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A COMPARISON OF THE METHODS USED IN DETERMINING THE SPIN STABILITY OF SMALL CALIBER PROJECTILES

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Ronald Andrew Mlinarchik

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FOREWORD

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Approved

PIO

For the Commandant

James L. Arnett
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ABSTRACT

This paper presents the various methods used in determining the spin stability of projectiles and attempts to familiarize the reader with how typical test facilities function. The origin and early history of aerodynamics are discussed to acquaint the reader with the field of exterior ballistics. This leads to an analysis of the motion of a projectile under the influence of a linear system of forces and moments and a constant gravitational field. The conditions necessary for static and dynamic stability are examined followed by a basic introduction to the testing methods available.

The presentation of each method includes a description of the equipment and operation of the respective facility as well as a discussion of the factors affecting its use and the accuracy to be expected. The methods are then critically compared for their accuracy and utility. Finally, possible alternative methods of analysis are suggested in the hope that they may stimulate areas worthy of further inquiry.

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CHAPTER I

INTRODUCTION TO BALLISTICS

Origin of Exterior Ballistics

Ballistics is the science that deals with the motion of projectiles and their associated discharge machines and propulsion mechanisms. Modern writers divide the subject into three parts described as interior, exterior, and terminal ballistics which are concerned, respectively, with the propulsion, motion, and destructive action of projectiles. The modern theory of exterior ballistics developed from the study of the analytical dynamics of rigid bodies moving under the influence of gravitational and aerodynamic forces.

The origin of exterior ballistics, which concerns itself with the flight performance of projectiles, may be traced back to the times of the ancient civilizations three centuries before Christ where men such as Aristotle, Philon, Uzziah, Empedocles, Philiponos, et al. (Nicolaides, 1970) were the ballisticians, astronomers, and mathematicians of the day. Galileo is generally regarded as the first modern exterior ballistician; however, famous men such as Newton, Torricelli, Robbins, Bernoulli, Euler, Taylor, D'Alembert, Lagrange, Laplace, Legendre, Mach, and Zahm have followed in his steps.

It was Galileo's experiments with cannon balls on inclined planes which led him to the formulation of rigid body motion in three parts, the first concerning uniform motion, the second dealing with accelerated motion, and the third combining the first two to treat the motion of projectiles. These experiments allowed him to deduce that, for any one body, gravity (g) was a constant and was independent of the mass. Having examined uniform horizontal and vertical motion, he combined the two to describe the form of the trajectory of a projectile, which he found to be a parabola. If Galileo had been furnished observed values of gravitational acceleration, the angle of elevation, and the muzzle velocity, he would have had sufficient knowledge to compute the range of a projectile in vacuo. He was well aware of the retardation or drag on a body by the air. He said that drag was a function of weight, velocity, and of the form or shape of the body. From these considerations, he developed the notion of limiting velocity and gave the modern definition for momentum. The reader may not recognize the significance of these accomplishments at this time unless he recalls that Galileo died in 1642, coincidentally the same year another rather noted ballistician was born-Sir Isaac Newton!

The problem of original concern to all the early flight dynamicists was the particle trajectory and subsequent drag forces. This is the classical problem which has rather predominated the study of exterior ballistics over the years. For an excellent history of the theoretical and experimental work achieved by these early investigators, see the last chapter of the book by McShane et al. (1953) given in the list of references following the paper. It is interesting to note that " the modern high speed computer was originally developed to compute the particle trajectory" (Nicolaides, 1970).

Newton had recognized that the drag on a body would be a function of its orientation (or yaw angle) with respect to its direction of motion (the reader who is unfamiliar with aerodynamic terminology may find it helpful to refer to the Glossary at the end of the paper). The discovery of the shape and roughness numbers augmented the conception of the functional relationship existing between drag and the aerodynamic numbers.

Fourier initiated the theory of dimensional analysis from which aerodynamic numbers are derived. One of the most significant aerodynamic numbers discovered was the Reynolds number, which explained the physical property of a gas known as viscosity. Maxwell continued along these lines with the kinetic theory to establish relations between viscosity and various other physical quantities describing gas flow. However, all the theoreticians remained incapable of measuring the drag on a projectile with any accuracy.

During the nineteenth century, the need for accurate measurements of drag became evident as projectile speeds increased. Several methods were employed to determine the retardation coefficient and these results were compared to those obtained by the ballistic pendulum, with little success. The Germans accurately determined the retardation function late in the nineteenth century by the intersected drum method (McShane et al., 1953).

With the advent of rifled artillery, there arose a need for a treatment of the projectile under aerodynamic forces other than the drag. The flight of a spinning shell introduced some interesting peculiarities to the artillerist, one of which was the tendency of some designs to tumble in flight creating large drags which appreciably shortened the range. Other designs followed their trajectories well while still others tended to return to their initial direction at muzzle exit. These three types of motion became known as unstable, stable, and superstable, respectively.

Originally, the tendency of the spin stabilized shell to wobble or even tumble was characterized by the motion of a spinning top. The first shell mentioned suffers from a case of under spin, that is, the static overturning moment is too large for the gyroscopic term resulting in a wobbling motion as a slow spinning football. This condition also causes the shell to fall short, possibly injurying one's own troops in a battlefield situation.

In the third instance, the shell with too much spin, the gyroscopic term overwhelms the static moment and the shell attempts to remain parallel to its original line of fire. For example, if a football is kicked with too much spin, it will actually point its nose toward the sky and hit tail first!

Another phenomenon which concerned the late nineteenth century investigators was the right deflection of a shell fired from a gun with a right-handed twist of rifling. This was first believed to be due to the rotation of the earth for firing in the northern hemisphere, but was found to be too large to be the Coriolis acceleration. Also, it was shown that the deflection of a shell fired with a left-handed spin was indeed to the left, hence it was caused by an aerodynamic effect of spin which became known as drift. The first theoretical treatment of the drift was by Fowler, Gallop, Lock, and Richmond (1919).

Recent Developments

The drag, lift, overturning moment, damping moment, and spin retarding moment appeared in the equations of Fowler et al. (1919). This lead to the effective use of wind tunnel measurements of aerodynamic forces and torques in planning the design of a shell. The linear theory was developed and perfected; first, by yaw card firings and, later, by the development of the Aeroballistic Range.

Wind tunnel measurements of drag, lift, and overturning moment were examined extensively during the beginning of the twentieth century. It was noted that the equations of Fowler et al. included the drag, lift, overturning moment, damping moment, and spin retarding moment; they also found solutions which held for short distances along the trajectory. Solutions to Fowler's equations applicable to special cases of projectile motion were obtained by Kent and Hitchcock between World War I and World War II (Hitchcock, 1947).

During 1942, John Synge introduced a complete force system to describe projectiles with small yaw and velocity. The groundwork for this was the recognition of a new force not noted by earlier ballisticians; it has been called the pitching force and has been measured experimentally (Kelly et al., 1944).

After an accurate method of determining the loss of spin was developed, Kent and Charters derived a relation between the spin retarding torque and the drag due to skin friction (Charters, 1947). Following World War II, it was Charters who made the major contribution to exterior ballistics by designing, developing, and perfecting the Aeroballistic Range.

The range permits the determination of the complete motion of the projectile by means of spark photographs; it allowed the first accurate measurements of projectile performance and the determination of the magnus and damping moments.

Problem Definition

The present literature in this field abounds with discussions of and formulae for assessing the gyroscopic stability of military projectiles, including small caliber bullets and artillery shells. The methods presented include the use of spark-shadowgraph photography, wind tunnel measurements, and yaw card firings. The standard calculations range from the elementary consideration of the precessional period to elegant computer analyses involving the location of the center of pressure and overturning moment.

The essential shortcomings among the various sources (according to the U.S. Army Munitions Command) are as follows:

1. The fragmentation of the material resulting in the lack of a cohesive presentation of the entire process.

2. The absence of a treatment related exclusively to small caliber bullets.

3. The lack of a critical comparison of the various methodologies and of their relative merits.

Approach to Solution

To predict the performance of a given design, the probable pattern of the surrounding air flow must be known dur- the flight of the projectile. The aerodynamic coefficients are the mathematical description of this flow; hence they must be estimated or measured. Estimation may be adequate in the design stages; however, if the coefficients are not well established before an actual round is made, there is a good chance that the flight performance will be totally unacceptable. Also, if the designer wishes to examine the possibility of trade-offs (as an example, increasing the time of flight to improve stability by employing a high drag configuration) he must be aware of the existing coefficient values so as not to go below the minimums possible for an acceptable design.

This paper presents the various methodologies used in projectile stability tests and attempts to familiarize the reader with how typical test facilities function. As each method is presented, the relative advantages and disadvantages and the accuracy to be expected are discussed.

As a means of introduction, Chapter II begins with an analysis of the forces acting on a projectile in flight and determines the conditions required for stability. A broad discussion of the testing methods is followed by a sampling of the work of recent investigators.

Ballistic range techniques are described in Chapter III; included are the yaw card and spark range methods of analysis. Factors to be considered in selecting the range facility for testing are presented, and the accuracy achieved in the determination of each aerodynamic coefficient is included.

The description and operation of the wind tunnel is discussed in Chapter IV. Both static and dynamic wind tunnel test procedures are treated and the equipment necessary to perform each test is described. The tunnel is then analyzed based on the same arguments presented for the range in Chapter III.

The factors affecting the selection of either method and the accuracies achieved by each are summarized in Chapter V. A critical comparison of the methods is followed by a general discussion of areas worthy of further inquiry.

The previous discussion was intended as a background for the reader whose aerodynamic knowledge is limited. A survey of the recent theoretical and experimental progress in ballistics is presented in the following chapter.

CHAPTER II

ANALYSIS OF PROJECTILE MOTION

Aerodynamic Forces and Moments

A projectile in free flight is under the influence of a force system resulting from the transfer of energy between it and the surrounding medium. This system may be related to three orthogonal axes to determine the components of the forces and moments acting on the projectile. Many different coordinate systems are defined in the literature usually to facilitate the development of the mathematics involved in a particular study (Murphy and Nicolaides, 1955).

Practically all systems agree on choosing the center of gravity of the shell as the origin since any motion may be described by its translation and rotation about it. In this paper, the z-axis points in the direction of the tangent to the trajectory and the x-axis and y-axis form a plane normal to the z-axis (see Figure 1).

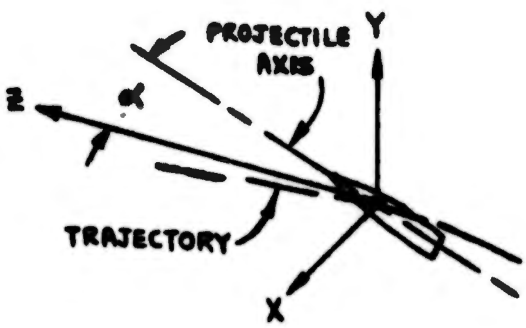

Fig. 1. TYPICAL COORDINATE SYSTEM

The force system on the projectile is determined by the pressure distribution on its surface but a resultant set of these forces acting at the proper point of application may be used. The point through which the resultant acts is called the center of pressure and is assumed to lie along the longitudinal axis. Its position is a function of the air speed, axial spin rate, and shape of the shell for small values of yaw.

The yaw is defined as the angle between the tangent to the trajectory and the axis of the shell(defined as α in Figure 1). The angle is varying continuously throughout the flight. It varies rapidly at first for a spin stabilized shell then settles to a constant called the yaw of repose. When the axis of the shell is projected onto the x-y plane, the components are related by the sine and cosine of the yaw orientation angle (Thomas, 1952).

When a projectile spins, the distribution of pressure is altered such that the resultant force is no longer in the yaw plane. The aerodynamicist introduces a force component normal to the yaw plane which he calls the magnus force. He then subtracts this force from the total force to yield a force in the yaw plane which may be resolved into lift and drag. Lift is parallel to the x-y plane and drag is parallel to the z-axis (Kelley, 1954).

The static moment is the product of the normal force and the distance between the center of pressure (cp) and center of gravity (cg). It is positive when the cp is forward of the cg as is generally the case for spin stabilized shells. If the cp is aft of the cg, the static moment opposes an increase in yaw. This is known as a " restoring moment."

