau) + f(\tau)] \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \ddot{x}(i-1)\tau & \ddot{x}(i-2)\tau & \ddot{x}(i-3)\tau & [f(i\tau) - 2f(i-1)\tau + f(i-2)\tau] & [f(i-1)\tau - 2f(i-2)\tau + f(i-3)\tau] \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} = \begin{bmatrix} \ddot{x}(3\tau) \\ \ddot{x}(4\tau) \\ \vdots \\ \ddot{x}(i\tau) \end{bmatrix} \quad (5)$$

This equation can be solved by least squares for the unknown coefficients a_1 through a_5 by letting i take on a sufficient number of values over the data segment of interest. The roots λ_1 , λ_2 , and λ_3 , can be obtained from a_1 , a_2 , and a_3 ; the remaining unknowns, a_4 and a_5 , can be used to obtain λ and $\phi_v/\tau m$.

The derivation, to this point, has assumed that $f(t)$ is such that a transducer can measure the applied force directly. Actually, the shakers used in flight flutter testing typically do not satisfy this assumption. To avoid this difficulty, a transfer function relating transducer signal to force is assumed and the difference equations are derived using a model similar to Figure 1(B). For example, a vane shaker has inertial and unsteady aerodynamic force terms:

$$f_v(t) = m_v \ddot{\beta} + v b_v \dot{\beta} + v^2 c_v \beta + \frac{\partial}{\partial t} \int_0^t [v b_{v0} \dot{\beta}(t_1) + v^2 c_{v0} \beta(t_1)] e^{\gamma_v(t-t_1)} dt_1$$

The symbology in this equation corresponds to that in Equation (1) but a "v" subscript refers to the vane and β refers to vane angular position. The transfer function between $\bar{f}_v(s)$ and $\bar{\beta}(s)$ is:

$$\bar{f}_v(s) = m_v(s - \zeta_1)(s - \zeta_2)(s - \zeta_3) \frac{1}{(s + \gamma_v)} \bar{\beta}(s)$$

where the ζ 's are the roots of a cubic polynomial similar to (3). If the aerodynamics are quasi-steady instead of unsteady, as would be the case when the vane is very small relative to the surface, we would write

$$\bar{f}_{vqs}(s) = m_v(s - \zeta_1)(s - \zeta_2) \bar{\beta}(s)$$

If the shaker is inertial only, with no aerodynamics, we would write

$$\bar{f}_{vi}(s) = m_v s^2 \bar{\beta}(s)$$

A difference equation would now be derived which would not only have the unknowns a_1 through a_5 as in Equation (5), but would contain additional unknowns corresponding to the ζ 's and γ_v .

3.2 Description of the Computer Program and Automated Telemetry Station

The Grumman computer program for model matching is written with the assumption that the forcing function will be a sinusoidal frequency sweep from low frequency to high. The program takes the sampled shaker position signal and continuously calculates frequency. The test engineer provides a starting and stopping frequency for each data segment to be analyzed. The computer program finds the data sequence within this segment, filters it, and stores the information into the matrix array described in the last section. Before storing the data into this matrix, a decimation is accomplished which reduces the number of data points/cycle to five at the high frequency end of the band being analyzed. This reduction is done to reduce computation time.

Options are provided to use any of three types of digital filters with various corner frequencies and roll-off characteristics. Options are also provided to use matrices conforming to 2nd, 3rd, 4th, 5th, and 6th order models. The program then solves the matrix equation by inversion, solves for the frequency and damping using a polynomial solution method, and displays the results on a cathode ray tube together with appropriate identification data.

The program provides for five separate frequency sweeps at each test point. These correspond, typically, to symmetric and antisymmetric wing and horizontal tail, and antisymmetric fin. For each sweep, provision has been made to obtain solutions for as many as seven record segments with separate model orders for each. Any of approximately 20 response channels can be used to supply the data for the 35 solutions. A primary and secondary transducer are used for each solution. Normally, the test engineers supply the necessary clues to the computer in advance of the test point by extrapolating from previous data. If, after the sweeps are analyzed, it is found that some of the clues are not correct, changes can be made while the flight is still in progress.

The data can be analyzed in nearly real time* in one mode of operation. This mode, however, requires a large amount of computer storage and prohibits other disciplines (stability and control, engine performance, etc.) from obtaining their test data in real time. The preferred mode, for high speed test points, is one wherein the five shaker sweeps are done quickly, one right after the other, while the data is digitized and stored directly on a disk. After all test objectives are completed, the aircraft slows down and waits. The flutter disk is then called up and the data analyzed in less than five minutes.

The data flow through the Automated Telemetry Station (ATS) for flutter testing is shown in Figure 2. In the RF DEMODULATOR, the data from the microwave stream are demodulated into 26,500 words per second of pulse code (PCM) data and two carrier bands each carrying 14 subcarriers of FM flutter test information. These three data streams are recorded on magnetic tape to form a complete record of all raw telemetered data acquired during a flight.

The PCM signal is aligned in the BIT SYNCHRONIZER, and transferred to the PREPROCESSOR. The 28 FM subcarriers are further demodulated in DISCRIMINATORS to obtain the raw analog flutter data.

The 12-bit digital output words from the Analog - Digital Converter (ADC) are passed to the PREPROCESSOR in parallel with the PCM data. The PREPROCESSOR performs the following functions:

- Converts PCM data to parallel format from its serial telemetry format
- Applies calibration to the data
- Buffers the data for transfer to the Computer (CPU) at 0.1 second intervals

* Nearly real time means, for example, that the answers for seven record segments, primary and secondary transducers, will be displayed in less than 30 sec, after the completion of a 15-second shaker sweep from 5-60 Hz.

- Records all data, after calibration, on magnetic tape in a format comparable with the CPU
- Transfers to the CPU up to 15,000 words per second of PCM and digital data.

The computer is a CDC 6400 with 98,000 60-bit words of central memory and a disk storage unit capable of storing 16,500,000 60-bit words. The command logic of this computer enables it to serve three Data Analysis Station (DAS) terminals simultaneously. During flight testing, one (or more) of these DAS terminals is dedicated to the data stream from each aircraft. These terminals are shared in turn by various test disciplines, so that more than one aircraft can fly simultaneously, and each aircraft may acquire data to satisfy various test objectives on the same flight.

In parallel with the digital data flow, 16 of the FM discriminator outputs are passed to two 8-channel BRUSH RECORDERS. These displays are monitored for safety of flight during acceleration to the next envelope expansion point and to evaluate data quality which may influence program results (e.g., telemetry interruption during a sweep).

3.3 Results of Model Matching When Applied To Theoretical Response Data

Prior to using the model matching program on a new airplane, a rather elaborate checkout was undertaken. The program was first used to analyze digitally generated response data for a spring-mass-damper system with six degrees of freedom and then it was used to analyze digitally generated response data for a two mode aeroelastic system.

In the analysis of the six degree of freedom system it was found that for a noise free signal, the model matching technique would give results which were almost exact for one or two modes. When white noise was introduced in the response signal (noise meaning response which was not due to the known force input), the accuracy was somewhat degraded. It became apparent during this early checkout that in a noise environment it was desirable to keep the record segment as short as possible and confined to the area where the response signal reached a peak value.

The analysis of the two mode aeroelastic system was undertaken to make a quantitative evaluation of the ability of the model matching program to obtain frequency and damping values from data generated during a fast sinusoidal sweep. The influence of unsteady aerodynamics, which manifests itself in the modal response and in the generation of the forcing function, was included in the evaluation as well as the effect of atmospheric turbulence.

The theoretical model chosen was one with a 9.2 Hz bending mode and a 28.9 Hz torsion mode. The basic parameters were chosen so that an explosive flutter condition occurred at 1065 kts. The unsteady aerodynamic forces were represented using an exponential approximation to Wagner's indicial lift function. One term was used in the approximation: $\phi(s) = 1 - Se^{-134s}$. A vane shaker was located at the trailing edge of the surface and its hinge line unbalance was made variable so that the frequency at which the aerodynamic and inertial forces canceled each other (the crossover frequency) could be varied. The same approximation to Wagner's indicial lift function was used for the vane aerodynamics.

