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13. ABSTRACT

This is the first of two volumes reporting the research activities on analytic modeling of air cavalry combat performed by Vector Research, Incorporated, under contract number DAAG25-71-C-0407. This volume contains an introduction to and summary of the research and modeling performed, a summary description of the differential models of attack helicopters supporting a battalion task force (which are fully documented in Volume II), and a description of the stochastic models of attack helicopters independently attacking ground targets, the program implementing these models, and a sample run of this program.

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1.0 INTRODUCTION

This is the first of two volumes reporting the research activities on analytic modeling of air cavalry combat performed by Vector Research, Incorporated, under contract number DAAG25-71-C-0407. This volume contains an introduction to and summary of the research and modeling performed, a summary description of the differential models of attack helicopters supporting a battalion task force (which are fully documented in Volume II), and a description of the stochastic models of attack helicopters independently attacking ground targets, the program implementing these models, and a sample run of this program.

1.1 Project Background

The importance of applying quantitative approaches to military planning activities is well recognized. Central to many of these activities, and of importance to weapons systems planning studies (selection, tactical doctrine, force mix, etc.) is the requirement for methods or models to predict the effectiveness of combat units when employing different mixes of equipment. This stems primarily from the fact that, in the planning phase, the systems of interest are not available to determine their effectiveness experimentally. Even when experimentation is feasible, methods are needed to provide guidance in conducting the experiments, in analyzing the data, and in interpolating or extrapolating the data to systems and environments not considered in the experiment. It is important that the effectiveness estimating methods be related to decision variables under the control of the military planner in a way such that the effect of their variations may be readily observed.

The development of methods to measure or predict effectiveness of military units, and identification of the variables which significantly contribute to combat effectiveness, has been limited for a number of reasons. By definition, measures of a tactical unit's effectiveness should reflect the degree to which the unit accomplishes its mission. Additionally, it is well known that mission accomplishment is highly dependent on a complex interaction among weapon system characteristics, threat variables, organizational structures, tactics employed, and environmental conditions. One approach used has been to develop simple "indicators" of combat effectiveness such as the "firepower score," indices of "combat effectiveness," and "single-shot kill probabilities." These indicators (a) do not measure accomplishment of unit missions, (b) essentially ignore most of the above factors which affect mission accomplishment, and (c) oftentimes bear little relation to the physical combat or other process under study.

A second and most heavily used approach to predict the effectiveness of military units is that of Monte Carlo simulation. This approach is essentially one of modeling the detailed behavior of the military process, explicitly including weapon system capabilities, threat, environment, and other factors which affect mission accomplishment. The activities of each system, its movement, the acquisition of each target, etc., are recorded during the course of a battle and eventually analyzed.

Solution of such models is essentially an experiment in which each individual activity of the process is sampled and replicated a large number of times to obtain a statistical stability. The literature reflects the existence of a large number of Monte Carlo simulations used to analyze defense planning problems (Adams, 1961; Roberts, 1963; Quade, 1964; USACDC, 1968; USACDC, 1969; Bishop and Clark, 1969).

Although Monte Carlo simulations are employed heavily in military planning circles, there are some meaningful drawbacks to their sole use as effectiveness assessment tools which are well recognized in the systems analysis community. Immediately evident is the loss in generality or flexibility since a new simulation must be developed for each class of weapon system or level of organization examined. Associated with a simulation is the large expenditure of time and financial resources for the development and utilization of the model. It is not unreasonable to expect to spend 10 - 15 man-years just developing a simulation of a specific combat or other military process. Additionally, it would not be unreasonable to expect each replication of the simulation to require 10 - 20 minutes of computer time,[1] and anywhere from 10 - 60 replications for statistical stability of the results. The large number of variables usually included in simulations makes it extremely difficult to perform parametric sensitivity analysis over the simulation assumptions

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[1] Tests conducted with the Carmonette ground combat simulation required two minutes of computer time to simulate one minute of battle in a single replication (Adams, 1961, p. 35).

and input data. This is due to both the statistical experimental design problems and money constraints which prohibit the number of replications needed to determine the distribution of outcomes. Finally, and perhaps most importantly, the large amount of detail contained in a simulation makes it difficult to use by itself as a tool for analysis, i.e., to single out the weapon system capabilities, and/or doctrine, which significantly contribute to the combat effectiveness.

In contrast to the Monte Carlo simulation approach, a limited amount of effort has been devoted to developing and using analytic (mathematical) models to predict the effectiveness of military units. In this approach the physical combat or other military situation is studied and decomposed into its basic elements; mathematical descriptions of these elements are developed and these element descriptions are integrated in an assumed overall mathematical structure of the process dynamics. Solutions are obtained by consistent mathematical operations giving rise to relationships between independent variables and the dependent ones of mission effectiveness. This approach has a number of obvious advantages both in its own right and as a powerful supplement to Monte Carlo simulations. Time and financial resources for development and utilization are usually markedly reduced. In analytic formulations the relationship between independent factors of the process and the process output is usually explicitly presented, facilitating both sensitivity analysis and determination of those independent variables which significantly contribute to combat effectiveness. Analytic structures are usually more general, thus facilitating more generalized use of the models across different combat organization levels and equipment. Experience has shown that analytic models are extremely valuable as supplements to Monte Carlo simulations in defense planning studies by aiding in the simulation debugging process, facilitating extensive sensitivity analyses, and in interpreting the study results (e.g., University of Michigan, 1969; VRI, 1970).

For these reasons, it is necessary to develop analytical models of relevant military processes that can be efficiently and effectively used to analyze division-level combat and its component activities in mid-intensity warfare. This objective requires the development of analytic structures for each of the relevant component processes (such as combat, reconnaissance, etc.) and research on methods of integrating them to describe division-level activities. Some of the component activities that eventually will have to be analytically structured include

1. Armored battalion task forces
    (a) Mounted assault role
    (b) Fixed defense role
2. Air cavalry activities
    (a) Aerial reconnaissance and surveillance
    (b) Independent attack of ground targets
    (c) Aerial fire support for battalion task forces
3. Dismounted infantry engagements
4. Armored cavalry combat surveillance
5. Forward area (short-range) air defense
6. Indirect fire support

7. Intelligence activities
8. Command and control activities
9. Logistical processes.

Although this project is specifically directed at items 2b and 2c, it is worthwhile to review the background of Army analytic modeling in some detail in order to put the work of this project in context.

Recognizing the difficulties associated with the sole reliance on Monte Carlo simulation, the Office of Naval Research (under contract N00014-67-A-0181-0012) and the Directorate, Weapon Systems Analysis, Hq., Department of the Army (under contract DAHC15-68-C-0314) jointly sponsored a research program with the Systems Research Laboratory, the University of Michigan, in 1968-69 to develop combat prediction methods that alleviated many of the above deficiencies of Monte Carlo simulations and could be used to supplement them in planning studies. The objective was to develop methods that were inexpensive in time and dollars to employ and, more importantly, readily interpretable to provide insight to those factors that produce effective and ineffective combat units.

The initial analytic methodology was completed in August 1969, with the focus on its applicability to the combat activities of battalion task force organizations. The methods developed are mathematical descriptions of combat in the form of sets of differential equations and other mathematical procedures required to generate appropriate inputs to the differential equations. The mathematical structures describe basic combat functions of firepower, maneuver, logistics, weapons assignment, and intelligence.[1]

Because of the complexity and abstractness of the differential combat analysis methodology, questions were raised in 1969 regarding the applicability of the methods to real planning problems. These discussions gave rise to a separate contract (contract DAHC15-70-C-0151) with Vector Research, Incorporated, (VRI) to compare the combat predictions generated by the differential model of combat to those predicted by more detailed Monte Carlo simulation methods. Under this study, the general methodology was applied to a set of tactical situations used in the TATAWS-III study, which is part of the overall main battle tank (MBT-70) study program (USACDC, 1968). The Individual Unit Action Monte Carlo simulation of ground combat was used in the MBT-70 program to evaluate candidate main battle tank systems in proposed battle task force structures. Using the same input data for the weapon system capabilities and force structures, the output (combat effectiveness) predictions of both models were compared. The comparisons were made on short-range defense and long-range attack engagements for different weapon systems and force structures. The comparisons indicated that predictions were essentially the same from both models. The computer program written to test the model when applied to tank-antitank combat situations is currently running at the Pentagon computer facility.[2]

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[1] The methods developed and associated computational programs are documented in Bonder and Farrell, 1970, and Spaulding, 1971.

[2] These programs, as revised by the user agency, are documented in Spaulding, 1970.

In summary, analytic combat-effectiveness prediction methods capable of describing armored battalion task force assault and fixed defense engagements have been developed. These methods have been shown to produce effectiveness predictions similar to those of more detailed Monte Carlo simulations. The benefits associated with these methods are that they are flexible (in that they can readily be applied to many combat situations), responsive (the application to the tank-antitank spectrum of engagements was accomplished in less than 90 days), and efficient to employ (one minute of computer time can simulate anywhere from 5 - 50 minutes of real battle). Perhaps most importantly, the simple analytic structure of the model makes it relatively easy to determine logically which factors contribute significantly to the effectiveness of the units. That is, one can readily ascertain the doctrine, force composition, or weapon system capabilities that contribute most heavily to the effectiveness of combat units.

This background, coupled with interest in the selection, employment doctrine, and force mix problems associated with helicopters led to this project to develop analytic structures which describe activities of the air cavalry squadron. Eventually, such structures should describe the complete activities of a squadron in its aerial reconnaissance role, its combat role as a quick reaction force against ground targets, and its combat role as fire support for battalion task forces in order to analyze the following types of questions:

(a) What are the acquisition, mobility, and protection tradeoffs that influence the effectiveness of light observation helicopters (LOH)?

(b) What are the acquisition, mobility, protection, and firepower trade-offs that influence the effectiveness of aerial fire-support systems (AFSS)?

(c) What is an appropriate force mix between LOH and AFSS gunships in an air cavalry squadron?

(d) What are appropriate employment tactics for an AFSS with an associated number of LOH's?

(e) How wide a front should an AFSS cover with an associated number of LOH's?

(f) What degree of mobility must an AFSS possess to respond adequately to intelligence provided by LOH's?

In this research program, analytic models were developed to represent two specific kinds of air cavalry activities:

(1) The direct fire support of engaged ground units, and

(2) The independent attack of enemy ground targets.

Two entirely different models were developed to represent these activities. To model the first activity, a deterministic differential model of battalion task force combat, based upon the differential models in use in studying armor and anti-tank problems, was designed to include supporting attack helicopters and air defense weapons. To model the second activity, a new stochastic analytic model was developed. Both mathematical models were implemented in computer programs.

Chapter 2 provides a general description of the differential model programs (the AIRCAV programs) developed in this study. It parallels more detailed material from Volume II of this report. There are two differential model programs incorporating AH and ADW activities, differing principally in the detailed assumptions and logic of their *ground* scenarios and the format of their data bases. Both models treat a battalion-level engagement between Red and Blue forces, with Blue forces including attack helicopters (AH's) in direct support and Red forces including air defense weapons (ADW's). These programs were constructed as modifications of existing differential model programs treating ground combat without AH support.

The scenarios represent a Blue armored battalion task force, which can be either attacker or defender, in combat with an appropriate Red force. The defending force is deployed in fixed, defilade positions while the opponent conducts the attack along three major axes with up to four pre-determined routes of advance per axis. Maneuver weapons on each route consist of tanks and armored personnel carriers, which are supported by long-range and medium-range antitank weapons employed in fixed positions as overwatch weapons. Medium-range antitank weapons are allowed to dismount from maneuvering APC's at point along the attack routes. Defending weapons also consist of tanks, APC's and antitank weapons. Indirect fire from artillery is played for both sides.

A complete description of all ground force movement, terrain line-of-sight, and concealment is taken as input by the programs. The maneuver weapons follow the pre-planned routes in accordance with this description, and may fire when they are not moving and are inside a range chosen for allowing fire, or if they are moving, have a moving-fire capability, and are within a range chosen for allowing moving fire. Attack helicopters mask and unmask in patterns dependent on their firing activities.

Fire is directed only at targets which have been acquired, with acquisitions occurring as a result of either pinpoint or non-firing detection. Targets are chosen from those which have been acquired in the order of a priority which is fixed throughout the battle and is based on target type and location. Firers are allowed more than one round type, and chose among them as a function of the target type and the range. For a given round-target combination the accuracy, lethality, and timing of the fire are dependent on such firer-target statuses as velocity, cover, range, etc. The AIRCAV5 model treats two processes not treated in AIRCAV1 - direct-fire suppression and area-effects kills of weapons with exposed crews.

The underlying ground combat scenarios and their detailed assumptions are based on situations developed for use in various studies using the Independent Unit Action (IUA) Monte Carlo simulation model. The two programs use scenarios and assumptions from different studies which have used the IUA. Specifically, the two programs are

(a) AIRCAV1 which plays the IUA scenarios used in the TATAWS III study (USACDC, 1968), but also uses data from the MBT-70 Producibility/Cost Reduction Study (Battelle, 1969). There are four basic scenarios presently available: two Blue attack and two Blue defense scenarios at opening ranges of 1.0 and 2.5 km.

(b) AIRCAV5, which plays the scenarios and weapons mixes of the Antitank Weapons Systems Requirements (ATMIX) Study (USACDC, 1970). There are presently available three Blue defense scenarios at opening ranges of 0.5 km., 1.0 km., and 2.5 km.

The outputs of the differential models include a complete time-step history of the firing activities, casualties, and survivors[1] of the engagement.

The independent helicopter attack model (implemented in the IHA program) depicts an engagement between a Blue attack helicopter unit acting independently of Blue ground elements and a Red armored ground unit; a situation which might ensue during a screening mission. The scenario begins with the attack helicopters at the assault position.[1] The information gathering elements have completed their reconnaissance of the target area and have selected assault and firing positions[2] for the attack helicopters.

The attack helicopters attack in mass, employing mask cresting and standoff techniques in an attempt to gain a tactical advantage over the ground elements. As nearly as possible, they unmask from their assault positions in unison, select targets, and fire their TOW missiles. Following missile impact each attack helicopter remasks. Successive volleys follow the same pattern.

The model does not explicitly consider activities of the Blue intelligence element (scout helicopters), but rather, reflects their performance in the target acquisition and assignment input data. It is assumed that the combat occurs in rolling or mountainous terrain, which allows the selection of fully covered assault positions and

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[1] A position where the attack helicopter makes its final coordination, deploys before engaging the target, and from which it moves directly to the firing position.

[2] A firing position is a position from which an attack helicopter will engage the enemy.

alternate firing positions. All firing positions have clear line of sight to the target area. Blue helicopters and Red ground elements may detect their targets visually or may employ any other appropriate sensor system. The meteorological visibility allows target detection at the Blue helicopter standoff range for all targets.

The independent helicopter attack models and the associated IHA program are documented in chapter 3 through 6 of this volume.

2.0 SUMMARY DESCRIPTION OF THE DIFFERENTIAL MODELS

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The ground combat scenarios represent a Blue armored battalion task force, which can be either attacker or defender, in combat with an appropriate Red force. The defending force is deployed in fixed, defilade positions while the opponent conducts the attack along three major axes with up to four pre-determined routes of advance per axis. Maneuver weapons on each route consist of tanks and armored personnel carriers, which are supported by long-range and medium-range antitank weapons employed in fixed positions as overwatch weapons. Medium-range antitank weapons are allowed to dismount from maneuvering APC's at point along the attack routes. Defending weapons also consist of tanks, APC's and antitank weapons. Indirect fire from artillery is played for both sides.

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The models developed in this program are not Monte-Carlo simulations, but are deterministic analytic models of the situations represented as discussed in chapter 1. They are based upon similar previously existing models of the ground combat situations (documented by the user agency in Spaulding, 1971). In this chapter we will present the assumptions used to include attack helicopters and air defense weapons in the differential models in section 2.1, and summarize the methodology of the models in section 2.2.

2.1 Assumptions Used to Model AH and ADW Activities

This section reviews specific assumptions concerning the scenario used to incorporate attack helicopter and air defense weapon activities into the differential models of combat. Areas discussed are: participating weapons; possible firer-target pairs; weapon deployment; and weapon exposure. Following this list of scenario assumptions, the tactical assumptions will be reviewed. This list was arrived at by VRI in conjunction with USACDCSAG personnel and with reference to air cavalry training manuals (USACDC Armor Agency, 1971).

(a) Participating Weapons

(i) Up to three AH types and up to three ADW types may participate in a battle, with the actual number to be input for each combat assessment.

(ii) AH ordnance will be automatic cannon and TOW missile or equivalent, with only one ordnance type in use at one time by one AH.

(b) Possible Firer-Target Combinations

(i) Red ground weapons in addition to air defense weapons will be able to engage AH's.

(ii) ADW's will be allowed to engage Blue ground weapons as well as AH's, but will fire only while stationary.

(c) Weapon Deployment

For AH operations a set of firing positions will be specified as input. All helicopter exposure and firing will be from locations in the immediate vicinity of these points. AH's may fly from one firing position area to another, but will be fully covered when doing so. Flight times for such movements will be provided as input if such movements are permitted in the specific combat. AH's will not be permitted to engage in a general advance on the Red positions, due to the impossibility of using present data bases to generate exposure histories for the AH's in such an advance.

(d) Weapon Exposure (Cover and Visibility)

(i) The AIRCAV model user will have a choice of two options for AH cover and concealment statuses which can be specified by input. When masked in the assault position all AH's are considered completely covered. When unmasked in the firing position, either

1. their cover will be that of a ground weapon at that coordinate, or
2. they will be completely exposed and unconcealed with respect to all Red elements.

Alternatives with different cover statuses for AH's were eliminated due to computer storage requirements believed necessary.

(ii) The cover and concealment statuses of attacking Red elements with respect to AH's will be full exposure, and defending Red elements will be in defilade with respect to AH's, but no AH in a mask period will be able to see or fire at any Red element.

(iii)The cover and concealment of ADW's with respect to Blue ground weapons will be the same as any other Red ground weapon in the same position. All other alternatives were excluded because of implementation difficulties in changing the basis cover and concealment logic.

(iv) The cover and concealment of Blue ground weapons with respect to ADW's is the same as it would be to any other Red ground weapon in the same location. All other alternatives were excluded because of implementation difficulties in changing the basic cover and concealment logic.

The tactics assumptions for employment of AH and ADW units will now be reviewed in the same format used to review the scenario assumptions. Areas discussed are: maneuver; target selection and priority of fires; firing doctrine; and round selection.

(a) Maneuver

(i) AH's will operate independently of each other in their masking and unmasking maneuvers. Coordinated maskings and unmaskings are ruled out.

(ii) The decisions for AH's to move from axis to axis are preplanned and described in a table of inputs.

(iii) In Red attacks ADW's may advance with the maneuver elements or be co-located with an overwatch position, the choice being input dependent. ADW's may not move along routes and terrains not otherwise used in the models. To do otherwise would have introduced problems in analyzing cover and concealment conditions.

(b) Target Selection and Priority of Fires

(i) AH's will independently attempt to acquire targets and will fire at the highest priority target found in a fixed period of time (with the priority list and time period inputs, and the input time period representing the time from mask-cresting to readiness for weapon launch), or if no target has been acquired the AH remasks.

(ii) ADW's can fire at either AH's or ground weapons, giving priority to targets on their own axis (unless an input list of target priorities supplied by the user specifies otherwise).

(iii) Priority ties in target selection are allowed for all firer weapon types. The ATMIX1 and ATMIX5 models previously assumed that targets could be ranked in strict order of priority with no ties. The AIRCAV models will allow priority ties (and a means of breaking them) as an option. A physical realization of priority ties occurs when it is impossible to discriminate between two or more target types.

(iv) The user will have the option of giving the ADW's on each axis the capability to share acquisitions (if one ADW has acquired a target, then all on that axis have acquired it). The other option will be the independent mode of acquisition (each system has to search for and detect its own targets).

(c) Firing Doctrines

(i) AH's will fire only one missile per exposure. Secondary armament is used in burst mode, and a single target is chosen for all rounds until it is killed or the AH remasks.

(ii) All three types of ADW's can fire either bursts or independent rounds, the choice being input-dependent for each type, such fire occurring only while the ADW is stationary.

(d) Round Selection

(i) AH ordnance is chosen by a rule referring only to firer and target type and target range, and it can be varied as input data.

