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FLUERICS 35. JET DYNAMICS AND ITS APPLICATION TO THE BEAM-DEFLECTION AMPLIFIER

Joseph M. Kirshner

Harry Diamond Laboratories
Washington, D.C.

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ABSTRACT

This report shows how the response of a jet to a pressure gradient is used to derive the impedance of the jet.

The results are used to obtain the transfer function, for the variation of the output pressure with variation of input pressure, for a beam-deflection amplifier, including the effects of the vent and of output loading.

The input impedance is shown in general, to depend upon the vent and output impedances.

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1. INTRODUCTION

The usual treatment of the proportional amplifier makes the quasistatic assumption that the instantaneous dynamic jet deflection has the same magnitude as the steady-state deflection of the jet under the same pressure. The transport time is then incorporated into the solution as a pure time delay. Where the impedance of the jet has been considered[1,2,3], it is assumed to be a capacitance and is determined in terms of the change in the volumes on either side of the jet, resulting from the jet deflection in a pressure field.

Under the assumption that the jet retains its shape during the deflection process, we derive in this report an expression for the jet impedance that is valid for all frequencies; for which this assumption is approximately correct. This result is then applied to an amplifier configuration to obtain an analytic expression relating the output pressure difference to the input pressure difference. This expression includes the effects of the vent, cavity, and jet impedances. The transport delay need not be included separately but appears naturally.

2. THE TRANSFER FUNCTION FOR A JET

The response of a jet to a pressure gradient (equation 7) has been obtained by Kirshner.[4] For the sake of clarity, the gist of the discussion is repeated in the following paragraphs.

Consider an incompressible two-dimensional jet having a velocity distribution u(x, y) in the axial direction x and a velocity distribution v(x, y) in the direction y; x is the axial distance from the power jet nozzle and y is the perpendicular distance from the jet axis.

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[1] A. J. Healey, "Vent Effects on the Response of a Proportional Fluid Amplifier," ASME Paper No. 67-WAFE 12, 1967.
[2] F.M. Manion, "Dynamic Analysis of Flueric Proportional Amplifiers," ASME Paper No. 68-FE-49, 1968.
[3] F. T. Brown and R. A. Humphrey, "Dynamics of a Proportional Amplifier," ASME Paper No. 69-WA/Flcs-2, and 69-WA/Flcs-3, 1969.
[4] J.M. Kirschner, "Response of a Jet to a Pressure Gradient and Its Relation to Edgetones, "The Second International JSME Symposium on Fluid Machinery and Fluidics, Tokyo, Sept 1972.

A transverse pressure gradient, $dp/dy = -g(x,t)$ is initiated at $t = 0$, exerting a force on the particles. This force will give rise to a transverse acceleration of the particles given by

$$g(x,t) = \rho \frac{d^2 y_o}{dt^2} \tag{1}$$

where $y_o = y_o(x)$ is the transverse displacement of a particle as a result of the pressure gradient and ρ is the density.

Rather than follow the motion of a particular particle, we will choose instead to consider the motion of particles passing through a given point. We therefore write

$$\frac{dy_o}{dt} = \frac{\partial y_o}{\partial t} + u(x,y) \frac{\partial y_o}{\partial x} \tag{2}$$

where we have assumed that

$$v(x,y) \frac{\partial y_o}{\partial y} << u(x,y) \frac{\partial y_o}{\partial x}$$

We assume that the jet deflects as a whole and that we can therefore restrict the discussion to the motion of the axis of the jet

$$u(x,o) = u(x)$$

so that equation (2) becomes

$$\frac{dy_o}{dt} = \frac{\partial y_o}{\partial t} + u(x) \frac{\partial y_o}{\partial t} \tag{3}$$

Letting

$$\tau = \int_o^x \frac{dx'}{u(x')} \tag{4}$$

equation (3) becomes

$$\frac{dy_o}{dt} = \frac{\partial y_o}{\partial t} + \frac{\partial y_o}{\partial \tau} \tag{5}$$

Inserting this into equation (1)

$$\frac{\partial^2 y_o}{\partial t^2} + 2\frac{\partial^2 y_o}{\partial t \partial t} + \frac{\partial 2_{y_o}}{\partial \tau 2} = \frac{g[x(\tau),\ t]}{\rho} \tag{6}$$

Taking the Laplace transform we find after some manipulation

$$Y(\tau, s) = \frac{1}{\rho} \int_0^{\tau} \tau' e^{-s\tau'} \ G[x - x'(\tau'), s] d\tau' \tag{7}$$

where

$$Y(\tau, s) = \mathcal{L}\{y_o\ (\tau, t)\} \tag{8a}$$

$$G(x, s) = \mathcal{L}\{g\ (x, t)\} \tag{8b}$$

$$g(x, t) = -\frac{dp}{dy}$$

and it is assumed that the jet starts from rest at $t = 0$.

Expression (7) is convenient in determining the jet impedance.

2.1 Transverse Impedance of a Jet

The impedance of the jet in the vicinity of the controls (fig. 1) is derived by finding the change in transverse volume flow δq_1 for a given pressure difference change $\delta(p_2 - p_1)$ across the jet. The impedance is then given by

$$Z_1 = \lim_{s \to j\omega} \frac{\mathcal{L}\{\delta(p_2 - p_1)\}}{\mathcal{L}\{\delta q_1\}} \tag{9}$$

