

AD-764 265

DESIGN, INDUSTRIAL PRODUCTION AND
EVALUATION OF IMPROVED DUCTILE CAST
IRON ALLOYS USING COMPUTER DERIVED,
MATHEMATICAL MODELS. PART I. MECHANICAL
PROPERTY ASSESSMENT

John Zotos

Northeastern University

Prepared for:

Army Materials and Mechanics Research Center

March 1973

DISTRIBUTED BY:

NTIS

National Technical Information Service
U. S. DEPARTMENT OF COMMERCE
5285 Port Royal Road, Springfield Va. 22151

Unclassified

Security Classification

AD-764265

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)

Northeastern University

2a. REPORT SECURITY CLASSIFICATION

Unclassified

2b. GROUP

3. REPORT TITLE

DESIGN, INDUSTRIAL PRODUCTION AND EVALUATION OF IMPROVED DUCTILE CAST IRON
ALLOYS USING COMPUTER DERIVED, MATHEMATICAL MODELS
PART I - MECHANICAL PROPERTY ASSESSMENT

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Final Report

5. AUTHOR(S) (First name, middle initial, last name)

John Zotos

6. REPORT DATE

March 1973

7a. TOTAL NO. OF PAGES

40 42

7b. NO. OF REFS

12

8a. CONTRACT OR GRANT NO.

DAAG46-71-C-0116

8b. PROJECT NO.

AMCMS Code 4932.05.3010.1

c.

d. Foundry Engineering for Ductile Iron

9a. ORIGINATOR'S REPORT NUMBER(S)

AMMRC 73-11

9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT

This document has been approved for public release and sale; its distribution is unlimited.

11. SUPPLEMENTARY NOTES

12. SPONSORING MILITARY ACTIVITY

AMMRC

Watertown, Massachusetts 02172

13. ABSTRACT

This investigation attempts to implement a complete scientific analysis of industrial, mechanical property data on ductile cast iron alloys and design, produce, and evaluate some improved alloys possessing predictable property magnitudes. With the assistance of AMMRC and the Lunenburg Foundry Company, a total of thirty-two (32) multiple regression, mechanical property, mathematical models were derived via computer analyses from a total of three-hundred and two (302) complete data sets describing variations in tensile strength, yield strength, per cent elongation and Brinell hardness number. The sixteen (16) best selection equations were used as the basis for the alloy design to produce the ductile cast iron alloys possessing predictable property levels. Metallurgically, 74 out of 140 major, independent, elemental variables contained in these 16 refined models, i.e., 52.8 per cent, are in agreement with theory as to how they should contribute towards the magnitude change of these dependent properties. The initial test run produced cast test bars whose actual mechanical property levels were within less than two standard errors of estimate of the predicted values in only 15 out of 32 specimens, i.e., 46.9 per cent. The final design test data, derived from the AMMRC coupons, were superior in achievement in that 29 out of 32 test pieces, i.e., 90.6 per cent, were actually within two S.E.E.'s of the predicted values, thus fulfilling the design objectives of phase one of this study. Since this scientific development program was successful in producing the desired products, it is recommended that the same analytical tools and techniques be applied to other ferrous shell alloy systems.

DD FORM 1473

1 NOV 66

REPLACES DD FORM 1473, 1 JAN 66, WHICH IS OBSOLETE FOR ARMY USE.

Unclassified

Security Classification

42

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
CAST IRON DUCTILE OR NODULAR IRON MATHEMATICAL ANALYSIS MECHANICAL PROPERTIES MATHEMATICAL MODELS THERMAL TREATMENTS IRON ALLOYS SHELLS (STRUCTURAL FORMS) DESIGN INDUSTRIAL PRODUCTION EVALUATION TESTING FRAGMENTATION						

AD-764265

AMRC CTR-73-11

DESIGN, INDUSTRIAL PRODUCTION AND EVALUATION OF IMPROVED DUCTILE
CAST IRON ALLOYS USING COMPUTER DERIVED, MATHEMATICAL MODELS
PART I - MECHANICAL PROPERTY ASSESSMENT
PROFESSOR JOHN ZOTOS

Department of Mechanical Engineering
NORTHEASTERN UNIVERSITY
360 Huntington Avenue
Boston, Massachusetts 02115

March 1973

FINAL REPORT - CONTRACT NUMBER DAAG 46-71-C-0116

AMCMS CODE NO. 4932.05.3010.1

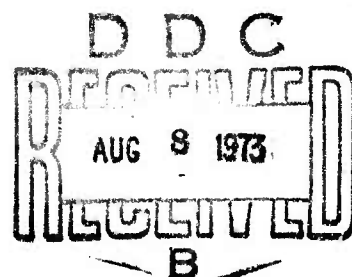
Approved for public release; distribution unlimited.

Reproduced by
NATIONAL TECHNICAL
INFORMATION SERVICE
U S Department of Commerce
Springfield VA 22151

Prepared for

ARMY MATERIALS AND MECHANICS RESEARCH CENTER
Watertown, Massachusetts 02172

ish



The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents.

Mention of any trade names or manufacturers in this report shall not be construed as advertising nor as an official indorsement or approval of such products or companies by the United States Government.

DISPOSITION INSTRUCTIONS

Destroy this report when it is no longer needed.
Do not return it to the originator.

Foreword

This report was prepared by Northeastern University under Army Contract DAAG46-71-C-0116. The contract was administered under the direction of the U.S. Army Materials and Mechanics Research Center with Kenneth D. Holmes providing technical supervision.

This project has been accomplished as part of the U.S. Army Manufacturing Methods and Technology Program, which has as its objective the timely establishment of manufacturing processes, techniques or equipment to insure the efficient production of current or future defense programs.

DESIGN, INDUSTRIAL PRODUCTION AND EVALUATION OF IMPROVED DUCTILE CAST IRON ALLOYS
USING COMPUTER DERIVED, MATHEMATICAL MODELS

PART I - MECHANICAL PROPERTY ASSESSMENT

by JOHN ZOTOS

ABSTRACT

This investigation attempts to implement a complete scientific analysis of industrial, mechanical property data on ductile cast iron alloys and design, produce and evaluate some improved alloys possessing predictable property magnitudes. With the assistance of AMMRC and the Lynchburg Foundry Company, a total of thirty-two (32), multiple regression, mechanical property, mathematical models were derived via computer analyses from a total of three-hundred and two (302) complete data sets describing variations in tensile strength, yield strength, per cent elongation and Brinell hardness number. The sixteen (16) best selection equations were used as the basis for the alloy design to produce the ductile cast iron alloys possessing predictable property levels. Metallurgically, 74 out of 140 major, independent, elemental variables contained in these 16 refined models, i.e., 52.8 per cent, are in agreement with theory as to how they should contribute towards the magnitude change of these dependent properties. The initial test run produced cast test bars whose actual mechanical property levels were within less than two standard errors of estimate of the predicted values in only 15 out of 32 specimens, i.e., 46.9 per cent. The final design test data, derived from the AMMRC coupons, were superior in achievement in that 29 out of the 32 test pieces, i.e., 90.6 per cent, were actually within two S.E.E.'s of the predicted values, thus fulfilling the design objectives of phase one of this study. Since this scientific development program was successful in producing the desired products, it is recommended that the same analytical tools and techniques be applied to other ferrous shell alloy systems.

TABLE OF CONTENTS

SECTION NUMBER	SECTION	PAGE NUMBER
	FOREWORD	i
	TITLE PAGE	ii
	ABSTRACT	iii
	TABLE OF CONTENTS	iv
I.	BACKGROUND	1
I. A.	PREFACE	1
I. B.	OBJECTIVE	1
I. C.	GENERAL PROCEDURE	1
I. D.	BACKGROUND	2
II.	STATISTICAL METHODS	3
III.	DESCRIPTION OF MECHANICAL PROPERTY DATA	5
IV.	MECHANICAL PROPERTY MATHEMATICAL MODELS	8
IV. A.	INTRODUCTION	8
IV. B.	AS CAST (80-55-06) SERIES	9
IV. C.	ANNEALED (60-40-18) SERIES	12
IV. D.	AIR QUENCHED AND TEMPERED (100-70-03) SERIES	14
IV. E.	OIL QUENCHED AND TEMPERED (120-90-02) SERIES	17
IV. F.	SUMMARY OF METALLURGICAL SIGNIFICANCE	20
IV. G.	AMRC DATA VERSUS LYNCHBURG FOUNDRY DATA	23
V.	UTILIZATION OF THESE MATHEMATICAL MODELS FOR CASTING DESIGN	26
VI.	INITIAL ALLOY DESIGN AND TEST RESULTS	26
VI. A.	INTRODUCTION	26
VI. B.	AMRC TEST COUPON	27
VI. C.	TEST RESULTS	29
VI. D.	UTILIZATION OF TEST RESULTS	29
VII.	FINAL ALLOY DESIGN AND TEST RESULTS	31
VII. A.	INTRODUCTION	31
VII. B.	TEST RESULTS	31
VIII.	CONCLUSIONS	34
IX.	RECOMMENDATIONS	34
X.	ACKNOWLEDGEMENTS	34
XI.	REFERENCES	35
	DISTRIBUTION LIST	36

DESIGN, INDUSTRIAL PRODUCTION AND EVALUATION OF IMPROVED DUCTILE CAST IRON
ALLOYS USING COMPUTER DERIVED, MATHEMATICAL MODELS

PART I - MECHANICAL PROPERTY ASSESSMENT

I. BACKGROUND

I. A. Preface

Three recently completed Department of the Army investigations dealing with the computer analyses of ductile cast iron data yielded promising results towards achieving the ultimate production of these ferrous alloy castings possessing predictable property magnitudes.(1,2,3) This study focused upon the design, evaluation and production of cast shells within the facilities of the Lynchburg Foundry Company, located in Lynchburg, Virginia, and dealt with both mechanical properties and fragmentation characteristics. This report contains a complete assessment of the mechanical property phase of the investigation and a sequel report will analyze the fragmentation results achieved.

I. B. Objective

This phase of the investigation utilized the most significant mechanical property data obtained from the files of the Lynchburg Foundry Company. It was agreed at the outset that the ultimate aim of the project was to manufacture both test coupons and rocket shells possessing:

- a. Maximum Strength with Maximum Ductility (Series - I); and
- b. Optimum Fragmentation Characteristics (Series - II);

using four specific thermal treatments, i.e., as cast, annealed, air quenched and tempered and oil quenched and tempered. For each strength and/or fragmentation condition and specific heat treatment, AMMRC double test coupons, modified keel blocks test bars, spectographic test pieces and 2.75" x 21" rocket shells were manufactured at Lynchburg and tested accordingly.

I. C. General Procedure

There are several steps required in the scientific development of improved ductile cast iron alloys, namely:

1. Evaluate the variables affecting the properties of ductile cast irons such as casting history, section size, grain size, thermal history, and alloy content.
2. Develop statistical models or equations which show the contributions of each of these variables towards the magnitudes of properties exhibited by these alloys.
3. Analyze the metallurgical and statistical significance, and validity of the developed models in predicting properties.

4. Design an improved ductile cast iron alloy composition and process history which should exhibit improved properties.
5. Produce the newly designed alloy in agreement with the prescribed process history.
6. Test the developed ductile cast iron alloy and assess its level of attainment of design objectives.
7. Redevelop new statistical models using the new data and repeat steps 3,4,5 and 6.

Research being conducted at Northeastern University has indicated the significance of this scientific approach towards the development of improved cast metal alloys having predictable chemical, mechanical and physical properties.⁽¹⁻⁹⁾

I. D. Background

The first three steps of this type of scientific development program were initiated for AMMRC in 1966⁽¹⁾ for some limited, reliable, ductile cast iron, mechanical property data. Two series of mathematical models were evaluated, i.e., Series 1, based on microstructural data and Series 2, based on alloy content data. Statistically, only the last four (4) of the eighteen (18) equations generated proved significant at the 0.001 confidence level, or less. However, metallurgically, seventeen (17) out of twenty-four (24) independent, elemental variables in these four (4) models, or 71 percent, agreed with theory as to their contribution towards the magnitude of the dependent mechanical properties.

The second AMMRC project on ductile cast iron alloys was completed in January, 1970,⁽²⁾ and attempted to expand the reliable data base and evaluate a multitude of properties. Some of the dependent properties being studied included tensile strength, yield strength, percent elongation, percent reduction in area, impact strength, hardness and fragmentation parameters. The independent variables measured included alloy content (twelve elements), Nodule count and microstructural content. The data base was adequate for a complete analysis and was categorized into the following three (3) series:

- SERIES I: MECHANICAL PROPERTY AND MICROSTRUCTURAL DATA USING AVERAGE VALUES
- SERIES II: FRAGMENTATION AND MECHANICAL PROPERTY DATA USING AVERAGE VALUES
- SERIES III: MECHANICAL PROPERTY, THERMAL TREATMENT AND ALLOY CONTENT DATA USING INDIVIDUAL TEST BAR RESULTS

A total of ninety-four (94) multiple, linear regression, mathematical models which describe the effect of several independent variables on the magnitude of each dependent fragmentation and mechanical property, were de-

rived from data supplied by AMMRC. Sixty-one out of the 64 mechanical property equations produced, or 95.3 percent, and 21 out of the 30 fragmentation property models developed, or 70 percent, were statistically significant at the 0.001 confidence level, or less. Two separate computer analyses were conducted within each series evaluation, i.e., an initial run utilizing all the independent variables in the data set plus a sequel computation which refined the initial results by deleting the less significant variables from the initial equations. Sixteen out of the 24 independent, microstructural variables retained in the refined, Series-I, mechanical property models, or 66.7 percent, and 128 out of the 218 independent, elemental variables retained in the "best-selection," Series III, mechanical property equations, or 58.7 percent were in agreement with metallurgical theory. Microstructural and elemental variables quantitatively described at least three dependent, fragmentation properties.

