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THE SPECTRUM OF LASER-INDUCED TURBULENCE

By

Chan Mou Tchen

Edward Collett

Atmospheric Sciences Laboratory
US Army Electronics Command
White Sands Missile Range, New Mexico 88002

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The structure of turbulence as induced by the propagation of a high energy laser beam in a fluid is investigated. An analytical theory is developed predicting the spectral laws of temperature fluctuations. The theory involves a hierarchy of equations, which is closed by the method of cascade, as has been developed and applied by Tchen to hydrodynamic and plasma turbulences. The inertia and dissipation subranges are found to follow a modified Kolmogorov law and a modified -13/3 power law, respectively. The critical

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wavenumber separating the two subranges has been determined to occur at a lower wavenumber than that in the velocity spectrum. In addition, the magnitude of the inertia spectrum is increased by the laser beam as a result of the enhanced conduction. The parameters and the numerical coefficients entering into the spectrum are also determined analytically.

FOREWORD

Professor Tchen is with the City College of the City University of New York.

CONTENTS

INTRODUCTION

The linear theory of the propagation of a laser beam in a random or turbulent medium, which is valid for a low power laser, has been widely investigated [1,2]. However, a high energy laser beam will modify the medium by various effects, such as inducing turbulence in a quiescent medium or changing the state of previously existing turbulence. Such a modified medium will in turn change the mode of propagation. Experiments have shown that a laser beam can indeed induce turbulence, and some criteria for the onset of turbulence have been developed [3,4,5]. In the present work, we shall develop a theory of thermal turbulence as induced by a laser beam. Here the laser acts as a source of heat which is deposited into the fluid medium by conduction and convection, due to absorption and beam inhomogeneity, respectively; in a self-consistent way, the thermal fluctuations also modify the heat source. The nonlinear equation describing the fluctuations of temperature gives rise to a hierarchy which needs to be closed. The method of cascade decomposition, which was developed by Tchen to close the hierarchy and which has been successfully applied to hydrodynamic and plasma turbulence, will be extended to laser induced turbulence [6,7].

In the present work we neglect the acoustic effects, as is permitted in all problems of thermal oscillations under the Boussinesq approximation. The buoyancy effect which is retained in the general formulation is found to be negligible in the inertia and dissipation subranges of the spectrum.

FUNDAMENTAL EQUATIONS

The fundamental equations which describe the fluide motion in the atmosphere in the presence of laser heating are the Navier-Stokes equation of momentum,

$$\rho\left(\frac{\partial \underset{\sim}{u}}{\partial t} + \underset{\sim}{u}\cdot\nabla \underset{\sim}{u}\right)$$

$$= -\nabla p + \nabla(\rho\ \nu\ \nabla\ \underset{\sim}{u}) + \frac{1}{3}\nabla(\rho\ \nu\ \nabla\ \underset{\sim}{u}) - \beta\ \underset{\sim}{g}(T-T_o) \qquad (1)$$

the equation of continuity,

$$\frac{\partial \rho}{\partial t} + \nabla(\rho\ \underset{\sim}{u}) = 0 \qquad (2)$$

and the equation of heat,

$$\rho c_V \frac{\partial T}{\partial t} + u\nabla T = -\ p\ \nabla\ \underset{\sim}{u} + k\ \nabla^2 T + \Phi + \alpha\ I \qquad (3)$$

Here ρ is the density, $\underset{\sim}{u}$ is the fluid velocity, T is the temperature, c_v is the specific heat at constant volume, Φ is the Rayleigh dissipation function, I is the intensity of the laser beam, and α is the absorption coefficient. The buoyancy force is represented in Eq. (1) by

$$(\rho - \rho_o)\, \underset{\sim}{g} \equiv - \beta(T-T_o)\, \underset{\sim}{g} \qquad (4)$$

where g is the acceleration of gravity, and β is the coefficient of expãnsion.

It is seen that the effect of the laser resides in (a) the heat disposition term αI in Eq. (3) which modified the temperature distribution, and (b) the buoyancy force in the momentum equation (1), modifying the velocity distribution.

In problems of thermal convection and diffusion, it is customary to introduce the Boussinesq approximation, which neglects the acoustic or compressibility effect in all terms except the buoyancy term of Eq. (1). This approximation reduces Eq. (2) to

$$\nabla \cdot \underset{\sim}{u} = 0 \qquad (5)$$

We shall also neglect the Rayleigh dissipation function in Eq. (3). The system of Eqs. (1), (3), and (5) determine the modifications of the atmospheric motions by the heat deposition from the laser. It is to be remarked that in the absence of such a heat deposition, the above system of equations degenerates to the one governing the classical problem of thermal instability.

In the present study it is necessary to specify the heat deposition term αI of Eq. (3). We note that I is related to the index of refraction n on the basis of photon conservation, and subsequently is related to the temperature by an equation of state. In principle, these relations can be derived from time-dependent microscopic models. However, their governing time scale is much smaller than the time scale of the turbulent motion, and consequently a quasi-stationary approximation, known as the adiabatic approximation, can be used.

