WHO ACCEPTS SAVAGE'S AXIOM

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Prepared for:

Office of Naval Research

February 1974

Unclassified
Security Classification

AD 774 927

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)*
Oregon Research Institute
Eugene, Oregon

2a. REPORT SECURITY CLASSIFICATION
Unclassified

2b. GROUP

3. REPORT TITLE
Who accepts Savage's axiom?

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*
Technical Report

5. AUTHOR(S) *(First name, middle initial, last name)*
Paul Slovic and Amos Tversky

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
February, 1974	18	8

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
N00014-73-C-0438	Oregon Research Institute Research Bulletin, Volume 14, No. 2.
b. PROJECT NO. NR(197-026)	
c. ARPA Order No. 2449	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*
d.	

10. DISTRIBUTION STATEMENT
This document has been approved for public release and sale. Its distribution is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Office of Naval Research Code 455 Arlington, Virginia 22217

13. ABSTRACT

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DD FORM 1 NOV 1473 REPLACES DD FORM 1473, 1 JAN 64, WHICH IS OBSOLETE FOR ARMY USE.

Unclassified
Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Decision Theory						
Rational Choice						
Savage's Independence Principle						

ABSTRACT

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WHO ACCEPTS SAVAGE'S AXIOM?[1]

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Individuals' actions in the face of uncertainty often violate the axioms of rational decision theory. Nevertheless, many decision theorists maintain that these violations should be treated as errors of judgment due to carelessness, lack of proper incentives, or simple misunderstandings. These theorists assert that if the axioms were presented in a clear and understandable fashion, no reasonable person would wish to violate them (Borch, 1968; Savage, 1954; Raiffa, 1961).

According to this view, the axioms of decision theory are treated on a par with principles of logic (Marschak, 1968), that are sometimes violated in ordinary discourse but are never rejected because of such violations. One decision principle that enjoys this status is transitivity, perhaps because it underlies the notion of ordering that is inherent in our conception of a preference relation. Thus, although transitivity is violated under some conditions, individuals are quick to revise their preference and conform to the principle once they are made aware of their intransitivity (MacCrimmon, 1968; Tversky, 1969).

While transitivity is generally accepted, other essential axioms of expected utility theory have met with some resistance, notably Savage's (1954) independence principle (SIP), also called the sure-thing principle. This axiom asserts that if two alternatives

have a common outcome under a particular state of nature, then the ordering of the alternatives should be independent of the value of that common outcome. Objections to SIP, in the form of counter-examples, have been proposed by Allais (1953) and Ellsberg (1961). These counter-examples have been the subject of much discussion among decision theorists, most of whom were initially tempted to accept them but eventually reaffirmed their belief in SIP (see, for example, Savage, 1954; Raiffa, 1961; Ellsberg, 1961).

An empirical study by MacCrimmon (1968) examined the degree to which business executives, participating in a training program for top management positions, accepted the basic postulates of expected utility theory. These individuals were given hypothetical decision problems especially designed to test these postulates. After their initial responses, the managers were given an opportunity to reflect upon their answers. They were given prepared arguments both conforming with and conflicting with the postulates and they were asked to provide a critique of each answer and to select the one that was more logical. Finally, each subject participated individually in a thirty minute discussion in which he was questioned about his decisions. The results showed that the subjects' initial choices often violated the postulates. Furthermore, subjects' ratings of the prepared arguments also indicated resistance to the postulates. During the presumably neutral discussion, however, almost all opposition to the postulates vanished, and the subjects tended to regard their violations as "mistakes". SIP was violated by about 40% of the subjects on the initial choices, but, nevertheless, was accepted by an overwhelming majority in the course of the discussion.

MacCrimmon's conclusion that the axioms were acceptable is open to question. The second phase of the experiment, in which arguments pro and con were presented, seemed to provide a fair test of the attractiveness of the axioms. Nevertheless, considerable resistance tc the axioms was in evidence. MacCrimmon was later able to get people to accept the axioms during the discussion, but it is not clear what was needed to obtain this agreement, beyond an intelligible presentation of the arguments. Despite the intent to keep the discussion neutral, subtle pressures, in combination with the cooperativeness of subjects participating in a training course for a prestigious job, may have influenced the subjects to conform to the axioms. In the absence of social pressure, do reasonable people who understand the competing arguments, accept SIP? The present study is addressed to this que ion.

Experiment I

In the first experiment, 29 male students from the University of Oregon participated as paid subjects. They were presented with a booklet describing the following two decision problems. Problem I was formulated by Allais and Problem II was formulated by Ellsberg.

