

AFFDL-TR-73-123

AD-774-047

ON SPECKLE DIFFRACTION INTERFEROMETRY FOR MEASURING WHOLE FIELD DISPLACEMENTS AND STRAINS

*FRANK D. ADAMS
GENE E. MADDUX*

TECHNICAL REPORT AFFDL-TR-73-123

DECEMBER 1973

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WRIGHT-PATTERSON AIR FORCE BASE, OHIO 45433

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by

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and

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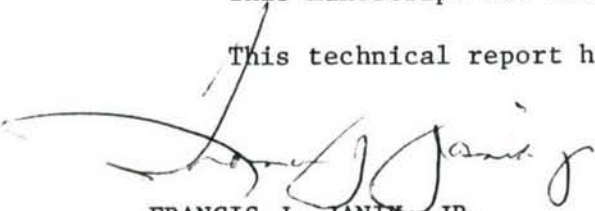
FOREWORD

This report is the result of an in-house effort conducted under Project 1467, "Structural Analysis Methods", Task 146702, "Thermo-elastic Structural Analysis Methods". Experimental work was performed in the Air Force Flight Dynamics Laboratory Photomechanics Facility.

The work reported herein was conducted from June 1972 to June 1973 by Dr. Frank D. Adams, Physicist, and Mr. Gene E. Maddux, Aerospace Engineer, both in the Analysis Group, Structures Division, Air Force Flight Dynamics Laboratory. Mr. Robert M. Bader, AFFDL(FBR), is the Technical Manager of the Analysis Group.

This manuscript was released by the authors in August 1973.

This technical report has been reviewed and approved.



FRANCIS J. JANIK, JR.
Chief, Solid Mechanics Branch
Structures Division

ABSTRACT

The procedure for using laser speckle diffraction interferometry to measure small displacements and strains is described. The variation of basic experimental parameters is investigated. A typical application of the method is discussed and data is presented for the displacement distribution around an interference fit fastener.

TABLE OF CONTENTS

SECTION		PAGE
I	INTRODUCTION	1
II	BASIC PHENONEMA	3
III	EXPERIMENTAL APPARATUS AND CONFIGURATION	9
IV	PARAMETER INVESTIGATION	12
V	A TYPICAL APPLICATION	24
APPENDIX		
A	LENS RESOLUTION AND SPECKLE SIZE	33
B	YOUNG'S FRINGES	36
REFERENCES		42

ILLUSTRATIONS

Figures		Page
1	Speckle Appearance of Resolution Test Chart Illuminated with Laser Light	4
2	Photograph of Speckle Pattern with 300X Magnification	5
3	Speckle Diffraction Halo with Young's Fringes	7
4	Experimental Arrangement to Make a Specklegram	10
5	Experimental Arrangement to View a Specklegram	10
6	Photomicrographs (150X) which show the Variation of Speckle Sizes with Magnification and f/number	14
7	Holographic Fringes on a Rectangular Plate Subjected to a Point Load. The Plate has a Square "cut out" in the Lower Right Quadrant	19
8	Speckle Correlation Fringes on a Compact Tension Specimen	20
9	Whole Field Speckle Diffraction Fringes on a Compact Tension Specimen	22
10	Viewing "Whole Field" Fringes Through a Specklgram	23
11	Tapered Bolt Fastener and Aluminum Plate with a Tapered Hole	26
12	Tapered Bolt Interference Fit Fastener. The Lower Bolt has the Head Removed	27
13	Photo Print of Specklegram	28
14	Displacement vs Radial Distance Along a Horizontal Line	29
15	Displacement vs Radial Distance Along a Vertical Line	29

SECTION I

INTRODUCTION

Two technical areas which have received considerable attention in recent years are fracture mechanics and the fatigue performance of joints utilizing interference fit fasteners. Investigations performed here at the Flight Dynamics Laboratory have required that measurements be made of the displacement/strain distributions in the vicinity of cracks and around the edge of fasteners. These data are used to validate analytical predictions of the strain pattern or to gain physical insight into structural behavior for the purpose of formulating new predictive models.

Some of the experimental data were obtained using a relatively new technique called "speckle diffraction interferometry". This method which employs lasers and coherent optics is the subject of this report.

In Section II, the basic phenomena is briefly explained. Detailed equations have been purposely deferred until later so that this section can serve as an elementary introduction to the subject if the reader has not encountered speckle diffraction interferometry in the past. Section III provides a succinct description of the apparatus required and how to configure it to perform speckle interferometry. In Section IV, the speckle phenomena is examined in more detail. Basic parameters and the effects of their variation are investigated. The final section characterizes a typical application

of speckle diffraction interferometry. Displacement measurements in the vicinity of an interference fit fastener (tapered bolt) are described. Two appendices have been included to supplement material presented in Section IV.

Throughout this report, the authors have attempted to simplify mathematical descriptions of physical optics phenomena. It has been our experience that those people who have the most need to use this technique are often not familiar with the rather complex mathematical formulations used in physical optics. Many of our arguments are inexact and call upon physical intuition for understanding. This was done, without apology, to encourage a wide spectrum of experimental stress analysis people to consider this new technique.

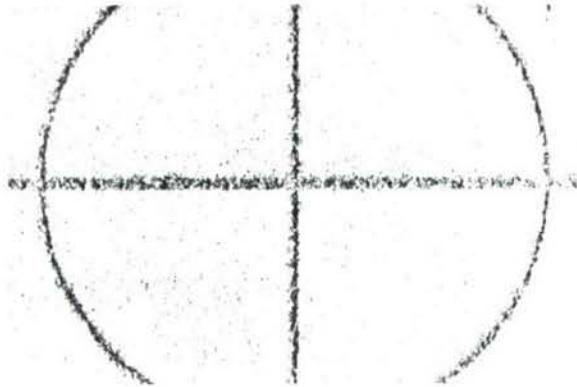
SECTION II

BASIC PHENOMENA

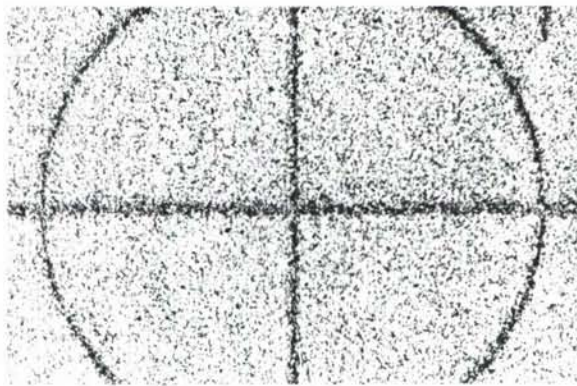
When using coherent laser light for illumination, a uniform diffuse surface has a speckled or grainy appearance due to random interference within the resolution limit of the eye (or photographic system). The speckled appearance of an object is illustrated by the photographs in Figure 1. It should be noted that the speckle size is dependent on the lens aperture. A magnified view of the speckle pattern is shown in Figure 2.

The speckle phenomenon provides a sensitive method for making measurements of displacements or strains in the surface of an object. Two basic procedures have been developed. The first employs correlation techniques to produce displacement contour fringes within the field of view (References 1, 2 and 3). The second procedure, and the one of interest in this report, exploits light diffraction to measure displacements (References 2 and 4). A double exposure "specklegram" (photo-negative) is made of the speckle surface. Exposures are made before and after a displacement occurs. Thus, each bright "speckle point" is recorded on the specklegram as two dark points - one in the undisplaced position and the other in the displaced position. Only knowledge of the photographic magnification factor is required to determine a local displacement vector once the length and direction of the line segment between corresponding speckle points are known.