This amounts to assuming that the heat deposition is spontaneous and localized. This requires that the approach to equilibrium of the heating process is faster than that of the turbulent process. This condition is usually fulfilled. Under such a circumstance we wish to find a relation analogous to that between the energy flux and the gradient from the thermodynamics of irreversible processes. A satisfactory derivation of such a relation requires the analysis of the equation of wave propagation in a medium of variable index of refraction and the study of the absorption or heat deposition during the propagation. A diffusion theory of absorption of the laser beam intensity on the basis of a stochastic theory will be developed in a later report. In this paper we shall present a heuristic argument. To that end, we know that a variation of temperature gradient $\delta(\nabla T)$, which corresponds to a variation in refractive index gradient $\delta(\nabla n)$, will cause a thermal distortion of the beam intensity, expressed in the form

$$\frac{\delta I}{I} = \ell^2 D \cdot \frac{\nabla \delta n}{n} \qquad (6a)$$

This is analogous to the relation between flux and gradient in the thermodynamics of irreversible processes, as mentioned earlier. Here ℓ is a scale length characteristic of the propagation pathlength in the turbulent medium (it should not be confused with the inner and outer scales of turbulence), and D is a differential operator

$$D = \nabla + \nabla \ell n \, I \qquad (6b)$$

Formula (6a) is a variant of the equation developed by Gebhardt and Smith [8]. We may consider that Eq. (6a) serves as a relation of state which, when combined with the constituitive equation

$$\delta n = \left(\frac{\partial n}{\partial T}\right)_\rho \delta T \qquad (7)$$

will give the equivalent value of the temperature increase δT corresponding to the energy deposition δI, as follows:

$$\delta I = A \, D \cdot \nabla \, \delta \, T \qquad (8a)$$

with

$$A = - \frac{\ell^2}{n} \left(\frac{\partial n}{\partial T}\right)_\rho I \qquad (8b)$$

We shall assume that a turbulent fluctuation follows such a variation, and shall rewrite Eq. (8a) as

$$I' = \overline{A} \, \overline{D} \cdot \nabla \, \theta \qquad (9)$$

Here I' and θ are turbulent fluctuations of radiation intensity and temperature, respectively, while I_o and T_o are the corresponding mean quantities; further,

$$\overline{D} = \nabla + \nabla \ln I_o, \quad \overline{A} = - \left[\frac{\ell^2}{n} \left(\frac{\partial n}{\partial T} \right)_\rho \right]_o I_o \tag{10}$$

It is to be noted that A and $\overline{A} > 0$, because $(\partial n/\partial T)_\rho$ and $(\partial \overline{n}/\partial T)_{\rho_o}$ are negative for most fluids.

The relationship between I' and θ, as expressed by Eq. (9), is analogous to the formula for the beam intensity developed by Gebhardt and Smith for the geometric optics regime [8]. This formula has been applied by Collett, et. al., to the determination of intensity correlations of a laser beam in a turbulent medium [9]. The results obtained by this method are identical to the more rigorous method of wave propagation used by Ho, when the effects of diffraction are omitted [10]. That approximation is valid, because the optical length scale is very small compared to the scale of the turbulence.

The system of Eqs. (3) and (9) forms the fundamental differential equations governing the interaction of a laser beam with the atmospheric medium, as described by the variables u, p, T, and I'.

EQUATIONS OF TURBULENT MOTIONS

We decompose the four variables p, T, u, and I into a mean part and a fluctuation:

$$\begin{aligned} \underset{\sim}{u} &= \underset{\sim}{U} + \underset{\sim}{u} \\ p &= p_o + p' \\ T &= T_o + \theta \\ I &= I_o + I' \end{aligned} \tag{11}$$

In the absence of a mean wind, $\underset{\sim}{u}$ represents a velocity fluctuation.

By substituting Eq. (11) into Eqs. (1), (3), and (5), we find the following four equations determining the four variables $\underset{\sim}{u}'$, p', θ, and I':

$$\frac{\partial \underset{\sim}{u}}{\partial t} + \underset{\sim}{u} \nabla \underset{\sim}{u} = - \frac{1}{\rho} \nabla p' + \nu \nabla^2 \underset{\sim}{u} - \beta g \theta + \overline{u \nabla u} \tag{12}$$

$$\nabla \underset{\sim}{u} = 0 \tag{13}$$

$$\frac{\partial \theta}{\partial t} + \underset{\sim}{u} \nabla\theta = -u \nabla T_o + \overline{u \nabla\theta} + (k/\rho c_v) \nabla^2 \theta + (\alpha/\rho c_v) I' \tag{14}$$

$$I' = \overline{A}\,(\nabla^2\theta - \underset{\sim}{\Gamma}\,\nabla\theta) \tag{15}$$

with

$$\Gamma \equiv -\nabla \ln I_o \tag{16a}$$

where Γ^{-1} represents the scale of the laser beam.

As a result of the turbulent heating of the atmospheric medium by a beam, we expect a distribution of the intensity of temperature fluctuation, with

$$\Lambda = -\nabla \ln \overline{\theta^2} \tag{16b}$$

where Λ^{-1} is a scale in the profile of $\overline{\theta^2}$ analogous to Eq. (16a).

It is well known that the buoyancy controls the large scale motions of the velocity spectrum. This effect is negligible for the inertia and dissipation subranges under the present investigation, enabling us to decouple Eqs. (12) and (13) from Eq. (14) and (15). Hence, we need only to be concerned with the thermal turbulence, and regard the velocity spectrum as a known quantity. In such a circumstance, and with a substitution of Eq. (15) into Eq. (14), we find the thermal turbulence governed by the following equation of heat transfer:

$$\frac{\partial\theta}{\partial t} + \underset{\sim}{u}\cdot\nabla\theta = -\underset{\sim}{u}\cdot\nabla T_o + \nabla\cdot\overline{\underset{\sim}{u}\theta} + (\lambda + \mu)\,\nabla^2\theta - \mu\underset{\sim}{\Gamma}\cdot\nabla\theta \tag{17}$$

The left hand side of Eq. (17) represents the time rate of change and the convection of the temperature fluctuation. The right hand side contains a generation of temperature fluctuation from the velocity fluctuations in the presence of a mean temperature gradient, a conduction from the thermal flux by eddy motions, a molecular conduction with a molecular conductivity

$$\lambda = k/\rho c_v \tag{18}$$

a conductivity

$$\mu = \alpha\overline{A}/\rho c_v \tag{19}$$

from the absorption of radiation, and finally a convection associated with the beam inhomogeneity.

