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# THE USE OF THE JOINT GENERALIZED LEAST SQUARES ESTIMATION TECHNIQUE IN THE AGGREGATION OF SUBSYSTEM COST ESTIMATING RELATIONSHIPS FOR SHIPS

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Naval Postgraduate School
Monterey, California

September 1973

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# NAVAL POSTGRADUATE SCHOOL

## Monterey, California





# THESIS

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by

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September 1973

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## ABSTRACT

This study addresses the problem of uncertainty in the aggregation of subsystem cost estimating relationships (CERs) for destroyer type naval ships. The case of correlations between subsystems is discussed with respect to total cost estimates, total cost variance, and prediction intervals. The joint generalized least squares method (JGLS) is developed and applied to the ship cost estimating problem. The models analyzed were developed by D. M. Hernon and R. R. McCumber, Estimation of Destroyer Type Naval Ship Procurement Costs, utilizing the data base from Resource Management Corporation Report CR-058, the Navships Cost Model. One model utilizes 9 subsystem CERs to obtain a total cost estimate, while the second model utilizes 4 subsystem CERs. Three solution methods were used for purposes of comparison; the JGLS method assuming correlated subsystems, the least squares method assuming correlated subsystems, and the least squares method assuming independent subsystems. The JGLS method is shown to provide the most meaningful results for correlated subsystems. A computer program and user's guide are provided to conduct JGLS analysis.

TABLE OF CONTENTS

LIST OF TABLES

# I. INTRODUCTION

Due to cost overruns, Congressional concern, and a continuing need for better planning estimates, it is imperative that new techniques be developed and old techniques refined in order to obtain better estimates for major weapon system procurement and construction. Along with these techniques a better understanding of the factors and forces that determine cost is required.

The area of concern in this paper is with Naval ship procurement and construction costs. The cost of a ship is a result of many factors and forces all of which it would be impossible to define and measure. Thus this cost is subject to variation. When estimating the cost of a ship it is necessary to provide both the estimate itself and as clear a picture as possible of the variability surrounding the estimate.

The general objective of this paper is to analyze the variability of ship cost estimates with respect to the Joint Generalized Least Squares (JGLS) estimation technique, and the Least Squares (LS) regression estimation technique. It is hypothesized that the Joint Generalized Least Squares Estimation technique takes advantage of additional information available and allows a more meaningful statement of the variance of a ship cost estimate.

## A. NAVAL SHIP PROCUREMENT AND CONSTRUCTION COSTS

Naval ships may be thought of as major weapon systems consisting of numerous subsystems including:

Propulsion
Hull
Weapons and fire control systems
Auxiliary

There are two basic methods used to predict ships costs. One method is to estimate the construction costs for each subsystem, which are then summed to obtain total ship cost, [Ref. 4 and 6]. The other method is to estimate total ship cost by using the performance and operational characteristics such as, speed or number of missile launchers as explanatory variables, [Ref. 2-3].

The first method is the one of concern in this paper. Here the subsystem costs are estimated by using the physical and performance characteristics of the ship as explanatory variables, normally utilizing the Standard Linear Least Squares multiple regression technique. Weight is often used as a primary explanatory variable in addition to many other characteristics. In order to make meaningful statements about the variability of the total cost of the ship some aggregation technique must be employed on the individual subsystem relations. The aggregation technique used depends upon the assumptions concerning the correlation of disturbances that exist between the subsystem relations. If the assumption of independence between subsystem relations can be made ( a very restrictive assumption) straightforward linear combinations may be used to estimate total cost variance and prediction intervals; call this method LS (independence). This would seem a highly suspect assumption due to the interdependencies of subsystems aboard ships. A more reasonable assumption would be that there are correlations between subsystems. If these interdependencies are recognized, methods do exist to make more meaningful statements about the uncertainty surrounding total cost. An extension of the least squares method may be made by constructing an estimated covariance matrix of the least squares estimates, assuming correlated disturbances; call this method LS (correlated). Finally, if correlations between subsystems are assumed, the joint generalized least squares technique allows this information to be used in determining a set of joint estimates and in constructing an estimated covariance matrix of the joint estimates. This assumption is less restrictive than the independence assumption and in fact should provide more meaningful results; call this method JGLS. Discussion and development of these three methods is contained in Section II, Nature of the Problem and Section III, Methodology of Joint Generalized Least Squares Estimation.

## II. NATURE OF THE PROBLEM

### A. RECENT SHIP COST ESTIMATING MODELS AND THE TREATMENT OF UNCERTAINTY IN AGGREGATE PREDICTIONS.

#### 1. Resource Management Corporation Report CR-058 to NAVSHIPS

Resource Management Corporation derived a statistical cost model for NAVSHIPS in which ship construction costs are estimated by subsystem and then aggregating these costs to arrive at Basic Contract Cost, and Total Ship End cost. The following subsystem breakdown was used:

| | | | |
|---|---|---|---|
| 1 | Hull | 6 | Outfitting |
| 2 | Propulsion | 7 | Armament |
| 3 | Electrical | 3 | Design and Engineering |
| 4 | Communication and Control | 9 | Construction Services |
| 5 | Auxiliary | | |

The summation of these nine cost categories plus profit and overhead was defined as basic contract cost and formed the nine-subsystem cost model. RMC also developed a condensed four-subsystem cost model composed of the following categories:

| | | | |
|---|---|---|---|
| 1 | Hull Group | : | Hull, Design and Engineering, Construction Services |
| 2 | Propulsion Group | : | Propulsion, Auxiliary |
| 3 | Armament Group | : | Armament, Electrical, Communication and Control |
| 4 | Outfitting Group | : | Outfitting |

Again, the summation of these four cost categories plus profit and overhead was defined as basic contract cost.

In determining total end costs RMC used NAVSHIPS records of the Shipbuilding and Conversion, Navy (SCN) fund. NAVSHIPS hardcore cost was subsequently defined as basic contract cost plus miscellaneous end costs and electronics end costs. The total end cost then became NAVSHIPS hardcore cost plus weapons end cost. CERs were developed from the data base for predicting basic contract cost and miscellaneous end cost. No CERs were developed for electronics or weapons end costs.

The models were developed for six different classes of ships:

1. Aircraft Carriers
2. Destroyer
3. Submarine
4. Auxiliary
5. Amphibious
6. Patrol/minesweeping

The area of interest in the Hernon and McCumber paper, which will be described next, and also in this paper is with the destroyer models developed. Reference 6 gives the full details of the RMC study and results.

Taking now a point of departure from the general RMC study development, their treatment of uncertainty will be discussed. The treatment of uncertainty involves basically two steps. The first step is to develop the best set of CERs from the data available based on a regression strategy which outlines the basic statistical properties to be used as criteria in the choice. The second step is to analyze and report as thoroughly as possible the variance which surrounds the CERs developed and, particularly, their aggregation to obtain total cost.

Two points arise from the RMC study with respect to the treatment of uncertainty. In the first, RMC uses the coefficient of determination, $R^2$, as a measure of the goodness of fit of the regression equation to the data, where

$$R^2 = \frac{\sum_i (\hat{Y}_i - \bar{Y})^2}{\sum_i (Y_i - \bar{Y})^2}$$

The ratio of the sum of squares explained to the total sum of squares of Y adjusted for the mean.

This statistic tends to overstate the goodness of fit as it does not take into consideration the degrees of freedom. A more acceptable statistic would have been the adjusted coefficient of determination (adjusted for degrees of freedom) where $R^2_{ADJ}$ is defined below [see Ref. 7]

$$R^2_{ADJ} = \frac{\frac{1}{n-k} \Sigma e_\alpha^2}{\frac{1}{n-1} \Sigma (Y_\alpha - \bar{Y})^2} - 1$$

where $e_\alpha = (\hat{Y}_\alpha - Y_\alpha)$

$\alpha$ is the observation

The second point concerning the reporting of total cost variance deals with prediction intervals on predicted total cost. The RMC study implicitly assumes that the errors in the estimates of costs for the individual subsystems are independent of one another (non-correlated), though they fail to state this assumption directly. This seems highly unlikely since one shipyard produces several subsystems for a ship and the factors that contribute to the errors in the estimates of one subsystem may very well be the same or partially the same as those that contribute to the errors in the estimates of another subsystem. The allocation of overhead is one example. The factors that contribute to the errors in estimating costs for the propulsion and auxiliary subsystems might very well be the same since these subsystems are in actuality very much dependent upon one another in the ship itself.

Though it is not reported directly in the RMC study, Total cost variance is treated as the summation of the variance not explained in the individual subsystem CERs.

$$V = \sum_{i=1}^{L} S_i^2$$

where $V$ is total cost

$S_i^2$ is the variance of subsystem i

$L$ is the number of subsystems in the model

Prediction intervals on total cost are reported in the RMC study as the summation of the individual subsystem prediction intervals, where predicted cost is the summation of the subsystem predicted costs.

$$\sum_{i=1}^{L} \hat{c}_i \pm t_{\alpha/2} \sqrt{\sum_{i=1}^{L} A_i}$$



where $\hat{c}_i$ = predicted cost for subsystem i

$A_i = S_i^2(1 + X_0'(X'X)^{-1}X_0)$

the contribution for the ith subsystem

Acting as if the subsystems are independent may lead to substantial overstatements of the confidence one should have in any given total cost estimate as this paper will endeavor to show.

2. Estimation of Destroyer Type Naval Ship Procurement Costs, by D. M. Hernon and R. R. McCumber Jr.

This paper is essentially an extension of the study direction of the RMC study. The objective as stated was,

> "... To develop a model for the prediction of total procurement cost of destroyer type naval ships that increases in precision as input data is refined and, hopefully, approaches a level of quality acceptable in cost-effectiveness studies and eventually for fiscal planning purposes."

Their approach was to first correct numerous deficiencies noted in the RMC study. Because their concern was the prediction of total cost rather than the identification of basic contract costs and separate end cost, data was aggregated with the basic contract cost data as follows:

a. Electronic end cost was added to command and control cost
b. Weapons end cost was added to armament cost
c. Miscellaneous end cost was added to construction services cost.

Next they examined three models; a nine-subsystem cost model, a four-subsystem cost model and finally a single cost estimation equation. The final step was to use the models developed to estimate the total procurement cost of a destroyer type ship under development and compare the predictions to the best available NAVSHIPS estimate.

The study used as a primary criteria for comparing the prediction values of these models the estimate of total cost variance associated with each model. They used two methods to estimate total cost variance. The first method was called the summation method. This is the same as the treatment used by RMC in their study, i.e., the summation of the variance not explained in the individual subsystem CERs. Again, this method requires the assumption that the errors of the individual CERs are independent of one another. Hernon and McCumber acknowledge the difficulties with this assumption and use this method to obtain a minimum total cost variance that may be attained by utilizing that particular set of model CERs.

The second method of total cost variance estimation involves the calculation of a total cost mean square residual value (MSR) for each model. The following method was used to calculate this value:

$$MSR = \frac{\sum_{i=1}^{N} \left[ \sum_{j=1}^{L} (\text{Residual}_{ij}) \right]^2}{N - M - L}$$

where N is the number of ships

M is the number of variables utilized in all CERs of the model

L is the number of CERs utilized in the model

Essentially, the residual values produced for each CER for a given ship are summed producing an aggregate of the individual CER residual values. The total cost residuals are squared and summed for all observations (ships). If this quantity is then corrected for degrees of freedom, an estimate of variance is produced for a given model. Hernon and McCumber considered the MSR method to produce an upper bound on the total cost variance estimate, a value below which the estimate of total cost variance is expected to lie.

Hernon and McCumber did not report prediction intervals in their paper. Their treatment of total cost variance did not permit utilization of the standard linear least squares methodology for handling prediction intervals. The coefficient of determination which they reported was the same as the one which the RMC study used and is subject to the comments made earlier concerning it.

3. <u>RAND Report, "Confidence in Estimated Airframe Costs: Assessment in Aggregate Predictions", F.S. Timson and D.P. Tihansky</u>

This report addresses the general problem of confidence measures for multi-equation prediction models utilizing in the development specific results from airframe cost estimation. The report describes how prediction intervals can be calculated for sums of component regressions. The regressions can be treated as independent and either Student t or normal statistics used (the same treatment as the RMC study), or the regressions can be treated as correlated and normal statistics used.

A simple method permits the determination of the degree of correlation between the individual regressions. It is assumed that there exists a correlated normal distribution for total costs. Consider two regressions derived from a common data base consisting of the same number of observations. Assume the error terms of the two regressions are distributed bivariate normal with zero mean then

$$(\varepsilon_{i1}, \varepsilon_{i2}) \sim N(0,\Omega)$$

$$\Omega = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2 \end{bmatrix}$$

where i refers to the observation number and $\Omega$ is the variance-covariance matrix.

Then if $\rho$ is the correlation between $\varepsilon_{i1}$ and $\varepsilon_{i2}$ over all observations $\rho = \dfrac{\sigma_{12}}{\sigma_1 \sigma_2}$ . $\rho$ may be estimated from the sample as

$$\hat{\rho}_{12} = \frac{\sum_i (Y_{i1} - \hat{Y}_{i1})(Y_{i2} - \hat{Y}_{i2})}{\sqrt{\sum_i (Y_{i1} - \hat{Y}_{i1})^2 \sum_i (Y_{i2} - \hat{Y}_{i2})^2}}$$

When cost models are significantly correlated a method for treating the situation is described. This method assumes normality as a reasonably good approximation to the actual probability distribution. The true variance, W, of the sum of normal costs is expressed as

$$W = \sum_i \sum_j \sigma_{ij} (1 + X'_0 (X'X)^{-1} X_0)$$

where $X_0$ is a specified set of values of the independent variables

$X$ is an observed n x K matrix of rank K consisting of values taken by K explanatory variables

$\sigma_{ij}$ is the covariance between models i and j. An approximation for which is

$$s_{ij} = \frac{\sum_{q=1}^{N} e_{qi}\, e_{qj}}{N - K - 1}$$

the sums of products of the residuals of models i and j over N observations for observation q.

Prediction interval bounds for the aggregate cost are thus approximated at some confidence level α by

$$\hat{C}_0 \pm N(\alpha)\sqrt{\sum_i \sum_j s_{ij}(1 + X_0'(X'X)^{-1} X_C)}$$

where $\hat{C}_0$ the predicted total cost, is the sum of the individual predicted costs of the subsystems.

In comparing prediction interval widths for the airframe cost model for independent normal and correlated normal assumptions at a fixed level of confidence, correlated normal distributions generally give the widest intervals.

It should be noted that in this formulation each subsystem CER uses exactly the same explanatory variables putting a very limiting constraint on the use of this method in multi-equation prediction models. In Naval ship cost models it is highly probable that there do exist correlations between subsystem CERs. The CERs developed in the RMC study and by Hernon and McCumber involve different explanatory variables in the subsystem CERs, hence the above described technique may not be applied. The Joint Generalized Least Squares Estimation Technique is a method which will produce more accurate results than the least squares (independence) technique under the circumstances of correlated residuals and different explanatory variables.

## B. THESIS OBJECTIVES

The objectives of the thesis are twofold:

1. To apply the method of joint generalized least squares to the ship cost problem.

2. Develop computer routines to carry out the calculations associated with the joint generalized least squares estimation technique.

## III. METHODOLOGY OF JOINT GENERALIZED LEAST SQUARES ESTIMATION

Joint generalized least squares is a method for taking into consideration information that may be available when one is interested in the combination of several linear relations which are related either because their coefficients are partly the same or because their disturbances are correlated. In the latter case one can say that the variables that are neglected in the various equations are partly the same or at least correlated. The statistic used to measure the correlation between relations or subsystem errors is

$$\hat{\rho}_{12} = \frac{\sum_i (Y_{i1} - \hat{Y}_{i1})(Y_{i2} - \hat{Y}_{i2})}{\sqrt{\sum_i (Y_{i1} - \hat{Y}_{i1})^2 \; \sum (Y_{i2} - \hat{Y}_{i2})^2}}$$

as was defined earlier in the RAND study on uncertainty in aggregate predictions.

### A. MODEL AND ASSUMPTIONS

A complete development of the Joint Generalized Least Squares Method is contained in Theil [Ref. 7]. In using the joint generalized least squares method it is desired to formulate the estimates of the parameter vectors of several subsystem equations simultaneously. Suppose there are L equations of the form

$$y_j = X_j\beta_j + \varepsilon_j \qquad j = 1,\ldots,L$$

where $y_j$ is a column vector of n observations

$X_j$ is an n x $K_j$ matrix of values taken by the $K_j$ explanatory variables of the $j^{th}$ subsystem relation

$\beta_j$ is the corresponding parameter vector to be estimated

$\varepsilon_j$ is a disturbance vector

Suppose also that each of these L subsystem equations meets the assumptions of the standard linear regression model. It is desired to combine the L equations in the following form:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_L \end{bmatrix} = \begin{bmatrix} X_1 & \cdots & 0 \\ \cdot & X_2 & \cdot \\ \vdots & \ddots & \vdots \\ 0 & \cdots & X_L \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_L \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_L \end{bmatrix}$$

For notational ease let

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_L \end{bmatrix} \quad X = \begin{bmatrix} X_1 & \cdots & 0 \\ \cdot & X_2 & \cdot \\ \vdots & \ddots & \vdots \\ 0 & \cdots & X_L \end{bmatrix} \quad \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_L \end{bmatrix} \quad \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_L \end{bmatrix}$$

Then the joint generalized least squares formulation becomes:

$$Y = X\beta + \varepsilon$$

1. <u>Assumptions for Joint Generalized Least Squares</u>

a. The standard linear model holds for all of the L subsystem relations under consideration.

b. The disturbances of each subsystem equation are homescedastic and uncorrelated,

$$E(\varepsilon_j \varepsilon_j') = \sigma_{jj} I \qquad j = 1,\ldots,L$$

where $\sigma_{jj}$ is the variance of the disturbance vector of the $j^{th}$ equation.

c. Disturbances of different observations but of the same linear relation are assumed to be zero,

$$E(\varepsilon_{\alpha j} \varepsilon_{nj}) = 0 \qquad \alpha \neq n \quad j = 1,\ldots,L$$

d. With respect to the covariance matrix of the disturbances of two different equations:

$$E(\varepsilon_j \varepsilon_\ell') = \begin{bmatrix} E(\varepsilon_{1j}\varepsilon_{1\ell}) & E(\varepsilon_{1j}\varepsilon_{2\ell}) & \cdots & E(\varepsilon_{1j}\varepsilon_{n\ell}) \\ E(\varepsilon_{2j}\varepsilon_{1\ell}) & E(\varepsilon_{2j}\varepsilon_{2\ell}) & \cdots & E(\varepsilon_{2j}\varepsilon_{n\ell}) \\ \vdots & \vdots & \ddots & \vdots \\ E(\varepsilon_{nj}\varepsilon_{1\ell}) & E(\varepsilon_{nj}\varepsilon_{2\ell}) & \cdots & E(\varepsilon_{nj}\varepsilon_{n\ell}) \end{bmatrix}$$

The diagonal elements are contemporaneous covariances of the form $(\varepsilon_{\alpha j}\varepsilon_{\alpha\ell})$ and are assumed to be constant in the sense of being independent of $\alpha$. These will be denoted by $\sigma_{j\ell}$. The off diagonal elements correspond to different observations and are assumed to be zero:

$$E(\varepsilon_{\alpha j}\varepsilon_{n\ell}) = 0 \qquad \begin{matrix} \alpha \neq n \\ j \neq \ell \end{matrix} \quad j, \; = 1,\ldots,L$$

Note we obta.

$$E(\varepsilon_j \varepsilon_\ell') = \sigma_{j\ell} I \qquad j,\ell = 1,\ldots,L$$

which includes $E(\varepsilon_j \varepsilon_j) = \sigma_{jj} I$ as a special case.

The disturbances of the L linear relations for the same observation are random drawings from a multivariate population with zero mean and constant covariance matrix. Thus the covariance of the complete vector $\varepsilon$ is:[1]

$$\mathrm{VAR}(\varepsilon) = \begin{bmatrix} \sigma_{11}I & \sigma_{12}I & \cdots & \sigma_{1L}I \\ \sigma_{21}I & \sigma_{22}I & \cdots & \sigma_{2L}I \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{L1}I & \sigma_{L2}I & \cdots & \sigma_{LL}I \end{bmatrix} = \Sigma \otimes I$$

where $\Sigma = [\sigma_{j\ell}]$

The L x L matrix $\Sigma$ is the covariance matrix of $[\varepsilon_{\alpha 1} \ldots \varepsilon_{\alpha L}]$ for any $\alpha$.

The complete $\beta$ vector may now be estimated by generalized least squares. The Aitken estimator for $\beta$ is,

$$\hat{\beta} = [X'(\Sigma^{-1} \otimes I)X]^{-1} X'(\Sigma^{-1} \otimes I)Y$$

---

[1]The Kronecker product of the m x n matrix A and the p x q matrix B is defined as the mp x nq matrix

$$A \otimes B = \begin{bmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & & \vdots \\ a_{n1}B & \cdots & a_{mn}B \end{bmatrix}$$

For further properties of the Kronecker product see [Ref. 7, p. 305].

for which the covariance matrix is,

$$VAR(\hat{\beta}) = [X'(\Sigma^{-1} \otimes I)X]^{-1}$$

Since Σ is usually unknown, it is estimated by the matrix of mean squares and products of the least squares residuals.

$$S = \frac{1}{n} \begin{bmatrix} e_1' \\ \vdots \\ e_L' \end{bmatrix} [e_1 \ . \ . \ . \ e_L]$$

where $e_j$ is the least squares residual vector of the $j^{th}$ equation and n is the number of observations.

Replacing Σ with S we obtain the joint generalized least squares estimate of the parameter vector β

$$b_j = [X'(S^{-1} \otimes I)X]^{-1}X'(S^{-1} \otimes I)Y$$

with the estimator for the covariance matrix

$$VAR(b_j) = [X'(S^{-1} \otimes I)X]^{-1}$$

Thus we have the essential information to carry out the joint generalized least squares estimation technique.

## B. TOTAL COST VARIANCE

Consider the estimator for Σ:

$$S = \frac{1}{n} \begin{bmatrix} e_1' \\ \vdots \\ e_L' \end{bmatrix} [e_1 \ . \ . \ . \ e_L]$$

An estimate for total cost variance may be obtained by taking the summation of all the elements of S

$$\text{Var}(C) = i' S i \qquad \text{where } i \text{ is a column vector of } L \text{ ones}$$

This estimate will be used in the analysis of the ship cost problem.

## C. PREDICTION INTERVALS

### 1. Prediction Intervals Utilizing Joint Generalized Least Squares

A method for determining a prediction interval utilizing joint generalized least squares may be developed. It must be assumed that the disturbances of the L linear relations for the same observation are random drawings from a multivariate normal distribution with zero mean vector and a constant covariance matrix. Let $C = \sum_{j=1}^{L} Y_j$ where C is total cost; then

$$C \sim N\left(\sum_j X_{*j}\beta_j,\ i'\Sigma i\right)$$

where $X_*$ is a row of explanatory variables $[X_{*1},\ldots,X_{*L}]$ for the L subsystem relations

Now if $\hat{C} = \sum_{j=1}^{L} \hat{Y}_j$ where $\hat{C}$ and $\hat{Y}_j$ are predicted costs, then

$$\hat{C} \sim N\left(\sum_j X_{*j}\beta_j,\ \text{VAR}\left(\sum X_{*j}\beta_j\right)\right)$$

Therefore

$$C - \hat{C} \sim N\left(0,\ i'\Sigma i + \text{VAR}\left(\sum X_{*j}\beta_j\right)\right)$$

If the estimates utilizing the sample observations are $i'Si$ for $i'\Sigma i$ and $b_j$, the joint generalized least squares estimate vector, for $\beta$, a prediction interval may be derived as

$$C \pm n_{(a/2)} \sqrt{i'Si + X_* \, VAR(b_j) X_*'}$$

2. Prediction Intervals for Correlated Subsystems Utilizing Least Squares Estimation Techniques

It is possible to provide prediction intervals by single equation least squares estimation technique under the circumstances of correlated residuals and different explanatory variables for each subsystem. When substantial correlations exist between subsystems in the model the below described methodology can be expected to give more accurate statements about uncertainty in aggregate cost predictions than would the normal practice of disregarding the correlations and without utilizing simultaneous estimates.

An estimated covariance matrix of the least squares estimates of the coefficients is obtained as will now be described. Consider the L subsystem equations of the form

$$y_j = X_j \beta_j + \varepsilon_j \qquad j = 1,\ldots,L$$

Postmultiply the sampling error of the least squares estimator, $\hat{\beta}_i$, of $\beta_i$ by the transpose of the sampling error of the estimator, $\hat{\beta}_j$, of $\beta_j$. Then the covariance matrix for selections i and j may be described as

$$E[(\hat{\beta}_i - \beta_i)(\hat{\beta}_j - \beta_j)] = \sigma_{ij}(X_i'X_i)^{-1}X_i'X_j(X_j'X_j)^{-1}$$

where $\sigma_{12}$ is the contemporaneous covariance,

$X_i, X_j$ are observation matrices

$(\hat{\beta}_i - \beta_i)$ are sampling errors.

This result will hold if all of the assumptions concerning disturbances that were made for the joint generalized least squares method are accepted. These assumptions are much less restrictive than those of the least squares (independence) method.

Utilizing this concept the covariance matrices for all combinations of subsystem selections may be determined. The estimates for $\hat{\sigma}_{ij}$ for all subsystems are contained in S, the matrix of mean squares and products of the residuals. Letting $E_{ij} = E[(\hat{\beta}_i - \beta_i)(\hat{\beta}_j - \beta_j)]$ a covariance matrix, denoted VAR($\beta$), similar to the covariance matrix of the joint estimates may be constructed in the form shown below

$$\begin{bmatrix} E_{11} & E_{12} & \cdots & E_{1L} \\ E_{21} & E_{22} & \cdots & E_{2L} \\ \vdots & \vdots & E_{ij} & \vdots \\ & & \ddots & \\ E_{L1} & E_{L2} & \cdots & E_{LL} \end{bmatrix}$$

If it is assumed that the disturbances of the L linear relations for the same observation are random drawings from a multivariate normal distribution with zero mean vector and a constant covariance matrix, a method for determining a prediction interval may be developed paralleling the prediction interval method utilizing joint generalized least squares estimates. The resulting prediction interval becomes

$$\hat{C} \pm \eta_{(\alpha/2)} \sqrt{i'Si + X_* VAR(\hat{\beta}) X_*'}$$

where $\hat{C}$ is estimate of total cost

$X_*$ is a row vector of explanatory variables $[X_{*1}, \ldots X_{*L}]$ for the L subsystem relations

$VAR(\hat{\beta})$ is the covariance matrix of the least squares estimates.

## IV. DATA BASE

There are three subsets of the data base that require discussion; contractor bid cost data, end cost data, and the independent variables. Contractor bid data was used for predicting subsystem costs. This was a compromise, the reason for which was that bid cost data was the most meaningful data available for the period under study (1954-1966). Cost accounting systems differed greatly among the various contractors and NAVSHIPS, making it impossible to obtain data on a uniform level of aggregation from any other sources. Bid prices themselves are subject to fluctuations due to factors in the shipbuilding industry such as overhead distribution, workload, level of expertise at a particular shipyard. The source of the bid data was the file of NAVSHIPS Form 4282.2, UNIT PRICE ANALYSIS-BASIC CONSTRUCTION, which lists contractor estimates for the nine different construction cost groups, subdivided into three categories; direct labor, direct material and overhead costs. The source of end cost data was the NAVSHIPS records of Shipbuilding and Conversion, Navy (SCN) fund. There are forty-one physical and performance characteristics for each ship which are contained in the RMC data base. These characteristics are essentially design parameters such as maximum speed, maximum draft, hull, propulsion weights, etc. They are utilized as candidates for independent variables in the models developed. Reference 4 and reference 6 give more detailed descriptions of the entire data base. Appendix A gives a description of Ship's Characteristics used as explanatory variables. Appendix B gives a description of basic contract cost categories.

## A. DATA ADJUSTMENTS

### 1. Bid Cost Data

Contractor raw bid data was adjusted in four specific ways to remove cost variations due to factors other than ship's characteristics as follows:

a. Learning effect - when the cost of ship construction decreases progressively with each ship in a procurement lot.

b. Temporal effect - which takes into consideration variations of prices, productivity and wage overtime. 1965 indices were used.

c. Installation of government furnished equipment.

d. Cost of plans from external sources.

The order of adjustment was as follows:

(1) Application of learning curves.

(2) Adjustments using 1965 indices.

(3) Addition of the cost of GFE and plans.

Details of these adjustments are found in Ref. 4 and Ref. 6.

### 2. End Cost Data

End cost data was adjusted in much the same way that contractor bid data was adjusted. Three specific adjustments were made to the raw end cost data.

a. Treatment for learning effect.

b. Adjustment for temporal effect using 1965 indices.

c. An adjustment for nuclear technology with propulsion costs.

Details of these adjustments are found in Ref. 4 and Ref. 6. The data base is published separately as APPENDIX H of Ref. 4 and is a CONFIDENTIAL document.

B. DATA BASE STRATIFICATION AND COST GROUP MODELS

Hernon and McCumber noted that some DLG type ships had significantly higher costs in the areas of hull, outfitting, construction services, weapons end cost, and electronics end cost. In addition the DLG ship was considered to have a different operational mission than the smaller, less expensive destroyer type ships. Thus, two basic data base stratification levels were examined:

1. General Data Base: DD/DDG/DE/DEG/DLG (36 Ships)
2. Escort Data Base : DD/DDG/DE/DEG (27 Ships)

Using both of the data bases, CERs were developed using two different methods of cost disaggregation schemes which are examined in this paper. The coding system as developed by Hernon and McCumber is contained in Table I and will be used for easy reference.

TABLE I

DATA BASE STRATIFICATION LEVELS AND COST GROUP MODELS

| | CER CODE General Level | CER CODE Escort Level |
|---|---|---|
| <u>MODELS B/C - 9 COST GROUP CERS</u> | | |
| Hull Cost | B-1 | C-1 |
| Propulsion Cost | B-2 | C-2 |
| Electrical Cost | B-3 | C-3 |
| Communication & Control + Electronics End Costs | B-4 | C-4 |
| Auxiliary Cost | B-5 | C-5 |
| Outfitting Cost | B-6 | C-6 |
| Armament + Weapons End Costs | B-7 | C-7 |
| Design & Engineering Cost | B-8 | C-8 |
| Construction Services + Miscellaneous End Costs | B-9 | C-9 |
| <u>MODELS D/E - 4 COST GROUP CER</u> | | |
| Base Cost = Hull + Outfitting | D-1 | E-1 |
| Engineering Cost = Propulsion + Electrical + Auxiliary | D-2 | E-2 |
| Payload Cost = C & C + Armament + Electronics End Cost + Weapons End Cost | D-3 | E-3 |
| Construction Cost = D & E + Construction Services + Miscellaneous End Cost | D-4 | E-4 |

# V. ANALYSIS

The general scheme of analysis was to apply the joint generalized least squares method as has been developed to the ship cost problem. In doing so computer routines were developed to carry out the indicated calculations associated with joint generalized least squares. Computer routines were also developed to solve the problem utilizing the least squares method in order to compare the results of the two methods and more clearly determine the nature of gains by using the joint generalized least squares method.

## A. GENERAL COMMENTS

### 1. Computer Programs

A computer program was developed in order to perform the joint generalized least squares analysis. Essentially, a data set is read in and then transformations to the data as programmed are conducted. The program then does the number of standard multiple linear regressions required (one for each subsystem CER). The residuals from the single regressions are saved and utilized to compute a matrix of correlations between selections. Next the program does the joints generalized least squares calculations; initially S is computed from the least squares residuals, next $b_j$, the parameter estimate vector is computed along with the co-variance matrix $V(b_j)$. Finally, the joint generalized least squares prediction interval values are calculated.

Details of the program and a users manual are contained in APPENDIX D. The program was designed to be easy to use for analysis utilizing the joint generalized least squares method. Double precision was used to improve accuracy by reducing roundoff errors. The program will handle up to thirty-six observations and nine subsystem relations. Each subsystem relation may have up to four explanatory variables (including the constant term). It should be noted that larger dimensioned problems may be possible but computer capacity becomes a constraint to be concerned with.

A computer program was also developed to perform the calculations required to compute a least squares prediction interval assuming independence between subsystem CERs. Essentially a data set is read in and then transformations to the data as programed are conducted. The program then does the number of standard multiple linear regressions required (one for each subsystem CER). Then the information required from each individual regression is used in the calculation of the prediction interval. This computer program is provided in Appendix D.

Finally, a computer program was developed to perform the calculations required to compute a least squares prediction interval assuming correlated disturbances between subsystem CERs. Essentially, a data set is read in and transformations as programed performed. Next the covariance matrix of the least squares estimates as described in

Section III.C.2 is constructed. Finally the calculations are carried out to compute least squares correlated prediction interval values. This computer program is provided in Appendix D.