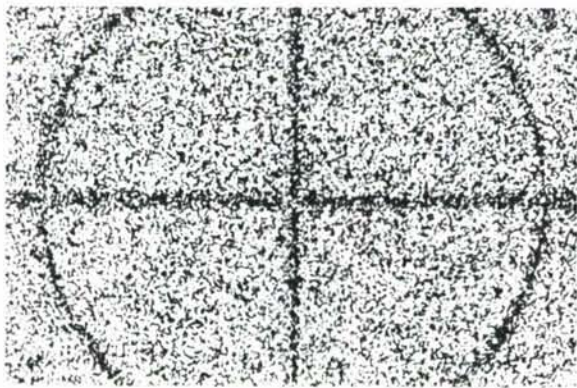
LENS APERTURE



f/8



f/22



f/45

Figure 1 Speckled Appearance of Resolution Test Chart Illuminated with Laser Light

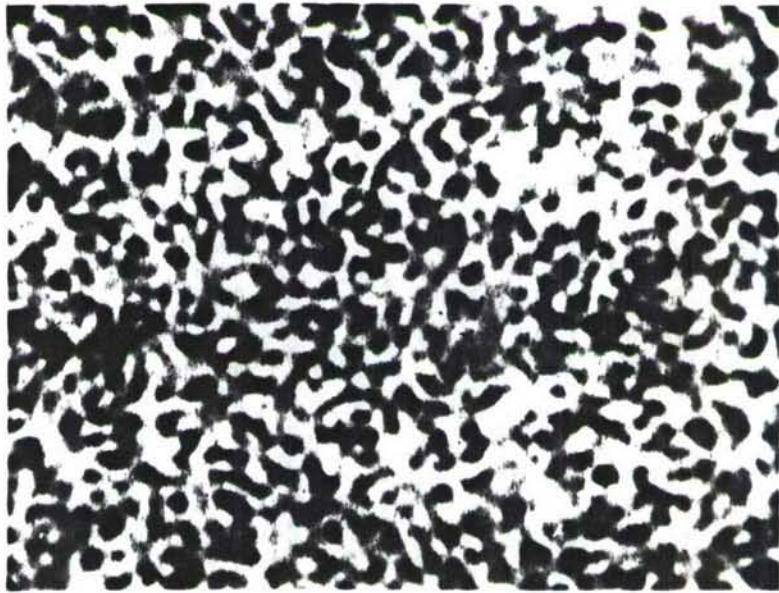


Figure 2 Photograph of Speckle Pattern with
300X Magnification

A direct point to point measurement on a specklegram is virtually impossible or at best extremely tedious because of randomness in the speckle field. Fortunately, this is not necessary. If a narrow collimated laser beam (monochromatic and coherent) is directed through the specklegram, diffraction will modify the emerging light rays into a cone. This "diffraction halo" is the result of diffraction from the random distribution of small speckles. Since the speckles are recorded in pairs, however, a parallel fringe pattern (Young's fringes) also occurs in the emerging light cone. Figure 3 is a photograph of a typical speckle diffraction halo with a displacement induced fringe pattern.

For small diffraction angles, Young's fringes are perpendicular to the displacement vector and the spacing is inversely proportional to the displacement magnitude. This provides a measurement of local displacement since, in general, a narrow laser beam is used to interrogate only a small area of the specklegram. The resolution depends on the laser beam diameter and magnification used in the photographic set up. The displacement distribution can be obtained by scanning the laser beam over the entire image area and taking measurements at selected points.

The sensitivity of the speckle diffraction fringe technique for measuring small displacements is comparable to holographic interferometry. It should be noted, however, that this technique is not a substitute, but rather supplements holographic methods. Although in principle, holographic interferometry may be used to measure three

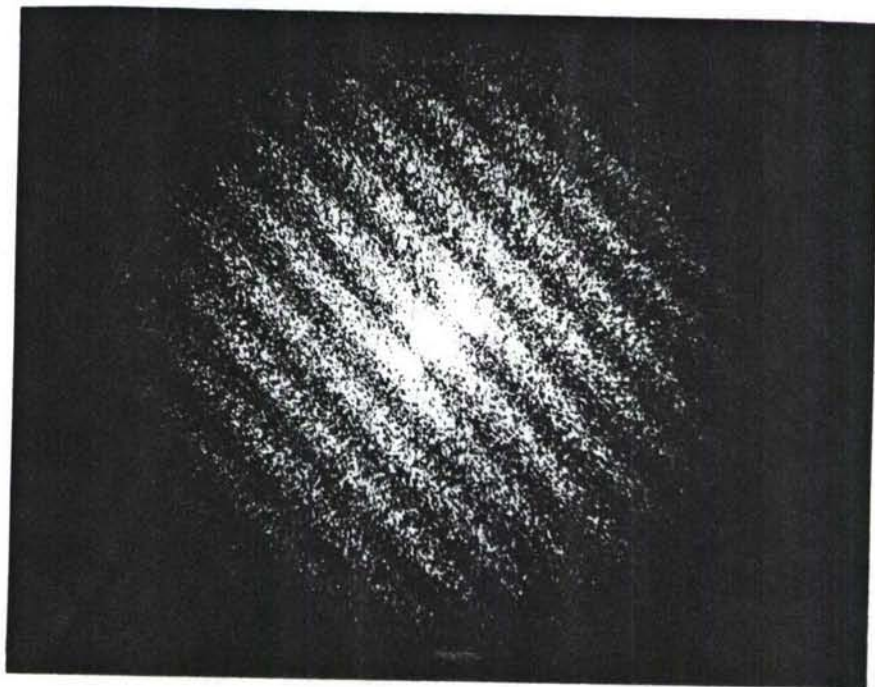


Figure 3 Speckle Diffraction Halo With Young's Fringes

dimensional displacements, it is not very sensitive for determining in-plane displacement components. In addition, measurement procedures are relatively complicated (Reference 5). The speckle diffraction method measures displacement components which lie in a plane perpendicular to the observation vector. Thus, this technique is particularly well suited to determining in-plane displacements and strains. Three dimensional displacements are best resolved by employing a combination of speckle and holographic techniques.

SECTION III

EXPERIMENTAL APPARATUS AND CONFIGURATION

The experimental arrangement for making a double exposure specklegram is shown in Figure 4. It is, in fact, a simple camera configuration. Monochromatic and coherent laser light is used to illuminate the specimen which provides for a speckle appearance on a diffuse surface. The lens focal length and object distance are often selected to yield an enlarged image; however, this is not always necessary or desirable and depends on the particular measurement to be made. As will be discussed later, the speckles can be made very small (less than one micron in diameter) so that high resolution photographic film is normally required. Thin emulsion holographic films such as Agfa 10E75 are commonly used. At AFFDL, specklegrams have also been made using Kodak "High Contrast Copy Film".

Figure 5 is a diagram of the arrangement used to measure speckle separation. A helium-neon laser with one to fifteen milliwatts of output will provide adequate illumination. To obtain good resolution of changes in displacement with respect to location, a small diameter input beam is preferred (one millimeter is typical). For a given beam diameter, the spatial resolution is determined by the selection of an image magnification factor. The fringe pattern is observed on a screen or through a frosted glass plate. A "main beam stop" is used to block the undiffracted portion of the laser beam in

