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IMAGING OF EXTENDED OBJECTS THROUGH A TURBULENT ATMOSPHERE

Hal T. Yura, et al

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Imaging of Extended Objects Through a Turbulent Atmosphere

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The Aerospace Corporation

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Laboratory Operations
THE AEROSPACE CORPORATION

Prepared for SPACE AND MISSILE SYSTEMS ORGANIZATION
AIR FORCE SYSTEMS COMMAND
LOS ANGELES AIR FORCE STATION
Los Angeles, California

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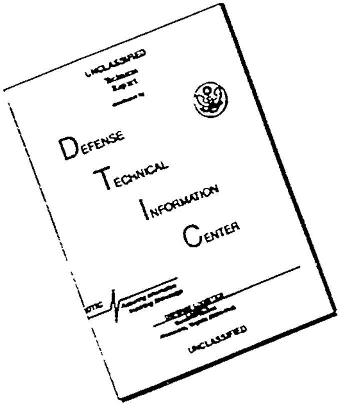

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FOREWORD

This report is published by The Aerospace Corporation, El Segundo, California, under Air Force Contract No. F04701-72-C-0073.

This report, which documents research carried out from October through December 1972, was submitted 16 April 1973 to Lieutenant Ernest L. Lockwood, DYAE, for review and approval.

A.H. Silver, Director
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Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

Ernest L. Lockwood
1st Lt., U.S. Air Force
Project Officer

ABSTRACT

The extended Huygens-Fresnel principle is used to derive an explicit expression for the image plane illuminance distribution of an extended object with an arbitrary luminance emittance distribution. The combined simultaneous effects of attenuation, background luminance, and atmospheric turbulence are given. A quantitative comparison of these effects is made, and their contribution to the overall loss in resolution is given. In particular, we derive a quantitative expression for the point spread function of the combined atmospheric-optical system. Explicit expressions are derived for both the atmospheric-optical system modulation transfer function (MTF) and image plane modulation. Numerical results for the image plane modulation are presented for imaging both up and down along an atmospheric path under various viewing conditions.

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I. INTRODUCTION

The effects of the atmosphere significantly degrade the imaging capability of optical systems in many applications. The mechanisms that produce image degradation include the attenuation of the light from the object of interest, light scattered into the imaging system from aerosols and unwanted objects (i.e., background luminance), and image blurring due to atmospheric turbulence. The limitations on imaging capability resulting from each of these mechanisms acting separately has been treated previously.[1] However, to the best of our knowledge no derivation of the image plane illuminance of an extended object, which includes the combined, simultaneous degrading effects of the above mechanisms, exists in the literature.

We present a systematic derivation of image plane illuminance and the corresponding limits of resolution, using the extended Huygens-Fresnel principle.[2] This technique permits a quantitative comparison of the contribution of each of the above mechanisms to the overall loss of resolution. In particular, an explicit expression is obtained for the point spread function, defined as the image plane illuminance due to a point source located in the object plane, of the combined atmospheric-optical system.

The extension of the vacuum Huygens-Fresnel principle to the atmosphere is that the optical field at an arbitrary point in space can again be represented as a superposition of spherical waves emitted from the primary wavefront existing at the object, but each spherical wavelet is now determined by the manner in which it propagates through the medium. As a result, a physical interpretation of the solution, in any given specific case, is easily obtained. The use of this principle gives directly the solutions to many problems of interest.[2-5] As an additional illustration of the usefulness of this principle, we apply it here to the calculation of image formation through the atmosphere. The extended Huygens-Fresnel principle is particularly suited for this application, since it permits the separation of the geometry of the problem, i.e., the extended source distribution, from the propagation problem, which is determined completely by the manner in which spherical waves propagate from various elements of the object through the atmosphere to the imaging optics.

In Sec. II, the extended Huygens-Fresnel principle is used to derive an explicit expression for the image plane illuminance of an extended object, which includes all of the degradations given earlier. In Sec. III, an expression for the optical transfer functions (OTF) of the atmospheric-optical system is obtained. In particular, we derive explicit expressions for both the atmospheric-optical system modulation transfer function (MTF) and image plane modulation. Finally, in Sec. IV, we present numerical results for the image plane modulation for imaging both up and down along an atmospheric path under various viewing conditions.

II. GENERAL CONSIDERATIONS: IMAGE PLANE ILLUMINANCE

We consider the problem of imaging an extended object with an arbitrary luminance emittance distribution through a turbulent atmosphere. Under natural illumination conditions, the object can be considered to be an incoherent radiator; i.e., the light emitted either by reflection or direct emission from each point of the object is uncorrelated with that emitted from any other point. Even when an object is illuminated by laser light, the reflected light is emitted as essentially incoherent radiation when the scale of irregularities of the object is small compared with the coherence length of the light beam. (This is the case for almost all objects of practical interest.)