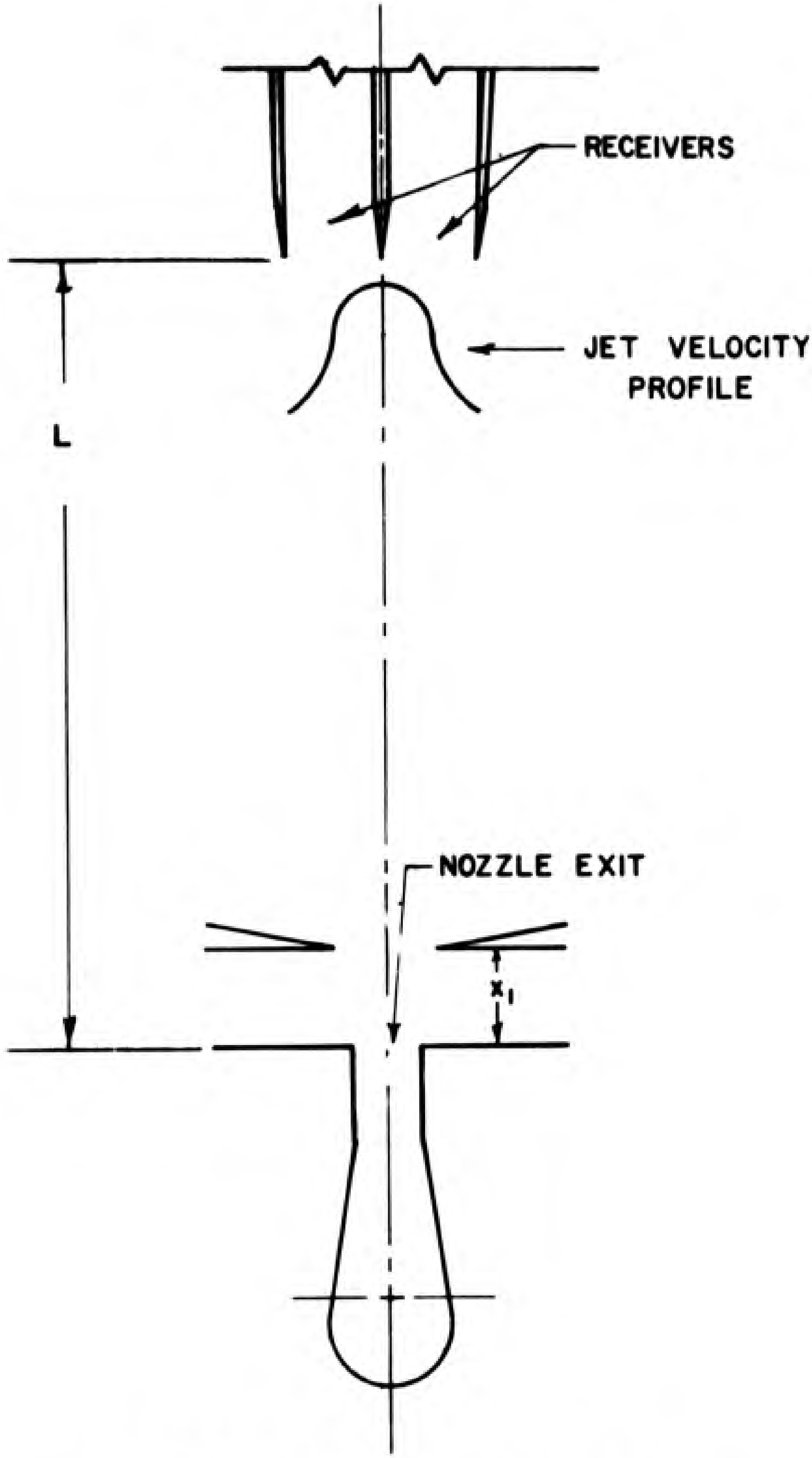

Figure 1. Interaction region parameters.

Assume that the small deflections all take place about zero, so that the change in pressure difference $\delta(p_2 - p_1)$ is the same as the pressure difference $p_2 - p_1$ and the deflection from zero is y_o

$$\delta q_1(t) = h \int_0^{x_1} \frac{\partial}{\partial t} y_o(\tau, t)\, dx \qquad (10)$$

where x_1 is the control width (fig. 1) and h is the distance between top and bottom plates.

In terms of the Laplace transform

$$\delta Q_1(s) = \mathcal{L}\{\delta q_1(t)\} = \mathcal{L}\{h \int_0^{x_1} \frac{\partial}{\partial t} y_o(\tau, t)\, dx$$

or

$$\delta Q_1(s) = h \int_0^{x_1} sY(\tau, s)\, dx \qquad (11)$$

[Note that the use of Eulerian coordinates makes the choice of limits on the integral much simpler than if Lagrangian coordinates (material or total derivative) had been used, since in those cases the effect of transport time must appear in the limits.]

$$Y(\tau, s) = \mathcal{L}\{y_o(\tau, t)\}$$

If we assume that the pressure gradient is uniform across the jet and that p is a function of t only then

$$g(t) = -\frac{dp}{dy} = \frac{p_2 - p_1}{b} = \frac{\Delta p}{b} \qquad (12a)$$

where b is the nominal width of the jet and

$$G(s) = \frac{\Delta P(s)}{b} \qquad (12b)$$

where

$$\Delta P(s) = \mathcal{L}\{\Delta p(t)\}$$

Equation (7) then reduces to

$$Y = \frac{\Delta P}{\rho b s^2}\{1 - e^{-s\tau}(1 + s\tau)\} \tag{13a}$$

Inserting this into equation (11)

$$\delta Q_1(s) = \frac{h\Delta P}{\rho b s}\int_0^{x_1}\{1 - e^{-s\tau}(1 + s\tau)\}dx \tag{13b}$$

In the vicinity of the nozzle the jet width and velocity are approximately constant, hence we let

$$\tau = \frac{x}{u_o}\,;\; u_o \equiv u(o) \tag{14}$$

Inserting equation (14) into (13) and integrating

$$\frac{\Delta P}{\delta Q_1} = \frac{\rho b s}{h\left(x_1 + \frac{2u_o}{s} e^{-\frac{sx_1}{u_o}} - \frac{2u_o}{s} + x e^{-\frac{sx_1}{u_o}}\right)} \tag{15}$$

The impedance is then given by

$$Z_1 = \lim_{s\to j\omega}\frac{\Delta P}{\delta Q_1} = \frac{\rho b\omega^2(\sin\beta - j\cos\beta)}{4hu_o(\sin\beta - \beta\cos\beta)} \tag{16a}$$

where

$$\beta = \frac{\omega x_1}{2u_o} \tag{16b}$$

For $\beta << 1$, equation (16) becomes

$$Z_{11} = R_{11} + \frac{1}{j\omega C_1} \qquad (17a)$$

where

$$R_{11} = \frac{3\rho b u_o}{h x_1^{\,2}} \qquad (17b)$$

$$C_1 = \frac{h x_1^{\,3}}{6\rho b u_o^{\,2}} \qquad (17c)$$

For $\beta >> 1$ equation (16) becomes

$$Z_{12} = R_{12} + j\omega L_1 \qquad (18a)$$

where

$$R_{12} = -\frac{\rho b \omega}{2hx_1} \tan\frac{\omega x_1}{2u_o} \qquad (18b)$$

$$L_1 = \frac{\rho b}{2hx_1} \qquad (18c)$$

Equations (17) and (18) indicate that the jet acts as a resistance and capacitance in series when β is small, and as a resistance and an inertance in series when β is large. For intermediate values, the impedance is partially capacitive, partially inertive and partially resistive.

