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ABSTRACT

Recently, considerable interest has been generated in efficient quadratic assignment problem algorithms as a result of computer-aided design automation system projects. Currently available QAP algorithms can be characterized as being computationally complex and requiring medium to large scale computers for implementation. Computer-aided design applications, however, are frequently centered around small process control computers with limited available memory so that the more sophisticated QAP procedures cannot be used ~~utilized~~. This paper presents a computationally simplified pair-exchange algorithm which has proven to be comparable with the currently available QAP algorithm and which is implementable on a small compute~~d~~r. A C[illegible] FORTRAN 4 [illegible].

I INTRODUCTION

The problem of interest can be described as follows: Given a set $S = (1,\ldots,m)$ of components and a circuit board with component location set $L = (1,\ldots,n)$, $n \geq m$. Determine the assignment of components to locations that will result in the minimum total length of interconnecting wire to electrically satisfy all required circuit connections. Stated in another manner, the problem is to determine the one-to-one mapping of the set S into the set L which will result in the minimum total wire length.

Associated with S is an n^2 interconnection matrix $F = |f_{ik}|, (i,k = 1,\ldots,n)$ with $f_{ik} \geq 0$. Each f_{ik} representing the number of wires connecting components s_i with component s_k. Associated with L is an n^2 distance matrix $D = |d_{jq}|$, $(j,q = 1,\ldots,n)$ with $d_{jq} \geq 0$. Each d_{jq} representing the distance between location j and location q. The length of the interconnecting wires is determined by forming the permuted dot product of F and D for a given mapping of S into L.

There are n! unique one-to-one mappings of S into L, thus the feasible region of the problem's solution space contains n! points.

The assignment of components to locations can be recorded by means of an n^2 permutation matrix $X = |x_{ij}|$, $(i, j = 1,\ldots,n)$ with $x_{ij} = 0$ or 1. Alternately, the mapping can be recorded as a permutation of length n. Thus, $V_i = (v_1, v_2,\ldots,v_n)$ is a permutation of the integers $(1, 2,\ldots,n)$ or component numbers with the position in the permutation designating its assigned location.

Using the terminology from above, the component placement problem can be stated mathematically as follows: Given F and D, find X so as to

minimize $$Z = \sum_i \sum_j \sum_k \sum_q f_{ik}\, d_{jq}\, x_{ij}\, x_{kq} \qquad (1)$$

for $(i, j, k, q = 1,\ldots,n)$*

subject to $$\sum_j x_{ij} = 1 \quad \text{for } (i = 1,\ldots,n) \qquad (2)$$

and $$\sum_i x_{ij} = 1 \quad \text{for } (j = 1,\ldots,n) \qquad (3)$$

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* With no loss of generality it can be assumed that m = n. If not, then m - r imaginary components may be added with associated $f_{ik} = 0$.

also letting

$$x_{ij} = \begin{cases} 1 & \text{if component } i \text{ is assigned to location } j \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

In some placement problems an initial fixed cost is often incorporated. This cost is represented by the n^2 matrix $C = |c_{ij}|$, $(i, j = 1,\ldots,n)$ with $c_{ij} \geq 0$. Here c_{ij} is the cost of assigning component i to location j and is independent of any other component or location. This cost is exactly the cost considered in the linear assignment problem. If the fixed assignment cost is to be included then the objective function becomes:

$$\text{minimize} \sum_i \sum_j c_{ij}\, x_{ij} + \sum_i \sum_j \sum_k \sum_q f_{ik}\, d_{jq}\, x_{ij}\, x_{kq} \tag{5}$$

The matrix C is frequently used to influence the final placement of the components. For example, if component i is constrained from being placed on location j then the coefficient c_{ij} would be set artificially high so as to discourage this possibility. Equation (5) along with equations (2), (3), and (4) represent a more general formulation of the placement problem which has come to be known as the <u>quadratic assignment problem</u>.

The quadratic assignment problem has been extensively investigated in recent years and several workable algorithms have been proposed, but as yet, no optimal algorithm has been produced. The available algorithms generate "good", but sub-optimal, solutions with the more successful procedures employing tree-search techniques, also known as branch and bound*. In general, the available algorithms are computationally complex and require medium to large scale computers, for implementation, depending of course, on the size of problem being solved.

