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DIPOLE RECEIVE CHARACTERISTICS IN THE PRESENCE OF SEA WATER

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Accepted for the Air Force
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Acting Chief, Lincoln Laboratory Liaison Office

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DIPOLE RECEIVE CHARACTERISTICS
IN THE PRESENCE OF SEA WATER

I. INTRODUCTION

The difficulty of satellite-to-submarine communication at UHF frequencies is readily apparent when sea-water UHF parameters and the complex sea-water reflection coefficient are investigated. The skin depth[1] of sea water for a frequency of 250 MHz is 0.625 inch, so the loss is about 14 dB/inch, which is too much for any useful application of a submerged receiving dipole antenna.

The reflection coefficient of a sea-water plane surface for radiation polarized perpendicular to the plane of incidence is

$$R_{\perp} = (\cos\Theta - \sqrt{N^2 - \sin^2\Theta})/(\cos\Theta + \sqrt{N^2 - \sin^2\Theta}) = |R_{\perp}|\, e^{j\varphi_{\perp}} \qquad (1)$$

where $|R_{\perp}|$ is the modulus and $\varphi_{\perp}$ is the phase retardation of $R_{\perp}$, Θ is the angle of incidence measured from the positive y-axis [Fig. 1(a)], and $N = \sqrt{\epsilon_r - j\sigma/\omega\epsilon_o}$, the index of refraction. $|R_{\perp}|$ and $\varphi_{\perp}$ are plotted in Fig. 1(a) as a function of Θ.

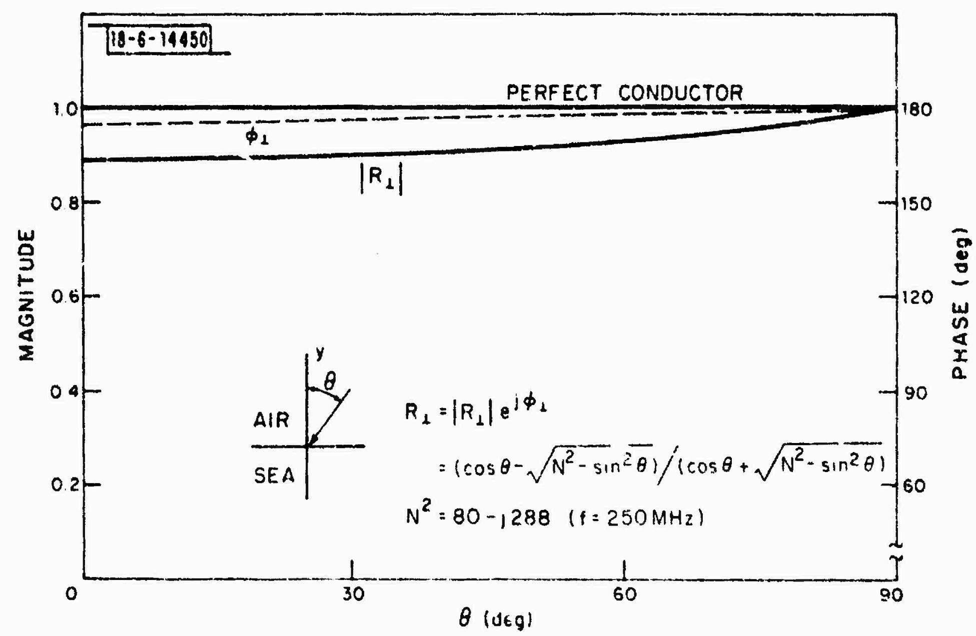

Fig. 1(a). Magnitude and phase of reflection coefficients $R_{\perp}$.

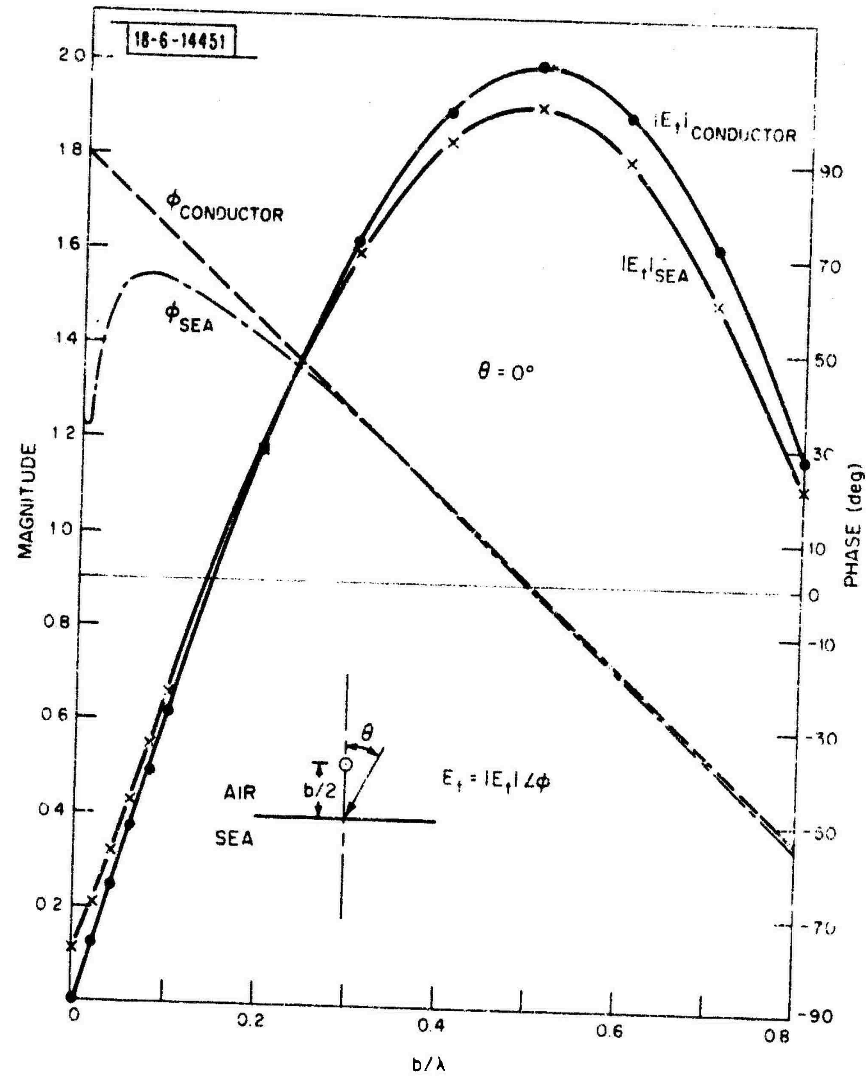

Fig. 1(b). Resultant electric field at low heights above sea water.

