

BRL CR III

BRL

AD

CONTRACT REPORT NO. 111

Research Report 72-7

PROPAGATION OF DISCONTINUITIES IN
HETEROGENEOUS ANISOTROPIC PLATES

Prepared by

Drexel University
Philadelphia, Pennsylvania

July 1973

Approved for public release; distribution unlimited.

USA BALLISTIC RESEARCH LABORATORIES
ABERDEEN PROVING GROUND, MARYLAND

Destroy this report when it is no longer needed.
Do not return it to the originator.

Secondary distribution of this report by originating
or sponsoring activity is prohibited.

Additional copies of this report may be obtained
from the National Technical Information Service,
U.S. Department of Commerce, Springfield, Virginia
22151.

The findings in this report are not to be construed as
an official Department of the Army position, unless
so designated by other authorized documents.

CONTRACT REPORT NO. 111

JULY 1973

PROPAGATION OF DISCONTINUITIES
IN HETEROGENEOUS ANISOTROPIC PLATES

by

A. S. D. Wang and D. L. Tuckmantel

Mechanics and Structures Advanced Study Group
Research Report 72-7

Drexel University
Philadelphia, Pennsylvania

Approved for public release; distribution unlimited..

This work was partially supported by the U.S. Army, Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland, under Contract No. DAAD05-70-C-0175.

PROPAGATION OF DISCONTINUITIES
IN HETEROGENEOUS ANISOTROPIC PLATES

By

A. S. D. Wang¹ and D. L. Tuckmantel²

ABSTRACT

Elastic stress waves propagating in thin, laminated composite plates are analyzed on the basis of a lamination theory. The theory is based on the Kirchhoff assumptions, but it includes the effects of shear deformation and rotary inertia, similar to Mindlin's theory for homogeneous isotropic plates. The individual layers comprising the plate are assumed to possess different thicknesses and material properties. In particular, each layer may be arbitrarily anisotropic. Thus, a general coupling in shear, bending, twisting and extensional effects is present in the plate constitutive relations. This coupling results in simultaneously coupled stress waves propagating in the plane of the plate. Several numerical examples involving laminated fiber-reinforced composite plates are presented.

¹Associate Professor of Applied Mechanics, College of Engineering, Drexel University, Philadelphia, Pennsylvania 19104.

²Research Associate, College of Engineering, Drexel University, Philadelphia, Pennsylvania 19104.

I. INTRODUCTION

The dynamics of heterogeneous anisotropic solids has been an important field of study in recent years. Increasing popularity in the use of modern composites as a structural material has necessitated intensive research into their material characterization. Their intrinsic properties are being investigated from both a phenomenological and microscopic point of view. This effort has resulted in the development of various lamination theories describing laminated anisotropic plate structures. The papers by Yang, Norris and Stavsky [1], and Whitney and Pagano [2] are among the most notable works. The former concerns plates made of layers possessing arbitrary anisotropy; the latter, with some modifications*, is valid for plates made of monoclinic layers, i.e., each layer of the plate possesses a mid-plane material symmetry. Both of these theories follow the basic approach set forth by Mindlin [4] for homogeneous isotropic plates, and include transverse shear deformation and the effect of rotary inertia. Recently, Chou and Carleone [5] improved the assumptions concerning the transverse shear deformation.

Such lamination theories, however, describe the macro-characteristics of the plate rather than the micro-characteristics that are effected by the inhomogeneous nature of the composite. Generally speaking, the macro-theory is accurate for plates subjected to static loading, but is deemed inaccurate when applied to stress wave problems. This is true in so far as the wave lengths are short compared with, say, the thickness of the material layers of the plate. But for low frequency waves with length longer than, say, the plate's thickness, the macro-theory may be regarded as a valid basis of analysis. Based on this assumption, Moon [6] recently investigated stress waves in a specially laminated fiber-reinforced composite plate, using the effective modulus theory which uncouples the transverse, bending and extensional displacements. A wave surface approach was used to describe the propagation of plane acceleration waves. This method yields the wave velocity surfaces in the plane of the plate.

*A comparison of the two theories is contained in Ref. [3].

The present paper is concerned with stress waves in laminates that are composed of layers possessing arbitrary anisotropy. General shear, bending, twisting and extensional coupling is present in the plate constitutive relations, resulting in simultaneously coupled wave surfaces in the plane of the plate. We follow general laminated plate theory and apply a control volume approach for the analysis [7]. Explicit solutions for the coupled wave surfaces and their velocities are obtained. Several numerical problems involving laminated fiber-reinforced composite plates are presented and their unique features discussed.

II. ANALYSIS

Let us consider a thin laminated plate of thickness h , Figure 1. The laminae comprising the plate are assumed to be individually homogeneous and anisotropic. Thus the in-homogeneity of the plate occurs only in the thickness direction. The constitutive relations for any one of the laminae are given by

$$\sigma_i = C_{ij} e_j \quad i, j = 1, 2, 3, 4, 5, 6. \quad (1)$$

where the C_{ij} 's are the elements of the stiffness matrix, the stresses, σ_i , are defined as $\sigma_1 = \sigma_x$, $\sigma_2 = \sigma_y$, $\sigma_3 = \sigma_z$, $\sigma_4 = \sigma_{yz}$, $\sigma_5 = \sigma_{zx}$, $\sigma_6 = \sigma_{xy}$, and the strains, e_j , are defined in the same manner as the stresses.

Following the theory developed by Yang, Norris and Stavsky [1], we assume the displacement field,

$$\begin{aligned} u &= u^0(x, y, t) + z \psi_x(x, y, t) \\ v &= v^0(x, y, t) + z \psi_y(x, y, t) \\ w &= w^0(x, y, t) \end{aligned} \quad (2)$$

where the coordinate system (x, y, z) is shown in Figure 1 and u , v and w are the displacements in the x , y and z directions, respectively, u^0 , v^0 and w^0 are the displacement components at $z = 0$, in the x , y and z directions, respectively and ψ_x and ψ_y are rotations about the y

and x axis, respectively.