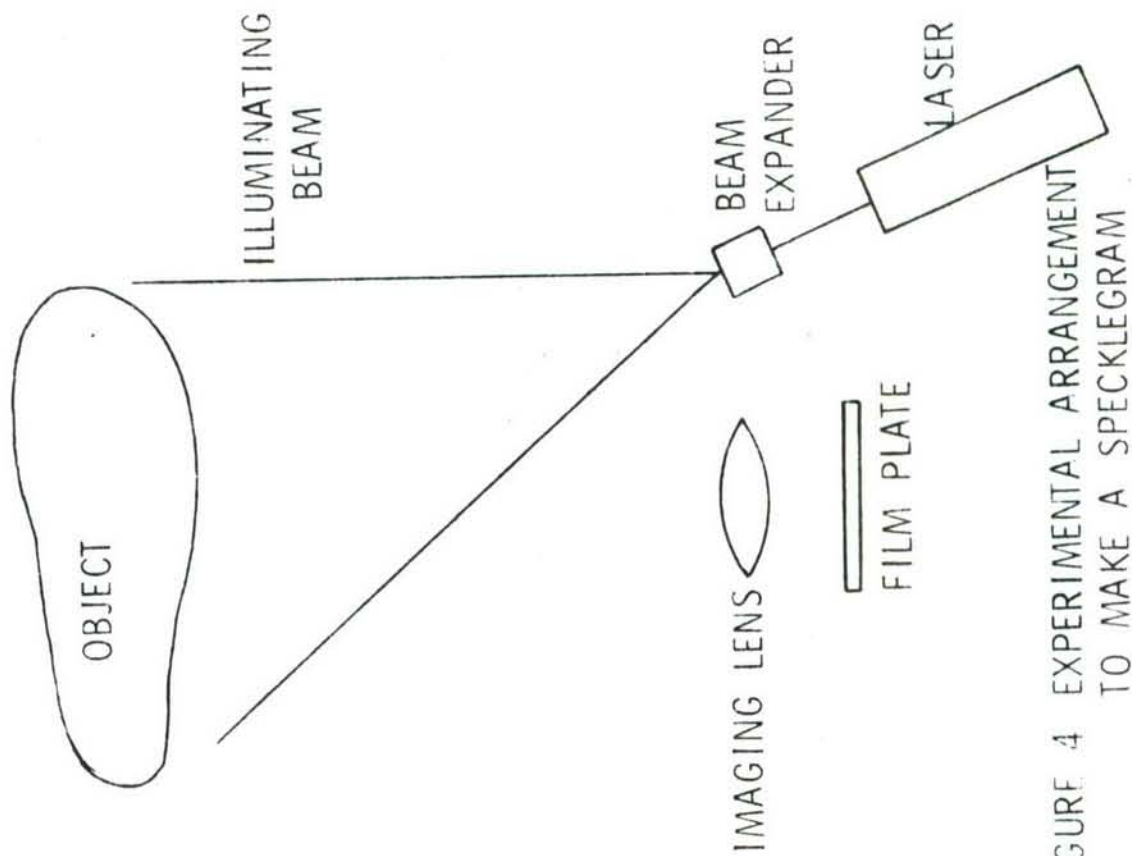


FIGURE 4 EXPERIMENTAL ARRANGEMENT TO MAKE A SPECKLEGRAM

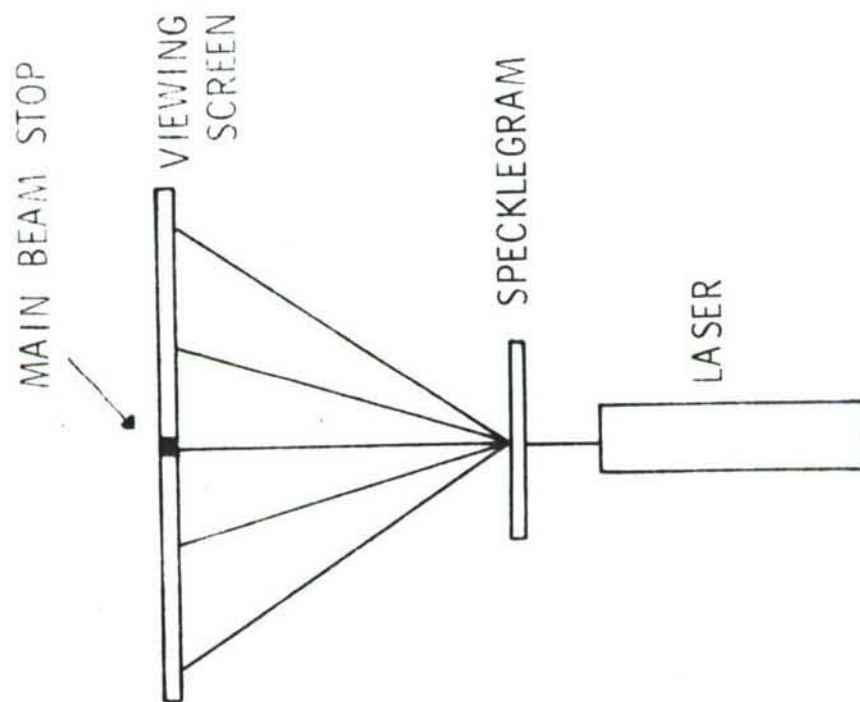


FIGURE 5 EXPERIMENTAL ARRANGEMENT TO VIEW A SPECKLEGRAM

order to obtain maximum contrast. If desired, an optional lens can be used to collimate the divergent fringe pattern.

SECTION IV

PARAMETER INVESTIGATION

A complete description of the speckle phenomenon requires knowledge of physical optics and some rather sophisticated mathematics. Some insight can be obtained, however, using a few simple arguments. Consider laser light (monochromatic with spatial and temporal coherence) to be incident on a diffuse, matte white surface. The scattered light will have a cosine intensity distribution and a random phase distribution. This destroys spatial coherence but not temporal coherence. As a result, interference occurs in the scattered light field and a random speckle pattern is produced. If no modifying optics are present, the far field is radially diverging. Thus, by intuition, one can conjecture that the speckle size in the far field must be proportional to $\lambda \ell / D$, where λ is the wavelength of light, D is the diameter of the illuminated area, and ℓ is the distance from the surface to the point of measurement. The far field approximation requires that $\ell / D \gg 1$. Note also that if the illuminated area has one dimension greater than the other, the speckles will be elongated in a direction perpendicular to the long axis. What may be less obvious is that the constant of proportionality in the speckle size equation is nearly unity. A formal derivation of this relationship can be obtained by calculating the diffraction pattern resulting from a continuous distribution of sources of random phase distribution in an aperture D . Of more interest to this study, however, is the speckle produced in the image plane of a lens used to photograph the illuminated surface. This will be discussed next.

The ultimate purpose of any imaging system is to produce a one to one correspondence between points in the object and image planes. To do this would require a "perfect" lens having an infinite aperture (all light emanating from a point on the object is collected by the lens and focused to the corresponding image point). With coherent illumination, no speckles would be produced (in the image plane) since interference would not be possible with adjacent source points on the object. With any real lens, however, a finite aperture limits the resolution. In other words, an infinitesimal point on the object is mapped into an area in the image plane. The converse is also true; an infinitesimal point in the image plane intercepts light from a finite area in the object plane. From either point of view, it is obvious that interference will take place if the light is temporally coherent and that a speckle pattern will be generated.

The approximate size of a primary speckle cell is just the dimension of an area in the image plane into which an object point has been mapped. An estimate of this parameter can be obtained by computing the "resolution" of the imaging lens. A derivation is given in Appendix A and yields a speckle diameter α given by

$$\alpha \approx \lambda F (M + 1) \quad (1)$$

where M is the image magnification factor and F is the numerical value of the lens aperture for an object located at infinity (i.e., for an $f/4.5$ aperture, $F = 4.5$). Figure 6 contains nine single exposure photomicrographs of speckle patterns in a three by three