2. Criteria

The principle criteria used for evaluating the joint generalized least squares method were the reduction in the covariance matrix of the estimate, the prediction intervals computed and the mean square residual values obtained from each observation. The first, a comparison of the diagonal elements of the estimated covariance matrix of the joint estimates and the estimated covariance matrix of the least squares estimates gives an impression of the gain obtained by applying the joint generalized least squares method rather than the least squares method. The second, the prediction intervals computed, gives the effect of the assumption of independent subsystem CER disturbances as compared to the assumption of correlated subsystem CER disturbances, and also a feel for the effect of using the joint generalized least squares estimation technique. In the third, mean squared residual values (MSR) are obtained from each observation, as the difference between observed and total cost. When these total cost residuals are squared, summed for all observations (ships), and corrected for the number of observations, a statistic is obtained for comparison of a set of models.[2]

3. Heteroscedasticity

Hernon and McCumber used a log linear model for their propulsion and engineering CERs in their nine CER group model and four CER group model respectively. This was done in order to correct for heteroscedasticity encountered with the linear model. The joint generalized least squares method will not allow calculation of prediction intervals when individual CERs are of mixed linear and log linear form, therefore, an alternate method was used to correct for heteroscedasticity in these relationships.

Consider the relation $y_\alpha = \beta_0 + \beta_1 X_{\alpha 1} + \varepsilon_\alpha$ for $\alpha = 1,\ldots,n$ where $\varepsilon_\alpha$ is a random disturbance with zero expectation. If it is assumed that the disturbance variance is proportional to the square of the independent variable then $\sigma_\alpha^2 = kX_{\alpha 1}^2$ for $\alpha = 1,\ldots,n$ and for some positive constant

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[2]Reference is made to mean squared residual (MSR) values in two contexts in this paper. MSR values refer to the sum of the squared total cost residuals for each observation divided by the number of observations. MSR values adjusted for degrees of freedom refer to the sum of the squared total cost residuals for each observation divided by the number of observations minus the number of parameters estimated minus the number of CERs utilized in the model. The first is a statistic for comparison between models, where as the second is an upper bound estimate on total cost variance as described by Hernon and McCumber [Ref. 4].

k. Dividing the values of the $\alpha^{th}$ observation by $X_{\alpha 1}$ is thus equivalent to the following reformulation:

$$\frac{y_\alpha}{X_{\alpha 1}} = \frac{\beta_0}{X_{\alpha 1}} + \beta_1 + \frac{\varepsilon_\alpha}{X_{\alpha 1}}$$

The disturbances $\varepsilon_1/X_{11} \ldots \varepsilon_n/X_{n1}$ have a scalar covariance matrix (kI) under the formulation so that the assumptions of the standard linear model are now satisfied. Scatter plots of residuals versus independent explanatory variables were used to determine the variable causing the heteroscedasticity in each relation and the above described reformulation was used to correct for it. A full description of the method is found in Theil [Ref. 7].

4. Number of Observations

Hernon and McCumber conducted an analysis of residuals to verify the normality assumption of the least squares regression technique and to identify any significant outliers. This resulted in the deletion of outliers in the development of several subsystem CERs. This caused the observations of a particular ship to be used to calculate some subsystem CERs, while not being used in others. Joint generalized least squares requires the same number of observations for all subsystem relations. Therefore, all observations were used in the joint generalized least squares analysis. In Appendix B of Ref. 4 Hernon and McCumber report the CERs developed including outliers in the data base. For purposes of comparison with joint generalized least squares results these CERs were used instead of the CERs developed by the deletion of outliers. Thus the general data base has thirty-six observations (ships) and the escort data base has twenty-seven observations.

## B. COST GROUP CER MODELS ANALYSIS

The nine cost group CER model consists of CERs as indicated and coded in Table 1. The regression strategy used in the development of these CERs along with a discussion of the results is found in Ref. 4. It was necessary to use a scale factor of 1/100 for the variables ENGPAY, PWRLD, ELEWGT, C&CWGT, PRAXWT, LSW, ARMWGT and PROLSW (the weight variables used) to enable inversion of the matrices required in calculating the joint generalized least squares estimates. These factors are reflected in the computer output in Appendix C for the mine group CER models, but have been removed in the results reported in this section.

The four cost group CER model consists also of CERs as indicated and coded in Table 1. No scale factor was required. In deriving the condensed model an attempt was made to make the subsystem categories as independent as possible, first with respect to the nature of ship design, and secondly with respect to the differences reported earlier in shipyard and contractor accounting practices. Details of the regression strategy and development of the subsystem CERs are contained in Hernon and McCumber,[Ref. 4].

## 1. Results

The following tables present four different sets of CERs - one set for each of the two models employing the general data base, and one set for each of the same two models employing the reduced escort data base. Following each set of CERs are tables containing correlations between selections for that particular model and selective comparisons of covariance matrices of the least squares and the joint generalized least squares estimates. The complete covariance matrices are contained in Appendix C, computer output.

TABLE II: MODEL B - 9 SUBSYSTEM CERs - GENERAL DATA BASE

| CODE | METHOD | COST SUB-CATEGORY | CER |
|---|---|---|---|
| B-1 | LS<br>JGLS | Hull | Cost = .0406 + .00139 [ENGPAY]<br>Cost = .0831 + .00136 [ENGPAY] |
| B-2 | LS<br>JGLS | Propulsion | Cost = -6.0916 + .5867 [PWRLD] + .000729 [RANGE]<br>Cost = -5.925 + .5870 [PWRLD] + .000697 [RANGE] |
| B-3 | LS<br>JGLS | Electrical | Cost = .1486 + .00386 [ELEWGT] + .2616 [NO-GEN]<br>Cost = .0310 + .00352 [ELEWGT] + .3099 [NO-GEN] |
| B-4 | LS<br>JGLS | C&C + Electronics End Cost | Cost = -.9037 + .032 [C&C WGT] + .348 [PROTO]<br>Cost = -.6208 + .029 [C&C WGT] + .627 [PROTO] |
| B-5 | LS<br>JGLS | Auxiliary | Cost = .0263 + .00224 [PRAXWGT]<br>Cost = -.0055 + .00227 [PRAXWGT] |
| B-6 | LS<br>JGLS | Outfitting | Cost = .436 + .000386 [LSW]<br>Cost = .549 + .000350 [LSW] |
| B-7 | LS<br>JGLS | Armament + Weapons End Cost | Cost = -2.606 + .0376 [ARMWGT] + 9.178 [MS-END]<br>Cost = -2.576 + .0884 [ARMWGT] + 3.859 [MS-END] |
| B-8 | LS<br>JGLS | Design and Engineering | Cost = -1.0125 + .00654 [ARMWGT] + .00151 [PROLSW]<br>Cost = -.9260 + .00638 [ARMWGT] + .00144 [PROLSW] |
| B-9 | LS<br>JGLS | Construction Services + Miscellaneous End Cost | Cost = 3.138 + .0394 [C&CWGT] - 59.846 [AR/LSW] + .00059 [PROLSW]<br>Cost = 2.520 + .0366 [C&CWGT] - 42.153 [AR/LSW] + .00052 [PROLSW] |

TABLE III: MATRIX OF CORRELATIONS BETWEEN SELECTIONS
MODEL B - 9 CERs - GENERAL DATA BASE

| Hull | Prop | Elect | C&C | Auxiliary | Out | Arm | D&E | Const | |
|---|---|---|---|---|---|---|---|---|---|
| 1.0 | .084 | -.227 | -.141 | .359 | -.021 | -.094 | .106 | .102 | Hull |
| | 1.0 | -.109 | -.105 | -.238 | .424 | .263 | .160 | .252 | Propulsion |
| | | 1.0 | .194 | .234 | -.050 | -.158 | -.023 | .180 | Electrical |
| | | | 1.0 | -.071 | -.405 | .139 | .115 | .402 | C&C + Elect. |
| | | | | 1.0 | .221 | -.152 | .057 | .101 | Auxiliary |
| | | | | | 1.0 | .322 | .021 | .059 | Outfitting |
| | | | | | | 1.0 | .252 | .362 | Arm + Weapons |
| | | | | | | | 1.0 | .234 | D&E |
| | | | | | | | | 1.0 | Const. Serv. |

## TABLE IV

## BLOCK DIAGONAL COVARIANCE MATRIX COMPARISONS
## MODEL B - 9 CERs - GENERAL DATA BASE

| JGLS | | | LS | | |
|---|---|---|---|---|---|
| **HULL** | | | | | |
| .7293D-01 | -.3523D-02 | | .7392D-01 | -.3578D-02 | |
| | .1960D-03 | | | .1991D-03 | |
| **PROPULSION** | | | | | |
| .1244D 02 | -.1994D 01 | .1330D-03 | .1499D 02 | -.2270D 01 | .1294D-03 |
| | .5012D 00 | -.5009D-04 | | .5734D 00 | -.5755D-04 |
| | | .6428D-08 | | | .7885D-08 |
| **ELECTRICAL** | | | | | |
| .3025D-01 | -.1964D-02 | -.7572D-02 | .3377D-01 | -.1597D-03 | -.9330D-02 |
| | .7613D-02 | -.2417D-02 | | .9372D-02 | -.3647D-02 |
| | | .3206D-02 | | | .4214D-02 |
| **COMMUNICATION AND CONTROL** | | | | | |
| .2177D 01 | -.1032D 01 | -.4362D 00 | .2381D 01 | -.1112D 01 | -.7143D 00 |
| | .6008D 00 | .9698D-01 | | .6405D 00 | .1518D 00 |
| | | .1095D 01 | | | .1839D 01 |
| **AUXILIARY** | | | | | |
| .6227D-01 | -.5399D-02 | | .6439D-01 | -.5612D-02 | |
| | .5425D-03 | | | .5639D-03 | |
| **OUTFITTING** | | | | | |
| .1808D-01 | -.4824D-03 | | .1978D-01 | -.5352D-03 | |
| | .1510D-04 | | | .1675D-04 | |

TABLE V: MODEL C - 9 SUBSYSTEM CERs - ESCORT DATA BASE

| CODE | METHOD | COST SUB-CATEGORY | CER |
|---|---|---|---|
| C-1 | LS<br>JGLS | Hull | Cost = .5524 + .00098 [ENGPAY]<br>Cost = .46877 + .00103 [ENGPAY] |
| C-2 | LS<br>JGLS | Propulsion | Cost = -.49215 + .38066 [PWRLD]<br>Cost = -.54098 + .38494 [PWRLD] |
| C-3 | LS<br>JGLS | Electrical | Cost = -.0151 + .00657 [ELEWGT] + .23057 [NO-GEN]<br>Cost = .07492 + .00524 [ELEWGT] + .24605 [NO-GEN] |
| C-4 | LS<br>JGLS | C&C + Electronics End Cost | Cost = -.4459 + .03429 [C&CWGT] - 1.2848 [MS-END]<br>Cost = -.9609 + .03851 [C&CWGT] - 1.4058 [MS-END] |
| C-5 | LS<br>JGLS | Auxiliary | Cost = -.0007 + .00229 [PRAXWT]<br>Cost = -.20962 + .00254 [PRAXWT] |
| C-6 | LS<br>JGLS | Outfitting | Cost = .5713 + .00332 [OUTWGT]<br>Cost = .6213 + .00311 [OUTWGT] |
| C-7 | LS<br>JGLS | Armament + Weapons End Cost | Cost = -1.00 + 7.9916 [MS-END] + .02814 [ARMWGT]<br>Cost = -1.392 + 7.7317 [MS-END] + .03110 [ARMWGT] |
| C-8 | LS<br>JGLS | Design and Engineering | Cost = -.3065 + .00407 [ARMWGT] + .00094 [PROLSW]<br>Cost = -.1182 + .00306 [ARMWGT] + .00092 [PROLSW] |
| C-9 | LS<br>JGLS | Construction Services + Miscellaneous End Cost | Cost + 3.7417 + .0394 [C&CWGT] - 68.6435 [AR/LSW] + .00039 [PROLSW]<br>Cost = 3.2207 + .0374 [C&CWGT] - 57.5419 [AR/LSW] + .00051 [PROLSW] |

TABLE VI: MATRIX OF CORRELATIONS BETWEEN SELECTIONS
MODEL C - 9 CERs - ESCORT DATA BASE

| Hull | Prop | Elect | C&C | Auxiliary | Out | Arm | D&E | Const | |
|---|---|---|---|---|---|---|---|---|---|
| 1.0 | -.035 | .029 | .001 | .382 | -.418 | -.060 | .0884 | -.104 | Hull |
| | 1.0 | .004 | .009 | -.449 | -.291 | -.013 | .148 | .000 | Propulsion |
| | | 1.0 | -.040 | .283 | .123 | -.230 | .034 | .187 | Electrical |
| | | | 1.0 | -.010 | -.50 | .062 | .063 | .577 | C&C + Elect. |
| | | | | 1.0 | -.206 | -.058 | -.019 | .049 | Auxiliary |
| | | | | | 1.0 | .019 | .006 | -.193 | Outfitting |
| | | | | | | 1.0 | .049 | .306 | Arm + Weapons |
| | | | | | | | 1.0 | .196 | D&E |
| | | | | | | | | 1.0 | Construction Services |

TABLE VII

BLOCK DIAGONAL COVARIANCE MATRIX COMPARISONS
MODEL C - 9 CERs - ESCORT BASE

| JGLS | | | LS | | |
|---|---|---|---|---|---|
| HULL | | | | | |
| .4421D-01 | -.2579D-02 | | .4831D-01 | -.2850D-02 | |
| | .17130D-03 | | | .1894D-03 | |
| PROPULSION | | | | | |
| .5893D 01 | -.5540D 00 | | .7706D 01 | -.7609D 00 | |
| | .6322D-01 | | | .8684D-01 | |
| ELECTRICAL | | | | | |
| .2340D-01 | -.1152D-01 | -.3137D-02 | .2498D-01 | -.1232D-01 | -.3373D-02 |
| | .2160D-01 | -.3461D-02 | | .2770D-01 | -.5219D-02 |
| | | .2137D-02 | | | .2792D-02 |
| COMMUNICATION AND CONTROL | | | | | |
| .3781D 01 | -.2604D 01 | .5018D 00 | .4243D 01 | -.3150D 01 | .1083D 01 |
| | .2230D 01 | -.8792D 00 | | .3007D 01 | -.1932D 01 |
| | | .1436D 01 | | | .3200D 01 |
| AUXILIARY | | | | | |
| .8039D-01 | -.8081D-02 | | .9440D-01 | -.9740D-02 | |
| | .9571D-03 | | | .1154D-02 | |
| OUTFITTING | | | | | |
| .2786D-01 | -.1039D-03 | | .3586D-01 | -.1363D-03 | |
| | .4208D-06 | | | .5520D-06 | |

TABLE VIII: MODEL D - 4 SUBSYSTEM CERs - GENERAL DATA BASE

| CODE | METHOD | COST SUB-CATEGORY | CER |
|---|---|---|---|
| D-1 | LS | Base | Cost = .4708 + .00115 [LSW] |
| | JGLS | | Cost = .5979 + .00111 [LSW] |
| D-2 | LS | Engineering | Cost = -.6515 + .00522 [PWRLD] + .2864 [ENGWGT] |
| | JGLS | | Cost = -.5748 + .00529 [PWRLD] + .2745 [ENGWGT] |
| D-3 | LS | Payload | Cost = -6.7857 + .00585 [LSW] + 6.6641 [MS-END] |
| | JGLS | | Cost = -6.3078 + .00558 [LSW] + 7.1679 [MS-END] |
| D-4 | LS | Construction | Cost = -1.0140 + .00555 [HULWGT] + .00201 [PROLSW] |
| | JGLS | | Cost = -.6801 + .00540 [HULWGT] + .00183 [PROLSW] |

MATRIX OF CORRELATIONS BETWEEN SELECTIONS
MODEL D - 4 CERs - GENERAL DATA BASE

| Base | Engin. | Pay | Const | |
|---|---|---|---|---|
| 1.0 | .441 | -.106 | -.055 | Base |
| | 1.0 | .085 | .279 | Engineering |
| | | 1.0 | .555 | Payload |
| | | | 1.0 | Construction |

TABLE IX

BLOCK DIAGONAL COVARIANCE MATRIX COMPARISONS
MODEL D - 4 CERs - GENERAL DATA BASE

| JGLS | | | LS | | |
|---|---|---|---|---|---|
| BASE D-1 | | | | | |
| .1197D 00 | -.3214D-04 | | .1251D 00 | -.3384D-04 | |
| | .1006D-07 | | | .1059D-07 | |
| ENGINEERING D-2 | | | | | |
| .1539D-06 | -.1136D-04 | .9565D-05 | .2336D-06 | -.1905D-04 | .3201D-04 |
| | .1745D-02 | -.9690D-02 | | .3165D-02 | -.1898D-01 |
| | | .1022D 00 | | | .1882D 00 |
| PAYLOAD D-3 | | | | | |
| .5081D 01 | -.1726D-02 | .1422D 01 | .5498D 01 | -.1991D-02 | .2016D 01 |
| | .7672D-06 | -.1004D-02 | | .9495D-06 | -.1444D-02 |
| | | .2470D 01 | | | .3598D 01 |
| CONSTRUCTION D-4 | | | | | |
| .7468D 00 | -.4696D-03 | .7886D-06 | .7844D 00 | -.4966D-03 | -.1791D-05 |
| | .3680D-06 | -.2255D-07 | | .3946D-06 | .3358D-07 |
| | | .3938D-07 | | | .6266D-07 |

TABLE X: MODEL E - 4 SUBSYSTEM CERs - ESCORT DATA BASE

| CODE | METHOD | COST SUB-CATEGORY | CER |
|---|---|---|---|
| E-1 | LS | Base | Cost = 1.2072 + .00083 [LSW] |
| | JGLS | | Cost = 1.0728 + .00088 [LSW] |
| E-2 | LS | Engineering | Cost = -.7559 + .66217 [PWRLD] |
| | JGLS | | Cost = -.9403 + .67833 [PWRLD] |
| E-3 | LS | Payload | Cost = -4.1883 + .00468 [LSW] + 7.1489 [MS-END] |
| | JGLS | | Cost = -4.7791 + .00478 [LSW] + 7.8049 [MS-END] |
| E-4 | LS | Construction | Cost = -2.6274 + .00736 [HULWGT] + .00156 [PROLSW] |
| | JGLS | | Cost = -2.0501 + .00679 [HULWGT] + .00156 [PROLSW] |

MATRIX OF CORRELATIONS BETWEEN SELECTIONS
MODEL E - 4 CERs - ESCORT DATA BASE

| Base | Engin. | Pay | Const | |
|---|---|---|---|---|
| 1.0 | -.255 | -.280 | -.318 | Base |
| | 1.0 | -.043 | .070 | Engineering |
| | | 1.0 | .490 | Payload |
| | | | 1.0 | Construction |

## TABLE XI

### BLOCK DIAGONAL COVARIANCE MATRIX COMPARISONS
### MODEL E - 4 CERs - ESCORT DATA BASE

| JGLS | | | LS | | |
|---|---|---|---|---|---|
| BASE E-1 | | | | | |
| .1056D 00 | -.3761D-04 | | .1108D 00 | -.396[illegible]D-04 | |
| | .1456D-07 | | | .1533D-07 | |
| ENGINEERING E-2 | | | | | |
| .1479D-02 | -.1452D-01 | | .1533D-02 | -.1513D-01 | |
| | .1657D 00 | | | .1727D 00 | |
| PAYLOAD E-3 | | | | | |
| .1052D 02 | -.4401D-02 | .3108D 01 | .1179D 02 | -.5108D-02 | .4255D 01 |
| | .2065D-05 | -.1943D-02 | | .2472D-05 | -.2658D-02 |
| | | .3970D 01 | | | .5429D 01 |
| CONSTRUCTION E-4 | | | | | |
| .2226D 01 | -.1993D-02 | -.5692D-04 | .2374D 01 | -.2120D-02 | -.8667D-04 |
| | .1932D-05 | .3906D-08 | | .2050D-05 | .1330D-07 |
| | | .8972D-07 | | | .1238D-06 |

The following table indicates correlations of greater than ± .4 for the four models:

TABLE XII

| MODEL | CORRELATION | SUBSYSTEMS |
|---|---|---|
| B | .424 | Outfitting and Propulsion |
| | - .405 | Outfitting and C&C + Electrical |
| | .402 | Construction and C&C + Electrical |
| C | - .449 | Auxiliary and Propulsion |
| | - .418 | Outfitting and Hull |
| | - .50 | Outfitting and C&C + Electrical |
| | .577 | Construction and C&C + Electrical |
| D | .441 | Engineering and Base |
| | .555 | Construction and Payload |
| E | .490 | Construction and Payload |

These correlation estimates do indicate significant degrees of correlation between subsystem CERs and would tend to provide evidence against a hypothesis of independence between subsystem CERs.

The block diagonal covariance matrix comparisons show a reduction in the variance (diagonal) elements in all cases and a reduction in the covariance (off-diagonal) elements in the large majority of cases by the joint generalized least squares estimates as compared with the least squares estimates. This may be attributed to the fact that the joint generalized least squares method takes into consideration the correlated subsystem disturbances in computing the joint estimates whereas the least squares method computes the individual subsystem estimates separately.

2. Total Cost Variance

Total cost variance for each model was computed utilizing three methods. The first method was to take the summation of the individual subsystem CER standard errors assuming independent subsystems. The second method was to take the summation of the entries in the S matrix of mean squares and products of all sets of subsystem CER residuals, the estimate for $\Sigma$. The third estimate was obtained by summing the residual values produced by each CER for a given ship which gives the difference between observed and predicted cost for that ship as an aggregate of the individual CER values. The total cost residuals are then squared and summed for all observations (ships) and divided by the degrees of freedom, defined as the number of observations minus the number of parameters estimated minus the number of subsystem CERs, to obtain the mean squared residual value (MSR). This method was used by Hernon and McCumber [Ref. 4] to form an upper bound on total cost variance. In essence, the summation method assuming independence provides a lower bound on total cost variance. The summation of the S matrix elements (S matrix summation), assuming correlated subsystems provides a midrange, more realistic estimate, between the upper bound (MSR method) and lower bound (LS Summation method). The values for these three methods are contained in the following table:

TABLE XIII

| | | METHOD | |
|---|---|---|---|
| MODEL | (LS SUMMATION) | (S MATRIX SUMMATION) | (MSR) |
| B | 19.64 | 123.9 | 163.0 |
| C | 16.12 | 58.38 | 526.0 |
| D | 7.86 | 36.54 | 58.3 |
| E | 7.04 | 27.47 | 43.6 |

3. Prediction Interval and Predicted Costs

Three prediction intervals were obtained for each model in the manner described in Section III.C, e.g., least squares with independence, least squares with correlations and joint generalized least squares with correlations. The estimated total ship cost was computed with the least squares estimates in the first two cases and the joint estimates in the third. The results are summarized in the following tables. There is a table for each model which contains the prediction interval length for each observation by each method. Also contained in each table are the absolute differences between actual and estimated cost obtained by using the least squares estimates and the joint estimates. The actual prediction intervals are contained in Appendix C. The signs of the value of the differences between actual

## TABLE XIV

### MODEL B: PREDICTION INTERVAL SUMMARY

| OBS | LENGTH LS(IND) | LENGTH JGLS | LENGTH LS(CORR) | \|Δ\|LS ACTUAL-ESTIMATED | \|Δ\|JGLS ACTUAL-ESTIMATED |
|---|---|---|---|---|---|
| 1 | 24.64 | 28.76 | 29.19 | 10.99 | 11.69 |
| 2 | 24.23 | 28.54 | 29.10 | 3.47 | 4.33 |
| 3 | 24.28 | 28.55 | 28.82 | 3.75 | 4.62 |
| 4 | 23.82 | 28.37 | 28.58 | 4.09 | 3.98 |
| 5 | 23.37 | 28.12 | 28.15 | 1.92 | 2.19 |
| 6 | 23.39 | 28.12 | 28.15 | 1.89 | 2.23 |
| 7 | 23.38 | 28.11 | 28.15 | .65 | .32 |
| 8 | 23.39 | 28.12 | 28.15 | 2.84 | 3.19 |
| 9 | 23.38 | 28.12 | 28.15 | 7.68 | 7.93 |
| 10 | 23.39 | 28.12 | 28.15 | 5.13 | 5.47 |
| 11 | 23.39 | 28.12 | 28.15 | 9.94 | 9.60 |
| 12 | 23.39 | 28.12 | 28.15 | 3.70 | 4.04 |
| 13 | 23.37 | 28.12 | 28.15 | 4.83 | 5.04 |
| 14 | 23.37 | 28.12 | 28.14 | 8.36 | 8.56 |
| 15 | 23.97 | 28.85 | 29.02 | .46 | 1.23 |
| 16 | 23.78 | 28.74 | 28.84 | 2.80 | 2.01 |
| 17 | 23.78 | 28.74 | 28.84 | 3.37 | 2.58 |
| 18 | 23.78 | 28.74 | 28.84 | 3.78 | 2.99 |
| 19 | 24.07 | 29.12 | 29.33 | 2.15 | 2.37 |
| 20 | 23.89 | 28.99 | 29.08 | 5.33 | 5.18 |
| 21 | 23.84 | 28.84 | 29.04 | 2.31 | 2.26 |
| 22 | 23.70 | 28.51 | 28.72 | 1.49 | 1.16 |
| 23 | 23.34 | 28.27 | 28.34 | 3.43 | 3.60 |
| 24 | 23.33 | 28.27 | 28.34 | 24.15 | 24.30 |
| 25 | 23.36 | 28.29 | 28.36 | 1.08 | .78 |
| 26 | 24.22 | 28.78 | 29.13 | .78 | 1.53 |
| 27 | 23.79 | 28.48 | 28.68 | 4.81 | 4.22 |
| 28 | 23.56 | 28.34 | 28.40 | 18.37 | 18.39 |
| 29 | 23.56 | 28.34 | 28.40 | 3.0 | 2.99 |
| 30 | 24.56 | 29.44 | 29.59 | 18.28 | 19.39 |
| 31 | 24.12 | 28.99 | 29.15 | 6.45 | 7.29 |
| 32 | 23.85 | 28.69 | 28.81 | 5.37 | 4.76 |
| 33 | 23.85 | 28.69 | 28.81 | 1.77 | 1.16 |
| 34 | 24.48 | 29.16 | 29.60 | 10.82 | 12.26 |
| 35 | 23.19 | 28.83 | 28.93 | .20 | 1.25 |
| 36 | 23.92 | 28.84 | 28.85 | 2.81 | 3.87 |

TABLE XV

MODEL C: PREDICTION INTERVAL SUMMARY

| OBS | LENGTH LS(IND) | LENGTH JGLS | LENGTH LS(CORR) | \|Δ\|LS ACTUAL-ESTIMATED | \|Δ\|JGLS ACTUAL-ESTIMATED |
|---|---|---|---|---|---|
| 1 | 22.02 | 23.38 | 23.90 | 6.96 | 8.0 |
| 2 | 21.73 | 23.17 | 23.56 | 2.02 | 2.79 |
| 3 | 21.76 | 23.17 | 23.57 | 2.19 | 2.93 |
| 4 | 20.71 | 23.02 | 23.35 | 10.58 | 10.05 |
| 5 | 20.92 | 22.88 | 22.98 | 1.89 | 1.62 |
| 6 | 20.92 | 22.82 | 22.95 | 1.83 | 1.59 |
| 7 | 20.91 | 22.81 | 22.95 | 4.32 | 4.10 |
| 8 | 20.92 | 22.82 | 22.95 | .89 | .64 |
| 9 | 20.92 | 22.87 | 22.98 | 3.9 | 4.17 |
| 10 | 20.92 | 22.82 | 22.95 | 1.41 | 1.65 |
| 11 | 20.92 | 22.82 | 22.95 | 13.67 | 13.42 |
| 12 | 20.92 | 22.82 | 22.95 | .03 | .22 |
| 13 | 20.91 | 22.86 | 22.97 | 1.07 | 1.31 |
| 14 | 20.91 | 22.88 | 22.97 | 4.62 | 4.86 |
| 15 | 21.30 | 23.46 | 23.58 | 1.43 | 1.18 |
| 16 | 21.29 | 23.45 | 23.56 | .41 | .79 |
| 17 | 21.29 | 23.45 | 23.56 | .98 | 1.36 |
| 18 | 21.29 | 23.45 | 23.56 | 1.39 | 1.77 |
| 19 | 21.38 | 23.66 | 23.77 | 3.04 | 2.35 |
| 20 | 21.38 | 23.68 | 23.79 | 3.01 | 3.48 |
| 21 | 20.87 | 23.07 | 23.17 | .24 | .76 |
| 22 | 21.19 | 22.97 | 23.35 | 1.80 | 1.75 |
| 23 | 21.05 | 22.87 | 23.17 | .60 | .95 |
| 24 | 21.04 | 22.86 | 23.17 | 21.46 | 21.77 |
| 25 | 21.30 | 23.01 | 23.40 | 3.96 | 3.58 |
| 26 | 21.28 | 23.15 | 23.38 | 2.95 | 3.31 |
| 27 | 21.11 | 23.05 | 23.24 | 5.05 | 4.44 |

## TABLE XVI

MODEL D: PREDICTION INTERVAL SUMMARY

| OBS | LENGTH LS(IND) | LENGTH JGLS | LENGTH LS(CORR) | \|Δ\|LS ACTUAL-ESTIMATED | \|Δ\|JGLS ACTUAL-ESTIMATED |
|---|---|---|---|---|---|
| 1 | 22.49 | 25.63 | 25.75 | 7.33 | 6.70 |
| 2 | 22.37 | 25.55 | 25.68 | .44 | .33 |
| 3 | 22.36 | 25.55 | 25.68 | .51 | .39 |
| 4 | 22.07 | 25.38 | 25.45 | 7.01 | 7.30 |
| 5 | 21.97 | 25.32 | 25.35 | .69 | .43 |
| 6 | 21.95 | 25.32 | 25.35 | .39 | .15 |
| 7 | 21.93 | 25.32 | 25.36 | 3.02 | 2.74 |
| 8 | 21.97 | 25.32 | 25.35 | .54 | .81 |
| 9 | 21.99 | 25.31 | 25.35 | 5.05 | 5.34 |
| 10 | 21.97 | 25.31 | 25.35 | 2.84 | 3.10 |
| 11 | 21.97 | 25.31 | 25.35 | 12.22 | 11.97 |
| 12 | 21.97 | 25.31 | 25.35 | 1.40 | 1.67 |
| 13 | 21.99 | 25.31 | 25.33 | 2.63 | 2.86 |
| 14 | 21.97 | 25.31 | 25.34 | 6.07 | 6.31 |
| 15 | 22.07 | 25.86 | 25.90 | .84 | 1.05 |
| 16 | 22.04 | 25.85 | 25.88 | 1.90 | 1.46 |
| 17 | 22.04 | 25.85 | 25.88 | 2.47 | 2.03 |
| 18 | 22.04 | 25.85 | 25.88 | 2.89 | 2.44 |
| 19 | 22.71 | 25.82 | 25.86 | 2.65 | 2.88 |
| 20 | 21.99 | 25.80 | 25.82 | 4.33 | 3.86 |
| 21 | 21.88 | 25.58 | 25.92 | .27 | .24 |
| 22 | 22.04 | 25.53 | 25.65 | 4.81 | 4.27 |
| 23 | 21.93 | 25.46 | 25.55 | .19 | .14 |
| 24 | 21.93 | 25.46 | 25.54 | 20.54 | 20.57 |
| 25 | 21.95 | 25.46 | 25.55 | 4.0 | 3.96 |
| 26 | 22.00 | 25.43 | 25.58 | 1.49 | 1.54 |
| 27 | 21.88 | 25.42 | 25.50 | 4.86 | 5.29 |
| 28 | 22.10 | 25.41 | 25.44 | 19.61 | 19.67 |
| 29 | 22.10 | 25.41 | 25.44 | 1.76 | 1.69 |
| 30 | 23.56 | 26.27 | 26.52 | 18.78 | 19.54 |
| 31 | 23.82 | 26.23 | 26.39 | 9.08 | 8.98 |
| 32 | 23.26 | 26.16 | 26.34 | 5.74 | 5.88 |
| 33 | 23.26 | 26.16 | 26.34 | 2.14 | 2.28 |
| 34 | 24.39 | 27.04 | 27.40 | 4.0 | 5.59 |
| 35 | 24.09 | 25.87 | 27.11 | 5.25 | 4.68 |
| 36 | 24.09 | 25.87 | 27.11 | 2.59 | 2.01 |

## TABLE XVII

### MODEL E: PREDICTION INTERVAL SUMMARY

| OBS | LENGTH LS(IND) | LENGTH JGLS | LENGTH LS(CORR) | \|Δ\|LS ACTUAL-ESTIMATED | \|Δ\|JGLS ACTUAL-ESTIMATED |
|---|---|---|---|---|---|
| 1 | 21.39 | 21.77 | 22.03 | 4.70 | 4.59 |
| 2 | 21.15 | 21.58 | 21.79 | .88 | 1.04 |
| 3 | 21.20 | 21.55 | 21.79 | .48 | .63 |
| 4 | 20.80 | 21.47 | 21.59 | 9.35 | 8.91 |
| 5 | 20.49 | 21.32 | 21.35 | 2.07 | 1.79 |
| 6 | 20.49 | 21.29 | 21.33 | 1.68 | 1.27 |
| 7 | 20.48 | 21.29 | 21.34 | 2.74 | 3.37 |
| 8 | 20.47 | 21.29 | 21.33 | .74 | .32 |
| 9 | 20.49 | 21.32 | 21.36 | 3.39 | 3.79 |
| 10 | 20.47 | 21.29 | 21.33 | 1.56 | 1.97 |
| 11 | 20.47 | 21.29 | 21.33 | 13.53 | 13.10 |
| 12 | 20.47 | 21.29 | 21.33 | .12 | .53 |
| 13 | 20.50 | 21.34 | 21.36 | .90 | 1.30 |
| 14 | 20.50 | 21.34 | 21.37 | 4.54 | 4.93 |
| 15 | 20.88 | 22.05 | 22.11 | 2.56 | 2.29 |
| 16 | 20.88 | 22.04 | 22.11 | .48 | .20 |
| 17 | 20.88 | 22.04 | 22.11 | .07 | .37 |
| 18 | 20.88 | 22.04 | 22.11 | .50 | .78 |
| 19 | 20.59 | 21.94 | 22.00 | 2.52 | 2.16 |
| 20 | 20.59 | 21.92 | 21.99 | 3.85 | 4.24 |
| 21 | 20.13 | 21.39 | 21.44 | .99 | 1.34 |
| 22 | 20.71 | 21.59 | 21.80 | 3.25 | 2.85 |
| 23 | 20.44 | 21.39 | 21.53 | .14 | .57 |
| 24 | 20.42 | 21.38 | 21.53 | 20.77 | 21.21 |
| 25 | 20.42 | 21.38 | 21.53 | 3.65 | 3.20 |
| 26 | 20.57 | 21.46 | 21.61 | 2.72 | 2.47 |
| 27 | 20.31 | 21.34 | 21.45 | 4.71 | 4.92 |

and estimated cost for both the least squares and the joint generalized least squares estimates are the same in all cases and have been dropped in the table for simplification.

