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A HISTORY DEPENDENT PARAMETER FOR THE CYCLIC STRESS-STRAIN BEHAVIOR OF METALS

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A HISTORY DEPENDENT PARAMETER FOR THE CYCLIC STRESS-STRAIN BEHAVIOR OF METALS

by

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FOREWORD

Recent requirements for increased strength and service life of machines and structures have been met by the use of higher strength materials and new fabrication and joining methods. Simultaneously, failures due to fracture have increased relative to those resulting from excessive deformation. Frequently service conditions are such that low temperature brittle fracture, fatigue fracture, and high temperature creep rupture must be considered in a single system. National concern with increased safety, reliability, and cost has focused attention upon these problems.

Methods are now available to predict both fatigue crack initiation life and crack propagation life. Paradoxically the materials properties required for long fatigue crack initiation life are incompatible with the requirements of high fracture toughness. Thus, the conflicting design approaches and requirements placed on the material are confusing and often impossible to satisfy.

Numerous publications dealing with a variety of fracture problems have led to many new and useful developments. However, the synthesis of the concepts into methods for design, testing and inspection has lagged.

This program of study is intended to contribute to the integration, correlation, and organization of mechanics and materials concepts and research information into a form that will permit enlightened decisions to be made regarding fracture control. Reports are in preparation in three categories:

1. Research reports designed to explore, study and integrate isolated and/or conflicting concepts and methods dealing with life prediction,
2. Reports to introduce and summarize the state-of-the-art concepts and methods in particular areas, and
3. Example problems and solutions intended to illustrate the use of these concepts in decision making.

H. T. Corten
Principal Investigator

SUMMARY

Transient and steady state stress-strain hysteresis behavior of several structural metals is analyzed. The study shows that a stress parameter, defined as the "Yield Range Increment," uniquely denotes the various transient phenomena including cyclic hardening, softening, relaxation and creep and the steady state cyclic stress-strain behavior. All transient and steady state hysteresis branches of a given material appear to be identical in shape, after their "Yield Range Increments" which are suitable portions of the initial "elastic" parts are deleted. A mathematical model incorporating the "Yield Range Increment" is proposed. With the determination of the functional relationship between the newly proposed parameter and the several input variables of cyclic loading, this approach should lead to a simple and unified model for describing the cyclic stress-strain response of materials.

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INTRODUCTION

Present formulations to describe the cycle dependent phenomena of transient hardening, softening, relaxation and creep schematically illustrated in Fig. 1, essentially use three independent concepts. Considering a constant strain cycling situation as shown in Fig. 1 (a) and (b), the stress amplitude, in general, either increases or decreases with cycles depending on the material, its initial heat treatment, and sometimes on the strain amplitude. This cycle dependent hardening or softening is transient and leads to a stable response much before failure occurs by fatigue. Such a behavior is visualized as being due to a cycle dependent change in the stress-strain relationship of the material, thus necessitating the stress-strain parameters as functions of cycles. This concept, together with the well known Masing's (1) hypothesis that the hysteresis curve is geometrically similar to the stress-strain curve but magnified by a factor of two, aid in the description of hysteresis behavior. Some structural metals including steels considerably depart from Masing's hypothesis (2) and, therefore, we shall distinguish the various materials as "Masing" and "Non-Masing" materials. While cycle dependent mean stress relaxation is denoted by a decrease in absolute mean stress with cycles under constant strain cycling situations as shown in Fig. 1 (c), cycle dependent creep is manifested as an increase in the absolute mean strain with cycles under constant stress cycling situations as illustrated in Fig. 1 (d). In one approach for describing this relaxation and creep, the tangent modulus is considered as a function of the absolute stress level (3) and in another approach these two phenomena are empirically treated as functions of mean stress, strain amplitude and cycles (2). While there is some physical basis for the concepts used for hardening or softening and hysteresis response (Masing's hypothesis), the basis for relaxation and creep is essentially mathematical. However, these approaches have been necessitated by a lack of complete understanding of the material stress-strain response under cyclic situations.

A few interesting phenomenological observations made recently, suggest a parameter which governs not only all the transient phenomena including cyclic hardening, softening, relaxation and creep but also is capable of accommodating both "Masing" and "Non-Masing" materials. This report presents these observations and an initial contribution towards the development of a unified approach for the description of cyclic stress-strain response of structural metals.

STABLE CYCLIC STRESS-STRAIN RESPONSE

Whereas during the transient hardening, softening, relaxation or creep period, the hysteresis loops are not closed, (see Fig. 1) stable conditions are associated with closed loops with their branches being "mirror images" of each other and hence identical in shape. Halford and Morrow (4, 5) observed that for structural metals, the cyclic stress-strain curve defined as the locus of the tips of stable fully reversed hysteresis loops of various amplitudes, when magnified by a scale factor of two, adequately described the stable hysteresis loop shape. This shows that Masing's hypothesis is a good approximation for the stable cyclic stress-strain response of those metals. It is interesting to note that the commonly used rheological model consisting of springs and frictional sliders, such as the one shown in Fig. 2, also leads to the same relationship between the monotonic and the hysteresis curve as Masing's postulation.

A previous investigation (2) showed that certain metals, notably some soft steels, considerably depart from Masing's hypothesis. For example, in Fig. 3 fully reversed stabilized hysteresis loops of Man-Ten steel* are superimposed with their lower tips coinciding with the origin. While tracing several loops the scatter in the linear parts was too small to be reproduced. Throughout this study, whenever various hysteresis branches are compared by superposition, zones of negligible scatter are shown by a single trace. Referring to Fig. 3 if the material followed Masing's postulation, the upper branches of these loops should have coincided, which is clearly not the case. In this case the cyclic stress-strain curve magnified by a scale factor of two does not accurately represent the loop shape, as it describes the locus of upper tips of loops

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*Sources of material data presented in this report are listed in Tables 1-3.

as shown in the figure. On the other hand, similar plots for 2024-T4 and 7075-T6 aluminum shown in Fig. 4 and 5 demonstrate that Masing's postulation is adequate for those metals. For materials such as Man-Ten steel, which depart from Masing's postulation, it was shown (2) that the hysteresis branches could be accurately matched by suitable translation of these branches along the initial elastic slope as illustrated for Man-Ten steel in Fig. 6. Similar plots for SAE 1018 normalized steel, A-36 steel and OFHC copper shown in Figs. 7 to 9 support these observations. While Figs. 3 to 9 illustrate a study of the upper branches, similar comparisons for the lower branches are also possible.

These observations show that the various sized stable hysteresis branches are identical in shape after suitable portions of their "elastic" ranges are deducted. We shall designate this parameter as the "Yield Range Increment," (YRI) because it is a direct measure of the change in the yield stress. For all practical purposes of mathematical modeling, this increment can be assumed to be a linearly elastic part and readily expressed in stress or strain units. For materials which obey Masing's postulation, the YRI is constant and independent of the size of the hysteresis loop (strain or stress amplitude). But for those materials which depart from Masing's postulation (as illustrated earlier), the YRI is a function of the hysteresis loop size.

It will be seen, in what follows, that the newly defined YRI parameter also plays an important unifying role in the description of cycle dependent hardening, softening, relaxation and creep.

CYCLIC HARDENING AND SOFTENING BEHAVIOR

Available data on four metals namely, Ausformed H-11 steel, a quenched and tempered SAE 1045 steel, 2024-T4 aluminum alloy and normalized SAE 1018 mild steel are analyzed. At the outset it is necessary to define certain terms used in analyzing the transient hysteresis loops. A given hysteresis loop has two branches, the upper one corresponding to the tensile loading direction and the lower one corresponding to the compressive direction. As the hysteresis loop is not closed under transient conditions, the two branches are not identical. Therefore, each of these branches is designated as a "Reversal" and the cyclic progress is denoted in terms of the number of reversals. The first reversal is defined as the unloading branch immediately following the static (or monotonic) loading at the beginning of a test. As a rule, the first static loading path is treated separately from the rest of the cyclic paths, and no reversal number is assigned to it.

The analysis involved here essentially consists of comparing the various branches of hysteresis plots by physical superposition. In comparing the hysteresis plots obtained on different graph papers, it is necessary to orient the papers such that their coordinate axes are parallel to each other. By doing so, it was found in most cases there was a slight mismatching of the elastic slopes of the hysteresis loops. In such cases a slight relative rotation of the coordinate axes provided a distinctly better match between different branches. This is illustrated for the data on Aus H-11 steel in Figs. 10 and 11. Assuming that the scale calibrations did not change between different recordings, there are two possible causes. One is that the Young's modulus 'E', varied with cycles and the size of the loop. The second cause could be due to the original slight mismatch between the coordinate axes on different papers. As the graph paper is manually set in the X-Y plotter and is held in position during plotting by an "air suction" or an "electro-static" condition slight misorientation of the observed order of magnitude (about few percent) between different settings is possible, unless one followed extremely careful setting up procedure. This was not a critical factor during the past tests as one was essentially interested in measuring the overall heights and widths of the hysteresis loops. On the other hand, there was also evidence of the variation in the measured 'E' in one case which will be discussed later. With this ambiguity in view, hysteresis branches of the same graph paper were matched by pure translation and those of the different papers were matched by both translation and slight rotation if necessary.