Magnification

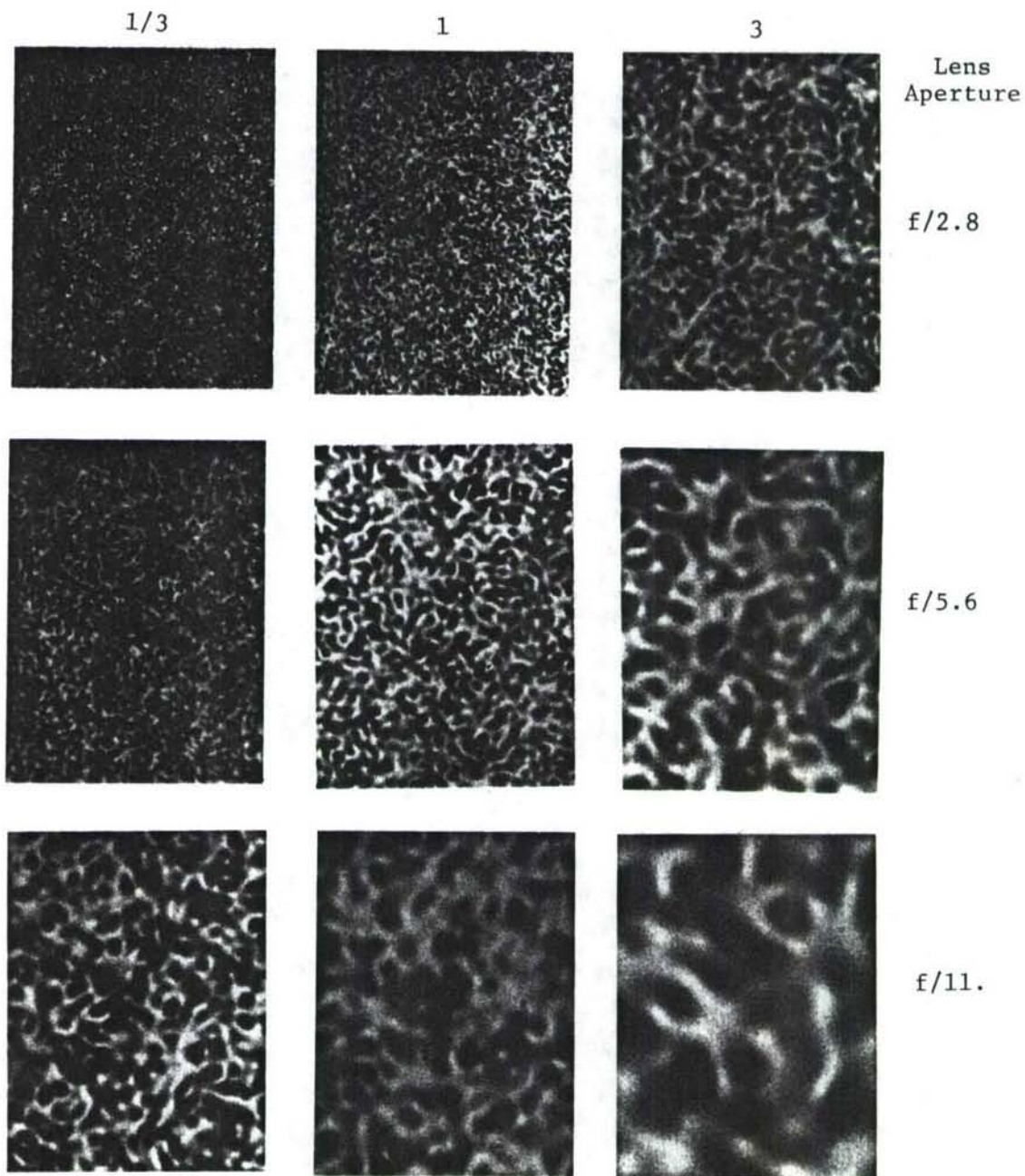


Figure 6 Photomicrographs (150X) which show the Variation of Speckle Size with Magnification and f/number

matrix format. These illustrate the changes in speckle size with variations of magnification (horizontal) and aperture (vertical). These speckle patterns were generated using $\lambda = .6328$ microns.

There is of course, some variation in size from speckle to speckle; however, the range is not large. The appearance of larger irregular dark or light areas on the photographs in Figure 5 is due to overlapping of individual primary speckles. This occurs because the spacing between speckles is a random function.

If a narrow laser beam is passed through a transparency of the speckle pattern, part of the transmitted light is diffracted. In practice, most of the diffracted light emerges within well defined angular limits. These angular limits roughly correspond to Airy's disk generated by a speckle size aperture. The far field intensity distribution is a diverging circular cone with an intense center spot from the undiffracted main beam. The center spot is usually eliminated with a main beam stop as is illustrated in Figure 5. The intensity distribution in this diffraction pattern is often called a "diffraction halo". There are, of course, no Young's fringes in the diffraction halo when the speckle pattern is photographed using a single exposure. However, if a double exposure is made with displacements or strain occurring between exposures, then an interferometric phenomenon is observed and the diffraction halo is modulated with Young's fringes. A more detailed discussion of Young's fringes is presented in Appendix B.

Equation (1) will now be used to examine the sensitivity of the speckle diffraction fringe technique for measuring small displacements. It should first be recognized that Young's fringes are readily visible only inside the diffraction halo. Fringe minima have an angular displacement Θ given by

$$(n - 1/2) \lambda = \Delta_i \sin \Theta$$

$$n = 1, 2, 3 \dots \quad (2)$$

where Δ_i is the distance between speckle pairs on the double exposure specklegram. From the previous discussion and Appendix A and B, a reasonable approximation for the angular limits of the diffraction halo is given by:

$$\lambda/2 \approx a \sin \beta$$

$$(3)$$

where β is the halo half angle. If for a small displacement, the first minima occur at the edge of the diffraction halo (i.e., $\Theta = \beta$, $n = 1$), then

$$\Delta_i = a$$

$$(4)$$

This indicates that displacements which produce speckle pairs separated by less than one speckle diameter will not generate Young's fringes within the diffraction halo.

Now, a speckle pair separation Δ_i is related to the true displacement on the object Δ by

$$\Delta_i = M \Delta \quad (5)$$

The minimum true displacement that can be measured is, therefore,

$$\Delta_{min} = \lambda / M \quad (6)$$

By substituting Equation (1) for λ into Equation (6), the expression for Δ_{min} becomes

$$\Delta_{min} = \lambda F (1 + 1/M) \quad (7)$$

Equation (7) shows that greatest sensitivity is obtained using a large aperture lens (small F) which is configured to produce a highly magnified image. Note, however, that values of M greater than 3 or 4 do not result in much improvement in the sensitivity. Without the aid of special techniques, values of F much smaller than unity are difficult to obtain. Thus, the ultimate sensitivity appears to be on the order of the wavelength λ . This is comparable to interferometric techniques and also the speckle correlation methods referenced in Section I. One notable advantage to the speckle diffraction fringe method is that it can be desensitized for measuring larger displacements. This is simply achieved by using a smaller aperture with the imaging lens. Holographic and speckle correlation methods are much more difficult to desensitize.

The desensitization of the speckle diffraction fringe method provides a bonus not shared with other interferometric techniques. It is possible to make measurements without a stable platform. Both holographic and speckle correlation fringe techniques require a platform which is vibration free to less than the wavelength of light when continuous wave laser illumination is employed. Our experience has shown that vibration amplitudes up to about 20% of the smallest displacement measured can be tolerated using the speckle diffraction method if the system is appropriately desensitized. It should be noted here that small values of M less than unity will also increase the minimum measurable displacement Δ_{min} . This does not really desensitize the technique, however, since the speckle size is nearly unchanging with M if its value is less than unity (compare Equations 1 and 7). A true desensitization is achieved only if the speckle diameter is made large with respect to any extraneous vibration displacements.

One very valuable characteristic of the holographic interferometry and the speckle correlation fringe method is that they provide the observer with a "whole field" view of the displacement distribution. In general, these are contour maps of one component of the displacement distribution. Figure 7 is a photograph taken through a holographic interferogram. The fringes form a contour map of the normal displacement component in a plate subject to a point load. Figure 8 shows speckle correlation fringes on a compact tension specimen. These fringes can be used to measure the horizontal displacement components due to a wedge load.

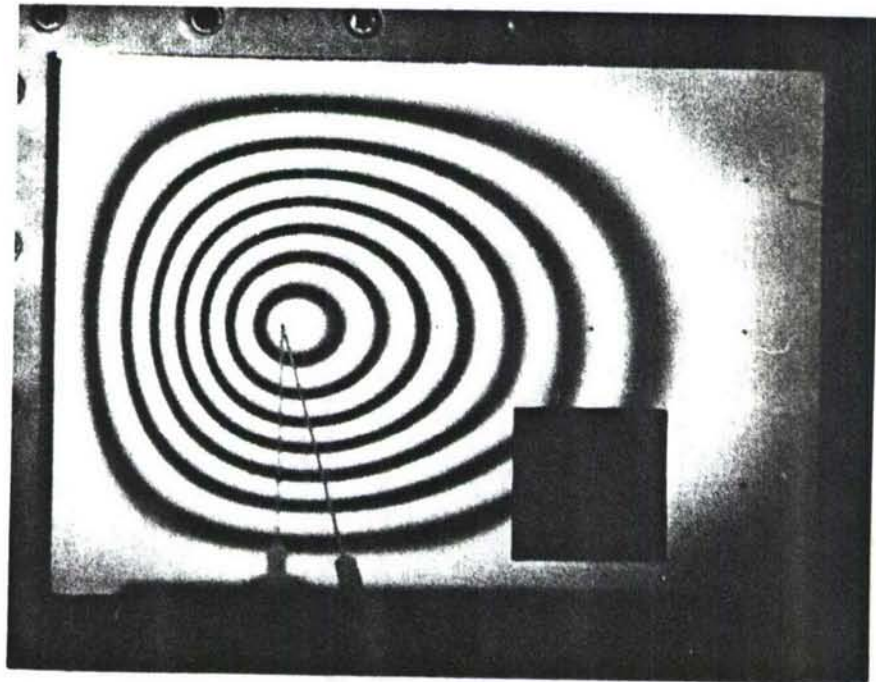


Figure 7 Holographic Fringes on a Rectangular Plate Subjected to a Point Load. The Plate has a Square "cut out" in the Lower Right Quadrant

