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THE DISTRIBUTION OF SUBMUNITION ARRIVAL TIMES

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In assessing the effectiveness of an artillery volley using Improved Conventional Munitions (ICM) in a situation where the personnel in the target area may react to seek protective cover, one must consider both the distribution of arrival time of submunitions in the target area and the reaction time distribution for the target personnel. A methodology is devised to quantitatively address the first of these sources of variability. This methodology is then applied to several specific fuze-munition configurations.

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U.S. ARMY MATERIEL SYSTEMS ANALYSIS AGENCY
ABERDEEN PROVING GROUND, MARYLAND

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U. S. ARMY MATERIEL SYSTEMS ANALYSIS AGENCY

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July 1973

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ABSTRACT

In assessing the effectiveness of an artillery volley using Improved Conventional Munitions (ICM) in a situation where the personnel in the target area may react to seek protective cover, one must consider both the distribution of arrival time of submunitions in the target area and the reaction time distribution for the target personnel. A methodology is devised to quantitatively address the first of these sources of variability. This methodology is then applied to several specific fuze-munition configurations.

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THE DISTRIBUTION OF SUBMUNITION ARRIVAL TIMES

1. INTRODUCTION

Recently, interest has been revived in a comparison of the antipersonnel effectiveness of an artillery shell equipped with a mechanical time (MT) fuzed improved conventional munition (ICM) versus one which uses a proposed proximity (VT) fuze. For such a comparison one must recognize that in reality an enemy target will react to the audible detection of an incoming projectile by seeking protective cover in order to reduce his exposure and therefore enhance his chance of survival. Thus, in addressing the effectiveness of volley fire in this situation one must examine for each candidate fuze, subsequent to warning, both the distribution of arrival time of submunitions in the target area and the target personnel reaction time distribution in order to be able to realistically assess the ability of the target personnel to achieve adequate cover before a significant number of submunitions have functioned.

This report considers a theoretical investigation of the first of these distributions, the arrival time of submunitions in the target area. The approach which is used accounts for three primary sources of variability, variation in time to payload ejection (due to the fact that volley rounds will not function simultaneously), variation in burst height (due to the fact that these rounds may function at different altitudes), and finally variation in submunition time to fall (due to the fact that given ejection, all submunitions from a given round will also not function simultaneously). The assumptions required concerning the specific functional form for the probability distributions are based on actual test data. The distribution of time to payload ejection is assumed to be a negative exponential which reasonably approximates the test results of the HELBAT I* firings conducted by the Human Engineering Laboratory.

*Human Engineering Laboratory, TM 24-70, "HEL Battalion Artillery Test," Sep 1970, G. L. Harley, D. J. Giordano.

Similarly, the uniform assumption for the distribution of submunition time to fall over the interval from first to last submunition arrival from a given round does not appear to be inconsistent with existing firing record data.* Details concerning the rationale used in choosing specific parameter values can be found in Section 4. Section 2 contains a brief account of the general methodology which includes a description of the basic underlying assumptions. This is then followed, in Section 3, by a detailed description of the mathematics involved in the derivation along with the final general mathematical expression for the distribution in question. Finally, Section 4 is devoted to a specific application which compares the submunition arrival time distributions for several munition configurations of current interest.

2. GENERAL METHODOLOGY

2.1 Basic Assumptions.**

- Time is initialized to the time of ejection of the submunitions from that round of the volley which is first to function. The noise signature associated with this initial ejection is assumed to constitute first warning.***

- Given warning, the time to ejection for any subsequent round is a continuous random variable with probability density function (p.d.f.) $g_3(t_e)$, $0 < t_e < \infty$. For the results presented in this report this p.d.f. is assumed to be of the form

$$g_3(t_e) = \frac{1}{\theta} e^{-\frac{t_e}{\theta}} \qquad 0 < t_e < \infty, \ \theta > 0.$$

- Burst height is a continuous random variable with p.d.f.

*FR # 5296, FR # 5419 Yuma Proving Ground.

**For further amplification of the practical significance of these assumptions the reader is referred to Section 4.2.

***Note that no correction has been made for the time required for the sound of the initial burst to reach the ground. If necessary, however, the results of this effort can easily be adjusted to reflect the uncertainty in when warning actually occurred.

$g_1(h)$, $a < h < b$. The specific parameter values of this distribution will depend on fuze type and will also generally depend on the angle of fall of the projectile. Note that in this report it is assumed that there exists a mean burst height for the volley fire and $g_1(h)$ represents the distribution about this mean. No attempt has been made to address the distribution of mean burst heights over a series of occassions.

- Given submunition ejection, submunition time to fall is a continuous random variable with conditional p.d.f. $g_2(t_f/h)$, $c(h) < t_f < d(h)$. For the results presented in this report this p.d.f. is assumed to be uniform where the time to first submunition function, $c(h)$, and the spread, $d(h)-c(h)$, are both linearly dependent on burst height. Let $c(h) = \alpha_1 + \beta_1 h$ and $d(h)-c(h) = \alpha_2+\beta_2 h$ ($\alpha_1 \geq 0, \alpha_2 > 0,\ \ \beta_1 > 0,\ \ \beta_2 \geq 0$).* Then the p.d.f. of submunition time to fall is given by

$$g_2(t_f/h) = \frac{1}{\alpha_2+\beta_2 h} \qquad \alpha_1+\beta_1 h < t_f/h < (\alpha_1+\alpha_2)+(\beta_1+\beta_2)h.$$

2.2 <u>General Theory</u>.

Based on the previous assumptions the time to fall distribution can be obtained by using the following relationship:

$$g_4(t_f) = \int_{R(t_f)} g_2(t_f/h)g_1(h)\ dh,$$

where $R(t_f)$ is the range of integration. Basically this is the expression for submunition time of fall averaged over height. Now the total time from initial warning to submunition burst, T, is a random variable which can be expressed as the sum of two independent random variables, T_e and T_f. That is, $T = T_e+T_f$ where T_e is the time to round function after initial warning with p.d.f. $g_3(t_e)$ and T_f is the time to submunition burst with p.d.f. $g_4(t_f)$. Since these variables

*Note that for the derivations provided in Section 3 only the constant spread case (i.e., $\beta_2 = 0$) was considered.

are independent their joint p.d.f. is given by $g_6(t_e, t_f) = g_3(t_e) \cdot g_4(t_f)$. Using the change of variable technique the joint p.d.f. of T and T_e, $g_7(t, t_e)$ is $g_6(t_e, t-t_e)$ multiplied by the appropriate Jacobian of the transformation. The marginal p.d.f. of T, the total time, can then be obtained by integrating out the time to round function variable. Thus, the p.d.f. of total submunition arrival time (after first ejection) is

$$g_5(t) = \int_{R(t)} g_6(t_e, t-t_e)\, dt_e.$$

Its corresponding cumulative probability function,

$$F(t) = \int_0^t g_5(x)\, dx,$$

depicts the percent of submunitions which have burst within t seconds after initial warning.

The following section provides the details involved in combining the basic assumptions of Section 2.1 with the general theory in order to obtain specific expressions for those distributions of interest.

3. THEORETICAL DETAILS

3.1 The Distribution of Submunition Time to Fall.

The probability density function of submunition time to fall is given by the expression $g_4(t_f) = \int_{R(t_f)} g_2(t_f/h) g_1(h)\, dh$. The range of integration, $R(t_f)$, is functionally dependent on t_f and can be described explicitly by examining the composition of the two dimensional space over which the product, $g_2(t_f/h) \cdot g_1(h)$, takes on non-zero values. This space consists of all two dimensional points (t_f, h) such that both $a < h < b$ and $\alpha_1+\beta_1 h < t_f < (\alpha_1+\alpha_2)+(\beta_1+\beta_2)h$. Two distinct cases must be considered in describing $R(t_f)$. These are:

a. $(\alpha_1+\alpha_2)+(\beta_1+\beta_2)a \leq \alpha_1+\beta_1 b$ (Case A),

b. $(\alpha_1+\alpha_2)+(\beta_1+\beta_2)a > \alpha_1+\beta_1 b$ (Case B).

The geometry pertaining to these two cases is pictured in Figure 3.1.

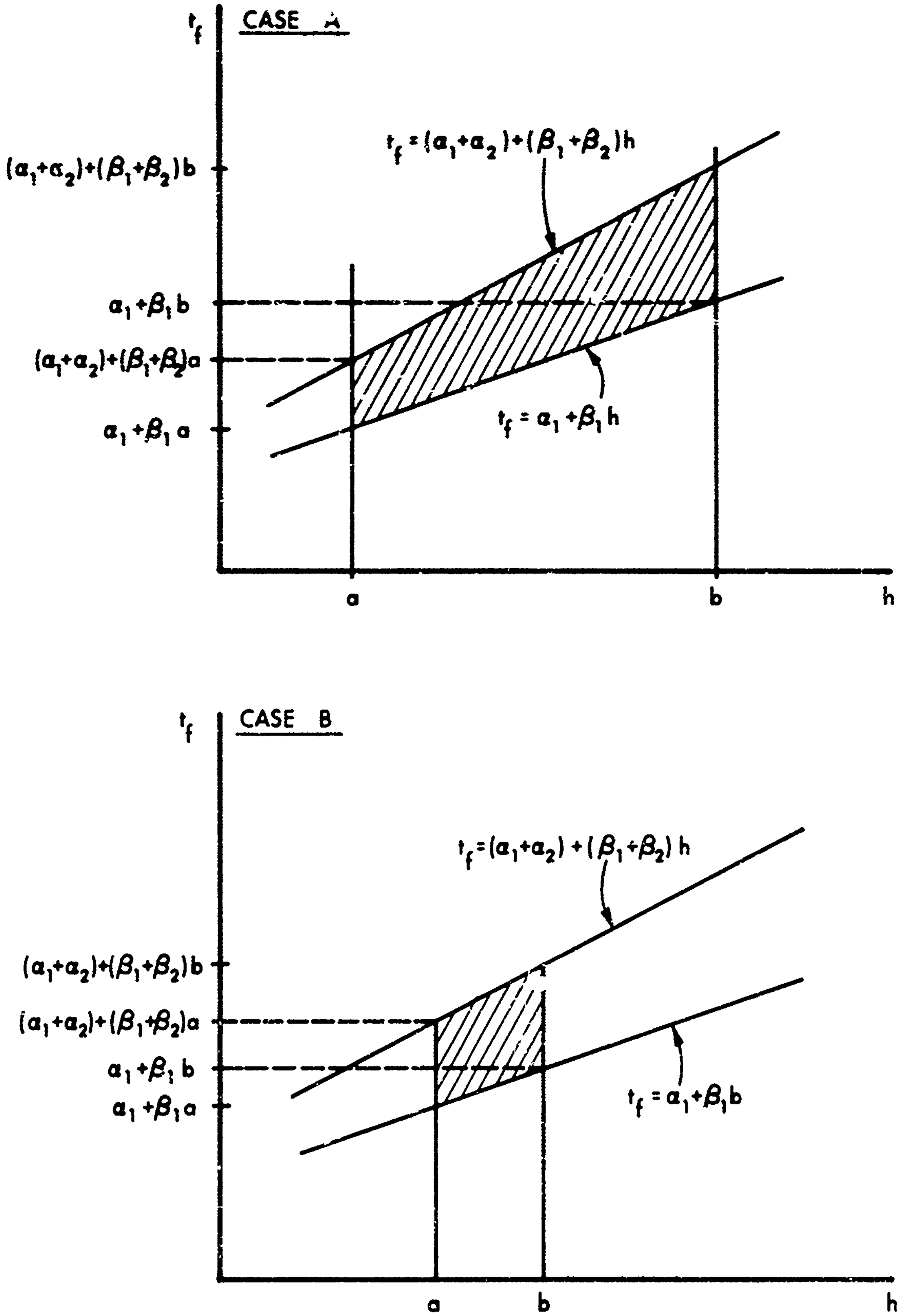

Figure 3.1 Geometry

3.1.1 Case A. For those conditions when $(\alpha_1+\alpha_2)+(\beta_1+\beta_2)a \leq \alpha_1+\beta_1 b$, $g_4(t_f)$ is given by one of the following expressions depending on the range of t_f.

