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SOME COMMENTS ON THE MODELING OF THE  
COLLAPSING WAKE

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Ocean and Atmospheric Science, Incorporated

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13. ABSTRACT

The problem of scaling the phenomena of the collapse of the wake of a self-propelled body in a stratified fluid is examined. Conditions are derived such that the Froude number (based on body speed, length and Vaisala period) is equal for both model and prototype. Conditions are also obtained for the existence of buoyancy and inertial subranges in the wake turbulence. These conditions are applied to determine the smallest size model for which the phenomenon of wake growth and collapse is properly scaled from the prototype.

## Abstract

The problem of scaling the phenomena of the collapse of the wake of a self-propelled body in a stratified fluid is examined. Conditions are derived such that the Froude number (based on body speed, length and Väisälä period) is equal for both model and prototype. Conditions are also obtained for the existence of buoyancy and inertial subranges in the wake turbulence. These conditions are applied to determine the smallest size model for which the phenomenon of wake growth and collapse is properly scaled from the prototype.

In this note, we consider a self-propelled body of length  $L$  traveling at a speed  $U$  in a stratified fluid which is characterized by a Väisälä frequency  $N$ . The diameter of the body is  $D$ , the diameter of the propeller is  $d$  and it will be assumed that  $L/D = 10$  and  $D/d = 2$  so that  $L/d = 20$ . For the full scale or prototype body, typical numerical values are  $U = 6$  knots  $\approx 3$  m/sec,  $L = 100$  m, and  $N = 10^{-2}$  sec $^{-1}$ .

The scaling parameter is the Froude number

$$F = U / (ND)$$

which is, of course, equivalent to Richardson number scaling since

$$Ri = 1 / F^2.$$

In the usual way,  $U$ ,  $D$  and  $N$  are chosen so that

$$F_P = \left( \frac{U}{ND} \right)_P = \left( \frac{U}{ND} \right)_M = F_M,$$

where the subscripts  $P$  and  $M$  refer to the values of the parameters for the prototype and model respectively.

In both the model and prototype, the physics must be the same for a meaningful model experiment. The propeller produces turbulence. The wake grows by entraining fluid with a consequent decrease in the turbulent energy and increase in the potential energy of the fluid within the wake. Simultaneously there is viscous dissipation of the turbulence energy. Finally, when the turbulent energy drops to a sufficiently low level, the wake collapses.

One of the consequences of Ko's model of the growing and collapsing wake (Ko, 1971) is that the time for the wake to collapse,  $t_c$ , is nearly

independent of the Froude number and is

$$t_c \approx 2/N$$

This is, of course, in agreement with the available experimental data (Schooley and Stewart, 1963; Vander Watering, 1966).

Another time scale is that of the turbulence (Batchelor, 1967)

$$t_T = l/u$$

where  $l$  is the integral scale of the turbulence and  $u$  is the turbulent intensity.

Clearly the ratio between these times must be equal for both model and prototype if the phenomenon of wake growth and collapse is to be correctly modeled. This ratio is

$$t_c/t_T = 2u/(Nl)$$

The turbulent intensity is a fraction of  $U$ ,

$$u = \alpha U, \quad 0 < \alpha < 1.$$

Ko suggests that  $\alpha \approx 0.25$  just behind the propeller and this is in rough agreement with the results obtained by Naudascher (1965). It is also to be expected that the integral scale is a fraction of the propeller diameter,  $d$ , i. e.,

$$l = \beta d, \quad 0 < \beta < 1$$

With these assumptions

$$\begin{aligned} \left(\frac{t_c}{t_T}\right)_M &= \left(\frac{2u}{Nl}\right)_M = 2\left(\frac{\alpha U}{\beta N d}\right)_M \\ &= 4\left(\frac{\alpha}{\beta}\right)_M \left(\frac{U}{ND}\right)_M = 4\left(\frac{\alpha}{\beta}\right)_M F_M, \end{aligned}$$

and

$$\begin{aligned}\left(\frac{t_c}{t_T}\right)_P &= \left(\frac{2U}{Nl}\right)_P = 2 \left(\frac{\alpha U}{\beta N d}\right)_P \\ &= 4 \left(\frac{\alpha}{\beta}\right)_P \left(\frac{U}{ND}\right)_P = 4 \left(\frac{\alpha}{\beta}\right)_P F_P\end{aligned}$$