PROBLEM I

Imagine the following two decision situations - - each involving a pair of gambles.

Situation A:

	Probability of winning	Amount to win
Gamble 1.	100%	$1,000,000
Gamble 2.	10%	$5,000,000
	89%	$1,000,000
	1%	$ 0

Situation B:

	Probability of winning	Amount to win
Gamble 3.	11%	$1,000,000
	89%	$ 0
Gamble 4.	10%	$5,000,000
	90%	$ 0

PROBLEM II

Imagine an urn known to contain 90 balls. Thirty of the balls are red, the remaining 60 are black and yellow in unknown proportion. One ball is to be drawn at random from the urn. Consider the following actions and payoffs:

Situation A:

	30	60	
	Red	Black	Yellow
Act 1. Bet on red	$100	$ 0	$ 0
Act 2. Bet on black	$ 0	$100	$ 0

If you bet on red (ActI) you will win $100 if a red ball is drawn and nothing if a black or yellow ball is drawn.

If you bet on black (Act 2) you will win $100 if a black ball is drawn and nothing if a red or yellow ball is drawn.

Now we consider the following two actions, under the same circumstances:

Situation B:

	30	60	
	Red	Black	Yellow
Act 3. Bet on red or yellow	$100	$ 0	$100
Act 4. Bet on black or yellow	$ 0	$100	$100

Here, if you bet on red or yellow (Act 3) you will win $100 if either a red or a yellow ball is drawn and win nothing if black is drawn.

If you choose Act 4, you win $100 if either a black or yellow ball is drawn and win nothing if red is drawn.

The subjects were asked to read each problem carefully and indicate their preferences for the acts or gambles under consideration. Thus, in Problem I, they had to choose between Gambles 1 and 2 in Situation A, and between Gambles 3 and 4 in Situation B. Likewise, they had to make a choice in each of the two situations in Problem II.

As will be shown later, SIP implies that one must choose either 1 and 3 or 2 and 4 in both problems. On the other hand, the choice of 1 and 4 is recommended by Allais (in Problem I) and by Ellsberg (in Problem II). After the subjects had responded to both problems, they were given an argument pertaining to Problem I which advocated decisions different from their own. The subjects were told:

> "Now that you have made your choices you might be interested in reading what a prominent decision theorist has to say about this problem."

Subjects who satisfied SIP were given the response of Dr. A. advocating Allais' position. Subjects who chose according to Allais's recommendation were given the response of Dr. S., advocating Savages's independence principle. The two arguments read as follows.

Response of Dr. A.

> "I would choose Gamble 1 over Gamble 2 in Situation A and Gamble 4 over Gamble 3 in Situation B.
>
> In Situation A, I have a choice between $1,000,000 for certain and a gamble where I might end up with nothing. Why gamble? The small probability of missing the chance of a lifetime to become rich seems very unattractive to me.
>
> In Situation B, there is good chance that I will end up with

nothing no matter what I do . The chances of getting $5,000,000 are almost as good as getting $1,000,000 so I might as well go for the $5,000,000 and choose Gamble 4 over Gamble 3."

Response of Dr. S.

"One way in which Gambles 1,2,3, and 4 could be played is by means of a lottery. Suppose we had 100 numbered tickets in a bowl where 1 ticket would be selected at random to determine the outcome. The four gambles can thus be represented as in the table below. The payoffs are the amounts that would be won if a ticket whose number appears at the top of the column is drawn.

		Ticket Number		
		1	2-11	12-100
Situation A:	Gamble 1.	$1,000,000	$1,000,000	$1,000,000
	Gamble 2.	$ 0	$5,000,000	$1,000,000

		1	2-11	12-100
Situation B:	Gamble 3	$1,000,000	$1,000,000	$ 0
	Gamble 4	$ 0	$5,000,000	$ 0

Now, if one of the tickets numbered from 12 through 100 is drawn, it will not matter, in either situation, which Gamble I choose. I therefore focus on the possibility that one of the tickets numbered 1-11 will be drawn, in which case Situations A and B are exactly parallel. My decision in both situations depends on whether I would rather have an outright gift of $1,000,000 or gamble to win $5,000,000.

(a) If I prefer the gift of $1,000,000. I should choose Gamble 1 over Gamble 2 and Gamble 3 over Gamble 4.

(b) If I prefer the gamble for $5,000,000. I should choose Gamble 2 over Gamble 1 and Gamble 4 over Gamble 3.

No other pairs of choices are logical. I imagine that I would choose Gamble 1 over Gamble 3."