$$
(1) \quad g_4(t_f) = \begin{cases}
\displaystyle\int_a^{\frac{t_f-\alpha_1}{\beta_1}} g_2(t_f/h)g_1(h)\,dh & \text{when } \alpha_1+\beta_1 a \leq t_f < (\alpha_1+\alpha_2)+(\beta_1+\beta_2)a \\[2ex]
\displaystyle\int_{\frac{t_f-(\alpha_1+\alpha_2)}{\beta_1+\beta_2}}^{\frac{t_f-\alpha_1}{\beta_1}} g_2(t_f/h)g_1(h)\,dh & \text{when } (\alpha_1+\alpha_2)+(\beta_1+\beta_2)a \leq t_f < \alpha_1+\beta_1 b \\[2ex]
\displaystyle\int_{\frac{t_f-(\alpha_1+\alpha_2)}{\beta_1+\beta_2}}^{b} g_2(t_f/h)g_1(h)\,dh & \text{when } \alpha_1+\beta_1 b \leq t_f \leq (\alpha_1+\alpha_2)+(\beta_1+\beta_2)b.
\end{cases}
$$

Now for the constant spread case (i.e., $\beta_2 = 0$), $g_2(t_f/h) = \frac{1}{\alpha_2}$ for all points (t_f,h) critical to the evaluation of the integrals defined in equation (1).

Thus in this case

$$
(2)\quad g_4(t_f) = \begin{cases}
\dfrac{1}{\alpha_2} \displaystyle\int_a^{\frac{t_f-\alpha_1}{\beta_1}} g_1(h)\, dh & \text{when } \alpha_1+\beta_1 a \leq t_f < (\alpha_1+\alpha_2)+\beta_1 a \\[2ex]
\dfrac{1}{\alpha_2} \displaystyle\int_{\frac{t_f-(\alpha_1+\alpha_2)}{\beta_1}}^{\frac{t_f-\alpha_1}{\beta_1}} g_1(h)\, dh & \text{when } (\alpha_1+\alpha_2)+\beta_1 a \leq t_f < \alpha_1+\beta_1 b \\[2ex]
\dfrac{1}{\alpha_2} \displaystyle\int_{\frac{t_f-(\alpha_1+\alpha_2)}{\beta_1}}^{b} g_1(h)\, dh & \text{when } \alpha_1+\beta_1 b \leq t_f \leq (\alpha_1+\alpha_2)+\beta_1 b.
\end{cases}
$$

Let $G_1(y) = \int_a^y g_1(h)\, dh$, $a \leq y \leq b$ denote the cumulative probability function of height of burst. Equation (2) can then be expressed as:

$$
(3)\quad g_4(t_f) = \begin{cases}
\dfrac{1}{\alpha_2} G_1\left(\dfrac{t_f-\alpha_1}{\beta_1}\right) & \text{when } \alpha_1+\beta_1 a \leq t_f < (\alpha_1+\alpha_2)+\beta_1 a \\[2ex]
\dfrac{1}{\alpha_2}\left[G_1\left(\dfrac{t_f-\alpha_1}{\beta_1}\right) - G_1\left(\dfrac{t_f-(\alpha_1+\alpha_2)}{\beta_1}\right)\right] & \text{when } (\alpha_1+\alpha_2)+\beta_1 a \leq t_f < \alpha_1+\beta_1 b \\[2ex]
\dfrac{1}{\alpha_2}\left[1 - G_1\left(\dfrac{t_f-(\alpha_1+\alpha_2)}{\beta_1}\right)\right] & \text{when } \alpha_1+\beta_1 b \leq t_f \leq (\alpha_1+\alpha_2)+\beta_1 b.
\end{cases}
$$

3.1.2 Case B. Similarly for those conditions when $(\alpha_1+\alpha_2)+(\beta_1+\beta_2)\,a > \alpha_1+\beta_1 b$, $g_4(t_f)$ is given by one of the following expressions depending on the range of t_f.

$$
(4) \quad g_4(t_f) = \begin{cases}
\displaystyle\int_a^{\frac{t_f-\alpha_1}{\beta_1}} g_2(t_f/h)g_1(h)\,dh & \alpha_1+\beta_1 a \le t_f < \alpha_1+\beta_1 b \\[2ex]
\displaystyle\int_a^{b} g_2(t_f/h)g_1(h)\,dh & \alpha_1+\beta_1 b \le t_f < (\alpha_1+\alpha_2)+(\beta_1+\beta_2)a \\[2ex]
\displaystyle\int_{\frac{t_f-(\alpha_1+\alpha_2)}{\beta_1+\beta_2}}^{b} g_2(t_f/h)g_1(h)\,dh & (\alpha_1+\alpha_2)+(\beta_1+\beta_2)a \le t_f \le (\alpha_1+\alpha_2)+(\beta_1+\beta_2)b.
\end{cases}
$$

For the constant spread case (i.e., $\beta_2=0$) equation (4) reduces to

$$
(5) \quad g_4(t_f) = \begin{cases}
\dfrac{1}{\alpha_2}\, G_1\!\left(\dfrac{t_f-\alpha_1}{\beta_1}\right) & \text{when} \quad \alpha_1+\beta_1 a \le t_f < \alpha_1+\beta_1 b \\[2ex]
\dfrac{1}{\alpha_2} & \text{when} \quad \alpha_1+\beta_1 b \le t_f < (\alpha_1+\alpha_2)+\beta_1 a \\[2ex]
\dfrac{1}{\alpha_2}\left[1 - G_1\!\left(\dfrac{t_f-(\alpha_1+\alpha_2)}{\beta_1}\right)\right] & \text{when} \quad (\alpha_1+\alpha_2)+\beta_1 a \le t_f < (\alpha_1+\alpha_2)+\beta_1 b,
\end{cases}
$$

where $G_1(y) = \int_a^y g_1(h)\,dh$, $a \le y \le b$.

3.2 The Distribution of Total Submunition Arrival Time (Constant Spread Case: $\beta_2=0$).

3.2.1 Introduction. As described in Section 2.2 the marginal p.d.f. of total time to fall, T, is given by:

$$(6)\quad g_5(t) = \int_{R(t)} g_6(t_e, t-t_e)\, dt_e$$

$$= \int_{R(t)} g_3(t_e)\cdot g_4(t-t_e)\, dt_e,$$

where the range of integration, R(t), is functionally dependent on t and can be described explicitly for a given value of t. In order to describe R(t) it is helpful to examine the two-dimensional region over which the joint probability of (T, T_e), $g_3(t_e)\cdot g_4(t-t_e)$, takes on non-zero values. Noting that, for the constant spread case (i.e., $\beta_2=0$), $g_3(t_e) > 0$ for $0 \le t_e < \infty$, $g_4(t_f) > 0$ for $\alpha_1+\beta_1 a \le t_f \le (\alpha_1+\alpha_2)+\beta_1 b$, and $T = T_f+T_e$, it follows that the region in question consists of all points, (t, t_e), such that simultaneously $0 \le t_e < \infty$ and $\alpha_1+\beta_1 a+t_e \le t \le (\alpha_1+\alpha_2)+\beta_1 b+t_e$. This region is depicted in Figure 3.2 and consists of all points in the shaded region. Using this figure it can be seen that the range of integration for $t=t_o$ is given by: $R(t_o) = \{t_e: 0 \le t_e < t_o-\alpha_1-\beta_1 a\}$. Similarly, for $t = t_1$, the range of integration is $R(t_1) = \{t_e: t_1-\alpha_1-\alpha_2-\beta_1 b \le t_e < t_1-\alpha_1-\beta_1 a\}$. Before the integration involved in equation (6) can be performed the region specified in Figure 3.2 must be further partitioned into subregions corresponding to the various definitions of $g_4(t_f)$. As in Section 3.1 both Case A and Case B must be considered. These subregions are shown in Figure 3.3. For Case A - Region I, $\alpha_1+\beta_1 a+t_e \le t \le \alpha_1+\alpha_2+\beta_1 a+t_e$ and thus $\alpha_1+\beta_1 a \le t-t_e < \alpha_1+\alpha_2+\beta_1 a$. Now using the fact that $t_f = t-t_e$ and the definition of $g_4(t_f)$ contained

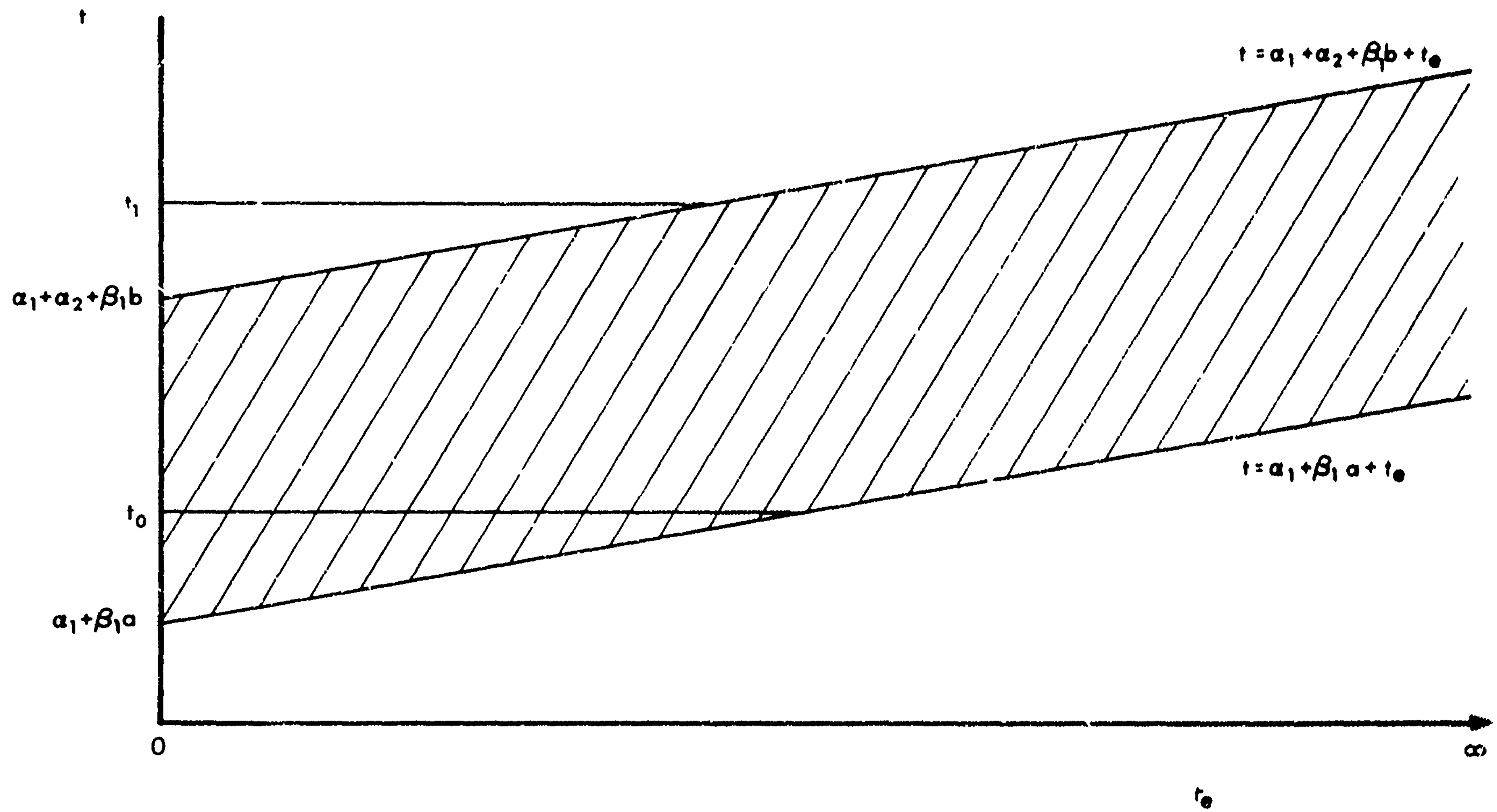

Figure 3.2 Points of Positive Probability.