In the configuration depicted in Fig. 1, the object is located a distance z from a thin positive lens of focal length f, and we want to determine the resulting image illuminance distribution in the image plane of the lens. The distance f' of the image plane from the lens plane is given, for a focused system, by

$$\frac{1}{z} + \frac{1}{f'} = \frac{1}{f} \tag{1}$$

The turbulent medium between the object and lens is characterized by the spectral density of the index of refraction fluctuations Φ_n[6] and a wavelength-dependent attenuation constant α, which is the sum of the absorption coefficient and a Rayleigh, Mie, and nonselective scattering coefficient.[7] To account for this attenuation, quantities proportional to the

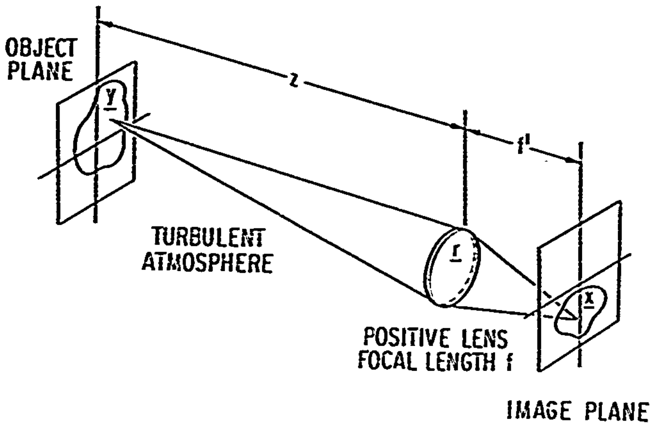

Fig. 1. Atmospheric optical imaging system

square of the field contain the transmission factor $\exp(-\alpha z)$. In addition to the light emanating from the object, we assume that the lens is illuminated by some background light that is incoherent with respect to the light coming from the object. The resulting optical illuminance entering the lens is then the sum of that coming from the object and the background light.

In addition, we assume, for simplicity, a two-dimensional object, which is described by a given field distribution $U_o(\underline{y})$, where $\underline{y}$ is an object coordinate. Coordinates in the lens and image plane will be denoted by the two-dimensional vectors $\underline{r}$ and $\underline{x}$, respectively.

In the following, the resulting image illuminance $U(\underline{r})$ at the entrance pupil of the lens is obtained from a direct application of the extended Huygens-Fresnel principle. The modification of the results of Ref. 2 to include atmospheric attenuation is[5]

$$U(\underline{r}) = \exp\left(-\frac{\alpha z}{2}\right)\left(-\frac{ik}{2\pi}\right)\int G(\underline{y},\underline{r})\, U_o(\underline{y})\, d^2\underline{y} \tag{2}$$

where the integration extends over the object distribution, G is the Green's function

$$G(\underline{y},\underline{r}) \cong \left(\frac{1}{z}\right)\exp[iks(\underline{y},\underline{r}) + \psi(\underline{y},\underline{r})] \tag{3}$$

$s(\underline{y},\underline{r})$ is the geometrical distance between the object point $\underline{y}$ and the lens point $\underline{r}$, ψ is the complex phase perturbation due to the turbulent medium,[2] and k is the optical wave number. We take the spatial extent of the object (and lens) to be much less than the propagation distance z, from which it follows that

$$s(\underline{y},\underline{r}) \cong z + \frac{(\underline{y}-\underline{r})^2}{2z} \tag{4}$$

The field $V(\underline{r})$ at the exit surface of the lens is given by $V(\underline{r}) = W(\underline{r}) \exp[-ikr^2/2f]\, U(\underline{r})$. The quantity $W(\underline{r})$, the lens factor, is unity for $|\underline{r}| \leq D/2$, where D is the diameter of the lens, and zero otherwise, and the exponential factor is just the transmission factor of a positive lens of focal length f. The instantaneous field distribution in the image plane is given by the vacuum Huygens-Fresnel principle

$$\begin{aligned} U_i(\underline{x}) &= \left(-\frac{ik}{2\pi}\right)\int V(\underline{r})\, G_o(\underline{r},\underline{x})\, d^2\underline{r} \\ &= \left(-\frac{ik}{2\pi}\right)\int W(\underline{r}) \exp\left[-\frac{ikr^2}{2f}\right] U(\underline{r})\, d^2\underline{r} \end{aligned} \tag{5a}$$

and substituting Eq. (2) into Eq. (5a) yields

$$\begin{aligned} U_i(\underline{x}) = \left(-\frac{ik}{2\pi}\right)^2 \exp\left(-\frac{\alpha z}{2}\right)\int d^2\underline{y}\int d^2\underline{r}\; U_o(\underline{y})\, G(\underline{y},\underline{r})\, G_o(\underline{r},\underline{x}) \\ \times W(\underline{r}) \exp\left[-\frac{ikr^2}{2f}\right] \end{aligned} \tag{5b}$$

where the $\underline{r}$ integration extends over the lens and

$$G_o(\underline{r},\underline{x}) \cong \left(\frac{1}{f'}\right) \exp[iks(\underline{r},\underline{x})]$$