A quasi-steady gust loading was applied to the model to simulate atmospheric turbulence in the following manner: a computer program which generates random white noise was used as a downwash input to a first order differential equation which shaped this downwash into a reasonable approximation to the gust spectrum shown in Reference 14. This gust was then multiplied by the appropriate constant which converted it to a force which was applied at the 20% chord of the surface. The rms value of gust intensity was treated as a variable.

The damping and frequency curves for the system are shown in Figure 3. They were obtained by taking the Laplace transform of the homogeneous equations and solving the resulting characteristic polynomial for the roots. Forced response curves for the system are shown in Figures 4-10. These were calculated by numerically integrating the equations of motion treating vane position as the following known input function: $\beta = 1 - \cos(150e^{-2t})$. The velocity response at a specific point on the surface and the vane position were written on a magnetic tape using the same time increment as that used by the analog/digital converter for telemetered signals. This tape was then analyzed with the model matching program.

Frequency and damping values obtained with the model matching program are summarized in the tables included with Figures 4 through 10. Three air speeds are presented: 1) 400 kts. which is a relatively low speed for the system and one wherein very little aerodynamic coupling exists between modes; 2) 840 kts. which is sufficiently high to insure significant aerodynamic coupling while being well short of the coalescence speed and flutter speed; and 3) 1000 kts. which is the coalescence speed and is just below the flutter speed. At each of these, results are presented for noise free data and for data which includes atmospheric turbulence. The turbulence levels chosen are fairly severe (4.2 ft./sec. RMS at 400 kts., 6.3 ft./sec. at 840 kts., and 8.4 ft./sec. at 1000 kts.) and would be encountered only occasionally during flutter testing. In order to give some insight into the range of answers which might be obtained with model matching, results are presented for each of 4 record segments (windows) corresponding to each peak in the response traces. For the two lower speeds, four windows were also chosen to include the two response peaks which occur in each sweep. Some of these segments were intentionally chosen to be bad selections. For each of the segments, frequency and damping values are given for 2nd, 3rd, 4th, 5th and 6th order models. The system used to generate the data is actually 5th order but, depending on the particular record segment in question, an adequate match may be accomplished with a lower order.

The results given in the table accompanying Figure 4 demonstrate that, for a straightforward case such as this, model matching readily calculates an excellent damping and frequency value for each of the separated peaks; slightly better answers are obtained for 3rd order analysis than for 2nd order. Higher order models produce the correct results for record segments containing one peak but the additional calculated roots are difficult to interpret. The results for record segments containing both peaks are naturally poor for 2nd and 3rd order models but give good results for higher orders.

The effect of noise can be appraised by comparing the table of Figure 5 with that of Figure 4. A slight degradation in accuracy has occurred and only in the higher-order models. This noise case, and the two subsequent ones, were analyzed after the data was passed through digital bandpass filters with a 12/db octave roll-off characteristic. The corner frequencies were set at values corresponding to the shaker instantaneous frequency at the beginning and end of each record segment.

Figures 6 and 7 present the results at 840 kts. As above, 3rd order results are generally better than 2nd order when a single peak is chosen for study and the 4th, 5th and 6th order results are generally better than at 400 kts. Noise has degraded the data more than at 400 kts. It is noted that the real root listed in the tables is seldom accurate; an explanation for this is that it is necessary to choose a window at the beginning of the shaker sweep to obtain this root. Although no data is presented, good results have been obtained when this is done.

The tables with Figures 8 and 9 give results at 1000 kts. for two types of shaker forces: one which has a slight variation in vector amplitude with frequency; and the other with a crossover frequency essentially the same as the coalescence frequency. Analysis of the system at this speed would be a very severe test for any flight flutter testing method, particularly with the second force input, which makes the single peak appear to be two separate peaks.

Only 4 record segments were chosen at this speed because one true peak appears instead of two. The tables show that more scatter occurs in these answers than at the lower speeds; the scatter is mainly in the heavily damped mode; the damping of the critical mode is fairly well predicted. Other analyses, which are not reported here, have also indicated that whenever two modes couple and one is lightly damped and the other is heavily damped, the model matching analysis gives acceptably good results for the lightly damped mode.

Figure 10 gives results at 1000 kts in the presence of rather severe atmospheric turbulence. Inspection of the response trace shows a considerable distortion of the signal. As expected, the results are somewhat inconsistent.

Some of the record segments used in the above examples are not representative of what would be used in actual practice. The preferred ones are double starred at the left of each table. The answers from these segments have been plotted in Figure 3. The resulting bands represent the degree of scatter which might be expected using model matching. It can be seen that as the modes become more coupled and one gets closer to the coalescence speed, the scatter band becomes wider. It is the authors' view, however, that these results are at least as accurate as those which could be obtained by other methods now in use.

3.4 Typical Flight Test Results

After gaining confidence that the model matching program would predict accurate damping and frequency values, the program was used to do actual flight flutter testing. The final evaluation was accomplished by checking the results with those obtained by the old shake-and-stop method. Satisfactory agreement was obtained.

Figure 11 presents the model matching program results for a vertical tail. These results, together with comparable data for the wing and horizontal tail, were obtained in only six flights; this is nine less than would have been required to obtain this quantity of data in the past.

4. CONCLUSION

Although flight flutter testing methods in current use in the United States are by no means standardized, they represent significant advances over those employed in the past. It is rather encouraging to note that more sophisticated analysis techniques producing essentially real time results are now being widely used. The advances result from the availability of large high speed computers which are becoming an integral part of flight test data management. Much more remains to be done to further improve present methods and to develop new ones. With a more universal use of dedicated computers, we should expect a continuing proliferation of rapid and safe testing techniques.

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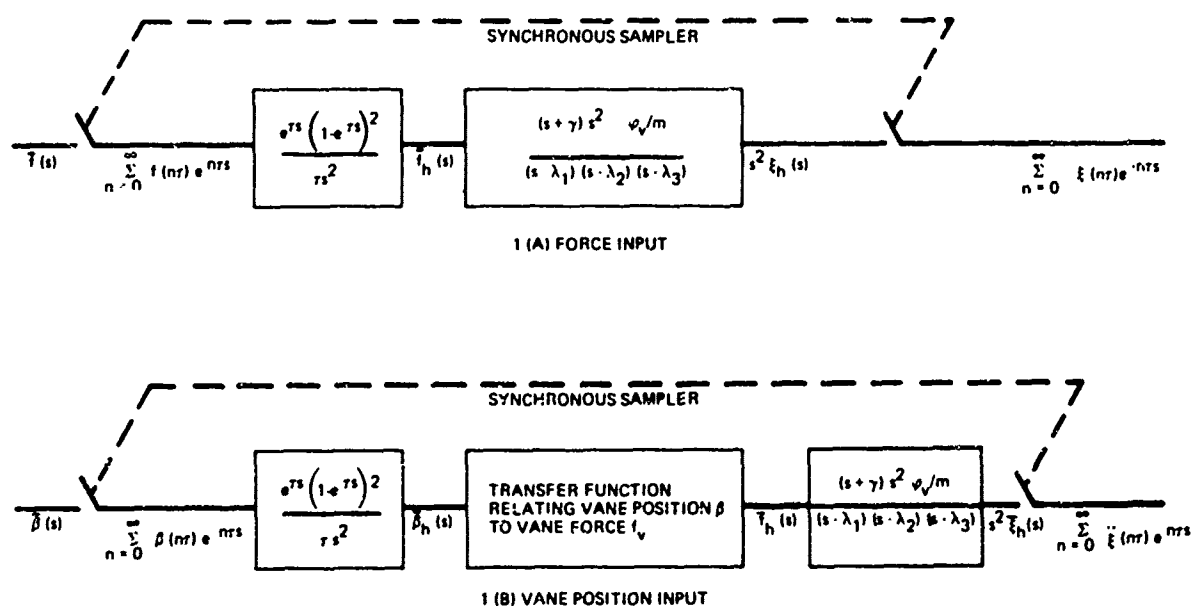