(ii) It is assumed that any one type of ADW will carry only one type of ordnance in a specific combat.

2.2 *Differential Methodology*

This section describes the mathematical methodology of the differential models of combat and an overview of the program structure of the AIRCAV models. A more detailed discussion of the mathematics of the differential models of combat is given in Bonder and Farrell (1970).

For each side in a battle it is hypothesized that in a short period of time,

(a) locations change due to tactical movement,

(b) weapon systems are attritted by enemy activity,

(c) resources are expended, and

(d) personnel become casualties due to enemy activity.

Focusing on the loss of weapon systems and personnel, it is assumed that, if the state of the battle at the beginning of the small interval is known, the rate at which weapons systems and personnel are attritted during this small interval can be predicted. It is because of this rate focus that the mathematical structure employed to model the combat activity is that of differential equations.

For convenience, names are assigned to the numbers of different groups of systems in each force. Let

m_i = the number of surviving Blue units of the i^{th} group ($i = 1,2,\ldots, I$), and

n_j = the number of surviving Red units of the j^{th} group ($j = 1,2,\ldots,J$),

where different groups are determined by their differing abilities to attrit weapon systems of an opposing group. Thus, a missile weapon system and a rapid-fire machine gun would be in different groups since the rates at which they can attrit targets of an opposing group are different. Additionally, similar weapon system types will form different groups if they are at different ranges to the target and this range difference affects their ability to attrit it. Thus, a tank platoon 1000 meters from the target is a different group than another tank platoon 3000 meters from the target.

The overall analytic structure of the combat activity is based on assumptions that

(a) the rate of loss of units in the j^{th} Red group due to the i^{th} Blue group is proportional to the number of units in the i^{th} Blue group with a proportionality factor called the <u>attrition coefficient</u>, and,

(b) the rate of loss of units in the j^{th} Red group is the sum of the rates of losses due to different i^{th} Blue groups.

Mathematically, these assumptions take the form of the following coupled sets of differential equations:

$$\frac{dn_j}{dt} = - \sum_{i=1} A_{ij}(t)m_i \qquad \text{for } j = 1,2,\ldots,J$$

$$\frac{dm_i}{dt} = - \sum_{j=1} B_{ji}(t)n_j \qquad \text{for } i = 1,2,\ldots,I$$

where

$A_{ij}(t)$ = the utilized per system effectiveness of systems in the i^{th} Blue group against the j^{th} Red group at time t. This is called the Blue attrition coefficient.

$B_{ji}(t)$ = the utilized per system effectiveness of systems in the j^{th} Red group against the i^{th} Blue group at time t. This is called the Red attrition coefficient.

Although not explicitly shown, resources expended are explicitly contained in the development of the A_{ij} and can be determined directly from the model. The coefficients $A_{ij}(t)$ and $B_{ji}(t)$ are explicitly dependent upon such indices of battle status at time t as weapon position, velocities, angular aspect, and cover and concealment. In the computational program these differential equations are approximated by the difference equations

$$m_i(t+\Delta t) = \max\{0, m_i(t) - \sum_{j=1}^{J} B_{ji}(t)n_j(t)\Delta t\} \quad \text{for } i = 1,2,\ldots,I$$

$$n_j(t+\Delta t) = \max\{0, n_j(t) - \sum_{i=1}^{I} A_{ij}(t)m_i(t)\Delta t\} \quad \text{for } j = 1,2,\ldots,J$$

where Δt is the computational time step.

It is noted that this formulation is a deterministic one which treats the numbers of surviving forces (m_i and n_j) as continuous variables, while clearly the actual battle activity is a random phenomenon and m_i and n_j are integer-valued variables. Although many probabilistic arguments are contained in this formulation, the output of the model is a deterministic trajectory of the surviving numbers of forces.

The attrition coefficients (A_{ij} and B_{ji}) are, as one would expect, complex functions of the weapon capabilities, target characteristics, distribution of the targets, allocation procedures for assigning weapons to targets, etc. The model attempts to reflect these complexities by partitioning the total attrition process into four distinct ones:

1. The effectiveness of weapons systems while firing on live targets,
2. The allocation procedure of assigning weapons to targets,
3. The inefficiency of fire when other than live targets are engaged, and,

4. The effect of terrain on limiting the firing activity and on mobility of the systems.

The first three effects are included in the attrition coefficient as

$$A_{ij}(t) = \alpha_{ij}(t)e_{ij}(t)I_{ij}(t) \tag{3}$$

$$B_{ji}(t) = \beta_{ji}(t)h_{ji}(t)K_{ji}(t) , \tag{4}$$

where

$\alpha_{ij}(t)$ = the <u>attrition rate</u>, the rate at which an individual system in the i^{th} Blue group destroys j^{th} group Red targets at time t when it is firing at them.

$e_{ij}(t)$ = the <u>allocation factor</u>, the proportion of the i^{th} Blue group systems assigned to fire on the j^{th} group Red targets at time t.

$I_{ij}(t)$ = the intelligence factor, the proportion of the i^{th} group firing Blue weapons allocated to the j^{th} Red group which are actually engaging live j^{th} group Red targets at time t.

Similar definitions exist for the components of the Red attrition coefficient, B_{ji}. The intelligence factor has not been considered in any applications to date, i.e., $I_{ij} = 1.0$ for all i,j.[1]

The methodological basis for calculating the allocation factor, $e_{ij}(t)$, is situation-dependent, since it depends on what assumptions are made concerning target selection doctrines in a given application. The factor not merely reflects target availability, as limited by line-of-sight and acquisition, but also reflects any attempts made by a firer or group of firers to

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[1] The IUA simulation, which the VRI differential models are designed to emulate, makes the same assumption of perfect intelligence.

allocate their fire to targets which are in some sense valuable and against which their weapons are effective. The question of optimum allocation strategies has been examined by Sternberg (1971) for some simple differential-type engagements. The allocation factors used in the AIRCAV models are derived later in this chapter.

Sections 2.2.1 and 2.2.2 are devoted to examining the prediction of the attrition rates and allocation factors displayed in equations 3 and 4. Sections 2.2.3 and 2.2.4 discuss these topics with relation to attack helicopter modeling. Section 2.2.5 briefly reviews the logic of the AIRCAV programs.

2.2.1 The Attrition Rate

If the stochastic sequence of times between successive kills by a weapon system (against a passive target array) is a renewal process, it has been shown that the appropriate attrition rate for use in the differential models is the reciprocal of the mean time between kills. (See Bonder and Farrell (1970) and Barfoot (1969)). Symbolically,

$$\alpha_{ij}(t) = \frac{1}{E(T_{ij}|t)} \quad , \qquad [5]$$

where $E(T_{ij}|t)$ is the expected time for a single Blue system of the i^{th} group to destroy a passive j^{th} group Red target, given the battle conditions of time t. Formulae for the determination of attrition rates from more elementary weapon system data have been published elsewhere (Bonder and Farrell, 1970) for systems using several firing doctrines. The formulae for those firing doctrines employed by weapons in the AIRCAV models, referred to as Markov fire, independent fire, and burst fire are discussed below.

The assumptions of Markov fire weapon systems models are

(a) lethality is due to an impact, as opposed to area lethality,

(b) the probability of kill given an impact is identical for every round fired,

(c) the probability that a round fired after a preceding hit or miss results in a hit or miss is not influenced by the knowledge of other history of the engagement (such as the number of rounds fired or the number of previous hits), and

(d) the engagement terminates immediately on a kill.

Markov fire is used to describe the fire of a tank main gun using a "burst-on-target" aim adjustment doctrine, in which accuracy refinements are made on the basis of the previous round fired. Let

P_1 = first round hit probability,

p = conditional probability of a hit given the preceding round missed the target,

u = conditional probability of a hit given the preceding round hit the target,

P_k = probability of a kill given a hit,

τ_a = mean time to acquire targets,

τ_1 = mean time to fire the first round,

τ_h = mean time to fire a round given the preceding round was a hit,

τ_m = mean time to fire a round given the preceding round was a miss,

τ_f = mean projectile flight time.

Under this doctrine the mean time to kill can be shown to be

$$E(T) = \tau_a + \tau_1 - \tau_h + \left(\frac{\tau_h+\tau_f}{P_k}\right) + \left(\frac{\tau_m+\tau_f}{p}\right)\left[\frac{1-u}{P_k} + u - P_1\right]. \qquad [6]$$

It should be noted that, although the symbols denoting dependence on i, j, and t were dropped in equation 6, the parameters may be dependent on both the firer-target pair and the battle conditions at time t.

Independent fire is a special case of the Markov doctrine arising when accuracy and timing are independent of the results of the preceding round. Such a situation occurs in the AIRCAV models with guided missile systems. Let $P_1 = p = u$ be denoted θ and $\tau_h = \tau_m$ be denoted τ_s. The expected time to kill is then

$$E(T) = \tau_a + \tau_1 - \tau_s + \left(\frac{\tau_s+\tau_f}{\theta P_k}\right) . \qquad [7]$$

For weapon systems with the capability for burst fire let

P_1 = single burst kill probability for the first burst

P_2 = single burst kill probability for subsequent bursts.

The result is a special case of equation 6:

$$E(T) = \tau_a + \tau_1 - \tau_s + (\tau_s+\tau_f)\left[1 + \left(\frac{1 - P_1}{P_2}\right)\right], \qquad [8]$$

where τ_s is the time to fire a burst, rather than a single round.

The parameter τ_a in these formulae is designed to measure target acquisition delays only in cases where acquisitions and firing periods alternate (serial acquisition). In cases where acquisition goes on during firing, as in the IUA simulation, $\tau_a = 0$. Therefore in the IUA versions of the differential models, from which the AIRCAV models have been constructed, $\tau_a = 0$.

2.2.2 The Allocation Factor

The attrition coefficient was defined above as the product of an attrition rate and an allocation coefficient, $e_{ij}(t)$, the fraction of group i allocated to fire at group j at time t. This section presents a method for calculating allocation factors based on parallel acquisition and a target priority list which allows more than one target to be tied at the same level of priority to a firing weapon. This is a generalization of the differential models from which the AIRCAV models were adapted, which lacked the capability of treating priority ties and computed allocations on the basis of a strictly ordered list of targets (weapon aggregates).

Let each weapon aggregate i have an ordered set of target collections, $\{C_{i1}, \ldots, C_{iK}\}$ where C_{ik} is a collection of all target groups at the k^{th} highest level of priority and where K is the number of such collections. It is assumed that the priority of targets in C_{i1} is higher than that of targets in C_{i2}, etc. Assume all weapon-target acquisitions conditional on exposure statuses and histories are independent. Let $q_{ij}(t)$ be the probability that a single weapon in group i has acquired no target in group j at time t. We assume that at any time, a weapon will fire at the highest priority target it has acquired, and we define $e_{ij}(t)$ as the expected fraction of group i firing at a group-j target in accordance with this discipline.

These assumptions give immediately the fact that

$$\sum_{j\varepsilon C_{ik}} e_{ij}(t) = \left(1 - \sum_{h<k} \sum_{j\varepsilon C_{ih}} e_{ij}(t)\right) \left(1 - \prod_{j\varepsilon C_{ik}} q_{ij}(t)\right).$$

This determines the amount of fire at each priority class as a whole, and leaves the problem of allocating fire array equal priority groups. For consistency, it is required that for any subset S of C_{ik}

$$\sum_{j\in S} e_{ij}(t) \le \left(1 - \prod_{j\in S} q_{ij}(t)\right)\left(1 - \sum_{h<k}\ \sum_{j\in C_{ih}} e_{ij}(t)\right)$$

that is, that no more firers can be allocated to a set of targets than make appropriate acquisitions. Within this constraint, two methods of allocation have been developed. One is based on the assumptions about the physical acquisition process and the other is approximate and requiring significantly less computation.

The detailed allocation method is based on the following assumption: any weapon which has acquired more than one highest priority target will fire at a randomly selected target from these acquisitions. Then for any $j \in C_{ik}$

$$\frac{e_{ij}(t)}{1 - \sum_{h<k}\ \sum_{j\in C_{ih}} e_{ij}(t)}$$

$$= p_{ij}(t) - \frac{1}{2} \sum p_{ij}(t)p_{is}(t) + \frac{1}{3} \sum p_{ij}(t)p_{is}(t)p_{ir}(t)$$

$$- + \dots$$

where $p_{ij}(t) = 1 - q_{ij}(t)$ and the summations are over the sets of distinct pairs, triples, etc. of target groups in C_{ik}. This form requires a great deal of computer time to use if there are many tied groups, and a simpler approximation has been developed and is used in the AIRCAV models.

In the approximation,

$$\frac{e_{ij}(t)}{1 - \sum_{h<k} \sum_{j \in C_{ih}} e_{ij}(t)} = \frac{p_{ij}(t)}{\sum p_{ij}(t)} .$$

The approximation form does satisfy the inequalities given above, and is thus partially consistent with the acquisition model.

The q_{ij}'s are determined in the models by continual solution of the basic acquisition process. If

$Q_{ij}(t)$ = one minus the single round pinpoint probability for a weapon in group i observing a group j weapon under the conditions of combat at time t,

$\lambda_{ij}(t)$ = non-firing acquisition rate for a group i weapon observing a fully-exposed group j weapon. The time to acquire is assumed negative exponentially distributed. The rate is zero if the target is in defilade or fully covered,

$r_j(t)$ = rate-of-fire of one group j weapon, and

$N_j(t)$ = number of survivors in group j at time t,

$q_{ij}(t)$ is determined by the equations

$$q_{ij}(0) = 1$$

$$q_{ij}(t) = 1 \qquad \text{if there is no line of sight}$$

$$f_{ij}(t) = \lambda_{ij}(t)N_j(t) + r_j(t)N_j(t)\ln(1-Q_{ij}(t))$$

for fully exposed targets, and

$$f_{ij}(t) = r_i(t)N_j(t)\ln(1-Q_{ij}(t))$$

for partially exposed targets,

where

$$f_{ij}(t) = -\frac{d}{dt}\ln q_{ij}(t).$$

2.2.3 AH Attrition Coefficients

Earlier, the Blue attrition coefficient, $A_{ij}(t)$, was shown to be the product of an attrition rate and an allocation factor. The purpose of this section is to derive attrition rate and allocation factor for an AH behaving according to the assumptions listed below.

A full list of assumptions concerning the incorporation of AH's and ADW's into the IUA scenarios was given in section 2.1. The AH attrition rate can be derived from the following three of those assumptions:

(a) Maneuver. AH's will operate independently of each other in their masking and unmasking maneuvers. Coordinated, simultaneous maskings and unmaskings are ruled out.

(b) Target selection. AH's will independently attempt to acquire targets and will fire at the highest priority target found in a fixed period of time (with the priority list and time period as inputs, and the input time period representing the time from mask-cresting to readiness for weapon launch), or if no target has been acquired the AH remasks.

(c) Firing doctrine. AH's will fire only one missile per exposure. Secondary armament is used in burst mode, and a single target is chosen for all rounds until it is killed or the AH remasks.

To reiterate the assumed helicopter firing doctrine: each AH acts independently of the others, alternating between a masked assault position (completely hidden from the enemy) and an unmasked firing position, from which it can fire its ordnance. Each AH spends a fixed amount of time searching for a target after unmasking. If no target is acquired during the given interval, the AH remasks. Otherwise he selects the highest priority target acquired, launches a missile, guides it to the target, and immediately remasks. If the automatic cannon is used instead of the missile, it is fired for a fixed length of time (possibly dependent on target range), and the AH remasks without engaging another target.

Attrition rates are used in the models to calculate the expected attrition of one group of targets by another group. In the old (ground weapon-oriented) models the attrition rates are taken to be a function of the weapon systems' statuses and are varied as the cover, velocity, etc. vary. In modeling the AH's attrition of ground targets another approach will be taken.

In the old approach it was not possible to use average attrition rates over the various status conditions since the conditions varied in a complex deterministic manner which could not reasonalby be evaluated by any technique but direct computation. In the AH case, the AH statuses vary in a random, but much more foreseeable way, and their variation may be reasonably analyzed before combat computations. For this analysis it is necessary to use AH tactical rules, AH performance data, and terrain maps to generate a description of assault position-to-firing position flight times and times between exposure periods.

Accordingly, the AH attrition rates will be average attrition rates over the complete AH maneuver pattern. In the new approach the effects of acquisition, target selection, and exposure are included in the computation of the attrition rates, rather than being calculated by letting the attrition rates vary as the weapon statuses are simulated in detail.

The mean time-to-kill involves the following quantities to be determined from the interaction of AH tactics, AH performance data, and the battlefield terrain:

t_s = time spent in search during each exposure period

t_f = time to launch and fly-out the missile to the target

T_c = expected length of a masked period

p_E = probability of acquiring and engaging a target during t_s. An eligible target is one which has been acquired, is on the target selection list of the AH, and is within range of the ordnance of the AH.

SSKP = missile single-shot kill probability (dependent on target type, range, and other engagement statuses).

The search time and the fly-out time, t_s and t_f, are required to be constants for any given situation, but the duration of the covered state may be a random variable and only its mean is required for the derivation. The derivation is simplified through the application of the following three random variables:

TBM = time between arrivals of successive missiles at the target range (time between firings)

N_k = number of missiles needed to kill the target

T_k = time to kill the target.

It is assumed that P_k is the same for every launch against the given target. That is, we are interested in calculating the attrition rate for fixed target status conditions and for fixed launch conditions.[1] With these assumptions the number of missiles required to kill is a geometrically-distributed random variable with expected value

$$E(N_k) = {}^{1}/_{SSKP} \quad . \qquad [1]$$

Similarly, it is assumed that the engagement probability P_E is constant over all unmasked periods under fixed firer and target statuses. Consequently, the number of exposure periods between missile launchings is also geometrically distributed (with mean $1/P_E$) and the expected time between missiles is

$$E(TBM) = t_f + \frac{1}{P_E}(\bar{T}_c + t_s). \qquad [2]$$

Since N_k and TBM are independent as defined above, the expected time-between-kills is

$$E(T_k) = E(N_k)E(TBM). \qquad [3]$$

$$E(T_k) = \frac{1}{SSKP}\{t_f + \frac{1}{P_E}(\bar{T}_c + t_s)\}. \qquad [4]$$

If the automatic cannon is used instead of the missile, it is necessary to define t_f as the time required to fire a fixed number of bursts and to remask. Then SSKP must be defined as the probability of killing the target in that fixed number of bursts. This formula for $E(T_k)$ determines the attrition rates for attack helicopters. This leaves the allocation factor to be determined.

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[1] These conditions will, of course, vary during a battle.

From the above it can be seen that for attack helicopters, unlike the other weapons, the expected time-to-kill formula includes the time spent in acquiring a target. Since the attrition rate for a helicopter is not conditioned on an allocation being made, the allocation factor for the helicopter need only give the distribution of fire over the targets, *given that these targets are engaged*. That is, the helicopter allocation factors, unlike the e_{ij} for all other weapon types, must be normalized so that their sum over j is unity, rather than summing to the fraction of a group allocated to fire. This is the only change required in the derivation of the e_{ij}'s in chapter 1.

2.2.4 Attrition of Attack Helicopters

Earlier, the Red attrition coefficient, $B_{ji}(t)$, was defined as the product of an attrition rate and an allocation factor. Whether a Red ground weapon is engaging a Blue helicopter or another ground weapon, the attrition rates and the methodology for allocating firers to targets described previously are also applicable when targets included helicopters. The allocation factor, however, is a function of the probabilities of acquisition against all a firer's target groups, and the algorithm discussed previously and used in the differential models for acquisitons between ground weapons is not applicable to calculating the probability of acquiring helicopters that behave according to the modeling assumptions of the AIRCAV models. The purpose of this section is to derive the acquisition probabilities for ground weapons observing attack helicopters.

Between ground weapons the AIRCAV models treat line-of-sight (LOS) deterministically -- given the coordinates of observer and target the cover status of the target is determined uniquely from an analysis of the terrain.