The stress and moment resultants are related to the displacements by (c.f. Equation 14, Ref. [1]).

$$\begin{bmatrix} N_x \\ N_y \\ Q_y \\ Q_x \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{14} & A_{15} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{24} & A_{25} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{14} & A_{24} & A_{44} & A_{45} & A_{46} & B_{14} & B_{24} & B_{46} \\ A_{15} & A_{25} & A_{45} & A_{55} & A_{56} & B_{15} & B_{25} & B_{56} \\ A_{16} & A_{26} & A_{46} & A_{56} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{14} & B_{15} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{24} & B_{25} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{46} & B_{56} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} u_{,x}^0 \\ v_{,y}^0 \\ w_{,y}^0 + \psi_y \\ w_{,x}^0 + \psi_x \\ u_{,y}^0 + v_{,x}^0 \\ \psi_{x,x} \\ \psi_{y,y} \\ \psi_{x,y} + \psi_{y,x} \end{bmatrix} \quad (3)$$

where

$$(N_x, N_y, Q_y, Q_x, N_{xy}) = \int_{-h/2}^{+h/2} (\sigma_1, \sigma_2, \sigma_4, \sigma_5, \sigma_6) dz \quad (4)$$

$$(M_x, M_y, M_{xy}) = \int_{-h/2}^{+h/2} (\sigma_1, \sigma_2, \sigma_6) z dz$$

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{+h/2} C_{ij} (1, z, z^2) dz, \quad i, j = 1, 2, 4, 5, 6 \quad (5)$$

and the notation $\psi_{x,y}$, e.g., represents partial differentiation of ψ_x with respect to y .

We now consider a wave which originates at an arbitrary point in the plate, for convenience let us say at the origin of the (x, y, z) system, and propagates in the x, y plane. At any given instant, the wave surface is denoted by S , as shown in Figure 1. Let $\tilde{n}$ be the normal of S at a point, A , on S , and let $\tilde{s}$ be the tangent of S at

the same point. The wave surface S is assumed to propagate in the direction $\bar{n}$ with a constant speed c.

Let $\bar{u}, \bar{v}, \bar{w}, \bar{u}^0, \bar{v}^0, \bar{w}^0, \psi_n$ and ψ_s be the displacements (see Equations (2)) referred to the local coordinates (n,s,z). The kinematic (continuity) conditions across S at any given point on S require (see [7]),

$$[\bar{u}^0] = [\bar{v}^0] = [\bar{w}^0] = [\psi_n] = [\psi_s] = 0$$

$$[\bar{u}_n^0, \bar{v}_n^0, \bar{w}_n^0, \psi_{n,n}, \psi_{s,n}] = \frac{1}{c} [\bar{u}_t^0, \bar{v}_t^0, \bar{w}_t^0, \psi_{n,t}, \psi_{s,t}] \quad (6)$$

$$\text{and } (\bar{u}, \bar{v}, \bar{w}, \psi_n, \psi_s)_{,s} = 0$$

where [] represent a discontinuity of the enclosed quantity across S.

With these conditions, the plate constitutive relations, Equations (3), when transformed to the local coordinates (n,s,z) yield**

$$\begin{bmatrix} N_n \\ N_{ns} \\ M_n \\ M_{ns} \\ Q_n \end{bmatrix} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{16} & \bar{B}_{11} & \bar{B}_{16} & \bar{A}_{15} \\ \bar{A}_{16} & \bar{A}_{66} & \bar{B}_{16} & \bar{B}_{66} & \bar{A}_{56} \\ \bar{B}_{11} & \bar{B}_{16} & \bar{D}_{11} & \bar{D}_{16} & \bar{B}_{15} \\ \bar{B}_{16} & \bar{B}_{66} & \bar{D}_{16} & \bar{D}_{66} & \bar{B}_{56} \\ \bar{A}_{15} & \bar{A}_{56} & \bar{B}_{15} & \bar{B}_{56} & \bar{A}_{55} \end{bmatrix} \begin{bmatrix} \bar{u}_n^0 \\ \bar{v}_n^0 \\ \psi_{n,n} \\ \psi_{s,n} \\ \bar{w}_n^0 \end{bmatrix} \quad (7)$$

To establish the dynamic relations across the wave surface, let us define a control volume which is located on S at point A,

*Quantities with a bar on top are referred to the (n,s,z) coordinate system.

**For the transformation of A_{ij} , B_{ij} and D_{ij} from the (x,y,z) system to the (n,s,z) system, refer ij to ij_{Ref} . ij [8].

as shown in Figure 1. Since the control volume moves with the wave front, an observer fixed with it sees a normal influx of mass entering with a speed U_1 , and a normal efflux of mass leaving with a speed U_2 ,

$$U_1 = c - \bar{u}_{1,t} \quad (8)$$

$$U_2 = c - \bar{u}_{2,t}$$

where the subscripts 1 and 2 refer to the properties in front of and behind the wave front, respectively. Thus the steady-state conservation of mass for the control volume yields,

$$\int_{-h/2}^{+h/2} (\rho_2 U_2 - \rho_1 U_1) dz = 0 \quad (9)$$

where ρ is the mass density of the material, U is the particle velocity relative to the wave front and the subscripts 1 and 2 refer to properties in front of and behind the wave, respectively.

It is noted that condition (9) is satisfied by a more restrictive condition, resulting from the classical thin plate assumptions, namely;

$$\rho_2 U_2 = \rho_1 U_1 \quad (10)$$

The force and moment resultants acting upon the control volume must satisfy the equations of balanced momenta. These are,

$$[N_n] = \int_{-h/2}^{+h/2} \rho_1 U_1 (U_2 - U_1) dz = P c^2 [\bar{u}_n^0] + R c^2 [\psi_{n,n}]$$

$$[N_{ns}] = \int_{-h/2}^{+h/2} \rho_1 U_1 (\bar{v}_2 - \bar{v}_1) dz = P c^2 [\bar{v}_n^0] + R c^2 [\psi_{s,n}]$$

$$[M_n] = \int_{-h/2}^{+h/2} \rho_1 U_1 (U_2 - U_1) z dz = R c^2 [\bar{u}_n^0] + I c^2 [\psi_{n,n}] \quad (11)$$

$$[M_{ns}] = \int_{-h/2}^{+h/2} \rho_1 U_1 (\bar{v}_2 - \bar{v}_1) z dz = R c^2 [\bar{v}_n^0] + I c^2 [\psi_{s,n}]$$

$$[Q_n] = \int_{-h/2}^{+h/2} \rho_1 U_1 (\bar{w}_2^0 - \bar{w}_1^0) dz = P c^2 [\bar{w}_n^0]$$

In obtaining the above relations, we have retained only the linear terms of the displacements, consistent with the plate theory. In addition, we have assumed density to be a function of z alone in presenting the following quantities

$$(P, R, I) = \int_{-h/2}^{+h/2} \rho_1(1, z, z^2) dz \approx \int_{-h/2}^{+h/2} \rho_0(1, z, z^2) dz \quad (12)$$

where ρ_0 is the undisturbed density of the laminae.