When the yaw of the projectile is changing, the swinging of the projectile about its cg changes the pressure distribution so as to produce a couple about an axis through the cg normal to the yawing velocity plane. This couple usually opposes the yawing velocity, and is called the damping moment.

The roll damping moment is a moment related to the friction between the shell and the surrounding medium about the longitudinal axis of the projectile.

It is now known that the aerodynamic forces and the static moment are proportional to the dynamic pressure of the resisting medium, to the yaw, and to the physical measurements of the shell. Also the three rotational moments are proportional to their respective velocities. These proportionality constants are the aerodynamic coefficients. To call them constants is actually a misnomer since they are functions of Mach number, Reynolds number, spin rate, and yaw. A list of the significant force and moment coefficients appears in Appendix I.

Requirements for Projectile Stability

The early flight dynamicists recognized the necessity for a projectile to be statically or gyroscopically stable; the gyroscopic stability of spin stabilized shells can be assessed by computing s_g, the stability factor, given by

$$s_g = \frac{I_x^2 \, p^2}{4 \, I_y \, \mu}$$

where I_x is the axial moment of inertia, slug-ft^2, I_y is the transverse moment of inertia, slug-ft^2, p is the angular velocity, rad/sec and μ is the static moment factor, $\frac{\text{lb-ft}}{\text{rad}}$.

If one assumes the yaw and static moment vary linearly, the formula for the static moment factor is

$$\mu = (\pi / 8) \rho d^3 V^2 C_{M\alpha}$$

where ρ is the air density, slug/ft^3, d is the maximum diameter, ft, V is the airspeed, ft/sec and $C_{M\alpha}$ is the static moment coefficient, per radian.

If the value of the factor s_g is between zero and one, the projectile is unstable and will " tumble " during flight; however if s_g is greater than one, the shell is gyroscopically stable and the dynamic stability should be investigated, as later described.

Noting that s_g is inversely proportional to the density of air, it is obvious that a projectile which is stable at standard atmospheric conditions may be unstable under other temperatures and pressures. This and other uncertain factors which affect s_g have caused some designers to set 1.3 as the lower limit in preliminary design calculations. An example of the estimation of the gyroscopic stability factor is given in Appendix II.

The stability factor at the muzzle can be expressed in terms of p and V as

$$s_g = c_1 \, (p/V)^2$$

where c_1 is a constant. However p/V is equal to $2\pi / nd$, where n is the twist of the rifling at the muzzle, in calibers per turn. Hence the initial stability of the projectile depends only on the twist and only indirectly on the muzzle velocity; this is due to the dependence of C_{M_α} on Mach number. Since almost all projectiles lose velocity much more rapidly than spin, the value of s_g nearly always increases as the shell flies down the range.

As previously mentioned, the yaw of repose permits the axis of the shell to follow the tangent to the trajectory. When the spin is clockwise from the rear, the equilibrium requirement causes the projectile to point to the right and a lift force causing a drift to the right results. Estimates of the magnitude of this drift are given the firing tables; the designer must maintain the drift as uniform as possible. If the yaw of repose becomes large, the projectile may become dynamically unstable with a resulting loss in accuracy.

The yaw of a symmetric missile acted upon by a linear force and moment system is given by

$$\delta = K_{10} \, e^{\lambda_1 s} \, e^{i\phi_1} + K_{20} \, e^{\lambda_2 s} \, e^{i\phi_2} - \delta_e$$

where K_{10} = initial magnitude of nutation vector,

K_{20} = initial magnitude of precession vector,

λ_1 = nutation damping exponent, per caliber,

λ_2 is the precession damping exponent, per caliber,

s is the travel of the projectile, calibers,

ϕ_j are the phase angles of the modal vectors (j=1,2)

and δ_e is the equilibrium yaw.

The magnitudes and sign of the λ values determine the magnitudes of the modal vectors; the larger the value of a λ the more rapid is the increase in the magnitude of the respective modal vector. The e term simply oscillates sinusoidally between +1 and -1, and between +i and -i. If neither of the two modal vectors (K_1 and K_2) grows in magnitude as the projectile travels down the range, the projectile is said to be dynamically stable. For dynamic stability, therefore, both λ_1 and λ_2 must be less than or equal to zero.

An exact mathematical treatment of the solutions for the λs can be found in <u>The Free Flight Motion of Symmetric Missiles</u> by Murphy (1963). Murphy introduces the dynamic stability factor s_d, and arrives at the identity

$$1/s_g = s_d (2 - s_d).$$

Plotting this expression as a curve with $1/s_g$ and s_d as coordinates, one can define the regions of stability and instability as in Figure 2.

Methods of Testing Projectiles

Probably the oldest method of measuring the characteristics of projectiles is by yaw cards. This method consists of firing the projectile through a series of thin papers at fixed intervals, The perforation that the shell makes is then analyzed through geometrical considerations of the

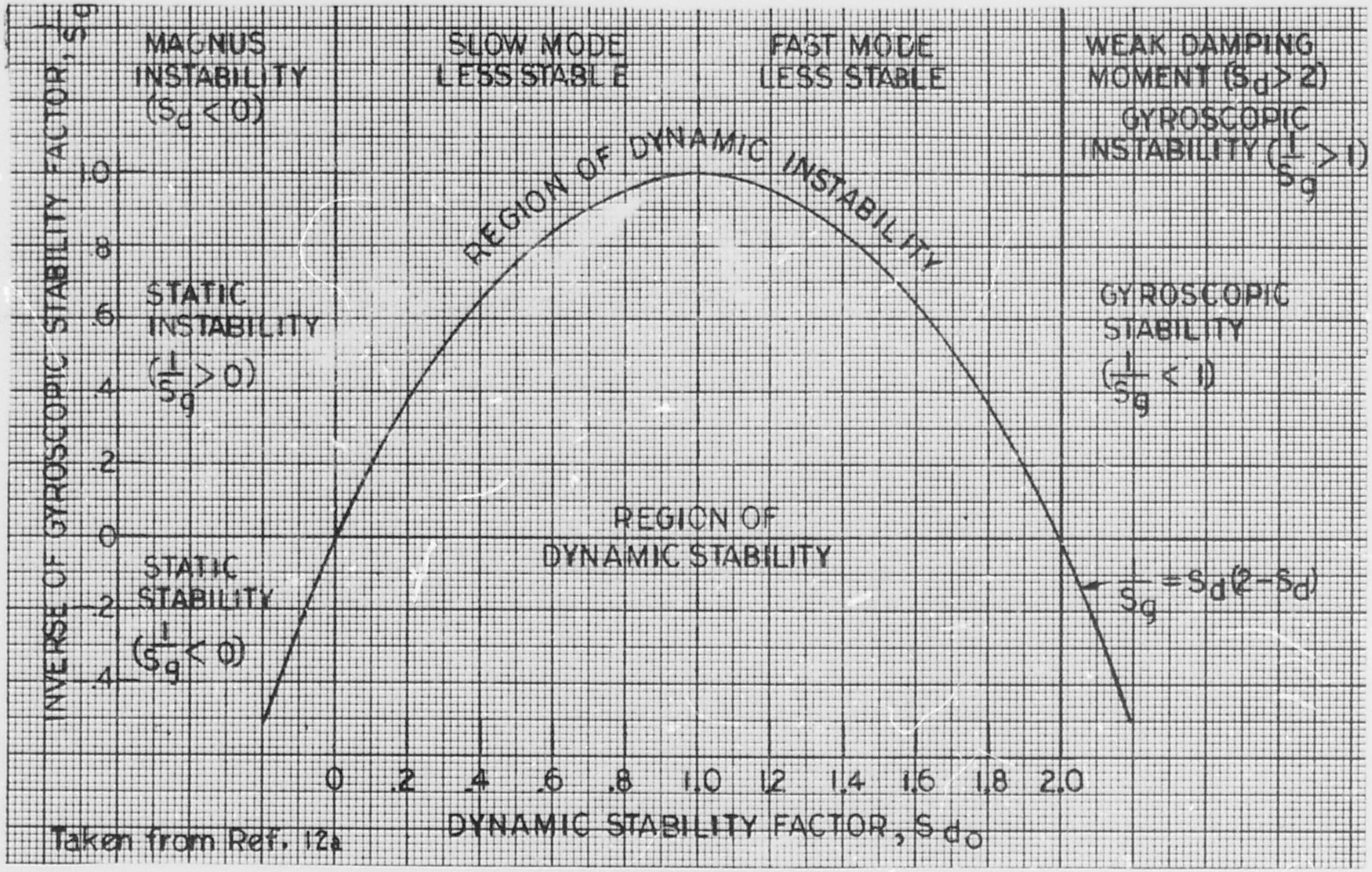

Fig. 2. GRAPH OF $1/S_g$ vs. S_d
(reprinted from Murphy, 1963)

bullet and various factors and coefficients can be calculated. The experimenter places a small quantity of stain on the shell before firing; traces of this stain appear on each card and further analysis yields the spin history of the shell along its trajectory.

The immediate objection the reader may have is that the papers affect the trajectory of the shell and, indeed, the point is well taken. It is also immediately obvious that the smaller the shell, the greater the difficulty in accurately measuring the perforations and predicting the performance of the shell. Further details concerning the accuracy and utility of this method are included in Chapter III(for a mathematical treatment, see Karpov, 1953).

Within the last 30 years a new facility, the aerodynamic range, has been developed for the measurement of the properties of a shell in free flight. Basically, the experiment consists of recording the positions, orientations, and times of the model at a series of stations along its flight path through the range. The medium for recording is photography; it is precise and in no way interferes with the flight of the model.

An electrical spark discharge provides the light for the photography. This light is of such a short duration that the resulting picture is almost instantaneous in relation to the movement of the model during the exposure time. The time at which this photograph is taken is recorded by a high precision chronograph. In addition, there are numerous measurements made of the model's physical properties,that is, weight, dimensions, center of gravity, and moments of inertia, before it is fired on the range (Dickinson, 1954).

The aerodynamics range is unique among test facilities in its measurement techniques and test conditions. The flight path is limited and the region of space under observation varies from 15 feet with a 1 square foot cross section up to 750 feet with a 25 square foot cross section in the largest range (Charters, 1955). Testing is limited to nearly straight line trajectories with small changes in the inclination and velocity of the model. Most models are either simple bodies of revolution, as small caliber ammunition, or they are finned but have 90 degree rotational symmetry (Loeb, 1968).

To obtain the aerodynamic characteristics of a model from the test data, one " fits " the experimental measurements into the equations of motion by the evaluation of the constants appearing in the equations. Then the aerodynamic coefficients are computed from certain of these constants (Murphy, 1963). An explanation of the test procedures and equipment of the range is given in Chapter III.

Over the past several years there has been an increase in the study of the so-called " dynamic derivatives." These are the rates of change of the angular position of a body. If the designer has an incomplete knowledge of the forces and moments which result from these angular changes, his chances of developing a projectile without many flight difficulties are very slim.

The wind tunnel facility is very useful in making dynamic stability measurements which allow the computation of the dynamic derivatives. These derivatives deal with the pitch and roll damping as well as the magnus moments. Very accurate determinations can be made if the need for such accuracy justifies the cost.

A wind tunnel test is usually made on a scale model which may be hollow to allow for suitable supporting apparatus. If the model is to spin, it may be mounted by ball bearings on a transverse support through the center of gravity. There are usually internal strain gage devices to measure the aerodynamic forces and moments;the damping derivative is obtained by high speed motion pictures (Shantz, 1960).

The determination of the magnus derivatives presents a problem since very small yaws must be measured in the presence of large lift forces. There are sensitive balances which partially overcome the problem of induced vibrations (Regan and Iandolo, 1966). An excellent treatment of the effect of the magnus force on dynamic stability is given in a paper presented at the Advisory Group for Aerospace Research and Development (AGARD) in 1966 (Regan, 1966).

It is rather obvious that construction of a projectile (or a model) for firing or wind tunnel testing which has little chance of success is both time consuming and wasteful, not to mention the possibility of damage to the range instrumentation upon firing. It is for this reason that preliminary estimates of the aerodynamic coefficients are made before testing (Hitchcock, 1947).