The instantaneous illuminance distribution in the image plane is given by

$$E_i(\underline{x}) = |U_i(\underline{x})|^2$$

$$= \left(-\frac{ik}{2\pi}\right)^2 \exp(-\alpha z) \int d^2\underline{y} \int d^2\underline{y}' \int d^2\underline{r} \int d^2\underline{r}' \, U_o(\underline{y}) \, U_o^*(\underline{y}') \, G(\underline{y}, \underline{r})$$

$$\times G^*(\underline{y}', \underline{r}') \, G_o(\underline{r}, \underline{x}) \, G_o^*(\underline{r}', \underline{x}) \, W(\underline{r}) \, W(\underline{r}') \exp\left[-\frac{ik(r^2 - r'^2)}{2f}\right]$$

Since the object radiates incoherently, the only contribution to the integrals in the above are obtained for $\underline{y} = \underline{y}'$. We have

$$\langle E_i(\underline{x})\rangle = A \exp(-\alpha z) \int d^2\underline{y} \int d^2\underline{r} \int d^2\underline{r}' \, E_o(\underline{y}) \langle G(\underline{y}, \underline{r}) \, G^*(\underline{y}, \underline{r}')\rangle G_o(\underline{r}, \underline{x}) \, G_o^*(\underline{r}', \underline{x})$$

$$\times W(\underline{r}) \, W(\underline{r}') \exp\left[-\frac{ik(r^2 - r'^2)}{2f}\right] \tag{7}$$

where $E_o(\underline{y}) = |U_o(\underline{y})|^2$ is the luminous emittance (lm/m^2) distribution of the object, and since we are dealing with a random medium, an ensemble average (denoted by angular brackets) has been taken. In the appendix, we show that by considering the vacuum limit of the illuminance when the object is reduced to a point, the constant can be determined as $A = k^2/8\pi^3$.

Setting $G = G_o \exp(\psi)$, we obtain

$$\langle G(\underline{y}, \underline{r}) \, G^*(\underline{y}, \underline{r}')\rangle = G_o(\underline{y}, \underline{r}) \, G_o^*(\underline{y}, \underline{r}') \, \langle \exp[\psi(\underline{y}, \underline{r}) + \psi^*(\underline{y}, \underline{r}')]\rangle \tag{8}$$

where

$$G_o(\underline{y}, \underline{r}) = \left(\frac{1}{z}\right) \exp[iks(\underline{y}, \underline{r})] \tag{9}$$

The quantity $\langle \exp[\psi(\underline{y},\underline{r}) + \psi^*(\underline{y},\underline{r}')]\rangle$ is recognized as the mutual coherence function (MCF) of a spherical wave propagating in the turbulent medium from the point $\underline{y}$ on the object to the lens points $\underline{r}$ and $\underline{r}'$.[2] We assume here that the MCF is independent of source coordinate $\underline{y}$; that is, we assume the region of isoplanatism is larger than the spatial extent of the object. Further, for the case of homogeneous isotropic turbulence the MCF can be expressed as $M_s(\rho)$, where $\rho = |\underline{r} - \underline{r}'|$. The turbulence properties of the medium are contained in the spherical wave MCF, and thus a knowledge of M_s is sufficient for the determination of the resulting average image illuminance characteristics.

Substituting Eqs. (6), (8), and (9) into Eq. (7) and changing variables from $\underline{r}$ and $\underline{r}'$ to $\underline{\rho} = \underline{r} - \underline{r}'$ and $\underline{R} = (\underline{r} + \underline{r}')/2$, the average illuminance can be written in the form

$$\langle E_i(x)\rangle = \int S(\underline{y} + \underline{x}z/f')\, E_o(\underline{y})\, d^2\underline{y} \tag{10a}$$

where the point source response function S is given by

$$S(\underline{y}) = \frac{k^2 D^2}{32\pi^2 f'^2 z^2} \exp(-\alpha z) \int M_L\left(\frac{\rho}{D}\right) M_s(\rho) \exp\left[-\frac{ik\, \underline{\rho}\cdot\underline{y}}{z}\right] d^2\underline{\rho} \tag{10b}$$

and M_L is the lens transfer function [Eq. (17)].

Equations (10a) and (10b) reveal that the average illuminance can be expressed as a convolution of the point-source response function with the luminous emittance distribution of the object, the point source response function is proportional to the Fourier-transform of the product of the lens transfer function and the spherical wave MCF.

In addition to the illuminance distribution resulting from the object, we must add the corresponding illuminance distribution due to the background light. Background light can be characterized by a uniform luminance $B_b(lm/m^2 - sr)$, resulting in an image plane illuminance E_b.

$$E_b = B_b \frac{(\text{area of lens})}{(\text{distance lens to image plane})^2}$$

$$\approx \frac{B_b(\pi D^2/4)}{(f')^2} \cong \frac{\pi B_b}{4F^2} \quad (11)$$

where the F number of the lens is the ratio of the focal length to the lens diameter. The background image brightness is uniform over the image plane and is independent of the distance to the object. The F number of the lens determines the image brightness; the smaller the F number, the brighter the image. From Eq. (10b), this applies to object light as well as the background light.