The third AMMRC investigation on ductile cast iron was completed in January, 1971,⁽³⁾ and focused on a scientific analysis of industrial, mechanical property data. Several firms donated their data banks for this study and 24 out of the 40 mathematical models generated were derived from information supplied by the Lynchburg Foundry Company.

Seventy-five percent of the 24 equations were statistically significant at the 0.001 confidence level, or less, and 53.7 percent of the 82 statistically significant, independent, elemental variables which appeared in the 12 refined models of this data set behaved in accordance with metallurgical theory. Comparison of these industrial results with previous AMMRC findings indicated that, overall, the Army and Lynchburg metallurgical significance levels were within 10 percentage points of one another.

The high quality statistical and metallurgical results achieved in this latest investigation justified the implementation of this final study to design, industrially produce and evaluate some improved ductile cast iron alloys possessing predictable property magnitudes.

II. STATISTICAL METHODS

This investigation used a multiple regression computer program to derive a series of mathematical models or equations which describe the contributions of several independent variables towards the magnitude of a dependent property.

A linear equation explicitly defining the property was used of the form,

$$\text{Property} = A + B (\%X_1) + C (\%X_2) + \dots$$

where A is a pure constant used to adjust the hypersurface to the proper range of inspection of the nodular cast iron's property. This constant is the mean value of the iron's property minus the sum of the products of the means of the independent variables with their respective coefficients. B,

C, D, ... are net regression coefficients (sometimes called partial regression coefficients), so called since they indicate the average change observed in the property due to a unit change of their respective independent variable while holding all other variables constant.

The regression equations can be justified only within the range specified by the observations used to derive the equation, cannot reflect any phenomena that might occur outside the inspected range. However, it can be assumed that the functional relationship between the chemical compositions of an alloy system, the process variables and the resultant property is a continuous one and some extrapolation beyond the observed range may be permitted with some degree of accuracy. A priori knowledge of the metal system then can justify some extrapolation of the regression equation beyond the observed range. The range of application of the data used for the derivation of each equation in this report is tabulated as is the arithmetic mean values of each variable. The total alloy content of the system is also given and any analysis of a system with alloy content exceeding this maximum will be an extrapolation beyond the intended range.

When an equation is derived by a regression system it must be justified as to its reliability and analyzed for its accuracy of estimate and its correlation with the given data. These parameters are (1) the standard error of estimate (σ_e), (2) the coefficient of multiple correlation (R) and (3) the "F-ratio." These statistical indicators can be used to show how closely the estimated values of the property can be expected to agree with the actual values, and what portion of the variance has been left unexplained. An indication is also given as to which dependent variables are most poorly represented by assuming a linear relationship.

The above statistical parameters are tabulated for each equation generated and proper conclusions are drawn. The level of significance (L. of S.) based on the "F-ratio" criteria is also recorded.

After deriving the initial, multiple regression, mathematical models, a sequel series of computer runs refined these equations by testing the independent variables in the order of decreasing significance. The general form of these "best selection" mathematical models varied due to the deletion of some of the less significant variables.

Once the validity of the refined equations has been established, quantitative and qualitative methods of analysis are presented and analyzed. To find the qualitative effect of the independent variables on the dependent variables simply inspect the signs of the constants (net regression coefficients). If a positive constant is associated with a particular variable then the equation infers that a positive addition of the variable will increase the value of the iron's property. Likewise, the addition of negative contributors will decrease the iron's property.

After the qualitative and quantitative results are established and discussed, a general conclusion as to the validity and predictability of the equation as well as its agreement with known experimental results is presented.

These refined mathematical models were then used to design the compositions for the two alloy series being produced to meet the objectives of this investigation.

III. DESCRIPTION OF THE MECHANICAL PROPERTY DATA

During the first few months of this investigation, the scope and responsibilities of the project were changed, with the full approval of the AMMRC project monitor, and at no added cost to the Army. Instead of focusing attention to only two thermal treatments, i.e., as cast and annealed, two additional conditions were assessed, i.e., air quenched and tempered and oil quenched and tempered. Also, all the data reduction and design responsibilities were turned over to the N.U. staff. Lynchburg Foundry Company agreed not to charge N.U. for any of the production costs, etc., and these funds were then used by the N.U. Staff to cover the added cost of their increased data reduction and design loads.

One-hundred and forty-three (143) complete data sets were available for the first computer run of the as cast (80-55-06) alloy series and the high, low and mean values of the four dependent and fourteen independent variables are listed in Table 1.

One-hundred and eighteen (118) complete data sets were available for the computer run of the annealed (60-40-18) alloy series and the high, low and mean values of the variables are also listed in Table 1.

Only nineteen (19) and twenty-two (22) complete data sets were available for the air quenched and tempered and oil quenched and tempered alloy series, respectively, i.e., the (100-70-03) and (120-90-02) data groups. Table 2 lists the high, low and mean values of both these series for each dependent and independent variable.

All 302 complete data sets were derived from 0.505 inch diameter test bars machined from one inch diameter, modified keel blocks (ASTM-A445) and poured during production runs at the Lynchburg Foundry Company. These same type test bars were used to assess the attainment of the design objectives throughout this investigation.

TABLE 1: HIGH, LOW AND MEAN VALUES OF THE DEPENDENT AND INDEPENDENT VARIABLES IN THE AS CAST (80-55-06) AND ANNEALED (60-40-18), LYNCHBURG FOUNDRY DATA BANKS

VARIABLE	AS CAST (80-55-06) SERIES			ANNEALED (60-40-18) SERIES		
	HIGH	LOW	MEAN	HIGH	LOW	MEAN
TENSILE STRENGTH = (T.S.)	133,500	75,500	98,325	71,000	61,000	65,727
YIELD STRENGTH = (Y.S.)	103,500	51,750	61,009	57,500	43,000	48,316
PERCENT ELONGATION = (% E.)	13.5	1.5	7.3	25.0	8.5	20.5
BRINELL HARDNESS NUMBER = (BHN)	304	179	226	189	156	175
PERCENT TOTAL CARBON = (% T.C.)	4.00	3.64	3.82	4.08	3.66	3.83
PERCENT SILICON = (% SI)	2.80	2.03	2.44	2.87	2.20	2.48
PERCENT NICKEL = (% NI)	1.01	0.06	0.26	0.87	0.06	0.26
PERCENT MANGANESE = (% MN)	0.61	0.25	0.38	0.53	0.21	0.36
PERCENT PHOSPHORUS = (% P)	0.100	0.025	0.066	0.150	0.020	0.069
PERCENT SULFUR = (% S)	0.027	0.004	0.012	0.027	0.005	0.012
PERCENT ALUMINUM = (% AL)	0.047	0.017	0.035	0.045	0.020	0.036
PERCENT COPPER = (% CU)	0.40	0.07	0.21	0.40	0.06	0.21
PERCENT CHROMIUM = (% CR)	0.15	0.05	0.10	0.15	0.05	0.10
PERCENT MAGNESIUM = (% MG)	0.063	0.026	0.042	0.072	0.022	0.043
PERCENT MOLYBDENUM = (% MO)	0.44	0.01	0.042	0.13	0.01	0.04
PERCENT TIN = (% SN)	0.064	0.002	0.013	0.080	0.002	0.016
PERCENT TITANIUM (% TI)	0.054	0.019	0.027	0.054	0.019	0.027
PERCENT CERIUM = (% CE)	0.015	0.005	0.0115	0.015	0.005	0.012

TABLE 2: HIGH, LOW AND MEAN VALUES OF THE DEPENDENT AND INDEPENDENT VARIABLES IN THE AIR QUENCHED AND TEMPERED (100-70-03) AND OIL QUENCHED AND TEMPERED (120-90-02), LYNCHBURG FOUNDRY DATA BANKS

VARIABLE	A.Q. & T (100-70-03) SERIES			O.Q. & T. (120-90-02) SERIES		
	HIGH	LOW	MEAN	HIGH	LOW	MEAN
TENSILE STRENGTH = (T.S.)	127,500	83,500	105,355	152,500	98,000	124,341
YIELD STRENGTH = (Y.S.)	82,500	55,500	68,671	129,750	83,250	106,352
PERCENT ELONGATION = (% E.)	12.5	4.0	7.7	7.0	2.0	4.2
BRINELL HARDNESS NUMBER (BHN)	269	197	226	293	227	254
PERCENT TOTAL CARBON = (% T.C.)	3.96	3.70	3.84	3.99	3.72	3.85
PERCENT SILICON = (% Si)	2.86	2.35	2.53	2.81	2.30	2.55
PERCENT NICKEL = (% Ni)	0.70	0.06	0.28	0.88	0.08	0.34
PERCENT MANGANESE = (% Mn)	0.50	0.29	0.36	0.54	0.28	0.38
PERCENT PHOSPHORUS = (% P)	0.110	0.025	0.072	0.110	0.040	0.075
PERCENT SULFUR = (% S)	0.013	0.007	0.011	0.017	0.005	0.012
PERCENT ALUMINUM = (% Al)	0.046	0.012	0.036	0.045	0.014	0.036
PERCENT COPPER = (% Cu)	0.27	0.08	0.19	0.32	0.08	0.19
PERCENT MAGNESIUM = (% Mg)	0.066	0.026	0.046	0.068	0.033	0.050
PERCENT MOLYBDENUM = (% Mo)	0.11	0.02	0.05	0.12	0.03	0.06
PERCENT TIN = (% Sn)	0.066	0.002	0.019	0.046	0.004	0.019
PERCENT TITANIUM = (% Ti)	0.033	0.021	0.026	0.048	0.022	0.029
PERCENT CERIUM = (% Ce)	0.015	0.005	0.012	0.015	0.005	0.010
PERCENT CHROMIUM = (% Cr)	0.12	0.06	0.08	0.16	0.06	0.09

IV. MECHANICAL PROPERTY MATHEMATICAL MODELS

IV. A. INTRODUCTION

Since the Lynchburg Foundry data were derived from keel blocks subjected to four thermal treatments, the initial computer analyses yielded four sets of multiple linear regression, mathematical models for each of the four dependent mechanical properties as a function of all their independent, elemental variables, listed in Tables 1 and 2. Each of these equations had this general form:

$$\begin{aligned} \text{MECHANICAL PROPERTY} = & A_0 + A_1(\%T.C.) + A_2(\%Si) + A_3(\%Ni) \\ & + A_4(\%Mn) + A_5(\%P) + A_6(\%S) + A_7(\%Al) \\ & + A_8(\%Cu) + A_9(\%Cr) + A_{10}(\%Mg) + A_{11}(\%Mo) \\ & + A_{12}(\%Sn) + A_{13}(\%Ti) + A_{14}(\%Ce) \end{aligned}$$

Four sequel computer runs refined these initial models by testing the independent variables in each equation in the order of decreasing statistical significance. Within all four data sets, some of the elemental variables were not contained in the "best-selection" models due to the elimination of the less significant parameters, but all these sequel equations proved to be more statistically significant than the primary ones.

IV. B. AS CAST (80-55-06) SERIES

IV. B.1. LINEAR REGRESSION MODELS

The four initial as cast (80-55-06) series, mathematical models describing the tensile strength, yield strength, percent elongation and Brinell Hardness number were derived from 143 complete sets of data and are listed in mathematical Model Set I. Solving for the fifteen constants required by the general equation, leaves 128 degrees of freedom for these first four regression analyses.

These mathematical models were generated to explain the variation in the ductile cast iron's strength, ductility and hardness properties. Also listed within these mathematical model sets are the correlation coefficient, i.e., R ; the F-ratio; the standard error of estimate, i.e., σ_E ; and the level of statistical significance, i.e., α , for each equation generated.

The four refined equations describing the same mechanical properties were also derived from 143 complete sets of data and are listed in Mathematical Model Set II. These four "best-selection" models deleted anywhere from four to six of the independent variables used in the initial analyses and thus not only increased the number of degrees of freedom in this sequel evaluation, but also improved the statistical significance of each equation.