In most cases of interest β is small. This is fortunate since one cannot place any great reliance on the results of equation (18) at high frequencies because the jet profile, and in particular the jet centerline velocity, undoubtedly changes appreciably which violates a previous assumption.

The loss of energy implied by a resistive component needs to be explained because viscosity has not been taken into account. This loss is, of course, due to power-jet particles being deflected and, thus, acquiring energy within the control region and then moving out of that region carrying the energy with them. At large values of β, the particles may make cycles of oscillation within the field and may leave the field moving in either the same or opposite sense as the field. Consequently, as β varies, the resistance goes through positive and negative values.

2.2 The Input Impedance

For the dynamic analysis of the amplifier, assume that the changes of pressure are very small, so that a linear analysis holds. All of the variables are then given in terms of deviation from the steady state value.

Consider figure 2 in which the flows and pressures shown are functions of time, and the transformed variables, P, Q, are functions of s.

The equivalent dynamic circuit is given in figure 3 for the interaction region where the returned flows δQ_r are shown as originating from sources whose strengths are proportional to the deflection.

We use upper case P's and Q's to designate the Laplace transforms of the respective p's and q's.

The symbol δ is used to indicate small changes.

It will be assumed that the control pressures are applied antisymmetrically, i.e.,

$$\delta P_{cL} = -\delta P_{cR} \tag{19a}$$

As a result it follows that

$$\delta P_2 = -\delta P_1 \tag{19b}$$

$$\delta Q_{cL} = \delta Q_{cR} = \delta Q_c \tag{20}$$

For the controls then

$$\frac{\delta P_{cL} - \delta P_{cR} + \delta P_1 - \delta P_2}{\delta Q_c} = 2Z_c \tag{21}$$

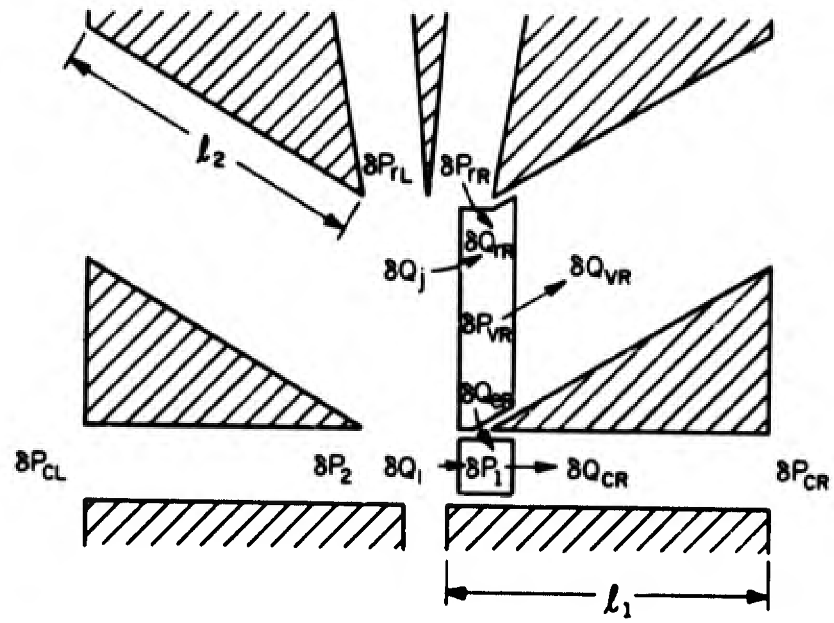

Figure 2. Small signal dynamics.

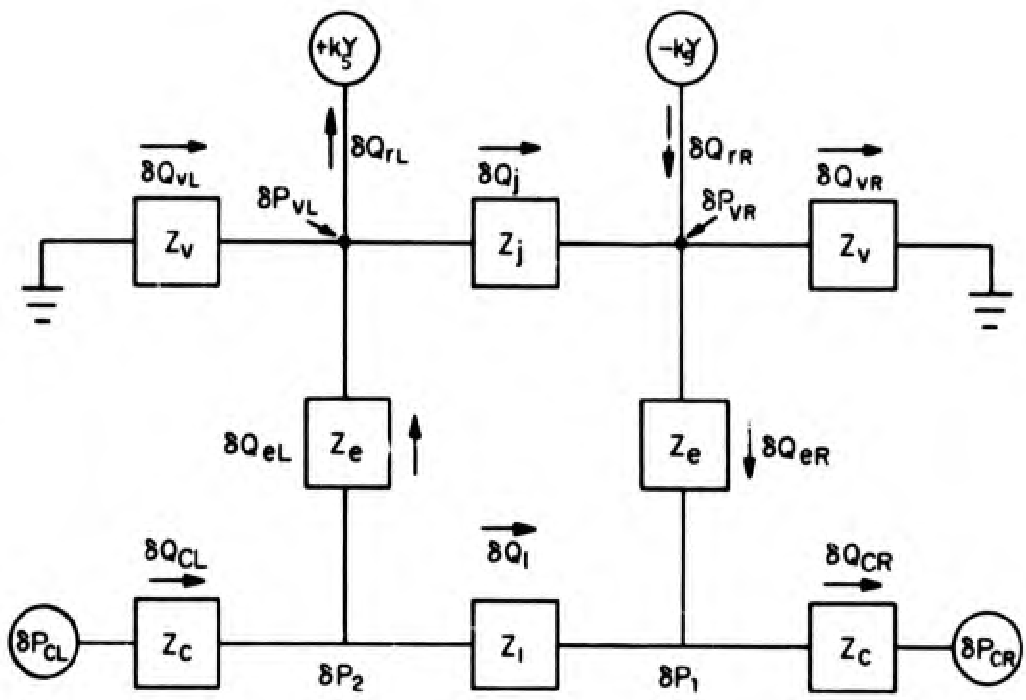

Figure 3. Equivalent dynamic circuit.

