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RESEARCH ON STRUCTURAL DYNAMIC TESTING BY IMPEDANCE METHODS. VOLUME III. FREE-BODY RESPONSE.

Alex Berman, et al

Kaman Aerospace Corporation

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VOLUME III

FREE-BODY RESPONSE

By

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Nicholas Giansante

November 1972

EUSTIS DIRECTORATE
U. S. ARMY AIR MOBILITY RESEARCH AND DEVELOPMENT LABORATORY
FORT EUSTIS, VIRGINIA

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KAMAN AEROSPACE CORPORATION
BLOOMFIELD, CONNECTICUT

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This program was conducted under Contract DAAJ02-70-C-0012 with Kaman Aerospace Corporation.

This report contains the theoretical derivation and the presentation of a methodology for system identification of structures. Computer experiments were run to verify this methodology.

The report has been reviewed by this Directorate and is considered to be technically sound. It is published for the exchange of information and the stimulation of future research.

This program was conducted under the technical management of Mr. Arthur J. Gustafson, Technology Applications Division.

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13. ABSTRACT

A method is presented for determining free-body dynamic responses from data taken on a constrained structure. The method requires the matrix of complex amplitudes of the deflection at N points on the structure and the sinusoidal forces of excitation at frequency ω applied at M points on the structure. The forces of constraint are considered to be "applied" forces. There is no need for the force points to coincide with or be equal to the response points. The displacements can also, with no change in the analysis, represent displacements, linear or angular, in two or three directions at one geometrical point. Further, the method is not restricted to displacement response, and the same formulation is applicable to either velocity or acceleration response.

A digital computer program was prepared for the IBM Model 360/40 computer using FORTRAN IV language to test the practicality of the subject theory. Computer experiments were conducted to test the sensitivity of the theory to measurement errors in the simulated test data and to numerically test the theory.

The method was proven to be a practical means of predicting the free-body dynamic responses from simulated experimental data derived from a constrained structure. The theoretical development was shown to be numerically sound and insensitive to measurement errors.

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FORT EUSTIS, VIRGINIA

FOREWORD

The work presented in this report was performed by Kaman Aerospace Corporation under Contract DAAJ02-70-C-0012 (Task 1F162204AA4301) for the Eustis Directorate, U. S. Army Air Mobility Research and Development Laboratory, Fort Eustis, Virginia. The program was implemented under the technical direction of Mr. Joseph H. McGarvey of the Reliability and Maintainability Division* and Mr. Arthur J. Gustafson of the Structures Division.** The report is presented in four volumes, each describing a separate phase of the basic theory of structural dynamic testing using impedance techniques.

Volume I presents the results of an analytical and numerical investigation of the practicality of system identification using fewer measurement points than there are degrees of freedom. The parameters in Lagrange's equations of motion, mass, stiffness, and damping for a mathematical model having fewer degrees of freedom than the linear elastic structure it represents may be determined directly from measured mobility data. Volume II describes the method of system identification wherein the necessary impedance data are experimentally determined by applying a force excitation at a single point on the structure. Volume III presents a method of determining the free-body dynamic responses from data obtained on a constrained structure. Volume IV describes a method of obtaining the equations for the combination of measured mobility matrices of a helicopter and its subsystems. The response of the combination of a helicopter and its subsystems is determined from data based on the experimental results of the main system and subsystems separately.

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*Division name changed to Military Operations Technology Division.

**Division name changed to Technology Applications Division.

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LIST OF SYMBOLS

$\{\bar{f}_j\}$	Vector of complex amplitude of applied force
$[F]$	Matrix of applied force vectors
M	Number of points at which force excitation is applied
N	Number of points at which response measurements are made
$\{\bar{y}_j\}$	Vector of complex amplitudes of deflection
$[y]$	Matrix of complex amplitudes of deflection vectors
$[Y]$	Complex displacement mobility matrix relating forces and responses (NxM)
ω	Frequency of applied sinusoidal forces

BRACKETS

$[\ ]$	Matrix
$\lceil\ \rfloor$	Diagonal matrix
$\{\ \}$	Column or row vector

SUPERSCRIPTS

-1	Inverse

INTRODUCTION

The dynamic testing of helicopters, both full scale and model, is required to anticipate the response of the vehicle to the vibratory load spectrum to which it is subjected in flight. These responses were computed from an analytical model which is partly developed from, and verified by, data obtained from testing, such as resonant frequencies, resonant responses, and damping.

The uncertainties in the experimental determination of mode shapes and resonant frequencies for unconstrained structures relate to the inherent difficulties involved in supporting the structure so as to simulate free-body conditions. During vibration tests the model or full-scale vehicle must be supported in a manner to prevent interference with the response of the structure. In order to simulate the free-body boundary conditions of a helicopter in flight, the usual procedure has been to support the structure on a system which is relatively soft to negate the effect of rigid-body modes on the elastic modes of the helicopter. A commonly used technique consists of supporting the fuselage on cables and bungee attached to the hub. As helicopters become larger and heavier, it will be necessary to construct massive structures capable of supporting the total weight of the vehicle. This situation, which is expensive and requires substantial design effort, will continue to deteriorate. In addition, there is some uncertainty in the effects of the suspension dynamics and nonlinearities on the helicopter response.

A practical method which eliminates the need for soft suspensions in order to simulate in-flight boundary conditions will be of great help in the dynamic testing of helicopters. Such a method would yield better correlation between experimental and flight results and significantly reduce the cost of this testing.

The method described in this report has several desirable features. It uses the measured forces of constraint to convert the measured constrained helicopter responses to free-body responses. The structure being tested is considered to be supported on real supports, but the characteristics of the supports themselves are not required since only their measured reactions are used. The procedure uses only data which is actually measured, and no quantitative assumptions are employed.

THEORY

For sinusoidally varying forces at a frequency ω, applied at M points on a structure, a vector $\{\bar{f}_j\}$ can be defined which represents the complex amplitude of applied force at each of the points. Similarly, $\{\bar{y}_j\}$ is a vector representing the complex amplitudes of the deflection at each of N points resulting from the force vector $\{\bar{f}_j\}$. There is no necessity for the force excitation points to coincide with the points at which the deflection response is measured. The complex displacement mobility matrix, [Y], of order NxM represents the relationship between the applied forces and responses. The vectors $\{\bar{f}_j\}$, $\{\bar{y}_j\}$ and the matrix [Y] and the relationship among them are written

$$\{\bar{y}_j\} = \begin{Bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{Bmatrix} \qquad \{\bar{f}_j\} = \begin{Bmatrix} f_1 \\ f_2 \\ \vdots \\ f_M \end{Bmatrix} \qquad [Y] = \begin{bmatrix} \frac{\partial y_1}{\partial f_1} & \frac{\partial y_1}{\partial f_2} \cdots & \frac{\partial y_1}{\partial f_M} \\ \frac{\partial y_2}{f_1} & & \\ \vdots & & \\ \frac{\partial y_N}{\partial f_1} & \cdots\cdots\cdots & \frac{\partial y_N}{\partial f_M} \end{bmatrix}$$

and

$$\{\bar{y}_j\} = [Y]\{\bar{f}_j\} \tag{1}$$

The previous expressions are not restricted to displacement response; exactly the same relationships apply for velocity and acceleration response. The displacements can also, with no modification in the analysis, represent displacements or rotations in two or three directions at one geometrical point by alloting one element in each vector for each generalized displacement. Similar considerations apply to the forces or moments. There is no necessity for the matrix, [Y], to be square; it will contain one row for each measured displacement, one column for each point at which an excitation is applied.

Consider a matrix, [F], containing several applied vectors and a matrix, [y], containing the corresponding deflections, as follows:

$$[F] = [\bar{f}_1 \mid \bar{f}_2 \mid \; \ldots.] \qquad [y] = [\bar{y}_1 \mid \bar{y}_2 \mid \; \ldots.]$$

and then

$$[y] = [Y][F] \tag{2}$$

If [F] is a nonsingular matrix, then the desired result, the response of the points of interest to single forces, may be written

$$[Y] = [y][F]^{-1} \tag{3}$$

In Equation (3), the matrices [y] and [F] consist of measured data. When the externally applied loads only are included in [F], then [Y] is the mobility of the structure as tested on the actual supports. If [F] includes any of the forces of constraint, then [Y] is the mobility of the structure with those constraints removed. If all the forces of constraint are included in [F], then [Y] is the mobility of the unconstrained system or free body.

As indicated previously, [F] must be nonsingular and thus have an inverse. If there are M forces to be considered, including the forces at the constraints, then M sets of vectors of applied force, $\{\bar{f}_j\}$, must be applied and all of these vectors must be linearly independent. The independence criterion may be achieved by applying an external force at each constraint.

FORCES AT CONSTRAINTS

If an exciting force is applied at the j-th constraint, the force vector will be of the form

$$\{\bar{f}_j\} = \begin{Bmatrix} f_{r_1} \\ f_{r_2} \\ \vdots \\ f_{r_j} + f \end{Bmatrix} \tag{4}$$

where the f_{r_j} are the forces at the constraints due to the applied force f.

For an optimally designed support system, the maximum number of constraint forces is six and the [F] matrix relating the support forces will be of the following form:

$$[F_c] = \begin{bmatrix} f_{r_1} + f_1 & f_{r_1} & \cdots\cdots & f_{r_1} \\ f_{r_2} & f_{r_2} + f_2 & & \vdots \\ \vdots & \vdots & & \vdots \\ f_{r_6} & f_{r_6} & & f_{r_6} + f_6 \end{bmatrix} \tag{5}$$

where the elements in each column represent the actual measurements taken for the force applied at each of the constraints. Because the loads are independent, the preceding matrix will always be nonsingular.

In conjunction with the force measurements, the displacements (or accelerations) are recorded at the points of interest on the structure and one column of [y] is formed for each column of $[F_c]$.

Then, as in Equation (3),

$$[Y_{FB}] = [y][F_c]^{-1} \tag{6}$$

where $[Y_{FB}]$ represents the deflection of each point of interest on the structure due to each of the applied loads at the supports, and is the free-body mobility matrix. The $[F_c]$ matrix was obtained at one particular frequency, and the same procedure must be implemented over the frequency bandwidth of interest.

Normally it is required to determine the response of the helicopter due to force excitation at points other than the supports. Therefore, the structure must also be forced at these points and the forces at the constraints must be measured. If $[F_{cA}]$ is a matrix representing the forces of constraint for each point of excitation and $[F_A]$ is a matrix representing the applied loads, then the [F] matrix becomes

$$[F] = \left[\begin{array}{c|c} [F_A] & 0 \\ \hline [F_{cA}] & [F_c] \end{array}\right] \tag{7}$$

However, if unit forces are applied, the matrix $[F_A]$ becomes the unit matrix and Equation (7) becomes

$$[F] = \left[\begin{array}{c|c} [I] & 0 \\ \hline [F_{cA}] & [F_c] \end{array}\right] \tag{8}$$

The inverse of this matrix is readily obtained and is given by

$$[F]^{-1} = \left[\begin{array}{c|c} [I] & 0 \\ \hline [F_c]^{-1}[F_{cA}] & [F_c]^{-1} \end{array}\right] \tag{9}$$

or by simply inverting the [F] matrix in its entirety by numerical means.