The AH exposure status must be treated probabilistically, however, since the durations of the masked and unmasked intervals are random variables. Consider one ADW observer and one AH target and assume that acquisition is independent of acquisitions of all other targets. Let T_E denote the duration of one AH exposure period. T_E is a two-valued random variable with probability distribution

$$P\,\{T_E = t_s + t_f\} = P_E$$

$$P\,\{T_E = t_s\} = 1 - P_E$$

Although t_s and t_f are constant for a given situation, in practice the AIRCAV computer programs compute mean values of t_s and t_f averaged all the members of an AH aggregate allocated to different targets, since the flyout time may vary from target to target. The time under cover is assumed to be sampled from a renewal process with mean T_c. Consequently, the total AH masking-unmasking process is an alternating renewal process, the masked and unmasked intervals being independent renewal processes.

Figure 1 shows one realization of the exposure process, where the points marked E are unmasking times and those marked M are remasking times. In figure 2 a Red ADW's acquisition process has been superimposed over the exposure process. In accordance with the assumptions of the IUA model every weapon's acquisition process proceeds in parallel with and independently of its firing process, so that the ADW can search for targets at the same time it is engaging another. The time-to-acquire for the ADW is taken as negative exponentially distributed with probability density $f_D(t) = \mu \exp(-\mu t)$. This density is in accordance with the IUA assumption that non-firing (visual)

Realization of AH Exposure Process:

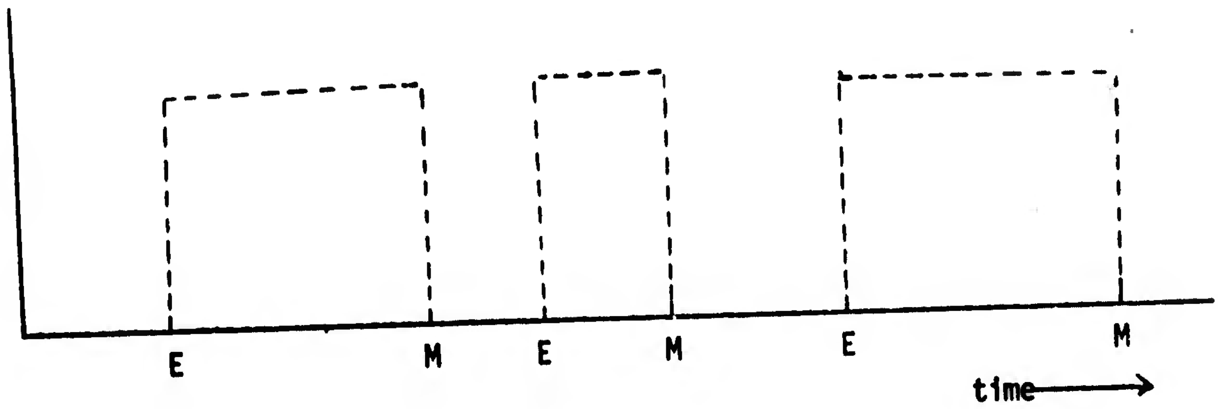

Figure 1

The Target Availability Process:

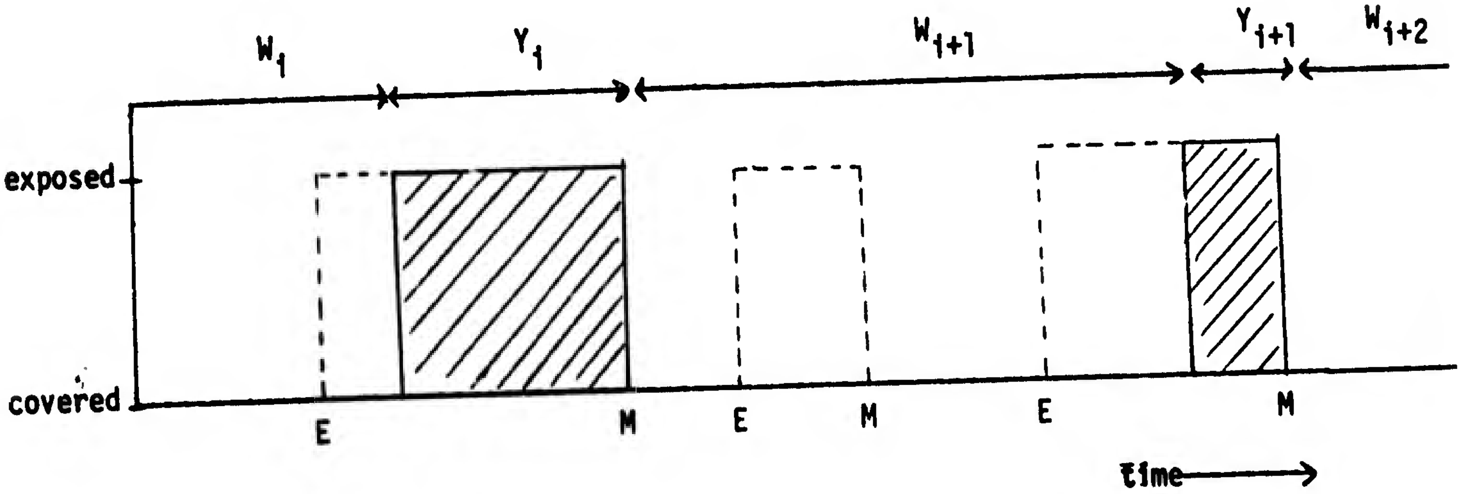

Figure 2

detection times are negative exponential, but it further assumes that pinpoint acquisitions can be smoothed over the exposure period well enough to be subsumed into a negative exponential density.

Of interest is the proportion of time the AH is in the acquired state (denoted by Y_i) versus the unacquired state (denoted by W_i) with respect to the given ADW observer. It will now be shown that the long run fraction of time the AH is in the acquired state is just π, where

$$\pi = \frac{E(Y)}{E(W) + E(Y)} \quad . \tag{3}$$

Once π is known it is possible to calculate the probability the ADW has acquired at least one AH out of a group of N as $1 - (1-\pi)^N$, since attack helicopters act independently. It is known that the sequence of exposure times and cover times $\{(T_{c,i}, T_{E,i})\}$ forms an alternating renewal process, but the sequence $\{(W_i, Y_i)\}$ is not as well behaved. Although Y_i and W_{i+1} are independent, the length of the acquired state Y_i is stochastically dependent on the length of the preceding unacquired interval W_i, so that $\{(W_i, Y_i)\}$ is not an alternating renewal process.

Let $X_i = W_i + Y_i$. This is the duration of one acquired state/unacquired state cycle. The $\{X_i\}$ do form a renewal process, since the process parameters are assumed stationary and succeeding X_i are independent. It is now useful to look at the new process $\{(X_i, Y_i)\}$, which is referred to as a renewal reward process. Y_i and X_i are obviously dependent, but their independence is not required in what follows. Let

$N(t)$ = the number of X_i cycles in $(0,t)$

$$Y(t) = \sum_{i=1}^{N(t)} Y_i \quad .$$

Then $Y(t)$ is the cumulative amount of time spent in the acquired state in $(0,t)$. We are interested in the fraction of time spent in the acquired state, which is $Y(t)/t$. Theorem 3.16 in Ross (1970) gives the following result:

Theorem. If $E(|Y_1|)$ and $E(X_1)$ are finite, then with probability 1,

$$D(t)/t \rightarrow E(Y_1)/E(X_1) \quad \text{as } t \rightarrow \infty \, . \tag{4}$$

Since $\pi = D(t)/t$, by definition equation 3 follows from equation 4. The remainder of this section is devoted to deriving $E(Y_1)$ and $E(W_1)$.

Let T_D denote the Red random time to detect the AH. Since the AH is detected during an exposure period only if $T_D < T_E$ (Red detects before AH remasks), equation 6 follows:

$$E(Y_1) = E(T_E|T_D<T_E) - E(T_D|T_D<T_E) \tag{5}$$

The first term of [5] is

$$E(T_E|T_D<T_E) = (\frac{t_s}{P_D})(1-P_E)(1-e^{-\mu ts}) + (\frac{t_s+t_f}{P_D})P_E(1-e^{-\mu(t_s+t_f)}) \, , \tag{6}$$

where

$$P_D = Pr\{T_D<T_E\} = (1-P_E)(1-e^{-\mu ts}) + P_E(1-e^{-\mu(t_s+t_f)}) \, . \tag{7}$$

The second term of [5] is $E(T_D|T_D<T_E) =$

$$\frac{1-P_E}{\mu(1-e^{-\mu ts})}[1 - (\mu ts+1)e^{-\mu ts}] + \frac{P_E}{\mu(1-e^{-\mu(t_s+t_f)})}[1 - (\mu ts+\mu t_f+1)e^{-\mu(t_s+t_f)}] \tag{8}$$

Then $E(Y_1)$ is just [6] minus [8]. Since the number of maskings needed to detect the AH is geometrically distributed with mean $1/P_D$, the expected time spent in the unacquired state is

$$E(W_1) = T_c/P_D + (\frac{1}{P_D} - 1)E(T_E|T_E<T_D) + E(T_D|T_D<T_E) \quad . \qquad [9]$$

The only term of equation 9 not yet derived is given by equation 10:

$$E(T_E|T_E<T_D) = \frac{t_s(1-P_E)e^{-\mu ts}}{1-P_D} + \frac{(t_s+t_f)P_E e^{-\mu(t_s+t_f)}}{1-P_D} \quad . \qquad [10]$$

With this, we have derived all the mathematics necessary to describe a combat. A brief description of the logic of the AIRCAV programs implementing this mathematics is given in section 2.2.5.

2.2.5 AIRCAV Program Description

This section presents a brief overview of the structure of the AIRCAV1 program. AIRCAV5 does not differ significantly at the level of detail presented here. More detailed descriptions are available in Volume II of this report.

Approximately half of the program is concerned with reading input data and initializing various arrays. The data provides information on maneuver element mobility, cover and concealment statuses, weapon dimensions and locations, and firepower performance data. The input data is described in detail in Volume II, sections 5.3 and 5.4. The next section of the programs computes a table of expected times-to-kill for stationary firers versus stationary firers for several values of range, cover, and other engagement status indices. This is followed by a short section setting up the initial locations of all the groups in the game. The remainder of the main program computes a numerical, time step solution to the set of differential equations discussed above.

The program next proceeds to the numerical, time step solution of the differential equations of combat. The solution is a trace of the number of survivors by group and by weapon type versus time. The solution procedure consists of the following steps:

(1) Update loops

First, the location, velocity, terrain roughness type, cover and concealment statuses of all maneuver groups are updated, with the exception of any ADW elements which may be maneuver groups. Next, attritions due to any artillery fires terminating in the current time increment are computed.

In a separate loop changes in the statuses of AH and ADW elements are registered. Since AH and ADW units are co-located with already existing weapon aggregates, their statuses are updated on the basis of the new statuses of their associated groups. Their statuses can change even if they are not associated with maneuver groups, since their line-of-sight to maneuver groups may change.

(2) Ground weapon firing loop

Fire by GW's (ground weapons, exclusive of ADW's) is computed as follows:

(a) Consider each group K in sequence as a firing group (K not an AH or ADW group).

(b) For each firer K take its targets in order of priority level, J. At each level there can be a collection of targets tied at the same level of priority.

(c) Edit the J-level target collection to exclude any ineligible targets, such as those beyond the maximum range of the firer.

(d) Allocate to each target group in the J-level collection a fraction of firing group K according to the allocation scheme described in section 2.1.

(e) Consider each target group IA in the J-level collection. If both K and IA are stationary, and IA is not an AH or an ADW, the attrition of group IA is computed using the table of expected times-to-kill. If either group is moving, the value of the expected time-to-kill against AH and ADW targets was not tabulated and must be calculated here.

(f) For each firing event the results are printed out in a firing event table that gives the group numbers of firer and target, their coordinates and velocities, the mean time-to-kill, the fraction of the firer allocated to that target, and the attrition suffered by the target.

(3) ADW firing loop

Considering each ADW group in sequence as a firer, the attrition of targets is computed as described in (2), except that new data structures were introduced for ADW's and that no table of expected times-to-kill was stored for ADW firers.

(4) AH firing loop

Considering each AH group in sequence as a firer, the attrition of targets is computed much as is done for ADW firers. Acquisitions of AH targets are also computed here, whereas the acquisitions by other firers are computed outside their attrition loops.

(5) Acquisition of non-AH targets

For each combination of non-AH observer and non-AH target acquisitions are computed as follows:

(a) Pinpoint acquisitions are computed based on the amount of firing done by the group being looked at and on the parameters of the optical system of the acquiring weapon type.

(b) Visual acquisitions (non-pinpoint) are computed using a rate of acquisition based on range and other statuses of the acquiring weapon and the target weapon.

(6) Acquisitions of AH targets

This is a double loop, iterating on the target group number, J, and the observer group number, I, as is done in the preceding acquisition loop. A separate loop is needed for these acquisitions, since a different data base is used, and since the acquisition probabilities of AH targets are computed differently than for group weapon systems. The method is described in section 2.3.

(7) Computation of survivors

Survivors are computed by weapon group and by weapon type as the number at the start of the time increment minus the sum of all attritions by opposing weapons during the time increment. If the result is negative, the number of survivors is set to zero.

(8) Output

The status of the battle at the current game time is printed, giving

(a) The survivors by group.

(b) The surviving fraction of attacker and defender "break-type" weapons. These values are used in determining whether or not the end of the game has been reached and the time at which the force effectiveness index (FEI) is computed.

(c) A killer-victim scoreboard showing the cumulative kills by each weapon type on each opposing weapon type.

(d) The survivors by weapon type.

(9) Analysis point check

If the Primary Analysis Point or a Blue Break occurs during the time increment, the FEI is computed and printed. The Primary Analysis Point is defined as the point at which 50 percent of the Red tanks have been killed, if this point is reached before a Blue Break. A Blue Break is defined as the point at which 70 percent of the Blue break-type weapons have been killed. The break-type weapons are the Blue tanks, if any, or all Blue antitank weapons if there are no Blue tanks in the game. If the user has specified a special scenario with no Blue group weapons the FEI is not computed. The FEI is the ratio of the fraction of Red firepower killed by Blue to the fraction of Blue firepower killed by Red and is calculated in the subprogram FEI.

(10) End of game check

The game ends when either

(a) The surviving fraction of break-type weapons on either side is .01 or less, or

(b) the mobility data indicate that the attacker has reached a maximum time limit, or

(c) if no Blue ground weapons are participating, and there are fewer than .5 AH or ADW survivors.

3.0 MATHEMATICAL MODELS OF INDEPENDENT HELICOPTER ATTACKS

This chapter describes the mathematical models developed on this project to describe the probabilities of survival of weapons on both sides in an independent attack on ground targets by attack helicopters. Section 1 describes the basic situation being modeled and section 2 describes the basic mathematical structure developed to describe it. The basic model is in fact a hierarchy of related models at different levels of detail. The models as programmed in the independent helicopter attack (IHA) program require data which is a significant level of abstraction above the basic hardware detail of the weapons, and the methods available for deriving this data from more basic parameters are presented in section 3. Section 4 presents an introduction to and overview of the IHA program, which is presented in detail in chapter 4.

3.1 The Military Situation

The independent helicopter attack model depicts an attack by a Blue attack helicopter unit, acting independently of Blue ground elements, on a Red armored or mechanized information; a situation which might arise, for instance, in the performance of a screening mission. (The differential combat models described in chapter 2 and in more detail in Volume II model the activities of attack helicopters in direct support of ground forces.) The scenario treated in this model begins with the attack helicopters at an assault position -- a position where the helicopters deploy before engaging the target forces -- and after the information and command/control elements have made their reconnaissance of the target area and selected firing positions and targets.

Although the model was principally designed for the case in which the helicopters attack in mass, employing mask cresting and standoff techniques, it can be used for cases in which the helicopters employ running fire. In the mask cresting situation for which the model was originally designed, the helicopters emerge from covered positions simultaneously (or as nearly simultaneously as possible) and acquire (or fail to acquire) their targets. They then fire their missiles and remask or fire their cannon for a limited time and then remask. The intelligence and command/control elements then coordinate the continuation of the attack using the same basic tactics. In the running fire situation, the helicopters use cannon only and maneuver while firing. The engagement or firing periods are typically somewhat longer.

There is no basic model limitation on the types of weapon systems which participate, although the model data may be generated by different techniques for different kinds of weapons. The helicopters are assumed to fire at and be fired at by up to nine different groups of weapons,[1] and also to fire at weapons which are not effective against them, and are treated only as targets. The total numbers of weapons is limited only by available computer storage. As written, the programs provide room for scenarios involving about 15 active weapons (as distinguished from the Red weapons which are ineffective against helicopters or beyond their effective ranges).[2] A simple change in three program statements,

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[1] Use of all the groups will require additional computer storage to that presently assigned.

[2] The exact formulas which relate the numbers of weapons to the required storage are in section 4.3. Engagements with numbers of weapons significant orders of magnitude greater than 15 might be better treated with the deterministic models.

clearly identified in chapter 4, will increase the storage and the allowable weapons.

3.2 *The Mathematical Model*

Because of the possible small numbers of attack helicopters (AH's) in operating units, a probabilistic (stochastic) description of the AH combat activity was selected for the analytic model developed in this study. Accordingly, the model describes the history of the combat activity in terms of the joint probability distribution of all surviving forces (AH and all active ground types) and the fire delivered to other targets. That is, the model is designed to answer the questions: "At time t, what is the value $F_t\ (x_1,\ldots,x_{10})$ of the probability that there are x_1 AH survivors and x_2 Red survivors from group two, x_3 from group 3,..., and x_{10} from group 10?" and "Given that at time t there are x_1 AH survivors and x_2 through x_{10} Red survivors from groups 2 through 10, what is $E_t(x_1,\ldots,x_{10})$, the conditional expected amount of AH fire which has been directed at Red targets other than the active weapons in groups 2 through 10?"

The remainder of this section is divided into two sub-sections. Section 3.2.1 presents the structure of the basic process model which was developed during the project, and section 3.2.2 explains the approximations which were made in order to make computation possible within reasonable constraints. Appendix A to this volume documents the initial investigatory research conducted to obtain as complete as possible a description of the basic underlying processes in attack helicopter attacks. The material presented there is at a significantly lower level of abstraction than the analytic model as finally developed in this project. The increased abstraction inherent in analytic models is one of the properties which enables them to provide a greater "transparency" in use (that is, easier identification and understanding of significant cause-effect relations).

3.2.1 The Process Model

The basic features of the mask-cresting tactic which were used to suggest the basic model structure are:

(1) That combat occurs in distinct periods during each of which the behavior of the engaged forces, conditioned upon their surviving strengths, is governed by an identical process, and

(2) That each AH can kill more than one target in a period only with negligible probability. The first of these features suggested a Markov chain model of the process, and the second was used in determining the form of the transition probabilities.

In describing the model, we will use and extend the notation used in the previous section. Specifically, let:

$F_t(x_1,\ldots,x_{10})$ be the probability that there are x_1 AH's and x_i Red weapons in group i surviving (for $i = 2,\ldots,10$).

$E_t(x_1,\ldots,x_{10})$ be the conditional expectation of the amount of fire which has been directed at other targets (measured as the number of periods in which an AH has directed its fire at such targets), conditioned on the survival of exactly x_1 AH's and x_i Red weapons in group i.

t_j (for $j = 1, 2, \ldots$) be the times at which the AH's are just unmasking for the j^{th} time, with $t_1 = 0$.

$T(x_1, \ldots, x_{10}; y_1, \ldots, y_{10})$ be the conditional probability that there are exactly $x_1, \ldots, x_{10}$ survivors at the end of an exposure period given that there were $y_1, \ldots, y_{10}$ at the opening of the period.

$L(x_1, \ldots, x_{10}; y_1, \ldots, y_{10})$ be the conditional expectation of the number of AH's directing their fire at targets other than those in groups 1 through 10, given that there were $y_1, \ldots, y_{10}$ survivors at the beginning of the interval and $x_1, \ldots, x_{10}$ surviving at the end.

Then from elementary probability theory, we know that

$$F_{t_{j+1}}(x_i) = \sum_{(y_i)} F_{t_j}(y_i)\, T(x_i; y_i)$$

and

$$E_{t_{j+1}}(x_i) = \frac{\sum_{(y_i)} F_{t_j}(y_i) \left\{ E_{t_j}(y_i) + L(x_i; y_i) \right\} T(x_i; y_i)}{F_{t_{j+1}}(x_i)}$$

where the compressed notations $F_t(x_i)$, etc., have the meaning $F_t(x_1, \ldots, x_{10})$, etc.