Thus, the total illuminance distribution in the image plane is

$$E_T(\underline{x}) = \langle E_i(\underline{x}) \rangle + E_b \quad (12)$$

Background luminance (or path luminance) depends on the relative positions of the sun, the observer, and the scene. It also depends on the observer's altitude and the condition of the atmosphere; it is different for an observer on the ground looking at an airplane than for the reverse situation. Figure 2 is a schematic of the viewing geometry. The parameters indicated in the figure are: observer's altitude H, zenith angle between the local vertical and the light of sight θ, azimuth angle between the line of sight and the observer sun line measured in a horizontal plane ϕ, and solar zenith angle β. Standard or average values of the path luminance have been tabulated for a solar zenith angle of $\beta = 41.5$ deg and an "average" atmosphere.[8] Figure 3 is adapted from tables in Ref. 8.

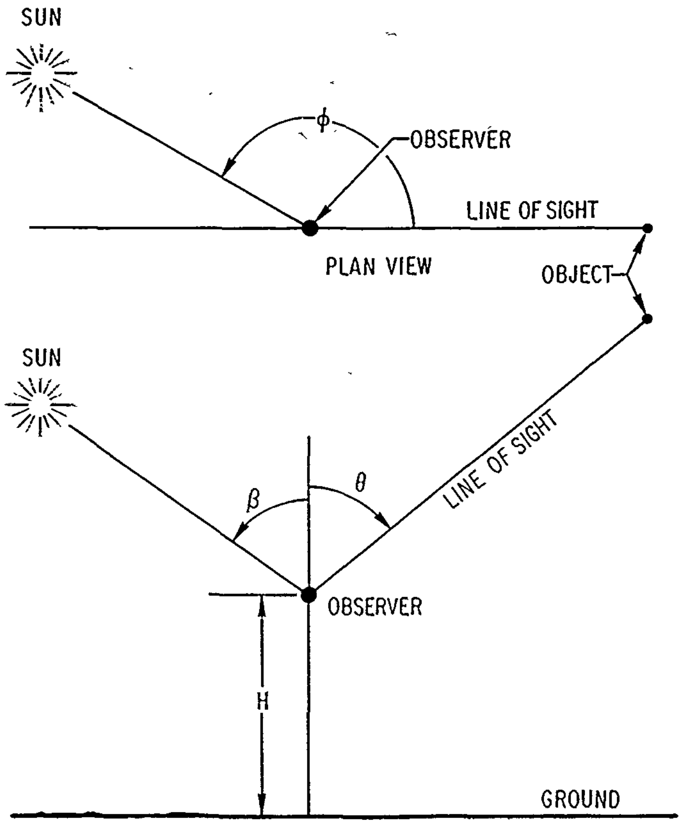

Fig. 2. Viewing geometry

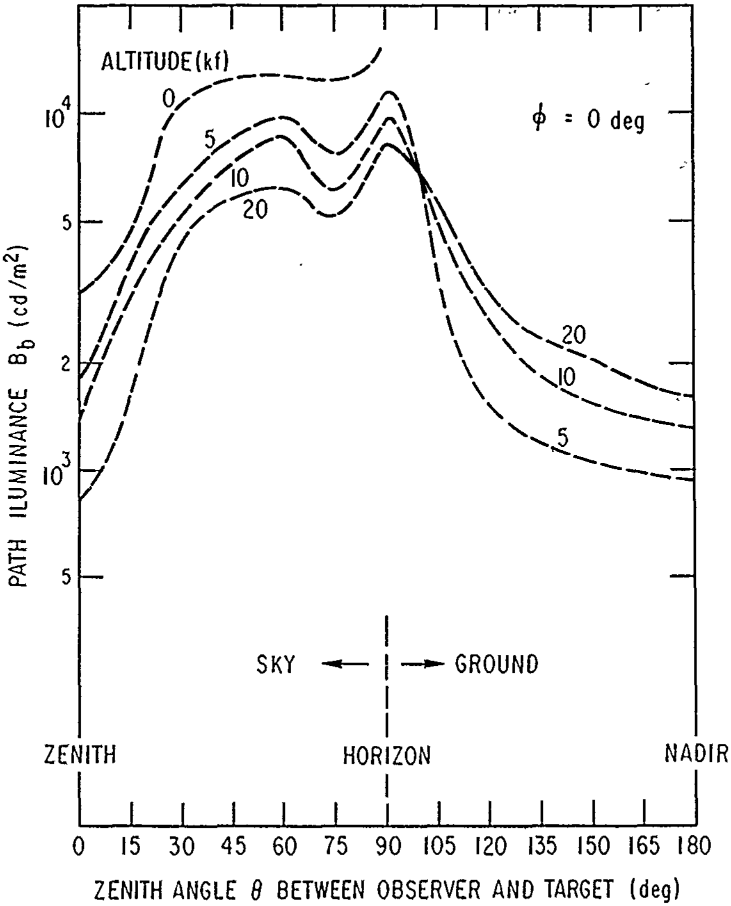

Fig. 3a. Average path luminance as a function of altitude and viewing angle for β = 41.5 deg and ϕ = 0 deg