The prediction interval length becomes longer progressing from least squares (independent) to joint generalized least squares and finally to least squares (correlated). Both least squares (correlated) and joint generalized least squares provide intervals which are significantly longer than those produced by the least squares (independent) method, while the joint generalized least squares method provides a small decrease in prediction interval length over the least squares (correlated) method. The latter two methods represent the prediction interval length more accurately assuming that there are correlations between subsystem CERs. Both of these methods require approximately the same degree of computational difficulty and thus it would appear that the joint generalized least squares technique offers the best alternative in accurately stating prediction intervals.

The mean square residual [MSR] value, not adjusted for degrees of freedom, was computed for the least squares estimates as compared to actual cost as was the same value for the joint estimates. It was intended to use these values as a way of comparing the two methods to see which provided the best estimates. But further examination led to the conclusion that there is no guarantee in using this technique that one method will be consistently better than the other.

In the joint generalized least squares case with heteroscedasticity the formulation of the problem is such that the length of the residual vector, after a transformation, is being minimized. Hence the solution we obtain is for the problem:

$$\text{Min } (Y - X\hat{\beta})(\Sigma^{-1} \otimes I)(Y - X\hat{\beta})$$

where $(\Sigma^{-1} \times I)$ is an unknown nonsingular matrix which is estimated by $(S^{-1} \times I)$

and for which $b_j = [X'(S^{-1} \otimes I)X]^{-1}X'(S^{-1} \otimes I)Y$ is the joint generalized least squares estimator as developed in Section III. This estimator, $b_j$, in fact does not minimize the length of the residual vector. Therefore the MSR method will not discriminate between the two methods of estimation in this case. For a complete discussion of this problem see Theil [Ref. 7].

The MSR results are contained in the following table:

TABLE XVIII

MODEL

| | B | C | D | E |
|---|---|---|---|---|
| LS MSR | 40.4 | 27.5 | 49.7 | 58.3 |
| JGLS MSR | 50.4 | 32.0 | 61.2 | 36.2 |

In models B, C and D the least squares estimates come out ahead whereas in model E the joint estimates come out ahead.

The total cost predictions by the least squares estimates and the joint estimates are very close when compared to each other. The Summary Tables present these values for comparison. On a worst case basis the differences between these two methods for each model, indicating the particular observation, are as follows:

MODEL/OBSERVATION

| | B/34 | C/1 | D/36 | E/7 |
|---|---|---|---|---|
| SIZE OF DIFFERENCE $/MILLION | 1.44 | 1.04 | 1.59 | .63 |

In no case is this difference greater than three percent of total estimated cost.

Utilizing the additional information available due to the correlations between subsystems, joint generalized least squares estimates should provide a more accurate weighting of the variables in each CER. This is important in assessing the contribution to subsystem cost of a particular variable. For example, consider the CER B-6:

OUTFITTING COST = .436 + .000386 [LSW] (Least Squares)

OUTFITTING COST = .549 + .000350 [LSW] (Joint Generalized Least Squares)

The coefficient of light ship weight (LSW) reflects the change in outfitting cost per pound for a change in light ship weight. The joint generalized least squares coefficient should provide a better reflection of the contribution of light ship weight to outfitting cost for this CER.

## VI. CONCLUSIONS

It has been demonstrated that the joint generalized least squares estimation technique, by taking advantage of the additional information which is provided when correlations exist between the residuals of subsystem CERs in aggregate cost estimating, provides more accurate and meaningful statements about predictions, the total cost variance surrounding the predictions, and the prediction intervals. The method involves an increase in computational difficulty when compared to one assuming independence between subsystem CERs, but gives a more accurate statement concerning the estimates and their uncertainty. A computer program has been provided which is readily usable in conducting joint generalized least squares analysis and the additional time required would be minimal.

The predictions utilizing the joint estimates provide gains in a more accurate weighting of the variables in the subsystem CERs. This allows a better factor analysis of the contributions to subsystem cost and hence to total cost.

The estimate for total cost variances used in the joint generalized least squares method provide a more meaningful statement taking into consideration the correlated disturbances. The least squares estimate assuming independence tends to substantially understate the total cost variance in the situation analyzed.

The prediction intervals obtained utilizing the joint generalized least squares method represented a slight gain over those obtained by the least squares method, assuming correlated disturbances. Both these methods provided more accurate prediction intervals under the assumptions made; however, with equal computational difficulty the joint generalized least squares method appears the superior of the two.

It should be emphasized that the joint generalized least squares method involves less restrictive assumptions than the least squares method. The assumption of correlations between subsystem CERs is a more reasonable statement of the true relationship as was shown by the tables of correlations between subsystem CERs. The least squares method does in fact tend to understate the uncertainty surrounding the cost estimate. Joint generalized least squares by utilizing this additional information gives more meaningful results in the derivation of the estimate and the statement of uncertainty surrounding the estimate.

The overall gains achieved by the joint generalized least squares method overshadow the small additional cost. This application to destroyer type ship models may be extended to any other situation where one is interested in aggregate predictions of the same nature.

## APPENDIX A

### DESCRIPTION OF SHIP'S CHARACTERISTICS USED AS EXPLANATORY VARIABLES IN THESIS

| | Characteristic | Symbol | Units | Definition |
|---|---|---|---|---|
| 1. | Light Ship Weight | LSW | long tons | Weight of ship complete with all items of outfit, equipment, and machinery but excluding cargo, stores, crew, etc. Includes lead ballast for surface ships but not for submarines. |
| 2. | Hull Weight | HULWGT | long tons | |
| 3. | Propulsion Weight | PROWGT | long tons | These are the weights of the seven groups as described in <u>Bureau of Ships Consolidated Index of Drawings, Materials, and Services Related to Construction Conversion.</u> |
| 4. | Electrical Weight | ELEWGT | long tons | |
| 5. | Communication and Control | C+CWGT | long tons | |
| 6. | Auxiliary Weight | AUXWGT | long tons | |
| 7. | Outfitting Weight | OUTWGT | long tons | |
| 8. | Armament Weight | ARMWGT | long tons | |
| 9. | Complement | COMP | integer | Allowance for all officers and men on board. |
| 10. | Maximum Shaft Horsepower | MAXSHP | | Total power that can be applied continuously to the shafts under designed operating conditions. For subs this applies to surface operation. |
| 11. | Range | RANGE | nautical miles | |

| | Characteristic | Symbol | Units | Definition |
|---|---|---|---|---|
| 12. | Series | SERIES | integer | A number that represents the position of a ship in a series of similar ships. For example, DD 936 to DDG 30 have the same basic design except for missiles and constitute a series of 39 similar ships. DD 936 would be assigned series number 1 and DDG 30 would be assigned series number 39. |
| 13. | DE or DEG | DEDEG | integer | 1 = DE or DEG; zero otherwise. |
| 14. | Prototype Dummy | PROTO | integer | 1 = protytype; zero otherwise. |
| 15. | Number of Generators | NO-GEN | integer | Number of ship service generators |
| 16. | Power Loading Factor | PWRLD | | Ratio of maximum shaft horsepower to full displacement. |
| 17. | Award Date | AWARD | years | Last two digits of the year in which the contract was awarded. |
| 18. | Generator Capacity | TKWCPY | kilowatts | Total maximum output of all generators. |
| 19. | Propulsion and Auxiliary Weight | PRAXWT | long tons | Propulsion weight plus auxiliary weight. |
| 20. | Prototype Light Ship Weight | PROLSW | | For prototypes, this takes on the value of LSW. For non-prototypes, it has the value zero. |
| 21. | Missile End Dummy | MS-END | integer | 0 = no launchers; 1 = a launcher at one end of the ship; 2 = a launcher at each end of the ship. |

| | Characteristic | Symbol | Units | Definition |
|---|---|---|---|---|
| 22. | Armament Weight-to-Light Ship Weight Ratio | AR/LSW | | Ratio of armament weight tc light ship weight. |
| 23. | Engineering Weight | ENGWGT | long tons | Total weight of all propulsion, electrical, and auxiliary equipment. |
| 24. | Payload Weight | PAYWGT | long tons | Total weight of all C+C, armament and outfitting equipment. |
| 25. | Engine and Payload Weight | ENGPAY | long tons | ENGWGT + PAYWGT |

## APPENDIX B

### BASIC CONTRACT COST CATEGORIES

| Category No. | Category Name | Includes |
|---|---|---|
| 1 | Hull Structure | Shell plating and planking, logitudinal and transverse frames, decks, superstructure, armor, etc. |
| 2 | Propulsion | Boilers and energy converters, propulsion units, uptakes, propulsion control equipment, feedwater and condensate system, etc. |
| 3 | Electric Plant | Electric power generators, power distribution switchboards and cables, lighting systems, etc. |
| 4 | Communication and Control | Navigation equipment, interior communication equipment, fire control systems, radar systems, radio communications systems, sonar systems, etc. |
| 5 | Auxiliary | Heating, ventilating, air conditioning, plumbing, elevators, arresting gears, rudders, etc. |
| 6 | Outfit and Furnishing | Hull fittings, nonstructural bulkheads, paintings, equipment for work shops, furnishings for quarters, etc. |
| 7 | Armament | Guns and gun mount, ammunition handling and storage systems, other weapon systems handling and storage systems, etc. |
| 8 | Design and Engineering Services | Contract drawings, working drawings, technical manuals, lofting, mock-up and models, etc. |
| 9 | Construction | Staging, scaffolding and cribbing, launching, trials, cleaning ship, drydocking, etc. |

## APPENDIX B (CONT.)

### END COST CATEGORIES

| | |
|---|---|
| Miscellaneous End Cost | Disaster costs; cost of hull, mechanical and electrical changes; post-delivery costs; etc. |
| Weapons End Cost | Weapons costs after contractor delivery; missile, ASROC systems; etc. |
| Electronics End Cost | Electronics costs after contractor delivery; radar, NTDS, fire control systems; etc. |

# APPENDIX C

## COMPUTER OUTPUT

### MODEL B - 9 CERs - 36 OBSERVATIONS

```
MULTIPLE REGRESSION.....GENRAL
     SELECTION..... 1

SELECTION CARD 0 2 114
          LEAST SQUARES REGRESSION COEFICIENTS

    0.040621    0.138546
STD. ERROR OF ESTIMATE=       0.605304

          COVARIANCE MATRIX OF THE ESTIMATES
     0.78264D-01 -0.37885D-02
    -0.37885D-02  0.21080D-03
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.720130

MULTIPLE REGRESSION.....GENRAL
     SELECTION..... 2

SELECTION CARD 0 3 23331
          LEAST SQUARES REGRESSION COEFICIENTS

   58.666710    -6.091649    0.000729
STD. ERROR OF ESTIMATE=       1.013978

          COVARIANCE MATRIX OF THE ESTIMATES
     0.30190D 00 -0.45721D-01  0.26075D-05
    -0.45721D-01  0.11551D-01 -0.11592D-05
     0.26075D-05 -0.11592D-05  0.15884D-09
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.754360
```

```
MULTIPLE REGRESSION.....GENRAL
     SELECTION..... 3

SELECTION CARD 0 4 21625
        LEAST SQUARES REGRESSION COEFICIENTS

    0.148642     0.386163     0.261561
STD. ERROR OF ESTIMATE=       0.293750

        COVARIANCE MATRIX OF THE ESTIMATES

    0.36839D-01 -0.17418D-03 -0.10179D-01
   -0.17418D-03  0.10224D-01 -0.39785D-02
   -0.10179D-01 -0.39785D-02  0.45967D-02
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.660927
```

```
MULTIPLE REGRESSION.....GENRAL
     SELECTION..... 4

SELECTION CARD 0 5 21729
        LEAST SQUARES REGRESSION COEFICIENTS

   -0.903701     3.173884     0.347513
STD. ERROR OF ESTIMATE=       3.643863

        COVARIANCE MATRIX OF THE ESTIMATES

    0.25973D 01 -0.12129D 01 -0.77927D 00
   -0.12129D 01  0.69871D 00  0.16562D 00
   -0.77927D 00  0.16562D 00  0.20063D 01
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.263167
```

```
MULTIPLE REGRESSION.....GENRAL
     SELECTION..... 5

SELECTION CARD C 6 118
         LEAST SQUARES REGRESSION COEFICIENTS

    0.026382    0.224177
STD. ERROR OF ESTIMATE=       0.570275

         COVARIANCE MATRIX OF THE ESTIMATES
    0.68175D-01 -0.59424D-02
   -0.59424D-02  0.59707D-03
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.703815


MULTIPLE REGRESSION.....GENRAL
     SELECTION..... 6

SELECTION CARD C 7 121
         LEAST SQUARES REGRESSION COEFICIENTS

    0.435838    0.038576
STD. ERROR OF ESTIMATE=       0.319834

         COVARIANCE MATRIX OF THE ESTIMATES
    0.20947D-01 -0.56669D-03
   -0.56669D-03  0.17737D-04
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.703134
```

```
MULTIPLE REGRESSION.....GENRAL
     SELECTION..... 7

SELECTION CARD 0 8 22820
               LEAST SQUARES REGRESSION COEFICIENTS

   -2.608855     9.178115     3.764265
STD. ERROR OF ESTIMATE=       3.793802

               COVARIANCE MATRIX OF THE ESTIMATES
       0.23557D 01  0.92404D 00 -0.12052D 01
       0.92404D 00  0.27109D 01 -0.13240D 01
      -0.12052D 01 -0.13240D 01  0.99298D 00
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.865614


MULTIPLE REGRESSION.....GENRAL
     SELECTION..... 8

SELECTION CARD 0 9 22023
               LEAST SQUARES REGRESSION COEFICIENTS

   -1.012537     0.653981     0.151729
STD. ERROR OF ESTIMATE=       1.307084

               COVARIANCE MATRIX OF THE ESTIMATES
       0.24949D 00 -0.88502D-01 -0.12751D-02
      -0.88502D-01  0.41244D-01 -0.17294D-03
      -0.12751D-02 -0.17294D-03  0.22420D-03
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.766887
```

```
MULTIPLE REGRESSION.....GENRAL
     SELECTION..... 9

SELECTION CARD 010 3172223
        LEAST SQUARES REGRESSION COEFICIENTS

    3.137940    3.949713  -59.846808    0.059936
STD. ERROR OF ESTIMATE=       1.643151

        COVARIANCE MATRIX OF THE ESTIMATES
    0.11594D 01 -0.12597D 00 -0.12864D 02 -0.40064D-02
   -0.12597D 00  0.15728D 00 -0.19555D 01 -0.12869D-02
   -0.12864D 02 -0.19555D 01  0.24024D 03  0.52447D-01
   -0.40064D-02 -0.12869D-02  0.52447D-01  0.36985D-03
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.776752
```

MATRIX OF RESIDUALS FOR SELECTIONS

| CASE NO. | SELECTION NUMBER 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | -0.57461 | 1.09332 | 0.13241 | -3.78724 | -1.51320 | 0.55974 | -0.58823 | -3.52181 | -2.71784 |
| 2 | -0.55461 | -1.84609 | 0.11241 | -2.84973 | -0.21320 | 0.19974 | -1.21823 | -0.26781 | 0.35812 |
| 3 | -0.15828 | -8.68408 | 0.47241 | -3.29973 | 0.81535 | 0.10574 | -1.15823 | 0.18219 | -1.07188 |
| 4 | -0.32745 | 1.82170 | -0.04897 | -0.87724 | -0.61215 | 0.00748 | -1.15521 | 3.12858 | 0.42541 |
| 5 | -0.64352 | -10.51763 | -0.12145 | -0.47060 | -0.29542 | -0.20721 | -1.22342 | 0.10181 | 1.06182 |
| 6 | 0.53172 | -9.36082 | -0.11759 | -0.45973 | 0.85939 | -0.37412 | -1.31635 | -0.00781 | -1.19838 |
| 7 | -0.26004 | -2.09861 | 0.22103 | 0.19027 | 0.87838 | -0.33442 | -1.13521 | 0.21451 | 0.63571 |
| 8 | -0.46828 | 4.97080 | 0.05241 | -0.86973 | -0.65061 | -0.38412 | -3.40635 | -0.22781 | 1.00162 |
| 9 | 0.22720 | 0.27929 | -0.10987 | -1.40233 | -0.53444 | -0.27148 | -4.74106 | -0.53473 | -1.98358 |
| 10 | -0.24828 | -0.36339 | 0.37241 | 0.06027 | 0.28939 | -0.21412 | -4.78635 | -0.48781 | -0.74838 |
| 11 | -0.37828 | -5.24771 | 0.01241 | 0.32027 | -0.02061 | -0.04412 | 7.97365 | -0.06781 | 1.33162 |
| 12 | 0.02172 | -9.10375 | 0.31241 | -0.99973 | 1.46539 | -0.05412 | -4.75635 | -0.11781 | 0.60162 |
| 13 | 0.40937 | -8.33254 | -0.26531 | -1.61060 | 0.14350 | -0.23575 | -1.71814 | -0.14511 | -0.88181 |
| 14 | -0.43075 | -7.88267 | 0.09330 | -1.03407 | -0.40835 | -0.15881 | -4.26521 | -0.12549 | -0.80073 |
| 15 | 0.74263 | -5.82074 | -0.09802 | -0.21206 | -0.21929 | 0.11268 | 0.50980 | -1.15966 | -0.48602 |
| 16 | 0.18263 | -0.57696 | 0.04198 | 0.19545 | 0.03071 | 0.23268 | 0.43980 | 0.82281 | 0.16314 |
| 17 | 0.29263 | -7.39824 | 0.11198 | 0.28545 | -0.02929 | 0.37268 | 0.63980 | 1.19281 | 0.65314 |
| 18 | 0.64263 | -0.38594 | 0.29198 | 0.51545 | -0.07929 | 0.25268 | 0.44980 | 1.33281 | 0.42314 |
| 19 | 0.32070 | -8.47721 | -0.50779 | -0.28858 | -0.32135 | -0.45433 | 1.36315 | -0.26774 | -2.00991 |
| 20 | 0.59070 | -1.44299 | -0.32779 | 0.18893 | -0.17135 | -0.28433 | 2.80315 | 1.23095 | 0.84121 |
| 21 | -0.61975 | 9.23266 | -0.29306 | 1.29665 | -0.11935 | 0.44398 | 1.59030 | -0.75318 | -0.65711 |
| 22 | -0.23756 | 1.16228 | -0.07268 | -0.17985 | 0.31078 | 0.08072 | 0.11638 | -1.84896 | 1.28331 |
| 23 | -0.56111 | -0.78136 | 0.14732 | -0.09234 | 0.27854 | 0.09499 | 0.60873 | 0.67932 | 1.54886 |
| 24 | -0.08955 | 8.35373 | 0.12732 | 17.86766 | -0.33474 | -0.54652 | 0.36581 | 0.62970 | 4.00863 |
| 25 | -0.27509 | 4.75800 | -0.05724 | -1.22668 | 0.19473 | -0.02537 | 0.81752 | 0.98818 | -2.60364 |
| 26 | -0.22464 | 3.33332 | -0.00724 | 1.30015 | 0.31922 | -0.06124 | 1.09241 | -3.31523 | 0.23002 |
| 27 | -0.32719 | 2.74623 | 0.07890 | 0.01419 | 0.36990 | 0.03140 | -4.09345 | 0.60738 | -2.09494 |
| 28 | 0.40487 | 18.16768 | 0.44359 | -1.64232 | -0.26255 | 0.63327 | 5.97216 | 0.68018 | 3.57978 |
| 29 | -0.09513 | 13.37890 | -0.49641 | -2.55232 | -0.47255 | 0.02327 | -2.93784 | 0.45018 | -0.89022 |
| 30 | 0.21293 | 17.53721 | -0.35306 | -1.48113 | 0.00356 | 0.56190 | 10.70899 | 2.51348 | 0.18506 |
| 31 | 0.32839 | -3.13824 | -0.56375 | -2.81404 | 0.17787 | 0.13641 | 5.60256 | -0.51183 | 1.98930 |
| 32 | 1.65253 | 5.51085 | -0.60306 | -3.32362 | 0.13356 | 0.36190 | -4.85101 | -0.51643 | -1.31276 |
| 33 | 1.63253 | 5.87560 | 0.30694 | -1.46362 | 0.71356 | 0.01190 | -4.72101 | -0.84643 | 1.36724 |
| 34 | -0.53580 | 1.47415 | 0.13316 | 4.22931 | -0.55233 | 0.12701 | 5.53688 | 0.86243 | 1.07835 |
| 35 | -0.94555 | -4.17831 | 0.14446 | 3.16249 | -0.79560 | 0.00890 | 0.64038 | -0.57712 | -2.01522 |
| 36 | 0.16559 | -3.89846 | 0.43446 | 3.31075 | 0.61785 | -0.71302 | 0.84038 | -0.31712 | -1.35070 |

MATRIX OF CORRELATIONS BETWEEN SELECTIONS

| SELECTION NO. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.000000 | 0.083952 | -0.226513 | -0.141016 | 0.359145 | 0.021498 | -0.094030 | 0.105750 | 0.102316 |
| 2 | 0.083952 | 1.000000 | -0.109229 | 0.105069 | -0.238084 | 0.424064 | 0.262607 | 0.160298 | 0.252176 |
| 3 | -0.226513 | -0.109229 | 1.000000 | 0.194853 | 0.234251 | -0.050094 | -0.158535 | -0.023243 | 0.179807 |
| 4 | -0.141016 | 0.105069 | 0.194853 | 1.000000 | -0.071872 | -0.405310 | 0.139125 | 0.115054 | 0.401507 |
| 5 | 0.359145 | -0.238084 | 0.234251 | -0.071872 | 1.000000 | -0.220777 | -0.152113 | 0.057277 | 0.101151 |
| 6 | 0.021498 | 0.424064 | -0.050094 | -0.405310 | -0.220777 | 1.000000 | 0.322134 | 0.021004 | 0.058955 |
| 7 | -0.094030 | 0.262607 | -0.158535 | 0.139125 | -0.152113 | 0.322134 | 1.000000 | 0.251701 | 0.362376 |
| 8 | 0.105750 | 0.160298 | -0.023243 | 0.115054 | 0.057277 | 0.021004 | 0.251701 | 1.000000 | 0.234359 |
| 9 | 0.102316 | 0.252176 | 0.179807 | 0.401507 | 0.101151 | 0.058955 | 0.362376 | 0.234359 | 1.000000 |

JOINT GENERALIZED LEAST SQUARES REGRESSION OUTPUT

COVARIANCE MATRIX OF THE JOINT ESTIMATES

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0.72531D-01 | -0.35232D-02 | -0.54559D-01 | 0.15208D-01 | -0.14361D-05 | -0.82822D-02 | 0.35059D-02 | 0.77179D-03 | -0.49378D-01 | 0.24547D-01 |
| -0.35232D-02 | 0.15604D-03 | 0.35833D-02 | -0.84615D-03 | 0.79908D-07 | 0.40282D-03 | -0.19508D-03 | -0.42944D-04 | 0.23002D-02 | -0.13658D-02 |
| -0.54559D-01 | 0.35833D-02 | 0.12443D 02 | -0.15941D 01 | 0.13295D-03 | 0.50157D-01 | -0.44530D-02 | -0.14981D-01 | -0.80776D-01 | 0.12568D 00 |
| 0.15208D-01 | -0.84615D-03 | -0.15941D 01 | 0.50123D 00 | -0.50087D-04 | -0.13930D-01 | 0.13504D-02 | 0.35963D-02 | 0.30547D-01 | -0.23523D-01 |
| -0.14361D-05 | 0.79908D-07 | 0.13295D-03 | -0.50087D-04 | 0.64277D-08 | 0.13741D-05 | -0.16207D-06 | -0.34493D-06 | -0.23663D-05 | 0.15763D-05 |
| -0.82822D-02 | 0.40282D-03 | 0.50157D-01 | -0.13930D-01 | 0.13741D-05 | 0.30250D-01 | -0.15635D-02 | -0.75724D-02 | 0.28565D-01 | -0.14059D-01 |
| 0.35059D-02 | -0.19508D-03 | -0.44530D-02 | 0.13504D-02 | -0.16207D-06 | -0.15635D-02 | 0.76126D-02 | -0.24168D-02 | -0.20777D-01 | 0.11466D-01 |
| 0.77179D-03 | -0.42944D-04 | -0.14981D-01 | 0.35963D-02 | -0.34493D-06 | -0.75724D-02 | -0.24168D-02 | 0.32057D-02 | 0.12660D-02 | -0.23736D-03 |
| -0.49378D-01 | 0.23002D-02 | -0.80776D-01 | 0.30547D-01 | -0.23663D-05 | 0.28565D-01 | -0.20777D-01 | 0.12660D-02 | 0.21768D 01 | -0.10316D 01 |
| 0.24547D-01 | -0.13658D-02 | 0.12568D 00 | -0.23523D-01 | 0.15763D-05 | -0.14059D-01 | 0.11466D-01 | -0.33736D-03 | -0.10316D 01 | 0.60081D 00 |
| 0.72947D-03 | -0.40585D-04 | -0.25562D 00 | 0.35569D-01 | -0.11066D-05 | 0.11876D-02 | 0.62095D-02 | -0.28012D-02 | -0.43628D 00 | 0.56977D-01 |
| 0.24563D-01 | -0.11858D-02 | 0.10050D 00 | -0.31163D-01 | 0.20561D-05 | 0.57159D-02 | -0.16134D-02 | -0.18741D-02 | -0.16252D-01 | 0.54765D-02 |
| -0.21412D-02 | 0.11914D-03 | -0.12729D-01 | 0.31312D-02 | -0.30707D-06 | -0.87432D-03 | 0.18220D-03 | 0.18830D-02 | 0.14501D-02 | -0.55217D-03 |
| 0.58454D-03 | -0.26450D-04 | -0.51443D-01 | 0.31115D-01 | -0.57490D-05 | -0.10450D-02 | -0.23559D-03 | 0.38615D-02 | -0.85652D-01 | 0.43150D-01 |
| -0.14678D-04 | 0.82785D-06 | 0.36808D-02 | -0.59160D-03 | 0.10165D-06 | 0.29025D-04 | 0.72235D-05 | -0.11523D-04 | 0.22995D-02 | -0.13560D-02 |
| -0.32456D-01 | 0.14554D-02 | -0.42781D 00 | 0.15046D 00 | -0.14627D-04 | -0.41314D-01 | 0.20190D-02 | 0.10157D-01 | 0.17384D 00 | -0.82804D-01 |
| 0.14213D-02 | -0.75083D-04 | 0.40251D-01 | -0.10759D-01 | 0.11372D-05 | -0.66727D-02 | -0.45513D-02 | 0.40124D-02 | 0.10302D-02 | -0.24068D-02 |
| 0.11875D-01 | -0.66077D-03 | 0.27016D 00 | -0.65513D-01 | 0.63424D-05 | 0.19194D-01 | 0.72656D-03 | -0.59978D-02 | -0.57706D-01 | 0.38640D-01 |
| 0.13252D-01 | -0.61928D-03 | -0.13460D 00 | 0.39683D-01 | -0.36107D-05 | -0.23554D-02 | 0.13757D-03 | 0.59068D-03 | 0.59905D-01 | -0.28038D-01 |
| -0.50314D-02 | 0.27956D-03 | 0.82381D-01 | -0.18352D-01 | 0.16329D-05 | 0.59731D-03 | -0.10161D-03 | -0.25667D-03 | -0.20036D-01 | 0.12476D-01 |
| -0.24153D-04 | 0.13439D-05 | -0.66578D-03 | 0.35697D-04 | 0.76678D-08 | -0.36261D-06 | 0.11254D-04 | -0.43445D-05 | -0.31812D-03 | 0.11975D-03 |
| 0.27753D-01 | -0.14024D-02 | -0.44355D 00 | 0.10755D 00 | -0.87479D-05 | 0.15260D-01 | -0.46625D-02 | -0.32213D-02 | 0.33650D 00 | -0.16815D 00 |
| -0.73224D-02 | 0.40795D-03 | 0.14155D 00 | -0.35044D-01 | 0.34476D-05 | -0.46054D-02 | 0.54547D-02 | -0.78010D-03 | -0.15881D 00 | 0.95598D-01 |
| -0.15267D 00 | 0.10776D-01 | 0.44209D 01 | -0.74867D 00 | 0.43872D-04 | -0.14021D 00 | -0.76116D-01 | 0.71723D-01 | 0.75335D-01 | 0.56655D-01 |
| -0.28140D-04 | 0.15658D-05 | -0.53379D-03 | 0.42092D-04 | 0.12232D-07 | -0.15428D-04 | 0.71814D-04 | -0.23718D-04 | -0.20220D-02 | 0.47670D-03 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0.72947D-03 | 0.24563D-01 | -0.21412D-02 | 0.58454D-03 | -0.14678D-04 | -0.32456D-01 | 0.14213D-02 | 0.11875D-01 | 0.13252D-01 | -0.50314D-02 |
| -0.40585D-04 | -0.11858D-02 | 0.11914D-03 | -0.26450D-04 | 0.82785D-06 | 0.14554D-02 | -0.75083D-04 | -0.66077D-03 | -0.61928D-03 | 0.27956D-03 |
| -0.25562D 00 | 0.10050D 00 | -0.12729D-01 | -0.51443D-01 | 0.36808D-02 | -0.42781D 00 | 0.40251D-01 | 0.27016D 00 | -0.13460D 00 | 0.82381D-01 |
| 0.35569D-01 | -0.31163D-01 | 0.31312D-02 | 0.31115D-01 | -0.59160D-03 | 0.15046D 00 | -0.10759D-01 | -0.65513D-01 | 0.39683D-01 | -0.18352D-01 |
| -0.11066D-05 | 0.20561D-05 | -0.30707D-06 | -0.57490D-05 | 0.10165D-06 | -0.14627D-04 | 0.11372D-05 | 0.63424D-05 | -0.36107D-05 | 0.16329D-05 |
| 0.11876D-02 | 0.57159D-02 | -0.87432D-03 | -0.10450D-02 | 0.29025D-04 | -0.41314D-01 | -0.66727D-02 | 0.19194D-01 | -0.23554D-02 | 0.59731D-03 |
| 0.62095D-02 | -0.16134D-02 | 0.18220D-03 | -0.23559D-03 | 0.72235D-05 | 0.20190D-02 | -0.45513D-02 | 0.72656D-03 | 0.13757D-03 | -0.10161D-03 |
| -0.28012D-02 | -0.18741D-02 | 0.18830D-02 | 0.38615D-02 | -0.11523D-04 | 0.10157D-01 | 0.40124D-02 | -0.59978D-02 | 0.59068D-03 | -0.25667D-03 |
| -0.43628D 00 | -0.16252D-01 | 0.14501D-02 | -0.85652D-01 | 0.22995D-02 | 0.17384D 00 | 0.10302D-02 | -0.57706D-01 | 0.59905D-01 | -0.20036D-01 |
| 0.56977D-01 | 0.54765D-02 | -0.55217D-03 | 0.43150D-01 | -0.13560D-02 | -0.82804D-01 | -0.24068D-02 | 0.38640D-01 | -0.28038D-01 | 0.12476D-01 |
| 0.10547D 01 | -0.58261D-01 | 0.56535D-03 | 0.45385D-02 | -0.14265D-03 | 0.55684D-01 | 0.12020D-01 | -0.29663D-01 | 0.42164D-02 | -0.35255D-02 |
| -0.58261D-01 | 0.62268D-01 | -0.13553D-02 | -0.62210D-01 | 0.16476D-03 | -0.38488D-01 | 0.31127D-02 | 0.12741D-01 | 0.78511D-02 | -0.30082D-02 |
| 0.56535D-03 | -0.13553D-02 | 0.54251D-03 | 0.52857D-03 | -0.16556D-04 | -0.30125D-02 | -0.31267D-03 | -0.12602D-02 | -0.67858D-03 | 0.30225D-03 |
| 0.45385D-02 | -0.62210D-01 | 0.52857D-03 | 0.11079D 00 | -0.48242D-05 | 0.47312D-01 | -0.55635D-02 | -0.15254D-01 | 0.43755D-03 | -0.16620D-03 |
| -0.14265D-03 | 0.16476D-03 | -0.16556D-04 | -0.48242D-05 | 0.11099D-05 | -0.11645D-02 | 0.17413D-03 | -0.47742D-03 | -0.66040D-05 | 0.52018D-05 |
| 0.55684D-01 | -0.38488D-01 | -0.30125D-02 | 0.47312D-01 | -0.11645D-02 | 0.25355D 01 | 0.55352D 00 | -0.50466D 00 | 0.14866D 00 | -0.54186D-01 |
| 0.12020D-01 | 0.31127D-02 | -0.31267D-03 | -0.55635D-02 | 0.17413D-03 | 0.55352D 00 | 0.16316D 01 | -0.79566D 00 | 0.14966D-02 | -0.11628D-02 |
| -0.29663D-01 | 0.12741D-01 | -0.12602D-02 | -0.15254D-01 | -0.47742D-03 | -0.50466D 00 | -0.79566D 00 | 0.67562D 00 | -0.54257D-01 | 0.25281D-01 |
| 0.42164D-02 | 0.78511D-02 | -0.67858D-03 | 0.43755D-03 | -0.66040D-05 | 0.14866D 00 | 0.14966D-02 | -0.54257D-01 | 0.22551D 00 | -0.80060D-01 |
| -0.35255D-02 | -0.30082D-02 | 0.30225D-03 | -0.16620D-03 | 0.52018D-05 | -0.54186D-01 | -0.11628D-02 | 0.25281D-01 | -0.80060D-01 | 0.37261D-01 |
| 0.46536D-03 | -0.27536D-04 | 0.28065D-05 | 0.20446D-04 | -0.64057D-06 | 0.12812D-03 | 0.14044D-03 | -0.10454D-03 | -0.10452D-02 | -0.14050D-03 |
| 0.23085D-01 | -0.27543D-01 | -0.25351D-02 | 0.52885D-02 | -0.14068D-03 | 0.38657D 00 | 0.52146D 00 | -0.16506D 00 | 0.85645D-01 | -0.33328D-01 |
| -0.25954D-02 | -0.75855D-02 | 0.80236D-03 | 0.62026D-03 | -0.15414D-04 | -0.85415D-01 | 0.16557D-01 | 0.35574D-01 | -0.22688D-01 | 0.10411D-01 |
| -0.68156D 00 | -0.17271D 00 | 0.17316D-01 | -0.54232D-01 | -0.22424D-03 | -0.27452D 01 | -0.12098D 01 | 0.16622D 01 | -0.51723D 00 | 0.24134D 00 |
| 0.48905D-02 | -0.73097D-04 | 0.73446D-05 | -0.21620D-05 | 0.47665D-07 | -0.48158D-04 | -0.85121D-04 | 0.50368D-04 | -0.14471D-03 | -0.67535D-06 |