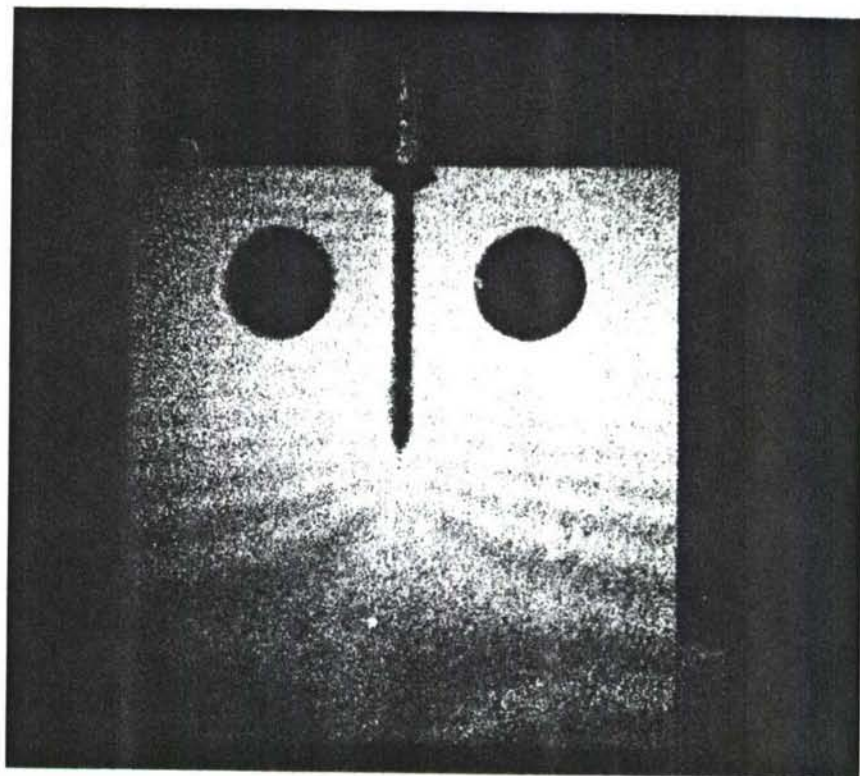


Figure 8 Speckle Correlation Fringes on a Compact
Tension Specimen

The speckle diffraction technique, as described thus far, provides only a point measurement of the displacement. A distribution is usually obtained by scanning the laser beam over the specklegram image. However, it is possible to generate "whole field" fringe patterns using specklegrams. If the entire specklegram image is illuminated with collimated light, an observer will see diffraction fringes superimposed on the image. Figure 9 is a photograph made using this procedure. These fringes are not contours in the usual sense, which makes interpretation somewhat more difficult. A simple rectilinear coordinate system cannot be used. The displacement component measured has a direction parallel to a line segment ($\vec{V}$) between a point on the fringe and a point on the observation ray. The observation ray is defined by the ray of collimated light passing through the observation point (see Figure 10). The natural coordinates are, therefore, associated with a two dimensional polar system centered on the observation ray. As is true with other interferometric methods, displacement magnitude is proportional to the fringe number.*

Quantitative data acquisition using this "read out" technique is accomplished only with difficulty as compared to the point by point method illustrated in Figure 5. However, qualitative and semi-quantitative inspection of a displacement distribution is easily achieved.

*Archbold and his colleagues (References 2 and 4) have demonstrated several other methods for observing the whole field displacement fringe pattern. These techniques are slightly more complicated than the one discussed here. We have not been able to achieve adequate spatial resolution which would allow us to obtain data on a small object when using other than the point read out technique illustrated in Figure 5.