Using the relations (7), (8) and (10), we obtain from Equations (11) a system of five linear algebraic equations relating the discontinuities in the normal derivatives of $\bar{u}^0$, $\bar{v}^0$, $\bar{w}^0$, ψ_n and ψ_s

$$\begin{bmatrix} \bar{A}_{11} - Pc^2 & \bar{A}_{16} & \bar{B}_{11} - Rc^2 & \bar{B}_{16} & \bar{A}_{15} \\ \bar{A}_{16} & \bar{A}_{66} - Pc^2 & \bar{B}_{16} & \bar{B}_{66} - Rc^2 & \bar{A}_{56} \\ \bar{B}_{11} - Rc^2 & \bar{B}_{16} & \bar{D}_{11} - Ic^2 & \bar{D}_{16} & \bar{B}_{15} \\ \bar{B}_{16} & \bar{B}_{66} - Rc^2 & \bar{D}_{16} & \bar{D}_{66} - Ic^2 & \bar{B}_{56} \\ \bar{A}_{15} & \bar{A}_{56} & \bar{B}_{15} & \bar{B}_{56} & \bar{A}_{55} - Pc^2 \end{bmatrix} \begin{bmatrix} [\bar{u}_{,n}^0] \\ [\bar{v}_{,n}^0] \\ [\psi_{n,n}] \\ [\psi_{s,n}] \\ [\bar{w}_{,n}^0] \end{bmatrix} = 0 \quad (13)$$

In order for a non-trivial solution to exist, the determinant of the coefficient matrix of Equations (13) must vanish. Thus, five possible wave front speeds, c , may be determined for any given direction $\bar{n}$.

Since the vanishing determinant represents a fifth order equation in c^2 , numerical, rather than analytical, techniques must be used to obtain a solution. However, it is of interest to note that, if the laminated plate has, for each lamina, a monoclinic symmetry (i.e. a plane symmetry with respect to the mid-plane of the lamina) the constants $\bar{A}_{15} = \bar{A}_{56} = \bar{B}_{15} = \bar{B}_{56} = 0$ for all directions in the x, y -plane. In such a case, $[\bar{w}_{,n}^0]$ is uncoupled from the system. Consequently,

$$c_5^2 = \bar{A}_{55}/P \quad (14)$$

where c_5 represents the propagation speed of the discontinuity $[\tilde{w}_n^0]$ in the direction $\tilde{n}$.

Furthermore, if, in addition, the plate has a symmetry with respect to the x,y -plane, the bending-extensional couplings $\tilde{B}_{ij}$ become zero for all i and j . Then, Equations (13) separate further yielding two quadratic equations whose roots are

$$c_{1,2}^2 = \frac{(\bar{A}_{11} + \bar{A}_{66}) \pm \sqrt{(\bar{A}_{11} - \bar{A}_{66})^2 + 4\bar{A}_{16}^2}}{2P} \quad (15)$$

and

$$c_{3,4}^2 = \frac{(\bar{D}_{11} + \bar{D}_{66}) \pm \sqrt{(\bar{D}_{11} - \bar{D}_{66})^2 + 4\bar{D}_{16}^2}}{2I} \quad (16)$$

In the particular case, such as that considered by Moon [6], the extensional and the bending rigidities are proportional, i.e.,

$$\bar{A}_{ij}/\bar{D}_{ij} = P/I = \text{constant}, \quad i,j = 1,2,6$$

Equations (14) and (15) are identical causing $c_{1,2}$ and $c_{3,4}$ to coincide (see Equations (17) and (23), Ref. [6]).

III. NUMERICAL ILLUSTRATIONS

For the numerical illustrations, plates which are laminated with unidirectional fiber-reinforced composite layers are considered. The material properties of these layers are described by the following engineering constants*:

$$E_L = 25 \times 10^6 \text{ psi}, \quad E_T = 10^6 \text{ psi}, \quad G_{LT} = 0.5 \times 10^6 \text{ psi}$$

$$\nu_{LT} = 0.25, \quad \nu_{TT} = 0.35, \quad \rho_0 = 0.073 \text{ pci}$$

*These values are typical of high modulus graphite-epoxy composites. Such material layers may be considered as being square symmetric. The computation for the plates' rigidities and their transformation to local coordinates was carried out following the outlines in Refs. [2] and [8].

where E is Young's modulus, G is the shear modulus, ν is Poisson's ratio, and the subscripts L and T indicate directions parallel and normal to the fibers, respectively.

We have considered four plates each of which is made of four layers having different lay-up angles and/or lamination sequences, namely: a) $(0^\circ/90^\circ/0^\circ/90^\circ)$, b) $(0^\circ/90^\circ/90^\circ/0^\circ)$, c) $(+30^\circ/-30^\circ/+30^\circ/-30^\circ)$ and d) $(+30^\circ/-30^\circ/-30^\circ/+30^\circ)$, where the angles are positive when measured counterclockwise from the x -axis to the fiber direction.

The roots of the vanishing determinant of Equations (13) are determined by the Newton Raphson technique. In general, five distinct roots representing five possible wave speeds exist in any given direction. However, in certain preferred directions, the material properties are such that repeated roots may exist.

Figures 2 - 5 show the wave velocity surfaces in the first quadrant of the x,y -plane, for $(0^\circ/90^\circ/0^\circ/90^\circ)$, $(0^\circ/90^\circ/90^\circ/0^\circ)$, $(30^\circ/-30^\circ/30^\circ/-30^\circ)$ and $(30^\circ/-30^\circ/-30^\circ/30^\circ)$ laminates, respectively. The velocity is non-dimensionalized by a factor of $(E_T/\rho_0)^{1/2}$. It is pointed out that the slowest velocity surface, which is associated with the transverse displacement $\bar{w}$, is uncoupled from the other four surfaces, since the material is monoclinic. However, all the other four velocity surfaces may be severely coupled. On each of the corresponding wave surfaces, discontinuities in normal forces, shear forces and bending and twisting moments exist simultaneously. Their relative magnitudes may be determined from Equations (13) once the wave speeds, c , have been determined.

Multiple coupled one-dimensional stress waves in a heterogeneous plate were first treated by Wang, Chou and Rose [9] using the method of characteristics. Subsequent experimental investigations in the same problem has not, as yet, confirmed the multiple wave nature associated with laminated plates. Recent experiments conducted at Drexel University using a low frequency ultrasonic transducer to produce normal-to-the-plate pulses showed only two distinct wave groups traveling in the plane of the plate. At present, a definitive conclusion cannot be drawn due to the limited data available.