Fig. 1 Model Matching Block Diagram

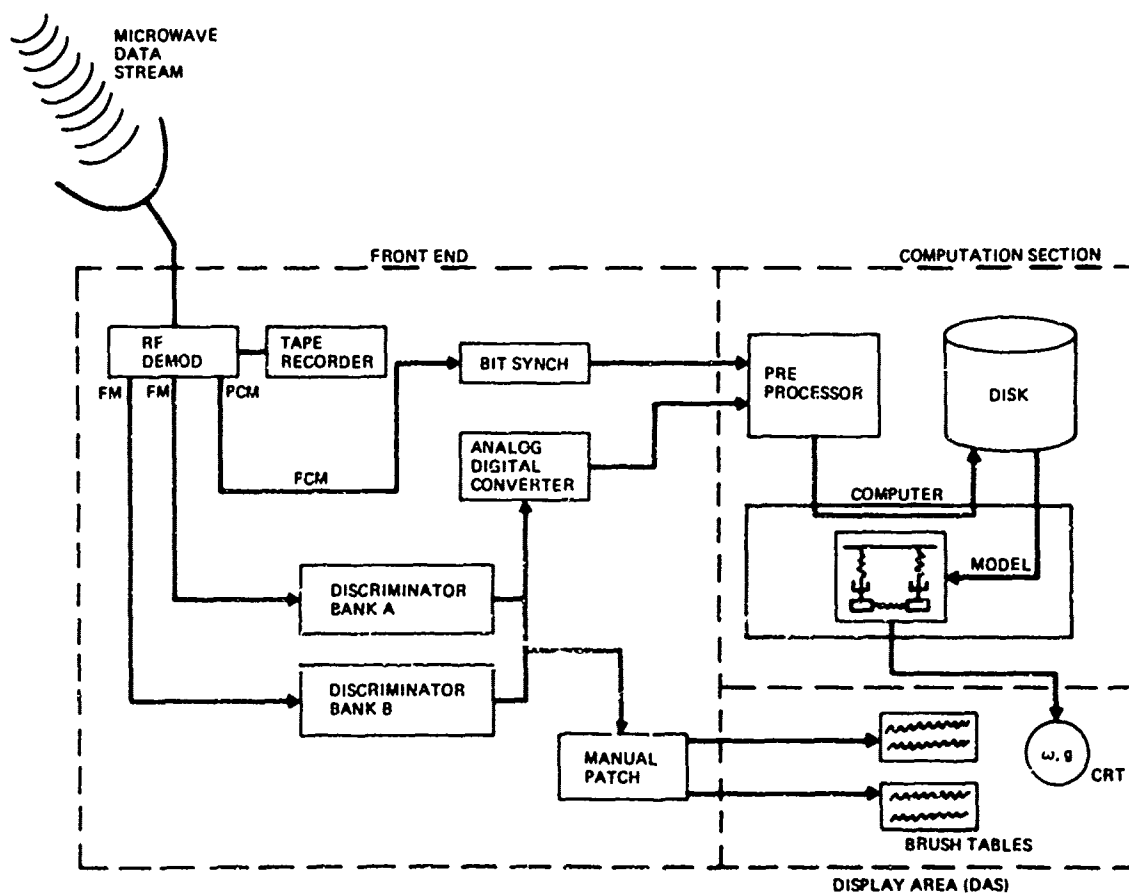


Fig. 2 Telemetry Station Data Flow

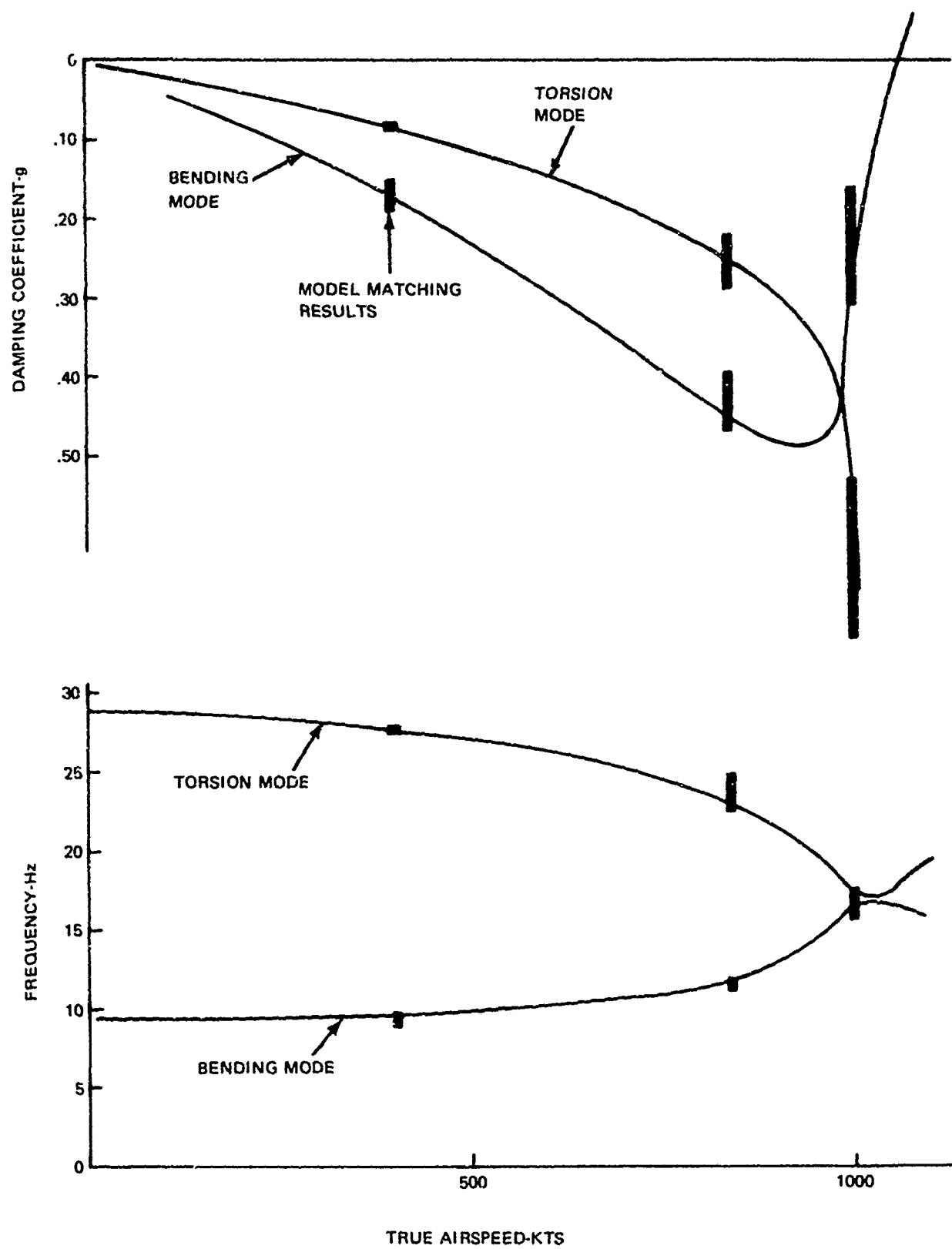


Fig. 3 Comparison of Theoretical Results with Model Matching Results

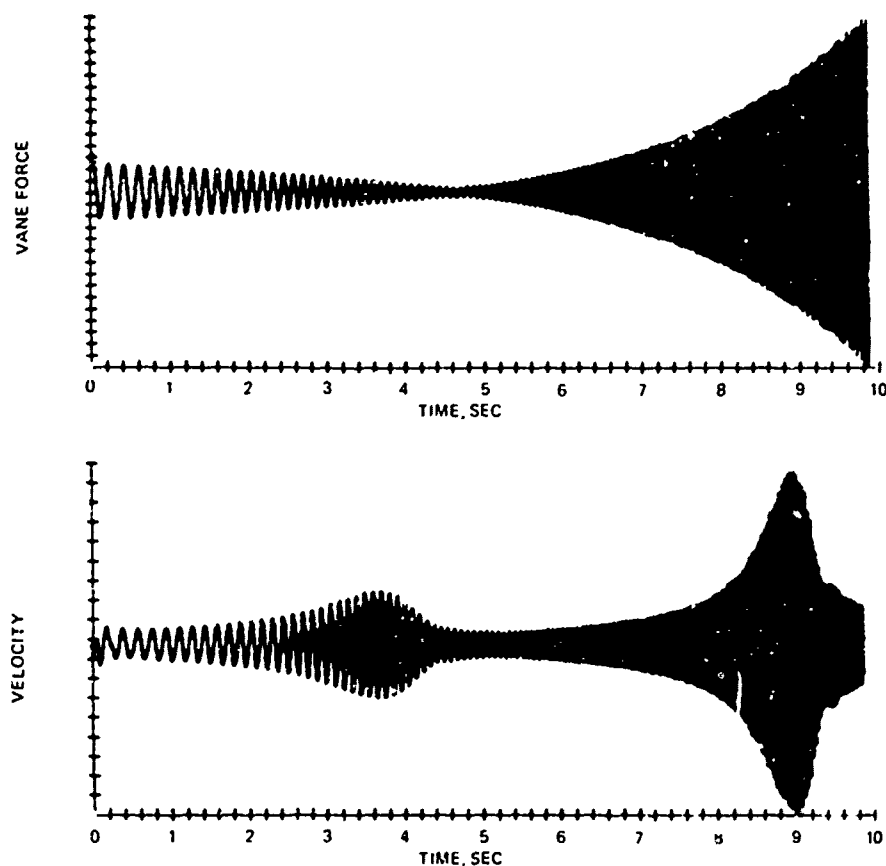


Fig. 4 Vane Force and Velocity Response of a Two-Mode Aeroelastic Model: 100 Kts (37% V_F); No Gust Force

RECORD SEGMENT, SECONDS	*FREQUENCY & DAMPING RESULTS FROM MODEL MATCHING PROGRAM FOR FIGURE 4					THEORETICAL RESULTS
	2ND ORDER	3RD ORDER	4TH ORDER	5TH ORDER	6TH ORDER MODEL	
30 40**	9 2/15	9 5/17 58 6	9 1/66 9 6/19	8 8/74 9 6/19 29 4	x x x x	$\omega_1 = 9.7 \text{ Hz}$ $\eta_1 = .17$
26 50**	9 6/11	9 5/18 x	9 5/18 20 1 x	9 6/48 9 5/18 x	11 8/13 9 5/18 x x	
20 60	10 0/09	9 5/18 x	9 5/18 21 3/1 36	27 1/33 9 5/18 22 5	x x x x	
10 70	10 6/11	9 5/20 308	9 5/18 28 2/45	27 7/28 9 5/18 22 5	x x x x	
86 94**	27 6/08	27 7/09 x	27 7/09 7 6/2 1	27 7/09 24 3/04 x	x x x x	$\omega_2 = 27.7 \text{ Hz}$ $\eta_2 = .089$