Thus, if one can determine the T and L functions, one can generate the survival probabilities for a battle at the times t_j from the initial strengths $X_1, \ldots, X_{10}$, which determine F_{t_1} as

$$F_{t_1}(x_i) = \begin{cases} 1 & \text{if } x_i = X_i \text{ for all } i \\ 0 & \text{otherwise.} \end{cases}$$

After one has the F_{t_j}'s, one can, of course, generate summary measures such as means, variances and co-variances, conditional means, and others at each t_j. The F_{t_j}'s and E_{t_j}'s thus constitute a complete description of the battle history.

For those familiar with Markov chain theory, it should be noted that T is the transition probability matrix for the Markov chain on the states (x_i). Complete, detailed mathematical methods to generate the exact form of T exist, but are not computationally feasible. In this project, approximations to T and L were developed to make computer programming possible. These approximations are described in section 3.2.2. Section 3.3 discusses the generation of the information needed for the models from more basic data on the tactics and weapon systems.

Although developed for the mask-cresting, standoff fire tactic, the model structure may be applied to other tactics by treating the "exposure" periods as continuous rather than separated by periods in which the AH's are behind terrain. The periods may then be taken to be simply short fixed-length time periods.

3.2.2 Mathematical Approximations Used to Determine T and L

The functions T and L are basically functions of three kinds of data: scenario data, tactics data, and weapon systems performance data. Appendix A gives some idea of the kinds and volumes of data which might be used if available. The most important tactics data describes the process of target selection, and the most important scenario and weapons performance data interact to determine the effectiveness of the participating weapons at attriting each other.

In determining the forms of T and L, we assume that two basic sets of data are known:

(1) the rule for allocating AH's to targets,[1] and

(2) the expected number of opposing weapons which a weapon system can destroy in an exposure period, given that they follow the tactics which they would follow in combat, but accomplish no attrition of the firing weapons (this is the total probability of achieving a kill for AH's).[2]

From these data, approximations to T and L may be developed as follows.

Since the superposition of independent renewal processes is, in the limit, a Poisson process, we approximate the distribution of the number of AH's which would be killed if no Red firers were killed (which we will later refer to as potential kills) as a Poisson distribution

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[1] And for programming purposes, we have assumed that the rule is a priority rule, identifying target groups in priority order, such that AH's will be assigned on a one AH per target basis in priority order.

[2] This is directly parallel to the attrition rate concept of the differential models.

with its mean the sum of the expected number of kills for the individual Red weapons alive at the commencement of a period.

For each Red group of active weapons, the distribution of potential kills is assumed to be binomial, with maximum number equal to the number of AH's allocated to fire on targets in the group and probability equal to the total or compound probability of kill of an AH on a target in this group, including all acquisition and reliability effects.

The important step in the approximation is the following: for each set of initial survivors (y_i) and each set of potential kills (k_i) with $i = 1,\ldots,10$, we approximate the conditional distribution of the number of actual kills and then take the distribution of final survivors (x_i) to be the marginal distribution of survivors (still conditioned on (y_i), but no longer on the (k_i)). Symbolically,

$$T(x_i;\, y_i) = \sum_{(k_i)} Pr_1\,(k_i;\, y_i)\; Pr_2\,(x_i;\, k_i;\, y_i)$$

where Pr_1 is the probability described earlier, specifically

$$Pr_1\,(k_i;\, y_i) = \begin{cases} e^{-\lambda}\,\dfrac{\lambda^{k_1}}{k_1!}\displaystyle\prod_{i=2}^{10}\binom{A_i}{k_i} p_i^{\,k_i}(1-p_i)^{A_i-k_i} & (k_1 < y_1) \\[2ex] \left(\displaystyle\sum_{j=k_1}^{\infty} e^{-\lambda}\,\dfrac{\lambda^{j}}{j!}\right)\displaystyle\prod_{i=2}^{10}\binom{A_i}{k_i} p_i^{\,k_i}(1-p_i)^{A_i-k_i} & (k_1 = y_1) \end{cases}$$

where

A_i is the number of AH's with targets in group i

λ is the total expected kills of AH's by Red $= \sum_{i=2}^{10} y_i \lambda_i$, with the λ_i the individual performance parameters for Red weapons in group i), and

p_i is the total probability of kill for an AH firing at a target in group i.

The approximation to T is thus reduced to the determination of a useful approximate form for Pr_2. The rule we have adopted is to take each $y_i - x_i$ to be binomially distributed with maximum k_i and probability a measure of the average strength of the enemy forces.

Specifically, we take R, the Red strengths, to be defined by

$$R = \frac{\lambda + \sum_{i=2}^{10} (y_i - k_i) \lambda_i}{2\lambda}$$

and B, the Blue strength ratio, to be defined analogously as

$$B = \frac{2y_1 - k_1}{2y_1} \quad .$$

The total of these approximations may be expressed as follows: one determines (stochastically) the potential kills which each force will produce, neglecting the attrition of the forces in determining the kills. Then one determines an approximate average strength ratio for each force as the mean of the initial strength and the strength if all potential kills were actual. Then one stochastically selects among the potential kills which are actual, using the appropriate force's strength ratio as the probability that a potential kill is actual.

The IHA programs which implement the methodology described in this and the previous subsection are discussed in section 4. Section 3 discusses the generation of the λ's and the p_i's.

3.3 Data Requirements

The data required for the model, the λ_i's and the p_i's, might be estimated from experimental or historical data if it becomes available, but must at this point be generated from more basic data which are either measurable or capable of estimation or prediction.

The p_i's may be predicted as a function of simpler probabilities in accordance with the following formula (or any analogy appropriate to a slightly different set of weapon system parameters)

$$p_i = p_A \, p_L \, p_F \, p_K$$

where

p_A is the probability that an AH actually acquires its target in group i within appropriate tactical limits on time and including the effects of range, target visibility, and possible operational failures,

p_L is the probability that an AH can and does launch its ordnance against its target after it acquires it, including all effects of reliability, weather, and operational failures,

p_F is the probability of successful flight (given a launch), of the AH's ordnance, taking all range and other effects into account, and

p_K is the probability that ordnance which flies successfully will kill the target, taking range and other effects into account (this parameter may sometimes be treated as the product of a hitting probability and a kill-given-a-hit probability).

The λ_i's, the expected numbers of passive AH's which a Red weapon can kill in an exposure period, can be determined from any detailed model of weapon performance. Possible methods include the use of the attrition rate models usually associated with the differential combat models or the use of the attrition rate from the air defense gun analytic model presented in the University of Michigan, 1969. The attrition rate formulae are presented briefly in chapter 2 of this volume, in some more detail in Volume II, and in great depth in Bonder and Farrell, 1970. Attrition rates, however determined, should be multiplied by the average exposure time of an AH in the engagement in order to determine λ_i.

It is worth noting that users may wish to include in their computations of λ_i the expected number of kills by one weapon of weapons other than its target. In determining this number, which may be significant for low-performance air defense weapons and any homing or self-guided ordnance, one should assume an appropriate spacing of AH's as dictated by doctrine and the assumed tactical situation and compute the kill probabilities for high-dispersion or homing weapons against an entire target array.

3.4 The IHA Program

The mathematical models presented in section 2 are implemented by the IHA program. The program accepts an initial force strength vector in terms of the number of AH's and of each Red weapon group from 2 to 10. The Red weapons may be grouped in any way in which each group has a reasonable degree of homogeneity in its performance data and the performance data of the AH's against it. It also reads the p_i's, the λ_i's, a target priority for selection of targets by AH's, and the number of periods for which it is to compute and display results.

The model first computes the transition probability matrix T and the transition lethality function L. It then displays the initial state information F_{t_1} and E_{t_1}, and computes F_{t_2} and E_{t_2} and displays them. It continues for the required number of periods, essentially performing a matrix multiplication ($F_{t_{i+1}} = T \cdot F_{t_i}$, where the F's and T are interpreted as state probability vectors and transition probability matrix, respectively) for each period.

The output from the model is a trace of the survival state probability F_{t_i}, the expected lethality vector, E_{t_i}, and the mean survivors for each period, t_i, up to the maximum the program is to run. The programming details are described in chapter 4. Chapter 6 contains a sample run of the model.

The case used in the sample run in Chapter 6 involves a single helicopter and two groups of ground weapons with one weapon per group. The helicopter probabilities of kill against the ground weapons are both .8 and the expected number of AH kills by the ground weapons are with .4. There are thus 8 states, which are denoted here on 000, 001, 010, 011, 100, 101, 110, 111, with the three digits representing the surviving helicopters, group 2 weapons, and group 3 weapons, respectively. It is assumed that group 3 is a higher priority target than group 2.

The Pr_1 matrix for this case is shown in figure 1, the Pr_2 function in table 1, and the resulting T matrix in figure 2. The case printout is in chapter 6.

FIGURE 1: THE Pr_1 MATRIX

Y_i-K_i Y_i	000	001	010	011	100	101	110	111
000	1.	0.	0.	0.	0.	0.	0.	0.
001	0.	1.	0.	0.	0.	0.	0.	0.
010	0.	0.	1.	0.	0.	0.	0.	0.
011	0.	0.	0.	1.	0.	0.	0.	0.
100	0.	0.	0.	0.	1.	0.	0.	0.
101	.264	.066	0.	0.	.536	.134	0.	0.
110	.264	0.	.066	0.	.536	0.	.134	0.
111	0.	0.	.440	.110	0.	0.	.359	.090

TABLE 1: THE Pr_2 FUNCTION

Pr_2	Y_i	Y_i-K_i	X_i	R	B
.25	101	000	000	.5	.5
.25	"	"	100	"	"
.25	"	"	101	"	"
.25	"	"	001	"	"
1.0	"	001	"	1.0	.5
0.0	"	"	101	"	"
1.0	"	100	100	.5	1.0
0.0	"	"	101	"	"
1.0	"	101	"	1.0	1.0
.25	110	000	000	.5	.5
.25	"	"	100	"	"
.25	"	"	110	"	"
.25	"	"	010	"	"
1.0	"	010	"	1.0	.5
0.0	"	"	110	"	"
1.0	"	100	100	.5	1.0
0.0	"	"	110	"	"
1.0	"	110	"	1.0	1.0
.375	111	010	010	.75	.5
.375			011	"	"
.125			110	"	"
.125			111	"	"
1.0		011	011	1.0	.5
0.0			111	"	"
1.0		110	110	.75	1.0
0.0			111	"	"
1.0		111	"	1.0	1.0

FIGURE 2: THE T MATRIX

	000	001	010	011	100	101	110	111
000	R	I	I	I	I	I	I	I
001	N	R	I	I	I	I	I	I
010	N	I	R	I	I	I	I	I
011	N	N	N	R	I	I	I	I
100	N	I	I	I	R	I	I	I
101	.066	.132	I	I	.602	.200	I	I
110	.066	I	.132	I	.602	I	.200	I
111	L	P	.165	.275	L	P	.414	.145

where

I = 0 because the transition represents increasing forces

N = 0 because the transition represents a kill with no enemy survivors to have caused it

L = 0 because the transition represents an AH killing more than one target

P = 0 because the transition would involve AH fire on non-priority targets

R = 1 because the transition is from an absorbing state in which no fire at active units will occur

4.0 THE IHA PROGRAMS

The computer program which implements the mathematical models of Chapter 3 is a set of interrelated subroutines. Section 1 of this chapter describes the logic of these routines, section 2 discusses the data formats used, and section 3 is a description of data storage techniques and a glossary of the principal variable names used in the programs. Chapter 5 is a program listing, and chapter 6 contains a sample output from a run of the programs in a simple situation.

4.1 Program Logic

The program consists of 10 related routines (including one utility routine which is presently unused, but may be useful if the programs are modified). The basic functions are described in the following list.

SUBROUTINE	FUNCTION PERFORMED
MAIN (cards 200-1600)	Reads the data from logical unit 5 and passes control to WORK. MAIN will terminate the run with a STOP 0 instruction when control is returned.
WORK (cards 1700-12000)	Manages the bookkeeping for and passes control to: (1) the subroutine MATRIX, which computes the transition probability matrix (T in chapter 3, PRTRAN in the programs) and the transition lethality matrix (L in chapter 3, EL in the programs); (2) the subroutine OUTPUT, which displays

SUBROUTINE	FUNCTION PERFORMED
	survival state probabilities (F_{t_i} in chapter 3 and NEWPR or PREVPR in the programs), conditional lethality delivery vectors (E_{t_i} in chapter 3 and NEWEL or PREVEL in the programs); and (3) the subroutine MULT which multiplies the transition probability matrix by the previous state probability vector to obtain the new state probabilities.
MATRIX (cards 14800-36200)	Computes the transition probability matrix (T in chapter 3, PRTRAN in the programs).
MULT (Cards 44900-49500	Multiplies a previous state probability vector by the transition probability matrix to abtain a new state probability vector ($F_{t_{i+1}} = T \cdot F_{t_i}$ in chapter 3, also written as $F_{t_i}(x_i)=\sum_{(y_i)} T(x_i;y_i)\ F(y_i)$ there, and NEWPR=PRTRAN PREVPR in the programs). This routine calls on the function PROBFN to compute the function

SUBROUTINE	FUNCTION PERFORMED
	denoted Pr_2 in chapter 3 (the conditional probability on a number of actual kills given a number of potential kills).
OUTPUT (cards 40700-44800)	Displays a state probability vector on logical unit 6.
PROBFN (cards 49600-56400)	Computes the function denoted Pr_2 in chapter 3 (the conditional probability of a number of actual kills given a number of potential kills).
PRIRTY (cards 12100-14700)	Computes AH allocations to targets from priority data.(Computes IALL from JRANKS, which is in common block PRIOR.)
INDEXF (cards 36300-37700)	Computes a linear subscript used in addressing transition matrices which are packed in storage (PRTAN and EL) from complete descriptions of the survival states transitted to and from (the y_i and x_i of chapter 3, which are N1 and N2 in the programs).
REVSUB (cards 3900-40600)	Computes a survival state description (N, N1, or N2-the x_i or y_i of chapter 3) from a linear subscript used in addressing state vectors (NEWPR, NEWEL, PREVPR,

SUBROUTINE	FUNCTION PERFORMED
	and PREVEL - the F_{t_i}'s and E_{t_i}'s of chapter 3).
SBSCPT (cards 37800-38900)	Computes a linear subscript from a survival state description. The reverse operation to REVSUB. This subroutine is not used at present, but may be useful to any user who wishes to modify the programs.

The following pages contain a narrative-style flow chart of the logic of the IHA programs.

FLOW CHART STEP	ACTION
1	Subroutine MAIN zeroes the working storage STRG. (seq. 1000-1100)
2	Subroutine MAIN reads the basic data NO,P,JRANKS, XP, and JTIMES. (seq. 1200)
3	Subroutine MAIN CALLs subroutine WORK (seq. 1400)
4	Subroutine WORK uses function INDEXF to compute storage table size for matrices. (seq. 3600)
5	Function INDEXF computes the size of a matrix by computing the linear subscript

FLOW CHART STEP	ACTION
	of the final element and RETURNS to WORK. (seq. 36300-37700)
6	Subroutine WORK computes the size of the state vectors (ISTATE), and the total storage needed. If this exceeds the available storage, WORK STOPs the programs. (seq. 4100-4900)
7	Subroutine WORK establishes valves for variables used in addressing arrays and initializes PREVPR, using an address in STRGE to do so. (seq. 5000-7300)
8	Subroutine WORK CALLs subroutine Matrix to compute PRTRAN, the transition probability matrix, and .EL, the transition lethality matrix. (seq. 7800)
9	Subroutine Matrix chooses a set of possible values for the initial survivors, potential kills, and actual survivors, using three nested sets of ten nested DO - loops each. In the process, it computes the condition of probability that the potential kills take the assigned values (Pr in chapter 3

FLOW CHART STEP	ACTION
	and GP in the program). In doing this, it computes the AH allocation to targets by CALLing the subroutine PRIRTY, (seq. 14800-32000)
10	Subroutine MATRIX uses function INDEXF to compute transition addressing information. (seq. 32400)
11	Subroutine INDEXF computes the addressing information and RETURNs to MATRIX. (seq. 36300-37700)
12	Subroutine MATRIX uses function PROBFN to compute, for the selected valves, conditional probability of the actual kills given the initial state and the potential kills (Pr_2 in chapter 3). (seq. 32500)
13	Function PROBFN computes the Blue and Red strength ratios BR and RR (B and R of chapter 3) checking to avoid division by zero at seq. 51700 and 52600. (seq. 49600-52800)

FLOW CHART STEP	ACTION
14	Function PROBFN computes the binomial probabilities defining Pr_2 of chapter 3 and multiplies them to obtain Pr_2. The array BINCFT of binomial coefficients is used in the computations. The function RETURNs to MATRIX. (seq. 52800-56400)
15	Subroutine MATRIX multiplies the returned valve by GP (computing $Pr_1 \cdot Pr_2$ of chapter 3) and adds this to the appropriate PRTRAN entry. (seq. 32500-32600)
16	Subroutine MATRIX computes the fraction of AH's which survive the engagement (FRACT) and multiplies this by the numbers of the AM's allocated to non-active targets (IALL (1)) to obtain the valve of EL for this transition. (seq. 32700-32900)
17	Subroutine MATRIX's DO-loops automatically go back to step 9 as long as there remain possible transitions to examine. When the possibilities are exhausted the subroutine RETURNs to WORK. (seq. 33000-36100)

FLOW CHART STEP	ACTION
18	Subroutine WORK CALLs subroutine OUTPUT to display the initial conditions. (seq. 8200)
19	Subroutine OUTPUT displays the initial conditions, first writing a heading including the current time step, then using a DO-loop to WRITE a record for each possible survival state, and finally WRITEing out a display of the mean survivors before RETURNing to WORK. (seq. 40700-44700)
20	Subroutine WORK CALLs subroutine MULT to compute new state probabilities by matrix multiplication as described in chapter 3. (seq. 9000)
21	Subroutine MULT uses two nested DO-loops to scan the possible states, using IA and IB as linear addresses. (seg. 44900-46900)
22	Subroutine MULT, for each different valve of IA or IB, CALLs subroutine REVSUB to compute an explicit survival state (N1

FLOW CHART STEP	ACTION
	or N2) corresponding to the linear subscript. (seq. 46500-47000)
23	Subroutine REVSUB computes the appropriate survival state vector, N, and RETURNs to MULT. (seq. 3900-40400)
24	Subroutine MULT skips steps 25 through 27 for any transitions with an increasing force component. (seq. 47500-47600)
25	Subroutine MULT uses function INDEXF to compute a transition address. (seq. 48100)
26	Subroutine INDEXF computes the address and RETURNs to MULT. (seq. 36300-37700)
27	Subroutine MULT bookkeeps this term of the matrix multiplication. (seq. 48200-48300)
28	Subroutine MULT's DO-loop returns to step 21 to continue scanning the possible transitions until they are exhausted. When thay have been exhausted, MULT now

FLOW CHART STEP	ACTION
	completes the computation of conditional lethality data by normalization and RETURNS to WORK. (seq. 48600-49400)
29	Subroutine WORK updates ICOUNT,the time-step counter and CALLs OUTPUT to display the new data. (seq. 9600-9700)
30	Subroutine OUTPUT displays the time step data (as in step 19) and RETURNs to WORK. (seq.40700-44700)
31	Subroutine WORK updates the previous probability and lethality data from the new data (using the STRGE array name to address the data). (seq. 10300-11100)
32	Subroutine WORK checks ICOUNT against JTIMES, the end-of-run count, and GOes TO step 20 (statement label 3) for the next step if the run is not over. When the run is over, WORK RETURNs to MAIN. (seq. 11600-11900)
33	MAIN STOPs (with a STOP 0). (seq. 1500)

4.2 *Inputs and Outputs*

The programs read five cards of data.

The first card contains ten one-digit weapon strengths for the weapon groups (group 1 is AH's, 2 through 10 arbitrary homogeneous groups of Red weapons) in columns 1 through 10. Column i data is associated with group i.

The second card contains 9 two-digit values for the Blue kill probabilities, the p_i's of chapter 3, in columns 3-4 through 19-20. The value in columns 2i-1, 2i is associated with group i. ZERO IS NOT AN ACCEPTABLE VALUE FOR ANY OF THESE VARIABLES; ITS USE WILL CAUSE THE PROGRAM TO ATTEMPT TO DIVIDE BY ZERO.