Observations of Hysteresis Loops

In Figs. 10 and 11, the upper hysteresis branches of Aus. H-11 steel corresponding to three strain amplitudes and each at two transient stages (shown by their reversal numbers) are matched by translation (along the elastic slope) and translation and slight rotation respectively. The material slightly hardens or is approximately stable at all the three strain levels. Consistent with this, the change in the YRI is not significant. The actual magnitudes of these increments are influenced by the matching method namely, pure translation or translation and rotation. The former method should be used if 'E' is assumed to be constant. The latter approach is used if variations in 'E' are also considered. The amount of rotation in this case determines the degree of variation in 'E'. Figure 12 shows that the lower branches can also be similarly matched. However, the match in the case of lower branches is distinctly inferior compared to that of the upper branches in Fig. 11. The reason becomes clearer in Fig. 13, where both the upper and lower branches are matched, by rotating the lower branches through 180 degrees. It can be seen that the branches corresponding to the first reversal depart from the rest of the branches. As all the three tests were started in tension, the first reversals happen to be the lower branches. It can be seen from Table 2 that this material is anisotropic in its initial condition as illustrated by the difference in its tensile and compressive yield strengths, as such it takes a few loadings and unloadings to "shake down" the material to a more isotropic condition. Therefore, it appears that the material is still closer to its "monotonic state" during the first reversal and thus the first reversal does not quite fall in line with subsequent reversals. A slight relative rotation appears to improve the matching of the first reversal branches with the subsequent ones, thus indicating a change in the measured value of 'E' between the monotonic and the cyclic state. Even if the Young's modulus is assumed to be constant, the matching of the transient loop shapes by only translation is still a good approximation.

Considerably better trends were observed in the case of SAE 1045 steel whose hysteresis branches are compared in Figs. 14-16. The lower branches in Fig. 15 match almost as well as the upper branches in Fig. 14, even though they include the first reversals. Figure 15 compares both the upper and the lower branches. This material cyclically softens and consistently the YRI decreases with reversals, an approximate trend of which is shown in Fig. 17. To obtain data for Fig. 17, the loop with the least elastic portion is used as reference, and its YRI is assigned a zero value. All other increments are measured relative to this reference.

Of the four strain amplitude data available on 2024-T4 aluminum, the three lower strain amplitude tests were started in tension and the larger strain amplitude test was started in compression, thus the first reversal in one case was an upper branch and in the other case was a lower branch. In Fig. 18, the upper branches are matched. The plot also includes a steady state loop corresponding to the larger amplitude showing that after a suitable deduction of YRI from each of the upper branches, the rest of the portions are essentially identical in shape and could be expressed by a single function. The material cyclically hardens at all strain levels and consistent with this the YRI also increases. As the material obeys Masing's postulation in its steady state (Fig. 4), the YRI reaches a constant steady state value independent of the size of the loop.

Available data on the normalized SAE 1018 mild steel were from non-fully reversed strain control tests, in which each test was started in tension as shown in Fig. 19. In a previous investigation (2) it was shown that the cyclic hardening or softening behavior of this material was not affected by the presence of these constant mean strains during the test. Therefore hysteresis loops of these tests will be compared without regard for the mean strains present. In Fig. 20 two plots of the transient hysteresis loops are shown. In one plot the lower tips are made coincident, and in the other upper branches are matched. It can be seen that the material cyclically softens at the two lower strain amplitudes and hardens at the higher strain amplitude. Consistent with this the YRI decreases with reversals at the two lower amplitudes and increases with reversals at the higher amplitude. In spite of the mixed behavior, the upper branches are well matched.

In general, the cyclic hardening or the softening behavior is therefore quantitatively denoted by the increase or decrease of the YRI respectively, while the rest of the hysteresis branches remain identical in shape. Depending on whether a material obeys Masing's postulation or not, the YRI approaches a stable value, which is respectively independent of or dependent on the size of the hysteresis loop. However, during the transient period the YRI is a function of the stress or strain amplitude and a measure of cyclic duration such as number of reversals. The potentiality of describing the cyclic hardening or softening behavior of metals by means of single parameter, namely the YRI is now evident after it has been consistently demonstrated for four different metals with quite different deformation characteristics (see Table 2).

CYCLE DEPENDENT RELAXATION AND CREEP BEHAVIOR

The available data on cyclic relaxation and creep is limited. Figure 21 (a) shows hysteresis loops obtained from typical cyclic relaxation and creep tests on normalized SAE 1018 steel which had a prior cyclic strain history. From a first look at the loops either in relaxation or creep, it appears that the loop shapes remain constant, suggesting that cycle dependent shifts in the loops may be caused by changes in the initial linear parts of consecutive reversals. A comparison by means of physical superposition of consecutive reversals shown in Fig. 21 (b), supports this observation, although the changes in the linear portions are appreciably small. A similar observation for the data on a SAE 1045 steel is shown in Fig. 22 (a) and (b).

In Fig. 23 (a) hysteresis loops from a relaxation test on normalized SAE 1018 steel are presented. After stabilizing the material with fully reversed strain cycles of large amplitude, mean stress is induced by stepping down to a lower strain amplitude as shown by sequence A-B-C in the figure. The hysteresis loop denoted by sequence D-E-F is that resulting after mean stress relaxation had virtually ceased. Thus the transient loops during relaxation which lie between sequence A-B-C and D-E-F are omitted in this figure. It is of interest to observe that during relaxation the upper loop tip has shifted down more than the lower tip, suggesting that the relaxation of mean stress was accompanied by cycle dependent softening (i.e., decrease in the stress range). Although at first sight the hysteresis branches appear to change shape in relaxation, Fig. 23 (b) which compares all the four branches shows that after an appropriate deduction of the YRI, the remaining non-linear portions of all the branches are almost identical in shape. The YRI of the branch AB is greater than that of BC, whereas those of DE and EF are equal, thus denoting mean stress relaxation during the event ABC and stable behavior during DEF. The reduction in YRI between the lower branches AB and DE or the upper branches BC and EF denotes softening.

In the case of 7075-T6 aluminum cyclic relaxation is accompanied by hardening (increase in stress range) as shown in Fig. 24 (a) which shows relaxation at two strain amplitudes. The shift in the upper tips is relatively small compared to that of the lower tips. The plot shows an apparent change in loop shape with relaxation. But when the various branches are compared by physical superposition and matched by translation along the initial "elastic" slope, as shown in Fig. 24 (b), it becomes clear that it is the YRI that varies, while the remaining portions of all the hysteresis branches are almost identical in shape. While relaxation is denoted by a decrease in the YRI between two consecutive reversals of a given cycle, the increase in its value with cycles, as compared between similar reversals (upper or lower branches) denotes hardening.

The phenomenon of transient hardening or softening associated with relaxation observed in Figs. 23 and 24 being an inherent material feature, prevents a study of pure relaxation behavior. It always occurs during step changes in strain or stress amplitude irrespective of whether the material has been previously stabilized by cyclic loading or not. Present formulations to simulate cyclic stress-strain response ignore this feature and assume that step changes in say, strain amplitude are accompanied by corresponding step changes in stress amplitude responses by the material (2, 3, 6). Consequently, these models continuously harden or soften to stabilization at a rate defined by a cumulative parameter such as the cumulative plastic strain (summed without regarding the sign) or number of cycles and cease to harden or soften once stabilization is reached.

Mechanics of Relaxation and Creep

Typical schematic diagrams of hysteresis loops in relaxation and creep are presented in Fig. 25. Considering the case of relaxation first, two possible variables governing relaxation appear. Firstly, as long as there is a mean stress and an accompanied relaxation, the plastic strain ranges of two consecutive reversals are different.

For the tensile mean stress situation in Fig. 25, the plastic strain range of an upper branch is always larger than its succeeding lower branch. In the case of a compressive mean stress, it is the other way. Thus, the YRI of a current branch could be assumed as a function of the plastic strain range of the previous branch and a relaxation model achieved. Unfortunately, it might be possible to show that relaxation occurs even under plastic strain control cycling, where the plastic strain range between loading and the unloading reversals remains the same. However, to the author's best knowledge such a test has not been done. A more logical variable appears to be the absolute value of the initial peak stress at each reversal. For example, as long as there is a tensile mean stress, the YRI of a current reversal being a function of the initial stress level (magnitude), will be larger in the lower branch than in the following upper branch which results in reversal by reversal relaxation until the stress limits are fully reversed. Also the rate of relaxation decreases continuously as the difference between the upper and lower stress limits decreases.

In the case of cyclic creep also both plastic strain range and absolute magnitude of peak stress are possible governing variables. In the case of tensile creep as shown in Fig. 25, the YRI is larger in the lower branch than in the following upper branch, thus causing the upper branch to be more nonlinear than the lower branch, which results in reversal by reversal increase in maximum, minimum and mean strain.

A cyclic creep test may lead to one of two situations. Either the creep rate continuously decreases thus reaching a stabilized state, or the creep continues indefinitely finally terminating in tensile fracture. An additional variable such as a peak strain or mean strain appears to be necessary to identify and simulate cyclic creep leading to stabilization or to a creep rupture situation.

Thus, in general, cycle dependent relaxation or creep can be denoted by the differences in the YRI between consecutive reversals. These differences decrease to zero as stabilization is reached. Admittedly, these conclusions are based on limited data and, therefore, may need further verification in terms of more materials tests.