CASCADE DECOMPOSITION

For the determination of the turbulent spectrum, it is necessary to make a Fourier analysis of Eq. (19). Instead of studying the behavior of each Fourier component, we separate the fluctuations θ and $\underset{\sim}{u}$ into two groups and write

$$\theta(t,x) = \theta^{(0)}(t,\underset{\sim}{x}) + \theta^{(1)}(t,\underset{\sim}{x}) \tag{20}$$

and

$$\underset{\sim}{u}(t,\underset{\sim}{x}) = \underset{\sim}{u}^{(0)}(t,\underset{\sim}{x}) + \underset{\sim}{u}^{(1)}(t,\underset{\sim}{x}) \tag{21}$$

where we define

$$\theta^{(0)}(t,\underset{\sim}{x}) = \int_0^{k} d\underset{\sim}{k}' \;\; \theta(t,\underset{\sim}{k}')\, e^{i\underset{\sim}{k}' \underset{\sim}{x}} \tag{22}$$

and

$$\theta^{(1)}(t,\underset{\sim}{x}) = \int_k^{\infty} d\underset{\sim}{k}' \;\; \theta(t,\underset{\sim}{k}')\, e^{i\underset{\sim}{k}' \underset{\sim}{x}} \tag{23}$$

Similarly,

$$\underset{\sim}{u}^{(0)}(t,\underset{\sim}{x}) = \int_0^{k} d\underset{\sim}{k}' \;\; \underset{\sim}{u}(t,\underset{\sim}{k}')\, e^{i\underset{\sim}{k}' \underset{\sim}{x}} \tag{24}$$

and

$$\underset{\sim}{u}^{(1)}(t,\underset{\sim}{x}) = \int_k^{\infty} d\underset{\sim}{k}' \;\; \underset{\sim}{u}(t,\underset{\sim}{k}')\, e^{i\underset{\sim}{k}' \underset{\sim}{x}} \tag{25}$$

where $\theta^{(0)}$, $\theta^{(1)}$, $\underset{\sim}{u}^{(0)}$ and $\underset{\sim}{u}^{(1)}$ are truncated within the limits indicated by Eqs. (22) to (25).

In order to separate $\theta^{(0)}$ and $\theta^{(1)}$ from θ in Eq. (20), and $\underset{\sim}{u}^{(0)}$ and $\underset{\sim}{u}^{(1)}$ from $\underset{\sim}{u}$ in Eq. (21), we make use of an ensemble average

$$\langle \cdot \cdot \cdot \rangle \tag{26}$$

which gives

$$\langle \theta \rangle^{(1)} = \theta^{(0)}, \quad \langle \theta^{(0)} \rangle^{(1)} = \theta^{(0)}, \quad \langle \theta^{(1)} \rangle^{(1)} = 0 \tag{27}$$

and

$$\langle \underset{\sim}{u} \rangle^{(1)} = \underset{\sim}{u}^{(0)}, \quad \langle \underset{\sim}{u}^{(0)} \rangle^{(1)} = \underset{\sim}{u}^{(0)}, \quad \langle \underset{\sim}{u}^{(1)} \rangle^{(1)} = 0 \tag{28}$$

The decomposition expressed by Eqs. (20) and (21) implies that the fluctuations θ and $\underset{\sim}{u}$ contain a zeroth rank and first rank, which represent macroscopic and random fluctuations, respectively, with the macroscopic quantity being of larger scale than the random quantity. The ensemble average, Eq. (26), of first rank is an average over many realizations with identical macroscopic conditions, as represented by $\theta^{(0)}$ and $\underset{\sim}{u}^{(0)}$, but with random $\theta^{(1)}$ and $\underset{\sim}{u}^{(1)}$. In this manner, the average over the random quantities vanishes and the same average over macroscopic quantities remains unaffected by Eqs. (27) and (28).

The ensemble average of zeroth rank

$$< \cdot \cdot \cdot > \tag{29}$$

is equivalent to the global average denoted by a bar in a homogeneous turbulence.

By means of the cascade ensemble average, Eq. (26), we can screen out the ranks $\theta^{(0)}$ and $\theta^{(1)}$ from θ in Eq. (20), and hence transform the equation of evolution of θ, Eq. (19), into corresponding equations of evolutions of $\theta^{(0)}$ and $\theta^{(1)}$. The details for this type of calculation can be found in the work of Tchen and will be omitted here [6]. The result of this calculation leads to

$$\frac{\partial \theta^{(0)}}{\partial t} + \underset{\sim}{u}^{(0)} \cdot \nabla \theta^{(0)} - \overline{\underset{\sim}{u}^{(0)} \cdot \nabla \theta^{(0)}}$$

$$= - \underset{\sim}{u}^{(0)} \cdot \nabla T_o + (\lambda + \mu) \nabla^2 \theta^{(0)} - \mu \underset{\sim}{\Gamma} \cdot \nabla \theta^{(0)} - < \underset{\sim}{u}^{(1)} \cdot \nabla \theta^{(1)} >^{(1)} \tag{30}$$

and

$$\frac{d\theta^{(1)}}{dt} \equiv \frac{\partial\theta^{(1)}}{\partial t} + \underset{\sim}{u}^{(0)} \nabla\theta^{(0)} \tag{31}$$

$$= -\underset{\sim}{u}^{(1)}\cdot\nabla(T_\circ+\theta^{(0)})+(\lambda+\mu)\nabla^2\theta^{(1)}-\mu\underset{\sim}{\Gamma}\cdot\nabla\theta^{(1)}+\langle\underset{\sim}{u}^{(1)}\cdot\nabla\theta^{(1)}\rangle^{(1)}$$

noting that the addition of Eqs. (30) and (31) recovers Eq. (19). The last term on the right hand side of Eq. (31) will be omitted in the following, since it does not contribute to the formulation of correlations.