| | | | | |
|---|---|---|---|---|
| -0.24153D-04 | 0.27753D-01 | -0.73224D-02 | -0.15267D 00 | -0.28140D-04 |
| 0.13439D-05 | -0.14024D-02 | 0.40795D-03 | 0.10776D-01 | 0.15658D-05 |
| -0.66578D-03 | -0.44355D 00 | 0.14155D 00 | 0.44209D 01 | -0.53379D-03 |
| 0.35697D-04 | 0.10755D 00 | -0.35044D-01 | -0.74867D 00 | 0.42092D-04 |
| 0.76678D-08 | -0.87479D-05 | 0.34476D-05 | 0.43872D-04 | 0.12232D-07 |
| -0.36261D-06 | 0.15260D-01 | -0.46054D-02 | -0.14021D 00 | -0.15428D-04 |
| 0.11254D-04 | -0.46625D-02 | 0.54547D-02 | -0.76116D-01 | 0.71814D-04 |
| -0.43445D-05 | -0.32213D-02 | -0.78010D-03 | 0.71723D-01 | -0.23718D-04 |
| -0.31812D-03 | 0.33650D 00 | -0.15881D 00 | 0.75335D-01 | -0.20220D-02 |
| 0.11975D-03 | -0.16815D 00 | 0.95598D-01 | 0.56655D-01 | 0.47670D-03 |
| 0.46536D-03 | 0.23085D-01 | -0.25954D-02 | -0.68156D 00 | 0.48905D-02 |
| -0.27536D-04 | -0.27543D-01 | -0.75855D-02 | -0.17271D 00 | -0.73097D-04 |
| 0.28065D-05 | -0.25351D-02 | 0.80236D-03 | 0.17316D-01 | 0.73446D-05 |
| 0.20446D-04 | 0.52885D-02 | 0.62026D-03 | -0.54232D-01 | -0.21620D-05 |
| -0.64057D-06 | -0.14068D-03 | -0.15414D-04 | -0.22424D-03 | 0.47665D-07 |
| 0.12812D-03 | 0.38657D 00 | -0.85415D-01 | -0.27452D 01 | -0.48158D-04 |
| 0.14044D-03 | 0.52146D 00 | 0.16557D-01 | -0.12098D 01 | -0.85121D-04 |
| -0.10454D-03 | -0.16506D 00 | 0.35574D-01 | 0.16622D 01 | 0.50368D-04 |
| -0.10452D-02 | 0.85645D-01 | -0.22688D-01 | -0.51723D 00 | -0.14471D-03 |
| -0.14050D-03 | -0.33328D-01 | 0.10411D-01 | 0.24134D 00 | -0.67535D-06 |
| 0.18409D-03 | -0.75415D-04 | 0.31736D-05 | -0.10455D-02 | 0.15846D-04 |
| -0.75415D-04 | 0.85553D 00 | -0.13645D 00 | 0.82782D 01 | -0.23071D-02 |
| 0.31736D-05 | -0.13645D 00 | 0.11783D 00 | -0.89063D 00 | -0.68853D-03 |
| -0.10455D-02 | 0.82782D 01 | -0.89063D 00 | 0.14495D 02 | 0.26843D-01 |
| 0.15846D-04 | -0.23071D-02 | -0.68853D-03 | 0.26843D-01 | 0.22103D-03 |

COMPUTATIONAL DETERMINANT CHECK= 0.99205140 48

JCINT LEAST SCUARES ESTIMATES

| SELECTICN NO. | CONSTANT | B1 | B2 | B3 |
| --- | --- | --- | --- | --- |
| | | COEFICIENTS | | |
| 1 | C.C83120 | 0.136181 | | |
| 2 | 58.7C4516 | -5.925497 | 0.000697 | |
| 3 | C.031C10 | 0.352351 | C.3C9889 | |
| 4 | -C.620830 | 2.963455 | 0.627307 | |
| 5 | -C.005553 | 0.227385 | | |
| 6 | 0.549161 | 0.035C29 | | |
| 7 | -2.576C43 | 8.845534 | 3.859541 | |
| 8 | -C.926C37 | 0.638683 | C.144506 | |
| 9 | 2.5204C5 | 3.662766 | -42.152667 | 0.C51540 |

TABLE OF PREDICTION INTERVAL VALUES

| | PI + | PREDICTED | ACTUAL | PI - |
|---|---|---|---|---|
| 1 | 53.343975 | 38.965281 | 27.270000 | 24.586588 |
| 2 | 46.997922 | 32.725161 | 28.390000 | 18.452401 |
| 3 | 47.295003 | 33.020052 | 28.400000 | 18.745101 |
| 4 | 61.820072 | 47.638389 | 51.610000 | 33.456706 |
| 5 | 56.293039 | 42.233032 | 40.040000 | 28.173025 |
| 6 | 55.445687 | 41.384644 | 39.150000 | 27.323601 |
| 7 | 54.759366 | 40.701178 | 41.020000 | 26.642991 |
| 8 | 55.445687 | 41.384644 | 38.200000 | 27.323601 |
| 9 | 56.566354 | 42.505990 | 34.570000 | 28.445627 |
| 10 | 55.445687 | 41.384644 | 35.910000 | 27.323601 |
| 11 | 55.445687 | 41.384644 | 50.980000 | 27.323601 |
| 12 | 55.445687 | 41.384644 | 37.340000 | 27.323601 |
| 13 | 56.425623 | 42.367731 | 37.320000 | 28.309840 |
| 14 | 56.314486 | 42.257164 | 33.690000 | 28.199842 |
| 15 | 26.982126 | 12.552976 | 11.320000 | -1.876175 |
| 16 | 23.871400 | 9.500342 | 11.510000 | -4.870715 |
| 17 | 23.871400 | 9.500342 | 12.080000 | -4.870715 |
| 18 | 23.871400 | 9.500342 | 12.490000 | -4.870715 |
| 19 | 25.638338 | 11.070105 | 8.700000 | -3.498128 |
| 20 | 22.258424 | 7.756960 | 12.930000 | -6.744504 |
| 21 | 30.654329 | 16.236835 | 18.490000 | 1.819341 |
| 22 | 40.509800 | 26.250017 | 25.090000 | 11.990233 |
| 23 | 34.528324 | 20.390131 | 23.990000 | 6.251937 |
| 24 | 34.540087 | 20.406466 | 44.700000 | 6.272846 |
| 25 | 35.180142 | 21.035734 | 20.250000 | 6.891326 |
| 26 | 51.356770 | 36.968139 | 38.490000 | 22.579509 |
| 27 | 45.738793 | 31.497752 | 27.270000 | 17.256711 |
| 28 | 63.377046 | 49.206398 | 67.590000 | 35.035749 |
| 29 | 63.394785 | 49.224009 | 46.230000 | 35.053232 |
| 30 | 89.087878 | 74.517103 | 93.900000 | 59.946327 |
| 31 | 82.519059 | 68.023626 | 75.310000 | 53.528193 |
| 32 | 78.410976 | 64.069823 | 59.300000 | 49.728670 |
| 33 | 78.410976 | 64.069823 | 62.900000 | 49.728670 |
| 34 | 82.366152 | 67.784101 | 80.040000 | 53.202050 |
| 35 | 72.664380 | 58.251049 | 59.500000 | 43.837718 |
| 36 | 72.721310 | 58.304188 | 62.170000 | 43.887065 |

MODEL B

JOINT GERERALIZED LEAST SQUARES

PREDICTION INTERVAL VALUES

LEAST SQUARES COVARIANCE MATRIX

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0.73516D-01 | -0.35780D-02 | -0.52589D-01 | 0.14596D-01 | -0.13660D-05 | -0.91919D-02 | 0.31201D-02 | 0.11545D-02 | -0.52045D-01 | 0.26448D-01 |
| -0.35780D-02 | 0.19909D-03 | 0.34715D-02 | -0.81218D-03 | 0.76005D-07 | 0.45343D-03 | -0.17361D-03 | -0.66467D-04 | 0.24486D-02 | -0.14716D-02 |
| -0.52589D-01 | 0.34715D-02 | 0.14987D 02 | -0.22698D 01 | 0.12944D-03 | 0.43734D-01 | -0.81438D-02 | -0.11615D-01 | -0.21773D 00 | 0.18947D 00 |
| 0.14596D-01 | -0.81218D-03 | -0.22698D 01 | 0.57343D 00 | -0.57549D-04 | -0.11844D-01 | 0.30680D-02 | 0.23145D-02 | 0.75944D-01 | -0.47042D-01 |
| -0.13660D-05 | 0.76005D-07 | 0.12944D-03 | -0.57549D-04 | 0.78854D-08 | 0.11255D-05 | -0.39370D-06 | -0.17965D-06 | -0.78391D-05 | 0.46511D-05 |
| -0.91919D-02 | 0.45343D-03 | 0.43734D-01 | -0.11844D-01 | 0.11255D-05 | 0.33769D-01 | -0.15966D-03 | -0.93303D-02 | 0.30947D-01 | -0.15851D-01 |
| 0.31201D-02 | -0.17361D-03 | -0.81438D-02 | 0.30680D-02 | -0.39370D-06 | -0.15966D-03 | 0.93717D-02 | -0.36470D-02 | -0.19941D-01 | 0.11090D-01 |
| 0.11545D-02 | -0.66467D-04 | -0.11615D-01 | 0.23145D-02 | -0.17965D-06 | -0.93303D-02 | -0.36470D-02 | 0.42136D-02 | 0.23363D-03 | 0.34399D-03 |
| -0.52045D-01 | 0.24486D-02 | -0.21773D 00 | 0.75944D-01 | -0.78391D-05 | 0.30947D-01 | -0.19941D-01 | 0.23363D-03 | 0.23805D 01 | -0.11118D 01 |
| 0.26448D-01 | -0.14716D-02 | 0.18947D 00 | -0.47042D-01 | 0.46511D-05 | -0.15851D-01 | 0.11090D-01 | 0.34399D-03 | -0.11118D 01 | 0.64045D 00 |
| -0.13504D-02 | 0.75137D-04 | -0.10880D 00 | 0.11718D-01 | 0.16282D-06 | 0.37633D-02 | 0.53826D-02 | -0.32416D-02 | -0.71423D 00 | 0.15182D 00 |
| 0.24631D-01 | -0.11896D-02 | 0.14353D 00 | -0.38950D-01 | 0.35755D-05 | 0.93984D-02 | -0.25406D-02 | -0.14929D-02 | -0.23792D-01 | 0.12053D-01 |
| -0.21481D-02 | 0.11952D-03 | -0.17053D-01 | 0.39136D-02 | -0.35926D-06 | -0.84242D-03 | 0.25527D-03 | 0.15001D-03 | 0.20027D-02 | -0.12111D-02 |
| 0.79704D-03 | -0.38273D-04 | -0.10794D 00 | 0.35415D-01 | -0.36847D-05 | -0.89151D-03 | 0.45346D-03 | 0.50293D-04 | -0.81236D-01 | 0.40926D-01 |
| -0.21529D-04 | 0.11979D-05 | 0.41971D-02 | -0.11084D-02 | 0.11533D-06 | 0.24096D-04 | -0.14193D-04 | -0.15741D-05 | 0.21605D-02 | -0.12809D-02 |
| -0.33590D-01 | 0.15585D-02 | -0.92314D 00 | 0.23414D 00 | -0.19517D-04 | -0.36181D-01 | 0.57584D-02 | 0.71559D-02 | 0.21988D 00 | -0.10583D 00 |
| 0.40505D-03 | -0.22761D-04 | -0.25325D 00 | 0.11547D-01 | 0.32933D-05 | -0.60374D-02 | -0.51186D-02 | 0.38141D-02 | -0.25753D-01 | 0.13387D-01 |
| 0.12732D-01 | -0.70843D-03 | 0.59510D 00 | -0.11140D 00 | 0.78735D-05 | 0.16558D-01 | -0.94716D-03 | -0.45531D-02 | -0.69975D-01 | 0.44176D-01 |
| 0.13218D-01 | -0.61514D-03 | -0.17150D 00 | 0.48520D-01 | -0.44564D-05 | -0.17108D-02 | 0.43452D-03 | 0.26994D-03 | 0.70557D-01 | -0.32954D-01 |
| -0.49905D-02 | 0.27769D-03 | 0.99750D-01 | -0.22228D-01 | 0.19784D-05 | 0.68924D-03 | -0.16655D-03 | -0.13941D-03 | -0.22788D-01 | 0.14246D-01 |
| -0.26133D-04 | 0.14541D-05 | -0.78922D-03 | -0.18591D-04 | 0.20374D-07 | -0.22620D-05 | -0.98286D-05 | 0.45476D-05 | -0.95058D-03 | 0.75408D-03 |
| 0.22542D-01 | -0.11325D-02 | -0.78573D 00 | 0.15770D 00 | -0.10613D-04 | 0.27513D-01 | -0.22616D-02 | -0.66467D-02 | 0.37092D 00 | -0.18685D 00 |
| -0.76359D-02 | 0.42488D-03 | 0.13743D 00 | -0.40415D-01 | 0.45397D-05 | -0.45245D-02 | 0.49879D-02 | -0.62016D-03 | -0.18186D 00 | 0.11071D 00 |
| -0.11187D 00 | 0.62250D-02 | 0.97011D 01 | -0.13489D 01 | 0.41805D-04 | -0.26799D 00 | -0.99064D-01 | 0.11878D 00 | 0.29047D 00 | -0.61735D-01 |
| -0.28792D-04 | 0.16021D-05 | 0.78958D-04 | -0.19952D-03 | 0.35270D-07 | -0.23722D-04 | 0.53740D-04 | -0.14127D-04 | -0.33639D-02 | 0.71493D-03 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| -0.13504D-02 | 0.24631D-01 | -0.21481D-02 | 0.79704D-03 | -0.21529D-04 | -0.33590D-01 | 0.40505D-03 | 0.12732D-01 | 0.13218D-01 | -0.49905D-02 |
| 0.75137D-04 | -0.11896D-02 | 0.11952D-03 | -0.38273D-04 | 0.11979D-05 | 0.15585D-02 | -0.22761D-04 | -0.70843D-03 | -0.61514D-03 | 0.27769D-03 |
| -0.10880D 00 | 0.14353D 00 | -0.17053D-01 | -0.10794D 00 | 0.41971D-02 | -0.92314D 00 | -0.25325D 00 | 0.59510D 00 | -0.17150D 00 | 0.99750D-01 |
| 0.11718D-01 | -0.38950D-01 | 0.39136D-02 | 0.35415D-01 | -0.11084D-02 | 0.23414D 00 | 0.11547D-01 | -0.11140D 00 | 0.48520D-01 | -0.22228D-01 |
| 0.16282D-06 | 0.35755D-05 | -0.35926D-06 | -0.36847D-05 | 0.11533D-06 | -0.19517D-04 | 0.32933D-05 | 0.78735D-05 | -0.44564D-05 | 0.19784D-05 |
| 0.37633D-02 | 0.93984D-02 | -0.84242D-03 | -0.89151D-03 | 0.24096D-04 | -0.36181D-01 | -0.60374D-02 | 0.16558D-01 | -0.17108D-02 | 0.68924D-03 |
| 0.53826D-02 | -0.25406D-02 | 0.25527D-03 | 0.45346D-03 | -0.14193D-04 | 0.57584D-02 | -0.51186D-02 | -0.94716D-03 | 0.43452D-03 | -0.16655D-03 |
| -0.32416D-02 | -0.14929D-02 | 0.15001D-03 | 0.50293D-04 | -0.15741D-05 | 0.71559D-02 | 0.38141D-02 | -0.45531D-02 | 0.26994D-03 | -0.13941D-03 |
| -0.71433D 00 | -0.23792D-01 | 0.20027D-02 | -0.81236D-01 | 0.21605D-02 | 0.21988D 00 | -0.25753D-01 | -0.69975D-01 | 0.70557D-01 | -0.22788D-01 |
| 0.15182D 00 | 0.12053D-01 | -0.12111D-02 | 0.40926D-01 | -0.12809D-02 | -0.10583D 00 | 0.13387D-01 | 0.44176D-01 | -0.32954D-01 | 0.14246D-01 |
| 0.18391D 01 | -0.11087D-02 | 0.11140D-03 | 0.16319D-02 | -0.51077D-04 | 0.26089D-01 | 0.13234D-01 | -0.16377D-01 | -0.54036D-02 | -0.43913D-02 |
| -0.11087D-02 | 0.64398D-01 | -0.56122D-02 | -0.74316D-02 | 0.19954D-03 | -0.51670D-01 | -0.11086D-02 | 0.20198D-01 | 0.67043D-02 | -0.25433D-02 |
| 0.11140D-03 | -0.56122D-02 | 0.56390D-03 | 0.64056D-03 | -0.20049D-04 | 0.43371D-02 | 0.11139D-03 | -0.20295D-02 | -0.56275D-03 | 0.25555D-03 |
| 0.16319D-02 | -0.74316D-02 | 0.64056D-03 | 0.19784D-01 | -0.53521D-03 | 0.53372D-01 | -0.74156D-02 | -0.17419D-01 | 0.12825D-02 | -0.46903D-03 |
| -0.51077D-04 | 0.19954D-03 | -0.20049D-04 | -0.53521D-03 | 0.16751D-04 | -0.13543D-02 | 0.23210D-03 | 0.54518D-03 | -0.33037D-04 | 0.14660D-04 |
| 0.26089D-01 | -0.51670D-01 | 0.43371D-02 | 0.53372D-01 | -0.13543D-02 | 0.21594D 01 | 0.84704D 00 | -0.11047D 01 | 0.16181D 00 | -0.59583D-01 |
| 0.13234D-01 | -0.11086D-02 | 0.11139D-03 | -0.74156D-02 | 0.23210D-03 | 0.84704D 00 | 0.24850D 01 | -0.12137D 01 | -0.12167D-02 | -0.16502D-03 |
| -0.16377D-01 | 0.20198D-01 | -0.20295D-02 | -0.17419D-01 | 0.54518D-03 | -0.11047D 01 | -0.12137D 01 | 0.91023D 00 | -0.59332D-01 | 0.27612D-01 |
| -0.54036D-02 | 0.67043D-02 | -0.56275D-03 | 0.12825D-02 | -0.33037D-04 | 0.16181D 00 | -0.12167D-02 | -0.59332D-01 | 0.22870D 00 | -0.81126D-01 |
| -0.43913D-02 | -0.25433D-02 | 0.25555D-03 | -0.46903D-03 | 0.14660D-04 | -0.59583D-01 | -0.16502D-03 | 0.27612D-01 | -0.81126D-01 | 0.37807D-01 |
| 0.20312D-02 | -0.87943D-05 | 0.88363D-06 | -0.46981D-05 | 0.14704D-06 | 0.72925D-04 | 0.21394D-03 | -0.10449D-03 | -0.11688D-02 | -0.15853D-03 |
| 0.10548D-01 | 0.21971D-01 | -0.19651D-02 | 0.56320D-02 | -0.15159D-03 | 0.48289D 00 | 0.10986D 00 | -0.23228D 00 | 0.10080D 00 | -0.39591D-01 |
| -0.15026D-01 | -0.65619D-02 | 0.65932D-03 | -0.25563D-02 | 0.80010D-04 | -0.92772D-01 | 0.33981D-01 | 0.31346D-01 | -0.22957D-01 | 0.10717D-01 |
| -0.74786D 00 | -0.12773D 00 | 0.12834D-01 | -0.75017D-02 | 0.23479D-03 | -0.40778D 01 | -0.25156D 01 | 0.27081D 01 | -0.70966D 00 | 0.33072D 00 |
| 0.86606D-02 | -0.24196D-04 | 0.24311D-05 | -0.88722D-05 | 0.27769D-06 | -0.44218D-03 | -0.25128D-03 | 0.28652D-03 | -0.41766D-03 | -0.93765D-05 |

| | | | | |
|---|---|---|---|---|
| -0.26133D-04 | 0.22542D-01 | -0.76359D-02 | -0.11187D 00 | -0.28792D-04 |
| 0.14541D-05 | -0.11325D-02 | 0.42488D-03 | 0.62250D-02 | 0.16021D-05 |
| -0.78922D-03 | -0.78573D 00 | 0.13743D 00 | 0.97011D 01 | 0.78958D-04 |
| -0.18591D-04 | 0.15770D 00 | -0.40415D-01 | -0.13489D 01 | -0.19952D-03 |
| 0.20374D-07 | -0.10613D-04 | 0.45397D-05 | 0.41805D-04 | 0.35270D-07 |
| -0.22620D-05 | 0.27513D-01 | -0.45245D-02 | -0.26799D 00 | -0.23722D-04 |
| -0.98286D-05 | -0.22616D-02 | 0.49879D-02 | -0.99064D-01 | 0.53740D-04 |
| 0.45476D-05 | -0.66467D-02 | -0.62016D-03 | 0.11878D 00 | -0.14127D-04 |
| -0.95058D-03 | 0.37092D 00 | -0.18186D 00 | 0.29047D 00 | -0.33639D-02 |
| 0.75408D-03 | -0.18685D 00 | 0.11071D 00 | -0.61735D-01 | 0.71493D-03 |
| 0.20312D-02 | 0.10548D-01 | -0.15086D-01 | -0.74786D 00 | 0.86606D-02 |
| -0.87943D-05 | 0.21971D-01 | -0.65619D-02 | -0.12773D 00 | -0.24196D-04 |
| 0.88363D-06 | -0.19651D-02 | 0.65932D-03 | 0.12834D-01 | 0.24311D-05 |
| -0.46981D-05 | 0.56320D-02 | -0.25563D-02 | -0.75017D-02 | -0.88722D-05 |
| 0.14704D-06 | -0.15159D-03 | 0.80010D-04 | 0.23479D-03 | 0.27769D-06 |
| 0.72925D-04 | 0.48289D 00 | -0.92772D-01 | -0.40778D 01 | -0.44218D-03 |
| 0.21394D-03 | 0.10986D 00 | 0.33981D-01 | -0.25156D 01 | -0.25128D-03 |
| -0.10449D-03 | -0.23228D 00 | 0.31346D-01 | 0.27081D 01 | 0.28652D-03 |
| -0.11688D-02 | 0.10080D 00 | -0.22997D-01 | -0.70966D 00 | -0.41766D-03 |
| -0.15853D-03 | -0.39591D-01 | 0.10717D-01 | 0.33072D 00 | -0.93765D-05 |
| 0.20552D-03 | -0.27178D-03 | -0.44937D-04 | -0.13867D-02 | 0.59471D-04 |
| -0.27178D-03 | 0.10306D 01 | -0.11157D 00 | -0.11435D 02 | -0.35613D-02 |
| -0.44937D-04 | -0.11157D 00 | 0.13980D 00 | -0.17382D 01 | -0.11439D-02 |
| -0.13867D-02 | -0.11435D 02 | -0.17382D 01 | 0.21355D 03 | 0.46620D-01 |
| 0.59471D-04 | -0.35613D-02 | -0.11439D-02 | 0.46620D-01 | 0.32876D-03 |

TABLE OF LS CORRELATED PREDICTION INTERVAL VALUES

| | PI + | PREDICTED | ACTUAL | PI - |
|---|---|---|---|---|
| 1 | 52.865238 | 38.269414 | 27.270000 | 23.673590 |
| 2 | 46.266816 | 31.861932 | 28.390000 | 17.457048 |
| 3 | 46.564809 | 32.156748 | 28.400000 | 17.748688 |
| 4 | 61.815729 | 47.526568 | 51.610000 | 33.237407 |
| 5 | 56.044064 | 41.969404 | 40.040000 | 27.894744 |
| 6 | 55.116949 | 41.040287 | 39.150000 | 26.963624 |
| 7 | 54.440909 | 40.367181 | 41.020000 | 26.293453 |
| 8 | 55.116949 | 41.040287 | 38.200000 | 26.963624 |
| 9 | 56.332498 | 42.257703 | 34.570000 | 28.182909 |
| 10 | 55.116949 | 41.040287 | 35.910000 | 26.963624 |
| 11 | 55.116949 | 41.040287 | 50.980000 | 26.963624 |
| 12 | 55.116949 | 41.040287 | 37.340000 | 26.963624 |
| 13 | 56.223151 | 42.151262 | 37.320000 | 28.079373 |
| 14 | 56.128873 | 42.057563 | 33.690000 | 27.986254 |
| 15 | 26.202169 | 11.688001 | 11.320000 | -2.826167 |
| 16 | 23.137449 | 8.712043 | 11.510000 | -5.713362 |
| 17 | 23.137449 | 8.712043 | 12.080000 | -5.713362 |
| 18 | 23.137449 | 8.712043 | 12.490000 | -5.713362 |
| 19 | 25.525223 | 10.853086 | 8.700000 | -3.819051 |
| 20 | 22.184774 | 7.605762 | 12.930000 | -6.973249 |
| 21 | 30.701544 | 16.185230 | 18.490000 | 1.668917 |
| 22 | 40.942925 | 26.583046 | 25.090000 | 12.223167 |
| 23 | 34.739514 | 20.566869 | 23.990000 | 6.394225 |
| 24 | 34.720124 | 20.552871 | 44.700000 | 6.385617 |
| 25 | 35.518926 | 21.338761 | 20.250000 | 7.158596 |
| 26 | 52.282515 | 37.716865 | 38.490000 | 23.151215 |
| 27 | 46.427530 | 32.083912 | 27.270000 | 17.740294 |
| 28 | 63.421959 | 49.222259 | 67.590000 | 35.022559 |
| 29 | 63.439720 | 49.239859 | 46.230000 | 35.039998 |
| 30 | 90.414639 | 75.620819 | 93.900000 | 60.827000 |
| 31 | 83.443143 | 68.867100 | 75.310000 | 54.291057 |
| 32 | 79.079522 | 64.671006 | 59.300000 | 50.262491 |
| 33 | 79.079522 | 64.671006 | 62.900000 | 50.262491 |
| 34 | 84.022361 | 69.223926 | 80.040000 | 54.425492 |
| 35 | 73.769632 | 59.301116 | 59.500000 | 44.832599 |
| 36 | 73.834053 | 59.361273 | 62.170000 | 44.888493 |

TABLE OF LS (INDEPEND) PREDICTION INTERVAL VALUES

| | PI + | PREDICTED | ACTUAL | PI - |
|---|---|---|---|---|
| 1 | 50.588425 | 38.268265 | 27.269989 | 25.948090 |
| 2 | 43.976730 | 31.860794 | 28.389999 | 19.744843 |
| 3 | 44.291870 | 32.155594 | 28.399994 | 20.019302 |
| 4 | 59.434250 | 47.525436 | 51.609985 | 35.616608 |
| 5 | 53.655869 | 41.968307 | 40.039993 | 30.280731 |
| 6 | 52.730240 | 41.039185 | 39.149994 | 29.348114 |
| 7 | 52.055588 | 40.366074 | 41.019989 | 28.676544 |
| 8 | 52.730240 | 41.039185 | 38.199997 | 29.348114 |
| 9 | 53.944000 | 42.256592 | 34.569992 | 30.569168 |
| 10 | 52.730240 | 41.039185 | 35.909988 | 29.348114 |
| 11 | 52.730240 | 41.039185 | 50.979996 | 29.348114 |
| 12 | 52.730240 | 41.039185 | 37.339996 | 29.348114 |
| 13 | 53.835571 | 42.150146 | 37.319992 | 30.464706 |
| 14 | 53.741562 | 42.056442 | 33.689987 | 30.371307 |
| 15 | 23.676498 | 11.687059 | 11.320000 | -0.302390 |
| 16 | 20.603622 | 8.711105 | 11.509999 | -3.181420 |
| 17 | 20.603622 | 8.711105 | 12.080000 | -3.181420 |
| 18 | 20.603622 | 8.711105 | 12.490000 | -3.181420 |
| 19 | 22.893280 | 10.851655 | 8.700000 | -1.189982 |
| 20 | 19.552795 | 7.604342 | 12.929999 | -4.344112 |
| 21 | 28.102158 | 16.184311 | 18.489990 | 4.266452 |
| 22 | 38.430557 | 26.582047 | 25.089996 | 14.733521 |
| 23 | 32.233276 | 20.565872 | 23.989990 | 8.898454 |
| 24 | 32.214645 | 20.551880 | 44.699997 | 8.889112 |
| 25 | 33.016861 | 21.337753 | 20.250000 | 9.658632 |
| 26 | 49.826096 | 37.715866 | 38.489990 | 25.605621 |
| 27 | 43.976547 | 32.082932 | 27.269989 | 20.189301 |
| 28 | 61.001038 | 49.220993 | 67.589996 | 37.440933 |
| 29 | 61.019516 | 49.238586 | 46.229996 | 37.457642 |
| 30 | 87.900284 | 75.618988 | 93.899994 | 63.337677 |
| 31 | 80.925568 | 68.865265 | 75.309998 | 56.804947 |
| 32 | 76.597549 | 64.669189 | 59.299988 | 52.740814 |
| 33 | 76.597549 | 64.669189 | 62.899994 | 52.740814 |
| 34 | 81.463455 | 69.222061 | 80.039993 | 56.980652 |
| 35 | 71.255280 | 59.299240 | 59.[illegible]00000 | 47.343185 |
| 36 | 71.319580 | 59.359375 | 62.169998 | 47.399155 |