PROPOSED MODEL

The foregoing observations and discussion lead to the following postulations:

1. All transient or steady state hysteresis branches of a given metal subjected to constant amplitude (stress or strain) cycling are identical in shape after deletion in each case of an appropriate initial "elastic" portion defined as the "Yield Range Increment."
2. The "Yield Range Increment," a history dependent stress parameter, by its variation or constancy quantitatively denotes the cycle dependent transient phenomena of hardening, softening, creep and relaxation or the steady state hysteresis behavior respectively.
3. The portions of hysteresis branches identical in shape are termed "Basic Hysteresis Curves" and their shape is independent of cyclic history at constant amplitudes. Therefore, they can be described by history independent material parameters.

It must be noted that the foregoing postulations do not describe the relationship between the monotonic stress-strain curve and the hysteresis branch as the Masing's postulation does, but consider the relationships between the hysteresis branches under constant amplitude transient and steady state cyclic situations. As mentioned earlier, the stress-strain relationships between the monotonic and the cyclic situations have to be treated separately. Additional formulation incorporating, "memory" effect (2) on cyclic stress-strain response under variable amplitude situations is also necessary. This subject will be treated in another forthcoming report.

Now considering the upper branch of a hysteresis loop in Fig. 26, let the portion 'oa' denote the "Yield Range Increment," and 'ab' the "Basic Hysteresis Curve." Let us further assume that the Basic Hysteresis Curve can be described by an expression commonly used for the current cyclic stress-strain curves after Morrow (5), as follows:

$$\Delta\epsilon_b = \frac{\Delta\sigma_b}{E} + \left(\frac{\Delta\sigma_b}{K}\right)^{1/n} \qquad (1)$$

where $\Delta\sigma_b$ and $\Delta\epsilon_b$ are basic parts of stress and strain ranges. As the total ranges $\Delta\sigma$ and $\Delta\epsilon$ are more readily measurable in a test, Eq. 1 can be modified as follows:

$$\Delta\epsilon_b = \frac{\Delta\sigma - \delta\sigma_y}{E} + \left(\frac{\Delta\sigma - \delta\sigma_y}{K}\right)^{1/n} \quad \text{for } \Delta\sigma > \delta\sigma_y \qquad (2)$$

where '$\delta\sigma_y$' is the Yield Range Increment. Assuming, $\delta\epsilon = \delta\sigma_y/E$, Eq. 2 reduces to

$$\Delta\epsilon = \frac{\Delta\sigma}{E} + \left(\frac{\Delta\sigma - \delta\sigma_y}{K}\right)^{1/n} \quad \text{for } \Delta\sigma \geq \delta\sigma_y$$

$$\text{and} \qquad \Delta\epsilon = \frac{\Delta\sigma}{E} \quad \text{for } \Delta\sigma \leq \delta\sigma_y \qquad (3)$$

Assuming that the Young's modulus E, remains constant during cyclic conditions, the only parameter that varies during the transient hardening, softening, relaxation and creep periods is the "Yield Range Increment," $\delta\sigma_y$ or $\delta\epsilon_y$. The parameters K and n which describe the "Basic Hysteresis Curve," remain unchanged throughout the transient and steady state periods and, therefore, can be defined as material parameters.

Based on past experience with the analysis of cyclic stress-strain behavior of metals, it appears that four variables including stress or strain amplitude, maximum stress, maximum strain and a measure of cyclic duration such as number of reversals may be sufficient to describe the history dependent YRI. Investigation is in progress to determine the functional form of its history dependence.

DISCUSSION

The basis for the newly proposed postulation of the hysteresis behavior is essentially phenomenological and, so far, no physical justification for the predominantly non-linear part of the hysteresis curve denoted as the "Basic Hysteresis Curve" to remain unchanged in shape during cyclic conditions is conceived. Therefore, the validity of using this approach should be ascertained for new materials before application. However, it appears logical that the various transient phenomena which essentially are different manifestations of strain hardening or softening mechanisms could be characterized by a parameter such as the YRI, which is a macroscopic (engineering) measure of hardening or softening. Physically, the YRI can be looked upon as a measure of the history dependent change in intrinsic resistance of the material to onset of macroscopic plastic deformation at the beginning of each stress or strain reversal.

Current formulations use expressions similar to Eq. 3, except that the parameter $\delta\sigma_y$ or $\delta\epsilon_y$ is equated to zero. In these formulations the hardening and softening is accomplished by varying both K and n. Cyclic relaxation or creep is accomplished by further changing the shape of the hysteresis curve through the tangent modulus thus necessitating at least one additional parameter. In spite of this multi-parameter approach, success in simulating cyclic stress-strain behavior is limited to materials which obey Masing's postulation. The YRI parameter, on the other hand, appears to account for all the transient phenomena including hardening, softening, relaxation and creep and successfully describe the hysteresis curves of materials irrespective of whether they do or do not obey Masing's postulation, and thus lead to a simpler and a unified approach.

CONCLUSIONS

The presented study of available data on several "different" structural metals leads to the new postulation that, all transient and steady state hysteresis branches of a given metal are identical in shape, provided appropriate portions of their initial "elastic" parts are deleted. Further, it appears that this elastic portion, defined as the "Yield Range Increment," by its variation or constancy, uniquely denotes the transient phenomena including hardening, softening, relaxation and creep or the steady state cyclic stress-strain behavior respectively, of both "Masing" and "Non-Masing" structural metals.

A mathematical model incorporating the "Yield Range Increment" parameter is proposed. Strain amplitude, absolute maximum stress, maximum strain and a suitable measure of the cyclic duration such as number of reversals may be initially considered as relevant variables governing the "Yield Range Increment" for seeking the necessary functional relationships. It is believed that these observations and guide lines will lead to a relatively simple and unified approach for describing the cyclic stress-strain behavior of structural metals.

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TABLE 1

MATERIAL AND DATA SOURCE FOR STABLE CYCLIC STRESS-STRAIN BEHAVIOR

No.	Material	Typical Mechanical Properties: Young's Modulus 10^3 ksi	Yield Strength ksi	Ultimate Strength ksi	Hardness BHN	Type of Test	Source of Loop Data if Unpublished	Reference
1	ASTM 440-63T Man-Ten Steel	30	46	82	150	Single specimen multi-block, fully reversed strain control	P. C. Rosenburger graduate research 1967-68	7
2	2024-T4 Aluminum	10	55	72	---	Multi-specimen, constant amplitude, fully reversed strain control	JoDean Morrow graduate course Spring, 1973	8
3	SAE 1018 steel normalized	30	33	60	80	Single specimen multi-block, fully reversed strain control	---	2
4	ASTM A-36 steel	30	40	65	---	"	F. B. Plummer, Jr. graduate research 1973	9
5	OFHC copper annealed	16	03	30	---	"	H. R. Jhansale graduate research 1970	2
6	7075-T6 aluminum	10	68	84	---	"	Present investigation	-

TABLE 2

MATERIAL AND DATA SOURCE FOR CYCLIC HARDENING OR SOFTENING

No.	Material	Typical Mechanical Properties: Young's Modulus 10^3 ksi	Yield Strength ksi	Ultimate Strength ksi	Hardness BHN	Type of Test	Deformation Behavior	Source of Loop Data if Unpublished	Reference
1	Aus. H-11 steel Ausformed	30	295 (T) 265 (C)	375	660	Fully reversed constant amplitude, strain control	Slightly hardens	J. E. Methany, Jr. graduate research 1968	10
2	SAE 1045 steel, quenched & Temp. (720°F)	30	185	195	390	"	Softens	R. W. Landgraf graduate research 1968	11
3	2024-T4 aluminum	10	55	72	---	"	Hardens	JoDean Morrow graduate course Spring, 1973	8
4	SAE 1018 steel normalized	30	33	60	80	Non-fully reversed constant amplitude strain control	Softens & hardens	H. R. Jhansale graduate research 1970	2

TABLE 3

MATERIAL AND DATA SOURCE FOR CYCLIC RELAXATION AND CYCLIC CREEP

No.	Material	Typical Mechanical Properties				Source of Loop Data if Unpublished	Reference
		Young's Modulus 10^3 ksi	Yield Strength ksi	Ultimate Strength ksi	Hardness BHN		
1	SAE 1045 steel, quenched and Temp. (1200°F)	30	92	105	225	---	11
2	SAE 1018 steel normalized	30	33	60	80	H. R. Jhansale graduate research 1970	2
3	7075-T6 aluminum	10	68	84	---	Present investigation	-