The methods for making such estimates vary from author to author, ranging from hand calculations (Wood, 1954 or Bolz and Nicolaides , 1949) to elegant computer analyses (Whyte, 1969). Either approach is fundamentally an interpolation of data from many wind tunnel and ballistic range tests of a variety of shapes and sizes. Linear aerodynamic theory is used in the construction of the formulae required to perform the interpolations.

Recently the angular motion of full scale projectiles has been obtained over the entire flight of the shell on the proving grounds. Special telemetry designed and developed in England by Hayden is now being used by Haseltine and Kline (Haseltine, 1967 and Nicolaides, 1970). These data reveal that wobbling grows and damps over the entire flight of a projectile and is critical at high angles of fire. There are many investigators continuing the analysis of this excellent flight data on projectiles.

The wobbling motion was first linked to other flight performance problems in the mid-fifties by improved theories of nonlinear flight dynamics. Turetsky analyzed aeroballistic range data on cone cylinder projectiles and found nonlinear magnus and static moments as early as 1950. Similar experience and problems all lead to a quasi-linear method for determining the stability of shells acted upon by nonlinear forces and moments. An explanation of the theory and utility of the method is given in Two Nonlinear Problems in the Flight Dynamics of Modern Ballistic Missiles, a report by Nicolaides, 1959.

While this method was confined to single arm motions (either pure nutation or precession), an excellent non-linear method for handling two armed motions was set forth by Leitmann and Murphy (Murphy, 1956). Murphy's mathematical development even included a nonlinear damping moment; he is largely responsible for the present methods for handling static and magnus nonlinearities (Kryvoruka, 1969).

The advent of the high speed computer aided immeasureably in the further developments in this area. If given the force and moment system, the computer can integrate the differential equations of motion to yield the six degree of freedom motion of a projectile. While the increased precision afforded by such programs is obvious, the lengthy table preparation and long computation and turn around time often discourages their use.

A three degree of freedom program can be used to make a rapid evaluation of a projectile's stability under the assumptions of linear aerodynamic loads and a constant gravitational field. Such an approximation may not be too severe in preliminary design where the linear aerodynamics is all that can be estimated. An example of such a program is now being published by the Naval Ordnance Laboratories.

This program was written primarily for determining the spin stability of projectiles, although it has limited applications to fin stabilized weapons. It operates from three basic sets of equations: the first is a trajectory program which solves two nonlinearly coupled differential equations by numerical integration yielding the dynamic pressure, velocity, and Mach number; next, an equation describing the spin rate as a function of arc length is piecewise integrated over constant values of the Mach number to determine the yaw of repose and the reduced frequency; then the angular motion of the body is described by two coupled second order differential equations whose solutions are used to formulate the gyroscopic and damping stability parameters (Regan and Edwards, 1969).

Now that the reader has become familiar with the general principles of projectile testing methods, the author wishes to direct him toward further examination and detailed analysis of selected test facilities. In this regard, Chapter III begins with a description of the operation and characteristics of the ballistic range.

CHAPTER III

BALLISTIC RANGE TECHNIQUES

Before the advent of the spark range facility, the yaw card method was relied upon heavily to analyze the angular motion of projectiles. This chapter begins with an explanation of the yaw card method and continues with a description of the equipment and operation of the spark range facility. Following the presentation of each method is an analysis of its accuracy and utility.

The Yaw Card Method

The method of yaw card analysis was devised by Fowler, Gallop, Lock, and Richmond (Fowler et al., 1920) to observe the angular motion of a projectile and make deductions from the observation. A series of cards are spaced at intervals and the projectile is fired through them. From the shape of the hole, an estimation can be made of the yaw at that point; hence, a continuous record of the yaw at given distances is obtained. A comparison is made with the theoretically known motion of the shell and constants are evaluated. Data reduction is a rather tricky operation and the accuracy of the results depends a great deal on the analyst. One author suggests that the only way to get optimum results is to have H. P. Hitchcock examine the data (McShane et al., 1953).

Any analysis of the shape of the hole and the actual yaw of the shell will depend on the shape of the shell. By geometric considerations, the maximum diameter of the hole can be obtained, and from this, the magnitude of the yaw. The line drawn along the maximum diameter is the line of symmetry; this line determines the orientation of the yaw. Any single determination of yaw is usually accurate to about 1 degree. This is very good when one considers that it is possible for a shell to pass through a yaw card and leave a perforation with a smaller diameter than that of the shell!

Once the magnitude of the yaw (δ) and the yaw orientation angle (θ) are known as a function of range coordinate (z) , one determines the overturning moment, stability factor, and the damping characteristics of the shell. The common practice is to plot δ^2 versus z and draw a curve through the points from which the period of δ^2 can be calculated. A simplified explanation of the determination of the period of δ^2 from the analysis of the nutational and precessional rates is given in Appendix III.

The clever reader may argue that the cards themselves have an effect on the motion of the shell and, unfortunately he is correct. There are certain means available to correct for this effect; for example, if one assumes that each card imparts a certain amount of angular momentum to the shell firings can be made with different distributions of cards and the change in the shell's behavior observed. Generally this method is useful but limited in that each shell is presupposed to exhibit uniformity in yawing motion from round to round.

Hitchcock has shown that the cards change the relationship of the yaw vectors so as to increase the period. The correction to the period must be obtained from the firings and is expressed by the relation

$$p = p_0 + (c/n) \sum (\delta / \delta_{max})^2$$

where p is the observed period, c is the card constant, δ is the observed yaw on the card, δ_{max} is the maximum yaw at the given z read off the curve of δ^2 vs. z , and n is the number of cards.

Many investigators have shown that the effect of yaw cards on damping rates is small. When small caliber ammunition is fired through photographic paper screens, the effect is negligible though it tends to decrease the damping rates. However, this low sensitivity to damping rates may lead to the erroneous conclusion that the amplitudes of the precessional and nutational components of yaw are the same. This is also due to the low resolution of yaw cards at angles near zero.

The literature generally agrees that the overturning moment coefficient and the gyroscopic stability factor can be calculated to an accuracy of around 10 %. This is sufficiently accurate for many practical applications as those associated with this field know. However, the determination of the damping factors is poor and unless an adequate statistical sample is used, the reliability of this measurement should be considered only as to the order of magnitude (Karpov, 1953).

If the range has sufficient instrumentation, there is good control of Mach number, temperature, velocity, and pressure. Models are usually full scale, or, in the case of small caliber ammunition, the actual shell itself is fired. One test firing will cover a range of Mach numbers but there is little control of the attitude of the model.

Since this method utilizes a free flight facility, there are no struts or other apparatus to interfere with the flow about the shell. The data reduction is, in theory, quite simple; however, the analysis of the results is quite tricky and complicated.

The Spark Range Facility

Introduction

The spark range facility is an instrument for producing data from wh'ch the aerodynamic coefficients can be obtained. It consists of apparatus that fires the shell and records its position, orientation, and time of flight at many stations along the trajectory.

Development of the range stemmed from the inadequate experimental facilities available for measurements just before World War II. As was previously stated, it was Dr. A.C. Charters who further developed the spark photography technique and its integration into a precision facility adequate fot the determination of the aerodynamic coefficients. The culmination of this effort was an experimental facility and method of analysis that determined ballistic data that was otherwise unobtainable.

The author knows of some facilities which presently perform large scale ballistic range tests. These facilities are: The Ballistics Research Laboratories (BRL) located at Aberdeen Proving Ground, Maryland; The Canadian Armament Research and Development Establishment in Quebec City, Can.; and The NASA Ames Research Center at Moffett Field, Calif. The Aberdeen range (BRL) does a large portion of the small caliber testing today so it will be used as a basis for discussion.

Description of the Range

The Aberdeen facility consists of a housed firing range instrumented to launch a projectile and record its motion over about 300 feet of its trajectory. The observed motion allows the determination of the aerodynamic coefficients from the retardation, the swerve, the yaw, and the rolling motion. The unusual accuracy required for angles and distances is obtained through the use of spark shadow photography. Distances are accurate to 0.001 feet, angles to 2 minutes of arc. Roll angle can be measured to less than 1 degree while a cycle counter chronograph measures time intervals to an accuracy of 1 microsecond.

The range itself consists of the firing room with the launching tube, the isolated blast chamber, the range gallery with 45 spark stations, and the control room where time measurements are recorded. A schematic diagram of the range appears in Figure 3. The steps in obtaining the aerodynamic

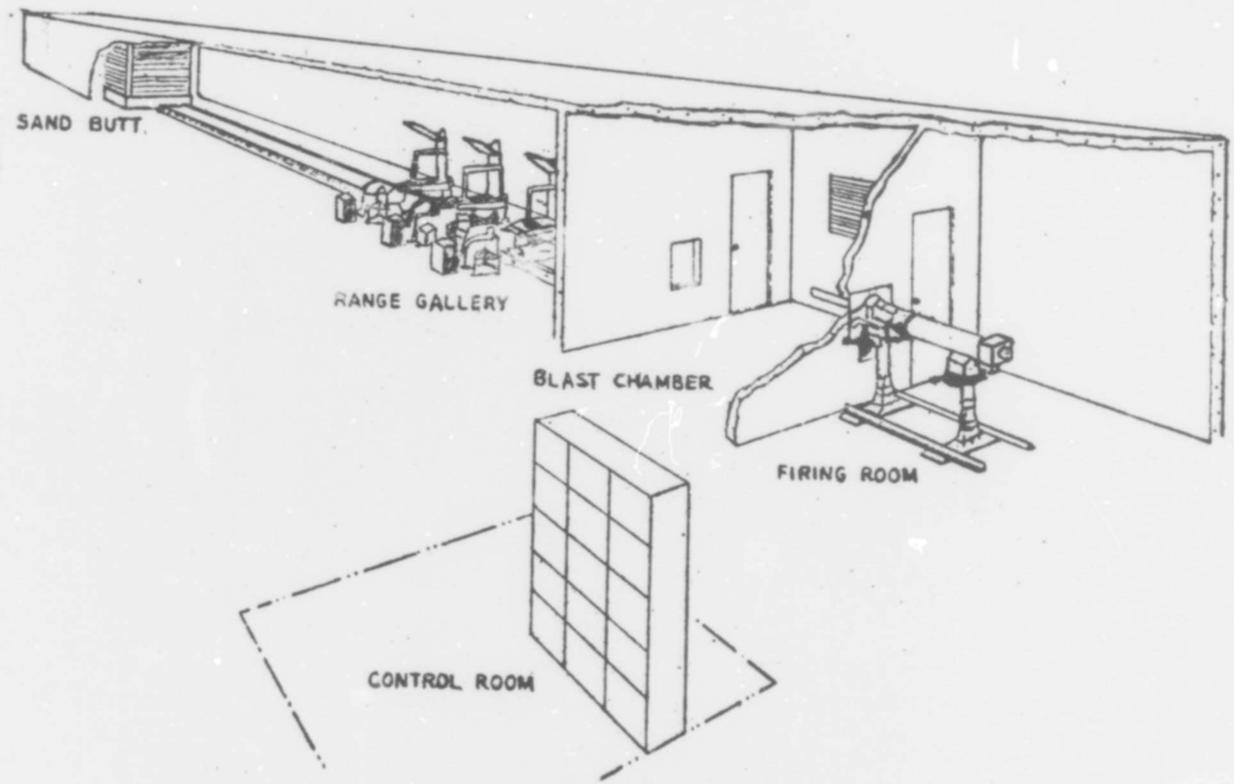

Fig. 3. SCHEMATIC DIAGRAM OF RANGE
(reprinted from Murphy, 1963)

coefficients are planning the program, construction and measurement of the shell or model, firing the rounds in the range, the measurement of the photographs and analysis and data reduction.

In planning the firing program, the capabilities and limitations of the range must be considered. The dispersion of the shell at the muzzle must be less than 1.7 mils or the missile will leave the field of the stations. If the dispersion exceeds this amount, the missile is stopped by armor plate which protects the apparatus.

Aerodynamic coefficients are obtained over a range of Mach numbers by firing the same projectile at several velocities. To obtain complete information, each round is fired at a minimum of seven velocities: three in the supersonic range, two in the transonic, and two in the subsonic. There are practical limits to the size, weight, and velocity that can be tested, however, these limitations do not affect the testing of small caliber ammunition.