MODEL C - 9 CERs - 27 OBSERVATIONS

```
MULTIPLE REGRESSICN.....ESCCRT
     SELECTICN..... 1

SELECTICN CARC 0 2 114
        LEAST SCUARES REGRESSICN CCEFICIENTS

   C.552360    C.C97619
STC. ERRCR CF ESTIMATE=       0.396638

        CCVARIANCE MATRIX CF THE ESTIMATES
   C.52175C-C1 -C.3C789C-C2
  -C.30789C-C2  C.20454D-C3
ACJUSTED MULTIPLE CCRRELATICN COEFICIENT 0.636825


MULTIPLE REGRESSICN.....ESCCRT
     SELECTICN..... 2

SELECTICN CARC 0 3 133
        LEAST SCUARES REGRESSICN COEFICIENTS

  38.C66134   -C.49215C
STC. ERRCR CF ESTIMATE=       0.714668

        CCVARIANCE MATRIX CF THE ESTIMATES
   C.14C36C CO -C.13859C-C1
  -C.13859C-C1  C.15817D-C2
ACJUSTED MULTIPLE CCRRELATICN COEFICIENT 0.827510
```

```
MULTIPLE REGRESSICN.....ESCCRT
     SELECTICN..... 3

SELECTICN CARC C 4 21625
          LEAST SQUARES REGRESSICN COEFICIENTS

   -0.C15067    C.657699    C.23C566
STD. ERRCR CF ESTIMATE=       0.218241

          CCVARIANCE MATRIX CF THE ESTIMATES

    C.28C98D-C1 -C.13865D-C1 -0.37943D-02
   -0.13865D-C1  C.3117CD-C1 -0.58711D-02
   -0.37943D-C2 -C.58711C-C2  0.314C6D-02
ADJUSTED MULTIPLE CCRRELATICN COEFICIENT 0.739007


MULTIPLE REGRESSICN.....ESCCRT
     SELECTICN..... 4

SELECTICN CARC 0 5 21728
          LEAST SQUARES REGRESSION CCEFICIENTS

   -C.445966    3.4290C6   -1.284769
STC. ERRCR CF ESTIMATE=       3.851717

          CCVARIANCE MATRIX CF THE ESTIMATES

    C.47734C C1 -C.35441C C1  0.12189D 01
   -0.35441D C1  C.33823D C1 -0.21745D 01
    C.12189C C1 -C.21745C C1  0.35989D 01
ADJUSTED MULTIPLE CCRRELATICN COEFICIENT 0.C66788
```

```
MULTIPLE REGRESSICN.....ESCCRT
     SELECTICN..... 5

SELECTICN CARC 0 6 118
          LEAST SCUARES REGRESSICN CCEFICIENTS

   -C.CCC657    C.229295
STC. ERRCR CF ESTIMATE=       0.595721

          CCVARIANCE MATRIX CF THE ESTIMATES
    C.1C195C CC -C.10519C-C1
   -C.1C519C-C1  C.12459C-C2
ADJUSTED MULTIPLE CCRRELATICN COEFICIENT 0.613091


MULTIPLE REGRESSICN.....ESCCRT
     SELECTICN..... 6

SELECTICN CARC C 7 119
          LEAST SCUARES REGRESSION CCEFICIENTS

    0.571337    C.C03315
STC. ERRCR CF ESTIMATE=       0.253202

          CCVARIANCE MATRIX CF THE ESTIMATES
    0.38726C-C1 -C.14722C-C3
   -0.14722C-C3  C.5962CD-C6
ADJUSTED MULTIPLE CCRRELATICN CCEFICIENT 0.401430
```

```
MULTIPLE REGRESSION.....ESCORT
     SELECTION..... 7

SELECTION CARD 0 8 22820
        LEAST SQUARES REGRESSION COEFICIENTS

   -1.000013     7.991636     2.813978
STD. ERROR OF ESTIMATE=        2.504982

        COVARIANCE MATRIX OF THE ESTIMATES
    0.120290 01  0.291470 00 -0.636120 00
    0.291470 00  0.165580 01 -0.623450 00
   -0.636120 00 -0.623450 00  0.536160 00
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.851478


MULTIPLE REGRESSION.....ESCORT
     SELECTION..... 8

SELECTION CARD 0 9 22023
        LEAST SQUARES REGRESSION COEFICIENTS

   -0.306542     0.406525     0.094358
STD. ERROR OF ESTIMATE=        1.237198

        COVARIANCE MATRIX OF THE ESTIMATES
    0.324310 00 -0.137240 00 -0.474380-02
   -0.137240 00  0.753260-01  0.965850-03
   -0.474380-02  0.965850-03  0.518580-03
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.379376
```

MULTIPLE REGRESSION.....ESCCRT

SELECTION..... 9

SELECTION CARD C1C 3172223

LEAST SQUARES REGRESSION COEFICIENTS

3.741737 3.940985 -68.643523 0.038777

STD. ERROR OF ESTIMATE= 1.560890

COVARIANCE MATRIX OF THE ESTIMATES

| | | | |
|---|---|---|---|
| 0.12337D 01 | -0.26698D 00 | -0.11320D 02 | -0.91207D-02 |
| -0.26698D 00 | 0.41816D 00 | -0.45499D 01 | -0.14559D-02 |
| -0.11320D 02 | -0.45499D 01 | 0.26394D 03 | 0.95685D-01 |
| -0.91207D-02 | -0.14559D-02 | 0.95685D-01 | 0.84052D-03 |

ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.617770

MATRIX OF RESIDUALS FOR SELECTIONS

| CASE NO. | SELECTION NUMBER 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | -0.66986 | 7.77760 | 0.08068 | -4.31331 | -1.54477 | 0.52741 | -0.12633 | -1.94190 | -1.90289 |
| 2 | -0.24986 | 4.83818 | 0.06068 | -3.72331 | -0.24477 | 0.16741 | -0.35633 | -0.33042 | 0.56729 |
| 3 | 0.13869 | -1.50078 | 0.42068 | -4.17331 | 0.78880 | 0.07741 | -0.29633 | 0.11958 | -0.81271 |
| 4 | -0.06322 | 7.51324 | -0.07355 | -0.11855 | -0.64095 | 0.17692 | 0.81715 | 4.85604 | 1.20605 |
| 5 | -0.34205 | -4.82610 | -0.17590 | -0.09003 | -0.33273 | 0.06581 | 0.79645 | 0.03177 | 1.17616 |
| 6 | 0.82869 | -3.68928 | -0.16922 | -0.04855 | 0.82680 | -0.27259 | 0.73203 | -0.07042 | -1.07542 |
| 7 | -0.00686 | 3.59293 | 0.19645 | 0.60145 | 0.85096 | -0.17308 | 0.83715 | 0.13210 | 0.75340 |
| 8 | -0.17131 | 10.66234 | 0.00068 | -0.45855 | -0.68120 | -0.28259 | -1.35797 | -0.29042 | 1.12458 |
| 9 | 0.54914 | 5.57082 | -0.15617 | -1.02432 | -0.56922 | -0.05313 | -2.71169 | -0.60229 | -1.87444 |
| 10 | 0.04869 | 5.32815 | 0.32068 | 0.47145 | 0.25880 | -0.11259 | -2.73797 | -0.55042 | -0.62542 |
| 11 | -0.08131 | 0.44383 | -0.03932 | 0.73145 | -0.05120 | 0.05741 | 10.02203 | -0.13042 | 1.45458 |
| 12 | 0.31869 | -3.41221 | 0.26068 | -0.58855 | 1.43880 | 0.04741 | -2.70797 | -0.18042 | 0.72458 |
| 13 | 0.72476 | -2.64101 | -0.32247 | -1.23003 | 0.10912 | 0.02339 | 0.28273 | -0.22010 | -0.78859 |
| 14 | -0.13010 | -2.19113 | 0.06330 | -0.65861 | -0.44190 | 0.11992 | -2.29285 | -0.20790 | -0.71173 |
| 15 | 0.51943 | -7.83075 | -0.02438 | -0.43709 | -0.21145 | -0.00870 | 0.07069 | -0.91540 | -0.26279 |
| 16 | -0.04057 | -2.13347 | 0.11562 | -0.37709 | 0.03851 | 0.11130 | -0.39931 | 0.31726 | 0.11131 |
| 17 | 0.06543 | -8.91475 | 0.18562 | -0.28709 | -0.02149 | 0.25130 | -0.19931 | 0.68726 | 0.60131 |
| 18 | 0.41943 | -1.94245 | 0.36562 | -0.05709 | -0.07149 | 0.13130 | -0.38931 | 0.82726 | 0.37131 |
| 19 | 0.06557 | 2.65915 | -0.35971 | -0.50851 | -0.31315 | -0.51223 | 0.13442 | -0.08877 | -2.06321 |
| 20 | 0.33557 | 9.69337 | -0.17971 | -0.37851 | -0.16315 | -0.34223 | 1.57442 | 0.62393 | 0.49803 |
| 21 | -0.73000 | -5.71978 | -0.39479 | 0.96702 | -0.12046 | 0.51140 | 0.55163 | -0.25451 | -0.57709 |
| 22 | -0.21848 | -4.32318 | -0.14296 | -0.68041 | 0.30419 | 0.07355 | -0.49466 | -0.75816 | 1.61055 |
| 23 | -0.53507 | -6.26681 | 0.07704 | -0.94041 | 0.27190 | 0.02282 | 0.00720 | 0.23563 | 1.31116 |
| 24 | -0.07579 | 2.86827 | 0.05704 | 17.01959 | -0.34122 | -0.52672 | -0.20722 | 0.19343 | 3.78413 |
| 25 | -0.25969 | -0.72743 | -0.11665 | -2.11047 | 0.16887 | 0.06291 | 0.13045 | 0.52221 | -2.86651 |
| 26 | -0.17998 | -2.33081 | -0.06665 | 2.02568 | 0.31145 | -0.10618 | 1.72486 | -2.19181 | 0.57566 |
| 27 | -0.26391 | -2.91790 | 0.01677 | 0.38710 | 0.36101 | -0.04259 | -3.40398 | 0.19091 | -2.30934 |

MATRIX OF CORRELATIONS BETWEEN SELECTIONS

| SELECTION NO. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.000000 | -0.035010 | 0.029758 | 0.001132 | 0.328492 | -0.418906 | -0.060438 | 0.088406 | -0.104664 |
| 2 | -0.035010 | 1.000000 | 0.004470 | 0.009239 | -0.449550 | -0.291248 | -0.012811 | 0.148321 | 0.000102 |
| 3 | 0.029758 | 0.004470 | 1.000000 | -0.040627 | 0.282828 | 0.122766 | -0.229566 | 0.033536 | 0.187222 |
| 4 | 0.001132 | 0.009239 | -0.040627 | 1.000000 | -0.009807 | -0.499669 | 0.062083 | 0.063300 | 0.577040 |
| 5 | 0.328492 | -0.449550 | 0.282828 | -0.009807 | 1.000000 | -0.206360 | -0.058409 | -0.018841 | 0.049356 |
| 6 | -0.418906 | -0.291248 | 0.122766 | -0.499669 | -0.206360 | 1.000000 | 0.018820 | 0.006415 | -0.192862 |
| 7 | -0.060438 | -0.012811 | -0.229566 | 0.062083 | -0.058409 | 0.018820 | 1.000000 | 0.049265 | 0.305964 |
| 8 | 0.088406 | 0.148321 | 0.033536 | 0.063300 | -0.018841 | 0.006415 | 0.049265 | 1.000000 | 0.195850 |
| 9 | -0.104664 | 0.000102 | 0.187222 | 0.577040 | 0.049356 | -0.192862 | 0.305964 | 0.195850 | 1.000000 |

JOINT GENERALIZED LEAST SQUARES REGRESSION OUTPUT

COVARIANCE MATRIX OF THE JOINT ESTIMATES

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0.44211D-01 | -0.25786D-02 | 0.23822D-01 | -0.30177D-02 | 0.15972D-02 | 0.80557D-03 | -0.87248D-03 | -0.63503D-02 | -0.93154D-04 | 0.13572D-01 |
| -0.25786D-02 | 0.17120D-03 | -0.17566D-02 | 0.20047D-03 | -0.12693D-03 | -0.53515D-04 | 0.57559D-04 | 0.42572D-03 | 0.61882D-05 | -0.90162D-03 |
| 0.23822D-01 | -0.17566D-02 | 0.58525D 01 | -0.55394D 00 | 0.43355D-02 | 0.14637D-01 | -0.61450D-02 | -0.50564D-01 | 0.53379D-01 | -0.31804D-01 |
| -0.30177D-02 | 0.20047D-03 | -0.55394D 00 | 0.63218D-01 | -0.47424D-03 | -0.16704D-02 | 0.70129D-03 | 0.65215D-02 | -0.60918D-02 | 0.36256D-02 |
| 0.15972D-02 | -0.12693D-03 | 0.43355D-02 | -0.47424D-03 | 0.23398D-01 | -0.11520D-01 | -0.31367D-02 | -0.60627D-02 | 0.33916D-02 | 0.65509D-03 |
| 0.80557D-03 | -0.53515D-04 | 0.14637D-01 | -0.16704D-02 | -0.11520D-01 | 0.21559D-01 | -0.34606D-02 | -0.23445D-02 | 0.24770D-02 | -0.21137D-02 |
| -0.87248D-03 | 0.57559D-04 | -0.61450D-02 | 0.70129D-03 | -0.31367D-02 | -0.34606D-02 | 0.21372D-02 | 0.23415D-02 | -0.18913D-02 | 0.47559D-03 |
| -0.63503D-02 | 0.42572D-03 | -0.50564D-01 | 0.65215D-02 | -0.60627D-02 | -0.23445D-02 | 0.23415D-02 | 0.37812D 01 | -0.26038D 01 | 0.50184D 00 |
| -0.93154D-04 | 0.61882D-05 | 0.53379D-01 | -0.60918D-02 | 0.33916D-02 | 0.24770D-02 | -0.16538D-02 | -0.26038D 01 | 0.22301D 01 | -0.87921D 00 |
| 0.13572D-01 | -0.50162D-03 | -0.31804D-01 | 0.36256D-02 | 0.65509D-03 | -0.21137D-02 | 0.47559D-03 | 0.50184D 00 | -0.87921D 00 | 0.14364D 01 |
| 0.17412D-01 | -0.57965D-03 | 0.16276D 00 | -0.24343D-01 | 0.12786D-01 | -0.35660D-02 | -0.28044D-02 | -0.10691D-01 | 0.84061D-03 | 0.18263D-01 |
| -0.17471D-02 | 0.11606D-03 | -0.25265D-01 | 0.28832D-02 | -0.14865D-02 | 0.42475D-03 | 0.33218D-03 | 0.11767D-02 | -0.55568D-04 | -0.21633D-02 |
| -0.12487D-01 | 0.73369D-03 | 0.54558D-01 | -0.78147D-02 | 0.15879D-02 | -0.30672D-02 | 0.45202D-03 | -0.15525D 00 | 0.10885D 00 | -0.10176D-01 |
| 0.44726D-04 | -0.29713D-05 | -0.27731D-03 | 0.31648D-04 | -0.71276D-05 | 0.12421D-04 | -0.18306D-05 | 0.57877D-03 | -0.44100D-03 | 0.41218D-04 |
| -0.11437D-01 | 0.62576D-03 | -0.23350D-01 | 0.15876D-02 | -0.33065D-01 | 0.64117D-02 | 0.64176D-02 | 0.72057D-01 | -0.26612D-01 | -0.33758D-01 |
| -0.85637D-03 | 0.56885D-04 | -0.71417D-02 | 0.81503D-03 | -0.61109D-03 | 0.19057D-02 | -0.43409D-03 | -0.88834D-01 | 0.12834D 00 | -0.17733D 00 |
| 0.56302D-02 | -0.37402D-03 | 0.11942D-01 | -0.13629D-02 | 0.16737D-01 | -0.53423D-02 | -0.35553D-02 | -0.54559D-02 | -0.20147D-01 | 0.69223D-01 |
| 0.11197D-01 | -0.64698D-03 | -0.63759D-01 | 0.13431D-01 | 0.15118D-02 | -0.22520D-02 | 0.35664D-03 | 0.43848D-01 | -0.22753D-01 | -0.62564D-02 |
| -0.55684D-02 | 0.36951D-03 | 0.68915D-01 | -0.78652D-02 | -0.65358D-03 | 0.12851D-02 | -0.20330D-03 | -0.18075D-01 | 0.11522D-01 | 0.50466D-02 |
| -0.25850D-05 | 0.17172D-06 | -0.45136D-03 | 0.51511D-04 | -0.42824D-06 | 0.13504D-05 | -0.30514D-06 | -0.39655D-03 | 0.44534D-03 | -0.42685D-03 |
| -0.20627D-01 | 0.12287D-02 | -0.13206D-01 | 0.15106D-02 | 0.24445D-01 | -0.15677D-01 | -0.15435D-02 | 0.80573D 00 | -0.49100D 00 | -0.65284D-01 |
| 0.52176D-02 | -0.61233D-03 | 0.19844D-01 | -0.22647D-02 | -0.54175D-02 | 0.17333D-01 | -0.27224D-02 | -0.53381D 00 | 0.40666D 00 | -0.37625D-01 |
| 0.54430D-01 | -0.62731D-02 | -0.21067D 00 | 0.24042D-01 | -0.14502D 00 | -0.12777D 00 | 0.88183D-01 | 0.45431D 00 | -0.10158D 01 | 0.19202D 01 |
| -0.13245D-04 | 0.87987D-06 | -0.30076D-04 | 0.34324D-05 | -0.50467D-04 | 0.59486D-04 | -0.35786D-05 | -0.42250D-03 | 0.72605D-03 | -0.11654D-02 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0.17412D-01 | -0.17471D-02 | -0.12487D-01 | 0.44726D-04 | -0.11437D-01 | -0.85637D-03 | 0.56302D-02 | 0.11197D-01 | -0.55684D-02 | -0.25850D-05 |
| -0.97965D-03 | 0.11606D-03 | 0.73369D-03 | -0.29713D-05 | 0.62576D-03 | 0.56885D-04 | -0.37402D-03 | -0.64698D-03 | 0.36951D-03 | 0.17172D-06 |
| 0.16276D 00 | -0.25265D-01 | 0.54558D-01 | -0.27731D-03 | -0.23350D-01 | -0.71417D-02 | 0.11942D-01 | -0.63759D-01 | 0.66915D-01 | -0.45136D-03 |
| -0.24343D-01 | 0.28832D-02 | -0.78147D-02 | 0.31648D-04 | 0.19876D-02 | 0.81502D-03 | -0.13629D-02 | 0.13431D-01 | -0.78652D-02 | 0.51511D-04 |
| 0.13786D-01 | -0.14865D-02 | 0.15879D-02 | -0.71276D-05 | -0.33065D-01 | -0.61109D-03 | 0.16737D-01 | 0.15118D-02 | -0.65358D-03 | -0.42824D-06 |
| -0.35660D-02 | 0.42475D-03 | -0.30672D-02 | 0.12421D-04 | 0.64117D-02 | 0.19057D-02 | -0.53423D-02 | -0.22520D-02 | 0.12851D-02 | 0.13504D-05 |
| -0.28044D-02 | 0.33216D-03 | 0.45202D-03 | -0.18306D-05 | 0.64176D-02 | -0.43409D-03 | -0.35553D-02 | 0.35684D-03 | -0.20330D-03 | -0.30514D-06 |
| -0.10691D-01 | 0.11767D-02 | -0.15525D 00 | 0.57877D-03 | 0.72057D-01 | -0.88834D-01 | -0.54559D-02 | 0.43848D-01 | -0.18075D-01 | -0.39655D-03 |
| 0.84061D-03 | -0.99568D-04 | 0.10889D 00 | -0.44100D-03 | -0.26612D-01 | 0.12834D 00 | -0.20147D-01 | -0.22753D-01 | 0.11526D-01 | 0.44534D-03 |
| 0.16263D-01 | -0.21633D-02 | -0.10178D-01 | 0.41216D-04 | -0.33758D-01 | -0.17733D 00 | 0.68223D-01 | -0.62564D-02 | 0.50466D-02 | -0.42685D-03 |
| 0.80391D-01 | -0.80806D-02 | -0.10091D-01 | 0.36543D-04 | -0.16757D-01 | -0.22787D-02 | 0.85470D-02 | 0.21605D-02 | -0.15063D-02 | -0.28290D-05 |
| -0.80806D-02 | 0.95712D-03 | 0.10686D-02 | -0.43284D-05 | 0.16379D-02 | 0.26551D-03 | -0.10124D-02 | -0.31354D-03 | 0.17841D-03 | 0.33505D-06 |
| -0.10091D-01 | 0.10688D-02 | 0.27856D-01 | -0.10391D-03 | 0.34158D-02 | 0.12345D-02 | -0.20650D-02 | 0.91521D-03 | -0.33734D-03 | -0.44545D-04 |
| 0.36543D-04 | -0.43284D-05 | -0.10391D-03 | 0.42031D-06 | -0.12225D-04 | -0.45554D-05 | 0.83790D-05 | -0.34452D-05 | 0.13661D-05 | 0.18040D-06 |
| -0.16757D-01 | 0.16379D-02 | 0.34158D-02 | -0.12225D-04 | 0.10504D 01 | 0.21880D 00 | -0.54354D 00 | 0.24244D-01 | -0.10962D-01 | -0.12576D-04 |
| -0.22787D-02 | 0.26551D-02 | 0.12345D-02 | -0.45554D-05 | 0.21880D 00 | 0.11517D 01 | -0.45385D 00 | 0.32316D-02 | -0.18856D-02 | 0.11575D-04 |
| 0.85470D-02 | -0.10124D-02 | -0.20650D-02 | 0.83790D-05 | -0.54354D 00 | -0.45385D 00 | 0.43625D 00 | -0.11655D-01 | 0.67983D-02 | 0.40100D-05 |
| 0.21605D-02 | -0.31354D-03 | 0.91521D-03 | -0.34452D-05 | 0.24244D-01 | 0.32316D-02 | -0.11855D-01 | 0.28172D 00 | -0.11927D 00 | -0.39055D-02 |
| -0.15063D-02 | 0.17841D-03 | -0.33734D-03 | 0.13661D-05 | -0.10962D-01 | -0.18856D-02 | 0.67983D-02 | -0.11927D 00 | 0.65557D-01 | 0.79952D-03 |
| -0.28290D-05 | 0.33505D-06 | -0.44545D-04 | 0.18040D-06 | -0.12576D-04 | 0.11575D-04 | 0.40100D-05 | -0.39055D-02 | 0.79954D-03 | 0.42628D-03 |
| 0.12361D-01 | -0.12654D-02 | -0.24348D-01 | 0.88449D-04 | 0.28685D 00 | 0.25377D-01 | -0.14518D 00 | 0.90534D-01 | -0.41937D-01 | -0.93472D-03 |
| -0.16153D-02 | 0.19132D-03 | 0.18335D-01 | -0.74253D-04 | -0.56247D-01 | 0.64946D-01 | 0.14302D-01 | -0.22751D-01 | 0.12490D-01 | 0.15949D-03 |
| -0.13173D 00 | 0.15604D-01 | -0.46767D-01 | 0.18540D-03 | -0.26829D 01 | -0.17123D 01 | 0.20084D 01 | -0.68201D 00 | 0.37713D 00 | 0.39736D-02 |
| -0.34477D-04 | 0.40837D-05 | -0.77584D-05 | 0.31420D-07 | 0.45478D-04 | -0.61542D-03 | 0.14361D-03 | -0.69293D-03 | 0.13157D-03 | 0.78567D-04 |

| | | | |
|---|---|---|---|
| -0.20627D-01 | 0.52176D-02 | 0.54430D-01 | -0.13245D-04 |
| 0.12287D-02 | -0.61233D-03 | -0.62731D-02 | 0.87987D-06 |
| -0.13206D-01 | 0.19844D-01 | -0.21067D 00 | -0.30076D-04 |
| 0.15106D-02 | -0.22647D-02 | 0.24042D-01 | 0.34324D-05 |
| 0.24445D-01 | -0.54175D-02 | -0.14502D 00 | -0.50467D-04 |
| -0.15677D-01 | 0.17333D-01 | -0.12777D 00 | 0.59486D-04 |
| -0.15439D-02 | -0.27224D-02 | 0.88183D-01 | -0.35786D-05 |
| 0.80573D 00 | -0.53381D 00 | 0.45431D 00 | -0.42250D-03 |
| -0.49100D 00 | 0.40666D 00 | -0.10158D 01 | 0.72605D-03 |
| -0.65284D-01 | -0.37625D-01 | 0.19202D 01 | -0.11654D-02 |
| 0.12361D-01 | -0.16153D-02 | -0.13173D 00 | -0.34477D-04 |
| -0.12654D-02 | 0.19132D-03 | 0.15604D-01 | 0.40837D-05 |
| -0.24348D-01 | 0.18335D-01 | -0.46767D-01 | -0.77584D-05 |
| 0.88445D-04 | -0.74253D-04 | 0.18940D-03 | 0.31420D-07 |
| 0.28665D 00 | -0.56247D-01 | -0.26829D 01 | 0.45478D-04 |
| 0.25377D-01 | 0.64946D-01 | -0.17123D 01 | -0.61542D-03 |
| -0.14518D 00 | 0.14302D-01 | 0.20084D 01 | 0.14364D-03 |
| 0.90534D-01 | -0.22751D-01 | -0.68201D 00 | -0.35293D-03 |
| -0.41937D-01 | 0.12490D-01 | 0.37713D 00 | 0.13157D-03 |
| -0.93472D-03 | 0.15949D-03 | 0.39736D-02 | 0.78567D-04 |
| 0.85837D 00 | -0.27053D 00 | -0.60948D 01 | -0.39115D-02 |
| -0.27053D 00 | 0.28458D 00 | -0.17531D 01 | -0.54835D-03 |
| -0.60948D 01 | -0.17531D 01 | 0.12842D 03 | 0.39154D-01 |
| -0.39115D-02 | -0.54838D-03 | 0.39154D-01 | 0.36308D-03 |

COMPUTATIONAL DETERMINANT CHECK= 0.2419779D 41

JOINT LEAST SQUARES ESTIMATES

| SELECTION NO. | CONSTANT | COEFICIENTS B1 | B2 | B3 |
|---|---|---|---|---|
| 1 | 0.468767 | 0.103172 | | |
| 2 | 38.494036 | -0.540984 | | |
| 3 | 0.074922 | 0.524027 | 0.246046 | |
| 4 | -0.960958 | 3.851363 | -1.405892 | |
| 5 | -0.209615 | 0.254045 | | |
| 6 | 0.621279 | 0.003113 | | |
| 7 | -1.392054 | 7.731775 | 3.110125 | |
| 8 | -0.118280 | 0.306717 | 0.091989 | |
| 9 | 3.220697 | 3.745272 | -57.541979 | 0.051347 |

TABLE OF PREDICTION INTERVAL VALUES

| | PI + | PREDICTED | ACTUAL | PI - |
|---|---|---|---|---|
| 1 | 46.961441 | 35.272428 | 27.270000 | 23.583414 |
| 2 | 42.753668 | 31.168716 | 28.390000 | 19.586765 |
| 3 | 42.922090 | 31.339508 | 28.400000 | 19.756926 |
| 4 | 53.073715 | 41.566213 | 51.610000 | 30.058711 |
| 5 | 49.861508 | 38.425092 | 40.040000 | 26.988675 |
| 6 | 48.972870 | 37.563356 | 39.150000 | 26.153842 |
| 7 | 48.323039 | 36.920174 | 41.020000 | 25.517308 |
| 8 | 48.972870 | 37.563356 | 38.200000 | 26.153842 |
| 9 | 50.186199 | 38.747780 | 34.570000 | 27.309360 |
| 10 | 48.972870 | 37.563356 | 35.910000 | 26.153842 |
| 11 | 48.972870 | 37.563356 | 50.980000 | 26.153842 |
| 12 | 48.972870 | 37.563356 | 37.340000 | 26.153842 |
| 13 | 50.063905 | 38.632108 | 37.320000 | 27.200311 |
| 14 | 49.988179 | 38.552669 | 33.690000 | 27.117159 |
| 15 | 24.232777 | 12.503649 | 11.320000 | 0.774521 |
| 16 | 22.451310 | 10.721117 | 11.510000 | -1.009075 |
| 17 | 22.451310 | 10.721117 | 12.080000 | -1.009075 |
| 18 | 22.451310 | 10.721117 | 12.490000 | -1.009075 |
| 19 | 22.894977 | 11.058243 | 8.700000 | -0.778491 |
| 20 | 20.939221 | 9.094539 | 12.930000 | -2.750144 |
| 21 | 29.266917 | 17.731768 | 18.490000 | 6.196619 |
| 22 | 38.334671 | 26.840069 | 25.090000 | 15.345467 |
| 23 | 34.482818 | 23.049254 | 23.990000 | 11.615689 |
| 24 | 34.362989 | 22.933478 | 44.700000 | 11.503967 |
| 25 | 35.338190 | 23.832306 | 20.250000 | 12.326423 |
| 26 | 46.757843 | 35.181267 | 38.490000 | 23.604691 |
| 27 | 43.244188 | 31.718632 | 27.270000 | 20.193076 |