MATHEMATICAL MODEL SET I - AS CAST 980-55-06) SERIES, MULTIPLE REGRESSION EQUATIONS

$$\begin{aligned} (T.S.) = & + 97,710 + 531,436 (\% \text{ Mg}) + 90,593 (\% \text{ Mo}) + 62,441 (\% \text{ Cu}) \\ & + 20,257 (\% \text{ Mn}) - 279,482 (\% \text{ Al}) - 13,412 (\% \text{ Ni}) + 77,618 (\% \text{ P}) \\ & - 693,052 (\% \text{ Ce}) - 5,862 (\% \text{ Si}) - 106,840 (\% \text{ Ti}) + 20,762 (\% \text{ Cr}) \\ & - 3,984 (\% \text{ T.C.}) + 12,517 (\% \text{ Sn}) + 3,945 (\% \text{ S}) \end{aligned} \quad (1)$$

$$R_{(1)} = 0.6629; F\text{-Ratio}_{(1)} = 7,168; \sigma_{E(1)} = 7,376; \alpha_{(1)} = 0.001$$

$$\begin{aligned} (Y.S.) = & + 75,430 + 98,036 (\% \text{ Mo}) - 824,377 (\% \text{ Ce}) - 232,603 (\% \text{ Al}) \\ & + 25,136 (\% \text{ Cu}) + 97,350 (\% \text{ Mg}) + 8,491 (\% \text{ Mn}) + 3,472 (\% \text{ Si}) \\ & - 5,970 (\% \text{ Ni}) - 4,576 (\% \text{ T.C.}) - 82,478 (\% \text{ Ti}) - 10,511 (\% \text{ Cr}) \\ & + 12,339 (\% \text{ P}) - 40,306 (\% \text{ S}) - 860 (\% \text{ Sn}) \end{aligned} \quad (2)$$

$$R_{(2)} = 0.7344; F\text{-Ratio}_{(2)} = 10,707; \sigma_{E(2)} = 4,061; \alpha_{(2)} = 0.001$$

$$\begin{aligned} (\% \text{ E.}) = & - 1.371 - 19.142 (\% \text{ Cu}) - 7.556 (\% \text{ Ni}) + 54.255 (\% \text{ Mg}) \\ & + 60.946 (\% \text{ Ti}) + 36.230 (\% \text{ Sn}) - 7.785 (\% \text{ Mo}) - 3.423 (\% \text{ Mn}) \\ & + 1.603 (\% \text{ Si}) + 55.762 (\% \text{ S}) + 2.129 (\% \text{ T.C.}) + 13.393 (\% \text{ P}) \\ & - 28.501 (\% \text{ Al}) - 4.738 (\% \text{ Cr}) - 31.307 (\% \text{ Ce}) \end{aligned} \quad (3)$$

$$R_{(3)} = 0.5266; F\text{-Ratio}_{(3)} = 3.508; \sigma_{E(3)} = 1.806; \alpha_{(3)} = 0.001$$

$$\begin{aligned} (\text{BHN}) = & + 248.139 + 196.780 (\% \text{ Cu}) + 47.464 (\% \text{ Ni}) + 45.554 (\% \text{ Mn}) \\ & + 99.625 (\% \text{ Mo}) - 115.074 (\% \text{ Cr}) - 602.875 (\% \text{ S}) + 281.143 (\% \text{ Mg}) \\ & - 12.568 (\% \text{ Si}) - 15.371 (\% \text{ T.C.}) - 86.617 (\% \text{ P}) + 169.801 (\% \text{ Al}) \\ & - 569.980 (\% \text{ Ce}) + 172.746 (\% \text{ Ti}) + 44.641 (\% \text{ Sn}) \end{aligned} \quad (4)$$

$$R_{(4)} = 0.6364; F\text{-Ratio}_{(4)} = 6.223; \sigma_{E(4)} = 13.737; \alpha_{(4)} = 0.001$$

MATHEMATICAL MODEL SET II - AS CAST (80-55-06) SERIES, BEST SELECTION EQUATIONS

$$\begin{aligned} (T.S.) = & + 81,338 + 536,213 (\% \text{ Mg}) + 93,840 (\% \text{ Mo}) + 62,396 (\% \text{ Cu}) \\ & + 20,739 (\% \text{ Mn}) - 292,535 (\% \text{ Al}) - 13,634 (\% \text{ Ni}) + 82,990 (\% \text{ P}) \\ & - 651,221 (\% \text{ Ce}) - 5,991 (\% \text{ Si}) \end{aligned} \quad (5)$$

$$R_{(5)} = 0.6599; F\text{-Ratio}_{(5)} = 11.399; \sigma_{E(5)} = 7,621; \alpha_{(5)} = 0.001$$

$$\begin{aligned} (Y.S.) = & + 74,450 + 97,630 (\% \text{ Mo}) - 830,943 (\% \text{ Ce}) - 237,562 (\% \text{ Al}) \\ & + 25,532 (\% \text{ Cu}) + 97,898 (\% \text{ Mg}) + 8,816 (\% \text{ Mn}) + 3,856 (\% \text{ Si}) \\ & - 5,547 (\% \text{ Ni}) - 4,550 (\% \text{ T.C.}) - 88,662 (\% \text{ Ti}) \end{aligned} \quad (6)$$

$$R_{(6)} = 0.7331; F\text{-Ratio}_{(6)} = 15.340; \sigma_{E(6)} = 4,008; \alpha_{(6)} = 0.001$$

$$\begin{aligned} (\% \text{ E.}) = & + 6.562 - 18.916 (\% \text{ Cu}) - 6.413 (\% \text{ Ni}) + 51.508 (\% \text{ Mg}) \\ & + 57.385 (\% \text{ Ti}) + 35.315 (\% \text{ Sn}) - 9.715 (\% \text{ Mo}) - 3.630 (\% \text{ Mn}) \\ & + 1.348 (\% \text{ Si}) + 49.992 (\% \text{ S}) \end{aligned} \quad (7)$$

$$R_{(7)} = 0.5129; F\text{-Ratio}_{(7)} = 5.275; \sigma_{E(7)} = 1.789; \alpha_{(7)} = 0.001$$

$$\begin{aligned} (\text{BHN}) = & + 182.125 + 181.796 (\% \text{ Cu}) + 50.439 (\% \text{ Mn}) + 50.675 (\% \text{ Ni}) \\ & + 108.717 (\% \text{ Mo}) - 118.935 (\% \text{ Cr}) - 604.999 (\% \text{ S}) + 260.192 (\% \text{ Mg}) \\ & - 9.508 (\% \text{ Si}) \end{aligned} \quad (8)$$

$$R_{(8)} = 0.6273; F\text{-Ratio}_{(8)} = 10.869; \sigma_{E(8)} = 13.554; \alpha_{(8)} = 0.001$$

IV. B.2. STATISTICAL SIGNIFICANCE

The level of significance of each equation and coefficient generated, i.e., α , was determined on the basis of the following parameters:

1. R-Correlation coefficient;
2. F-ratio calculated;
3. t-Test calculated; and
4. the degrees of freedom.

Usually those equations whose level of significance, i.e., α , is 0.010 or less are considered significant enough for detailed evaluation. In addition, individual α values for each coefficient computed for the refined mathematical models are considered somewhat significant when they are 0.2000 or less in magnitude.

All four mechanical property models listed in Mathematical Model Set I, i.e., equations (1) through (4), are significant at the 0.001 confidence level, or less, and their four correlation coefficients ranged from 0.5266 to 0.7344. In addition, the four refined equations listed in Mathematical Model Set II, i.e., equations (5) through (8), were also significant at the 0.001 confidence level, or less, and their correlation coefficients range from 0.5129 to 0.7331. This small drop in the R values is more than offset by an increase in the F-RATIO magnitude ranges from (3.508 to 10.707) to (5.275 to 15.340), and verifies that these latter models are the statistically superior ones.

Table 3 lists the detailed qualitative, quantitative and statistical results from the refined, as cast (80-55-06) series, mechanical property equations (5) through (8). All thirty-six independent, elemental variables in these four models had alpha values of 0.200, or less, and the standard error of estimate was reduced to minimum value for each of the four dependent properties.

IV. B.3 METALLURGICAL SIGNIFICANCE

The sign of the coefficients in each mathematical model gives a qualitative judgement as to the independent variable's contribution towards the magnitude of the dependent variable.

All the equations are unique in that they provide quantitative as well as qualitative results. The quantitative contribution for each independent variable is simply the product of the regression coefficient and the independent variable mean. The percentage contribution of each independent variable can also be computed, and is the ratio of each individual product to the algebraic sum of all the products, including the contribution of the constant, expressed in percent.

TABLE 3: QUALITATIVE, QUANTITATIVE AND STATISTICAL RESULTS OF THE REFINED, AS CAST (80-55-06) SERIES, MECHANICAL PROPERTY, MATHEMATICAL MODELS (5) THROUGH (8)

INDEP. VARIABLE	EQUATION (5) - (T.S.)			EQUATION (6) - (Y.S.)			EQUATION (7) - (% E.)			EQUATION (8) - (BHN)		
	MEAN CONTRI.	PERCENT CONTRI.	ALPHA	MEAN CONTRI.	PERCENT CONTRI.	ALPHA	MEAN CONTRI.	PERCENT CONTRI.	ALPHA	MEAN CONTRI.	PERCENT CONTRI.	ALPHA
(% T.C.)	---	---	---	- 17,373	- 28.47	0.200	---	---	---	---	---	---
(% Si)	- 14,611	- 14.86	0.200	+ 8,746	+ 14.33	0.200	+ 3,289	+ 45.33	0.200	- 23.188	- 10.28	0.200
(% Ni)	- 3,570	- 3.63	0.100	- 1,452	- 2.38	0.200	- 1,679	- 23.14	0.0005	+ 13.268	+ 5.88	0.0005
(% Mn)	+ 7,801	+ 7.93	0.025	+ 3,316	+ 5.44	0.100	- 1,366	- 18.82	0.100	+ 18.973	+ 8.41	0.005
(% P)	+ 5,491	+ 5.58	0.100	---	---	---	---	---	---	---	---	---
(% S)	---	---	---	---	---	---	+ 0,612	+ 8.43	0.200	- 7.405	- 3.28	0.050
(% Al)	- 10,201	- 10.37	0.025	- 8,284	- 13.58	0.005	---	---	---	---	---	---
(% Cu)	+ 12,933	+ 13.15	0.0005	+ 5,292	+ 8.67	0.005	- 3,921	- 54.04	0.0005	+ 37.681	+ 16.70	0.0005
(% Mg)	+ 22,703	+ 23.09	0.0005	+ 4,145	+ 6.79	0.050	+ 2,181	+ 30.06	0.025	+ 11.017	+ 4.88	0.100
(% Mo)	+ 3,931	+ 4.00	0.0005	+ 4,090	+ 6.70	0.0005	- 0.407	- 5.61	0.050	+ 4.554	+ 2.02	0.005
(% Sn)	---	---	---	---	---	---	+ 0.456	+ 6.28	0.100	---	---	---
(% Ti)	---	---	---	- 2,361	- 3.87	0.200	+ 1,528	+ 21.06	0.100	---	---	---
(% Ce)	- 7,489	- 7.62	0.100	- 9,556	- 15.66	0.005	---	---	---	---	---	---
(% Cr)	---	---	---	---	---	---	---	---	---	- 11.411	- 5.06	0.025
Constant	+ 81,338	+ 82.72	---	+ 74,450	+122.02	---	+ 6,562	+ 90.45	---	+182.125	+ 80.72	---
Mean Property	+ 98,326	+100.00	---	+ 61,013	+100.00	---	7.255	+100.00	---	+225.613	+100.00	---

The metallurgical significance of the refined, as cast, (80-55-06) series equations (5) through (8) can be assessed using the following criteria of judgment with respect towards the elemental variables' contribution towards the magnitudes of the dependent properties: (10,11,12)

1. The molybdenum, phosphorus, manganese, magnesium, aluminum, cerium, chromium, titanium and tin variables should contribute positively to strength and hardness and negatively towards percent elongation;
2. The carbon, silicon, nickel and copper variables should contribute negatively towards strength properties and positively towards ductility; and
3. The sulfur contributes negatively towards both strength and ductility properties' magnitudes.

Thus, examination of Table 3 shows that nineteen out of the thirty-six significant, independent, elemental variables in these four refined equations, or 53 percent, are in agreement with metallurgical theory. Furthermore, the silicon, nickel, manganese, copper, magnesium and molybdenum variables appear in all four models. The sulfur, aluminum, titanium and cerium terms are present in at least two of the four equations, while the total carbon, phosphorus, tin and chromium show up only one time. Of major significance is the fact that the six, major, repeating, elemental variables make up 24 of the 36 terms in equations (5) through (8), and 16 out of the 24, or 66.7 percent, are in agreement with metallurgical theory.

It should also be noted that 63 percent of the time, each mechanical property model can predict the property's value within ± 1 standard error of estimate of its mean level, and 95 percent of the time its magnitude should be within $\pm$ two σ_E 's of its mean value.

IV. C. ANNEALED (60-40-18) SERIES

IV. C.1. LINEAR REGRESSION MODELS

The four initial, annealed (60-40-18) series, mathematical models describing the tensile strength, yield strength, percent elongation and Brinell hardness number were derived from 118 complete sets data and listed in Mathematical Model Set III. Solving for the fifteen constants required by the general equation leaves 103 degrees of freedom for these first four regression analyses.

The four sequel equations describing the same mechanical properties were also derived from these 118 data sets and are listed in Mathematical Model Set IV. From four to eight of the independent, elemental variables were eliminated during the refining runs and thus not only increased the degrees of freedom in this second assessment, but also improved the statistical significance of each equation.