As the bullet traverses the range, it passes through 45 spark stations covering 285 feet of the trajectory. These stations provide the position and orientation of the missile by means of a photographic technique with a special camera. Each station consists of two mutually perpendicular plates arranged so that the shell passes above the horizontal plate and to the left of the vertical plate. To the left of the trajectory is a point light source generated by a short duration spark gap and above is a mirror attached to the station frame.

The range gallery is dark when the shell is fired but as it approaches each station it passes through an antenna at the leading edge discharging a condenser through a spark gap. The spark leaves a silhouette of the shell on the vertical plate and the overhead mirror (at approximately 45 degrees) reflects the spark and yields a silhouette on the horizontal plate. Since the position of the spark source is known and the locations of the silhouettes are known, a little projective geometry will orient the projectile in space. A sketch of the coordinate reference system for the range is given in Figure 4.

These measurements result in one knowing to about 0.01 inches the position of the center of mass of the projectile at 45 positions down the range, as well as the angular position of the axis of the projectile to about 0.0015 rad. At certain stations a record is made of the time that the spark occurs so that the time that the projectile passed these stations is known, usually to about one microsecond. Finally at the end of the range the missile is stopped by a sand butt.

Another very important type of data to be obtained by the spark photograph is the spin of the missile. Pins are placed in the base of the projectile, one of which is pointed to provide a means of identification. Since the distance between pins is known, their orientation in space on the photograph can be used to determine the roll angle at each station (Bolz and Nicolaides, 1949).

It is necessary to know the velocity of sound and the

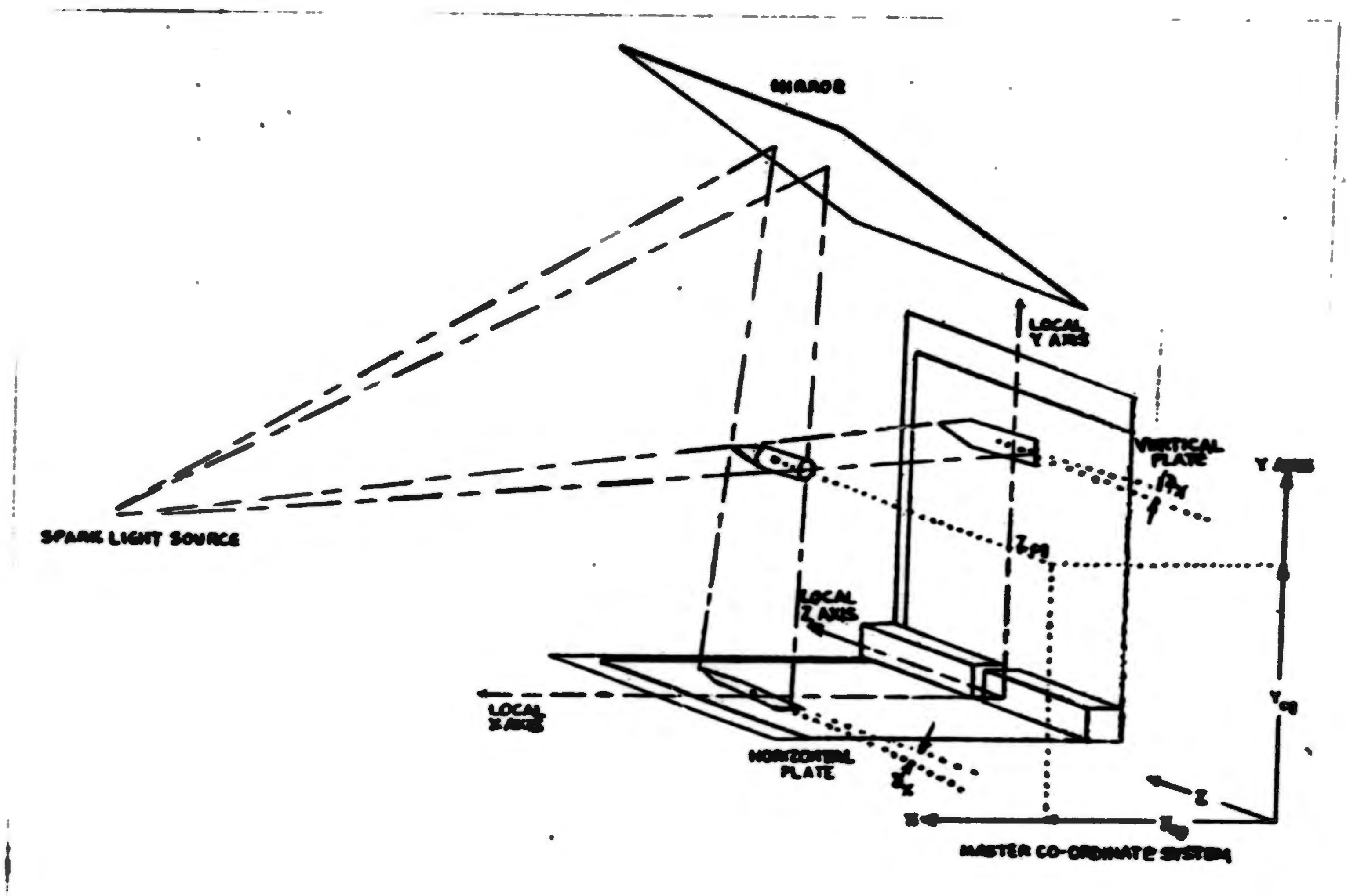

Fig. 4. COORDINATE REFERENCE SYSTEM FOR THE RANGE
(reprinted from Murphy, 1963)

density of the air in the range in order to analyze the data. Following the firing of each round, the air pressure, relative humidity, and the temperature (at three stations) are measured. These factors are kept reasonably constant by the air conditioning in the range.

The procedure for measuring the photographic plates and for the reduction of the range data is given in Data Reduction for the Free Flight Spark Ranges by Murphy (1956).

Accuracy and Utility

The advent of the free flight test facility caused the usual ballistics problem to be worked in reverse. Previously one would obtain the aerodynamic coefficients in the wind tunnel. By fitting this data into the theoretical equations, one would determine the motion and the trajectory. However, in the range technique, one observes the trajectory and the motions and with proper ballistic theory determines the forces and moments that would have been necessary to produce such a motion.

This points out the major advantage of the ballistics range: the observation of the projectile in a free flight condition. There are no struts to interfere with the flow around the traveling projectile. A full scale model or the actual projectile is fired. As with yaw cards, the attitude of the model can not be controlled; however, there is good control of Mach number, velocity, temperature, and pressure.

The model must be statically or gyroscopically stable to be fired. This also extends to the shape of the model since most ballistics range operators would refuse to fire unusual shapes. Preliminary estimates must be made of such shapes and they should be tested in a wind tunnel.

The Reynolds number can be varied quite simply by varying the model size. Since the projectile is observed over the entire flight path, one test covers a range of Mach numbers.

Data is obtained from photographs and shadowgraphs and some data is telemetered. The data reduction required is lengthy and quite complicated.

A survey of the literature and a few personal contacts at the Ballistics Research Labs yielded the following estimates of the accuracy of aerodynamic forces and moments obtained by ballistic range testing. See Table 1.

Table 1. Typical Limits for Achievable Accuracy Obtained by Ballistic Range Testing

Coefficient	Symbol	Est. error in percent
Damping moment	$C_{M_q} + C_{M_{\dot{\alpha}}}$	± 10.
Drag	C_D	± 0.5
Lift	C_{L_α}	± 5.
Magnus force	$C_{N_{p\alpha}}$	± 25
Magnus moment	$C_{M_{p\alpha}}$	± 15
Roll damping moment	C_{l_p}	± 1.
Separation	cp-cg	± .10 cal.
Static moment	C_{M_α}	± 2.

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Static moment	C_{M_α}	± 2.

All the coefficients listed can be determined in the ballistic range; however, the damping moment must be determined as the sum of C_{M_q} and $C_{M_{\dot{\alpha}}}$. This is one respect in which the range and the wind tunnel differ; the two damping moments can be determined separately in the wind tunnel, if desired.

The wind tunnel operates in just the opposite fashion from the ballistic range; instead of firing the projectile through the atmosphere, it is held fixed and the air moved past it. The following chapter is devoted to describing and analyzing the wind tunnel facility.

CHAPTER IV

WIND TUNNEL TEST METHODS

One of the most useful experimental methods of measuring the aerodynamic forces on a projectile is essentially a reversal of the previously discussed method. This chapter deals with the equipment, techniques, and analysis one would typically encounter in static and dynamic wind tunnel testing. The methods for obtaining pitch and roll damping derivatives as well as magnus force and moment coefficients are described. The accuracy to be expected in estimating coefficients is presented and the capabilities and limitations of wind tunnel testing are discussed.

Static Tests

A wind tunnel is designed to send air through a test section in such a manner that the vector velocity of the air at every point is the same. If one desires to examine subsonic flow, there is a larger section preceeding the test section where the velocity is lower. The air density, velocity, and temperature are computed from various measurements. The basic measurements are made by means of orifices in the tunnel walls; however, the temperature is difficult to measure when the velocity is high so a direct measurement is inaccurate.

The density in the pre-test section can be computed by using the gas law $P/\rho = KT$, where K is a constant depending on the gas, P is the pressure, ρ is the density, and T is the temperature. The velocity and temperature in the test section is obtained through the laws of conservation of matter, momentum, and energy. This is a very sketchy theoretical approach; in actuality, the procedure is modified considerably based on comprehensive calibration tests. An excellent description of wind tunnel testing is available in W.F. Durand's Aerodynamic Theory (Berlin: Julius Springer Tracts, 1934). The concern in this paper is only with the general principles.

The model to be tested is suspended in the test section by means of a strut which is connected to a balance system consisting of a table free to make small movements. The force and torque necessary to restrain the model and strut are measured; then the test is repeated without the model and the force and torque required to restrain the strut alone are measured and subtracted from the total force. The corrections resulting from the interference of the strut with the flow should be kept as small as possible.

For subsonic work, the best suspension is probably a network of fine wires. By removing various wires and measuring the resulting force, the entire force system can be calculated. This suspension slows the work considerably and one of the big advantages of the tunnel is lost, that is, the speed of the operation (McShane et al., 1953).

A typical test program would begin with the selection of the speeds, which would depend on the expected speed in flight and the capabilities of the tunnel. The model would be varied through angular positions at each speed, and the drag D, lift L, and moment M would be measured. The results could then be graphed as functions of yaw at each speed.

Rather than assume that the lift and moment are linear functions of yaw, one may examine the data and decide over what range the linear approximation is valid. It is still desirable to examine the dimensionless ballistic coefficients

$$K_D = \text{drag}/\rho d^2 u^2,$$
$$K_L = \text{lift}/\rho d^2 u^2 \text{ and}$$
$$K_M = \text{moment}/\rho d^3 u^2.$$

The last two are related to the coefficients K_L and K_M in the following manner

$$K_L = k_L \sin\delta \text{ and}$$
$$K_M = k_M \sin\delta$$

where the use of $\sin\delta$ instead of δ is hardly justified since the departures of K_L and K_M from linearity are more serious than the difference in $\sin\delta$ and δ. The most satisfactory method to obtain k_L and k_M is to select an interval about zero degrees yaw on which K_L and K_M appear to be linear then fit a straight line to the data over this range.

The biggest advantage of the wind tunnel is that one can obtain direct and very rapid measurements. This direct observation is also useful in deducing the probable performance of a model by analysis of certain phenomena like separation of the air flow. The flow around the front of a streamlined projectile (not too blunt) will be steady; that is, the vector velocity at points near the shell will not change with time. The flow about the tail of the shell will be unsteady and vary with time, as the flow observed in the wake of a boat. The separation point is defined by the interface between the two types of flow. It is highly desirable to design a projectile where separation occurs as far to the rear as possible. An interesting example of what happens when one attempts to over-design is given in Aerodynamic Properties of A Caliber 0.50 Bullet With Reflex Boattail (Piddington, 1957).