The third card contains priority information. The Red group numbers should be listed in the order in which they will be selected as targets, one number per column, starting in column 1. Group numbers may be omitted (and these groups will not be fired on). The list must terminate with a "1." LISTS WHICH DO NOT TERMINATE WITH A "1" OR WHICH CONTAIN A ZERO OR BLANK WILL CAUSE PROGRAM ERRORS WHICH MAY NOT BE APPARENT IN THE RUN OUTPUT.

The fourth card contains the expected kill numbers, the λ_i's of chapter 3 for the Red groups in F5.2 format in colums 6-10 through 46-50. The datum in columns 5i-4 through 5i is associated with group i.

The fifth card contains a single one-digit number of periods for which data is to be computed in column 1.

The program output contains for each time step (including 0) a heading with the time step count, a list of possible states and their probabilities and lethality data, and a second heading followed by the vector of mean survivors. The sample run in chapter 6 shows the formats clearly. They have been designed to fit 70-character carriages in case the model is to be used interactively in a time shared manner.

4.3 Data Storage and Variable Names

The major data storage in these programs is used for the matrix tables and the state probability vectors. These data are stored in what is reserved as a single large variable array, named STRGE in MAIN and WORK. (This array is presently 50000 variables long. If it is desired to charge its size, four statements in the program must be changed: the two DIMENSION statements at sequences 500-510 and 2300, the DO-loop at 1000 and the size check in WORK at sequence 4900.) References to these data in these programs require use of pointers to the effective initial points for the various arrays - IMKEL, IMKPPR, IMKPEL, IMKNPR, and IMKNEL. In subroutines, since the appropriate areas' initial points are passed to the routine as parameters, the data may be referred to by the names PRTRAN, EL, PREVPR, PREVEL, NEWPR, and NEWEL. The data is in fact the same data.

The arrays are stored in the STRGE area in the order listed above. Within the individual arrays, the storage is as follows:

(1) For the two matrix arrays no storage is reserved for transitions with initial force components less than final ones. For transitions from (y_i) to (x_i) with $y_i \geq x_i$ for all i, the area is stored as if a standard 10-dimensional array with subscripts S_1, S_2, ..., S_{10} where S_i is computed as the linear address of the y_i, X_i pair in a ragged array starting with X_i, X_i, letting the second component varying fastest, and ending

with 0,0. The ten subscript maxima are then $(X_i+1)\ (X_i+2)/2$.

(2) For the four survivor state vector arrays, standard 10 dimensional subscripting is used, the first component varying fastest, and the i^{th} conponent taking the X_i+1 values $0,1,\ldots,X_i$.

The total storage required for a run is thus

$$2\prod_{i=1}^{10} \frac{(X_i+1)(X_i+2)}{2} + 4\prod_{i=1}^{10} (X_i+1).$$

The only other general information the user should require concerning the data storage is that, as much as possible, variable names for the same data in different programs are identical. The following pages contain a glossary of the main program variables. Chapter 5 is a listing of the programs, and chapter 6 contains a simple sample run.

GLOSSARY

The following is a list of major program variables and their meanings.

VARIABLE(S)	MEANING
BR	The Blue strength ratio, B in chapter 3.
EL	The transition lethality matrix, L in chapter 3. NOTE: This name is used in OUTPUT (only) to denote the lethality vector for display.
GP	The probability of a number of potential kills, Pr_1 in chapter 3.
I1,...,I10	The values of the potential kills, k_i in chapter 3.
IEQ	A vector EQUIVALENCED to I1,...,I10.
IMKEL, IMKNEL, IMKNPR, IMKPEL, and IMKPPR	The addresses of the initial points of the matrix and vector information in the STRGE block.
ISTATE	The size of a survival state vector.
JRANKS	The list of Red groups in priority order.
JTIMES	The number of periods for which data is to be computed.
NO	The initial strength vactor at period 0, X_i in chapter 3.

VARIABLE(S)	MEANINGS
N1	The initial strength vector at the current period, y_i in chapter 3.
N2	The final strength vector of the current period, x_i in chapter 3.
NEWEL, NEWPR	The survival state probability and expected lethality vectors, $F_{t_{i+1}}$ and $E_{t_{i+1}}$ in chapter 3.
P	The array of Blue kill probabilities, p_i in chapter 3.
PREVEL, PREVPR	The survival state probability and expected lethality vectors, F_{t_i} and E_{t_i} in chapter 3.
PRTRAN	The transition probability matrix, T in chapter 3.
RR	The Red strength ratio, R in chapter 3.
SCALES	An array of bookkeeping values used in computing subscripts.
SIZE	The size of a transition matrix.
STRGE	The main storage block in which all transition matrix and state probability vector information is stored.
XP	The Red weapon lethalities, λ_i in Chapter 3.

5.0 PROGRAM LISTINGS

This chapter contains the program listings.

```
C
C THIS IS THE MAIN PROGRAM FOR THE IHA PROGRAMS.  IT SERVES ONLY
C TO READ IN DATA AND PASS CONTROL TO OTHER SUBROUTINES.
C
      DIMENSION NO(10),N1(10),N2(10),STRGE(50000),P(10),
     1 JRANKS(10),XP(10),IALL(10)
      INTEGER SIZE
      COMMON /PROBS/ P,XP
      COMMON /PRIOR/ JRANKS
      DO 1 IA=1,50000
    1 STRGE(IA)=0.
      READ (5,51) NO,P,JRANKS,XP,JTIMES
   51 FORMAT (10I1/10F2.2/10I2/10F5.2/I2)
      WRITE (6,61) JTIMES,NO,(P(II),II=2,10),(XP(II),II=2,10),JRANKS
   61 FORMAT (I2,18H RUNS WILL BE MADE  /15HINITIAL FORCES:/
     1 I3,3H    ,9I3/22HAH KILL PROBABILITIES:  /6H           ,
     2 9F5.2/22HDATA ON KILLS OF AHS:   /6H      ,9F5.2/
     3 16HPRIORITY ORDER:    /10I3)
      CALL WORK(NO,STRGE,JTIMES)
      STOP 0
      END
```

```
      SUBROUTINE WORK(NO,STRGE,JTIMES)
C
C THIS SUBROUTINE PERFORMS THE BASIC MANAGEMENT OF THE IHA PROGRAM
C GROUP.  IT PERFORMS NO INPUT OR OUTPUT, AND PERFORMS ONLY MINOR
C BOOKKEEPING CALCULATIONS
C
      DIMENSION NO(10),N1(10),N2(10),STRGE(50000)
      INTEGER SIZE
      DIMENSION JRANKS(10)
      DIMENSION P(10),XP(10),IALL(10)
      COMMON /PROBS/ P,XP
      INTEGER SCALES
      COMMON /PRIOR/ JRANKS
      DIMENSION NZEROS(10),SCALES(10)
      DATA NZEROS/10*0/
C
C SIZE WILL CONTAIN THE LENGTH OF THE STORAGE ARRAY NEEDED FOR
C THE TRANSITION PROBABILITY MATRIX ENTRIES.  SUBROUTINE INDEXF IS USED.
C
      SIZE=INDEXF(NO,NZEROS,NZEROS)
C
C ISTATE WILL CONTAIN THE LENGTH OF THE STORAGE ARRAY NEEDED
C FOR STATE PROBABILITIES.
C
      ISTATE=1
      DO 1 IA=1,10
    1 ISTATE=ISTATE*(NO(IA)+1)
C
C THE ARRAY SCALES IS SET UP TO STORE ENTRIES BY WHICH SUBSCRIPT
C INTERPRETATION CAN BE MADE.  IT IS USED BY THE SUBROUTINE REVSUB.
C
      IA=2*SIZE+4*ISTATE
      IF (IA.GT.50000) GO TO 33
      SCALES(1)=1
      DO 2 IA=2,10
      SCALES(IA)=SCALES(IA-1)*(NO(IA-1)+1)
    2 CONTINUE
      ICOUNT = 0
C
C THE NEXT SECTION SETS UP THE MARKERS FOR THE BEGINNINGS OF THE
C DIFFERENT SECTIONS OF THE WORKING STORAGE ARRAY STRGE.  IMKEL
C POINTS TO THE BEGINNING OF THE AREA WHERE DATA ON THE LETHALITY
C DELIVERED DURING A TRANSITION IS KEPT.  IMKPPR AND IMKNPR POINT
C TO THE AREAS WHERE THE PREVIOUS AND NEW PROBABILITY VECTORS FOR
C THE STATES ARE KEPT.  IMKPEL AND IMKPEL ARE ANALOGOUS POINTERS
C FOR THE PREVIOUS AND CURRENT CONDITIONAL DELIVERIES OF
C LETHALITY.
C
      IMKPPR=2*SIZE+1
      IMKEL=SIZE+1
      IMKPEL=2*SIZE+ISTATE+1
      IMKNPR=2*SIZE+2*ISTATE+1
      IMKNEL=IMKNPR+ISTATE
C
```

```
C THE NEXT STATEMENT INITIALIZES THE PROBABILITY VECTOR.
C
      STRGE(IMKPEL-1)=1.
C
C THE SUBROUTINE MATRIX IS CALLED TO COMPUTE THE TRANSITIONS
C PROBABILITIES AND EXPECTED DELIVERIES OF LETHALITY PER TRANSITION.
C
      CALL MATRIX(NO,STRGE,STRGE(IMKEL),SIZE)
C
C OUTPUT IS CALLED TO PRINT THE INITIAL PROBABILITY INFORMATION.
C
      CALL OUTPUT(STRGE(IMKPPR),STRGE(IMKPEL),ICOUNT,SCALES,NO,ISTATE)
    3 CONTINUE
C
C THE SUBROUTINE MULT IS CALLED TO USE THE TRANSITION MATRIX
C INFORMATION TO UPDATE THE SURVIVING FORCE PROBABILITY VECTORS
C FOR THE NEXT TIME STEP.  IT ALSO UPDATES THE LETHALITY DELIVERY
C VECTORS.
C
      CALL MULT(NO,STRGE,STRGE(IMKEL),STRGE(IMKPPR),STRGE(IMKPEL),
     1 STRGE(IMKNPR),STRGE(IMKNEL),SIZE,ISTATE,SCALES)
C
C ICOUNT, THE TIME STEP COUNTER, IS UPDATED, AND THE OUTPUT
C SUBROUTINE IS CALLED TO PRINT THE NEW SURIVIAL PROBABILITIES.
C
      ICOUNT=ICOUNT+1
      CALL OUTPUT(STRGE(IMKNPR),STRGE(IMKNEL),ICOUNT,SCALES,NO,ISTATE)
C
C THE NEW INFORMATION IS PLACED IN THE STORAGE FOR THE PREVIOUS
C INFORMATION AND THE NEW INFORMTION STORAGE IS ZEROED IN
C PREPARATEION FOR THE NEXT TIME STEP.
C
      DO 4 IA=1,ISTATE
      IB=IMKPPR+IA-1
      IC=IMKPEL+IA-1
      ID=IMKNPR+IA-1
      IE=IMKNEL+IA-1
      STRGE(IB)=STRGE(ID)
      STRGE(IC)=STRGE(IE)
      STRGE(ID)=0.
    4 STRGE(IE)=0.
C
C IF THE REQUIRED TIME HISTORY HAS BEEN PRODUCED, THE PROGRAM WILL
C END; OTHERWISE, ANOTHER TIME STEP WILL BE INITIATED AT THIS POINT.
C
      IF (ICOUNT.GE.JTIMES) GO TO 1111
      GO TO 3
 1111 CONTINUE
      RETURN
33    WRITE (6,66666)
66666 FORMAT (40H0 THE STORAGE PROVIDED IS TOO SMALL                   )
      RETURN
      END
```

```
      SUBROUTINE PRIRTY(N1,IALL)
C
C THIS SUBROUTINE ALLOCATES THE HELICOPTERS TO TARGETS ON A
C PRIORITY BASIS IN ACCORDANCE WITH INPUTS.
C
      DIMENSION N1(10),IALL(10),JRANKS(10)
      COMMON /PRIOR/ JRANKS
C
C ITOGO REPRESENTS UNALLOCATED AHS
C
      ITOGO=N1(1)
      DO 101 IA=1,10
  101 IALL(IA)=0
C
C IN THIS LOOP, AHS SELECT TARGETS IN PRIORITY ORDER.
C
      DO 1 IA=1,10
      IB=JRANKS(IA)
      IF (IB.EQ.1) GO TO 3
      IALL(IB)=MIN0(ITOGO,N1(IB))
      ITOGO=ITOGO-IALL(IB)
      IF (ITOGO.EQ.0) GO TO 2
    1 CONTINUE
    3 IALL(1)=ITOGO
    2 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MATRIX(NO,PRTRAN,EL,SIZE)
C
C THIS SUBROUTINE COMPUTES TRANSITION PROBABILITIES AND EXPECTED
C DELIVEIES OF LETHALITY PER TRANSITION.
C
      INTEGER SIZE
      DIMENSION NO(10),N1(10),N2(10),PRTRAN(SIZE),EL(SIZE)
      DIMENSION G(10)
      DIMENSION P(10),XP(10),IALL(10)
      COMMON /PROBS/ P,XP
      DIMENSION IEQ(10)
      EQUIVALENCE (IEQ(1),I1),(IEQ(2),I2),(IEQ(3),I3),(IEQ(4),I4),
     1 (IEQ(5),I5),(IEQ(6),I6),(IEQ(7),I7),(IEQ(8),I8),(IEQ(9),I9),
     2 (IEQ(10),I10)
      DIMENSION GP(10)
C
C THE OUTER SET OF 10 DO-LOOPS ALLOW THE INITIAL STATE (N1) TO
C VARY THROUGH THE LEGAL RANGE.  III1 THROUGH III10 ARE USED AS
C DO-LOOP VARIABLES, WITH KKK1 THROUGH KKK10 AS BOUNDS (WHICH
C ARE SET ON THE BASIS OF THE DATA ARRRAY NO).
C
      KKK1=NO(1)+1
      KKK2=NO(2)+1
      KKK3=NO(3)+1
      KKK4=NO(4)+1
      KKK5=NO(5)+1
      KKK6=NO(6)+1
      KKK7=NO(7)+1
      KKK8=NO(8)+1
      KKK9=NO(9)+1
      KKK10=NO(10)+1
      DO 1001 III1=1,KKK1
      N1(1)=III1-1
      DO 1002 III2=1,KKK2
      N1(2)=III2-1
      DO 1003 III3=1,KKK3
      N1(3)=III3-1
      DO 1004 III4=1,KKK4
      N1(4)=III4-1
      DO 1005 III5=1,KKK5
      N1(5)=III5-1
      DO 1006 III6=1,KKK6
      N1(6)=III6-1
      DO 1007 III7=1,KKK7
      N1(7)=III7-1
      DO 1008 III8=1,KKK8
      N1(8)=III8-1
      DO 1009 III9=1,KKK9
      N1(9)=III9-1
      DO 10010 III10=1,KKK10
      N1(10)=III10-1
C
C INSIDE THE OUTER DO-LOOPS, THE ALLOCATION SUBROUTINE PRIRTY IS
C CALLED TO ALLOCATE AH FIRE, AND A SET OF INNER DO-LOOPS USING
```

```
C VARIABLES I1 THROUGH I10 ARE INITIATED TO ALLOW THE FIRST
C APPROXIMATION TO THE LOSSES TO BE REPRESENTED (BY THE
C DO-LOOP VARIABLES.
C
      K1=N1(1)+1
      CALL PRIRTY(N1,IALL)
      K2=IALL(2)+1
      K3=IALL(3)+1
      K4=IALL(4)+1
      K5=IALL(5)+1
      K6=IALL(6)+1
      K7=IALL(7)+1
      K8=IALL(8)+1
      K9=IALL(9)+1
      K10=IALL(10)+1
      TOTAL=0.
      DO 101 IA=2,10
      TOTAL=TOTAL+XP(IA)*FLOAT(N1(IA))
      GP(IA)=1.
  101 G(IA)=(1.-P(IA))**(IALL(IA)+1)/P(IA)
      CUM=0.
      GP(1)=1.
      IFACT=1
      DO 1101 I1=1,K1
      IF (I1.GT.1) IFACT=IFACT*(I1-1)
      IA=I1-1
C
C INSIDE THESE DO LOOPS, THE INITIIAL PROBABILITY ESTIMATES ARE
C MADE AS DESCRIBED IN THE DESCRIPTION OF THE MATHEMATICAL MODELS
C IN THIS PROGRAM.
C
      IF (TOTAL.LT.0.00001) GO TO 2
      G(1)=EXP(-TOTAL)*TOTAL**FLOAT(I1-1)/FLOAT(IFACT)
      CUM=CUM+G(1)
      IF (I1.EQ.K1) G(1)=G(1)+1.-CUM
      GO TO 3
    2 CONTINUE
      IF (I1.EQ.1) G(1)=1.
      IF (I1.NE.1) G(1)=0.
    3 CONTINUE
      GP(1)=G(1)
      XX2=1.
      G(2)=(1.-P(2))**(IALL(2)+1)/P(2)
      DO 1102 I2=1,K2
      IF (I2.GT.1) XX2=(FLOAT(IALL(2)-I2+2))/FLOAT(I2-1)
      G(2)=G(2)*P(2)/(1.-P(2))*(XX2)
      GP(2)=GP(1)*G(2)
      XX3=1.
      G(3)=(1.-P(3))**(IALL(3)+1)/P(3)
      DO 1103 I3=1,K3
      IF (I3.GT.1) XX3=(FLOAT(IALL(3)-I3+2))/FLOAT(I3-1)
      G(3)=G(3)*P(3)/(1.-P(3))*(XX3)
      GP(3)=GP(2)*G(3)
      XX4=1.
```

```
      G(4)=(1.-P(4))**(IALL(4)+1)/P(4)                                  25600
      DO 1104 I4=1,K4                                                   25700
      IF (I4.GT.1) XX4=(FLOAT(IALL(4)-I4+2))/FLOAT(I4-1)                25800
      G(4)=G(4)*P(4)/(1.-P(4))*(XX4)                                    25900
      GP(4)=GP(3)*G(4)                                                  26000
      XX5=1.                                                            26100
      G(5)=(1.-P(5))**(IALL(5)+1)/P(5)                                  26200
      DO 1105 I5=1,K5                                                   26300
      IF (I5.GT.1) XX5=(FLOAT(IALL(5)-I5+2))/FLOAT(I5-1)                26400
      G(5)=G(5)*P(5)/(1.-P(5))*(XX5)                                    26500
      GP(5)=GP(4)*G(5)                                                  26600
      XX6=1.                                                            26700
      G(6)=(1.-P(6))**(IALL(6)+1)/P(6)                                  26800
      DO 1106 I6=1,K6                                                   26900
      IF (I6.GT.1) XX6=(FLOAT(IALL(6)-I6+2))/FLOAT(I6-1)                27000
      G(6)=G(6)*P(6)/(1.-P(6))*(XX6)                                    27100
      GP(6)=GP(5)*G(6)                                                  27200
      XX7=1.                                                            27300
      G(7)=(1.-P(7))**(IALL(7)+1)/P(7)                                  27400
      DO 1107 I7=1,K7                                                   27500
      IF (I7.GT.1) XX7=(FLOAT(IALL(7)-I7+2))/FLOAT(I7-1)                27600
      G(7)=G(7)*P(7)/(1.-P(7))*(XX7)                                    27700
      GP(7)=GP(6)*G(7)                                                  27800
      XX8=1.                                                            27900
      G(8)=(1.-P(8))**(IALL(8)+1)/P(8)                                  28000
      DO 1108 I8=1,K8                                                   28100
      IF (I8.GT.1) XX8=(FLOAT(IALL(8)-I8+2))/FLOAT(I8-1)                28200
      G(8)=G(8)*P(8)/(1.-P(8))*(XX8)                                    28300
      GP(8)=GP(7)*G(8)                                                  28400
      XX9=1.                                                            28500
      G(9)=(1.-P(9))**(IALL(9)+1)/P(9)                                  28600
      DO 1109 I9=1,K9                                                   28700
      IF (I9.GT.1) XX9=(FLOAT(IALL(9)-I9+2))/FLOAT(I9-1)                28800
      G(9)=G(9)*P(9)/(1.-P(9))*(XX9)                                    28900
      GP(9)=GP(8)*G(9)                                                  29000
      XX10=1.                                                           29100
      G(10)=(1.-P(10))**(IALL(10)+1)/P(10)                              29200
      DO 11010 I10=1,K10                                                29300
      IF (I10.GT.1) XX10=(FLOAT(IALL(10)-I10+2))/FLOAT(I10-1)           29400
      G(10)=G(10)*P(10)/(1.-P(10))*(XX10)                               29500
      GP(10)=GP(9)*G(10)                                                29600
C                                                                       29700
C THE NEXT SET OF DO-LOOPS ALLOWS THE REPRESENTATION OF THE             29800
C SECOND APPROXIMATION TO THE TRANSITION PROBABILITY MATRIX.            29900
C                                                                       30000
      DO 1201 II1=1,I1                                                  30100
      N2(1)=III1-II1                                                    30200
      DO 1202 II2=1,I2                                                  30300
      N2(2)=III2-II2                                                    30400
      DO 1203 II3=1,I3                                                  30500
      N2(3)=III3-II3                                                    30600
      DO 1204 II4=1,I4                                                  30700
      N2(4)=III4-II4                                                    30800
      DO 1205 II5=1,I5                                                  30900
```

```
      N2(5)=III5-II5
      DO 1206 II6=1,I6
      N2(6)=III6-II6
      DO 1207 II7=1,I7
      N2(7)=III7-II7
      DO 1208 II8=1,I8
      N2(8)=III8-II8
      DO 1209 II9=1,I9
      N2(9)=III9-II9
      DO 12010 III10=1,I10
      N2(10)=IIII10-III10
C
C THIS CODE ACTUALLY COMPUTES THE TRANSITION INFORMATION.
C
      INDEX=INDEXF(N0,N1,N2)
      PROBXX=GP(10)*PROBEN(N0,N1,N2,IEQ,TOTAL)
      PRTRAN(INDEX)=PRTRAN(INDEX)+PROBXX
      IF (N1(1).GT.0) FRACT=FLOAT(N2(1))/FLOAT(N1(1))
      IF (N1(1).EQ.0) FRACT=0.
      EL(INDEX)=FLOAT(IALL(1))*FRACT
12010 CONTINUE
 1209 CONTINUE
 1208 CONTINUE
 1207 CONTINUE
 1206 CONTINUE
 1205 CONTINUE
 1204 CONTINUE
 1203 CONTINUE
 1202 CONTINUE
 1201 CONTINUE
11010 CONTINUE
 1109 CONTINUE
 1108 CONTINUE
 1107 CONTINUE
 1106 CONTINUE
 1105 CONTINUE
 1104 CONTINUE
 1103 CONTINUE
 1102 CONTINUE
 1101 CONTINUE
10010 CONTINUE
 1009 CONTINUE
 1008 CONTINUE
 1007 CONTINUE
 1006 CONTINUE
 1005 CONTINUE
 1004 CONTINUE
 1003 CONTINUE
 1002 CONTINUE
 1001 CONTINUE
C
      RETURN
      END
```

```
      FUNCTION INDEXF(NO,N1,N2)
C
C THIS SUBROUTINE COMPUTES AN ADDRESS FOR TRANSITION INFORMATION
C USING THE STORAGE ALLOCATION ALGORITHM DESCRIBED IN THE MANUAL.
C
      DIMENSION NO(10),N1(10),N2(10)
      INDEX=1
      IPROD=1
      DO 1 I=1,10
      INDEX=INDEX+IPROD*((NO(I)+1)*(NO(I)-N1(I))-
     1 ((NO(I)-N1(I))*(NO(I)-N1(I)-1))/2 +N1(I)-N2(I))
    1 IPROD=IPROD*((NO(I)+1)*(NO(I)+2))/2
      INDEXF=INDEX
      RETURN
      END
```

```
      INTEGER FUNCTION SBSCPT(N,SCALES)                                 37800
C                                                                       37900
C THIS SUBROUTINE COMPUTES A LINEAR SUBSCRIPT FOR REFERENCING           38000
C THE SURVIVING FORCE PROBABILITY ARRAYSAND SIMILAR INFORMATION.        38100
C                                                                       38200
      DIMENSION N(10),SCALES(10)                                        38300
      INTEGER SCALES                                                    38400
      SBSCPT=1                                                          38500
      DO 101 IA=1,10                                                    38600
  101 SBSCPT=SBSCPT+N(IA)*SCALES(IA)                                    38700
      RETURN                                                            38800
      END                                                               38900
```

```
      SUBROUTINE REVSUB(NO,N,ISUB,SCALES)
C
C THIS SUBROUTINE COMPUTES A SURVIVOR STATUS FROM A LINEAR SUBSCRIPT.
C
      INTEGER SCALES
      DIMENSION NO(10),N(10),SCALES(10)
      ISUBZ=ISUB-1
      DO 101 IA=1,10
      IB=11-IA
      IF (NO(IB).EQ.0) GO TO 102
      N(IB)=ISUBZ/SCALES(IB)
      ISUBZ=ISUBZ-N(IB)*SCALES(IB)
      GO TO 101
  102 N(IB)=0
  101 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE OUTPUT(PR,EL,ICOUNT,SCALES,NO,ISTATE)
C
C THIS SUBROUTINE WRITES OUT THE INFORMATION PASSED TO IT
C DESCRIBING THE FORCE PROBABILITIES OF SURVIVAL.
C
      DIMENSION PR(ISTATE),EL(ISTATE),SCALES(10),NO(10)
      DIMENSION MARGIN(10,10),N(10)
      REAL MARGIN
C
C THE HEADING IS WRITTEN HERE
C
      WRITE (6,62) ICOUNT
   62 FORMAT (10H0 THESE AF,10HE THE RESU,11HLTS AT THE ,
     1 10HEND OF PER,3HIOD,I3)
      WRITE (6,63)
   63 FORMAT (48HTHE FIRST TEN NUMBERS ARE THE INITIAL STRENGTHS,/
     1 52HTHE NEXT TEN DESCRIBE A POSSIBLE SURVIVOR STATE, AND       /
     2 54HTHE NEXT NUMBERS ARE THE PROBABILITY OF BEING IN THIS
     3 9HSTATE AND  /42HTHE EXPECTED LETHALITY DELIVERED TO REMOTE
     4 20H OR PASSIVE TARGETS.     /)
      DO 101 IA=1,10
      DO 101 IB=1,10
  101 MARGIN(IA,IB)=0.
C
C THE NEXT DO LOOP WRITES OUT THE DETAILED PROBABILITIES OF
C SURVIVAL OF THE VARIOUS POSSIBLE FORCE MIXES.
C
      DO 1 IA=1,ISTATE
      CALL REVSUB(NO,N,IA,SCALES)
      WRITE (6,61) NO,N,PR(IA),EL(IA)
   61 FORMAT (10I2,I4,9I3,2(2X,F6.2))
      DO 111 IB=1,10
      MARGIN(IB,1)=MARGIN(IB,1)+PR(IA)*FLOAT(N(IB))
  111 CONTINUE
    1 CONTINUE
C
C INFORMATION ON MEANS IS WRITTEN OUT BELOW
C
      WRITE (6,64) (MARGIN(J,1),J=1,10)
   64 FORMAT (///10HTHE MEAN S,10HURVIVORS A,10HRE:           //10F6.3)
      RETURN
      END
```

```
      SUBROUTINE MULT(NO,PRTRAN,EL,PREVPR,PREVEL,NEWPR,NEWEL,SIZE,ISTATE
     1,SCALES)
C
C THIS SUBROUTINE MULTIPLIES THE SURVIVAL PROBABILITY VECTOR BY
C THE TRANSITION PROBABILTIY MATRIX IN ORDER TO COMPUTE THE
C SURVIVAL PROBABILITY VECTOR FOR THE NEXT TIME STEP.
C
      INTEGER SIZE,SCALES
      REAL NEWPR,NEWEL
      DIMENSION NO(10),SCALES(10),PRTRAN(SIZE).EL(SIZE),PREVPR(ISTATE)
      DIMENSION PREVEL(ISTATE),NEWPR(ISTATE),NEWEL(ISTATE)
      DIMENSION N1(10),N2(10)
C
C IA SCANS THE POSSIBLE INITIAL STATES FOR A TRANSITION
C
      DO 1 IA=1,ISTATE
      CALL REVSUB(NO,N1,IA,SCALES)
C
C IB SCANS THE POSSIBLE TERMINAL STATES FOR A TRANSITION
C
      DO 2 IB=1,ISTATE
      CALL REVSUB(NO,N2,IB,SCALES)
C
C THE NEXT LOOP REJECTS TRANSITIONS WHICH ARE IMPOSSIBLE BECAUSE
C THEY WOULD INVOLVE INCREASING FORCES.
C
      DO 3 IC=1,10
      IF (N1(IC).LT.N2(IC)) GO TO 2
    3 CONTINUE
C
C THE ACTUAL BOOKKEEPING IS DONE BELOW
C
      INDEX=INDEXF(NO,N1,N2)
      NEWPR(IB)=NEWPR(IB)+PREVPR(IA)*PRTRAN(INDEX)
      NEWEL(IB)=NEWEL(IB)+PRTRAN(INDEX)*PREVPR(IA)*(EL(INDEX)+PREVEL(IA)
     1)
    2 CONTINUE
    1 CONTINUE
C
C THE NEXT LOOP NORMALIZES THE LETHALITY DELIVERY FIGURES.
C
      DO 4 IA=1,ISTATE
      IF (NEWPR(IA).LT.0.000001) GO TO 4
      NEWEL(IA)=NEWEL(IA)/NEWPR(IA)
    4 CONTINUE
      RETURN
      END
```

```
      FUNCTION PROBGN(NO,N1,N2,IEQ,TOTAL)
C
C THIS SUBROUTINE COMPUTES THE BINOMIAL PROBABILITES USED TO
C GENERATE THE SECOND APPROXIMATION TO THE DISTRIBUTION OF
C SURVIVORS
C
      DIMENSION NO(10),N1(10),N2(10),IEQ(10)
      DIMENSION BINCFT(10,10)
      DATA BINCFT/1.,9*0.,2.,1.,8*0.,3.,3.,1.,7*0.,4.,6.,4.,1.,6*0.,
     1 5.,10.,10.,5.,1.,5*0.,6.,15.,20.,15.,6.,1.,4*0.,
     2 7.,21.,35.,35.,21.,7.,1.,3*0.,8.,28.,56.,70.,56.,28.,8.,1.,
     3 2*0.,9.,36.,84.,126.,126.,84.,36.,9.,1.,0.,
     4 10.,45.,120.,210.,252.,210.,120.,45.,10.,1./
      COMMON /PROBS/ P,XP
      DIMENSION P(10),XP(10)
      PROBGN=1.
      SUMR=0.
      X1=FLOAT(N1(1))
      X2=FLOAT(N2(1))
      X3=FLOAT(IEQ(1))
      BR=1.
      IF (X1.LE.0.) GO TO 113
      BR=(X1-X3+1.)/X1
  113 CONTINUE
      BR=BR*0.5+0.5
      DO 1 IA=2,10
      IB=N1(IA)-IEQ(IA)+1
      SUMR=SUMR+XP(IA)*FLOAT(IB)
    1 CONTINUE
      RR=1.
      IF (TOTAL.LE.0.) GO TO 112
      RR=SUMR/TOTAL
  112 CONTINUE
      RR=RR*0.5+0.5
      X3=X3-1
      X4=X1-X2
      IF (RR.GT.1.) PROBGN=0.
      IF (X2.GT.X1) PROBGN=0.
      IF (X4.GT.X3) PROBGN=0.
      IF (PROBGN.EQ.0.) GO TO 111
      IC=IEQ(1)-1
      ID=N1(1)-N2(1)
      IF ((RR.GT..9999).AND.(IC.EQ.ID)) GO TO 2
      PROBGN=PROBGN*RR**X4
      PROBGN=PROBGN*(1.-RR)**(X3-X4)
      IF (IC.EQ.0) GO TO 2
      IF (ID.EQ.0) GO TO 2
      PROBGN=PROBGN*BINCFT(ID,IC)
    2 CONTINUE
      DO 3 IA=2,10
      X1=FLOAT(N1(IA))
      X2=FLOAT(N2(IA))
      X4=X1-X2
      X3=FLOAT(IEQ(IA))
```

```
      X3=X3-1.
      IC=IEQ(IA)-1                                                      55000
      ID=N1(IA)-N2(IA)                                                  55100
      IF ((BR.GT..9999).AND.(IC.EQ.ID)) GO TO 3                         55200
      PROBGN=PROBGN*BR**X4                                              55300
      PROBGN=PROBGN*(1.-BR)**(X3-X4)                                    55400
      IF (IC.LE.0) GO TO 3                                              55500
      IF (IC.LE.0) GO TO 3                                              55600
      IF (ID.GT.IC) PROBGN=0.                                           55700
      PROBGN=PROBGN*BINCFT(ID,IC)                                       55800
    3 CONTINUE                                                          55900
  111 CONTINUE                                                          56000
      PROBFN=PROBGN                                                     56100
      RETURN                                                            56200
      END                                                               56300
                                                                        56400
```

```
PY 3  619272----------------619272----------------619272----------------619272----------------619272----------------619272-------

JOB NO. 619272          UNIVERSITY OF MICHIGAN TERMINAL SYSTEM  (MODEL PR293)                        13:00.37     05-10-73




                        MMMMM                   MMMMM      TTTTTTTTTTTTTTTTTTTTTTTTT           SSSSSSSSS
                        MMMMMM                 MMMMMM      TTTTTTTTTTTTTTTTTTTTTTTTT         SSSSSSSSSSSSS
                        MMMMMMM               MMMMMMM      TTTTTTTTTTTTTTTTTTTTTTTTT       SSSSSSSSSSSSSSSSS
                        MMMMMMMM             MMMMMMMM                TTTTT               SSSSSS        SSSSSSS
                        MMMMMMMMM           MMMMMMMMM                TTTTT              SSSSS            SSSSS
                        MMMMMMMMMM         MMMMMMMMMM                TTTTT              SSSSS
                        MMMMM MMMMM       MMMMM MMMMM                TTTTT               SSSSSS
                        MMMMM  MMMMM     MMMMM  MMMMM                TTTTT                 SSSSSSSSSSSSS
                        MMMMM   MMMMM   MMMMM   MMMMM                TTTTT                   SSSSSSSSSSSSS
                        MMMMM    MMMMMMMMMMM    MMMMM                TTTTT                     SSSSSSSSSSSSS
                        MMMMM     MMMMMMMMM     MMMMM                TTTTT                             SSSSSSS
                        MMMMM      MMMMMMM      MMMMM                TTTTT                               SSSSS
                        MMMMM       MMMMM       MMMMM                TTTTT                               SSSSS
                        MMMMM        MMM        MMMMM                TTTTT              SSSSS            SSSSS
                        MMMMM                   MMMMM                TTTTT              SSSSSSS        SSSSSSS
                        MMMMM                   MMMMM                TTTTT                SSSSSSSSSSSSSSSSS
                        MMMMM                   MMMMM                TTTTT                  SSSSSSSSSSSSS
                        MMMMM                   MMMMM                TTTTT                    SSSSSSSSS
```