REFERENCES

1. Yang, P. C., Norris, C. H. and Stavsky, Y., "Elastic Wave Propagation in Heterogeneous Plates," Int. Journal of Solids and Structures, Vol. 2, 1966, pp. 665-684.
2. Whitney, J. M. and Pagano, N. J., "Shear Deformation in Heterogeneous Anisotropic Plates," Journal of Applied Mechanics, Vol. 37, 1970, pp. 1031-1036.
3. Wang, A. S. D. and Chou, P. C., "A Comparison of Two Laminated Plates Theories," Journal of Applied Mechanics, Vol. 39, 1972, pp. 611-613.
4. Mindlin, R. D., "Influence of Rotatory Inertia and Shear on Flexural Motions of Isotropic Elastic Plates," Journal of Applied Mechanics, Vol. 18, 1951, pp. 31.
5. Chou, P. C. and Carleone, J., "On Transverse Shear Effects in Laminated Plates," Mechanics & Structures Advanced Study Group Research Report No. 73-4, Drexel University, Philadelphia, Pa., 1973.
6. Moon, F. C., "Wave Surfaces Due to Impact on Anisotropic Plates," Journal of Composite Materials, Vol. 6, 1972, pp. 62-79.
7. Chou, P. C. and Wang, A. S. D., "Control Volume Analysis of Elastic Wave Front in Composite Materials," Journal of Composite Materials, Vol. 4, 1970, pp. 444-461.
8. Tsai, S. W., "Mechanics of Composite Materials," Parts I and II, AFML-TR-66-149, 1966.
9. Wang, A. S. D., Chou, P. C. and Rose, J. L., "Strongly Coupled Stress Waves in Heterogeneous Plates," AIAA Journal, Vol. 10, No. 2, 1972, pp. 1088-1090.

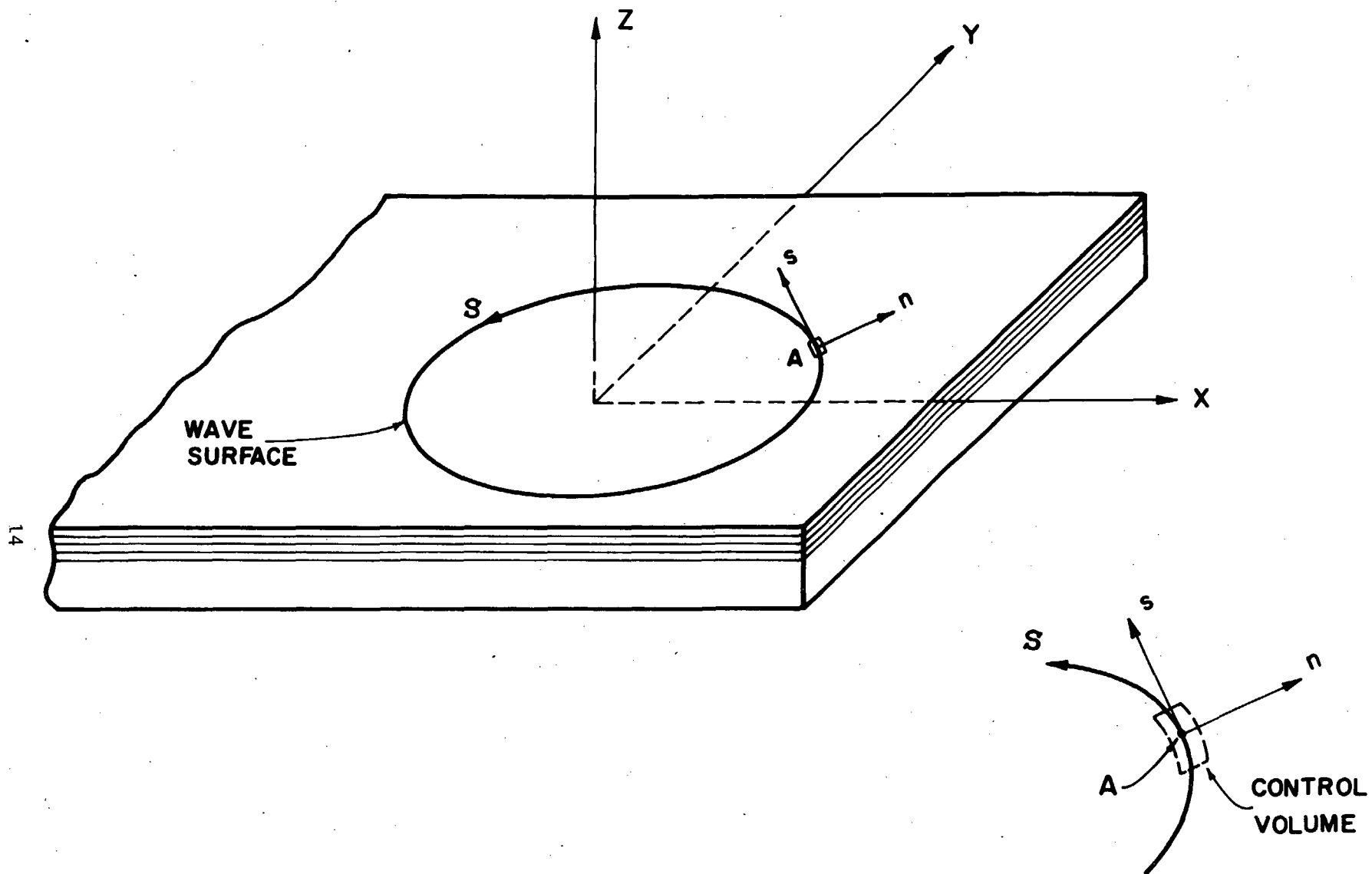


Figure 1. Geometry of the plate and the control volume.

