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PREDICTION OF NORMAL FORCE, PITCHING MOMENT, AND YAWING FORCE ON BODIES
OF REVOLUTION AT ANGLES OF ATTACK UP TO 50 DEGREES USING A CONCENTRATED
VORTEX FLOW-FIELD MODEL

M < 0.9

BY
Andrew B. Wardlaw, Jr.

24 OCTOBER 1973

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less than 0.9. For models with smaller nose fineness ratios, and at higher Mach numbers, accuracy deteriorates and side force predictions are too high.

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A CONCENTRATED VORTEX FLOW-FIELD MODEL

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ROBERT WILLIAMSON II
Captain, USN

L. H. Schindel
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By direction

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INTRODUCTION

As a body of revolution is exposed to an increasing angle of attack, the flow on its leeward side separates and rolls up to form vortices. Under these conditions, normal force and pitching moment can no longer be predicted by slender body theory. For relatively small angles of attack ($\alpha < \tan^{-1}(4.5/\ell)$) two symmetric vortices form as shown in Figure 1a. At larger angles of attack wake vortices grow unsymmetrically and break away or are shed from the leeward side, as illustrated in Figure 1b. The resulting flow-field asymmetries induce a yawing force and moment.

The analysis of bodies of revolution at angles of attack has been approached by many workers with varying degrees of rigor. Simple engineering calculations can be used to determine normal forces and pitching moments (Refs. (1)-(3)), but extensive numerical computations must be employed if detailed flow-field information is desired (Refs. (4), (5)). The objective of this report is to predict the yawing force component. Since this force arises from asymmetries in the vortex wake, the appropriate level of analysis must be sufficiently rigorous to provide an approximate wake description. Engineering level calculations do not generally provide this description and, therefore, are not amenable to these ends. While detail flow-field calculations can, in principle, achieve this purpose, they have only been applied up to very moderate angles of attack (10-15 degrees), too low for yawing forces to occur. Semi-empirical methods, such as those by Bryson (Ref. (6)), Schindel (Ref. (7)), and Angelucci (Ref. (8)), provide an approximate wake description, but assume a symmetric vortex wake and are not capable of predicting yawing forces. Also, normal forces and pitching moments can only be calculated up to moderate angles of attack (20-30 degrees).

The theory developed in this paper extends the model originally proposed by Bryson (Ref. (6)) by removing the symmetric wake assumption. This allows the side forces to be predicted, and increases angle of attack for which the theory is valid to about 50 degrees. In the current theory, the flow field is approximated using the impulsive flow analogy. This analogy assumes that the crossflow field is swept down the length of the body at the rate $U \cos \alpha$. At each axial station the flow field is taken to be analogous to flow about a cylinder in crossflow, whose radius is equal to the body radius at that axial location. The developing crossflow field is thus similar to flow about an impulsively started cylinder in crossflow. The development of the viscous flow field is modeled by superimposing point vortices on the potential flow solution for flow about a cylinder. Analysis is limited to ogive cylinders with sharp or blunt nose tips.

Using experimental data for a body with a laminar boundary layer ($2.5 < Re/ft \times 10^{-6} < 5.6$) and covering a range in Mach number from 0.5 to 1.1, free parameters inherent in the model are determined.

Calculated force and moment values are compared to experiment and computed vortex paths are compared to those determined from schlieren photographs. This type of approach allows conclusions to be drawn concerning the validity of the current problem formulation.

SYMBOLS

C_{DC}	local drag coefficient, $\frac{\pi}{2} \frac{r}{\sin^2 \alpha} \frac{dC_N}{dx}$
C_M	pitching moment coefficient, $\frac{M_Y}{Dq_\infty \pi r^2}$, referenced to model nose
C_M'	yawing moment coefficient, $\frac{M_Z}{Dq_\infty \pi r^2}$
C_N	normal force coefficient, $\frac{F_N}{q_\infty \pi r^2}$
C_Y	yawing force coefficient, $\frac{F_Y}{q_\infty \pi r^2}$
D	diameter of body of revolution
F_N	normal force
F_Y	yawing force
k	number of vortices in wake
L	body length
L_N	nose length
ℓ	body fineness ratio
ℓ_n	nose fineness ratio
M	Mach number
M_c	crossflow Mach number, $M \sin \alpha$
M_Y	pitching moment
M_Z	yawing moment
n	frequency for shedding vortices of like sign

q_∞	free-stream dynamic pressure
R_n	nose tip radius of curvature
r	local body radius
r_0	maximum body radius
Δr	difference in the radial location of the two initial vortices at which the first vortex will be shed.
r_n	the distance from the separation point, measured along the feed sheet at which nascent vortices are introduced
Re	Reynolds number
Δr_I	perturbation in the radial location of the first two vortices
S	Strouhal number
t	time
U	free-stream velocity
$W(\zeta)$	crossflow plane velocity at ζ , $\frac{d\bar{\phi}(\zeta)}{d\zeta}$
w	dimensionless crossflow plane velocity, $W/U \sin \alpha$
w_j	dimensionless crossflow plane velocity at the location of the j th vortex. The influence of the j th vortex is neglected.
x	distance along missile body (see Fig. 1)
Δx	x distance between points where successive vortices are shed.
y, z	crossflow plane coordinates (see Fig. 2)
y_j, z_j	y and z coordinates of the j th vortex
α	angle of attack
Γ	vortex circulation
δ	rate of radius change as a function of x , $\frac{dr}{dx}$
ζ	complex crossflow plane coordinate, $y + iz$
ζ_{01}, ζ_{02}	crossflow plane separation points (see Fig. 2)

θ_{01}, θ_{02}	crossflow plane separation angles (see Fig. 2)
λ	dimensionless circulation, $\frac{\Gamma}{2\pi r \sin \alpha U}$
λ_n	dimensionless circulation of newly introduced vortice
$\Delta \lambda_I$	initial vortex strength
ϕ	crossflow velocity potential
ϕ_N	nose roll angle
γ	dimensionless crossflow velocity potential, $\phi/U \sin \alpha$

LITERATURE SURVEY

The complete analysis of a body of revolution at angle of attack is a three-dimensional problem involving the solutions to the Navier-Stokes equations. This problem may be approached numerically in its entirety requiring an extensive computational effort. For relatively small angles of attack (12 degrees), numerical solutions have been obtained for cones using an approximate set of Navier-Stokes equations (Ref. (4)).

The device most commonly used to simplify the problem formulation is the impulsive flow analogy. This reduces the complete problem to a two-dimensional time varying analysis of a viscous fluid. By this analogy, the crossflow field is equivalent to that about an impulsively started, expanding cylinder, where $t = x/U \cos \alpha$. The expanding cylinder represents the increase in cross-sectional radius which occurs along the length of the body. Often the additional assumption is made that the flow about an impulsively started cylinder is incompressible. Throughout the remainder of this report the term impulsive flow analogy will be assumed to imply this assumption.

On the most rigorous level, the impulsive flow analogy allows the crossflow drag to be deduced from a numerical solution to the Navier-Stokes equations for flow over an impulsively started cylinder in crossflow. Several examples of this type of procedure are available for cylinders of constant radius (Refs. (9), (10)). For direct application to a body at angle of attack Walitt and Trullio (Ref. (5)) have carried out this analysis with a cylinder of varying radius in compressible flow. However, the computational requirements are prohibitive and solutions have only been obtained for angles of attack of 15 degrees.

A less rigorous analysis of impulsively started flow over a cylinder involves superposition of a large number of point vortices on the potential solution for incompressible flow about a cylinder. Typically, several hundred discrete vortices may be used. After formation, each point vortex is assumed to drift with the local fluid velocity. The schemes for predicting rate of vorticity

formation differ extensively. Although many such studies have been carried out, most are restricted to bodies of fixed cross-sectional dimensions (Refs. (11), (12) and (13)). However, Angelucci (Ref. (8)) has made a direct application to bodies at angle of attack, assuming a symmetric vortex wake. Good agreement is obtained for normal forces and moments.

A somewhat simpler method uses a single vortex to describe each wake vortex. In a procedure originally proposed by Bryson (Ref. (6)), vortex motion is determined by a balance of forces on a vortex and its feeding sheet. The model was originally limited to circular bodies with a wake pattern containing two symmetric vortices. This approach has been extended by Schindel (Ref. (7)) to bodies of elliptic cross section using conformal mapping techniques. In addition, changes in empirical parameters allow laminar and turbulent boundary effects to be accounted for. Results compare well with experimental data for normal forces. However, agreement of the pitching-moment calculation with experimental data is not as good. Range of validity for the results is $\alpha < \tan^{-1}(5/\ell)$. Davis (Ref. (14)) has extended Bryson's methods to a cylinder in crossflow with an asymmetric wake. But this formulation is valid only for cylinders of fixed radius and, therefore, cannot be directly applied to a body at angle of attack.

The most approximate theories available consider forces on a body at angle of attack to be made up of a viscous and an inviscid contribution which are additive. Initial attempts along these lines were by H. J. Allen (Ref. (1)). The inviscid force expression applied is similar to that obtained from slender body theory. The viscous contribution at each axial station is computed by applying the steady-state drag force for a cylinder in crossflow. This report was followed by that of Kelly (Ref. (2)) who attempted to improve the method by using drag data of an impulsively started cylinder in crossflow. His results were somewhat better than those of Allen, giving a better prediction of the pitching moment. Kelly's method has been refined by others, most recently by Thomson (Ref. (15)) to include angles of attack up to 90 degrees. In this work advantage is taken of impulsive flow data obtained by Sarpkaya (Ref. (16)). Correction tables allow Mach number, end effects, nose shapes, and the transition from laminar to turbulent flow to be taken into consideration. In addition, an experimental wake description is developed which allows side forces to be estimated. In a slightly different vein, L. H. Jorgensen (Ref. (3)) in an extension of Allen's work, presents a method of estimating normal forces on bodies at an angle of attack varying from 0 to 180 degrees and Mach numbers varying from subsonic to hypersonic.

BRYSON'S METHOD FOR IMPULSIVELY STARTED CYLINDERS IN INCOMPRESSIBLE FLOW

As pointed out above, our analysis is essentially that of an impulsively started cylinder in incompressible, viscous flow. Such flow can be simulated by superimposing point vortices on the potential solution for flow about a cylinder. In the present approach, a single point vortex is used to simulate each wake vortex. To satisfy the boundary conditions on a cylinder, each vortex in the wake is required to have a corresponding image vortex. The resulting configuration is shown in Figure 2 and the complex velocity potential is given as follows:

$$\phi(\zeta) = U \left[-\left(\zeta - \frac{r^2}{\zeta}\right)i - \frac{i}{2\pi} \sum_{j=1}^k \Gamma_j \ln \left\{ \frac{(\zeta - \zeta_j)}{(\zeta - r^2/\bar{\zeta}_j)} \right\} + \frac{r}{\tan \alpha} \frac{dr}{d\alpha} \ln(\zeta) \right] \quad (1)$$

The term, $-(\zeta - \frac{r^2}{\zeta})i U$, is the velocity potential solution for flow over a cylinder. Each of the logarithmic terms within the summation represent the velocity potential for a vortex and its image within the cylinder. The remaining term, which is the expression for a source, takes into account the changing cylinder radius.

The equations governing the motion of the vortices remain to be determined. Each logarithmic term within the summation in Equation (1) is a double valued function and has a branch line which connects the described vortex to its image. The portion of the branch line of physical importance is the part which crosses the flow field. It is considered to extend from the vortex location to the nearer of the points, ζ_{0_1} and ζ_{0_2} , which are on the cylinder surface (see Fig. 2).

These two points correspond to the position where flow is assumed to separate from the cylinder surface. The branch line of a growing vortex ($d\Gamma/dt \neq 0$) is known as a feeding sheet. Across it, a pressure jump of $\frac{\rho(d\Gamma/dt)}{\text{unit length}}$ occurs, which can be determined from the momentum conservation equation. The total force on the feeding sheet is $i\rho(\zeta_k - \zeta_{0_k})\frac{d\Gamma}{dt}$. This force on the feeding sheet is balanced by a lift force on the vortex of $-i\rho\Gamma(\frac{d\zeta_k}{dt} - W_k)$ given by the Kutta-Joukowski theorem. Here $(\frac{d\zeta_k}{dt} - W_k)$ is the vortex velocity with

respect to the local fluid velocity. Thus, the motion of each vortex is governed by the equation

$$\frac{d\mathcal{I}_j}{dt} + \frac{(\mathcal{I}_j - \mathcal{I}_{0j})}{\Gamma_j} \frac{d\Gamma_j}{dt} = W_j \quad (2)$$

The vortex circulations are determined by applying the Kutta conditions at the crossflow separation points:

$$W(\mathcal{I}_{01}) = W(\mathcal{I}_{02}) = 0 \quad (3)$$

This allows the circulation of two different vortices at a time to be specified. This does not preclude the existence of other vortices in the flow whose circulations are known.

Left undetermined by the Bryson method are the crossflow separation points, ζ_{01} and ζ_{02} , and boundary conditions in t . The boundary conditions in t include the points where vortices are introduced and shed. Also, the location of vortices in the crossflow plane and their strengths must be specified when they are introduced.

In applying the Bryson method the boundary conditions in t must be specified in such a manner that the resulting wake conforms to observations. When the wake behind the cylinder is assumed to become asymmetric, the following type of procedure appears to be required. Initially, two vortices are introduced. Using the governing equations the growth and the motion of these vortices are traced numerically. At some point, to be empirically determined, the circulation of one of these vortices is fixed and this vortex is considered to be shed. A new vortex is then introduced on the same side of the cylinder that the shed vortex originally emanated from. This shedding process is repeated at later points in time, vortices being shed from alternate sides of the cylinder.

APPLICATION OF THE BRYSON METHOD TO A MISSILE AT ANGLE OF ATTACK

The impulsive flow analogy implies the relations $x = U \cos \alpha t$ and $W(\infty) = U \sin \alpha$. Using these expressions and the definitions $\Gamma = 2\pi U r \sin \alpha \lambda$ and $W_j = w_j U \sin \alpha$, Equation (2) becomes

$$\frac{d\mathcal{I}_j}{d\lambda} + \frac{(\mathcal{I}_j - \mathcal{I}_{0j})}{\lambda_j} \frac{d\lambda_j}{d\lambda} = \tan \alpha w_j - \frac{(\mathcal{I}_j - \mathcal{I}_{0j})}{r} \frac{dr}{d\lambda}; \quad j=1,2 \quad \text{growing vortices}$$

$$\frac{d\mathcal{I}_j}{d\lambda} = \tan \alpha w_j; \quad j=3,4,\dots,K \quad \text{shed vortices}$$

The Kutta condition for a cylinder whose radius may be expanding becomes:

$$\frac{1}{U \sin \alpha} \frac{d\bar{\phi}(\zeta_{01})}{d\zeta} = \omega(\zeta_{01}) = \frac{r}{\tan \alpha} \frac{dr}{dz} \frac{1}{\bar{\zeta}_{01}} \quad (5)$$

$$\frac{1}{U \sin \alpha} \frac{d\bar{\phi}(\zeta_{02})}{d\zeta} = \omega(\zeta_{02}) = \frac{r}{\tan \alpha} \frac{dr}{dz} \frac{1}{\bar{\zeta}_{02}}$$

Substituting Equation (1) into Equations (5), it becomes evident that λ_1 and λ_2 are determined by solving two simultaneous equations. It is also evident in this formulation that satisfaction of the inviscid boundary condition on the cylinder is sufficient to ensure that both λ_1 and λ_2 are real.

In solving Equations (4) subject to conditions (5), λ_1 and λ_2 have been eliminated from (4) reducing the number of equations involved. The actual derivation is shown in Appendix A.

The formulated method is applied in the following manner. Near the nose of the missile two nascent vortices are introduced. The x coordinate of introduction is determined by satisfying the relation $\frac{1}{2} \sin \theta \tan \alpha = \tan \delta$. This relationship moves the point of vortex² introduction toward the nose of the model with increasing angle of attack. In cases where this relation places the starting point in front of the nose, the problem is automatically started at the nose. This relationship was derived by Bryson (Ref. (6)) by considering a cone. The slope δ represents the half angle of the largest cone for which vortex separation occurs. The two vortices are each assumed to be located along the directions $\theta_{01} + \pi/3$ and $\theta_{02} - \pi/3$ from their respective separation points

δ_{01} and δ_{02} . This criterion is also due to Bryson and was determined by the analysis of the trajectory of a vortex near the crossflow separation point.

Asymmetry is introduced at this point by slightly perturbing the radial location* of the two vortices. The initial strengths of the two vortices, as well as the magnitude of the perturbation applied, are free parameters which are empirically determined. They will be discussed in more detail later.

As the numerical solution to the governing equations progresses, the asymmetry in the problem grows. When the difference in radial location of the two vortices reaches a specified value, the outermost vortex is shed and a new vortex is introduced on the same

*the radial location of the jth vortex is defined to be $\sqrt{y_j^2 + z_j^2}$

side of the body. In order to introduce a new vortex whose circulation is other than zero, the existing vortex pattern must be perturbed. This is accomplished by reducing the circulation of the shed vortex a few percent. It can be shown that this procedure will always lead to a new vortex with a circulation of the correct sign. The new vortex is assumed to be located along the feeding sheet of the vortex to be shed. Its initial strength is uniquely determined by its crossflow field location and the decrease in circulation ascribed to the shed vortex.

Subsequent vortices are shed at intervals specified by the Strouhal frequency. Initial conditions for succeeding vortices are identical to those just described.

Application of the method described above requires specification of the following free constants:

1. Initial vortex strength (λ_1)
2. Initial radial perturbation (Δr_p)
3. The difference in radial location of the two initial vortices at which the first vortex is shed (Δr)
4. The distance from the separation point, measured along the feeding sheet of the shed vortex, at which a nascent vortex is introduced (r_n)
5. Circulation reduction of the shed vortex ($\Delta \lambda_n$)
6. Crossflow separation angles (θ_{01} and θ_{02})
7. Strouhal number (S)

The Strouhal number is determined experimentally using schlieren photographs of a tangent ogive cylinder at angle of attack. Although measured values showed a large degree of fluctuation, an average value of .17 was established.

The crossflow separation angle can also be determined in the wind tunnel, however, use of measured values leads to unstable numerical results. This may be due to the approximate nature of the flow field model. Simulation of a wake vortex using a single point vortex may not be giving an accurate flow field description in the vicinity of the separation points. To circumvent this problem, the lead of Bryson is followed and the separation angle is set near 45 degrees.

There is no direct way of readily determining the remaining parameters. However, it should be pointed out that the numerical solution is much less sensitive to these remaining parameters. Their actual values were established by a comparison with experimental measurements of normal forces and schlieren pictures of the leeward flow field.

NORMAL AND YAWING FORCES

The forces acting on any section of a body can be determined by enclosing it in a cylindrical control volume and balancing the momentum flux through the volume. Such a procedure is adequately discussed elsewhere (Ref. (17)) and results in the following expression:

$$C_N + i C_Y = 2 \sin \alpha \cos \alpha \left(\frac{r}{r_0} \right)^2 \left[1 + 2 \sum_{j=1}^K \lambda_j \left\{ \frac{r_j}{r} - \frac{r}{r_j} \right\} \right]$$

The first term in the above equation reduces to the slender body theory result of 2α at small angles of attack. The second term represents the crossflow drag due to the presence of the vortices, or the viscous contribution.

EVALUATION OF THE PROBLEM FORMULATION

The validity of the current formulation hinges on several points which are difficult to resolve in a conclusive manner. First and foremost, is the validity of the impulsive flow analogy. This analogy cannot be justified in its entirety. However, for inviscid flow about a slender body at small angle of attack, it is a consequence of slender body theory. At low Mach numbers where the flow is approximately incompressible, the restriction on small angle of attack can be relaxed. This does not, of course, shed any light on the validity of the impulse flow analogy for fluid flows in which viscous effects are important. All that can really be said in these cases is that the observed development of a vortex wake on the leeward side of a body at angle of attack bears at least a qualitative relation to the wake which develops behind a cylinder in crossflow (Ref. (18)).

Another point of question in the current problem formulation is the practice of simulating viscous flow by superimposing point vortices on inviscid, potential flow solutions. Even though this is an ad-hoc problem formulation, previous success in studies for cylinders in crossflow using a large number of point vortices indicate that this manner of approach provides an accurate wake description - in one case, good enough to predict the shedding frequency (Ref. (11)). The use of a single-point vortex to describe each wake vortex, as is done in this report, is more questionable. Such an approach is only capable of describing the gross flow-field features. Pressure calculations around the circumference of a cylinder using this type of description show an unrealistic peak near each point vortex. Despite this shortcoming, previous studies using this approach indicate that the integrated circumferential pressure distribution gives a good approximation of the normal forces at low angles of attack (Refs. (6), (7)).

The present approach is thus the extension of other studies which have proved to be accurate at lower angles of attack. Free parameters available in the current model allow adjustments to be made for effects which are not directly accounted for.

NUMERICAL CALCULATIONS

The governing equations are integrated using an Adams-Moulton predictor-corrector scheme which adjusts step size based on estimated error. This routine is started using the Runge-Kutta technique. The adjustable step size feature is extremely important in reducing running time. Small step sizes are required near the model nose where the local radius is small and at points at which new vortices are introduced. A listing of the program is provided in Appendix B.

Correct use of the outlined method requires the determination of a set of free parameter values that not only give a correct qualitative description of the wake, but also produce realistic forces and moments. There does not appear to be a unique set of parameters that best satisfy these requirements. However, certain sets of parameter values produce unacceptable results in that the computed crossflow wake does not develop into a Karman vortex street type of pattern. In some instances newly introduced vortices may move with diminishing circulation toward the cylinder surface. In any event, unless an orderly wake is formed, point vortices eventually approach one another resulting in numerical instability.

Numerical results were initially calculated for an impulsively started cylinder in crossflow. Suitable results were obtained using the set of parameters shown in Table 1. As indicated in this table, calculations were carried out assuming a symmetric wake containing two vortices, an unsymmetric wake still containing only a vortex pair, and an asymmetric wake in which vortices were periodically shed. In Figure 3 the calculated drag curves are plotted as a function of dimensionless time. Also shown in this graph is the experimental drag curve determined on an impulsively started cylinder

Table 1
FREE PARAMETER VALUES

Parameter	Cylinder in Crossflow			Missile at angle of attack Mach no. taken into account
	Symmetric Vortex Pair	Asymmetric Vortex Pair no shedding	Asymmetric wake with vortex shedding	
S	-	-	.1666	.1666

$$\theta_{01}, \theta_{02} \leftarrow \left[\begin{array}{ll} 40^\circ & \text{if } \frac{u_t}{r} < 10.2 \\ 44.6^\circ - 4.6 e^{-.7(\frac{u_t}{r} - 10.2)} & \text{if } \frac{u_t}{r} \geq 10.2 \end{array} \right] \rightarrow 42^\circ + 7.5(F)^\circ$$

λ_I	.08	.08	.08	.04
$\Delta r_p/D$	0.0	.006	.006	.006
$\Delta r_s/D$	-	∞	.1	.1 + .05(F)
$\Delta \lambda_n$	-	-	-2.5%	-2.5%
r_n/D			.075	.075

$$F = -.835 + 1.67M - .244 \sin\left[\frac{\pi}{6} (M-.5)\right] \quad .5 \leq M \leq 1.1$$

in crossflow by Sarpkaya (Ref. (16)). Only the calculation which provides for vortex shedding is comparable to experimental results over wide ranges in dimensionless time. The accompanying wake development is illustrated in Figure 4.

In our initial attempt to apply this method to a body at angle of attack, the same set of free parameter values used for the impulsively started cylinder was assumed. Results were compared to experimental tests on a tangent ogive cylinder ($\ell = 12$ and $\ell_n = 4$). These tests are further described in Appendix C. The only adjustment to the set of free parameter values concerned the crossflow separation angles. As can be seen in Table 1 for the cylinder in crossflow, the separation angles are decreased at the dimensionless time $\frac{Ut}{r} = 10.2$. In application to the ogive cylinder at angle of attack, the point in the computation where this decrease occurs must be adjusted, since the change in radial dimensions at the missile nose disturbs the time sequence of the flow development. It is found that the point at which $\frac{Ut}{r} = 10.2$ occurs two-thirds of the distance in time between the point where the first and second vortices are shed. This criterion is used to determine where θ_{01} and θ_{02} will be decreased. As suggested by Schindel (Ref. (7)), the smooth transition from one separation angle to another is achieved by solving the equation:

$$\frac{d\theta_{0j}}{dx} = \frac{.7}{r} \tan \alpha (44^\circ - \theta_{0j}) \quad ; \quad j = 1, 2 \quad (6)$$

and subject to the boundary condition: $x \rightarrow \infty$ as $\theta_{0j} \rightarrow 48^\circ$. In

the special case of a cylinder in crossflow, r is constant and the solution to 16 reduces to that shown in Table 1 for a cylinder in crossflow.

The calculated forces and pitching moments are compared to experimental results in Figures 5 and 6. In each of these figures experimental data for a range in Mach number from .5 to 1.1 are shown. The computed normal force coefficient (Fig. 5) compares well to the experimental data being closest to the curve generated at Mach .9. The calculated pitching moment coefficient (Fig. 6) is slightly high, matching best with the Mach 1.1 curve. The predicted yawing force coefficient, shown in Fig. 7, is somewhat larger than those measured and is closest to the Mach .5 curve.

The current theory is independent of Mach number. However, as can be seen from experimental data in Figures 5, 6, and 7, force and moment data is Mach number dependent. In order to include a Mach number effect in the theory, the set of free constants is assumed to be a weak function of Mach number. The relation between Mach number and free constant value is determined empirically by adjusting the set of constants to give the measured normal force curve at each Mach number. In addition, the sets of

free parameter values are modified somewhat to obtain better correspondence between observed and calculated vortex trails. The final set of constants is shown in the right-hand column of Table 1. These constant values yield stable vortex streets up to angles of attack near 50 degrees. The upper limit in each case is dependent on the actual nose configuration. Generally, the upper limit on angles of attack increases with decreasing nose fineness.

Computations using the Mach number dependent set of constants are compared to experiment in Figures 8 through 19. The close agreement between experimental and computed normal force coefficients are shown in Figure 8 while the pitching moment results are compared in Figure 9. The computed yawing force coefficient is compared to experimental measurements at Mach .5, .7, .9, and 1.1, in Figures 10, 11, 12, and 13, respectively. In each of these graphs four different yawing force coefficient curves are shown. They were generated by rolling the model nose. From these graphs it is evident that the yawing force coefficient is reasonably well predicted for Mach numbers less than .9. For larger Mach numbers the measured magnitudes of the yaw force coefficients decreases. This trend is not followed by the computed results. At the highest Mach number tested, 1.1, the yawing force coefficient is over-estimated by a factor of four.

In Figures 14 through 19, comparisons between the computed and observed vortex wakes are shown for angles of attack varying from 20 degrees to 45 degrees and Mach numbers of .5 and .9. It is evident from these pictures that the wake is qualitatively well reproduced. More consistent agreement is obtained at lower Mach numbers. At an angle of attack of 45 degrees and a Mach number of .9, the wake width is clearly over-estimated.

In order to assess the general applicability of this model, a comparison is made between calculated and experimental results for differently shaped ogive cylinders. Results at Mach .5 are shown in Figures 20 through 22. Good agreement is obtained for normal force. Although the pitching moment coefficient comparison is not as good as the normal force comparison, the experimental trend is followed. Figure 22 reveals that the decreasing yawing force coefficient observed on blunter bodies is not predicted by the theory.