ERROR ANALYSIS

The technique of structural dynamics testing utilizing impedance methods, to be of any practical engineering significance, must be functional with a reasonable degree of experimental error. Measurements of the complex amplitudes of displacement required to implement the method will be subject to experimental errors of various types. In general, all errors can be classified as either random or bias.

In the present analysis, the simulated test data were polluted with measurement error, and computer experiments were conducted to test the sensitivity of the method to error. The 20-degree-of-freedom "beam-type" representation of a helicopter and the 18-degree-of-freedom, three-dimensional mathematical model were analyzed incorporating measurement error. The simulated test data in the form of acceleration mobility and reaction force measurements were both polluted with a 5 percent bias error and a ±5 percent random error on amplitude and a ±1 degree error on phase angle. The random amplitude phase angle errors were assumed to be uniformly distributed. The resulting distribution of random error between the selected limits is conservative compared to limits applied to a normal distribution.

APPLICATION OF THE THEORY

The method has several attributes which make it especially suitable for practical application, including the use of only measured data, the lack of quantitative assumptions regarding the mass or stiffness distributions or the assumption of infinitely rigid supports. There are, however, as in all procedures, certain considerations involved in planning an efficient and accurate application of the method.

At each frequency it is necessary to conduct one test for each constraint. It is possible to constrain all rigid-body motions with six constraining forces. There is no necessity for such complete constraint, however. During the design of a test, test configurations should be considered which allow freedom of motion in the horizontal plane and around the vertical axis. Such an arrangement for a typical helicopter might consist of three supports at the base, each resisting only vertical motion and mounted on a device which allows unconstrained, but limited, motion in the horizontal plane. For sinusoidal force excitation at the rotor hub, it is necessary to measure the complex amplitude of the response of the points of interest on the structure as well as the forces induced at the constraint points. It is also necessary to apply vertical excitation at each support and measure each of the vertical forces of constraint. In addition, any other shaker position or orientation could be used while the three vertical forces are measured.

Theoretically, the stiffness characteristics of the supports are immaterial. However, these characteristics do affect the magnitudes of the forces and responses which will be measured. The performance of transducers such as accelerometers or load cells is dependent on the magnitude and frequency of the quantity being measured. Thus, for the most reliable results, the supports should be designed and the transducers selected for optimum operation.

A computer simulation of alternative test configurations can be an invaluable aid in the preliminary design stage of an actual test program. Prior to implementation of a test plan, a computer simulation applied to this method using an intuitive analytical model of the structure can be used to determine the sensitivity to error and the expected accuracy of the results of the various arrangements considered. It should include realistic experimental errors, approximate constraint characteristics and a frequency spectrum bounding the range of interest.

COMPUTER SIMULATION RESULTS

A computer simulation study was conducted to determine the practicality of the theory described in the subject report. The digital computer program listing in FORTRAN IV language and a description of the program input cards are presented in the appendix. Two basic configurations were analyzed, a 20-degree-of-freedom "beam-type" representation of a helicopter and an 18-degree-of-freedom, three-dimensional structure. Table I presents a description of the "beam-type" model including mass and stiffness distributions. The discrete masses, coordinate locations, member internal spring rates and ground springs are given in Table II for the 18-degree-of-freedom analytical model. For the models investigated, the simulated test data, acceleration mobility amplitude and reaction forces, were separately polluted with measurement errors of 5 percent bias error, ±5 percent random error and ±1 degree error on phase angle. Tables III and IV present a compilation of a portion of the computer experiments performed in implementing the theory presented in this report.

The effect of measurement error on the free-body real and imaginary acceleration frequency response for the 20-degree-of-freedom "beam-type" representation of a helicopter is shown in Figures 1 and 2, respectively. In each instance the exact curve represents the result of the analysis utilizing simulated experimental data with zero error. Figures 3 and 4 present similar data for the 18-degree-of-freedom, three-dimensional mathematical model. As evidenced by the aforementioned figures, the theoretical development presented herein is essentially insensitive to the degree of error incorporated in the analysis.

Figures 5 and 6 portray the results of implementation of the method using the measured forces of constraint to yield the free-body response of the 20-degree-of-freedom "beam-type" structure. Two simulated constrained modes are shown on the figures with the specimen supported on real supports of 10,000 lb/in. and 50,000 lb/in. spring rates, respectively. The free-body response of the structure achieved by deleting the effects of supports for both constrained boundary conditions is also presented. It is important to note that regardless of support system, application of the method yields identical unconstrained or free-body boundary conditions for the structure. Many computer experiments were conducted, including the effect of measurement error, to obtain the data presented in Figures 5 and 6. Comparison of the simulated "exact" free response and the response obtained incorporating realistic measurement error indicates the results

TABLE I. 20-DEGREE-OF-FREEDOM MODEL, "BEAM TYPE"

Sta No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Sta (In.)	0		60		120		160		200		240		280		320		370		430	
		30		100		140		180		220		260		300		340		400		460
Mass (Lb-Sec^2/In.)	.029		3.67		2.18		2.385		2.08		.910		.170		.070		.095		.210	
		1.05		3.71		2.18		2.59		1.56		.260		.085		.060		.120		.150
EI (Lb-In.^{2} x 10^{10})	.35		.35		1.95		4.37		5.80		4.425		3.07		2.05		.975		.55	
		.35		1.20		3.00		5.70				3.6		2.60		1.60		.65		.50
Springs to Ground (Lb/In.)						10,000												10,000		
						50,000												50,000		

TABLE II. THREE-DIMENSIONAL 18-DEGREE-OF-FREEDOM MODEL

Mass No.	Mass Lb-Sec^2/In.	Coordinate Numbers			Coordinate Locations*			Member Internal Spring Rate x 10^{-3}						Ground Springs		
															Configuration	
		X	Y	Z	X	Y	Z	1	2	3	4	5	6	Coordinate	H-1	H-2
1	2	1	2	3	0	0	-50	-	200	100	100	100	-	4	1000	2000
2	2	4	5	6	100	-50	0	200	-	100	20	100	50	5	5000	2000
3	6	7	8	9	100	-50	-100	100	100	-	100	150	200	6	10,000	50,000
4	10	10	11	12	100	120	-120	100	20	100	-	50	500	13	1000	5000
5	3	13	14	15	100	50	0	100	100	150	50	-	50	15	10,000	20,000
6	1	16	17	18	300	20	-50	-	50	200	500	50	-	18	500	2000

* Note Conventional Right-Hand Coordinate System

TABLE III. 20-DEGREE-OF-FREEDOM "BEAM TYPE" CONFIGURATION

	Error - Mobility				Error - Force				Constraint*		Applied Force** At Sta 1,6,10, and 18	
Case No.	Rand %	Bias %	Phase Deg	Seed	Rand %	Bias %	Phase Deg	Seed	Spring	Lb/In.	Force	Lb
1	0	0	0	0	0	0	0	0	10,000	10,000	10 100	10 100
2	5	5	1	4316	5	5	1	-	10,000	10,000	10 100	10 100
3	5	5	1	529591	5	5	1	-	10,000	10,000	10 100	10 100
4	5	5	1	75123	5	5	1	-	50,000	50,000	10 100	10 100
5	5	5	1	514272	5	5	1	-	50,000	50,000	10 100	10 100
6	5	5	1	7580101	5	5	1	-	50,000	50,000	10 100	10 100
7	5	5	1	124	5	5	1	-	50,000	10,000	10 100	10 100
8	5	5	1	16	5	5	1	-	100,000	10,000	10 100	10 100
9	5	5	1	893	5	5	1	-	10,000	10,000	10 100	10 100
10	5	5	1	11171	5	5	1	-	10,000	10,000	10 100	10 100
11	5	5	1	12	5	5	1	-	10,000	10,000	10 100	10 100
12	0	0	0	0	0	0	0	-	50,000	50,000	10 100	10 100

* Constraints located at Sta 6 and 18

** Applied forces at Sta 1, 6, 10 and 18

TABLE IV. THREE-DIMENSIONAL CONFIGURATION

		Error on Force and Mobility									
Case No.	Results	Rand %	Bias %	Phase Deg	Seed	Constraints* Applied Forces**					
13	Conf H-1	0	0	0	-	1000	5000	10,000	1000	10,000	500
14	Conf H-1	0	0	0	-	1000	5000	10,000	1000	10,000	500
15	Conf H-1	5	5	1	243012	1000	5000	10,000	1000	10,000	50
16	Conf H-1	5	5	1	4911	1000	5000	10,000	1000	10,000	500
17	Conf H-2	0	0	0	-	2000	2000	50,000	5000	20,000	2000
18	Conf H-2	0	0	0	-	2000	2000	50,000	5000	20,000	2000
19	Conf H-2	5	5	1	821	2000	2000	50,000	5000	20,000	2000
20	Conf H-2	5	5	1	5632	2000	2000	50,000	5000	20,000	2000

* Constraint attachment numbers 4, 5, 6, 13, 15 and 18

**Applied forces at above stations 10, 10, 100, 10, 100 and 10 (pounds)

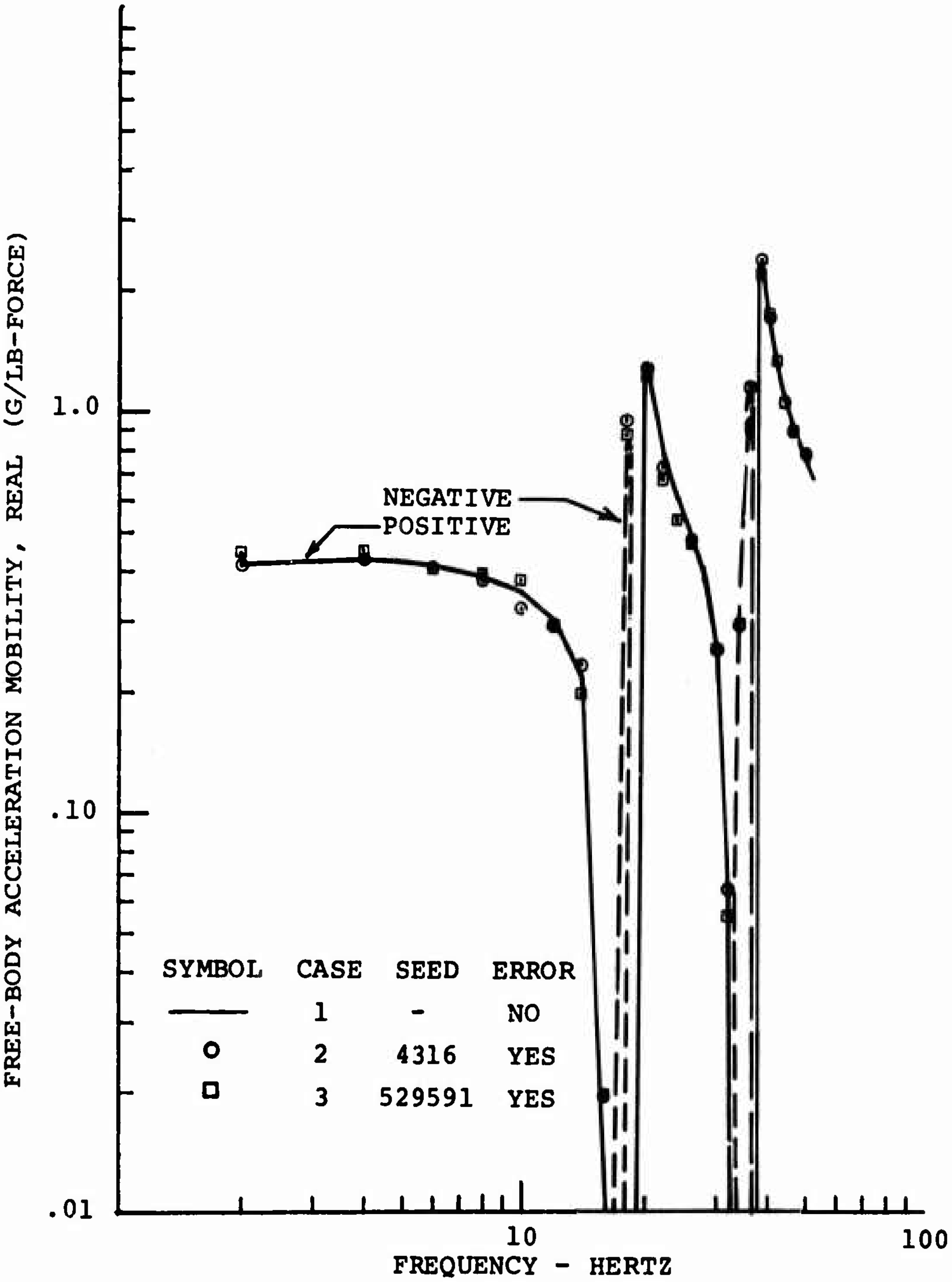

Figure 1. The Effect of Error on "Beam-Type" Model; Driving Point Real Acceleration Response.