6.0 SAMPLE RUN

This chapter contains a sample run of the program.

```
5 RUNS WILL BE MADE
INITIAL FORCES:
 1    1  1  0  0  0  0  0  0  0
AH KILL PROBABILITIES:
     0.80 0.80 0.80 0.80 0.80 0.80 0.80 0.80 0.80
DATA ON KILLS OF AHS:
     0.40 0.40 0.40 0.0  0.0  0.0  0.0  0.0  0.0
PRIORITY ORDER:
 3  2  1  0  0  0  0  0  0

 THESE ARE THE RESULTS AT THE END OF PERIOD  0
THE FIRST TEN NUMBERS ARE THE INITIAL STRENGTHS,
THE NEXT TEN DESCRIBE A POSSIBLE SURVIVOR STATE, AND
THE NEXT NUMBERS ARE THE PROBABILITY OF BEING IN THIS STATE AND
THE EXPECTED LETHALITY DELIVERED TO REMOTE OR PASSIVE TARGETS.

1 1 1 0 0 0 0 0 0 0   0  0  0  0  0  0  0  0  0  0    0.0     0.0
1 1 1 0 0 0 0 0 0 0   1  0  0  0  0  0  0  0  0  0    0.0     0.0
1 1 1 0 0 0 0 0 0 0   0  1  0  0  0  0  0  0  0  0    0.0     0.0
1 1 1 0 0 0 0 0 0 0   1  1  0  0  0  0  0  0  0  0    0.0     0.0
1 1 1 0 0 0 0 0 0 0   0  0  1  0  0  0  0  0  0  0    0.0     0.0
1 1 1 0 0 0 0 0 0 0   1  0  1  0  0  0  0  0  0  0    0.0     0.0
1 1 1 0 0 0 0 0 0 0   0  1  1  0  0  0  0  0  0  0    0.0     0.0
1 1 1 0 0 0 0 0 0 0   1  1  1  0  0  0  0  0  0  0    1.00    0.0


THE MEAN SURVIVORS ARE:

1.000 1.000 1.000 0.0   0.0   0.0   0.0   0.0   0.0   0.0

 THESE ARE THE RESULTS AT THE END OF PERIOD  1
THE FIRST TEN NUMBERS ARE THE INITIAL STRENGTHS,
THE NEXT TEN DESCRIBE A POSSIBLE SURVIVOR STATE, AND
THE NEXT NUMBERS ARE THE PROBABILITY OF BEING IN THIS STATE AND
THE EXPECTED LETHALITY DELIVERED TO REMOTE OR PASSIVE TARGETS.

1 1 1 0 0 0 0 0 0 0   0  0  0  0  0  0  0  0  0  0    0.0     0.0
1 1 1 0 0 0 0 0 0 0   1  0  0  0  0  0  0  0  0  0    0.0     0.0
1 1 1 0 0 0 0 0 0 0   0  1  0  0  0  0  0  0  0  0    0.17    0.0
1 1 1 0 0 0 0 0 0 0   1  1  0  0  0  0  0  0  0  0    0.41    0.0
1 1 1 0 0 0 0 0 0 0   0  0  1  0  0  0  0  0  0  0    0.0     0.0
1 1 1 0 0 0 0 0 0 0   1  0  1  0  0  0  0  0  0  0    0.0     0.0
1 1 1 0 0 0 0 0 0 0   0  1  1  0  0  0  0  0  0  0    0.23    0.0
1 1 1 0 0 0 0 0 0 0   1  1  1  0  0  0  0  0  0  0    0.14    0.0


THE MEAN SURVIVORS ARE:

0.559 1.000 0.420 0.0   0.0   0.0   0.0   0.0   0.0   0.0

 THESE ARE THE RESULTS AT THE END OF PERIOD  2
THE FIRST TEN NUMBERS ARE THE INITIAL STRENGTHS,
THE NEXT TEN DESCRIBE A POSSIBLE SURVIVOR STATE, AND
THE NEXT NUMBERS ARE THE PROBABILITY OF BEING IN THIS STATE AND
THE EXPECTED LETHALITY DELIVERED TO REMOTE OR PASSIVE TARGETS.

1 1 1 0 0 0 0 0 0 0   0  0  0  0  0  0  0  0  0  0    0.03    0.0
1 1 1 0 0 0 0 0 0 0   1  0  0  0  0  0  0  0  0  0    0.25    0.0
```

```
1 1 1 0 0 0 0 0 0 0   0  1  0  0  0  0  0  0  0  0    0.24    0.0
1 1 1 0 0 0 0 0 0 0   1  1  0  0  0  0  0  0  0  0    0.14    0.0
1 1 1 0 0 0 0 0 0 0   0  0  1  0  0  0  0  0  0  0    0.0     0.0
1 1 1 0 0 0 0 0 0 0   1  0  1  0  0  0  0  0  0  0    0.0     0.0
1 1 1 0 0 0 0 0 0 0   0  1  1  0  0  0  0  0  0  0    0.32    0.0
1 1 1 0 0 0 0 0 0 0   1  1  1  0  0  0  0  0  0  0    0.02    0.0


THE MEAN SURVIVORS ARE:

0.414 0.723 0.336 0.0   0.0   0.0   0.0   0.0   0.0   0.0

 THESE ARE THE RESULTS AT THE END OF PERIOD  3
THE FIRST TEN NUMBERS ARE THE INITIAL STRENGTHS,
THE NEXT TEN DESCRIBE A POSSIBLE SURVIVOR STATE, AND
THE NEXT NUMBERS ARE THE PROBABILITY OF BEING IN THIS STATE AND
THE EXPECTED LETHALITY DELIVERED TO REMOTE OR PASSIVE TARGETS.

1 1 1 0 0 0 0 0 0 0   0  0  0  0  0  0  0  0  0  0    0.04    0.0
1 1 1 0 0 0 0 0 0 0   1  0  0  0  0  0  0  0  0  0    0.34    0.74
1 1 1 0 0 0 0 0 0 0   0  1  0  0  0  0  0  0  0  0    0.27    0.0
1 1 1 0 0 0 0 0 0 0   1  1  0  0  0  0  0  0  0  0    0.04    0.0
1 1 1 0 0 0 0 0 0 0   0  0  1  0  0  0  0  0  0  0    0.0     0.0
1 1 1 0 0 0 0 0 0 0   1  0  1  0  0  0  0  0  0  0    0.0     0.0
1 1 1 0 0 0 0 0 0 0   0  1  1  0  0  0  0  0  0  0    0.32    0.0
1 1 1 0 0 0 0 0 0 0   1  1  1  0  0  0  0  0  0  0    0.00    0.0


THE MEAN SURVIVORS ARE:

0.376 0.628 0.324 0.0   0.0   0.0   0.0   0.0   0.0   0.0

 THESE ARE THE RESULTS AT THE END OF PERIOD  4
THE FIRST TEN NUMBERS ARE THE INITIAL STRENGTHS,
THE NEXT TEN DESCRIBE A POSSIBLE SURVIVOR STATE, AND
THE NEXT NUMBERS ARE THE PROBABILITY OF BEING IN THIS STATE AND
THE EXPECTED LETHALITY DELIVERED TO REMOTE OR PASSIVE TARGETS.

1 1 1 0 0 0 0 0 0 0   0  0  0  0  0  0  0  0  0  0    0.04    0.0
1 1 1 0 0 0 0 0 0 0   1  0  0  0  0  0  0  0  0  0    0.36    1.63
1 1 1 0 0 0 0 0 0 0   0  1  0  0  0  0  0  0  0  0    0.27    0.0
1 1 1 0 0 0 0 0 0 0   1  1  0  0  0  0  0  0  0  0    0.01    0.0
1 1 1 0 0 0 0 0 0 0   0  0  1  0  0  0  0  0  0  0    0.0     0.0
1 1 1 0 0 0 0 0 0 0   1  0  1  0  0  0  0  0  0  0    0.0     0.0
1 1 1 0 0 0 0 0 0 0   0  1  1  0  0  0  0  0  0  0    0.32    0.0
1 1 1 0 0 0 0 0 0 0   1  1  1  0  0  0  0  0  0  0    0.00    0.0


THE MEAN SURVIVORS ARE:

0.367 0.603 0.322 0.0   0.0   0.0   0.0   0.0   0.0   0.0

 THESE ARE THE RESULTS AT THE END OF PERIOD  5
THE FIRST TEN NUMBERS ARE THE INITIAL STRENGTHS,
THE NEXT TEN DESCRIBE A POSSIBLE SURVIVOR STATE, AND
THE NEXT NUMBERS ARE THE PROBABILITY OF BEING IN THIS STATE AND
THE EXPECTED LETHALITY DELIVERED TO REMOTE OR PASSIVE TARGETS
```

```
1 1 1 0 0 0 0 0 0 0   0  0  0  0  0  0  0  0  0  0    0.04    0.0
1 1 1 0 0 0 0 0 0 0   1  0  0  0  0  0  0  0  0  0    0.36    2.60
1 1 1 0 0 0 0 0 0 0   0  1  0  0  0  0  0  0  0  0    0.27    0.0
1 1 1 0 0 0 0 0 0 0   1  1  0  0  0  0  0  0  0  0    0.00    0.0
1 1 1 0 0 0 0 0 0 0   0  0  1  0  0  0  0  0  0  0    0.0     0.0
1 1 1 0 0 0 0 0 0 0   1  0  1  0  0  0  0  0  0  0    0.0     0.0
1 1 1 0 0 0 0 0 0 0   0  1  1  0  0  0  0  0  0  0    0.32    0.0
1 1 1 0 0 0 0 0 0 0   1  1  1  0  0  0  0  0  0  0    0.00    0.0


THE MEAN SURVIVORS ARE:

0.365 0.597 0.322 0.0   0.0   0.0   0.0   0.0   0.0   0.0
```