The impedance across the control edges is

$$Z_e = Z_{eR} = \frac{\delta P_{vR} - \delta P_1}{\delta Q_{eR}} \tag{22a}$$

and

$$Z_e = Z_{eL} = \frac{\delta P_2 - \delta P_{vL}}{\delta Q_{eL}} \tag{22b}$$

But

$$\delta Q_{eL} = \delta Q_{eR} \equiv \delta Q_e$$

Thus from equations (22a) and (22b)

$$2Z_e = \frac{\delta P_{vR} - \delta P_{vL} + \delta P_2 - \delta P_1}{\delta Q_e} \tag{23a}$$

Except for resonant frequencies of the vents

$$\delta P_{vR} - \delta P_{vL} \ll \delta P_2 - \delta P_1$$

hence for nonresonant frequencies

$$\frac{\delta P_2 - \delta P_1}{\delta Q_e} \cong 2Z_e \tag{23b}$$

Thus for nonresonant frequencies the input impedance to the controls may be found as follows:

Since

$$Z_1 = \frac{\delta P_2 - \delta P_1}{\delta Q_1}$$

and

$$\delta Q_e = \delta Q_c - \delta Q_1$$

then from equations (21) and (23b)

$$\delta P_2 - \delta P_1 = \frac{Z_1 Z_e}{2Z_e Z_c + Z_1 Z_c + Z_1 Z_e} (\delta P_{cL} - \delta P_{cR}) \tag{24}$$

and, again using equation (21), the input impedance Z_{in} is given by

$$Z_{in} = \frac{\delta P_{cL} - \delta P_{cR}}{\delta Q_c} = \frac{2(2Z_e Z_c + Z_1 Z_c + Z_1 Z_e)}{2Z_e + Z_1} \tag{25a}$$

For the general case when $\delta P_{vL} - \delta P_{vR}$ must be considered the expression for Z_{in} becomes much more complicated. We indicate the more general solution in section 3 of this report.

It is worth pointing out that for large Z_e (small edge clearances), equation (25a) becomes

$$Z_{in1} = 2Z_c + Z_1 \tag{25b}$$

whereas for small Z_e (large clearances)

$$Z_{in2} = 2Z_c \tag{25c}$$

Thus the impedance of the jet in the interaction region has negligible effect when the clearance is large and an appreciable effect when the clearance is small.

2.3 The Pressure Gain

We now find how the change in pressure difference at the receivers, $\delta P_{rL} - \delta P_{rR}$, is related to the change in pressure differences across the controls $\delta P_{cL} - \delta P_{cR}$.

From equation (7)

$$Y(\tau, s) = \frac{1}{\rho b} \int_o^{\tau_1} (\tau - \tau') e^{-s(\tau - \tau')} \Delta P \, d\tau' + \frac{1}{\rho b} \int_{\tau_1}^{\tau} (\tau - \tau') e^{-s(\tau - \tau')} \Delta P_v \, d\tau' \qquad (26a)$$

where

$$\Delta P = \delta P_2 - \delta P_1$$

$$\Delta P_v = \delta P_{vL} - \delta P_{vR}$$

$$\tau_1 = \int_o^{x_1} \frac{dx}{u(x)} \qquad (26b)$$

$$\tau_1 \leq \tau \leq \tau_L \qquad (26c)$$

$$\tau_L = \int_o^L \frac{dx}{u(x)}$$

L is the distance from the power jet nozzle to the splitter (fig. 1).

Carrying through the integration of equation (26a) we obtain

$$Y(\tau, s) = \frac{\Delta P}{\rho b s^2} \left\{ e^{-s(\tau - \tau_1)} \left[1 + s(\tau - \tau_1) \right] - e^{-s\tau} (1 + s\tau) \right\} + \frac{\Delta P_v}{\rho b s^2} \left\{ 1 - e^{-s(\tau - \tau_1)} \left[1 + s(\tau - \tau_1) \right] \right\} \qquad (27)$$

Now the transverse jet flow, δQ_j, corresponding to the jet deflection is given by

$$\delta Q_j = h \int_{x_1}^{L} s\, Y(\tau, s)\, dx \tag{28}$$

As before if we assume that the velocity u(x) is constant, we obtain

$$\tau = \frac{x}{u_o}\,;\ \tau_1 = \frac{x_1}{u_o}$$

$$\delta Q_j = \frac{hu_o}{\rho b s^2} \left\{ A_1 \Delta P_v + A_2 \Delta P \right\} \tag{29a}$$

where

$$A_1 = \alpha_2 - \alpha_1 + 2\left[e^{-(\alpha_2 - \alpha_1)} - 1\right] + \alpha_2\, e^{-(\alpha_2 - \alpha_1)} - \alpha_1\, e^{-(\alpha_2 - \alpha_1)} \tag{29b}$$

$$A_2 = -2\left[e^{-(\alpha_2 - \alpha_1)} - 1\right] - (\alpha_2 - \alpha_1)\, e^{-(\alpha_2 - \alpha_1)} + 2(e^{-\alpha_2} - e^{-\alpha_1}) + \alpha_2 e^{-\alpha_2} - \alpha_1 e^{-\alpha_1} \tag{29c}$$

and

$$\alpha_1 = \frac{x_1 s}{u_o}\,;\ \alpha_2 = \frac{Ls}{u_o} \tag{29d}$$

Then rewriting equation (27) using (29d) and with

$$\tau = \tau_L = \frac{L}{u_o}$$

we find

$$Y(L, s) = \frac{1}{\rho h s^2}\{B_1 \Delta P_v + B_2 \Delta P\} \tag{30a}$$

where

$$B_1 = 1 - (1 + \alpha_2 - \alpha_1)\ e^{-(\alpha_2 - \alpha_1)} \tag{30b}$$

and

$$B_2 = (1 + \alpha_2 - \alpha_1)\ e^{-(\alpha_2 - \alpha_1)} - (1 + \alpha_2)\ e^{-\alpha_2} \tag{30c}$$

For the volume involving δP_{vR} of figure 2 (or the junction point on the equivalent circuit diagram of figure 3) we obtain

$$\delta Q_v + \delta Q_e = \delta Q_r + \delta Q_j \tag{31}$$

where

$$\delta Q_v = \frac{\delta P_{vR} - \delta P_{vL}}{2Z_v} \tag{32a}$$

and similarly

$$\delta Q_e = \frac{\delta P_{vR} - \delta P_{vL} + \delta P_2 - \delta P_1}{2Z_e} \tag{32b}$$

We show in section 3 that for small deflections the returned flow, δQ_r, is proportional to the deflection:

$$\delta Q_r = k_5 Y(L,s) \tag{32c}$$

where k_5 depends on the output impedance and therefore on the frequency.