MATHEMATICAL MODEL SET III - ANNEALED (60-40-18) SERIES, MULTIPLE REGRESSION EQUATIONS

$$\begin{aligned} (\text{T.S.}) = & + 34,258 + 10,324 (\% \text{ Si}) + 5,456 (\% \text{ Ni}) + 25,985 (\% \text{ Cr}) \\ & + 13,368 (\% \text{ Cu}) - 6,596 (\% \text{ Mn}) - 137,644 (\% \text{ Ce}) + 50,947 (\% \text{ Ti}) \\ & - 55,259 (\% \text{ S}) - 11,833 (\% \text{ P}) + 15,121 (\% \text{ Mg}) + 26,132 (\% \text{ Al}) \\ & + 11,332 (\% \text{ Sn}) - 4,790 (\% \text{ Mo}) + 451 (\% \text{ T.C.}) \end{aligned} \quad (9)$$

$$R_{(9)} = 0.8162; \text{F-Ratio}_{(9)} = 14,677; \sigma_{E(9)} = 1,438; \alpha_{(9)} = 0.001$$

$$\begin{aligned} (\text{Y.S.}) = & + 22,502 + 11,177 (\% \text{ Si}) + 6,784 (\% \text{ Ni}) - 224,098 (\% \text{ Ce}) \\ & + 78,998 (\% \text{ Ti}) - 6,019 (\% \text{ Mn}) - 49,611 (\% \text{ S}) + 16,629 (\% \text{ Sn}) \\ & - 9,949 (\% \text{ P}) + 3,459 (\% \text{ Cu}) + 5,027 (\% \text{ Cr}) - 11,377 (\% \text{ Mg}) \\ & - 3,604 (\% \text{ Mo}) - 143 (\% \text{ T.C.}) + 537 (\% \text{ Al}) \end{aligned} \quad (10)$$

$$R_{(10)} = 0.8980; \text{F-Ratio}_{(10)} = 30.655; \sigma_{E(10)} = 1,697; \alpha_{(10)} = 0.001$$

$$\begin{aligned} (\% \text{ E.}) = & + 48.987 + 99.774 (\% \text{ Mg}) - 7.676 (\% \text{ T.C.}) - 3.998 (\% \text{ Ni}) \\ & - 31.148 (\% \text{ Sn}) - 17.271 (\% \text{ Mo}) - 7.506 (\% \text{ Cu}) + 55.713 (\% \text{ Al}) \\ & - 67.032 (\% \text{ Ce}) + 19.832 (\% \text{ Ti}) - 0.510 (\% \text{ Si}) + 1.799 (\% \text{ Cr}) \\ & + 1.625 (\% \text{ P}) - 0.400 (\% \text{ Mn}) - 4.644 (\% \text{ S}) \end{aligned} \quad (11)$$

$$R_{(11)} = 0.3911; \text{F-Ratio}_{(11)} = 1.329; \sigma_{E(11)} = 2.530; \alpha_{(11)} = 0.001$$

$$\begin{aligned} (\text{BHN}) = & + 77.423 + 92.191 (\% \text{ Cr}) + 12.994 (\% \text{ Si}) + 129.350 (\% \text{ Sn}) \\ & - 21.217 (\% \text{ Mn}) + 14.062 (\% \text{ T.C.}) + 65.562 (\% \text{ Mo}) + 165.947 (\% \text{ Al}) \\ & + 57.258 (\% \text{ Mg}) - 12.672 (\% \text{ Cu}) - 93.669 (\% \text{ Ti}) + 2.900 (\% \text{ Ni}) \\ & + 27.882 (\% \text{ S}) + 32.704 (\% \text{ Ce}) + 3.657 (\% \text{ P}) \end{aligned} \quad (12)$$

$$R_{(12)} = 0.6641; \text{F-Ratio}_{(12)} = 5,804; \sigma_{E(12)} = 5.620; \alpha_{(12)} = 0.001$$

MATHEMATICAL MODEL SET IV - ANNEALED (60-40-18) SERIES, BEST SELECTION EQUATIONS

$$\begin{aligned} (\text{T.S.}) = & + 36,478 + 10,645 (\% \text{ Si}) + 4,918 (\% \text{ Ni}) + 26,519 (\% \text{ Cr}) \\ & + 13,583 (\% \text{ Cu}) - 7,265 (\% \text{ Mn}) - 153,808 (\% \text{ Ce}) + 53,558 (\% \text{ Ti}) \\ & - 58,850 (\% \text{ S}) - 11,981 (\% \text{ P}) + 19,659 (\% \text{ Mg}) \end{aligned} \quad (13)$$

$$R_{(13)} = 0.8129; \text{F-Ratio}_{(13)} = 20.843; \sigma_{E(13)} = 1,422; \alpha_{(13)} = 0.001$$

$$\begin{aligned} (\text{Y.S.}) = & + 22,757 + 10,890 (\% \text{ Si}) + 5,929 (\% \text{ Ni}) - 197,932 (\% \text{ Ce}) \\ & + 76,711 (\% \text{ Ti}) - 5,814 (\% \text{ Mn}) - 52,379 (\% \text{ S}) \end{aligned} \quad (14)$$

$$R_{(14)} = 0.8954; \text{F-Ratio}_{(14)} = 74.845; \sigma_{E(14)} = 1,655; \alpha_{(14)} = 0.001$$

$$\begin{aligned} (\% \text{ E.}) = & + 47.045 + 101.510 (\% \text{ Mg}) - 7.562 (\% \text{ T.C.}) - 3.557 (\% \text{ Ni}) \\ & - 28.150 (\% \text{ Sn}) - 16.223 (\% \text{ Mo}) - 8.631 (\% \text{ Cu}) + 55.777 (\% \text{ Al}) \end{aligned} \quad (15)$$

$$R_{(15)} = 0.3863; \text{F-Ratio}_{(15)} = 2.757; \sigma_{E(15)} = 2.454; \alpha_{(15)} = 0.005$$

$$\begin{aligned} (\text{BHN}) = & + 77.368 + 87.499 (\% \text{ Cr}) + 12.980 (\% \text{ Si}) + 123.495 (\% \text{ Sn}) \\ & - 21.482 (\% \text{ Mn}) + 14.238 (\% \text{ T.C.}) + 64.451 (\% \text{ Mo}) + 153.529 (\% \text{ Al}) \\ & + 69.742 (\% \text{ Mg}) - 16.533 (\% \text{ Cu}) \end{aligned} \quad (16)$$

$$R_{(16)} = 0.6608; \text{F-Ratio}_{(16)} = 9.301; \sigma_{E(16)} = 5.510; \alpha_{(16)} = 0.001$$

IV. C. 2. STATISTICAL SIGNIFICANCE

Three out of the four mechanical property models listed in Mathematical Model Set III are significant at the 0.001 confidence level, or less, and the four correlation coefficients range from 0.3911 to 0.8980. Of the four refined equations listed in Mathematical Model Set IV, i.e., models (13) through (16), all but the (% E.), equation (15), are significant at the 0.001 confidence level, or less, and their correlation coefficients range from 0.3863 to 0.8954. This small drop in the R values is more than offset by an increase in the F-Ratio magnitude ranges from (0.3294 to 30.655) to (2.757 to 74.845) and again verifies that these latter models are the statistically superior ones.

Table 4 lists the detailed qualitative, quantitative and statistical results from the refined, annealed (60-40-18) series, mechanical property equations (13) through (16). All thirty-two independent, elemental variables in these four models had alpha values of 0.200, or less, and minimum values were again achieved for each dependent property's standard error of estimate.

IV. C. 3. METALLURGICAL SIGNIFICANCE

The metallurgical significance of the refined, annealed (60-40-18) series equations (13) through (16) can be assessed using the exact same criteria of judgment delineated in Section IV B.3. of this report, with one exception. In annealed, ductile cast iron alloys, silicon contributes positively towards strength and hardness properties and negatively towards the percent elongation magnitude,⁽¹²⁾ i.e., just the opposite of its behavior in the as cast alloys. Thus, examination of Table 4 shows that 17 out of the 32 significant, independent, elemental variables in these four refined equations, or 53 percent, are in agreement with metallurgical theory. In addition, none of the fourteen variables appear in all four models but the manganese, silicon, copper, nickel and magnesium variables are present in at least three of the four equations. Of major significance is the fact that the five, major, repeating, independent variables make up 15 of the 32 parameters in equations (13) through (16) and only 6 out of the 15, or 40 percent, are in agreement with metallurgical theory.

IV. D. AIR QUENCHED AND TEMPERED (100-70-03) SERIES

IV. D.1. LINEAR REGRESSION MODELS

The four initial, air quenched and tempered (100-70-03) series, mathematical models describing the strength and ductility properties were derived from only 19 complete sets of data and are listed in Mathematical Model Set V. Solving for the fifteen constants required by the general equation leaves only four degrees of freedom for these first four regression analyses.

The four sequel equations describing the same mechanical properties were also derived from these same 19 data sets and are listed in Mathematical Model Set VI. From two to five of the independent, elemental variables were eliminated during the refining runs and caused not only an increase in the degrees of freedom in this second assessment, but also improved the statistical significance of each equation.

TABLE 4: QUALITATIVE, QUANTITATIVE AND STATISTICAL RESULTS OF THE REFINED, ANNEALED (60-40-18) SERIES, MECHANICAL PROPERTY, MATHEMATICAL MODELS (13) THROUGH (16)

INDEP. VARIABLE	EQUATION (13) - (T.S.)			EQUATION (14) - (Y.S.)			EQUATION (15) - (% E.)			EQUATION (16) - (BHR)		
	MEAN CONTRI.	PERCENT CONTRI.	ALPHA	MEAN CONTRI.	PERCENT CONTRI.	ALPHA	MEAN CONTRI.	PERCENT CONTRI.	ALPHA	MEAN CONTRI.	PERCENT CONTRI.	ALPHA
(% T.C.)	---	---	---	---	---	---	- 28.980	-141.10	0.010	+ 54.564	+ 31.26	0.025
(% Si)	+ 26,379	+ 40.13	0.0005	+ 26,986	+ 55.86	0.0005	---	---	---	+ 32.165	+ 18.43	0.005
(% Ni)	+ 1,273	+ 1.94	0.0005	+ 1,534	+ 3.18	0.0005	- 0.921	- 4.48	0.100	---	---	---
(% Mn)	- 2,587	- 3.94	0.005	- 2,070	- 4.29	0.025	---	---	---	- 7.650	- 4.38	0.025
(% P)	- 830	- 1.26	0.100	---	---	---	---	---	---	---	---	---
(% S)	- 719	- 1.09	0.100	- 640	- 1.32	0.200	---	---	---	---	---	---
(% Al)	---	---	---	---	---	---	+ 1.965	+ 9.57	0.200	+ 5.456	+ 3.13	0.200
(% Cu)	+ 2,826	+ 4.30	0.001	---	---	---	- 1.796	- 8.74	0.100	- 3.440	- 1.97	0.050
(% Mg)	+ 845	+ 1.28	0.200	---	---	---	+ 4.361	+ 21.23	0.001	+ 2.996	+ 1.72	0.200
(% Mo)	---	---	---	---	---	---	- 0.682	- 3.32	0.200	+ 2.709	+ 1.55	0.025
(% Sn)	---	---	---	---	---	---	- 0.454	- 2.21	0.200	+ 1.993	+ 1.14	0.025
(% Ti)	+ 1,451	+ 2.21	0.050	+ 2,078	+ 4.30	0.025	---	---	---	---	---	---
(% Ce)	- 1,930	- 2.94	0.025	- 2,332	- 4.83	0.010	---	---	---	---	---	---
(% Cr)	+ 2,539	+ 3.86	0.0005	---	---	---	---	---	---	+ 8.379	+ 4.80	0.001
Constant	+ 36,478	+ 55.50	---	+ 22,757	+ 47.10	---	+ 47.045	+229.06	---	+ 77.368	+ 44.33	---
Mean Property	+ 65,725	+100.00	---	+ 48,313	+100.00	---	+ 20.54	+100.00	---	+174.540	+100.00	---