82 96**	27 6/08	27 7/09 x	x x	27 7/09 17 3/05 x	x x x x	
75 96	27 5/08	27 7/09 x	x x	27 7/09 11 3/57 x	x x x x	
50 96	27 3/09	27 5/08 83 0	27 7/09 32 0 240	27 7/09 9 8/23 x	x x x x	
36 91	17 8/42	25 8/27 18 1	10 3/23 27 4/10	x x x	27 7/09 9 5/17 x x	$\omega_1 = 9.7 \text{ Hz}$ $\omega_2 = 27.7 \text{ Hz}$ $\eta_1 = .17$ $\eta_2 = .089$ $\lambda_{\text{real}} = .161$
30 94	25 9/08	27 0/10 x	10 3/18 27 5/08	x x x	x x x x	
25 96	25 6/08	26 9/09 12 6	27 5/08 10 4/15	x x x	x x x x	
10 96	24 4/09	26 8/09 15 3	27 5/08 10 0/09	27 7/09 9 5/17 x	27 7/09 x 25 2 x	

*Results are presented as follows $\omega_1/\eta_1, \omega_2/\eta_2, \lambda_{\text{real}}$

**Preferred record segments

x Program results were not meaningful

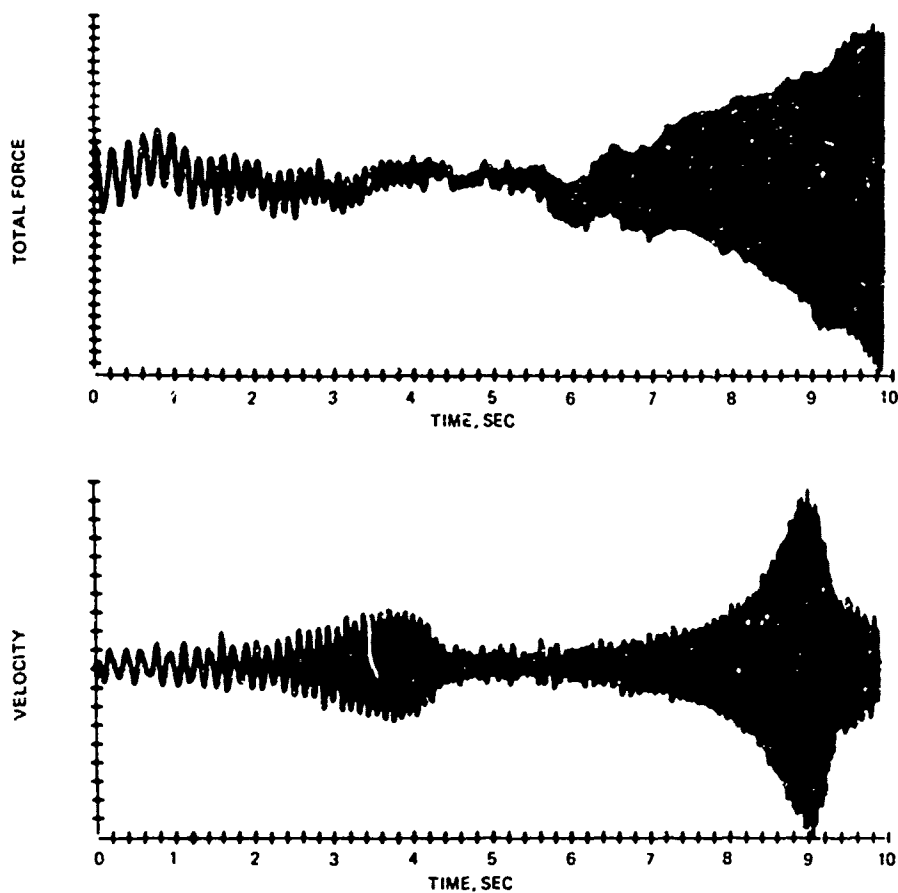


Fig. 5 Total Force & Velocity Response of a Two-Mode Aeroelastic Model at 400 Kts; Total Force is Comprised of a Vane Force and a Gust Force with an RMS Intensity of 4.2 ft/sec