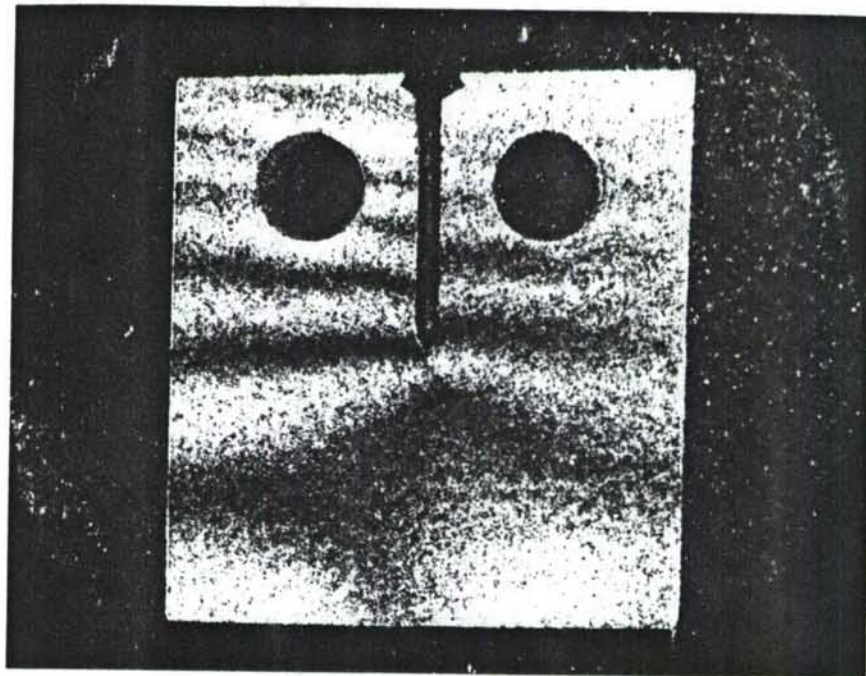


Figure 9 Whole Field Speckle Diffraction Fringes
on a Compact Tension Specimen

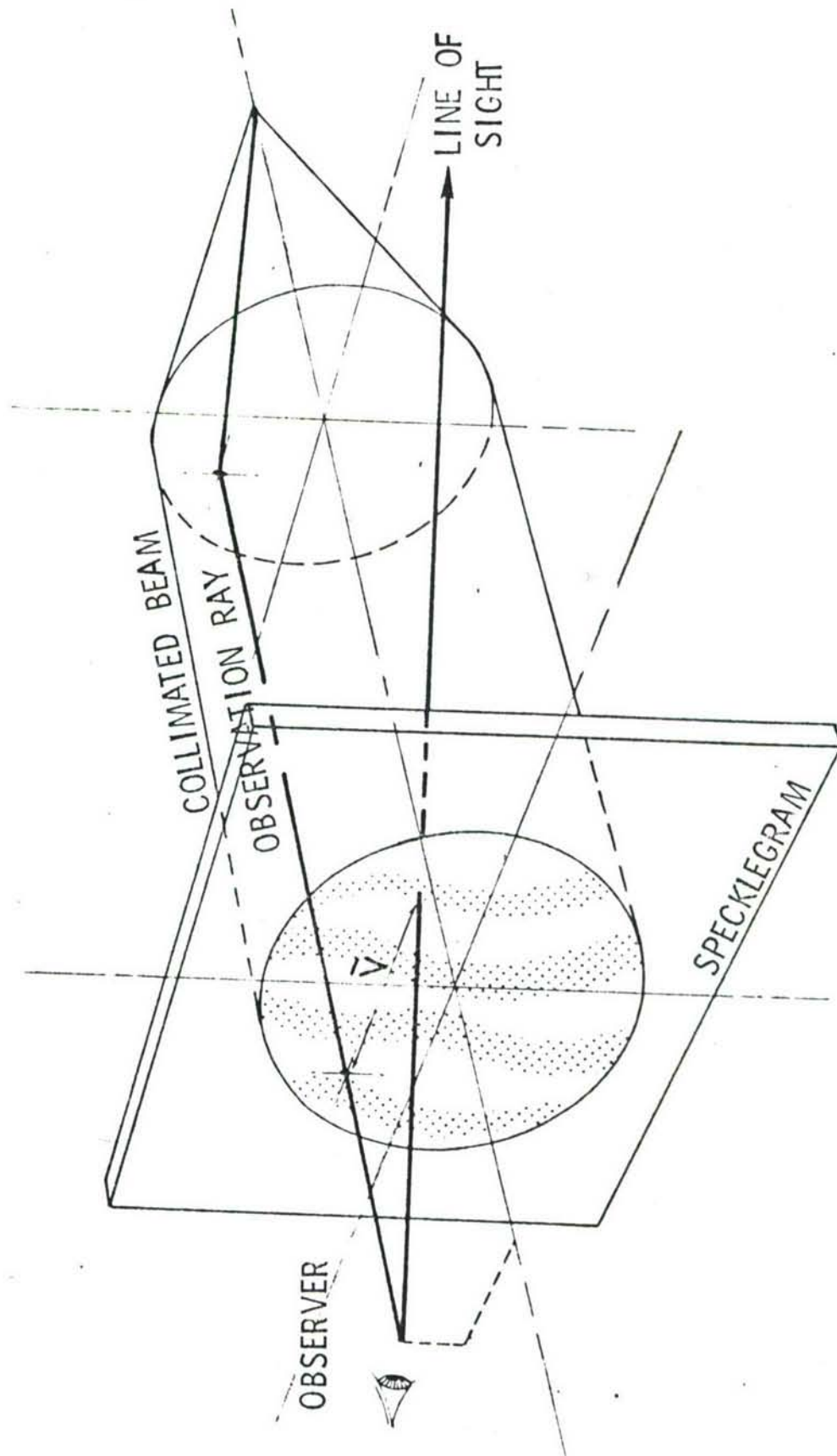


FIGURE 10 VIEWING "WHOLE FIELD" FRINGES THROUGH A SPECKLEGRAM

SECTION V

A TYPICAL APPLICATION

An interesting and mission relevant problem for the Air Force Flight Dynamics Laboratory concerns the fatigue resistance of structural joints utilizing IFF (Interference Fit Fasteners). These IFF's are employed extensively in advanced aircraft where structural fatigue is critical to the life expectancy of the system. One class of IFF is essentially an oversized tapered bolt which is force-fit into a tapered hole. A one-quarter inch per foot taper is standard for most applications. In a typical installation, a $\frac{1}{4}$ inch hole diameter is increased by .003 inches when the bolt is forced into place. This usually causes "work hardening" of the material in the immediate vicinity of the bolts and also induces a residual stress pattern. Under certain conditions, both of these results can improve fatigue performance of a structural joint (Reference 6). However, recent experience with IFF installations in new aircraft systems has indicated that the expected improvement in fatigue performance is not always obtained. Although many fatigue failures can be directly traced to faulty installation procedure, or more precisely, inadequate quality control standards, the ordeal has brought to light the fact that the physical mechanisms involved with the employment of IFF's to improve fatigue performance are not fully understood. As a result, the Air Force Flight Dynamics Laboratory has introduced research on a broad front to learn more about this subject. One facet of this

research program involves measuring the strain pattern induced by the installation of an IFF. The "speckle diffraction" technique is a natural candidate for performing this task.

Figure 11 is a photograph of a $\frac{1}{4}$ inch steel "taper lok" IFF and an aluminum plate containing a tapered hole. In a normal installation, the bolt head is drawn down until it just makes contact with the plate. Since this head would normally shadow the material very near the bolt-plate interface, the head was removed from the IFF (Figure 12). A specklegram was made with exposures taken before and after the IFF installation. A photo-print of the specklegram transparency is shown in Figure 13. The shadow on one side of the bolt is due to the illuminating beam being incident at an angle away from the normal.

Using the techniques described in Section II, displacement measurements were obtained along a horizontal line passing through the bolt center. The results are plotted in Figure 14. Similar measurements were obtained along a vertical line also passing through the bolt center (Figure 15). The data were fit to a curve of the form

$$\Delta = k / r \quad (8)$$

where k is a "fit" constant and r is the radial distance from the bolt center. Equation (8) is the two dimensional elastic displacement in the vicinity of a circular hole which is loaded radially on its periphery in a uniform manner. The "fit" constant k was determined by the "least-squares" method to be .00012 square inches for both sets of data.

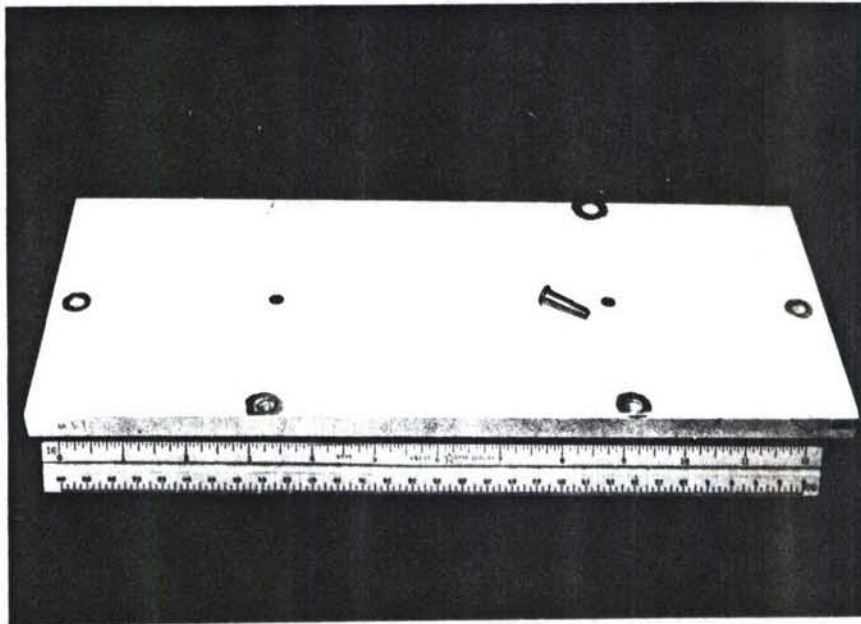


Figure 11 Tapered Bolt Fastener and Aluminum
Plate with a Tapered Hole

P27



Figure 12 Tapered Bolt Interference Fit Fastener.
The Lower Bolt has the Head Removed.

(28)