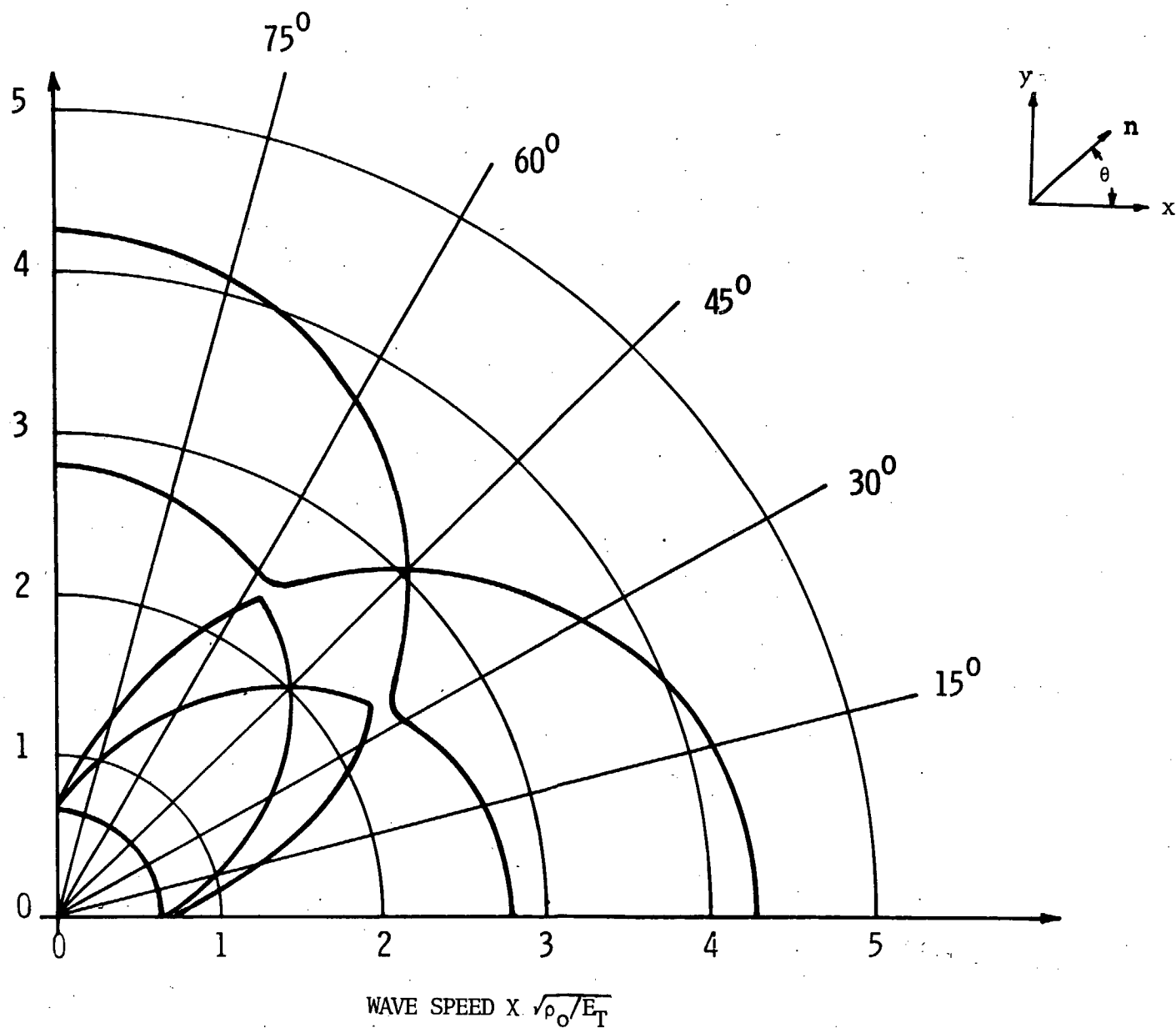


Figure 2. Wave velocity surfaces for the $(0^\circ/90^\circ/0^\circ/90^\circ)$ plate.

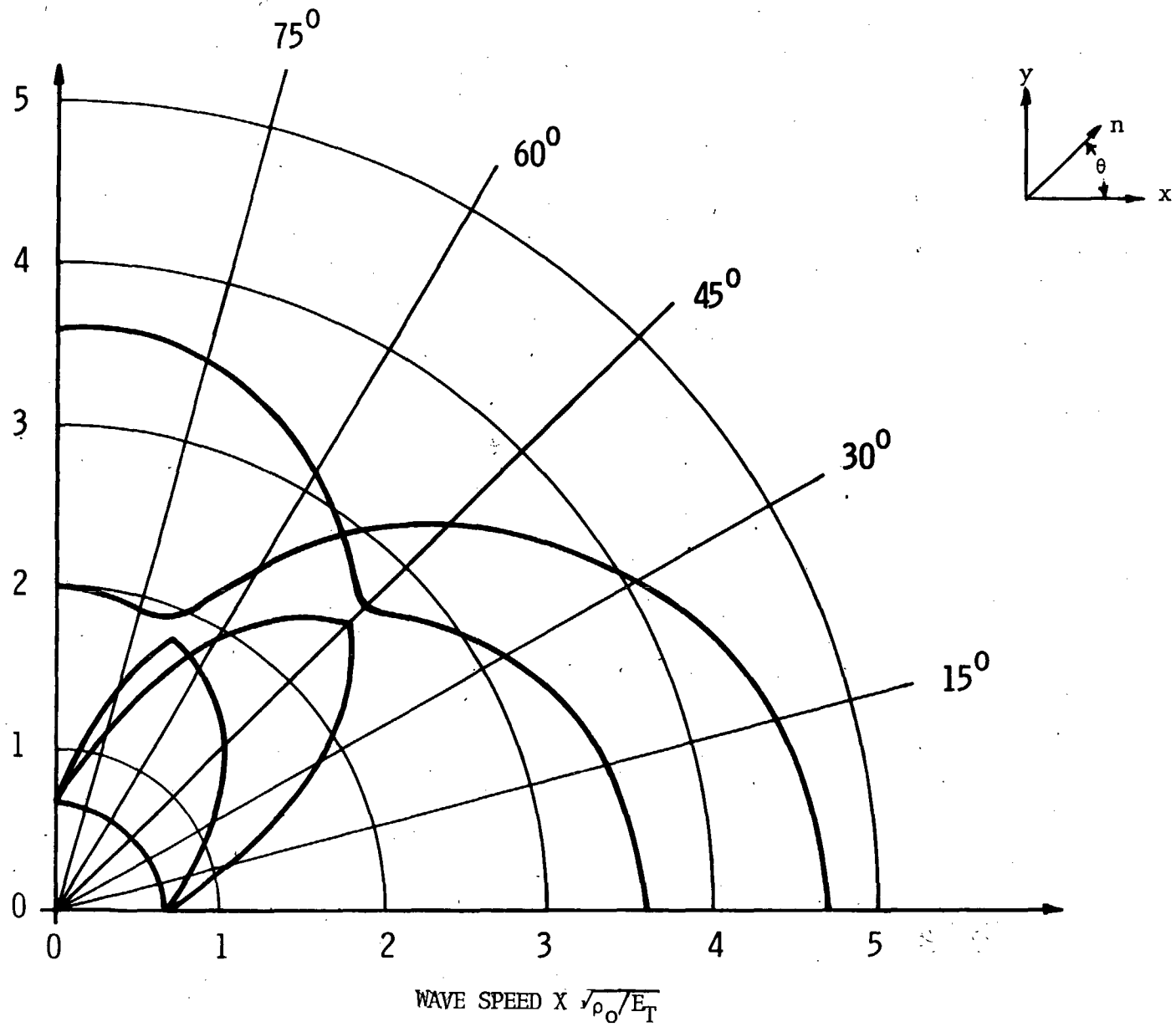


Figure 3. Wave velocity surfaces for the $(0^\circ/90^\circ/90^\circ/0^\circ)$ plate.