EQUATION OF SPECTRAL BALANCE

We multiply Eq. (30) by $\theta^{(0)}$ and take an ensemble average of zeroth rank, as defined by Eq. (29). We then find

$$\frac{1}{2}\frac{\partial}{\partial t}\langle\theta^{(0)2}\rangle^{(0)} + \frac{1}{2}\langle\underset{\sim}{u}^{(0)}\cdot\nabla\theta^{(0)2}\rangle^{(0)}$$

$$= -\langle\underset{\sim}{u}^{(0)}\cdot\theta^{(0)}\rangle^{(0)}\,\nabla T_\circ - \langle\theta^{(0)}\nabla\cdot\langle\underset{\sim}{u}^{(1)}\,\theta^{(1)}\rangle^{(1)}\rangle^{(0)}$$

$$+ (\lambda+\mu)\,\langle\theta^{(0)}\nabla^2\theta^{(0)}\rangle^{(0)} - \frac{1}{2}\mu\underset{\sim}{\Gamma}\cdot\nabla\langle\theta^{(0)2}\rangle^{(0)} \tag{32}$$

or, rewritten as

$$\frac{1}{2}\frac{\partial}{\partial t}\langle\theta^{(0)2}\rangle^{(0)} + \{\frac{1}{2}\nabla\cdot\langle\underset{\sim}{u}^{(0)}\theta^{(0)2}\rangle^{(0)}\}$$

$$= -\langle\underset{\sim}{u}^{(0)}\theta^{(0)}\rangle^{(0)}\cdot\nabla T_\circ - \langle\theta^{(0)}\nabla\cdot\langle\underset{\sim}{u}^{(1)}\theta^{(1)}\rangle^{(1)}\rangle^{(0)}$$

$$-(\lambda+\mu)\,\langle(\nabla\theta^{(0)})^2\rangle^{(0)} + \{\frac{1}{2}(\lambda+\mu)\nabla^2\langle\theta^{(0)2}\rangle^{(0)}\}$$

$$+\frac{1}{2}\mu\,\underset{\sim}{\Gamma}\cdot\underset{\sim}{\Lambda}^{(0)}\langle\theta^{(0)2}\rangle^{(0)} \tag{33}$$

where

$$\Lambda^{(0)} \equiv \nabla\,\ell n\,\langle\theta^{(0)2}\rangle^{(0)} \tag{34}$$

In order to simplify Eq. (33), we assume that the turbulent motions are locally homogeneous. This implies that the spectral distributions are similarly dependent on position through the intermediary of the parameters, which are given by the mean quantities, e.g., profiles of beam, mean temperature and intensity of thermal turbulence, and the rate of dissipation of the thermal turbulence. Under that hypothesis, we can neglect the terms between {...} in Eq. (33). The term proportional to $\underset{\sim}{\Gamma}\cdot\underset{\sim}{\Lambda}^{(0)}$ is a coupling between the laser beam and thermal turbulence, and is therefore proportional to the product of the gradients in the beam profile and the turbulent intensity profile. In this connection, we approximate $\Lambda^{(0)} \cong \Lambda$, where Λ is defined by Eq. (16b), since the rank of Eq. (22) up to a wavenumber k lying in the inertia subrange embodies the major portion of the thermal energy.

With the above approximations, we reduce Eq. (34) to the following form:

$$\frac{1}{2}\frac{\partial}{\partial t}\langle\theta^{(0)2}\rangle^{(0)} = -\langle\underset{\sim}{u}^{(0)}\theta^{(0)}\rangle^{(0)}\cdot\nabla T_o - \langle\nabla\cdot\langle u^{(1)}\theta^{(1)}\rangle^{(1)}\theta^{(0)}\rangle^{(0)}$$

$$-(\lambda+\mu)\,\langle(\nabla\theta^{(0)})^2\rangle^{(0)} + \gamma\langle\theta^{(0)2}\rangle^{(0)} \tag{35a}$$

with

$$\gamma \equiv \mu\,\underset{\sim}{\Gamma}\cdot\underset{\sim}{\Lambda} \tag{35b}$$

We conclude that the rate of change of thermal energy $\langle\theta^{(0)2}\rangle^{(0)}$ is governed by a production function $-\langle u^{(0)}\theta^{(0)}\rangle^{(0)}\nabla T_o$, a transfer function $-\langle\nabla\langle u^{(1)}\theta^{(1)}\rangle^{(1)}\theta^{(0)}\rangle^{(0)}$, a dissipation function $-(\lambda+\mu)\langle(\nabla\theta^{(0)})^2\rangle^{(0)}$, combining the effects of molecular conduction and the absorption of radiation, and finally a source function $\gamma\langle\theta^{(0)2}\rangle^{(0)}$ from the laser beam.