MODEL C

JOINT GENERALIZED LEAST SQUARES

PREDICTION INTERVAL VALUES

LEAST SQUARES COVARIANCE MATRIX

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0.48310D-01 | -0.28509D-02 | 0.12466D-01 | -0.17217D-02 | 0.98083D-03 | -0.50977D-03 | -0.11602D-03 | 0.41680D-03 | -0.24035D-03 | -0.67354D-04 |
| -0.28509D-02 | 0.18938D-03 | -0.10022D-02 | 0.11437D-03 | -0.59407D-04 | 0.33864D-04 | 0.77074D-05 | -0.23828D-04 | 0.15967D-04 | 0.44743D-05 |
| 0.12466D-01 | -0.10022D-02 | 0.77062D 01 | -0.76094D 00 | -0.10401D-02 | 0.10087D-02 | 0.55164D-04 | -0.22240D-01 | 0.19732D-01 | 0.42277D-02 |
| -0.17217D-02 | 0.11437D-03 | -0.76094D 00 | 0.86841D-01 | 0.13929D-03 | -0.11512D-03 | -0.62955D-05 | 0.32891D-02 | -0.22519D-02 | -0.48248D-03 |
| 0.98083D-03 | -0.59407D-04 | -0.10401D-02 | 0.13929D-03 | 0.24976D-01 | -0.12324D-01 | -0.33727D-02 | -0.95531D-02 | 0.56134D-02 | 0.16804D-02 |
| -0.50977D-03 | 0.33864D-04 | 0.10087D-02 | -0.11512D-03 | -0.12324D-01 | 0.27707D-01 | -0.52187D-02 | 0.10469D-01 | -0.89530D-02 | 0.34971D-02 |
| -0.11602D-03 | 0.77074D-05 | 0.55164D-04 | -0.62955D-05 | -0.33727D-02 | -0.52187D-02 | 0.27917D-02 | -0.77371D-03 | 0.11682D-02 | -0.16866D-02 |
| 0.41680D-03 | -0.23828D-04 | -0.22240D-01 | 0.32891D-02 | -0.95531D-02 | 0.10469D-01 | -0.77371D-03 | 0.42431D 01 | -0.31503D 01 | 0.10834D 01 |
| -0.24035D-03 | 0.15967D-04 | 0.19732D-01 | -0.22519D-02 | 0.56134D-02 | -0.89530D-02 | 0.11682D-02 | -0.31503D 01 | 0.30065D 01 | -0.19329D 01 |
| -0.67354D-04 | 0.44743D-05 | 0.42277D-02 | -0.48248D-03 | 0.16804D-02 | 0.34971D-02 | -0.16866D-02 | 0.10834D 01 | -0.19329D 01 | 0.31990D 01 |
| 0.22034D-01 | -0.12869D-02 | 0.21058D 00 | -0.29800D-01 | 0.13106D 00 | -0.55922D-01 | -0.19272D-01 | -0.47251D-02 | 0.25663D-02 | 0.10085D-02 |
| -0.22946D-02 | 0.15243D-03 | -0.30929D-01 | 0.35298D-02 | -0.14060D-01 | 0.56238D-02 | 0.22827D-02 | 0.47012D-03 | -0.30397D-03 | -0.11946D-03 |
| -0.15404D-01 | 0.92748D-03 | 0.85257D-01 | -0.11318D-01 | 0.28500D-02 | -0.25345D-02 | 0.26108D-05 | -0.17565D 00 | 0.11865D 00 | -0.36575D-02 |
| 0.56542D-04 | -0.37561D-05 | -0.40163D-03 | 0.45836D-04 | -0.10619D-04 | 0.10264D-04 | -0.10573D-07 | 0.64504D-03 | -0.48051D-03 | 0.14974D-04 |
| -0.12749D-01 | 0.71287D-03 | 0.19350D-01 | -0.28855D-02 | -0.32964D-01 | 0.96174D-02 | 0.59881D-02 | 0.88444D-01 | -0.48105D-01 | -0.71022D-02 |
| 0.82202D-03 | -0.54607D-04 | 0.23308D-02 | -0.26600D-03 | 0.35296D-03 | -0.27926D-02 | 0.80847D-03 | 0.11376D-01 | -0.47151D-01 | 0.10930D 00 |
| 0.59184D-02 | -0.39316D-03 | -0.15121D-01 | 0.17257D-02 | 0.16413D-01 | -0.47374D-02 | -0.36519D-02 | -0.42490D-01 | 0.40550D-01 | -0.26070D-01 |
| 0.94511D-02 | -0.53366D-03 | -0.10759D 00 | 0.16151D-01 | 0.24777D-02 | -0.89874D-03 | -0.39606D-03 | 0.46728D-01 | -0.23252D-01 | -0.10760D-01 |
| -0.45353D-02 | 0.30128D-03 | 0.81304D-01 | -0.92787D-02 | -0.12131D-02 | 0.45216D-03 | 0.23629D-03 | -0.19891D-01 | 0.12140D-01 | 0.70862D-02 |
| -0.19256D-04 | 0.12792D-05 | -0.77677D-04 | 0.88647D-05 | -0.10364D-04 | 0.18514D-04 | -0.28112D-05 | -0.34966D-03 | 0.35474D-03 | -0.27387D-03 |
| -0.21590D-01 | 0.12926D-02 | -0.19821D-03 | 0.25917D-04 | 0.25793D-01 | -0.12163D-01 | -0.35307D-02 | 0.84198D 00 | -0.49043D 00 | -0.13387D 00 |
| 0.79739D-02 | -0.52971D-03 | 0.57518D-04 | -0.65641D-05 | -0.91130D-02 | 0.14697D-01 | -0.19500D-02 | -0.59333D 00 | 0.46017D 00 | -0.65009D-01 |
| 0.13472D 00 | -0.89455D-02 | 0.23266D-02 | -0.26552D-03 | -0.17396D 00 | -0.12582D 00 | 0.96735D-01 | 0.12578D 01 | -0.22440D 01 | 0.37139D 01 |
| -0.21676D-05 | 0.14400D-06 | -0.54604D-07 | 0.62316D-08 | -0.34173D-04 | 0.49204D-04 | -0.53651D-05 | -0.93498D-03 | 0.16680D-02 | -0.27607D-02 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0.22034D-01 | -0.22946D-02 | -0.15404D-01 | 0.56542D-04 | -0.12749D-01 | 0.82202D-03 | 0.59184D-02 | 0.94511D-02 | -0.45353D-02 | -0.19256D-04 |
| -0.12869D-02 | 0.15243D-03 | 0.92748D-03 | -0.37561D-05 | 0.71287D-03 | -0.54607D-04 | -0.39316D-03 | -0.53366D-03 | 0.30128D-03 | 0.12792D-05 |
| 0.21058D 00 | -0.30929D-01 | 0.85257D-01 | -0.40163D-03 | 0.19350D-01 | 0.23308D-02 | -0.15121D-01 | -0.10759D 00 | 0.81304D-01 | -0.77677D-04 |
| -0.29800D-01 | 0.35298D-02 | -0.11318D-01 | 0.45836D-04 | -0.28855D-02 | -0.26600D-03 | 0.17257D-02 | 0.16151D-01 | -0.92787D-02 | 0.88647D-05 |
| 0.13106D 00 | -0.14060D-01 | 0.28500D-02 | -0.10619D-04 | -0.32964D-01 | 0.35296D-03 | 0.16413D-01 | 0.24777D-02 | -0.12131D-02 | -0.10364D-04 |
| -0.55922D-01 | 0.56238D-02 | -0.25345D-02 | 0.1026[illegible] 04 | 0.96174D-02 | -0.27926D-02 | -0.47374D-02 | -0.89874D-03 | 0.45216D-03 | 0.18514D-04 |
| -0.19272D-01 | 0.22827D-02 | 0.26108D-05 | -0.10573[illegible]07 | 0.59881D-02 | 0.80847D-03 | -0.36519D-02 | -0.39606D-03 | 0.23629D-03 | -0.28112D-05 |
| -0.47251D-02 | 0.47012D-03 | -0.17565D 00 | 0.64504D-03 | 0.88444D-01 | 0.11376D-01 | -0.42490D-01 | 0.46728D-01 | -0.19891D-01 | -0.34966D-03 |
| 0.25663D-02 | -0.30397D-03 | 0.11865D 00 | -0.48051D-03 | -0.48109D-01 | -0.47151D-01 | 0.40550D-01 | -0.23252D-01 | 0.12140D-01 | 0.35474D-03 |
| 0.10085D-02 | 0.11946D-03 | -0.36575D-02 | 0.14974D-04 | -0.71022D-02 | 0.10930D 00 | -0.26070D-01 | -0.10760D-01 | 0.70862D-02 | -0.27387D-03 |
| 0.94397D-01 | -0.97395D-02 | -0.99236D-02 | 0.35866D-04 | -0.17555D-01 | 0.82224D-03 | 0.81487D-02 | -0.28532D-02 | 0.13544D-02 | 0.36528D-05 |
| -0.97395D-02 | 0.11536D-02 | 0.10490D-02 | -0.42482D-05 | 0.17324D-02 | -0.97392D-04 | -0.96519D-03 | 0.28269D-03 | -0.16042D-03 | -0.43266D-06 |
| -0.99236D-02 | 0.10490D-02 | 0.35857D-01 | -0.13631D-03 | 0.27195D-02 | -0.34465D-03 | -0.12326D-02 | 0.49203D-03 | -0.23424D-03 | -0.26241D-05 |
| 0.35866D-04 | -0.42482D-05 | -0.13631D-03 | 0.55203D-06 | -0.93892D-05 | 0.13958D-05 | 0.49918D-05 | -0.17192D-05 | 0.94861D-06 | 0.10627D-07 |
| -0.17555D-01 | 0.17324D-02 | 0.27195D-02 | -0.93892D-05 | 0.10693D 01 | 0.25909D 00 | -0.56544D 00 | 0.24974D-01 | -0.11398D-01 | -0.72593D-05 |
| 0.82224D-03 | -0.97392D-04 | -0.34465D-03 | 0.13958D-05 | 0.25909D 00 | 0.14719D 01 | -0.55418D 00 | 0.37725D-03 | -0.76810D-04 | -0.41240D-04 |
| 0.81487D-02 | -0.96519D-03 | -0.12326D-02 | 0.49918D-05 | -0.56544D 00 | -0.55418D 00 | 0.47659D 00 | -0.11527D-01 | 0.65482D-02 | 0.15528D-04 |
| -0.28532D-02 | 0.28269D-03 | 0.49203D-03 | -0.17192D-05 | 0.24974D-01 | 0.37725D-03 | -0.11527D-01 | 0.28328D 00 | -0.12155D 00 | -0.42167D-02 |
| 0.13544D-02 | -0.16042D-03 | -0.23424D-03 | 0.94861D-06 | -0.11398D-01 | -0.76810D-04 | 0.65482D-02 | -0.12199D 00 | 0.66956D-01 | 0.85854D-03 |
| 0.36528D-05 | -0.43266D-06 | -0.26241D-05 | 0.10627D-07 | -0.72593D-05 | -0.41240D-04 | 0.15528D-04 | -0.42167D-02 | 0.85854D-03 | 0.46096D-03 |
| 0.14156D-01 | -0.14980D-02 | -0.29665D-01 | 0.10958D-03 | 0.29676D 00 | 0.25074D-01 | -0.15477D 00 | 0.10396D 00 | -0.48294D-01 | -0.12609D-02 |
| -0.46065D-02 | 0.54562D-03 | 0.16626D-01 | -0.67333D-04 | -0.51848D-01 | 0.66350D-01 | 0.11397D-01 | -0.21466D-01 | 0.11782D-01 | 0.15108D-03 |
| -0.10003D 00 | 0.11848D-01 | 0.69543D-01 | -0.28163D-03 | -0.29213D 01 | -0.17104D 01 | 0.21444D 01 | -0.88539D 00 | 0.48597D 00 | 0.62313D-02 |
| 0.44980D-05 | -0.53278D-06 | 0.21281D-04 | -0.86184D-07 | -0.51937D-04 | -0.90781D-03 | 0.28004D-03 | -0.98353D-03 | 0.18767D-03 | 0.11124D-03 |

| | | | |
|---|---|---|---|
| -0.21590D-01 | 0.79735D-02 | 0.13472D 00 | -0.21676D-05 |
| 0.12926D-02 | -0.52971D-03 | -0.89495D-02 | 0.14400D-06 |
| -0.19821D-03 | 0.57518D-04 | 0.23266D-02 | -0.54604D-07 |
| 0.25917D-04 | -0.65641D-05 | -0.26552D-03 | 0.62316D-08 |
| 0.25793D-01 | -0.91130D-02 | -0.17396D 00 | -0.34173D-04 |
| -0.12163D-01 | 0.14697D-01 | -0.12582D 00 | 0.49204D-04 |
| -0.35307D-02 | -0.19500D-02 | 0.96735D-01 | -0.53651D-05 |
| 0.84198D 00 | -0.59333D 00 | 0.12578D 01 | -0.93498D-03 |
| -0.49043D 00 | 0.46017D 00 | -0.22440D 01 | 0.16680D-02 |
| -0.13387D 00 | -0.65009D-01 | 0.37139D 01 | -0.27607D-02 |
| 0.14156D-01 | -0.46065D-02 | -0.10003D 00 | 0.44980D-05 |
| -0.14980D-02 | 0.54562D-03 | 0.11848D-01 | -0.53278D-06 |
| -0.29665D-01 | 0.16626D-01 | 0.69543D-01 | 0.21281D-04 |
| 0.10958D-03 | -0.67333D-04 | -0.28163D-03 | -0.86184D-07 |
| 0.29676D 00 | -0.51848D-01 | -0.29213D 01 | -0.51937D-04 |
| 0.25074D-01 | 0.66350D-01 | -0.17104D 01 | -0.90781D-03 |
| -0.15477D 00 | 0.11397D-01 | 0.21444D 01 | 0.28004D-03 |
| 0.10396D 00 | -0.21466D-01 | -0.88539D 00 | -0.98353D-03 |
| -0.48294D-01 | 0.11782D-01 | 0.48597D 00 | 0.18767D-03 |
| -0.12609D-02 | 0.15108D-03 | 0.62313D-02 | 0.11124D-03 |
| 0.10509D 01 | -0.22743D 00 | -0.96515D 01 | -0.77695D-02 |
| -0.22743D 00 | 0.35621D 00 | -0.38759D 01 | -0.12402D-02 |
| -0.96515D 01 | -0.38759D 01 | 0.22484D 03 | 0.81509D-01 |
| -0.77695D-02 | -0.12402D-02 | 0.81509D-01 | 0.71600D-03 |

TABLE OF LS CORRELATED PREDICTION INTERVAL VALUES

| | PI + | PREDICTED | ACTUAL | PI - |
|---|---|---|---|---|
| 1 | 46.179220 | 34.234324 | 27.270000 | 22.289429 |
| 2 | 42.205896 | 30.422669 | 28.390000 | 18.639443 |
| 3 | 42.380366 | 30.596566 | 28.400000 | 18.812766 |
| 4 | 52.642602 | 41.020917 | 51.610000 | 29.399232 |
| 5 | 49.639078 | 38.147348 | 40.040000 | 26.655619 |
| 6 | 48.794931 | 37.317595 | 39.150000 | 25.840259 |
| 7 | 48.169940 | 36.699251 | 41.020000 | 25.228561 |
| 8 | 48.794931 | 37.317595 | 38.200000 | 25.840259 |
| 9 | 49.955463 | 38.462943 | 34.570000 | 26.970424 |
| 10 | 48.794931 | 37.317595 | 35.910000 | 25.840259 |
| 11 | 48.794931 | 37.317595 | 50.980000 | 25.840259 |
| 12 | 48.794931 | 37.317595 | 37.340000 | 25.840259 |
| 13 | 49.868924 | 38.382012 | 37.320000 | 26.895100 |
| 14 | 49.789118 | 38.300706 | 33.690000 | 26.812293 |
| 15 | 24.544679 | 12.757103 | 11.320000 | 0.969526 |
| 16 | 22.891935 | 11.106287 | 11.510000 | -0.679361 |
| 17 | 22.891935 | 11.106287 | 12.080000 | -0.679361 |
| 18 | 22.891935 | 11.106287 | 12.490000 | -0.679361 |
| 19 | 23.634649 | 11.745668 | 8.700000 | -0.143313 |
| 20 | 21.821871 | 9.921718 | 12.930000 | -1.978434 |
| 21 | 29.830509 | 18.248583 | 18.490000 | 6.666657 |
| 22 | 38.584367 | 26.911125 | 25.090000 | 15.237882 |
| 23 | 34.969159 | 23.383477 | 23.990000 | 11.797795 |
| 24 | 34.821636 | 23.241512 | 44.700000 | 11.661388 |
| 25 | 35.865046 | 24.165638 | 20.250000 | 12.466230 |
| 26 | 47.221375 | 35.535042 | 38.490000 | 23.848708 |
| 27 | 43.930199 | 32.312108 | 27.270000 | 20.694018 |

## TABLE OF LS (INDEPEND) PREDICTION INTERVAL VALUES

| | PI + | PREDICTED | ACTUAL | PI - |
|---|---|---|---|---|
| 1 | 45.244064 | 34.234222 | 27.269989 | 23.224365 |
| 2 | 41.287399 | 30.422546 | 28.389999 | 19.557678 |
| 3 | 41.473557 | 30.596436 | 28.399994 | 19.719299 |
| 4 | 51.648346 | 41.020844 | 51.609985 | 30.393326 |
| 5 | 48.606689 | 38.147217 | 40.039993 | 27.687729 |
| 6 | 47.776276 | 37.317474 | 39.149994 | 26.858658 |
| 7 | 47.151108 | 36.699127 | 41.019989 | 26.247131 |
| 8 | 47.776276 | 37.317474 | 38.199997 | 26.858658 |
| 9 | 48.924454 | 38.462814 | 34.569992 | 28.001160 |
| 10 | 47.776276 | 37.317474 | 35.909988 | 26.858658 |
| 11 | 47.776276 | 37.317474 | 50.979996 | 26.858658 |
| 12 | 47.776276 | 37.317474 | 37.339996 | 26.858658 |
| 13 | 48.839310 | 38.381882 | 37.319992 | 27.924438 |
| 14 | 48.758224 | 38.300552 | 33.689987 | 27.842865 |
| 15 | 23.405258 | 12.757086 | 11.320000 | 2.108913 |
| 16 | 21.751877 | 11.106257 | 11.509999 | 0.460636 |
| 17 | 21.751877 | 11.106257 | 12.080000 | 0.460636 |
| 18 | 21.751877 | 11.106257 | 12.490000 | 0.460636 |
| 19 | 22.433395 | 11.745620 | 8.700000 | 1.057842 |
| 20 | 20.619278 | 9.921659 | 12.929999 | -0.775964 |
| 21 | 28.684082 | 18.248499 | 18.489990 | 7.812836 |
| 22 | 37.507278 | 26.911041 | 25.089996 | 16.314789 |
| 23 | 33.909637 | 23.383377 | 23.989990 | 12.857104 |
| 24 | 33.762192 | 23.241409 | 44.699997 | 12.720620 |
| 25 | 34.811674 | 24.165512 | 20.250000 | 13.519141 |
| 26 | 46.174057 | 35.534973 | 38.489990 | 24.895874 |
| 27 | 42.866684 | 32.312012 | 27.269989 | 21.757324 |

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MODEL D - 4 CERs - 36 OBSERVATIONS

```
MULTIPLE REGRESSICN.....GENRAL
     SELECTICN..... 1

SELECTICN CARD 037 121
          LEAST SQUARES REGRESSION COEFICIENTS

    0.470823     0.001158
STD. ERROR OF ESTIMATE=        0.804216

          COVARIANCE MATRIX CF THE ESTIMATES
   0.13244D 00 -0.35830D-04
  -0.35830D-04  0.11214D-07
ADJUSTED MULTIPLE CCRRELATICN COEFICIENT 0.772147


MULTIPLE REGRESSICN.....GENRAL
     SELECTION..... 2

SELECTION CARD 038 23341
          LEAST SQUARES REGRESSION COEFICIENTS

    0.005228     0.286414    -0.651530
STD. ERRCR OF ESTIMATE=        1.070823

          COVARIANCE MATRIX CF THE ESTIMATES
   0.26784D 00 -0.21842D 02  0.36659D 02
  -0.21842D 02  0.36286D 04 -0.21771D 05
   0.36659D 02 -0.21771D 05  0.21581D 06
ADJUSTED MULTIPLE CORRELATICN COEFICIENT 0.891135
```

MULTIPLE REGRESSION.....GENRAL

SELECTION..... 3

SELECTION CARD 039 22128

LEAST SQUARES REGRESSION COEFICIENTS

-6.785734    0.005850    6.664107

STD. ERROR OF ESTIMATE=    4.824125

COVARIANCE MATRIX OF THE ESTIMATES

| | | |
|---|---|---|
| 0.59974D 01 | -0.21718D-02 | 0.21987D 01 |
| -0.21718D-02 | 0.10359D-05 | -0.15753D-02 |
| 0.21987D 01 | -0.15753D-02 | 0.39246D 01 |

ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.844043

MULTIPLE REGRESSION.....GENRAL

SELECTION..... 4

SELECTION CARD 040 21423

LEAST SQUARES REGRESSION COEFICIENTS

-1.014043    0.005553    0.002013

STD. ERROR OF ESTIMATE=    2.233319

COVARIANCE MATRIX OF THE ESTIMATES

| | | |
|---|---|---|
| 0.85570D 00 | -0.54173D-03 | -0.19542D-05 |
| -0.54173D-03 | 0.43047D-06 | -0.36635D-07 |
| -0.19542D-05 | -0.36635D-07 | 0.68360D-07 |

ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.824401

MATRIX OF RESIDUALS FOR SELECTIONS

| CASE NO. | SELECTION NUMBER 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0.14339 | -0.00057 | -2.94288 | -5.43512 |
| 2 | 0.20339 | 0.00005 | -2.58288 | 0.59742 |
| 3 | 0.48339 | 0.00041 | -2.97288 | -0.33258 |
| 4 | -0.12363 | -0.00005 | 0.31065 | 3.61752 |
| 5 | -0.57775 | -0.00155 | 0.49625 | 1.27806 |
| 6 | -0.57848 | -0.00041 | 0.19305 | -1.34539 |
| 7 | -1.41931 | 0.00099 | 1.17350 | 0.52246 |
| 8 | -0.58848 | 0.00030 | -2.30695 | 0.63461 |
| 9 | 0.26940 | -0.00048 | -4.10020 | -2.2442[illegible] |
| 10 | -0.19848 | 0.00064 | -2.75695 | -1.375[illegible] |
| 11 | -0.15848 | -0.00050 | 10.26305 | 1.12[illegible] |
| 12 | 0.23152 | 0.00045 | -3.78695 | 0.34[illegible]6 |
| 13 | 0.43655 | -0.00109 | -1.64665 | -0.7430[illegible] |
| 14 | -0.36260 | -0.00116 | -3.56135 | -0.63637 |
| 15 | 0.83358 | -0.00115 | 1.19070 | -3.01340 |
| 16 | 0.39358 | 0.00114 | 0.78070 | -0.51681 |
| 17 | 0.64358 | -0.00048 | 1.07070 | 0.34319 |
| 18 | 0.87358 | 0.00151 | 1.11070 | 0.25319 |
| 19 | -0.31749 | -0.00056 | -0.44880 | -2.12091 |
| 20 | 0.12251 | 0.00116 | 1.12120 | 2.08657 |
| 21 | -0.23298 | -0.00123 | 0.72584 | -0.66180 |
| 22 | -0.42351 | -0.00064 | -3.30648 | 0.47806 |
| 23 | -0.68065 | -0.00066 | -2.82003 | 3.01800 |
| 24 | -0.87518 | -0.00025 | 15.13867 | 5.20251 |
| 25 | -0.53170 | -0.00037 | -3.49378 | -0.29083 |
| 26 | -0.48941 | -0.00036 | 2.43806 | -1.28208 |
| 27 | -0.47153 | -0.00033 | -4.35519 | -0.08743 |
| 28 | 1.95286 | 0.00212 | 5.04403 | 4.84444 |
| 29 | -0.15714 | 0.00077 | -4.77597 | 0.14444 |
| 30 | 0.69801 | 0.00134 | 10.34475 | 1.63506 |
| 31 | 0.37146 | -0.00015 | 5.04655 | 1.43833 |
| 32 | 1.93801 | 0.00042 | -7.40525 | -3.41301 |
| 33 | 1.56801 | 0.00130 | -5.41525 | -1.06301 |
| 34 | -0.75720 | -0.00024 | 4.30830 | 1.10806 |
| 35 | -1.30152 | -0.00069 | -1.21455 | -2.53988 |
| 36 | -0.91731 | 0.00029 | -0.86380 | -1.56988 |

MATRIX OF CORRELATIONS BETWEEN SELECTIONS

| SELECTION NO. | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 1.000000 | 0.441009 | -0.106147 | -0.054891 |
| 2 | 0.441009 | 1.000000 | 0.084968 | 0.278739 |
| 3 | -0.106147 | 0.084968 | 1.000000 | 0.555159 |
| 4 | -0.054891 | 0.278739 | 0.555159 | 1.000000 |

JOINT GENERALIZED LEAST SQUARES REGRESSION OUTPUT

COVARIANCE MATRIX OF THE JOINT ESTIMATES

```
 0.11967D 00 -0.32144D-04 -0.32757D-04  0.18977D-02  0.15322D-01 -0.85815D-01
-0.32144D-04  0.10061D-07  0.12835D-07 -0.59397D-06 -0.47957D-05  0.23528D-04
-0.32757D-04  0.12835D-07  0.15394D-06 -0.11363D-04  0.95645D-05 -0.32109D-04
 0.18977D-02 -0.59397D-06 -0.11363D-04  0.17446D-02 -0.96895D-02  0.35733D-02
 0.15322D-01 -0.47957D-05  0.95645D-05 -0.96895D-02  0.10216D 00 -0.33900D-02
-0.85815D-01  0.23528D-04 -0.32109D-04  0.35733D-02 -0.33900D-02  0.50814D 01
 0.23743D-04 -0.74313D-08  0.13648D-07 -0.11461D-05  0.78328D-06 -0.17264D-02
-0.95107D-03  0.29768D-06 -0.29105D-05  0.12235D-03  0.12288D-02  0.14220D 01
-0.26754D-01  0.75761D-05 -0.46303D-04  0.41524D-02  0.72026D-02  0.98139D 00
 0.18501D-04 -0.57906D-08  0.45298D-07 -0.29554D-05 -0.71024D-05 -0.62762D-03
-0.32204D-06  0.10079D-09  0.97977D-09 -0.33628D-06  0.29607D-05  0.13747D-06

 0.23743D-04 -0.95107D-03 -0.26754D-01  0.18501D-04 -0.32204D-06
-0.74313D-08  0.29768D-06  0.75761D-05 -0.57906D-08  0.10079D-09
 0.13648D-07 -0.29105D-05 -0.46303D-04  0.45298D-07  0.97977D-09
-0.11461D-05  0.12235D-03  0.41524D-02 -0.29554D-05 -0.33628D-06
 0.78328D-06  0.12288D-02  0.72026D-02 -0.71024D-05  0.29607D-05
-0.17264D-02  0.14220D 01  0.98139D 00 -0.62762D-03  0.13747D-06
 0.76717D-06 -0.10035D-02 -0.26667D-03  0.19983D-06  0.36243D-08
-0.10035D-02  0.24703D 01  0.31761D-01 -0.14995D-04 -0.16224D-04
-0.26667D-03  0.31761D-01  0.74682D 00 -0.46959D-03  0.78863D-06
 0.19983D-06 -0.14995D-04 -0.46959D-03  0.36801D-06 -0.22551D-07
 0.36243D-08 -0.16224D-04  0.78863D-06 -0.22551D-07  0.39376D-07
```

COMPUTATIONAL DETERMINANT CHECK= 0.1366244D 44

JOINT LEAST SQUARES ESTIMATES

| SELECTION NO. | CONSTANT | B1 | B2 | B3 |
|---|---|---|---|---|
| | | COEFICIENTS | | |
| 1 | 0.597865 | 0.001118 | | |
| 2 | 0.005293 | 0.274581 | -0.574847 | |
| 3 | -6.307849 | 0.005587 | 7.167954 | |
| 4 | -0.680097 | 0.005401 | 0.001832 | |

TABLE OF PREDICTION INTERVAL VALUES

| | PI + | PREDICTED | ACTUAL | PI - |
|---|---|---|---|---|
| 1 | 46.779931 | 33.966494 | 27.270000 | 21.153057 |
| 2 | 41.496805 | 28.721677 | 28.390000 | 15.946549 |
| 3 | 41.567747 | 28.791345 | 28.400000 | 16.014943 |
| 4 | 57.000953 | 44.309232 | 51.610000 | 31.617510 |
| 5 | 52.267304 | 39.608704 | 40.040000 | 26.950104 |
| 6 | 51.668665 | 39.009967 | 39.150000 | 26.351269 |
| 7 | 50.944936 | 38.280008 | 41.020000 | 25.615080 |
| 8 | 51.668665 | 39.009967 | 38.200000 | 26.351269 |
| 9 | 52.574953 | 39.918795 | 34.570000 | 27.262637 |
| 10 | 51.668665 | 39.009967 | 35.910000 | 26.351269 |
| 11 | 51.668665 | 39.009967 | 50.980000 | 26.351269 |
| 12 | 51.668665 | 39.009967 | 37.340000 | 26.351269 |
| 13 | 52.836926 | 40.183513 | 37.320000 | 27.530101 |
| 14 | 52.655075 | 40.000924 | 33.690000 | 27.346774 |
| 15 | 25.302176 | 12.369640 | 11.320000 | -0.562896 |
| 16 | 22.971987 | 10.045793 | 11.510000 | -2.880401 |
| 17 | 22.971987 | 10.045793 | 12.080000 | -2.880401 |
| 18 | 22.971987 | 10.045793 | 12.490000 | -2.880401 |
| 19 | 24.486919 | 11.580702 | 8.700000 | -1.325514 |
| 20 | 21.968723 | 9.070958 | 12.930000 | -3.826807 |
| 21 | 31.529251 | 18.733967 | 18.490000 | 5.938682 |
| 22 | 42.126864 | 29.364184 | 25.090000 | 16.601505 |
| 23 | 36.868823 | 24.138036 | 23.990000 | 11.407248 |
| 24 | 36.853816 | 24.125882 | 44.700000 | 11.397948 |
| 25 | 36.939294 | 24.211309 | 20.250000 | 11.483325 |
| 26 | 49.691832 | 36.953751 | 38.490000 | 24.215669 |
| 27 | 45.270486 | 32.558521 | 27.270000 | 19.846556 |
| 28 | 60.623622 | 47.915886 | 67.590000 | 35.208151 |
| 29 | 60.631846 | 47.924124 | 46.230000 | 35.216402 |
| 30 | 87.494562 | 74.356561 | 93.900000 | 61.218559 |
| 31 | 79.445994 | 66.331784 | 75.310000 | 53.217575 |
| 32 | 78.264738 | 65.180421 | 59.300000 | 52.096105 |
| 33 | 78.264738 | 65.180421 | 62.900000 | 52.096105 |
| 34 | 87.973501 | 74.452855 | 80.040000 | 60.932210 |
| 35 | 77.611894 | 64.175077 | 59.500000 | 50.738260 |
| 36 | 77.615738 | 64.176842 | 62.170000 | 50.737946 |

MODEL D

JOINT GENERALIZED LEAST SQUARES

PREDICTION INTERVAL VALUES

LEAST SQUARES COVARIANCE MATRIX

| | | | | | |
|---|---|---|---|---|---|
| 0.12508D 00 | -0.33839D-04 | -0.36173D-04 | 0.21077D-02 | 0.16007D-01 | -0.78464D-01 |
| -0.33839D-04 | 0.10591D-07 | 0.13904D-07 | -0.65968D-06 | -0.50100D-05 | 0.21227D-04 |
| -0.36173D-04 | 0.13904D-07 | 0.23358D-06 | -0.19049D-04 | 0.32005D-04 | -0.42120D-04 |
| 0.21077D-02 | -0.65968D-06 | -0.19049D-04 | 0.31645D-02 | -0.18987D-01 | 0.27176D-02 |
| 0.16007D-01 | -0.50100D-05 | 0.32005D-04 | -0.18987D-01 | 0.18821D 00 | 0.15430D-01 |
| -0.78464D-01 | 0.21227D-04 | -0.42120D-04 | 0.27176D-02 | 0.15430D-01 | 0.54976D 01 |
| 0.21227D-04 | -0.66438D-08 | 0.16507D-07 | -0.97927D-06 | -0.37052D-05 | -0.19908D-02 |
| 0.98412D-16 | -0.31542D-19 | -0.17017D-05 | 0.56932D-03 | -0.49736D-02 | 0.20155D 01 |
| -0.16762D-01 | 0.44489D-05 | -0.54705D-04 | 0.44311D-02 | 0.11613D-01 | 0.10210D 01 |
| 0.10857D-04 | -0.33981D-08 | 0.51459D-07 | -0.30981D-05 | -0.11034D-04 | -0.65541D-03 |
| -0.17567D-06 | 0.54984D-10 | 0.13540D-08 | -0.45879D-06 | 0.40236D-05 | -0.38304D-05 |

| | | | | |
|---|---|---|---|---|
| 0.21227D-04 | 0.53176D-16 | -0.16762D-01 | 0.10857D-04 | -0.17567D-06 |
| -0.66433D-08 | -0.15579D-19 | 0.44489D-05 | -0.33981D-08 | 0.54984D-10 |
| 0.16507D-07 | -0.17017D-05 | -0.54705D-04 | 0.51459D-07 | 0.13540D-08 |
| -0.97927D-06 | 0.56932D-03 | 0.44311D-02 | -0.30981D-05 | -0.45879D-06 |
| -0.37052D-05 | -0.49736D-02 | 0.11613D-01 | -0.11034D-04 | 0.40236D-05 |
| -0.19908D-02 | 0.20155D 01 | 0.10210D 01 | -0.65541D-03 | -0.38304D-05 |
| 0.94953D-06 | -0.14440D-02 | -0.27963D-03 | 0.20776D-06 | 0.69809D-08 |
| -0.14440D-02 | 0.35975D 01 | 0.34172D-01 | -0.11604D-04 | -0.25579D-04 |
| -0.27963D-03 | 0.34172D-01 | 0.78440D 00 | -0.49659D-03 | -0.17913D-05 |
| 0.20776D-06 | -0.11604D-04 | -0.49659D-03 | 0.39459D-06 | -0.33582D-07 |
| 0.69809D-08 | -0.25579D-04 | -0.17913D-05 | -0.33582D-07 | 0.62664D-07 |

## TABLE OF LS CORRELATED PREDICTION INTERVAL VALUES

| | PI + | PREDICTED | ACTUAL | PI - |
|---|---|---|---|---|
| 1 | 47.496196 | 34.597153 | 27.270000 | 21.698110 |
| 2 | 41.673262 | 28.833934 | 28.390000 | 15.994607 |
| 3 | 41.753755 | 28.912179 | 28.400000 | 16.070603 |
| 4 | 57.334560 | 44.605451 | 51.610000 | 31.876342 |
| 5 | 52.027124 | 39.350252 | 40.040000 | 26.673380 |
| 6 | 51.422412 | 38.745332 | 39.150000 | 26.068252 |
| 7 | 50.687839 | 38.001443 | 41.020000 | 25.315047 |
| 8 | 51.422412 | 38.745332 | 38.200000 | 26.068252 |
| 9 | 52.340885 | 39.667734 | 34.570000 | 26.994582 |
| 10 | 51.422412 | 38.745332 | 35.910000 | 26.068252 |
| 11 | 51.422412 | 38.745332 | 50.980000 | 26.068252 |
| 12 | 51.422412 | 38.745332 | 37.340000 | 26.068252 |
| 13 | 52.614181 | 39.944989 | 37.320000 | 27.275797 |
| 14 | 52.431483 | 39.760984 | 33.690000 | 27.090484 |
| 15 | 25.116778 | 12.164660 | 11.320000 | -0.787458 |
| 16 | 22.551253 | 9.607907 | 11.510000 | -3.335440 |
| 17 | 22.551253 | 9.607907 | 12.080000 | -3.335440 |
| 18 | 22.551253 | 9.607907 | 12.490000 | -3.335440 |
| 19 | 24.283593 | 11.355201 | 8.700000 | -1.573191 |
| 20 | 21.514075 | 8.597391 | 12.930000 | -4.319294 |
| 21 | 31.594248 | 18.775287 | 18.490000 | 5.956326 |
| 22 | 42.729326 | 29.902177 | 25.090000 | 17.075028 |
| 23 | 36.954224 | 24.177540 | 23.990000 | 11.400856 |
| 24 | 36.935584 | 24.163164 | 44.700000 | 11.390745 |
| 25 | 37.018018 | 24.245725 | 20.250000 | 11.473432 |
| 26 | 49.791322 | 37.002960 | 38.490000 | 24.214598 |
| 27 | 44.883587 | 32.133265 | 27.270000 | 19.382943 |
| 28 | 60.699060 | 47.980272 | 67.590000 | 35.261484 |
| 29 | 60.707653 | 47.988864 | 46.230000 | 35.270076 |
| 30 | 88.383230 | 75.122716 | 93.900000 | 61.862201 |
| 31 | 79.428899 | 66.233432 | 75.310000 | 53.037964 |
| 32 | 78.209221 | 65.039599 | 59.300000 | 51.869977 |
| 33 | 78.209221 | 65.039599 | 62.900000 | 51.869977 |
| 34 | 89.737422 | 76.038214 | 80.040000 | 62.339005 |
| 35 | 78.311529 | 64.757111 | 5[illegible].500000 | 51.202693 |
| 36 | 78.317571 | 64.760783 | 62.170000 | 51.203995 |