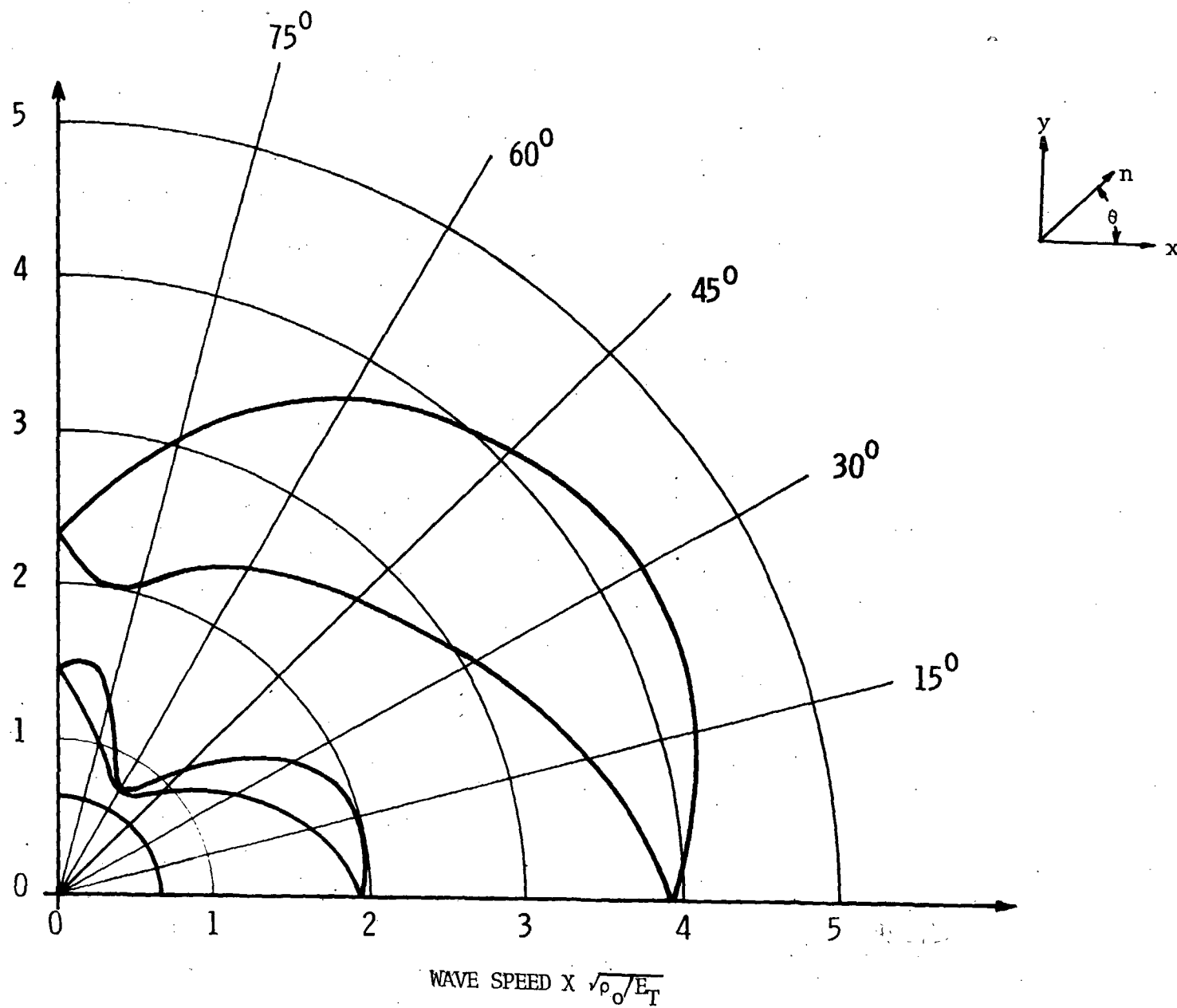


Figure 4. Wave velocity surfaces for the $(30^\circ/-30^\circ/30^\circ/-30^\circ)$ plate.