Observe that $|R_{\perp}|$ lies between 0.9 and 1 and the total variation of the phase retardation $\varphi_{\perp}$ occurs between +176° and +180°, while those same parameters for a perfect conductor are $|R_{\perp}| = 1$ and $\varphi_{\perp} = 180°$. The resultant electric field E_t, polarized perpendicularly, on and above a sea-water plane surface is shown in Fig. 1(b) where the modulus $|E_t|$ and the phase are plotted for normal incidence at 250 MHz. The corresponding values for a perfect conductor surface are also included for purposes of comparison. It may be noted that on the sea-water surface $|E_t|$ drops to 1/10 that of the incident wave so that, apart from other factors, a dipole very close to the surface would have its received power much reduced from its free-space value.

It appears that the only possibility for reception of UHF signals with reasonable antenna gain (>20 dB below isotropic) lies in the use of an antenna floating on or above the sea surface. Previous theory[1] indicated a sharp drop in gain as a dipole antenna was brought close to the sea surface; in fact, this theory showed the gain tending to zero and the input resistance tending to infinity. Thus, the question of whether reception with a dipole floating close to the sea surface was even theoretically feasible is still open. To refine this theory, especially for dipoles within 1 to 2 cm of the surface at 250 MHz, a theoretical approach based on image theory was used. The close analogy between the field over sea water and over a perfect conductor suggests that at UHF, sea water should act like a good conductor for a wave polarized perpendicularly and that image theory may be applied.

The problem to be solved is to determine how the power in the load of an isolated dipole used to receive a UHF signal from a satellite is altered when brought in proximity to a sea-water plane. For simplicity, the assumption is made that the dipole is far away from the transmitting source, that the amplitude of the electric vector is sensibly constant over the length of the antenna and is directed tangential to its axis.[2] The possible effect of a transmission line is excluded in this analysis. In the practical case, the transmission line connecting the load impedance to the antenna is part of the antenna and must be analyzed integrally with the antenna.

The method used to determine the receive characteristics of a dipole in the presence of sea water is to dispense with the conducting medium by introducing the image configuration of both dipole and field. Integral equations are then set up for the current along both the unloaded dipole and its image, which are now equivalent to two coupled antennas with different driving voltages. Thus, the simultaneous solution of the integral equations can be carried out by resolving[3] the current into a symmetric and an antisymmetric pair. The short-circuit current at the terminals, multiplied by the driving point impedance of the dipole when used for transmission, gives the open-circuit voltage. The equivalent circuit of the receiving antenna is then used to obtain the current in, or voltage across, the load impedance.

II. MATHEMATICAL FORMULATION

A dipole of radius "a" and its image [Fig. 2(a-b)] are spaced $\beta_0 b$ radians apart, measured from center-to-center, and their axes are parallel to the z-coordinate with ends at $z = \pm h$. The incoming wave is represented as rays in the diagram and the corresponding image rays are altered by the reflection coefficient. Ray 1 and Image Ray 2 excite the dipole, and the image dipole is excited by Ray 2 and Image Ray 1. The phase difference between the arrival of Ray 1 and the Image Ray 2 at the dipole is simply the sum of the net difference in path length of Ray 1 and Image Ray 2 and the phase retardation of the reflection coefficient, i.e., $\varphi_\perp - \beta_0 b \cos\Theta$. Having dispensed with the ocean surface via the image, the dipole in the presence of sea water bears great resemblance to the case of two identical, nonstaggered parallel receiving antennas. Therefore, the integral equation for the current along the unloaded dipole can now be written as

$$\int_{-h}^{h} I_1(z')\, K_a(z, z')\, dz' + \int_{-h}^{h} I_2(z')\, K_b(z, z')\, dz' = -j \frac{4\pi}{\zeta_0} [C_1 \cos\beta_0 z + U(1 + R_\perp e^{-j\gamma})] \qquad (2)$$