Comparison between experimental and calculated results on differently shaped bodies at Mach 1.1 (Figures 23 through 25) shows that general trends found in experimental data for normal force and pitching moments are followed. However, agreement is not as close as in the case of lower Mach numbers. Also the side forces (Fig. 26) are consistently over-estimated and do not follow experimental trends.

SUMMARY AND CONCLUSIONS

The flow field surrounding an ogive cylinder at high angles of attack is simulated using an extension of the method initially proposed by Bryson (Ref. (6)). This method employs the impulsive flow analogy and models each wake vortex using a single point vortex. Free parameters inherent in the problem formulation are initially determined by comparison to drag force coefficient data on an impulsively started cylinder in crossflow.

Final parameter values are obtained by matching the normal force measured on a tangent ogive cylinder ($\ell_n = 4.0$, $\ell = 12$) over a range in Mach number of .5 to 1.1. These parameter values are made weakly Mach number dependent to reflect the experimental data. The resulting pitching moments compare well in trend with experiment. The calculated yaw force coefficient is of the correct magnitude at Mach numbers less than .9, but is too large at higher Mach numbers.

At a Mach number of .5 calculated and experimental results are compared for ogive cylinders with different nose fineness ratios. Predicted normal force and pitching moment coefficients are in good agreement with experimental results. On ogive cylinders with small slenderness ratios ($\ell_n \lesssim 3$) yawing force calculations over-estimated the experimental results.

The same type of comparison is carried out at a Mach number of 1.1. Agreement between calculated normal force and pitching moment coefficients was not as good as at Mach .5. The yawing force coefficient was consistently lower than predicted.

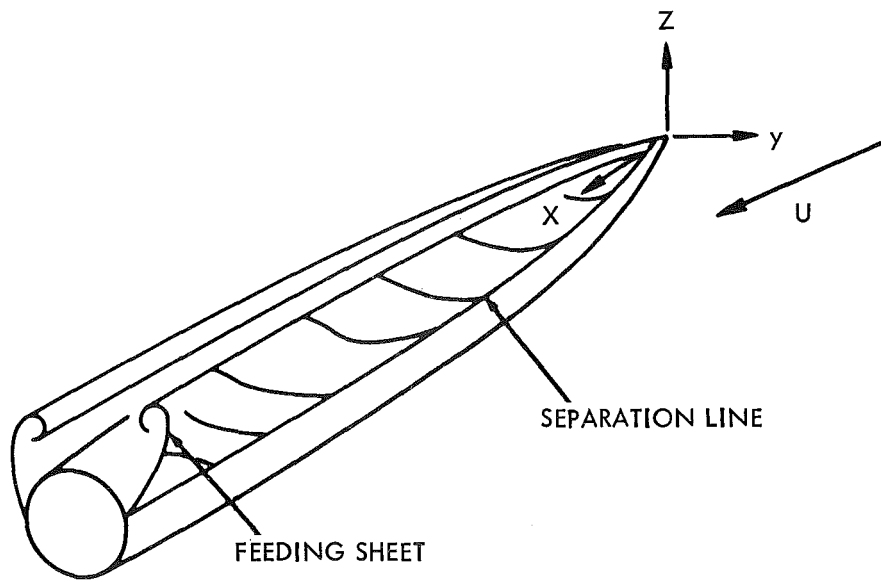
The current results indicate that on slender bodies and at low Mach numbers, the impulsive flow analogy combined with the Bryson method can be used to predict normal force, pitching moment and yawing force. For nose fineness ratios less than 4 and at Mach numbers near unity, accuracy deteriorates markedly and side forces are over-estimated.

Apparently the separation parameters are sensitive to Mach number in the transonic range. A more thorough investigation of this effect would be required to improve the accuracy of side force predictions in this regime.

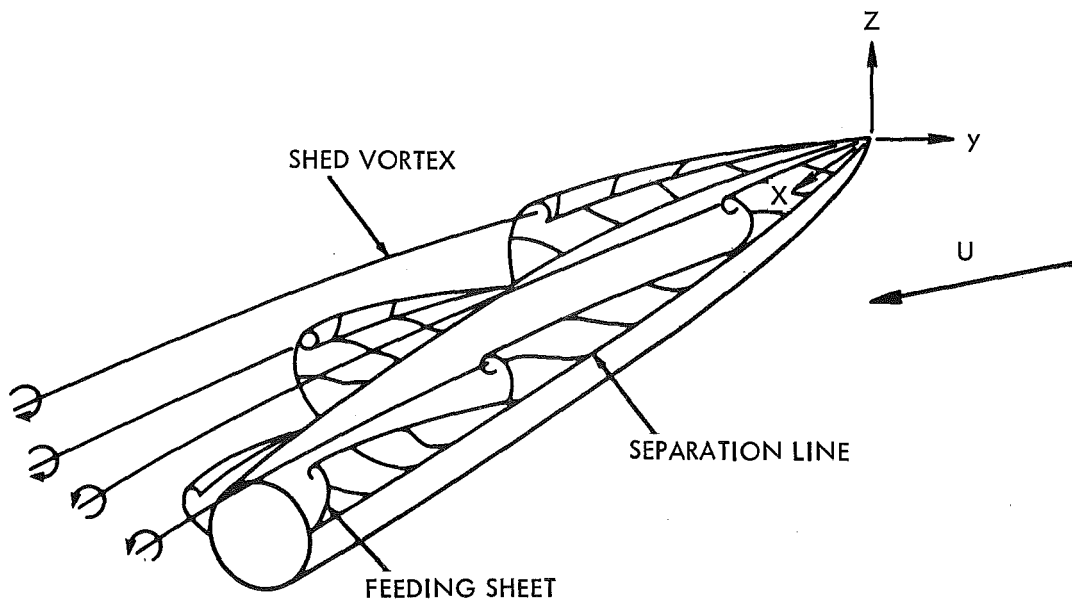
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(a) SYMMETRICAL VORTEX SHEDDING



(b) ASYMMETRICAL VORTEX SHEDDING

FIG. 1 VORTEX FORMATION ON A BODY OF REVOLUTION AT ANGLE OF A ATTACK

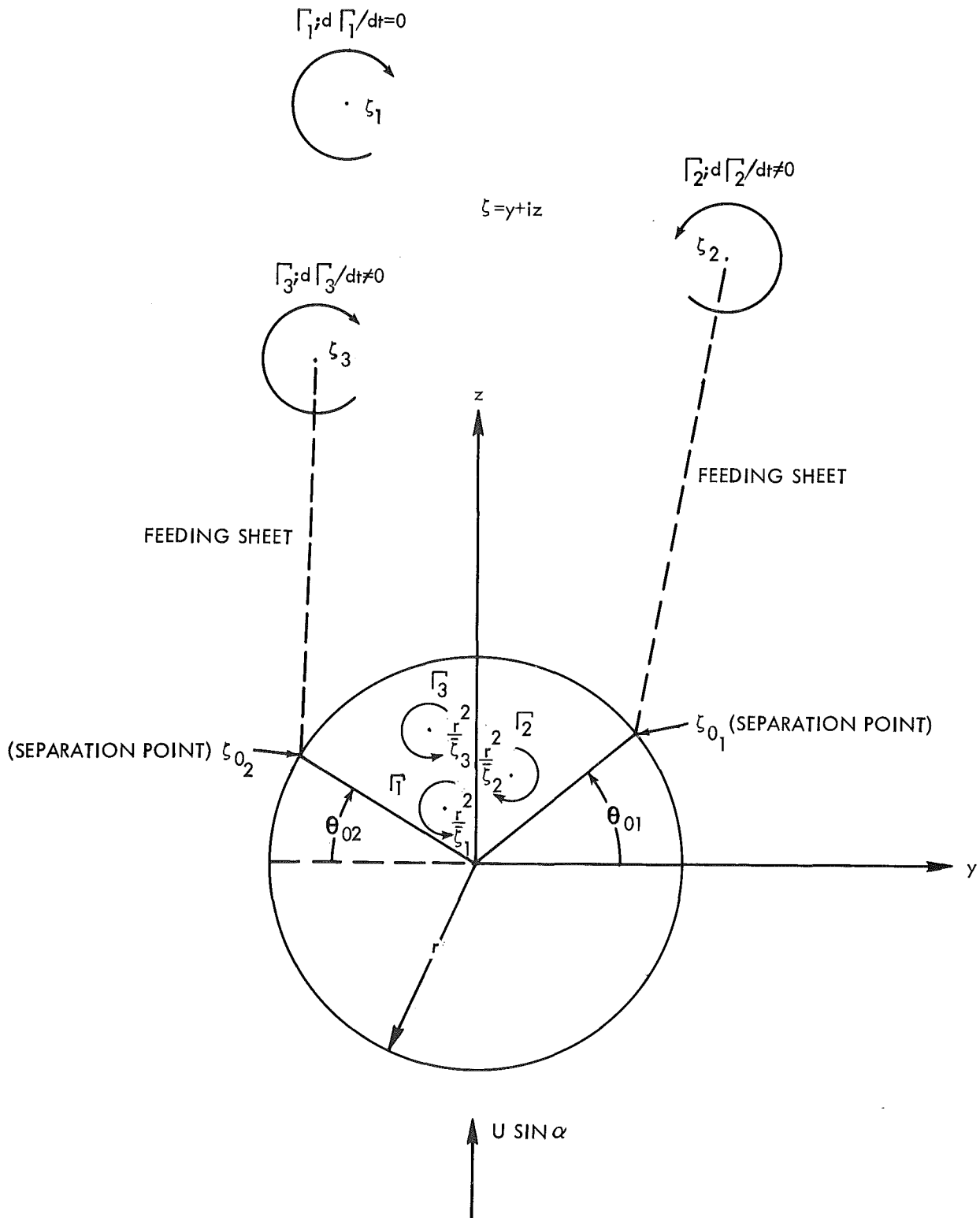


FIG. 2 DIAGRAM OF THE CROSSFLOW PLANE.

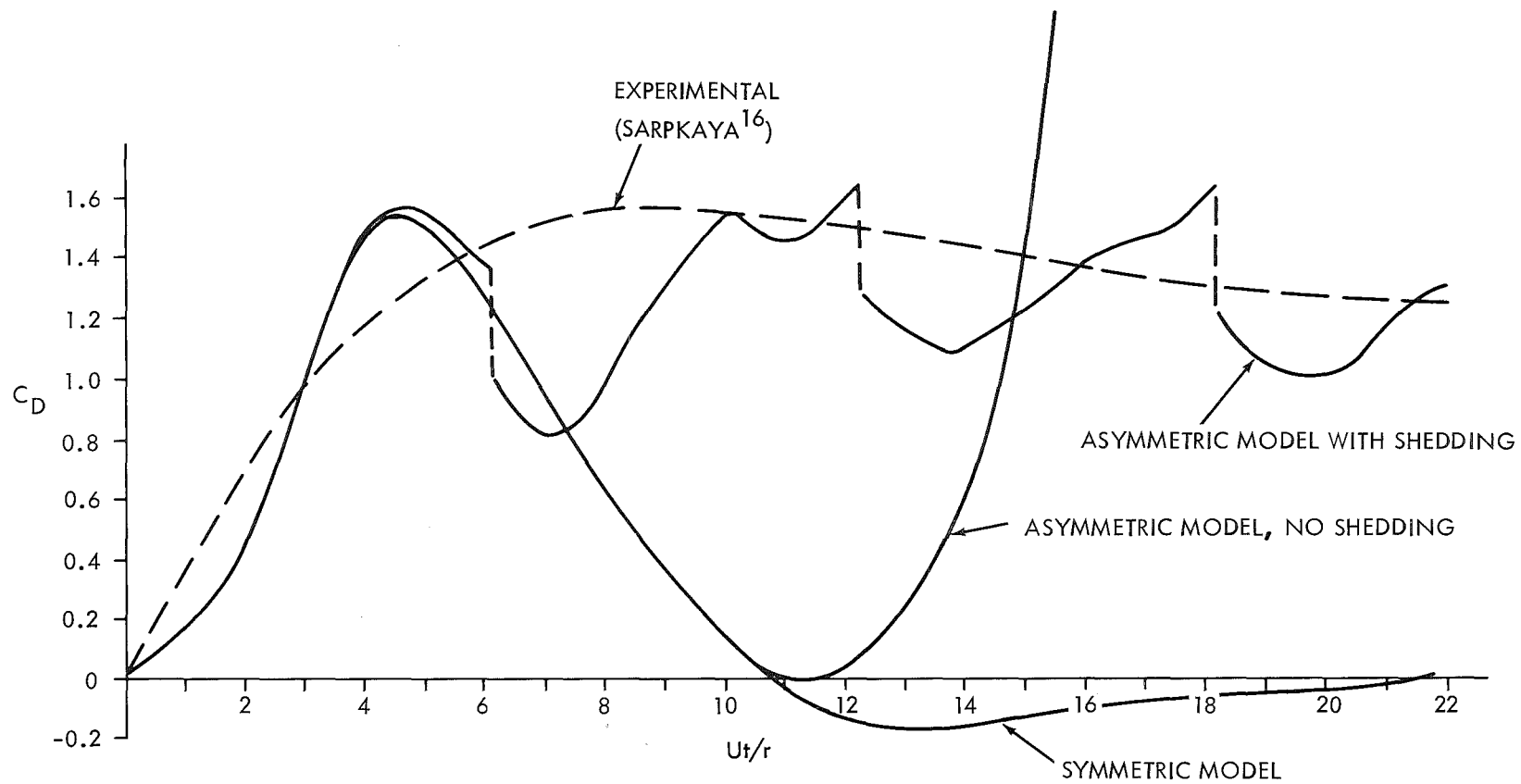


FIG. 3 DRAG FORCE COEFFICIENT AS A FUNCTION OF DIMENSIONLESS TIME FOR AN IMPULSIVELY STARTED CYLINDER IN CROSSFLOW.

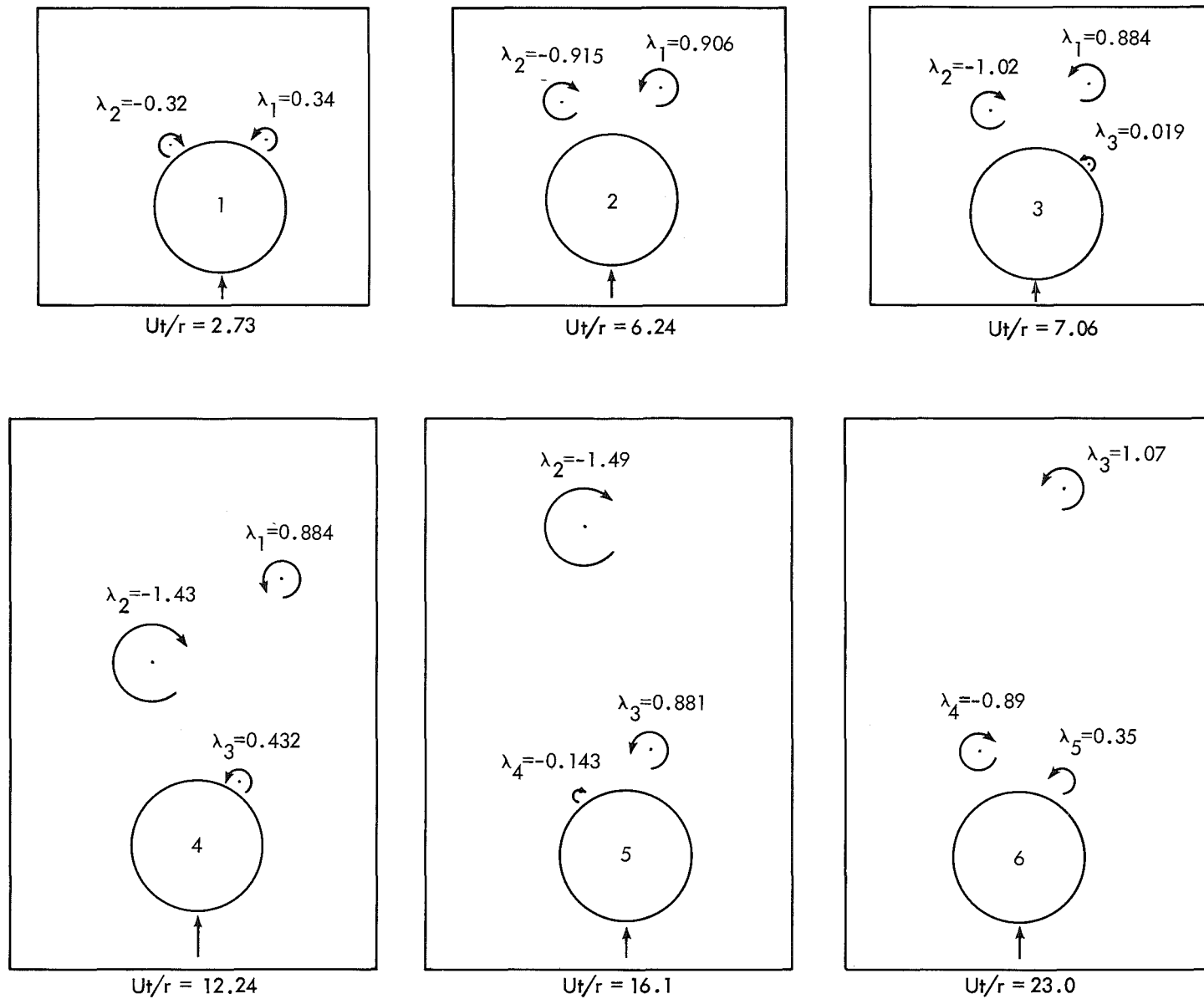


FIG. 4 DEVELOPMENT OF THE FLOW FIELD BEHIND AN IMPULSIVELY STARTED CYLINDER IN CROSSFLOW.

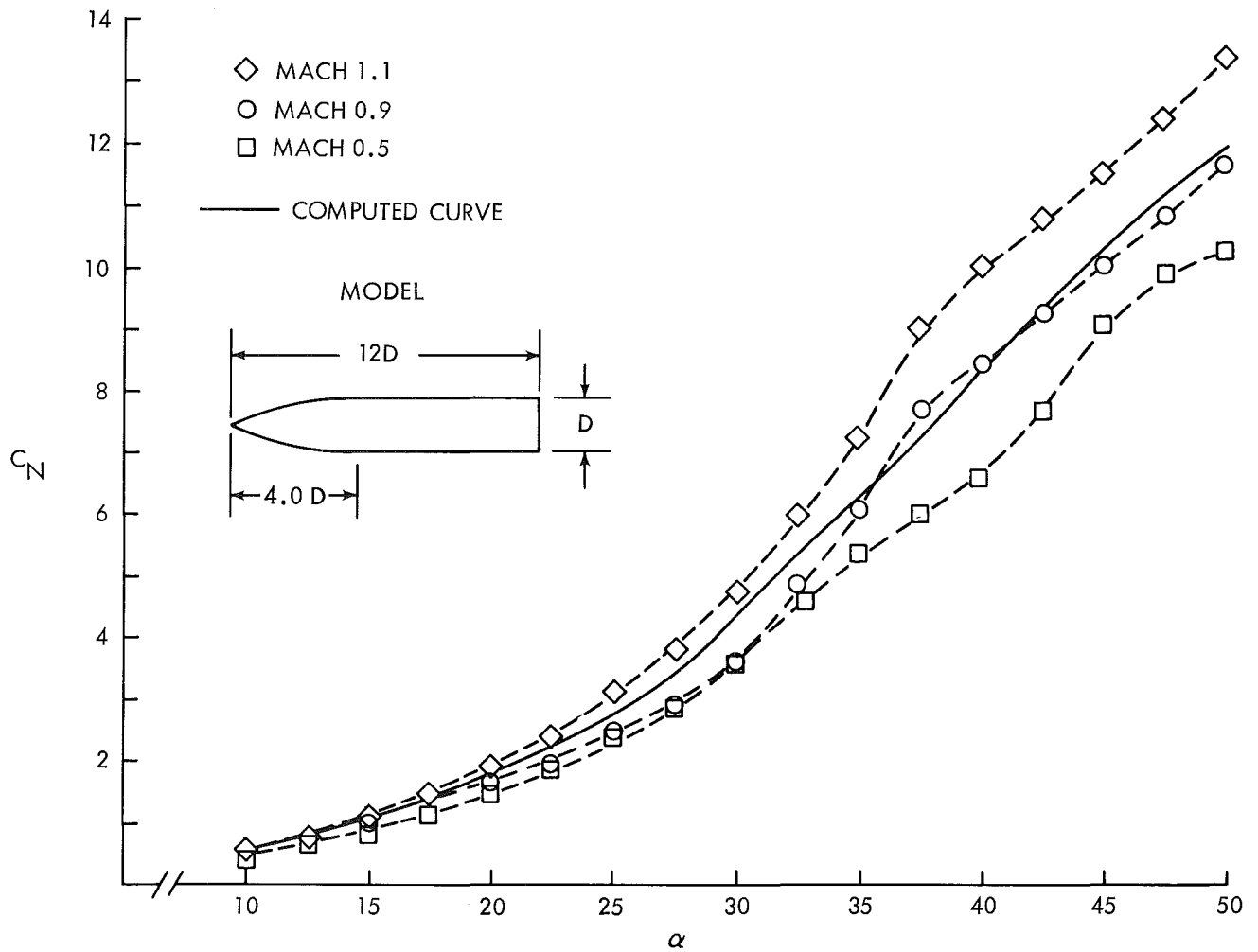


FIG. 5 NORMAL FORCE COEFFICIENT AS A FUNCTION OF ANGLE ATTACK. COMPUTED CURVE BASED ON FREE PARAMETER VALUES USED FOR A CYLINDER IN CROSSFLOW.

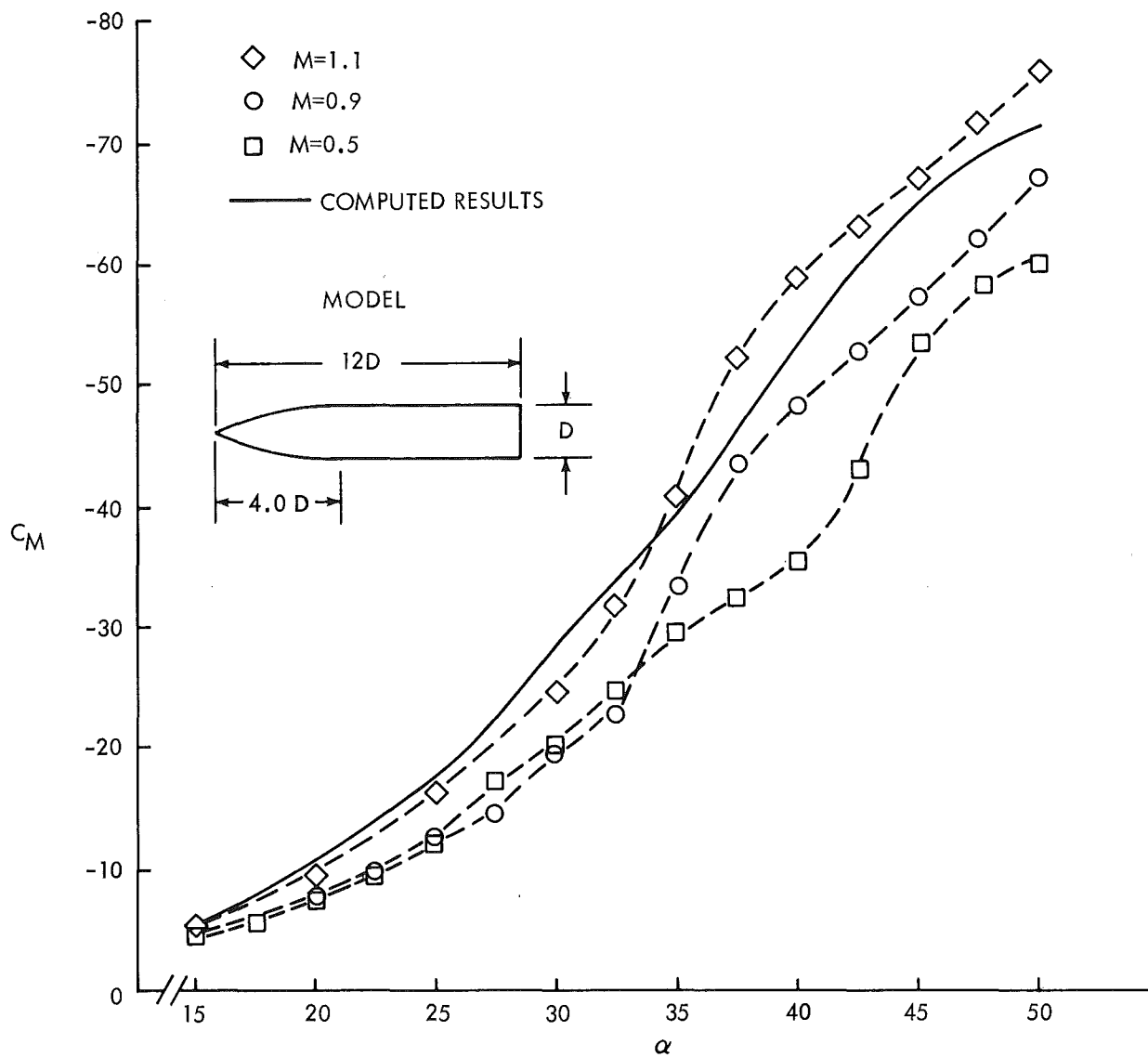


FIG. 6 PITCHING MOMENT COEFFICIENT AS A FUNCTION OF ANGLE OF ATTACK. COMPUTED CURVE BASED ON FREE PARAMETER VALUES USED FOR A CYLINDER IN CROSSFLOW.