APPENDIX A

This Appendix documents the initial work done on this project to study the real process which was to be modelled. In the process of constructing analytic models of a real world process, as in the process of constructing any other models, the first step involves acquiring a detailed understanding of the process. Such an understanding is often documented by a logic diagram of one variety or another, representing what one believes to be a complete understanding of the process. It should be noted that such a logic diagram is not strictly equivalent to the logic diagram associated with a simulation model of the process. Only to the extent that no special assumptions have been made in the process of simulating the real world, either for the sake of model simplicity or computational ease, are the diagrams equivalent.

Following the construction of the logic diagram representing his beliefs about the real process, the modeler now constructs a mathematical structure which he believes represents the real process and can be used in the computations required for his problems. In the case of simulatory models, the model structure is essentially parallel to that of the assumed underlying process. In analytic models, the structure of the mathematical formulae and equations may not be directly analogous, although the intent is always to represent the actual dynamics of the real process in the same kind of detail. One may take as an illustrative example two alternative models of a simple orbital motion process. A simulatory model is the differential equation of gravitational force (together with a numerical solution technique), while the equation of a conic section, a method for determining the coefficients from the problem data, and a numerical evaluation technique constitute an analytic model. In this particular example, the models are perfectly equivalent, although their structures are very different.

This Appendix documents the work done on this project which was designed to generate an understanding of the processes involved when independent attack helicopters attack a ground target unit. It is incomplete in two areas:

(1) it reflects a lack of detailed understanding concerning the details of Command, Control, Communications, and Intelligence activities and their interaction with the other combat activities, and

(2) it does not contain the detailed numerical and logical models used in determining the timing and probabilistic consequences of the various events. Rather, it displays the logical interactions of the elementary combat activities taking place without presenting complete details. The detailed models for the processes identified are available in the literature, and documenting the full set of accepted models of each process was unnecessary in the development of the analytic model.

The logic model as here presented is essentially a listing of all the activities in which a participating system may engage. Associated with each activity are sets of system statuses which are either altered by the activity or which may affect the progress of the activity.

Table 1 is a tentative list of the activities considered in the detailed logic model. These activities may be broadly classified into six categories: (1) movement, (2) acquisition, (3) target selection, (4) firing, (5) damage, and (6) command, control, communications and intelligence. Activities listed in Table 1 may apply to only one type of element in the combat process, for example, Attack Helicopter Hovers in Firing position, or, the listed activity may apply to various participating elements, e.g., Command and Intelligence Elements Coordinate Information.

Activities 1 through 27 of Table 1 are structured in detail in the tables at the end of this Appendix. Each sheet contains seven headings, with entries under these headings as follows:

(1) Normal Events -- those events which will or may occur as a direct result of an element engaging in the listed activity. These normal events denote the beginning or end of the listed activity, or serve to initiate or terminate a concurrent activity.

(2) Status Parameters Influenced -- those system statuses (state variables) which are directly influenced by the listed activity. In most cases the list of status variables influenced pertain to the element engaging in the listed activity.

(3) Performance Data Base -- data which describes the capabilities of the element or elements engaged in the activity. (Example: Probability of a kill given a hit is part of the performance data base for the activity -- Warhead Detonates Which Possibly Damages Target(s)).

Table 1

TENTATIVE LIST OF MODEL ACTIVITIES

1. Attack helicopter moves from holding position to assault position
2. Attack helicopter moves from assault position to firing position
3. Attack helicopter hovers in firing position
4. Attack helicopter moves from firing position to alternate firing position
5. Attack helicopter engages in running fire
6. Attack helicopter withdraws from firing position to assault position
7. Attack helicopter withdraws from running fire to assault position
8. Attack helicopter hovers at assault position
9. Attack helicopter withdraws from assault position to holding position
10. Attack helicopter performs evasive maneuvers
11. Ground element continues normal maneuver pattern (independent of helicopter attack)
12. Ground element seeks covered position
13. Ground element seeks firing position
14. Ground element moves from cover to firing position (or changes "posture")
15. Ground element moves from firing position to cover (or changes "posture")
16. Ground element maintains covered position
17. Ground element maintains firing position
18. Ground element moves from covered (or firing) position to alternate
19. Ground element attempts to withdraw
20. Weapon element searches for target(s)

Table 1 (Continued)

21. Weapon element attempts to identify target(s)

22. Weapon element tracks target(s)

23. Weapon element selects target

24. Weapon element prepares to fire ordnance (orienting, aiming, loading)

25. Weapon element guides ordnance

26. Ordnance flies

27. Warhead detonates which possibly damages target(s)

28. Command and intelligence elements assess battle status

29. Command and intelligence elements coordinate information

30. Command elements make tactical decisions

31. Weapon element attempts to counter enemy acquisition capability

(4) Tactics Data Base -- Data which describes the tactical behavior of an element engaged in the listed activity. (Example: Weapon selection rules in the activity -- Weapon Element Prepares to Fire Ordnance).

(5) Concurrent Activities Influenced -- the list of concurrent activities which may be affected by

(a) status parameter changes brought about by the listed activity which may influence a concurrent activity. (Example: "Attack Helicopter Performs Evasive Maneuver" changes the status parameter. "AH Location" which influences the concurrent activity. "Weapon Element Tracks Target.")

(b) Normal Events during the listed activity which may disrupt a concurrent activity (Example: the event "operational failure of mobility system" during the activity "AH hovers in firing position" influences the concurrent activity "Weapon Element Guides Ordnance").

(6) Events which May Disrupt this Activity -- Normal events in the course of concurrent activities which may prematurely terminate this activity.

(7) Status Parameters Influencing the Course of This Activity -- those system statuses (state variables) which directly influence the course of the listed activity.

Activity (1)   Attack helicopter moves from holding position to assault position

Normal Events

(1) Leave holding position
(2) Arrive assault position
(3) Operational failure of mobility system

Status Parameters Influenced

(1) Location
(2) Exposure (cover)
(3) Visibility (background and concealment)
(4) Orientation
(5) Velocity
(6) Flight roughness and acceleration
(7) Operational status of mobility system

Performance Data Base

(1) Helicopter mobility capabilities

Tactics Data Base

(1) Selection of flight path, velocity

Concurrent Activities Influenced

(1) All acquisition activities involving this AH
(2) Ground element target selection of AH
(3) Ordnance preparation, guidance, flight, and detonation (for ordnance with this AH as target)
(4) Battle status assessment
(5) Command element tactical decisions

Events Which May Disrupt This Activity

(1) Damage to this AH
(2) Identification of ground elements
(3) Ground element firing
(4) AH receiving fire
(5) AH receiving warning of being tracked
(6) Command element changes this AH's assignment
(7) Operational failure (other than mobility)
(8) Fuel Expended

Status Parameters Influencing the Course of This Activity

(1) Environment (map, weather, etc.)
(2) Locations of other AH's.

Activity (2) Attack helicopter moves from assault position to firing position

Normal Events

(1) Unmask as assault position
(2) Arrive firing position
(3) Operational failure of mobility system

Status Parameters Influenced

(1) Location
(2) Exposure (cover)
(3) Visibility (background and concealment)
(4) Orientation
(5) Velocity
(6) Flight roughness and acceleration
(7) Operational status of mobility system

Performance Data Base

(1) Helicopter mobility capabilities

Tactics Data Base

(1) Flight path selection

Concurrent Activities Influenced

(1) All acquisition activities involving this AH
(2) Ground element target selection of AH
(3) Ordnance preparation, guidance, flight, and detonation (for ordnance with this AH as target)
(4) Battle status assessment
(5) Command element tactical decisions

Events Which May Disrupt This Activity

(1) Damage to this AH
(2) Identification of ground elements
(3) Ground element firing
(4) AH receiving fire
(5) AH receiving warning of being tracked
(6) Command element changes this AH's assignment
(7) Operational failure (other than mobility)
(8) Fuel expended

Status Parameters Influencing the Course of This Activity

(1) Environment (map, weather, etc.)
(2) Firing positions selected
(3) Assault positions selected

Activity (3) Attack helicopter hovers in firing position

Normal Events

(1) Arrives firing position
(2) Departs firing position
(3) Operational failure of mobility system

Status Parameters Influenced

(1) Location
(2) Exposure (cover)
(3) Visibility (background and concealment)
(4) Orientation
(5) Velocity
(6) Flight roughness and acceleration
(7) Operational status of mobility system

Performance Data Base

(1) Helicopter mobility capabilities

Tactics Data Base

(1) Orientation and altitude selection

Concurrent Activities Influenced

(1) All acquisition activities involving this AH
(2) Ground element target selection of AH
(3) Ordnance preparation, guidance, flight, and detonation (for ordnance with this AH as target)
(4) Battle status assessment
(5) Command element tactical decisions
(6) AH prepares to fire ordnance, ordnance flies, is guided, detonates

Events Which May Disrupt This Activity

(1) Damage to this AH
(2) Identification of ground elements
(3) Ground element firing
(4) AH receiving fire
(5) AH receiving warning of being tracked
(6) Command element changes this AH's assignment
(7) Operational failure (other than mobility)
(8) Fuel expended
(9) Ammunition expended

Status Parameters Influencing the Course of This Activity

(1) Environment (map, weather, etc.)
(2) Firing position selected

Activity (4) Attack helicopter moves from firing position to alternate firing position

Normal Events

(1) Depart firing position
(2) Arrive alternate firing position
(3) Operational failure of mobility system

Status Parameters Influenced

(1) Location
(2) Exposure (cover)
(3) Visibility (background and concealment)
(4) Orientation
(5) Velocity
(6) Flight roughness and acceleration
(7) Operational status of mobility system

Performance Data Base

(1) Helicopter mobility capabilities

Tactics Data Base

(1) Flight path selection
    (a) Locations and capabilities of identified ground elements
    (b) Locations of other AH's

Concurrent Activities Influenced

(1) All acquisition activities involving this AH
(2) Ground element target selection of AH
(3) Ordnance preparation, guidance, flight, and detonation (for ordnance with this AH as target)
(4) Battle status assessment
(5) Command element tactical decisions
(6) AH prepares ordnance, guides ordnance, flight and detonation

Events Which May Disrupt This Activity

(1) Damage to this AH
(2) Identification of ground elements
(3) Ground element firing
(4) AH receiving fire
(5) AH receiving warning of being tracked
(6) Command element changes this AH's assignment
(7) Operational failure (other than mobility)
(8) Fuel expended

Continued----

Activity 4 Continued

Status Parameters Influencing the Course of This Activity

(1) Environment (map, weather, etc.)
(2) Firing positions selected

Activity (5)   Attack helicopter engages in running fire

Normal Events

(1) Begin firing run
(2) End firing run
(3) Operational failure of mobility system

Status Parameters Influenced

(1) Location
(2) Exposure (cover)
(3) Visibility (background and concealment)
(4) Orientation
(5) Velocity
(6) Flight roughness and acceleration
(7) Operational status of mobility system

Performance Data Base

(1) Helicopter mobility capabilities

Tactics Data Base

(1) Selection of flight path

Concurrent Activities Influenced

(1) All acquisition activities involving this AH
(2) Ground element target selection
(3) Ordnance preparation, guidance, flight, and detonation
(4) Battle status assessment
(5) Command element tactical decisions

Events Which May Disrupt This Activity

(1) Damage to this AH
(2) Operational failure (other than mobility)

Status Parameters Influencing the Course of This Activity

(1) Environment (map, weather, etc.)
(2) Locations of weapon elements
(3) Selection of firing run tactic

Activity (6) Attack helicopter withdraws from firing position to assault position

Normal Events

(1) Leave firing position
(2) Remask at assault position (with respect to individual GW's)
(3) Operational failure of mobility system

Status Parameters Influenced

(1) Location
(2) Exposure (cover)
(3) Visibility (background and concealment)
(4) Orientation
(5) Velocity
(6) Flight roughness and acceleration
(7) Operational status of mobility system

Performance Data Base

(1) Helicopter mobility capabilities

Tactics Data Base

(1) Flight path selection

Concurrent Activities Influenced

(1) All acquisition activities involving this AH
(2) Ground element target selection of AH
(3) Ordnance preparation, guidance, flight, and detonation (for ordnance with this AH as target)
(4) Battle status assessment
(5) Command element tactical decisions

Events Which May Disrupt This Activity

(1) Damage to this AH
(2) AH receiving fire
(3) AH receiving warning of being tracked
(4) Command element changes this AH's assignment
(5) Operational failure (other than mobility)
(6) Fuel expended

Status Parameters Influencing the Course of This Activity

(1) Environment (map, weather, etc.)
(2) Firing and assault positions selected

Activity (7) Attack helicopter withdraws from running position to assault position

Normal Events

(1) Begin withdrawal (end of firing run)
(2) Remask at assault position
(3) Operational failure of mobility system

Status Parameters Influenced

(1) Location
(2) Exposure (cover)
(3) Visibility (background and concealment)
(4) Orientation
(5) Velocity
(6) Flight roughness and acceleration
(7) Operational status of mobility system

Performance Data Base

(1) Helicopter mobility capabilities

Tactics Data Base

(1) Flight path selection

Concurrent Activities Influenced

(1) All acquisition activities involving this AH
(2) Ground element target selection of AH
(3) Ordnance preparation, guidance, flight, and detonation (for ordnance with this AH as target)
(4) Battle status assessment
(5) Command element tactical decisions

Events Which May Disrupt This Activity

(1) Damage to this AH
(2) Identification of ground elements
(3) Ground element firing
(4) AH receiving fire
(5) AH receiving warning of being tracked
(6) Command element changes this AH's assignment
(7) Operational failure (other than mobility)
(8) Fuel expended

Status Parameters Influencing the Course of This Activity

(1) Environment (map, weather, etc.)
(2) AH location at end of firing run
(3) Assault position selected

Activity (8) Attack helicopter hovers at assault position

Normal Events

(1) Arrive assault position
(2) Unmask (with respect to specific ground elements)
(3) Operational failure of mobility system

Status Parameters Influenced

(1) Location
(2) Exposure (cover)
(3) Visibility (background and concealment)
(4) Orientation
(5) Velocity
(6) Flight roughness and acceleration
(7) Operational status of mobility system

Performance Data Base

(1) Helicopter mobility capabilities

Tactics Data Base

(1) Selection of hovering location and altitude

Concurrent Activities Influenced

(1) All acquisition activities involving this AH
(2) Ground element target selection of AH
(3) Ordnance preparation, guidance, flight, and detonation (for ordnance with this AH as target)
(4) Battle status assessment
(5) Command element tactical decisions

Events Which May Disrupt This Activity

(1) Damage to this AH
(2) Identification of ground elements
(3) Ground element firing
(4) AH receiving fire
(5) AH receiving warning of being tracked
(6) Command element changes this AH's assignment
(7) Operational failure (other than mobility)
(8) Fuel expended

Status Parameters Influencing the Course of This Activity

(1) Environment (map, weather, etc.)

Activity (9) Attack helicopter withdraws from assault position to holding position

Normal Events

(1) Depart assault positions
(2) Arrive holding positions
(3) Operational failure of mobility system

Status Parameters Influenced

(1) Location
(2) Exposure (cover)
(3) Visibility (background and concealment)
(4) Orientation
(5) Velocity
(6) Flight roughness and acceleration
(7) Operational status of mobility system

Performance Data Base

(1) Helicopter mobility capabilities

Tactics Data Base

(1) Flight path selection

Concurrent Activities Influenced

(1) All acquisition activities involving this AH
(2) Ground element target selection of AH
(3) Ordnance preparation, guidance, flight, and detonation (for ordnance with this AH as target)
(4) Battle status assessment
(5) Command element tactical decisions

Events Which May Disrupt This Activity

(1) Damage to this AH
(2) Identification of ground elements
(3) Ground element firing
(4) AH receiving fire
(5) AH receiving warning of being tracked
(6) Command element changes this AH's assignment
(7) Operational failure (other than mobility)
(8) Fuel expended

Status Parameters Influencing the Course of This Activity

(1) Environment (map, weather, etc.)