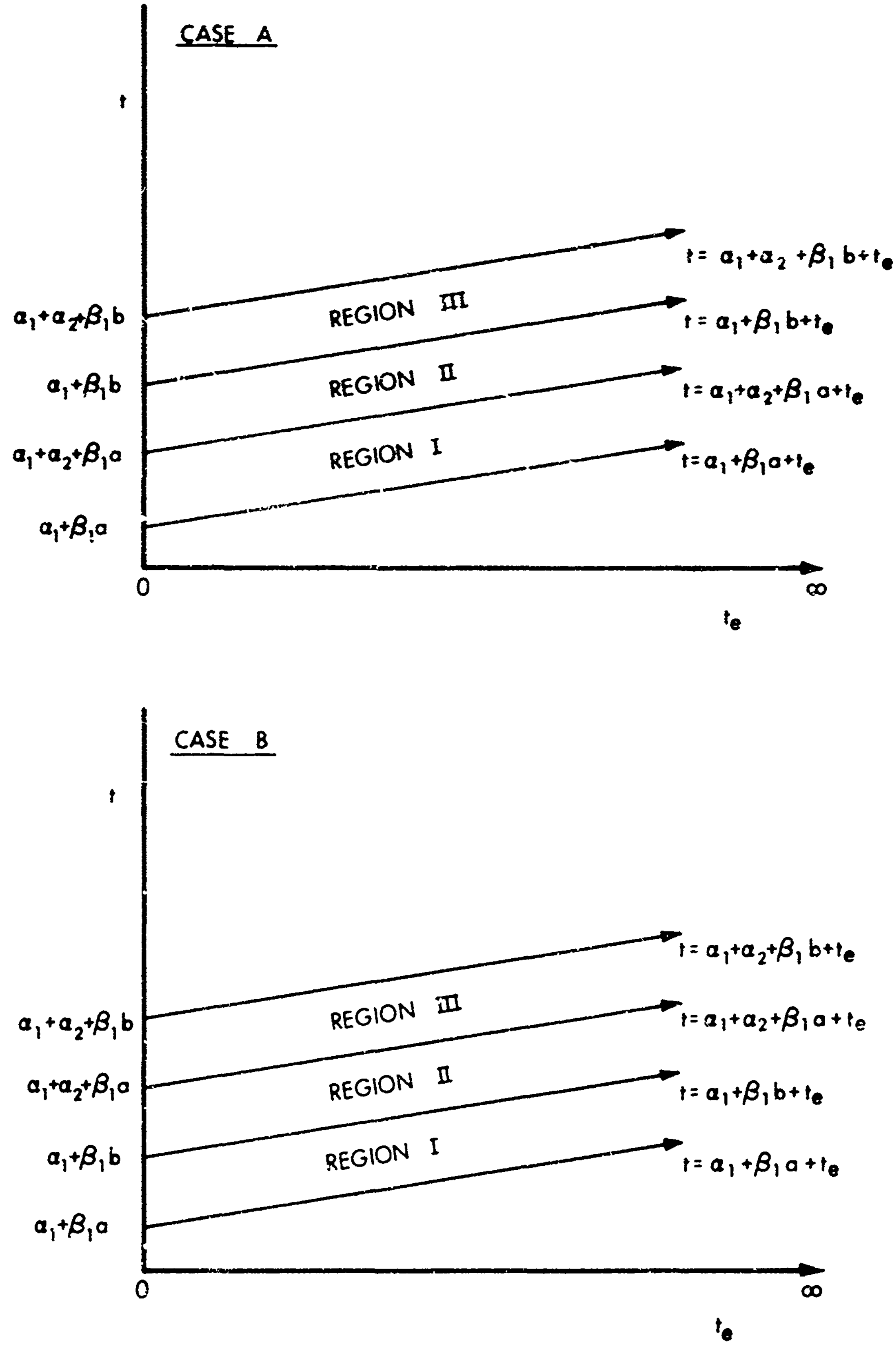

Figure 3.3 Points of Positive Probability.

in equation (3) it follows that $g_4(t-t_e) = \frac{1}{\alpha_2} G_1\left(\frac{t-t_e-\alpha_1}{\beta_1}\right)$ in Region I - Case A. A similar argument can be used for each of the five remaining special cases. The results of each of these investigations is contained in Table 3.2.

Having obtained the above partition one can now proceed with the integration involved in equation (6). Cases A and B will be considered separately.

<u>3.2.2 Case A.</u> The integration in question is that involved in the following expression:

$$g_5(t) = \int_{R(t)} g_3(t_e) \cdot g_4(t-t_e)\, dt_e .$$

For the sake of clarity this integration will be performed in stages, each stage representing the integration over a specific range of t. In all cases Figure 3.4 and Table 3.2 will be useful guides.

For $\alpha_1+\beta_1 a \leq t < \alpha_1+\alpha_2+\beta_1 a$,

$$g_5(t) = \int_0^{t-(\alpha_1+\beta_1 a)} g_3(t_e) g_4(t-t_e)\, dt_e .$$

Now for the specific distributional assumption made in Section 2.1 this expression reduces to:

$$g_5(t) = \int_0^{t-(\alpha_1+\beta_1 a)} \frac{1}{\theta} e^{-t_e/\theta} \frac{1}{\alpha_2} G_1\left(\frac{t-t_e-\alpha_1}{\beta_1}\right) dt_e .$$

TABLE 3.2 DEFINITION OF $g_4(t-t_e)$

Case	Region	$g_4(t-t_e)$
A	I	$\frac{1}{\alpha_2} G_1\left(\frac{t-t_e-\alpha_1}{\beta_1}\right)$
A	II	$\frac{1}{\alpha_2}\left[G_1\left(\frac{t-t_e-\alpha_1}{\beta_1}\right)-G_1\left(\frac{t-t_e-(\alpha_1+\alpha_2)}{\beta_1}\right)\right]$
A	III	$\frac{1}{\alpha_2}\left[1-G_1\left(\frac{t-t_e-(\alpha_1+\alpha_2)}{\beta_1}\right)\right]$
B	I	$\frac{1}{\alpha_2} G_1\left(\frac{t-t_e-\alpha_1}{\beta_1}\right)$
B	II	$\frac{1}{\alpha_2}$
B	III	$\frac{1}{\alpha_2}\left[1-G_1\left(\frac{t-t_e-(\alpha_1+\alpha_2)}{\beta_1}\right)\right]$

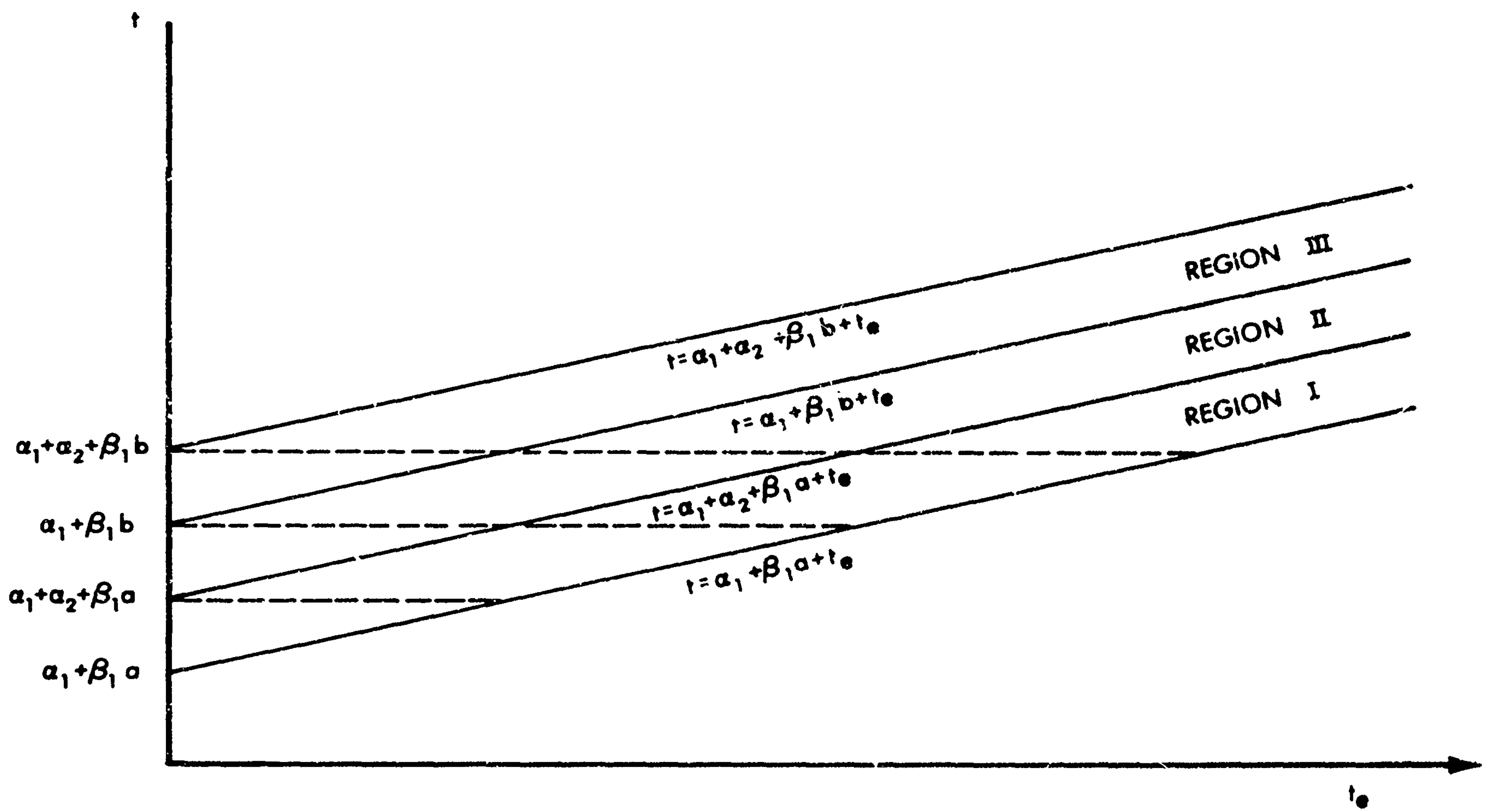

Figure 3.4 Points of Positive Probability — Case A

Similarly, for $\alpha_1+\alpha_2+\beta_1 a \leq t < \alpha_1+\beta_1 b$,

$$g_5(t) = \int_0^{t-(\alpha_1+\alpha_2+\beta_1 a)} g_3(t_e)g_4(t-t_e)\, dt_e + \int_{t-(\alpha_1+\alpha_2+\beta_1 a)}^{t-(\alpha_1+\beta_1 a)} g_3(t_e)g_4(t-t_e)\, dt_e$$

$$= \int_0^{t-(\alpha_1+\alpha_2+\beta_1 a)} \left(\frac{1}{\theta} e^{-t_e/\theta}\right) \frac{1}{\alpha_2} \left[G_1\left(\frac{t-t_e-\alpha_1}{\beta_1}\right) - G_1\left(\frac{t-t_e-(\alpha_1+\alpha_2)}{\beta_1}\right)\right] dt_e + \int_{t-(\alpha_1+\alpha_2+\beta_1 a)}^{t-(\alpha_1+\beta_1 a)} \left(\frac{1}{\theta} e^{-t_e/\theta}\right) \frac{1}{\alpha_2} G_1\left(\frac{t-t_e-\alpha_1}{\beta_1}\right) dt_e.$$