TABLE OF LS (INDEPEND) PREDICTION INTERVAL VALUES

| | PI + | PREDICTED | ACTUAL | PI - |
|---|---|---|---|---|
| 1 | 45.842148 | 34.597580 | 27.269989 | 23.352997 |
| 2 | 40.020111 | 28.835068 | 28.389999 | 17.650009 |
| 3 | 40.093536 | 28.913330 | 28.399994 | 17.733109 |
| 4 | 55.642090 | 44.605728 | 51.609985 | 33.569351 |
| 5 | 50.345749 | 39.351334 | 40.039993 | 28.356903 |
| 6 | 49.722260 | 38.746384 | 39.149994 | 27.770493 |
| 7 | 48.965347 | 38.002487 | 41.019989 | 27.039612 |
| 8 | 49.722260 | 38.746384 | 38.199997 | 27.770493 |
| 9 | 50.668243 | 39.668839 | 34.569992 | 28.669418 |
| 10 | 49.722260 | 38.746384 | 35.909988 | 27.770493 |
| 11 | 49.722260 | 38.746384 | 50.979996 | 27.770493 |
| 12 | 49.722260 | 38.746384 | 37.339996 | 27.770493 |
| 13 | 50.939285 | 39.946075 | 37.319992 | 28.952850 |
| 14 | 50.747025 | 39.762070 | 33.689937 | 28.777100 |
| 15 | 23.195160 | 12.164678 | 11.320000 | 1.134190 |
| 16 | 20.629440 | 9.608246 | 11.509999 | -1.412958 |
| 17 | 20.629440 | 9.608246 | 12.080000 | -1.412958 |
| 18 | 20.629440 | 9.608246 | 12.490000 | -1.412958 |
| 19 | 22.365707 | 11.355088 | 8.700000 | 0.344466 |
| 20 | 19.597290 | 8.597610 | 12.929999 | -2.402081 |
| 21 | 29.731567 | 18.775330 | 18.489990 | 7.819076 |
| 22 | 40.925278 | 29.902405 | 25.089996 | 18.879517 |
| 23 | 35.157039 | 24.178421 | 23.989990 | 13.204800 |
| 24 | 35.132599 | 24.164032 | 44.699997 | 13.195461 |
| 25 | 35.219788 | 24.246597 | 20.250000 | 13.273404 |
| 26 | 48.000046 | 37.003036 | 38.489990 | 26.006012 |
| 27 | 43.078568 | 32.134003 | 27.269989 | 21.189423 |
| 28 | 59.034042 | 47.981674 | 67.529996 | 36.929291 |
| 29 | 59.042648 | 47.990265 | 46.229996 | 36.937866 |
| 30 | 86.897797 | 75.123077 | 93.899994 | 63.348343 |
| 31 | 77.875992 | 66.235046 | 75.309998 | 54.594086 |
| 32 | 76.669571 | 65.041183 | 59.299988 | 53.412781 |
| 33 | 76.669571 | 65.041183 | 62.899994 | 53.412781 |
| 34 | 88.234818 | 76.038618 | 80.039993 | 63.842804 |
| 35 | 76.781921 | 64.759079 | 59.500000 | 52.736221 |
| 36 | 76.785767 | 64.762741 | 62.169998 | 52.739700 |

MODEL E - 4 CERs - 27 OBSERVATIONS

```
MULTIPLE REGRESSION.....ESCORT
     SELECTION..... 1

SELECTION CARD 037 121
          LEAST SQUARES REGRESSION COEFICIENTS

    1.207170    0.000825
STD. ERROR OF ESTIMATE=       0.495120

          COVARIANCE MATRIX OF THE ESTIMATES
   0.11966D 00 -0.42793D-04
  -0.42793D-04  0.16566D-07
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.606372


MULTIPLE REGRESSION.....ESCORT
     SELECTION..... 2

SELECTION CARD 038 133
          LEAST SQUARES REGRESSION COEFICIENTS

    0.662174   -0.755852
STD. ERROR OF ESTIMATE=       0.912000

          COVARIANCE MATRIX OF THE ESTIMATES
   0.22857D 00 -0.22570D 01
  -0.22570D 01  0.25757D 02
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.894681
```

```
MULTIPLE REGRESSION.....ESCORT
     SELECTION..... 3

SELECTION CARD 039 22128
         LEAST SQUARES REGRESSION COEFICIENTS

   -4.188301    0.004682    7.148997
STD. ERROR OF ESTIMATE=       4.414830

         COVARIANCE MATRIX OF THE ESTIMATES

    0.13266D 02 -0.57465D-02  0.47872D 01
   -0.57465D-02  0.27811D-05 -0.29960D-02
    0.47872D 01 -0.29960D-02  0.61073D 01
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.687568


MULTIPLE REGRESSION.....ESCORT
     SELECTION..... 4

SELECTION CARD 040 21423
         LEAST SQUARES REGRESSION COEFICIENTS

   -2.627433    0.007357    0.001560
STD. ERROR OF ESTIMATE=       2.051149

         COVARIANCE MATRIX OF THE ESTIMATES

    0.26712D 01 -0.23852D-02 -0.97503D-04
   -0.23852D-02  0.23061D-05  0.14967D-07
   -0.97503D-04  0.14967D-07  0.13923D-06
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.593167
```

MATRIX OF RESIDUALS FOR SELECTIONS

| CASE NO. | SELECTION NUMBER 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0.36221 | -0.01455 | -2.19580 | -4.25904 |
| 2 | 0.42221 | 0.03284 | -1.83580 | 0.47754 |
| 3 | 0.70221 | 0.03799 | -2.22580 | -0.45246 |
| 4 | 0.19227 | 0.03229 | 0.91279 | 4.47984 |
| 5 | -0.24050 | -0.06026 | 1.17315 | 0.67120 |
| 6 | -0.24390 | 0.01815 | 0.86060 | -1.86567 |
| 7 | -1.11041 | 0.10234 | 1.75111 | -0.05373 |
| 8 | -0.25390 | 0.07535 | -1.63940 | 0.11433 |
| 9 | 0.61900 | 0.03679 | -3.38008 | -2.82580 |
| 10 | 0.13610 | 0.10298 | -2.08940 | -1.89567 |
| 11 | 0.17610 | 0.01108 | 10.93060 | 0.60433 |
| 12 | 0.56610 | 0.08756 | -3.11940 | -0.17567 |
| 13 | 0.79849 | -0.01591 | -0.88331 | -1.33730 |
| 14 | -0.00667 | -0.02876 | -2.81903 | -1.25765 |
| 15 | 0.53094 | -0.19377 | 0.11191 | -1.85765 |
| 16 | 0.09094 | -0.10238 | -0.29809 | 0.05048 |
| 17 | 0.34094 | -0.16973 | -0.00809 | 0.91048 |
| 18 | 0.57094 | -0.08759 | 0.03191 | 0.82048 |
| 19 | -0.59677 | 0.00624 | -1.44582 | -0.97135 |
| 20 | -0.15677 | 0.13932 | 0.12418 | 2.61599 |
| 21 | -0.35512 | -0.00047 | 0.27904 | 0.34926 |
| 22 | -0.26608 | -0.02409 | -2.77434 | 1.10732 |
| 23 | -0.53557 | -0.02506 | -2.33112 | 2.45983 |
| 24 | -0.73743 | 0.00409 | 15.60188 | 4.59384 |
| 25 | -0.39496 | 0.00603 | -3.03407 | -0.89409 |
| 26 | -0.32164 | 0.00865 | 2.52152 | -0.66404 |
| 27 | -0.28874 | 0.02137 | -4.21916 | -0.74479 |

MATRIX OF CORRELATIONS BETWEEN SELECTIONS

| SELECTION NO. | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 1.000000 | -0.255531 | -0.280213 | -0.317896 |
| 2 | -0.255531 | 1.000000 | -0.043023 | 0.070142 |
| 3 | -0.280213 | -0.043023 | 1.000000 | 0.490192 |
| 4 | -0.317896 | 0.070142 | 0.490192 | 1.000000 |

JOINT GENERALIZED LEAST SQUARES REGRESSION OUTPUT

COVARIANCE MATRIX OF THE JOINT ESTIMATES

| | | | | |
|---|---|---|---|---|
| 0.10560D 00 | -0.37611D-04 | 0.17057D-02 | -0.23309D-01 | -0.24853D 00 |
| -0.37611D-04 | 0.14555D-07 | -0.79041D-06 | 0.90205D-05 | 0.88216D-04 |
| 0.17057D-02 | -0.79041D-06 | 0.14789D-02 | -0.14520D-01 | 0.35658D-02 |
| -0.23309D-01 | 0.90205D-05 | -0.14520D-01 | 0.16571D 00 | -0.46347D-01 |
| -0.24853D 00 | 0.88216D-04 | 0.35658D-02 | -0.46347D-01 | 0.10517D 02 |
| 0.88042D-04 | -0.34071D-07 | -0.16387D-05 | 0.18701D-04 | -0.44006D-02 |
| 0.93678D-03 | -0.36253D-06 | 0.35994D-03 | -0.41077D-02 | 0.31079D 01 |
| -0.13655D 00 | 0.48646D-04 | -0.16152D-02 | 0.22715D-01 | 0.19153D 01 |
| 0.12291D-03 | -0.47567D-07 | 0.19748D-05 | -0.22537D-04 | -0.17163D-02 |
| -0.16259D-05 | 0.62922D-09 | -0.75664D-07 | 0.86350D-06 | -0.10190D-05 |

COMPUTATIONAL DETERMINANT CHECK= 0.8418093D 34

| | | | | |
|---|---|---|---|---|
| 0.88042D-04 | 0.93678D-03 | -0.13655D 00 | 0.12291D-03 | -0.16259D-05 |
| -0.34071D-07 | -0.36253D-06 | 0.48646D-04 | -0.47567D-07 | 0.62922D-09 |
| -0.16387D-05 | 0.35994D-03 | -0.16152D-02 | 0.19748D-05 | -0.75664D-07 |
| 0.18701D-04 | -0.41077D-02 | 0.22715D-01 | -0.22537D-04 | 0.86350D-06 |
| -0.44006D-02 | 0.31079D 01 | 0.19153D 01 | -0.17163D-02 | -0.10190D-05 |
| 0.20649D-05 | -0.19425D-02 | -0.68831D-03 | 0.66468D-06 | 0.57203D-08 |
| -0.19425D-02 | 0.39702D 01 | 0.19585D-01 | -0.26538D-05 | -0.28584D-04 |
| -0.68831D-03 | 0.19585D-01 | 0.22262D 01 | -0.19933D-02 | -0.56919D-04 |
| 0.66468D-06 | -0.26538D-05 | -0.19933D-02 | 0.19321D-05 | 0.39063D-08 |
| 0.57203D-08 | -0.28584D-04 | -0.56919D-04 | 0.39063D-08 | 0.89723D-07 |

JOINT LEAST SQUARES ESTIMATES

| SELECTION NO. | COEFICIENTS CONSTANT | B1 | B2 | B3 |
|---|---|---|---|---|
| 1 | 1.072839 | 0.000877 | | |
| 2 | 0.678334 | -0.940282 | | |
| 3 | -4.779052 | 0.004788 | 7.804857 | |
| 4 | -2.050096 | 0.006788 | 0.001575 | |

TABLE OF PREDICTION INTERVAL VALUES

| | PI + | PREDICTED | ACTUAL | PI - |
|---|---|---|---|---|
| 1 | 42.744578 | 31.861400 | 27.270000 | 20.978222 |
| 2 | 38.144302 | 27.352879 | 28.390000 | 16.561456 |
| 3 | 38.566898 | 27.773446 | 28.400000 | 16.979995 |
| 4 | 53.430695 | 42.696682 | 51.610000 | 31.962670 |
| 5 | 48.907651 | 38.248580 | 40.040000 | 27.589510 |
| 6 | 48.522766 | 37.877430 | 39.150000 | 27.232095 |
| 7 | 48.297413 | 37.651677 | 41.020000 | 27.005941 |
| 8 | 48.522766 | 37.877430 | 38.200000 | 27.232095 |
| 9 | 49.023746 | 38.363142 | 34.570000 | 27.702538 |
| 10 | 48.522766 | 37.877430 | 35.910000 | 27.232095 |
| 11 | 48.522766 | 37.877430 | 50.980000 | 27.232095 |
| 12 | 48.522766 | 37.877430 | 37.340000 | 27.232095 |
| 13 | 49.289304 | 38.620256 | 37.320000 | 27.951207 |
| 14 | 49.290316 | 38.620112 | 33.690000 | 27.949909 |
| 15 | 24.638061 | 13.611625 | 11.320000 | 2.585188 |
| 16 | 22.729269 | 11.706895 | 11.510000 | 0.684520 |
| 17 | 22.729269 | 11.706895 | 12.080000 | 0.684520 |
| 18 | 22.729269 | 11.706895 | 12.490000 | 0.684520 |
| 19 | 21.843423 | 10.869270 | 8.700000 | -0.104882 |
| 20 | 19.679577 | 8.711857 | 12.930000 | -2.255862 |
| 21 | 27.853401 | 17.156160 | 18.490000 | 6.458919 |
| 22 | 38.739543 | 27.946116 | 25.090000 | 17.152688 |
| 23 | 34.119318 | 23.422719 | 23.990000 | 12.726120 |
| 24 | 34.179619 | 23.488163 | 44.700000 | 12.796707 |
| 25 | 34.140610 | 23.450805 | 20.250000 | 12.760999 |
| 26 | 46.752112 | 36.022949 | 38.490000 | 25.293786 |
| 27 | 42.864222 | 32.193560 | 27.270000 | 21.522899 |

MODEL E

JOINT GERERALIZED LEAST SQUARES

PREDICTION INTERVAL VALUES

LEAST SQUARES COVARIANCE MATRIX

| | | | | |
|---|---|---|---|---|
| 0.11079D 00 | -0.39623D-04 | 0.18428D-02 | -0.24873D-01 | -0.27123D 00 |
| -0.39623D-04 | 0.15334D-07 | -0.84344D-06 | 0.96256D-05 | 0.97001D-04 |
| 0.18428D-02 | -0.84344D-06 | 0.15326D-02 | -0.15133D-01 | 0.28119D-02 |
| -0.24873D-01 | 0.96256D-05 | -0.15133D-01 | 0.17271D 00 | -0.37743D-01 |
| -0.27123D 00 | 0.97001D-04 | 0.28119D-02 | -0.37743D-01 | 0.11792D 02 |
| 0.97001D-04 | -0.37538D-07 | -0.13039D-05 | 0.14881D-04 | -0.51080D-02 |
| 0.38512D-15 | -0.14550D-18 | 0.12907D-03 | -0.14730D-02 | 0.42552D 01 |
| -0.14794D 00 | 0.53053D-04 | -0.17310D-02 | 0.24036D-01 | 0.20186D 01 |
| 0.13400D-03 | -0.51855D-07 | 0.21069D-05 | -0.24045D-04 | -0.18106D-02 |
| -0.16806D-05 | 0.65039D-09 | -0.11017D-06 | 0.12573D-05 | -0.11404D-04 |

| | | | | |
|---|---|---|---|---|
| 0.97001D-04 | 0.56295D-15 | -0.14794D 00 | 0.13400D-03 | -0.16806D-05 |
| -0.37538D-07 | -0.20522D-18 | 0.53053D-04 | -0.51855D-07 | 0.65039D-09 |
| -0.13039D-05 | 0.12907D-03 | -0.17310D-02 | 0.21069D-05 | -0.11017D-06 |
| 0.14881D-04 | -0.14730D-02 | 0.24036D-01 | -0.24045D-04 | 0.12573D-05 |
| -0.51080D-02 | 0.42552D 01 | 0.20186D 01 | -0.18106D-02 | -0.11404D-04 |
| 0.24721D-05 | -0.26583D-02 | -0.73074D-03 | 0.70197D-06 | 0.12506D-07 |
| -0.26583D-02 | 0.54287D 01 | 0.32746D-01 | -0.69297D-05 | -0.43433D-04 |
| -0.73074D-03 | 0.32746D-01 | 0.23744D 01 | -0.21202D-02 | -0.86669D-04 |
| 0.70197D-06 | -0.69297D-05 | -0.21202D-02 | 0.20499D-05 | 0.13304D-07 |
| 0.12506D-07 | -0.43433D-04 | -0.86669D-04 | 0.13304D-07 | 0.12376D-06 |

## TABLE OF LS CORRELATED PREDICTION INTERVAL VALUES

| | PI + | PREDICTED | ACTUAL | PI - |
|---|---|---|---|---|
| 1 | 42.956416 | 31.976923 | 27.270000 | 20.957429 |
| 2 | 38.402354 | 27.510643 | 28.390000 | 16.618931 |
| 3 | 38.815021 | 27.921190 | 28.400000 | 17.027360 |
| 4 | 53.056183 | 42.264562 | 51.610000 | 31.472982 |
| 5 | 48.544981 | 37.866001 | 40.040000 | 27.187021 |
| 6 | 48.133818 | 37.468809 | 39.150000 | 26.803801 |
| 7 | 47.942283 | 37.272837 | 41.020000 | 26.603392 |
| 8 | 48.133818 | 37.468809 | 38.200000 | 26.803801 |
| 9 | 48.645678 | 37.966762 | 34.570000 | 27.287847 |
| 10 | 48.133818 | 37.468809 | 35.910000 | 26.803801 |
| 11 | 48.133818 | 37.468309 | 50.980000 | 26.803801 |
| 12 | 48.133818 | 37.468809 | 37.340000 | 26.803801 |
| 13 | 48.908250 | 38.222020 | 37.320000 | 27.535791 |
| 14 | 48.921407 | 38.233249 | 33.690000 | 27.545092 |
| 15 | 24.942192 | 13.883649 | 11.320000 | 2.825107 |
| 16 | 23.050856 | 11.994706 | 11.510000 | 0.938556 |
| 17 | 23.050856 | 11.994706 | 12.080000 | 0.938556 |
| 18 | 23.050856 | 11.994706 | 12.490000 | 0.938556 |
| 19 | 22.225079 | 11.221966 | 8.700000 | 0.218853 |
| 20 | 20.082272 | 9.084766 | 12.930000 | -1.912739 |
| 21 | 28.220243 | 17.501517 | 18.490000 | 6.782790 |
| 22 | 39.242649 | 28.342673 | 25.090000 | 17.442697 |
| 23 | 34.622194 | 23.856676 | 23.990000 | 13.091159 |
| 24 | 34.699258 | 23.941518 | 44.700000 | 13.183779 |
| 25 | 34.658616 | 23.902926 | 20.250000 | 13.147237 |
| 26 | 46.575587 | 35.767393 | 38.490000 | 24.959200 |
| 27 | 42.711111 | 31.986247 | 27.270000 | 21.261384 |

TABLE OF LS (INDEPEND) PREDICTION INTERVAL VALUES

| | PI + | PREDICTED | ACTUAL | PI - |
|---|---|---|---|---|
| 1 | 42.668549 | 31.974792 | 27.269939 | 21.281021 |
| 2 | 38.081787 | 27.508209 | 28.389999 | 16.934616 |
| 3 | 38.517990 | 27.918762 | 28.399994 | 17.319519 |
| 4 | 52.661728 | 42.262314 | 51.609985 | 31.862885 |
| 5 | 48.105057 | 37.863342 | 40.039993 | 27.621613 |
| 6 | 47.700073 | 37.466171 | 39.149994 | 27.232254 |
| 7 | 47.505875 | 37.270248 | 41.019989 | 27.034607 |
| 8 | 47.700073 | 37.466171 | 38.199997 | 27.232254 |
| 9 | 48.207306 | 37.964081 | 34.569992 | 27.720840 |
| 10 | 47.700073 | 37.466171 | 35.909988 | 27.232254 |
| 11 | 47.700073 | 37.466171 | 50.979996 | 27.232254 |
| 12 | 47.700073 | 37.466171 | 37.339996 | 27.232254 |
| 13 | 48.468903 | 38.219299 | 37.319992 | 27.965681 |
| 14 | 48.479965 | 38.230545 | 33.689987 | 27.981110 |
| 15 | 24.318436 | 13.882758 | 11.320000 | 3.447076 |
| 16 | 22.433441 | 11.993677 | 11.509999 | 1.553909 |
| 17 | 22.433441 | 11.993677 | 12.080000 | 1.553909 |
| 18 | 22.433441 | 11.993677 | 12.490000 | 1.553909 |
| 19 | 21.518036 | 11.221060 | 8.700000 | 0.924072 |
| 20 | 19.377182 | 9.083716 | 12.929999 | -1.209761 |
| 21 | 27.563980 | 17.500229 | 18.489990 | 7.436467 |
| 22 | 38.695038 | 28.340729 | 25.089996 | 17.986404 |
| 23 | 34.076553 | 23.854492 | 23.989990 | 13.632421 |
| 24 | 34.152878 | 23.939346 | 44.699997 | 13.725811 |
| 25 | 34.112030 | 23.900742 | 20.250000 | 13.689447 |
| 26 | 46.046143 | 35.765503 | 38.489990 | 25.484848 |
| 27 | 42.138733 | 31.984024 | 27.269989 | 21.829300 |

## APPENDIX D

## COMPUTER PROGRAMS

### A. JOINT GENERALIZED LEAST SQUARES PROGRAM

#### 1. Program Description

The joint generalized least squares computer program is designed to read in a set of data and then carry out joint generalized least squares computations as specified on a control card, and selection cards. The program uses a number of IBM 360 Scientific Subroutine packages to do the calculations. It is programmed using FORTRAN IV language much like the Multiple Regression program in the scientific subroutine package (REGRE). The program conducts a number of standard multiple linear regressions (one for each subsystem CER). The residuals from the single regresions are saved and utilized to compute a matrix of correlations between selections. Both of these items are listed in the output. Next the program does the joint generalized least squares calculations; initially S, the matrix of mean squares and products of the least squares residuals, is computed, next $b_j$, the joint parameter estimate vector is computed along with $V(b_j)$, the covariance matrix of the joint estimates. Finally, a set of joint generalized least squares prediction interval values is calculated. The program was designed to be easy to use for analysis utilizing the joint generalized least squares method. Double precision was used to improve accuracy by reducing roundoff errors. The program will handle up to thirty-six observations and nine subsystem relations. Each subsystem CER may have up to three explanatory variables. The program takes 280 K of core storage space to run with a maximum run time of about thirty seconds. It should be noted that larger dimensional problems may be possible but computer capacity rapidly becomes a constraint to be concerned with. The scientific subroutine ARRAY was used to allow variable dimensioning so that those subroutines that deal with matrices can operate on any size array, limited only by the maximum program dimensions.

2. JGLS Program Users Guide

The program listed herein is straightforward to use. The user must supply as input a data deck of independent and dependent variables. The data array is presently programed to store up to forty-one variables and up to thirty-six observations for each variable. The user should supply the appropriate read statements in the program to accommodate the data format he is using. After the data is read in any transformations desired may be made.

a. Control Card

One control card is required for each program and is read before the data deck. This card is prepared as follows:

| COLUMNS | CONTENTS | SAMPLE |
| --- | --- | --- |
| 1-6 | Problem Name | Escort |
| 7-11 | Nu ber of Observations | 36 |
| 12-13 | Number of Variables | 41 |
| 14-15 | Number of Selection Cards | 04 |

b. Selection Card

The selection card is used to specify the type of transformation desired, the dependent variable and a set of independent variables in a subsystem multiple linear regression analysis. One selection card is required for each subsystem in the model. The variables are designated as columns in the data array. The selection card is prepared as follows:

| COLUMNS | CONTENTS | SAMPLE |
| --- | --- | --- |
| 1-2 | Transformation Code<br>00 Linear<br>01 Log Linear | 00 |
| 3-4 | Dependent Variable Designated for the Regression | 37 |
| 5-6 | Number of Independent Variables Included in Regression | 01 |
| 7-8 | 1st Independent Variable Included | 21 |
| 9-10 | 2nd Independent Variable Included | |
| 11-12 | 3rd Independent Variable Included | |

The input format 36I2 is used for the selection card.

c. Deck Setup

The deck setup to run the program for a number of selections is as follows:

1. Main Program
2. Control Card
3. Data deck (Read statements must be supplied in the main program.
4. Selection Cards (One selection card for each subsystem multiple linear regression).