MATHEMATICAL MODEL SET V - AIR QUENCHED AND TEMPERED (100-70-03) SERIES, MULTIPLE REGRESSION EQUATIONS

$$\begin{aligned} (T.S.) = & -446,191 - 185,36 (\% Ni) - 600,957 (\% Sn) + 166,127 (\% T.C.) \\ & + 5,608,723 (\% Ti) - 456,385 (\% Mo) - 5,134,103 (\% S) - 181,682 (\% Mn) \\ & - 1,641,139 (\% Al) - 534,035 (\% Cu) + 38,063 (\% Si) + 239,852 (\% Cr) \\ & + 654,275 (\% Mg) - 961,682 (\% Ce) + 47,590 (\% P) \end{aligned} \quad (17)$$

$$R_{(17)} = 0.9405; F\text{-Ratio}_{(17)} = 2.190; \sigma_{E(17)} = 10,658; \alpha_{(17)} = 0.100+$$

$$\begin{aligned} (Y.S.) = & -144,942 - 98,369 (\% Ni) - 316,819 (\% Sn) + 35,591 (\% Si) \\ & + 3,032,200 (\% Ti) - 3,019,567 (\% S) + 60,006 (\% T.C.) - 936,230 (\% Al) \\ & - 96,923 (\% Mn) - 270,413 (\% Cu) - 195,494 (\% Mo) + 350,284 (\% Mg) \\ & + 71617 (\% P) - 476,792 (\% Ce) - 47,355 (\% Cr) \end{aligned} \quad (18)$$

$$R_{(18)} = 0.8909; F\text{-Ratio}_{(18)} = 1.099; \sigma_{E(18)} = 7,288; \alpha_{(18)} = 0.100+$$

$$\begin{aligned} (\% E.) = & -83.228 + 100.764 (\% P) + 52.775 (\% Mn) + 1,119.482 (\% S) \\ & + 91.52 (\% Cr) + 19.922 (\% T.C.) + 15.078 (\% Ni) - 244.114 (\% Mg) \\ & + 265.476 (\% Al) - 7.137 (\% Si) + 526.759 (\% Ce) - 570.794 (\% Ti) \\ & + 43.190 (\% Sn) - 52.884 (\% Mo) - 33.013 (\% Cu) \end{aligned} \quad (19)$$

$$R_{(19)} = 0.9139; F\text{-Ratio}_{(19)} = 1.447; \sigma_{E(19)} = 1.774; \alpha_{(19)} = 0.100+$$

$$\begin{aligned} (BHN) = & +125.22 - 474.84 (\% Mn) - 12,662.34 (\% S) - 797.17 (\% Sn) \\ & - 183.42 (\% Ni) + 9,599.17 (\% Ti) - 3,657.68 (\% Al) + 104.26 (\% Si) \\ & - 8,261.92 (\% Ce) - 694.87 (\% P) + 2,490.25 (\% Mg) - 378.04 (\% Mo) \\ & - 298.44 (\% Cr) + 36.78 (\% T.C.) + 197.46 (\% Cu) \end{aligned} \quad (20)$$

$$R_{(20)} = 0.8496; F\text{-Ratio}_{(20)} = 0.741; \sigma_{E(20)} = 22.910; \alpha_{(20)} = 0.100+$$

MATHEMATICAL MODEL SET VI - AIR QUENCHED AND TEMPERED (100-70-03) SERIES, MULTIPLE REGRESSION EQUATIONS

$$\begin{aligned} (T.S.) = & -473,500 - 194,719 (\% Ni) - 660,783 (\% Sn) + 185,793 (\% T.C.) \\ & + 3,808,957 (\% Ti) - 370,256 (\% Mo) - 4,159,810 (\% S) - 164,377 (\% Mn) \\ & - 1,107,353 (\% Al) - 745,203 (\% Cu) + 45,158 (\% Si) + 308,904 (\% Cr) \end{aligned} \quad (21)$$

$$R_{(21)} = 0.9287; F\text{-Ratio}_{(21)} = 3.990; \sigma_{E(21)} = 8,796; \alpha_{(21)} = 0.050$$

$$\begin{aligned} (Y.S.) = & -124,591 - 89,719 (\% Ni) - 336,268 (\% Sn) + 41,152 (\% Si) \\ & + 1,071,286 (\% Ti) - 1,588,820 (\% S) + 57,353 (\% T.C.) - 684,306 (\% Al) \\ & - 66,800 (\% Mn) - 319,525 (\% Cu) \end{aligned} \quad (22)$$

$$R_{(22)} = 0.8560; F\text{-Ratio}_{(22)} = 2.742; \sigma_{E(22)} = 5,529; \alpha_{(22)} = 0.100$$

$$\begin{aligned} (\% E.) = & -65.227 + 75.005 (\% P) + 53.865 (\% Mn) + 1,292.056 (\% S) \\ & + 74.675 (\% Cr) + 13.641 (\% T.C.) + 20.198 (\% Ni) - 210.079 (\% Mg) \\ & + 226.508 (\% Al) - 5.552 (\% Si) + 445.988 (\% Ce) - 820.096 (\% Ti) \\ & 44.117 (\% Sn) \end{aligned} \quad (23)$$

$$R_{(23)} = 0.8943; F\text{-Ratio}_{(23)} = 1.998; \sigma_{E(23)} = 1.595; \alpha_{(23)} = 0.100+$$

$$\begin{aligned} (BHN) = & +294.52 - 348.46 (\% Mn) - 8,688.16 (\% S) - 622.83 (\% Sn) \\ & - 165.35 (\% Ni) + 4,831.98 (\% Ti) - 2,706.98 (\% Al) + 92.97 (\% Si) \\ & - 6,110.51 (\% Ce) - 746.83 (\% P) + 1,699.33 (\% Mg) \end{aligned} \quad (24)$$

$$R_{(24)} = 0.8159; F\text{-Ratio}_{(24)} = 1.593; \sigma_{E(24)} = 17.759; \alpha_{(24)} = 0.100+$$

IV. D.2. STATISTICAL SIGNIFICANCE

Although the correlation coefficients of equations (17) through (20), listed in Mathematical Model Set V, range from 0.8496 to 0.9405, not one of the four models is significant because their alpha values are all greater than 0.100. However, the "best-selection" equations listed in Mathematical Model Set VI show some improvement in that the tensile strength model has an alpha value of less than 0.050 and the yield strength one has an alpha magnitude of less than 0.100. The correlation coefficients of these sequel equations range from 0.8159 to 0.9287. Once again, the small decrease in R values is offset by an increase in the F-Ratio magnitude ranges from (0.741 to 2.190) to (1.593 to 3.990) and again verifies that these sequel models are the statistically superior ones.

Table 5 lists the detailed qualitative, quantitative and statistical data for the refined, air quenched and tempered (100-70-03) series, mechanical property equations (21) through (24). All forty-two independent, elemental variables in these four models had alpha values of 0.200, or less, and minimum values were again achieved for each dependent property's standard error of estimate.

IV. D.3 METALLURGICAL SIGNIFICANCE

The metallurgical significance of the refined, air quenched and tempered (100-70-03) series equations (21) through (24) can also be assessed using the exact same criteria of judgment described in Section IV.B.3 of this report. Thus, examination of Table 5 shows that only 17 out of the 42 significant, independent, elemental variables in the four "best-selection" models, or 40.5 percent, are in agreement with metallurgical theory. This poor showing is due to the limited data base available on this specific thermal history.

IV. E. OIL QUENCHED AND TEMPERED (120-90-02) SERIES

IV. E.1. LINEAR REGRESSION MODELS

The four, initial oil quenched and tempered (120-90-02) series, mathematical models describing the tensile strength, yield strength, percent elongation and Brinell hardness number were developed from 22 complete data sets and are listed in Mathematical Model Set VII. Solving for the fifteen constants required by the general equation leaves only seven degrees of freedom for these first four regression runs.

The four sequel equations describing the same strength and ductility properties were also developed from these same 22 data sets and are listed in Mathematical Model Set VIII. From five to ten of the independent, elemental terms were eliminated during the "best-selection" run and again increased the degrees of freedom and improved each equation's statistical significance.

TABLE 5: QUALITATIVE, QUANTITATIVE AND STATISTICAL RESULTS OF THE REFINED, AIR QUENCHED AND TEMPERED (100-70-03) SERIES, MECHANICAL PROPERTY, MATHEMATICAL MODELS (21) THROUGH (24)

INDEP. VARIABLE	EQUATION (21) - (T.S.)			EQUATION (22) - (Y.S.)			EQUATION (23) - (% E.)			EQUATION (24) - (BHN)		
	MEAN CONTRI.	PERCENT CONTRI.	ALPHA	MEAN CONTRI.	PERCENT CONTRI.	ALPHA	MEAN CONTRI.	PERCENT CONTRI.	ALPHA	MEAN CONTRI.	PERCENT CONTRI.	ALPHA
(% T.C.)	+713,640	+677.46	0.005	+220,296	+320.82	0.050	+ 52.396	+680.96	0.100	---	---	---
(% Si)	+114,345	+108.55	0.050	+104,201	+151.75	0.025	- 14.058	-182.71	0.200	+235.410	+104.17	0.100
(% Ni)	- 55,137	- 52.34	0.005	- 25,403	- 36.99	0.005	+ 5.719	+ 74.33	0.025	- 46.821	- 20.72	0.025
(% Mn)	- 59,523	- 56.50	0.100	- 24,189	- 35.23	0.100	+ 19.505	+253.50	0.025	-126.181	- 55.84	0.050
(% P)	---	---	---	---	---	---	+ 5.388	+ 70.03	0.050	- 53.552	- 23.74	0.050
(% S)	- 45,758	- 43.44	0.100	- 17,477	- 25.45	0.200	+ 14.213	+184.71	0.025	- 95.570	- 42.29	0.050
(% Al)	- 39,632	- 37.62	0.050	- 24,491	- 35.67	0.050	+ 8.107	+105.36	0.100	- 96.883	- 42.87	0.100
(% Cu)	-141,589	-134.67	0.005	- 60,710	- 88.41	0.005	---	---	---	---	---	---
(% Mg)	---	---	---	---	---	---	- 9.741	-126.60	0.025	+ 78.798	+ 34.87	0.025
(% Mo)	- 17,539	- 16.65	0.100	---	---	---	---	---	---	---	---	---
(% Sn)	- 12,628	- 11.99	0.010	- 6,426	- 9.36	0.025	+ 0.843	+ 10.96	0.200	- 11.902	- 5.27	0.100
(% Ti)	+ 97,624	+ 92.67	0.050	+ 27,457	+ 39.99	0.100	- 21.019	-273.17	0.025	+123.944	+ 54.80	0.050
(% Ce)	---	---	---	---	---	---	+ 5.517	+ 71.70	0.100	- 75.587	- 33.45	0.025
(% Cr)	+ 25,037	+ 23.77	0.100	---	---	---	+ 6.052	+ 78.66	0.100	---	---	---
Constant	-473,500	-449.49	---	-124,591	-181.44	---	- 65.227	-847.72	---	+294.520	+130.33	---
Mean Property	+105,341	+100.00	---	+ 68,667	+100.00	---	+ 7.694	+100.00	---	+225.976	+100.00	---

MATHEMATICAL MODEL SET VII - OIL QUENCHED AND TEMPERED (120-90-02) SERIES, MULTIPLE REGRESSION EQUATIONS

$$\begin{aligned} (\text{T.S.}) = & + 235,757 - 84,598 (\% \text{ Ni}) + 742,217 (\% \text{ Ti}) - 207,456 (\% \text{ Cr}) \\ & - 97,573 (\% \text{ Cu}) - 267,349 (\% \text{ P}) - 1,325,548 (\% \text{ S}) + 906,183 (\% \text{ Ce}) \\ & - 18,134 (\% \text{ Si}) + 198,896 (\% \text{ Sn}) + 381,120 (\% \text{ Mg}) + 26,815 (\% \text{ Mn}) \\ & - 211,615 (\% \text{ Al}) - 5,489 (\% \text{ T.C.}) + 15,042 (\% \text{ Mo}) \end{aligned} \quad (25)$$

$$R_{(25)} = 0.7950; \quad F\text{-Ratio}_{(25)} = 0.859; \quad \sigma_{E(25)} = 16,641; \quad \alpha_{(25)} = 0.100+$$

$$\begin{aligned} (\text{Y.S.}) = & + 204.078 - 69,264 (\% \text{ Ni}) - 169,158 (\% \text{ Cr}) - 90,641 (\% \text{ Cu}) \\ & + 41,666 (\% \text{ Mn}) - 297,294 (\% \text{ Al}) + 216,805 (\% \text{ Sn}) - 8,786 (\% \text{ Si}) \\ & + 234,749 (\% \text{ Ti}) + 46,246 (\% \text{ Mo}) - 10,088 (\% \text{ T.C.}) + 193,692 (\% \text{ Ce}) \\ & - 42.310 (\% \text{ P}) + 55,356 (\% \text{ Mg}) - 77,429 (\% \text{ S}) \end{aligned} \quad (26)$$

$$R_{(26)} = 0.8162; \quad F\text{-Ratio}_{(26)} = 1.003; \quad \sigma_{E(26)} = 12,633; \quad \alpha_{(26)} = 0.100+$$

$$\begin{aligned} (\% \text{ E.}) = & + 7.815 + 6.203 (\% \text{ Ni}) + 57.475 (\% \text{ Al}) - 193.422 (\% \text{ S}) \\ & - 35.566 (\% \text{ Sn}) + 8.746 (\% \text{ Cu}) - 3,757 (\% \text{ Mn}) - 1.563 (\% \text{ Si}) \\ & - 11.863 (\% \text{ Mo}) + 11.860 (\% \text{ Cr}) - 24.291 (\% \text{ Ti}) + 15.425 (\% \text{ Ce}) \\ & - 0.323 (\% \text{ T.C.}) + 1.649 (\% \text{ P}) + 2.235 (\% \text{ Mg}) \end{aligned} \quad (27)$$

$$R_{(27)} = 0.8991; \quad F\text{-Ratio}_{(27)} = 2.110; \quad \sigma_{E(27)} = 0.864; \quad \alpha_{(27)} = 0.100+$$

$$\begin{aligned} (\text{BHN}) = & + 479.27 - 51.27 (\% \text{ Ni}) + 696.24 (\% \text{ Sn}) + 1,160.67 (\% \text{ Ti}) \\ & - 285.48 (\% \text{ Mo}) - 403.37 (\% \text{ P}) + 246.30 (\% \text{ Cr}) + 869.03 (\% \text{ Mg}) \\ & - 54.96 (\% \text{ T.C.}) - 371.55 (\% \text{ Al}) - 14.96 (\% \text{ Si}) - 1,058.37 (\% \text{ S}) \\ & + 4.91 (\% \text{ Mn}) + 2.62 (\% \text{ Cu}) - 9.29 (\% \text{ Ce}) \end{aligned} \quad (28)$$

$$R_{(28)} = 0.8335; \quad F\text{-Ratio}_{(28)} = 1.138; \quad \sigma_{E(28)} = 15.135; \quad \alpha_{(28)} = 0.100+$$