RECORD SEGMENT, SECONDS	*FREQUENCY AND DAMPING RESULTS FROM MODEL MATCHING PROGRAM FOR FIGURE 5					THEORETICAL RESULTS
	2ND ORDER	3RD ORDER	4TH ORDER	5TH ORDER	6TH ORDER MODEL	
30 40	9.2/ 14	9.8/ 18 8 45	9.8/ 24 12 2/ 10	10.2/ 31 12.5/ 20 22 0	6.4/ 72 12 8/ 09 10 4/ 17	$\omega_1 = 9.7 \text{ Hz}$ $\eta_1 = 17$
26 50	9.6/ 10	9.6/ 17 47 0	9.4/ 15 11 9/ 14	9.6/ 17 12 4/ 04 56 5	13 7/ 25 9 6/ 20 11 3/ 15	
20 60	10.0/ 08	9.6/ 19 98.2	13 3/ 50 9 3/ 14	13 1/ 16 9.6/ 17 104 0	18 7/ 30 9 4/ 10 11 4/ 15	
10 70	10.5/ 10	9.6/ 20 174 0	15 3/ 1.1 9 4/ 14	16.9/ 38 9.7/ 17 92 0	x x x	
86 94	27.6/ 08	27 7/ 09 47.3	27 1/ 90 27.7/ 08	27 7/ 09 25 1/ 11 180 0	x x x	$\omega_2 = 27.7 \text{ Hz}$ $\eta_2 = 089$
82 96	27.6/ 08	27.7/ 09 x	27 7/ 09 246 290	27.7/ 09 23 2/ 18 246	37.9/ 50 27 6/ 09 22 9/ 02	
75 96	27.5/ 08	27.7/ 09 614	27.7/ 09 350 149	27 7/ 09 21 3/ 10 376	28 9/ 11 27 7/ 09 21 5/ 15	
50 96	27.3/ 09	27.5/ 08 52.4	13 6/ 97 27 7/ 08	27 7/ 09 12.1/ 07 337	14 7/ 16 27 7/ 09 13 6/ 13	
36 91	16.7/ 47	25.2/ 31 17.9	10.3/ 18 27 4/ 10	27.5/ 09 9.4/ 06 76	27.5/ 08 17.3/ 54 10.3/ 12	$\omega_1 = 9.7 \text{ Hz}$ $\omega_2 = 27.7 \text{ Hz}$ $\eta_1 = 17$ $\eta_2 = 089$ $\lambda_{\text{real}} = 16.1$
30 94	25.6/ 09	26.9/ 10 10.7	10.3/ 16 27.5/ 08	x x x	16.1/ 13 27 7/ 09 9.9/ 09	
25 96	25.2/ 09	26.7/ 09 12.1	27.5/ 08 10.3/ 13	27 7/ 08 9.2/ 15 154	13.3/ 16 27 7/ 09 9.7/ 08	
10 96	23.9/ 10	26.6/ 09 14.6	27.5/ 08 10.0/ 08	27 7/ 08 9.3/ 20 143	8.4/ 19 27 7/ 09 9.4/ 10	

*Results are presented as follows $\omega_1/\eta_1, \omega_2/\eta_2, \lambda_{\text{real}}$

x Program results were not meaningful

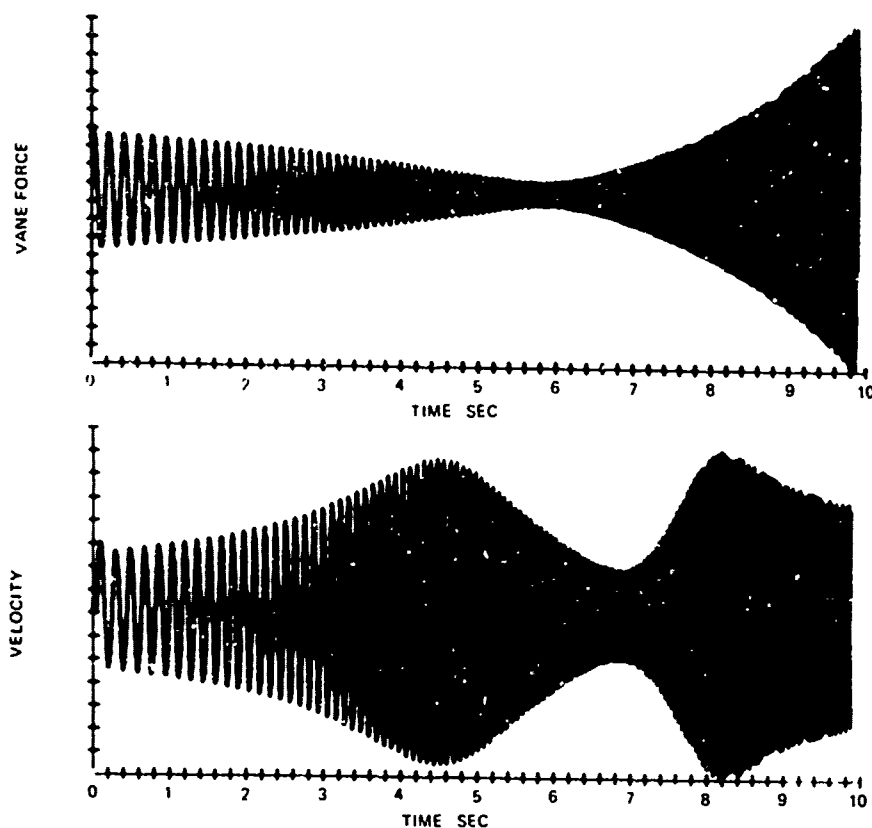


Fig. 6 Vane Force and Velocity Response of a Two-Mode Aeroelastic Model; 840 Kts Airspeed (79% V_F); No Gust Force.

RECORD SEGMENT, SECONDS	*FREQUENCY AND DAMPING RESULTS OF MODEL MATCHING PROGRAM FOR FIGURE 6										THEORETICAL RESULTS
	2ND ORDER		3RD ORDER		4TH ORDER		5TH ORDER		6TH ORDER MODEL		
40 50 35 55** 35 65** 20 - 70	11 1/45 11 .43 11 2/31 10 8/28	11 6/45 163 11 6/46 300 12 0/47 138 12 4/43 126	11 5/47 19/74 11 6/46 32 240 22 7/2 11 5/47 22 4/3 11 5/50	28 1/53 11 6/46 75 11 5/46 11 8/8 x 22/05 11 5/46 19.3 22 1/12 11 5/46 13.2	25 4/012 11 5/46 x x 19 9/06 11 5/46 x x 21 6/08 11 5/46 x x x x x x	$\omega_1 = 11.8 \text{ Hz}$ $\eta_1 = .46$					
78 - 86 75 90** 75 95** 65 - 97	24 1/24 24 4/22 24 8/27 22 4/1	22 6/25 171 22 5/25 189 22 5/23 275 22 3/26 164	22 7/25 x 500 22 7/25 x 42 22 7/25 x 54 22 5/24 x x	x x x x x x x x x 22 7/25 11 5/45 x	22 7/25 152 x x x x x 22 7/25 9 8/51 106/44 22 6/25 12 3/36 x x	$\omega_2 = 22.9 \text{ Hz}$ $\eta_2 = .25$					
45 82 45 95 35 90 20 97	17 9/24 19 2/04 16 8/04 14 8/13	18 6/22 22 21 5/13 75 19 3/03 19.7 19 8/07 16	23 2/23 11 5/52 22 7/33 11 3/77 23 3/29 11 8/54 23 /36 12 2/56	x x x 22 7/26 11 4/44 22 7/26 11 3/46 x x x x	22 7/25 11 4/45 x x 22 7/26 11 4/45 x x x x x x x x x x	$\omega_1 = 11.8 \text{ Hz}$ $\omega_2 = 22.9 \text{ Hz}$ $\eta_1 = .46$ $\eta_2 = .25$ $\lambda_{\text{real}} = 18.7$					

*Results are presented as follows: $\omega_1/\eta_1, \omega_2/\eta_2, \lambda_{\text{real}}$