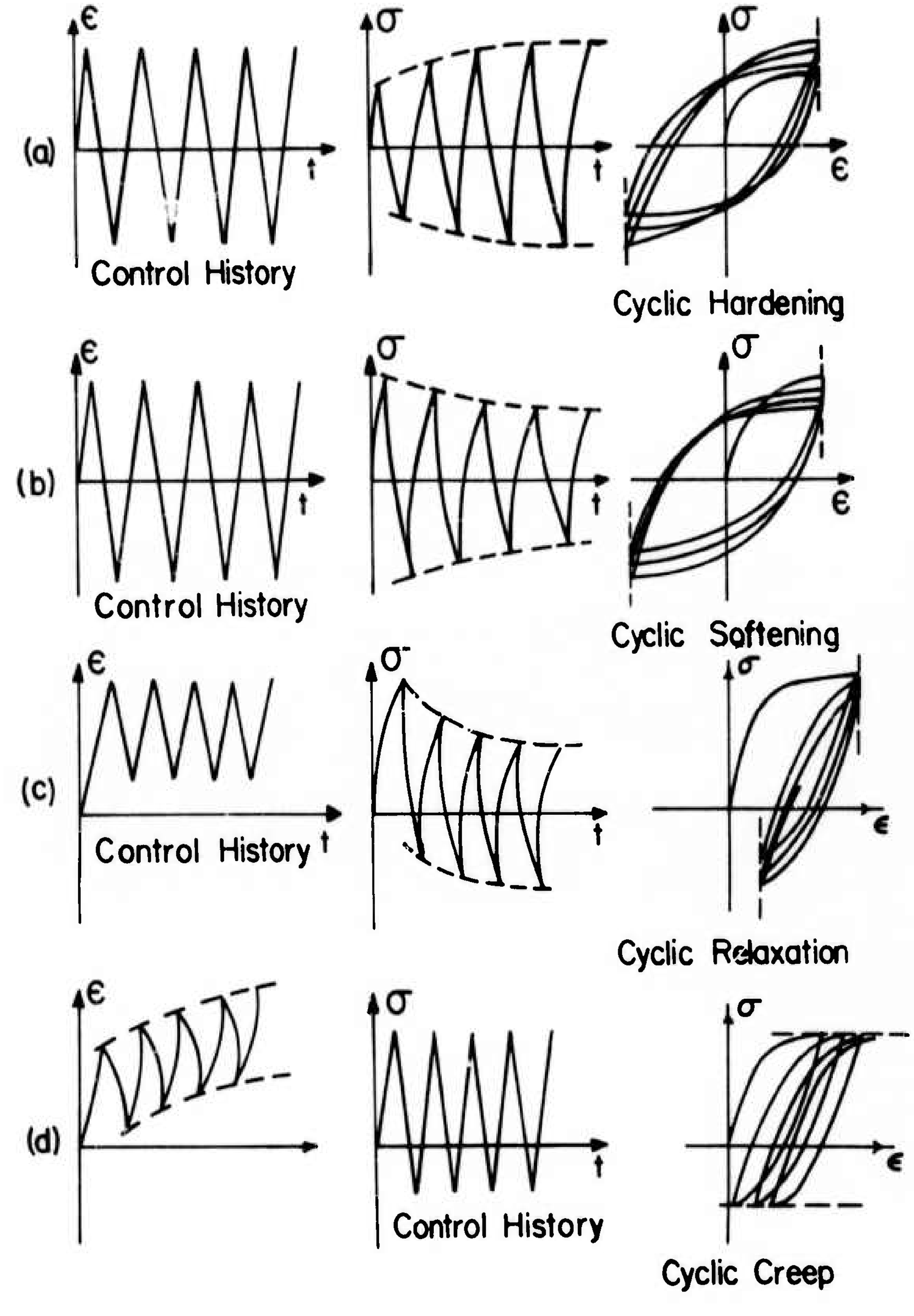


Fig. 1 Schematic Illustrations of Cyclic Transient Phenomena

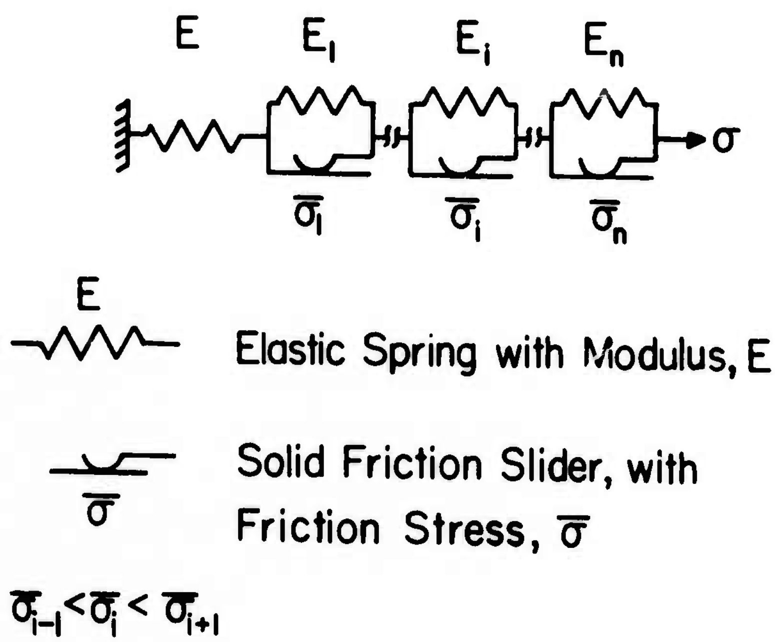


Fig. 2 Rheological Model Commonly Used to Describe Stress–Strain Behavior

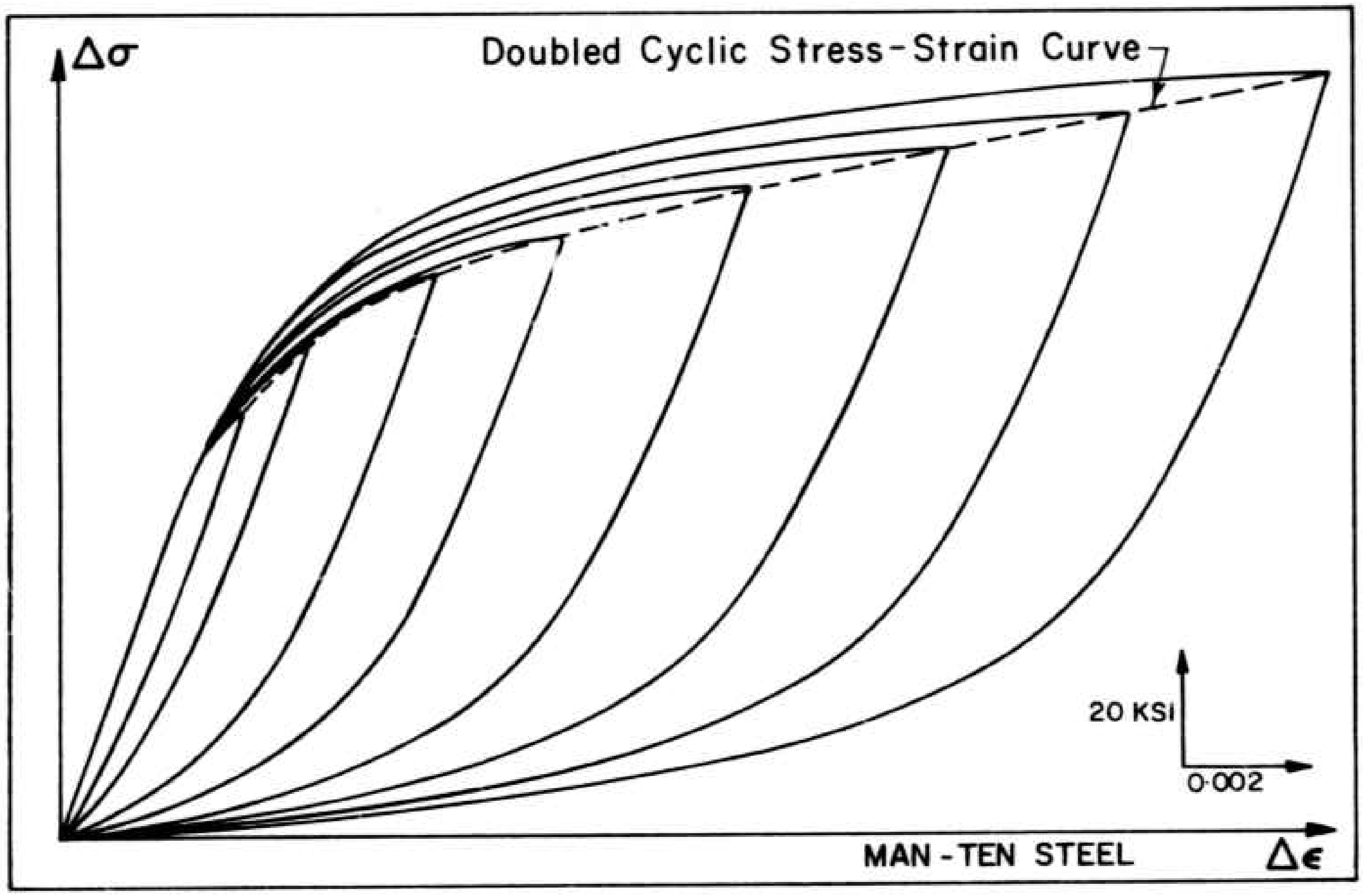