The methods described up to this point are characteristic of any wind tunnel test facility. There are six facilities which do a large amount of projectile testing (to the knowledge of the author). These facilities are: The NASA Ames Research Center at Moffett Field, California; The NASA Lewis Research Center in Cleveland, Ohio; The NASA Langley Research Center at Langley Field, Virginia; The Naval Ordnance Laboratory in White Oak, Maryland; The Arnold Engineering Development Center, Arnold Air Force Station, Tennessee; and the Ballistic Research Laboratories at Aberdeen Proving Ground in Maryland. The Naval Ordnance Labs is the typical facility which is the basis of the following discussion of the instrumentation, methods, and analysis of data employed in dynamic wind tunnel testing.

Dynamic Tests

Many of the flight difficulties in ballistics that are caused by the rate of change of a body's angular position have brought to light the importance of the so-called " dynamic derivatives." In particular, the derivatives measured include the damping-in-pitch derivative, $C_{mq} + C_{m\dot{\alpha}}$, the damping-in-roll derivative, C_{lp}, and the magnus derivative, $C_{np\dot{\alpha}}$.

The techniques employed almost always involve some oscillatory motion about a fixed axis. In the free oscillation technique (used at NOL), the model is mounted on a transverse support shaft connected by ball bearings, or an air bearing, to the center of gravity. The model is held at some angle of attack, and, when released, undergoes a damped oscillatory motion (see Figure 5). The mathematics of the situation is omitted since it can be found in any standard reference (Regan and Iandolo, 1966).

Until recently, the motion was recorded on movie film; now it is read by an E-core electromagnetic transducer mounted within the model. Since the E-core is an integral part of the model, it also oscillates and this oscillation is related to changes in inductance and reluctance of the circuits. The E-core is calibrated with the pitch-damping calibration rig (see Figure 5) by manually rotating the apparatus through various increments and recording calibration counts on magnetic tape. During the test, the transducer " reads " the electrical signal 300 times per second

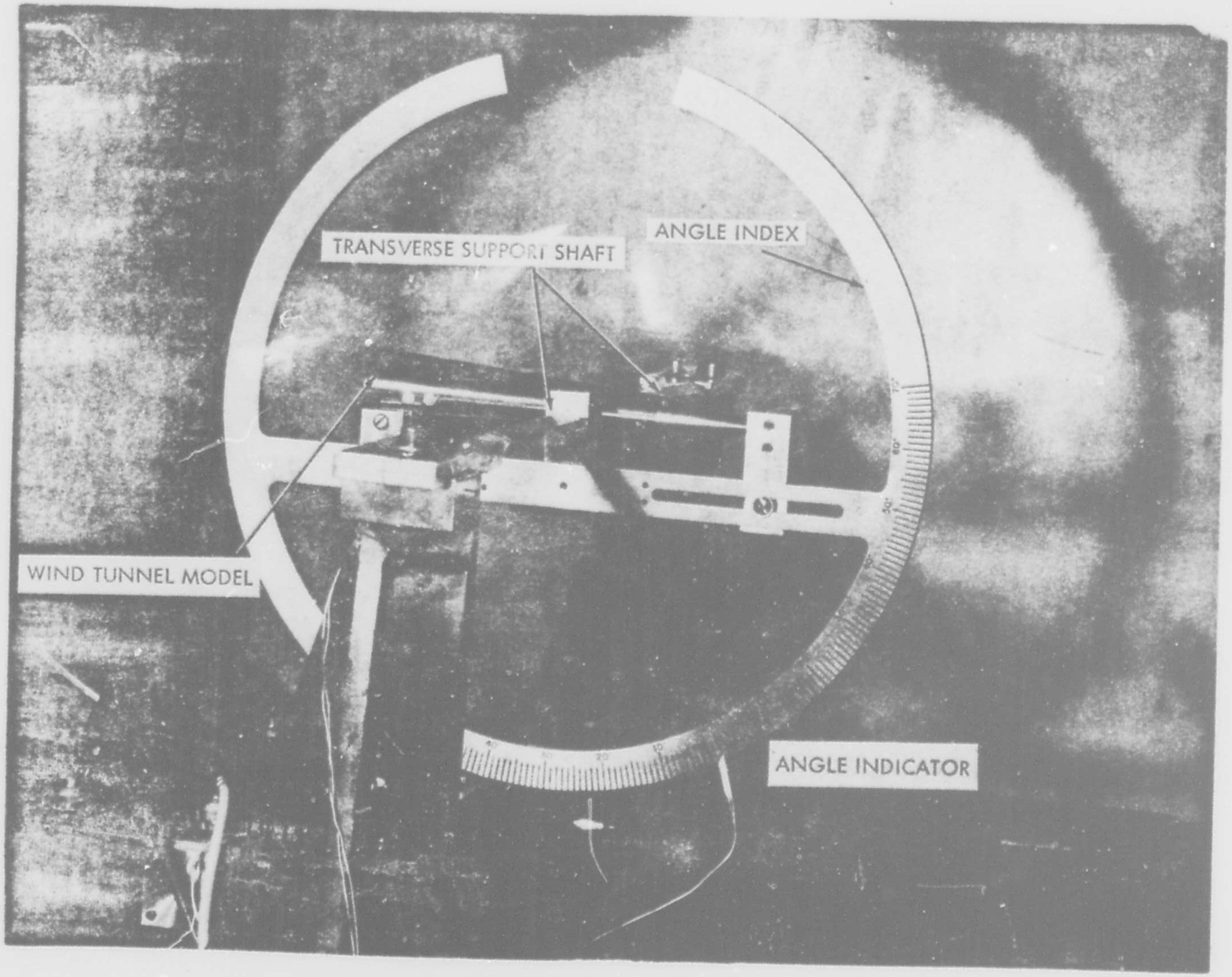

Fig. 5. PITCH-DAMPING CALIBRATION RIG
(photo courtesy U.S. Naval Ordnance Laboratory)

and records it, yielding an angle of attack versus time record.

In a roll damping test, the model is attached to the rotating shaft and spun by a sliding vane air motor (see Figure 6) until the desired spin rate is achieved. A magnetic clutch disconnects the motor and a tachometer provides a signal equal to the spin rate. This signal recorded as a function of time provides a roll decay history which is reduced to give the damping-in-roll derivative as a function of the spin parameter.

Perhaps the most elusive of the dynamic forces and moments is the magnus force. The stability of spin stabilized projectiles depends immensely on the cross derivatives C_{Np} and C_{yp} which are generally referred to as the magnus force and moment derivatives. The mechanical features of the magnus balance are shown in Figure 7.

Spin is imparted to the model by means of the air turbine shown and the angular rate is measured by the tachometer at the forward portion of the balance. In operation, the model is spun to the desired rate, the air supply terminated, and gage readings are recorded on magnetic tape. The data reduction is similar to the static-stability test with the results presented as side force and moment coefficients , $\dot{C}_y$ and $\dot{C}_n$, versus spin parameter.

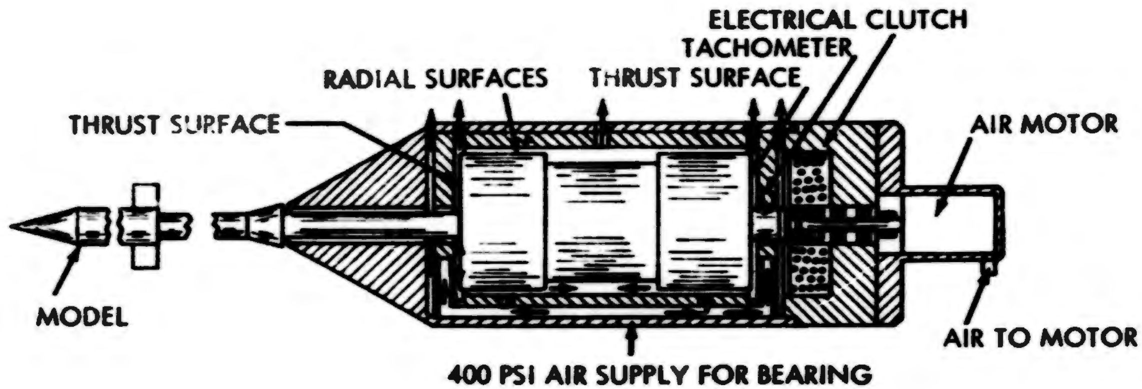

FIG.6. AIR BEARING ROLL DAMPING MECHANISM
(diagram courtesy Regan and Iandolo, 1966)

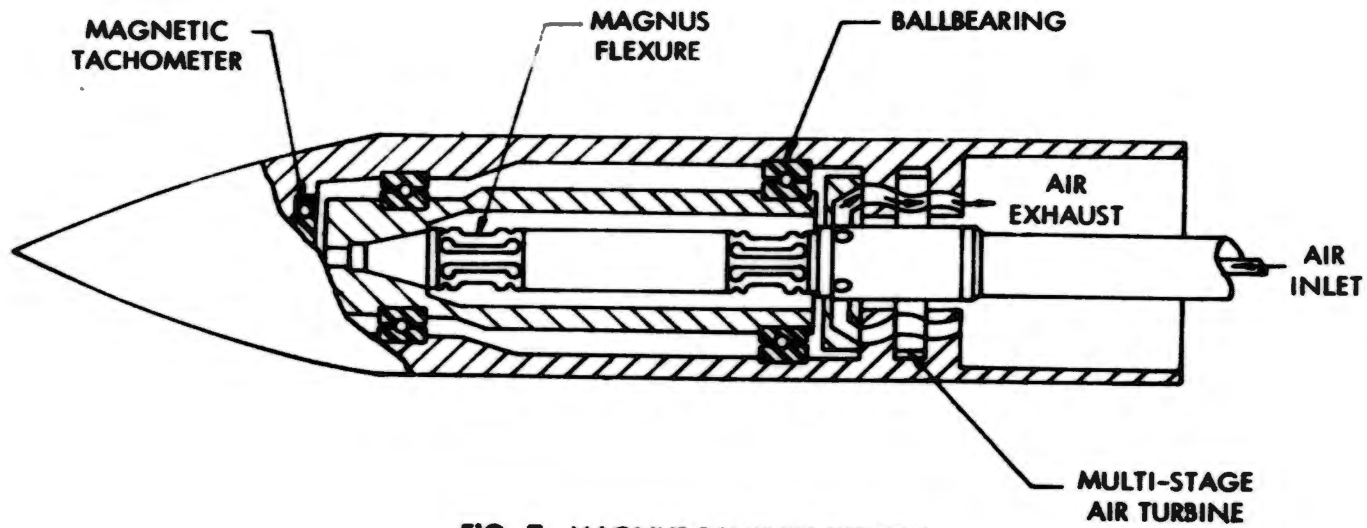

FIG. 7. MAGNUS BALANCE DETAILS
(diagram courtesy Regan and Iandolo, 1966)

Accuracy and Utility

The tremendous advantage of the wind tunnel method is the directness and rapidity of the measurement and the simplicity of the data reduction. There is excellent control of the model's attitude, Mach number, velocity, temperature, and pressure. On the other hand, the models are usually reduced in size and only one Mach number can be run at a time.

Another important advantage is that the shell or model need not be gyroscopically stable; hence, data can be obtained on both stable and unstable configurations. The data can be obtained from force and moment balances, pressure taps, schlieren photographs and shadowgraphs.

The Reynolds number can be varied by changing the tunnel pressure, however, it may not be possible to test at free-flight Reynolds number. It is also possible that the model suppor may interfere with the base flow.

A list of the estimated measuring accuracies of the aerodynamic forces and moments to be expected in wind tunnel testing is given in Table 2. Note that these accuracies are very comparable in magnitude to those obtained in the ballistic range.

Table 2. Typical Limits for Achievable Accuracy Obtained in Wind Tunnel Tests.

Coefficient	Symbol	Est. error in percent
Damping moment	$C_{Mq} + C_{M\dot{\alpha}}$	± 10.
Drag	C_D	± 2.
Lift	$C_{L\alpha}$	± 1.
Magnus force	$C_{Np\alpha}$	± 10.
Magnus moment	$C_{Mp\alpha}$	± 10.
Roll damping moment	C_{lp}	± 1.
Separation	cp-cg	± .10 cal.
Static moment	$C_{M\alpha}$	± 1.

