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THE ENTROPY PROBLEM IN PASSIVE NETWORKS

J. Edmund Fitzgerald

Utah University

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IN

PASSIVE NETWORKS

J. E. Fitzgerald

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University of Utah
Civil Engineering Department
Salt Lake City, Utah 84112

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Meixner (1972) recently reiterated his discussion of the nonuniqueness of a nonequilibrium entropy using electrically equivalent passive RC-networks. These circuits, part of a problem originally posed by Slepian (1948) are indistinguishable by classical electrical measurements at the input terminals.

It is shown herein that the two circuits can be distinguished and their RC-components uniquely determined using simultaneous electrical and thermal observations. The concept of an empirical nonequilibrium temperature such as Meixner posed is essential.

It is concluded that while the dissipation function (entropy) is uniquely determined in nonequilibrium conditions for each given RC-circuit, the RC-circuits are not necessarily uniquely determined for each impedance and entropy function. Rather, each nonequilibrium impedance and entropy function defines an equivalence class of RC-networks. This definition is unique to within the equivalence relation.

The author believes that this concept is compatible with Meixner's (1972) statements.

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THE ENTROPY PROBLEM IN PASSIVE NETWORKS

(Comments on Slepian's and Meixner's Paradox)

by

J. E. Fitzgerald, Professor of Applied Mechanics
and Civil Engineering

INTRODUCTION

Meixner (1972) recently reiterated his discussion of the nonuniqueness of a nonequilibrium entropy using a passive electrical network. The argument was based upon a comparison between the two circuits shown in Fig. 1.

These circuits, part of a problem originally posed by Slepian (1948) are electrically equivalent and are therefore indistinguishable with any electrical measurements made at the input terminals of the idealized "black boxes".

It is shown herein, however, that the two circuits can be distinguished and their respective components uniquely determined by using simultaneous electrical and thermal observations. To do so, however, one must accept the empirical concept of a nonequilibrium temperature along the lines proposed by Meixner in his fundamental inequality.

It then follows that while a nondistinguishing entropy function applies between equilibrium states, a unique entropy is implied for nonequilibrium states which defines the circuit components to within a unique equivalence class.

Circuit Analysis

Elementary circuit analysis shows that the input impedance, Z^N, of the RCL network is given in the Laplace domain by

$$Z_{1N}^{N} = \frac{S^2RCL + S(R^2C + L) + R}{S^2CL + 2S\,RC + 1} = \frac{\varepsilon_{1N}(s)}{i(s)} \qquad (1)$$

while that for the R-circuit is obviously given by

$$Z_{1N}^{R} = \frac{\epsilon_{1N}(s)}{i(s)} = R \quad . \tag{2}$$

Setting the value of the inductance in the network as

$$L = R^2C \tag{3}$$

the network impedance given by (1) becomes

$$Z_{1N}^{N} = \frac{\epsilon_{1N}(s)}{i(s)} = R \quad . \tag{4}$$

Apply now an arbitrary but identical voltage input to each of the circuits of Fig. 1. Without loss of generality, let the input voltage be a finite duration step such that

$$\epsilon_{1N}(t) = \epsilon_o\left[U(t) - U(t-t^*)\right] \tag{5}$$

where t^* is the duration.

The work input as well as the dissipation between equilibrium states in each circuit is given by

$$W = \frac{\epsilon_o^2}{R}\int_{-\infty}^{+\infty}\left[U(t) - U(\tau-t^*)\right]d\tau = \frac{\epsilon_o^2 t^*}{R} \tag{6}$$

Thus, for example, if each circuit were enclosed by an adiabatic wall, the temperature difference between the initial and final equilibrium state will be

$$\Delta\Theta = \frac{\varepsilon_0^2 t^*}{MRc_p} \tag{7}$$

where M is the, identical, mass of each "black box" and c_p is the specific heat at constant pressure of each "box".

It is thus apparent that at the lowest level of macroscopic descriptions the two circuits are thermoelectrically equivalent with respect to classical equilibrium thermodynamics. That is,

(1) no test involving electrical measurements at the terminal can distinguish the two circuits [N.B. This is Slepian's (1948, 1949) problem in which it is assumed that the R, L and C components are idealized] and

(2) no thermal measurement between two equilibrium states can distinguish the two circuits [N.B. The classicist may wish to measure the total energy dissipated by using a calorimeter rather than the temperature, but the point of difference is trivial in this instance since only equilibrium states are being so far considered.]