Inserting equation (32) into (31)

$$\frac{\delta P_{vR} - \delta P_{vL}}{2Z_v} + \frac{\delta P_{vR} - \delta P_{vL}}{2Z_e} + \frac{\delta P_2 - \delta P_1}{2Z_e} = k_5 Y(L,s) + \delta Q_j \tag{33}$$

Then using equations (29a) and (30a) we obtain

$$\Delta P_v = \frac{\frac{1}{2Z_e} - \frac{hu_o A_2}{\rho bs^2} - \frac{k_5 B_2}{\rho bs^2}}{\frac{1}{2Z_v} + \frac{1}{2Z_e} + \frac{hu_o A_1}{\rho bs^2} + \frac{k_5 B_1}{\rho bs^2}} \Delta P \tag{34}$$

and from equations (30a) and (34)

$$Y(L,s) = \frac{\Delta P}{\rho bs^2} \left\{ B_2 + \frac{B_1 \left(\frac{1}{2Z_e} - \frac{hu_o A_2}{\rho bs^2} - \frac{k_5 B_2}{\rho bs^2} \right)}{\frac{1}{2Z_v} + \frac{1}{2Z_e} + \frac{hu_o A_1}{\rho bs^2} + \frac{k_5 B_1}{\rho bs^2}} \right\} \tag{35}$$

We insert the relation between Δp_c and Δp (equation 24) and obtain the relationship between the input pressure difference ΔP_c and the jet deflection $Y(L,s)$ in the vicinity of the splitter.

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13. ABSTRACT

This report shows how the response of a jet to a pressure gradient is used to derive the impedance of the jet.

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14. KEY WORDS	LINK A		LINK B		LINK C	
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Fluerics						
Fluidics						
Jet Dynamics						
Fluid Amplifier						

$$Y(L,s) = \frac{Z_1 Z_e \Delta P_c}{\rho b s^2 (2Z_e Z_c + Z_1 Z_c + Z_1 Z_e)} \tag{36}$$

$$\left\{ B_2 + \frac{B_1 \left(\frac{1}{2Z_e} - \frac{hu_o A_2}{\rho b s^2} - \frac{k_5 B_2}{\rho b s^2} \right)}{\frac{1}{2Z_v} + \frac{hu_o A_1}{\rho b s^2} + \frac{k_5 B_1}{\rho b s^2} + \frac{1}{2Z_e}} \right\}$$

We proceed to find the relation between the output Δp_r and the jet deflection. In general this relation is of the form shown in figure 4. Since we are concerned only with small deflections, for which the curve is quite linear, we need know only the slope of the curve, $\frac{\partial \Delta p_r}{\partial y_o}$, in the vicinity of $y_o = 0$. The relationship between Δp_r and y_o for small values of y_o is then

$$\Delta p_r = \left. \frac{\partial \Delta p_r}{\partial y_o} \right|_0 y_o$$

We assume that the recovered pressure depends on the amount of the jet dynamic pressure intercepted by the splitters, hence, if the velocity profile at the distance L from the nozzle is given by $u(L, y)$, then for a jet deflected by an amount y_o, for receivers of widths b_2:

$$\Delta p_r = \frac{\rho}{2b_2} \int_{-b_2}^{o} u^2(L, y - y_o) dy - \frac{\rho}{2b_2} \int_{o}^{b_2} u^2(L, y - y_o) dy \tag{37}$$

$$\frac{\partial \Delta p_r}{\partial y_o} = \frac{\rho}{2b_2} \int_{-b_2}^{o} \frac{\partial u^2}{\partial y_o} (L, y - y_o)\, dy - \frac{\rho}{2b_2} \int_{o}^{b_2} \frac{\partial u^2}{\partial y_o} (L, y - y_o) dy$$

but for any function $f(y - y_o)$

$$-\frac{\partial f}{\partial y_o} = \frac{\partial f}{\partial y}$$

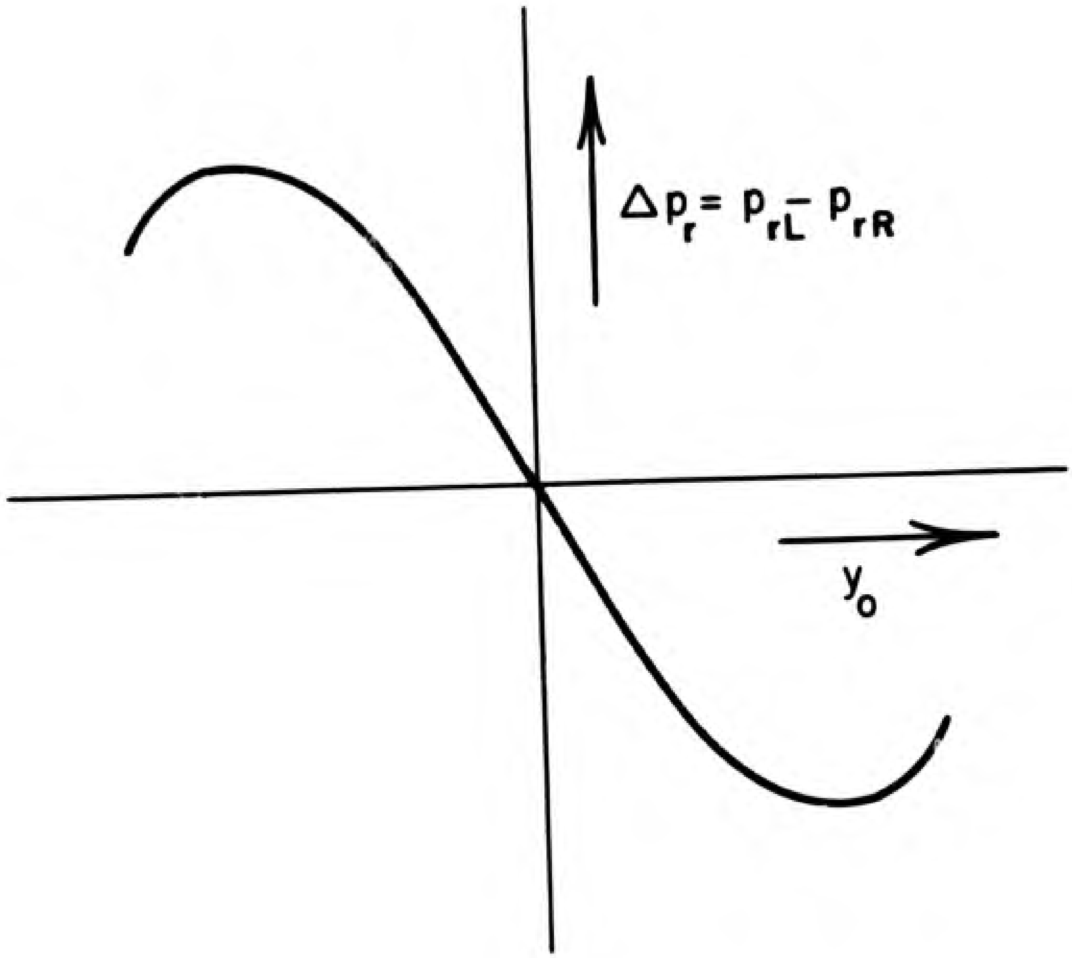

Figure 4. Recovered pressure difference as a function jet deflection.