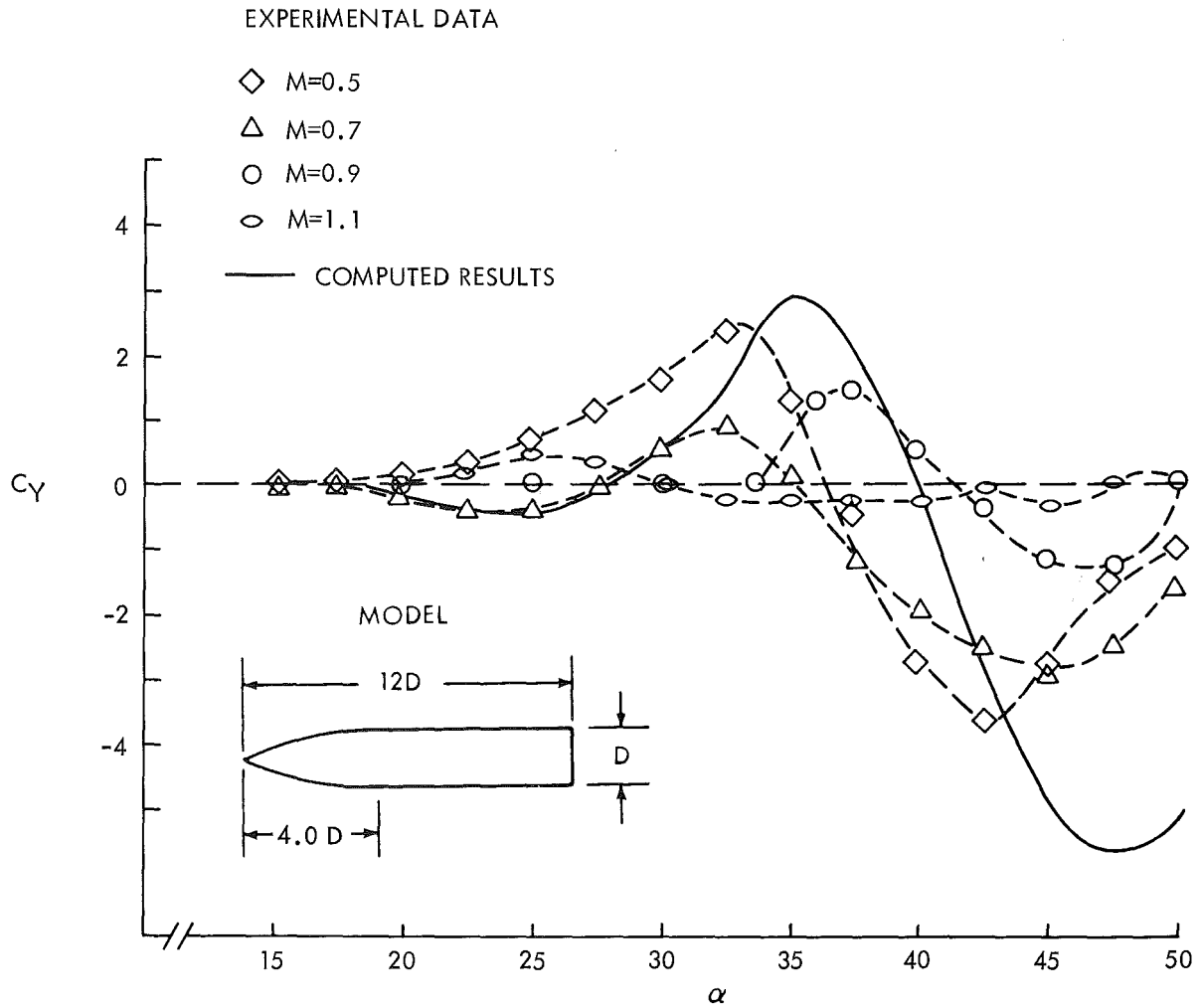


FIG. 7 YAWING FORCE COEFFICIENT AS A FUNCTION OF ANGLE OF ATTACK. COMPUTED CURVE BASED ON THE FREE PARAMETER VALUES USED FOR A CYLINDER IN CROSSFLOW.

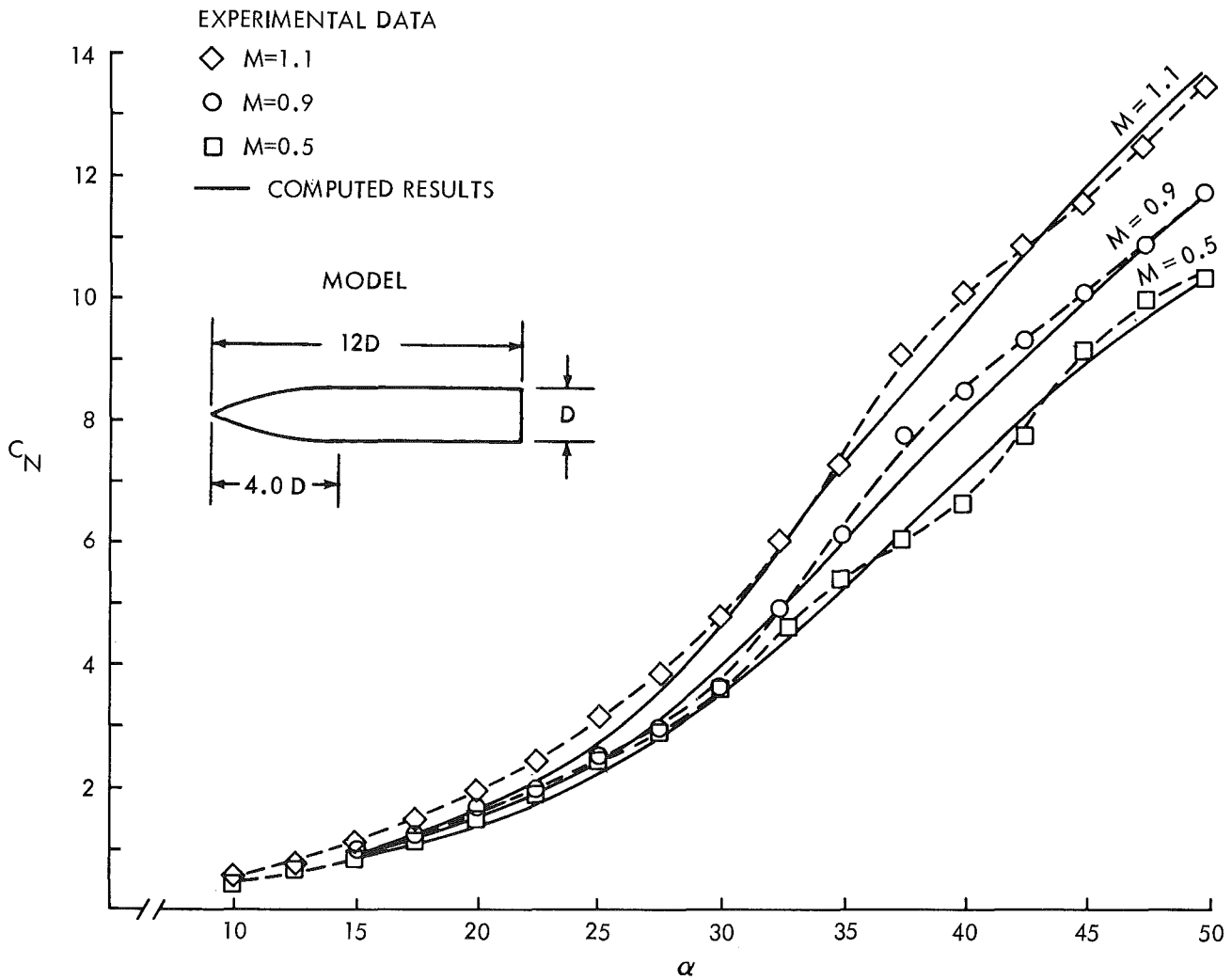


FIG. 8 NORMAL FORCE COEFFICIENT AS A FUNCTION OF ANGLE OF ATTACK. COMPUTED CURVES ARE BASED ON A SET OF PARAMETER VALUES WHICH ARE MACH NUMBER DEPENDENT.

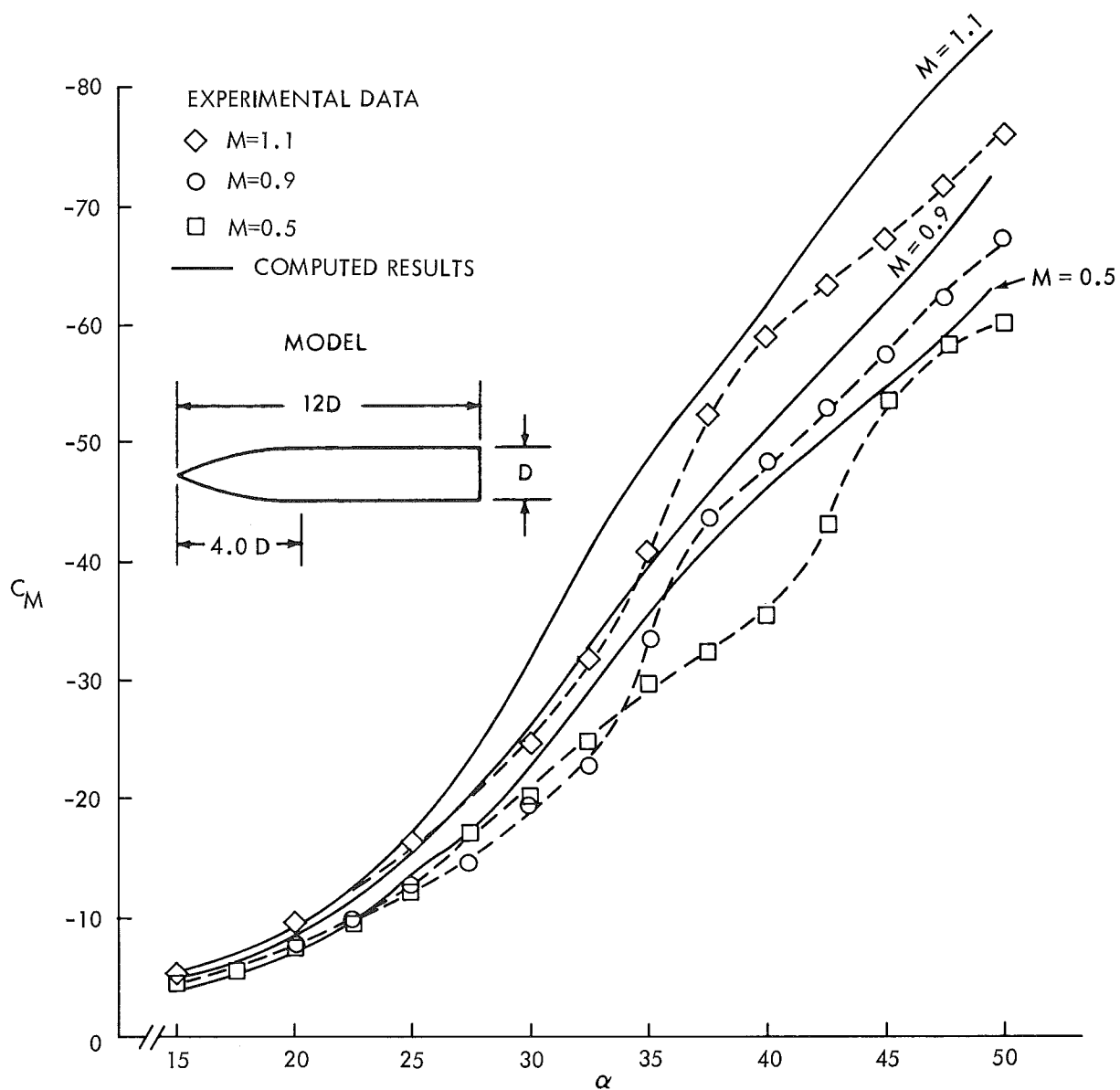


FIG. 9 PITCHING MOMENT COEFFICIENT AS A FUNCTION OF ANGLE OF ATTACK. COMPUTED CURVES ARE BASED ON A SET OF PARAMETER VALUES WHICH ARE MACH NUMBER DEPENDENT.

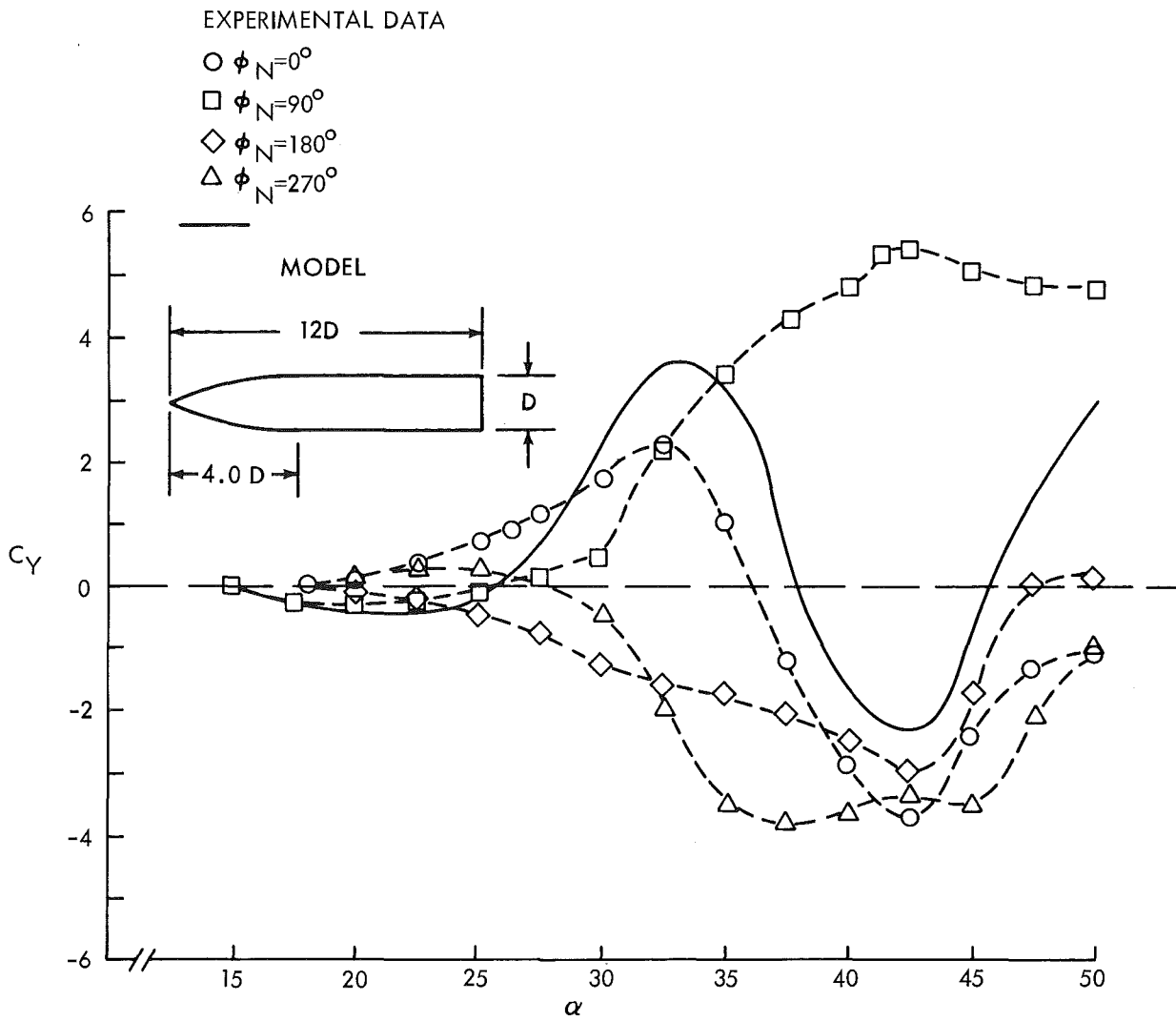


FIG. 10 YAW FORCE COEFFICIENT AS A FUNCTION OF ANGLE OF ATTACK FOR $M=0.5$. COMPUTED CURVES BASED ON FREE PARAMETER VALUES WHICH ARE MACH NUMBER DEPENDENT.

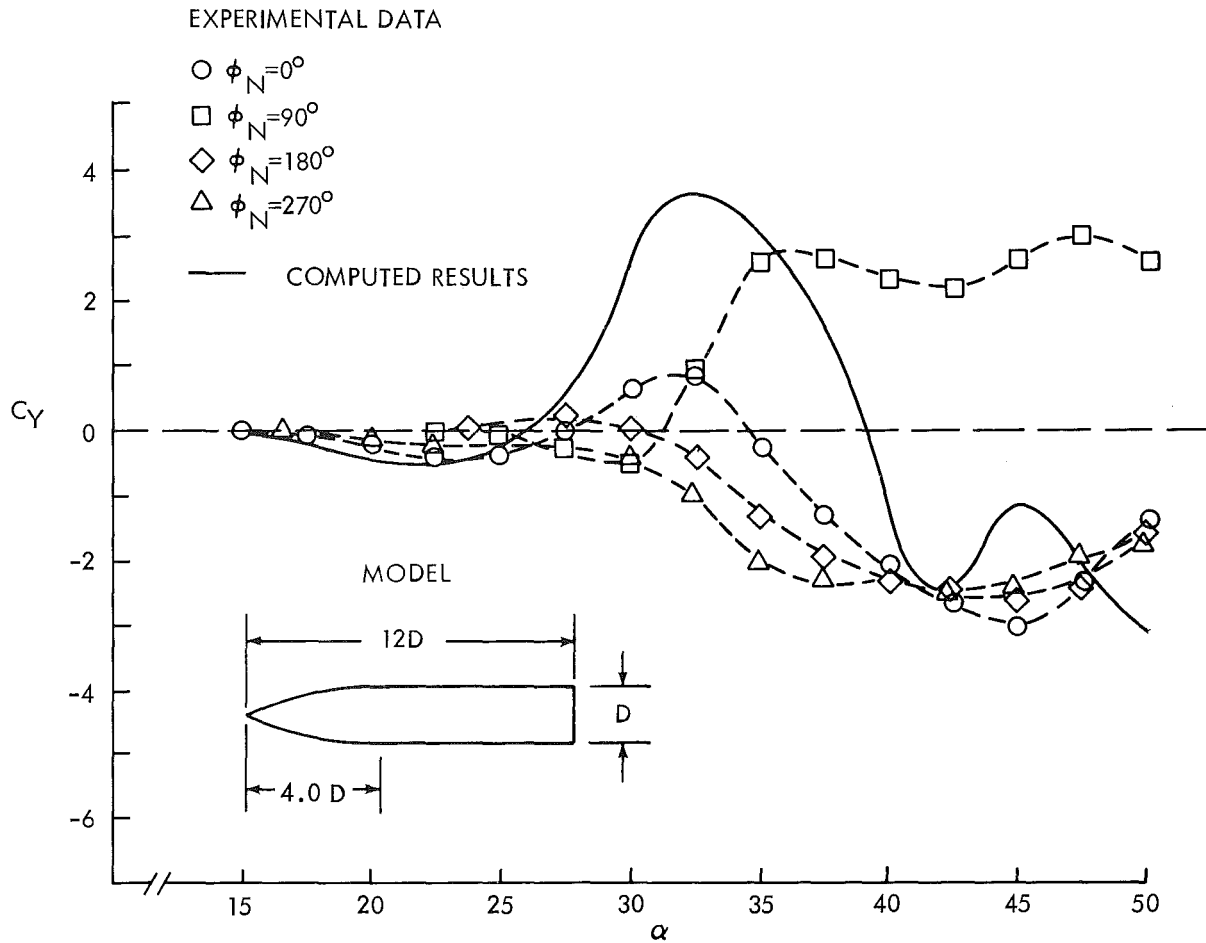


FIG. 11 YAW FORCE COEFFICIENT AS A FUNCTION OF ANGLE OF ATTACK FOR $M=0.7$. COMPUTED CURVE BASED ON FREE PARAMETER VALUES WHICH ARE MACH NUMBER DEPENDENT.

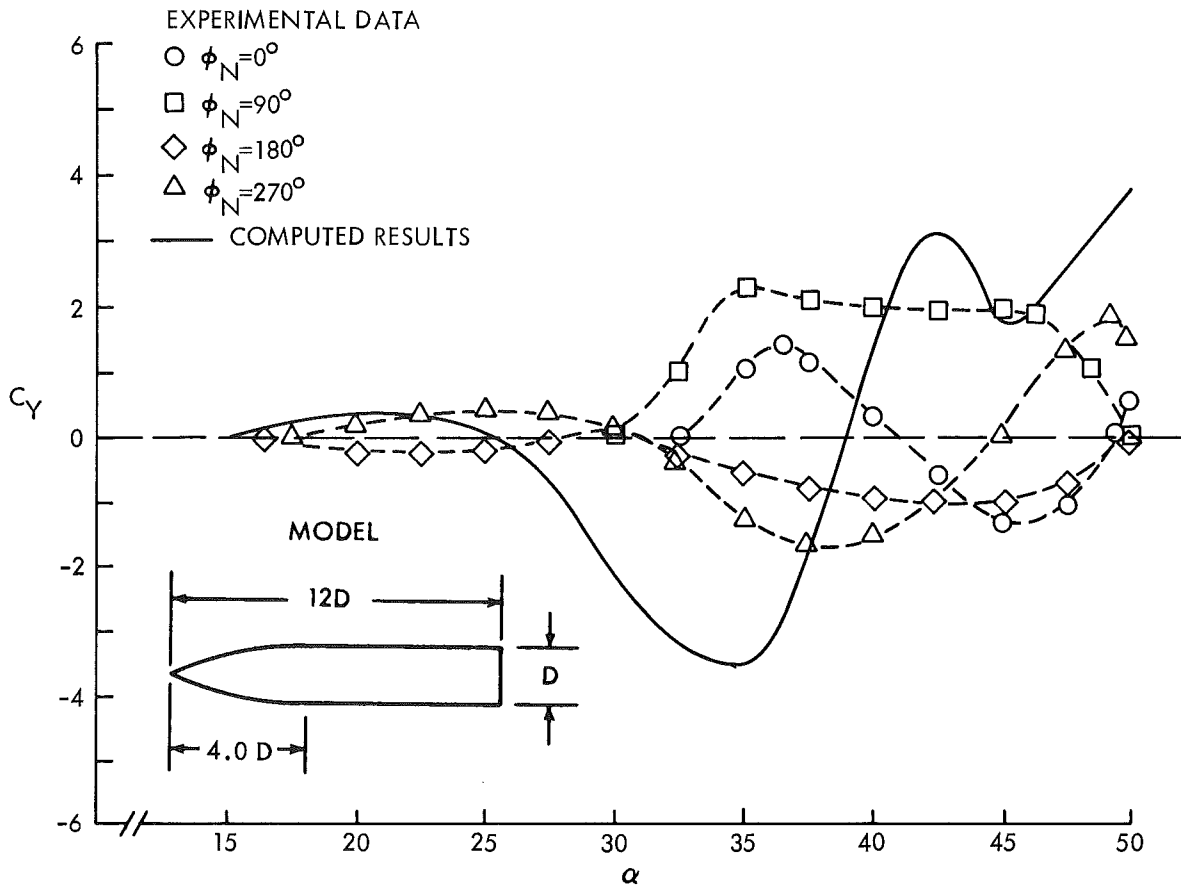


FIG. 12 YAW FORCE COEFFICIENT AS A FUNCTION OF ANGLE OF ATTACK FOR $M=0.9$. COMPUTED CURVE BASED ON FREE PARAMETER VALUES WHICH ARE MACH NUMBER DEPENDENT.

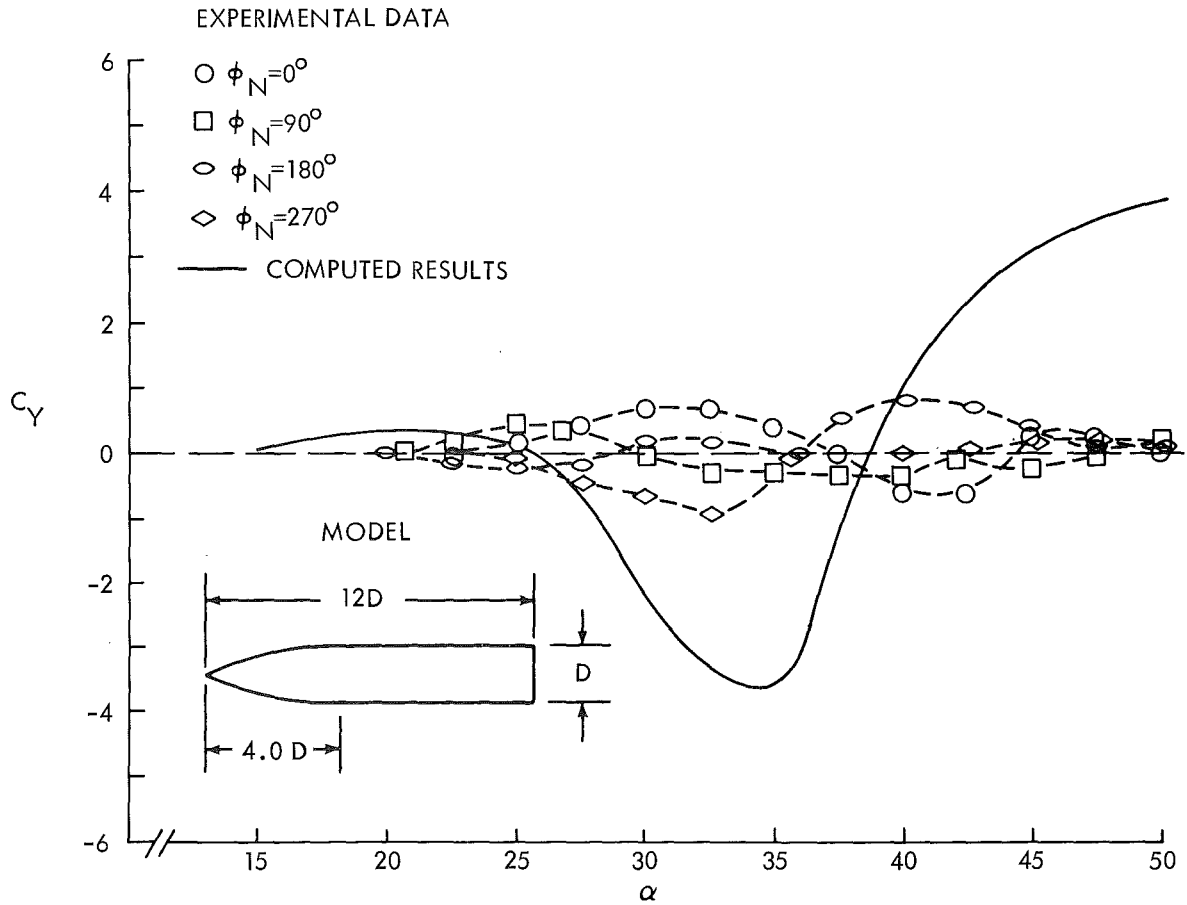


FIG. 13 YAW FORCE COEFFICIENT AS A FUNCTION OF ANGLE OF ATTACK FOR $M=1.1$.
 COMPUTED CURVE BASED ON FREE PARAMETER VALUES WHICH ARE MACH
 NUMBER DEPENDENT.

▨▨▨ CORRIDOR IN WHICH VORTICES ARE OBSERVED.