MATHEMATICAL MODEL SET VIII - OIL QUENCHED AND TEMPERED (120-90-02) SERIES, BEST SELECTION EQUATIONS

$$\begin{aligned} (\text{T.S.}) = & + 247,761 - 74,062 (\% \text{ Ni}) + 752,970 (\% \text{ Ti}) - 219,001 (\% \text{ Cr}) \\ & - 113,805 (\% \text{ Cu}) - 201,503 (\% \text{ P}) - 1,630,250 (\% \text{ S}) + 927,979 (\% \text{ Ce}) \\ & - 21,479 (\% \text{ Si}) \end{aligned} \quad (29)$$

$$R_{(29)} = 0.7710; \quad F\text{-Ratio}_{(29)} = 2.382; \quad \sigma_{E(29)} = 12.819; \quad \alpha_{(29)} = 0.100$$

$$\begin{aligned} (\text{Y.S.}) = & + 135,091 - 61,414 (\% \text{ Ni}) - 165,788 (\% \text{ Cr}) - 59,728 (\% \text{ Cu}) \\ & + 48,051 (\% \text{ Mn}) \end{aligned} \quad (30)$$

$$R_{(30)} = 0.7777; \quad F\text{-Ratio}_{(30)} = 6,506; \quad \sigma_{E(30)} = 9,834; \quad \alpha_{(30)} = 0.005$$

$$\begin{aligned} (\% \text{ E.}) = & + 5.819 + 6.420 (\% \text{ Ni}) + 815.79 (\% \text{ Sn}) + 1,178.50 (\% \text{ Ti}) \\ & - 34.747 (\% \text{ Sn}) + 10.777 (\% \text{ Cu}) - 3,439 (\% \text{ Mn}) - 1.416 (\% \text{ Si}) \\ & - 13.782 (\% \text{ Mo}) + 11.070 (\% \text{ Cr}) \end{aligned} \quad (31)$$

$$R_{(31)} = 0.8920; \quad F\text{-Ratio}_{(31)} = 5.192; \quad \sigma_{E(31)} = 0.681; \quad \alpha_{(31)} = 0.005$$

$$\begin{aligned} (\text{BHN}) = & + 475.27 - 50.05 (\% \text{ Ni}) + 815.79 (\% \text{ Sn}) + 1,178.50 (\% \text{ Ti}) \\ & - 258.20 (\% \text{ Mo}) - 406.22 (\% \text{ P}) + 191.51 (\% \text{ Cr}) + 793.95 (\% \text{ Mg}) \\ & - 64.60 (\% \text{ T.C.}) - 451.20 (\% \text{ Al}) \end{aligned} \quad (32)$$

$$R_{(32)} = 0.8188; \quad F\text{-Ratio}_{(32)} = 2.711; \quad \sigma_{E(32)} = 12.011; \quad \alpha_{(32)} = 0.100$$

IV. E.2. STATISTICAL SIGNIFICANCE

Once again, although equations (25) through (28)'s correlation coefficients range from 0.7950 to 0.9991 (Mathematical Model Set VII), not one of the models is significant because all the alphas are greater than 0.100. The four refined models (29) through (32), listed in Mathematical Model Set VIII, however, show marked improvements. For example, the tensile strength equation (29) and Brinell Hardness equation (32) have alpha values of less than 0.100, while the yield strength model (30) and percent elongation model (31) have alpha values less than 0.005. The correlation coefficients of the sequel expressions range from 0.7710 to 0.8920. Just as in the previous tests, the small drop in the values of K is again offset by an increase in the F-Ratio magnitude range from (0.859 to 2.110) to (2.382 to 6.006), and proves once more that the "best-selection" models are the best, statistically.

Table 6 lists the detailed qualitative, quantitative and statistical data for the refined oil quenched and tempered (120-90-02) series, mechanical property equations (25) through (32). All thirty, independent, elemental terms in these four models had alpha values of 0.200, or less, and each of the four standard errors of estimate was again of minimum magnitude.

IV. E.3. METALLURGICAL SIGNIFICANCE

The metallurgical significance of the refined oil quenched and tempered (120-90-02) series equations (29) through (32) can also be assessed using the exact same criteria of judgment described in Section IV.3.3 of this report. Thus, examination of Table 6 indicates that 21 out of the 30 significant, independent, elemental terms in these four "best-selection" expressions or 70 percent, are in agreement with metallurgical theory. Also, the nickel and chromium terms appear in all four models, while the copper variables is in 3 out of the 4 equations. Of major significance is the fact that the three, major, replating variables make up 11 out of the 30 terms in equations (29) through (32) and 8 out of the 11, or 72.7 percent, are in agreement with the metallurgical theory.

IV. F. SUMMARY OF METALLURGICAL SIGNIFICANCE

The overall assessment of the metallurgical significance of the sixteen "best-selection" mechanical property, mathematical models generated during this study can be accomplished by examination of Table 7. The best agreement with metallurgical theory occurs in the Brinell Hardness number equations, i.e., 22 out of 36, or 61.2 percent. Second best are the tensile strength models, i.e., 22 out of 38, or 57.9 percent, followed by the yield strength equations, i.e. 15 out of 29, or 51.8 percent, and the percent elongation models, i.e., 15 out of 37, or 40.6 percent. The four equations generated for the oil quenched and tempered series were the most outstanding metallurgically in that 21 out of the 30 statistically significant variables, i.e., 70 percent, behave in accordance with the theory. Thus, overall, 74 out of a total of 140 major independent variables, or 52.8 percent, show agreement with metallurgical prediction.

TABLE 6: QUALITATIVE, QUANTITATIVE AND STATISTICAL RESULTS OF THE REFINED, OIL QUENCHED AND TEMPERED (120-90-02) SERIES, MECHANICAL PROPERTY, MATHEMATICAL MODELS (29) THROUGH (32)

INDEP. VARIABLE	EQUATION (29) - (T.S.)			EQUATION (30) - (Y.S.)			EQUATION (31) - (% E.)			EQUATION (32) - (B.IN)		
	MEAN CONTRI.	PERCENT CONTRI.	ALPHA	MEAN CONTRI.	PERCENT CONTRI.	ALPHA	MEAN CONTRI.	PERCENT CONTRI.	ALPHA	MEAN CONTRI.	PERCENT CONTRI.	ALPHA
(% T.C.)	---	---	---	---	---	---	---	---	---	-248.622	- 97.72	0.100
(% Si)	54,781	- 44.06	0.200	---	---	---	- 3.611	- 86.40	0.200	---	---	---
(% Ni)	- 24,911	- 20.04	0.005	- 20,657	- 19.42	0.001	+ 2.159	+ 51.66	0.0005	- 16.835	- 6.62	0.025
(% Mn)	---	---	---	+ 18,107	+ 17.03	0.100	- 1.296	- 31.00	0.100	---	---	---
(% P)	- 15,167	- 12.20	0.200	---	---	---	---	---	---	- 30.576	- 12.02	0.100
(% S)	- 18,829	- 15.14	0.200	---	---	---	- 2.366	- 56.59	0.025	---	---	---
(% Al)	---	---	---	---	---	---	+ 1.837	+ 43.94	0.025	- 16.099	- 6.33	0.200
(% Cu)	- 21,727	- 17.47	0.200	- 11,403	- 10.72	0.200	+ 2.057	+ 49.22	0.025	---	---	---
(% Mg)	---	---	---	---	---	---	---	---	---	+ 39.443	+ 15.50	0.200
(% Mo)	---	---	---	---	---	---	- 0.758	- 18.13	0.100	- 14.201	- 5.58	0.100
(% Sn)	---	---	---	---	---	---	- 0.643	- 15.38	0.100	+ 15.092	+ 5.93	0.025
(% Ti)	+ 21,701	+ 17.45	0.200	---	---	---	---	---	---	+ 33.964	+ 13.35	0.050
(% Ce)	+ 3,697	+ 7.80	0.200	---	---	---	---	---	---	---	---	---
(% Cr)	- 19,412	- 15.61	0.100	- 14,695	- 13.82	0.100	+ 0.981	+ 23.47	0.200	+ 16.975	+ 6.67	0.200
Constant	+247,761	+199.28	---	+135,001	+126.94	---	+ 5.819	+139.21	---	+475.270	+186.81	---
Mean Property	+124,331	+100.00	---	+106,352	+100.00	---	+ 4.180	+100.00	---	+254.412	+100.00	---

TABLE 7: SUMMARY OF THE REFINED, LYNCHBURG FOUNDRY COMPANY, MECHANICAL PROPERTY, MATHEMATICAL MODELS' METALLURGICAL SIGNIFICANCE

MECHANICAL PROPERTY	THERMAL TREATMENT CONDITION				TOTALS	PERCENT AGREEMENT
	AS CAST (80-55-06)	ANNEALED (60-40-18)	AIR QUENCHED AND TEMPERED (700-70-03)	OIL QUENCHED AND TEMPERED (120-90-02)		
TENSILE STRENGTH	6/9	5/10	5/11	6/8	22/38	57.9 %
YIELD STRENGTH	5/10	3/6	4/9	3/4	15/29	51.8 %
PERCENT ELONGATION	3/9	2/7	4/12	6/9	15/37	40.6 %
BRINELL HARD. NO.	5/8	7/9	4/10	6/9	22/36	61.2 %
TOTALS	19/36	17/32	17/42	21/30	74/140	52.8 %
PERCENT AGREEMENT	52.8 %	53.1 %	40.5 %	70.0 %	52.8 %	X

IV. G. AMMRC DATA VERSUS LYNCHBURG FOUNDRY DATA

A comparison between these industrial production data reduction results and previous findings of investigations which used only AMMRC supplied data⁽²⁾ is not in order.

IV. G.1. STATISTICAL SIGNIFICANCE

Since the AMMRC and Lynchburg Foundry Company data banks were different in their make-up, only the common features will be examined, herein.

The AMMRC information was generated from as cast, annealed and air quenched tempered test pieces, while the industrial data came from specimens subjected to all three of these heat treatments plus a fourth one, i.e., oil quench and tempered. Table 8 lists the important characteristics of both data sets derived from only the three, common denominator, process histories and offers the best means of assessing the comparative statistical significance differences of the two sets of results. For example, although about 100 percent of all the refined models generated from both data banks describing all four mechanical properties were significant at the 0.001 confidence level, or less, the correlation coefficient (R) and F-Ratio ranges were much greater for the AMMRC results, contrasted to the Lynchburg output. Thus, the AMMRC data is statistically better than the industrial, production information but the latter output is still very desirable for future design purposes, in fact, it is far better than originally anticipated.

Furthermore, Table 8 shows that the ratios of the data range to mean values for all but two of the twelve common models covered are almost a fraction of two greater for the Lynchburg equations, compared to the AMMRC results. In addition, the elemental variables that appeared at least once in both data sets include carbon, silicon, manganese, sulfur, magnesium, tin and titanium.

IV.G.2. METALLURGICAL SIGNIFICANCE

Examination of all the AMMRC and Lynchburg data banks shows that a total of nine, independent, elemental variables were common to both populations and included carbon, manganese, silicon, phosphorous, sulfur, magnesium, cerium, tin and titanium.