Recently, interest in the quadratic assignment problem and its variate the component placement problem, has been generated by computer-aided design automation engineers. This application typically centers around a small process control computer driving automatic drafting equipment which is capable of producing photo masters for electronic systems directly from master wiring lists. These installations do not have the luxury of the large scale memories generally associated with

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*Refer to references [1], [3] and [5] for information on tree-search algorithms.

scientific computation centers and therefore many of the sophisticated placement algorithms cannot be implemented.

This paper presents a computationally simplified pair-exchange algorithm which has proved to be competitive with the more complex algorithms. A comparative analysis is made between the pair-exchange algorithm and the most efficient tree-search algorithm found to date, the N-step, 2-variable Search Algorithm [1]. The evaluation is based on test results obtained using the Steinberg [6] test problem.

II A COMPUTATIONALLY SIMPLIFIED PAIR-EXCHANGE ALGORITHM FOR THE QUADRATIC ASSIGNMENT PROBLEM

The concept of a pair-exchange algorithm is perhaps the least sophisticated approach to solving the quadratic assignment short of complete enumeration*. The procedure which begins with an initial starting solution, is simply to position swap component pairs always seeking to identify exchanges which produce reductions in the current criterion function value. The initial solution can be obtained either at random or through the use of another placement algorithm. One

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*A pair-exchange procedure has also been used by Graves [4] possibility, for example, is to sort components in ascending order according to link density and to sort locations in descending order according to interlocation distance sums. The components can then paired according to this ordering.

The trick in designing a pair-exchange algorithm is the manner in which the criterion function value is evaluated following each pair-exchange. One option, for example, would be to evaluate the expression given by equation 1, a task requiring N^4 multiply operations. However, when the number of iterations required in pair-exchanging is considered, the use of equation 1 would clearly be prohibitive in terms of computational costs.

It has been demonstrated in reference [1] that the criterion function of equation 1 could be transformed by using efficient indexing schemes and through the construction of a symmetric F matrix. This equation resulting from these simplifications is as follows:

$$\text{minimize } Z = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} F_{ij}\, d_{\ell(i)\ell(j)}^{*} \qquad (6)$$

---

[*] read $\ell(i)$ as the location of i

The transformation reduces the number of multiplications and additions required to evaluate the objective function from n^4 to $((n^2 - n)/2)$. Although this is a substantial improvement, it is still prohibitive for practical application in pair-exchanging.

Fortunately, further simplification is possible as it is only necessary to compute those terms in equation 6 that are affected by the pair swap. Thus the computational requirements are further reduced to 4(n-1) multiplications and additions. The significance of these computational simplifications can be observed by considering the number of multiplications required in each case for one iteration of the Steinberg test problem (N = 36).

Case	Equation	Multiplications Required per Iteration
1	N^4	1,679,616
2	$((N^2 - N)/2)$	630
3	$4\ (N - 1)$	140

A second consideration in pair-exchanging is the swapping strategy. An orderly procedure is required and all possible exchanges must be considered at some point in the procedure. The pair exchange algorithm to be described always beings with the components occupying the first and second positions of the component vector as the initial candidates for exchange. Sequential indexing is used and pairs are consecutively evaluated for possible swapping until a criterion function value reduction is achieved. The associated candidate swap is then made permanent and the indexing continues. When a complete pass through all indexing combinations is made without encountering a reduction producing pair-exchange, the procedure is terminated. The current value of the criterion function is kept along with the associated component and location vectors as the best solution. The solution found by pair-exchanging is a local optima which cannot be improved upon by continued pair-exchanging. Additional improvement can only be obtained by backtracking to a different starting point or by considering triplet exchanges, etc. While no statement concerning global optimality can be made regarding the outcome of the pair-exchange procedure, the solutions have generally been found to be near-optimal and competitive with the solutions generated by the more complex tree-search algorithms*.

III DETAILED PAIR-EXCHANGE PROCEDURE

The detailed procedure for the pair-exchange procedure is as follows:

Step 1. Obtain an initial starting solution either through stochastic enumeration or by other means. Evaluate the criterion function value Z(S'(L')) where S' is the component vector and L' the location vector of the initial placement. That is, the component occupying position 1 of S' is considered to be assigned to the location specified in position 1 of L', etc. Set switch indicator $I_{sw} = 0$.

Step 2. Initialize the indexing parameters which point to the current candidate component pair (i, j) to the starting pair (1, 2).