**Preferred record segments

x Program results were not meaningful

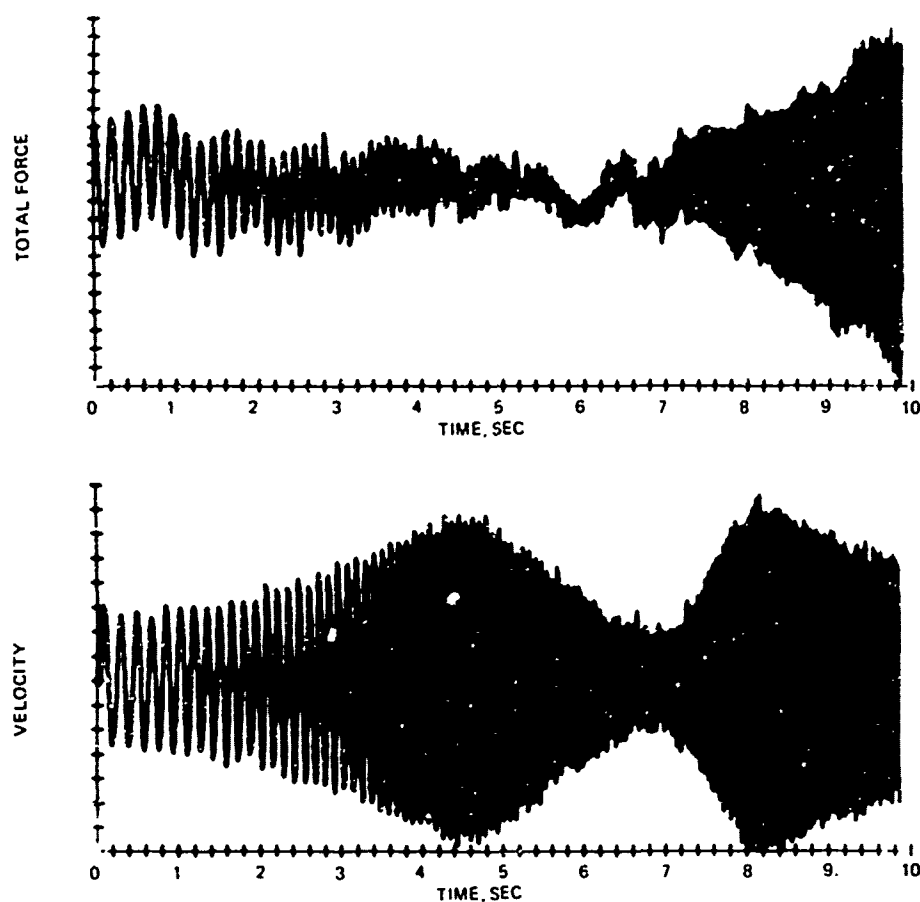


Fig. 7 Total Force and Velocity Response of a Two-Mode Aeroelastic System at 840 Kts, Total Force is Comprised of a Vane Force and a Gust Force with an RMS Intensity of 6.3 ft/sec

RECORD SEGMENT, SECONDS	*FREQUENCY & DAMPING RESULTS FROM MODEL MATCHING PROGRAM FOR FIGURE 7					THEORETICAL RESULTS
	2ND ORDER	3RD ORDER	4TH ORDER	5TH ORDER	6TH ORDER MODEL	
40 50	11 3/38	11 7/34 53.2	12 3/59 12 6/02	14 3/29 12 6/002 64.2	18 2/38 9 9/16 13 5/008	$\omega_1 = 11.8 \text{ Hz}$ $\eta_1 = 46$
35 55	11 1/44	11 5/44 42.2	11 2/57 12 4/14	13 2/37 12 5/19 96.9	19 7/14 11 3/52 11 4/06	
35 65	11 3/33	12 1/45 93.8	11 2/44 17 8/19	18 9/13 11 5/44 55.6	22 1/06 11 6/49 12 5/15	
20 70	11 0/34	12 3/46 155.0	11 1/53 18 3/36	19 7/13 11 6/44 42.2	22 5/08 11 3/58 11 5/23	
78 86	24 0/19	22 6/25 101.3	23 1/13 25 7/02	22 8/17 25 7/002 73.8	27 3/01 21 7/04 24 0/30	$\omega_2 = 22.9 \text{ Hz}$ $\eta_2 = 25$
75 90	24 5/22	22 5/27 115.7	24 4/50 21 5/15	22 6/28 25 7/05 107.5	29 3/14 21 2/10 24 3/22	
75 95	24 9/30	22 4/23 259.1	12 7/18 22 4/21	22 2/27 27 1/06 168.9	25 9/63 21 6/20 28 5/15	
65 97	24 9/17	22 2/26 155.5	11 9/35 22 4/25	22 3/28 27 0/06 138.1	13 2/14 30 0/12 22 3/30	
45 8.2	18 0/28	18 4/24 47.3	11 8/50 23 0/21	22 9/23 11 7/41 104.5	11 7/40 24 2/13 20 6/24	$\omega_1 = 11.8 \text{ Hz}$ $\omega_2 = 22.9 \text{ Hz}$ $\eta_1 = 46$ $\eta_2 = 25$ $\lambda_{\text{real}} = 18.7$
45 95	21 2/01	22 2/20 90.5	11 4/91 22 5/34	x x x	11 8/59 29 3/35 23 3/2	
35 90	18 5/08	20 4/08 41.2	23 3/29 11 8/57	x x x	11 7/47 27 5/31 22 6/19	
20 97	18 0/06	21 4/09 23.1	22 8/36 12 2/62	23 1/27 10 6/42 193.8	33 9/40 11 7/51 23 2/24	

*Results are presented as follows $\omega_1/\eta_1, \omega_2/\eta_2, \lambda_{\text{real}}$

x Program results were not meaningful

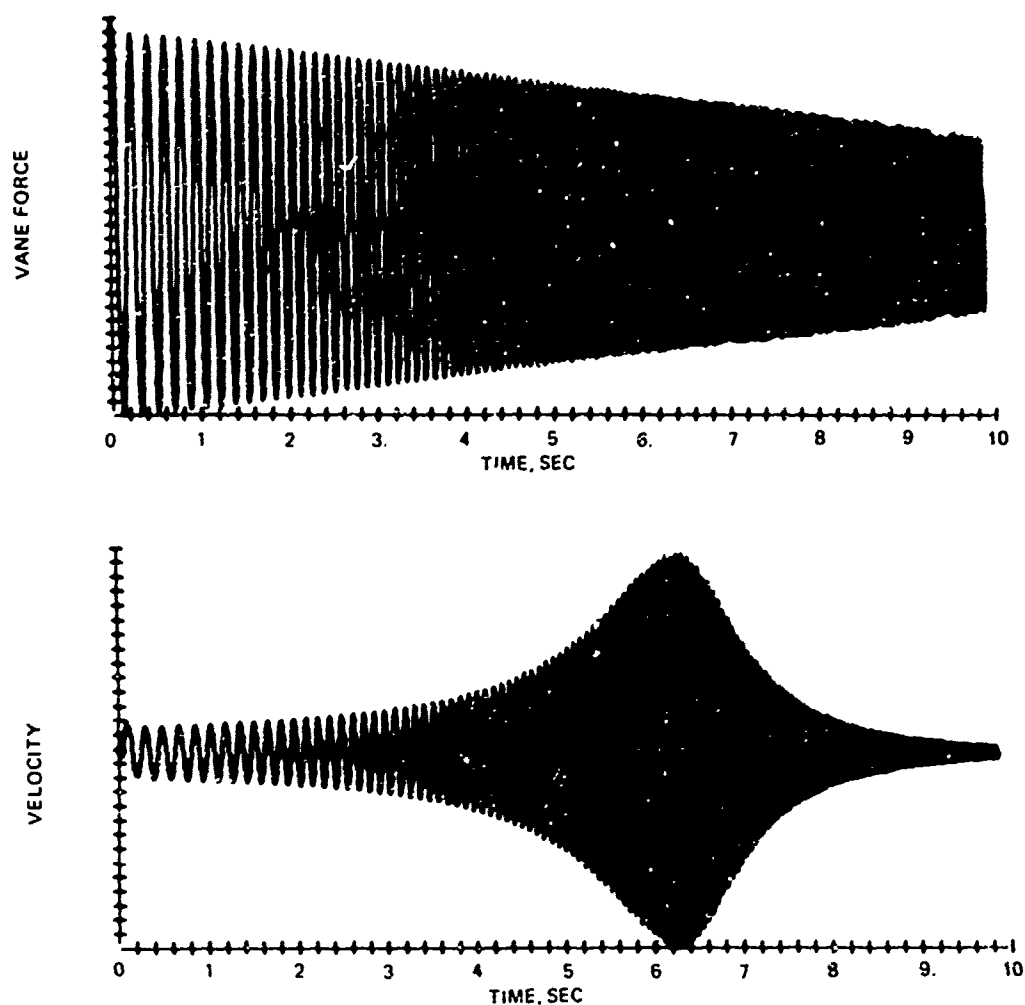


Fig. 8 Vane Force and Velocity Response of a Two-Mode Aeroelastic Model at the Coalescence Speed, 1000 Kts (94% V_F); Vane Unbalance Chosen to Produce a High Crossover Frequency; No Gust Force.