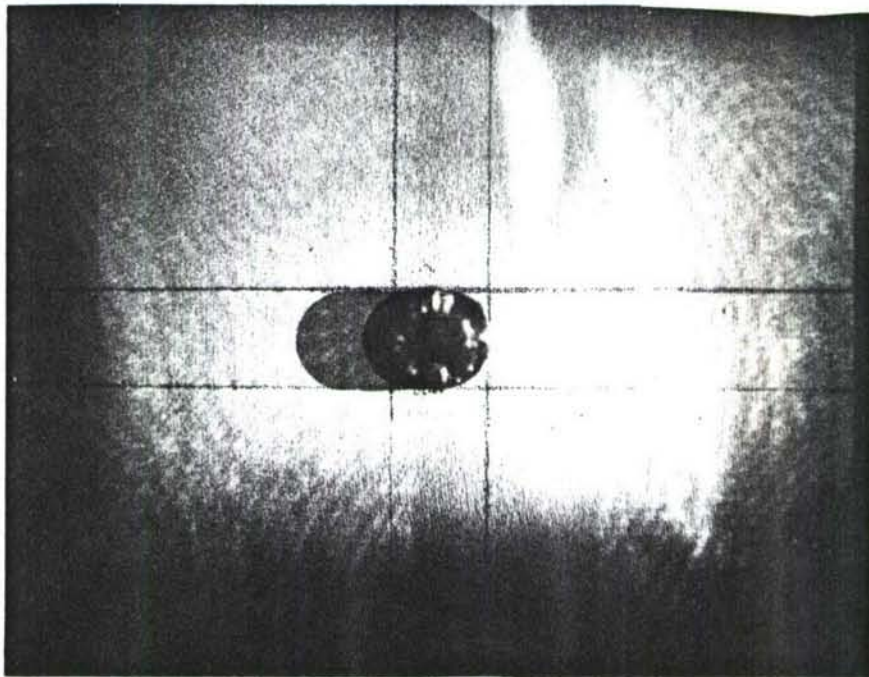


Figure 13 Photo Print of Specklegram

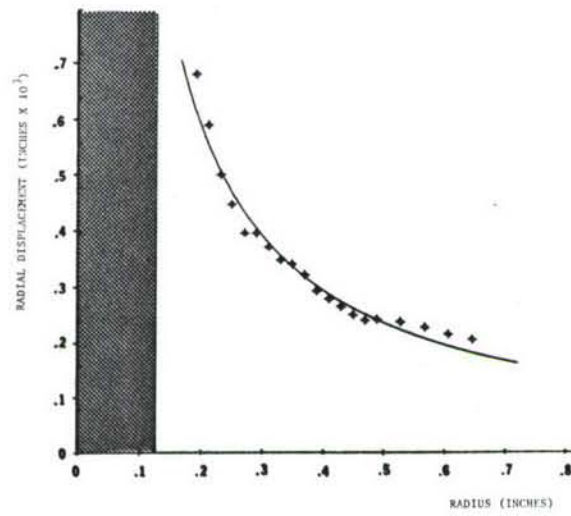


Figure 14 Displacement vs Radial Distance Along a Horizontal Line

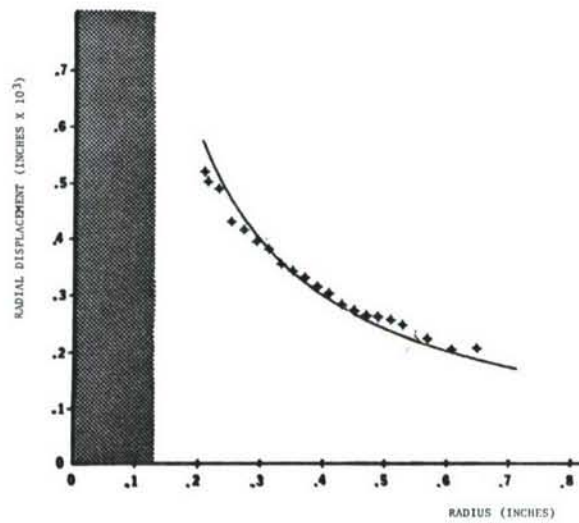


Figure 15 Displacement vs Radial Distance Along a Vertical Line

The data plotted in Figures 14 and 15 are for radii greater than .2 inches. No Young's fringes were found when the specklegram was interrogated at lesser radii. The probable explanation for this is that insertion of a tapered bolt causes plastic deformation of the joint material near the edge of the fastener hole. Since plastic deformation is accompanied by rearranging of grain structure, the surface characteristics, and therefore the speckle pattern, are changed substantially. As a result, one should not expect to find speckle "pairs" on a double exposure specklegram in a region that has been plastically deformed.

The above explanation is also consistent with another feature in the data. A value of $k = .00012$ square inches implies a diametral interference of about .002 inches assuming there is no plastic deformation. The actual fastener was installed to an interference of between .003 and .004 inches. These numbers are based on measurements made of the amount of protrusion found before and after the fastener was installed (the amount of taper is .25 inches per foot). These measurements strongly suggest that plastic deformation did occur near the fastener hole edge. Analytical work done by Allen and Ellis (Reference 7) and others also predicts a plastic zone in the near vicinity of properly installed interference fit fasteners.

Although the same value of the "fit" constant k was found in the horizontal and vertical data scans, some asymmetry in the displacement distribution can be detected. In Figure 14, the data appear to fall off slightly faster than k/r while in Figure 15 the opposite is true. This suggests that the hole is not uniformly loaded.

Since the Taper Lok is a precision fastener, manufactured to very close tolerances, the probable cause of non-uniform loading is an out-of-round hole. This explanation is consistent with other observations we have made while experimenting with interference fit fasteners. In particular, assymetric fringe contours have been often observed using "real time" holographic interferometry during the installation of Taper Loks. This has been attributed to out-of-round holes (unpublished work done by the authors at AFFDL),

Before concluding, a few comments about using this technique for measuring displacements in a plastically deformed region are applicable. The disappearance of Young's fringes, as one scans from an elastically strained to a plastically deformed region recorded on a specklegram, is not abrupt. This suggests that very small plastic strains do not completely decorrelate the speckle pattern. In other words, gross changes in the grain structure are not incurred until plastic strains reach some minimum value. As a result, we have been able to use speckle diffraction interferometry within a plastic zone by applying the strain inducing load in small increments. A double exposure specklegram was made for each load increment and the resulting displacement distribution measured. The final displacement distribution can be obtained by superposition (unpublished work done by the authors at AFFDL).

The typical application discussed above demonstrates that speckle diffraction interferometry has a wide potential for measuring displacement and strain distributions in loaded structural components.

The data presented in Figures 14 and 15 would normally require the installation of more than forty conventional strain gages. To obtain data equivalent to that recorded on the one specklegram shown in Figure 13 would require at least fifty thousand strain gages. In addition, speckle interferometry is rather easy to use and does not require calibration. Other features include: high ultimate sensitivity, variable sensitivity, and the fact that no physical contact is required with the loaded specimen. This last point suggests that speckle technology may be applicable to non-destructive inspection of "strain critical" components.

APPENDIX A

LENS RESOLUTION AND SPECKLE SIZE

Two points on an object can be resolved if the two corresponding areas into which these points are mapped in the image plane are, themselves, resolved. The traditional physical optics derivation involves the Fraunhofer approximation of Kirchhoff's diffraction integral and a calculation of the "Airy disk" diameter (Reference 8). However, practically the same results can be obtained using a few simple physical arguments.

The essence of diffraction phenomena involves the fact that information about spatial distribution is encoded in the diffraction pattern. For example, if only the zero-order wave from a simple transmission grating is intercepted, an observer cannot ascertain information about the grating except for a relative intensity of the incident illumination. If he gains knowledge about positions of the first order maxima, he can now relate the slit spacing. The position and intensity of higher order maxima provide the observer with information about details concerning the shape of individual slits. A lens, therefore, can resolve two points on an object only if the aperture is wide enough to intercept at least the zero order wave and one of the first order maxima created by diffraction.

Let Θ be the angular separation of the zero and first order maxima, and d be the minimum resolvable spacing of two points on the object. Then Θ and d are related by

$$\lambda = d \sin \Theta \quad (A-1)$$

If O is the object to lens distance, D the lens aperture diameter and Θ is small, the ratio $D/O \approx \sin \Theta$ so that Equation (A-1) can be written

$$\lambda = d D / O \quad (A-2)$$

In the image plane, the spacing should be

$$d_i = M d \quad (A-3)$$

where M is a magnification for an image plane located at a distance I behind the lens. From simple geometrical optics come equations

$$M = I / O \quad (A-4)$$

and

$$\frac{1}{f} = \frac{1}{I} + \frac{1}{O}$$

(A-5)

where f is the focal length of the lens.

Equation (A-3) can be rewritten in terms of Equations (A-2), (A-4) and (A-5) and become

$$d_s \approx \lambda (M+1) f/D$$

(A-6)

Note that f/D is just the numerical value of the lens aperture for an object located at infinity and can be denoted by F (i.e., for an $f/4.5$ aperture, $F = 4.5$). Previous arguments indicated that d_s is a reasonable estimate of the speckle diameter which will now be denoted by α . Thus, Equation (A-1) can be written as

$$\alpha \approx \lambda F (M+1)$$

(A-7)

A formal derivation of Equation (A-6) yields a multiplicative constant of 1.2. In view of the approximate nature of other assumptions, the difference between 1.2 and unity is not significant.

APPENDIX B
YOUNG'S FRINGES

"Scalar Wave Theory" provides that the far field intensity distribution of diffracted light is closely related to the Fourier transformation integral (Reference 8). In particular, the intensity I is proportional to a scalar product $U U^*$ where

$$U \approx \int_w dy dx \frac{1}{S} e^{i k s} \quad (B-1)$$

Equation (B-1) represents a summation over an aperture W contained by the x-y plane at $z = 0$. The wave number $k = 2\pi/\lambda$ and S is the distance from a point in the aperture to the point of observation.