For, $\alpha_1+\beta_1 b \leq t < \alpha_1+\alpha_2+\beta_1 b$,

$$g_5(t) = \int_0^{t-(\alpha_1+\beta_1 b)} g_3(t_e)g_4(t-t_e)\, dt_e + \int_{t-(\alpha_1+\beta_1 b)}^{t-(\alpha_1+\alpha_2+\beta_1 a)} g_3(t_e)g_4(t-t_e)\, dt_e + \int_{t-(\alpha_1+\alpha_2+\beta_1 a)}^{t-(\alpha_1+\beta_1 a)} g_3(t_e)g_4(t-t_e)\, dt_e.$$

This expression reduces to

$$g_5(t) = \int_{0}^{t-(\alpha_1+\beta_1 b)} \left(\frac{1}{\theta} e^{-te/\theta}\right)\frac{1}{\alpha_2}\left[1-G_1\left(\frac{t-t_e-(\alpha_1+\alpha_2)}{\beta_1}\right)\right]dt_e + \int_{t-(\alpha_1+\beta_1 b)}^{t-(\alpha_1+\alpha_2+\beta_1 a)} \left(\frac{1}{\theta} e^{-te/\theta}\right)\frac{1}{\alpha_2}\left[G_1\left(\frac{t-t_e-\alpha_1}{\beta_1}\right) - G_1\left(\frac{t-t_e-(\alpha_1+\alpha_2)}{\beta_1}\right)\right]dt_e$$

$$+ \int_{t-(\alpha_1+\alpha_2+\beta_1 a)}^{t-(\alpha_1+\beta_1 a)} \left(\frac{1}{\theta} e^{-te/\theta}\right)\frac{1}{\alpha_2} G_1\left(\frac{t-t_e-\alpha_1}{\beta_1}\right) dt_e .$$

Finally, for $\alpha_1+\alpha_2+\beta_1 b \leq t < \infty$

$$g_5(t) = \int_{t-(\alpha_1+\alpha_2+\beta_1 b)}^{t-(\alpha_1+\beta_1 b)} g_3(t_e)g_4(t-t_e)\, dt_c + \int_{t-(\alpha_1+\beta_1 b)}^{t-(\alpha_1+\alpha_2+\beta_1 a)} g_3(t_e)g_4(t-t_e)\, dt_e + \int_{t-(\alpha_1+\alpha_2+\beta_1 a)}^{t-(\alpha_1+\beta_1 a)} g_3(t_e)g_4(t-t_e)\, dt_e .$$

Thus, for this range of t,

$$g_5(t) = \int_{t-(\alpha_1+\alpha_2+\beta_1 b)}^{t-(\alpha_1+\beta_1 b)} \left(\frac{1}{\theta} e^{-t_e/\theta}\right)\frac{1}{\alpha_2}\left[1-G_1\left(\frac{t-t_e-(\alpha_1+\alpha_2)}{\beta_1}\right)\right]dt_e + \int_{t-(\alpha_1+\beta_1 b)}^{t-(\alpha_1+\alpha_2+\beta_1 a)} \left(\frac{1}{\theta} e^{-t_e/\theta}\right)\frac{1}{\alpha_2}\left[G_1\left(\frac{t-t_e-\alpha_1}{\beta_1}\right) - G_1\left(\frac{t-t_e-(\alpha_1+\alpha_2)}{\beta_1}\right)\right] dt_e$$

$$+ \int_{t-(\alpha_1+\alpha_2+\beta_1 a)}^{t-(\alpha_1+\beta_1 a)} \left(\frac{1}{\theta} e^{-t_e/\theta}\right)\frac{1}{\alpha_2} \; G_1\left(\frac{t-t_e-\alpha_1}{\beta_1}\right) dt_e .$$

Note that each of the integrals involved in the above representation of $g_5(t)$ is of the form

$$(7) \qquad \int_r^s \frac{1}{\theta} e^{-t_e/\theta} \frac{1}{\alpha_2} G_1(m+nt_e)\, dt_e.$$

Consider, for example, the first integral involved in the expression which is

$$(8) \qquad \int_0^{t-(\alpha_1+\beta_1 a)} \frac{1}{\theta} e^{-t_e/\theta} \frac{1}{\alpha_2} G_1\left(\frac{t-t_e-\alpha_1}{\beta_1}\right) dt_e.$$

In this case, $r = 0$, $s = t-(\alpha_1+\beta_1 a)$, $m = \frac{t-\alpha_1}{\beta_1}$, and $n = -\frac{1}{\beta_1}$.

Using the technique of integration by parts the following general solution for (7) results:

$$\int_r^s \frac{1}{\theta} e^{-t_e/\theta} \frac{1}{\alpha_2} G_1(m+nt_e)\, dt_e$$

$$= \frac{1}{\alpha_2}\left[G_1(m+nr)e^{-r/\theta} - G_1(m+ns)e^{-s/\theta}\right] + \frac{1}{\alpha_2} e^{\left(\frac{m}{n\theta}\right)} \int_{m+nr}^{m+ns} e^{-\left(\frac{w}{n\theta}\right)} g_1(w)\, dw.$$

For the special case, (8), it follows that

$$\int_0^{t-(\alpha_1+\beta_1 a)} \frac{1}{\theta} e^{-t_e/\theta} \frac{1}{\alpha_2} G_1\left(\frac{t-t_e-\alpha_1}{\beta_1}\right) dt_e$$

$$= \frac{1}{\alpha_2}\left[G_1\left(\frac{t-\alpha_1}{\beta_1}\right) - G_1(a)e^{\left(\frac{-[t-(\alpha_1+\beta_1 a)]}{\theta}\right)}\right] + \frac{1}{\alpha_2} e^{\left(\frac{-(t-\alpha_1)}{\theta}\right)} \int_{\frac{t-\alpha_1}{\beta_1}}^{a} e^{\left(\frac{\beta_1}{\theta}\right)w} g_1(w)\, dw.$$

Similarly, each of the integrals involved in the previous expression for $g_5(t)$ can be reduced and after some algebraic simplification they form the following expression for $g_5(t)$.

For $\alpha_1+\beta_1 a < t < (\alpha_1+\alpha_2)+\beta_1 a$,

$$g_5(t) = \frac{1}{\alpha_2}\left[G_1\left(\frac{t-\alpha_1}{\beta_1}\right) - G_1(a) - e^{-\left(\frac{t-\alpha_1}{\theta}\right)} \int_a^{\frac{t-\alpha_1}{\beta_1}} e^{\left(\frac{\beta_1}{\theta}\right)w} g_1(w)\, dw\right].$$

Similarly,

$$
g_5(t) = \begin{cases}
\dfrac{1}{\alpha_2}\left[G_1\left(\dfrac{t-\alpha_1}{\beta_1}\right) - G_1\left(\dfrac{t-(\alpha_1+\alpha_2)}{\beta_1}\right) - e^{-\left(\frac{t-\alpha_1}{\theta}\right)} \displaystyle\int_a^{\frac{t-\alpha_1}{\beta_1}} e^{\left(\frac{\beta_1}{\theta}\right)w} g_1(w)\, dw + e^{\left(\frac{t-(\alpha_1+\alpha_2)}{\theta}\right)} \displaystyle\int_a^{\frac{t-(\alpha_1+\alpha_2)}{\beta_1}} e^{\left(\frac{\beta_1}{\theta}\right)w} g_1(w)\, dw \right] \\
\qquad \text{for } (\alpha_1+\alpha_2)+\beta_1 a < t < \alpha_1+\beta_1 b \\
\dfrac{1}{\alpha_2}\left[G_1(b) - G_1\left(\dfrac{t-(\alpha_1+\alpha_2)}{\beta_1}\right) + e^{-\left(\frac{t-(\alpha_1+\alpha_2)}{\theta}\right)} \displaystyle\int_a^{\frac{t-(\alpha_1+\alpha_2)}{\beta}} e^{\left(\frac{\beta_1}{\theta}\right)w} g_1(w)\, dw - e^{-\left(\frac{(t-\alpha_1)}{\theta}\right)} \displaystyle\int_a^{b} e^{\left(\frac{\beta_1}{\theta}\right)w} g_1(w)\, dw \right] \\
\qquad \text{for } \alpha_1+\beta_1 b < t < (\alpha_1+\alpha_2)+\beta_1 b \\
\dfrac{1}{\alpha_2}\left[e^{-\left(\frac{t-(\alpha_1+\alpha_2)}{\theta}\right)} \displaystyle\int_a^{b} e^{\left(\frac{\beta_1}{\theta}\right)w} g_1(w)\, dw - e^{-\left(\frac{t-\alpha_1}{\theta}\right)} \displaystyle\int_a^{b} e^{\left(\frac{\beta_1}{\theta}\right)w} g_1(w)\, dw \right] \\
\qquad \text{for } (\alpha_1+\alpha_2)+\beta_1 b < t < \infty.
\end{cases}
$$

3.2.3 Case B. By repeating the methodology presented in Section 3.2.2, using the geometry and functional definitions appropriate for Case B, the following formulation for $g_5(t)$ results:

$$
g_5(t) = \begin{cases}
\dfrac{1}{\alpha_2}\left[G_1\left(\dfrac{t-\alpha_1}{\beta_1}\right) - G_1(a) - e^{\left(\frac{t-\alpha_1}{\theta}\right)} \displaystyle\int_a^{\frac{t-\alpha_1}{\beta_1}} e^{\left(\frac{\beta_1}{\theta}\right)w} g_1(w)\, dw\right] \\
\qquad\qquad \alpha_1+\beta_1 a < t < \alpha_1+\beta_1 b \\[2ex]
\dfrac{1}{\alpha_2}\left[G_1(b)-G_1(a) - e^{\left(\frac{t-\alpha_1}{\theta}\right)} \displaystyle\int_a^{b} e^{\left(\frac{\beta_1}{\theta}\right)w} g_1(w)\, dw\right] \\
\qquad\qquad \alpha_1+\beta_1 b < t < (\alpha_1+\alpha_2)+\beta_1 a \\[2ex]
\dfrac{1}{\alpha_2}\left[G_1(b)-G_1\left(\dfrac{t-(\alpha_1+\alpha_2)}{\beta_1}\right) + e^{\left(\frac{t-(\alpha_1+\alpha_2)}{\theta}\right)} \displaystyle\int_a^{\frac{t-(\alpha_1+\alpha_2)}{\beta_1}} e^{\left(\frac{\beta_1}{\theta}\right)w} g_1(w)\, dw\right. \\
\qquad\qquad \left. - e^{\left(\frac{t-\alpha_1}{\theta}\right)} \displaystyle\int_a^{b} e^{\left(\frac{\beta_1}{\theta}\right)w} g_1(w)\, dw\right] \\
\qquad\qquad (\alpha_1+\alpha_2)+\beta_1 a < t < (\alpha_1+\alpha_2)+\beta_1 b.
\end{cases}
$$

Continuing for $(\alpha_1+\alpha_2)+\beta_1 b < t < \infty$,

$$g_5(t) = \left[\frac{1}{\alpha_2} \, e^{-\frac{t-(\alpha_1+\alpha_2)}{\beta}} \int_a^b e^{\left(\frac{\beta_1}{\beta}\right)w} g_1(w) \, dw - e^{-\left(\frac{t-\alpha_1}{\beta}\right)} \int_a^b e^{\left(\frac{\beta_1}{\beta}\right)w} g_1(w) \, dw. \right]$$

4. SPECIFIC APPLICATION

4.1 Introduction.

The following is a description of a specific application of the theory described in Sections 2 and 3. Note that where possible existing data were used to construct the underlying distributions. As further data become available the results can easily be updated to reflect this additional information. In any case, this description should provide the reader with a comparison of the distribution of submunition arrival times for several specific munition configurations of current interest, a clearer understanding of how the methodology might be applied, and some indication of the sensitivity of the results to variations in the basic parameter values.