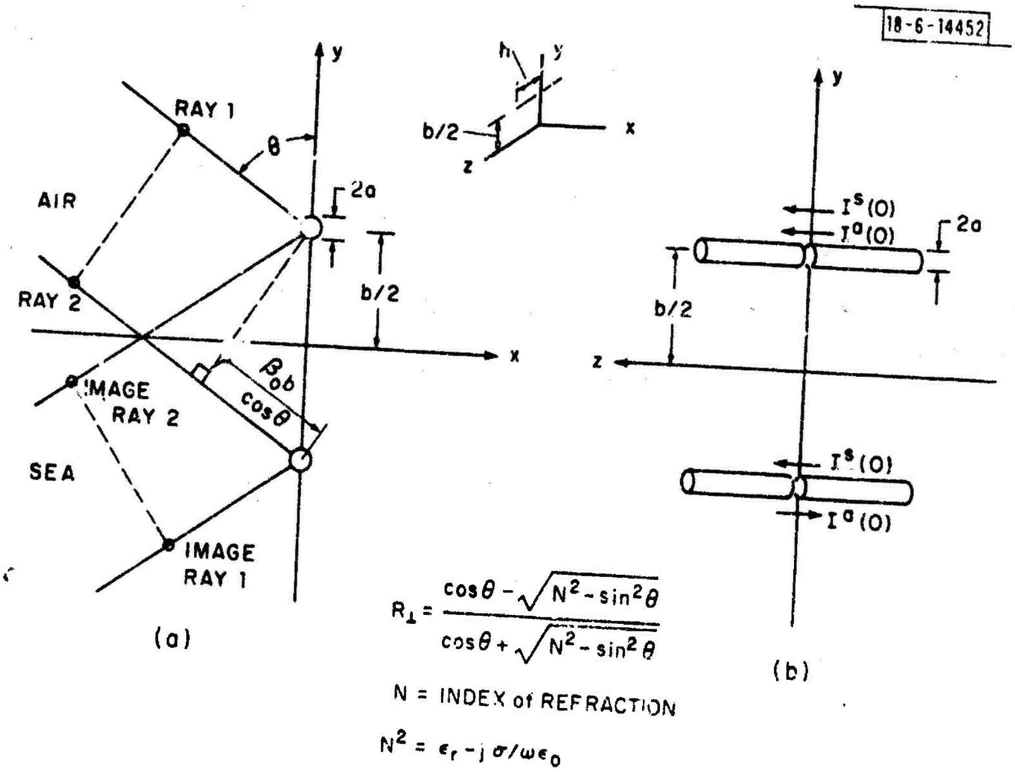

Fig. 2(a-b). Mathematical model.

where C_1 is an arbitrary constant and

$$K_a(z, z') = \exp\left[\frac{-j\beta_0 \sqrt{(z - z')^2 + a^2}}{\sqrt{(z - z')^2 + a^2}}\right] \tag{3}$$

$$K_b(z, z') = \exp\left[\frac{-j\beta_0 \sqrt{(z - z')^2 + b^2}}{\sqrt{(z - z')^2 + b^2}}\right] \tag{4}$$

$$\gamma = \beta_0 b \cos\Theta \tag{5}$$

and ζ_0 is the free-space characteristic resistance and

$$U = -E/\beta_0 \quad . \tag{6}$$

The reference of the phase is taken at the center of the dipole. The corresponding integral equation for the image dipole is

$$\int_{-h}^{h} I_2(z')\, K_a(z, z')\, dz' + \int_{-h}^{h} I_1(z')\, K_b(z, z')\, dz' = -j\, \frac{4\pi}{\zeta_o} [C_2 \cos\beta_o z + U(e^{-j\gamma} + R_{\perp})] \tag{7}$$

where C_2 is an arbitrary constant.

The currents $I_1(z)$ and $I_2(z)$ may be written as

$$I_1(z) = I^s(z) + I^a(z) \tag{8}$$

$$I_2(z) = I^s(z) - I^a(z) \tag{9}$$

where $I^s(z)$ and $I^a(z)$ are the symmetrical and antisymmetrical components of current in either antenna. The assumed direction of these currents is indicated in Fig. 2(b).

By substituting Eqs. (8) and (9) into Eqs. (2) and (3), the following independent integral equations for the currents $I^s(z)$ and $I^a(z)$ are obtained:

$$\int_{-h}^{h} I^s(z')\, K^s(z, z')\, dz' = -j\, \frac{4\pi}{\zeta_o} (C^s \cos\beta_o z + U^s) \tag{10}$$

$$\int_{-h}^{h} I^a(z')\, K^a(z, z')\, dz' = -j\, \frac{4\pi}{\zeta_o} (C^a \cos\beta_o z + U^a) \tag{11}$$

where

$$K^s(z, z') = K_a(z, z') + K_b(z, z') \tag{12}$$

$$K^a(z, z') = K_a(z, z') - K_b(z, z') \tag{13}$$

$$U^s = U(1 + e^{-j\gamma})\,(1 + R_{\perp})/2 \tag{14}$$

$$U^a = U(1 - e^{-j\gamma})\,(1 - R_{\perp})/2 \tag{15}$$

$$C^s = (C_1 + C_2)/2 \tag{16}$$

$$C^a = (C_1 - C_2)/2 \quad . \tag{17}$$

Equations (10) and (11) are identical in form to the integral equation for the unloaded current in an isolated receiving antenna.[3] Therefore, the solution for Eqs. (10) and (11) can be obtained by investigating the isolated receiving dipole with appropriate substitution for the Kernel and U-functions.

The current at the load terminals for a short-circuited isolated dipole is

$$I_i(0) = -4\pi U(h_e/\lambda)(1/Z_o) \tag{18}$$

where h_e is the effective half-length, Z_o is the driving point impedance of the dipole when used for transmission, and $U = -E/\beta_o$. Thus, the corresponding solution for Eqs. (10) and (11) is

$$I^s(0) = -4\pi U^s\left(\frac{h_e^s}{\lambda}\right)\left(\frac{1}{Z^s}\right) = -2\pi U\left(\frac{h_e^s}{\lambda}\right)\left[\frac{(1 + e^{-j\gamma})(1 + R_\perp)}{Z^s}\right] \quad . \tag{19}$$

Similarly,

$$I^a(0) = -4\pi U^a\left(\frac{h_e^a}{\lambda}\right)\left(\frac{1}{Z^a}\right) = -2\pi U\left(\frac{h_e^a}{\lambda}\right)\left[\frac{(1 - e^{-j\gamma})(1 - R_\perp)}{Z^a}\right] \quad . \tag{20}$$

In Eqs. (19) and (20),

$$Z^s = Z_{11} + Z_{12} \tag{21}$$

$$Z^a = Z_{11} - Z_{12} \tag{22}$$

where Z_{11} is the self-impedance of the dipole in proximity to its image, and Z_{12} is the mutual impedance.