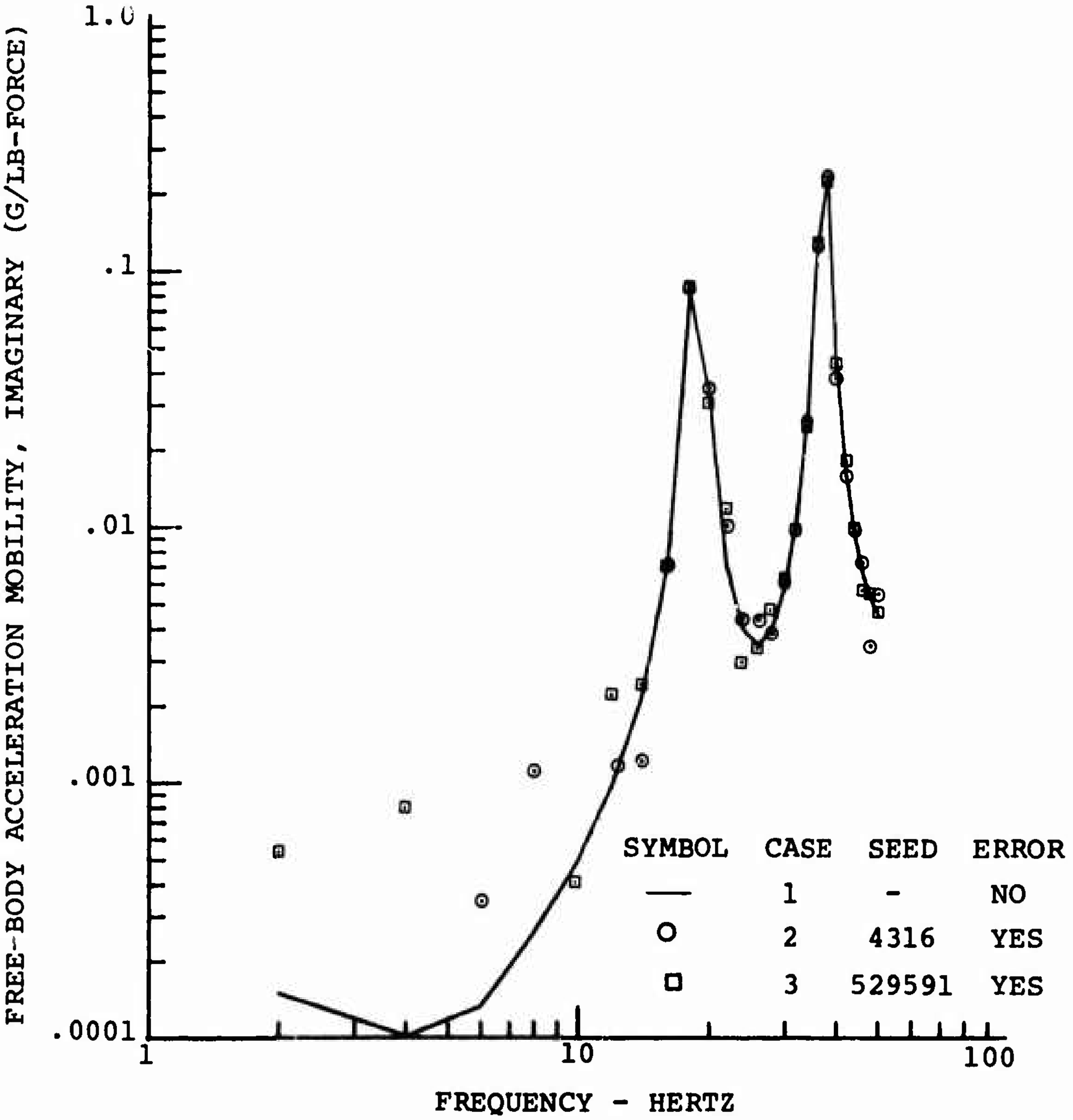

Figure 2. The Effect of Error on "Beam-Type" Model; Driving Point Imaginary Acceleration Response.

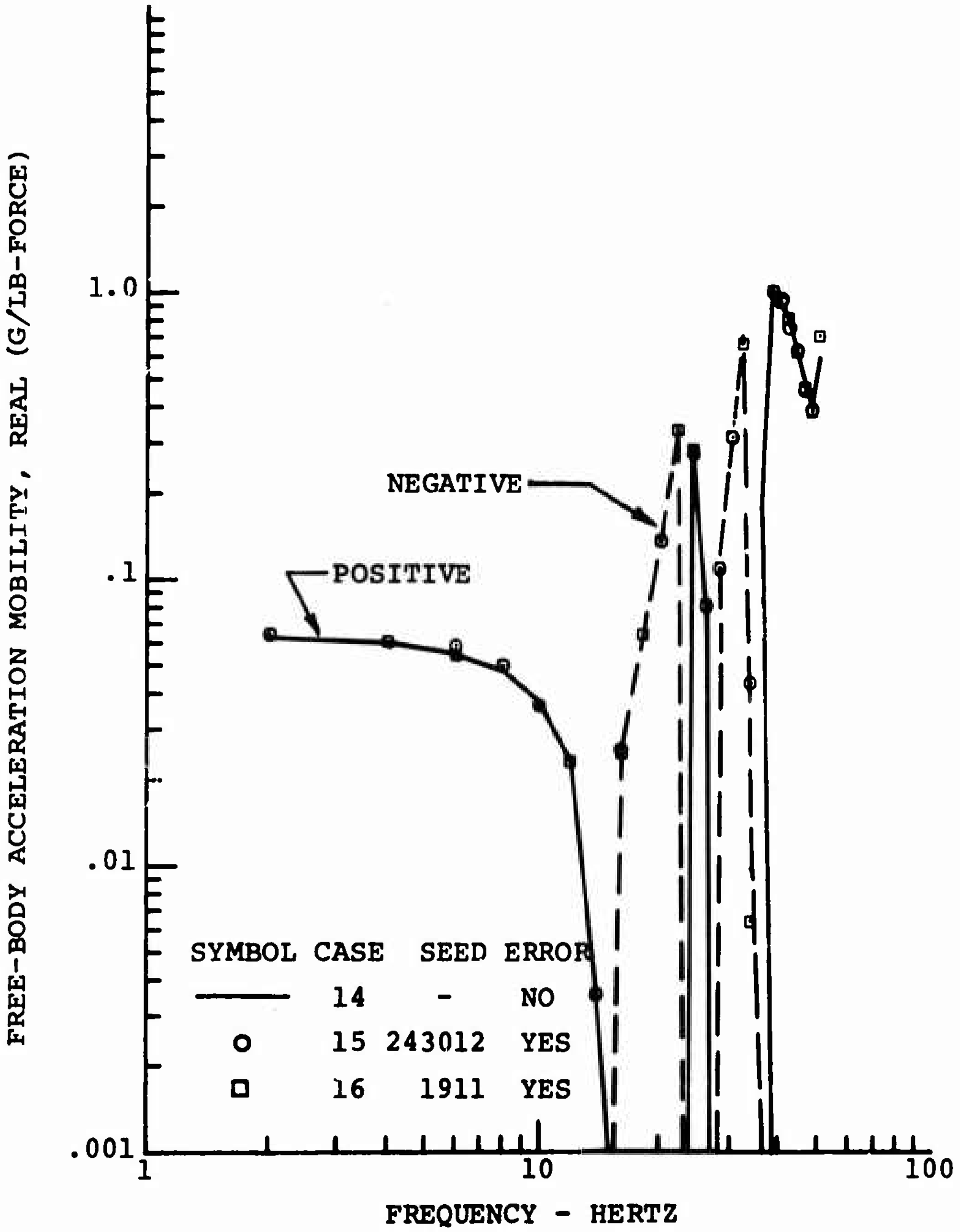

Figure 3. The Effect of Error on Three-Dimensional Model; Driving Point Real Acceleration Response.