Further, since the input impedances of the two circuits are identical, no purely electrical measurement at the input terminals can differentiate the two "black boxes even under nonequilibrium conditions. [N.B. Slepian (1949) refers to a solution offered by J. A. Hutcheson of Westinghouse Research Labs who suggested noise vs. frequency measurements at the terminals. The thermal noise in the electrical components is independent of frequency in the R-circuit and decreases in amplitude as frequency increases in the RLC network. Macklem presents a complete noise differentiating solution in the Proc. I.E.E.E., V. 51, p. 1269, Sept., 1963. None of these solutions, however, is within the "gentlemanly" set problem involving only idealized R, L, C components.]

These results have led Meixner (1972) to draw his conclusion stated at the beginning of this article.

Network Determination

Nevertheless, there is a method for differentiating between the two circuits which permits specifically determining the values of the RCL components involved.

Consider, without loss of generality, the following input conditions for each circuit, Fig. 2. At time t = o close the switch to terminal A producing a step-input voltage of amplitude ε_o .

Conventional circuit analysis techniques then give the current flow at the input terminals of each box as

$$i_{1N} = \frac{\varepsilon_o}{B} \quad . \tag{8}$$

The current i_2 flowing in the network loop, Fig. 1, is given by

$$i_2(t) = \frac{\varepsilon_o}{R}\left(1 - e^{\frac{-t}{RC}}\right). \tag{9}$$

However, the current $i_2(t)$ cannot be determined by purely electrical measurements at the terminals.

The current flowing in the resistor in series with the inductance is then

$$i_2(t) = \frac{\varepsilon_o}{R}\left(1 - e^{\frac{-t}{RC}}\right) \tag{10}$$

and the current flowing in the resistor in series with the capacitor is

$$i_1 - i_2 = \frac{\varepsilon_o}{R} e^{\frac{-t}{RC}} \quad . \tag{11}$$

The rate of dissipation, P, in the R-circuit is then given by the product i^2R or

$$P_R = \frac{\varepsilon_o^2}{R} \tag{12}$$

and the total energy dissipated up to time, t, is

$$W_R = \frac{\varepsilon_o^2 t}{R} \quad . \tag{13}$$

In the RCL network circuit the rate of dissipation is

$$P_N = \frac{\varepsilon_o^2}{R}\left(1 - 2e^{\frac{-t}{RC}} + 2e^{\frac{-2t}{RC}}\right) \tag{14}$$

and the energy dissipated is

$$W_N = e_o^2 C\left[\frac{t}{RC} + 2e^{\frac{-t}{RC}} - e^{\frac{-2t}{RC}} - 1\right] \tag{15}$$

It may be noted that the initial rate of dissipation is identical for each box. As the duration increases such that t>>RC where the product RC is the time constant of the network from (12), (13), (14), (15), it is seen that the rates of dissipation in the two circuits approach equality,

$$P_R \rightarrow P_N \quad \forall \quad t >> RC \quad . \tag{16}$$

The rates of dissipation are then equal but the total energy dissipated is

$$W_N = W_R - \epsilon_o^2 C \quad \forall \quad t >> RC \quad . \tag{17}$$

Thus, the stored energy in the network is given asymptotically by $\epsilon_o^2 C$.

Assuming adiabatic conditions again, the dissipated energy being thermal in nature, the results of Fig. 3 are obtained. Dissipation in this instance means the conversion of electrical energy input into heat stored in the system.