Circuital Relations:- Once the symmetric and antisymmetric short-circuit currents are obtained, the equivalent circuit for a receiving dipole can be applied to find the voltage V_{1L} that is developed across the load impedance Z_L

connected across the terminals of the antennas, namely,[4]

$$I_L^s(0) = I^s(0) \frac{Z^s}{Z^s + Z_L} \tag{23}$$

$$I_L^a(0) = I^a(0) \frac{Z^a}{Z^a + Z_L} \quad . \tag{24}$$

Hence,

$$V_{1L} = Z_L \left[I_L^s(0) + I_L^a(0)\right]$$

$$= \frac{Z_L}{(Z^s + Z_L)(Z^a + Z_L)} \left[I_1(0)\left(Z_{11}^2 + Z_{11}Z_L - Z_{12}^2\right) + I_2(0)\, Z_{12}Z_L\right] \quad . \tag{25}$$

III. NUMERICAL SOLUTIONS FOR $\lambda/2$ DIPOLE

Consider a dipole for which $\beta_o h = \pi/2$ and $\Omega = 10$. The self-impedance of this dipole, when isolated, is $Z_o = 86.5 + j41.7$ ohms. Let it be center-loaded by an impedance $Z_L = 86.5 - j41.7$ ohms so that a conjugate match is obtained. An investigation of the receive properties of this dipole when its axis is parallel and in proximity to the ocean should be made.

The close resemblance of Eqs. (10) and (11) to the corresponding equation for an isolated dipole provides a simple method for obtaining V_{1L} with appropriate substitution of expansion parameters.

The effective length of a receiving antenna with radius "a" when oriented in parallel to a distant transmitter dipole is

$$h_e/\lambda = \frac{Z_o}{4\pi} U_o(\beta_o h) \tag{26}$$

where Z_o is the input impedance of the dipole when used for a transmitting antenna and

$$U_o(\beta_o h) = j4\pi(1 - \cos\beta_o h)\Big/\zeta_o \left[\Psi_{du} \cos\beta_o h - \Psi_u(h)\right] \quad . \tag{27}$$

For a half-wave dipole, i.e., $\beta_o h = \pi/2$,

$$U_o(\pi/2) = -j4\pi/\zeta_o \, [C_s(A, \pi) - jS_s(A, \pi)] \tag{28}$$

where $A = \beta_o a$, and C_s and S_s are generalized sine and cosine integrals. Hence,

$$\frac{h_e(h = \lambda/4)}{\lambda} = -jZ_o/\zeta_o \, [C_s(A, \pi) - jS_s(A, \pi)] \quad . \tag{29}$$

Note that the effective lengths $2h_e^s$ and $2h_e^a$ are like $2h_e$ for an isolated dipole except that the radii are A^s and A^a instead of A. The effective radii A^s and A^a can be obtained readily by examining the integral equations for which the corresponding expansion parameters are defined when $\beta_o h = \pi/2$, i.e., for an isolated dipole,[3]

$$\Psi_{K1} = \left|C_a\left(\frac{\pi}{4}, 0\right)\right| = \left|2 \sinh^{-1} \frac{h}{a} - [\gamma = \ln \pi - Ci(\pi)] - jSi(\pi)\right| \quad . \tag{30}$$

Therefore, for the symmetrical case,

$$\Psi^s = \left|C_a\left(\frac{\lambda}{4}, 0\right) + C_b\left(\frac{\lambda}{4}, B\right)\right|$$

$$= \left|2 \sinh^{-1} \frac{h}{a} - [\gamma + \ln \pi - Ci(\pi)] - jSi(\pi) + 2Cci(B, \pi/2) - j2Sc(B, \pi/2)\right| \tag{31}$$

and for the antisymmetric case,

$$\Psi^a = \left|2 \sinh^{-1} \frac{h}{a} - [\gamma + \ln \pi - Ci(\pi)] - jSi(\pi) - 2Cci(B, \pi/2) + j2Sc(B, \pi/2)\right| \quad . \tag{32}$$

For a half-wavelength isolated dipole,

$$\Psi_{K1} = \Omega - K \tag{33}$$

where $\Omega = 2 \ln (2h/a)$ and $K = 1.445$. Henceforth, it is reasonable to put

$$\Psi^s = \Omega^s - K \tag{34}$$

$$\Psi^a = \Omega^a - K \quad . \tag{35}$$

TABLE I

EFFECTIVE LENGTH

b/λ	Ω	$\lvert h_e^a \rvert/\lambda$	$\lvert h_e^s \rvert/\lambda$
0.02	10	0.209	0.167
0.03	10	0.2077	0.1679
0.04	10	0.204	0.1686
0.05	10	0.2001	0.1693
0.06	10	0.198	0.17
0.07	10	0.196	0.173
0.08	10	0.195	0.177
0.09	10	0.193	0.179
0.1	10	0.192	0.1796
0.2	10	0.186	0.1817
0.3	10	0.183	0.185
0.4	10	0.183	0.1865
0.5	10	0.182	0.187
0.6	10	0.182	0.186
0.7	10	0.183	0.185
0.8	10	0.184	0.185
0.9	10	0.185	0.183
1.0	10	0.186	0.1825

From Eqs. (34) and (35), the effective symmetric and antisymmetric radii are

$$A^s = 2\beta_o h_e \exp[-\Omega^s/2] \tag{36}$$

$$A^a = 2\beta_o h_e \exp[-\Omega^a/2] \quad . \tag{37}$$

Therefore, the values of h_e^s/λ and h_e^a/λ are then calculated directly from Eq. (29). Note that h_e^s/λ and h_e^a/λ are real provided $\beta_o h = \pi/2$. Computations of the corresponding input impedances for a $\lambda/2$ dipole of radii A^s and A^a were carried out by the polynomial-exponential-product method. The computed values of the symmetric and antisymmetric effective lengths for $\beta_o h = \pi/2$ and $\Omega = 10$ are listed in Table I.