Step 3. Compute the criterion function value decrement associated with the candidate pair (i, j) prior to executing the swap. For component i

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*See reference [1] for an assessment of the question of near-optimal solutions to the quadratic assignment problem.

$$\Delta_i = \sum_{k=1}^{n} F(S'(i),S'(k))*D(L'(i),L'(k))$$

For component j

$$\Delta_j = \sum_{k=1}^{n} F(S'(j),S'(k))*D(L'(j),L'(k))$$

Now perform the swap

$T = S'(i)$

$S'(i) = S'(j)$

$S'(j) = T$

Compute the criterion function value increment associated with the new positioning of the candidate pair (i,j).

For component i

$$I_i = \sum_{k=1}^{n} F(S'(i),S'(k))*D(L'(i)L'(k))$$

For component j

$$I_j = \sum_{k=1}^{n} F(S'(j),S'(k))*D(L'(j),L'(k))$$

Compute Z_{swap} the criterion function value for the candidate swap pair (i,j).

$$Z_{swap} = Z - \Delta_i - \Delta_j + I_i + I_j$$

Step 4. Determine the effectiveness of the potential swap.

If $Z_{swap} < Z$ then the swap is justifiable.

Set $Z = Z_{swap}$

Set switch indicator $I_{sw} = 1$

Proceed to Step 5.

If $Z_{swap} \geq Z$, the swap is not justifiable.

Return pair (i,j) to their pre-swap positions.

$T = S'(i)$

$S'(i) = S'(j)$

$S'(j) = T$

Discard Z_{swap}.

Step 5. Continue pair candidate indexing.

If $j < n$, set $j = j+1$ and return to step 3.

If $j = n$ but $i < (n - 1)$, set $i = i+1$ and then $j = i+1$. Return to step 3.

If $i = (n - 1)$ and $I_{sw} = 1$, set $i = 1$, $j = 2$ and $I_{sw} = 0$. Return to step 3.

If $i = (n - 1)$ and $I_{sw} = 0$. Proceed to step 6.

Step 6. Terminate swapping procedure. The current value of Z is the best value encountered and is associated with the current component vector S' and location vector L'.

IV EXPERIMENTAL RESULTS

The pair-exchange algorithm has been tested using the problem originally presented by Steinberg in reference [6]. A CDC 3800 computer was used with programming being performed in FORTRAN IV. (A program listing is contained in the appendix).

The Steinberg problem is reportedly an actual computer backboard layout problem for a UNIVAC computer and is concerned with finding the optimal assignment of 34 component modules to 36 backboard locations to minimize the total interconnecting wire length among components.

The F matrix for the components in the Steinberg problem is given in figure 1. A symmetrical matrix was formed using

$$\sum_{i} (f_{ij} + f_{ji}) \qquad (i,j = 1,\ldots,n)$$

and only the part above the diagonal was retained. This particular F matrix is sparse, that is, about 70% zeros. The number of interconnecting wires varies from 0 to 316 with mean equal to 15.26. The extreme variation in F creates a large range in potential solution values. For example, the lower bound on all possible solution values is 3001 and the mean

	E 1	E 2	E 3	E 4	E 5	E 6	E 7	E 8	E 9	E 10	E 11	E 12	E 13	E 14	E 15	E 16	E 17	E 18	E 19	E 20	E 21	E 22	E 23	E 24	E 25	E 26	E 27	E 28	E 29	E 30	E 31	E 32	E 33	E 34	Total	↓
E1				2	1	7	9		4	75	7	12	22	7	1					23															170	12
E2							4	16		8			16					6		4															54	6
E3						4	16	20					20							4															64	5
E4					29	5	18	47	23	2	4		48		4					25															207	11
E5						18	12	25			4		25		3					18		3													138	10
E6							4	2		1	23	2	19						2	19															106	12
E7									14	72	7	8	39	8	40	8		8	4	7								28	8						314	19
E8									10	71	2							41									7	8							249	11
E9										14			18																						83	6
E10											11	1	17		1			17		15															305	13
E11												316	33	8	2				8	34			6				10			6					481	16
E12													157	25	4			1							22		1								549	11
E13														11	6			6		5	8	3	10				9	11	2			1			486	22
E14															3			1	1	21		1		2			5			3	2	5	5	4	112	17
E15																19		2	2	12								7	3						109	15
E16																		6		1															34	4
E17																		40																	40	1
E18																				26															154	11
E19																				13	9		7					27	16	3		20		4	116	13
E20																					11	4	36				16	18	9	10	1	28	6	2	368	26
E21																						36	6		8		2								80	7
E22																											4								51	6
E23																										12	9								86	7
E24																									26		5								33	3
E25																										35	2								93	5
E26																											4								51	3
E27																																			74	12
E28																													10	22	4	6	4	12	157	12
E29																														19	12				79	8
E30																															19	4	5	8	99	10
E31																																	3	13	54	7
E32																																	18	24	106	8
E33																																		20	61	7
E34																																			87	8