Now consider an aperture consisting of two small pinholes located at $x = \pm \Delta/2, y = 0$ where Δ is the pinhole spacing. The pinholes can be mathematically represented by two dimensional Dirac delta functions. A first order approximation for the distance variation is $S = S_0 + x \sin \theta$ where S_0 is the distance from the origin to the observation point and θ is the angle this line segment projects in the x-z plane as measured from the z axis. This approximation is valid when the distance from the observation point to the z axis is small compared to S_0 . With this aperture, Equation (B-1) can be written

$$U \approx \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dx \frac{1}{S_0 + x \sin \theta} e^{ik(S_0 + x \sin \theta)} \left[\int_{(x - \frac{\Delta}{2}, y)} + \int_{(x + \frac{\Delta}{2}, y)} \right]$$

(B-2)

If Δ is very small compared to S_0 , the quantity $x \sin \theta$ can be neglected in the $1/(S_0 + x \sin \theta)$ factor. Thus:

$$U \approx \frac{e^{ikS_0}}{S_0} \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dx e^{ikx \sin \theta} \left[\int_{(x - \frac{\Delta}{2}, y)} + \int_{(x + \frac{\Delta}{2}, y)} \right]$$

(B-3)

or

$$U \approx 2 \frac{e^{ikS_0}}{S_0} \cos(k \frac{\Delta}{2} \sin \theta)$$

(B-4)

By employing $I \propto U U^*$, the intensity function for Young's fringes is obtained and is

$$I \propto 1 + \cos(k \Delta \sin \theta)$$

(B-5)

By using $k = 2\pi / \lambda$ and noting that minima occur for $\cos(k \Delta \sin \theta) = -1$ the conditions for fringe minima are

$$(n - \frac{1}{2}) \lambda = \Delta \sin \theta$$

$$n = 1, 2, 3 \dots$$

(B-6)

Now consider a case where the aperture contains a speckle pattern. Let $R(x, y)$ be a distribution function describing the spatial character of the speckle pattern. The diffraction integral is then

$$U_h \approx \frac{1}{s_o} \int_w dy dx e^{ik S(x, y)} R(x, y)$$

(B-7)

The function $U_h U_h^*$ as calculated from (B-7) is the diffraction halo for a single exposure speckle pattern as described in Section II.

It follows that the U function for a double exposure specklegram can be written as

$$U \approx \frac{1}{s_o} \int_w dy dx e^{ik S(x, y)} \left[R(x - \frac{\Delta}{2}, y) + R(x + \frac{\Delta}{2}, y) \right]$$

(B-8)

It is now useful to use a series expansion

$$R_{(x \pm \frac{\Delta}{2}, y)} = R_{(x, y)} \pm R'_{(x, y)} \left(\frac{\Delta}{2} \right) + \frac{R''_{(x, y)}}{2!} \left(\frac{\Delta}{2} \right)^2 \pm \dots \quad (\text{B-9})$$

where primes indicate a differential with respect to x . Also, an appropriate linear first order approximation for $S_{(x, y)}$ is

$S_0 + V(y) + x \sin \theta$ where $V(y)$ carries the functional dependence on y . Now Equation (B-8) can be written

$$U \approx \frac{e^{i k S_0}}{S_0} \int_{-\infty}^{\infty} dy e^{i k V(y)} \int_{-\infty}^{\infty} dx e^{i k x \sin \theta} \Phi_{(x, y)} \quad (\text{B-10})$$

where

$$\Phi_{(x, y)} = 2 \left[R_{(x, y)} + \frac{R''_{(x, y)}}{2!} \left(\frac{\Delta}{2} \right)^2 + \frac{R''''_{(x, y)}}{4!} \left(\frac{\Delta}{2} \right)^4 + \dots \right] \quad (\text{B-11})$$

The integration limits have been set at $\pm \infty$ since it is assumed that $R_{(x, y)} = 0$ outside the laser beam. Now, the second

integral in Equation (B-10) is essentially a Fourier transformation

with a spatial variable of x and a frequency variable of $-k \sin \theta$.

Hence, apart from a constant the Fourier transform is:

$$\mathcal{F}_{(\omega, y)} \{ \Phi_{(x, y)} \} = \int_{-\infty}^{\infty} dx e^{-i \omega x} \Phi_{(x, y)}$$

(B-12)

where $\omega = -k \sin \theta$

The linear properties of a Fourier transform provide that

$$\mathcal{F}_{(\omega)} \{ g_{(x)}^{(n)} \} = (i \omega)^n \mathcal{F}_{(\omega)} \{ g_{(x)} \}$$

(B-13)

where $g_{(x)}^{(n)}$ is the (n)th derivative of a function $g_{(x)}$. Thus,

by using Equation (B-11) and (B-13), Equation (B-12) becomes

$$\mathcal{F}_{(\omega, y)} \{ \Phi_{(x, y)} \} = 2 \mathcal{F}_{(\omega, y)} \{ R_{(x, y)} \} \left[1 - \frac{1}{2!} \left(\omega \frac{\Delta}{2} \right)^2 + \frac{1}{4!} \left(\omega \frac{\Delta}{2} \right)^4 - \dots \right]$$

(B-14)

or

$$\mathcal{F}_{(\omega, y)} \{ \Phi_{(x, y)} \} = 2 \mathcal{F}_{(\omega, y)} \{ R_{(x, y)} \} \left[\cos \left(k \frac{\Delta}{2} \sin \theta \right) \right]$$

(B-15)

If Equation (B-15) is substituted for the second integral in Equation (B-10), it should be recognized that apart from the factor $2 \cos(k \frac{\Delta}{2} \sin \theta)$, Equation (B-10) and Equation (B-7) are the same. That is

$$U = U_h 2 \cos(k \frac{\Delta}{2} \sin \theta)$$

(B-16)

and the intensity is

$$I \propto U_h U_h^* [1 + \cos(k \Delta \sin \theta)]$$

(B-17)

Thus, the diffraction pattern from a double exposure specklegram is a diffraction halo modulated with Young's fringes which are intimately related to the displacement Δ by Equation (B-6).

REFERENCES

1. J.A. Leendertz, J. Phys. E. (J. Scient. Instrum.), 3, 214, (1970).
2. E. Archbold, J. M. Burch, and A. E. Ennos, Optics Acta, 17, Nr 12, 883, (1970).
3. F. D. Adams. "A Study of the Parameters Associated with Employing Laser Speckle Correlation Fringes to Measure In-Plane Strain", AFFDL-TR-72-20, (Mar 1972).
4. E. Archbold, and A. E. Ennos, Optics Acta, 19, Nr 4, 253, (1972).
5. F. D. Adams, and R. R. Corwin, "A Technique for Measuring In-Plane Displacements by Holographic Interferometry", AFFDL-TR-72-5, (Feb 1972).
6. C. R. Smith, "Tapered Bolts - Their Influence on Fatigue of Airplane Structures", Convair Rpt Nr ZR 659-053, (May 1960).
7. M. Allen, and J. A. Ellis "Stress and Strain Distribution in the Vicinity of Interference Fit Fasteners", AFFDL-TR-72-153, (Jan 1973).
8. M. Born, and E. Wolf, Principles of Optics, London; Pergamon Press LTD, (1959).

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(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Air Force Flight Dynamics Laboratory Solid Mechanics Branch (FBR) Wright-Patterson AFB, Ohio 45433		2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	
		2b. GROUP	
3. REPORT TITLE (U) On Speckle Diffraction Interferometry For Measuring Whole Field Displacements And Strains			
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Final Report, June 1972 to June 1973			
5. AUTHOR(S) (Last name, first name, initial) ADAMS, FRANK D. (DR) MADDUX, GENE E. (MR)			
6. REPORT DATE December 1973	7a. TOTAL NO. OF PAGES 46	7b. NO. OF REFS 8	
8a. CONTRACT OR GRANT NO.		9a. ORIGINATOR'S REPORT NUMBER(S) AFFDL-TR-73-123	
b. PROJECT NO. 1467			
c. Task 146702		9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	
d.			
10. AVAILABILITY/LIMITATION NOTICES Approved for public release, distribution unlimited			
11. SUPPLEMENTARY NOTES		12. SPONSORING MILITARY ACTIVITY AFFDL (FBR) WPAFB, OHIO 45433	
13. ABSTRACT The procedure for using laser speckle diffraction interferometry to measure small displacements and strains is described. The variation of basic experimental parameters is investigated. A typical application of the method is discussed and data is presented for the displacement distribution around an interference fit fastener.			

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Laser Speckle In-Plane Strain Measurements Speckle Interferometry Speckle Speckle Photography Holography Holographic Interferometry						

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