Activity (10) Attack helicopter performs evasive maneuvers

Normal Events

(1) Break from normal movement activity
(2) Return to original activity or alternative
(3) Operational failure of mobility system

Status Parameters Influenced

(1) Location
(2) Exposure (cover)
(3) Visibility (background and concealment)
(4) Orientation
(5) Velocity
(6) Flight roughness and acceleration
(7) Operational status of mobility system

Performance Data Base

(1) Helicopter mobility capabilities
(2) Pilot skill to perform evasion

Tactics Data Base

(1) Evasion tactics

Concurrent Activities Influenced

(1) All acquisition activities involving this AH
(2) Ground element target selection of AH
(3) Ordnance preparation, guidance, flight, and detonation (for ordnance with this AH as target and as firer)
(4) Battle status assessment
(5) Command element tactical decisions

Events Which May Disrupt This Activity

(1) Damage to this AH
(2) AH receiving fire (changes evasion tactic)
(3) Operational failure (other than mobility)
(4) Fuel expended

Status Parameters Influencing the Course of This Activity

(1) Environment (map, weather, etc.)

Activity (11) Ground element continues normal maneuver pattern (independent of helicopter attack)

Normal Events

(1) Operational failure

Status Parameters Influenced

(1) Location
(2) Exposure
(3) Visibility
(4) Orientation
(5) Velocity
(6) Terrain roughness and acceleration
(7) Operational status

Performance Data Base

(1) Ground element mobility capabilities

Tactics Data Base

(1) Selection of normal maneuvers

Concurrent Activities Influenced

(1) All wcquisition activities involving this ground element
(2) Target selection by AH's
(3) AH prepares to fire ordnance

Events Which May Disrupt This Activity

(1) Identification of AH
(2) AH fires
(3) Ground element receives fire
(4) Ground element damaged
(5) Ground command element changes this ground element's assignment

Status Parameters Influencing the Course of This Activity

(1) Environment (map, weather, etc.)
(2) Selection of normal maneuver pattern

Activity (12) Ground element seeks covered position

Normal Events

(1) Begin maneuver
(2) Position attained
(3) Operational failure of mobility system

Status Parameters Influenced

(1) Location
(2) Exposure (cover)
(3) Visibility (background and concealment)
(4) Posture
(5) Velocity
(6) Terrain roughness and acceleration
(7) Operational status of mobility system

Performance Data Base

(1) Ground element mobility capabilities

Tactics Data Base

(1) Selection of covered position
(2) Route selection to covered position

Concurrent Activities Influenced

(1) All acquisition activities involving this ground element
(2) AH target selection of this ground element
(3) AH ordnance preparation, guidance, flight, and detonation (with this ground element as target)
(4) Battle status assessment
(5) Command element tactical decisions

Events Which May Disrupt This Activity

(1) Damage to this ground element
(2) Identification of AH
(3) AH firing
(4) Ground element receiving fire
(5) Command element changes this ground element's assignment
(6) Operational failure (other than mobility)
(7) Covered position becomes unavailable or untenable

Status Parameters Influencing the Course of This Activity

(1) Environment (map, weather, etc.)
(2) Covered position selected
(3) Present position

Activity (13) Ground element seeks firing position

Normal Events

(1) Begins movement to firing position
(2) Firing position attained
(3) Operational failure of mobility system

Status Parameters Influenced

(1) Location
(2) Exposure (cover)
(3) Visibility (background and concealment)
(4) Posture
(5) Velocity
(6) Terrain roughness and acceleration
(7) Operational status of mobility system

Performance Data Base

(1) Ground element mobility capabilities

Tactics Data Base

(1) Route selection
(2) Firing position selection

Concurrent Activities Influenced

(1) All acquisition activities involving this ground element
(2) All target selection of this ground element
(3) All ordnance preparation, guidance, flight, and detonation (with this ground element as target)
(4) Battle status assessment
(5) Command element tactical decisions

Events Which May Disrupt This Activity

(1) Damage to this ground element
(2) Identification of AH
(3) AH firing
(4) Ground element receiving fire
(5) Command element changes this ground element's assignment
(6) Operational failure (other than mobility)

Status Parameters Influencing the Course of This Activity

(1) Environment (map, weather, etc.)
(2) Present position (at start of activity)
(3) Firing position selected

Activity (14) Ground element moves from cover to firing position (or changes "posture")

Normal Events

(1) Leaves covered position
(2) Arrives firing position
(3) Operational failure of mobility system

Status Parameters Influenced

(1) Location
(2) Exposure (cover)
(3) Visibility (background and concealment)
(4) Posture
(5) Velocity
(6) Terrain roughness and acceleration
(7) Operational status of mobility system

Performance Data Base

(1) Ground element mobility capabilities

Tactics Data Base

(1) Route selection

Concurrent Activities Influenced

(1) All acquisition activities involving this ground element
(2) AH target selection of this ground element
(3) AH ordnance preparation, guidance, flight, and detonation (with this ground element as target)
(4) Battle status assessment
(5) Command element tactical decisions

Events Which May Disrupt This Activity

(1) Damage to this ground element
(2) Identification of AH
(3) AH firing
(4) Ground element receiving fire
(5) Command element changes this ground element's assignment
(6) Operational failure (other than mobility)

Status Parameters Influencing the Course of This Activity

(1) Environment (map, weather, etc.)
(2) Covered position location
(3) Firing position selected

Activity (15) Ground element moves from firing position to cover (or changes "posture")

Normal Events

(1) Ground element departs firing position
(2) Ground element arrives at covered position
(3) Operational failure of mobility system

Status Parameters Influenced

(1) Location
(2) Exposure (cover)
(3) Visibility (background and concealment)
(4) Posture
(5) Velocity
(6) Terrain roughness and acceleration
(7) Operational status of mobility system

Performance Data Base

(1) Ground element mobility capabilities

Tactics Data Base

(1) Route selection

Concurrent Activities Influenced

(1) All acquisition activities involving this ground element
(2) AH target selection of this ground element
(3) AH ordnance preparation, guidance, flight, and detonation (with this ground element as target)
(4) Battle status assessment
(5) Command element tactical decisions

Events Which May Disrupt This Activity

(1) Damage to this ground element
(2) Identification of AH changes estimated exposure status of covered position
(3) Command element changes this ground element's assignment

Status Parameters Influencing the Course of This Activity

(1) Environment (map, weather, etc.)
(2) Location of firing position
(3) Covered position selected

Activity (16)* Ground element maintains covered position

Normal Events

(1) Ground element arrives covered position
(2) Ground element departs covered position
(3) Operational failure of mobility system

Status Parameters Influenced

(1) Location
(2) Exposure (cover)
(3) Visibility (background and concealment)
(4) Posture
(5) Velocity
(6) Terrain roughness and acceleration
(7) Operational status of mobility system

Performance Data Base

(1) Ground element mobility capabilities

Tactics Data Base

(1) Selection of covered positions

Concurrent Activities Influenced

(1) All acquisition activities involving this ground element
(2) AH target selection of this ground element
(3) AH ordnance preparation, guidance, flight, and detonation (with this ground element as target)
(4) Battle status assessment
(5) Command element tactical decisions

Events Which May Disrupt This Activity

(1) Damage to this ground element
(2) Ground element receiving fire (changes this elements estimate of exposure status)
(3) Command element changes this ground element's assignment
(4) Operational failure (other than mobility)

Status Parameters Influencing the Course of This Activity

(1) Environment (map, weather, etc.)
(2) Current exposure status
(3) Selection of cover as a function of helicopter deployment

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*Ground element may be required to maneuver in order to maintain cover with respect to changing threat deployment.

Activity (17)* Ground element maintains firing position

Normal Events

(1) Arrive firing position
(2) Depart firing position
(3) Operational failure of mobility system

Status Parameters Influenced

(1) Location
(2) Exposure (cover)
(3) Visibility (background and concealment)
(4) Posture
(5) Velocity
(6) Terrain roughness and acceleration
(7) Operational status of mobility system

Performance Data Base

(1) Ground element mobility capabilities

Tactics Data Base

(1) Selection of location and posture in firing position

Concurrent Activities Influenced

(1) All acquisition activities involving this ground element
(2) Target selection activities
(3) All ordnance preparation, guidance, flight, and detonation activities
(4) Battle status assessment
(5) Command element tactical decisions
(6) Countermeasure activities

Events Which May Disrupt This Activity

(1) Damage to this ground element
(2) Identification of AH
(3) AH firing
(4) Ground element receiving fire
(5) Command element changes this ground element's assignment
(6) Operational failure (other than mobility)
(7) Ground element loses target track
(8) Target destroyed by another ground element
(9) Target seeks cover

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*Ground element may maneuver or may be required to maneuver in firing position.

Continued----

Activity 17 Continued

Status Parameters Influencing the Course of This Activity

(1) Environment (map, weather, etc.)
(2) Selection of location and posture in firing position

Activity (18) Ground element moves from covered (or firing) position to alternate

Normal Events

(1) Depart present position
(2) Arrive alternate position
(3) Operational failure of mobility system

Status Parameters Influenced

(1) Location
(2) Exposure (cover)
(3) Visibility (background and concealment)
(4) Posture
(5) Velocity
(6) Terrain roughness and acceleration
(7) Operational status of mobility system

Performance Data Base

(1) Ground element mobility capabilities

Tactics Data Base

(1) Rules for selecting alternate covered (or firing) positions
(2) Flight path selection

Concurrent Activities Influenced

(1) All acquisition activities involving this ground element
(2) AH target selection of this ground element
(3) AH ordnance preparation, guidance, flight, and detonation (with this ground element as target)
(4) Ground element ordnance preparation, firing, flight, detonation*
(5) Battle status assessment
(6) Command element tactical decisions

Events Which May Disrupt This Activity

(1) Damage to this ground element
(2) Identification of AH
(3) AH firing
(4) Ground element receiving fire
(5) Command element changes this ground element's assignment
(6) Operational failure (other than mobility)
(7) Alternate position becomes undesirable

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*For ground elements which fire while moving.

Continued----

Activity 18 Continued

Status Parameters Influencing the Course of This Activity

(1) Environment (map, weather, etc.)
(2) Alternate position and flight path selected

Activity (19) Ground element attempts to withdraw

Normal Events

(1) Ground element begins withdrawing
(2) Ground element arrives withdrawn position
(3) Operational failure of mobility system

Status Parameters Influenced

(1) Location
(2) Exposure (cover)
(3) Visibility (background and concealment)
(4) Posture
(5) Velocity
(6) Terrain roughness and acceleration
(7) Operational status of mobility system

Performance Data Base

(1) Ground element mobility capabilities

Tactics Data Base

(1) Selection of routes of withdrawal

Concurrent Activities Influenced

(1) All acquisition activities involving this ground element
(2) AH target selection of this ground element
(3) AH ordnance preparation, guidance, flight, and detonation (with this ground element as target)
(4) Battle status assessment
(5) Command element tactical decisions

Events Which May Disrupt This Activity

(1) Damage to this ground element
(2) Ground element receiving fire
(3) Command element changes this ground element's assignment
(4) Operational failure (other than mobility)

Status Parameters Influencing the Course of This Activity

(1) Environment (map, weather, etc.)
(2) Withdrawal positions and routes selected
(3) Locations of AH's

Activity (20)   Weapon element searches for target(s)

Normal Events

(1) Search commences
(2) Target found
(3) False alarm
(4) Search terminates
(5) Operational failure

Status Parameters Influenced

(1) Detected target list of searching element
(2) Weapon element damage status

Performance Data Base

(1) Detection behavior data (detection rates, false alarm rates, reliability)

Tactics Data Base

(1) Search tactic
(2) Turn-on, turn-off tactic
(3) Threshold or equivalent setting or doctrine

Concurrent Activities Influenced

None

Events Which May Disrupt This Activity

(1) Damage to this element
(2) Warhead detonation (near this element)
(3) Element loses exposure

Status Parameters Influencing the Course of This Activity

(1) Exposure statuses
(2) Visibility statuses
(3) Location and velocity statuses
(4) Environment (map, weather, etc.)
(5) Sector of responsibility assignments
(6) Readiness (early warning status)
(7) Identified target locations

Activity (21) Weapon element attempts to identify target(s)

Normal Events

(1) Weapon element detects target
(2) Weapon element modifies search scan and search tactic for identification
(3) Weapon element begins scan of target area with auxiliary sensors
(4) Target identified as false alarm
(5) Friendly target identified
(6) Enemy target identified
(7) Activity terminates without identification
(8) Weapon element communicates identification of enemy target
(9) Weapon element begins tracking target
(10) Weapon element begins search
(11) Operational failure

Status Parameters Influenced

(1) Identified target list of searching element
(2) Detected target list of searching element
(3) Identified target list of command and intelligence element
(4) Status of this weapon element's acquisition systems (searching, identifying, tracking)
(5) Weapon element operational status

Performance Data Base

(1) Identification behavior data base

Tactical Data Base

(1) Selection of auxiliary sensors
(2) End activity without identification
(3) Begin tracking
(4) Report identified target

Concurrent Activities Influenced

(1) Weapon element movement
(2) Weapon element searches for targets
(3) Weapon element tracks targets
(4) Weapon element selects target
(5) Weapon element prepares to fire ordnance
(6) Weapon element guides ordnance
(7) Weapon element employs countermeasures
(8) Battle status assessment
(9) Information coordination
(10) Tactical decisions made

Continued----

Activity 21 Continued

Events Which May Disrupt This Activity

(1) Damage to this element
(2) Element receiving fire
(3) Loss or change in visibility of this element or target

Status Parameters Influencing the Course of This Activity

(1) Environment (map, weather, etc.)
(2) Visibility statuses
(3) Exposure statuses
(4) Location and velocity statuses
(5) This element's list of identified targets

Activity (22) Weapon element tracks target(s)

Normal Events

(1) Begin tracking (end of identification)
(2) Begin tracking with auxiliary sensors
(3) Lose track
(4) End track (target destroyed or ignored)
(5) Operational failure

Status Parameters Influenced

(1) Estimated location of tracked target for this tracker
(2) Tracker operational status

Performance Data Base

(1) Tracking performance

Tactical Data Base

(1) On-off tactic

Concurrent Activities Influenced

(1) Ordnance preparation (including aiming), flight, guidance, detonation
(2) Countermeasures against radar track

Events Which May Disrupt This Activity

(1) Loss of visibility
(2) AH damage
(3) AH receiving fire
(4) Target destroyed by another AH
(5) ECM or IRCM

Status Parameters Influencing the Course of This Activity

(1) Tracker and target locations, velocities, etc.
(2) Environment (map, weather, etc.)
(3) Tracker and target exposures and visibilities

Activity (23)* Weapon element selects target

Normal Events

(1) Begin selection
(2) Target selected (including null target)

Performance Data Base

(1) Firepower capabilities

Tactical Data Base

(1) Battle drill target selection tactics
(2) Target selection rules

Concurrent Activities Influenced

None

Events Which May Disrupt This Activity

None

Status Parameters Influencing the Course of This Activity

(1) List of identified targets for this weapon element and their statuses and locations
(2) List of friendly weapon elements and their statuses and locations
(3) Mission objective
(4) Available firepower and ammunition for this weapon element
(5) Target assignments from command elements

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*Viewed as an activity of very short duration.

Activity (24) Weapon element prepares to fire ordnance (orienting, aiming, loading)

Normal Events

(1) Begin firing preparation
(2) Load ordnance*
(3) Ordnance loaded
(4) Weapon element oriented
(5) Weapon aimed
(6) Weapon fired
(7) Extract shell
(8) Operational failure of weapon or ordnance (does not fire)

Status Parameters Influenced

(1) Orientation, exposure, visibility of this firer
(2) Operational status of this weapon element's firepower systems
(3) Aiming errors (bias)

Performance Data Base

(1) Sequence of ordnance preparation activities for particular weapon and weapon element pair
(2) Sequence of activities in the event of operational failure
(3) Aiming errors

Tactical Data Base

(1) Weapon selection rules
(2) Firing process rules
(3) Aiming tactics (estimation of lead angle, etc.)

Concurrent Activities Influenced

(1) Movement by this firer
(2) Acquisition and tracking by this firer

Events Which May Disrupt This Activity

(1) Operational failure of systems other than firepower
(2) Identification of other targets
(3) Weapon element receiving fire
(4) Element receives warning of being tracked
(5) Command element changes this weapon element's assignment
(6) Lose tracking of target
(7) Target status changes (out of range, moves to cover, etc.)

Status Parameters Influencing the Course of This Activity

(1) Target statuses affecting aiming (movement, cover, exposure, etc.)
(2) Firer statuses affecting aiming and orientation

Activity (25) Weapon element guides ordnance

Normal Events

(1) Ordnance fired (begin guidance)
(2) Ordnance detonates (end guidance)
(3) Guidance operational failure

Status Parameters Influenced

(1) This firer's "busy" state
(2) Ordnance "under guidance" state
  (a) Ordnance hit probability
(3) Guidance system operational status

Performance Data Base

(1) Hit or kill probabilities vs. guidance time, guidance range, guider skill, etc.

Tactics Data Base

None

Concurrent Activities Influenced

(1) Location, orientation of firer and target
(2) All acquisition activities involving this firer

Events Which May Disrupt This Activity

(1) Firer damaged
(2) Target destroyed by another firer
(3) Target maneuvers to cover or out of range
(4) Firer receiving fire
(5) Firer re-aims missile at another target

Status Parameters Influencing the Course of This Activity

(1) Target and firer locations
(2) Target and firer exposure and visibility
(3) Target orientation

Activity (26) Ordnance flies

Normal Events

(1) Ordnance fired
(2) Ordnance misses target
(3) Ordnance hits target
(4) Operational failure

Status Parameters Influenced

(1) Amm-nition supply of firer
(2) Ordnance operational status
(3) Number of rounds (bursts) en route to target
(4) Target hit by previously arrived round
(5) Target missed by previously arrived round
(6) Initial rounds (bursts) en route
(7) Weapon element firing
(8) Target receiving fire

Performance Data Base

(1) Ordnance reliability
(2) Hit probabilities as a function of
  (a) Firer type
  (b) Firer status (moving, suppressed, etc.)
  (c) Ordnance type
  (d) Target type
  (e) Target status (moving, cover, evading, suppressed, etc.)
  (f) Target range
  (g) Target aspect (to arriving round)
  (h) Markov fire parameters
(3) Time-of-flight as a function of
  (a) Ordnance type
  (b) Target range

Concurrent Activities Influenced

(1) Target movement
(2) All acquisition activites involving this firer
(3) Target selection (selection of this firer as a target)
(4) All firing activities involving this target
(5) Battle status assessment

Events Which May Disrupt This Activity

(1) Missile guidance disabled
  (a) Guiding weapon element damaged
  (b) TOW wire cut
  (c) ECM
  (d) IRCM

Continued----

Activity 26 Continued

Status Parameters Influencing the Course of This Activity

(1) Firer location and status
(2) Target location and status

Activity Process Models

(1) n-order Markov firing process model
(2) Time-of-flight model
(3) Ammunition expenditure model

Activity (27) Warhead detonates which possibly damages target(s)

Normal Events

(1) Warhead detonates
(2) Possible change in target(s)' damage status
(3) Operational failure of warhead

Status Parameters Influenced

(1) Warhead operational status
(2) Warhead status (warhead detonated)
(3) Target(s) damage status (list of target damage)
(4) Round fired hit or miss status

Performance Data Base

(1) Damage probabilities as a function of
  (a) Warhead type
  (b) Detonation location
  (c) Target(s)' location
  (d) Target posture
  (e) Target type
  (f) Target cover
  (g) Other target statuses
(2) Detonation location for fuze types, target types, target postures, etc.

Tactics Data Base

None

Concurrent Activities Influenced

(1) Target movement
(2) All acquisition activities for this target
(3) All firing activities for this target
(4) Battle status assessment
(5) Command element information coordination and tactical decisions (if command element is target)
(6) Countermeasure activities of this target

Events Which May Disrupt This Activity

(1) Countermeasures against fuzed warheads?

Status Parameters Influencing the Course of This Activity

(1) Target(s)' locations, postures, cover, aspect to detonating round
(2) Detonation location
(3) Previous damage status of target (for cumulative damage)

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