Observe that, as the dipole is brought close to the ocean surface, the symmetric radius A^s becomes smaller and corresponds to a thinner dipole, whereas A^a becomes so large that the validity of this approach is in doubt. However, it is reasonable to assume that for $b/\lambda \geqslant 0.02$, i.e., the dipole is situated at 1.2 cm or more above the sea water, the calculated results are generally accurate.

It is now a simple matter to obtain V_{1L} as given by Eq. (25) in terms of $I_1(0)$ and $I_2(0)$. The results, in which the power in the load of an isolated dipole is used as a reference, are plotted in Fig. 3.

IV. DISCUSSION

For comparison purposes, Fig. 4, obtained principally from Fig. 3, is a plot of the relative power received with the dipole at $b/2 = 0.15\lambda$ as a reference. The generally good agreement with the experimental data obtained by A. Sotiropoulos suggests that the mathematical model is valid, at least for $b/2 \geqslant 0.01\lambda$.

It is interesting to compare the receive properties of a $\lambda/2$ dipole in the presence of a perfectly conducting sheet with those for a sea-water plane.

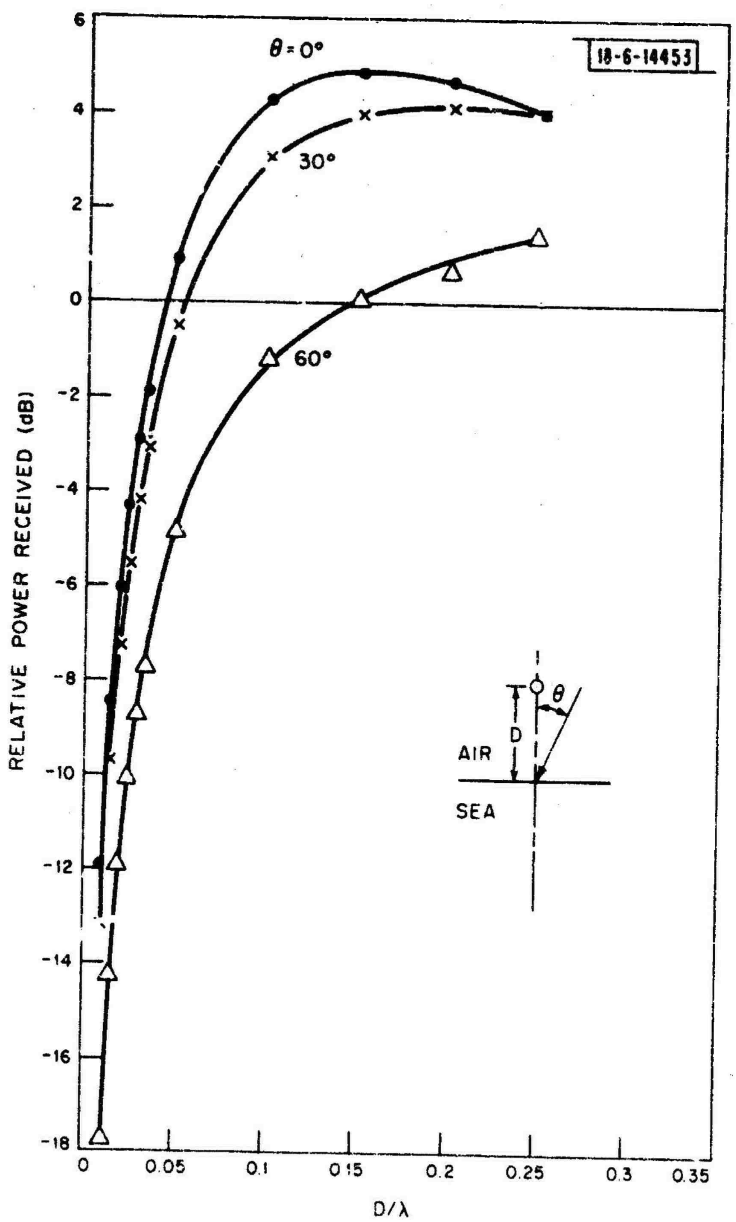

Fig. 3. Relative power received vs dipole heights D/λ.

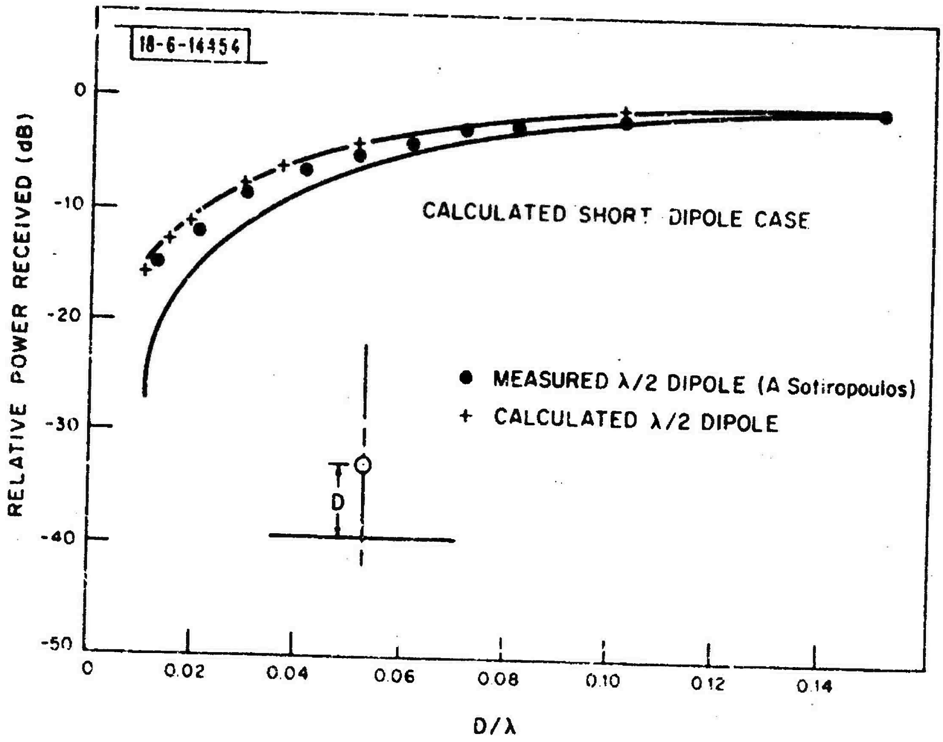

Fig. 4. Relative power received vs dipole position.