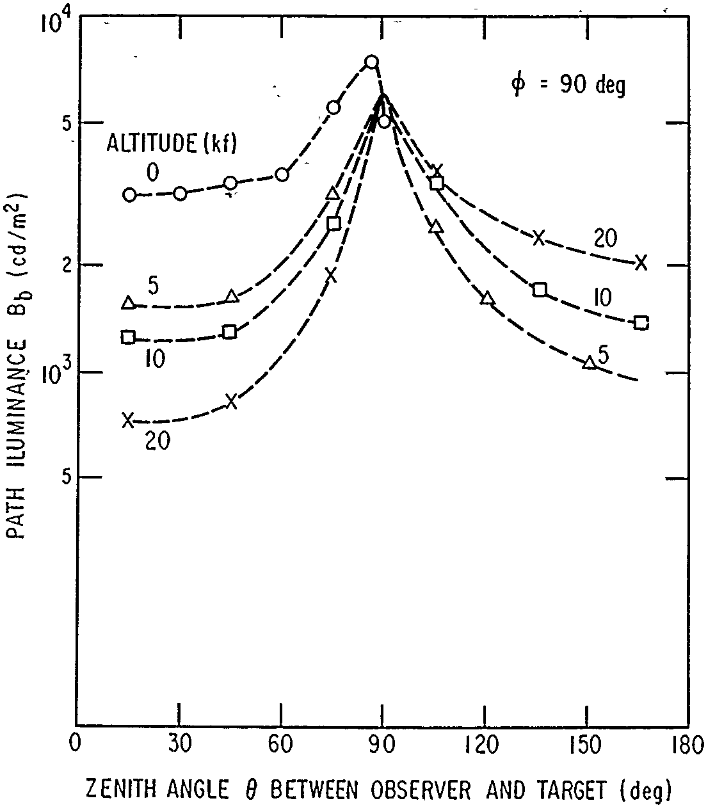

Fig. 3b. Average path luminance as a function of altitude and viewing angle for β = 41.5 deg and ϕ = 90 deg

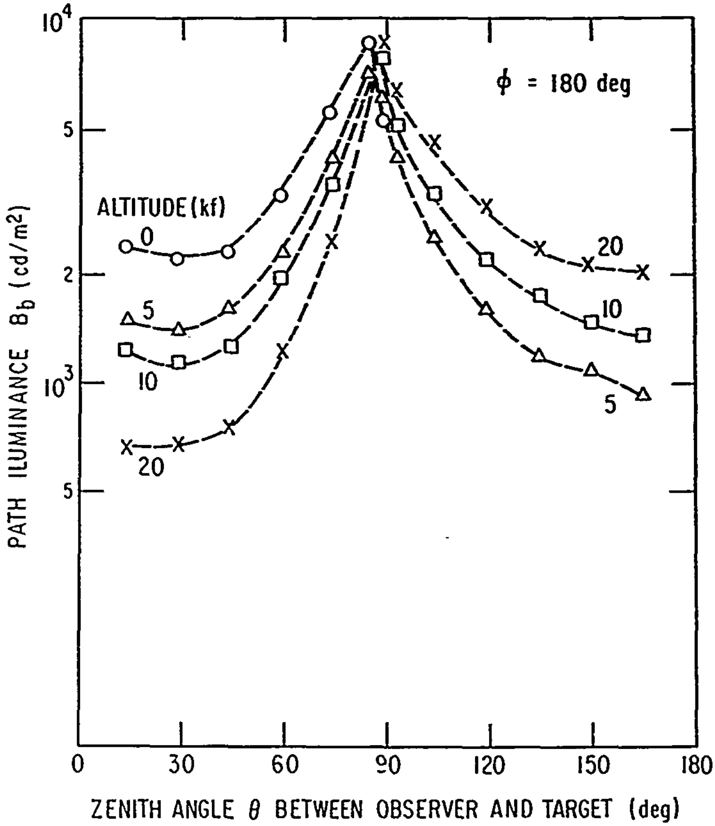

Fig. 3c. Average path luminance as a function of altitude and viewing angle for β = 41.5 deg and ϕ = 180 deg

III. OPTICAL TRANSFER FUNCTION

A quantity of particular interest is the optical transfer function (OTF) of the system. This quantity is defined as the complex (image) illuminance distribution corresponding to a unit amplitude (object) luminous emittance of a single spatial frequency. The OTF describes how both the amplitude and phase of the image distribution change as a function of spatial frequency. The resolution of the viewing system is determined directly from the OTF. When only amplitude changes are considered, as is often the case, the OTF is equal to the MTF.