Following the presentation of the arguments, subjects were asked to reconsider Problems I and II and make their decisions again. They did not receive any arguments pertaining to Problem II so as to permit a test of the effect of the presented arguments.

INSERT TABLE 1 ABOUT HERE

On Problem I, only a few individuals switched their choices after exposure to counter arguments. Moreover, the argument of Dr. S was slightly less effective than the argument of Dr. A. Table 1 indicated that on the first choices, 17 persons chose according to A and 12 chose according to S. The frequences shifted only slightly on the second choices (A 19, S 10). In fact, fewer persons changed on Problem I than on Problem II where no counter-arguments were presented. Note that in Problem I the majority of subjects violated SIP on both the first and second choices.

The failure of the counter-arguments to influence subjects' preferences in Problem I is consistent with MacCrimmon's finding that subjects selected as 'more logical' the argument that was compatible

with their initial preferences. Nevertheless, this resistance to change contrasts greatly with the effects of the discussion session in MacCrimmon's study and it raises questions regarding the nature of that discussion.

Experiment II

The results of the first experiment showed that the counter-examples to SIP had strong appeal. These findings prompted a second experiment, to examine the attractiveness of SIP in a situation where special efforts were made to present the relevant arguments in a clear and compelling fashion.

Experiment II differs from the first experiment in the following ways. First, subjects were exposed to the arguments for and against SIP prior to making any choices on both Problems I and II. Second, the subjects were run in small groups and, after they had read the argument for the two opposing positions, the experimenter went through each argument, step by step, in an attempt to make certain that everyone understood the reasoning. Third, prior to their choices, subjects were asked to rate the arguments of the opposing decision theorists in terms of how compelling they were. A scale ranging from 1 (not compelling at all) to 11 (very compelling) was used for this purpose.

Forty-nine male students from the University of Oregon served as paid subjects. Some groups of subjects worked on Problem I first; others started with Problem II.

The arguments presented by Dr. A. and Dr. S. for Problem I were the same as those used in Experiment I. For Problem II, subjects

were presented with two additional arguments, one by Dr. S. promoting SIP, and one by Dr. A. advocating an opposing position. These arguments read as follows:

Response of Dr. A.

"I would choose Act 1 over Act 2 and Act 4 over Act 3. The expected payoff for Act 1 is definite and satisfactory (i.e. 1/3 chance of winning $100). The expected payoff for Act 2 might be greater, but I know the risk for Act 1 and I do not know it for Act 2. The expectation for Act 2 is ambiguous. It might be better or it might be worse. To be on the safe side, I'll choose Act 1. By the same token, Act 4 looks better than Act 3. I know what my chances are with Act 4 whereas they might be much poorer with Act 3.

I know this is a rather pessimistic view. An optimist might choose Act 2 over Act 1 and Act 3 over Act 4."

Response of Dr. S.

"If a yellow ball is drawn it will not matter, in either situation, which act I choose. So let us concentrate our attention upon what would happen if a red or black ball was chosen , in which case A and B are exactly parallel. If I felt that red was more probable than black, I would choose Act 1 over Act 2 and Act 3 over Act 4. If I felt black was more likely than red, I would choose Act 2 over Act 1 and Act 4 over Act 3. If I felt that red and black were equally likely, I would be indifferent between Act 1 and Act 2 and between Act 3 and Act 4 ."

To permit the subjects to consult the arguments while making their responses, written statements of the problem and the decision theorists' views were kept in front of them.

INSERT TABLE 2 ABOUT HERE

Table 2 presents the results of this experiment. Looking first at Problem I we see that A's argument was rated slightly higher than S's argument. However, the majority of choices conformed to SIP. The lack of agreement between the ratings of logical persuasiveness and the choices on Problem I replicates MacCrimmon's findings. His results, as well as the comments of subjects in the present study, indicated that some subjects satisfied SIP because the actions recommended by A appeared too conservative, and not because the argument of S was more compelling.

No disagreement between ratings and choices was observed in Problem II. Both responses indicated overwhelming preference for the position advocated by A.

DISCUSSION

The following is a hypothetical dialogue between our two contenders, Dr. S. and Dr. A.

Dr. A.: It seems to me the results of these experiments show that the subjects violated SIP and did not find your reasoning

particularly compelling. Furthermore, my arguments were at least as convincing as yours. Would you not agree, then, that SIP cannot be regarded as a generally accepted principle of choice?