hence

$$\frac{\partial \Delta p_r}{\partial y_o} = \frac{-\rho}{2b_2}\Big[u^2(L, -y_o) - u^2(L, -b_2 - y_o)$$

$$- u^2(L, b_2 - y_o) + u^2(L, -y_o)\Big]$$

For $y_o = 0$

$$\left.\frac{\partial \Delta p_r}{\partial y_o}\right|_{y_o=0} = \frac{-\rho}{2b_2}\Big[2u^2(L,O) - u^2(L, -b_2) - u^2(L, b_2)\Big]$$

Since u is an even fuction, the slope may be written as

$$\left.\frac{\partial \Delta p_r}{\partial y_o}\right|_{y_o=0} = \frac{-\rho}{b_2}\Big[u^2(L,0) - u^2(L,b_2)\Big]$$

Hence for small deflections

$$\Delta p_r = \frac{-y_o\rho}{2}\Big[u^2(L,0) - u^2(L,b_2)\Big] \tag{38a}$$

or assuming that the relation also holds dynamically

$$\Delta P_r = \frac{\ddot{Y}\rho}{b_2}\Big[u^2(L,0) - u^2(L,b_2)\Big] \tag{38b}$$

Inserting this into equation (36)

$$\Delta P_r = -\frac{\left[u^2(L,0) - u^2(L,b_2)\right] Z_1 Z_e \Delta P_c}{bb_2 s^2 (2Z_e Z_c + Z_1 Z_c + Z_1 Z_e)} \qquad (39)$$

$$\left\{ B_2 + \frac{B_1\left(\frac{1}{2Z_e} - \frac{hu_o A_2}{\rho b s^2} - \frac{k_5 B_2}{\rho b s^2}\right)}{\frac{1}{2Z_e} + \frac{1}{2Z_v} + \frac{hu_o A_1}{\rho b s^2} + \frac{k_5 B_1}{\rho b s^2}} \right\}$$

Equation (39), which relates the pressure difference across the receivers to the pressure difference across the controls, is the desired transfer function.

It should be noted that in addition to the impedances appearing explicitly, the result also depends on the outlet impedance through k_5.

The predominant poles appear in the factor

$$\frac{1}{2Z_e} + \frac{1}{2Z_v} + \frac{hu_o A_1}{\rho b s^2} + \frac{k_5 B_1}{\rho b s^2}$$

where A_1 and B_1 are functions of $\frac{sx_1}{u_o}$ and $\frac{sL}{u_o}$. It is seen, therefore, that resonances can be obtained from the vent impedance term Z_v that can be primarily inductive and resistive (wide open), primarily capacitive (closed) or some combination of inductive, capacitive, and resistive as discussed in Section 3.2.

The edge impedance Z_e is essentially resistive (the inductive effect is negligible in the frequency range of interest) and hence provides only damping. The terms in $\frac{sL}{u_o}$ tend to provide resonant peaks at frequencies proportional to $\frac{u_o}{L}$ and are thus similar to edgetone effect frequencies.

There is a similar effect at higher frequencies due to the term $\frac{sx_1}{u_o}$.

Finally resonance can be obtained from the outlet impedance. In this connection the remarks made for the vents apply. Various combinations of inductance, resistance, and capacitance are possible.

3. CALCULATION METHOD

In this section we obtain a more general expression for the input impedance and indicate how the various parameters may be obtained.

3.1 The General Input Impedance

The previous expression (25a) obtained for the input impedance assumed that ΔP_v was small compared with ΔP. If this is not true, the general expression can be obtained by inserting into equation (23a) the value of ΔP_v given in equation (34) resulting in

$$2Z_e = \frac{\Delta P}{\Delta Q_e}\left[1 + \frac{\frac{1}{2Z_e} - \frac{hu_oA_2}{\rho bs^2} - \frac{k_5B_2}{\rho bs^2}}{\frac{1}{2Z_v} + \frac{1}{2Z_e} + \frac{hu_oA_1}{\rho bs^2} + \frac{k_5B_1}{\rho bs^2}}\right] \tag{40}$$

This equation can then be used instead of equation (23b) to obtain the more general form of the input impedance.

3.2 Evaluation of the Impedances and of k_5

In calculating the resistance portion of the impedance, the approach to be used for the proper small pressure fluctuation approximations depends on the geometry associated with the particular impedances. For shapes that are essentially parallel wall ducts, we can use the Hagen-Poiseuille resistance (as modified for rectangular ducts).

We linearize around the nonlinear resistance associated with an orifice. The Hagen-Poiseuille resistance, associated with a short duct terminated with an orifice, can usually be neglected in comparison with the orifice resistance. But if the duct is sufficiently long, both resistances should be considered.

In the following we assume that only the control-edge resistance and the outlet resistance are of the orifice type, but, in practice, the particular configuration must be taken into account.

3.2.1 The Control-Duct Impedance

The impedance Z_c is given by

$$Z_c = R_c + sL_c \tag{41a}$$

where

$$R_c \cong \frac{12\mu \ell_1 (x_1^2 + h^2)}{x_1^3 h^3} \qquad (41b)$$

$$L_c = \frac{\rho \ell 1}{x_1 h} \qquad (41c)$$

Where ℓ_1 is the length of the control (fig. 2) and

$\ell_1 \gg x_1, h$

3.2.2 <u>The Vent Impedance</u>

The vent impedance for a wide open vent, as in figure 2, is also an inertance in series with a resistance and is evaluated similarly.

$$Z_v = R_v + s L_v \qquad (42a)$$

$$R_v = \frac{12\mu \ell_{2e} (b_v^2 + h^2)}{(b_v h)3} \qquad (42b)$$

$$L_v = \frac{\rho \ell 2e}{b_v h} \qquad (42c)$$

$\ell_{2e} \gg b_v h$

where ℓ_{2e} is the effective length of the vent and b_v is its width.