In proximity to a conducting sheet, the integral equation for the current along the unloaded dipole can then be written from Eq. (2) by putting $R_{\perp} = -1$:

$$\int_{-h}^{h} I_1(z')\,[K_a(z, z') - K_b(z, z')]\,dz' = -j\,\frac{4\pi}{\zeta_o}\,[C_1 \cos\beta_o z + U(1 - e^{-j\gamma})] \quad . \tag{38}$$

The current in the image dipole is obviously the negative of the current in the dipole, and no integral equation need be written for it. The same numerical calculations for Eq. (2) can be carried out for Eq. (38).

The terminal currents of a $\lambda/2$ receiving dipole in proximity to either a perfectly conducting sheet or sea water are plotted in Fig. 5. The symmetric currents are much smaller in the presence of sea water than the corresponding antisymmetric currents, which have the same order of magnitude as the terminal currents, for the antenna in the presence of a perfect conductor.

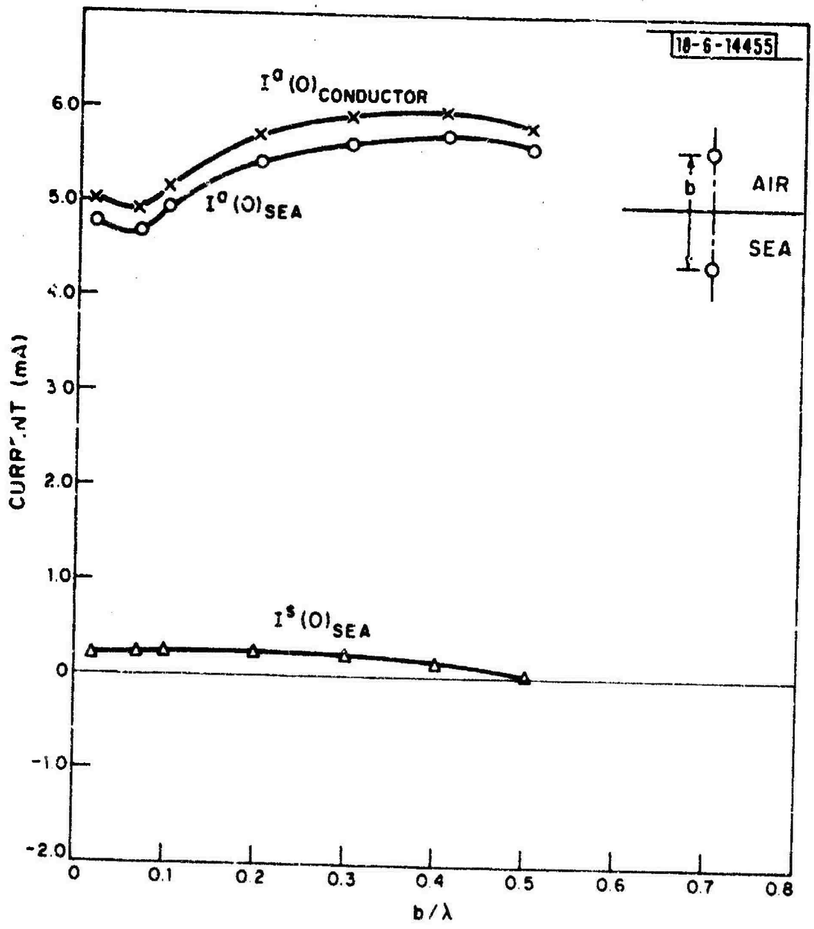

Fig. 5. Short-circuit current.

It is also observed that, except for a small amount of symmetric terminal current (which is approximately 1/20 of the corresponding antisymmetric current), the terminal current of a $\lambda/2$ dipole in the presence of sea water is practically the same as that for the same dipole in the presence of a perfect conductor. The relative power received at low dipole heights for both sea water and a perfect conductor is plotted in Fig. 6.

The general similarity of the curves for $b/2 \geqslant 0.01\lambda$ suggests that, for practical purposes, the receive properties of a $\lambda/2$ dipole over the sea can be obtained readily by looking at the same dipole over a perfectly conducting plane.

When the dipole is very close to the sea surface ($b/2 < 1.2$ cm), the power loss of a $\lambda/2$ dipole is unknown. An extrapolation of the available calculated data is possible by using the following facts:

(a) The relative power loss at low dipole heights is approximately proportional to the relative reduction of the electric field strength.

(b) The wavelength along the antenna with moderately thick insulation (the ratio of diameters of insulation and antenna is at least greater than four) is comparable to that in air or a dielectric such as rubber or polystyrene. A fairly smooth change in properties would then be expected as the antenna is brought in contact with the sea-water plane.

When the dipole is very close to the sea surface, the antisymmetric impedance is closely approximated by the conventional transmission line theory when applied to a section of lossless line of length 2h with an equal and opposite point generator connected in series with the two conductors at its centers. Therefore, for a $\lambda/2$ dipole, its symmetric impedance should be very small as shown in Table I, while the antisymmetric impedance is close to twice the input impedance of an isolated antenna. Therefore, one would expect at least an additional 6-dB loss from $b/2 = 1.2$ cm to 0. It appears that a $\lambda/2$ dipole might have a reasonable gain (>−20 dB).