In this regard, we are led to consider the two-dimensional Fourier transform of the image illuminance distribution. The illuminance consists of two terms, the background term and that due to light from the object. Since the background term E_b is constant, its Fourier transform is just a dc term at zero spatial frequency. The Fourier transform of $\langle E_i \rangle$ is

$$\tilde{E}_i(\underline{K}) = \int \langle E_i(\underline{x}) \rangle \exp(-i\underline{K} \cdot \underline{x})\, d^2\underline{x} \tag{13}$$

where K is the spatial frequency (rad/length). By substituting Eq. (10b) into Eq. (13), we find

$$\begin{aligned} \tilde{E}_o(\underline{K}) = {} & \frac{1}{16F^2}\, \tilde{E}_o\left(-\frac{f'\underline{K}}{z}\right) \exp(-\alpha z)\, M_s(\rho) \\ & \times \left[\left(\frac{\pi D^2}{4}\right)^{-1} \int d^2\underline{R}\; W\left(\underline{R} + \frac{\rho}{2}\right) W\left(\underline{R} - \frac{\rho}{2}\right)\right] + \frac{\pi}{4}\frac{B_b}{F^2}\,\delta(\underline{K}) \end{aligned} \tag{14}$$

where

$$\underline{\rho} = \frac{f'\underline{K}}{k} \tag{15}$$

and

$$\widetilde{E}_o(\underline{Q}) = \int E_o(\underline{y}) \exp(-i\underline{Q}\cdot\underline{y})\, d^2\underline{y} \tag{16}$$

is the Fourier transform of the object distribution. The minus sign appearing in $\widetilde{E}_o$ in Eq. (14) corresponds to the inversion of the image by the lens while the factor f'/z corresponds to the linear magnification of the spatial distribution of the object. For cases of interest here, z is large compared to the focal length f, and thus $f' \approx f$.

The integral in the bracket in Eq. (14) is the convolution of the lens pupil function with itself and is given by

$$\left(\frac{\pi D^2}{4}\right)^{-1} \int d^2\underline{R}\; W\left(\underline{R} + \frac{\underline{\rho}}{2}\right) W\left(\underline{R} - \frac{\underline{\rho}}{2}\right) = M_L\left(\frac{\rho}{D}\right) \tag{17}$$

where M_L, the MTF of the circular lens, is given by

$$M_L\left(\frac{\rho}{D}\right) = \frac{2}{\pi}\left\{\cos^{-1}\left(\frac{\rho}{D}\right) - \left(\frac{\rho}{D}\right)\left[1 - \left(\frac{\rho}{D}\right)^2\right]^{1/2}\right\} \tag{18}$$

and ρ is given by Eq. (15). In the general case, M_L, as defined by Eq. (17), is the MTF of a lens specified by an arbitrary pupil function $W(\underline{r})$.

The OTF is given by the ratio of the object to image Fourier components. In this case, the OTF is real, and is identical to the MTF. From Eqs. (14) and (17), we find

$$\mathrm{MTF} = \left(\frac{1}{16F^2}\right) \exp(-\alpha z)\, M_s(\rho)\, M_L\left(\frac{\rho}{D}\right) \tag{19}$$

where $\rho \simeq Kf/k = K'f\lambda$ where $K' = K/2\pi$ has the dimensions cycles per unit length. In this case, the MTF of the system is the product of the spherical wave MCF of the medium, evaluated at $\rho = Kf/k$, and the MTF of the lens, both of which are functions of spatial frequency. In addition, the MTF contains the factor $\exp(-\alpha z)$, which is independent of spatial frequency and corresponds to the loss of signal illuminance due to large angle scattering and absorption. The decomposition of the overall system MTF into a product of independent terms (e.g., medium, lens) is possible only if the spherical wave MTF of the medium is independent of the origin of the spherical wave source, i.e., the entire object lies in an isoplanatic patch.

As an illustrative example, we calculate the resulting degradation of contrast of a sinusoidal object. We consider an object that consists of a series of black and white lines or some other repetitive luminance pattern of a given spatial frequency. For mathematical simplicity, we model such a pattern by the following one-dimensional function

$$E_o(y) = \frac{E_o}{2} (1 + \cos Ky) \tag{20}$$

Here, with the standard photometric notation used,[9] E_o represents the maximum luminous emittance of the pattern. The modulation contrast M of a pattern is defined as

$$M = \frac{\langle E \rangle_{max} - \langle E \rangle_{min}}{\langle E \rangle_{max} + \langle E \rangle_{min}} \tag{21}$$

where E is the image plane illuminance.

The image plane illumination distribution, including that due to the background, resulting from an object distribution described by Eq. (20) is

$$E_T(x) = \left(\frac{1}{8F^2}\right)\left(\frac{E_o}{2}\right) \exp(-\alpha z) \left[1 + M_s(\rho)\, M_L\left(\frac{\rho}{D}\right) \cos\left(\frac{Kfx}{z}\right)\right] + \left(\frac{\pi}{4F^2}\right) B_b \tag{22}$$

Hence, from Eqs. (21) and (22), we obtain

$$(\mathrm{Mod})_{image} = \frac{M_s(\rho)\, M_L(\rho/D)}{1 + [4\pi B_b / E_o \exp(-\alpha z)]} \tag{23}$$

Thus, for $4\pi B_b \ll E_o \exp(-\alpha z)$, the modulation of the image pattern is independent of α and is given by the product of the spherical wave and lens MTF, respectively. In the general case, the image contrast will be degraded over and above that due to the diffraction limit of the lens whenever the spherical wave MTF cuts off sharper than the lens MTF.