Total number of wires: 2625

FIG. 1: COMPONENT INTERCONNECTION MATRIX FOR STEINBERG PROBLEM

solution value is 9378.58. Therefore, the minimum feasible solution is located between these values, that is, 3001 < minimum Z << 9378.58, most likely much closer to 3001 than to 9378.58. On the average, each component is connected to 10 other components with the maximum number being 26 and the minimum 1. The graph which results from considering the components as nodes and the interconnections as links is continuous in that no disjoint subset exists. Therefore, the problem cannot be decomposed into several independent problems of lesser size. A total of 2625 wires connect the components.

The Steinberg backboard has 36 positions arrange to form a 4 x 9 grid as illustrated in figure 2. The distance matrix is constructed by determining the 2-dimensional Euclidean distance using,

$$d_{jq} = \sqrt{(x_j - x_q)^2 + (y_j - y_q)^2}$$

The distance from location 1 to location 36 in figure 2 is, for example,

$$\sqrt{3^2 + 8^2} \sim 8.54$$

units.

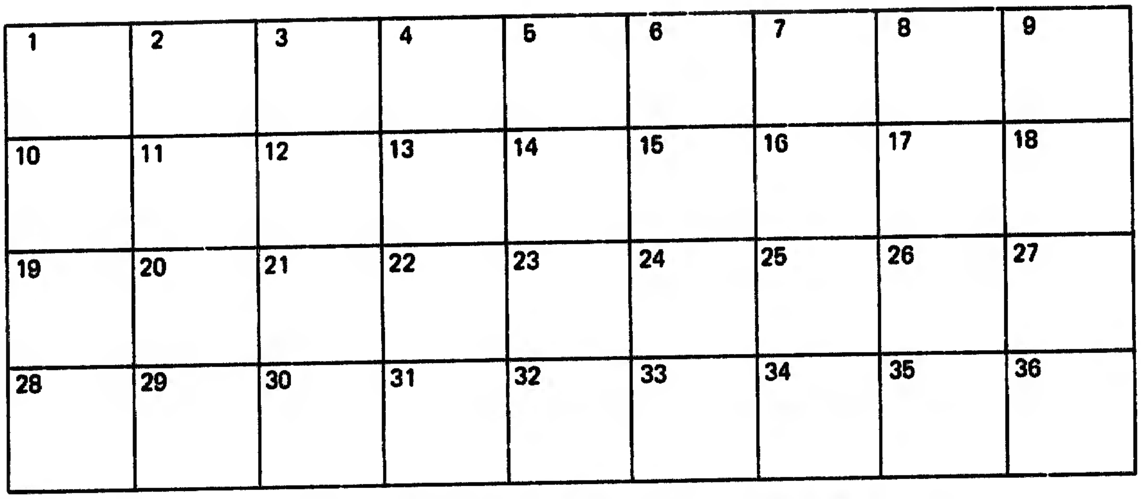

FIG. 2: 4 x 9 CIRCUIT BOARD CONFIGURATION

A summary of previously published Steinberg test problem results is presented in table 1.

TABLE 1

SUMMARY OF PUBLISHED STEINBERG PROBLEM RESULTS

Algorithm	Solution	Execution Time
1. Gilmore [8]	4547.54	60 seconds
2. Graves & Whinston [3]	4612.27	1.5 seconds* IBM 360/91
3. Graves [4]	4186.__	13.4 seconds IBM 360/91
4. Graves [4]	4176.__	30.5 seconds IBM 360/91
5. Heider [1]	4419.49	38 seconds CDC 3800
6. Heider [2]	4184.57	60 seconds CDC 3800
7. Gaschutz & Ahrens [7]	4141.94	several minutes

*Equivalent CDC 3800 execution time is 35 seconds.