Table 9 summarizes the comparative differences between the AMMRC and the Lynchburg results' metallurgical significance, i.e., their fraction agreement with theory. For example, the totals column shows that while the industrial tensile strength test results are better than the Army's, i.e., 16 out of 30 (53.3%) to 9 out of 17 (52.9%), both Brinell Hardness data sets have equal, as well as the highest metallurgical agreement, i.e., 16 out of 27 (59.3%). In addition, the AMMRC totals for both the yield strength and percent elongation models are in better agreement with metallurgical theory than the industrial tabulations. In fact, the AMMRC percent elongation

TABLE 8: COMPARISON BETWEEN THE COMMON, REFINED, AMRC AND LYNCHBURG MECHANICAL PROPERTY, MATHEMATICAL MODELS' STATISTICAL SIGNIFICANCE

ITEM	COMMON THERMAL TREATMENT CONDITION					
	AS CAST (80-55-06)		ANNEAL ¹ (60-40-18)		AIR Q & T (100-70-03)	
	AMRC	LYNCHBURG	AMRC	LYNCHBURG	AMRC	LYNCHBURG
RANGE OF C.C., I.e., R's	0.3 - 0.67	0.5129 - 0.6599	0.516 - 0.998	0.3863 - 0.8954	0.888 - 0.976	0.8159 - 0.9287
RANGE OF F-RATIOS	8.459 - 33.97	5.275 - 15.340	11.57 - 3,662.2	2.757 - 74.845	6.939 - 27.77	1.593 - 3.990
RATIO OF DATA RANGE TO MEAN VALUES FOR:						
T.S.	0.327	0.590	0.239	0.152	0.329	0.4175
Y.S.	0.205	0.848	0.115	0.300	0.250	0.393
% E.	0.827	1.645	0.474	0.805	1.02	1.103
BHN	0.368	0.553	0.076	0.1884	0.140	0.319
MOST SIGNIFICANT VARIABLES	Mn, S, Sn	Mn, Si, Ni, Cu, Mg, Mo	Mn, S, Sn, C, Ti, P	Mn, Si, Ni, Cu, Mg	Mn, S, Sn, Si, Mg, Ti, Co	Mn, S, Sn, Si, Ni, C, Ti, Al

equations were the best of all for these three treatments as shown by the 21 out of 26, or 80.8 percent agreement. Comparison of the overall totals delineated in Table 9 and the major variable agreement ratios indicates in both cases that the Army data is superior in terms of metallurgical significance to the Lynchburg results, i.e., 57 out of 92 (61.9%) to 53 out of 110 (48.2%), and 36 out of 55 (65.5%) to 30 out of 71 (42.2%), respectively.

TABLE 9: COMPARISON BETWEEN THE COMMON, REFINED, AMMRC AND LYNCHBURG MECHANICAL PROPERTY, MATHEMATICAL MODELS' METALLURGICAL SIGNIFICANCE

STATISTICAL SIGNIFICANCE

MECH. PROP.	COMMON THERMAL TREATMENT COND.						TOTALS		PERCENT AGREEMENT	
	AS CAST (80-55-06)		ANNEALED (60-40-18)		AIR QUENCH & T. (100-70-03)					
	AMMRC	LYNCH.	AMMRC	LYNCH.	AMMRC	LYNCH.	AMMRC	LYNCH.	AMMRC	LYNCH.
T.S.	2/4	6/9	5/9	5/10	2/4	5/11	9/17	16/30	52.9%	53.3%
Y.S.	3/6	5/10	6/10	3/6	2/6	4/9	11/22	12/25	50.0%	48.0%
% E.	5/6	3/9	10/11	2/7	6/9	4/12	21/26	9/28	80.8%	32.2%
BHN	3/5	5/8	7/13	7/9	6/9	4/10	16/27	16/27	59.3%	59.3%
TOTAL	13/21	19/36	28/43	17/32	16/28	17/42	57/92	53/110	61.9%	48.2%
% AGR.	61.9%	52.8%	65.2%	53.1%	57.2%	40.5%	61.9%	48.2%		
MAJOR VAR.A. RATIO	8/11	15/24	14/20	3/16	14/24	12/31	36/55	30/71	65.5%	42.2%
% MVAR	72.7%	62.5%	70.0%	18.8%	58.3%	38.7%	65.5%	42.2%		

V. UTILIZATION OF THESE MATHEMATICAL MODELS FOR CASTING DESIGN

The refined mechanical property, mathematical models derived during this investigation, coupled with the "best-selection" fragmentation property equations produced in a previous AMMRC project,⁽²⁾ have sufficient statistical and metallurgical significance to be used in designing and producing improved ductile cast iron alloys.

Both the mechanical and fragmentation properties must be adjusted accordingly, using only the refined expressions, to produce ductile cast iron alloys which should exhibit predictable strength, ductility and fragmentation levels.

Boundary conditions had to be established, however, to maintain some degree of control, and they were as follows:

1. The independent variables and thermal treatments had to be the same as those used in generating the refined models;
2. Extrapolation may be permitted to a small degree beyond the range of the original data base if metallurgical theory predicts no certain pitfalls; and
3. Only the more significant independent variables could be varied within the equations, i.e., only those whose α values are 0.200 or less.

As stated previously, only mechanical property aspects will be discussed in the report.

VI. INITIAL ALLOY DESIGN AND TEST RESULTS

VI. A. INTRODUCTION

After joint discussions with the AMMRC and Lynchburg technical staffs, two objectives were agreed to as realistic and attainable, i.e., the manufacture of ductile cast iron one-inch modified keel block test specimens, AMMRC test coupons and 2.75" x 21" rocket shells possessing:

1. Maximum strength with maximum ductility; and
2. Optimum fragmentation characteristics,

using four specific thermal treatments, i.e., as cast, annealed, air quench and tempered, and oil quench and tempered.

A review of the chemical analyses from a total of three-hundred and two (302) complete data sets derived from all four, specific thermal treatments, i.e., as cast (143), annealed (118), air quenched and tempered (19) and oil quenched and tempered (22) showed that the high, low and mean values of the fourteen independent, elemental variables have the magnitudes listed in Table 10.

Table 10: HIGH, LOW AND MEAN VALUES OF THE INDEPENDENT, ELEMENTAL VARIABLES FROM ALL FOUR THERMAL TREATMENTS

ELEMENT	HIGH	LOW	MEAN	ELEMENT	HIGH	LOW	MEAN
(ZT.C.)	4.08	3.54	3.84	(% Cu)	0.40	0.06	0.20
(% Si)	2.87	2.03	2.50	(% Cr)	0.15	0.05	0.09
(% Ni)	1.01	0.06	0.28	(% Mg)	0.072	0.022	0.045
(% Mn)	0.61	0.21	0.37	(% Mo)	0.44	0.01	0.05
(% P)	0.150	0.020	0.070	(% Sn)	0.080	0.002	0.017
(% S)	0.027	0.004	0.012	(% Ti)	0.054	0.019	0.027
(% Al)	0.047	0.012	0.036	(% Ce)	0.015	0.005	0.011

The initial test casting were then manufactured at Lynchburg in accordance with the following specifications:

1. SERIES I - This group of specimens should contain maximum % Mg and % Si, minimum % Ni, and mean levels of the remaining eleven, elemental variables. Thus, aiming for 0.070 % Mg, 2.80 % Si, 0.06 % Ni, etc., should have yielded maximum strength with maximum ductility for all four heat treated conditions; and
2. SERIES II - This group of specimens should contain maximum % Ti and % Sn, minimum % Mg, % Si, % P and % S, and mean levels of the remaining eight elemental variables. Thus aiming for 0.050 % Ti, 0.050 % Sn, 0.022 % Mg, 2.10 % Si, 0.020 % P and 0.004 % S, etc., should have yielded optimum fragmentation characteristics for all four heat treated conditions.

VI. B. AMMRC TEST COUPON

The Lynchburg Foundry Company tested their ability to produce the AMMRC test coupon and achieved immediate success. Figure 1 shows the as cast double test coupons, along with their attached ingates, down sprue and pour cup. Figure 2 illustrates the sectioned coupons, plus their risers, and they came out to be very sound and clean.

A total of twelve (12), 0.394 inch square, Charpy blanks; five (5), 0.357 inch tensile blanks; three (3) fragmentation test cylinders; and several test pieces for microscopic examination, were all machined out of the AMMRC double test coupons. Subsequent testing of as cast, 0.357 inch diameter test bars from these coupons produced average mechanical properties of 86,250 psi tensile strength, 61,500 psi yield strength, 12.5 per cent elongation and a Brinell hardness number of 207, i.e., well within the range levels of the 0.505 inch

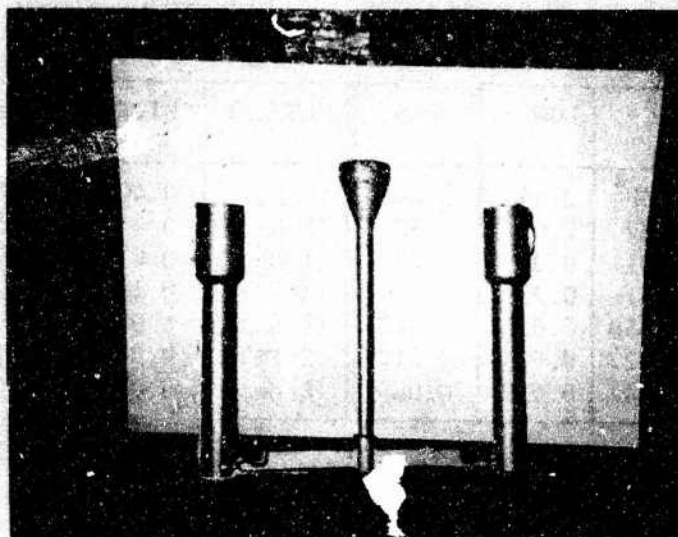


**FIGURE 1 : THE AS CAST, AMMRC DOUBLE TEST COUPON
CASTINGS WITH ATTACHED IN-GATES, DOWN
SPRUE AND POUR CUP**

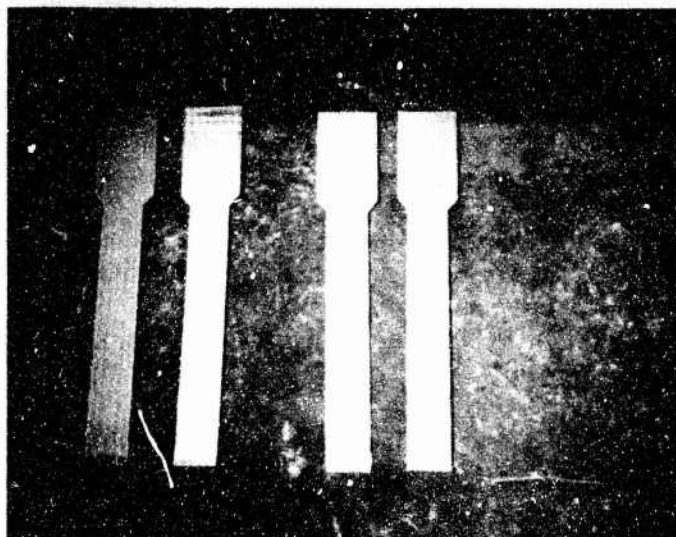


**FIGURE 2 : THE TWO, AS CAST, AMMRC DOUBLE TEST
COUPONS, SECTIONED FOR MACROSCOPIC EXAM-
INATION**

as cast, test bar data used during the mathematical model phase of this investigation.

VI. C. TEST RESULTS

The average of two chemical tests for the SERIES I and II samples are listed in Table 11. The results achieved met the desired goals outlined in Section VI. A., so thermal treatment of the one inch keel blocks test bars then commenced.

Four sets of 0.505 tensile test bars from both SERIES I and II were mechanically tested to determine their tensile strength, yield strength, per cent elongation and Brinell hardness number. In addition, the chemical analyses results listed in Table 11 were plugged into the sixteen (16), refined, mechanical property equations listed in Mathematical Model Sets II, IV, VI and VIII, to compute the predicted property magnitudes. Both these actual and predicted, mechanical property magnitudes are listed in Tables 12, along with the number of standard errors of estimate the actual test results are from the magnitudes predicted by the computer generated, mathematical models.

Examination of Table 12 shows that the SERIES I equations were far superior to the SERIES II models in predicting the magnitudes of the test castings' mechanical properties. In fact, in ten out of the sixteen (62.5%) SERIES I equations the actual property magnitudes were less than to two standard errors of estimate away from the predicted values, compared to only five out of the sixteen (31.25%) SERIES II models.

Why did this difference occur?

As previously mentioned, the prime objective of the SERIES I tests was to design and manufacture test casting possessing maximum strength with maximum ductility and that goal was accomplished because the compositions were well within the data bank's magnitude range for almost all the independent variables. The SERIES II tests, however, dealt with the analysis of AMMRC data translated to an industrial, production environment to achieve optimum fragmentation results and not to optimize mechanical property attainment. In addition, some of the SERIES II compositions were at the extremities of the data bank's magnitude range, and even beyond it in several variable, thus explaining some of the poor predictions within this group of results.

Table 13 lists the Charpy impact results and some supplemental Brinell hardness data derived from two SERIES I and three SERIES II, machineable test bar sets.

VI. D. UTILIZATION OF INITIAL TEST RESULTS

These initial test results, from the mechanical property point of view, were quite satisfactory and acted as the starting point of the refined test run. Some modifications had to be made to reduce the number of poor predictions of mechanical property magnitudes, especially in the case of the SERIES II analyses.