HEAT FLUX AND EDDY CONDUCTIVITY

In Eq. (35) there occurs a term,

$$\langle\underset{\sim}{u}^{(1)}\theta^{(1)}\rangle^{(1)} \tag{36}$$

called eddy "thermal flux" of first rank, and a similar term of zeroth rank,

$$\langle\underset{\sim}{u}^{(0)}\theta^{(0)}\rangle^{(0)} = \langle u\theta\rangle - \langle\underset{\sim}{u}^{(1)}\theta^{(1)}\rangle^{(1)} \tag{37}$$

The statistical study of molecular transport processes can be based upon a Langevin equation, which describes the time evolution of a fluctuation in the Lagrangian representation. We shall generalize the above Langevin equation to apply to a turbulent transport. In order to do this, we note that Eq. (31) may serve this purpose by considering the variable t in the Lagrangian time derivative as a single variable, since this is the requirement for the calculation of the transport properties. Effectively, we regard the Fourier form of Eq. (31) as a turbulent Langevin equation with $\underset{\sim}{k}$ as a parameter. It will be written as follows:

$$\frac{d\theta^{(1)}(t,\underset{\sim}{k})}{dt} = - \underset{\sim}{u}^{(1)}(t,\underset{\sim}{k})\ \nabla(T_o+\theta^{(0)}) - \lambda^* k^2 \theta^{(1)}(t,\underset{\sim}{k}) \tag{38}$$

where T_o and $\theta^{(0)}$ have weaker inhomogeneities than $\underset{\sim}{\theta}^{(1)}$ and $\underset{\sim}{u}^{(1)}$, and $\lambda^* k^2$ represents

$$\lambda^* k^2 = \lambda k^2 + \mu(k^2 + \underset{\sim}{\Gamma}\cdot\underset{\sim}{\Lambda}) \tag{39}$$

A formal solution of Eq. (38) is

$$\theta^{(1)}(t,\underset{\sim}{k}) = - \nabla(T_o+\theta^{(0)})\cdot\int_o^t dt'\ \exp[-\lambda^* k^2(t-t')]\ \underset{\sim}{u}^{(1)}(t',\underset{\sim}{k})$$

$$+\ \theta^{(0)}(t=o,\underset{\sim}{k})\ \exp(-\lambda^* k^2 t) \tag{40}$$

Here we have put $\nabla(T_o + \theta^{(0)})$ outside of the integral, because it varies slowly with time. It follows from Eq. (40) that

$$\langle u_j^{(1)}(t,\underset{\sim}{k})\ \theta^{(1)}(t,\underset{\sim}{k})\rangle^{(1)} = - \eta_{sj}^{(1)}(\underset{\sim}{k})\ \frac{\partial(T_o + \theta^{(0)})}{\partial x_s} \tag{41}$$

where

$$\eta_{sj}^{(1)}(\underset{\sim}{k}) = \int_o^t dt'\ \exp[-\lambda^* k^2(t-t')]\ \langle u_s^{(1)}(t',\underset{\sim}{k})\ u_j^{(1)}(t,-\underset{\sim}{k})\rangle^{(1)} \tag{42}$$

noting that

$$\langle\theta^{(1)}(o,\underset{\sim}{k})\ \theta^{(1)}(t,-\underset{\sim}{k})\rangle^{(1)} = 0 \tag{43}$$

for t larger than the correlation time. By the same token we can replace the upper limit t by ∞ in Eq. (42) to give

$$\eta_{sj}^{(1)}(\underset{\sim}{k}) = \int_{0}^{\infty} dt' \exp[\lambda^* k^2 (t-t')] \langle u_s^{(1)}(t',k)\, u_j^{(1)}(t,-k)\rangle^{(1)} \quad (44)$$

For an isotropic eddy conductivity we have

$$\eta_{sj}^{(1)} = \eta^{(1)} \delta_{sj} \quad (45)$$

and

$$\eta^{(1)}(\underset{\sim}{k}) = \frac{1}{3} \int_{0}^{\infty} dt' \exp[\lambda^* k^2 (t-t')] \langle \underset{\sim}{u}^{(1)}(t',\underset{\sim}{k})\, \underset{\sim}{u}^{(1)}(t,-\underset{\sim}{k})\rangle^{(1)} \quad (46)$$

This reduces the thermal flux, Eq. (36), to

$$\langle u_j^{(1)}(t,-\underset{\sim}{k})\, \theta^{(1)}(t,\underset{\sim}{k})\rangle^{(1)} = -\, \eta^{(1)}(\underset{\sim}{k}) \frac{\partial(T_o + \theta^{(0)})}{\partial x_j} \quad (47)$$

in k space, or to

$$\langle u_j^{(1)}(t,x)\, \theta^{(1)}(t,x)\rangle^{(1)} = -\, \eta^{(1)} \frac{\partial(T_o + \theta^{(0)})}{\partial x_j} \quad (48)$$

where $\eta^{(1)}$ is the eddy conductivity in x space.