RECORD SEGMENT, SECONDS	*FREQUENCY & DAMPING RESULTS OF MODEL MATCHING PROGRAM FOR FIGURE 8					THEORETICAL PROGRAM RESULTS
	2ND ORDER	3RD ORDER	4TH ORDER	5TH ORDER	6TH ORDER MODEL	
5.5 7.0**	16.3/ 19	16.3/ 25 80	16.5/ 63 16.3/ 29	16.8/ 59 16.3/ 29 x	16.5/ 60 16.3/ 29 x x	$\omega_1 = 16.5 \text{ Hz}$
5.0 7.5**	16.5/ 17	16.4/ 23 67	16.4/ 64 16.3/ 29	16.6/ 59 16.3/ 29 41	16.5/ 59 16.3/ 29 x x	$\omega_2 = 17.3 \text{ Hz}$
5.0 9.0	16.8/ 17	16.6/ 20 28	16.2/ 62 16.3/ 29	16.5/ 61 16.3/ 29 25	16.5/ 59 16.3/ 29 x x	$\eta_1 = 29$
3.0 9.0	16.6/ 13	16.5/ 19 81	16.3/ 29 16.4/ 28	16.6/ 60 16.3/ 29 12	16.5/ 59 16.3/ 20 x x	$\eta_2 = 59$
						$\lambda_{\text{real}} = 11.8$

*Results are presented as follows $\omega_1/\eta_1, \omega_2/\eta_2, \lambda_{\text{real}}$

**Preferred record segments

x Program results were not meaningful

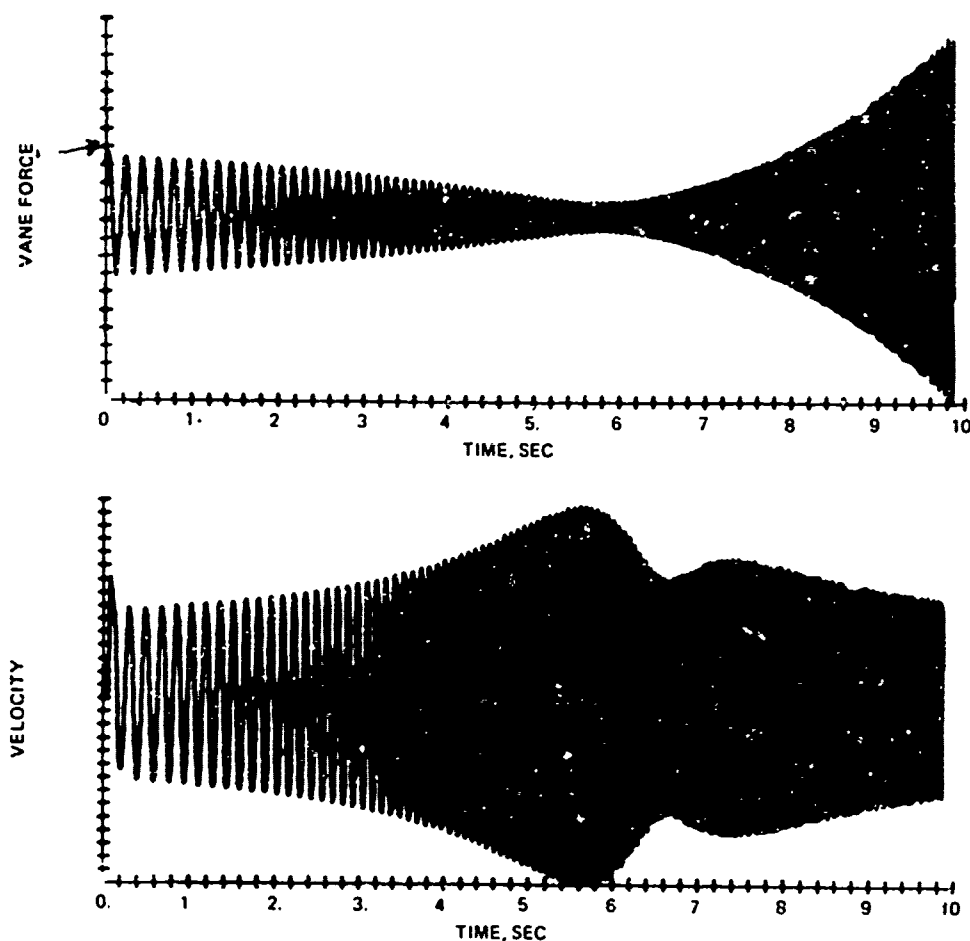


Fig. 9 Vane Force and Velocity Response of a Two-Mode Aeroelastic Model at the Coalescence Speed, 1000 Kts (94% V_F); Vane Unbalance Chosen to Produce a Crossover Frequency Near the Coalescence Frequency; No Gust Force.

RECORD SEGMENT, SECONDS	*FREQUENCY & DAMPING RESULTS OF MODEL MATCHING PROGRAM FOR FIGURE 9					THEORETICAL PROGRAM RESULTS
	2ND ORDER	3RD ORDER	4TH ORDER	5TH ORDER	6TH ORDER MODEL	
5.5 - 7.0	15.3/.22	15.3/.15 38.	17.3/ 68 16.2/ 29	16.5/ 73 16.2/ 28 28	16.3/ 30 16.7/.56 530 x	$\omega_1 = 16.5 \text{ Hz}$
5.0 - 7.5	15.0/.26	14.7/.07 80.	17.2/ 72 16.1/ 27	15.6/ 70 16.4/ 27 198	21.8/1.19 17.3/.53 16.2/ 30	$\omega_2 = 17.3 \text{ Hz}$
5.0 - 9.0**	13.2/.90	14.1/.012 78	15.1/1.08 16.1/ 16	15.9/ 61 16.5/ 29 x	16.4/.61 16.3/ 29	$\eta_1 = .29$
3.0 - 9.0**	13.4/.84	12.2/.25 225.	15.9/1.02 15.8/ 19	15.0/ 69 16.8/ 23 x	16.5/ 61 16.3/ 28	$\eta_2 = .59$
						$\lambda_{real} = 11.8$

*Results are presented as follows $\omega_1/\eta_1, \omega_2/\eta_2, \lambda_{real}$