Fig. 3 Stable Hysteresis Loops of Man-Ten Steel with Matched Lower Tips

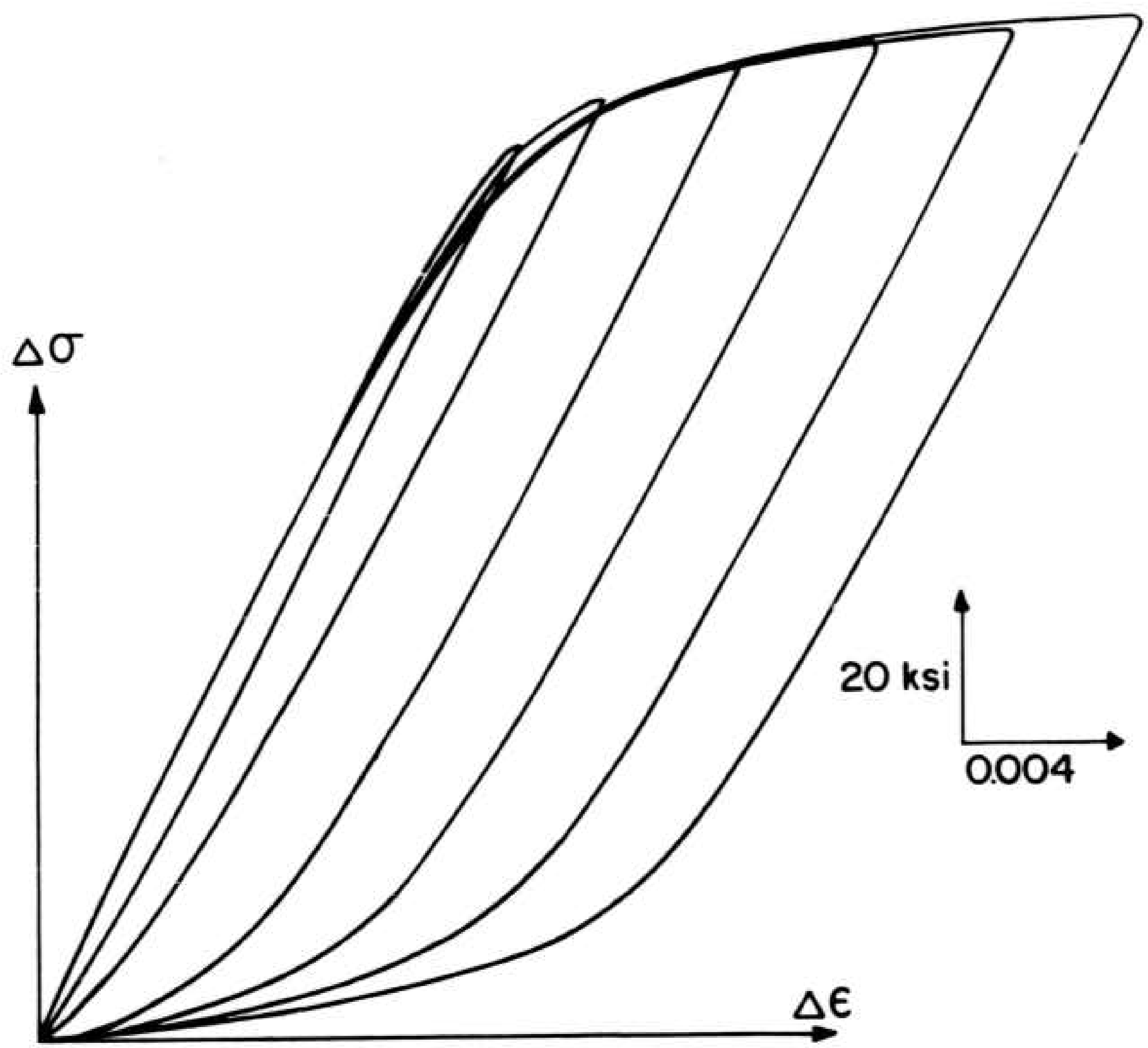


Fig. 4 Stable Hysteresis Loops of 2024-T4 Aluminum with Matched Lower Tips

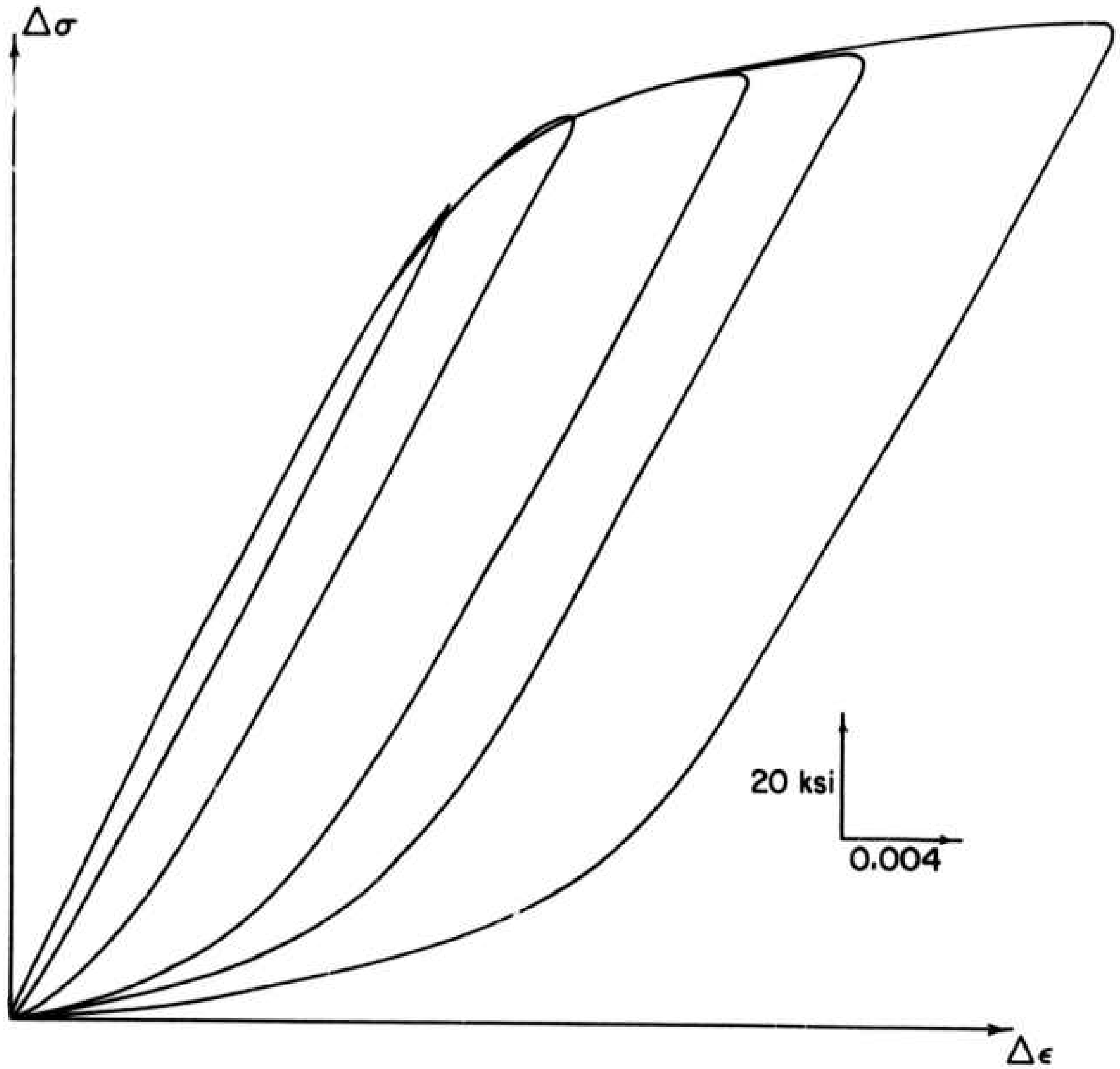


Fig. 5 Stable Hysteresis Loops of 7075-T6 Aluminum with Matched Lower Tips

