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AN INVESTIGATION OF MINIMUM TIME, LAUNCH-TO-RENDEZVOUS TRAJECTORIES

Steven H. Edelman

Air Force Institute of Technology
Wright-Patterson Air Force Base, Ohio

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13. ABSTRACT

Minimum time trajectories between the surface of the Earth and a 270 statute mile circular orbit, generated with the assumption of impulsive thrusting, are used to determine minimum time-to-rendezvous with a target in the specified orbit. The amount of impulse is fixed by specifying the ratio of the launch vehicle's final to initial masses. The study is divided into two principal phases. In the orbit injection portion of the study, a set of minimum time orbit injection trajectories, subject to certain launch constraints, is generated by the solution of a fourth order polynomial for various range angles. The effect on the number of admissible trajectories of changing the mass ratio is also investigated. During the rendezvous phase of the study, the generated set of trajectories is used to determine a window for direct launch to rendezvous. The trajectory resulting in minimum rendezvous time is then chosen as a function of the initial target position relative to the launch window. The dynamics of the launch window as the mass ratio changes are also investigated. Results indicate that for the baseline mass ratio of .05, the launch window is only 4.7% of the target's period; when the target is within the window, rendezvous will take from 96 to 413 seconds depending on initial target position. The study of various mass ratios indicates that a sizable tradeoff is available to the mission planner in terms of mass placed in orbit, time to rendezvous and rendezvous opportunities.

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AN INVESTIGATION OF MINIMUM TIME, LAUNCH-TO-RENDEZVOUS TRAJECTORIES

THESIS

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of the Air Force Institute of Technology

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by

Steven H. Edelman, B. S.

Captain USAF

Graduate Astronautics

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Preface

The subject of this thesis was prompted by a desire to learn more about trajectory optimization and thoughts about possible rescue missions to future manned space stations. My goal was to contribute something to the area of space rescue and other similar missions where time equates with lives or national security. I find it a bit ironic that while I was in the midst of this study, NASA was seriously considering a rescue mission to the Skylab workshop.

I must thank my advisor, Major Gerald M. Anderson, for helping me settle on a way of working towards the goal I set and for making that journey much smoother than it might have been. And I could not have completed this study without the constant support and understanding of my wife, Rochelle, who continually reminded me "It's later than you think!"

Steven H. Edelman

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Abstract

Minimum time trajectories between the surface of the Earth and a 270 statute mile circular orbit, generated with the assumption of impulsive thrusting, are used to determine minimum time-to-rendezvous with a target in the specified orbit. The amount of impulse is fixed by specifying the ratio of the launch vehicle's final to initial masses. The study is divided into two principal phases. In the orbit injection portion of the study, a set of minimum time orbit injection trajectories, subject to certain launch constraints, is generated by the solution of a fourth order polynomial for various range angles. The effect on the number of admissible trajectories of changing the mass ratio is also investigated. During the rendezvous phase of the study, the generated set of trajectories is used to determine a window for direct launch to rendezvous. The trajectory resulting in minimum rendezvous time is then chosen as a function of the initial target position relative to the launch window. The dynamics of the launch window as the mass ratio changes are also investigated. Results indicate that for the baseline mass ratio of .05, the launch window is only 4.7% of the target's period; when the target is within the window, rendezvous will take from 96 to 413 seconds depending on initial target position. The study of various mass ratios indicates that a sizable tradeoff is available to the mission planner in terms of mass placed in orbit, time to rendezvous and rendezvous opportunities.

AN INVESTIGATION OF MINIMUM TIME, LAUNCH-TO-RENDEZVOUS TRAJECTORIES

I. Introduction

Problem Statement

Emergency resupply of an orbiting space station; immediate inspection of an unidentified, and perhaps threatening, satellite; return of a planetary exploration vehicle to an orbiting command module after loss of life support equipment--these are all missions requiring trajectories with a minimum time span between launch and rendezvous. In general, the determination of such time-optimal trajectories, flown by vehicles with limited thrust capabilities, requires extensive and often difficult numerical procedures. Handelsman (Ref 9) has shown that a very good estimate of these trajectories can be generated with the use of the impulsive approximation, in which finite-thrust, finite-duration powered phases of flights are replaced by instantaneous changes of velocity (Ref 6:827). Under the proper conditions, the zero thrust trajectory arcs between impulses can then be described by orbital equations which are algebraic in nature. It is, therefore, the purpose of this study to investigate launch vehicle trajectories for minimum time between launch from a planet's surface and rendezvous with an orbiting target, using solutions generated with the impulsive approximation, in order to gain a better understanding of their characteristics, limitations, and associated systems (e.g., launch vehicle) requirements.

Background

There are three principal categories into which all planet-to-orbit and orbit-to-orbit problems fall. These are orbit injection, in which the transferring vehicle must achieve a predetermined radius and velocity vector relationship; interception, in which the vehicle must reach a specified point at the proper time; and rendezvous, in which the vehicle must attain the proper radius and velocity vectors at the proper time. To date, the vast majority of the work in all of these categories, using either finite thrust or the impulsive approximation, has dealt with the orbit-to-orbit, minimum time/fuel problem; this is evidenced by Gobetz and Doll's survey (Ref 6), NASA and DDC literature searches, and several theses accomplished at the Air Force Institute of Technology (Ref 1, 3-5, 8, 11, 12). The planet-to-orbit injection problem with a constant thrust rocket, under various conditions, is illustrated in Bryson and Ho (Ref 2:83-86). Direct launch from a planet's surface to rendezvous with an orbiting spacecraft, which usually encompasses a variety of logistics and operational considerations, is not generally thought of in terms of time-optimal trajectories, since preparation for the launch generally requires considerably more time than the flight itself; the preparations for a possible rescue mission to Skylab illustrate this. Consequently, relatively little work has been done specifically in this area. However, one can assume, particularly in the inspection and emergency return missions described earlier, that the vehicle is ready for launch and needs only target information to determine its proper trajectory.

One investigation of planet-to-orbit rendezvous trajectories was done by Wolaver (Ref 15); in it, he graphically determined admissible trajectories for various objectives and constraints. The analytical methods used in this study to generate orbit injection trajectories are extensions of work done by Tubbs (Ref 13), who investigated transfers between general coplanar orbits.

Study Outline

This study effort is divided into two major areas. First, a set of minimum time orbit injection trajectories are generated. Then, relationships are developed which determine the proper launch time and trajectory for any given initial target position (the position of the target when the decision to rendezvous is made). The study results are presented as follows.

Chapter II contains the development of the general, minimum time, finite-thrust orbit injection problem, using the calculus of variations. No attempt to solve this problem is made. Rather, Chapter III describes the substitution of impulses for the powered portions of the flight. The methods used for generating injection trajectories are described in Chapter IV, along with an interpretation of some early data, a graphical presentation of the desired trajectories and an analysis of treating parametrically the amount of impulse available. Chapter V poses the general rendezvous problem, as seen for this study, then describes the methods used for its solution and their results. The conclusions and recommendations resulting from this study are presented in Chapter VI.

II. The General Minimum Time, Orbit Injection Problem

In this chapter, the general minimum time, orbit injection problem is developed using the calculus of variations and the Pontryagin Minimum Principle (PMP). This particular method of development closely follows that used by Tubbs (Ref 13:5-11); other approaches can be found in Lawden (Ref 10) and Bryner (Ref 1).

Assumptions

1. All analyses are done in two dimensions, i.e., the initial launch vehicle (hereafter referred to as the vehicle) velocity, the orbit injection trajectory, and the target orbit are all coplanar.

2. The vehicle is treated throughout as a point of variable mass. The Earth as the central attracting body in the system is also considered a point mass. However, at launch, the vehicle is assumed to be located at a point on the equator of a spherical Earth of radius R_E, thus having a linear velocity of $R_E \omega_E$, where ω_E is the angular speed of Earth's rotation.

3. The only forces acting on the vehicle are the thrust of its motor and the force due to a time-invariant, inverse square gravitational field. Atmospheric drag and other aerodynamic forces are neglected.

4. The rocket motor operates only at maximum thrust or zero thrust; intermediate thrust levels are not considered.

5. The target is in a circular orbit at an altitude of 270 statute miles, which is comparable to the orbit of the Skylab space station. Since this information completely describes the motion of the target, the analyses preceding rendezvous can ignore the target.

6. The amount of fuel available for orbit injection and rendezvous, i.e., the initial and final vehicle masses, is known a priori.

Problem Set-up

First, a suitable coordinate system in which to represent the vehicle dynamics must be chosen. The Earth-centered coordinate frame and the variables used to describe the motion of the vehicle in this development are shown in Fig. 1.

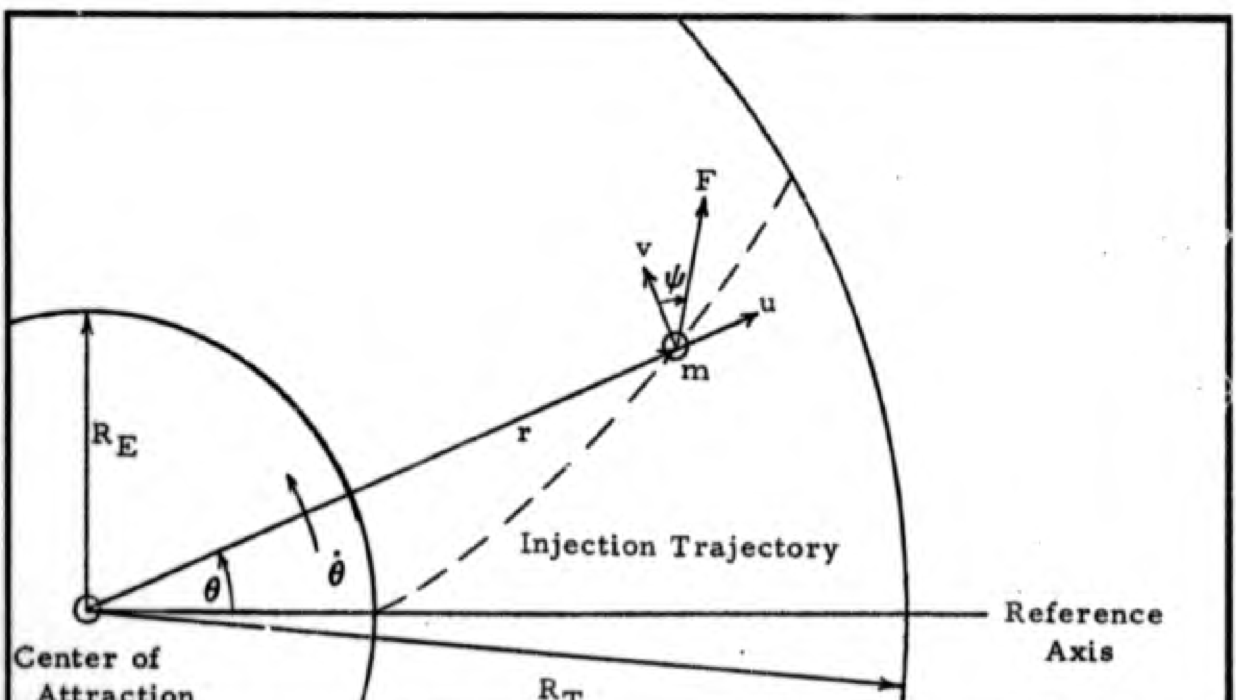

Fig. 1. Problem Variables and Coordinate System

The state vector, comprised of the variables describing the vehicle's motion, and the control vector, comprised of the control variables, are defined by

$$\overline{X} = \begin{bmatrix} u \\ v \\ r \\ \theta \\ m \end{bmatrix} \quad (1) \qquad \overline{U} = \begin{bmatrix} F \\ \psi \end{bmatrix} \quad (2)$$

where u is the radial velocity of the vehicle, v is the tangential velocity, r is the radial position, θ is the angular position, m is the vehicle's mass, F is the thrust magnitude and ψ is the thrust direction. The vehicle dynamics are described by the following state differential equations and known boundary conditions:

$$\dot{\overline{X}} = \overline{G}\,(\overline{X}, \overline{U}) \qquad (3)$$

or

$$\dot{u} = -\frac{\mu}{r^2} + \frac{v^2}{r} + \frac{F}{m}\sin\psi \qquad u(t = 0) = 0 \quad u(t = t_f) = 0 \qquad (4)$$

$$\dot{v} = -\frac{uv}{r} + \frac{F}{m}\cos\psi \qquad v(0) = R_E\,\omega_E \quad v(t_f) = \sqrt{\frac{\mu}{R_T}} \qquad (5)$$

$$\dot{r} = u \qquad r(0) = R_E \quad r(t_f) = R_T \qquad (6)$$

$$\dot{\theta} = \frac{v}{r} \qquad \theta(0) = 0 \quad \theta(t_f) = \theta_f \qquad (7)$$

$$\dot{m} = -\frac{F}{C} \qquad m(0) = 1 \quad m(t_f) = m_f \qquad (8)$$

where μ = gravitational constant of Earth

C = constant motor exhaust velocity

R_T = final, or target, orbit radius

The range angle, $\theta(t_f)$, may be free for a global minimum time solution or fixed for a particular transfer. In this study, $\theta(t_f)$ is initially specified and the corresponding minimum time trajectory is determined; θ_f is then varied to examine all possible range angles.

The objective of the problem is to determine F(t) and $\psi(t)$, subject to the constraint $0 \leq F \leq F_{max}$, so as to minimize the time from launch (t=0) to injection and rendezvous ($t = t_f$). The problem cost is therefore

$$J = t_f \tag{9}$$

The next step in applying the calculus of variations is to introduce the costate vector $\bar{\lambda}$ and the Hamiltonian H:

$$\bar{\lambda} = \begin{bmatrix} \lambda_u \\ \lambda_v \\ \lambda_r \\ \lambda_\theta \\ \lambda_m \end{bmatrix} \tag{10}$$

$$\begin{aligned} H &= \bar{\lambda}^T \bar{G} + L \\ &= \lambda_u \left(\frac{-\mu}{r^2} + \frac{v^2}{r} + \frac{F\sin\psi}{m} \right) + \lambda_v \left(\frac{-uv}{r} + \frac{F\cos\psi}{m} \right) \\ &\quad + \lambda_r(u) + \lambda_\theta \left(\frac{v}{r} \right) + \lambda_m \left(-\frac{F}{C} \right) \end{aligned} \tag{11}$$

The costate differential equations are then formed:

$$\dot{\bar{\lambda}} = -\frac{\partial H}{\partial \bar{X}} \tag{12}$$

or

$$\dot{\lambda}_u = \lambda_v \left(\frac{v}{r}\right) - \lambda_r \tag{13}$$

$$\dot{\lambda}_v = -2\lambda_u \left(\frac{v}{r}\right) + \lambda_v \left(\frac{u}{r}\right) + \lambda_\theta \left(\frac{1}{r}\right) \tag{14}$$

$$\dot{\lambda}_r = -\lambda_u \left(\frac{\mu}{r^3} - \frac{v^2}{r^2}\right) - \lambda_v \left(\frac{uv}{r^2}\right) + \lambda_\theta \left(\frac{v}{r^2}\right) \tag{15}$$

$$\dot{\lambda}_\theta = 0 \tag{16}$$

$$\dot{\lambda}_m = \frac{F}{m^2} \left(\lambda_u \sin\psi + \lambda_v \cos\psi\right) \tag{17}$$

If θ_f is free, then $\lambda_\theta(t_f) = 0$; otherwise, the transversality conditions provide no useful information about $\bar{\lambda}(t_f)$.