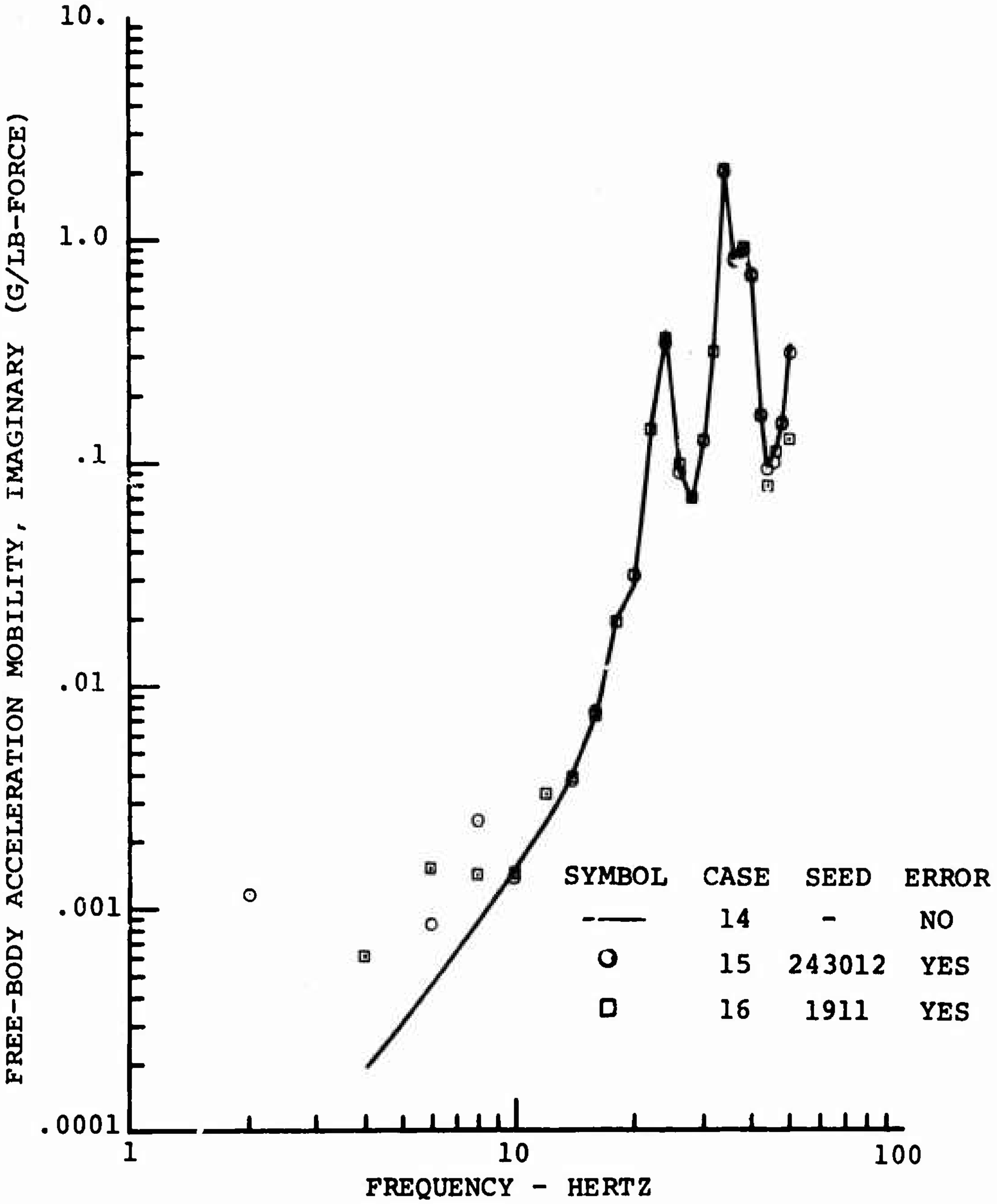

Figure 4. The Effect of Error on Three-Dimensional Model; Driving Point Imaginary Acceleration Response.

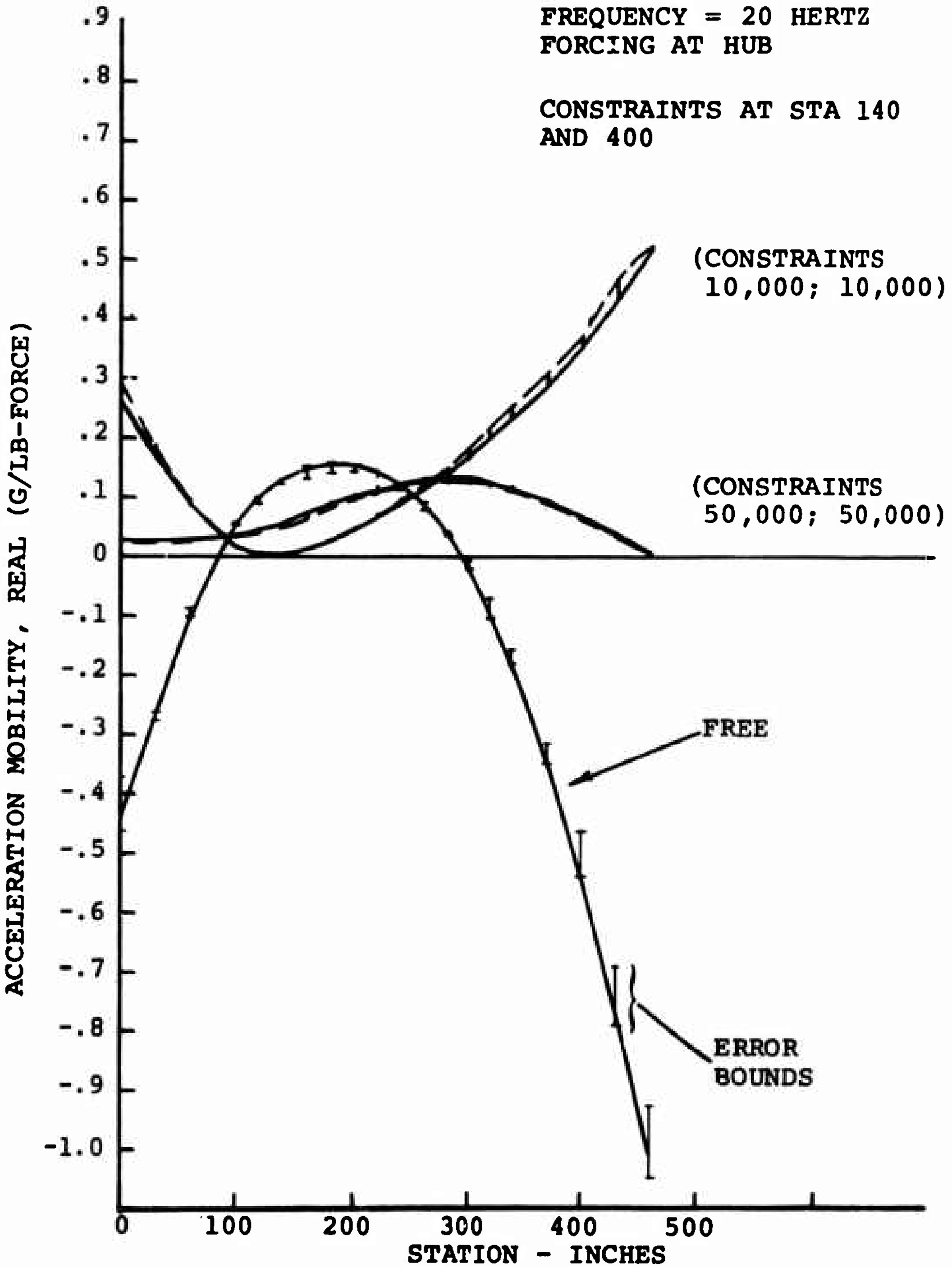

Figure 5. "Beam-Type" Model; Free-Body Real Acceleration Response.

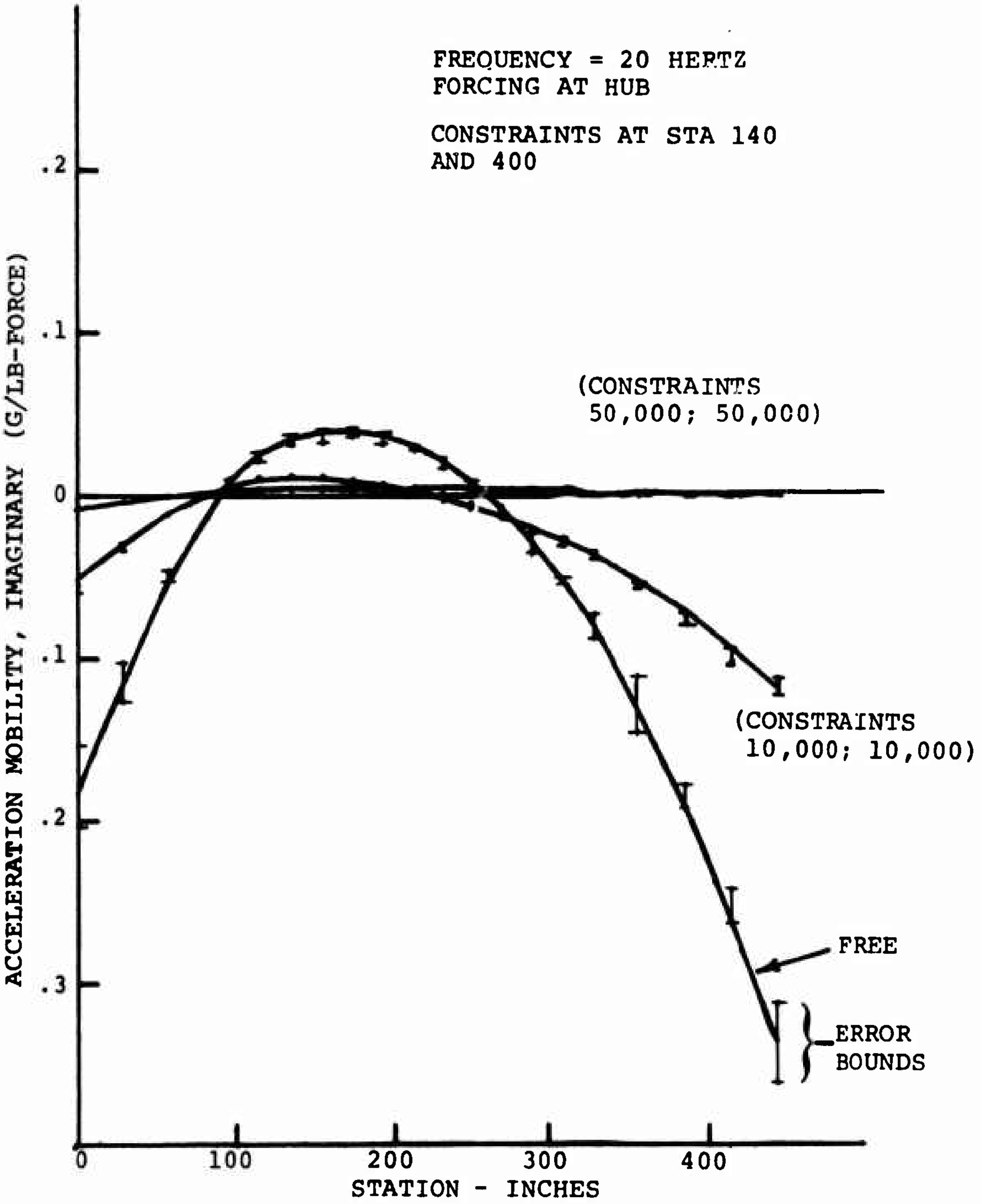

Figure 6. "Beam-Type" Model; Free-Body Imaginary Acceleration Response.

are effectively invariant with error and the method can be applied, with confidence, to yield the free-body characteristics of a structure.

The results of the investigation to determine the influence of measurement error on the methodology are shown in Figures 7, 8, 9, and 10. Figure 7 is a histogram obtained by considering the real acceleration mobility of a specific element on the constrained 20-degree-of-freedom "beam-type" model over the frequency range of interest. The percentage error was based on a comparison of the "exact" or zero error simulation and the conditions wherein measurement error was applied to acceleration mobility data and reaction force data. The solid line represents the theoretical cumulative frequency polygon and yields the probability of obtaining an error deviation below a specified value. Figure 8 shows similar data for the imaginary component of constrained acceleration mobility. Corresponding data for the free-body condition is given in Figures 9 and 10. Examining Figures 7, 8, 9, and 10 reveals an increase in error bandwidth accompanying removal of the system constraints. This is a consequence of polluting the reaction forces with measurement forces and the subsequent interaction with the acceleration mobility measurements also subjected to error. However, the errors are not prohibitively large and are within an acceptable range.