The application considers a battery volley of artillery fire using Improved Conventional Munitions. A comparison is made of the per cent of submunition bursts as a function of time after initial warning for several specific munition configurations. As specified in Section 2 initial warning occurs (i.e., time is initialized) at the time of ejection of the submunitions from that round of the volley which is first to function.* Three munition configurations were considered for the 155mm, M449A1 ICM with specific characteristics as listed in Table 4.1.

*Note that no correction has been made for the time required for the sound of the initial burst to reach the ground. If the uncertainty associated with actual warning is thought to be significant the warning time axis should be adjusted appropriately.

TABLE 4.1 MUNITION CONFIGURATIONS

Configuration	Fuze	Angle of Fall
I	Mechanical Time (MT) XM 577	30^o
II	MT XM 577	50^o
III	Proximity (VT)	Not Applicable

The distributional assumptions required by the theory of Sections 2 and 3 will now be discussed for each of these cases.

4.2 Distributional Assumptions.

4.2.1 $g_3(t_e)$ - Round Arrival Time. In practice there are many sources of error which contribute to the variation in round arrival time* when firing a battery volley. The major sources of variability are the inability of the firing battery to conduct a synchronized firing and delivery error sources such as tube-to-tube velocity variations, velocity variations within a given tube, aiming variations, etc. The contribution of differences in fuze type, on the other hand, will most likely be relatively insignificant compared to those of the factors mentioned above. Thus, for the purposes of this example $g_3(t_e)$ is assumed to be independent of fuze type.**

As stated in Section 2.1 the general form assumed for the p.d.f. of round arrival given warning is

$$g_3(t_e) = \frac{1}{\theta} e^{-\frac{t_e}{\theta}} \qquad 0 < t_e < \infty;\ \theta > 0.$$

*Round arrival time is considered to be the time to round function subsequent to the function of the first round of the volley.

**Note that this assumption was made by choice and should not be construed as a restriction inherent in the methodology.

For each of the cases specified in Table 4.1, results are obtained for two values of the parameter θ ($\theta = 1$ and $\theta = 2$). In the case where $\theta = 2$, $g_3(t_e)$ reasonably approximates the round arrival time data obtained from HELBAT I.* On the other hand, certain situations may exist where a slightly increased likelihood of shorter arrival times is more realistic. $\theta = 1$ reflects such a situation. In any case, consideration of both values provides insight into the sensitivity of the methodology to this parameter. The practical significance of the values $\theta = 1$ and $\theta = 2$ can be better appreciated by examining the cumulative distribution functions** displayed in Figure 4.1. Examining the curves it is seen that 90% of the rounds from the volley will function within a span of 2.3 seconds when $\theta = 1$ and within a span of 4.65 seconds when $\theta = 2$. As stated earlier, if other values of θ appear to be more realistic as further information is obtained then these cases should also be examined.

4.2.2 $g_1(h)$ - Burst Height. Projectile burst height is assumed to be a normal random variable with p.d.f.

$$g_1(h) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(h-\mu)^2}{2\sigma^2}} \qquad -\infty < h < \infty;\ -\infty < \mu < \infty;\ \sigma^2 > 0.$$

*Human Engineering Laboratory, TM 24-70, "Human Engineering Laboratories Battalion Artillery Test," Sep 1970, Gary L. Harley, Dominick J. Giordano.

$$**F(t_o) = \int_0^{t_o} g_3(t_e)\, dt_e = 1-e^{-t_e/\theta} \qquad \theta > 0;\ 0 < t_o < \infty$$

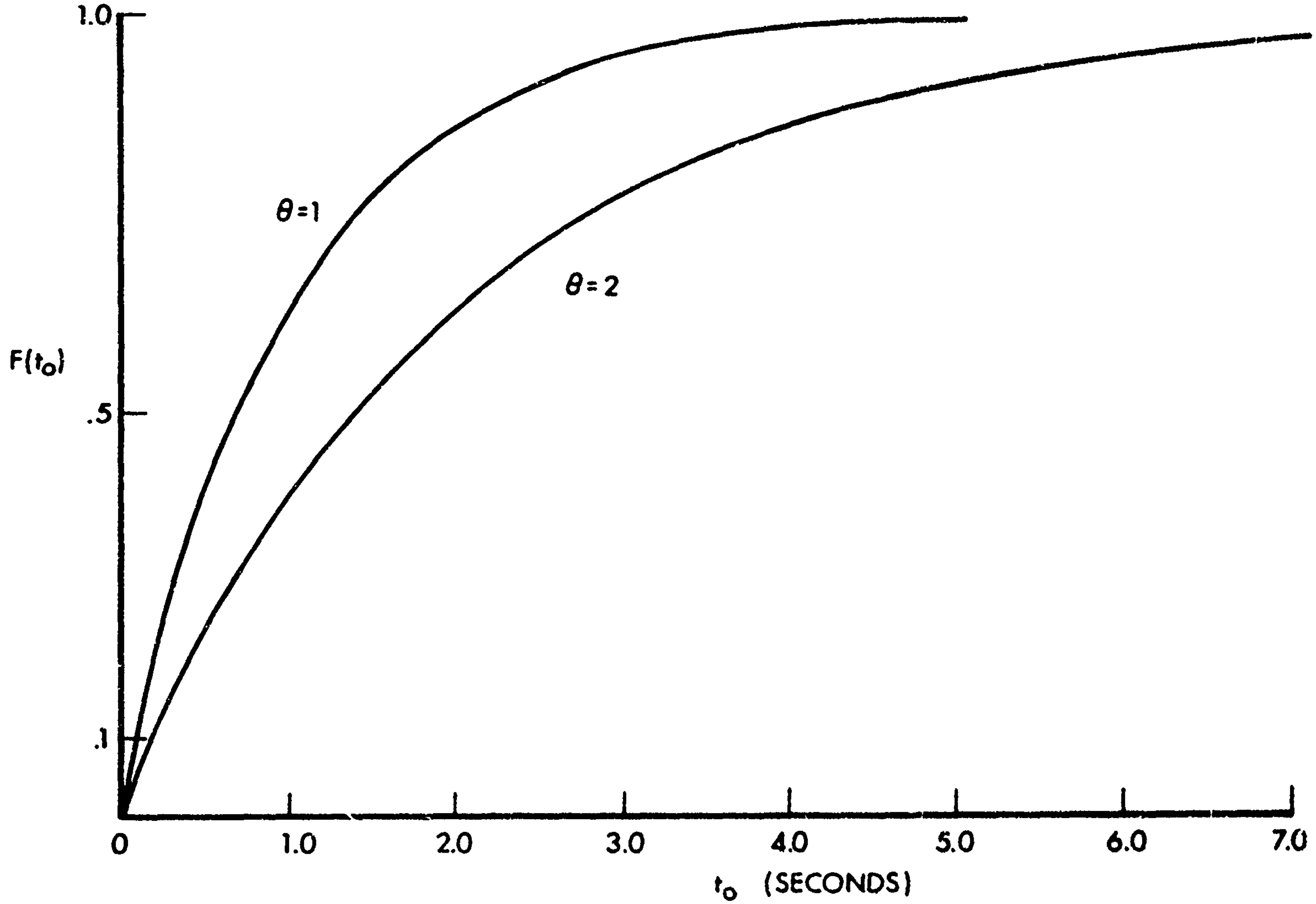

Figure 4.1 Cumulative Distribution of Round Arrival Times.

In practice burst heights are limited in that they will neither occur below the surface (i.e., $h \geq 0$) nor above some limit, say L, determined by the ballistic altitude capabilities of the gun/shell system (i.e., $h \leq L$). Thus the normal p.d.f., with infinite range, is not a precise representation of the true burst height distribution. In all cases considered, however, the normal approximation is adequate since points a and b ($a < b$; $a > 0$, $b < L$) do exist such that $\int_{-\infty}^{a} g_1(h)\,dh$ and $\int_{b}^{\infty} g_1(h)\,dh$ are both approximately equal to zero.

The form of the distribution (i.e., the values of μ and σ^2) will generally depend on the various components of the delivery error (precision, MPI, fuze), the fuze type, and the angle of fall of the projectile. Specific values for the three munition configurations considered here were taken from a recent report by the USAMUCOM ORG* and are listed in Table 4.2. Details concerning the rationale supporting these values can be found in the referenced document.

TABLE 4.2 NORMAL PARAMETERS

Configuration	Mean Burst Height (μ)	Standard Deviation (σ)
I	235 meters	25.7 meters
II	350 meters	76.3 meters
III	187 meters	4.8 meters

4.2.3 $g_2(c_f/h)$ - Submunition Time to Fall. As specified in Section 2.1, submunition time to fall (given round function) is assumed to be a uniform random variable where the time to first submunition function, c(h), and the spread, d(h)-c(h), are both linearly dependent on the height that the round functions. Specifically, the methodology of Section 2.3 uses $c(h) = \alpha_1 + \beta_1 h$ and $d(h) - c(h) = \alpha_2$ (constant - independent of burst height). For this example both a six second ($\alpha_2 = 6$) and a four second ($\alpha_2 = 4$) spread were considered. In addition results were

*"A Comparison of the Effectiveness of Improved Conventional Munitions Equipped with Mechanical Time and Proximity Fuzes (U)", DRAFT, USAMUCOM ORG, November 1972, CONFIDENTIAL.

obtained for two different relationships for time to first submunition burst. These are c(h) = .023h and c(h) = .018h where the time to first burst, c(h), is expressed in seconds and burst height, h, in meters. The .023 relationship was chosen since it reasonably approximates existing firing record data* for the M43 type bomblet. The .018 relationship, on the other hand, was examined since it reflects the rate of fall assumed in the previously referenced USAMUCOM ORG study and seemed to be a reasonable alternative to consider in examining the sensitivity of the methodology to this assumption.

4.3 Results.

The methodology of Section 3.2 has been exercised for eight specific cases. Each case represents a comparison of the distribution of submunition time to fall for three configurations of the 155mm, M449 A1 ICM. Configurations I and II involve the XM577 mechanical time fuze at a 30° and 50° angle of fall, respectively, while configuration III involves a proposed proximity fuze. The distributional parameter values considered in each of the eight cases are listed in Table 4.3.