The transversality condition for free final time applied to the Hamiltonian gives

$$H(t_f) = -1 \tag{18}$$

Since the Hamiltonian is not an explicit fuction of time, H is constant for all time so that

$$H(t) = -1 \tag{19}$$

The necessary conditions for the optimal thrust direction, ψ^*, are $\frac{\partial H}{\partial \psi} = 0$ and $\frac{\partial^2 H}{\partial \psi^2} \geq 0$. Applying these to Eq. (11),

$$\frac{\partial H}{\partial \psi} = \frac{F}{m} (\lambda_u \cos \psi - \lambda_v \sin \psi) = 0 \tag{20}$$

$$\frac{\partial^2 H}{\partial \psi^2} = \frac{F}{m} (-\lambda_u \sin \psi - \lambda_v \cos \psi) \geq 0 \tag{21}$$

To satisfy both Eq. (20) and Eq. (21), ψ^* must be determined by

$$\sin \psi^* = \frac{-\lambda_u}{\sqrt{\lambda_u^2 + \lambda_v^2}} \tag{22}$$

$$\cos \psi^* = \frac{-\lambda_v}{\sqrt{\lambda_u^2 + \lambda_v^2}} \tag{23}$$

assuming $F \neq 0$; ψ is undefined when $F = 0$.

Since F appears linearly in H, the PMP must be used to determine the optimal thrust magnitude, F^*. From Eq. (11), the switching function, K, is defined as

$$K = \frac{\lambda_u \sin \psi + \lambda_v \cos \psi}{m} - \frac{\lambda_m}{C} \tag{24}$$

Then H is minimized if

$$F^* = 0 \text{ when } K > 0 \tag{25}$$

$$F^* = F_{max} \text{ when } K < 0 \tag{26}$$

If $K = 0$ for a finite period of time, F^* must be some intermediate thrust level; however, assumption 4 does not allow such a situation. Therefore K = 0 only at a finite number of points, called switching points.

This concludes the set-up of the general minimum time problem. The solution of this problem requires complicated numerical techniques and will not be attempted in this study. Rather, an approximation to the solution will be found, using the simplifying assumptions discussed in the next chapter.

III. The Impulsive Approximation

The difficulties associated with the solution of the problem described in Chapter II may be circumvented and the solution made analytical by the use of impulsive approximations to the thrust arcs. A typical thrusting sequence for the orbit injection problem is $\left\{F_{max},\ 0,\ F_{max}\right\}$. The two-impulse approximation to this sequence, illustrated in Fig. 2, is used for this study. The zero thrust trajectory between impulses is a portion of an orbit, since the vehicle is then moving through an inverse-square gravitational field with no other external forces acting on it. This orbit is defined by the radius and velocity vectors at the terminal points (launch and injection). The velocity vectors are determined, in turn, by the ΔV, or impulse, applied at each terminal and the given initial or required final velocity. Since the direction of impulse application is generally unconstrained, the amount of total ΔV or impulse available to the vehicle determines the orbits which will connect the desired terminals. This amount, often called the characteristic velocity, depends on the ratio of the initial to final mass, where m_f represents the mass placed in orbit and $(m_0 - m_f)$ the fuel, and is determined by

$$\Delta V_1 + \Delta V_2 = I = C \ln \frac{m_0}{m_f} \qquad (27)$$

The problem of Chapter II may now be restated as follows:

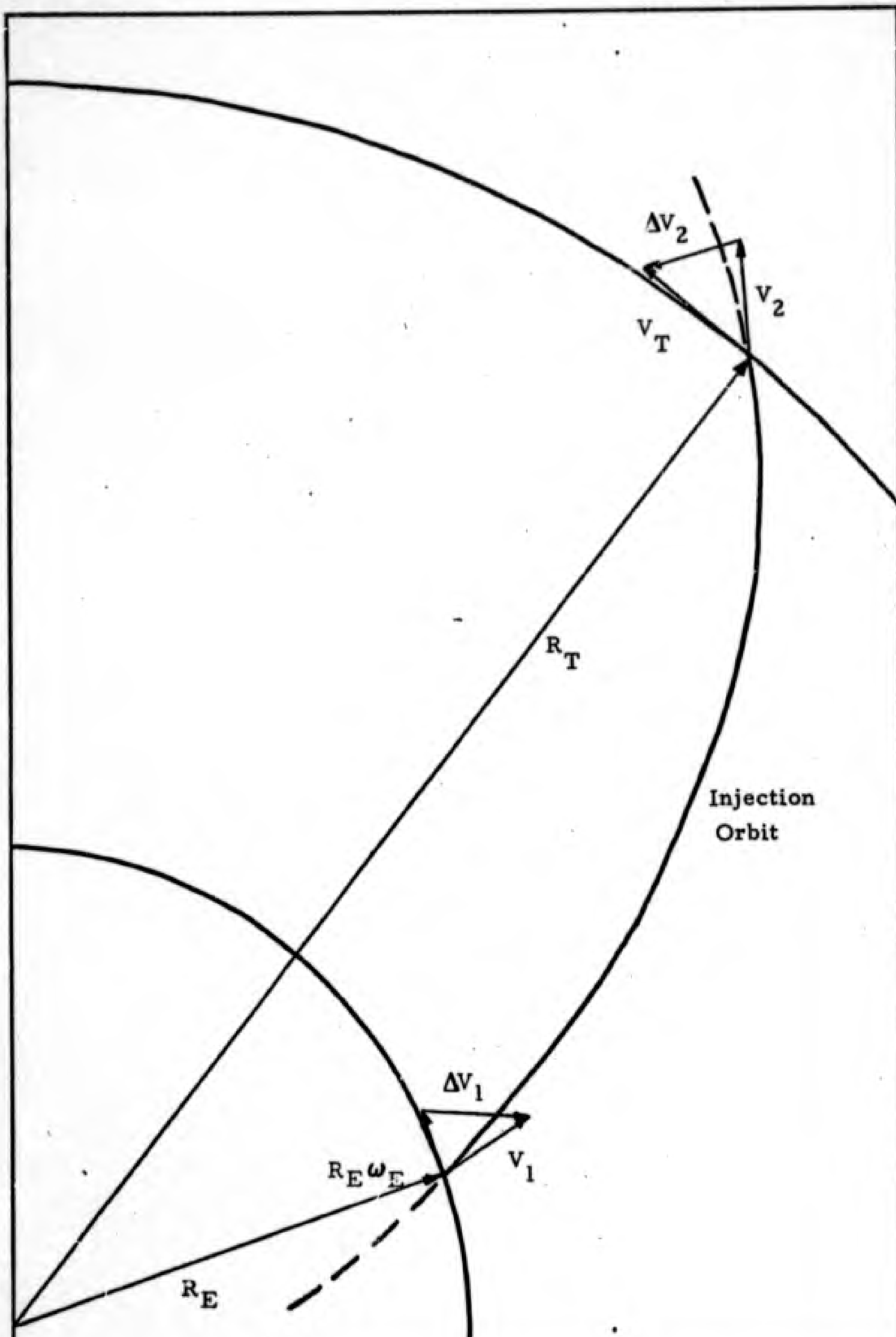

Fig. 2. The Two-Impulse Approximation

Determine all possible distributions of the total available ΔV between launch and injection and thereby the orbit or orbits that will put the vehicle into the target orbit in minimum time for either a fixed or free range angle. The solution of this problem is the subject of the next chapter.

IV. Impulsive Orbit Injection Trajectories

This chapter contains a description of the procedures used to generate the trajectories called for at the end of Chapter III and an analysis and discussion of those trajectories.

The Governing Polynomial

For the general two-impulse transfer problem, Tubbs developed a procedure to determine the minimum time trajectory for a given mass ratio and range angle. Through a geometrical analysis, he derived expressions for ΔV_1 and $\Delta \bar{V}_2$ in terms of the semi-latus rectum ℓ of the transfer orbit and, from these, reduced the problem to a fourth-order polynomial in $\sqrt{\ell}$, which he called the governing polynomial. The general procedure is to solve this polynomial and calculate the time of flight corresponding to each real root. The minimum time trajectory is selected and the associated root is used to calculate the other elements required to describe the orbit. A complete description of the derivation of the governing polynomial can be found in Appendix A of Reference 13; a listing of the polynomial is contained in Appendix A of this study.

Program RDVZ1

Program RDVZ1, listed in Appendix B, was written to perform the calculations of the procedure described above. It is basically

Tubbs' program for orbit-to-orbit transfers extended to include a variety of checks and constraints. The main program sets up the problem for an initial range angle, calculates the coefficients of the governing polynomial and then solves the polynomial using the library subroutine DMULR, a standard root-finding algorithm operating with complex algebra.

The real roots are isolated; an absence of real roots indicates that the vehicle has insufficient ΔV for orbit injection over the specified range angle. They are then passed to subroutine ELIM, which, for each root, calculates ℓ and checks its validity; the check is made by reconstructing the total $\Delta \bar{V}$ corresponding to the root in question using terms from the building blocks of the development of the governing polynomial, and comparing it to the ΔV available as computed in the main program to insure their equality. This procedure eliminates any spurious roots that may appear.

Next, ELIM determines whether the orbit violates the constraint imposed by launching the vehicle from the surface of the Earth. The original version of this program contained no restriction on the minimum altitude of the vehicle; hence, a trajectory could conceivably pass through perigee prior to termination. In this study, all orbits have a perigee radius less than R_E and therefore any trajectory which passes through perigee cannot be considered. This check is performed by examining the radial velocity at the launch point, which is determined in the course of forming the governing polynomial. If this quantity is negative, the trajectory passes through perigee and cannot be considered further; the number of such trajectories is passed back to the main program for later use. ELIM then computes the orbit's eccentricity, which dictates the proper equation for computing the time of flight; it should be noted here that ELIM can handle both elliptical and hyperbolic orbits. The elliptical time of flight equations used

$$\cos E = \frac{e + \cos f}{1 + e \cos f} \tag{28}$$

$$t = \sqrt{\frac{a^3}{\mu}} \, (E - e \sin E) \tag{29}$$

may be found in any standard text on astrodynamics; the hyperbolic time of flight equations,

$$\tan \frac{h}{2} = \sqrt{\frac{e - 1}{e + 1}} \, \tan \frac{f}{2} \tag{30}$$

$$t = \sqrt{\frac{(-a)^3}{\mu}} \left[e \tan h - \ln \tan \left(\frac{\pi}{4} + \frac{h}{2} \right) \right] \tag{31}$$

where h is an artificial variable, are taken from Greenwood (Ref 7: 207).

After these steps are repeated for all real roots, control is passed back to the main program. If all of the trajectories go underground, i. e. pass through perigee, the program prints out the data generated to that point and terminates. Otherwise, pertinent information from ELIM is stored, the range angle is increased and the procedure starts again; in this manner, orbit injection over all possible range angles is examined.

Initial Results

A mass ratio of .05, which is representative of the ratios for various U.S. launch vehicles (Ref 15:120), was chosen as the baseline ratio for this study; the effect of varying this ratio is examined later. Preliminary examination of early program output indicated that, for elliptical injection orbits (which characterized all orbits for $m_f = .05$), there are two possible routes to satisfy θ_f, the specified range angle--the direct route and the apogee route, illustrated in Fig. 3. The partial apogee route pictured in Fig. 4, which seemed to be a third possibility even though it did not cover θ_f, was found to be merely a direct route with θ_A the specified range angle. The partial apogee route range angles and times of flight were calculated by ELIM for each apogee route which appeared and provided more data about the direct routes.

The Minimum Injection Time Curve

Minimum orbit injection time as a function of range angle, for the baseline mass ratio, is shown in Figs. 5 and 6. Fig. 5 contains the entire set of admissible trajectories. Direct route trajectories result in minimum time for $\theta_f \leq 35^o$; beyond this range angle, the apogee routes must be used for orbit injection. The reason for the