— COMPUTED VORTEX PATH

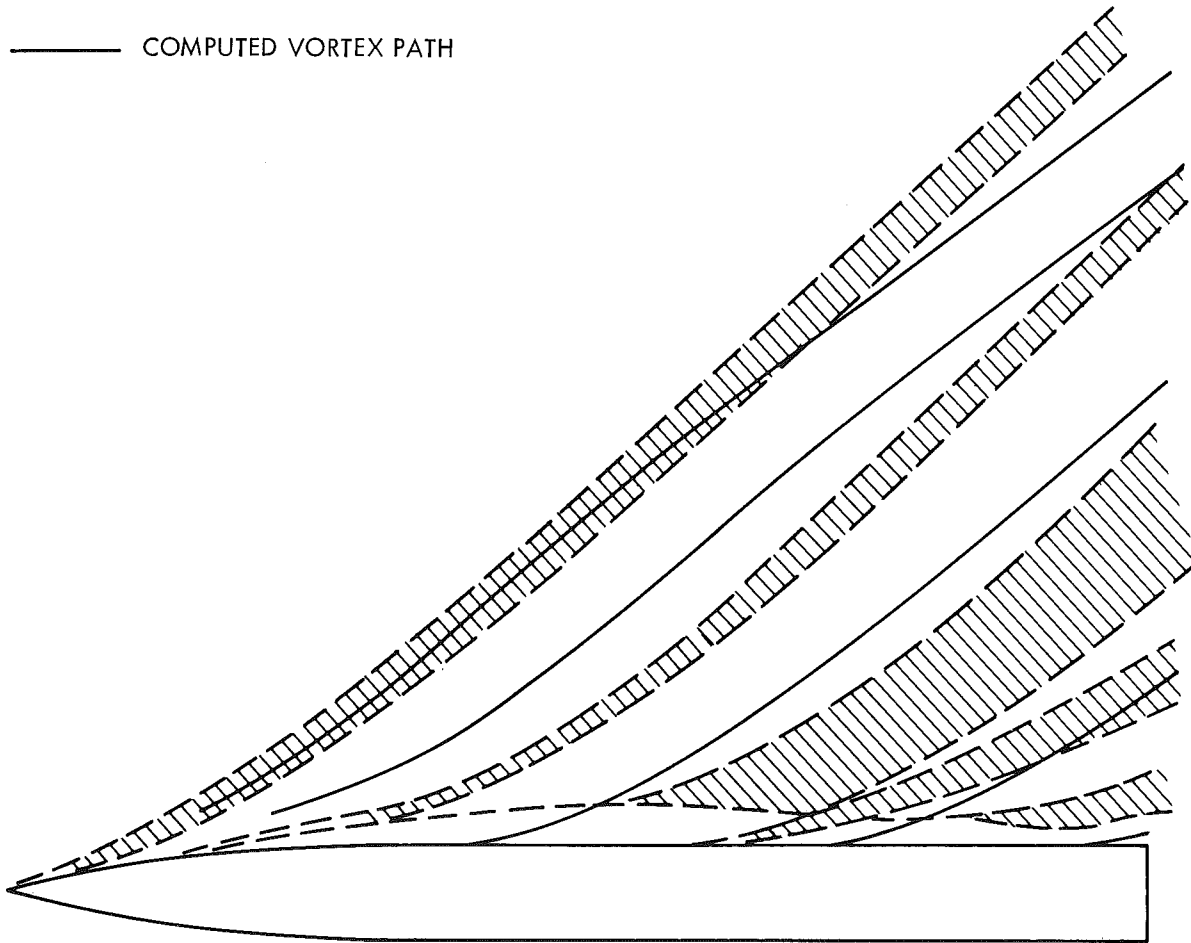


FIG. 14 COMPARISON OF OBSERVED AND COMPUTED VORTEX PATHS. $\alpha = 45^\circ$ MACH=0.5

 CORRIDOR IN WHICH VORTICES ARE OBSERVED.
 COMPUTED VORTEX PATH.

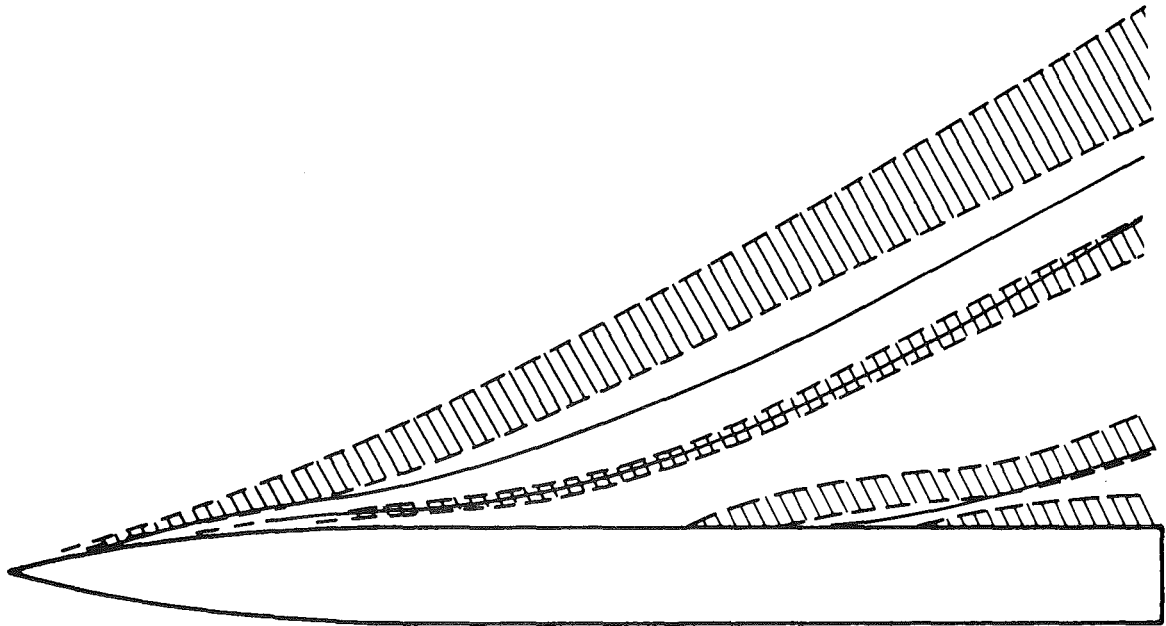


FIG. 15 COMPARISON OF OBSERVED AND COMPUTED VORTEX PATHS.
 $\alpha = 35^\circ$ MACH = 0.5

 OBSERVED VORTEX PATH.
 COMPUTED VORTEX PATH.

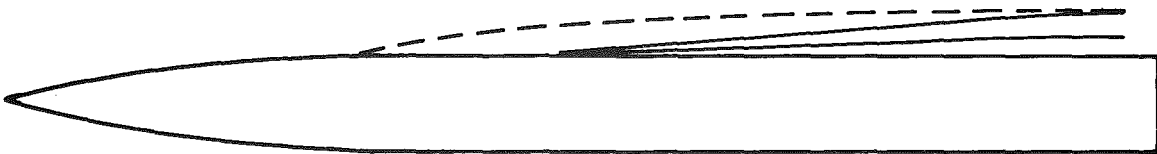


FIG. 16 COMPARISON OF OBSERVED AND COMPUTED VORTEX PATHS.
 $\alpha = 20^\circ$ MACH = 0.5

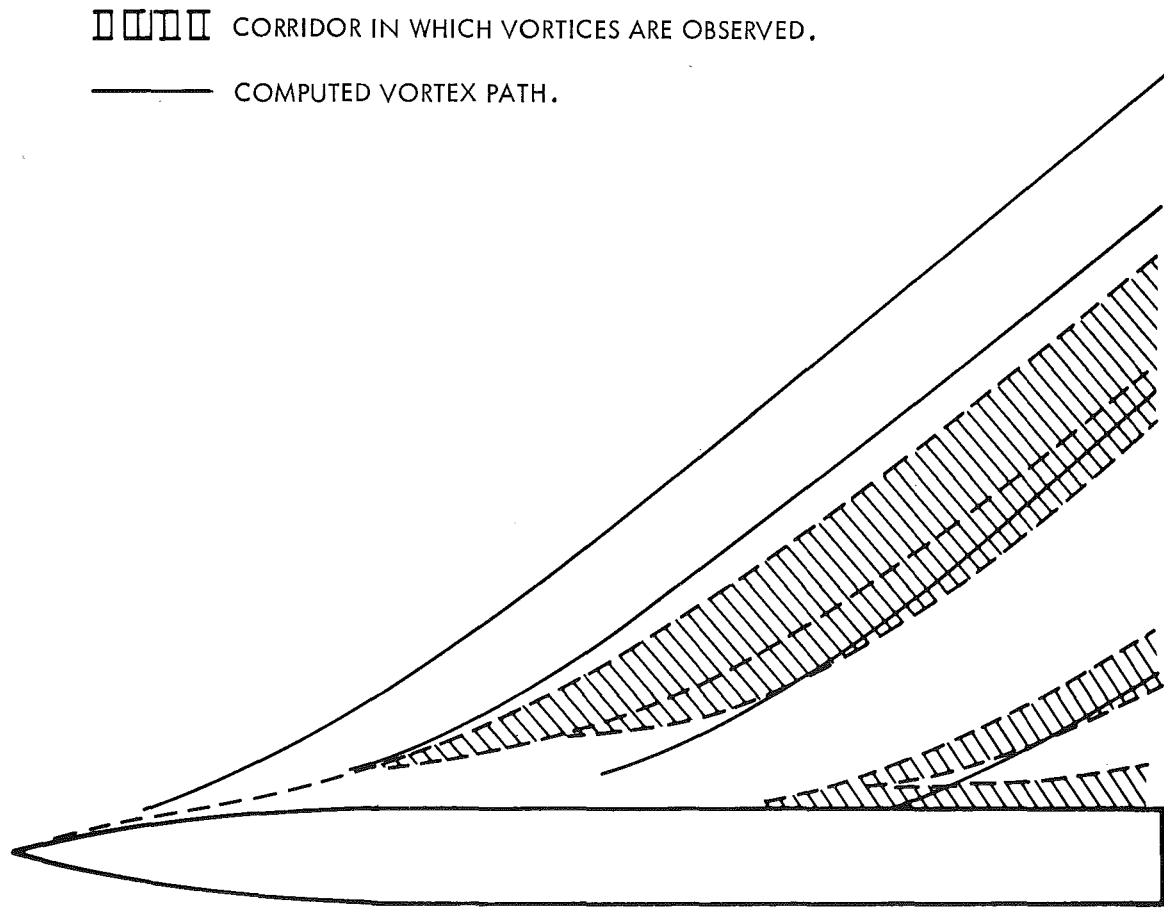


FIG. 17 COMPARISON OF OBSERVED AND COMPUTED VORTEX PATHS.
 $\alpha = 45^\circ$ MACH = 0.9

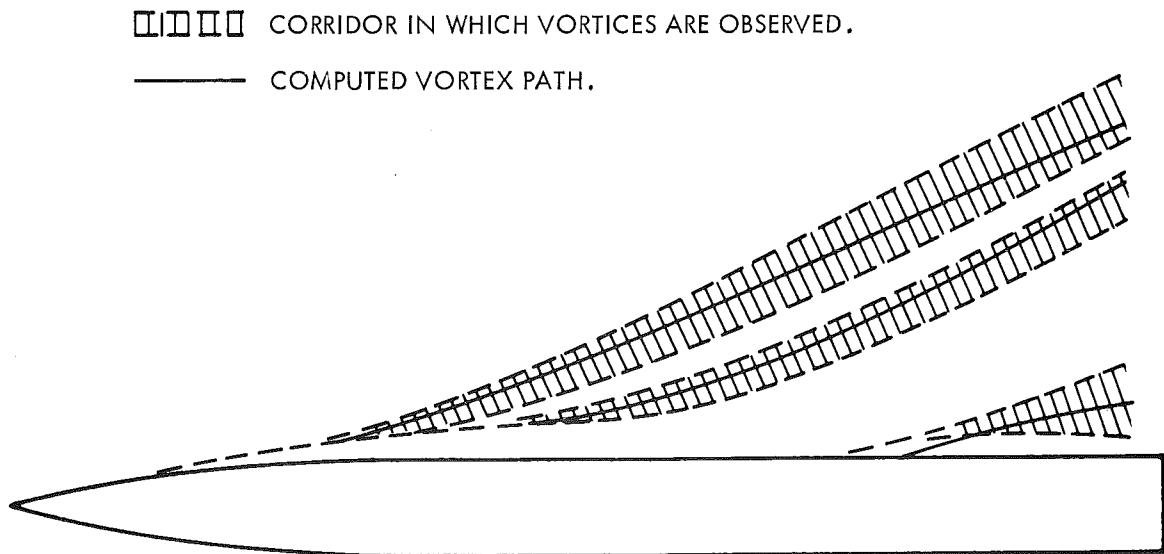


FIG. 18 COMPARISON OF OBSERVED AND COMPUTED VORTEX PATHS.
 $\alpha = 35^\circ$ MACH = 0.9

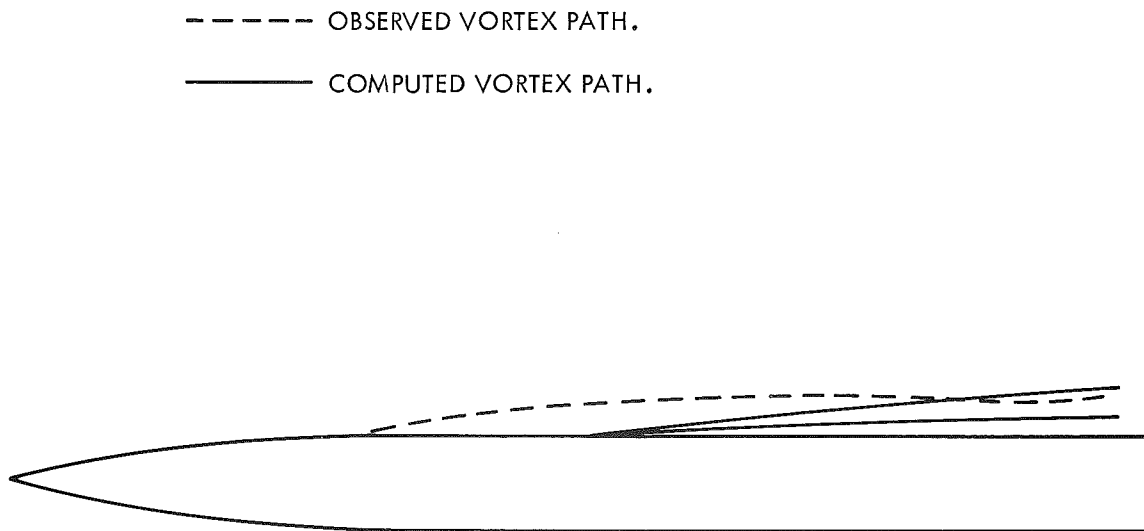


FIG. 19 COMPARISON OF OBSERVED AND COMPUTED VORTEX PATHS.
 $\alpha = 20^\circ$ MACH = 0.9

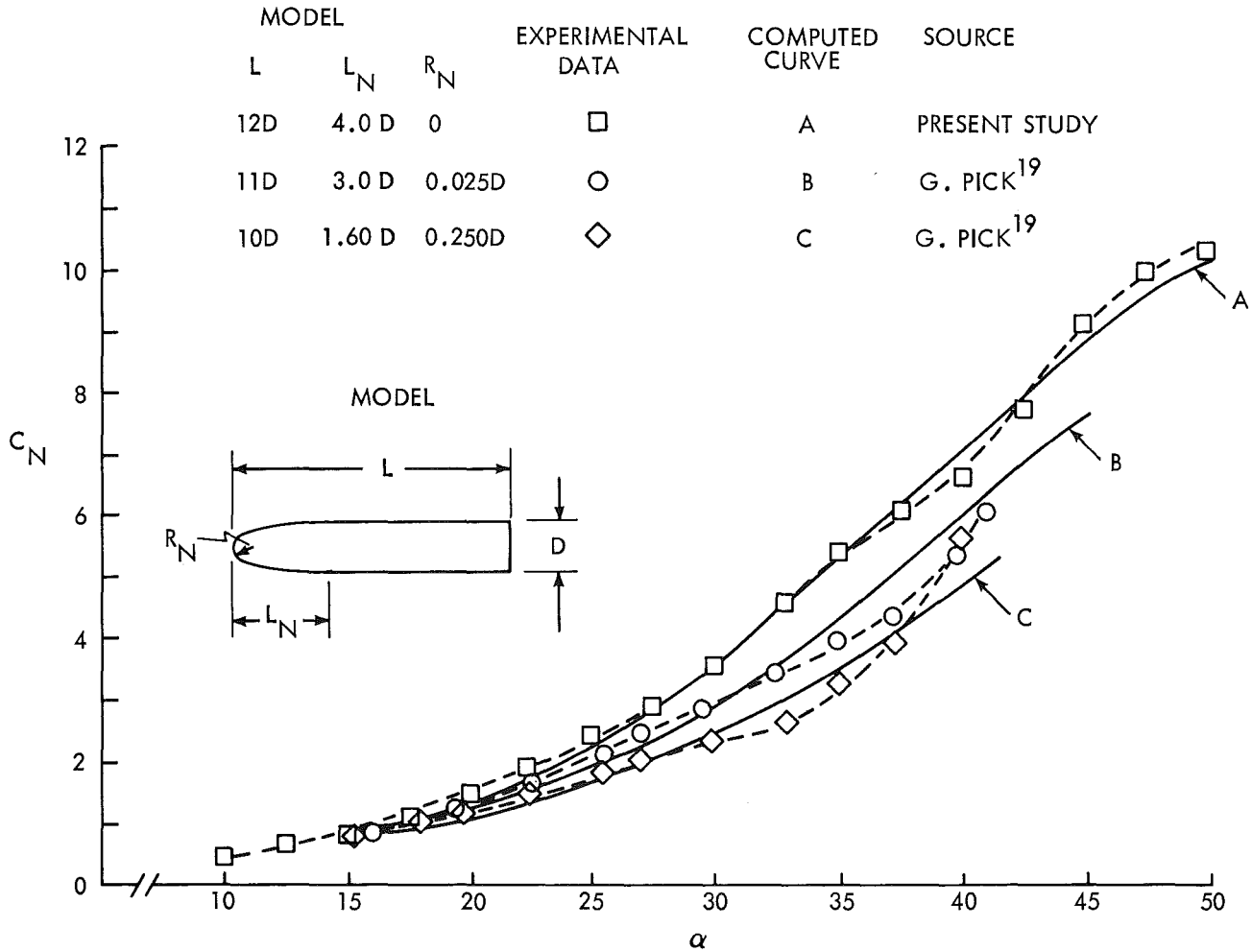


FIG. 20 NORMAL FORCE COEFFICIENT AS A FUNCTION OF ANGLE OF ATTACK FOR SEVERAL DIFFERENT MODELS, $M = 0.5$

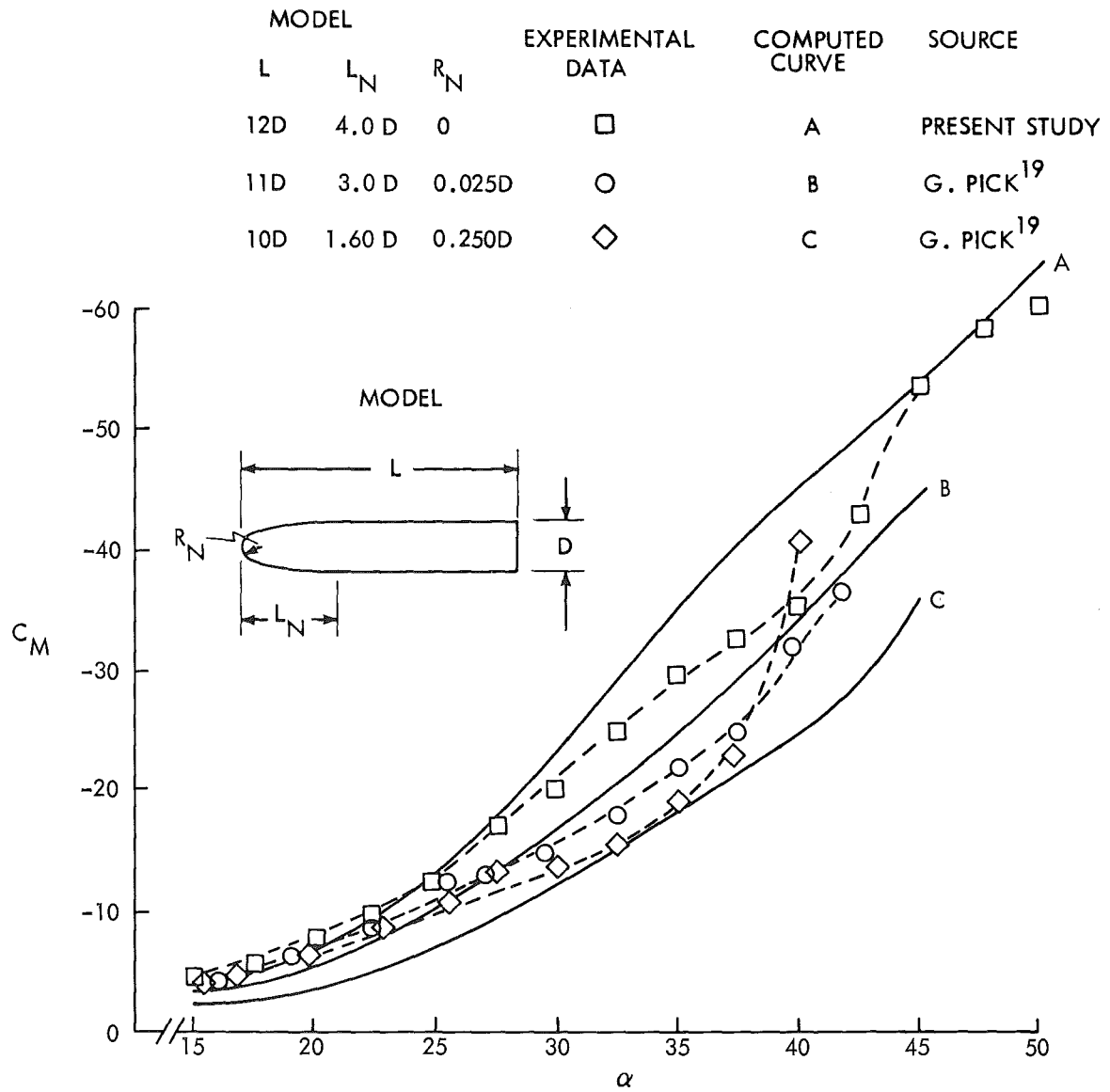


FIG. 21 PITCHING MOMENT COEFFICIENT AS A FUNCTION OF ANGLE OF ATTACK FOR SEVERAL DIFFERENT MODELS, $M = 0.5$.

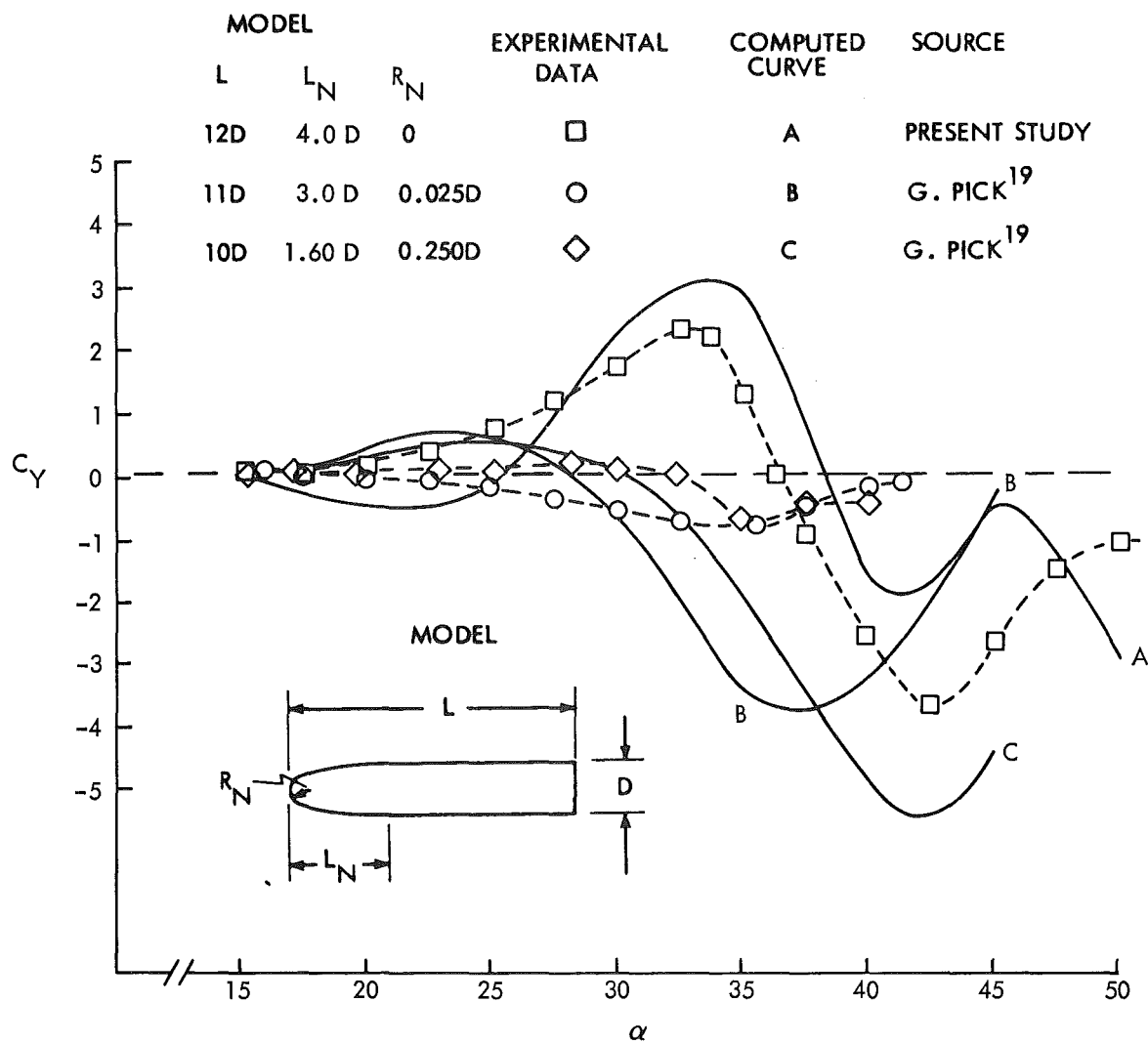


FIG. 22 YAW FORCE COEFFICIENT AS A FUNCTION OF ANGLE OF ATTACK FOR SEVERAL DIFFERENT MODELS, $M = 0.5$.

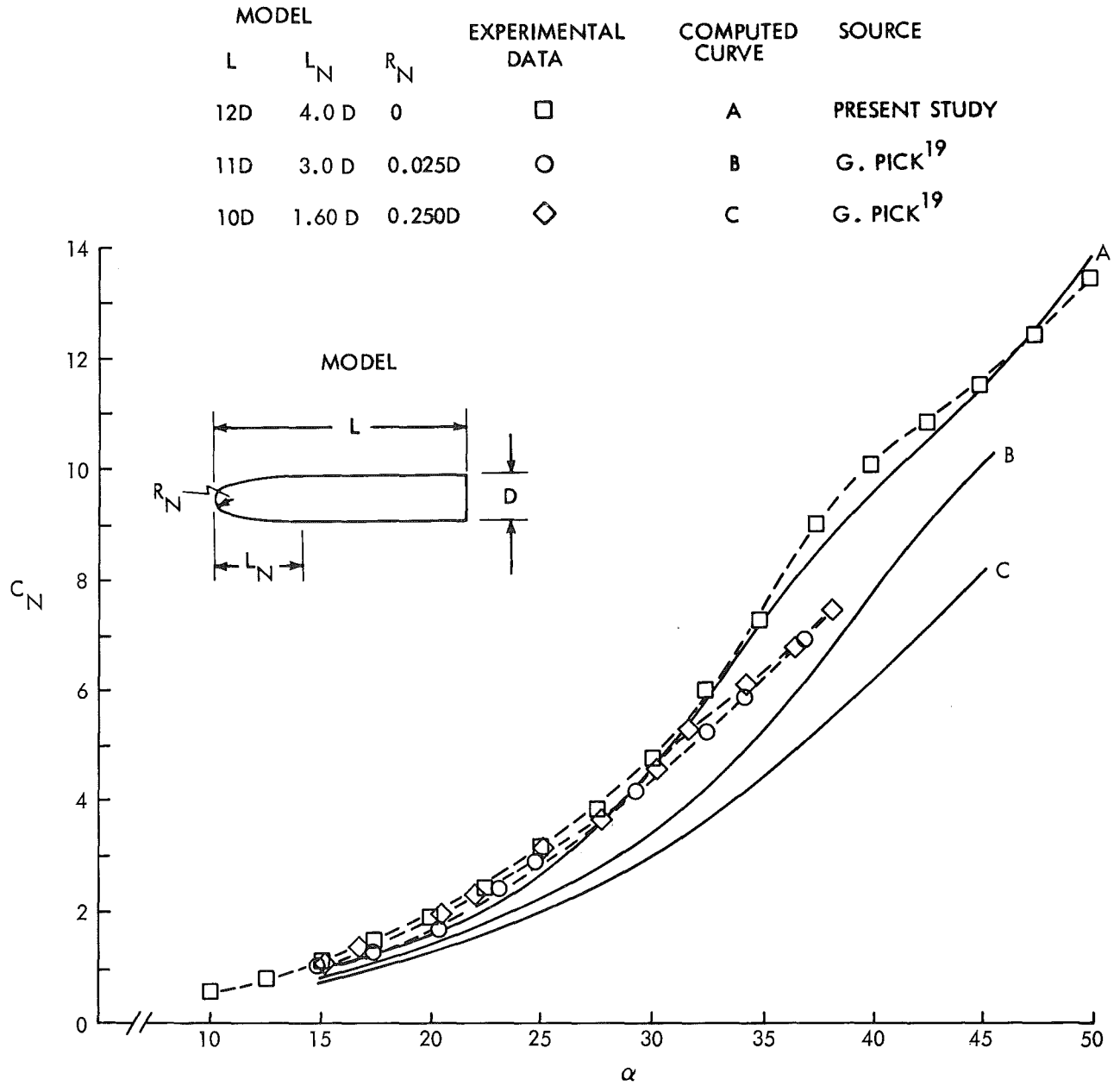


FIG. 23 NORMAL FORCE COEFFICIENT AS A FUNCTION OF ATTACK FOR SEVERAL DIFFERENT MODELS, $M = 1.1$.