TABLE 4.3 SPECIFIC CASES

Case	Time to Ejection Parameter (θ)	Spread From First to Last Submunition Function (α_2)	Height Dependence β_1
1	2	6 seconds	.023
2	2	4 seconds	.023
3	1	6 seconds	.023
4	1	4 seconds	.023
5	2	6 seconds	.018
6	2	4 seconds	.018
7	1	6 seconds	.018
8	1	4 seconds	.018

The rationale for the selection of these values of θ, α_2, and β_1 is contained

*FR #5296, FR #5419, Yuma Proving Ground.

in the discussion of Section 4.2. Note that values other than the nominal values have also been examined in order to get some indication of the sensitivity of the results to variation in the basic parameter values. Figures 4.2 thru 4.9 display the results of this evaluation. In each case the cumulative probability distribution* corresponding to the p.d.f. of total time to fall, $g_5(t)$, is provided. It is seen, for Case 1 for example, from Figure 4.2 that for the initial conditions specified in Table 4.3 only 35 per cent (Munition Configuration III) and 20.5 per cent (Munition Configuration I) of the volley submunitions will burst within the first eight seconds after initial warning (first round submunition ejection). In fact in all cases there will be considerable time (4-7 seconds) for personnel to react before many of the submunitions from the volley detonate. Additionally this estimate may even be conservative since in the operational environment there may possibly be an additional 2 to 3 seconds of warning due to the incoming whistling of the round prior to submunition ejection.

Another observation that can be made is that the results are quite sensitive to the parameters θ, α_2 and β_1. Thus the variability in round arrival time and variation in time to submunition impact do significantly affect the ultimate distribution of submunition arrival time. Failure to recognize and account for these sources of variability could seriously bias the results of any study evaluating weapons effects.

5. SUMMARY

In assessing the effectiveness of an artillery battery volley using Improved Conventional Munitions in a situation where the personnel in the target area may react to seek protective cover, one must consider both the distribution of arrival time of submunitions in the target area and the reaction time distribution for the target personnel. The first

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*The cumulative distribution function $F(t_o) = \int_0^{t_o} g_5(t)\,dt$ depicts the per cent of submunitions which have burst within t_o seconds after initial warning.

of these has been addressed in this report and a mathematical expression for the distribution of submunition arrival time is provided in Section 3. In the subsequent analysis, comparing several munition configurations of current interest, in Section 4 it is shown that variability in time to round function and variation in time to submunition fall do significantly affect the distribution of submunition burst time. Further even when employing the VT fuze there will in most cases be considerable time for personnel to react before many of the submunitions from the volley detonate.

To realistically assess the degree of surprise which can be achieved with alternative munition configurations results such as those provided should be coupled with information pertaining to the ability of target personnel to react to audible warning in seeking protective cover. However, the magnitude of the warning times alone make one seriously question whether any significant degree of surprise can be achieved by using any of the munition configurations considered in this report. This opinion is reinforced by the small quantity of test data available in HEL Technical Notes 7-68 and 1-71 where troop posture sequences were tested for dismounted (armored) infantrymen and for artillery crews respectively. These reports both indicated that with 4-6 seconds of time to react target personnel would be able to significantly decrease their total exposed presented area. As further data become available on target personnel posture distributions an additional analysis may be required to completely summarize the ICM effectiveness envelope for area personnel targets.

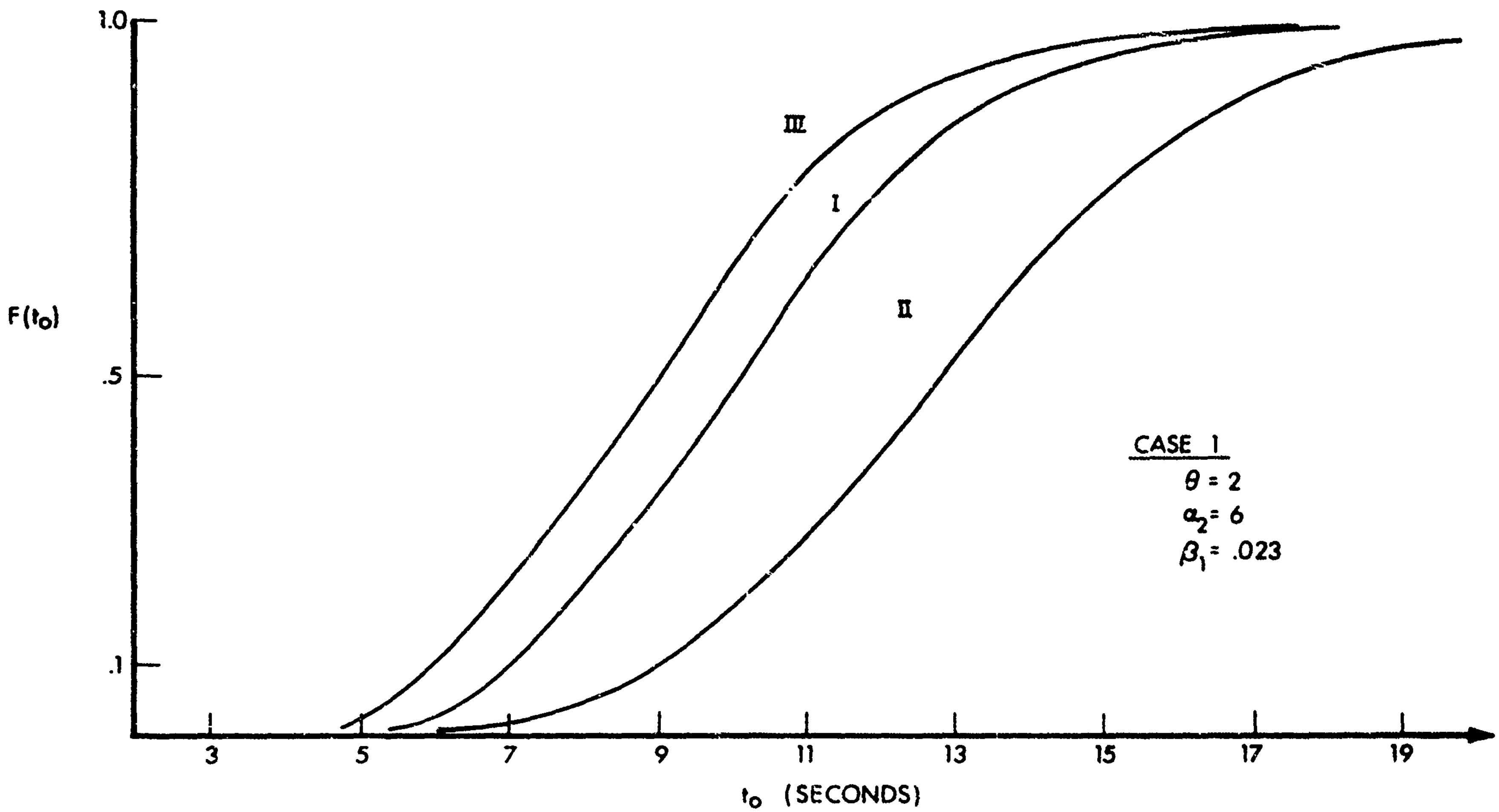

Figure 4.2 Cumulative Distribution of Submunition Arrival Times.

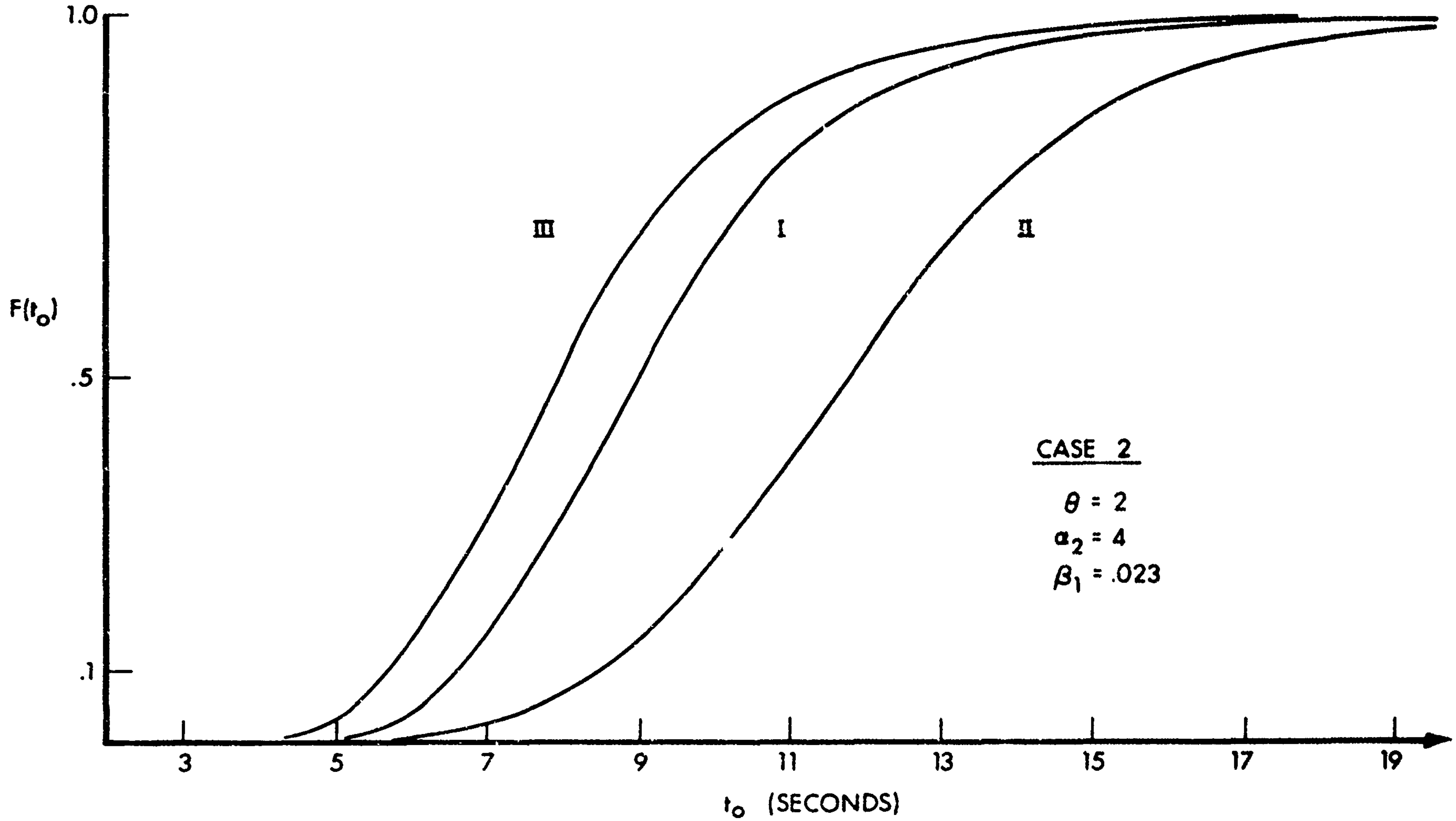

Figure 4.3 Cumulative Distribution of Submunition Arrival Times.