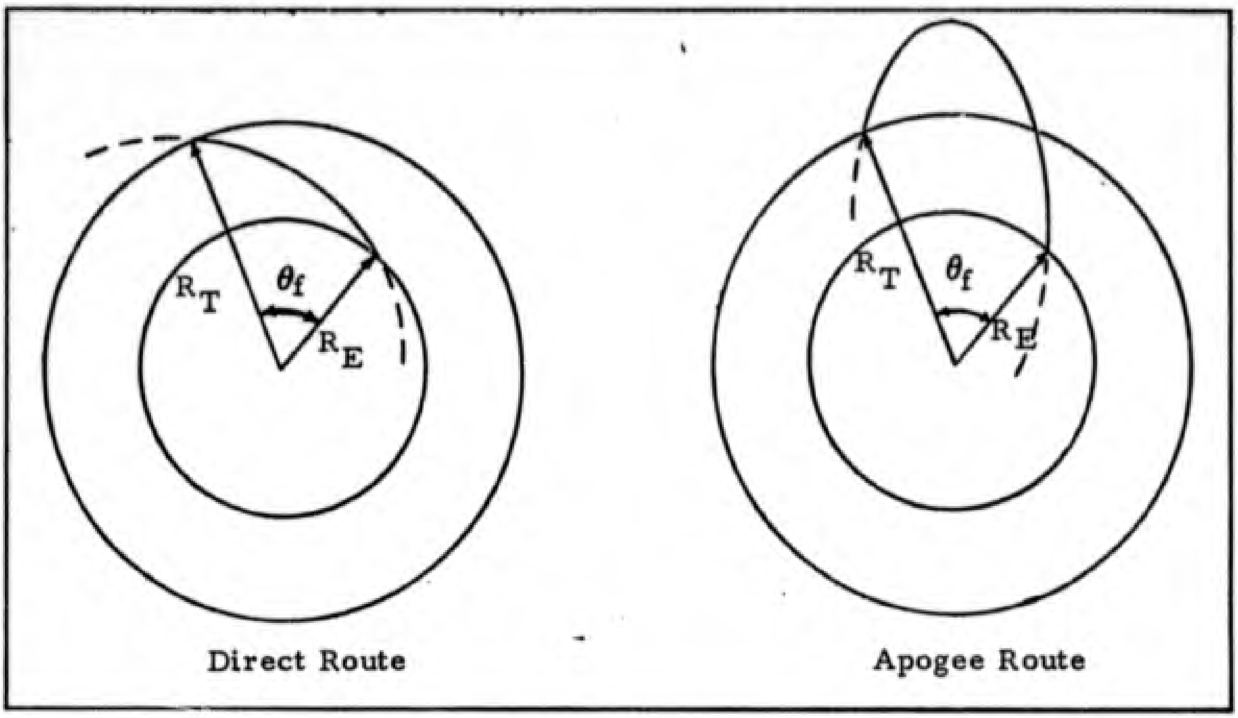

Fig. 3. Direct and Apogee Routes

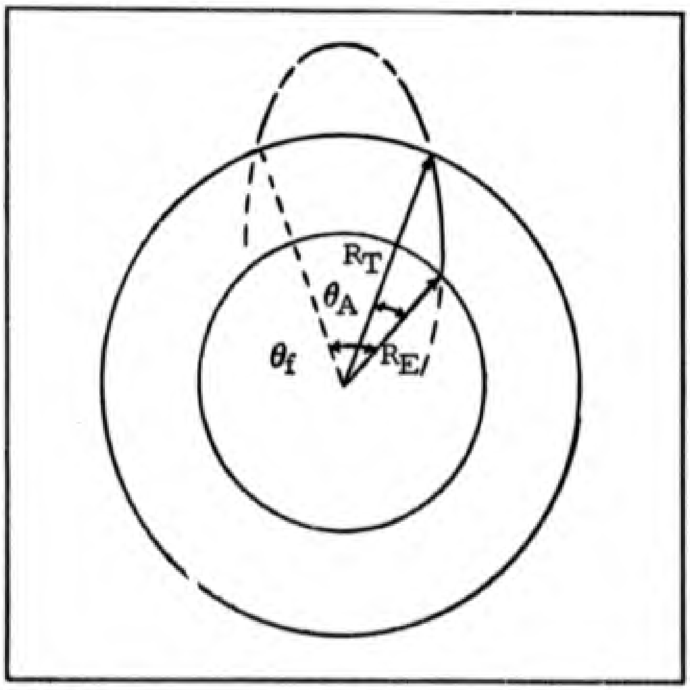

Fig. 4. Partial Apogee Route

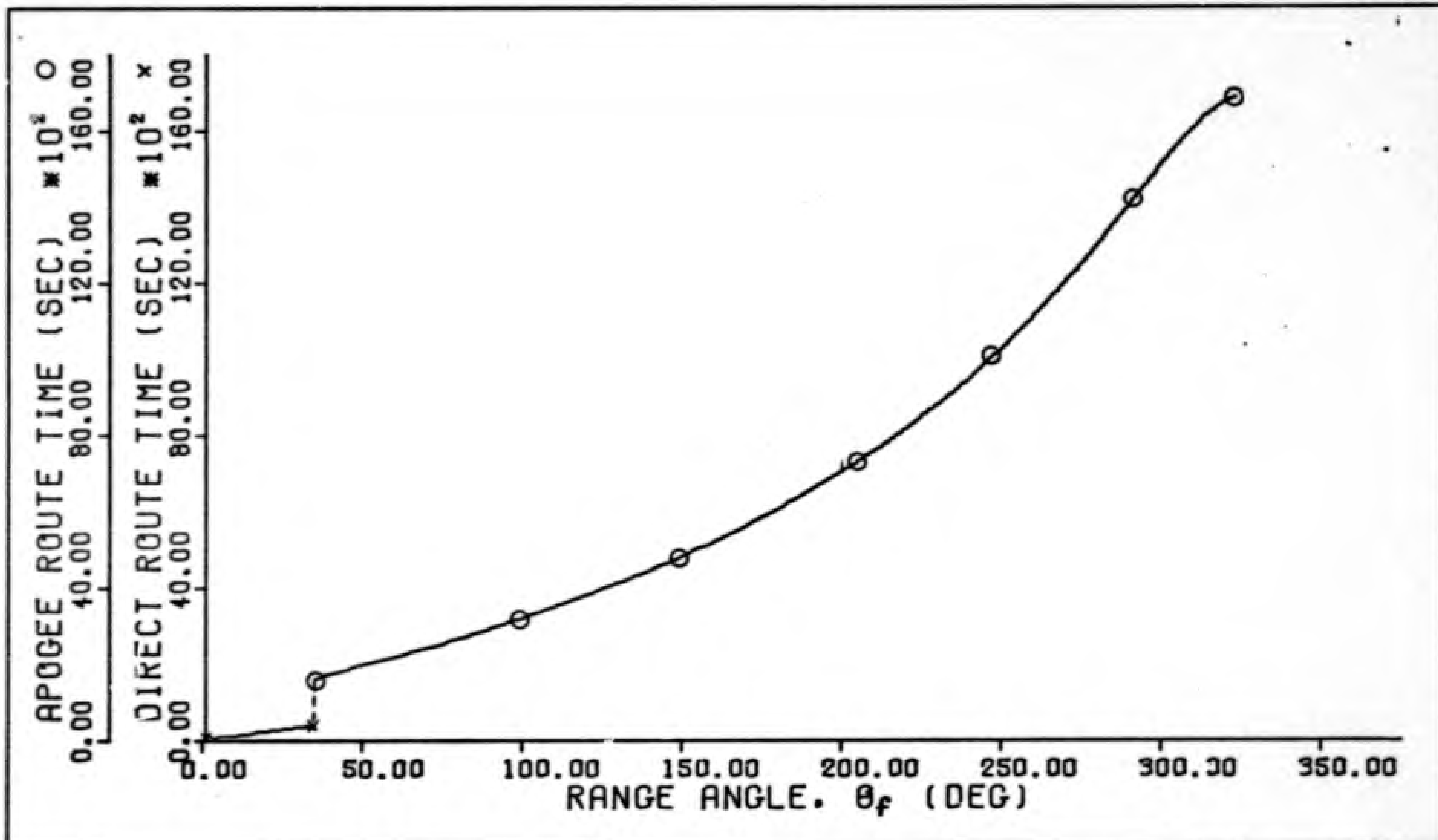

FIG. 5. MINIMUM INJECTION TIME. MASS RATIO = .05

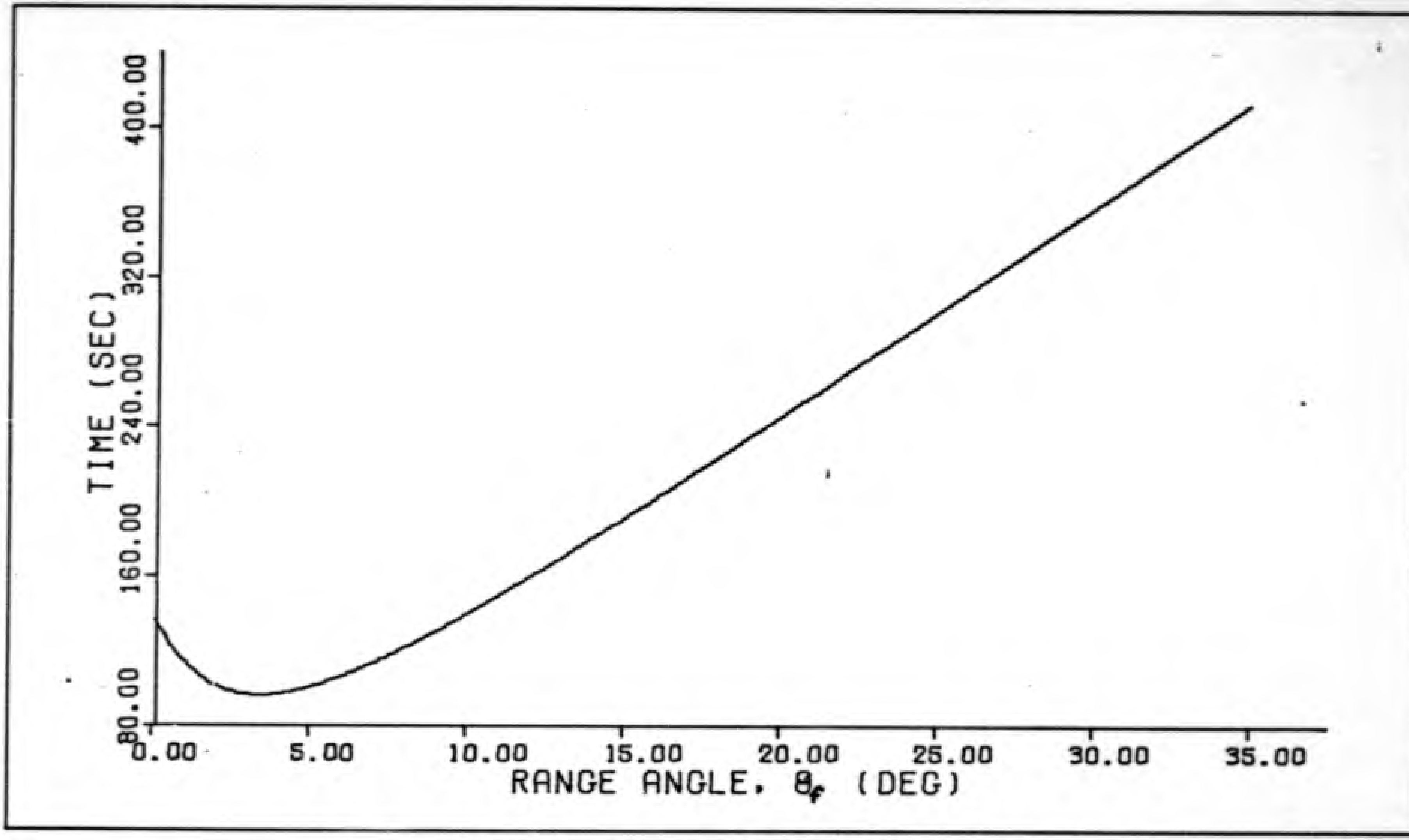

FIG. 6. DIRECT ROUTE MINIMUM INJECTION TIME

discontinuity is the launch constraint built into ELIM. For $\theta_f > 35^o$, the direct route trajectory begins prior to perigee and takes the vehicle underground. The apogee route is then the only trajectory which is allowable. The upper limit on achievable range angle may be determined by any of several factors. In Tubbs' orbit-to-orbit study, the amount of ΔV available fixed the limit (Ref 13:22). In this study, it is the physical launch constraint that sets the maximum θ_f at 124°, i.e., the apogee route initially goes below ground beyond this point.

The direct route portion of the minimum time curve is expanded into Fig. 6. In general, there is a minimum achievable range angle which the curve approaches asymptotically; this limit is fixed by the impulse available. For the baseline mass ratio, trajectories with range angles as low as .09° were observed, indicating that the limit is very close to the rectilinear orbit ($\theta_f = 0^o$). The curve exhibits an absolute, or global, minimum which does not correspond to the minimum range angle. The distribution of the total impulse between launch and injection offers at least a partial explanation. For very small range angles ($\theta_f < 1.0^o$), the injection orbits are nearly parabolic ($e > .999$), resulting in crossing angles at injection of nearly 90°. Therefore, a significantly large portion of the total ΔV (e.g. 65% for $\theta_f = .5^o$) must be reserved for injection, giving the vehicle a lower initial speed and resulting in a higher time of flight. For $0^o < \theta_f \leq 3.43^o$ the increase in range angle is more than offset by a decrease in the ΔV required at injection (47.6% at $\theta_f = 3.43^o$); thus the flight time decreases. Beyond $\theta_f = 3.43^{\circ}$, the decreasing crossing angle has less effect than the increasing range angle and the flight time begins to increase.

In Chapter V, which describes actual rendezvous, it will be shown that the direct route always provides a shorter rendezvous time over a smaller range angle, regardless of the initial position of the target in its orbit. In this light, the remaining discussions of injection trajectories will deal only with direct routes.

Parameterization of Mass Ratio

The launch envelope, which comprises all admissible injection trajectories, plays a very important part in the rendezvous mission. Knowledge of the effect on the launch envelope of varying the vehicle's mass ratio would be useful to a mission planner, as it provides information regarding the tradeoff between time of flight and mass which can be placed in orbit. It is natural to expect that the injection time varies directly with the mass ratio, or inversely with the available impulse, for a given θ_f. The dynamics of the launch envelope are not as readily apparent.

Launch Envelope Dynamics. Figs. 7 and 8 illustrate changes in the launch envelope as the mass ratio is changed. Minimum injection time curves, similar to Fig. 6, for various mass ratios are shown in Fig. 7. As expected, the flight times increase as the mass ratio does. For mass ratios above .12, the curves lose their