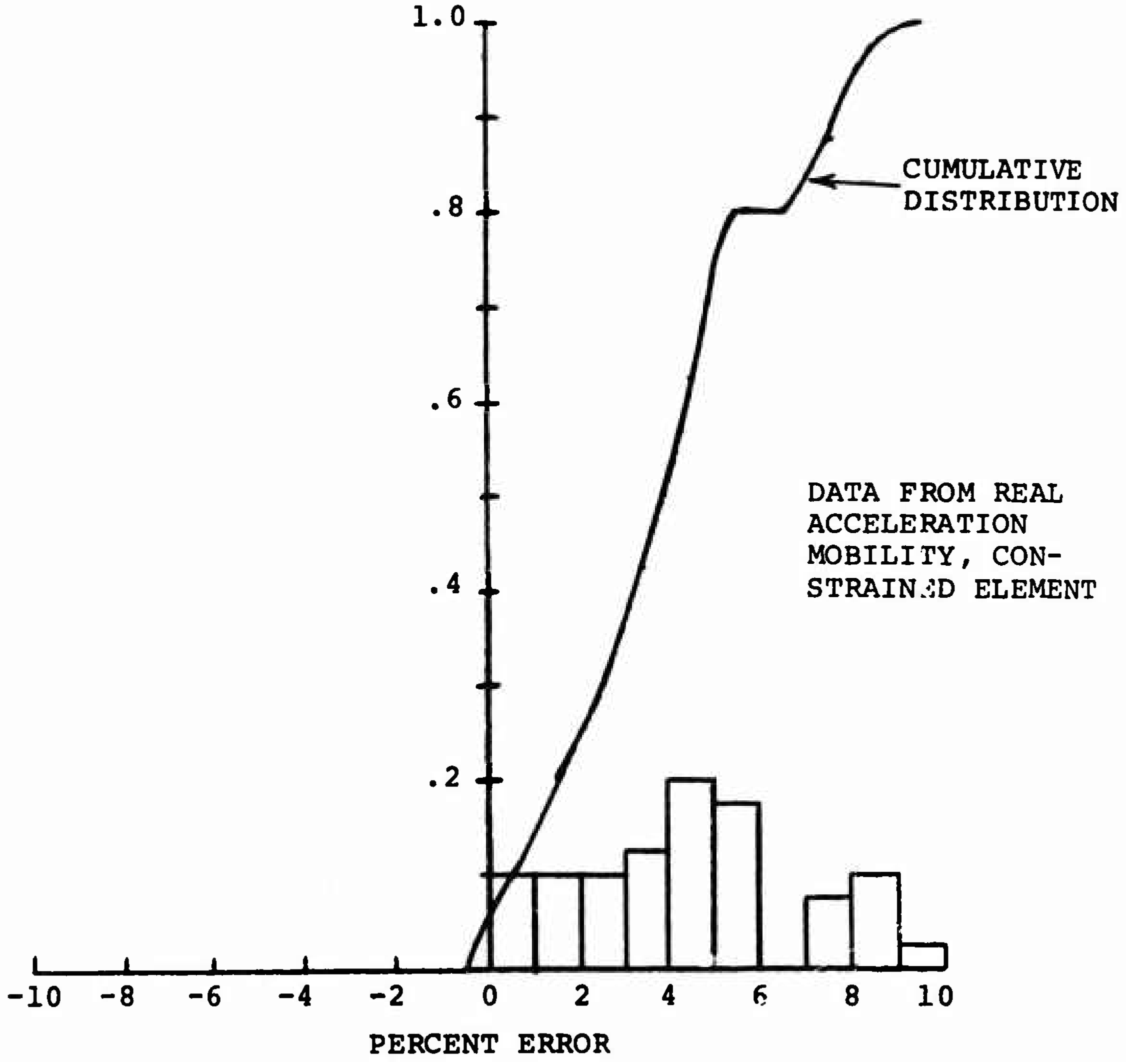

Figure 7. Error Distribution for Constrained Element Using Real Acceleration Mobility.

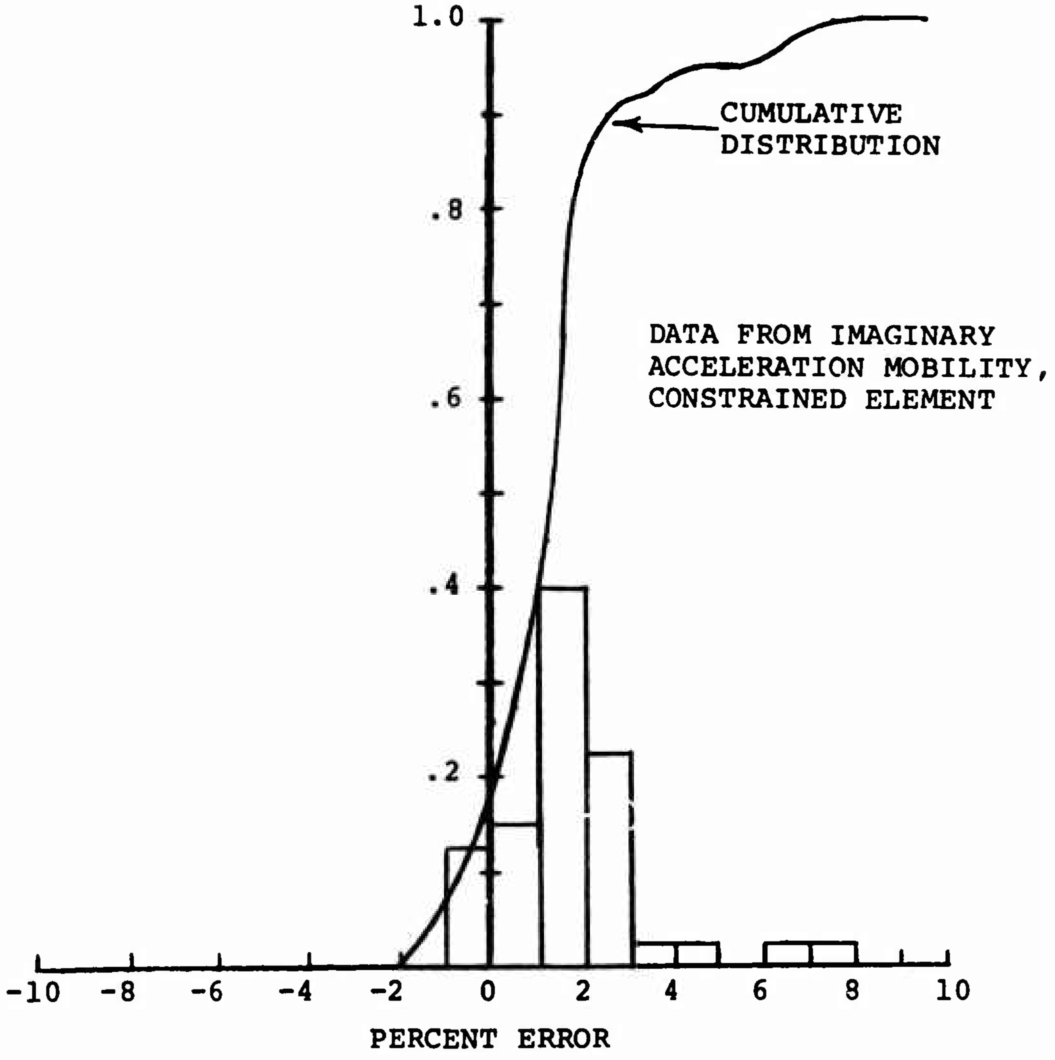

Figure 8. Error Distribution for Constrained Element Using Imaginary Acceleration Mobility.

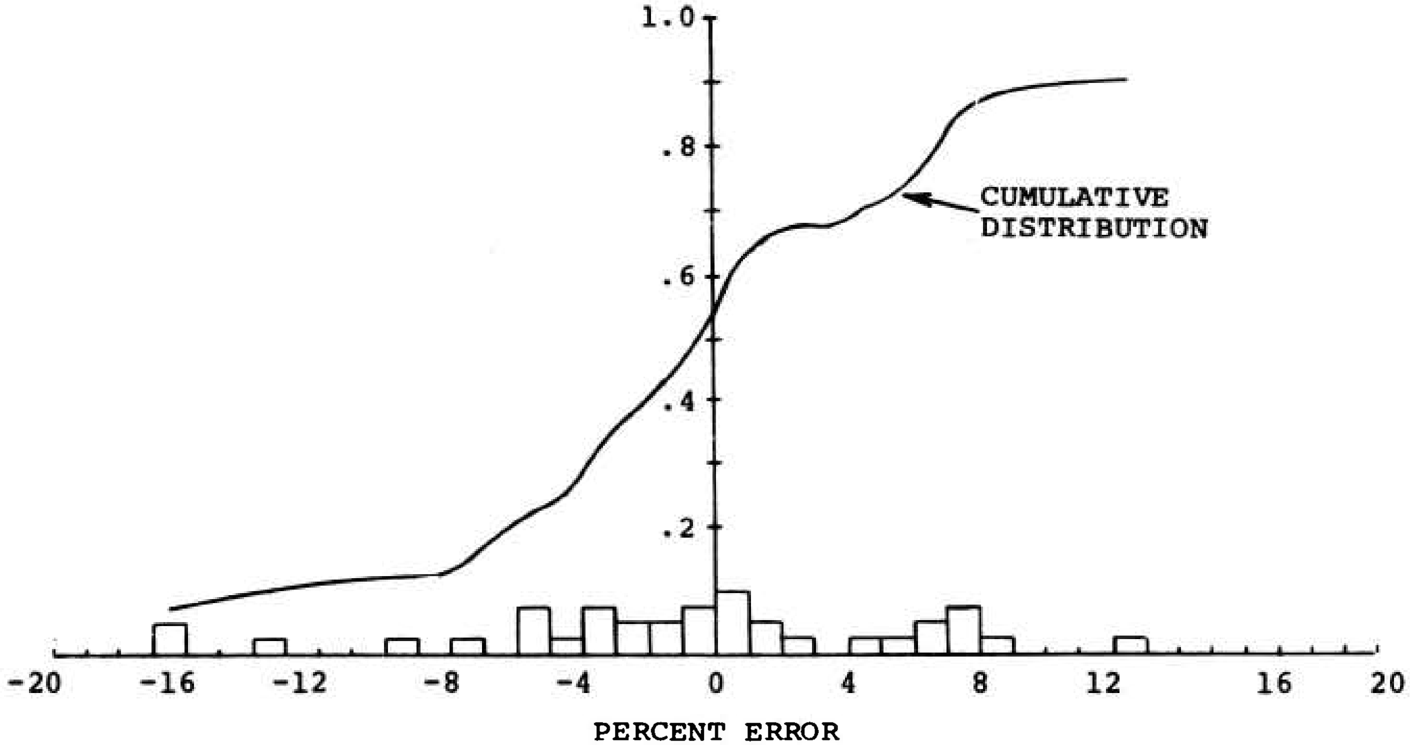

Figure 9. Error Distribution for "Beam-Type" Model Free Element Using Real Acceleration Mobility Data.