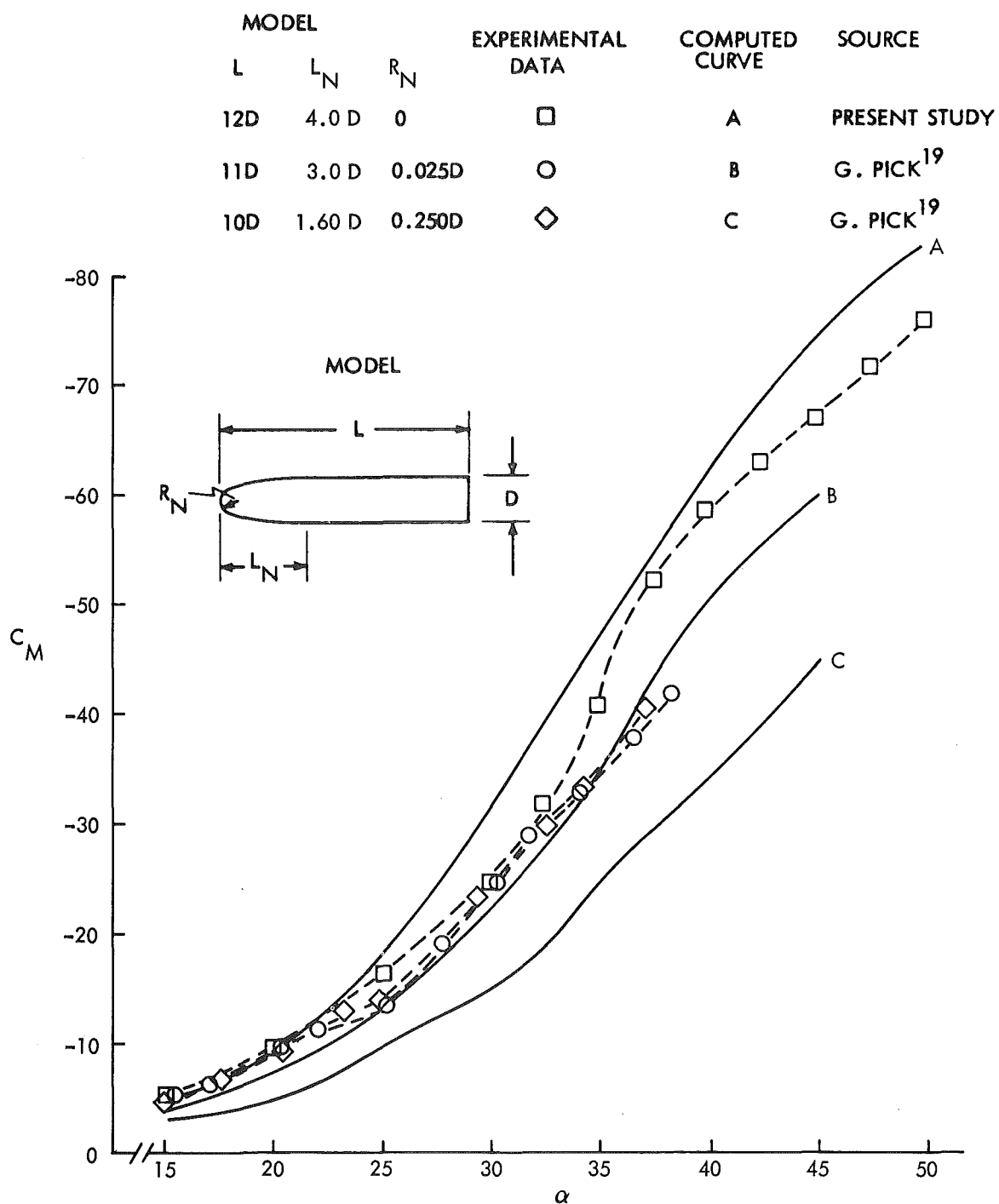


FIG. 24 PITCHING MOMENT COEFFICIENT AS A FUNCTION OF ANGLE OF ATTACK FOR SEVERAL DIFFERENT MODELS, $M = 1.1$

MODEL			EXPERIMENTAL DATA	COMPUTED CURVE	SOURCE
L	L_N	R_N			
12D	4.0 D	0	□	A	PRESENT STUDY
11D	3.0 D	0.025D	○	B	G. PICK ¹⁹
10D	1.60 D	0.250D	◇	C	G. PICK ¹⁹

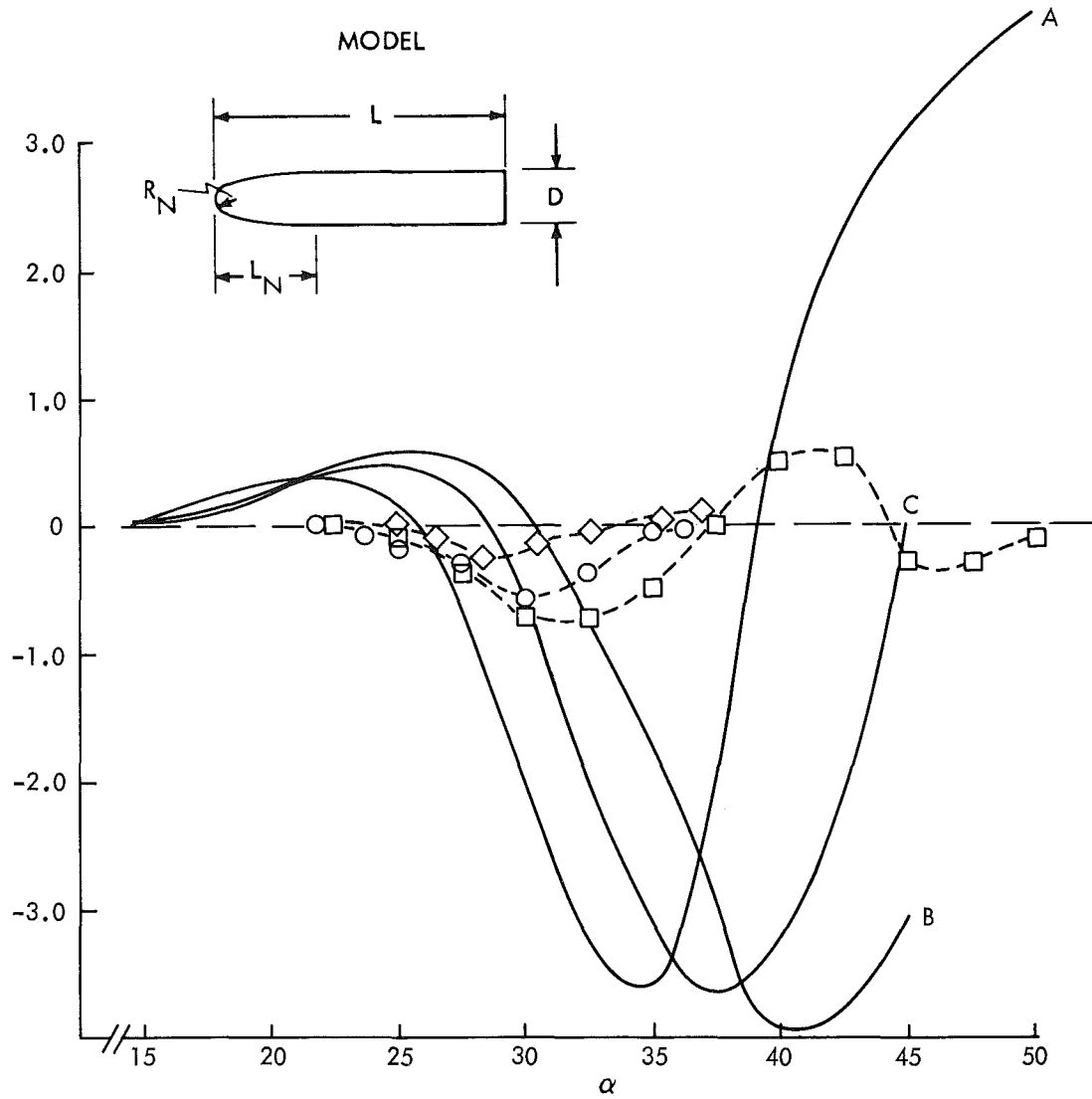


FIG. 25 YAW FORCE COEFFICIENT AS A FUNCTION OF ANGLE OF ATTACK FOR SEVERAL DIFFERENT MODELS, $M = 1.1$

APPENDIX A

DERIVATION OF PROGRAMMED EQUATIONS

The governing equations are:

$$\left. \begin{aligned} \frac{d\zeta_1}{d\chi} + \frac{(\zeta_1 - \zeta_{01})}{\lambda_1} \frac{d\lambda_1}{d\chi} &= \tan \alpha \omega_1 - \frac{(\zeta_1 - \zeta_{01})}{r} \frac{dr}{d\chi} \\ \frac{d\zeta_2}{d\chi} + \frac{(\zeta_2 - \zeta_{02})}{\lambda_2} \frac{d\lambda_2}{d\chi} &= \tan \alpha \omega_2 - \frac{(\zeta_2 - \zeta_{02})}{r} \frac{dr}{d\chi} \end{aligned} \right\} \begin{array}{l} \text{growing} \\ \text{vortices} \end{array}$$

$$\frac{d\zeta_j}{d\chi} = \tan \alpha \omega_j \quad j = 3, 4, \dots, k \quad \text{shed vortices} \quad (\text{A-1})$$

In order to define λ_1 and λ_2 the above equation must be augmented by the constraint that the fluid velocity be zero at the stagnation points relative to the changing radius:

$$\omega(\zeta_{01}) = \frac{r}{\tan \alpha} \frac{dr}{dz} \frac{1}{\bar{\zeta}_{01}} \quad \omega(\zeta_{02}) = \frac{r}{\tan \alpha} \frac{dr}{dz} \frac{1}{\bar{\zeta}_{02}} \quad (\text{A-2})$$

The dimensionless velocity potential, ϕ , is defined as follows:

$$\begin{aligned} \tilde{\phi}(\zeta) &= -\left(\zeta - \frac{r^2}{\bar{\zeta}}\right)^2 - ir \sum_{j=1}^k \lambda_j \ln \left[\frac{(\zeta - \zeta_j)}{(\zeta - r^2/\bar{\zeta}_j)} \right] \\ &+ \frac{r}{\tan \alpha} \frac{dr}{dz} \ln(\zeta) \end{aligned} \quad (\text{A-3})$$

Differentiating Equation (A-3), the dimensionless velocity is determined.

$$\begin{aligned} \omega &= \text{CONJUGATE} \left| -2\left(1 + \frac{r^2}{\bar{\zeta}^2}\right) - ir \sum_{j=1}^k \lambda_j \left\{ \frac{1}{(\zeta - \zeta_j)} - \frac{1}{(\zeta - r^2/\bar{\zeta}_j)} \right\} \right. \\ &\left. + \frac{r}{\tan \alpha} \frac{dr}{dz} \frac{1}{\bar{\zeta}} \right| \end{aligned} \quad (\text{A-4})$$

To simplify manipulations, the following functions are defined

$$a(s, s_j) = \left\{ \frac{1}{(s - s_j)} - \frac{1}{(s - r^2/\bar{s}_j)} \right\} \quad (\text{A-5a})$$

$$b(s) = \left(\frac{1}{r} + \frac{r}{s^2} \right) + \sum_{j=3}^k \lambda_j a(s, s_j) \quad (\text{A-5b})$$

Substituting Equations (A-5) into Equation (A-4):

$$\omega = -2r \text{ CONJUGATE} \left| b(s) + \lambda_1 a(s, s_1) + \lambda_2 a(s, s_2) + \frac{i}{\tan \alpha} \frac{dr}{dz} \frac{1}{s} \right| \quad (\text{A-6})$$

Substituting Equation (A-6) into constraints (A-2):

$$\begin{aligned} \lambda_1 a(s_{01}, s_1) + \lambda_2 a(s_{01}, s_2) &= -b(s_{01}) \\ \lambda_1 a(s_{02}, s_1) + \lambda_2 a(s_{02}, s_2) &= -b(s_{02}) \end{aligned} \quad (\text{A-7})$$

Solving the above:

$$\begin{aligned} \lambda_1 &= \frac{b(s_{02}) a(s_{01}, s_2) - b(s_{01}) a(s_{02}, s_2)}{a(s_{01}, s_1) a(s_{02}, s_2) - a(s_{02}, s_1) a(s_{01}, s_2)} \equiv \frac{A1}{C} \\ \lambda_2 &= \frac{b(s_{01}) a(s_{02}, s_1) - b(s_{02}) a(s_{01}, s_1)}{a(s_{01}, s_1) a(s_{02}, s_2) - a(s_{02}, s_1) a(s_{01}, s_2)} \equiv \frac{B1}{C} \end{aligned} \quad (\text{A-8})$$

Differentiating the above and substituting into the governing equations produces differential equations for z_j and y_j . The algebra involved is simplified by defining the following functions.

$$T(s, s_j) = \left[\frac{1}{(s - r^2/\bar{s}_j)^2} - \frac{1}{(s - s_j)^2} \right]$$

$$E(s, s_j) = (s - s_j)^{-2}$$

$$H(s, s_j) = \frac{-2r}{\bar{s}_j (s - r^2/\bar{s}_j)^2} \frac{dr}{dz}$$

$$XF(s, s_j) = \frac{r^2}{\bar{s}_j^2 (s - r^2/\bar{s}_j)^2}$$

$$G(r) = -\frac{Zr}{r^3} + \sum_{j=3}^K \lambda_j T(r, r_j)$$

$$P(r) = \frac{dr}{dz} \left(\frac{1}{r^2} - \frac{1}{r_z^2} \right) + \sum_{j=3}^K H(r, r_j) \lambda_j$$

$$C1 = [-a(r_{02}, r_2) E(r_{01}, r_1) + a(r_{01}, r_2) E(r_{02}, r_1)] / C$$

$$C2 = [-a(r_{02}, r_2) \chi F(r_{01}, r_1) + a(r_{01}, r_2) \chi F(r_{02}, r_1)] / C$$

$$C3 = [-a(r_{01}, r_1) E(r_{02}, r_2) + a(r_{02}, r_1) E(r_{01}, r_2)] / C$$

$$C4 = [-a(r_{01}, r_1) \chi F(r_{02}, r_2) + a(r_{02}, r_1) \chi F(r_{01}, r_2)] / C$$

$$C5 = [-a(r_{02}, r_2) T(r_{01}, r_1) + a(r_{02}, r_1) T(r_{01}, r_2)] / C$$

$$C6 = [-a(r_{01}, r_1) T(r_{02}, r_2) + a(r_{01}, r_2) T(r_{02}, r_1)] / C$$

$$C7 = [a(r_{02}, r_1) H(r_{01}, r_2) + a(r_{01}, r_2) H(r_{02}, r_1) - a(r_{01}, r_1) H(r_{02}, r_2) - a(r_{02}, r_2) H(r_{01}, r_1)] / C$$

$$K11 = C1(r_1 - r_{01}) + 1.$$

$$K12 = C2(r_1 - r_{01})$$

$$K13 = \left[C3 - \frac{E(r_{02}, r_2) b(r_{01})}{A1} + \frac{b(r_{02}) E(r_{01}, r_2)}{A1} \right] (r_1 - r_{01})$$

$$K14 = \left[C4 - \frac{\chi F(r_{02}, r_2) b(r_{01})}{A1} + \frac{b(r_{02}) \chi F(r_{01}, r_2)}{A1} \right] (r_1 - r_{01})$$

$$K15 = \left[C5 - \frac{a(r_{02}, r_2) G(r_{01})}{A1} + \frac{b(r_{02}) T(r_{01}, r_2)}{A1} \right] (r_1 - r_{01})$$

$$K16 = \left[C6 - \frac{a(r_{01}, r_2) G(r_{02})}{A1} + \frac{T(r_{02}, r_2) B(r_{01})}{A1} \right] (r_1 - r_{01})$$

$$KL(r_j) = [-a(r_{02}, r_2) E(r_{01}, r_j) + a(r_{01}, r_2) E(r_{02}, r_j)] (r_1 - r_{01}) / A1$$

$$KM(r_j) = [-a(r_{02}, r_2) \chi F(r_{01}, r_j) + a(r_{01}, r_2) \chi F(r_{02}, r_j)] (r_1 - r_{01}) / A1$$

$$K19 = [C7 - \{b(s_{01})H(s_{02}, s_2) + a(s_{02}, s_2)P(s_{01}) - b(s_{02})H(s_{01}, s_2) - a(s_{01}, s_2)P(s_{02})\} / B1] (s_1 - s_{01})$$

$$K21 = [C1 - \frac{b(s_{02})E(s_{01}, s_1)}{B1} + \frac{b(s_{01})E(s_{02}, s_1)}{B1}] (s_2 - s_{02})$$

$$K22 = [C2 - \frac{b(s_{02})XF(s_{01}, s_1)}{B1} + \frac{b(s_{01})XF(s_{02}, s_1)}{B1}] (s_2 - s_{02})$$

$$K23 = C3 (s_2 - s_{02}) + 1.$$

$$K24 = C4 (s_2 - s_{02})$$

$$K25 = [C5 - \frac{b(s_{02})T(s_{01}, s_2)}{B1} + \frac{a(s_{02}, s_1)G(s_{01})}{B1}] (s_2 - s_{02})$$

$$K26 = [C6 - \frac{b(s_{01})T(s_{02}, s_1)}{B1} + \frac{a(s_{01}, s_1)G(s_{02})}{B1}] (s_2 - s_{02})$$

$$KN(s_j) = [-E(s_{02}, s_j)a(s_{01}, s_j) + E(s_{01}, s_j)a(s_{02}, s_1)](s_2 - s_{02}) / B1$$

$$KO(s_j) = [-XF(s_{02}, s_j)a(s_{01}, s_j) + XF(s_{01}, s_j)a(s_{02}, s_1)](s_2 - s_{02}) / B1$$

$$K29 = [C7 - \{b(s_{02})H(s_{01}, s_1) + a(s_{01}, s_1)P(s_{02}) - b(s_{01})H(s_{02}, s_1) - a(s_{02}, s_1)P(s_{01})\} / B1] (s_2 - s_{02})$$

Using the above, explicit expressions for all derivatives can be obtained in the following form:

$$\frac{dy_j}{dx} = \text{Real} [w_j] \quad (A-9a)$$

$$j = 3, 4, \dots, K$$

$$\frac{dz_j}{dx} = \text{Imag.} [w_j] \quad (A-9b)$$

$$\begin{Bmatrix} \frac{dy_1}{dz} \\ \frac{dz_1}{dz} \\ \frac{dy_2}{dz} \\ \frac{dz_2}{dz} \end{Bmatrix} = [D]^{-1} \{Q\} \quad (A-9c)$$

The matrices D and Q are defined as follows:

$$D(1,1) = \text{Real}[K_{11} + K_{12}]$$

$$D(1,2) = \text{Imag.}[K_{12} - K_{11}]$$

$$D(1,3) = \text{Real}[K_{13} + K_{14}]$$

$$D(1,4) = \text{Imag.}[K_{14} - K_{13}]$$

$$D(2,1) = \text{Real}[K_{11} + K_{12}]$$

$$D(2,2) = \text{Imag.}[K_{11} - K_{12}]$$

$$D(2,3) = \text{Real}[K_{13} + K_{14}]$$

$$D(2,4) = \text{Imag.}[K_{13} - K_{14}]$$

$$D(3,1) = \text{Real}[K_{21} + K_{22}]$$

$$D(3,2) = \text{Imag.}[K_{22} - K_{21}]$$

$$D(3,3) = \text{Real}[K_{23} + K_{24}]$$

$$D(3,4) = \text{Imag.}[K_{24} - K_{23}]$$

$$D(4,1) = \text{Real}[K_{21} + K_{22}]$$

$$D(4,2) = \text{Imag.}[K_{21} - K_{22}]$$

$$D(4,3) = \text{Real}[K_{23} + K_{24}]$$

$$D(4,4) = \text{Imag.}[K_{23} - K_{24}]$$

$$Q(1) = \text{Real}(L_1)$$

$$Q(2) = \text{Imag}(L_1)$$

$$Q(3) = \text{Real}(L_2)$$

$$Q(4) = \text{Imag}(L_2)$$

APPENDIX B

COMPUTER PROGRAM FOR A BODY OF REVOLUTION AT
ANGLE OF ATTACK UP TO 50 DEGREES

The program listing and a sample run are included in this section. The program was developed on the CDC 6400. Running time is on the order of a minute. The sample run is for a sharp nose tangent ogive body of revolution at an angle of attack of 45 degrees.

The program is written to describe an ogive body of revolution with a sharp or blunt nose. If a sharp nose is to be specified, variables listed by the caption "nose tip dimensions" can be neglected. Allowable geometries for the blunted nose tip are restricted to ogive shapes of which a circle is a special case.

To use the program the following input is required.

	<u>Card</u>	<u>Variables</u>	<u>Format</u>
These cards must be repeated for each case	1	N	I5
	2	A	F10.0
	3	XL, $r_{\max}$, ZM2, TL, OLAP, XL1, R_N , ZM1	7F10.0
	4	XTR, AKT, AKL, λ_I , XST, S, $\Delta r_p/D$	7F10.0
	5	M, NE, NGB, G, Y(N+2)	3I5, 2F10.0
	6	$\Delta r_s/D$, GS, r_n/D , $\Delta \lambda_n$, NNE	4F10.0, I5

where:

N - number of cases to be run

A - angle of attack (degrees)

XL

$r_{\max}$

ZM2

ogive dimensions

TL

total missile length

OLAP

XL1

R_N

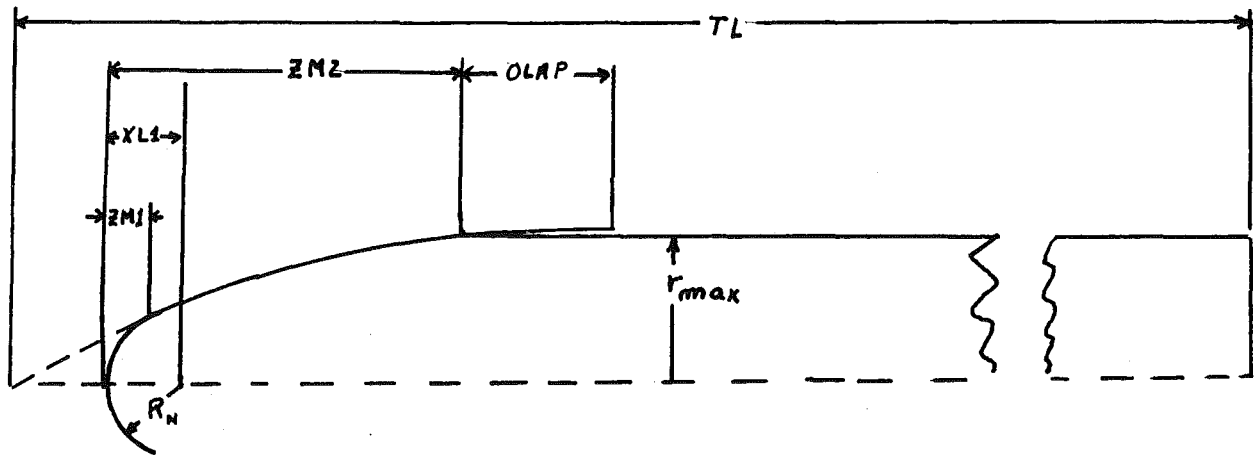
ZM1

nose tip dimensions

these parameters are
defined in the accom-
panying figure

XTR	fractional distance from missile nose to the point where transition from one separation angle to another takes place
AKT	final separation angle (radians)
AKL	initial separation angle (radians)
λ_I	initial vortex strength. If not specified, the value of .04 is assumed.
XST	nearest point to missile nose that integration is allowed to start. If not specified, the value of .1D is assumed.
S	Strouhal number
Δr_p	initial radial perturbation
M	number of integration steps between printing
NE	error bound. Step size is adjusted to keep estimated error between 10^{-NE} and $10^{-(NE+3)}$.
NGB	NGB-1 is the number of consecutive steps to be printed at the start or restart of integration.
G	initial step size
Y(N+2)	applies when M=0. It is the approximate distance in x between printing.
Δr_s	first vortex is shed when the radial location of the first two vortices, multiplied by r_{max} , differs by more than this amount.
r_n	this parameter, multiplied by r, represents the distance between the flow separation point, ζ_{0j} , and a newly introduced vortex. This distance is taken along the feeding sheet of the shed vortex.
$\Delta \lambda$	percent decrease in the circulation of a shed vortex
NNE	error bound in effect after first vortex is shed

Values for the parameters listed on this page which are used in the current study are found in the right-hand column of Table 1.