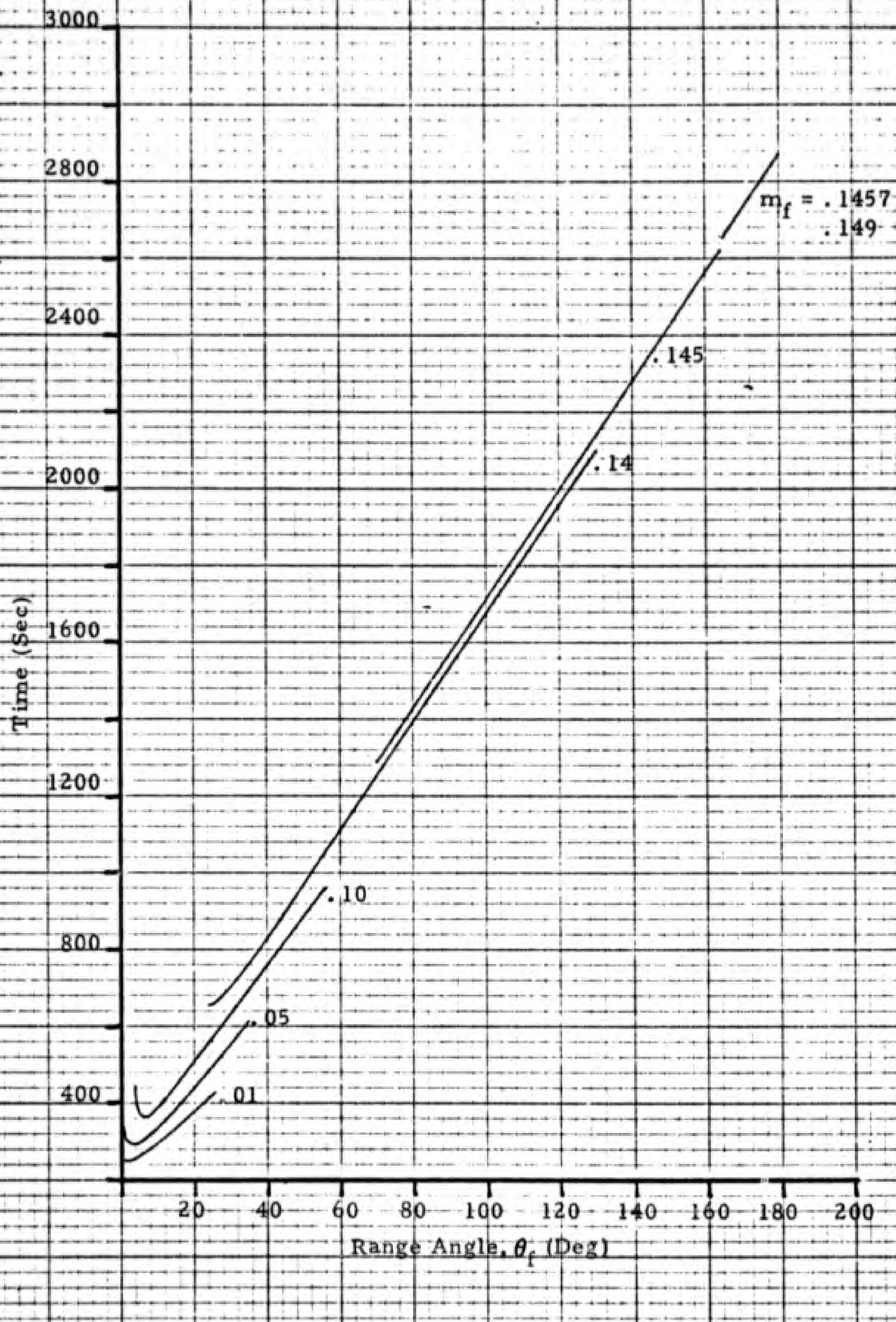

Fig. 7. The Direct Route Launch Envelope

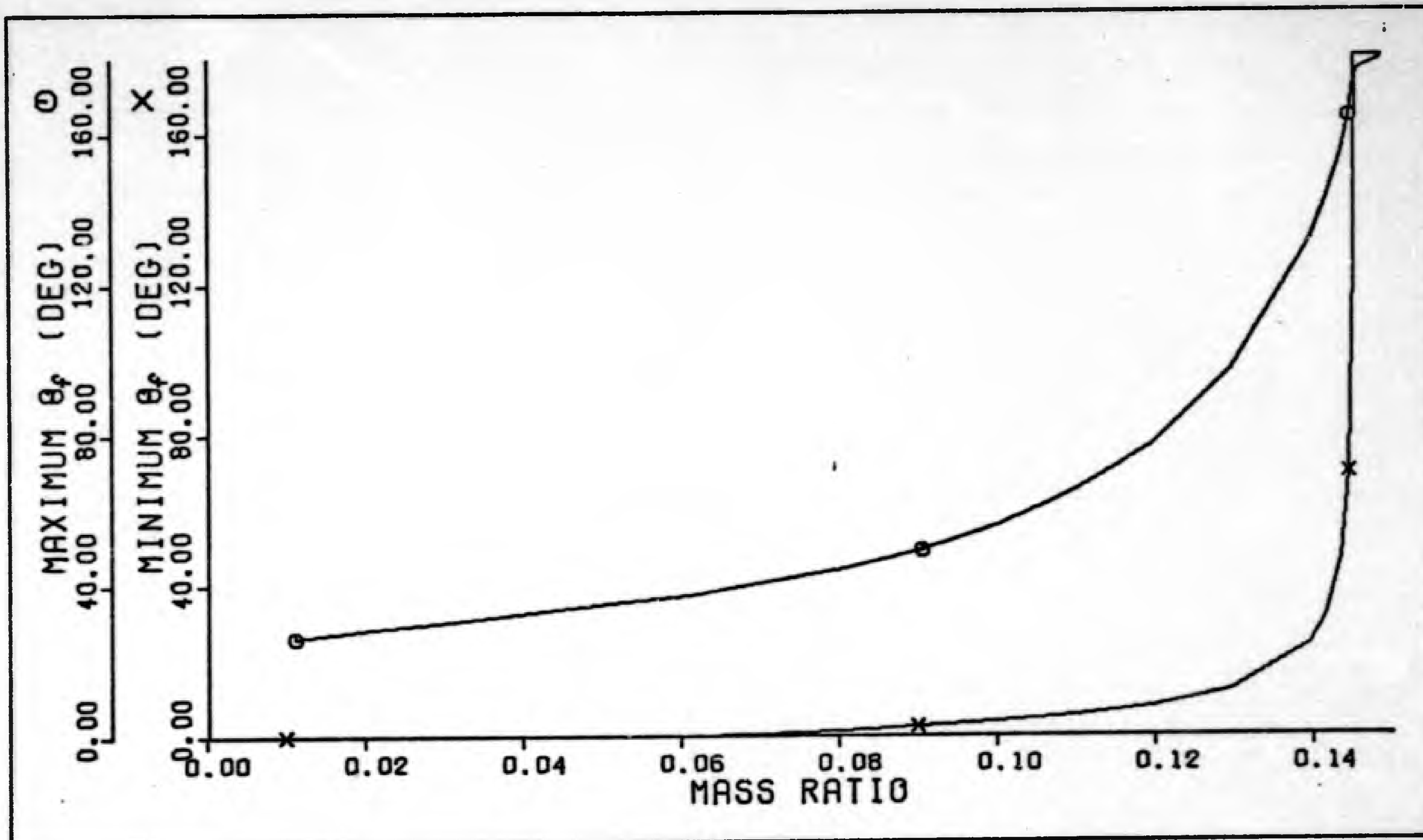

FIG.8. LAUNCH ENVELOPE DYNAMICS

diatonic feature and eventually become linear. The curves for $.1457 \leq m_f \leq .149$ are all colinear and no admissible trajectories are generated for $m_f > .149$, indicating that this is the maximum allowable mass ratio for injection into the target orbit. Flight times range from 56 sec for $m_f = .01$ to 2670 sec for $m_f = .1458$; this maximum value remains constant for higher mass ratios. The changes in the minimum and maximum range angles are pictured in Fig. 8. The minimum θ_f remains near 0° below $m_f = .08$ and then begins to increase because of insufficient ΔV to provide the high energy required for small range angle injection. It grows rapidly beyond $m_f = .14$ and reaches 179.5° for $m_f = .149$. The maximum range angle increases steadily with mass ratio, approaching and remaining near 180° for $m_f \geq .1458$. (It should be noted that trajectory data for $\theta_f = 180^\circ$ cannot be calculated because of singularities in several governing polynomial terms, e.g., $(\sin \theta_f)^{-1}$.)

The launch envelope reaches a maximum width of 110.5° at $m_f = .142$. This then is the mass ratio which provides the maximum number of admissible injection trajectories. However, as will be shown in Chapter V, this is not the mass ratio which allows the greatest number of rendezvous opportunities.

Additional Observations. Hyperbolic injection orbits appear for $m_f \leq .02$ and $4^\circ \leq \theta_f \leq 28^\circ$ as a result of the large impulse available. All other injection orbits are elliptical. Apogee routes are no longer generated for $m_f > .1457$; this occurrence is related to the behavior of the program, as discussed in the next paragraph.

Method Limitations. That both the minimum and maximum range angles tend towards 180° suggests a comparison with the Hohmann transfer, which is the minimum fuel, maximum time, two impulse transfer for $\theta_f = 180^{\circ}$. Calculation of the relevant Hohmann parameters indicated a maximum mass ratio (corresponding to minimum fuel) of .1457 and an injection time of 2670 sec. RDVZ1 generates a set of admissible trajectories for this mass ratio and continues to do so until $m_f = .149$, indicating that this method of generating trajectories gives poor results for mass ratios approaching the theoretical maximum. This can best be attributed to a loss of numerical accuracy near the maximum m_f.

This concludes the orbit injection phase of the study. The results of this phase serve as the basis for the rendezvous phase.

V. Rendezvous

The second major portion of this study deals with the rendezvous of the vehicle and a target whose motion is completely specified. This chapter describes the rendezvous problem, the solution method which resulted from the analysis and some results.

The General Problem

In this study, rendezvous is considered a point occurrence in terms of the total mission and is treated as the matching of the target's and the vehicle's inertial positions and velocities; that is to say, it is treated as occurring instantaneously at injection, rather than over a finite period of time. Since the previous portions of this study have determined minimum time trajectories for placing the vehicle into the target's orbit, the problem becomes one of proper phasing of the launch. The objective of this analysis may then be described as the determination, as a function of initial target position, of the launch time and injection trajectory resulting in minimum total time to rendezvous. The general procedure, for a given mass ratio, is to establish a launch window and use the target's initial position relative to this window to determine this functional relation. The variables used in this analysis are depicted in Fig. 9 and are described in the text that follows.

All angles are measured relative to the launch point, which is

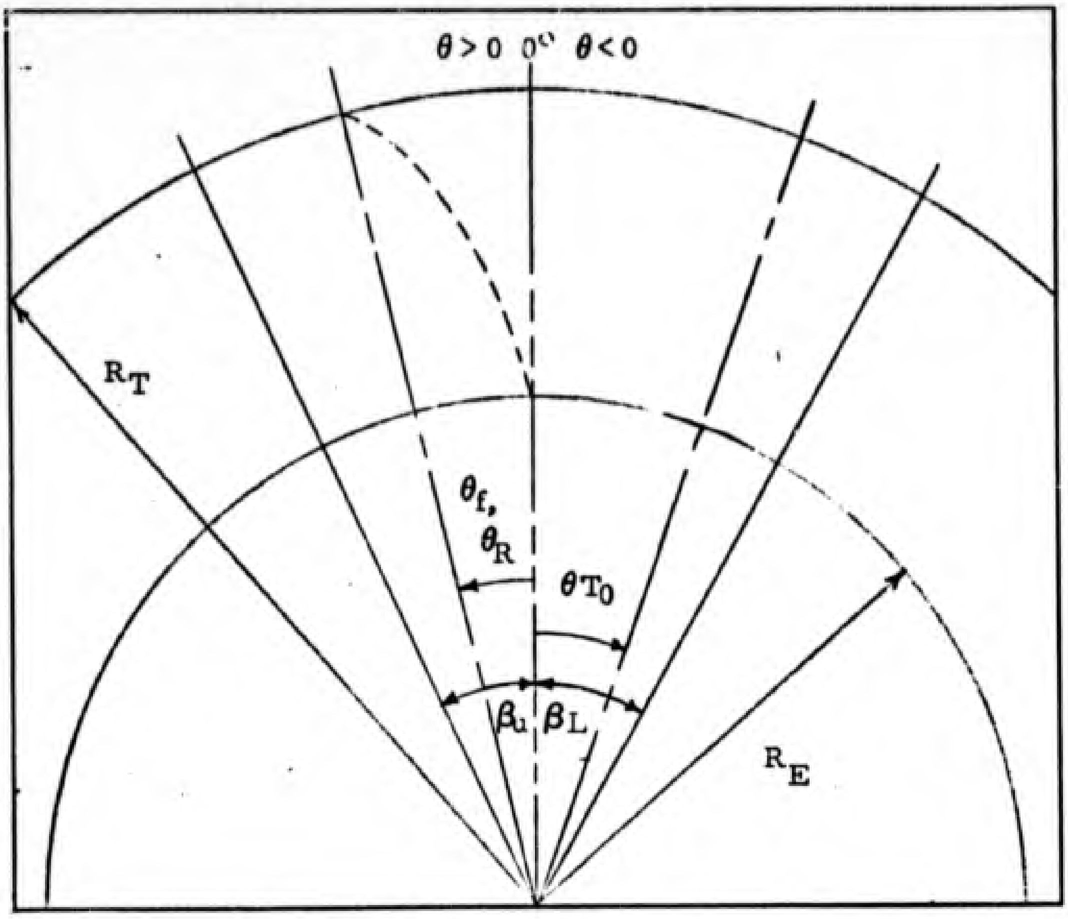

Fig. 9. Rendezvous Variables

0° for any given set of circumstances and is located at the intersection of the reference line and the earth's surface. It remains fixed in inertial space once launch has occurred. Angles measured counter-clockwise (the direction of motion) are 0° to 180°; those measured clockwise are 0° to -180°. Since rendezvous occurs at injection, the rendezvous position, θ_R, (or final vehicle/target position) is numerically equivalent to the range angle θ_f traversed by the vehicle; therefore, all of these terms are used interchangeably in the text.

Data for all examples used to complement the discussion are for the baseline mass ratio.

The Launch Window

The launch window is generally thought of as the (clock) times during which launch must occur to insure mission success. The term is used here in a slightly different way.

The lower window boundary β_L is an angle representing the initial position of the target θ_{T_0} for the first rendezvous opportunity. This opportunity corresponds to the use of the minimum range angle injection trajectory. Similarly, the upper boundary β_u represents the last target position for which immediate launch and rendezvous can occur and corresponds to the use of the maximum range angle trajectory. The lower (upper) boundary is calculated by subtracting from the minimum (maximum) range angle the angular distance covered by the target during the vehicle's time of flight $\omega_T t$; e.g.,

$$\beta_L = \theta_{f_{min}} - \omega_T t \tag{32}$$

$$= .09^\circ - \frac{180}{\pi} \sqrt{\frac{\mu}{R_T}} \frac{^\circ}{sec} \times 136.35 \text{ sec}$$

$$= -8.65^\circ$$

$$\beta_u = 8.48^\circ$$

Thus, the launch window comprises all target positions for which immediate launch on the proper trajectory will result in rendezvous.

Determination of Launch Time

Once the launch window is established, initial target positions from -180° to 180° are examined to establish the proper launch time. If a target is within the launch window when the decision to rendezvous is made, launch can occur immediately and the scheme described below is used to find the proper trajectory. If the target is outside the window, the procedure established for this study is to wait until the target arrives at the lower boundary and then launch the vehicle on the minimum range angle trajectory.

The reason for considering only the direct route trajectories can now be explained. The majority of the apogee route times, for mass ratios less than .1, equal or exceed the orbital period of the target, thereby being incompatible with the minimum rendezvous time objective. Furthermore, initial target positions required for immediate launch into the apogee routes put the target in positions where rendezvous can take place in a shorter time by delaying launch until the target is at β_L and flying the minimum range angle trajectory. For example, the maximum direct route range angle for $m_f = .05$ is 35.00°. The time of flight, and therefore rendezvous time, for an apogee route covering 60° (i.e., rendezvous occurs at 60°) is 2210.27 sec. Using a general form of Eq. (32), i.e.

$$\theta_{T_0} = \theta_f - \omega_T t \tag{33}$$

launch must occur when the target is located at -75.81°. But if launch is postponed until the target travels from this position to β_L and the minimum range angle trajectory is used, the total time for rendezvous (waiting time plus the minimum range angle time of flight) is only 1183.90 sec. It is obvious that the direct route always provides the minimum rendezvous time, assuming the absence of a requirement (e.g., the location of a tracking station) to rendezvous outside the positions defined by the direct route.