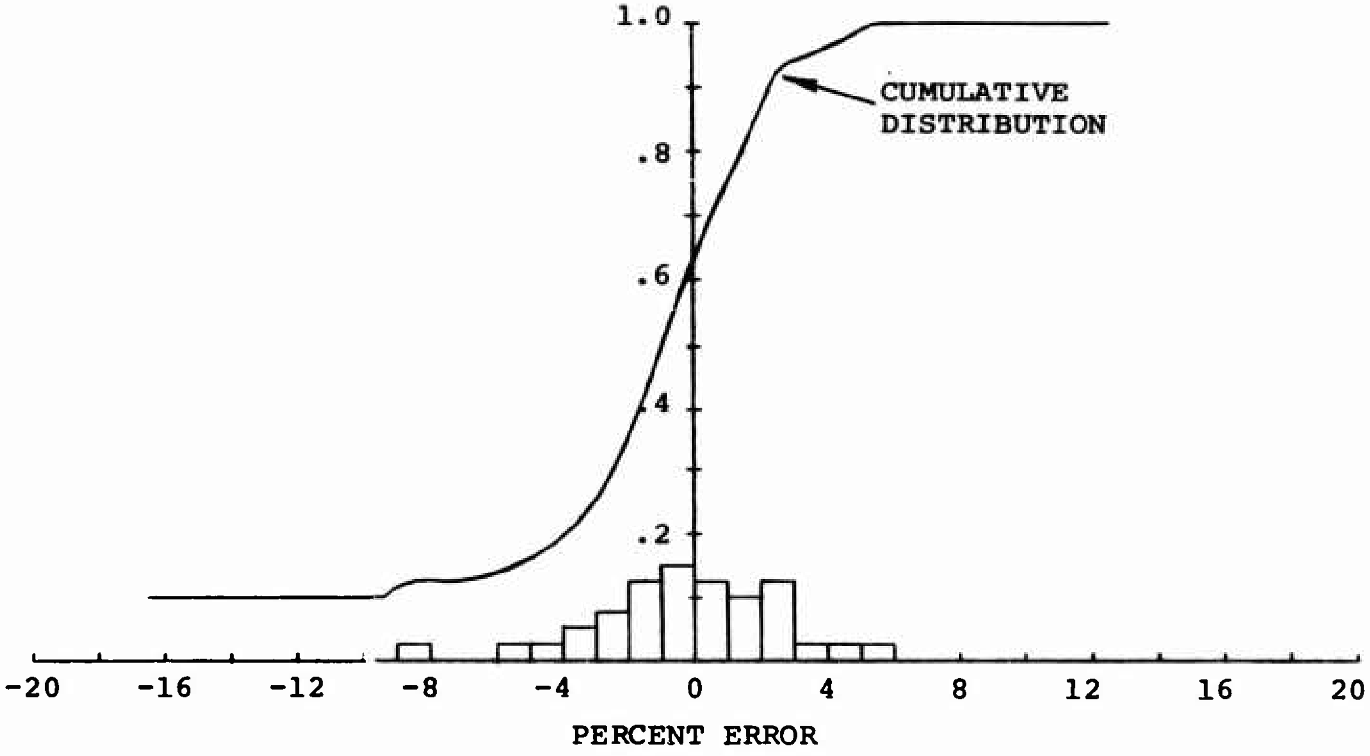

Figure 10. Error Distribution for "Beam-Type" Model Free Element Using Imaginary Acceleration Mobility Data.

CONCLUSIONS

1. The free-body dynamic response of a vehicle can be determined from experimental data taken on a constrained structure.
2. The methodology is insensitive to measurement error using simulated test data subjected to errors that are within the state of the measurement art.
3. The method can eliminate the need for soft suspension systems currently used in helicopter test techniques.
4. By eliminating the effect of the uncertain characteristics of the supporting structure, the accuracy and validity of all dynamic test results will be improved.
5. The method is inherently flexible and can be implemented using displacement, velocity or acceleration data, either linear or angular, in two or three directions at one geometrical point.
6. The theory was found to be experimentally practical and numerically sound.

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APPENDIX
COMPUTER PROGRAM

A digital computer program was generated to implement the theory presented in this report. The program was written for the IBM 360/40 operating system using FORTRAN IV language. A flow chart delineating the program logic is shown in Figure 11. A description of the input cards and a program source listing are included in this appendix.

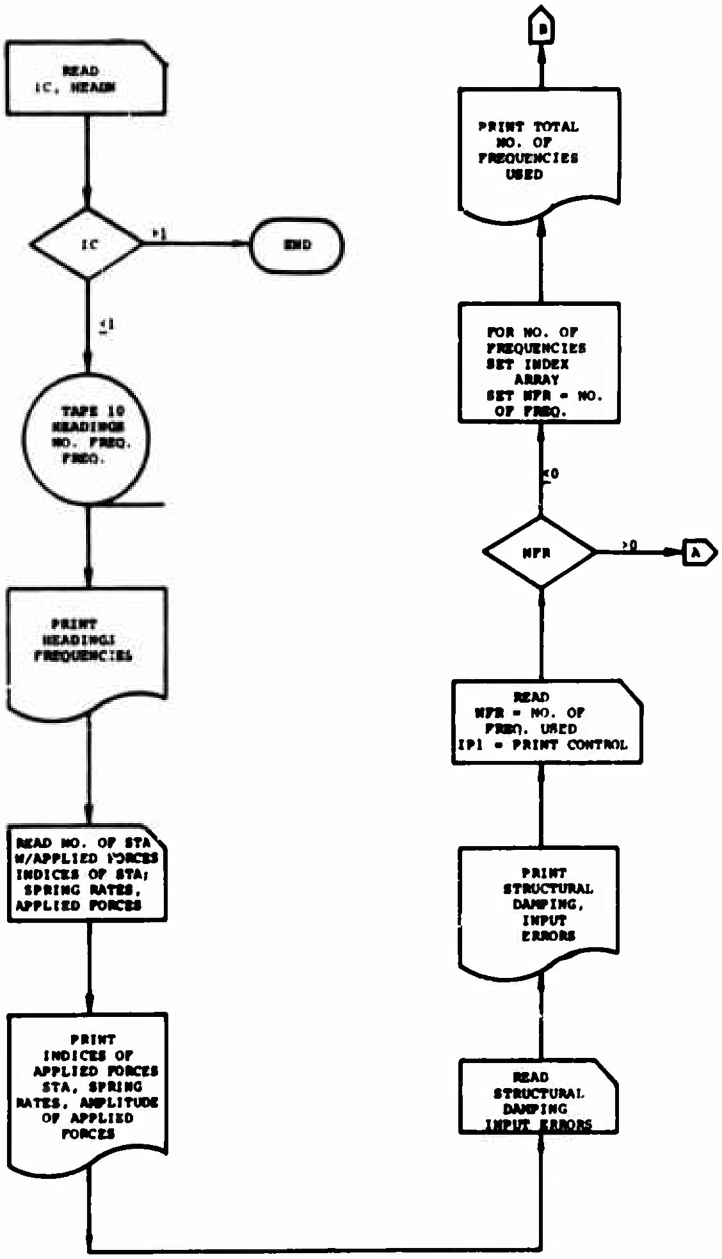

Figure 11. Computer Program Flow Chart.

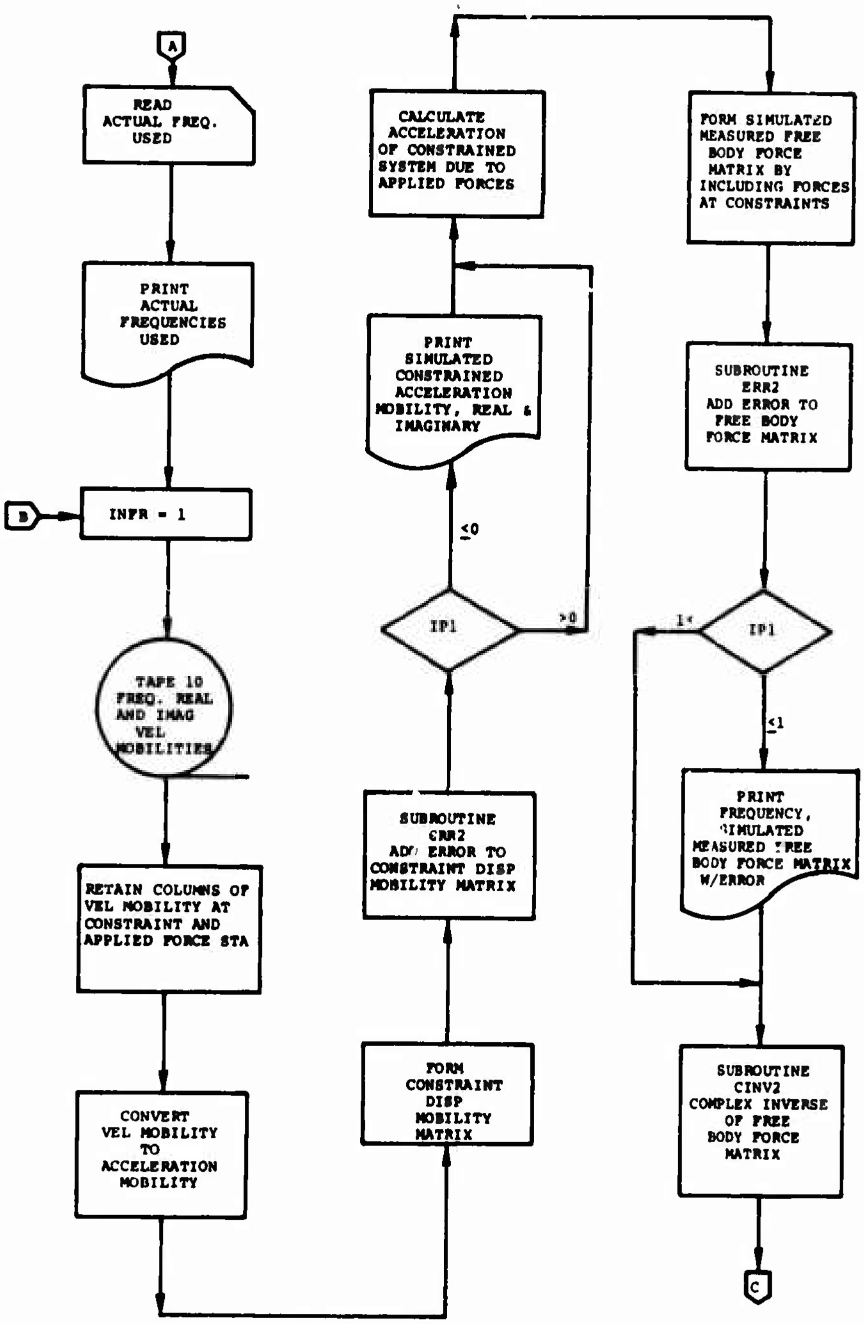

Figure 11 - Continued.