The symbols on the output page have the following meanings.

x distance along body axis

R local body radius

THETA0 θ_{01}, θ_{02}

VORTEX NUMBER - identification number for vortex

LAM circulation strength, λ

RADIUS } polar coordinates of vortex

ANGLE } see accompanying figure

Y } cartesian coordinates of vortex

Z } see accompanying figure

CL linear portion of normal force

CN nonlinear portion of normal force

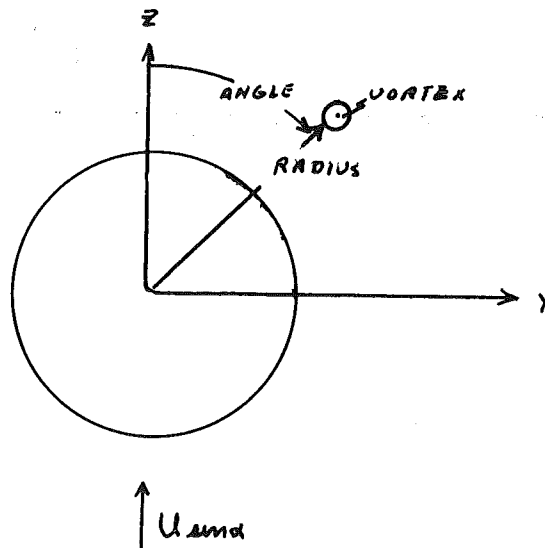
$$\left. \begin{array}{l} CL \\ CN \end{array} \right\} C_N = CL + CN$$

CML linear portion of pitching moment

CMN nonlinear portion of pitching moment

$$\left. \begin{array}{l} CML \\ CMN \end{array} \right\} C_M = CML + CMN$$

CD local normal force coefficient, $\frac{C_{DC}}{dx}$
 CY yaw force coefficient, C_Y
 CYM yawing moment coefficient, C_M'



PROGRAM	VORT	TRACE	CDC 6400 FTN V3.0-P316 OPT=0	01/07/74	14.28.01.	PAGE	1
		PROGRAM VORT(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)		VORT	2		
	C.....	MAIN PROGRAM		VORT	3		
		COMPLEX XC		VORT	4		
		DIMENSION Y(35),D(35),XL(15),XC(15),ERROR(31)		VORT	5		
5		COMMON/GAMM/WRAD,ZM,FLEN,CR,TL,TSL,WRAD2,ZM2,FLEN2,CR2,OLAP		VORT	6		
		COMMON/BG/N,DRT,PRAD,PRTU,TOP,XLP,PERR,STR,SHED,XAKT,XAKL,XXTR		VORT	7		
		COMMON/GARB/XL,R,DR,XC,TAL,FVORT		VORT	8		
		COMMON/STA/A,NGB		VORT	9		
		COMMON/VEL/U,IE		VORT	10		
10		COMMON/XLIF/MC,M,CDMN,CDML,XS,CYM,CDCL,CDCN,CY		VORT	11		
		COMMON/TRAN/XTR,X,AKT,AKL,ISD		VORT	12		
		EXTERNAL DERIV,OUT,TERM2,TERM3		VORT	13		
		CONVR=57.2957795		VORT	14		
		READ(5,1005)NDS		VORT	15		
15		DO 21 KT=1,NDS		VORT	16		
		READ(5,1000)A		VORT	17		
		WRITE(6,2000)A		VORT	18		
		ZM2=0.0		VORT	19		
		READ(5,1000)FLEN,WRAD,ZM,TL,OLAP,FLEN2,WRAD2,ZM2		VORT	20		
20		A=A/CONVR		VORT	21		
		TAL=TAN(A)		VORT	22		
		ISEC=0		VORT	23		
		IF(FLEN2.GT.0.0)ISEC=1		VORT	24		
		WRITE(6,2001)WRAD,FLEN,ZM,TL		VORT	25		
25		CR2=(WRAD2*WRAD2+FLEN2*FLEN2)/2./WRAD2		VORT	26		
		IF(ISEC.EQ.1)WRITE(6,2016)WRAD2,ZM2,FLEN2		VORT	27		
		CR=(WRAD*WRAD+FLEN*FLEN)/2./WRAD		VORT	28		
		READ(5,1000)XTR,AKT,AKL,XLP,XST,STR,PERR		VORT	29		
		XXTR=XTR		VORT	30		
30		XAKT=AKT		VORT	31		
		XAKL=AKL		VORT	32		
		ISD=0.0		VORT	33		
		IF(AKT.EQ.0.0)AKT=.9251		VORT	34		
		IF(AKL.EQ.0.0)AKL=13.12		VORT	35		
35		IF(AKL.LT.1.6)ISD=1		VORT	36		
		IF(XLP.LE.0.0)XLP=.04		VORT	37		
		IF(XST.LE.0)XST=.2*WRAD		VORT	38		
		J=2		VORT	39		
		IE=1		VORT	40		
40		E1=.000001		VORT	41		
		E2=E1		VORT	42		
		DT=.04		VORT	43		
	C.....	DEFINING COMMON QUANTITIES		VORT	44		
		ICOUNT=0		VORT	45		
45		SHED=0.0		VORT	46		
		IF(STR.NE.0)SHED=WRAD/STR/TAL		VORT	47		
		U=1.		VORT	48		
		N=5		VORT	49		
		FVORT=0.0		VORT	50		
50		TOP=0.0		VORT	51		
		DO 10 K=1,15		VORT	52		
	10	XL(K)=0.0		VORT	53		
	C.....	DETERMINE INITIAL INTEGRATION PARAMETERS		VORT	54		
		CALL START(A,DD1,DT,E1,E2,XLP,XST)		VORT	55		
55	C.....	SET UP INTEGRATION		VORT	56		

```

        CALL PROFILE(R,DR,X,DD1,1)
        Y(1)=E1*R*COS(E2)
        Y(2)=E1*R*SIN(E2)
        Y(3)=-Y(1)
        Y(4)=Y(2)
60      Y(1)=Y(1)*(1.+PERR)
        Y(2)=Y(2)*(1.+PERR)
        Y(3)=Y(3)*(1.-PERR)
        Y(4)=Y(4)*(1.-PERR)
65      Y(5)=THETI(DD1)
        XS=X
        Y(N+1)=X
        Y(N+3)=-1.
        READ(5,1005)M,NE,NGB,G,Y(N+2)
70      MC=M
        M=1
        CALL REST(X,Y,J,G,L,NE,D,0,MC)
        CYM=0.0
        CY =0
75      CDCN=SIN(A)*COS(A)*8.*R*Y(1)*XLP*(1.-R*R/(Y(1)*Y(1)+Y(2)*Y(2)))
        1/WRAD/WRAD
        CDMN=CDCN*X/2./WRAD
        CDCL=2.*SIN(A)*COS(A)*R*R/WRAD/WRAD
        X0=CR*CR-FLEN*FLEN
80      X2=(X0+2.*FLEN*X-X*X)**.5
        X0=X0**.5
        CDML=2.*SIN(A)*COS(A)*((WRAD-CR)*((X+FLEN)*X2/2.-FLEN/2.*X0-CR*CR/
        12.*(ASIN((X-FLEN)/CR)-ASIN(-FLEN/CR)))+X*X*(FLEN/2.-X/3.))/WRAD/WR
        2AD/WRAD
85      TOP=0.0
        FVORT=0.0
        18 CONTINUE
        CALL FNOL3(J,N,G,L,1,NE,X,Y,D,DERIV,TERM2,OUT,ERROR)
        M=1
90      CALL OUT(X,Y,D,ERROR,N,L,G)
        IF(X+.001*GE.TL)GO TO 20
        C..... INTRODUCING FIRST NEW VORTEX
        N=7
        IC=3
95      IF(Y(1)**2+Y(2)**2.GT.Y(3)**2+Y(4)**2)GO TO 25
        WRITE(6,2003)
        STOP
        25 CONTINUE
        Y(10)=Y(8)
100      Y(9)=Y(7)
        Y(8)=Y(6)
        Y(7)=Y(5)
        Y(6)=Y(2)
        Y(5)=Y(1)
105      XL(3)=XL(1)*(1.+PRTU)
        40 CALL REST(X,Y,J,G,L,NE,D,2,MC)
        TSL=X+SHED
        M=1
        CALL FNOL3(J,N,G,L,1,NE,X,Y,D,DERIV,TERM3,OUT,ERROR)
110      M=1

```

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VORT 57
VORT 58
VORT 59
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VORT 108
VORT 109
VORT 110
VORT 111

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PROGRAM	VORT	TRACE	CDC 6400 FTN V3.0-P316 OPT=0	01/07/74	14.28.01.	PAGE	3
		CALL OUT(X,Y,D,ERROR,N,L,H)		VORT	112		
		IF(X+.001.GE.TL)GO TO 20		VORT	113		
		C.....NEW VORTEX IS ADDED		VORT	114		
		N=N+2		VORT	115		
115		IF(N.GT.31)GO TO 20		VORT	116		
		Y(N+3)=Y(N+1)		VORT	117		
		Y(N+2)=Y(N)		VORT	118		
		Y(N+1)=Y(N-1)		VORT	119		
		Y(N)=Y(N-2)		VORT	120		
120		IC=(N-1)/2		VORT	121		
		IC2=IC-1		VORT	122		
		DO 50 IN=1,IC2		VORT	123		
		K=2*(IC-IN)		VORT	124		
		Y(K+2)=Y(K)		VORT	125		
125		Y(K+1)=Y(K-1)		VORT	126		
	50	XL(IC+1-IN)=XL(IC-IN)		VORT	127		
		XL(3)=XL(3)*(1.+PRTU)		VORT	128		
		ISTOP=1		VORT	129		
		ICOUNT=0		VORT	130		
130		GO TO 40		VORT	131		
	20	CNN=CDCL+CDCN		VORT	132		
		CMM=CDMN+CDML		VORT	133		
		CSP=CMM/CNN		VORT	134		
		CYP=0.0		VORT	135		
135		IF(DRT.NE.0.0)CYP=CYM/CY		VORT	136		
		WRITE(6,2004)		VORT	137		
		WRITE(6,2005)CNN		VORT	138		
		WRITE(6,2006)CMM		VORT	139		
		WRITE(6,2007)CSP		VORT	140		
140		WRITE(6,2008)CY		VORT	141		
		WRITE(6,2009)CYM		VORT	142		
		WRITE(6,2010)CYP		VORT	143		
	21	CONTINUE		VORT	144		
		STOP		VORT	145		
145		1000 FORMAT(8F10.0)		VORT	146		
		1001 FORMAT(6F10.0,I5)		VORT	147		
		1005 FORMAT(3I5,2F10.0)		VORT	148		
		2000 FORMAT(1H1,40X,16HANGLE OF ATTACK ,F5.2,2X,7HDEGREES)		VORT	149		
		2001 FORMAT (1H0,46X,16HOGIVE DIMENSIONS/1H ,6X,14HMAXIMUM RADIUS,6X,		VORT	150		
150		113H OGIVE LENGTH,18X,16HZ MATCHING POINT,5X,11HBODY LENGTH/1H ,		VORT	151		
		26X,F10.5,14X,F6.3,22X,F10.5,13X,F10.5)		VORT	152		
		2003 FORMAT(27H0VORTEX IDENTITIES REVERSED)		VORT	153		
		2004 FORMAT(1H0,40X,26H*****FINAL STATISTICS*****)		VORT	154		
		2005 FORMAT(1H ,37X,32HNORMAL FORCE COEFFICIENT=CL+CN= ,F10.5)		VORT	155		
155		2006 FORMAT(1H ,35X,37HPITCHING MOMENT COEFFICIENT=CML+CMN= ,F10.5)		VORT	156		
		2007 FORMAT(1H ,41X,23HCENTER OF NORMAL FORCE=,F10.5)		VORT	157		
		2008 FORMAT(1H ,42X,22HYAW FORCE COEFFICIENT=,F10.5)		VORT	158		
		2009 FORMAT(1H ,41X,23HYAW MOMENT COEFFICIENT=,F10.5)		VORT	159		
		2010 FORMAT(1H ,43X,20HCENTER OF YAW FORCE=,F15.5)		VORT	160		
160		2016 FORMAT(1H0,46X,15HNOSE DIMENSIONS/1H ,28X,6HRRADIUS,5X, 14HMATCHIN		VORT	161		
		1G POINT,5X,6HLENGTH/1H ,35X,3(F12.7,3X))		VORT	162		
		END		VORT	163		

	SUBROUTINE REST(X,Y,J,G,L,NE,D,IT,M)	REST	2
	COMMON/GARB/XL,R,DR,XC,TAL,FVORT	REST	3
	COMMON/BG/N,DRT,PRAD,PRTU,TOP,XLP,PERR,S,SHED,AKT,AKL,XTR	REST	4
	COMMON/GAMM/WRAD	REST	5
5	DIMENSION Y(1),D(1),XL(15),XLR(15),XC(15)	REST	6
	COMPLEX XC	REST	7
	COMMON/STA/A	REST	8
	CONVR=57.2957795	REST	9
	IC=(N-1)/2	REST	10
10	IF(IT.EQ.0)GO TO 10	REST	11
	NE=NNE	REST	12
	G=XG	REST	13
	ANG=Y(N)	REST	14
	IF((IC/2)*2.EQ.IC)ANG=3.1415926536-ANG	REST	15
15	P1=COS(ANG)*R	REST	16
	P2=SIN(ANG)*R	REST	17
	PAN=ABS((Y(6)-P2)/(Y(5)-P1))	REST	18
	PAN=ATAN(PAN)	REST	19
	IF(Y(5).LT.P1)PAN=3.1415926536-PAN	REST	20
20	Y(1)=P1+COS(PAN)*PRAD*R	REST	21
	Y(2)=P2+SIN(PAN)*PRAD*R	REST	22
	RETURN	REST	23
10	CONTINUE	REST	24
	READ(5,1001)DRT,XG,PRAD,PRTU,NNE	REST	25
25	WRITE(6,2000)XLP,PERR,DRT,PRTU,PRAD,S,SHED,AKL,AKT,XTR	REST	26
	PRTU=PRTU/100.	REST	27
	WRITE(6,2005)G,M,NE,Y(N+2),NNE	REST	28
	DRT=DRT*2.*WRAD	REST	29
	RETURN	REST	30
30	1001 FORMAT(4F10.0,I5)	REST	31
	2000 FORMAT(1H0,40X,25H*****FREE PARAMETERS*****/1H,40X,24HINITIAL VOR	REST	32
	1TEX STRENGTH ,F10.5/1H,38X,28HINITIAL RADIAL PERTURBATION ,F10.5/	REST	33
	21H,30X,47HRAIAL ASYMMETRY AT WHICH FIRST VORTEX IS SHED ,F10.5/	REST	34
	31H,28X,50HPER CENT DECREASE IN CIRCULATION OF SHED VORTICES ,F10.	REST	35
35	45/1H,25X,46HLOCATION FACTOR FOR NEWLY INTRODUCED VORTICES ,F10.5/	REST	36
	51H,18X,16HSTROUHAL NUMBER ,F10.5,2X,32HCORRESPONDING SHEDDING DIS	REST	37
	6TANCE ,F10.5/1H,12X,27HSEPARATION ANGLE CONSTANTS ,F10.5,2X,F10.5	REST	38
	7,2X,20HPOINT OF TRANSITION ,F10.5)	REST	39
	2005 FORMAT(1H0,42X,19HINTEGRATION OPTIONS/1H,18HINITIAL STEP SIZE ,F1	REST	40
40	10.7,5X,15HPRINT FREQUENCY, 15,5X,12HERROR BOUND ,15,2X,16HPRINT IN	REST	41
	2CREMENT ,F10.7/1H,42X,34HERROR BOUND AFTER VORTEX SHEDDING ,15)	REST	42
	END	REST	43

FUNCTION	THETI	TRACE	CDC 6400 FTN V3.0-P316 OPT=0	01/07/74	14.28.01.	PAGE	1
		FUNCTION THETI(DELTA)				THETI	2
		COMMON/STA/AL				THETI	3
		COMMON/GAMM/WRAD,ZM,FLEN,CR,TL				THETI	4
		COMMON/TRAN/XTR,X,AKT,AKL,ISD				THETI	5
5		CONVR=57.2957795				THETI	6
		IF(X/TL.LT.XTR)GO TO 11				THETI	7
		THETI=AKT				THETI	8
		RETURN				THETI	9
	11	CONTINUE				THETI	10
10		IF(ISD.EQ.0)GO TO 12				THETI	11
		THETI=AKL				THETI	12
		RETURN				THETI	13
	12	CONTINUE				THETI	14
		THETI=(AKL+CONVR*DELTA)*(3.13-(.116*AL*CONVR-1.16)**.5)				THETI	15
15		THETI=THETI/CONVR				THETI	16
		RETURN				THETI	17
	10	THETI=3.13*(AKL+CONVR*DELTA)				THETI	18
		THETI=THETI/CONVR				THETI	19
		RETURN				THETI	20
20		END				THETI	21

	SUBROUTINE PROFILE(R,DR,Z,DEL,IC)	PROFILE	2
	COMMON/GAMM/R02,ZM2,XL2,CR2,TL,TSL,R01,ZM1,XL1,CR1,OLAP	PROFILE	3
	IF(IC.EQ.1)GO TO 4	PROFILE	4
5	6 CONTINUE	PROFILE	5
	R0=R02	PROFILE	6
	CR=CR2	PROFILE	7
	ZM=ZM2+OLAP	PROFILE	8
	IF(Z.GT.ZM1)GO TO 3	PROFILE	9
10	R0=R01	PROFILE	10
	CR=CR1	PROFILE	11
	ZM=XL1	PROFILE	12
	GO TO 3	PROFILE	13
	4 SF=TAN(DEL)	PROFILE	14
	IF(ZM1.EQ.0.0)GO TO 5	PROFILE	15
15	Z=XL1-CR1*SF/((SF*SF+1.)**.5)	PROFILE	16
	IF(Z.LT.ZM1)GO TO 6	PROFILE	17
	5 Z=ZM2+OLAP-CR2*SF/((SF*SF+1.)**.5)	PROFILE	18
	IF(Z.LT.ZM1)Z=ZM1+.001	PROFILE	19
	GO TO 6	PROFILE	20
20	3 IF(Z.LT.ZM2)GO TO 20	PROFILE	21
	DES=CR*CR-(ZM-ZM2)**2	PROFILE	22
	R=R0-CR+DES**.5	PROFILE	23
	DR=0.0	PROFILE	24
	IF(IC.EQ.1)Z=ZM2	PROFILE	25
25	RETURN	PROFILE	26
	20 DES=CR*CR-(ZM-Z)**2	PROFILE	27
	IF(DES.GE.0.0)GO TO 30	PROFILE	28
	WRITE(6,200)R0,ZM,Z,DEL,XL1,CR1	PROFILE	29
	STOP	PROFILE	30
30	30 DES=DES**.5	PROFILE	31
	R=R0-CR+DES	PROFILE	32
	DR=(ZM-Z)/DES	PROFILE	33
	RETURN	PROFILE	34
35	200 FORMAT(1H ,25HPROBABLE INPUT DATA ERROR,6E16.8)	PROFILE	35
	END	PROFILE	36

	SUBROUTINE START(AL,DD1,DT,E1,E2,XLMS,XST)	START	2
	COMMON/GAMM/WRAD,ZM,FLEN,CR,TL	START	3
	COMMON/TRAN/XTR,X,AKT,AKL,ISH	START	4
	C.....INTERNAL FUNCTIONS	START	5
5	XR(R1)=((R1**.5-1./(R1**.5))**.2)/4.	START	6
	XL(R1,T1,T0)=8.*(XR(R1)+(SIN((T1-T0)/2.))**.2)*(XR(R1)	START	7
	1+(COS((T1+T0)/2.))**.2)/COS(T1)/(R1-1./R1)	START	8
	EN1(D)=(AKL/CONVR+D)*XK2-ASIN(XK1*TAN(D))	START	9
	CONVR=57.2957795	START	10
10	C.....DETERMINE DELTA CRITICAL	START	11
	TAL=TAN(AL)	START	12
	IF(ISH.EQ.0)GO TO 3	START	13
	D=ATAN(TAL*SIN(AKL)/1.5)	START	14
	GO TO 10	START	15
15	3 CONTINUE	START	16
	XK1=1.5/TAN(AL)	START	17
	XK2=3.13	START	18
	IF(AL.GT..17453)XK2=3.13- (.116*AL*CONVR-1.16)**.5	START	19
	D1=ATAN(1./XK1)	START	20
20	IF((AKL+D1*CONVR)*XK2.LT.90.)GO TO 4	START	21
	WRITE(6,2004)	START	22
	2004 FORMAT(21H0NO STARTING SOLUTION)	START	23
	STOP	START	24
	4 CONTINUE	START	25
25	DD1=D1/3.	START	26
	D=D1-DD1/2.	START	27
	DP=D	START	28
	IC=0	START	29
30	30 ER1=EN1(D)	START	30
	IF(ABS(ER1).LT.E1)GO TO 10	START	31
	IC=IC+1	START	32
	IF(IC.NE.1)GO TO 20	START	33
	ER2=ER1	START	34
	DP=D	START	35
35	D=D-DD1	START	36
	GO TO 30	START	37
	20 DH=D	START	38
	D=D-ER1*(DP-D)/(ER2-ER1)	START	39
	IF(D.GT.D1)D=(D1+DH)/2.	START	40
40	DP=DH	START	41
	ER2=ER1	START	42
	GO TO 30	START	43
	10 CONTINUE	START	44
	C.....TRIAL AND ERROR CALCUATION FOR R1	START	45
45	CALL PROFILE(R,DR,X,D,1)	START	46
	IF(X.LT.0.0)X=0.0	START	47
	IF(X/TL.LT.XTR)GO TO 12	START	48
	D=ATAN(TAL*SIN(AKT)/1.5)	START	49
	CALL PROFILE(R,DR,X,D,1)	START	50
50	12 IF(X.GT.XST)GO TO 31	START	51
	13 X=XST	START	52
	CALL PROFILE(R,DR,X,DEL,2)	START	53
	D=ATAN(DR)	START	54
	31 DDEG=D*CONVR	START	55
55	D1=D	START	56

	T0=THETI(D1)	START	57
	T1=T0+.001	START	58
	IT=0	START	59
	IP2=1	START	60
60	TP=T0+1.047198	START	61
	REX=SIN(T0)-TAN(TP)*COS(T0)	START	62
	TTP=TAN(TP)	START	63
70	FE2=FE1	START	64
	R1=REX/(SIN(T1)-TTP*COS(T1))	START	65
65	FE1=ABS(XL(R1,T1,T0))-XLMS	START	66
	IF(ABS(FE1).LT.E2) GO TO 90	START	67
	IT=IT+1	START	68
	IF(IT.NE.1) GO TO 100	START	69
	T2=T1	START	70
70	T1=T1+DT	START	71
	GO TO 70	START	72
100	TH=T1	START	73
	T1=T1-FE1*(T2-T1)/(FE2-FE1)	START	74
	IF(FE1*FE2.LT.0.0) IP2=0	START	75
75	IF(IP2.EQ.1) T1=TH+DT	START	76
	T2=TH	START	77
	GO TO 70	START	78
90	E1=R1	START	79
	IF(T1.GT.1.570796327) T1=3.141592654-T1	START	80
80	E2=T1	START	81
	DD1=D1	START	82
	RETURN	START	83
	END	START	84

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      SUBROUTINE FNOL3(J,NN,G,L,MPR,NE,X,Y,D,DERIV,TERM,OUTPUT,ERROR)  FNOL3      2
C 001 J=INTEGRATION METHOD CONTROL  FNOL3      3
C 002 NN=NUMBER OF SIMULTANEOUS DIFFERENTIAL EQUATIONS  FNOL3      4
C 003 G=FIRST INTERVAL OF INTEGRATION  FNOL3      5
5 C 004 L=NUMBER OF Y'S GREATER THAN N TO BE PRINTED  FNOL3      6
C 005 MPR=PRINT FREQUENCY  FNOL3      7
C 006 NE=CONTROL FOR INTERVAL OF INTEGRATION  FNOL3      8
C 007 X=INDEPENDENT VARIABLE  FNOL3      9
C 008 Y=DEPENDENT VARIABLE  FNOL3     10
10 C 009 D=ARRAY CONTAINING DERIVATIVES  FNOL3     11
C 010 DERIV=SUBROUTINE IN WHICH DIFFERENTIAL EQUATIONS ARE FOUND  FNOL3     12
C 011 TERM=SUBROUTINE FOR TERMINATION CONDITION  FNOL3     13
C 012 OUTPUT=SUBROUTINE FOR PRINTING OUTPUT  FNOL3     14
C      NRET=TERM LOOP COUNTER  FNOL3     15
15 C      NPR=COUNT OF STEPS SINCE LAST PRINT  FNOL3     16
C      PC=Y(N+1)=PRINT CONTROL OTHER THAN STEP COUNT  FNOL3     17
C      JJ=J-2=0 FOR AM,-1 FOR RK,+1 FOR RK2  FNOL3     18
C      MK=AM RK STEP COUNT  FNOL3     19
C      XD=DP FORM OF X  FNOL3     20
20 C DOUBLE PRECISION CARD HAS BEEN REMOVED IN ORDER TO RUN ON 6400  FNOL3     21
C THIS DECK RUNS ON THE 7090 WITH OR WITHOUT DOUBLE PRECISION  FNOL3     22
C      DOUBLE PRECISION XD,YD,YP,YC,YK,H,HC,XS,XDS,HS,HN,HD24,HD6  FNOL3     23
      DIMENSION C(3),Y(183),YD(183),YP(183),YC(183),D(203),DM(183,4)  FNOL3     24
      DIMENSION DK(183,4),ERROR(183),YK(183)  FNOL3     25
25      DATA EP6,EP11,M4/1.E-6,1.E-11,-4/  FNOL3     26
      DATA (C(K),K=1,3)/2*.5,1./  FNOL3     27
      DATA HMAX5/1.E35/  FNOL3     28
      NHTS=0  FNOL3     29
      N=NN  FNOL3     30
30      JJ=J-2  FNOL3     31
      H=G  FNOL3     32
      HN=H  FNOL3     33
      MK=1  FNOL3     34
      NRET=M4  FNOL3     35
35      EC=Y(N+3)  FNOL3     36
      JTEST=1  FNOL3     37
      IF (JJ .LT. 0) JTEST=4  FNOL3     38
      IF (NE.EQ.0) GO TO 15  FNOL3     39
      EUP=10.**(-NE)  FNOL3     40
40      ELO=EUP*.001  FNOL3     41
      EM=ELO*31.6227766  FNOL3     42
15      XD=X  FNOL3     43
      DO 20 I=1,N  FNOL3     44
      ERROR(I)=0.  FNOL3     45
45      20 YD(I)=Y(I)  FNOL3     46
      CALL DERIV(X,Y,D)  FNOL3     47
      CALL TERM (X,Y,D,F)  FNOL3     48
C      PRINT  FNOL3     49
50      CALL OUTPUT(X,Y,D,ERROR,N,L,H)  FNOL3     50
      IF (NRET) 65,60,55  FNOL3     51
60      RETURN  FNOL3     52
55      WRITE (6,3000) HN  FNOL3     53
3000 FORMAT(108H1EXECUTION TERMINATED BECAUSE INTERVAL OF INTEGRATION L  FNOL3     54
      LESS THAN 1.0E -6 TIMES INDEPENDENT VARIABLE (X).  H =,1PF15.7)  FNOL3     55
55      STOP  FNOL3     56