Program RDVZ2

Program RDVZ2 was written to perform the steps described above and determine the proper trajectory for an initial target position inside the launch window. It is listed in Appendix B and described below.

The trajectories comprising the direct route minimum injection time curve are input to the program in terms of range angles and corresponding times; these trajectories are irregularly spaced throughout the launch envelope. For each trajectory, the initial target position which will result in rendezvous at injection is determined by Eq. (33); the minimum and maximum θ_{T_0} define β_L and β_u. The functional relationship of Eq. (33) is then reversed in order to select the proper trajectory as a function of θ_{T_0}.

All target positions between -180° and 180° are examined. If the θ_{T_0} in question is outside the window, the program calculates the waiting time, adds it to the flight time for the minimum range angle trajectory and records the sum as the time to rendezvous; θ_R is the minimum θ_f. If the target is inside the window, first θ_R is determined by interpolating within the curve of θ_{T_0} versus θ_f (or θ_R) with the library function subprogram ATKN; since this curve is monotonic, θ_{T_0} may be treated as the independent variable even though the curve was first generated with θ_f in that role. This selects the proper trajectory. The corresponding time to rendezvous is found by interpolating within the range angle-time curve originally input to the program.

Results

A sample of the output of RDVZ2 is included in Appendix B and the results of the program for $m_f = .05$ are displayed graphically in Fig. 10, which shows the overall curve, and Fig. 11, which shows only the portion within the launch window. The shape of the curve within the launch window follows that of the injection time curve, as expected. Outside the window, the curve is linear since it represents the waiting time from θ_{T_0} to β_L plus the minimum range angle flight time. The discontinuity occurs as θ_{T_0} moves downrange of β_u, at which point the target must complete a major portion of its orbit before another rendezvous opportunity arises.

The size and location of the launch window and the corresponding rendezvous times were investigated for various mass ratios and are illustrated in Fig. 12. Following the pattern of the injection trajectories, increasing mass ratios lead to increasing rendezvous time.

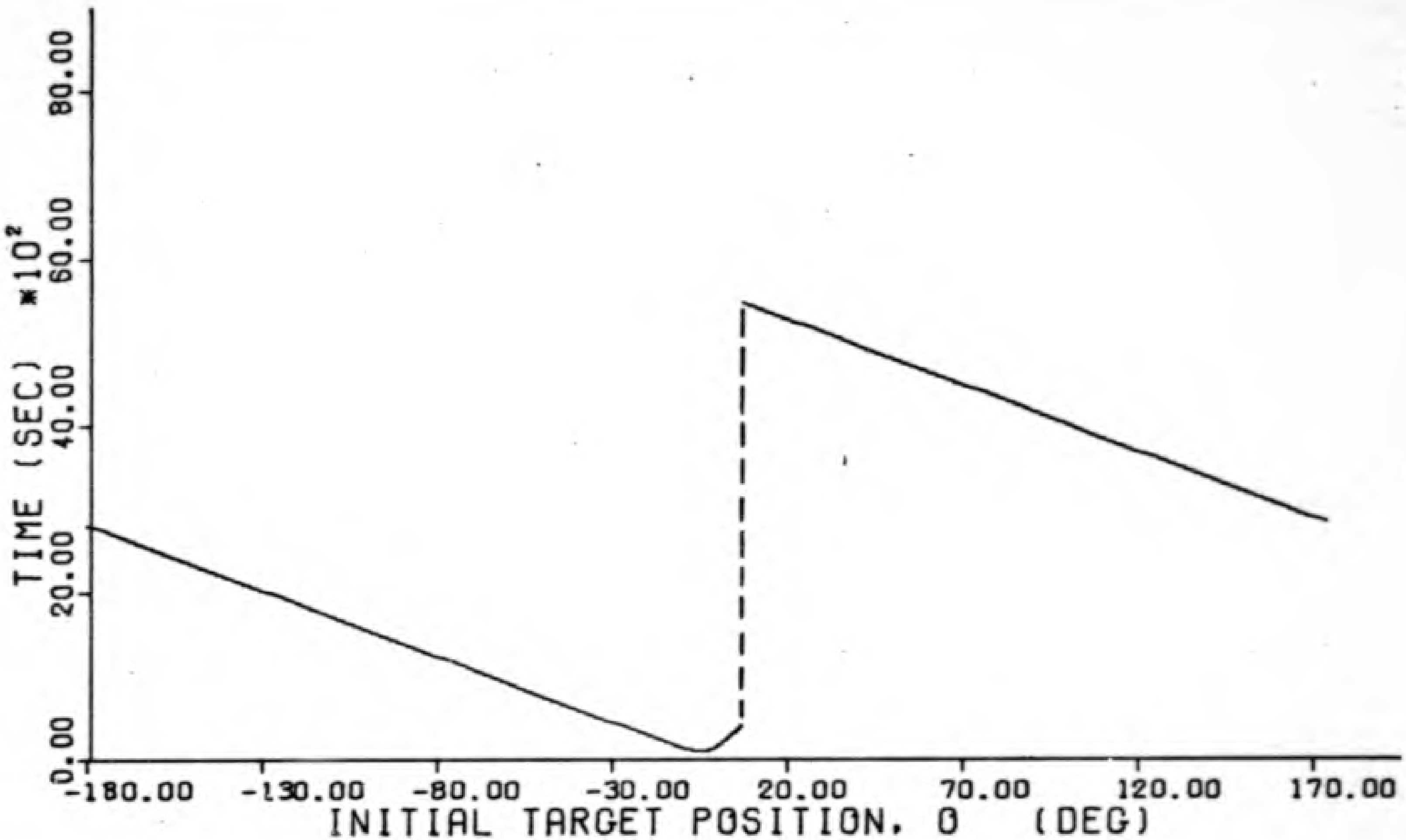

FIG. 10. MINIMUM RENDEZVOUS TIME, MASS RATIO = .05

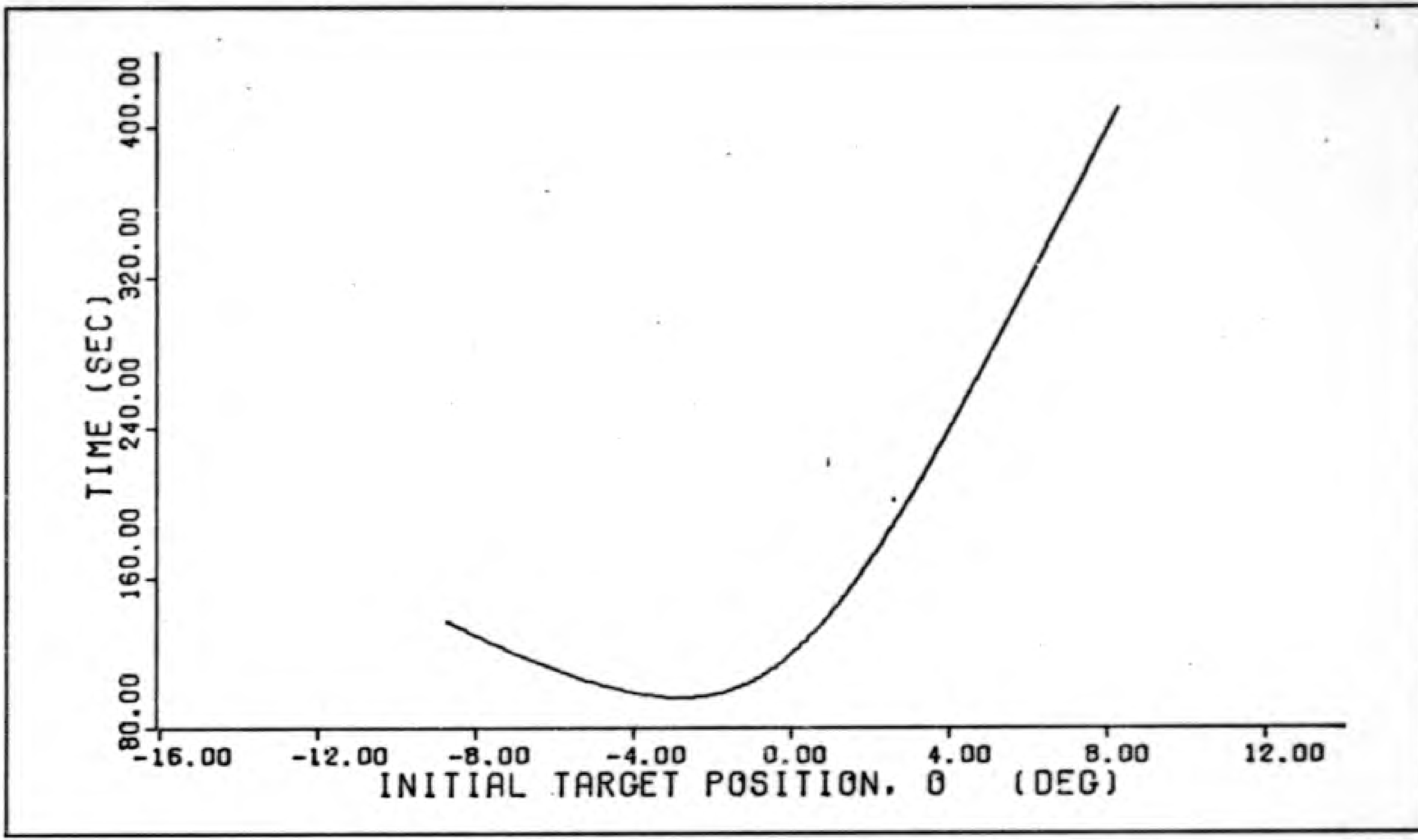

FIG. 11. MINIMUM RENDEZVOUS TIME WITHIN WINDOW

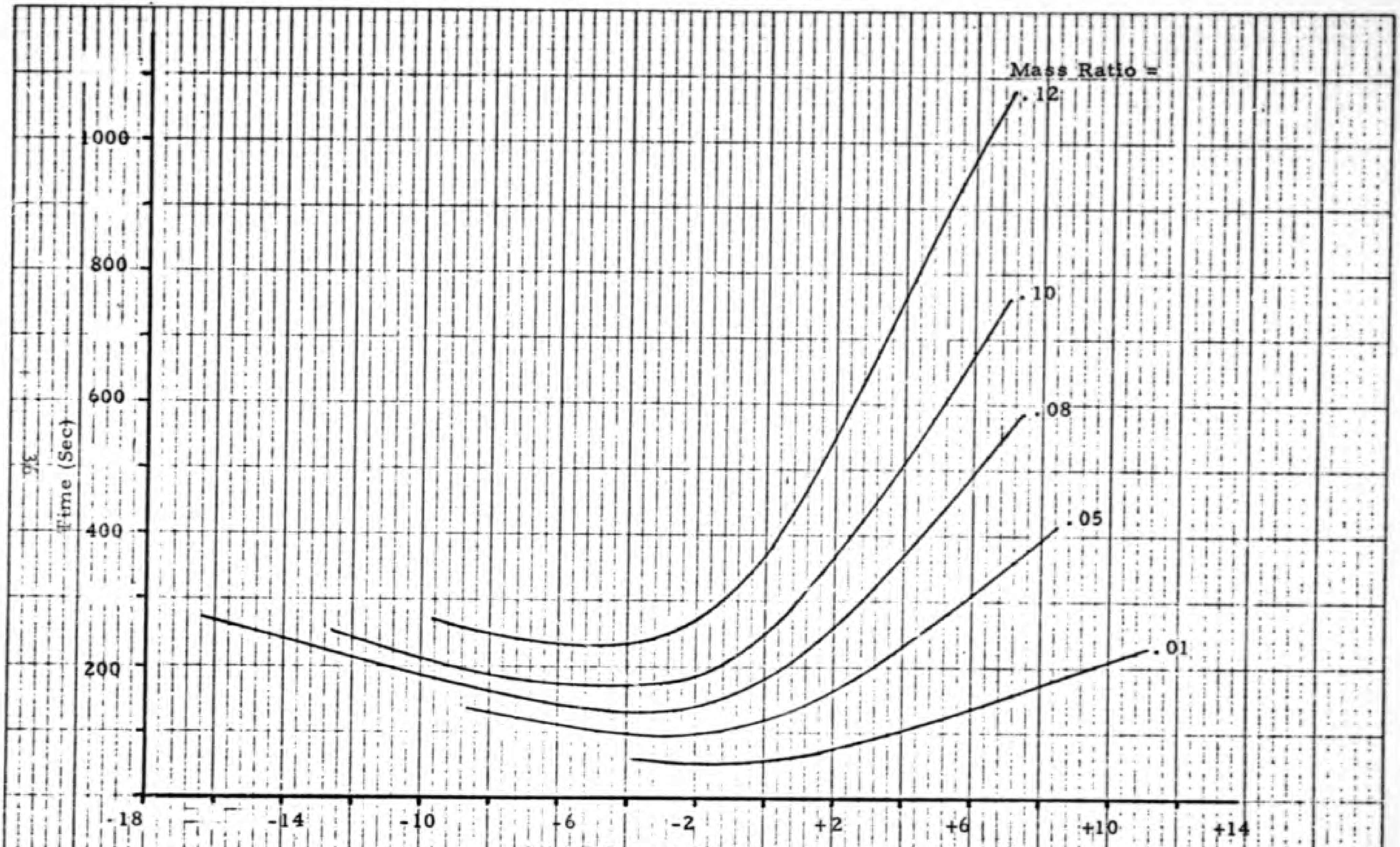

Fig. 12. Rendezvous Tradeoffs

However, the dynamics of the launch window are quite different. The boundaries move uprange and the size of the window goes up, thereby increasing the time span during which the vehicle must be launched, as m_f increases to .08. This is the m_f of maximum uprange movement, $\beta_L = -16.37°$, and maximum window width, 23.73° or 370.1 sec; it should be noted that the launch envelope is 43.33° wide for this mass ratio. As the mass ratio increases further, the window moves back downrange and its size diminishes. At $m_f = .142$, the mass ratio with the widest launch envelope, the window is only 12.92° wide. Beyond this value, the window decreases rapidly, approaching a point as the mass ratio approaches its theoretical limit. The movement of β_L can be explained by Eq. (32) and Fig. 8. As m_f increases initially, the second term in that equation dominates, since $\theta_{f_{min}}$ remains very close to 0°, and β_L becomes more negative, i.e., moves uprange; for $m_f \geq .08$, $\theta_{f_{min}}$ increases more rapidly than $\omega_T t$ and thus dominates, making β_L less negative. A similar explanation holds for β_u, although the minimum occurs at $m_f = .10$. Since β_L decreases and increases sharply around $m_f = .08$, while the change in β_u is much more gentle, the maximum window then occurs at $m_f = .08$.