The power gain vs angle of incidence is plotted in Fig. 7. It is observed that the power loss for the $\lambda/2$ dipole at a fixed dipole height for various angles of incidence is roughly proportional to $(\cos\Theta)^2$. This property imposes a

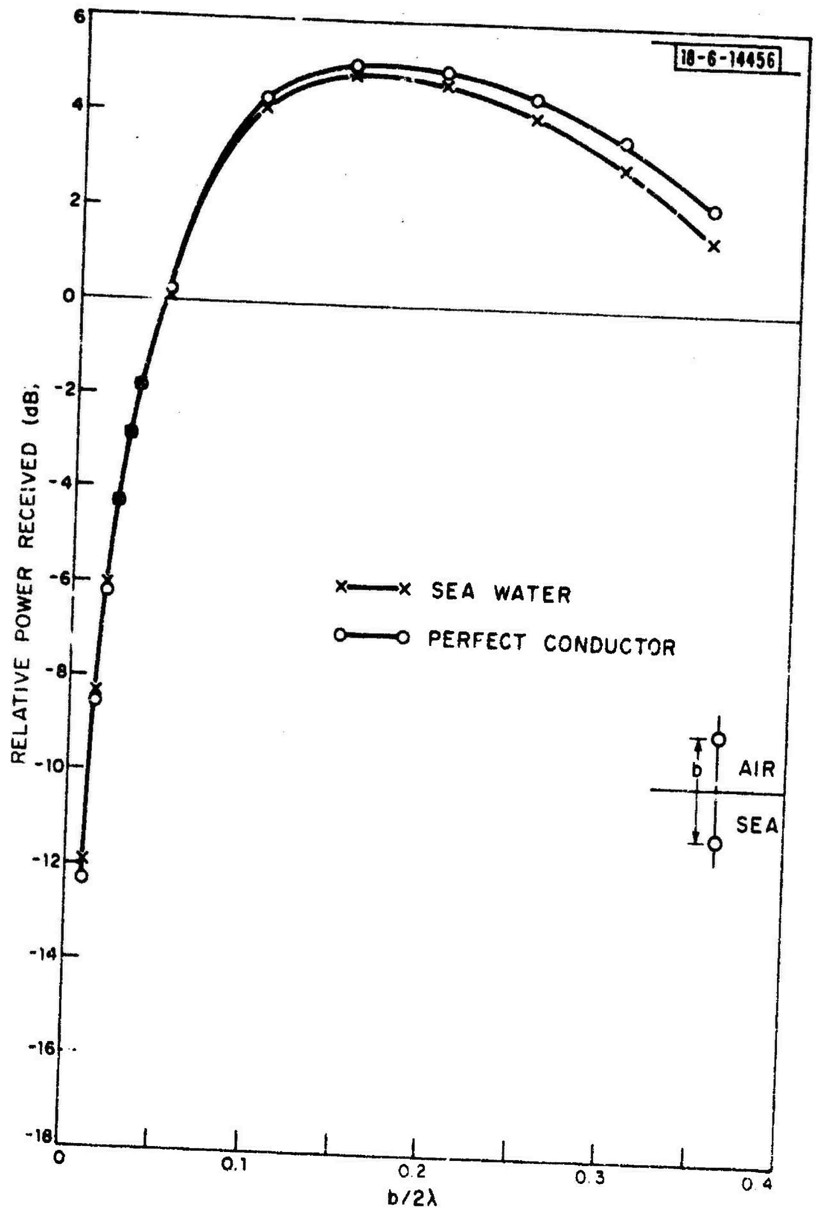

Fig. 6. Relative power received of λ/2 dipole over b/2λ sea-water plane or conducting sheet.

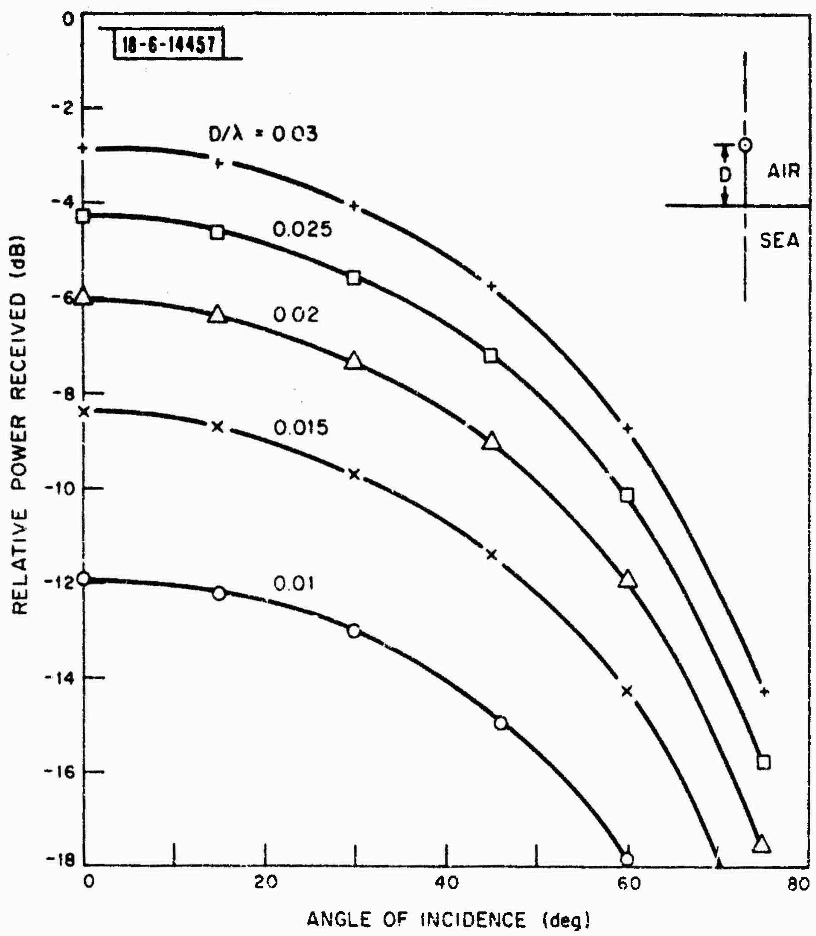

Fig. 7. Relative power received vs angle of incidence.

serious limitation on performance of the λ/2 dipole for reception in proximity to sea water in which 6-dB loss is expected at $\Theta = 60°$ as compared with normal incidence.