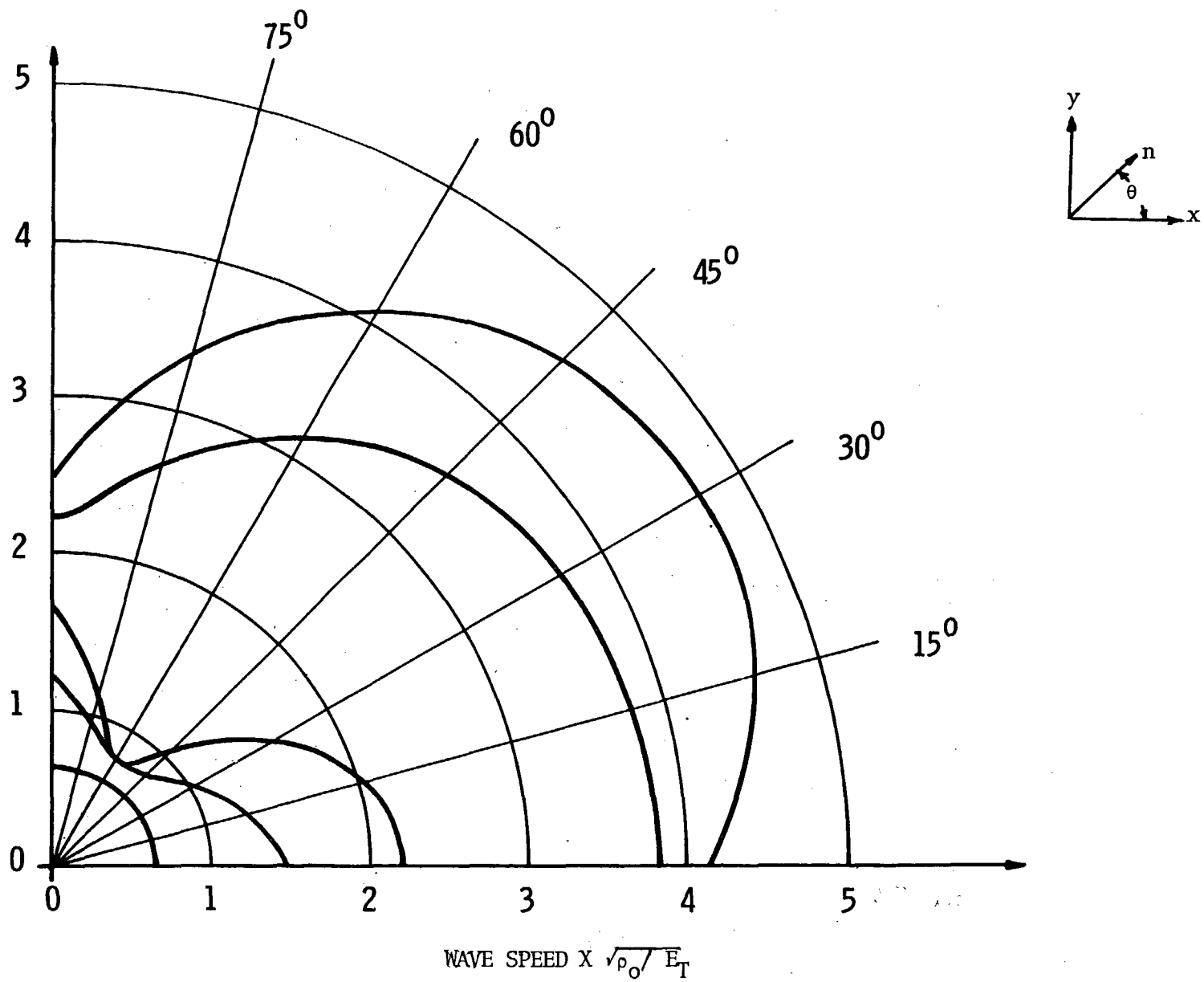


Figure 5, Wave velocity surfaces for the $(+30^\circ/-30^\circ/-30^\circ/30^\circ)$ plate.

DISTRIBUTION LIST

<u>No. of</u> <u>Copies</u>	<u>Organization</u>	<u>No. of</u> <u>Copies</u>	<u>Organization</u>
12	Commander Defense Documentation Center ATTN: TIPCR Cameron Station Alexandria, Virginia 22314	1	Commander U.S. Army Materiel Command ATTN: AMCRD, Dr.J.V.R.Kaufman 5001 Eisenhower Avenue Alexandria, Virginia 22304
1	Director Defense Advanced Research Projects Agency 1400 Wilson Boulevard Arlington, Virginia 22209	1	Commander U.S. Army Materiel Command ATTN: AMCRD-TE 5001 Eisenhower Avenue Alexandria, Virginia 22304
1	Director Weapons Systems Evaluation Group Washington, DC 20305	1	Commander U.S. Army Materiel Command ATTN: AMCRD-TP 5001 Eisenhower Avenue Alexandria, Virginia 22304
1	Director Institute for Defense Analysis 400 Army-Navy Drive Arlington, Virginia 22202	1	Commander U.S. Army Aviation Systems Command ATTN: AMSAV-E 12th and Spruce Streets St. Louis, Missouri 63166
1	Director Defense Nuclear Agency ATTN: STSP Washington, DC 20305	1	Director U.S. Army Air Mobility Research and Development Laboratory Ames Research Center Moffett Field, California 94035
1	Commander U.S. Army Materiel Command ATTN: AMCDL 5001 Eisenhower Avenue Alexandria, Virginia 22304	1	Commander U.S. Army Electronics Command ATTN: AMSEL-RD Fort Monmouth, New Jersey 07703
1	Commander U.S. Army Materiel Command ATTN: AMCDMA, MG J.R. Guthrie 5001 Eisenhower Avenue Alexandria, Virginia 22304	4	Commander U.S. Army Missile Command ATTN: AMSMI-R AMSMI-RSD AMCPM-PE AMCPM-LC Redstone Arsenal, Alabama 35809
1	Commander U.S. Army Materiel Command ATTN: AMCRD, MG S. C. Meyer 5001 Eisenhower Avenue Alexandria, Virginia 22304		

DISTRIBUTION LIST

<u>No. of Copies</u>	<u>Organization</u>	<u>No. of Copies</u>	<u>Organization</u>
1	Commander U.S. Army Tank Automotive Command ATTN: AMSTA-RHFL Warren, Michigan 48090	1	Commander U.S. Army Materials and Mechanics Research Center ATTN: AMXMR-ATL Watertown, Massachusetts 02172
3	Commander U.S. Army Mobility Equipment Research & Development Center ATTN: Tech Docu Cen, Bldg. 315 AMSME-RZT AMEFB-MW Fort Belvoir, Virginia 22060	1	Commander U.S. Army Natick Laboratories ATTN: AMXRE, Dr. D. Sieling Natick, Massachusetts 01762
1	Commander U.S. Army Armament Command Rock Island, Illinois 61202	3	HQDA (DARD-MS, Dr. W. Taylor; DARD-ARP; DARD-MD, Dr. P. Friel) Washington, DC 20310
1	Commander U.S. Army Frankford Arsenal ATTN: SMUFA-C2500 Philadelphia, Pennsylvania 19137	3	Commander U.S. Naval Air Systems Command ATTN: AIR-604 Washington, DC 20360
2	Commander U.S. Army Picatinny Arsenal ATTN: SMUPA-V SMUPA-VK Dover, New Jersey 07801	3	Commander U.S. Naval Ordnance Systems Command ATTN: ORD-9132 Washington, DC 20360
1	Director U.S. Army Advanced Materiel Concepts Agency 2461 Eisenhower Avenue Alexandria, Virginia 22314	1	Commander U.S. Naval Weapons Center China Lake, California 93555
1	Commander U.S. Army Harry Diamond Laboratories ATTN: AMXDO-TI Washington, DC 20438	1	Director U.S. Naval Research Laboratory ATTN: Code 6240, Mr. Atkins Washington, DC 20390
		1	AFATL (DLR) Eglin AFB Florida 32542
		1	AFATL (DLRD) Eglin AFB Florida 32542