A summary of the typical limits of accuracy obtained by each method and a critical analysis of the factors to be considered in selecting a particular method are given in Chapter V.

CHAPTER V

SUMMARY AND CONCLUSIONS

The yaw card method was essentially the only method available prior to the development of the spark range. It is used primarily to find the gyroscopic stability factor and overturning moment coefficient together with the gross features of the yawing motion. This method is very simple and rapid but it is not very accurate at small yaws. This low accuracy and the interference of the yaw cards with the motion are the most severe limitations.

The overturning moment coefficient and the stability factor can be obtained to within an accuracy of 10%. The accuracy deteriorates rapidly at small angles of yaw unless painstaking care is exercised in the interpretation of the perforations in the cards. An inference, as is usually drawn from yaw cards, that the two yaw damping rates are equal should be regarded with great caution since this apparent equality is due to very bad resolution of yaw card data at angles near zero.

Determination of the damping factors from yaw card firings is poor and should be relied upon only as to the order of magnitude.

The second method discussed was the spark photography technique which gives accuracies at least an order of magnitude higher than yaw cards. Very similar accuracies can be obtained by the wind tunnel test. Table 3 presents a comparison of the limits of accuracy for both the range and the wind tunnel test methods.

Table 3. Typical Limits for Achievable Accuracy Obtained in Wind Tunnel and Ballistic Range Tests.

Coefficient	Symbol	Est. error in percent: Range	Est. error in percent: Tunnel
Damping moment	$C_{Mq} + C_{M\dot{\alpha}}$	± 10.	± 10.
Drag	C_D	± 0.5	± 2.
Lift	$C_{L\alpha}$	± 5.	± 1.
Magnus force	$C_{Np\alpha}$	± 25.	± 10.
Magnus moment	$C_{Mp\alpha}$	± 15.	± 10.
Roll damping moment	C_{lp}	± 1.	± 1.
Separation (cal.)	cp-cg	± 0.10	± 0.10
Static moment	C_M	± 2.	± 1.

Both of these methods yield values which are adequate to permit reasonably confident design compromises. That is to say, they not only yield accurate values sufficient for a particular design, but also good estimates of the changes in coefficients that would result from a change in the design.

The method to be chosen in a particular case may depend on the technical factors compared in Table 4; if not, the choice may be based on factors of time and cost. The

Table 4. Factors to be Considered in Selection of Method

Control of...	Range	Tunnel
Mach number	good	excellent
velocity	good	excellent
temperature	good	excellent
pressure	good	excellent
model attitude	fair	excellent
data reduction	complicated	simple
model must be statically stable	yes	no
model support interferes	no	yes
Mach numbers per test	a few possibly	only one
can vary Reynolds number	by varying model size	by varying tunnel pressure
model size	full scale	reduced
data acquisition	shadowgraphs photographs yaw cards telemetry	balances pressure taps shadowgraphs photographs

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major considerations are the availability of the facility and the speed with which the necessary data reduction can be performed since costs are not widely different.

It is clear that one of the main disadvantages of the wind tunnel is the model support. Regardless of how carefully it is designed, it introduces an artificiality into the test. It is for this reason that studies have been undertaken to investigate the possibility of flying a model in a wind tunnel. This technique is not new, but it has recently received renewed attention (Regan and Iandolo, 1966).

At the Ames Laboratory, for example, the test chamber of the range is the working section of a supersonic wind tunnel. Thus the wind stream of the tunnel is combined with the measurement techniques of the ballistic range. The purpose of this combination is to carry out free-flight testing at higher Mach numbers than would be attainable in still air in a range test (Briggs, 1952 and Charters, 1955).

The Naval Ordnance Laboratory is developing launch systems for its wind tunnel facilities. The launcher for Supersonic Tunnel Number 1 at NOL is shown in Figure 8. Briefly, this device consists of an air cylinder which admits a regulated air supply and controls the motion of the piston rod. When the piston comes to an abrupt stop, an inertial mass inside the launch head moves forward and releases the sabot fingers, thus launching the model.

Although the launch head contains an air motor capable

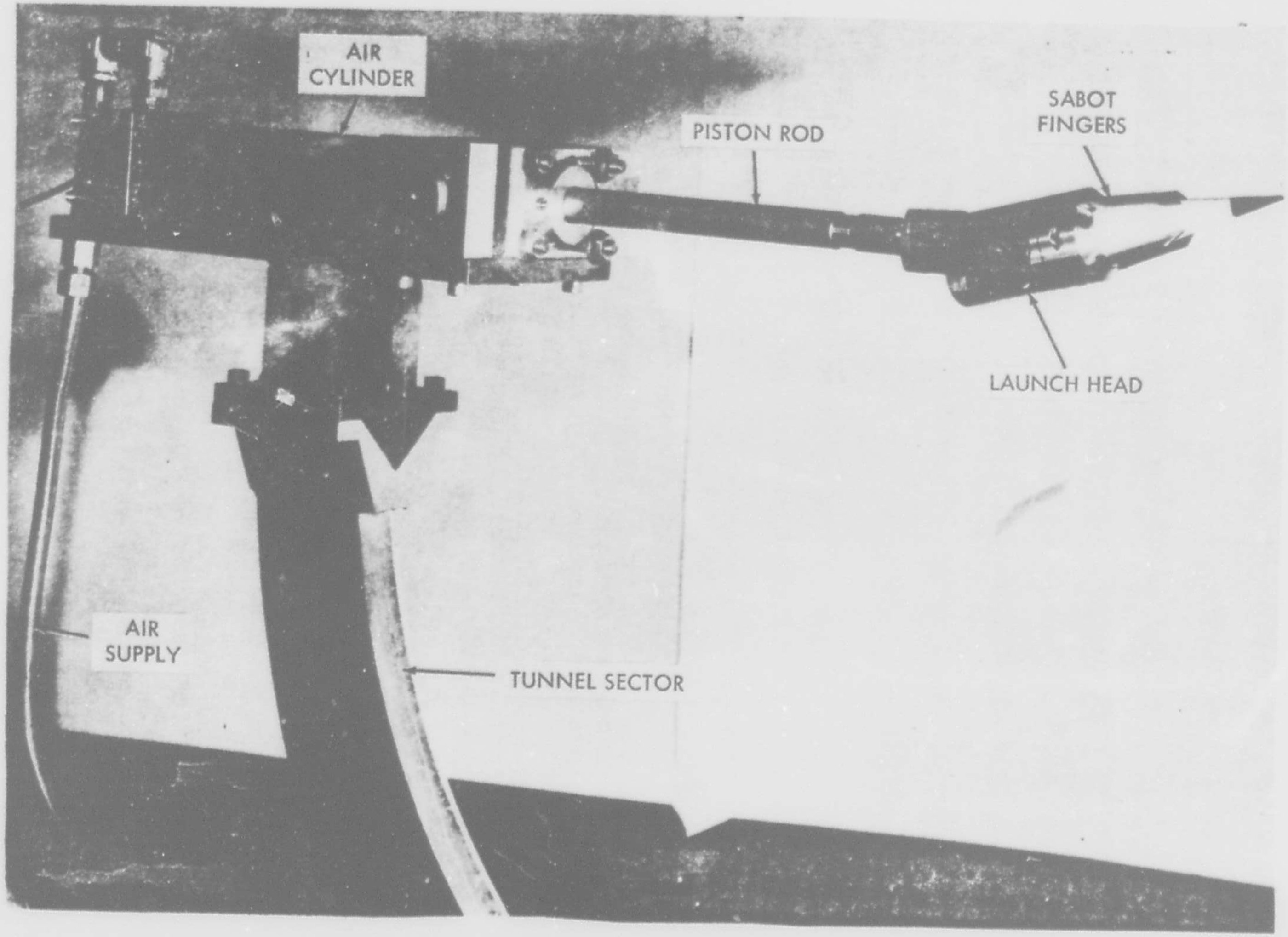


Fig. 8. FREE FLIGHT LAUNCHER FOR SUPERSONIC TUNNEL
(photo courtesy U.S. Naval Ordnance Laboratory)

of imparting a spin rate to the model, the author was unable to find any conclusive data or reports concerning attempts to utilize this type launching. The advantages of such a method are obvious; it is only presented here as an area worthy of further research and investigation.

There has been very little experimental work done to examine the effects of the magnus force on dynamic stability. Most of the studies that have been done were devoted to research shapes. While research shapes are similar to the operational rounds, the effects of surface modifications such as rotation bands, blunt noses, bourrelets, and alterations of base geometry are neglected. Using data obtained from research rounds to predict the performance of the operational rounds having the same general shape is poor practice indeed (Regan, 1966).

The designer of spin-stabilized projectiles must know the effects of various surface protuberances on the stability of his design. In this regard, experimental studies of base effects seem necessary since the slightest change in projectile base geometry affects the location of the center of pressure rather drastically.

One disadvantage of all the methods is that of obtaining data on the flow near the model, as in the shock layer. Perhaps the wind tunnel- free flight technique just discussed could be studied with the use of electromagnetic wave diagnostics, molecular beams, spectroscopy, x-rays or some other method of experimental physics.

The study of projectile flight dynamics seems to be following a trend which is becoming more and more prevalent in the advancement of technology: as the methods of analysis become more sophisticated and greater quantities of data are made available, more and more opportunities arise to improve the design and performance of projectiles.

GLOSSARY

GLOSSARY

ACCURACY- The quality of correctness or freedom from error of a measurement.

AERODYNAMIC JUMP- The deflection of the trajectory arising from the alternating lift forces on a yawing projectile.

AERODYNAMIC NUMBERS- The dimensionless power products of the physical quantities arising in the problem of fluid flow around a projectile.

AIRSPEED- The speed of the shell relative to the medium in which it travels.

ANGLE OF YAW- The angle defined by the axis of a projectile and the direction of motion.

AXIAL DRAG- The aerodynamic force on a body that acts along its axis of symmetry, opposing the motion.

BALLISTIC COEFFICIENT- A numerical measure of the ability of a projectile to overcome resistance to its motion. It is a function of the mass, diameter, and form factor of the body.

BALLISTIC RANGE- An area or enclosure instrumented so as to observe trajectories of projectiles, and by analysis, yield estimates of aerodynamic coefficients.

BOATTAIL- The base of a projectile, shaped like the frustrum of a cone.

BOUNDARY LAYER- A thin layer of air (or other fluid) next to a body where the main effects of viscosity are concentrated.

BOURRELET- The surface of a projectile on which it rests while in the bore of a weapon. Generally, the bourrelet is located just behind the ogive and has a slightly larger diameter than the main body.

CALIBER- The diameter of a projectile or of the bore of a gun. In rifled arms, the caliber designation is based on a nominal value representing a close approximation rather than an exact measurement. Caliber may be used as a unit of length: for example, a 6 inch 50 caliber gun (6"/50) would have a bore diameter of 6 inches and a tube length of 50 calibers or 25 feet.

CENTER OF PRESSURE- That point on the axis of a projectile through which the resultant of a set of aerodynamic forces passes.

DERIVATIVE- In projectile aerodynamics, the rate of change of an aerodynamic coefficient with respect to a change in the magnitude of the yaw angle, eg., the slope of the C_M vs. α curve gives the static moment derivative, $C_{M\alpha}$.

DIFFERENTIAL EFFECTS- The effects upon the elements of the trajectory due to variations from standard conditions.

DIVERGING YAW- If the angle of yaw increases from the initial value during the flight of a projectile, the yaw is diverging.

DRAG- Component of air resistance in the direction opposite to that of the motion of the center of gravity.

DRAG COEFFICIENT- A number which relates drag force to the dynamic pressure of the air stream and the frontal area of the projectile.

DRIFT- The lateral deviation of the trajectory of a spin stabilized shell, due to the equilibrium yaw.

DYNAMIC PRESSURE- The pressure exerted by a fluid only by its relative motion when it strikes an object. It is proportional to the density and the square of the velocity and is sometimes called the " velocity head."

EQUILIBRIUM YAW- The yaw angle to which the dynamically stable projectile decays, partly due to the asymmetry of the shell and the effect of gravity.

FINENESS RATIO- The ratio of the length of a projectile to its diameter (l/d).