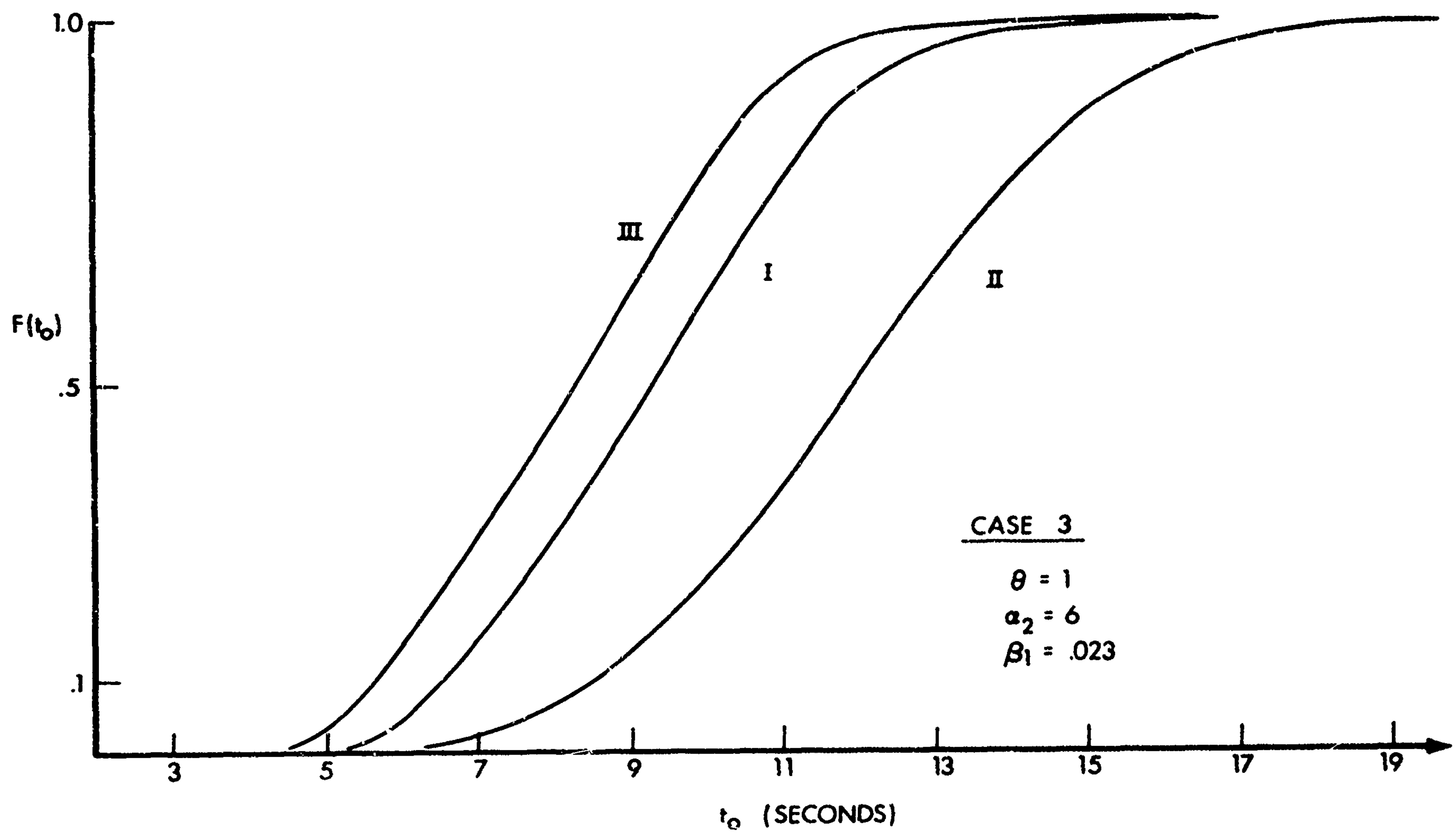

Figure 4.4 Cumulative Distribution of Submunition Arrival Times.

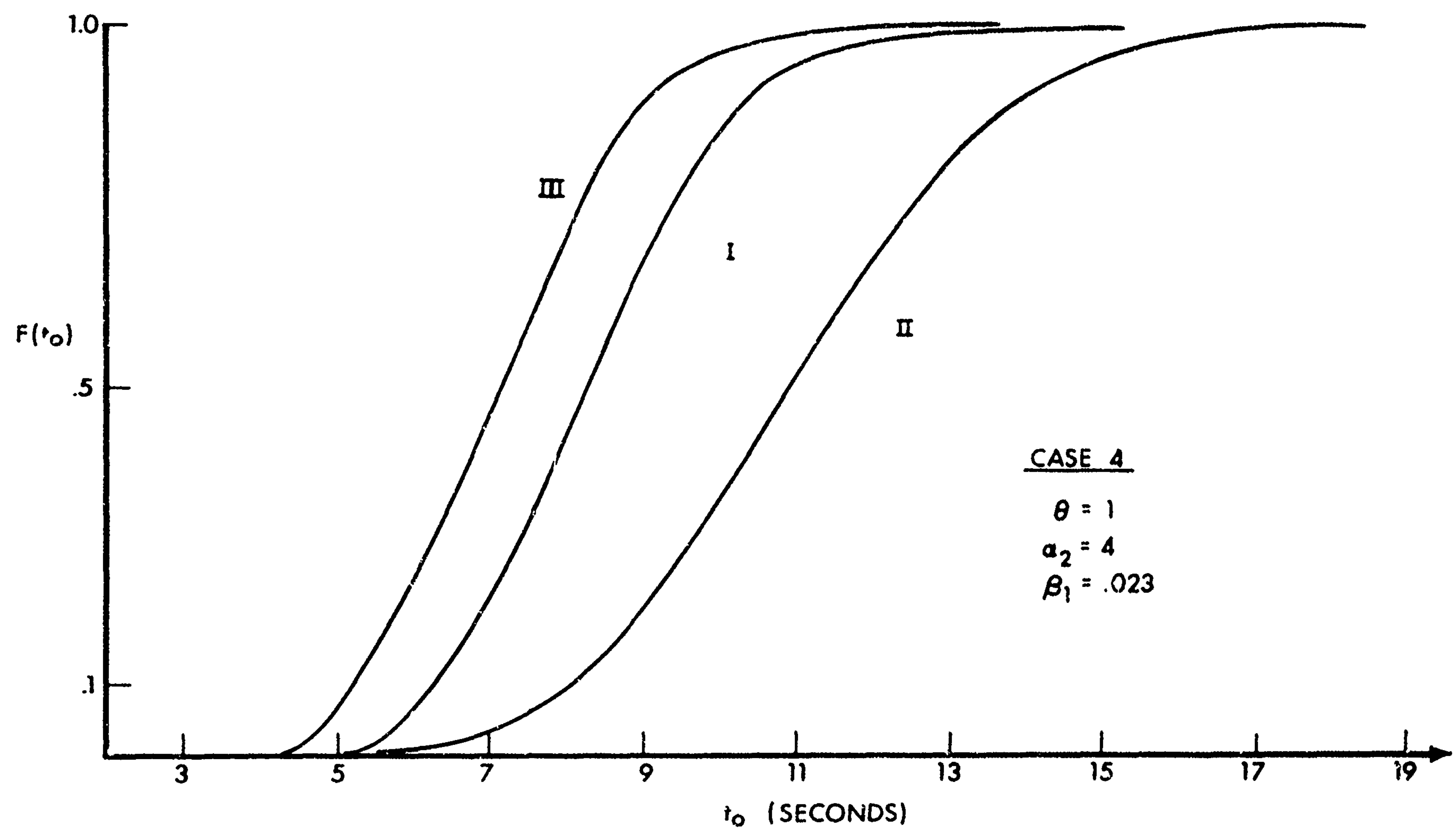

Figure 4.5 Cumulative Distribution of Submunition Arrival Times.

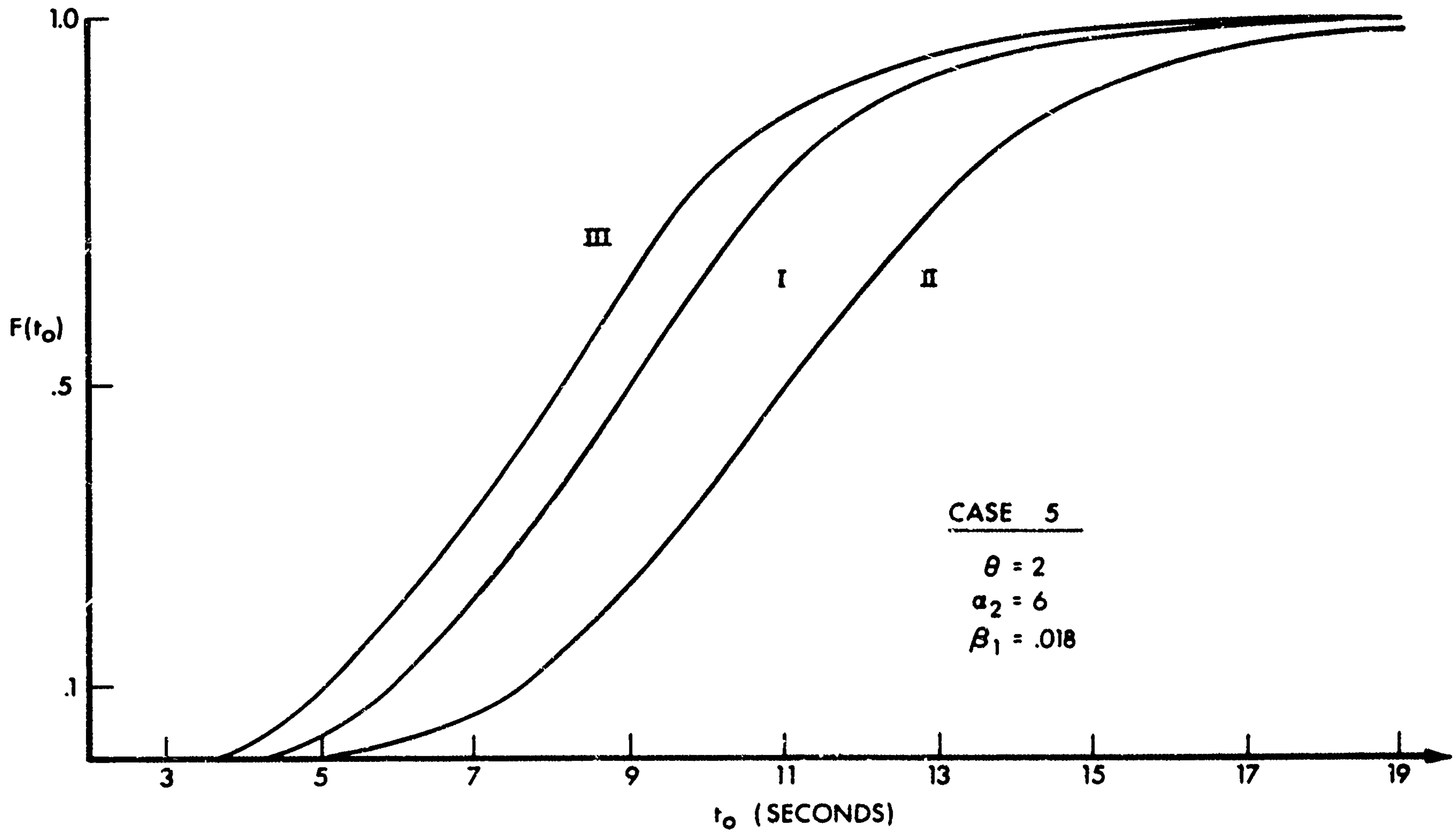

Figure 4.6 Cumulative Distribution of Submunition Arrival Times.