DISTRIBUTION LIST

<u>No. of</u> <u>Copies</u>	<u>Organization</u>	<u>No. of</u> <u>Copies</u>	<u>Organization</u>
1	AFATL (DLRV) Eglin AFB Florida 32542	1	AVCO Corporation Research & Advance Development 201 Lowell Street Wilmington, Massachusetts 01887
1	AFWL Wright-Patterson AFB Ohio 45433	1	Batelle Memorial Institute Defense Metals Info Ctr Columbus Laboratories 505 King Avenue Columbus, Ohio 43201
1	Director U.S. Bureau of Mines ATTN: Mr. R. Watson 4800 Forbes Street Pittsburg, Pennsylvania 15123	1	General Motors Corporation Manufacturing Development ATTN: Mr. W. Isbell Warren, Michigan 48090
1	Director Environmental Science Services Administration ATTN: Code R, Dr. J. Rinehart U.S. Department of Commerce Boulder, Colorado 80302	1	Gruman Aircraft Corporation Bethpage, New York 11714
1	Director Lawrence Livermore Laboratory ATTN: Mr. M. Wilkins P. O. Box 808 Livermore, California 94550	1	Hughes Aircraft Company Research & Development Laboratory Centinela & Teale Street Culver City, California 90232
1	Director Los Alamos Scientific Lab P. O. Box 1663 Los Alamos, New Mexico 87544	1	Kaman Nuclear Division Garden of the Gods Road Colorado Springs, Colorado 80907
1	Director National Aeronautics and Space Administration Langley Research Center Langley Station Hampton, Virginia 23365	1	Philco-Ford Corporation Aeroanautical Division ATTN: Dr. M. Boyer Ford Road Newport Beach, California 92663
1	Aerospace Corporation P. O. Box 95085 Los Angeles, California 90045	1	Physics International Corp ATTN: Dr. C. Godfrey 2700 Merced Street San Leandro, California 94577
		1	Sandia Corporation P. O. Box 5800 Albuquerque, New Mexico 87115

DISTRIBUTION LIST

<u>No. of</u> <u>Copies</u>	<u>Organization</u>	<u>No. of</u> <u>Copies</u>	<u>Organization</u>
3	Systems, Science & Software ATTN: Dr. J. Walsh Dr. J. Dienes Dr. T. Riney P. O. Box 1620 La Jolla, California 92037	1	Southwest Research Institute ATTN: Dr. W. Baker 8500 Culebra Road San Antonio, Texas 78228
1	Brown University Division of Engineering ATTN: Prof. P. Symonds Providence, Rhode Island 02912	1	Stanford Research Institute Poulter Laboratories ATTN: Dr. M. Cowperthwaite 333 Ravenswood Avenue Menlo Park, California 94025
1	Drexel Institute of Technology Wave Propagation Research Ctr ATTN: Prof. P. Chou Philadelphia, Pennsylvania 19104	1	Stanford University Department of Mechanical Engineering ATTN: Prof. E. H. Lee Stanford, California 94305
2	The Johns Hopkins University Mechanics Department ATTN: Prof. J. Bell Prof. R. Green 34th & Charles Street Baltimore, Maryland 21218	1	University of Dayton Research Institute ATTN: Mr. H. F. Swift Dayton, Ohio 45409
1	Lincoln Laboratory 244 Wood Street Lexington, Massachusetts 02173	1	University of Illinois Department of Aeronautical Engineering ATTN: Prof. R. Strehlow Urbana, Illinois 61803
1	Massachusetts Institute of Tech Aerolastic & Structures Lab 77 Massachusetts Avenue Cambridge, Massachusetts 92139	1	University of Notre Dame Department of Met. Engineering and Mat. Science ATTN: Prof. N. Fiore Notre Dame, Indiana 46556
1	Pennsylvania State University Department of Engineering Mechanics ATTN: Prof. N. Davids University Park, Pennsylvania 16802	1	University of Texas Department of Engineering Mechanics ATTN: Prof. H. Calvit Austin, Texas 78712

DISTRIBUTION LIST

<u>No. of Copies</u>	<u>Organization</u>
1	Washington State University Department of Physics ATTN: Prof. G. Duvall Pullman, Washington 99163

Aberdeen Proving Ground

Ch, Tech Lib
Marine Corp Ln Ofc
Cmdr, USAEA
ATTN: SMUEA-AG
Dir, USAMSAA

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)

Drexel University
Mechanics and Structures Advanced Study Group
Philadelphia, PA 19104

2a. REPORT SECURITY CLASSIFICATION

UNCLASSIFIED

2b. GROUP

3. REPORT TITLE

PROPAGATION OF DISCONTINUITIES IN HETEROGENEOUS ANISOTROPIC PLATES

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Technical Report

5. AUTHOR(S) (First name, middle initial, last name)

A.S.D. Wang
D. L. Tuckmantel

6. REPORT DATE

JULY 1973

7a. TOTAL NO. OF PAGES

23

7b. NO. OF REFS

9

8a. CONTRACT OR GRANT NO.

DAAD05-70-C-0175

b. PROJECT NO.

c.

d.

9a. ORIGINATOR'S REPORT NUMBER(S)

BRL CONTRACT REPORT NO. 111

9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

Research Report 72-7

10. DISTRIBUTION STATEMENT

Distribution of this document is unlimited.

11. SUPPLEMENTARY NOTES

12. SPONSORING MILITARY ACTIVITY

USA Ballistic Research Laboratories
Aberdeen Proving Ground, MD 21005

13. ABSTRACT

Elastic stress waves propagating in thin, laminated composite plates are analyzed on the basis of a lamination theory. The theory is based on the Kirchhoff assumptions, but it includes the effects of shear deformation and rotary inertia, similar to Mindlin's theory for homogeneous isotropic plates. The individual layers comprising the plate are assumed to possess different thicknesses and material properties. In particular, each layer may be arbitrarily anisotropic. Thus, a general coupling in shear, bending, twisting and extensional effects is present in the plate constitutive relations. This coupling results in simultaneously coupled stress waves propagating in the plane of the plate. Several numerical examples involving laminated fiber-reinforced composite plates are presented.

DD FORM 1473

1 NOV 66

REPLACES DD FORM 1473, 1 JAN 64, WHICH IS OBSOLETE FOR ARMY USE.

UNCLASSIFIED

Security Classification

UNCLASSIFIED

Security Classification

14.

KEY WORDS

LINK A

LINK B

LINK C

ROLE

WT

ROLE

WT

ROLE

WT

Composite materials
Wave propagation
Graphite-epoxy composites
Heterogeneous laminates
Coupled stress waves

UNCLASSIFIED

Security Classification