The effect of the sea-water plane obviously increases as the height of the horizontal dipole is decreased, and one of the main parameters of practical interest is the input impedance of such a dipole. As very little information is available on the behavior of a horizontal dipole radiator above the sea surface, a mathematical analysis of the problem is very difficult. However, advantage can be taken of the available calculated data to obtain the impedance of a horizontal dipole whose height above the sea surface is only a small fraction of a wavelength.

The power in the load is obtained from Eq. (25),

$$P_L(R_L, X_L) = R_L \Delta_L / 120 \Delta_s \Delta_a \tag{39}$$

where

$$\Delta_L = K_1 + K_2(R_L^2 + X_L^2) + K_3 R_L + K_4 X_L \tag{40a}$$

$$\Delta_s = (R_s^+)^2 + (X_s^+)^2 + R_L^2 + X_L^2 + 2R_s^+ + 2X_s^+ X_L \tag{40b}$$

$$\Delta_a = (R_s^-)^2 + (X_s^-)^2 + R_L^2 + X_L^2 + 2R_s^- R_L + 2X_s^- X_L \tag{40c}$$

and

$$K_1 = T|Z_s^-|^2 + S|Z_s^+|^2 + 2R_e [C \cdot (Z_s^+)* \cdot (Z_s^-)] \tag{41a}$$

$$K_2 = T + S + 2R_e(C) \tag{41b}$$

$$K_3 = 2TR_s^- + 2SR_s^+ + 2R_e \{C [(Z_s^+)* + (Z_s^-)]\} \tag{41c}$$

$$K_4 = 2TX_s^- + 2SX_s^+ - 2I_m \{C* [(Z_s^-)* + (Z_s^+)*]\} \tag{41d}$$

and

$$T = |I^s(0)\, Z_s^+|^2 \tag{42a}$$

$$S = |I^a(0)\, Z_s^-|^2 \tag{42b}$$

$$C = I^s(0)\, I^a(0)*\; Z_s^+(Z_s^-)* \tag{42c}$$

$$Z_s^+ = Z_{11} + Z_{12} \tag{42d}$$

$$Z_s^- = Z_{11} - Z_{12} \quad . \tag{42e}$$

Maximum power transfer will occur when the circuit is adjusted for a conjugate impedance match. Therefore, a method whereby P_L becomes maximum is used to find the values of R_L and X_L. The results are plotted in Fig. 8. As

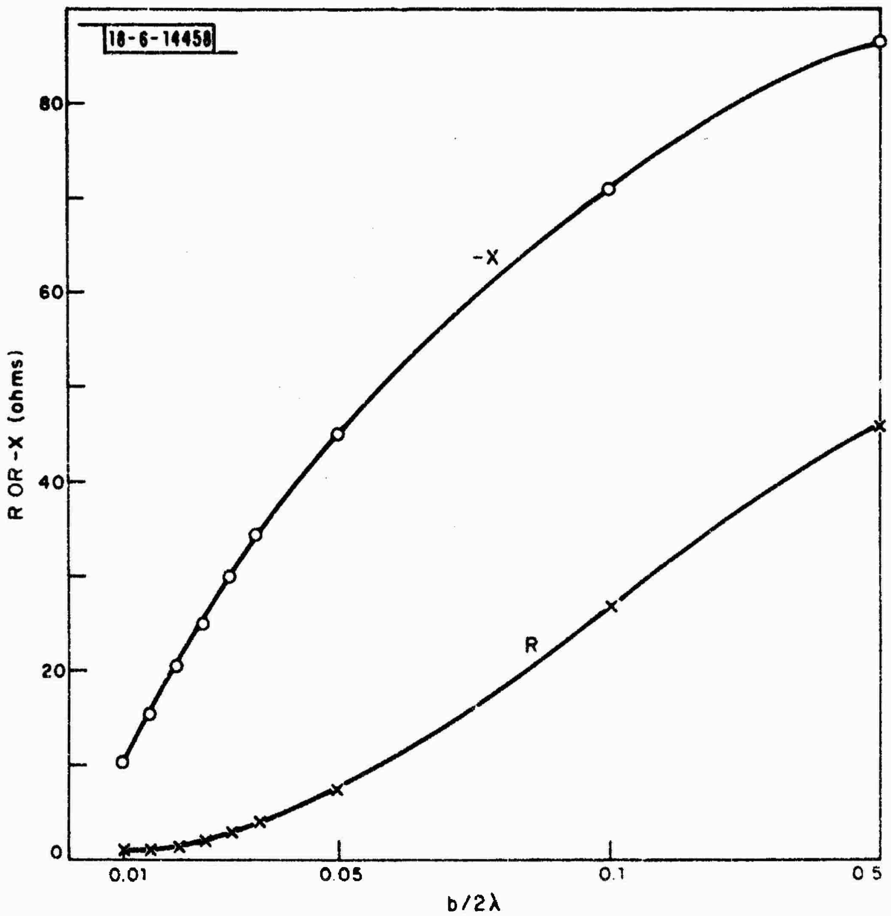

Fig. 8. Impedance of $\lambda/2$ dipole at low dipole heights.

the horizontal antenna is brought closer and closer to the sea surface, its radiation resistance decreases rapidly; this presents a serious problem in designing a matching circuit.

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