FIN-STABILIZED- A projectile made statically stable by the aerodynamic moment arising from the presence of lifting surfaces aft of the center of gravity.

FORM FACTOR- Factor introduced into the denominator of the ballistic coefficients based on the shape of the projectile.

FREE STREAM- The flow of air or other fluid undisturbed by the presence of a moving body; specifically, the flow of air ahead of a shock wave.

HYPERSONIC- Pertaining to speeds of Mach 5 or greater.

INITIAL YAW- The yaw of a projectile as it leaves the muzzle.

INITIAL YAWING VELOCITY- The rate of change of the yaw as it leaves the muzzle.

LIFT- The component of the aerodynamic force perpendicular to the relative wind, acting in the plane of the yaw.

LINE OF DEPARTURE- The path of a projectile as it leaves the muzzle.

MACH NUMBER- The ratio of the velocity of a body to that of sound in the medium being considered. At sea level in the U.S. Standard Atmosphere, a body moving at a Mach number of one would have a velocity of 1116.2 feet per second (the speed of sound in air under those conditions).

MAGNUS FORCE- The lateral thrust on a rotating body when acted upon by an airstream having a velocity component normal to the axis of rotation of the body.

MAGNUS MOMENT- The moment about the center of gravity of the body produced by the magnus force.

MODAL VECTORS- A pair of rotating arms, called the precession and nutation vectors, which when added together give the magnitude and orientation of the variable part of the yaw of the projectile at any instant. The precession vector is a tangent to the trajectory which rotates slowly; the outer end of this vector is taken as the origin of the nutation vector, which rotates more rapidly.

NORMAL FORCE- The component of the total aerodynamic force perpendicular to the longitudinal axis of the projectile, and acting in the plane of the yaw.

NUTATION- The oscillation of the axis of a rotating body, such as a spinning projectile. This oscillation is superimposed on the slower motion known as the precession.

OVERTURNING MOMENT- An aerodynamic moment tending to increase the yaw of the projectile.

PARTICLE TRAJECTORY- The theoretical trajectory determined by gravity and zero-lift drag which a particle would describe if it could maintain zero angle of yaw.

PLANE OF YAW- The plane containing both the longitudinal axis of the projectile and the tangent to the trajectory.

PRECESSION- A circular motion of the axis of rotation of a spinning body which is brought about by the application of a constant torque about an axis perpendicular to the axis of rotation.

PRECISION- The property of having small dispersion about the mean.

RELATIVE VELOCITY- The velocity of relative motion, especially with respect to a projectile and the airstream.

RESTORING MOMENT- A static moment which is negative when the angle of attack is positive, and vice versa.

REYNOLDS NUMBER- An index of similarity used in the analysis of fluid flow about scale models in wind tunnel tests to determine the results to be expected about full-scale models. The Reynolds number is expressed in a fraction equal to $\rho Vl/\mu$, where ρ is the density of the resisting medium, V is the velocity, l is a linear dimension of the body (such as a diameter), and μ is the coefficient of viscosity.

ROLL- An angular displacement about the longitudinal axis of a projectile.

ROLL RATE- The time rate of projectile rotation about its axis.

ROLLING MOMENT- An aerodynamic moment about the longitudinal axis of a projectile, tending to change the roll rate.

ROTATING BAND- Soft metal band around a projectile near its base.The band centers the projectile and makes it fit tightly on the bore preventing the escape of gas, and by engaging the rifling gives the projectile its spin.

SEPARATION- The phenomenon in which the boundary layer of the flow over a body in a moving stream of fluid separates from the surface of the body. This is also the name given to the point on the body at which the separation begins.

SLUG- The engineering unit of mass, chosen such that a force of one pound acting on a unit mass will produce an acceleration of one foot per second per second.

SPARK RANGE- A firing range in which projectiles in free flight can be photographed by the light from an electric spark which is triggered by the passage of the projectile.

SPIN- The product of roll rate and some characteristic dimension, as diameter, divided by the airspeed.

SPIN STABILIZATION- Method of stabilizing a projectile during flight by causing it to rotate about its own longitudinal axis.

STABILITY- A characteristic of a projectile that causes it, if disturbed from its condition of equilibrium or steady flight, to return to that condition.

STABILITY FACTOR, DYNAMIC- A number related to the yaw damping characteristics of a shell.

STABILITY FACTOR, GYROSCOPIC- A number relating the angular momentum of a projectile to the slope of its aerodynamic overturning moment. A necessary, but not sufficient, condition for stability is that this factor be greater than unity, or negative.

STABILITY, STATIC- Stability in the absence of spin; generally, a mechanism is statically stable if any displacement from a rest position creates a force or moment opposing the displacement.

STATIC MOMENT- An aerodynamic moment related only to the angle of yaw.

STATIC PRESSURE- The pressure that is exerted by a fluid at rest.

SUBSONIC- Pertaining to motion (relative) between a body and a surrounding fluid at a speed less than the speed of sound in that fluid.

SWERVING MOTION- In flight, the motion of the center of gravity of a projectile perpendicular to its particle (or zero lift) trajectory.

TRAJECTORY- The curve in space traced by the center of gravity of the projectile.

TRANSONIC RANGE- A range of speeds between the speed at which one point on a body reaches supersonic speed (relative to the air flow in the immediate vicinity of the point) and the speed at which the shock wave system is fully developed.

TRIM- The equilibrium attitude of the longitudinal axis of the projectile relative to the tangent to the trajectory.

TWIST (OF RIFLING)- The inclination of the spiral grooves of the rifling to the axis of the bore of the weapon; expressed as the number of calibers of length in which the rifling makes one complete turn.

VISCOSITY- The coefficient of viscosity is dependent on the fluid and its temperature and is the ratio of the shearing stress to the velocity gradient in a boundary layer.

YAW- The angle between the direction of motion of a projectile and its direction of the longitudinal axis.

YAW OF REPOSE- That part of the equilibrium yaw which is due to gravity.

YAW DRAG- Drag due to yaw.

YAWING VELOCITY- Time rate of change of yaw; the change may be in magnitude or direction, or both.

APPENDICES

APPENDIX I

Aerodynamic Forces and Moments

The significant aerodynamic force coefficients are:

$$C_N = \frac{N}{q\ S} \qquad C_L = \frac{L}{q\ S} \qquad C_D = \frac{D}{q\ S}$$

$$C_{Np} = \frac{N_p}{q\ S\ (\ pd/V\ )}$$

where $q = \frac{1}{2}\rho V^2$ is the dynamic pressure, $S = \pi d^2 / 4$ is the frontal area of the projectile, and α = the yaw in radians.

Note that ... ρ = air density (slug/ ft^2),

V = speed of shell (ft./sec.),

p = roll rate (rad./ sec.),

d = shell diameter (ft.),

N = normal force (lb.),

L = lift (lb.),

D = drag (lb.) , and

N_p = magnus force.

For small angles of yaw (α < 0.17 rad.), all coefficients except C_D are assumed to vary linearly with yaw. Now differentiating with respect to α , one can write:

$$N = \frac{d\ C_N}{d\ \alpha}\ q\ S\alpha = C_{N\alpha}\ q\ S\alpha ,$$

$$L = \frac{d\,C_L}{d\alpha}\, q\, S\alpha = C_{L\alpha}\, q\, S\alpha \quad \text{and}$$

$$N_p = \frac{d\,C_{Np}}{d\,\alpha}\, q\, S\, (\,p\, d\, /\, V\,)\, \beta\ .$$

Since the drag varies with the square of the yaw

$$D = (\,C_{D_o} + C_{D_{\alpha^2}}\, \alpha^2\,)\, q\, S$$

where C_{D_o} is the drag coefficient at zero yaw and $C_{D_{\alpha^2}}$ is the rate of change of C_D with α^2.

The moment coefficients are derivatives with respect to yaw or angular velocity that is,

$$\frac{d\,C_M}{d\,\alpha} = C_{M\alpha} = \text{static moment coefficient,}$$

$$\frac{1}{\frac{1}{2}\rho V^2\, S\, d}\left[\frac{\partial M_y}{\partial(\,qd/V\,)} + \frac{\partial M_y}{\partial(\,\dot{\alpha}\, d/V\,)}\right] = C_{Mq} - C_{M\dot{\alpha}} =$$

damping moment coefficient, and

$$\frac{d\,C_{Mp}}{d\,\alpha} = C_{Mp\alpha} = \text{magnus moment coefficient.}$$

The moment about the horizontal axis through the cg is

$$M_y = \frac{d\,C_M}{d\,\alpha}\, q\, S\, D\alpha + \frac{\partial M_y}{\partial(\,qd/\,V\,)}\,(\,qd/\,V\,) + \ldots$$

$$\ldots + \frac{\partial M_y}{\partial(\dot{\alpha} d/\,V\,)}\,(\dot{\alpha} d/\,V\,) + \frac{d\,C_{Mp}}{d\,\alpha}\,(\,pd/\,V\,)\, q\, S\, d\, \beta$$

where q in the second term is the angular velocity about the horizontal axis when $\dot{\alpha}$ (the yawing velocity about that axis) is zero; that is the total angular velocity about the horizontal axis is $q + \dot{\alpha}$.

In coefficient form, then,

$$M_y = \frac{1}{2}\rho V^2 S d \left[C_{M_\alpha}\alpha + C_{M_q}(qd/V) + C_{M_{\dot{\alpha}}}(\dot{\alpha}d/V) + C_{M_{p\alpha}}(pd/V)\beta \right]$$

where the first term is the static moment, the next two are damping moments, and the last is the magnus moment.

Similarly, one may obtain the moment about the vertical axis by interchanging α for β and r for q and noting that $r + \dot{\beta}$ is the angular velocity about the y-axis.

The moment about the longitudinal axis for a nonfinned shell is simply

$$M_z = C_{l_p} q S d \ (pd/V)$$

where C_{l_p} is called the roll damping moment coefficient. Note that pd/V is sometimes denoted by nu (ν), the spin in radians per caliber.

If one defines α as the component of yaw in the vertical direction and the horizontal component is β, then the total yaw angle δ is $\beta + i\alpha$ where $\tan^{-1}\alpha/\beta$ is the yaw orientation (Murphy, 1963).

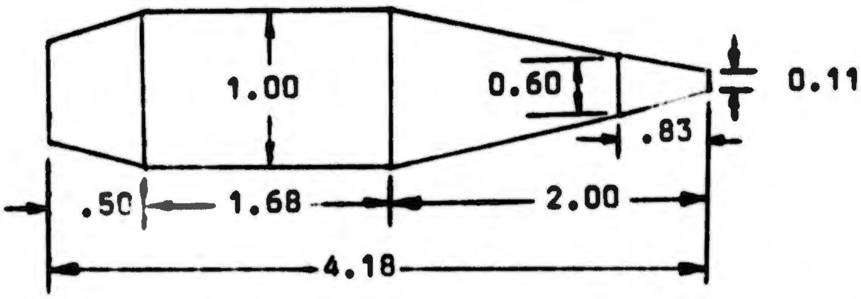
The aerodynamic coefficient slopes (or derivatives) can be defined in terms of α because of the rotational symmetry of a projectile; these values can be obtained by measurements on a model with a yaw in one plane, identified as the α- plane (Kelly et al., 1944).

If one views the projectile from the front, β is positive to the right and α is positive upward. A projectile with a right-handed spin (counterclockwise when viewed from the front) experiences a magnus force downward when β is positive. Note that the center of pressure of the force being aft of the cg of the projectile causes a positive magnus moment since it adds to the static moment produced by positive α and $C_{M\alpha}$.