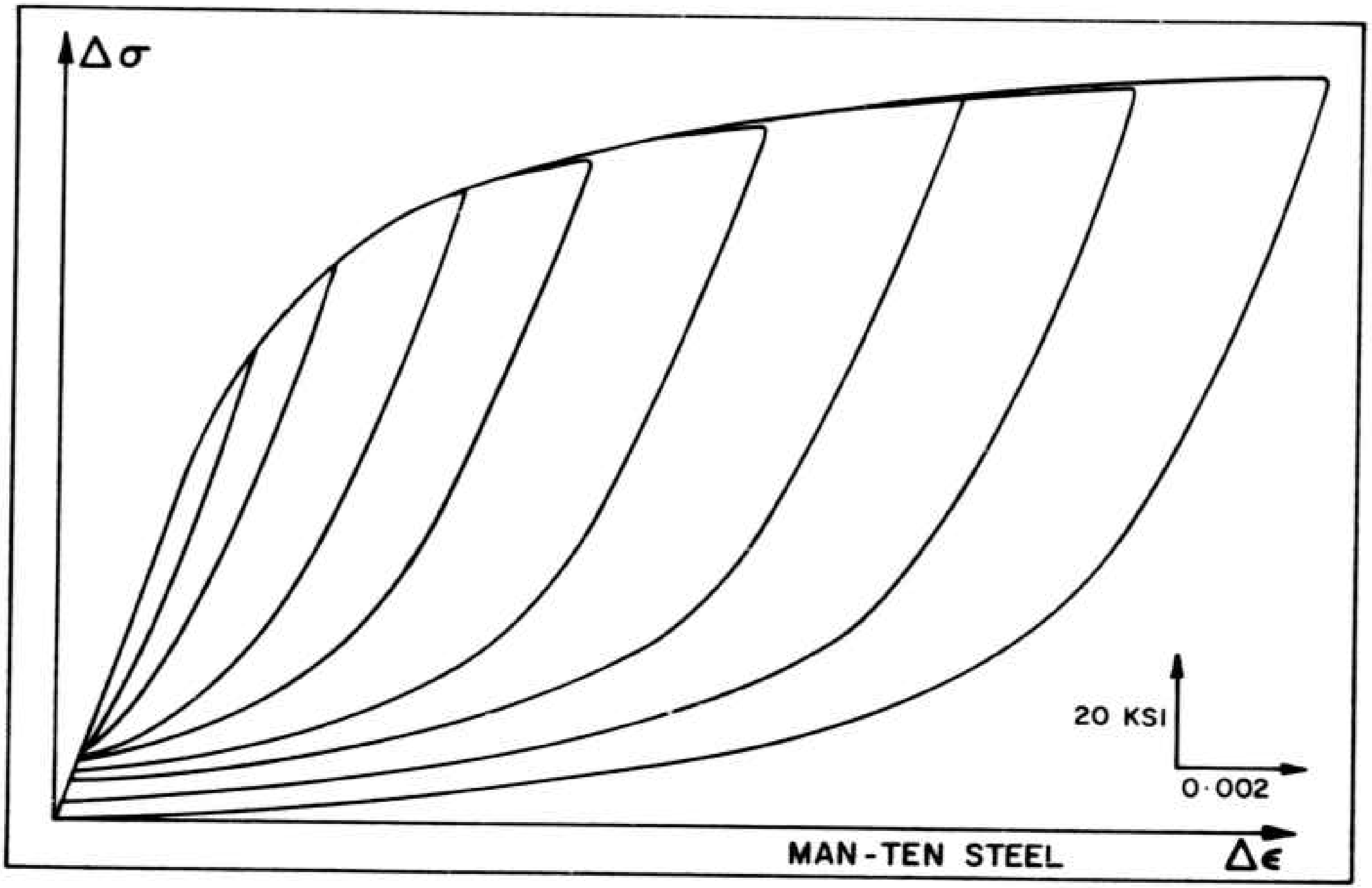


Fig. 6 Stable Hysteresis Loops of Man-Ten Steel with Matched Upper Branches

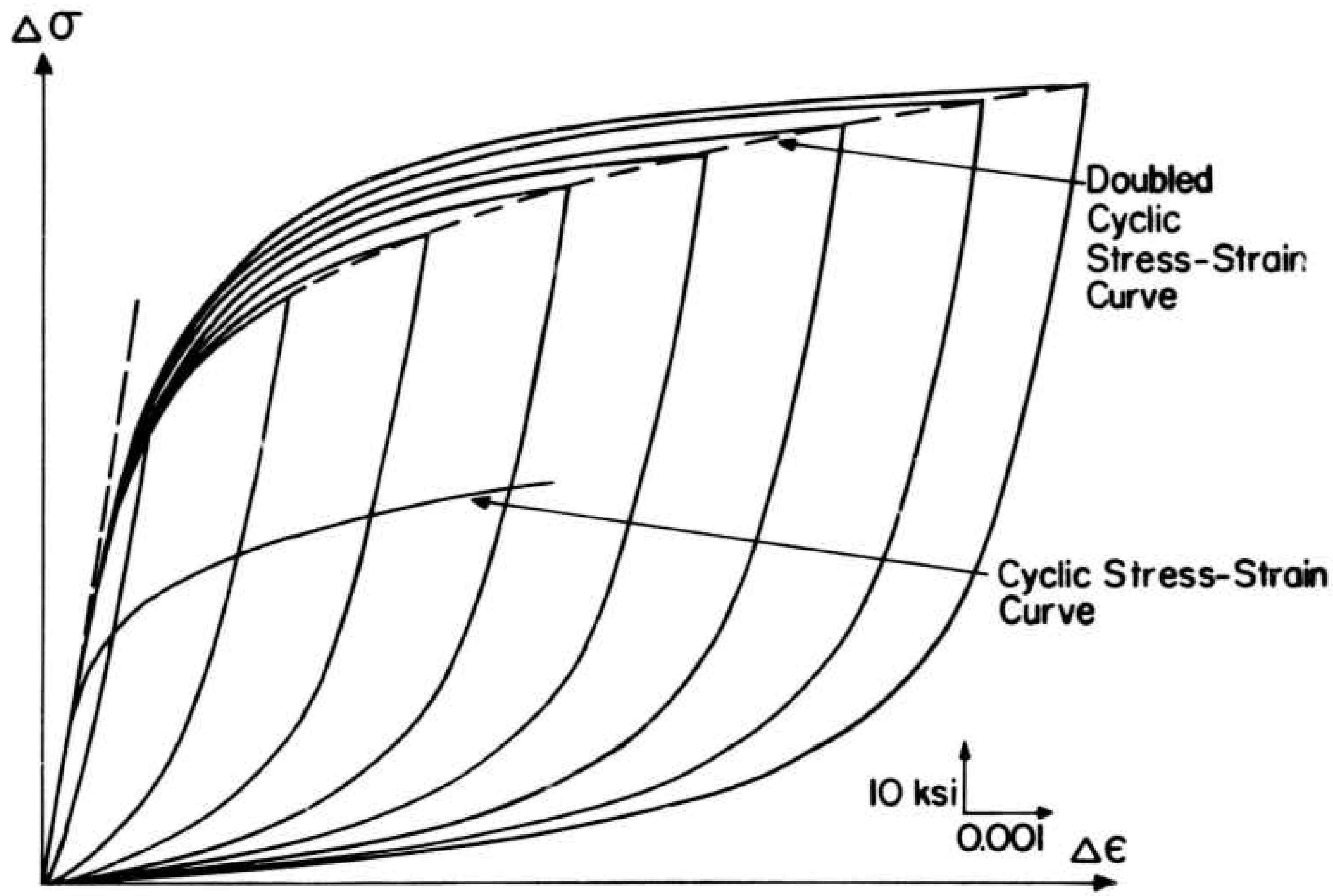


Fig. 7a Stable Hysteresis Loops of SAE 1018 Steel with Matched Lower Tips

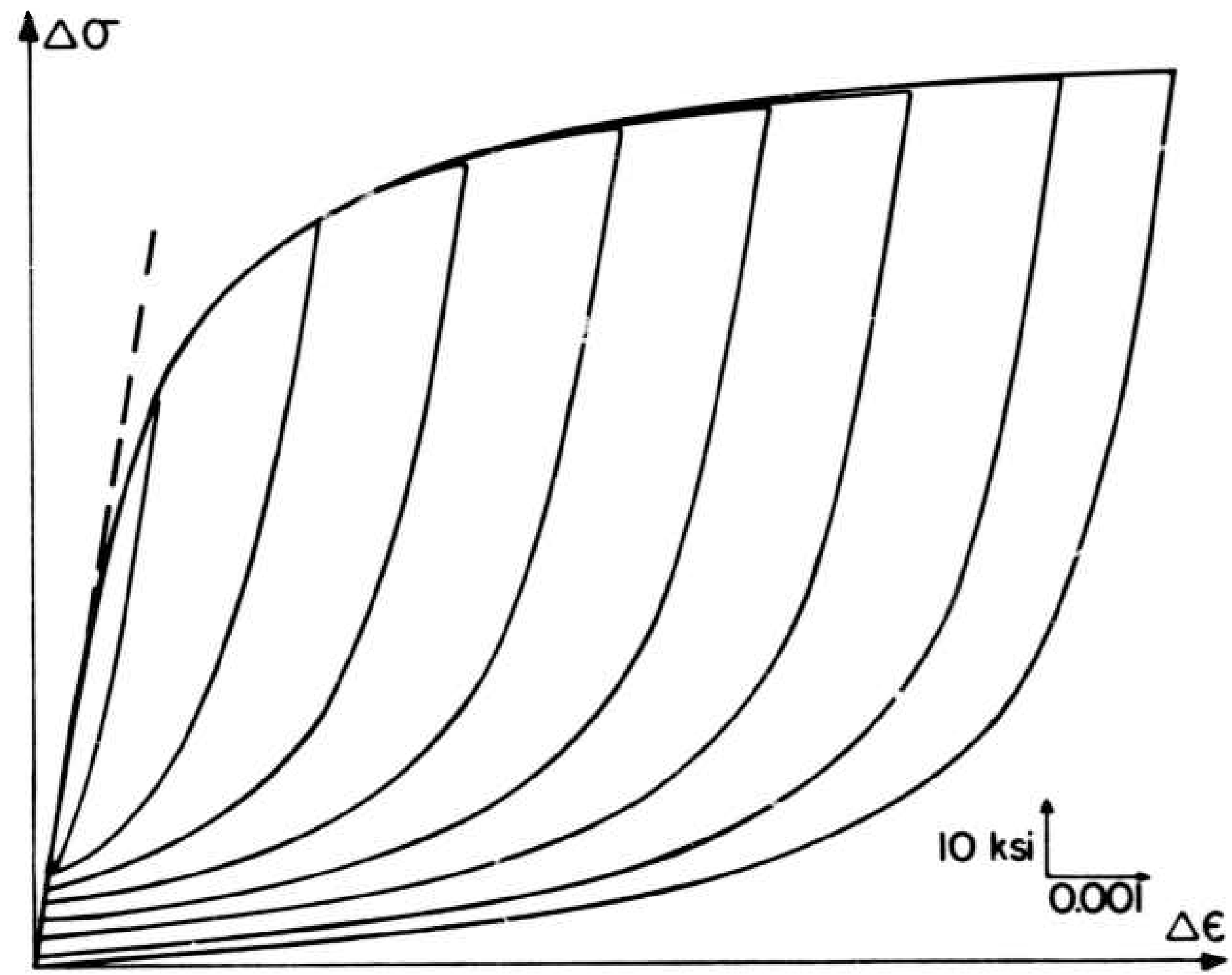