Thus one concludes that

(1) by measuring the input impedance it may be determined that the two "black boxes" are electrically equivalent, Eq. (2)

(2) by measuring the slope of the instantaneous (nonequilibrium) temperature the impedance or resistor value is again determined as in Step (1)

(3) by measuring the difference in temperature between the two nonequilibrium values at a time which is large compared to the circuit time constant RC (determined as

in Fig. 3) The value of the capacitor, C, may be determined since

$$\Theta_R - \Theta_N = \frac{\varepsilon_o^2 C}{Mc_p} \qquad \text{or} \tag{18}$$

$$C = \frac{Mc_p}{\varepsilon_o^2}\left(\Theta_R - \Theta_N\right) \quad . \tag{19}$$

It may also be noted from Fig. 3 that if the switch of Fig. 2 is moved to terminal B at a time $t = t^*$, the temperature of the resistive circuit remains constant while the temperature of the RLC network continues to rise until it asymptotically reaches that of the resistive circuit. This result is caused by the current i_2 which is then given by

$$i_2(\tau) = \frac{\varepsilon_o}{R} e^{\frac{-\tau}{RC}} \quad ; \qquad \tau = t - t^* \tag{20}$$

so that the rate of dissipation of the stored energy in the network is

$$P_N = 2i_2^2 R = \frac{2\varepsilon_o^{\ 2}}{R} e^{\frac{-2\tau}{RC}} \tag{21}$$

and the stored energy dissipated is then

$$W = \int_{\tau=0}^{\infty} P_n d\xi = \frac{2\varepsilon_o^{\ 2}}{R}\int_{\tau=0}^{\infty} e^{\frac{-2\xi}{RC}} d\xi = \varepsilon_o^2 C \tag{22}$$

which is consistent with (17).

Thus neither electrical nor thermal measurement *between equilibrium points* can distinguish the two "black boxes".

However, transient temperature measurements can not only distinguish between the two "black boxes" but the temperature difference under a step voltage input can uniquely determine the capacitance, C, of the RCL network. Since R is obtained electrically (or thermally) and $L = R^2C$, it is thus shown that coupled thermoelectric measurements can precisely and uniquely determine the characteristics of each black box to within an equivalence class of RC-networks.

Conclusions

The implications here are that

(1) on the lowest level of description a unique determination may be made of equivalences if the two dimensions (electrical and thermodynamic in the case described) are used.

(2) electrical (or mechanical) equivalence does not imply thermodynamic equivalence.

(3) electrical (or mechanical) and thermal equivalence implies complete equivalence within the dimensions used, i.e., electrical, mechanical, and thermal observations are needed in a coupled electromechanical system.

(4) the determination of coupled electro-(or mechanical-) thermal equivalence cannot in general be done by confining the measurements to changes between equilibrium states

(5) nonequilibrium test determinations are necessary and sufficient to determine full electro (mechanical)-thermal equivalences.

(6) to the extent that a nonequilibrium entropy concept is tied to a dissipation concept, the nonequilibrium entropy can be uniquely determined, and it may be determined only under nonequilibrium observations (a not too startling a result, since any transient or dynamic constitutive equation in general requires transient or dynamic observations except for certain linear constitutive assumptions).

It should be reiterated that the determination of the RC-network to within an equivalence class is the same as the determination of a spring-dashpot model in linear viscoelasticity to within an equivalence class. That is, there may exist many (infinitely) RC-circuits which produce the same input impedance and the same thermal dissipation (entropy) function. Thus, while *the dissipation function (entropy) is unique for each given RC-circuit, the RC-circuits are not necessarily unique for each impedance and entropy*. Rather each impedance and entropy defines an equivalence class of RC-circuits (or spring-dashpot models).

The author believes that this conclusion is compatible with Meixner's (1972) statements.

Thus, while Meixner's thermodynamic extension of Slepian's problem is correct when the restriction to electrical observations alone is postulated, the conclusions are reversed when both the electrical dimension, i(t) and the thermal dimension, θ(t), are observed during nonequilibrium conditions.

It should be mentioned that conceptual care must be taken when discussing the entropy of dissipative electrical systems. For example, the usual electrical functions of a system are such that heat generated is wasted. Thus the identification of entropy with electrical energy dissipated is usually made. If, however, the purpose of the system is to generate heat, then a purely resistive circuit (within idealized definitions of resistance) is, in fact, a perfectly (100%) efficient converter of electrical input into thermal output. The second law restrictions come into play then only when a subsequent conversion cycle is performed wherein the heat generated is used to again produce electrical energy such as utilizing the heat in the network by removing the adiabatic wall so as to generate steam to run a generator, for example. This latter step cannot be perfectly efficient with respect to the complete electrical-to-electrical conversion cycle.