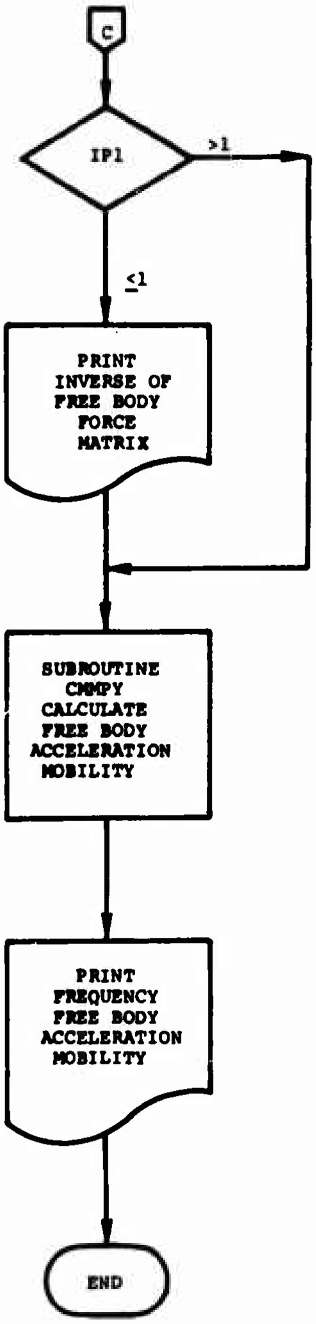

Figure 11 - Concluded.

DESCRIPTION OF INPUT CARDS

Note: All integer variables must be right justified with no decimal point.

Tape, Card Reader and Printer Assignments

1	Card Reader
3	Printer
10	Tape assignment. Contains mobility data, with zero error, for specified frequencies.

All input data must be in the following units:

Mass - lb-sec^2/in.

Stiffness - lb/in.

Frequencies - Hz

PROGRAM FRESIM
FREE-BODY TEST METHOD SIMULATION

Card 1	Columns	1	IC	Program Control
		2-80	HEADN	Heading
Card 2		1-5	NS	No. of stations with applied force.
		5-10	INDS	The indices of the stations at which the forces are applied, including all constraint stations (FORMAT I5, 7I10, Maximum of 8).
Card 3		1-80	SPR	Spring rates at the above stations, if at constraints, otherwise zero (FORMAT 8F10.0, Maximum of 8).
Card 4		1-80	FAMP	Amplitude of applied forces at the above stations (FORMAT 8F10.0, Maximum of 8).
Card 5		1-10	G	Structural damping coefficient
		11-20	PCT	Random error applied to mobility amplitude, uniform between - and + PCT* element amplitude.
		21-30	PCTB	Bias error applied to mobility amplitude PCTB* element amplitude.
		31-40	PHE	Random error in degrees applied to mobility phase angle. Uniform between - PHE and + PHE.
		41-50	FPCT	Random error applied to force amplitude, uniform between - and + FPCT* Force amplitude.
		51-60	FPCTB	Bias error applied to force amplitude FPCTB* Force amplitude.
		61-70	FPHE	Random error in degrees applied to force phase angle. Uniform between - PHE and + PHE.
		71-80	IZ	Random number seed.