We have

$$(\mathrm{Mod})_{image} = \frac{M_s(\rho)\, M_L(\rho/D)}{1 + \eta} \tag{24}$$

where

$$\eta = \frac{4\pi B_b}{E_o \exp(-\alpha z)} \tag{25}$$

Now, for a Lambertian surface of average reflectivity r, the average luminance E_o of the object is given by

$$E_o = \frac{rE_s}{\pi} \tag{26}$$

where E_s is the incident illuminance.[10] By substituting Eq. (26) into Eq. (25), we obtain

$$\eta = \frac{4\pi^2 B_b}{E_s r \exp(-\alpha z)} \tag{27}$$

The dimension loss quantity η is independent of spatial frequency and F number. The factor $(1 + \eta)^{-1}$ represents an overall reduction in image modulation. Only for the case $\eta \ll 1$ is the image modulation given by the product of the spherical wave and lens MTF.

IV. NUMERICAL EXAMPLES: SYSTEM RESOLUTION

As an illustrative example, we consider the case of looking both up or down with a 6-in. lens from an altitude of 3 km along a slant path of 10 km. To be definite, we consider viewing a solar illuminated horizontal object in the visible. We set ϕ, the azimuth angle between the line of sight and the observer sun line measured in a horizontal plane, equal to 90 deg, and β, the solar zenith angle, equal to 41.5 deg. From Fig. 4, we find that for looking down (θ = 108 deg) $B_b = 3 \times 10^3$ cd/m^2 and for looking up (θ = 72 deg) $B_b = 2 \times 10^3$ cd/m^2. For this viewing geometry, we find from Ref. 9 that $E_s \simeq 7.44 \times 10^4$ lm/m^2, and assuming an average reflectivity of 0.5 we obtain, from Eq. (27) and Ref. 9, Figs. 4a and 4b and the Table I.

In Figs. (4a) and (4b), we have plotted the image modulation looking down and looking up as a function of angular spatial frequency K'f (which is equal to ρ/λ). In obtaining these curves, we have used Coulman's data[11] for the temperature structure constant profile for average conditions during the midpart of sunny days.

Table I. η For Various Atmospheric Conditions

Looking	η		
	Extra Clear	Very Clear	Light Haze
Down	0.92	1.40	6.60
Up	0.62	0.93	4.40

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For any given receiving system, there is some threshold modulation m_o (<1), such that if the image modulation is less than m_o, resolution is not possible. Generally, m_o is a function of spatial frequency and level of illumination reaching the receiver. Here, for illustrative purposes, we arbitrarily assume m_o is a constant and equal to 5%. In this case, we see, from Fig. 4a, that for a very clear atmosphere the limiting angular resolution is ~0.5 μrad. Conversely, for looking up under these same conditions, the limiting resolution is ~3 μrad. (Both of these limits apply to resolution in a direction normal to the line of sight.)

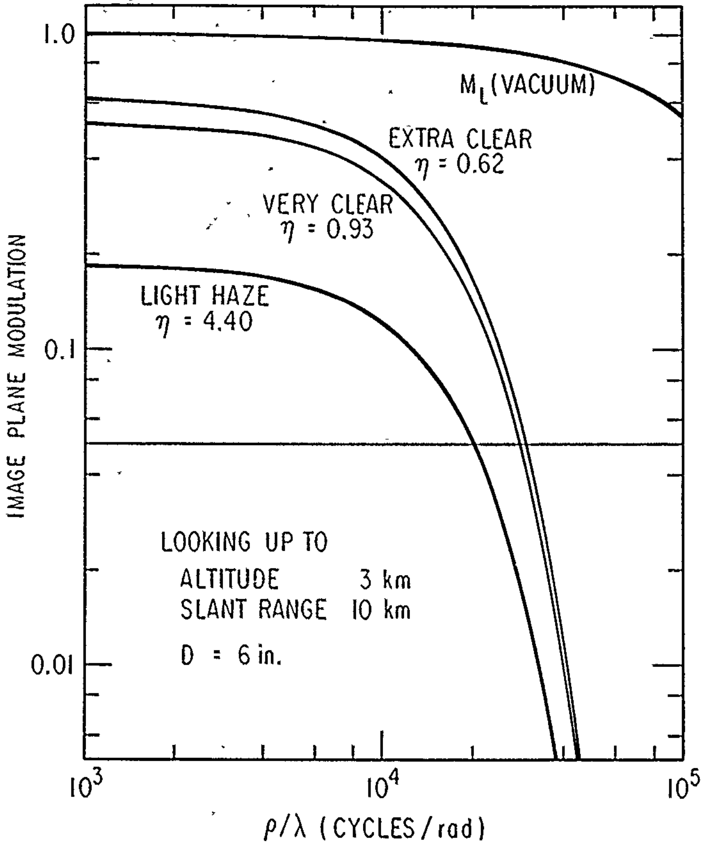

Fig. 4a. Image plane modulation for summer daytime turbulence conditions, looking up