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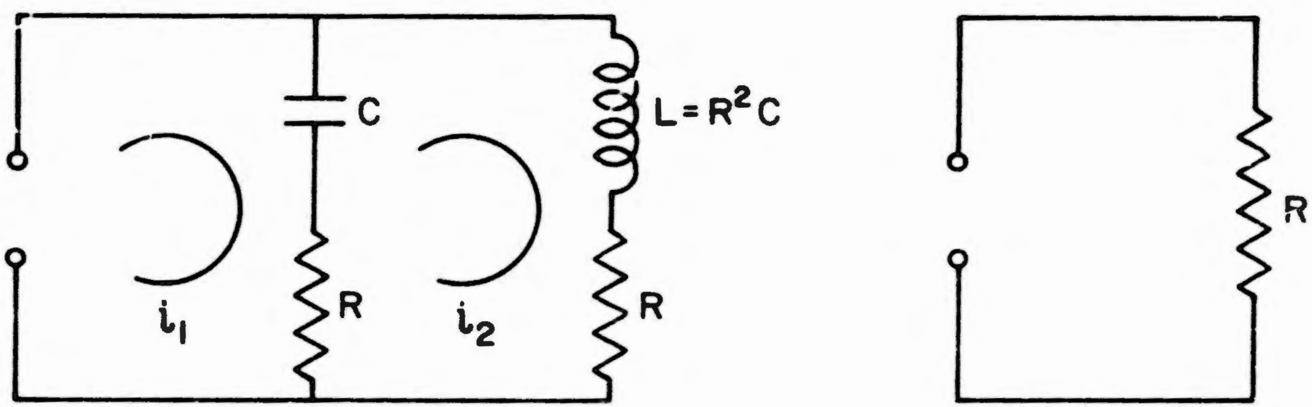

RCL — NETWORK

R — CIRCUIT

FIG. 1. EQUIVALENT ELECTRICAL NETWORKS.

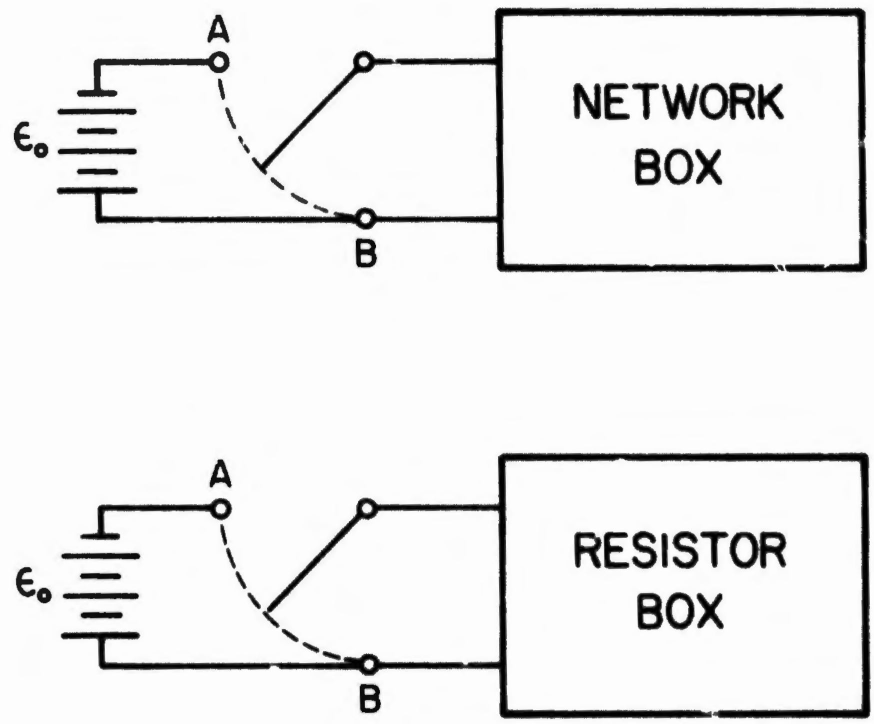

FIG. 2. INPUT CONDITIONS FOR EACH CIRCUIT OF FIG. 1.