TABLE 11: MEAN CHEMICAL ANALYSES OF THE INITIAL SERIES I AND II TEST BARS

SERIES	MEAN CHEMICAL COMPOSITION													
	%T.C.	% Si	% Ni	% Mn	% P	% S	% Al	% Cu	% Cr	% Mg	% Mo	% Sn	% Ti	% Co
I	3.55	2.80	.085	.320	.120	.011	.050	.130	.080	.0335	.100	.0055	.027	.010
II	3.93	2.10	.440	.310	.110	.006	.030	.140	.080	.0235	.050	.0155	.040	.030

TABLE 12: MECHANICAL PROPERTY MAGNITUDES ATTAINED BY THE INITIAL SERIES I AND II TEST BARS AND THEIR COMPARISON WITH THE VALUES PREDICTED BY THE BEST SELECTION EQUATIONS

SERIES & COND	TENSILE STRENGTH(psi)			YIELD STRENGTH(psi)			PERCENT ELONGATION			BRINELL HARD. NO.		
	PREDICT	ACTUAL	# S.E..	PREDICT	ACTUAL	# S.E..	PREDICT	ACTUAL	# S.E..	PREDICT	ACTUAL	# S.E..
I - A.C.	94,319	82,000	1.70	64,469	59,125	1.33	9.22	11.00	.993	204.2	179.5	1.8
I - ANN.	66,648	68,750	1.48	51,409	52,000	.357	23.16	22.00	.473	179.4	164.0	2.8
I - AQ&T	132,247	94,250	4.32	99,082	68,250	5.57	2.64	9.50	4.27	231.6	201.0	1.7
I - OQ&T	136,497	122,500	1.09	124,129	105,000	2.17	2.34	4.00	2.37	220.4	232.0	.96
II - A.C.	76,031	99,500	3.23	39,547	65,250	6.41	6.67	3.75	1.63	223.9	213.0	.80
II - ANN.	58,788	67,000	5.77	43,249	48,000	2.87	17.35	11.50	2.39	169.9	164.0	1.0
II - AQ&T	200,673	106,250	10.73	89,888	72,000	3.23	7.40	4.00	2.13	133.7	227.0	5.2
II - OQ&T	162,598	133,500	.707	101,250	115,500	1.61	6.09	2.00	5.85	220.0	267.0	3.9

TABLE 13: CHARPY IMPACT DATA FROM SEVERAL SERIES I AND II, INITIAL TEST BARS PLUS SUPPLEMENTAL BRINELL HARDNESS NOS.

SERIES & COND	BHN	AVG. C.I. @ R.T.	AVG. C.I. @ -10°F	SERIES & COND	BHN	AVG. C.I. @ R.T.	AVG. C.I. @ -40°F
I-AC	196	2.65	1.95	II-AC	269	2.05	1.6
I-ANN	166	4.4	2.3	II-ANN	196	2.35	1.65
I-AQ&T	TOO DIFFICULT TO MACHINE.			II-AQ&T	286	2.4	2.15
I-OQ&T	TOO DIFFICULT TO MACHINE.			II-OQ&T	TOO DIFFICULT TO MACHINE.		

VII. FINAL ALLOY DESIGN AND TEST RESULTS

VII. A. INTRODUCTION

After additional discussions with the AMMRC and Lynchburg technical staffs, the final test castings were then manufactured in accordance with the following specifications:

1. Series I - This group of specimens should contain maximum % Mg and % Si, Minimum % C, and mean levels of the remaining eleven elemental variables; and
2. Series II - This group of specimens should aim at the mean analysis listed in Table 10 on Page 27.

The first set of final test castings were lost during the floods generated by hurricane AGNES and had to be reproduced during the last quarter of 1972.

VII. B. TEST RESULTS

Eight sets of test castings were manufactured, i.e., one for each of the four thermal treatments within both Series I and II. Each of these sets contained:

1. Six (6), 2.75" x 21" rocket shells;
2. Two (2), AMMRC double test coupons;
3. Two (2), one inch, modified keel block test specimens; and
4. Two (2), spectographic test pieces.

The forty-eight (48), 2.75" x 21", ductile cast iron rocket shells were sent to MEDICO INDUSTRIES in Wilkes Barre, Pennsylvania, in the heat treated conditions, for finish machining and processing. Upon completion of these tasks, these shells were forwarded to AMMRC for storage and future testing.

The average of two chemical tests for the final SERIES I and II samples are listed in Table 14, and the results achieved met the goals outlined in Section VII. A. The AMMRC double test coupons were then heat treated and machined for final testing.

Four sets of 0.357" tensile test bars from both SERIES I and II, AMMRC coupons were machined and tested to determine their tensile strength, yield strength, per cent elongation and Brinell hardness number. The chemical analyses listed in Table 14 were then plugged into the sixteen (16), best-selection, mechanical property equations listed in Mathematical Model Sets II, IV, VI and VIII, to compute the predicted strength and ductility properties magnitudes. Both these actual and predicted, mechanical property magnitudes are listed in Table 15, along with the number of standard errors of estimate deviation between these two numbers.

Examination of Table 15 indicates the degree of success in attaining the design objectives in both the SERIES I and II test castings was outstanding. For example, for ten out of the sixteen (62.5 %) SERIES I equations, the actual mechanical property magnitudes were less than one standard error of estimate away from the predicted values, five out of the sixteen (31.25%) were between one and two standard errors of estimate away, and the last one out of the sixteen (6.25%) was within 2.09 S.E.E.'s. In addition, for seven out of the sixteen (43.75%) SERIES II models, the actual mechanical property magnitudes were less than one S.E.E. away from the predicted values, seven out of the sixteen (43.75%) were between one and two S.E.E.'s away, and two out of the sixteen (12.5%) were within two and three S.E.E.'s.

How do these final results compare to the initial test data reported in Section VI?

TABLE 14: MEAN CHEMICAL ANALYSES OF THE FINAL SERIES I AND II TEST BARS

SERIES	M E A N C H E M I C A L C O M P O S I T I O N													
	%T.C.	% Si	% Ni	% Mn	% P	% S	% Al	% Cu	% Cr	% Mg	% Mo	% Sn	% Ti	% Co
I	3.66	2.75	.200	.380	.084	.015	.042	.160	.110	.068	.060	.020	.027	.010
II	3.86	2.59	.140	.360	.085	.014	.043	.170	.090	.057	.070	.020	.027	.010

TABLE 15: MECHANICAL PROPERTY MAGNITUDES ATTAINED BY THE FINAL SERIES I AND II TEST BARS AND THEIR COMPARISON WITH THE VALUES PREDICTED BY THE BEST SELECTION EQUATIONS

SERIES & COND	TENSILE STRENGTH(psi)			YIELD STRENGTH(psi)			PERCENT ELONGATION			BRINELL HARD NO.		
	PREDICT	ACTUAL	# S.E.	PREDICT	ACTUAL	# S.E.	PREDICT	ACTUAL	#S.E.	PREDICT	ACTUAL	#S.E.
I-AC	110,265	99,500	1.48	65,819	63,000	.703	10.5	8.5	1.12	216.4	223	.485
I-ANN	68,322	69,250	.653	50,988	53,250	1.37	25.3	22.0	1.36	181.5	170	2.09
I-AQ&T	102,534	106,500	.451	73,666	75,000	.241	6.3	5.0	.791	250.4	262	.653
I-OQ&T	119,813	123,500	.288	113,184	105,750	.842	2.4	3.5	1.60	283.5	286	.211
II-AC	107,081	100,000	.976	64,408	63,500	.227	9.8	8.5	.751	216.9	228	.817
II-ANN	65,906	68,750	.200	49,058	53,250	2.57	22.7	19.0	1.515	180.8	179	.330
II-AQ&T	133,156	121,000	1.38	73,981	81,250	1.32	7.4	3.5	2.45	239.0	269	1.69
II-OQ&T	132,362	121,250	.862	118,626	106,500	1.38	2.3	3.5	1.72	257.5	277	1.62

The SERIES I data shows that 93.75 per cent of the actual, final mechanical property results, i.e., 15 out of the 16, were within less than two S.E.E.'s of their predicted values, compared to only 62.5 per cent, i.e., ten out of sixteen, of the initial test data. Furthermore, the SERIES II data shows that 37.5 per cent of the actual, final test results, i.e., 14 out of the 16 within less than two S.E.E.'s of their predicted values, compared to only 31.25 per cent, i.e., five out of sixteen, of the initial test data.

Table 16 lists the Charpy impact results and some supplemental Brinell hardness data from three out of the four, SERIES I and II, machineable test bar sets.

Thus, the final alloy designs for SERIES I and II achieved the objective of phase one of this investigation, i.e., the industrial production of ductile cast iron products possessing predictable mechanical property magnitudes.

TABLE 16: CHARPY IMPACT DATA FROM SEVERAL SERIES I AND II, FINAL TEST BARS PLUS SUPPLEMENTAL BRINELL HARDNESS NOS.

SERIES & COND	BHN	AVG. C.I. @ R.T.	AVG. C.I. @ -40°F	SERIES & COND	BHN	AVG. C.I. @ R.T.	AVG. C.I. @ -40°F
I-AC	228	1.7	1.0	II-AC	241	1.75	1.05
I-ANN	179	2.5	1.15	II-ANN	174	1.7	1.4
I-AQ&T	269	2.75	1.3	II-AQ&T	255	3.4	1.55
I-OQ&T	TOO DIFFICULT TO MACHINE.			II-OQ&T	TOO DIFFICULT TO MACHINE.		

VIII. Conclusions

In association with AMMRC and the Lynchburg Foundry Company, this investigation attempted to design, produce and evaluate improved ductile cast iron alloys within an industrial plant environment using computer derived, mathematical models. A total of three-hundred and two (302) complete data sets derived from four, specific thermal treatments, i.e., as cast (143), annealed (118), air quenched and tempered (19) and oil quenched and tempered (22), were thoroughly analyzed and resulted in the generation of thirty-two multiple regression, mechanical property, mathematical models which described variations in tensile strength, yield strength, per cent elongation and Brinell hardness number. The sixteen (16) refined equations were possessing predictable properties. Overall, 74 out of a total of 140 major, independent variables contained in these 16 best-selection models, or 52.8 per cent, are in agreement with metallurgical theory. The initial design test results produced cast test bars whose actual mechanical property magnitudes were less than two standard errors of estimate away from predicted values in only fifteen out of the 32 specimens, i.e., 46.9 per cent. The final design test data, derived from specimens machined from the AMMRC coupons, significantly improved this achievement in that 29 out of the 32 pieces, i.e., 90.6 per cent, were actually within two S.E.E.'s of the predicted values. Thus, the final alloy designs for phase one of this investigation achieved its objective of producing ductile cast iron products possessing predictable mechanical property magnitudes within an industrial facility.

IX. RECOMMENDATIONS

Since the implemented scientific development program achieved the objective of producing ductile cast iron products possessing predictable property levels within an industrial foundry, it is recommended that the same analytical tools and techniques be applied to other ferrous shell alloy systems.

X. ACKNOWLEDGEMENTS

The author wishes to express his sincerest thanks and appreciation to Messrs. K. D. Holmes (AMMRC), H. E. Henderson (LFC) and the late T. W. Curry, (LFC) for their cooperation and assistance in the data collection, testing, and production of the test castings. In addition, he would like to thank Mr. C. Uritz of Medico Industries for the finish machining of the 48 rocket shells. Mr. Juris Krumins, Northeastern University Research Assistant, who conducted the computer analyses and assisted in the data reduction, and Mrs. Keiko Fujiwara Oh and Miss Margaret M. R. Sullivan, both of whom helped type this report.

XI. REFERENCES

1. Zotos, J., "Design, Production and Evaluation of Improved Cast-Iron Using Mathematical Models," AMRA Contract No. DA-19-066-AMC-00317(x), AMMRC CR 68-07/1, June 28, 1968.
2. Zotos, J., "Design, Production and Evaluation of Improved Ductile Cast Iron Alloys Using Computer Derived, Mathematical Models," AMMRC Contract No. DAAG-46-68-0071, AMMRC CR 70-4, January, 1970.
3. Zotos, J., "Design, Industrial Production and Evaluation of Improved Ductile Cast Iron Alloys Using Computer Derived, Mathematical Models," AMMRC Contract No. DAAG-46-70-C-0007, AMMRC CR 71-1, January, 1971.
4. DeLuca, E., and Zotos, J., "Considerations in the Kinetics of Phosphorus and Sulfur Removal from Basic-Electric Arc Furnace Steels," Watertown Arsenal Laboratories Report No. 310.9/1, May, 1960.
5. Zotos, J., Confidential Itek Reports No. 61/1, 2, 62/1, 2, 3, 4, December, 1961 - April, 1962.
6. Zotos, J., et. al., "Optimum Steel Castings Through Computer Design," Modern Castings, vol. 44, no. 2, August, 1963.
7. Zotos, J., et. al., "Development of Improved Mathematical Models for Prediction of Steel Castings' Mechanical Properties," A.F.S. Transactions, vol. 68, 1964.
8. Zotos, J., "Computers Open New Horizons in Casting Technology," Modern Castings, vol. 47, no. 1, January, 1965, pp. 769-776.
9. Cooper, P.A., Kent, G.F. and Zotos, J., "Development of Mathematical Models to Predict the Properties of High Strength Steels," AMRA Technical Report No. AMRA CR 64-02/1, September, 1964.
10. Bates, C.E., Wallace, J.F., "Effects and Neutralization of Trace Elements in Gray and Ductile Iron," A.F.S. Research Project Report, 1969.
11. Loper, C.R., Heine, R.W., "The Effect of Processing Variables on Ductile Iron Quality," A.F.S. Transactions, vol. 73, pp. 488-496, 1965.
12. Abar, J.W., Celitti, R.A., Halgren, J.A., "Effect of Silicon Content on the Heat Treatment and Mechanical Properties of Nodular Iron," A.F.S. Transactions, vol. 73, pp. 344-354, 1965.