Fig. 7b Stable Hysteresis Loops of SAE 1018 Steel with Matched Upper Branches

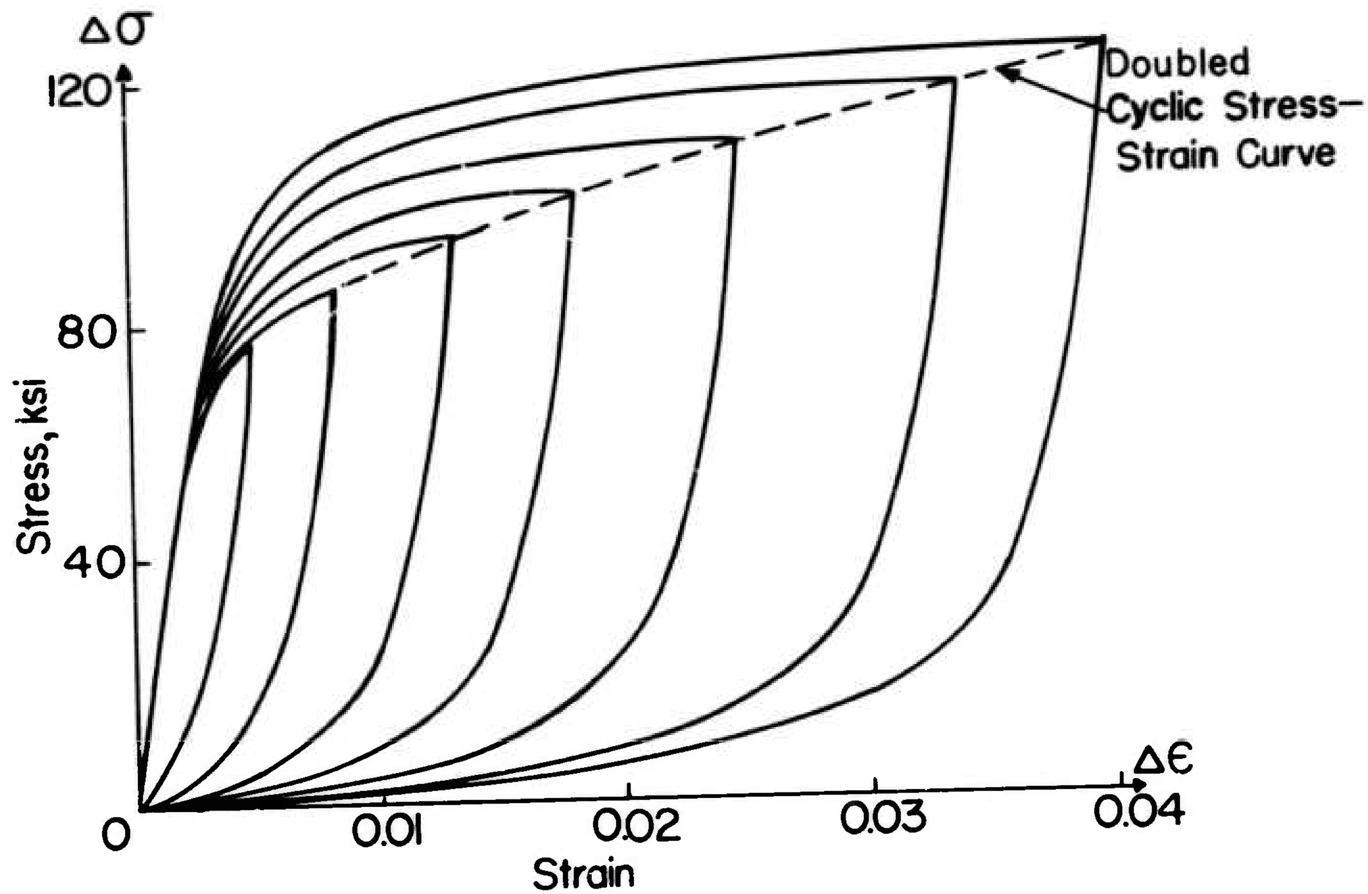


Fig. 8a Stable Hysteresis Loops of A-36 Steel with Matched Lower Tips

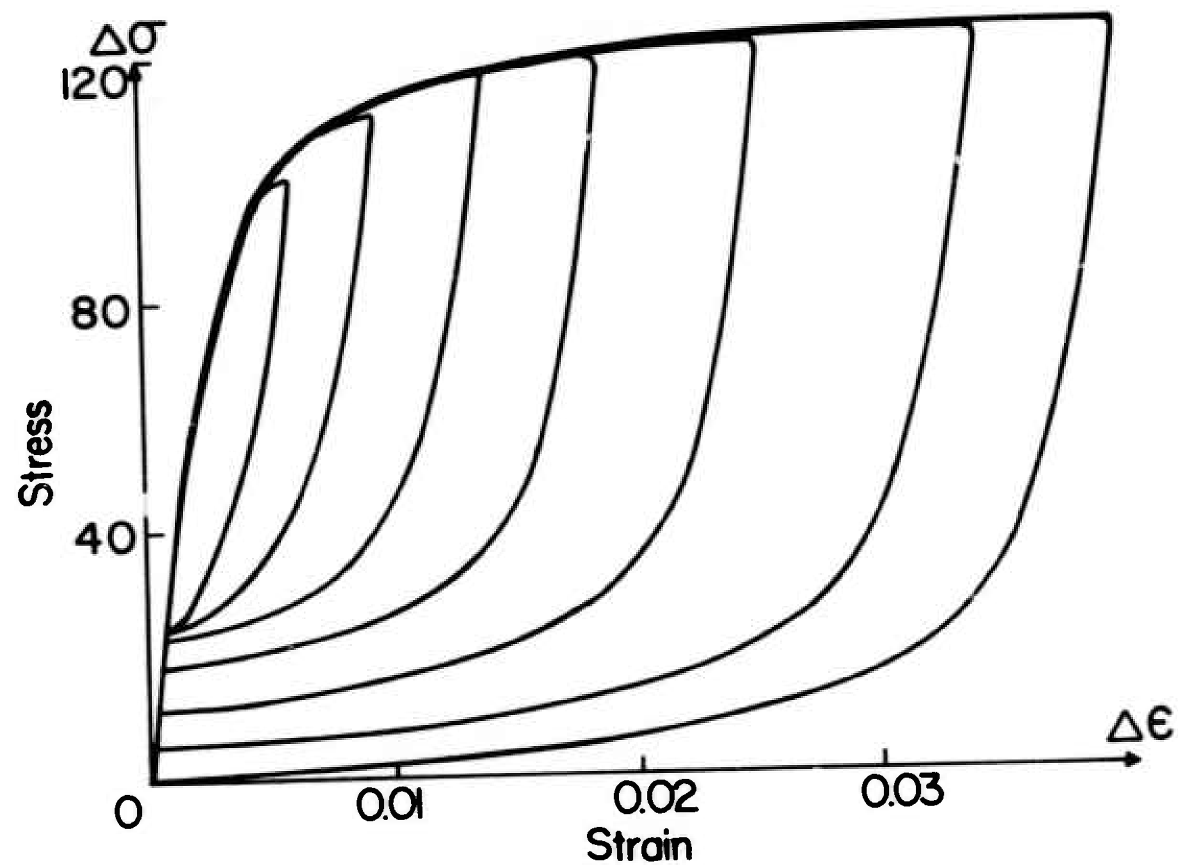


Fig. 8b Stable Hysteresis Loops of A-36 Steel with Matched Upper Branches.