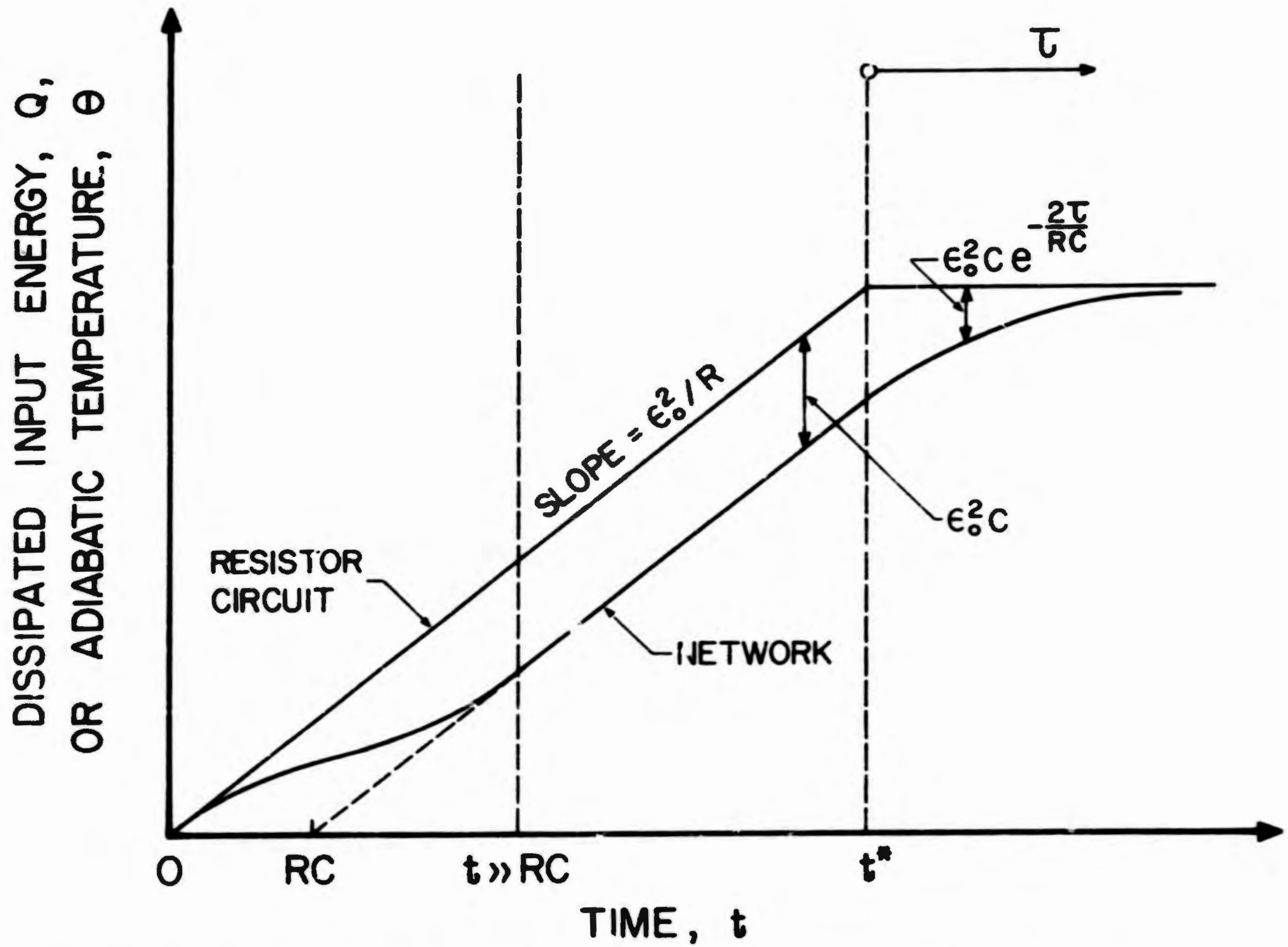

FIG. 3. DISSIPATION VS. TIME IN RESISTOR CIRCUIT & RCL-NETWORK.