The turbulent transport relations, Eqs. (41) and (48), take the form analogous to the phenomenological relations of Onsager in the thermodynamics of irreversible phenomena, stating that any flux is caused by the contribution of a force or gradient with a proportionality constant called the transport coefficient. Here we have derived such relations by means of a generalized Langevin equation, Eq. (38), applicable to turbulent motions; hence, we have determined the relevant forces and transport coefficient, as given by Eqs. (41), (44), (46), and (48). A precise determination of the structure of $\eta^{(1)}$ in terms of the spectral function $F(\boldsymbol{k})$, such that

$$\frac{1}{2} \langle \underset{\sim}{u}^{(0)2}\rangle^{(0)} = \int_{0}^{k} dk' \; F(k') \quad (49)$$

is given by Tchen [6]. We shall not enter into its detailed derivation, but simply write the following result:

$$\eta' = c_2 \int_{k}^{\infty} dk' \; \frac{F(k')}{w^{(1)}(k')} \tag{50}$$

Here the relaxation frequency is

$$\begin{aligned} w^{(1)}(k') &= k'^2 \quad (\lambda^* + \eta^{(1)}) \\ &\cong k'^2 \quad \eta^{(1)} \end{aligned} \tag{51}$$

since the molecular conductivity λ^* can be assumed to be smaller than the eddy conductivity. The integral equation, Eq. (50), with the relaxation frequency given by Eq. (51), leads to the following solution:

$$\eta^{(1)} = \left[2\, c_2 \int_{k}^{\infty} dk' \quad F(k')\; k'^{-2} \right]^{\frac{1}{2}} \tag{52}$$

where c_2 is a numerical coefficient evaluated for three-dimensional turbulence to be [6]

$$c_2 = \frac{4}{3} \tag{53}$$

EQUILIBRIUM RANGE OF THE SPECTRUM

Along with the spectral fluctuation distribution $F(k)$ for the velocity, Eq. (49), we introduce a spectral distribution $G(k)$ for the temperature which is defined to be

$$\frac{1}{2} \langle \theta^{(0)2} \rangle \quad (0) = \int_{0}^{k} dk' \; G(k') \tag{54}$$

The use of Eq. (48) permits the transfer function in Eq. (35) to be rewritten as

$$\langle \nabla\cdot\langle u^{(1)}\theta^{(1)}\rangle^{(1)}\ \theta^{(0)}\rangle^{(0)} = -\ \eta^{(1)}\ \langle\theta^{(0)}\nabla^2(T_o + \theta^{(0)})\rangle^{(0)}$$

$$= -\ \eta^{(1)}\ \langle\theta^{(0)}\nabla^2\theta^{(0)}\rangle^{(0)}$$

$$= \eta^{(1)}\ \langle(\nabla\theta^{(0)})^2\rangle^{(0)} - \frac{1}{2}\ \eta^{(1)}\nabla^2\langle\theta^{(0)2}\rangle^{(0)}$$

$$= \eta^{(1)}\ \langle(\nabla\theta^{(0)})^2\rangle^{(0)} \tag{55}$$

with the application of local homogeneity of turbulence.

If we introduce the notations

$$J^\circ \equiv \langle(\nabla\theta^{(0)})^2\rangle^{(0)} = 2\int_0^k dk'\ k'^2\ G(k') \tag{56}$$

and

$$\eta = \eta^{(0)} + \eta^{(1)}$$

we can rewrite the equation of spectral balance, Eq. (35a), in the form:

$$\frac{\partial}{\partial t}\int_0^k dk'\ G(k') = -\ (\eta-\eta^{(1)})\ (\nabla T_o)^2 - (\lambda+\mu)J^\circ - \eta^{(1)}J^\circ$$
$$+\ \gamma\int_0^k dk'\ G(k') \tag{58}$$

or

$$\frac{\partial}{\partial t}\int_0^k dk'\ G(k') = (\eta - \eta^{(1)})\bar{J} - (\lambda_o + \eta^{(1)})J^\circ$$
$$+\ \gamma\int_0^k dk'\ G(k') \tag{59}$$

where

$$\overline{J} = (\nabla T_o)^2 \tag{60a}$$

and

$$\lambda_o = \lambda + \mu \tag{60b}$$

are called thermal vorticity and total conductivity, respectively. The total conductivity includes a molecular conductivity λ and a conductivity μ enhanced by the beam. Use has been made of Eqs. (56) and (57).

For $k = \infty$, we reduce Eq. (59) to

$$\frac{\partial}{\partial t} \int_0^k dk' \; G(k') = \eta\overline{J} - \lambda_o J + \gamma \int_0^k dk' \; G(k') \tag{61}$$

as

$$\eta^{(1)}(k{=}\infty) = 0, \qquad J = J^{\circ}(k{=}\infty) \tag{62}$$

The equilibrium range of the spectrum is defined as that range covering sufficiently large wave numbers such that, in the integral term on the left hand side of Eq. (59), the upper limit k may be replaced by ∞. Under that circumstance, and by subtracting Eq. (59) from Eq. (61), we obtain the equation of spectal balance in the equilibrium range,

$$\eta^{(1)}\overline{J} + (\lambda_o + \eta^{(1)})J^{\circ} + \gamma \int_k^{\infty} dk' \; G(k') = \lambda_o J \tag{63}$$

THE SPECTRUM IN THE ABSENCE OF BUOYANCY

The universal range of the spectrum is usually divided into the following subranges in the increasing orders of wavenumbers: buoyancy, inertia, and dissipation. As mentioned earlier, the buoyancy subrange reigns over the largest scale of the spectrum, and will not be included in the present study. Consequently, the velocity spectrum in the inertial subrange is given as the Kolmogorov law,

$$F = A\,\varepsilon^{2/3}\,k^{-5/3} \tag{64a}$$

Here ε is the rate of energy dissipation and the numerical coefficient A is [6]

$$A = 1.58 \tag{64b}$$

The spectrum, Eq. (64a), is the most commonly adopted law for the description of the interaction of a propagating laser beam in a turbulent atmosphere. Following that law, we find the eddy conductivity from Eq. (52) to be

$$\eta^{(1)} = c_3\,\varepsilon^{1/3}\,k^{-4/3} \tag{65a}$$

with [6]

$$c_3 = \left(\frac{3}{4}Ac_2\right)^{\frac{1}{2}} \cong 1.26 \tag{65b}$$

For the derivation of the spectral function $G(k)$ for the temperature fluctuations, we consider Eq. (63), which we rewrite in the form

$$(\lambda_o + \eta^{(1)})\,(\overline{J} + J^o) + \gamma \int_k^\infty dk'\,G(k') = \lambda_o(\overline{J} + J) \tag{66}$$

or, upon dividing by $\lambda_o + \eta^{(1)}$,

$$\overline{J} + J^o + \gamma(\lambda_o + \eta^{(1)})^{-1} \int_k^\infty dk'\,G(k') = \lambda_o(\lambda_o+\eta^{(1)})^{-1}(\overline{J} + J) \tag{67}$$