An analysis of permitting a choice of mass ratio at launch in hopes of further reducing the rendezvous time was performed using the data presented in Fig. 12. For example, the time to rendezvous resulting from launching at the β_L for $m_f = .08$ was compared with that resulting from waiting until the target moved from the .08 β_L to the

β_L for $m_f = .05$ and then launching. All comparisons of using less fuel versus waiting and using more fuel resulted in shorter rendezvous time for the latter procedure. Therefore, from the standpoint of achieving minimum rendezvous time without regard to the mass placed in orbit or the location of rendezvous, the smallest mass ratio possible is the most desirable. On the other hand, if these other two factors are considered equally important, there are significant tradeoffs available. For example, the mass in orbit can be doubled, the maximum θ_R increased 60%, and the size of the launch window increased 15% by using the trajectories for $m_f = .10$ rather than .05 and accepting increases in rendezvous time ranging from 112.5% at $\theta_R = 4^o$ to only 18.3% at $\theta_R = 34^o$ (these values of θ_R being the limits within which both sets of trajectories can achieve rendezvous).

VI. Conclusions and Recommendations

Conclusions

This study has analyzed the use of two-impulse, fixed fuel trajectories to achieve minimum time rendezvous with an orbiting spacecraft. There are several important conclusions as a result of this analysis.

1. This study established a technique for generating and analyzing the desired rendezvous trajectories. Although this technique was applied only to a target in a circular, low altitude orbit, its extension to other situations should be straightforward. Results for mass ratios near that of the limiting Hohmann transfer were not completely reliable and operation with these values is not recommended.

2. The imposition of physical launch constraints, i.e., the restriction that the traversed portion of the injection orbit may not go below the surface of the earth, severely limited the number of admissible trajectories that the governing polynomial could generate. That the number of direct routes was thus limited, coupled with the, in general, unacceptably high flight times of the apogee routes, resulted in a relatively small opportunity (e.g., 4.7% of the target's period for $m_f = .05$) for direct launch to rendezvous.

3. Minimum time to rendezvous at a given position for a given mass ratio can be reduced only be decreasing the mass ratio, i.e.,

using more fuel. However, substantial tradeoffs exist among the mass placed in orbit, the location of rendezvous, the time during which launch may occur, and the time to rendezvous.

Recommendations

The results of this study may be extended in several ways:

1. A numerical solution of the general problem, as described in Chapter II, could be attempted using the impulsive solutions as initial estimates (c.f., Handelsman (Ref 9)).

2. The use of more than two impulses could be studied, including the use of intermediate parking orbits.

3. The methods used in this study could be extended to targets in other than circular orbits and at other than low altitudes.

4. The use of selected apogee injection routes to extend the launch windows, particularly for those mass ratios where the time difference between the direct and apogee routes is not great, is a possible area of investigation.

5. The application of constraints on rendezvous position, such as the requirement to rendezvous over selected ground stations, could be studied.

6. A verification that the generated trajectories are indeed minimum time trajectories could be done using Lawden's primer vector (Ref 10).

Bibliography

1. Bryner, G. H. Time-Optional Rendezvous from Circular Orbit Using a Thrust-Limited Rocket. Unpublished Thesis. Wright-Patterson AFB, Ohio: Air Force Institute of Technology, May 1969.

2. Bryson, A. E. and Y. C. Ho. Applied Optimal Control. Waltham, Massachusetts: Ginn and Company, 1969.

3. Childers, H. D. and D. Irion. Investigation of Minimum-Fuel Limited Thrust Orbital Transfer Trajectories. Unpublished Thesis. Wright-Patterson AFB, Ohio: Air Force Institute of Technology, March 1969.

4. Forster, T. J. and L. L. Gooch. Velocity and Time Requirements for Orbital Transfer and Rendezvous. Unpublished Thesis. Wright-Patterson AFB, Ohio: Air Force Institute of Technology, August 1964.

5. Gill, D. E. and G. D. Wambold. Improved Initial Value Estimates for Minimum Fuel Orbital Transfers. Unpublished Thesis. Wright-Patterson AFB, Ohio: Air Force Institute of Technology, April 1969.

6. Gobetz, F. W. and J. R. Doll. "A Survey of Impulsive Trajectories," AIAA Journal, 7: 802-834 (May 1969).

7. Greenwood, D. T. Principles of Dynamics. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1965.

8. Hamlin, J. R. Minimum Time Transfer between Close Coplanar Orbits. Unpublished Thesis. Wright-Patterson AFB, Ohio: Air Force Institute of Technology, March 1970.

9. Handelsman, M. "Optimal Free-Space Fixed-Thrust Trajectories Using Impulsive Trajectories as Starting Iteratives," AIAA Journal, 4: 1077-1082 (June 1966).

10. Lawden, D. G. Optimal Trajectories for Space Navigation. London: Butterworth & Co. Ltd., 1963.

11. Rapp, F. F. Time-Optimal Orbital Transfer with Limited Fuel. Unpublished Thesis. Wright-Patterson AFB, Ohio: Air Force Institute of Technology, May 1970.

12. Thompson, J. L. Total Impulse Minimization for Two-Impulse Orbital Transfer. Unpublished Thesis. Wright-Patterson AFB, Ohio: Air Force Institute of Technology, August 1962.

13. Tubbs, R. C. Some Necessary Conditions for Time-Optimal, Fixed-Fuel Transfers between Coplanar Orbits. Unpublished Thesis. Wright-Patterson AFB, Ohio: Air Force Institute of Technology, March 1971.

14. "U.S. Launch Vehicles." Aviation Week and Space Technology, 98: 119-120 (March 19, 1973).

15. Wolaver, L. E. Graphical Representation of Satellite Rendezvous and Interception Orbits. Working Paper. Wright-Patterson AFB, Ohio: Wright Air Development Center, June, 1959.

Appendix A

Development of the Governing Polynomial

The governing polynomial is

$$Ay^4 + By^3 + Cy^2 + Dy + E = 0 \qquad \text{where } y = \sqrt{\ell} \qquad \text{(A1)}$$

The coefficients are developed for the general case in the following manner.

$$V_i^2 = \mu \left(\frac{2}{R_i} - \frac{1}{a_i} \right) \qquad i = 1, 2 \qquad \text{(A2, A3)}$$

$$\bar{V}_i \cdot \hat{u}_i = \sqrt{\frac{\mu}{\ell_i}} \; e_i \sin f_i \qquad i = 1, 2 \qquad \text{(A4, A5)}$$

$$\bar{V}_i \cdot \hat{v}_i = \sqrt{\frac{\mu \ell_i}{R_i}} \qquad i = 1, 2 \qquad \text{(A6, A7)}$$

$$\omega_1 = \frac{\sqrt{\mu}\,(1 - \cos\theta)}{\sin\theta} \qquad \text{(A8)}$$

$$\omega_2 = \frac{\sqrt{\mu}\,(R_2 \cos\theta - R_1)}{R_1 R_2 \sin\theta} \qquad \text{(A9)}$$

$$\omega_3 = \frac{\sqrt{\mu}}{R_1} \qquad \text{(A10)}$$

$$\omega_4 = \frac{\sqrt{\mu}\,(R_2 - R_1 \cos\theta)}{R_1 R_2 \sin\theta} \qquad \text{(A11)}$$

$$\omega_5 = \frac{\sqrt{\mu}}{R_2} \qquad \text{(A12)}$$

$$\alpha_1 = \omega_2^2 + \omega_3^2 = \omega_4^2 + \omega_5^2 \qquad \text{(A13)}$$

$$\alpha_2 = -2\omega_2 \overline{V}_1 \cdot \hat{u}_1 - 2\omega_3 \overline{V}_1 \cdot \hat{v}_1 \qquad \text{(A14)}$$

$$\alpha_3 = -2\omega_1 \overline{V}_1 \cdot \hat{u}_1 \qquad \text{(A15)}$$

$$\alpha_4 = \omega_1^2 \qquad \text{(A16)}$$

$$\alpha_5 = V_1^2 + 2\omega_1 \omega_2 \qquad \text{(A17)}$$

$$\alpha_6 = -2\omega_4 \overline{V}_2 \cdot \hat{u}_2 - 2\omega_5 \overline{V}_2 \cdot \hat{v}_2 \qquad \text{(A18)}$$

$$\alpha_7 = 2\omega_1 \overline{V}_2 \cdot \hat{u}_2 \qquad \text{(A19)}$$

$$\alpha_8 = V_2^2 - 2\omega_1 \omega_4 \qquad \text{(A20)}$$

$$\beta_1 = \alpha_2 - \alpha_6 \qquad \text{(A21)}$$

$$\beta_2 = \alpha_3 - \alpha_7 \qquad \text{(A22)}$$

$$\beta_3 = \alpha_5 - \alpha_8 \quad \text{(A23)}$$

$$\beta_4 = 2\alpha_1 \quad \text{(A24)}$$

$$\beta_5 = \alpha_2 + \alpha_6 \quad \text{(A25)}$$

$$\beta_6 = \alpha_3 + \alpha_7 \quad \text{(A26)}$$

$$\beta_7 = 2\alpha_4 \quad \text{(A27)}$$

$$\beta_8 = \alpha_5 + \alpha_8 \quad \text{(A28)}$$

$$A = \beta_1^2 - 2I^2 \beta_4 \quad \text{(A29)}$$

$$B = 2\beta_1\beta_3 - 2I^2\beta_5 \quad \text{(A30)}$$

$$C = 2\beta_1\beta_2 + \beta_3^2 - 2I^2\beta_8 + I^4 \quad \text{(A31)}$$

$$D = 2\beta_2\beta_3 - 2I^2\beta_6 \quad \text{(A32)}$$

$$E = \beta_2^2 - 2I^2\beta_7 \quad \text{(A33)}$$

where I is the total impulse, as defined by Eq. (27), and the other variables are described by Fig. 13.

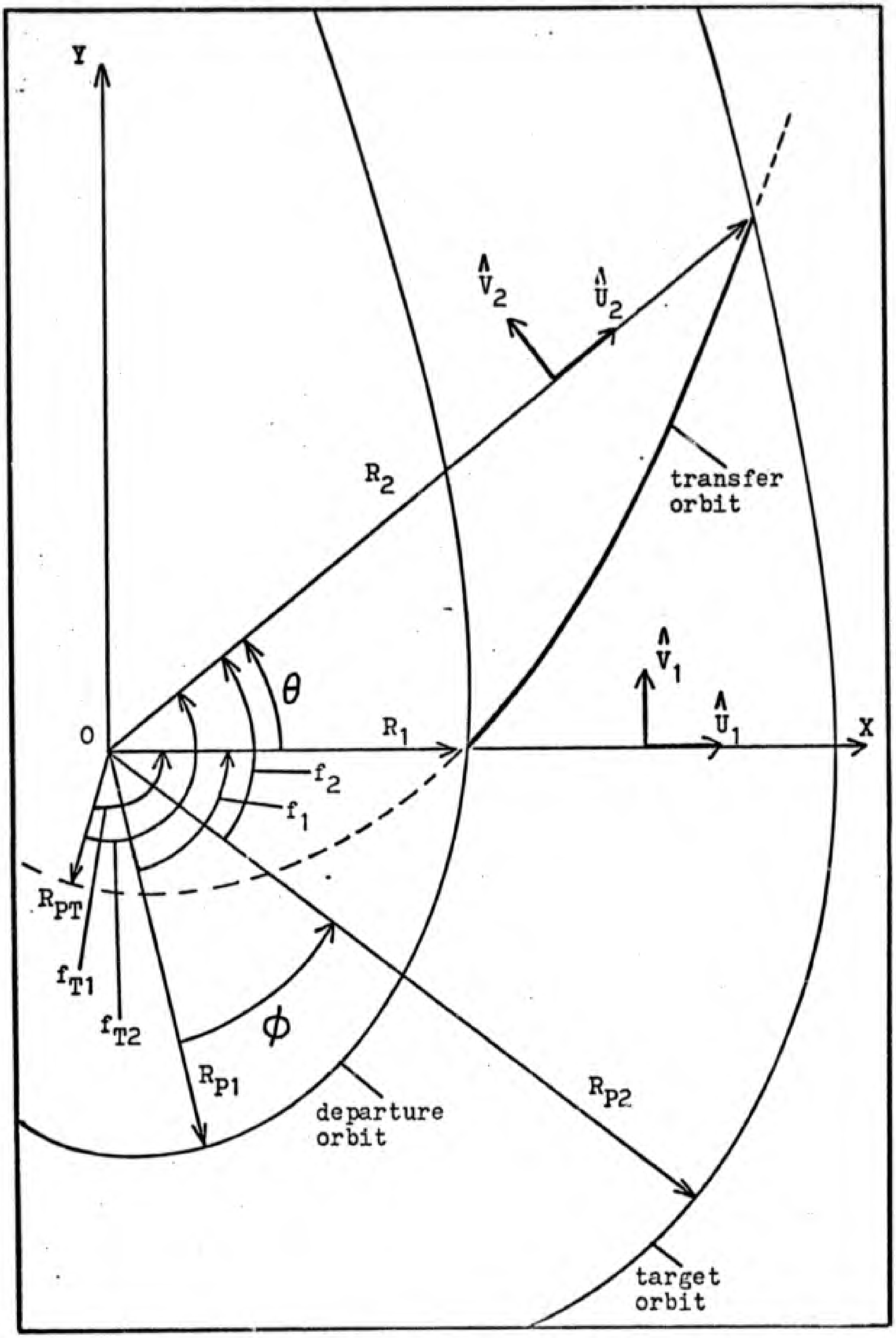

Fig 13. Geometry of the General Impulsive Transfer (from Ref 13:18)

Appendix B

Computer Program Listings and Sample Outputs

The following listings, with output, are located in this appendix:

Program RDVZ1

- Main program
- Subroutine ELIM
- Subroutine QUAD
- Sample output

Program RDVZ2

- Main program
- Sample output

```
      FFOGRAM  FEVZ1(INPUT,CLTFUT,TAPE5=INPUT,TAFE6=CLTFUT,FLOT,FUNCH)
C
C     VARIAFLE ENDING IN 1 USUALLY RELATED TO INITIAL CRBIT OR TERMINAL
C     VARIAFLE ENDING IN T USUALLY RELATED TO FINAL CRBIT OR TERMINAL
C     VARIAFLE ENDING IN X USUALLY RELATED TO INJECTION CRBIT
C     VARIAFLE ENDING IN D USUALLY DENOTES DIMENSIONED QUANTITY
C     EXAMPLE: SINFX1-SINE CF TRUE ANCMALY AT LAUNCH FCINT ON INJECTION
C     ORBIT
C     SEE THESIS FIG. 13 FOR EXPLANATION OF VARIABLES
C     ALL UNITS IN MKS SYSTEM UNLESS CTHERWISE SPECIFIED
C
      DOUBLE PRECISION COE(5),RCOTR(4),ROOTT(4)
      COMMON F(4),A1,A2,A3,A4,A5,A6,A7,A8,W1,W2,W4,V1S,V2S,R1,RT,TCFMIN,
     1 LXMIN,ML,FI,RAD,SPML,FUNIT,TUNIT,VUNIT,TIC,THETAPD,TOFAD,TCFDC,
     2 TOFFD,TCFFP,ECMIN,IFLAG
      REAL L1,LT,MU,LXMIN,MINT(350),ANGLE(350),LXMINC,ANGLEP(350)
      REAL LCNGT(350),DIRECT(350),PARTIAL(350)
      READ (5,1) C,RATIO,ECT,LT,FT,AT,RT
      WRITE(6,996)
  996 FORMAT(*1*)
      WRITE(6,16) C,RATIO,ECT,LT,FT,AT,RT
C
C     CONSTANTS
C
      RAD = 57.2957795131
      MU = 3.986032E5
      RE = 6378.165
      WE = 7.292115E-5
      PI = 3.14159265359
      RUNIT = RE
      TUNIT = SQRT(RE**3/MU)
      VUNIT = RUNIT/TUNIT
      R1=RE
      EC1 = C.
      L1=R1
      F1 = 0.
      AA1 = L1
      SRMU = SQRT(MU)
C
C     NON-DIMENSIONALIZATION
C
      MU = 1.
      SRMU = 1.
      RE = RE/RUNIT
      WE = WE*TUNIT
      R1 = R1/RUNIT
      RT = RT/RUNIT
      L1 = R1
      LT = LT/RUNIT
      AA1 = L1
      AT = AT/RUNIT
      C = C/VUNIT
C
C     CALCULATION OF TOTAL IMPULSE (TI) AVAILABLE
C
```

```
 88   TI = C*ALOG(1./RATIO)
      TIO = TI*VUNIT
      N = 0
C
C     INITIALIZATION OF RANGE ANGLE
C
C
      WRITE(6,100)
      THETA = 1.
 19   THETAR=THETA/RAD
C
C     CALCULATION OF COEFFICIENTS FOR GOVERNING POLYNOMIAL
C
      VV2 = SRML*SQRT(LT)/RT
      VU2 = SRML*ECT*SIN(FT)/SQRT(LT)
      V2S = ML*((2./RT)-(1./AT))
      VV1 = RE*WE
      VU1 = SRML*EC1*SIN(F1)/SQRT(L1)
      V1S = (RE*WE)**2
 90   W5 = SRML/RT
      W4 = SRML*(RT-R1*COS(THETAR))/(R1*RT*SIN(THETAR))
      W3 = SRML/R1
      W2 = SRML*(RT*COS(THETAR)-R1)/(R1*RT*SIN(THETAR))
      W1 = SRML* (1.-COS(THETAR))/SIN(THETAR)
      A8 = V2S-2.*W1*W4
      A7 = 2.*W1*VU2
      A6 = -2.*W4*VU2-2.*W5*VV2
      A5 = V1S+2.*W1*W2
      A4 = W1**2
      A3 = -2.*W1*VU1
      A2 = -2.*W2*VU1-2.*W3*VV1
      A1 = W2**2+W3**2
      B8 = A5+A8
      B7 = 2.*A4
      B6 = A3+A7
      B5 = A2+A6
      B4 = 2.*A1
      B3 = A5-A8
      B2 = A3-A7
      B1 = A2-A6
      TIS = TI**2
      COE(5) = B2**2-2.*TIS*B7
      COE(4) = 2.*B2*B3-2.*TIS*B6
      COE(3) = 2.*B1*B2+B3**2-2.*TIS*B8+TIS**2
      COE(2) = 2.*B1*B3-2.*TIS*B5
      COE(1) = B1**2-2.*TIS*B4
C
C     SOLUTION OF GOVERNING POLYNOMIAL AND ISOLATION OF REAL ROOTS
C
      CALL CPOLR(COE,4,ROOTR,ROOTI)
      J=0
      DO 20 M=1,4
      IF(DABS(ROOTR(M)) .EQ. 0.D0) GO TO 20
      IF(DABS(ROOTI(M)).GT. 1.D-4) GO TO 20
      J=J+1
```

```
      F(J)=SNGL(DABS(ROOTR(M)))
20    CONTINUE
      IF(J.EQ.0) 200,201
200   WRITE(6,15) THETA
      IF(THETA.GT.250..AND.J.EQ.0) GO TO 22
      GO TO 21
201   CALL ELIM(J)
      IF(IFLAG.GE.2) 210,211
210   WRITE(6,101) THETA
101   FORMAT(* ALL TRAJECTORIES FOR THETA=*,F6.2,* AND GREATER ARE INADM
     1ISSIBLE*)
      GO TO 22
C
C     STORAGE OF RELEVANT DATA
C
211   TOFMIND = TOFMIN*TUNIT
      LXMIND = LXMIN*RUNIT
      N=N+1
      MINT(N)=TOFMIND
      ANGLE(N)=THETA
      ANGLEP(N) = THETAPD
      LONGT(N) = TOFAD
      DIRECT(N) = TOFDD
      IF(ECMIN.GE.1.) DIRECT(N)=TOFMD
      PARTIAL(N) = TOFPD
      WRITE(6,30)ANGLE(N),DIRECT(N),LONGT(N),PARTIAL(N),ANGLEP(N),MINT(
     1N)
21    IF(THETA.LT.40.) 500,501
500    THETA=THETA+1.
      GO TO 190
501   IF(THETA.GT.39. .AND. THETA.LT.309.) 502,503
502   THETA=THETA+10.
      GO TO 190
503   THETA=THETA+1.
190   IF(THETA.EQ.160.) GO TO 21
      IF(THETA.EQ.360.) GO TO 22
      GO TO 19
1     FORMAT(8F10.0)
9     FORMAT(*//////////////////////////////////////////////////////////*)
10    FORMAT(1X,*POLYNOMIAL COEFFICIENTS: *,/5(1X,D15.8,4X))
11    FORMAT(*ROOTS: *,3X,*REAL*,16X,*IMAGINARY*,/10X,D15.8,5X,D15.8)
12    FORMAT(//*ONUMBER OF REAL ROOTS IS *,I3)
13    FORMAT(* THEIR VALUES ARE:*,/4(1X,E15.8,4X))
14    FORMAT(//* FOR THETA = *,F5.1,* DEGREES,*/15X,*MINIMUM TIME OF FLI
     1GHT IS *,E15.8,* SEC AND THE CORRESPONDING SLR IS *,F15.8,* KM*)
15    FORMAT(* THE ROCKET CAN-T ATTAIN TGT ALTITUDE FOR THETA = *,F6.2)
16    FORMAT(*1INPUT PARAMETERS: C,RATIO,ECT,LT,FT,AT,RT*,/4(1X,F12.6,4X
     1))
17    FORMAT(/*CTI = *,F12.6,* KM/SEC*)
18    FORMAT(1X,F 6.2,81X,F10.4)
100   FORMAT(//* THETA,*,4X,*DIRECT ROUTE*,3X,*APOGEE ROUTE*,3X,*PARTIAL
     1 APOGEE ROUTE*,5X,*ACTUAL THETA,*,12X,*MINIMUM*
     2         /* DEG*,7X,*TOF, SEC*,7X,*TOF, SEC*,7X,*TOF, SEC*,17X,*PA
     3RTIAL ROUTE, DEG*,7X,*TOF, SEC*)
30    FORMAT(1X ,F6.2,4X,F12.4,3X,F12.4,3X,F20.4,5X,F18.4,3X,F10.4)
```

```
C
C       DATA OUTPUT
C
  22    CONTINUE
  998   WRITE(6,997)
  997   FORMAT(*S*)
  999   STOP
        END
```

```
      SUBROUTINE ELIM(N)
      COMMON R(4),A1,A2,A3,A4,A5,A6,A7,A8,W1,W2,W4,V1S,V2S,R1,RT,TOFMIN,
     1 LXMIN,MU,PI,RAD,SRMU,RUNIT,TUNIT,VUNIT,TID,THETAFD,TOFAD,TOFCD,
     2 TOFFD,TOFHD,ECMIN,IFLAG
      REAL LX    ,MU,LXMIN,LXD
      EXTERNAL GLAD
C
C     ISOLATE A SEMI-LATUS RECTUM
C
      IFLAG = 0
      TOF1=1.E7
      DO 90 J=1,N
      IF(R(J).EQ.0.) GO TO 90
      LX    =R(J)**2
      HX = SQRT(MU*LX    )
C
C     CALCULATION OF VELOCITIES AND ANGLES AT EACH TERMINAL
C
      VX1S    =(A1*LX    )+(A4/LX    )+(2.*W1*W2)
      VX1=SQRT(VX1S    )
      VX1D = VX1*VUNIT
      V1 = SQRT(V1S)
      V1D = V1*VUNIT
      DV1S=A1*LX    +A2*R(J)+A3/R(J)+A4/LX    +A5
      DV1=SQRT(DV1S)
      DV1D = DV1*VUNIT
      VX2S=A1*LX    +A4/LX    -2.*W1*W4
      VX2=SQRT(VX2S)
      VX2D = VX2*VUNIT
      V2 = SQRT(V2S)
      V2D = V2*VUNIT
      DV2S=A1*LX    +A6*R(J)+A7/R(J)+A4/LX    +A8
      DV2=SQRT(DV2S)
      DV2D = DV2*VUNIT
      TDVD = DV1D + DV2D
      DIF = ABS(TDVD-TID)
      IF(DIF.GT..1) GO TO 90
      PSI1=PI-ACOS((DV1S+V1S-VX1S    )/(2.*DV1*V1))
      PSI2=   -ACOS((DV2S+V2S-VX2S    )/(2.*DV2*V2))
      PSI1D=PSI1*RAD
      PSI2D=PSI2*RAD
C
C     CHECK TO SEE IF TRAJECTORY VIOLATES LAUNCH CONSTRAINT
C
      VXRAD1=W1/R(J)+W2*R(J)
      IF(VXRAD1.LT.0.) 40,41
 40   IFLAG = IFLAG + 1
      GO TO 90
C
C     COMPUTE SEMI-MAJOR AXIS AND ECCENTRICITY
C
 41   AX    =1./((2./R1)-(VX1S    /MU))
      ECX    =SQRT(1.-(LX    /AX    ))
C
C     COMPUTE TRANSFER TIME
```

```
C
      SINFX1   =(1./ECX    )*( W1+W2*LX    )/SRMU
      COSFX1 = (LX    /R1-1.)/ECX
      CALL QUAD(SINFX1    ,COSFX1,FX1)
      SINFX2   =(1./ECX    )*(-W1+W4*LX    )/SRMU
      COSFX2 = (LX    /RT-1.)/ECX
      CALL QUAD(SINFX2    ,COSFX2,FX2)
      IF(ECX    .GT.1.) 30,31
 30   H1O2 = ATAN(SQRT((ECX    -1.)/(ECX    +1.))*TAN(FX1/2.))
      H2O2 = ATAN(SQRT((ECX    -1.)/(ECX    +1.))*TAN(FX2/2.))
      TOF     = SQRT(((-AX    )**3)/MU)*(ECX    *(TAN(H2O2*2.)-TAN(H1O2*2.)
     1)-ALOG(TAN(PI/4.+H2O2)/TAN(PI/4.+H1O2)))
      TOFHD = TOF    *TUNIT
      WRITE(6,20) TOFHD,ECX
      IF(TOFHD.LT.0.) GO TO 40
      GO TO 33
 31   SINEA1    =(R1*SINFX1    )/(AX    *SQRT(1.-ECX    **2))
      COSEA1=(ECX+COSFX1)/(1.+ECX*COSFX1)
      CALL QUAD(SINEA1,COSEA1,EA1)
      SINEA2    =(RT*SINFX2    )/(AX    *SQRT(1.-ECX    **2))
      COSEA2=(ECX+COSFX2)/(1.+ECX*COSFX2)
      CALL QUAD(SINEA2,COSEA2,EA2)
      DELSE    =SINEA2    -SINEA1
      EA1D=EA1    *RAD
      EA2D=EA2    *RAD
      DELEA    =EA2    -EA1
      TOF    =SQRT(AX    **3/MU)*(DELEA    -ECX    *DELSE    )
      IF(EA2D.LT.180.) TOFDD = TOF    *TUNIT
C
C     IF APOGEE ROUTE,CALCULATE PARTIAL APOGEE ROUTE RANGE ANGLE AND TIME
C     OF FLIGHT
C
      IF(EA2D.GT.180.) 50,33
 50   FX2P = 2.*PI - FX2
      SINEA2P = (RT*SIN(FX2P))/(AX    *SQRT(1.-ECX    **2))
      EA2P = ASIN(SINEA2P)
      IF(RT.GT.AX    ) EA2P = PI - EA2P
      THETAPD = (FX2P-FX1)*RAD
      TOFAD= TOF    *TUNIT
      TOF     = SQRT(AX    **3/MU)*((EA2P-EA1    )-ECX    *(SINEA2P-SINEA1))
      TOFPD = TOF    *TUNIT
 33   LXD = LX    *RUNIT
      AXD = AX    *RUNIT
C
C     SELECT AND STORE MINIMUM TIME ORBIT
C
      IF(VXPAD1.LT.0.) TOF     = 1.E8
      IF(TOF    .GT.TOF1) GO TO 90
      TOF1=TOF
      TOFMIN=TOF1
      LXMIN=LXD
      ECMIN=ECX
 90   CONTINUE
 20   FORMAT( 5X,F15.4,35X,F10.4,2X,F10.6)
 21   FORMAT(20X,F15.4,5X,F15.4,10X,F10.4,31X,F10.4,2X,F10.6)
```

```
22    FORMAT(/* XFER ORBIT HYPERBOLIC*)
      RETURN
      END
```

```
      SUBROUTINE QUAD(SARG,CARG,ANGLE)
C
C     INPUTS: SINE AND COSINE OF ARGUMENT; OUTPUT: ARGUMENT IN PROPER QUADRANT
C
      PI = 3.14159265359
      IF(SARG.GE.0..AND.CARG.GE.0.) ANGLE = ASIN(SARG)
      IF(SARG.GE.0..AND.CARG.LE.0.) ANGLE = ACOS(CARG)
      IF(SARG.LE.0..AND.CARG.LE.0.) ANGLE = ATAN(SARG/CARG) + PI
      IF(SARG.LE.0..AND.CARG.GE.0.) ANGLE = ASIN(SARG) + 2.*PI
      RETURN
      END
```