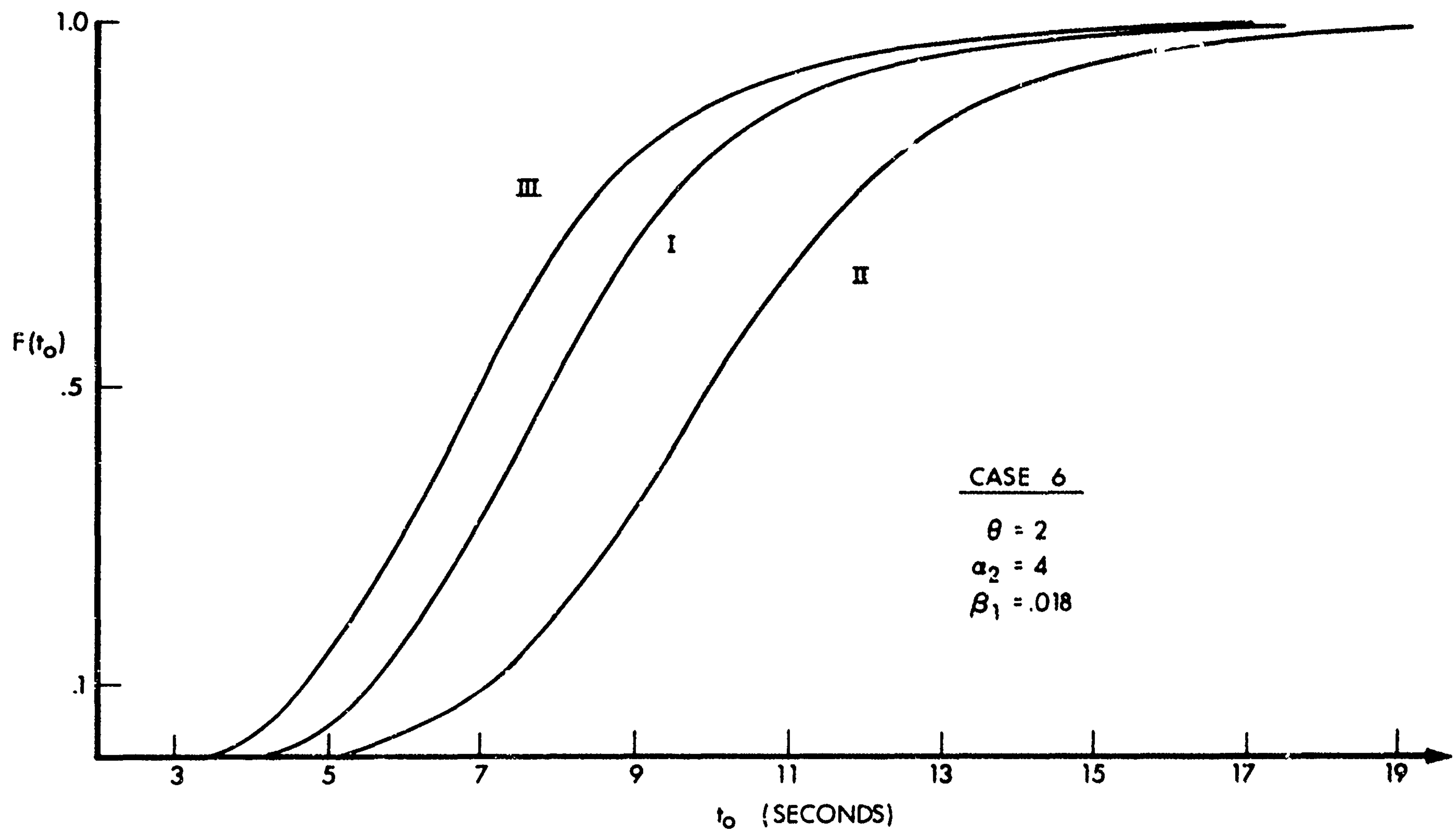

Figure 4.7 Cumulative Distribution of Submunition Arrival Times.

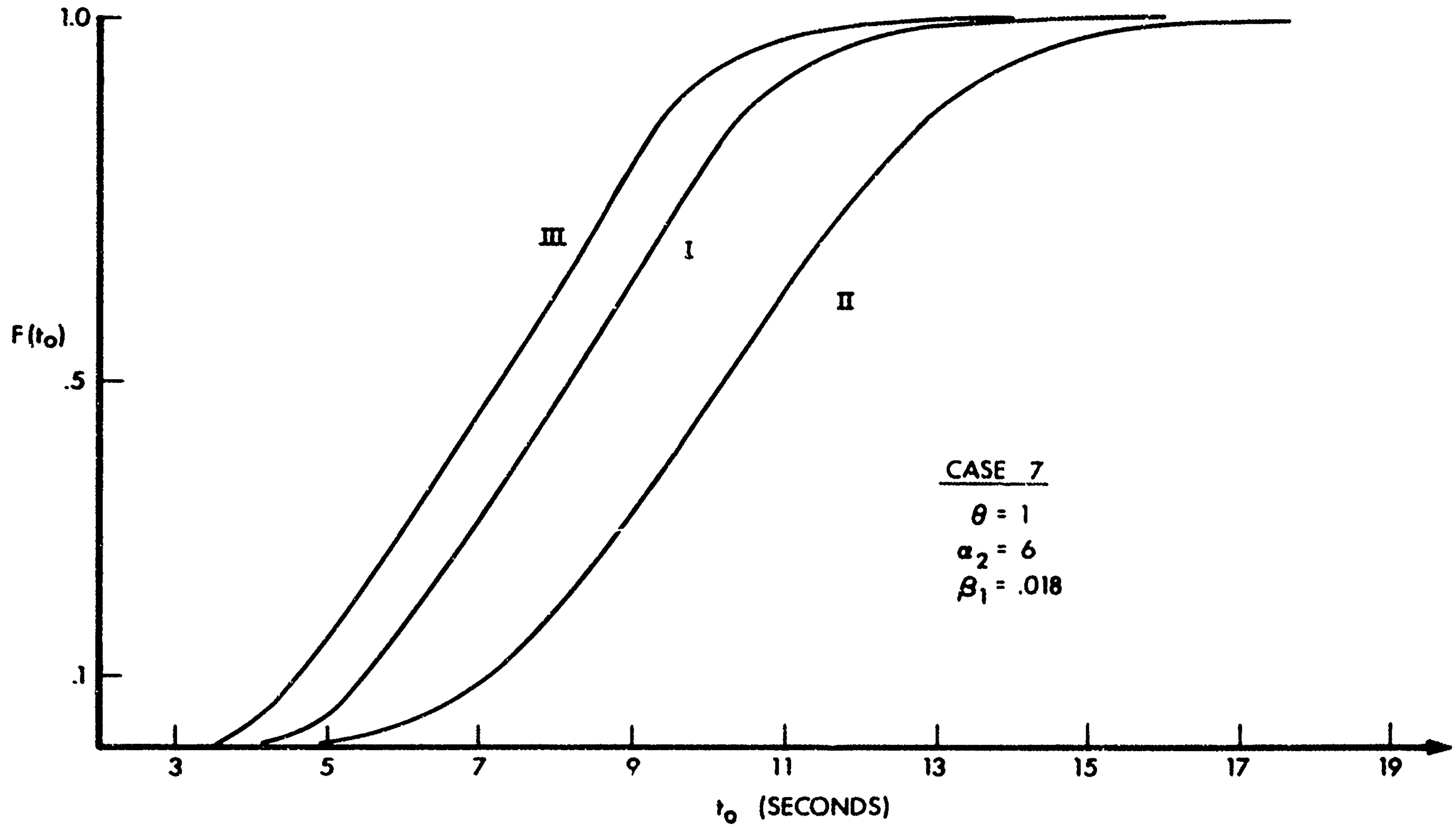

Figure 4.8 Cumulative Distribution of Submunition Arrival Times.

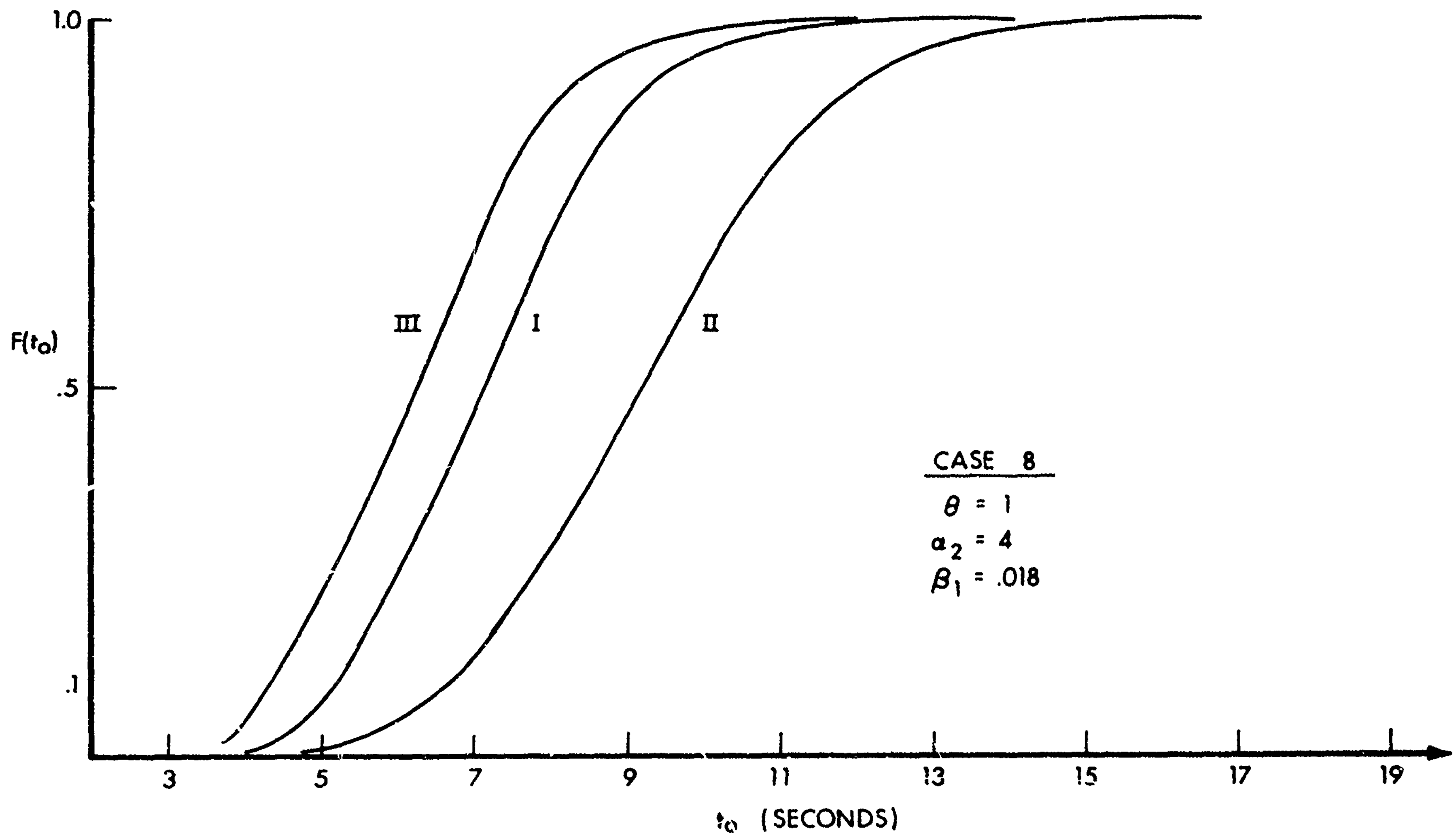

Figure 4.9 Cumulative Distribution of Submunition Arrival Times.

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