Therefore

$$\left(\frac{t_c}{t_T}\right)_M = \left(\frac{t_c}{t_T}\right)_P$$

if, and only if

$$\left(\alpha/\beta\right)_M = \left(\alpha/\beta\right)_P$$

That is, this ratio of the time for the wake to collapse to the characteristic time of the turbulence will have the same value for both the model and the prototype if the turbulent intensity is the same fraction of the body speed and the integral scale is the same fraction of the propeller diameter in both model and prototype. In both model and prototype the self-propulsion is accomplished by a propeller; both the model and prototype propellers are geometrically similar and the ratio of the rotational speed of the propeller to body speed is the same for both model and prototype. Since both the geometry and the velocity triangles of the blades are similar, one would expect, provided that the model propeller produces a turbulent flow, that the ratios of the integral scale to the diameter and the turbulent intensity to the body speed are invariant with scale. That is, we expect  $\alpha$  and  $\beta$  are truly constants. Then the ratio  $(t_c/t_T)$  is also invariant. Finally, we can estimate this ratio. If we take  $\alpha = 1/4$  and  $\beta = 1/2$ ,

$$(t_c/t_T) = 2\left(\frac{U}{ND}\right)_P \approx 100$$

These estimates suggest that the collapsing wake can be properly modeled at any scale provided that the wake is turbulent at that scale. We must therefore ask, is the wake turbulent behind the propeller and, say, up to the point of collapse? It is clear that the Reynolds number will be very different in model and prototype, but there must be a similarity in that a buoyancy range and an inertial range must be present in model scale if they are present in prototype scale.

We will therefore examine the flow at two sections; just behind the propeller and at the point of collapse. Taking the time of collapse as

$$t_c = 2/N$$

the point of collapse is

$$x_c = U t_c = 2U/N$$

Therefore

$$(x_c/d) = 2\left(\frac{U}{Nd}\right) = 2\left(\frac{D}{d}\right)\left(\frac{U}{ND}\right) \approx 120$$

using the values of  $U$ ,  $D$  and  $N$  for the prototype. So we must estimate the state of the flow at

$$x = 0 \quad \text{and} \quad x = 120 d.$$

In all of the numerical work below, it will be assumed that the speed of the model,  $U_M = 1 \text{ m/sec.}$

A necessary condition for the existence of an inertial range (Phillips, 1966) is that

$$R_{\ell}^{1/2} = \left( \frac{u \ell}{\nu} \right)^{1/2} \gg 1.$$

We will assume that, just behind the propeller ( $\alpha = 0$ ),

$$u = U/4, \quad \ell = d/2$$

then

$$R_{\ell_0}^{1/2} = \left( \frac{u \ell}{\nu} \right)^{1/2} = \left( \frac{U d}{8 \nu} \right)^{1/2} = \left( \frac{U L}{8 \nu} \right)^{1/2} \left( \frac{d}{L} \right)^{1/2}$$

With  $U = 1 \text{ m/sec}$ ,  $d/L = 1/20$ , and  $\nu = 10^{-2} \text{ cm}^2/\text{sec}$ ,

$$R_{\ell_0}^{1/2} \approx 8 L^{1/2}$$

with  $L$  in cm. It is clear that any model longer than about 50 cm easily satisfies the condition  $R_{\ell_0}^{1/2} \gg 1$ .

Next we will estimate  $R_{\ell}^{1/2}$  at  $\alpha = 120^\circ$ . We need estimates of  $u$  and  $\ell$  at  $\alpha = 120^\circ$ . We will use the only experimental results which we have, those of Naudascher (1965). Using the data in Naudascher's Table 3 and Figure 15\*

$$\ell/d \approx 0.5 \quad \text{and} \quad u/U \approx 7 \times 10^{-3}$$

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\* Naudascher uses  $D$  for the diameter of the turbulence generator and  $L$  for the integral scale; our  $d$  and  $\ell$ , respectively.

Therefore

$$\begin{aligned} R_{L/120}^{1/2} &= (uL/\nu)^{1/2} = (3.5 \times 10^{-3})^{1/2} (UL/\nu)^{1/2} \\ &= (3.5 \times 10^{-3})^{1/2} (UL/\nu)^{1/2} (d/L)^{1/2} \\ &\approx 1.3 L^{1/2} \end{aligned}$$

with  $L$  in cm. We thus need

$$L^{1/2} \gg 1$$

and it appears that  $L = 100$  cm will be adequate. Therefore there will be an equilibrium range at collapse if  $L \gtrsim 1$  meter.