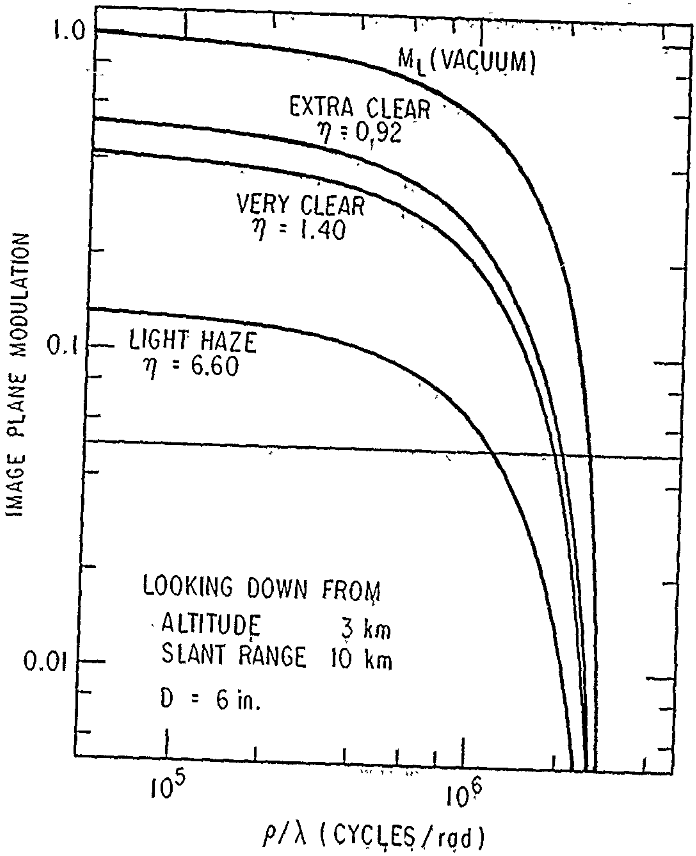

Fig. 4b. Image plane modulation for summer daytime turbulence conditions, looking down

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APPENDIX: EVALUATION OF THE CONSTANT A

To evaluate the constant A in Eq. (7), we consider the vacuum limit (i.e., $\alpha = 0$, $G = 1$) of the image plane illuminance as the diameter of the object Δy tends toward zero (positioned at $y = 0$) and of luminous strength tends to W_0; i.e.,

$$W_0 \equiv \lim_{\Delta y \to 0} \int E_o(\underline{y})\, d^2\underline{y} \tag{A-1}$$

In this case, Eq. (7) becomes

$$E_i(\underline{x}) = \frac{AW_0}{z^2} \left| \int G_o(\underline{r}, \underline{x})\, W(\underline{r})\, \exp\left[\frac{ikr^2(z^{-1} - f^{-1})}{2}\right] d^2\underline{r} \right|^2 \tag{A-2}$$

The vacuum propagator $G_o(\underline{r}, \underline{x})$ is given, in the paraxial approximation, by

$$G_o(\underline{r}, \underline{x}) \cong \left(\frac{1}{f'}\right) \exp\left\{ik\left[\frac{f' + (\underline{r} - \underline{x})^2}{2f'}\right]\right\} \tag{A-3}$$

Hence, from Eqs. (A-2) and (A-3), we obtain

$$\begin{aligned} E_i(\underline{x}) &= \frac{AW_0}{(zf')^2} \left| \int \exp\left(-\frac{ik\underline{x}\cdot\underline{r}}{f'}\right) W(\underline{r})\, d^2\underline{r} \right|^2 \\ &= \frac{AW_0}{(zf')^2} \left| 2\pi \int_0^a J_0\left(\frac{kxr}{f'}\right) r\,dr \right|^2 \\ &= \frac{AW_0(\pi a^2)^2}{(zf')^2} \left[\frac{2J_1(kax/f')}{(kax/f')}\right]^2 \end{aligned} \tag{A-4}$$

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where a is the radius of the lens.

The total luminous flux W_1 impinging on the image plane is obtained by integrating Eq. (A-4) over all $\underline{x}$. We obtain

$$W_1 = \int E_i(\underline{x})\, d^2\underline{x}$$

$$= 2\pi \int_0^\infty E_i(x)\, x dx$$

$$= \frac{\pi^3 A W_0 D^2}{k^2 z^2} \qquad \text{(A-5)}$$

where D = 2a is the diameter of the lens.

The quantity W_1 is equal to the total flux impinging on the lens, which for a point source radiating into a half space must also equal the source strength W_0 multiplied by the fractional area intercepted by the lens on a hemisphere of radius z:

$$W_1 = \left(\frac{\pi a^2}{2\pi z^2}\right) W_0 \qquad \text{(A-6)}$$

By comparing Eqs. (A-5) and (A-6), we obtain

$$A = \frac{k^2}{8\pi^3} \qquad \text{(A-7)}$$