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	65	IF (JTEST .EQ. 5 .AND. H .NE. HN) GO TO 455	FNOL3	57
		NPR=0	FNOL3	58
		PC=Y(N+1)	FNOL3	59
	100	IF (JTEST .EQ. 5 .AND. H .NE. HN) GO TO 455	FNOL3	60
60		IF (JJ .GE. 0) H=HN	FNOL3	61
		IF (MK .NE. 0 .OR. JJ .NE. 0) GO TO 300	FNOL3	62
		C-----THE ADAMS MOULTON METHOD	FNOL3	63
	200	HD24=H/24.	FNOL3	64
		JAM=0	FNOL3	65
65		DO 210 I=1,N	FNOL3	66
		YPI=(55.*DM(I,1)+37.*DM(I,2))-(59.*DM(I,3)+9.*DM(I,4))	FNOL3	67
		YP(I)=YD(I)+HD24*YPI	FNOL3	68
		Y(I)=YP(I)	FNOL3	69
	210	CONTINUE	FNOL3	70
70		X=XD+H	FNOL3	71
		CALL DERIV(X,Y,DM(1,4))	FNOL3	72
		DO 220 I=1,N	FNOL3	73
		YPI=(9.*DM(I,4)+19.*DM(I,1)+DM(I,2))-5.*DM(I,3)	FNOL3	74
		YC(I)=YD(I)+HD24*YPI	FNOL3	75
75		ERROR(I)=(YP(I)-YC(I))/14.	FNOL3	76
		C THIS ADDS IN A 2D CORRECTION	FNOL3	77
		YC(I)=YC(I)+ERROR(I)	FNOL3	78
	220	CONTINUE	FNOL3	79
		IF (NE.NE.0) GO TO 700	FNOL3	80
80		GO TO 455	FNOL3	81
		C-----THE RUNGE KUTTA METHOD	FNOL3	82
	300	GO TO (301,309,308,309,303),JTEST	FNOL3	83
	301	DO 302 I=1,N	FNOL3	84
		YK(I)=YD(I)	FNOL3	85
85	302	CONTINUE	FNOL3	86
		XDS=XD	FNOL3	87
		GO TO 309	FNOL3	88
	303	DO 304 I=1,N	FNOL3	89
		YK(I)=YC(I)	FNOL3	90
90	304	CONTINUE	FNOL3	91
		XDS=XD+H	FNOL3	92
	308	HS=H	FNOL3	93
		H=2.*H	FNOL3	94
		GO TO 320	FNOL3	95
95	309	X=XD	FNOL3	96
		JAM=1	FNOL3	97
		DO 310 I=1,N	FNOL3	98
		Y(I)=YD(I)	FNOL3	99
		DK(I,1)=D(I)	FNOL3	100
100	310	CONTINUE	FNOL3	101
		IF (JTEST .LE. 2) CALL DERIV(X,Y,DK)	FNOL3	102
		IF (MK .GT. 1 .OR. JTEST .GT. 1) GO TO 320	FNOL3	103
		DO 315 I=1,N	FNOL3	104
		DM(I,4)=DK(I,1)	FNOL3	105
105	315	CONTINUE	FNOL3	106
	320	DO 335 K=2,4	FNOL3	107
		HC=H*C(K-1)	FNOL3	108
		DO 330 I=1,N	FNOL3	109
		Y(I)=YD(I) + HC*DK(I,K-1)	FNOL3	110
110	330	CONTINUE	FNOL3	111

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	X=XD+HC	FNOL3	112
	CALL DERIV(X,Y,DK(1,K))	FNOL3	113
	335 CONTINUE	FNOL3	114
	HD6=H/6.	FNOL3	115
115	DO 340 I=1,N	FNOL3	116
	YPI=DK(I,1)+DK(I,4)+2.*(DK(I,2)+DK(I,3))	FNOL3	117
	YC(I)=YD(I)+HD6*YPI	FNOL3	118
	340 CONTINUE	FNOL3	119
	GO TO (360,390,370,455,370),JTEST	FNOL3	120
120	360 DO 365 I=1,N	FNOL3	121
	YP(I)=YC(I)	FNOL3	122
	365 CONTINUE	FNOL3	123
	JTEST=3	FNOL3	124
	GO TO 308	FNOL3	125
125	370 DO 380 I=1,N	FNOL3	126
	YD(I)=YP(I)	FNOL3	127
	YP(I)=YC(I)	FNOL3	128
	380 CONTINUE	FNOL3	129
	H=HS	FNOL3	130
130	XD=XD+H	FNOL3	131
	JTEST=2	FNOL3	132
	IF (MK .EQ. 1) GO TO 309	FNOL3	133
	GO TO 451	FNOL3	134
	390 DO 400 I=1,N	FNOL3	135
135	ERROR(I)=(YC(I)-YP(I))/15.	FNOL3	136
	YC(I)=YC(I)+ERROR(I)	FNOL3	137
	YP(I)=YC(I)	FNOL3	138
	400 CONTINUE	FNOL3	139
	JTEST=5	FNOL3	140
140	IF (NE.NE.0) GO TO 700	FNOL3	141
	C-----ACCEPT THE STEP SIZE	FNOL3	142
	450 IF (JAM .EQ. 0) GO TO 455	FNOL3	143
	IF (MK .EQ. 3 .AND. JJ .EQ. 0) GO TO 455	FNOL3	144
	IF (MK .NE. 1) GO TO 303	FNOL3	145
145	IF (JTEST .EQ. 1) GO TO 455	FNOL3	146
	451 DO 452 I=1,N	FNOL3	147
	Y(I)=YD(I)	FNOL3	148
	452 CONTINUE	FNOL3	149
	GO TO 466	FNOL3	150
150	455 DO 459 NQ=1,N	FNOL3	151
	YD(NQ)=YC(NQ)	FNOL3	152
	Y(NQ)=YD(NQ)	FNOL3	153
	459 CONTINUE	FNOL3	154
	IF (JJ .GE. 0) JTEST=1	FNOL3	155
155	IF (MK .NE. 0 .OR. JJ .NE. 0 .OR. NRET .NE. M4) GO TO 465	FNOL3	156
	DO 460 I=1,N	FNOL3	157
	DM(I,4)=DM(I,2)	FNOL3	158
	DM(I,2)=DM(I,3)	FNOL3	159
	DM(I,3)=DM(I,1)	FNOL3	160
160	460 CONTINUE	FNOL3	161
	465 XD=XD+H	FNOL3	162
	466 X=XD	FNOL3	163
	IF (MK .EQ. 3) MK=0	FNOL3	164
	CALL DERIV(X,Y,D)	FNOL3	165
165	DO 470 I=1,N	FNOL3	166

	DM(I,MK+1)=D(I)	FNOL3	167
	470 CONTINUE	FNOL3	168
	FP=F	FNOL3	169
	CALL TERM (X,Y,D,F)	FNOL3	170
170	C-----DO YOU TERMINATE	FNOL3	171
	500 IF (ABS(F) .LE. EP6) GO TO 800	FNOL3	172
	IF (FP .EQ. 0.) GO TO 510	FNOL3	173
	IF (NRET .NE. M4 .OR. F*FP .LT. EP11) GO TO 805	FNOL3	174
	510 XS=XD	FNOL3	175
175	IF (MK .NE. 0 .AND. H .EQ. HN) MK=MK+1	FNOL3	176
	C-----DO YOU PRINT	FNOL3	177
	600 NPR=NPR+1	FNOL3	178
	IF (MPR .GT. 0 .AND. NPR .GE. MPR) GO TO 50	FNOL3	179
	IF (MPR .LE. 0 .AND. ABS(Y(N+1)-PC) .GE. Y(N+2)) GO TO 50	FNOL3	180
180	GO TO 100	FNOL3	181
	C-----DETERMINING THE STEP SIZE	FNOL3	182
	700 HB = HMAX5	FNOL3	183
	DO 760 I = 1,N	FNOL3	184
	Z=ABS(ERROR(I))	FNOL3	185
185	IF (YC(I) .EQ. 0.) GO TO 720	FNOL3	186
	ZZ= ABS(YC(I))	FNOL3	187
	IF (EC) 720,710,705	FNOL3	188
	705 IF (EC .GT. ZZ) ZZ=EC	FNOL3	189
	710 Z=Z/ZZ	FNOL3	190
190	720 IF (Z.GT.ELO.AND.Z.LT.EUP) GOTO 750	FNOL3	191
	HB = AMIN1(HB,EM/(Z+EP11))	FNOL3	192
	GOTO760	FNOL3	193
	750 HB=AMIN1(HB,1.)	FNOL3	194
	760 CONTINUE	FNOL3	195
195	IF (HB.EQ.1.) GOTO 450	FNOL3	196
	HN=H*HB**.2	FNOL3	197
	IF (MK .NE. 1) JTEST=1	FNOL3	198
	MK=1	FNOL3	199
	IF (HB.LT.1.) GOTO 770	FNOL3	200
200	IF (ABS(HN) .GT. ABS(4.*H)) HN=4.*H	FNOL3	201
	NHTS=0	FNOL3	202
	GOTO 450	FNOL3	203
	770 HEPS=ABS(X*EP6) + EP11	FNOL3	204
	IF (ABS(HN) .LT. ABS(H/4.)) HN=H/4.	FNOL3	205
205	IF (ABS(HN) .GT. HEPS) GO TO 790	FNOL3	206
	NHTS = NHTS + 1	FNOL3	207
	IF (NHTS.LT.10) GOTO 780	FNOL3	208
	JTEST=1	FNOL3	209
	NRET = 1	FNOL3	210
210	GOTO 50	FNOL3	211
	780 HN=SIGN(HEPS,HN)	FNOL3	212
	790 IF (JAM .EQ. 0) GO TO 100	FNOL3	213
	DO 795 I=1,N	FNOL3	214
	YD(I)=YK(I)	FNOL3	215
215	795 CONTINUE	FNOL3	216
	XD=XDS	FNOL3	217
	JTEST=1	FNOL3	218
	GO TO 100	FNOL3	219
	C-----THE TERMINATION LOOP	FNOL3	220
220	800 NRET=0	FNOL3	221

SUBROUTINE FNOL3 TRACE

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      JTEST1=1
      805 IF (NRET .NE. 0) GO TO 806
      H=XD-XS
      GO TO 50
225    806 IF (F*FP.LT.0.) GOTO 810
      IF (F*FP2.LT.0.) GOTO 820
      GO TO 800
      810 FP2 =FP
      HP =H
230    GOTO 830
      820 FP =FP2
      HP =H + HP
      830 NRET=NRET+1
      H=HP*F/(FP-F)
235    JTEST=4
      GOTO 300
      END

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      FNOL3 222
      FNOL3 223
      FNOL3 224
      FNOL3 225
      FNOL3 226
      FNOL3 227
      FNOL3 228
      FNOL3 229
      FNOL3 230
      FNOL3 231
      FNOL3 232
      FNOL3 233
      FNOL3 234
      FNOL3 235
      FNOL3 236
      FNOL3 237
      FNOL3 238

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SUBROUTINE TERM2 TRACE

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5      SUBROUTINE TERM2(X,Y,D,F)
        DIMENSIONY(1),D(1)
        COMMON/GAMM/WRAD,ZM,FLEN,CR,TL
        COMMON/BG/N,DRT,PRAD,PRTU,TOP
        F=TL-X
        IF(DRT.EQ.0.0)RETURN
        R1=(Y(1)**2+Y(2)**2)**.5
        R2=(Y(3)**2+Y(4)**2)**.5
15     IF(ABS(R1-R2).GT.DRT)F=0.0
10     RETURN
        END
```

```
TERM2      2
TERM2      3
TERM2      4
TERM2      5
TERM2      6
TERM2      7
TERM2      8
TERM2      9
TERM2     10
TERM2     11
TERM2     12
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SUBROUTINE TERM3 TRACE

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	SUBROUTINE TERM3(X,Y,D,F)	TERM3	2
	COMMON/BG/N,DRT,PRAD,PRTU,TOP	TERM3	3
	COMMON/GAMM/WRAD,ZM,FLEN,CR,TL,TSL	TERM3	4
	DIMENSION Y(1),D(1)	TERM3	5
5	DELT=0.0	TERM3	6
	CALL PROFILE(R,DR,X,DELT,2)	TERM3	7
	R2=R*R	TERM3	8
	IF (Y(1)**2+Y(2)**2.GT.R2.AND.Y(3)**2+Y(4)**2.GT.R2)GO TO 2	TERM3	9
	TL=X	TERM3	10
10	WRITE(6,2000)	TERM3	11
	2 XFIN=TSL	TERM3	12
	IF (TL.LT.XFIN)XFIN=TL	TERM3	13
	F=XFIN-X	TERM3	14
	RETURN	TERM3	15
15	2000 FORMAT(51H0EXECUTION TERMINATED DUE TO NUMERICAL INSTABILITY.)	TERM3	16
	END	TERM3	17

	SUBROUTINE DERIV(X,Y,D)	DERIV	2
	COMPLEX A,B,T,E,XF,G,XZ,XF1,XF2,XF3,DZOP,XJ,XJC,C1,C2,C3,C4,C5,	DERIV	3
	IC6,K11,K12,K13,K14,K21,K22,K23,K24,K15,K16,K25,K26,DTH,DZ0,TH2,	DERIV	4
	2KN1,KN2,WN,XK,XKK,XWN,CIR,XC,L1,L2,X1,X2,X0,XOP,P,H,C7,K19,K29,XNP	DERIV	5
5	INTEGER Z0,ZOP,Z1,Z2,ZJ	DERIV	6
	COMMON/GARB/XLR,R,DR,XC,TAL,FVORT	DERIV	7
	COMMON/BG/N,DRT,PRAD,PRTU, TOP	DERIV	8
	COMMON/VEL/U,IE	DERIV	9
	DIMENSION Y(1),D(1),WN(15),DD(4,4),Q(4,1),XL(15),XC(15),XZ(2),	DERIV	10
10	1A(2,2),B(2),T(2,15),E(2,15),XF(2,15),G(2),P(2),H(2,2),XNP(15),	DERIV	11
	3XLR(15)	DERIV	12
	CALL PROFILE(R,DR,X,DELTA, 2)	DERIV	13
	XK=CMPLX(0.0,1.0)	DERIV	14
	XKK=XK*R	DERIV	15
15	IC=(N-1)/2	DERIV	16
	DO 22 KQ=1,IC	DERIV	17
	22 XL(KQ)=XLR(KQ)/R	DERIV	18
	C...OBTAIN THE STAGNATION POINTS	DERIV	19
	THETA=Y(N)*(1.-TOP)	DERIV	20
20	S1=R*COS(THETA)	DERIV	21
	S2=R*SIN(THETA)	DERIV	22
	XZ(1)= CMPLX(S1,S2)	DERIV	23
	S1=-S1	DERIV	24
	XZ(2)=CMPLX(S1,S2)	DERIV	25
25	IF(IC.EQ.2.OR.IC.NE.2*(IC/2))GO TO 6	DERIV	26
	DTH=XZ(1)	DERIV	27
	XZ(1)=XZ(2)	DERIV	28
	XZ(2)=DTH	DERIV	29
	6 CONTINUE	DERIV	30
30	C.....TRANSFORMS VORTEX POSITIONS TO COMPLEX COORDINATES	DERIV	31
	DO 5 I=1,IC	DERIV	32
	5 XC(I)=CMPLX(Y(2*I-1),Y(2*I))	DERIV	33
	C.....CALCULATE FUNCTIONAL VALUES	DERIV	34
	DO 4 L=1,2	DERIV	35
35	X1=XZ(L)	DERIV	36
	B(L)=(R*R/X1/X1+1.)/R+FVORT/X1	DERIV	37
	G(L)=-2.*R/X1/X1/X1-FVORT/X1/X1	DERIV	38
	P(L)=DR*(1./X1/X1-1./R/R)	DERIV	39
	DO 4 K=1,IC	DERIV	40
40	XJ=XC(K)	DERIV	41
	XJC=CONJG(XJ)	DERIV	42
	XF1=1./(X1-XJ)-1./(X1-R*R/XJC)	DERIV	43
	XF2=1./(X1-R*R/XJC)**2-1./(X1-XJ)**2	DERIV	44
	XF3=-DR*2.*R/((X1-R*R/XJC)**2)/XJC	DERIV	45
45	XF(L,K)=(R/(X1-R*R/XJC)/XJC)**2	DERIV	46
	E(L,K)=1./(X1-XJ)**2	DERIV	47
	IF(K.GT.2)GO TO 3	DERIV	48
	A(L,K)=XF1	DERIV	49
	T(L,K)=XF2	DERIV	50
50	H(L,K)=XF3	DERIV	51
	GO TO 4	DERIV	52
	3 B(L)=B(L)+XL(K)*XF1	DERIV	53
	G(L)=G(L)+XL(K)*XF2	DERIV	54
	P(L)=P(L)+XL(K)*XF3	DERIV	55
55	4 CONTINUE	DERIV	56

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C.....CALCULATE THE TWO UNKNOWN CIRCULATIONS
      Z0=1
      ZOP=2
      Z1=1
      Z2=2
      A1=REAL(-B(Z0)*A(ZOP,Z2)+B(ZOP)*A(Z0,Z2))
      B1=REAL(-B(ZOP)*A(Z0,Z1)+B(Z0)*A(ZOP,Z1))
      C=REAL(A(Z0,Z1)*A(ZOP,Z2)-A(Z0,Z2)*A(ZOP,Z1))
      XL(1)=A1/C
      XL(2)=B1/C
      XLR(1)=XL(1)*R
      XLR(2)=XL(2)*R
      IF(U.LT.0)RETURN
C.....CALCULATE THE FLUID VELOCITY AT EACH VORTEX LOCATION
      DO 20 K=1,IC
      X1=XC(K)
      XWN=R*DR/X1/TAL-XK*(1.+R*R/X1/X1)-XK*FVORT/X1
      DO 10 L=1,IC
      XJ=XC(L)
      IF(L.NE.K)GO TO 8
      XNP(L)=XKK*XL(L)/(X1-R*R/CONJG(XJ))
      GO TO 10
      8 XNP(L)=-XKK*XL(L)*(1./(X1-XJ)-1./(X1-R*R/CONJG(XJ)))
      10 CONTINUE
      WN(K)=XWN+XNP(K)
      DO 15 MBS=1,IC
      IF(MBS.EQ.K)GO TO 15
      WN(K)=WN(K)+XNP(MBS)
      15 CONTINUE
      20 WN(K)=CONJG(WN(K))
C.....DETERMINE THE DERIVATIVE OF THETA
      DELTA=ATAN(DR)
      THETA1=THET1(DELTA)
      D(N)=(.7*TAL/R)*(THETA1-Y(N))
C.....CALCULATE DERIVATIVES FOR ALL FREE VORTICES
      IF(IC.LT.3)GO TO 40
      DO 30 IL=3,IC
      RWN=REAL(WN(IL))
      XIWN=AIMAG(WN(IL))
      D(2*IL)=XIWN*TAL
      30 D(2*IL-1)=RWN*TAL
      40 CONTINUE
C.....CALCULATE DD MATRIX FOR ATTACHED VORTICES
      X2=XC(2)
      X1=XC(1)
      X0=XZ(1)
      XOP=XZ(2)
      C1=-(A(ZOP,Z2)*E(Z0,Z1)-A(Z0,Z2)*E(ZOP,Z1))/C
      C2=-(A(ZOP,Z2)*XF(Z0,Z1)-A(Z0,Z2)*XF(ZOP,Z1))/C
      C3=-(A(Z0,Z1)*E(ZOP,Z2)-A(ZOP,Z1)*E(Z0,Z2))/C
      C4=-(A(Z0,Z1)*XF(ZOP,Z2)-A(ZOP,Z1)*XF(Z0,Z2))/C
      K11=C1*(X1-X0)+1.
      K12=C2*(X1-X0)
      K13=(C3-(B(Z0)*E(ZOP,Z2)-B(ZOP)*E(Z0,Z2))/A1)*(X1-X0)
      K14=(C4-(B(Z0)*XF(ZOP,Z2)-B(ZOP)*XF(Z0,Z2))/A1)*(X1-X0)

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	K21=(C1-(B(ZOP)*E(ZO,Z1)-B(ZO)*E(ZOP,Z1))/B1)*(X2-XOP)	DERIV	112
	K22=(C2-(B(ZOP)*XF(ZO,Z1)-B(ZO)*XF(ZOP,Z1))/B1)*(X2-XOP)	DERIV	113
	K23=C3*(X2-XOP)+1.	DERIV	114
	K24=C4*(X2-XOP)	DERIV	115
115	DD(1,1)=REAL(K11+K12)	DERIV	116
	DD(1,2)=AIMAG(K12-K11)	DERIV	117
	DD(1,3)=REAL(K13+K14)	DERIV	118
	DD(1,4)=AIMAG(K14-K13)	DERIV	119
	DD(2,1)=AIMAG(K11+K12)	DERIV	120
120	DD(2,2)=REAL(K11-K12)	DERIV	121
	DD(2,3)=AIMAG(K13+K14)	DERIV	122
	DD(2,4)=REAL(K13-K14)	DERIV	123
	DD(3,1)=REAL(K21+K22)	DERIV	124
	DD(3,2)=AIMAG(K22-K21)	DERIV	125
125	DD(3,3)=REAL(K23+K24)	DERIV	126
	DD(3,4)=AIMAG(K24-K23)	DERIV	127
	DD(4,1)=AIMAG(K21+K22)	DERIV	128
	DD(4,2)=REAL(K21-K22)	DERIV	129
	DD(4,3)=AIMAG(K23+K24)	DERIV	130
130	DD(4,4)=REAL(K23-K24)	DERIV	131
	C.....CALCULATE THE Q VECTORS	DERIV	132
	C5=(-A(ZOP,Z2)*T(ZO,Z1)+A(ZOP,Z1)*T(ZO,Z2))/C	DERIV	133
	C6=(A(ZO,Z1)*T(ZOP,Z2)-A(ZO,Z2)*T(ZOP,Z1))/C	DERIV	134
	C7=(A(ZOP,Z1)*H(ZO,Z2)+A(ZO,Z2)*H(ZOP,Z1)-A(ZOP,Z2)*H(ZO,Z1)	DERIV	135
135	1-A(ZO,Z1)*H(ZOP,Z2))/C	DERIV	136
	K15=(C5-(A(ZOP,Z2)*G(ZO)-B(ZOP)*T(ZO,Z2))/A1)*(X1-XO)	DERIV	137
	K16=(C6+(T(ZOP,Z2)*B(ZO)-A(ZO,Z2)*G(ZOP))/A1)*(X1-XO)	DERIV	138
	K19=(C7+(A(ZO,Z2)*P(ZOP)+B(ZOP)*H(ZO,Z2)-A(ZOP,Z2)*P(ZO)-B(ZO)*	DERIV	139
	1H(ZOP,Z2))/A1)*(X1-XO)	DERIV	140
140	K25=(C5+(A(ZOP,Z1)*G(ZO)-T(ZO,Z1)*B(ZOP))/B1)*(X2-XOP)	DERIV	141
	K26=(C6+(A(ZO,Z1)*G(ZOP)-B(ZO)*T(ZOP,Z1))/B1)*(X2-XOP)	DERIV	142
	K29=(C7+(A(ZOP,Z1)*P(ZO)+B(ZO)*H(ZOP,Z1)-A(ZO,Z1)*P(ZOP)-B(ZOP)*	DERIV	143
	1H(ZO,Z1))/B1)*(X2-XOP)	DERIV	144
	DT2=D(N)*R*(1.-TOP)	DERIV	145
145	DTH=CMPLX(DR,DT2)	DERIV	146
	TH2=CMPLX(0.0,THETA)	DERIV	147
	DZO=CEXP(TH2)*DTH	DERIV	148
	DTH=CMPLX(-DR,DT2)	DERIV	149
	DZOP=-CEXP(-TH2)*DTH	DERIV	150
150	IF(IC.EQ.2.OR.IC.NE.2*(IC/2))GO TO 55	DERIV	151
	TH2=DZO	DERIV	152
	DZO=DZOP	DERIV	153
	DZOP=TH2	DERIV	154
55	CONTINUE	DERIV	155
155	L1=-K15*DZO-K16*DZOP+TAL*WN(1)-DR*(X1-XO)/R-K19	DERIV	156
	L2=-K25*DZO-K26*DZOP+TAL*WN(2)-DR*(X2-XOP)/R-K29	DERIV	157
	IF(IC.LT.3)GO TO 50	DERIV	158
	DO 60 M=3,IC	DERIV	159
	DTH=CMPLX(D(2*M-1),D(2*M))	DERIV	160
160	TH2=CONJG(DTH)	DERIV	161
	ZJ=M	DERIV	162
	CIR=XL(M)	DERIV	163
	KN1=(X1-XO)*(-A(ZOP,Z2)*E(ZO,ZJ)+A(ZO,Z2)*E(ZOP,ZJ))/A1*CIR	DERIV	164
	KN2=(X1-XO)*(-A(ZOP,Z2)*XF(ZO,ZJ)+A(ZO,Z2)*XF(ZOP,ZJ))/A1*CIR	DERIV	165
165	L1=L1-KN1*DTH-KN2*TH2	DERIV	166

SUBROUTINE DERIV TRACE

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	KN1=(-E(ZOP,ZJ)*A(ZO,Z1)+A(ZOP,Z1)*E(ZO,ZJ))*(X2-XOP)*CIR/B1	DERIV	167
	KN2=(-XF(ZOP,ZJ)*A(ZO,Z1)+XF(ZO,ZJ)*A(ZOP,Z1))*(X2-XOP)*CIR/B1	DERIV	168
	60 L2=L2-OTH*KN1-TH2*KN2	DERIV	169
	50 Q(1,1)=REAL(L1)	DERIV	170
170	Q(2,1)=AIMAG(L1)	DERIV	171
	Q(3,1)=REAL(L2)	DERIV	172
	Q(4,1)=AIMAG(L2)	DERIV	173
	II=4	DERIV	174
	IX=1	DERIV	175
175	CALL MATRIX(DD,II,II,Q,IX,DETERM,ID)	DERIV	176
	D(1)=Q(1,1)	DERIV	177
	D(2)=Q(2,1)	DERIV	178
	D(3)=Q(3,1)	DERIV	179
	D(4)=Q(4,1)	DERIV	180
180	RETURN	DERIV	181
	END	DERIV	182