We shall introduce the following notations:

$$H(k) = \int_k^\infty dk'\,G(k') \tag{68a}$$

$$P = \frac{\Phi}{1-\Phi}\,\frac{d}{dk}\,\ell n\,(\lambda_o + \eta^{(1)}) \tag{68b}$$

$$Q = \frac{\lambda_o(\bar{J} + J)}{2(\lambda_o + \eta^{(1)})k^2(1-\Phi)} \frac{d}{dk} \ln(\lambda_o + \eta^{(1)}) \tag{68c}$$

$$\Phi = \frac{\gamma}{2k^2(\lambda_o + \eta^{(1)})} \tag{68d}$$

and transform the integral equation, Eq. (67), into the following differential equation

$$\frac{dH}{dk} + P\,H = Q \tag{69}$$

with the solution

$$G(k) = \frac{d}{dk} \int_k^\infty dk'\, Q(k') \exp[- \int_{k'}^{k} dk''\, P(k'')]$$

$$= - Q(k) - P(k) \int_k^\infty dk'\, Q(k') \exp[- \int_{k'}^{k} dk''\, P(k'')] \tag{70a}$$

or, in an approximate form,

$$G(k) = - Q(k) - P(k) \int_k^\infty dk'\, Q(k') \tag{70b}$$

valid for a weak beam factor γ, or Φ.

Equation (70b), which is valid for all subranges, appears in a form too complex for interpretation. Therefore we shall consider them separately. We find:

(a) In the inertia subrange

$$G(k) = \frac{2}{3c_3} \varepsilon_\lambda\, \varepsilon^{-1/3}(1-\Phi_i)\, k^{-5/3} \tag{71}$$

with

$$\Phi_i = (2c_3)^{-1}(\gamma^3/\varepsilon)^{1/3}\, k^{-2/3} \tag{72}$$

(b) In the dissipation subrange

$$G(k) = \frac{2c_3}{3}\ \varepsilon^{1/3}\ \varepsilon_\lambda \lambda_o^{-2}(1+\Phi_d)k^{-13/3} \tag{73}$$

with

$$\Phi_d = (\gamma/_2\lambda_o)k^{-2} \tag{74}$$

In the above, ε and ε_λ are rates of dissipations

$$\varepsilon = \nu\ \overline{(\nabla \underset{\sim}{u})^2} \tag{75}$$

$$\varepsilon_\lambda = \lambda_o\ \overline{(\nabla\theta)^2}\ \equiv \lambda_o J \tag{76}$$

for the fluid and temperature fluctuations, respectively.

The transition from the inertia to the dissipation subranges occurs at a critical wavenumber

$$k_\lambda = c_3{}^{3/4}(\varepsilon/\lambda_o^3)^{1/4} \tag{77}$$

We conclude that the effect of the beam factor is to decrease the power of -5/3 in the inertia subrange and steepen the power of -13/3 in the dissipation subrange. The laser heating increases λ_o and J.

CONCLUSIONS

In the present work we have shown that a laser can induce temperature fluctuations which, because of their nonlinear nature, can lead to turbulence. Because the hydrodynamic system is nonlinear, it generates a "hierarchy," i.e., an infinite sequence of equations of ever increasing order of correlations. This is closed by the method of cascade.

The universal subrange, which is divided into an inertia subrange and a dissipation subrange, is not controlled by the buoyancy effect, so that the spectrum of velocity fluctuations follows the Kolmogorov law. The spectrum of temperature fluctuations is derived for the inertia and dissipation subranges, and includes the effects of enhanced diffusion and inhomogeneity of the laser beam. The resulting equation of spectral balance is in the form of a nonlinear integro-differential equation and determines the spectral distribution of the temperature fluctuations; it is solved for a turbulent state in statistical equilibrium. It contains a production function due to the beam inhomogeneity, a transfer function which describes the transfer of energy across the spectrum from large eddies into small eddies, and finally a dissipation function which determines the dissipation of small eddies by molecular motions.

If the temperature variable is considered as a passive scaler driven by a fluid turbulence obeying a Kolmogorov, it is known, by dimensional theories, that the temperature fluctuation will also follow the Kolmogorov law in the inertia subrange, and a $k^{-13/3}$ law in the dissipation subrange. By means of a cascade theory, we have derived analytically the temperature spectra which confirm the results of the dimensional theories. The heating by a laser beam introduces the following two effects:

(1) In view of the increased conductivity by the absorption, the rate of thermal dissipation ε_λ increases, and, consequently, the temperature spectrum has a higher value in the inertia subrange, and a lower value in the dissipation subrange. For the same reason, the critical wavenumber separating the two subranges occurs at a lower value.