**Preferred record segments

x Program results were not meaningful

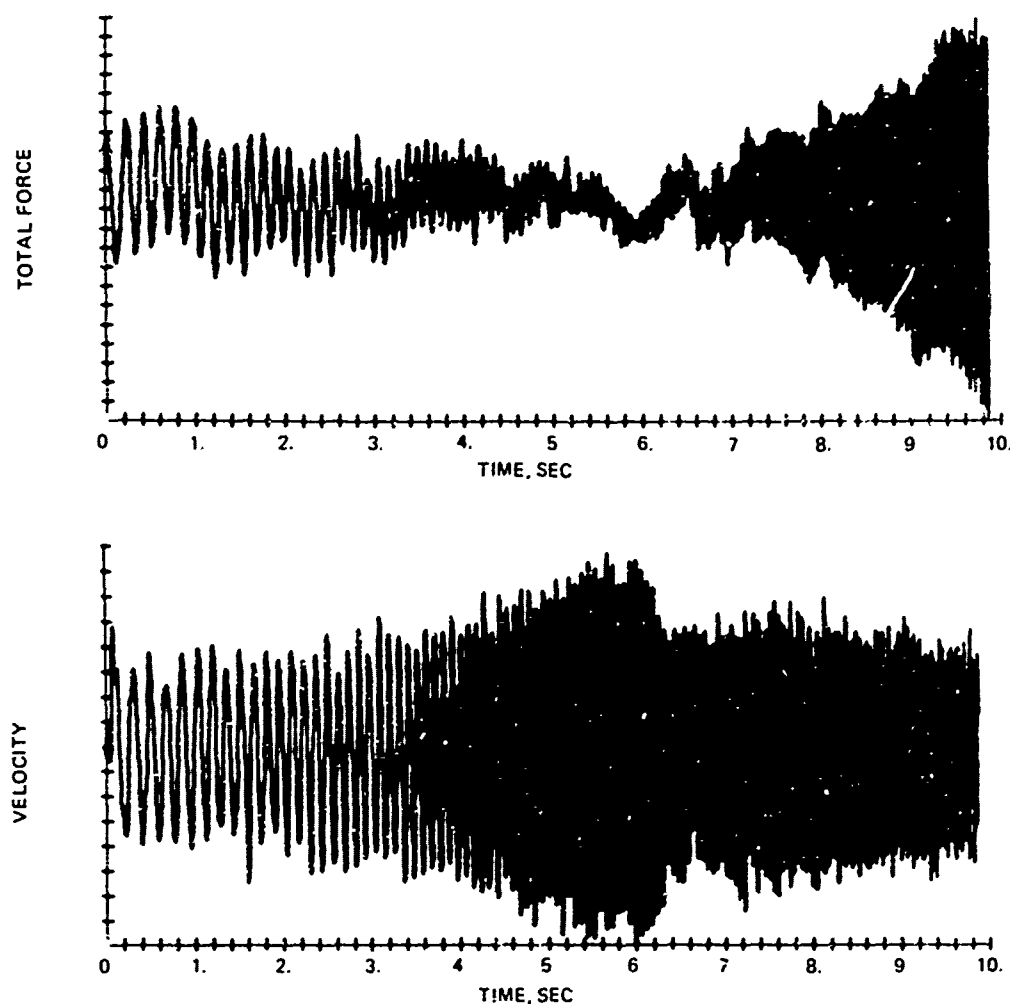


Fig. 10 Total Force and Velocity Response of a Two-Mode Aeroelastic Model at the Coalescence Speed; Total Force Is Comprised of a Vane Force and a Gust Force with an RMS Intensity of 8.4 ft/sec; 1000 Kts (94% V_F) at 9000 ft; Vane Unbalance Chosen to Produce a Crossover Frequency Near the Coalescence Frequency

RECORD SEGMENT, SECONDS	*FREQUENCY & DAMPING RESULTS OF MODEL MATCHING PROGRAM FOR FIGURE 10					THEORETICAL PROGRAM RESULTS
	2ND ORDER	3RD ORDER	4TH ORDER	5TH ORDER	6TH ORDER MODEL	
5.5 - 7.0	16.0/19	16.3/11 45	15.0/23 18.5/09	20.7/12 15.5/16 50.	26.9/17 12.7/13 18.2/12	$\omega_1 = 16.5 \text{ Hz}$
5.0 - 7.5	15.4/22	15.3/09 59	14.9/26 18.8/09	20.4/12 15.1/21 51	26.7/16 13.0/17 17.3/13	$\omega_2 = 17.3 \text{ Hz}$
5.0 - 9.0	14.4/73	14.9/02 192.	15.1/97 16.8/14	18.8/14 13.6/24 144	18.2/87 20.1/20 15.6/21	$\eta_1 = .29$
3.0 - 9.0	14.5/66	13.1/26 183.	16.3/83 16.6/17	12.9/63 18.2/14 125	18.2/74 20.5/24 15.7/23	$\eta_2 = .59$
						$\lambda_{\text{real}} = 11.8$

*Results are presented as follows: $\omega_1/\eta_1, \omega_2/\eta_2, \lambda_{\text{real}}$

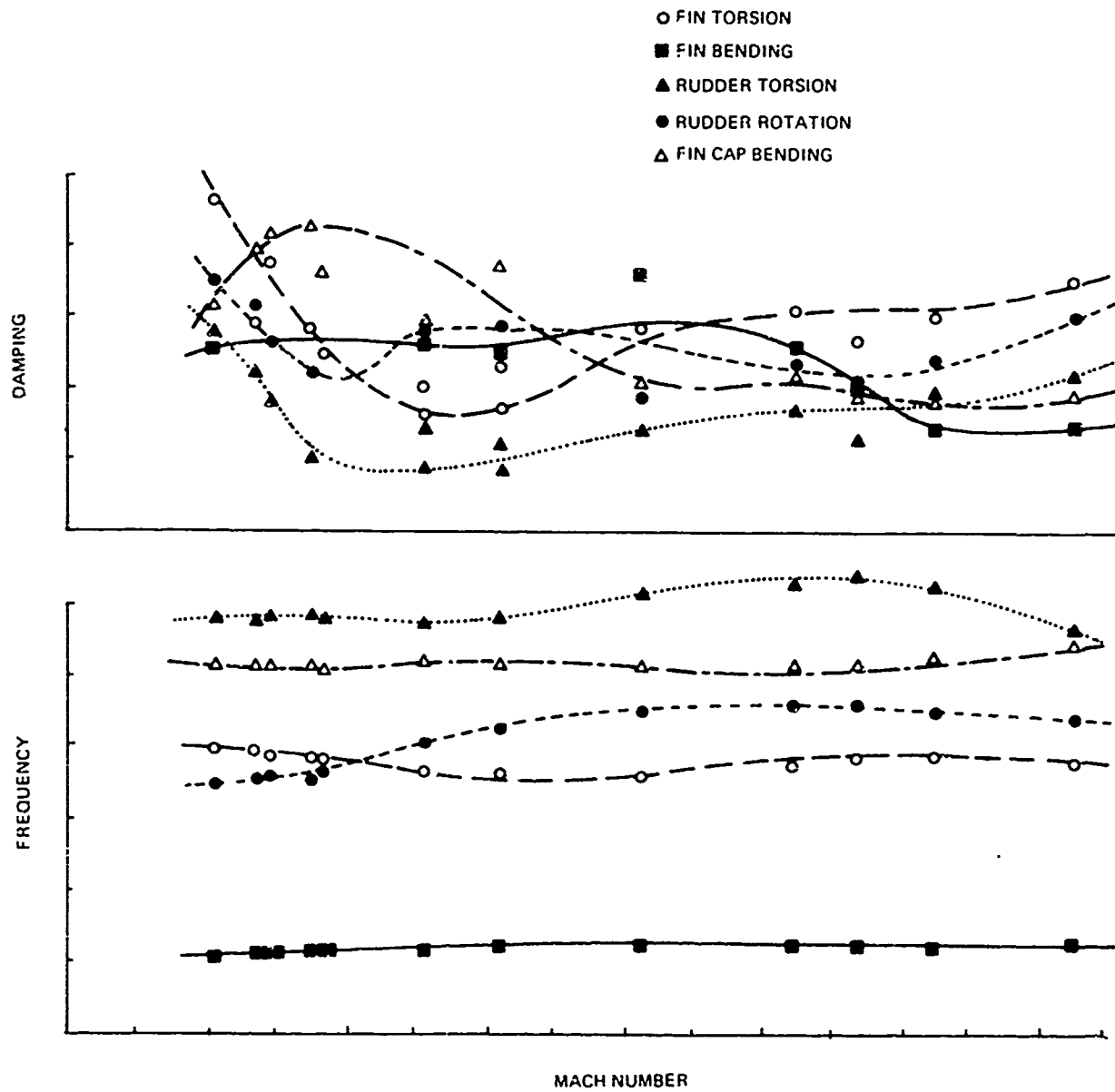


Fig. 11 Flutter Flight Test Results Using Model Matching

MANUAL ON AEROELASTICITY

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R. Mazet

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Report No.583	D.L.Woodcock	A Comparison of Methods Used in Lifting Surface Theory	Published June 1971
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