INPUT PARAMETERS: C,RATIO,ECT,LT,FT,AT,RT

4.000000	.050000	0.000000	6828.165000
0.000000	6828.165000	6828.165000	

THETA, DEG	DIRECT ROUTE TOF, SEC	APOGEE ROUTE TOF, SEC	PARTIAL APOGEE ROUTE TOF, SEC	ACTUAL THETA, PARTIAL ROUTE, DEG	MINIMUM TOF, SEC
1.00	114.6242	842.9983	133.7953	.1805	114.6242
2.00	101.5411	880.5939	129.4310	.3376	101.5411
3.00	96.5252	915.2908	125.8131	.4791	96.5252
4.00	98.7337	947.7704	122.7614	.6059	98.7337
5.00	100.2684	978.4966	120.1249	.7236	100.2684
6.00	105.9589	1007.8018	117.8328	.8333	105.9589
7.00	113.0995	1035.9335	115.8094	.9364	113.0995
8.00	121.2182	1063.0619	114.0113	1.0339	114.0113
9.00	130.0503	1089.3969	112.4025	1.1266	112.4025
10.00	139.[illegible]154	1114.9989	110.9548	1.2152	110.9548
11.00	149.1661	1139.9862	109.6450	1.3002	109.6450
12.00	159.2171	1164.4408	108.4579	1.3819	108.4579
13.00	169.[illegible]027	1188.4312	107.3797	1.4609	107.3797
14.00	179.3744	1212.0158	106.3872	1.5372	106.3872
15.00	190.5956	1235.2447	105.4820	1.6113	105.4820
16.00	201.7381	1258.1614	104.6516	1.6833	104.6516
17.00	212.1801	1280.8035	103.8834	1.7534	103.8834
18.00	223.1041	1303.2047	103.1959	1.8215	103.1959
19.00	234.0963	1325.3931	102.5388	1.8885	102.5388
20.00	245.1453	1347.3961	101.9421	1.9533	101.9421
21.00	256.2418	1369.2364	101.3916	2.0177	101.3916
22.00	267.3779	1390.9351	100.8836	2.0804	100.8836
23.00	278.5471	1412.5108	100.4147	2.1413	100.4147
24.00	289.[illegible]439	1433.9806	99.9819	2.2022	99.9819
25.00	300.7636	1455.3601	99.5927	2.2615	99.5927
26.00	312.2019	1476.6632	99.2146	2.3199	99.2146
27.00	323.4553	1497.9029	98.8754	2.3773	98.8754
28.00	334.7207	1519.0909	98.5634	2.4339	98.5634
29.00	345.9953	1540.2381	98.2766	2.4896	98.2766
30.00	357.2765	1561.3544	98.0135	2.5445	98.0135
31.00	368.[illegible]622	1582.4491	97.7727	2.5987	97.7727
32.00	379.3503	1603.5306	97.5527	2.6522	97.5527
33.00	391.1389	1624.6069	97.3524	2.7051	97.3524
34.00	402.4264	1645.6854	97.1707	2.7573	97.1707
35.00	413.7111	1666.7728	97.0054	2.8089	97.0054
36.00	413.7111	1687.8756	96.8587	2.8599	96.8587
37.00	413.7111	1708.9996	96.7266	2.9103	96.7266
38.00	413.7111	1730.1505	96.6093	2.9602	96.6093
39.00	413.7111	1751.3335	96.5061	3.0095	96.5061
40.00	413.7111	1772.5534	96.4152	3.0585	96.4152
50.00	413.7111	1987.6797	96.1237	3.5240	96.1237
60.00	413.7111	2210.2657	96.6422	3.9552	96.6422
70.00	413.7111	2442.4618	97.7000	4.3611	97.7000
80.00	413.7111	2665.7606	99.1365	4.7476	99.1365
90.00	413.7111	29[illegible]1.3518	100.8532	5.1197	100.8532
100.00	413.7111	3210.3179	102.7930	5.4812	102.7930
110.00	413.7111	3493.7531	104.9119	5.8360	104.9119
120.00	413.7111	3792.6447	107.2014	6.1573	107.2014
130.00	413.7111	4108.9349	109.6541	6.5386	109.6541
140.00	413.7111	4443.5749	112.2760	6.8933	112.2760
150.00	413.7111	4798.5773	115.0827	7.2551	115.0827

160.00	413.7111	5176.0718	119.0934	7.6279	119.0934
170.00	413.7111	5578.5665	121.3572	8.0163	121.3572
190.00	413.7111	6470.8582	125.7991	8.6517	125.7991
200.00	413.7111	6988.2589	133.1175	9.3325	133.1175
210.00	413.7111	7505.9384	137.9535	9.8474	137.9535
220.00	413.7111	8089.3174	143.4564	10.4187	143.4564
230.00	413.7111	8724.8036	149.7838	11.0523	149.7838
240.00	413.7111	9418.5343	157.1808	11.7495	157.1808
250.00	413.7111	10177.9952	165.9735	12.6597	165.9735
260.00	413.7111	11069.1081	176.6534	13.6345	176.6534
270.00	413.7111	11915.3927	189.8983	14.9347	189.8983
280.00	413.7111	12894.2481	205.7744	16.5024	206.7744
290.00	413.7111	13930.3929	223.9724	18.5346	223.9724
300.00	413.7111	14983.6536	259.3521	21.2796	259.3521
310.00	413.7111	15965.3416	303.1438	25.1041	303.1438
311.00	413.7111	16053.6553	308.5939	25.6791	308.5939
312.00	413.7111	16139.2826	314.2836	26.1859	314.2836
313.00	413.7111	16221.9253	320.2475	26.7151	320.2475
314.00	413.7111	16301.2775	326.4854	27.2691	326.4854
315.00	413.7111	16377.0037	333.0229	27.8493	333.0229
316.00	413.7111	16448.7482	339.8813	28.4578	339.8813
317.00	413.7111	16516.0916	347.0342	29.0965	347.0342
318.00	413.7111	16578.6292	354.6572	29.7679	354.6572
319.00	413.7111	16635.8850	362.6289	30.4743	362.6289
320.00	413.7111	16687.3499	371.0306	31.2187	371.0306
321.00	413.7111	16732.4676	379.8959	32.0041	379.8959
322.00	413.7111	16770.6299	389.2666	32.8341	389.2666
323.00	413.7111	16801.1708	399.1824	33.7126	399.1824
324.00	413.7111	16823.3595	409.6924	34.6435	409.6924

ALL TRAJECTORIES FOR THETA=325.00 AND GREATER ARE INADMISSIBLE

```
      PROGRAM REVZ2 (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,PLOT)
C
C     THIS PARTICULAR VERSION WRITTEN TO COMPARE RENDEZVOUS TIMES FOR MASS
C     RATIOS OF .05 AND .10
C
      DIMENSION THETAF(250),TMIN(250),TTO(250),TTRDV(250),THETAR(250)
      DIMENSION ITR(250)
      REAL ITP(250),MU
C     CONSTANTS
      MU = 3.986032E5      $       RAD = 57.2957795131
      RT = 6828.165        $       WT = SQRT(MU/RT**3)*RAD
 997  READ(5,1)RATIO,N
      WRITE(6,10) RATIO
C
C     READ IN TRAJECTORIES AS TABLE OF RANGE ANGLES AND TIMES
C
      READ(5,2) (THETAF(I),TMIN(I),I=1,N)
C
C     GENERATE TABLE OF INITIAL TARGET POSITION AS FUNCTION OF RANGE ANGLE
C     (OR RENDEZVOUS LOCATION) AND DETERMINE WINDOW BOUNDARIES
C
      DO 100 I=1,N
 100  ITP(I)=THETAF(I)-WT*TMIN(I)
      BETAL=ITP(1) $       TBETAL=TMIN(1)
      BETAU=ITP(N)     $       TBETAU=TMIN(N)
      WRITE(6,15) BETAL,TBETAL,THETAF(1),BETAU,TBETAU,THETAF(N)
C
C     EXAMINE ALL TARGET POSITIONS FROM -180 DEG TO 180 DEG
C
      THETATO=-180.
      DO 300 I=1,500
 200  IF(BETAL.LE.THETATO .AND. THETATO.LE.BETAU) GO TO 201
C
C     OUTSIDE WINDOW
C
      DTHETA=5.
      IF(THETATO.LT.BETAL) 400,401
 400  IF(RATIO.LT..07)        TTRDV(I)  = ((BETAL-THETATO)/WT)+TMIN(1)
      IF(RATIO.GT..07)        TTR   (I)  = ((BETAL-THETATO)/WT)+TMIN(1)
 401  IF(THETATO.GT.BETAU) 402,403
 402  IF(RATIO.LT..07)        TTRDV(I)  = ((360.+BETAL-THETATO)/WT)+TMIN(1)
      IF(RATIO.GT..07)        TTR   (I)  = ((360.+BETAL-THETATO)/WT)+TMIN(1)
 403  THETAR(I) = THETAF(1)
      TTO(I) = THETATO
      IF(BETAL.LT.(THETATO+DTHETA) .AND. (THETATO+DTHETA).LT.BETAU) DTHE
     1TA = .5
      THETATO=THETATO+DTHETA
      IF(THETATO.GT.180.) GO TO 999
      GO TO 300
C
C     INSIDE WINDOW
C
 201  THETAR(I) = ATKN(ITP,THETAF,N,1,THETATO)
      IF(RATIO.LT..07)TTRDV(I) = ATKN(THETAF,TMIN,N,3,THETAR(I))
      IF(RATIO.GT..07)TTR   (I) = ATKN(THETAF,TMIN,N,3,THETAR(I))
```

```
      TT0(I)=THETATO
      THETATC=THETATO+DTHETA
      IF(THETATO.GT.180.) GO TO 999
300   CONTINUE
1     FORMAT(F5.2,I5)
2     FORMAT(2F20.4)
10    FORMAT(*1FINAL MASS = *,F5.2)
11    FORMAT(    *0THETA-T-0*,T15* TTRDV *,T25,*THETA-R*,T35,*THETA-T-0*,
     1T49,* TTRDV *,T60,*THETA-R*/1X,*(DEG)*,T15,*(SEC)*,T25,*(DEG)*,
     2T35,*(DEG)*,T49,*(SEC)*,T60,*(DEG)* )
12    FORMAT(1X,2(F8.2,4X,F8.2,4X,F6.2,4X))
15    FORMAT(*0BETA-L = *,F8.4,* DEG; TIME OF FLIGHT FOR LAUNCH AT BETA-
     1L = *,F8.4,* SEC; THETA-F = *,F8.4,* DEG*/* BETA-U = *,F8.4,* DEG;
     2 TIME OF FLIGHT FOR LAUNCH AT BETA-U = *,F8.4,* SEC; THETA-F = *,
     3 F8.4,* DEG*)
999   WRITE(6,11)
      WRITE(6,12)(TT0(J),TTRDV(J),THETAR(J),J=1,I)
998   STOP      $  END
```

FINAL MASS = .05

BETA-L = -8.6482 DEG: TIME OF FLIGHT FOR LAUNCH AT BETA-L = 136.3539 SEC; THETA-R = .0937 DEG
BETA-U = 8.4764 DEG: TIME OF FLIGHT FOR LAUNCH AT BETA-U = 413.7100 SEC; THETA-R = 35.0000 DEG

THETA-T-0 (DEG)	TTRDV (SEC)	THETA-R (DEG)	THETA-T-0 (DEG)	TTRDV (SEC)	THETA-R (DEG)
-180.00	2809.06	.09	-175.00	2731.07	.09
-170.00	2653.09	.09	-165.00	2575.10	.09
-160.00	2497.11	.09	-155.00	2419.12	.09
-150.00	2341.13	.09	-145.00	2263.14	.09
-140.00	2185.15	.09	-135.00	2107.16	.09
-130.00	2029.17	.09	-125.00	1951.18	.09
-120.00	1873.20	.09	-115.00	1795.21	.09
-110.00	1717.22	.09	-105.00	1639.23	.09
-100.00	1561.24	.09	-95.00	1483.25	.09
-90.00	1405.26	.09	-85.00	1327.27	.09
-80.00	1249.28	.09	-75.00	1171.30	.09
-70.00	1093.31	.09	-65.00	1015.32	.09
-60.00	937.33	.09	-55.00	859.34	.09
-50.00	781.35	.09	-45.00	703.36	.09
-40.00	625.37	.09	-35.00	547.38	.09
-30.00	469.40	.09	-25.00	391.41	.09
-20.00	313.42	.09	-15.00	235.43	.09
-10.00	157.44	.09	-9.50	149.64	.09
-9.00	141.64	.09	-8.50	134.79	.15
-8.00	129.81	.32	-7.50	124.91	.52
-7.00	120.25	.72	-6.50	115.89	.93
-6.00	111.76	1.17	-5.50	107.92	1.42
-5.00	104.43	1.70	-4.50	101.49	2.01
-4.00	99.03	2.35	-3.50	97.25	2.73
-3.00	96.25	3.17	-2.50	96.22	3.67
-2.00	97.33	4.24	-1.50	99.79	4.90
-1.00	103.80	5.66	-.50	109.54	6.52
0.00	117.12	7.51	.50	126.59	8.62
1.00	137.90	9.84	1.50	150.90	11.17
2.00	165.42	12.61	2.50	181.23	14.12
3.00	198.13	15.70	3.50	215.92	17.34
4.00	234.41	19.03	4.50	253.46	20.75
5.00	272.96	22.50	5.50	292.77	24.27
6.00	312.84	26.06	6.50	333.08	27.85
7.00	353.43	29.66	7.50	373.84	31.47

8.00	334.27	33.28	8.50	5484.06	.09
13.50	5406.09	.09	18.50	5328.11	.09
23.50	5250.12	.09	28.50	5172.13	.09
33.50	5094.14	.09	38.50	5016.15	.09
43.50	4938.16	.09	48.50	4860.17	.09
53.50	4782.18	.09	58.50	4704.19	.09
63.50	4626.21	.09	68.50	4548.22	.09
73.50	4470.23	.09	78.50	4392.24	.09
83.50	4314.25	.09	88.50	4236.26	.09
93.50	4158.27	.09	98.50	4080.28	.09
103.50	4002.29	.09	108.50	3924.30	.09
113.50	3846.32	.09	118.50	3768.33	.09
123.50	3690.34	.09	128.50	3612.35	.09
133.50	3534.36	.09	138.50	3456.37	.09
143.50	3378.38	.09	148.50	3300.39	.09
153.50	3222.40	.09	158.50	3144.42	.09
163.50	3066.43	.09	168.50	2988.44	.09
173.50	2910.45	.09	178.50	2832.46	.09

Vita

Steven H. Edelman [redacted] and immediately entered the United States Air Force Academy. He graduated from USAFA in 1969 with a Bachelor of Science degree in Engineering Sciences and Astronautical Engineering and an Air Force commission as a Second Lieutenant. His first assignment was as a Project Officer in the Deputy for Development Plans, Headquarters Space and Missile Systems Organization, Los Angeles, California. He entered the Graduate Astronautics program at the Air Force Institute of Technology in June, 1972.

[redacted]