Dr. S.: I agree with your summary of the results, but I cannot accept your conclusions. When I first saw your problems, I, too, was tempted to choose in a way that violated the axiom and I still feel an intuitive attraction to those preferences. However, after further analysis and deliberation, I have learned to regard my initial responses as erroneous. Had the subjects fully understood the normative nature of the axiom, I believe they would have changed their preferences to conform to it.

Dr. A.: At first I thought that SIP should be satisfied by people's preferences. Then I was told that while it may be violated in practice, people will accept it in principle once it is presented to them clearly. Now, the present results show that the subjects rejected the axiom even after they were exposed to your lucid and persuasive arguments. But you still defend the axiom on the grounds that it was not properly understood by the subjects. Do you have any reason to believe that they did not properly understand the argument other than the fact that they have chosen to reject it?

Dr. S.: I agree that this is a difficult problem. However, while Slovic and Tversky stated my arguments well, I am not sure that they did all they could to make their subjects understand the issue. In my experience, it often takes a long time for people to appreciate the normative impact of axioms. They have to be educated before they are willing to live by the axioms of rational choice. MacCrimmon's data, for example, show that after he spent more time with each one of his

subjects, they eventually came around to seeing things my way.

Dr. A.: You seem to be saying that SIP enjoys normative status because under some conditions, which you have failed to specify, some people could convince other people that they should accept it. Even if I could accept the axiom, I certainly could not accept this criterion. Your ability to convince people to accept an axiom is not a sufficient basis for establishing its normative appeal. What you call education, others may call brainwashing. Why do yeu not simply accept the fact that, unlike transitivity, SIP lacks general appeal as a normative principle of choice?

Dr. S.: It is all too easy to adopt a crude view of human rationality. Transitivity of preferences is readily accepted because it is relatively simple and familiar. Children, incidentally, have difficulties with transitivity, and even adults find it less compelling when it is formulated in a non-transparent fashion. SIP is more complicated and less familiar than transitivity, so people do not accept it at first. Only after they have had ample opportunity to become acquainted with the axiom, do they appreciate its full normative impact.

Dr. A.: I am not convinced by your arguments. First, I think that I fully understand the axiom and yet I do not accept it, even though I am opposing the overwhelming majority of decision theorists. Second, you have no valid way to distinguish between outright rejection of the axiom and failure to "understand" it. Is there anything else you can say to convince me to accept SIP besides the fact that you believe it and you think that I should believe it too?

Dr. S.: It is not my belief in the axiom that is at issue, but rather the arguments upon which my belief is based. Your objections are well taken. Yet I have observed that, in general, the deeper the understanding of the axiom, the greater the readiness to accept it. Were it not for the cogency of the argument, I doubt that this would be the case.

AUTHORS' NOTE

This paper originates from a program of research addressed to the behavioral aspects of decision analysis. Since the paper is written in highly concise form, we append the following remarks in order to elaborate the implications of this work for decision analysis.

One implication is fairly obvious and is addressed directly in the paper. This is the fact that determining the acceptability of the normative principles underlying decision theory (and hence decision analysis) is not as simple a matter as many theorists would like to believe. At present there is no criteria for distinguishing between lack of understanding of an axiom and lack of acceptance.

Less directly elaborated in this paper, but nevertheless important, is the implication that the context within which a decision axiom is interpreted plays a critical role in determining the acceptability of the axiom. One may wish to accept SIP in principle, but not in the contexts proposed by Allais and Ellsberg where psychological factors such as regret and ambiguity become salient. An analysis which fails to appreciate the concern for regret and ambiguity is likely to violate the decision maker's preferences. Therefore, we must work to devise methods for incorporating such psychological variables into the decision analysis, despite the aesthetic and practical complications that will arise when utilities and preferences are context-dependent.

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FOOTNOTE

1. This research was supported by the Advanced Research Projects Agency of the Department of Defense (ARPA Order No. 2449), and was monitored by ONR under Contract N00014-73-C-0438 (NR197-026).

Table 1

Conformity with the Positions of A and S on First and Second Choices

First Choice	Second Choice	Number of Subjects Problem I	Problem II
A	A	16	17
A	S	1	2
S	S	9	6
S	A	3	4

Note: Prior to the second choice on Problem I, subjects received the argument in conflict with their preference on Choice 1. There was no argument presented between the two choices on Problem II.

Table 2

Summary of Responses in Experiment II.

	Problem I	Problem II
Mean Rating of A	6.9	7.0
Mean Rating of S	6.1	4.6
% Subjects rating A higher	51	78
% Subjects rating S higher	42	20
% Choices conforming to A	35	80
% Choices conforming to S	61	20