The joint generalized least squares program is listed in the following section of this Appendix.

```
C PROGRAM TO COMPUTE LEAST SQUARES COST ESTIMATING RELATIONSHIPS FOR A
C NUMBER OF DESIGNATED SELECTIONS. THE RESIDUAL VALUES FROM THESE
C SELECTIONS ARE THEN USED IN THE COMPUTATION OF JOINT GENERALIZED LEAST
C SQUARES ESTIMATES AND A COVARIANCE MATRIX OF THE JOINT ESTIMATES.
C FINALLY, PREDICTION INTERVAL VALUES ARE COMPUTED FOR THE GIVEN OBSER-
C VATIONS UTILIZING THE JOINT GENERALIZED LEAST SQUARES ESTIMATES.
C
C OUTPUT FOR THE PROGRAM CONSISTS OF:
C    1. INDIVIDUAL REGRESSION OUTPUT
C    2. LISTING OF RESIDUALS FOR THE SELECTIONS
C    3. MATRIX OF CORRELATIONS BETWEEN SELECTIONS
C    4. COVARIANCE MATRIX OF THE JOINT ESTIMATES
C    5. JOINT GENERALIZED LEAST SQUARES ESTIMATES
C    6. TABLE OF JGLS PREDICTION INTERVAL VALUES
C
C USER MUST SUPPLY DATA INPUT AND CORRESPONDING READ STATEMENTS IN THE
C MAIN PROGRAM. ALSO A CONTROL CARD AND SELECTION CARDS MUST BE PROVIDED
C FOR THE REGRESSIONS DESIRED AS DESCRIBED IN THE USER'S GUIDE.
C
C
// EXEC FORTCLG,REGION.GC=350K
//FORT.SYSIN DD *
      IMPLICIT REAL*8(A-H,O-V,X-Z), INTEGER*4(W)
      DIMENSION DATA(36,41)
      DIMENSION Y(36), X(36,4), XT(4,36), XTX(4,4), W1(4), W2(4),
     1XTXXT(4,36), B(4), ISAVE(10), XB(36), RESID(36), SS(1), YY(1),
     2RESIDM(36,9)
      DIMENSION YGLS(324), XGLS(324,36), BBB(36)
      DIMENSION COMP1(36,324), COMP2(36,36), BB(36), E(9,9), W3(9),
     1W4(9), W5(36), W6(36)
      DIMENSION SSAVE(9), ROE(9,9), NSAVE(9)
      DIMENSION XOBS(36), CHAT(1), XSAVE(36), VSAVE(1), PIP(36), PIM(36)
     1,PRED(36)
      DATA NNI/0/, NNIG/0/, NN/0/
      DATA XGLS/11664*0.0/, COMP1/11664*0.0/
      DATA XOBS/36*0.0/
C   READ PROBLEM PARAMETER CARD
      READ(5,1)PR, PR1, N, M, NS
    1 FORMAT(A4,A2,I5,2I2)
C
C PR,PR1....PROBLEM NUMBER
C  N........NUMBER OF OBSERVATIONS
C  M........NUMBER OF VARIABLES
C  NS.......NUMBER OF SELECTIONS
C
C   OBSERVATION MATRIX READ IN
      DO 6000 I=1,36
      READ(5,6010)(DATA(I,J),J=1,36)
```

```
 6000 CCNTINUE
 6010 FCRMAT(/12F6.0/12F6.0/5F6.0/7F6.0)
      READ(5,6020)(DATA(I,37),I=1,36)
      READ(5,6020)(DATA(I,38),I=1,36)
      READ(5,6020)(DATA(I,39),I=1,36)
      READ(5,6020)(DATA(I,40),I=1,36)
      READ(5,602C)(DATA(I,41),I=1,36)
 602C FORMAT(9F8.0)
C
C  TEST NUMBER OF SELECTICNS
      IF(NS) 108,108,109
  108 WRITE(6,4)
    4 FORMAT(53H1NUMBER OF SELECTIONS NCT SPECIFIEC.  JOB TERMINATED.)
      GC TO 300
C
C  COMMENCE MULTIPLE REGRESSION PROGRAM
  109 DC 300 I=1,NS
      WRITE(6,5)PR,PR1,I
    5 FCRMAT(25H1MULTIPLE REGRESSION.....A4,A2//6X,14HSELECTION.....I2//
     1)
C  REAC IN SUBSET SELECTICN CARD
C
      READ(5,3)NT,NDEP,NI,(ISAVE(J),J=1,NI)
    3 FORMAT(36I2)
      WRITE(6,6001)NT,NDEP,NI,(ISAVE(J),J=1,NI)
 6001 FCRMAT('0','SELECTION CARD',15I2)
C
C  NT.......MCDEL TRANSFCRMATION CODE CESIRED
C      O.....LINEAR
C      1.....LCG LINEAR
C  NI.......NUMBER OF INDEPENDENT VARIABLES INCLUDED
C  NCEP.....DEPENDENT VARIABLE
C  ISAVE....AVECTCR CONTAINING THE INCEPENDENT VARIABLES INCLUDED
C
C  TRANSFORM CATA AND LOAC THE X AND Y ARRAYS
      DC 110 J=1,N
      Y(J)=DATA(J,NDEP)
      IF(NT) 113, 112, 113
  112 GC TO 114
  113 Y(J)=DLCG(Y(J))
      GC TO 114
  114 YGLS(NN+J)=Y(J)
      X(J,1)=1.0
      XGLS(NN+J,NNIG+1)=1.0
  11C CONTINUE
      DC 120 J=1,NI
      MM=ISAVE(J)
      DC 130 JJ=1,N
```

```
      X(JJ,J+1)=DATA(JJ,MM)
  118 XGLS(NN+JJ,NNIG+J+1)=X(JJ,J+1)
  130 CONTINUE
  120 CONTINUE
      NNI=NI+1
      NSAVE(I)=NNI
      NNIG=NNIG+NNI
      NN=NN+N
C
C   CALCULATE VECTOR OF ESTIMATES
      CALL ARRAY(2,N,NNI,36,4,X,X)
      CALL ARRAY(2,NNI,NNI,4,4,XTX,XTX)
      CALL ARRAY(2,NNI,N,4,36,XT,XT)
      CALL ARRAY(2,NNI,N,4,36,XTXXT,XTXXT)
      CALL GTPRD(X,X,XTX,N,NNI,NNI)
      CALL DMINV(XTX,NNI,DET,W1,W2)
      CALL GMTRA(X,XT,N,NNI)
      CALL GMPRD(XTX,XT,XTXXT,NNI,NNI,N)
      CALL GMPRD(XTXXT,Y,B,NNI,N,1)
      WRITE(6,5040)
 5040 FORMAT('0',T10,'LEAST SQUARES REGRESSION COEFICIENTS'//)
      WRITE(6,6)(B(J),J=1,NNI)
    6 FORMAT(' ',10F12.6)
C
C   CALCULATE RESIDUAL VECTOR
      CALL GMPRD(X,B,XB,N,NNI,1)
      CALL GMSUB(Y,XB,RESID,N,1)
C
C   ENTER RESIDUALS INTO RESIDUAL MATRIX
      DO 290 J=1,N
      RESIDM(J,I)=RESID(J)
  290 CONTINUE
C
C   CALCULATE THE STANDARD ERROR OF THE ESTIMATE
  397 CALL GTPRD(RESID,RESID,SS,N,1,1)
      BN=DFLOAT(N-NNI)
      S1=SS(1)/BN
      S=DSQRT(S1)
      SSAVE(I)=S
      WRITE(6,8)S
    8 FORMAT('0','STD. ERROR OF ESTIMATE=  ',F13.6//)
C
C   CALCULATE THE COVARIANCE MATRIX OF THE ESTIMATES
      CALL ARRAY(1,NNI,NNI,4,4,XTX,XTX)
      DO 141 J=1,NNI
      DO 142 K=1,NNI
      XTX(J,K)=S1*XTX(J,K)
  142 CONTINUE
```

```
  141 CONTINUE
      WRITE(6,5050)
 5050 FORMAT('0',T10,'COVARIANCE MATRIX OF THE ESTIMATES'/)
      DO 143 J=1,NNI
      WRITE(6,144)(XTX(J,K),K=1,NNI)
  143 CONTINUE
  144 FORMAT(' ',2X,8D13.5)
C
C  CALCULATE ADJUSTED MULTIPLE CORRELATION COEFICIENT
  990 SUM=0.0
      DO 150 J=1,N
      SUM=SUM+Y(J)
  150 CONTINUE
  992 AN=DFLOAT(N)
      YBAR=SUM/AN
      DO 160 J=1,N
      Y(J)=Y(J)-YBAR
  160 CONTINUE
      CALL GTPRD(Y,Y,YY,N,1,1)
      CN=DFLOAT(N-1)
      R=1.0-(S1/(YY(1)/CN))
      WRITE(6,9)R
    9 FORMAT('0','ADJUSTED MULTIPLE CORRELATION COEFICIENT',F9.6)
  300 CONTINUE
      WRITE(6,5000)
 5000 FORMAT('1','MATRIX OF RESIDUALS FOR SELECTIONS'//2X,'CASE NO.',
     1T31,'SELECTION NUMBER'/18X,'1',T30,'2',T42,'3',T54,'4',T66,'5',
     2T78,'6',T90,'7',T102,'8',T114,'9'/)
      DO 291 J=1,N
      WRITE(6,292)J,(RESIDM(J,K),K=1,NS)
  291 CONTINUE
  292 FORMAT(' ',5X,I3,9F12.5)
C
C
C. THE ESTIMATE FOR SIGMA IS COMPUTED
      CALL ARRAY(2,N,NS,36,9,RESIDM,RESIDM)
      CALL ARRAY(2,NS,NS,9,9,E,E)
      CALL GTPRD(RESIDM,RESIDM,E,N,NS,NS)
      CALL ARRAY(1,NS,NS,9,9,E,E)
      DO 1000 I=1,NS
      DO 1010 J=1,NS
      E(I,J)=E(I,J)/AN
 1010 CONTINUE
 1000 CONTINUE
      SSUM=0.0
      DO 1001 I=1,NS
      DO 1011 J=1,NS
      ROE(I,J)=E(I,J)/(DSQRT(E(I,I)*E(J,J)))
```

```
      SSUM=SSUM+E(I,J)
 1011 CONTINUE
 1001 CONTINUE
      WRITE(6,6250)
 6250 FORMAT('1','MATRIX OF CORRELATIONS BETWEEN SELECTIONS'//1X,
     1'SELECTION NO.',5X,'1',T30,'2',T42,'3',T54,'4',T66,'5',T78,'6',
     2T90,'7',T102,'8',T114,'9'//)
      DO 6300 I=1,NS
      WRITE(6,6400)I,(ROE(I,J),J=1,NS)
 6300 CONTINUE
 6400 FORMAT(' ',5X,I3,9F12.6)
      WRITE(6,5010)
 5010 FORMAT('1','JOINT GENERALIZED LEAST SQUARES REGRESSION OUTPUT'/)
C
C   THE ESTIMATE FOR SIGMA IS INVERTED
      CALL ARRAY(2,NS,NS,9,9,E,E)
      CALL DMINV(E,NS,DET,W3,W4)
      CALL ARRAY(1,NS,NS,9,9,E,E)
C
C
C   COMPUTE X TRANSPOSE TIMES THE KRONECKER PRODUCT OF SIGMA INVERSE AND
C   THE IDENTITY MATRIX
      KKK=((N*(NS-1))+1)
      NEC=1
      NC=N+1
      JJ=0
      DO 900 J=1,NN
      JJ=JJ+1
      IF(J.NE.NC) GO TO 910
      JJ=1
      NEC=NEC+1
      NC=((NEC*N)+1)
  910 DO 920 I=1,NNIG
      NER=0
      CSAVE=0.0
      DO 930 M=1,KKK,N
      NER=NER+1
      CSAVE=CSAVE+XGLS(JJ+M-1,I)*E(NER,NEC)
  930 CONTINUE
      COMP1(I,J)=CSAVE
  920 CONTINUE
  900 CONTINUE
C
C   MATRIX MANIPULATIONS USED TO COMPUTE JOINT VECTOR
C   OF ESTIMATES
      CALL ARRAY(2,NN,NNIG,324,36,XGLS,XGLS)
      CALL ARRAY(2,NNIG,NN,36,324,COMP1,COMP1)
      CALL ARRAY(2,NNIG,NNIG,36,36,COMP2,COMP2)
```

```
      CALL GMPRD(COMP1,YGLS,BB,NNIG,NN,1)
      CALL GMPRD(COMP1,XGLS,CCMP2,NNIG,NN,NNIG)
      CALL DMINV(COMP2,NNIG,DET,W5,W6)
      CALL GMPRD(COMP2,BB,BBB,NNIG,NNIG,1)
      CALL ARRAY(1,NNIG,NNIG,36,36,COMP2,COMP2)
      WRITE(6,1091)
 1091 FORMAT(' ',5X,'COVARIANCE MATRIX OF THE JCINT ESTIMATES'/)
      IF(10-NNIG) 6500,6450,6450
 6450 DO 2000 I=1,NNIG
      WRITE(6,1071)(COMP2(I,J),J=1,NNIG)
 2000 CONTINUE
 1071 FORMAT(' ',2X,10D13.5)
      GO TO 6600
 6500 Z=1.0
      IF(20-NNIG) 6520,6510,6510
 6510 Z=1.0
      DO 6530 I=1,NNIG
      WRITE(6,1071)(COMP2(I,J),J=1,10)
 6530 CONTINUE
      WRITE(6,6561)
 6561 FORMAT(' ')
      DO 6540 I=1,NNIG
      WRITE(6,1071)(COMP2(I,J),J=11,NNIG)
 6540 CONTINUE
      GO TO 6600
 6520 Z=1.0
      DO 6550 I=1,NNIG
      WRITE(6,1071)(COMP2(I,J),J=1,10)
 6550 CONTINUE
      WRITE(6,6561)
      DO 6560 I=1,NNIG
      WRITE(6,1071)(COMP2(I,J),J=11,20)
 6560 CONTINUE
      WRITE(6,6561)
      DO 6570 I=1,NNIG
      WRITE(6,1071)(COMP2(I,J),J=21,NNIG)
 6570 CONTINUE
 6600 WRITE(6,5030)DET
 5030 FORMAT(' ','COMPUTATIONAL DETERMINANT CHECK=  ',E13.7//)
C
C  THE ESTIMATES ARE READ OUT
      WRITE(6,2011)
 2011 FORMAT('1',T10,'JCINT LEAST SQUARES ESTIMATES'/' ',2X,'SELECTION
     1NO.',20X,'COEFICIENTS'/' ',T20,'CONSTANT',T35,'B1',T48,'B2',T61,
     2'B3'/)
      LL=1
      DO 2013 I=1,NS
      LM=LL+NSAVE(I)-1
```

```
      WRITE(6,2012)I,(BBB(J),J=LL,LM)
      LL=LL+NSAVE(I)
 2013 CONTINUE
 2012 FORMAT('0',6X,I3,3X,5F13.6/)
C
C   CALCULATE PREDICTION INTERVAL
C
      CALL ARRAY(2,NNIG,NNIG,36,36,COMP2,COMP2)
      CALL ARRAY(1,NN,NNIG,324,36,XGLS,XGLS)
      DO 9100 I=1,N
      NSS=1
      NSSS=0
      L=0
      DO 9200 J=1,KKK,N
      L=L+1
      NSSS=NSSS+NSAVE(L)
      DO 9300 K=NSS,NSSS
      XOBS(K)=XGLS(I+J-1,K)
 9300 CONTINUE
      NSS=NSS+NSAVE(L)
 9200 CONTINUE
      CALL GTPRD(XOBS,BBB,CHAT,NNIG,1,1)
      PRED(I)=CHAT(1)
      CALL GMPRD(COMP2,XOBS,XSAVE,NNIG,NNIG,1)
      CALL GTPRD(XOBS,XSAVE,VSAVE,NNIG,1,1)
      SSSUM=1.96*DSQRT( SSUM+VSAVE(1))
      PIP(I)=CHAT(1)+SSSUM
      PIM(I)=CHAT(1)-SSSUM
 9100 CONTINUE
      WRITE(6,9005)
 9005 FORMAT('1',//////////19X,'TABLE OF PREDICTION INTERVAL VALUES'/'0'
     1,17X,'PI +',9X,'PREDICTED',4X,'ACTUAL',7X,'PI -'/'0')
      DO 9000 I=1,N
      XSAVE(I)=DATA(I,36)
      WRITE(6,9010)I,PIP(I),PRED(I),XSAVE(I),PIM(I)
 9000 CONTINUE
 9010 FORMAT(' ',11X,I2,4F13.6)
      STOP
      END

      SUBROUTINE ARRAY (MODE,I,J,N,M,S,D)
      DOUBLE PRECISION S,D
      DIMENSION S(1),D(1)
      NI=N-I
      IF(MODE-1) 100, 100, 120
  100 IJ=I*J+1
      NM=N*J+1
```

```
      DO 110 K=1,J
      NM=NM-NI
      DO 110 L=1,I
      IJ=IJ-1
      NM=NM-1
  110 D(NM)=S(IJ)
      GO TO 140
  120 IJ=0
      NM=0
      DO 130 K=1,J
      DO 125 L=1,I
      IJ=IJ+1
      NM=NM+1
  125 S(IJ)=D(NM)
  130 NM=NM+NI
  140 RETURN
      END

      SUBROUTINE GTPRD(A,B,R,N,M,L)
      DOUBLE PRECISION A,B,R
      DIMENSION A(1),B(1),R(1)
C
      IR=0
      IK=-N
      DO 10 K=1,L
      IJ=0
      IK=IK+N
      DO 10 J=1,M
      IB=IK
      IR=IR+1
      R(IR)=0
      DO 10 I=1,N
      IJ=IJ+1
      IB=IB+1
   10 R(IR)=R(IR)+A(IJ)*B(IB)
      RETURN
      END

      SUBROUTINE GMSUB(A,B,R,N,M)
      DOUBLE PRECISION A,B,R
      DIMENSION A(1),B(1),R(1)
C
C        CALCULATE NUMBER OF ELEMENTS
C
      NM=N*M
C
```

```
C         SUBTRACT MATRICES                                             GMSU 390
C                                                                       GMSU 400
      DO 10 I=1,NM                                                      GMSU 410
   10 R(I)=A(I)-B(I)                                                    GMSU 420
      RETURN                                                            GMSU 430
      END                                                               GMSU 440

      SUBROUTINE GMPRD(A,B,R,N,M,L)                                     GMPR 370
      DOUBLE PRECISION A,B,R
      DIMENSION A(1),B(1),R(1)                                          GMPR 380
C                                                                       GMPR 390
      IR=0                                                              GMPR 400
      IK=-M                                                             GMPR 410
      DO 10 K=1,L                                                       GMPR 420
      IK=IK+M                                                           GMPR 430
      DO 10 J=1,N                                                       GMPR 440
      IR=IR+1                                                           GMPR 450
      JI=J-N                                                            GMPR 460
      IB=IK                                                             GMPR 470
      R(IR)=0                                                           GMPR 480
      DO 10 I=1,M                                                       GMPR 490
      JI=JI+N                                                           GMPR 500
      IB=IB+1                                                           GMPR 510
   10 R(IR)=R(IR)+A(JI)*B(IB)                                           GMPR 520
      RETURN                                                            GMPR 530
      END                                                               GMPR 540

      SUBROUTINE GMTRA(A,R,N,M)                                         GMTR 300
      DOUBLE PRECISION A,R
      DIMENSION A(1),R(1)                                               GMTR 310
C                                                                       GMTR 320
      IR=0                                                              GMTR 330
      DO 10 I=1,N                                                       GMTR 340
      IJ=I-N                                                            GMTR 350
      DO 10 J=1,M                                                       GMTR 360
      IJ=IJ+N                                                           GMTR 370
      IR=IR+1                                                           GMTR 380
   10 R(IR)=A(IJ)                                                       GMTR 390
      RETURN                                                            GMTR 400
      END                                                               GMTR 410
```

```
C PROGRAM TO COMPUTE A COVARIANCE MATRIX OF THE LEAST SQUARES REGRESSION
C COEFFICIENT ESTIMATES UNDER THE ASSUMPTION OF CORRELATIONS BETWEEN IND-
C IVIDUAL SELECTIONS. NEXT THE PROGRAM UTILIZES THIS COVARIANCE MATRIX
C TO COMPUTE LEAST SQUARES CORRELATED PREDICTION INTERVALS FOR THE GIVEN
C OBSERVATIONS.
C
C
// EXEC FORTCLG,REGION.GO=100K
//FORT.SYSIN DD *
      IMPLICIT REAL*8(A-H,O-V,X-Z), INTEGER*4(W)
      DIMENSION DATA(36,41)
      DIMENSION X1(36,4),X2(36,4),ISAVE(10),X1TX1(4,4),W1(4),W2(4),
     1COMP1(4,4),X2TX2(4,4),COMP2(4,4),COMP3(4,4)
      DIMENSION XBIG(36,30)
      DIMENSION CVARB(30,30)
      DIMENSION XOBS(36),CHAT(1),XSAVE(36),VSAVE(1),
     1PIP(36),PIM(36),PRED(36),B(36)
      DATA MMI/0/, MSI/0/
      DATA XBIG/1080*0.0/
      DATA NNI/0/, NSI/0/
C READ PROBLEM PARAMETER CARD
      READ(5,1)PR,PR1,N,M,NS
    1 FORMAT(A4,A2,I5,2I2)
C READ IN OBSERVATION MATRIX
      DO 6000 I=1,36
      READ(5,6010)(DATA(I,J),J=1,36)
 6000 CONTINUE
 6010 FORMAT(/12F6.0/12F6.0/5F6.0/7F6.0)
      READ(5,6020)(DATA(I,37),I=1,36)
      READ(5,6020)(DATA(I,38),I=1,36)
      READ(5,6020)(DATA(I,39),I=1,36)
      READ(5,6020)(DATA(I,40),I=1,36)
      READ(5,6020)(DATA(I,41),I=1,36)
 6020 FORMAT(9F8.0)
C READ IN FIRST SUBSET SELECTION CARD
      DO 305 JK=1,NS
      READ(5,3)NT,NDEP,NI,(ISAVE(J),J=1,NI)
    3 FORMAT(36I2)
      DO 110 J=1,N
      X1(J,1)=1.0D0
  110 CONTINUE
      DO 120 J=1,NI
      MM=ISAVE(J)
      DO 130 JJ=1,N
      X1(JJ,J+1)=DATA(JJ,MM)
  130 CONTINUE
  120 CONTINUE
      NNI=NI+1
```

```
      CALL ARRAY(2,N,NNI,36,4,X1,X1)
      CALL GTPRD(X1,X1,X1TX1,N,NNI,NNI)
      CALL DMINV(X1TX1,NNI,DET,W1,W2)
C READ IN SECOND SUBSET SELECTION CARD
      NNIG=0
      DO 500 JJJ=1,NS
      READ(5,3)NT,NDEP,NI,(ISAVE(J),J=1,NI)
      DO 200 J=1,N
      X2(J,1)=1.0D0
      IF(JK.NE.1) GO TO 200
      XBIG(J,NNIG+1)=1.0D0
  200 CONTINUE
      DO 210 J=1,NI
      MM=ISAVE(J)
      DC 230 JJ=1,N
      X2(JJ,J+1)=DATA(JJ,MM)
      IF(JK.NE.1) GO TO 230
      XBIG(JJ,NNIG+J+1)=X2(JJ,J+1)
  230 CONTINUE
  210 CONTINUE
      NSI=NI+1
      NNIG=NNIG+NSI
C READ IN COVARIANCE ESTINATE
      READ(5,25)S
   25 FCRMAT(F10.0)
      IF(JK.NE.JJJ) GO TO 600
      CALL ARRAY(2,N,NSI,36,4,X2,X2)
      CALL GTPRD(X2,X2,COMP3,N,NSI,NSI)
      CALL DMINV(COMP3,NSI,DET,W1,W2)
      GC TO 610
  600 CALL ARRAY(2,N,NSI,36,4,X2,X2)
      CALL GTPRD(X1,X2,COMP1,N,NNI,NSI)
      CALL GTPRD(X2,X2,X2TX2,N,NSI,NSI)
      CALL DMINV(X2TX2,NSI,DET,W1,W2)
      CALL GMPRD(X1TX1,COMP1,COMP2,NNI,NNI,NSI)
      CALL GMPRD(COMP2,X2TX2,COMP3,NNI,NSI,NSI)
  610 CALL ARRAY(1,NNI,NSI,4,4,COMP3,CCMP3)
      DC 300 K=1,NNI
      DC 310 L=1,NSI
      COMP3(K,L)=S*COMP3(K,L)
  310 CCNTINUE
  300 CONTINUE
      WRITE(6,10)
   10 FCRMAT('0',3X,'OUTPUT')
      DO 320 K=1,NNI
      WRITE(6,11)(COMP3(K,J),J=1,NSI)
  320 CONTINUE
   11 FORMAT(' ',3X,4F13.6)
```

```
      DO 620 K=1,NNI
      DO 630 L=1,NSI
      CVARB(MMI+K,MSI+L)=COMP3(K,L)
  630 CONTINUE
  620 CONTINUE
      MSI=MSI+NSI
  500 CONTINUE
      MMI=MMI+NNI
      MSI=0
  305 CONTINUE
      DO 304 I=1,N
      WRITE(6,303)(XBIG(I,J),J=1,12)
  304 CONTINUE
  303 FORMAT(' ',2X,12F10.2)
      WRITE(6,1070)
 1070 FORMAT('1',5X,'LEAST SQUARES COVARIANCE MATRIX'/)
      IF(10-NNIG) 6500,6450,6450
 6450 DO 2000 I=1,NNIG
      WRITE(6,1071)(CVARB(I,J),J=1,NNIG)
 2000 CONTINUE
 1071 FORMAT(' ',2X,10D13.5)
      GO TO 6600
 6500 Z=1.0
      IF(20-NNIG) 6520,6510,6510
 6510 Z=1.0
      DO 6530 I=1,NNIG
      WRITE(6,1071)(CVARB(I,J),J=1,10)
 6530 CONTINUE
      WRITE(6,6561)
 6561 FORMAT(' ')
      DO 6540 I=1,NNIG
      WRITE(6,1071)(CVARB(I,J),J=11,NNIG)
 6540 CONTINUE
      GO TO 6600
 6520 Z=1.0
      DO 6550 I=1,NNIG
      WRITE(6,1071)(CVARB(I,J),J=1,10)
 6550 CONTINUE
      WRITE(6,6561)
      DO 6560 I=1,NNIG
      WRITE(6,1071)(CVARB(I,J),J=11,20)
 6560 CONTINUE
      WRITE(6,6561)
      DO 6570 I=1,NNIG
      WRITE(6,1071)(CVARB(I,J),J=21,NNIG)
 6570 CONTINUE
 6600 READ(5,632)(B(J),J=1,NNIG)
  632 FORMAT(8F10.0)
```

```
      READ(5,631)BSSUM
  631 FORMAT(F10.0)
      CALL ARRAY(2,NNIG,NNIG,30,30,CVARB,CVARB)
      DO 9100 I=1,N
      DO 9300 K=1,NNIG
      XOBS(K)=XBIG(I,K)
 9300 CONTINUE
      CALL GTPRD(XOBS,B,CHAT,NNIG,1,1)
      PRED(I)=CHAT(1)
      CALL GMPRD(CVARB,XOBS,XSAVE,NNIG,NNIG,1)
      CALL GTPRD(XOBS,XSAVE,VSAVE,NNIG,1,1)
      SSSUM=1.96*DSQRT(BSSUM+VSAVE(1))
      PIP(I)=CHAT(1)+SSSUM
      PIM(I)=CHAT(1)-SSSUM
 9100 CONTINUE
      WRITE(6,9005)
 9005 FORMAT('1'//////////16X,'TABLE OF LS CORRELATED PREDICTION INTERVA
     1L VALUES'/'0',17X,'PI +',9X,'PREDICTED',4X,'ACTUAL',7X,'PI -'/'0')
      DO 9000 I=1,N
      XSAVE(I)=DATA(I,36)
      WRITE(6,9010)I,PIP(I),PRED(I),XSAVE(I),PIM(I)
 9000 CONTINUE
 9010 FORMAT(' ',11X,I2,4F13.6)
      STOP
      END

      SUBROUTINE ARRAY (MODE,I,J,N,M,S,D)
      DOUBLE PRECISION S,D
      DIMENSION S(1),D(1)
      NI=N-I
      IF(MODE-1) 100, 100, 120
  100 IJ=I*J+1
      NM=N*J+1
      DO 110 K=1,J
      NM=NM-NI
      DO 110 L=1,I
      IJ=IJ-1
      NM=NM-1
  110 D(NM)=S(IJ)
      GO TO 140
  120 IJ=0
      NM=0
      DO 130 K=1,J
      DO 125 L=1,I
      IJ=IJ+1
      NM=NM+1
  125 S(IJ)=D(NM)
```

```
130 NM=NM+NI
140 RETURN
    END

    SUBROUTINE GTPRD(A,B,R,N,M,L)                                 GTPR 360
    DOUBLE PRECISION A,B,R
    DIMENSION A(1),B(1),R(1)                                      GTPR 370
C                                                                 GTPR 380
    IR=0                                                          GTPR 390
    IK=-N                                                         GTPR 400
    DO 10 K=1,L                                                   GTPR 410
    IJ=0                                                          GTPR 420
    IK=IK+N                                                       GTPR 430
    DO 10 J=1,M                                                   GTPR 440
    IB=IK                                                         GTPR 450
    IR=IR+1                                                       GTPR 460
    R(IR)=0                                                       GTPR 470
    DC 10 I=1,N                                                   GTPR 480
    IJ=IJ+1                                                       GTPR 490
    IB=IB+1                                                       GTPR 500
 10 R(IR)=R(IR)+A(IJ)*B(IB)                                       GTPR 510
    RETURN                                                        GTPR 520
    END                                                           GTPR 530

    SUBROUTINE GMSUB(A,B,R,N,M)                                   GMSU 320
    DCUBLE PRECISION A,B,R
    DIMENSICN A(1),B(1),R(1)                                      GMSU 330
C                                                                 GMSU 340
C      CALCULATE NUMBER OF ELEMENTS                               GMSU 350
C                                                                 GMSU 360
    NM=N*M                                                        GMSU 370
C                                                                 GMSU 380
C      SUBTRACT MATRICES                                          GMSU 390
C                                                                 GMSU 400
    DO 10 I=1,NM                                                  GMSU 410
 1C R(I)=A(I)-B(I)                                                GMSU 420
    RETURN                                                        GMSU 430
    END                                                           GMSU 440

    SUBROUTINE GMPRD(A,B,R,N,M,L)                                 GMPR 370
    DOUBLE PRECISION A,B,R
    DIMENSICN A(1),B(1),R(1)                                      GMPR 380
C                                                                 GMPR 390
    IR=0                                                          GMPR 400
    IK=-M                                                         GMPR 410
```

```
      DO 10 K=1,L                                                        GMPR 420
      IK=IK+M                                                            GMPR 430
      DO 10 J=1,N                                                        GMPR 440
      IR=IR+1                                                            GMPR 450
      JI=J-N                                                             GMPR 460
      IB=IK                                                              GMPR 470
      R(IR)=0                                                            GMPR 480
      DO 10 I=1,M                                                        GMPR 490
      JI=JI+N                                                            GMPR 500
      IB=IB+1                                                            GMPR 510
   10 R(IR)=R(IR)+A(JI)*B(IB)                                            GMPR 520
      RETURN                                                             GMPR 530
      END                                                                GMPR 540

      SUBROUTINE GMTRA(A,R,N,M)                                          GMTR 300
      DOUBLE PRECISION A,R
      DIMENSION A(1),R(1)                                                GMTR 310
C                                                                        GMTR 320
      IR=0                                                               GMTR 330
      DO 10 I=1,N                                                        GMTR 340
      IJ=I-N                                                             GMTR 350
      DO 10 J=1,M                                                        GMTR 360
      IJ=IJ+N                                                            GMTR 370
      IR=IR+1                                                            GMTR 380
   10 R(IR)=A(IJ)                                                        GMTR 390
      RETURN                                                             GMTR 400
      END                                                                GMTR 410
```

```
C PRCGRAM TO COMPUTE A NUMBER OF MULTIPLE LINEAR REGRESSIONS AND THEN TO
C COMPUTE A COMBINED LEAST SQUARES PRECICTION INTERVAL UNDER THE ASSUMP-
C TICN OF INDEPENDENCE BETWEEN ALL SELECTIONS.
C
C
      DIMENSION DATA(36,41)
      DIMENSION Y(36), X(36,4), XT(4,36), XTX(4,4), W1(4), W2(4),
     1XTXXT(4,36), B(4), ISAVE(10), XB(36), RESIC(36), SS(1), YY(1)
      DIMENSION SSAVE(9), ROE(9,9), NSAVE(9)
      DIMENSION YHAT(1),PSAVE(36),A(36),TCSAVE(36),PIP(36),PIM(36),
     1XOBS(4),PHAT(1)
      DATA NNI/0/, NNIG/0/, NN/0/
      DATA A/36*0.0/, TCSAVE/36*0.0/
C  READ PROBLEM PARAMETER CARD
      READ(5,1)PR, PR1, N, M, NS
    1 FORMAT(A4,A2,I5,2I2)
C
C PR,PR1....PROBLEM NUMBER
C  N........NUMBER OF OBSERVATIONS
C  M........NUMBER OF VARIABLES
C  NS.......NUMBER OF SELECTIONS
C
C  OBSERVATION MATRIX READ IN
      DO 6000 I=1,36
      READ(5,6010)(DATA(I,J),J=1,36)
 6000 CONTINUE
 6010 FCRMAT(/12F6.0/12F6.0/5F6.0/7F6.0)
      READ(5,6020)(DATA(I,37),I=1,36)
      READ(5,6020)(DATA(I,38),I=1,36)
      READ(5,6020)(DATA(I,39),I=1,36)
      READ(5,6020)(DATA(I,40),I=1,36)
      READ(5,6020)(DATA(I,41),I=1,36)
 6020 FORMAT(9F8.0)
C
C  NOTE  COL37 BASE CST, COL38 ENG CST, COL 39 PAYLOAD CST, COL 40 CONST CST
C  CCL 41 ENG WGT
C
C
C  TEST NUMBER OF SELECTIONS
      IF(NS) 108,108,109
  108 WRITE(6,4)
    4 FORMAT(53H1NUMBER OF SELECTIONS NOT SPECIFIED.  JOB TERMINATED.)
      GO TO 300
C
C  COMMENCE MULTIPLE REGRESSION PROGRAM
  109 DO 300 I=1,NS
      WRITE(6,5)PR,PR1,I
    5 FCRMAT(25H1MULTIPLE REGRESSION.....A4,A2//6X,14HSELECTION.....I2//
     1)
```

```
C
C   READ IN SUBSET SELECTION CARD
        READ(5,3)NT,NDEP,NI,(ISAVE(J),J=1,NI)
      3 FCRMAT(36I2)
        WRITE(6,6001)NT,NDEP,NI,(ISAVE(J),J=1,NI)
   6001 FORMAT('0','SELECTION CARD',15I2)
C
C   NT.......MODEL TRANSFORMATION CODE CESIRED
C       0.....LINEAR
C       1.....LOG LINEAR
C   NDEP.....DEPENDENT VARIABLE
C   NI.......NUMBER OF INDEPENDENT VARIABLES INCLUCED
C   ISAVE....AVECTOR CONTAINING THE INDEPENDENT VARIABLES INCLUDED
C
C   TRANSFORM DATA AND LOAD THE X AND Y ARRAYS
        DC 110 J=1,N
        Y(J)=DATA(J,NDEP)
        IF(NT) 113, 112, 113
    112 GC TO 114
    113 Y(J)=ALCG(Y(J))
        GO TO 114
    114 X(J,1)=1.0
    110 CONTINUE
        DC 120 J=1,NI
        MM=ISAVE(J)
        DC 130 JJ=1,N
        X(JJ,J+1)=DATA(JJ,MM)
    130 CONTINUE
    12C CONTINUE
        NNI=NI+1
        NSAVE(I)=NNI
        NNIG=NNIG+NNI
        NN=NN+N
C
C   CALCULATE VECTOR OF ESTIMATES
        CALL ARRAY(2,N,NNI,36,4,X,X)
        CALL ARRAY(2,NNI,NNI,4,4,XTX,XTX)
        CALL ARRAY(2,NNI,N,4,36,XT,XT)
        CALL ARRAY(2,NNI,N,4,36,XTXXT,XTXXT)
        CALL GTPRD(X,X,XTX,N,NNI,NNI)
        CALL MINV(XTX,NNI,DET,W1,W2)
        CALL GMTRA(X,XT,N,NNI)
        CALL GMPRD(XTX,XT,XTXXT,NNI,NNI,N)
        CALL GMPRD(XTXXT,Y,B,NNI,N,1)
        WRITE(6,5040)
   5040 FORMAT('0',T10,'LEAST SQUARES REGRESSION CCEFICIENTS'//)
        WRITE(6,6)(B(J),J=1,NNI)
      6 FORMAT(' ',10F12.6)
```

```
C
C  CALCULATE RESIDUAL VECTOR
      CALL GMPRD(X,B,XB,N,NNI,1)
      CALL GMSUB(Y,XB,RESID,N,1)
C
C
C  CALCULATE THE STANDARD ERROR OF THE ESTIMATE
      CALL GTPRD(RESID,RESID,SS,N,1,1)
      BN=N-NNI
      S1=SS(1)/BN
      S=SQRT(S1)
      SSAVE(I)=S
      WRITE(6,8)S
    8 FORMAT('0','STD. ERROR OF ESTIMATE=   ',F13.6//)
C
C  CALCULATE THE COVARIANCE MATRIX OF THE ESTIMATES
      CALL ARRAY(1,NNI,NNI,4,4,XTX,XTX)
      DO 141 J=1,NNI
      DO 142 K=1,NNI
      XTX(J,K)=S1*XTX(J,K)
  142 CONTINUE
  141 CONTINUE
      WRITE(6,5050)
 5050 FORMAT('0',T10,'COVARIANCE MATRIX OF THE ESTIMATES'/)
      DO 143 J=1,NNI
      WRITE(6,144)(XTX(J,K),K=1,NNI)
  143 CONTINUE
  144 FORMAT(2X,8E13.5)
C
C  CALCULATE ADJUSTED MUNTIPLE CORRELATION COEFICIENT
      SUM=0.0
      DO 150 J=1,N
      SUM=SUM+Y(J)
  150 CONTINUE
      AN=N
      YBAR=SUM/AN
      DO 160 J=1,N
      Y(J)=Y(J)-YBAR
  160 CONTINUE
      CALL GTPRD(Y,Y,YY,N,1,1)
      CN=N-1
      R=1.0-(S1/(YY(1)/CN))
      WRITE(6,9)R
    9 FORMAT('0','ADJUSTED MULTIPLE CORRELATION COEFICIENT',F9.6)
C
C  CONPUTE LEAST SQUARES (INDEPENDENT) PREDICTION INTERVAL
      CALL ARRAY(2,NNI,NNI,4,4,XTX,XTX)
      CALL ARRAY(1,N,NNI,36,4,X,X)
```

```
      DO 5100 K=1,N
      DO 5000 J=1,NNI
  990 XOBS(J)=X(K,J)
 5000 CONTINUE
      WRITE(6,5700)(XOBS(IK),IK=1,NNI)
 5700 FORMAT(' ',4F13.3)
      CALL GTPRD(XOBS,B,YHAT,NNI,1,1)
      TCSAVE(K)=TCSAVE(K)+YHAT(1)
      CALL GMPRD(XTX,XOBS,PSAVE,NNI,NNI,1)
      CALL GTPRD(XOBS,PSAVE,PHAT,NNI,1,1)
  991 ASAVE=S1+PHAT(1)
  992 A(K)=A(K)+ASAVE
 5100 CONTINUE
  300 CONTINUE
      DO 888 I=1,N
      A(I)=1.96*SQRT(A(I))
  888 CONTINUE
      DO 5200 L=1,N
      PIM(L)=TCSAVE(L)-A(L)
      PIP(L)=TCSAVE(L)+A(L)
 5200 CONTINUE
      WRITE(6,9005)
 9005 FORMAT('1'//////////16X,'TABLE OF LS (INDEPEND) PREDICTION INTERVA
     1L VALUES'/'0',17X,'PI +',9X,'PREDICTED',4X,'ACTUAL',7X,'PI -'/'0')
      DO 5300 J=1,N
      WRITE(6,5400)J,PIP(J),TCSAVE(J),DATA(J,36),PIM(J)
 5300 CONTINUE
 5400 FORMAT(' ',11X,I2,4F13.6)
      STOP
      END
```

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