For vents at right angles to the amplifier axis, the effective length ℓ_{2e} is the equal to the vent wall length, ℓ_2, but obviously when the angle is less than a right angle $\ell_{2e} < \ell_2$.

3.2.3 <u>The Control-Edge Impedance</u>

The control-edge impedance is essentially a resistance and can be evaluated in terms of the clearance between the jet and the control-edge.

$$Z_e = R_e = \frac{q_{eo}}{\left(\frac{D-b_1}{2}\right)^2 h^2} \qquad (43)$$

where q_{eo} is the flow past the control edge when the jet is undeflected. Because Z_e depends on Q_{eo}, it is a function of the bias pressure $\frac{P_{cL} + P_{cR}}{2}$.

3.2.4 <u>The Jet Impedance</u>

The impedance of the jet in the vicinity of the controls has already been evaluated in equation (16). In particular, for small values of $\frac{sx}{u_o}$

$$Z_{11} = R_{11} + \frac{1}{sC_1} \qquad (44a)$$

where

$$R_{11} = \frac{3\rho b u_o}{h x_1^2} \qquad (44b)$$

$$C_1 = \frac{h x_1^3}{6\rho b u_o^2} \qquad (44c)$$

3.2.5 <u>The Outlet Impedance</u>

We assume that the lines attached to the outlets have equal inductances L_o and see the same output load P_{LOAD}. The relation between the pressure and the flow at the right outlet, for example, is given by

$$P_{rR} - P_{LOAD} = K_o\, q_{oR}^2 \qquad (45a)$$

where

$$K_o = \frac{\rho}{2b_o^2 h^2 c_{do}} \tag{45b}$$

b_o = outlet orifice width

c_{do} = discharge coefficient.

At zero input across the controls

$$q_{oRo} = q_{oLo} \equiv q_{oo} = \frac{1}{K_o^{1/2}} \left(p_{ro} - p_{LOAD}\right)^{1/2} \tag{45c}$$

where p_{ro} is the dc pressure appearing at both the left and right receivers for zero input, i.e.,

$$p_{ro} = p_{rLo} = p_{rRo} \tag{46}$$

where q_{oRo}, q_{oLo}, p_{rRo}, and p_{rLo} are the left and right receiver flows and pressures respectively at zero deflection.

Since the outlets are considered here as lines terminated with an orifice, the impedance of the line* is given as[5] :

$$\frac{\delta P_{rL}}{\delta Q_{oL}} = \frac{R_o \cosh s\sqrt{L_o C_o} + \sqrt{\frac{L_o}{C_o}} \sinh s\sqrt{L_o C_o}}{R_o\sqrt{\frac{C_o}{L_o}} \sinh s\sqrt{L_o C_o} + \cosh s\sqrt{L_o C_o}} \tag{47a}$$

where R_o is the outlet orifice resistance, L_o is the inertance of the outlet, and C_o is the capacitance of the outlet.

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*Here, we use the expression for a circular line because it is much simpler than that of the rectangular line. The error in the combined expression should be small.

[5] J.M. Kirshner, Fluid Amplifiers, McGraw-Hill, 1966, pp 170.

For low frequencies the lumped circuit approximation for equation (47a) is

$$\frac{\delta P_{rL}}{\delta Q_{oL}} = \frac{R_o + sL_o + \frac{s^2 R_o L_o C_o}{2} + \frac{s^3 L_o^2 C_o}{6}}{1 + sR_o C_o + \frac{s^2 C_o L_o}{2}} \tag{47b}$$

The outlet line may therefore be approximated relatively well by the circuit of figure 5 for which the circuit equation is

$$\frac{\delta P_{rL}}{\delta Q_{oL}} = \frac{R_o + sL_o + \frac{s^2 R_o L_o C_o}{2} + \frac{s^3 L_o^2 C_o}{4}}{1 + sR_o C_o + \frac{s^2 C_o L_o}{2}} \tag{47c}$$

Inasmuch as the cubic term in s is small for low frequencies, the slight discrepancy is relatively unimportant.

If R_o is small, that is if the orifice opening is not much smaller than the tube, then at sufficiently low frequencies, equation (47b) or (47c) can be simplified to

$$\frac{\delta P_{rL}}{\delta Q_{oL}} = R_o + sL_o \tag{47d}$$

where the variational resistance R_o is given by

$$R_o = 2k_o q_{oo} \tag{48}$$

[q_{oo} is defined in equation (45)].

Then since

$$\delta q_o - \delta q_{oR} = \delta q_{oL} \equiv \delta q_o$$

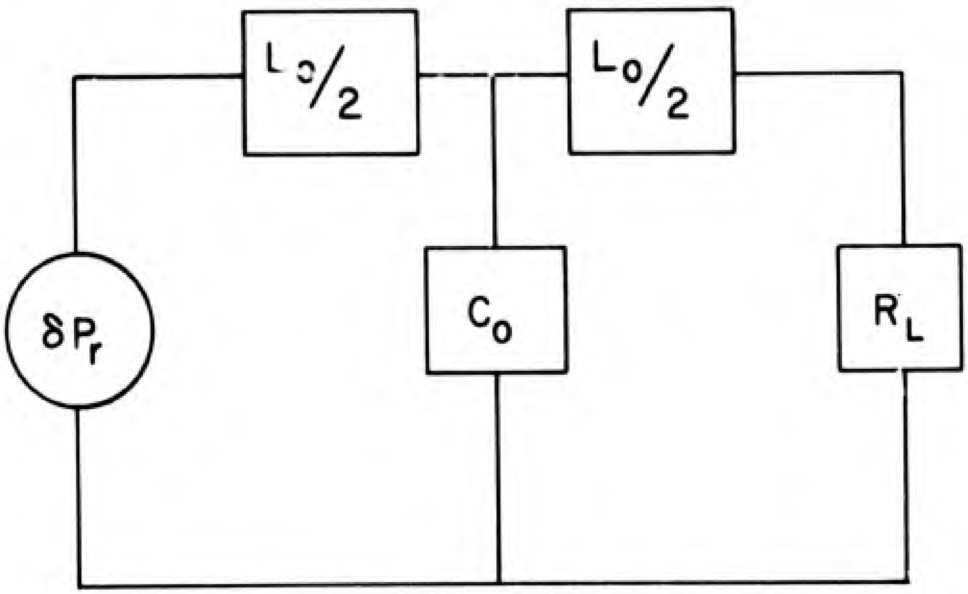

Figure 5. Equivalent input impedance circuit for receiver.