Next, we ask under what conditions a buoyancy range will exist. It is clear that, just as in considering  $R_L$ , the crucial conditions occur at  $\alpha = 120 d$ . The buoyancy wavenumber  $K_b$  which separates the buoyancy range and the equilibrium range is (Lumley, 1964; Phillips, 1966)

$$K_b \approx (N^3/\epsilon)^{1/2}$$

where  $\epsilon$  is the rate of energy dissipation. For a buoyancy range to exist

$$(1/\eta) \gg K_b$$

where

$$\eta = (\nu^3/\epsilon)^{1/4}$$

is the Kolmogorov micro-scale. Taking

$$\epsilon = u^3/l,$$

this condition is

$$u^3 / (\nu \ell N^2) \gg 1.$$

This can be written as

$$\left(\frac{u}{U}\right)^3 F^2 (d/\ell) (D/d) (D/L) (UL/\nu) \gg 1,$$

or with  $u/U = 7 \times 10^{-3}$ ,  $\ell/d = 0.5$  from Naudascher's results, and  $U = 1 \text{ m/sec.}$

$$L \gg 1$$

with  $L$  in cm. Again, it seems reasonable that  $L \approx 1$  meter will be adequate.\*

Finally, we might require that the Reynolds number based on the dissipation length scale

$$R_\lambda = (u \lambda / \nu) \gg 1.$$

Again we consider only the case  $\alpha = 120 d$ . Naudascher (Figure 13) gives curves of  $\lambda/\lambda_\infty$  ( $\lambda_\infty/d = 0.135$ ) as a function of  $\nu(\alpha - \alpha_0)/U \lambda_\infty^2$ . Then, taking  $\alpha_0 = 0$ ,

$$\begin{aligned} \left(\frac{\nu \alpha}{U \lambda_\infty^2}\right) &= \left(\frac{\nu}{U d}\right) \left(\frac{\alpha}{d}\right) \left(\frac{d}{\lambda_\infty}\right)^2 = \left(\frac{\nu}{U L}\right) \left(\frac{L}{d}\right) \left(\frac{\alpha}{d}\right) \left(\frac{d}{\lambda_\infty}\right)^2 \\ &\approx 2/L \end{aligned}$$

---

\* At  $\alpha = 120 d$ , the ratio  $u^3/\ell$  is actually larger than  $\epsilon$  by the amount of energy extracted in buoyancy range. Including this consideration, this condition is given by

$$L \gg \gg 1$$

Again, it seems reasonable that  $L \approx 100 \text{ cm}$  is adequate.

with  $L$  in cm. For  $1 \text{ m} \leq L \leq 10 \text{ m}$ ,  $\lambda/\lambda_\infty$  lies in the range 0.5 to 0.2. We will take

$$\lambda/\lambda_\infty = 0.4 \quad \text{and} \quad \lambda_\infty/d = 0.135$$

to obtain an estimate of  $R_\lambda$ . Therefore,

$$\begin{aligned} R_\lambda &= (u/U)(\lambda/\lambda_\infty)(\lambda_\infty/d)(d/L)(UL/\nu) \\ &= 0.16 L \end{aligned}$$

with  $L$  in cm. Again, it appears that  $L \approx 1 \text{ m}$  will be satisfactory.

We finally conclude that:

- $(t_c/t_T)$  will scale correctly with the Froude number  $F = U/(ND)$  if the wake flow is turbulent at model scale.
- For the model wake to be similar to that of the prototype, the requirements are:

for an inertial range,

$$R_\lambda^{1/2} = [(u/U)(l/d)(d/L) R_L]^{1/2} \gg 1 ;$$

for a buoyancy range,

$$(u/U)^3 (d/l)(D/d)(D/L) F^2 R_L \gg 1 ;$$

and finally,

$$R_\lambda = (u/U)(\lambda/\lambda_\infty)(\lambda_\infty/d)(d/L) R_L \gg 1$$

with

$$R_L = UL/\nu$$

Using Naudascher's results, we find that in all cases these inequalities are satisfied by taking  $R_L = 10^6$ , i. e.,  $U = 1 \text{ m/sec}$ .  $L = 1 \text{ m}$  with

$$\nu = 10^{-2} \text{ cm}^2/\text{sec}.$$