A number of test runs were conducted using the pair-exchange algorithm with different starting solutions obtained in a variety of ways. The resultant solutions along with the associated execution times are presented in table 2.

TABLE 2

PAIR-EXCHANGE ALGORITHM TEST RESULTS

Run	Starting Solution	Selection Process	Pair-exchange Solution	Execution Time (CDC 3800)
1	6223.36	Sorting & pre-ordering	4488.38	13 seconds
2	11116.95	Single random placement	4794.99	22
3	8500.99	Single random placement	4451.85	18
4	6923.06	Best of 100 random placements	4341.13	22*
5	7750.25	Best of 100 random placements**	4346.13	27*
6	6923.06	Best of 1000 random placements	4341.13	65*
7	4612.27	Graves & Whinston [3]	4301.01	17
8	4344.98	Graves & Whinston [3]	4257.88	8
9	4419.49	Heider-N-Step, 2-variable Search Algorithm [1]	4236.36	11
10	4141.94	Gaschutz & Ahrens [7]	4138.72***	8

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* includes time to select starting solution

** same random number seed as in run 4

*** best solution of record for the Steinberg test problem

V SUMMARY AND CONCLUSIONS

1. Quadratic assignment problem solutions generated with the pair-exchange algorithm are comparable to those generated with the more sophisticated algorithms including the tree-search or implicit enumeration procedures.

2. Computational requirements in terms of machine capacity and execution time are, however, much less than those required by the traditional quadratic assignment problem algorithms.

3. The pair-exchange algorithm can be used effectively to improve the solutions generated by other algorithms. This strategy has resulted in the location of the best Steinberg problem solution recorded to date (4138.72). This solution was obtained by pair-exchanging the Gaschutz & Ahrens solution (4141.94) which was the previous record solution.

4. One possible disadvantage with the pair-exchange algorithm is that the solutions generated are not predictable or reproducible given different initial starting solutions. However, for applications requiring only a good solution obtained at minimum cost this should not be a problem.

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APPENDIX: FORTRAN LISTING OF PAIR-EXCHANGE ALGORITHM

```
      SUBROUTINE SWITCH (F,D,S,L,CSMALL,N)
      DIMENSION F(100,100),D(100,100)
      INTEGER S(100),L(100)
      WRITE (6,20)
20    FORMAT (*1*)
      WRITE (6,19)
19    FORMAT (//*  ENTER PAIR EXCHANGE ALGORITHM*)
      WRITE (6,21)
21    FORMAT (//*  SWITCH LIST*)
      WRITE (6,22)
22    FORMAT (////)
      IC=1
      JC=2
      C=CSMALL
1     ISW=0
2     SUM1=0.
      SUM2=0.
      SUM3=0.
      SUM4=0.
      DO 3 I=1,N
      SUM1=SUM1+F(S(IC),S(I))*D(L(IC),L(I))
3     SUM2=SUM2+F(S(JC),S(I))*D(L(JC),L(I))
      C=C-SUM1-SUM2
      IA=S(IC)
      S(IC)=S(JC)
      S(JC)=IA
      DO 4 I=1,N
      SUM3=SUM3+F(S(IC),S(I))*D(L(IC),L(I))
4     SUM4=SUM4+F(S(JC),S(I))*D(L(JC),L(I))
      C=C+SUM3+SUM4
      IF (C.LT.CSMALL) GO TO 7
      IA=S(IC)
      S(IC)=S(JC)
      S(JC)=IA
      C=CSMALL
5     IF (JC+1.GT.N) GO TO 6
      JC=JC+1
      GO TO 2
6     IF (IC+2.GT.N) GO TO 8
      IC=IC+1
      JC=IC+1
      GO TO 2
7     CSMALL=C
      ISW=1
      WRITE (6,11) S(JC),S(IC),C
11    FORMAT (/*  COMPONENT*,I5,*  SWITCHED WITH COMPONENT*,I5,*  NEW SO
     1LUTION IS*,F8.2)
      GO TO 5
8     IF (ISW.EQ.0) GO TO 9
      IC=1
      JC=2
      GO TO 1
9     WRITE (6,10)
10    FORMAT (//*  SWITCH COMPLETED*)
      RETURN
      END
```

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