APPENDIX II

Estimate of Projectile Stability

The following is an outline of a sample calculation for the spin stabilized projectile shown below; the details of the calculations are omitted, but the methods for estimating coefficients are given in a report by Wood (1954).



dimensions in calibers   1 cal = 4.98 "

Velocity: $V = 1925$ fps

Twist: $n = 28$ cal/turn

Spin rate: $p = 165$ rps $= 1040$ rad/ sec

Max diam: $d = 0.415$ ft

Air density $\rho = 0.002378$ slug/ft^3

$$I_x^2 / I_y = 0.0049 \text{ slug/ft}^2$$

$$= (\pi / 8) \rho d^3 V^2 C_{M\alpha}$$

$$S_g = p^2 I_x^2 / 4 I_y$$

$$S_g = \frac{0.0049(1.0816 \times 10^6)}{4(\pi / 8)(0.002387)(0.0715)(3.705625 \times 10^6)(3.59)}$$

$$= 1.49$$

The value of S_g is an indication that the projectile is stable, at least statically; Murphy (1963) has shown that the dynamic stability factor is given by

$$S_d = \frac{2 (C_{L\alpha} + K_a^{-2} C_{Mp\alpha})}{C_{L\alpha} - C_D + K_t^{-2} (C_{Mq} + C_{M\dot{\alpha}})}$$

where $C_{L\alpha} = 2.70 \text{ rad}^{-1}$,

Mach = 1.72,

$m = (46.08/ 32.2)$ slugs,

$I_x = 0.0359$ slug-ft^2,

$I_y = 0.2640$ slug-ft^2,

$K_a^{-2} = md^2/ I_x = 6.854$,

$K_t^{-2} = md^2/ I_y = 0.933$,

$C_{Mp\alpha} = 0.20$

$C_{Mq} + C_{M\dot{\alpha}} = -9.0$ and

$C_D = 0.33$.

Substituting these values into the equation for S_d yields

$$S_d = \frac{2 [2.70 + 6.864(0.20)]}{2.70 - 0.33 - 0.933(-9.0)} = \frac{8.14}{10.767}$$

$= \underline{0.756}$

Recall that the identity for stability was given in Chapter II as

$$1/S_g < S_d (2.0 - S_d)$$

Substituting for S_g and S_d, one obtains $1/S_g$ equals 0.671 and $S_d (2.0 - S_d)$ equals 0.94.

Since $0.671 < 0.94$, the conclusion to be drawn is that the projectile is stable.

APPENDIX III

Determination of the Period of the Yawing Motion

The yawing motion of a projectile can be represented by the following expression:

$$\delta = K_1\, e^{(-\lambda_1 + i\,\phi_1')z} + K_2\, e^{(-\lambda_2 + i\,\phi_2')z}$$

where K_1 and K_2 are the amplitudes of the nutational and precessional components which change at the rates λ_1 and λ_2, respectively.

It can be shown from the above expression for yaw as a vector sum of two components that one can write two orthogonal components of yaw in the form

$$\delta_1 = K_1 \sin \phi_1' z + K_2 \sin \phi_2' z$$
$$\delta_2 = K_1 \cos \phi_1' z + K_2 \cos \phi_2' z$$

where the coordinate system is so oriented that z= zero corresponds to the maximum yaw $\delta_2 = K_1 + K_2$. The rates at which the arms K_1 and K_2 turn, ϕ_1' and ϕ_2', respectively, are defined by the following relations:

$$\phi_1' = \frac{A\,\delta}{2\,d\,B}\,(1 + \sigma)$$ which is the

nutational rate, rad/ft and

$$\phi_2' = \frac{A\,\delta}{2\,d\,B}\,(1 + \sigma)$$ which is the

precessional rate, rad/ft, where A is the axial moment of inertia, ν is the dimensionless spin, rad/ cal, B is the transverse moment of inertia, d is the diameter, σ is equal to the square root of (1 - 1/S) and S is the stability factor.

The amplitudes, K_1 and K_2, vary along the range as

$$K_1 = K_{10}\, e^{-\lambda_1 z} \quad \text{and}$$

$$K_2 = K_{20}\, e^{-\lambda_2 z}$$

where λ_1 and λ_2 are the damping rates and K_{10} and K_{20} are the initial values of amplitude and are positive real numbers.

From this it follows that

$$\delta^2 = \delta_1^2 + \delta_2^2 = K_1^2 + K_2^2 + 2K_1K_2 \cos (\phi_1' - \phi_2')z.$$

But at the maximum yaw

$$(\phi_1' - \phi_2') z = 2n\pi \qquad n = 0,1,2...$$

Therefore, the period of is

$$P = \frac{2\pi}{\phi_1' - \phi_2'} .$$

For a complete mathematical treatment and analysis, the reader is referred to Karpov (1953), Hitchcock (1947), and McShane et al. (1953).

REFERENCES

REFERENCES

1. Bolz, R.E. and Nicolaides, J.D. A Method of Determining Some Aerodynamic Coefficients from Supersonic Free Flight Tests of a Rolling Missile. Aberdeen Proving Ground, Md.: BRL Report No. 711, 1949.

2. Braun, Walter F. The Free Flight Aerodynamics Range. Aberdeen Proving Ground, Md.: BRL Report No. 1048, July, 1958.

3. Briggs, Robert O., Kerwin, W.J., and Schmidt, Stanley F. Instrumentation of the Ames Supersonic Free Flight Wind Tunnel. NACA RM A52A18, 1952.

4. Bull, Gerald V. " Some Aerodynamic Studies in the Canadian Armament Research and Development Establishment Aeroballistics Range," Canadian Aeronautical Journal, Vol. 2, No. 5, May, 1956.

5. Charters, A.C. " Some Ballistic Contributions to Aerodynamics," Journal of Aerodynamic Science, Vol.14, No. 3, March, 1947.

6. Charters, A.C. The Linearized Equations of Motion Underlying the Dynamic Stability of Aircraft, Spinning Projectiles, and Symmetric Missiles. NACA TN 3350, 1955.

7. DeMerrite, Fred J. The Correlation of Range and Wind Tunnel Dynamic Stability Measurements. US Naval Ordnance Laboratory, White Oak, Md.: NAVORD Report 6765, Dec. 1959.

8. Dickinson, Elizabeth R. Physical Measurements ofProjectiles. Aberdeen Proving Ground, Md.: BRLTN-874, Feb. 1954.

9. Durkin, John M. and O'Neill, Edwin B. A Method of Determining the Aerodynamic Properties of Symmetric Bodies in Free Flight. Navalship Research and Development Center: Report 2872, March, 1969.

10. Eikenberry, Robert S. Analysis of the Angular Motions of Missiles. Sandia Laboratory Contractor Report (being published).

11. Eikenberry, Robert S. Dynamic Wind Tunnel Testing Techniques. AIAA Paper No. 66-752, 1966.

12. Fowler, R.H., Gallop, E.G., Lock, C.N.H., and Richmond, H.W. The Aerodynamics of A Spinning Shell. Phil. Trans. Royal Soc., London, 1920.

13. Haseltine, W.R. Averaged Equations for the Slow Oscillatory Motion of A Spinning Shell. NOTS TP 4355, June, 1967.

14. Hitchcock, H.P. Aerodynamic Data for Spinning Projectiles. Aberdeen Proving Ground, Md.: BRL Report No. 620, 1947.

15. Ingram, C.W., Butler, C.B., Martin, J.M., and Vasquez, R.A. A Three Degree of Freedom Wind Tunnel Testing Procedure. Presented at the Conference on Dynamics and Aerodynamics of Bomblets, Eglin Air Force Base, Fla., Oct., 1967.

16. Karpov, B.G. A Comparison of Aerodynamic Data Obtained by ' Yaw Card ' and Spark Photography Methods. Aberdeen Proving Ground, Md.: BRL Memo Report No. 728, Oct., 1953.

17. Karpov, B.G. The Accuracy of Drag Measurements as a Function of Number and Distribution of Timing Stations. Aberdeen Proving Ground, Md.: BRL R-658, 1948.

18. Kelley, H.R. An Analytical Method for Predicting the Magnus Forces and Moments on Spinning Projectiles. China Lake, California: NAVORD Test Station Tech. Memo. 1634, 1954.

19. Kelly, J.L. and McShane, E.J. On the Motion of a Projectile With Small or Slowly Changing Yaw. Aberdeen Proving Ground: BRL Report No. 446, 1944.

20. Kryvoruka, J.K. Effects of Inadequate Mathematical Models on Dynamic Data Analysis. Sandia Report. SCL-DR-69-84, Sept., 1969.

21. Liepmann, H.W. and Puckett, A.E. Introduction to the Aerodynamics of a Compressible Fluid. New York: John Wiley and Sons, Inc., 1947.

22. Loeb, Alfred A. Discussion Paper on Fin Stabilized Munitions. Seventh meeting of Tripartite Panel O-7, May, 1968.

23. May, Albert J. and Williams, T.J. Free Flight Ranges at the Naval Ordnance Laboratory. Silver Spring, Md.: NAVORD R-4063, July, 1955.

24. McMullen, J.C. Wind Tunnel Testing Facilities at the Ballistics Research Laboratories. Aberdeen Proving Ground, Md.: BRL Memo Report 1292, 1960.

25. McShane, E.J., Kelly, J.L., and Reno, F. Exterior Ballistics. Denver Press, 1953.

26. Moore, Frank G. A Study to Optimize the Aeroballistic Design of Naval Projectiles. NWL Tech. Rpt. TR 2337, Sept., 1969.

27. Moulton, F.R. New Methods in Exterior Ballistics. Chic., Ill.: University of Chicago Press, 1926.

28. Murphy, Charles H. Free Flight Motion of Symmetric Missiles. Aberdeen Proving Ground, Md.: BRL Report No. 1216, July, 1963.

29. Murphy, Charles H. The Measurement of Nonlinear Forces and Moments by Means of Free Flight Tests. Aberdeen Proving Ground, Md.: BRL Rpt. 974, 1956.

30. Murphy, Charles H. Data Reduction for Free Flight Spark Ranges. Aberdeen Proving Ground, Md.: BRL Rpt. 900, Feb., 1956.

31. Murphy, Charles H. and Nicolaides, J.D. A Generalized Ballistic Force System. Aberdeen Proving Ground, Md. BRL Rpt. No. 933, 1955.

32. Nicolaides, J.D. Firing Range Method of Obtaining Aerodynamic Data. Dragonfly Summary Report (MX-802) General Electric Co., 1948.

33. Nicolaides, J.D. A History of Ordnance Flight Dynamics. Presented at AIAA Conference (no. 70-533) Tullahoma, Tenn., May, 1970.

34. Nicolaides, J.D. Two Nonlinear Problems in the Flight Dynamics of Modern Ballistic Missiles. IAS Report 59-17, Jan., 1959.

35. Piddington, Maynard J. Aerodynamic Properties of A Caliber 0.50 Bullet With Reflex Boattail. Aberdeen Proving Ground, Md.: BRL Memo Report No. 1108, 1957.

36. Regan, F.J. and Edwards, G.R. Basic and Fortran IV Computer Program for the Rapid Determination of Projectile Stability. U.S. Naval Ordnance Laboratory, Md.: NOL Files (being published), Sept., 1969.

37. Regan, F.J. and Iandolo, J.A. Instrumentation, Techniques, and Analysis Used at the Naval Ordnance Labs for the Determination of Dynamic Derivatives in the

Wind Tunnel. U.S. Naval Ordnance Laboratory, Md.: NOLTR 66-23, Aug., 1966.

38. Regan, F.J. Magnus Effects. Presented at AGARD Conference (Proceedings No. 10), Paris, France, Sept., 1966.

39. Rogers, Walter K. The Transonic Free Flight Range. Aberdeen Proving Ground, Md.: BRL Rpt. No. 1044, June, 1958.

40. Shantz, Irving and Groves, Robert T. Dynamic and Static Measurements of the Basic Finner at Supersonic Speeds. U.S. Naval Ordnance Labs, Md.: NAVORD Rpt. 4516, Jan., 1960.

41. Shapiro, Ascher H. The Dynamics and Thermodynamics of Compressible Fluid Flow. New York: The Ronald Press, Inc., 1953.

42. Thomas, J.L. The Theory of a Spinning Shell. Aberdeen Proving Ground, Md.: BRL Rpt. 839, Nov., 1952.

43. Whyte, R.H. ' Spinner ' A Computer Program for Predicting the Aerodynamic Coefficients of Spin Stabilized Projectiles. Genaral Electric Report Class 2 69APB3, Aug., 1969.

44. Wood, R.M. Quick Methods for Estimating the Static Aerodynamic Coefficients of Shell. Aberdeen Proving Ground, Md.: BRL Memo Rpt. 854, 1954.