(2) The laser beam has an inhomogeneous profile, denoted by the beam factor γ (see Eq. (35b)). Since the inhomogeneity is usually weak, the powers -5/3 and -13/3 are modified slightly, with a negative and a positive correction term in the inertia and dissipation laws, respectively (see Eqs. (71) and (73)).

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CG, US Army Missile Command
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Redstone Arsenal, AL 35809
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Commanding General
US Army Combat Devel Command
Intelligence & Control Sys Gr.
ATTN: INCSCSD-C
Ft Belvoir, VA 22060
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US Army Limited War. Lab
ATTN: CRDLWL-7C
Aberdeen Proving Ground, MD 21005

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US Army Ballistic Lab
ATTN: AMXRD-BSP (Dr. Reuyl)
Aberdeen Proving Ground, MD 21005

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ATTN: AMSTE-EL
Aberdeen Proving Ground, MD 21005

Commanding General
US Army Test & Eval Command
ATTN: AMSTE-FA
Aberdeen Proving Ground, MD 21005

Commanding General
US Army Test & Eval Command
ATTN: AMSTE-NBC
Aberdeen Proving Ground, MD 21005

Commanding General
US Army Munitions Cmd
ATTN: AMSMU-RE-R
Dover, NJ 07801

Director
US Army Munitions Command
Operations Research Group
Aberdeen Proving Ground, MD 21010

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ATTN: AMSMU-RE-P
Dover, NJ 07801

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Picatinny Arsenal
ATTN: SMUPA-TVI
Dover, NJ 07801

Commanding Officer
USA Garrison
ATTN: Tech Ref. Div.
Fort Huachuca, Arizona 85613

Chief, A.M. & EW Division
ATTN: USAEPG-STEEP-TD
Fort Huachuca, Arizona 85613

US Army Rsch. Off. - Durham
Box CM, DUKE Station
ATTN: CRDARD-IP
Durham, N. Carolina 27706

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ATTN: AMXAM
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USACDC CBR Agcy
Ft. McClellan, Ala 36201

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US Army Materiel Command
ATTN: AMCRD R (H. Cohen)
Washington, D. C. 20315

Commanding Officer
US Army Combat Dev Cmd
Communications-Elect Agcy
Ft Monmouth, NJ 07703

US Army Liaison Office
MIT-Lincoln Lab, Rm A-210
P. O. Box 73
Lexington, Mass. 02173

HQ, US Army Combat Dev Cmd
ATTN: CDCLN-EL
Ft Belvoir, VA 22060

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ATTN: AMSTA-Z, Mr. R. McGregor
Warren, Michigan 48090

Commanding General
US Army Elect Command
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Ft Monmouth, NJ 07703
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Ft Monmouth, NJ 07703

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US Army Elect Command
ATTN: AMSEL-SC
Ft Monmouth, NJ 07703

Commanding General
US Army Elect Command
ATTN: AMSEL-RD-D
Ft Monmouth, NJ 07703

Commanding General
US Army Combat Dev Cmd
ATTN: CDCMR-E
Ft Belvoir, VA 22060

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US Army Artillery Board
Ft Sill, OK 73503

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Defense Devel & Engineering Lab
ATTN: SMUEA-DDW-2, Mr. H. Tannenbaum
Edgewood Arsenal, MD 21010

Commanding Officer
Aberdeen Proving Ground
ATTN: STEAP-TL
Aberdeen Proving Ground, MD 21002

Director, Ballistic Rsch Labs
US Army Aberdeen Rsch & Dev Cen
ATTN: AMXRD-BTL, Mr. F. J. Allen
Aberdeen Proving Ground, MD 21005

Commanding Officer
US Army Ballistic Rsch Labs
ATTN: AMXBR-B & AMXBR-1A
Aberdeen Proving Ground, MD 21005
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US Army Land Warfare Lab
ATTN: CRDLWL-4A
Aberdeen Proving Ground, MD 21005

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Ft Huachuca, Arizona 85613

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Commanding Officer
USACDC Inst of Nuclear Studies
Ft Bliss, TX 79916

Commanding Officer
USADDC Inst of Systems Analysis
ATTN: CDISA-SSCI & CDISA-STANO
Ft Belvoir, VA 22060

Commanding General
US Army Missile Command
ATTN: AMSMI-RFG, Mr. N. Bell
Redstone Arsenal, AL 35809

Commanding Officer
Frankford Arsenal
ATTN: SMUFA-N-5300, Mr. M. Schoenfield
Bldg 201
Philadelphia, PA 19137

Commanding Officer
US Army Satellite Comm Agcy
ATTN: AMCPM-SC-3
Ft Monmouth, NJ 07703

Director
Night Vision Lab (USAECOM)
ATTN: ASMEL-NV-OR, Mr. S. Segal
Ft Belvoir, VA 22060

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Cameron Station (Bldg 5)
Alexandria, VA 22314
12

Director, USN Research Lab.
ATTN: Code 6530, H. Shenker
Washington, D. C. 20390

Mr. E. F. Corwin
Head, Met Research Branch
Meteorological Division
Naval Air Sys Cmd (AIR-5401)
Washington, D. C. 20360

Commanding Officer
US Army Combat Dev Cmd
Intelligency Agency
ATTN: CDCINT-P, Cpt Thoresen
Ft. Huachuca, Arizona

Director
US Army Advanced Materiel Concepts Agency
2461 Eisenhower Avenue
Alexandria, VA 22314

University of Oklahoma
Research Institute
Field Artillery Research Off
P. O. Box 3124
Ft. Sill, OK 73503

Dir US Naval Research Lab
ATTN: CODE 6530, H. Shenker
Washington, D. C. 20390