Card 6	Column	1-5	NFR	Number of frequencies to be used from Tape 10 (≤100; if 0 use all frequencies on Tape 10).
		6-10	IP1	= 0 Print constrained displacement mobility; force matrix; free displacement mobility. = 1 Print force matrix; free displacement mobility. = 2 Print free displacement mobility.
Card 7		1-80	INDX	Indices of frequencies to be used from Tape 10. Indices must be in ascending order. Five columns per value, 16 values per card (FORMAT 16I5).

```
C         FREE BODY TEST METHOD SIMULATION
C
C                 USES MOBILITY DATA FROM 'XACT'
C                 SIMULATES SHAKING AT EACH SUPPORT TO OBTAIN FREE BODY
C
C
C
      INTEGER HEADN(20),HEAD(20),HT(7),INDS(8),INDX(100)
      REAL HZ(100),YR(20,21),YI(20,21),SPR(8),FAMP(8),YDR(20,8),
     A  YDI(20,8),CYDR(8,9),CYDI(8,9),FINVR(8,9),FINVI(8,9),
     B   FYR(20,8),FYI(20,8)
C                 INPUT HEADING, XACT TAPE HEADING
  100 READ (1,110) IC,HEADN
  110 FORMAT ( I1,A3,19A4)
      IF (IC.NE.0) REWIND 10
      IF (IC.GT.1) CALL EXIT
      READ (10) HT,HEAD,NF,ND,(HZ(I),I=1,NF)
      WRITE (3,120) HEADN,HT,HEAD,ND,(HZ(I),I=1,NF)
  120 FORMAT ('1'/T10,17(' FREE ')//T25,A3,19A4//T10,'TAPE HEADING'/
     A T25,7A4/T25,A3,19A4/T25,I2,' DEGREES OF FREEDOM'/T25,'FREQUENCIES
     B(HZ) ON TAPE'//(T10,10F10.2))
C                 INPUT SUPPORT SPRINGS AND APPLIED FORCES
      READ (1,130) NS,INDS,SPR,FAMP
  130 FORMAT (2I5,7I10/(8F10.0))
      WRITE (3,140) (INDS(I),I=1,NS)
      WRITE (3,150) ( SPR(I),I=1,NS)
      WRITE (3,160) (FAMP(I),I=1,NS)
  140 FORMAT (/T10,'CONSTRAINTS AND FORCES'//T10,'STATION   ',5X,8I10)
  150 FORMAT (T10,'SPRING',10X,8F10.0)
  160 FORMAT (T10,'APPLIED FORCE   ',8F10.0)
C                 INPUT ERRORS, FREQUENCIES
      READ (1,170) G,PCT,PCTB,PHE,FPCT,FPCTB,FPHE,IZ
  170 FORMAT (    7F10.0,I10)
      WRITE (3,180) G
  180 FORMAT (/T10,'STRUCTURAL DAMPING',F8.4)
      IX=IZ*2+1
      WRITE (3,190) PCT,PCTB,PHE,IZ,FPCT,FPCTB,FPHE
  190 FORMAT (/T10,'MAX RAND ERROR =',F6.3,'   BIAS ERROR =',F6.3,' OF E
     ALEMENTS,  MAX RAND PHASE ERROR =',F5.2,' DEG.     SEED =',I10/
     B T10,'FORCE ERRORS',4X,F6.3,15X,F6.3,37X,F5.2/)
      READ (1,200) NFR,IP1
  200 FORMAT (16I5)
      IF (NFR.GT.0) GO TO 230
      DO 210 I=1,NF
  210 INDX(I)=I
      NFR=NF
      WRITE (3,220) NF
  220 FORMAT (/T10,'ALL ',I3,' FREQUENCIES USED')
      GO TO 250
  230 READ (1,200) (INDX(I),I=1,NFR)
      WRITE (3,240) (HZ(INDX(I)),I=1,NFR)
  240 FORMAT (/T10,'FREQUENCIES USED'//(T10,10F12.4))
  250 INFR=1
C                            START MAIN LOOP
      DO 410 L=1,NF
```

```
      READ (10) FREQ,((YR(I,J),YI(I,J),I=1,ND),J=1,ND)                  1FRE  56
      IF(L.NE.INDX(INFR)) GO TO 410                                     1FRE  57
C                   ELIMINATE COLUMNS AND CONVERT TO ACCL MOB           1FRE  58
      OMR=HZ(L)*6.2832                                                  1FRE  59
      OMRS=OMR*OMR                                                      1FRE  60
      DO 260 I=1,ND                                                     2FRE  61
      DO 260 J=1,NS                                                     3FRE  62
      YDR(I,J)=-YI(I,INDS(J))*OMR                                       3FRE  63
  260 YDI(I,J)= YR(I,INDS(J))*OMR                                       3FRE  64
C                   FORM CONSTRAINT DISP   MOB MATRIX ANC ADD ERROR     1FRE  65
      DO 270 I=1,NS                                                     2FRE  66
      DO 270 J=1,NS                                                     3FRE  67
      CYDR(I,J)=-YDR(INDS(I),J)/OMRS                                    3FRE  68
  270 CYDI(I,J)=-YDI(INDS(I),J)/OMRS                                    3FRE  69
      CALL ERR2 (YDR,YDI,PCT,PCTB,PHE,ND,NS,IX,20)                      1FRE  70
      IF (IP1.GT.0) GO TO 300                                           1FRE  71
      WRITE (3,280)   HZ(L)                                             1FRE  72
  280 FORMAT ('1',T20,'SIMULATED MEASURED CONSTRAINED ACCELERATION MOBIL1FRE  73
     AITY, REAL PART    F = ',F10.2,' HZ'/)                             1FRE  74
      CALL MOUT3 (YDR,ND,NS,20)                                         1FRE  75
      WRITE (3,290)                                                     1FRE  76
  290 FORMAT (      /T50,'IMAGINARY PART'/)                             1FRE  77
      CALL MOUT3 (YDI,ND,NS,20)                                         1FRE  78
C                 ACCELERATIONS OUE TO FORCES AND FORCES AT CONSTRAINTS 1FRE  79
C                                 WITH ERRORS ON TOTAL FORCES           1FRE  80
  300 DO 320 J=1,NS                                                     2FRE  81
      F=FAMP(J)                                                         2FRE  82
      DO 310 I=1,ND                                                     3FRE  83
      YDR(I,J)=YDR(I,J)*F                                               3FRE  84
  310 YDI(I,J)=YDI(I,J)*F                                               3FRE  85
      DO 320 I=1,NS                                                     3FRE  86
      YYYR=(-CYDR(I,J)+G*CYDI(I,J))*F*SPR(I)                            3FRE  87
      YYYI=(-CYDI(I,J)-G*CYDR(I,J))*F*SPR(I)                            3FRE  88
      CYDR(I,J)=YYYR                                                    3FRE  89
  320 CYDI(I,J)=YYYI                                                    3FRE  90
      DO 330 I=1,NS                                                     2FRE  91
  330 CYDR(I,I)=CYDR(I,I)+FAMP(I)                                       2FRE  92
      CALL ERR2 (CYDR,CYDI,FPCT,FPCTB,FPHE,NS,NS,IX,8)                  1FRE  93
      IF (IP1.GT.1) GO TO 360                                           1FRE  94
      WRITE (3,340)   HZ(L)                                             1FRE  95
  340 FORMAT ('1',T20,'SIMULATED MEASURED FREE BODY FORCE MATRIX, REAL P1FRE  96
     AART      F = ',F10.2,' HZ'/)                                      1FRE  97
      CALL MOUT3 (CYDR,NS,NS,8)                                         1FRE  98
      WRITE (3,350)                                                     1FRE  99
  350 FORMAT (     //T50,'IMAGINARY PART'/)                             1FRE 100
      CALL MOUT3 (CYDI,NS,NS,8)                                         1FRE 101
C                                 INVERT FORCE MATRIX   DIM 8 X 9       1FRE 102
  360 CALL CINV2 (CYDR,CYDI,NS,FINVR,FINVI,8)                           1FRE 103
      IF (IP1.GT.1) GO TO 380                                           1FRE 104
      WRITE (3,370)                                                     1FRE 105
  370 FORMAT (//T40,'INVERSE OF FORCE MATRIX, REAL PART'/)              1FRE 106
      CALL MOUT3 (FINVR,NS,NS,8)                                        1FRE 107
      WRITE (3,350)                                                     1FRE 108
      CALL MOUT3 (FINVI,NS,NS,8)                                        1FRE 109
C                   FREE BODY MOBILITY    FY = YD * FINV                1FRE 110
```

```
380 CALL CMMPY (YDR,YDI,FINVR,FINVI,ND,NS,NS,FYR,FYI,20,8,20)
    WRITE (3,390) HZ(L),(INDS(I),I=1,NS)
390 FORMAT ('1',T20,'FREE BODY ACC MOB  MATRIX, REAL PART, F =',F10.2
   A ,' HZ'/T5,10I12)
    CALL MOUT3 (FYR,ND,NS,20)
    WRITE (3,290)
    CALL MOUT3 (FYI,ND,NS,20)
400 INFR=INFR+1
    IF (INFR.GT.NFR) GO TO 100
410 CONTINUE
    REWIND 12
    GO TO 100
    END
```

```
      SUBROUTINE CMMPY (A,B,C,D,N1,N2,N3,E,F,NRA,NRC,NRE)
C
C        COMPLEX MATRIX MULT     OBJECT TIME DIMENSIONS
C        E + I*F =(A + I*B)*(C + I*D)       I = SQR(-1)
C        NRA IS NO OF ROWS IN DIMENSION OF A,B
C        NRC IS NO OF ROWS IN DIMENSION OF C,D
C        NRE IS NO OF ROWS IN DIMENSION OF E,F
C        A,B ARE N1 X N2      C,D ARE N2 X N3      E,F ARE N1 X N3
C
      REAL A(NRA,1),B(NRA,1),C(NRC,1),D(NRC,1),E(NRE,1),F(NRE,1),
     A  G(20,20)
      CALL MMPY2 (A,C,N1,N2,N3,E,NRA,NRC,NRE)
      CALL MMPY2 (B,D,N1,N2,N3,G,NRA,NRC,20)
      DO 100 I=1,N1
      DO 100 J=1,N3
  100 E(I,J)=E(I,J)-G(I,J)
      CALL MMPY2 (A,D,N1,N2,N3,F,NRA,NRC,NRE)
      CALL MMPY2 (B,C,N1,N2,N3,G,NRA,NRC,20 )
      DO 110 I=1,N1
      DO 110 J=1,N3
  110 F(I,J)=F(I,J)+G(I,J)
      RETURN
      END
```

```
      SUBROUTINE CINV2 (A,B,N,C,D,NR)                                   CIN   1
C                                                                       CIN   2
C        COMPLEX INVERSE  C + I*D = INV OF A + I*B        I = SQR (-1)  CIN   3
C        OBJECT TIME DIMENSIONS   NR IS NO OF ROWS IN DIM. OF A,B,C,D   CIN   4
C        THEY MUST BE DIMENSIONED WITH EXTRA COLUMN IN CALLING PROG.    CIN   5
C                                                                       CIN   6
C        A IS ASSUMED NON-SINGULAR               N MUST BE .LE. 20      CIN   7
C        USES  INVRS2, MMPY2                                            CIN   8
C                                                                       CIN   9
      REAL A(NR,1),B(NR,1),C(NR,1),D(NR,1),E(20,21)                     CIN  10
      CALL INVRS2 (C,N,A,NR)                                            CIN  11
      CALL MMPY2 (C,B,N,N,N,E,NR,NR,20)                                 CIN  12
      CALL MMPY2 (B,E,N,N,N,D,NR,20,NR)                                 CIN  13
      DO 100 I=1,N                                                     1CIN  14
      DO 100 J=1,N                                                     2CIN  15
      E(I,J)=-E(I,J)                                                   2CIN  16
  100 D(I,J)=D(I,J)+A(I,J)                                             2CIN  17
      CALL INVRS2 (C,N,D,NR)                                            CIN  18
      CALL MMPY2 (E,C,N,N,N,D,20,NR,NR)                                 CIN  19
      RETURN                                                            CIN  20
      END                                                               CIN  21
```

```
      SUBROUTINE MMPY2 (A,B,N1,N2,N3,C,NRA,NRB,NRC)                     MY2
C        OBJECT TIME DIMENSIONS   NRA IS NO OF ROWS IN A IN DIMENSION   MY2
C        STATEMENT IN MAIN PROG. ETC FOR NRB, NRC.                      MY2
C                             C = A * B                                 MY2
C                             A (N1 X N2)   B (N2 X N3)   C (N1 X N3)   MY2
C                                                                       MY2
      REAL A(NRA,1),B(NRB,1),C(NRC,1)                                   MY2
      DO 100 I=1,N1                                                    1MY2
      DO 100 J=1,N3                                                    2MY2
      C(I,J)=0.                                                        2MY2
      DO 100 K=1,N2                                                    3MY2
  100 C(I,J)=C(I,J)+A(I,K)*B(K,J)                                      3MY2
      RETURN                                                            MY2
      END                                                               MY2
```

```
      SUBROUTINE RANDU (IX,IY,YFL)                                      RAN    1
C                        THIS SUBROUTINE IS FROM  SSP VERS. II          RAN    2
      IY=IX*65539                                                       RAN    3
      IF(IY)100,110,110                                                 RAN    4
  100 IY=IY+2147483647+1                                                RAN    5
  110 YFL=IY                                                            RAN    6
      YFL=YFL*.4656613E-9                                               RAN    7
      RETURN                                                            RAN    8
      END                                                               RAN    9
```

```
      SUBROUTINE ERR2 (A,B,PCT,PCTB,PHE,N1,N2,IX,NR)                    ERR    1
C             OBJECT TIME DIMENSIONS                                    ERR    2
C             EACH ELEMENT OF A COMPLEX MATRIX, A + I*B,IS MODIFIED TO  ERR    3
C             INCLUDE A SMALL PHASE ERROR, PNE (DEG),  A BIAS ERROR,    ERR    4
C             PCTB (RATIO) ON AMPLITUDE, AND A UNIFORM RANDOM ERROR     ERR    5
C             HAVING A +/- MAXIMUM OF PCT (RATIO) ON AMPLITUDE.         ERR    6
C             THE PHASE ERROR IS ALSO RANDOMLY DISTRIBUTED              ERR    7
C     TE   NO SYMMETRIZATION IS PERFORMED                               ERR    8
C                                                                       ERR    9
C        USES RANDU                                                     ERR   10
      DIMENSION A(NR,1),B(NR,1)                                         ERR   11
      IF(PCT ) 120,100,120                                              ERR   12
  100 IF(PCTB) 120,110,120                                              ERR   13
  110 IF(PHE) 120,140,120                                               ERR   14
  120 P=PHE/57.296                                                      ERR   15
      DO 130 I=1,N1                                                    1ERR   16
      DO 130 J=1,N2                                                    2ERR   17
      CALL RANDU (IX,IY,YFL)                                           2ERR   18
      IX=IY                                                            2ERR   19
      E=2.0*P*(YFL-0.5)                                                2ERR   20
      A1=A(I,J)-E*B(I,J)                                               2ERR   21
      B(I,J)=B(I,J)+E*A(I,J)                                           2ERR   22
      A(I,J)=A1                                                        2ERR   23
      CALL RANDU (IX,IY,YFL)                                           2ERR   24
      IX=IY                                                            2ERR   25
      E=1.0+2.0*PCT*(YFL-0.5)+PCTB                                     2ERR   26
      A(I,J)=A(I,J)*E                                                  2ERR   27
  130 B(I,J)=B(I,J)*E                                                  2ERR   28
  140 RETURN                                                            ERR   29
      END                                                               ERR   30
```

```
      SUBROUTINE MOUT3 (A,M,N,NR)                                       MO3    1
C        OBJECT TIME DIMENSIONS    NR IS NO OF DIMENSIONED ROWS         MO3    2
C        N SHOULD NOT EXCEED 20                                         MO3    3
      REAL A(NR,1)                                                      MO3    4
      ID=MINO(N,10)                                                     MO3    5
      WRITE (3,100) (I,I=1,ID)                                          MO3    6
  100 FORMAT (/T5,10I12)                                                MO3    7
      WRITE (3,100)                                                     MO3    8
      DO 110 I=1,M                                                     1MO3    9
  110 WRITE (3,120) I,(A(I,J),J=1,ID)                                  1MO3   10
  120 FORMAT (I5,5X,1P10E12.4)                                          MO3   11
      IF (ID-N) 130,150,150                                             MO3   12
  130 WRITE (3,100) (I,I=11,N)                                          MO3   13
      WRITE (3,100)                                                     MO3   14
      DO 140 I=1,M                                                     1MO3   15
  140 WRITE (3,120) I,(A(I,J),J=11,N)                                  1MO3   16
  150 RETURN                                                            MO3   17
      END                                                               MO3   18
```

```
      SUBROUTINE INVRS2 (A,N,B,NR)
C         OBJECT TIME DIMENSIONS.     ARRAYS SHOULD BE DIMENSIONED
C         WITH ONE MORE COLUMN THAN ROW.        NR SHOULD BE .LE. 20
C     A = INVERSE OF B          B UNDISTURBED
C
      DIMENSION A(NR,1),D(20,21),IROW(21),ICOL(21),B(NR,1)
      DO 100 I=1,N
      DO 100 J=1,N
  100 A(I,J)=B(I,J)
      M=N+1
      DO 110  I=1,N
      IROW(I)=I
  110 ICOL(I)=I
      DO 260 K=1,N
      AMAX= A(K,K)
      DO 130 I=K,N
      DO 130 J=K,N
      IF(ABS( A(I,J))-ABS(AMAX))130,120,120
  120 AMAX= A(I,J)
      IC=I
      JC=J
  130 CONTINUE
      KI=ICOL(K)
      ICOL(K)=ICOL(IC)
      ICOL(IC)=KI
      KI=IROW(K)
      IROW(K)=IROW(JC)
      IROW(JC)=KI
      IF(AMAX) 160,140,160
  140 WRITE (3,150)
  150 FORMAT(' SOLUTION OF EXISTING MATRIX NOT POSSIBLE')
      GO TO 330
  160 DO 170 J=1,N
      E=A(K,J)
      A(K,J)=A(IC,J)
  170 A(IC,J)=E
      DO 180 I=1,N
      E=A(I,K)
      A(I,K)=A(I,JC)
  180 A(I,JC)=E
      DO 210 I=1,N
      IF(I-K) 200,190,200
  190 A(I,M)=1.
      GO TO 210
  200 A(I,M)=0.
  210 CONTINUE
      PVT=A(K,K)
      DO 220 J=1,M
  220 A(K,J)=A(K,J)/PVT
      DO 250 I=1,N
      IF(I-K)230,250,230
  230 AMULT=A(I,K)
      DO 240 J=1,M
  240 A(I,J)=A(I,J)-AMULT*A(K,J)
  250 CONTINUE
```

```
      DO 260 I=1,N                                                      2INV  56
260    A(I,K)=A(I,M)                                                    2INV  57
      DO 290 I=1,N                                                      1INV  58
      DO 270 L=1,N                                                      2INV  59
      IF(IROW(I)-L)270,280,270                                          2INV  60
270   CONTINUE                                                          2INV  61
280   DO 290 J=1,N                                                      2INV  62
290   D(L,J)=A(I,J)                                                     2INV  63
      DO 320 J=1,N                                                      1INV  64
      DO 300 L=1,N                                                      2INV  65
      IF(ICOL(J)-L) 300,310,300                                         2INV  66
300   CONTINUE                                                          2INV  67
310   DO 320 I=1,N                                                      2INV  68
320   A(I,L)=D(I,J)                                                     2INV  69
330   RETURN                                                             INV  70
      END                                                                INV  71
```