and

$$\delta p_{rR} = \delta p_{rL} \equiv \delta p_r,$$

we can write

$$Z_o \equiv \frac{\delta P_r}{\delta Q_o} \cong 2k_o q_{oo} + sL_o \qquad (49)$$

3.2.6 Evaluation of the Coefficient k_5

Equation (32c) assumed that

$$\delta Q_r = k_5 Y$$

In order to find k_5 we must recognize that the dynamic change in spilled (or returned) flow, δq_{rR}, for example, is equal and opposite to the change in output flow δq_{oR}, that is

$$\delta q_r = -\delta q_o \qquad (50a)$$

or

$$\delta Q_r = -\delta Q_o \qquad (50b)$$

Further, in keeping with our previous definition

$$\Delta P_r \equiv \delta P_{rL} - \delta P_{rR} = 2\delta P_r \qquad (51)$$

It follows from equations (49), (50), and (51) that

$$\delta Q_r = -\frac{\Delta P_r}{2Z_o} \qquad (52)$$

The relation between ΔP_r and Y is given by equation (38b). Inserting it into equation (52)

$$\delta Q_r = \frac{\rho \left[u^2(L, o) - u^2\left(L, b_2\right)\right] Y}{Z_o \; 2b_2} \tag{53}$$

hence

$$k_5 = \frac{\rho \left[u^2(L, o) - u^2\left(L, b_2\right)\right]}{Z_o \; 2b_2} \tag{54}$$

4. SUMMARY

In summary we have found an expression for the impedance due to the transverse motion of a jet in a pressure gradient.

We have also shown how the dynamic response of a jet can be incorporated into the expression for the relations between the pressure difference input and the pressure difference output for a beam deflection amplifier. The resulting transfer function incorporates the particle transport time in terms of the jet dynamics, instead of adding a time delay as is usually done. As a result the effect of frequency on the jet deflection and on the phase shift is taken into account more accurately.

In addition we have found (for small deflections) a simple relation between the jet deflection and both the output and returned flows. Knowledge of the velocity distribution is not necessary in order to obtain this relation, as it holds for any distribution.

5. LITERATURE CITED

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5. J.M. Kirshner, Fluid Amplifiers, McGraw-Hill, 1966, pp 170.

GLOSSARY

A_1	=	defined in equation (29b)
A_2	=	defined in equation (29c)
b	=	power jet nozzle width
b_o	=	outlet orifice width
b_v	=	vent width
b_2	=	receiver width
B_1	=	defined in equation (30b)
B_2	=	defined in equation (30c)
c_{do}	=	outlet discharge coefficient
C_o	=	outlet capacitance
C_1	=	transverse jet capacitance in control region
$g(x, t)$	=	force acting on a fluid particle due to transverse pressure gradient
$G(x, s)$	=	Laplace transform of $g(x, t)$
h	=	slit height (distance between top and bottom plates)
k_5	=	parameter giving proportionality between jet deflection and returned flow
K_o	=	resistance coefficient for output orifice
ℓ_1	=	length of control nozzle
ℓ_2	=	length of vent wall
ℓ_{2e}	=	effective vent length
L	=	distance between power jet nozzle and splitter
L_c	=	control line inductance
L_o	=	output receiver inductance
L_v	=	vent inductance

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L_1	=	jet transverse inductance in control region
p	=	pressure
p_c	=	control pressure
p_{cL}, p_{cR}	=	left and right control pressure respectively
p_{LOAD}	=	load (output) pressure
p_r	=	pressure at the receiver
p_{rL}, p_{rR}	=	left and right receiver pressures respectively
p_{rRo}	=	pressure at the right receiver when the jet is undeflected
p_{ro}	=	pressure at either receiver when the jet is undeflected
p_v	=	vent pressure acting on power jet
p_{vL}, p_{vR}	=	left and right vent pressures
$p(s) = \mathcal{L}\{p(t)\}$	=	Laplace transform of pressure
p_2, p_1	=	left and right interaction pressures
q_c	=	control flow
q_{cL}, q_{cR}	=	left and right control flows
q_e	=	flow between control edge and jet edge
q_{eo}	=	flow between control edge and jet edge when the jet is undeflected
q_j	=	transverse jet flow corresponding to jet deflection in vent region
q_o	=	output flow
q_{oR}	=	right output flow
q_{oRo}	=	right output flow when jet is undeflected
q_r	=	return flow
q_v	=	vent flow

GLOSSARY (Continued)

q_1 = transverse flow due to jet deflection in control region

$Q = \mathcal{L}\{q\}$ = Laplace transform of flow

R_c = control resistance

R_e = control edge resistance

R_o = outlet resistance

R_v = vent resistance

R_1 = jet transverse resistance in control region

R_{11} = jet transverse resistance in control region at low frequencies

R_{12} = jet transverse resistance in control region at high frequencies

s = Laplace transform of flow

t = time

$u_o = u(o)$ = power jet-nozzle exit velocity

$u(x, y)$ = axial velocity distribution

$u(x) = u(x, o)$ = centerline velocity

$v(x, y)$ = transverse velocity distribution

x = axial distance

x_1 = control nozzle width near power jet

y = transverse distance

$y_o(x, t)$ = jet particle displacement at distance x from nozzle

$Y = \mathcal{L}\{y_o\}$ = Laplace transform of jet deflection

Z_c = control nozzle impedance

Z_e = impedance of clearance between control edge and jet

Z_{in} = input impedance

GLOSSARY (Continued)

Z_v = vent impedance

Z_1 = jet impedance in control region

Z_{11} = jet impedance in control region at low frequencies

Z_{12} = jet impedance in control region at high frequencies

α_1 = $\frac{x_1 s}{u_o}$

α_2 = $\frac{Ls}{u_o}$

β = $\frac{\omega x_1}{2u_o}$

δ = small change in a variable

θ = angle vent axis makes with amplifier axis

μ = viscosity

ρ = density

τ_L = time for jet centerline particle to move the distance L

τ = time for jet centerline particle to move the distance x

τ_1 = time for jet centerline particle to move the distance x_1

ω = angular frequency