	SUBROUTINE OUT(X,Y,D,ERROR,N,L,H)	OUT	2
	COMPLEX XC,F1	OUT	3
	DIMENSION XC(15),XL(15),D(1),ERROR(1),Y(1),XLR(15)	OUT	4
	COMMON/STA/A,NGB	OUT	5
5	COMMON/XLIF/MC,M,CDMN,CDML,XS,CYM,CDCL1,CDCN1,CY1	OUT	6
	COMMON/GARB/XLR,R,DR,XC,TAL	OUT	7
	COMMON/VEL/U,IE	OUT	8
	COMMON/GAMM/RO,ZM,FLEN,CR,TL	OUT	9
	COMMON/BG/IBLANK,DRT	OUT	10
10	CONVR=57.2957795	OUT	11
	IF(DRT.NE.0.0)GO TO 8	OUT	12
	Y(1)=(Y(1)-Y(3))/2.	OUT	13
	Y(3)=-Y(1)	OUT	14
	Y(2)=(Y(2)+Y(4))/2.	OUT	15
15	Y(4)=Y(2)	OUT	16
	8 CONTINUE	OUT	17
	U=-U	OUT	18
	CALL DERIV(X,Y,D)	OUT	19
	U=-U	OUT	20
20	C.....CALCULATE THE FORCE COEFFICIENTS	OUT	21
	CDCL=2.*SIN(A)*COS(A)*R*R/RO/RO	OUT	22
	F1=CMPLX(0.0,0.0)	OUT	23
	IC=(N-1)/2	OUT	24
	DO 22 KQ=1,IC	OUT	25
25	22 XL(KQ)=XLR(KQ)/R	OUT	26
	DO 10 I=1,IC	OUT	27
	10 F1=XL(I)*(XC(I)-R*R/CONJG(XC(I)))+F1	OUT	28
	F1=4.*SIN(A)*COS(A)*F1*R/RO/RO	OUT	29
	CDCN=REAL(F1)	OUT	30
30	CY=AIMAG(F1)	OUT	31
	CD=0.0	OUT	32
	IF(X.NE.XS)CD=(CDCN-CDCN1)/(X-XS)	OUT	33
	CD=CD*3.1416*R/2./SIN(A)/SIN(A)	OUT	34
	C.....CALCULATE MOMENT COEFFICIENTS	OUT	35
35	CDML=(X+XS)*(CDCL-CDCL1)/4./RO +CDML	OUT	36
	CDMN=(XS+X)*(CDCN-CDCN1)/4./RO +CDMN	OUT	37
	CYM=(X+XS)*(CY-CY1)/4./RO +CYM	OUT	38
	C.....REINITIALIZE FORCE COEFFICIENTS	OUT	39
	CDCL1=CDCL	OUT	40
40	CDCN1=CDCN	OUT	41
	CY1=CY	OUT	42
	IF(X.EQ.XS)CIT=0	OUT	43
	XS=X	OUT	44
	M=M-1	OUT	45
45	IF(X.EQ.TL)GO TO 16	OUT	46
	CIT=CIT+1	OUT	47
	IF(CIT.LT.NGB)GO TO 16	OUT	48
	IF(MC.EQ.0)GO TO 15	OUT	49
	IF(M.EQ.0)GO TO 16	OUT	50
50	RETURN	OUT	51
	15 IF(M.EQ.0)GO TO 17	OUT	52
	IF(X-Y(N+1).LT.Y(N+2))RETURN	OUT	53
	Y(N+1)=X	OUT	54
	GO TO 17	OUT	55
55	16 M=MC	OUT	56

SUBROUTINE OUT

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17 DDEG=CONVR*Y(N) OUT 57
HPP=0.0 OUT 58
IF (ABS(H).LT.1.)HPP=H OUT 59
WRITE(6,2000)X,R,DDEG,HPP OUT 60
60 WRITE(6,2004) OUT 61
DO 20 KT=1,IC OUT 62
K=1+IC-KT OUT 63
IF (IC.EQ.2)K=KT OUT 64
XR2=(Y(2*K-1)**2+Y(2*K)**2)**.5 OUT 65
65 ANG2=CONVR*ATAN(Y(2*K-1)/Y(2*K)) OUT 66
IF (Y(2*K).LT.0.0)ANG2=Y(2*K-1)/ABS(Y(2*K-1))*(ABS(ANG2)+90.0) OUT 67
20 WRITE(6,2001)KT,XL(K),XR2,ANG2,Y(2*K-1),Y(2*K) OUT 68
WRITE(6,2002)CDCL,CDML,CDCN,CDMN,CD,CY,CYM OUT 69
IF (IE.EQ.0)RETURN OUT 70
70 WRITE(6,2003)(ERROR(II),II=1,N) OUT 71
RETURN OUT 72
2000 FORMAT(5H0X = ,F12.7,4X,4HR = ,F12.7,4X,9HTHETA0 = ,F10.5,4X,12HST OUT 73
1EP SIZE = ,E14.8) OUT 74
2001 FORMAT(1H ,20X,I3,6X,5(F14.7,3X)) OUT 75
75 2002 FORMAT(1H ,6X,2HCL,12X,4H CML,11X,2HCN,12X,3HCMN,11X,2HCD,12X,2HCY OUT 76
1,12X,3HCYM,/7F14.7) OUT 77
2003 FORMAT( 8H ERRORS ,8E14.6) OUT 78
2004 FORMAT(1H ,15X,14HVORTEX NUMBER ,7X,3HLAM,13X,6HRADIUS,11X,5HANGLE OUT 79
1,13X,1HY,16X,1HZ) OUT 80
80 END OUT 81

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		SUBROUTINE MATRIX(A,N1,I1,B,M1,DETERM,ID)	MATRIX	2
	C	MATRIX INVERSION WITH ACCOMPANYING SOLUTION OF LINEAR EQUATIONS	MATRIX	3
	C		MATRIX	4
	C	TEST FOR LOST OF DIGITS DUE TO SUBTRACTION	MATRIX	5
5	C		MATRIX	6
		DIMENSION A(I1,I1),B(I1,M1),INDEX(50,3)	MATRIX	7
		EQUIVALENCE (IROW,JROW), (ICOLUMN,JCOLUMN), (AMAX, T, SWAP)	MATRIX	8
		DATA EPS4/1.E-7/	MATRIX	9
	C		MATRIX	10
10		ID=1	MATRIX	11
		M=M1	MATRIX	12
		N=N1	MATRIX	13
		10 DETERM=1.0	MATRIX	14
		15 DO 20 J=1,N	MATRIX	15
15		20 INDEX(J,3) = 0	MATRIX	16
		30 DO 550 I=1,N	MATRIX	17
	C		MATRIX	18
	C	SEARCH FOR PIVOT ELEMENT	MATRIX	19
	C		MATRIX	20
20		40 AMAX=0.0	MATRIX	21
		45 DO 105 J=1,N	MATRIX	22
		IF (INDEX(J,3) .EQ. 1) GO TO 105	MATRIX	23
		60 DO 100 K=1,N	MATRIX	24
		IF (INDEX(K,3)-1) 80, 100, 100	MATRIX	25
25		80 IF (AMAX .GE. ABS(A(J,K))) GO TO 100	MATRIX	26
		85 IROW=J	MATRIX	27
		90 ICOLUMN=K	MATRIX	28
		AMAX = ABS(A(J,K))	MATRIX	29
		100 CONTINUE	MATRIX	30
30		105 CONTINUE	MATRIX	31
		INDEX(ICOLUMN,3) = INDEX(ICOLUMN,3) +1	MATRIX	32
		260 INDEX(I,1)=IROW	MATRIX	33
		270 INDEX(I,2)=ICOLUMN	MATRIX	34
		IF (BMAX.EQ.AMAX) ID=2	MATRIX	35
35	C		MATRIX	36
	C	INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL	MATRIX	37
	C		MATRIX	38
		130 IF (IROW .EQ. ICOLUMN) GO TO 310	MATRIX	39
		140 DETERM=-DETERM	MATRIX	40
40		150 DO 200 L=1,N	MATRIX	41
		160 SWAP=A(IROW,L)	MATRIX	42
		170 A(IROW,L)=A(ICOLUMN,L)	MATRIX	43
		200 A(ICOLUMN,L)=SWAP	MATRIX	44
		IF (M) 310, 310, 210	MATRIX	45
45		210 DO 250 L=1, M	MATRIX	46
		220 SWAP=B(IROW,L)	MATRIX	47
		230 B(IROW,L)=B(ICOLUMN,L)	MATRIX	48
		250 B(ICOLUMN,L)=SWAP	MATRIX	49
	C		MATRIX	50
50	C	DIVIDE PIVOT ROW BY PIVOT ELEMENT	MATRIX	51
	C		MATRIX	52
		310 PIVOT =A(ICOLUMN,ICOLUMN)	MATRIX	53
		DETERM=DETERM*PIVOT	MATRIX	54
		330 A(ICOLUMN,ICOLUMN)=1.0	MATRIX	55
55		340 DO 352 L=1,N	MATRIX	56

	IF (PIVOT .NE. 0.) GO TO 350	MATRIX	57
	A(ICOLUM,L) = 0.	MATRIX	58
	GO TO 352	MATRIX	59
60	350 A(ICOLUM,L)=A(ICOLUM,L)/PIVOT	MATRIX	60
	352 CONTINUE	MATRIX	61
	355 IF(M) 380, 380, 360	MATRIX	62
	360 DO 375 L=1,M	MATRIX	63
	IF (PIVOT .NE. 0.) GO TO 370	MATRIX	64
65	B(ICOLUM,L) = 0.	MATRIX	65
	GO TO 375	MATRIX	66
	370 B(ICOLUM,L)=B(ICOLUM,L)/PIVOT	MATRIX	67
	375 CONTINUE	MATRIX	68
	C	MATRIX	69
	C REDUCE NON-PIVOT ROWS	MATRIX	70
70	C	MATRIX	71
	BMAX=0.0	MATRIX	72
	380 DO 550 L1=1,N	MATRIX	73
	390 IF (L1 .EQ. ICOLUM) GO TO 550	MATRIX	74
	400 T=A(L1,ICOLUM)	MATRIX	75
75	420 A(L1,ICOLUM)=0.0	MATRIX	76
	430 DO 450 L=1,N	MATRIX	77
	SUB =A(ICOLUM,L)*T	MATRIX	78
	A(L1,L)=A(L1,L)-SUB	MATRIX	79
	C	MATRIX	80
80	C	MATRIX	81
	IF (INDEX(L1,3).EQ.1) GO TO 450	MATRIX	82
	IF (ABS(A(L1,L))-EPS4* ABS(SUB)) 449,449,450	MATRIX	83
	449 BMAX=AMAX1(BMAX,ABS(A(L1,L)))	MATRIX	84
	450 CONTINUE	MATRIX	85
85	455 IF(M) 550, 550, 460	MATRIX	86
	460 DO 500 L=1,M	MATRIX	87
	500 B(L1,L)=B(L1,L)-B(ICOLUM,L)*T	MATRIX	88
	550 CONTINUE	MATRIX	89
	C	MATRIX	90
90	C INTERCHANGE COLUMNS	MATRIX	91
	C	MATRIX	92
	600 DO 710 I=1,N	MATRIX	93
	610 L=N+1-I	MATRIX	94
	620 IF (INDEX(L,1) .EQ. INDEX(L,2)) GO TO 710	MATRIX	95
95	630 JROW=INDEX(L,1)	MATRIX	96
	640 JCOLUM=INDEX(L,2)	MATRIX	97
	650 DO 705 K=1,N	MATRIX	98
	660 SWAP=A(K,JROW)	MATRIX	99
	670 A(K,JROW)=A(K,JCOLUM)	MATRIX	100
100	700 A(K,JCOLUM)=SWAP	MATRIX	101
	705 CONTINUE	MATRIX	102
	710 CONTINUE	MATRIX	103
	DO 730 K = 1,N	MATRIX	104
	IF (INDEX(K,3) .NE. 1) GO TO 715	MATRIX	105
105	730 CONTINUE	MATRIX	106
	RETURN	MATRIX	107
	715 ID=2	MATRIX	108
	RETURN	MATRIX	109
	END	MATRIX	110

ANGLE OF ATTACK 45.00 DEGREES

OGIVE DIMENSIONS
 MAXIMUM RADIUS .50000
 OGIVE LENGTH 3.100
 Z MATCHING POINT 3.10000
 BODY LENGTH 11.00000

*****FREE PARAMETERS*****
 INITIAL VORTEX STRENGTH .04000
 INITIAL RADIAL PERTURBATION .00600
 RADIAL ASYMMETRY AT WHICH FIRST VORTEX IS SHED .10000
 PER CENT DECREASE IN CIRCULATION OF SHED VORTICES -2.50000
 LOCATION FACTOR FOR NEWLY INTRODUCED VORTICES .07500
 STROUHAL NUMBER .16660 CORRESPONDING SHEDDING DISTANCE 3.00120
 SEPARATION ANGLE CONSTANTS .73300 -0.00000 POINT OF TRANSITION 1.00000

INTEGRATION OPTIONS
 INITIAL STEP SIZE .0002500 PRINT FREQUENCY -0 ERROR BOUND 6 PRINT INCREMENT 1.0000000
 ERROR BOUND AFTER VORTEX SHEDDING 6

X = .1000000 R = .0325289 THETA0 = 41.99781 STEP SIZE = .25000000E-03
 VORTEX NUMBER LAM RADIUS ANGLE Y Z
 1 .0355897 .0331857 46.6493614 .0241315 .0227807
 2 -.0578202 .0327899 -46.6493614 -.0238437 .0225090
 CL CML CN CMN CD CY CYM
 .0042325 .0002809 .0000144 .0000014 0.0000000 .0000029 .0000003
 ERRORS 0. 0. 0. 0. 0. 0.

X = 1.1030758 R = .2956664 THETA0 = 41.99781 STEP SIZE = .21046273E-01
 VORTEX NUMBER LAM RADIUS ANGLE Y Z
 1 .2438641 .3327822 39.0463583 .2096358 .2584508
 2 -.2082865 .3244723 -40.5432500 -.2109141 .2465715
 CL CML CN CMN CD CY CYM
 .3496745 .2415966 .0430999 .0372341 .1302038 .0107881 .0094284
 ERRORS .607359E-09 .218640E-09 .236340E-08 -.652509E-09 0.

X = 2.1196964 R = .4511470 THETA0 = 41.99781 STEP SIZE = .35862999E-01
 VORTEX NUMBER LAM RADIUS ANGLE Y Z
 1 .4714942 .6214725 30.9960132 .3200449 .5327282
 2 -.4356448 .5754213 -32.8963095 -.3125230 .4831553
 CL CML CN CMN CD CY CYM
 .8141343 .9785404 .4469488 .7358462 .9504091 .1361153 .2284237
 ERRORS -.665432E-09 -.820530E-09 .471271E-10 -.646229E-09 0.

X = 2.8623315 R = .4971352 THETA0 = 41.99781 STEP SIZE = .25000000E-03
 VORTEX NUMBER LAM RADIUS ANGLE Y Z
 1 .6638951 .8170361 26.8023124 .3684126 .7292599
 2 -.6455558 .7118971 -27.9646625 -.3338277 .6287738
 CL CML CN CMN CD CY CYM
 .9885735 1.4009501 1.0517271 2.2520225 0.0000000 .3855470 .8595407
 ERRORS -.105708E-09 .329976E-08 -.291649E-08 -.409414E-08 0.

X = 2.8623315 R = .4971352 THETA0 = 41.99781 STEP SIZE = .10000000E-02
 VORTEX NUMBER LAM RADIUS ANGLE Y Z
 1 .6472977 .8170361 26.8023124 .3684126 .7292599
 2 -.6421862 .7118971 -27.9646625 -.3338277 .6287738
 3 .0022805 .5227481 44.9565092 .3693581 .3699192
 CL CML CN CMN CD CY CYM
 .9885735 1.4009501 1.0344401 2.2025413 0.0000000 .3598689 .7860415
 ERRORS 0. 0. 0. 0. 0. 0.

X = 3.1358214 R = .5000000 THETA0 = 41.99781 STEP SIZE = .17107098E-01
 VORTEX NUMBER LAM RADIUS ANGLE Y Z
 1 .6435889 .9293537 23.6852872 .3733331 .8510703
 2 -.7280791 .7461542 -26.2962073 -.3305552 .6689390
 3 .0139121 .5284685 46.1529236 .3811269 .3660892
 CL CML CN CMN CD CY CYM
 1.0000000 1.4345633 1.2155258 2.7449078 .9638301 .4855503 1.1628711
 ERRORS .166824E-07 .117700E-07 -.490723E-08 .449925E-08 .344445E-08 .805208E-08 0.

X = 4.1446835 R = .5000000 THETA0 = 41.99781 STEP SIZE = .35007074E-01
 VORTEX NUMBER LAM RADIUS ANGLE Y Z
 1 .6435889 1.4027342 17.7113434 .4267421 1.3362464
 2 -1.0237597 .8997440 -20.3545102 -.3129559 .8435625
 3 .1757541 .5636118 39.5839052 .3591377 .4343713
 CL CML CN CMN CD CY CYM
 1.0000000 1.4345633 1.8985781 5.2548438 1.3165386 .6803161 1.8155487
 ERRORS .371962E-08 .239884E-11 .809811E-10 -.652020E-09 -.393803E-09 -.422120E-09 0.

X = 5.1549628 R = .5000000 THETA0 = 41.99781 STEP SIZE = .29757162E-01
 VORTEX NUMBER LAM RADIUS ANGLE Y Z
 1 .6435889 1.9788370 16.2281326 .5530109 1.8999934
 2 -1.2720087 1.2138898 -16.4435008 -.3436154 1.1642409
 3 .3921419 .5724520 36.5638004 .3410197 .4597900
 CL CML CN CMN CD CY CYM
 1.0000000 1.4345633 2.9112933 9.9895735 1.8015862 -.1686913 -2.2403960
 ERRORS -.311141E-07 -.123495E-07 .649968E-10 -.286473E-09 -.973207E-12 -.383286E-10 0.

X = 5.8635320 R = .5000000 THETA0 = 41.99781 STEP SIZE = 0.
 VORTEX NUMBER LAM RADIUS ANGLE Y Z
 1 .6435889 2.4158047 15.6478473 .6516006 2.3262693
 2 -1.4302811 1.5175554 -15.0338253 -.3936376 1.4656138
 3 .5239926 .5833192 35.0541973 .3350300 .4775104
 CL CML CN CMN CD CY CYM
 1.0000000 1.4345633 3.7994491 14.8912346 0.0000000 -1.4771218 -9.4785667
 ERRORS -.165932E-06 -.758859E-07 .640592E-09 .155940E-08 .114145E-08 .486370E-09 0.

X = 5.8635320 R = .5000000 THETA0 = 41.99781 STEP SIZE = .10000000E-02
 VORTEX NUMBER LAM RADIUS ANGLE Y Z
 1 .6435889 2.4158047 15.6478473 .6516006 2.3262693
 2 -1.3945240 1.5175554 -15.0338253 -.3936376 1.4656138
 3 .5145310 .5833192 35.0541973 .3350300 .4775104
 4 -.0028980 .5263422 -45.0209574 -.3723162 .3720440
 CL CML CN CMN CD CY CYM
 1.0000000 1.4345633 3.7463173 14.5796946 0.0000000 -1.2954685 -8.4134367
 ERRORS 0. 0. 0. 0. 0. 0. 0.
 ERRORS 0.

X = 6.1615320 R = .5000000 THETA0 = 41.99781 STEP SIZE = .16000000E-01
 VORTEX NUMBER LAM RADIUS ANGLE Y Z
 1 .6435889 2.6042565 15.3967217 .6914326 2.5107913
 2 -1.3945240 1.7091872 -13.9904236 -.4132126 1.6584861
 3 .5714561 .5970385 33.7584075 .3317697 .4963707
 4 -.0076643 .5335495 -46.3339277 -.3859569 .3683915
 CL CML CN CMN CD CY CYM
 1.0000000 1.4345633 4.0499953 16.4051674 1.5663684 -1.8966131 -12.0277866
 ERRORS .353227E-08 .589238E-09 -.121830E-09 -.504559E-10 .452755E-11 .363392E-11 -.873714E-12 .946037E-12
 ERRORS 0.

X = 7.1721751 R = .5000000 THETA0 = 41.99781 STEP SIZE = .32094397E-01
 VORTEX NUMBER LAM RADIUS ANGLE Y Z
 1 .6435889 3.2348758 14.4416539 .8067585 3.1326605
 2 -1.3945240 2.4021354 -11.1526320 -.4646290 2.3567720
 3 .8646256 .7177918 25.8484516 .3129517 .6459770
 4 -.0825237 .5600676 -41.4797306 -.3709636 .4195971
 CL CML CN CMN CD CY CYM
 1.0000000 1.4345633 5.0887432 23.3413337 1.7434051 -3.5828642 -23.1921245

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ERRORS .132244E-08 -.211717E-08 .292732E-09 -.290729E-10 -.248214E-10 .382313E-10 -.172845E-10 .215497E-10
ERRORS 0.

X = 8.1991958 R = .5000000 THETA0 = 41.99781 STEP SIZE = .32094397E-01
      VORTEX NUMBER LAM RADIUS ANGLE Y Z
      1 .6435889 3.8595730 13.1682765 .8792562 3.7580862
      2 -1.3945240 3.1316880 -9.5957769 -.5220401 3.0878705
      3 1.1935511 1.0327161 18.2391696 .3232239 .9808307
      4 -.2689768 .5561387 -38.7058756 -.3477662 .4339919
      CL CML CN CMN CD CY CYM
      1.0000000 1.4345633 6.3164263 32.7899077 1.9881864 -3.7775881 -24.5257429
ERRORS .731932E-09 -.986816E-10 -.325549E-09 -.216827E-09 .257688E-10 .622232E-11 .236255E-11 -.817124E-12
ERRORS 0.

X = 8.8647325 R = .5000000 THETA0 = 41.99781 STEP SIZE = 0.
      VORTEX NUMBER LAM RADIUS ANGLE Y Z
      1 .6435889 4.2580371 12.1574910 .8967408 4.1625396
      2 -1.3945240 3.6060791 -9.1234539 -.5717880 3.5604585
      3 1.3749567 1.3339893 15.6128605 .3590245 1.2847680
      4 -.3515362 .5549913 -38.6311965 -.3464839 .4335485
      CL CML CN CMN CD CY CYM
      1.0000000 1.4345633 7.1938063 40.2793319 0.0000000 -2.9521614 -17.4474940
ERRORS .888596E-06 -.352818E-06 -.744009E-09 -.187193E-08 .596501E-11 -.720727E-10 -.225967E-10 -.219927E-10
ERRORS 0.

X = 8.8647325 R = .5000000 THETA0 = 41.99781 STEP SIZE = .10000000E-02
      VORTEX NUMBER LAM RADIUS ANGLE Y Z
      1 .6435889 4.2580371 12.1574910 .8967408 4.1625396
      2 -1.3945240 3.6060791 -9.1234539 -.5717880 3.5604585
      3 1.3405828 1.3339893 15.6128605 .3590245 1.2847680
      4 -.3449832 .5549913 -38.6311965 -.3464839 .4335485
      5 .0030946 .5254779 44.9261199 .3710896 .3720478
      CL CML CN CMN CD CY CYM
      1.0000000 1.4345633 7.1501010 39.8918962 0.0000000 -3.1014182 -18.7706156
ERRORS 0. 0. 0. 0. 0. 0. 0.
ERRORS 0. 0. 0. 0. 0. 0. 0.

X = 9.2130058 R = .5000000 THETA0 = 41.99781 STEP SIZE = .14707534E-01
      VORTEX NUMBER LAM RADIUS ANGLE Y Z
      1 .6435889 4.4654066 11.5864241 .8968582 4.3744144
      2 -1.3945240 3.8539932 -8.9834568 -.6017983 3.8067181
      3 1.3405828 1.5724424 13.6477708 .3710216 1.5280439
      4 -.3858415 .5614540 -37.9124832 -.3449894 .4429592
      5 .0106460 .5376864 46.1574387 .3878041 .3724441
      CL CML CN CMN CD CY CYM
      1.0000000 1.4345633 7.4810645 42.8818751 1.3880138 -2.5306527 -13.6101726
ERRORS -.165317E-08 .131929E-08 .843059E-11 -.463477E-11 -.817403E-11 .558131E-11 .408816E-12 -.119777E-12
ERRORS .639488E-13 -.178651E-12 0.

X = 10.2611996 R = .5000000 THETA0 = 41.99781 STEP SIZE = .63391534E-01
      VORTEX NUMBER LAM RADIUS ANGLE Y Z
      1 .6435889 5.0895694 9.7266230 .8598694 5.0164072
      2 -1.3945240 4.5943797 -8.8936108 -.7102921 4.5391420
      3 1.3405828 2.3492428 9.1053566 .3717686 2.3196400
      4 -.6270742 .6291921 -31.5159190 -.3289010 .5363831
      5 .0965087 .6233788 40.7982071 .4073138 .4719074
      CL CML CN CMN CD CY CYM
      1.0000000 1.4345633 8.3707261 51.5497577 1.4464884 -.7860946 3.3568254
ERRORS -.141296E-07 -.659408E-08 -.998941E-08 .161486E-07 -.408549E-09 .977546E-09 .201482E-09 -.208869E-09
ERRORS .377907E-10 -.231874E-09 0.

X = 11.0000000 R = .5000000 THETA0 = 41.99851 STEP SIZE = 0.
      VORTEX NUMBER LAM RADIUS ANGLE Y Z
      1 .6435889 5.5301893 8.3632984 .8043622 5.4713797
      2 -1.3945240 5.1088224 -8.9891053 -.7982364 5.0460762
      3 1.3405828 2.9151405 7.0659801 .3585980 2.8930005

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	4		-.8784763		.7550590		-24.7394574		-.3159866		.6857598
	5		.2409266		.6908357		36.0218935		.4062766		.5587426
	CL	CML	CN	CMN	CD	CY	CYM				
	1.0000000	1.4345633	9.1400144	59.7364535	0.0000000	.0524773	12.2374875				
ERRORS	.644419E-07	.419243E-06	.855520E-07	.533789E-06	.806359E-11	.303639E-11	.688279E-12	.253901E-11			
ERRORS	.160275E-11	.120698E-11	.892321E-06								

*****FINAL STATISTICS*****

NORMAL FORCE COEFFICIENT=CL+CN= 10.14001

PITCHING MOMENT COEFFICIENT=CML+CMN= 61.17102

CENTER OF NORMAL FORCE= 6.03264

YAW FORCE COEFFICIENT= .05248

YAW MOMENT COEFFICIENT= 12.23749

CENTER OF YAW FORCE= 233.19566

APPENDIX C

EXPERIMENTAL INVESTIGATION

The experiments referred to in this report were carried out in the NOL Supersonic Tunnel No. 1. This facility is an open jet, blowdown tunnel, 16 inches square, and is capable of subsonic as well as supersonic speeds.

The test model had an overall fineness of 12, a nose fineness of 4 and a diameter of 1.1 inches. It was tested at the Mach numbers of 0.5, 0.7, 0.9 and 1.1, which corresponded to a Reynolds number variation of $2(10^5)$ to $5(10^5)$ based on the body diameter. The model was swept from 0 to 70 degrees angle of attack. Four runs were made at each Mach number with the nose roll orientation being varied 90 degrees from run to run. Select runs were made using a boundary-layer trip. This consisted of a grit strip 1/8-inch wide along the leeward meridian of the body. Since the wind tunnel is an open jet type, it was possible to use a subsonic nozzle throughout the tests without encountering problems with reflected shock waves.

Force measurements were made using a four-component strain-gage balance. This type of information was augmented by schlieren photographs which were taken at increments of five degrees in angle of attack. Oil-flow studies were made at each Mach number at angles of attack of 20, 35, 50 and 70 degrees.

Results from the force tests indicate that normal force increases slightly with increasing Mach number, while the yaw force decreased strongly under these conditions. The measured forces and moments agree well with the data taken by Pick (Ref. (20)) using the same model. Pick's results, though, only go up to an angle of attack of 45 degrees. The Strouhal number measured from the schlieren photographs appears to be lower than that measured by Pick, although there is considerable scatter in the data obtained in the present study. The boundary-layer trip significantly lowered the normal force and greatly reduced the yaw force at the lower Mach numbers.

In order for force measurements to be meaningful at low crossflow Mach numbers, it is necessary to know whether the boundary layer is laminar or turbulent. It is well known that for $M_c \lesssim 0.5$, the transition of the boundary is accompanied by a large decrease in normal force. This results from an increase in the crossflow separation angle. At larger crossflow Mach numbers the transition of the boundary layer no longer has a large effect on normal force (Ref. (3)).

In the tests conducted, the crossflow Reynolds number ($Re \sin \alpha$) were of the order of 10^5 . Since transition is expected near this value, it is difficult to predict the state of the boundary layer. However, measurements of the crossflow separation angle taken from the oil-flow studies indicate that the boundary layer was laminar. This conclusion is supported by the reduction of force observed when the boundary-layer trip was used.

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