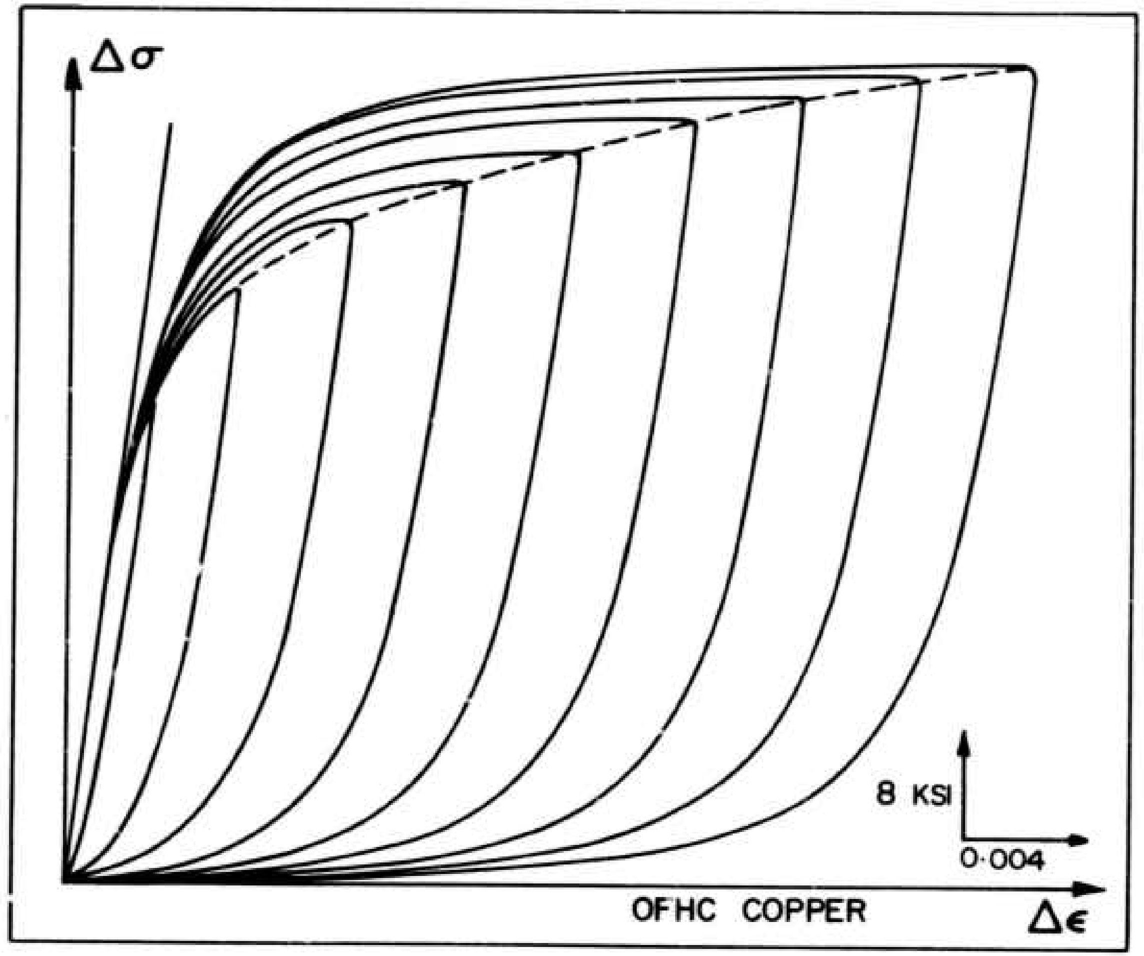


Fig. 9a Stable Hysteresis Loops of OFHC Copper with Matched Lower Tips

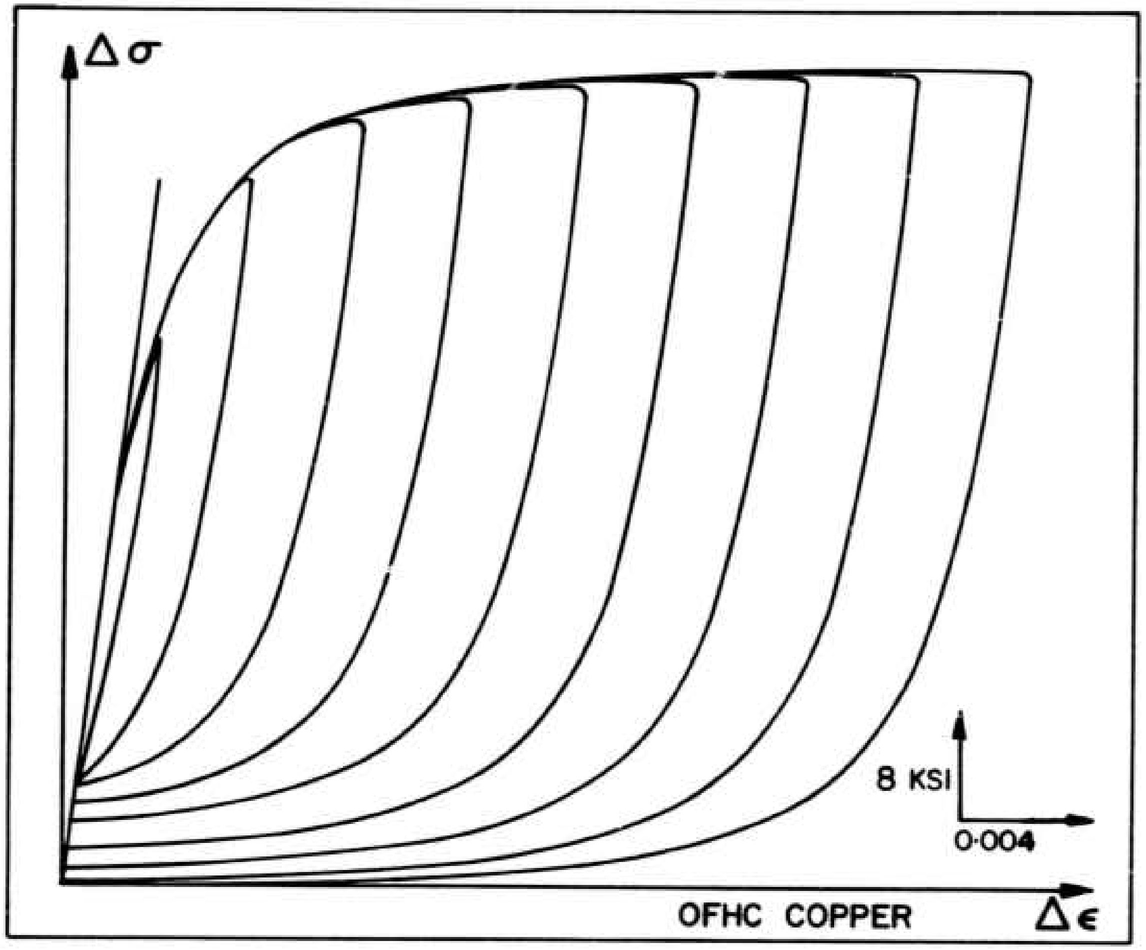


Fig. 9b Stable Hysteresis Loops of OFHC Copper with Matched Upper Branches

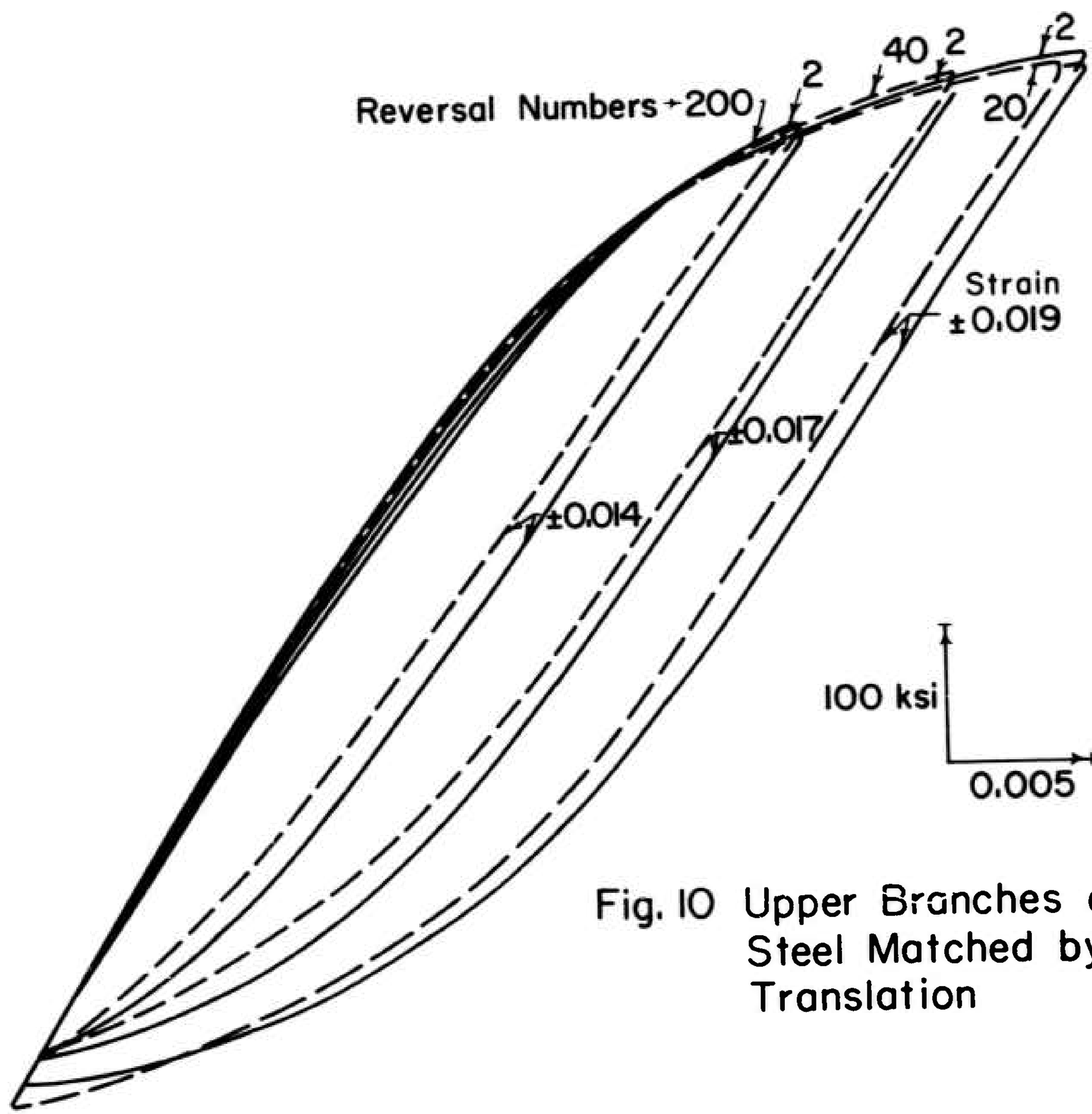


Fig. 10 Upper Branches of AUS. H-11 Steel Matched by Pure Translation

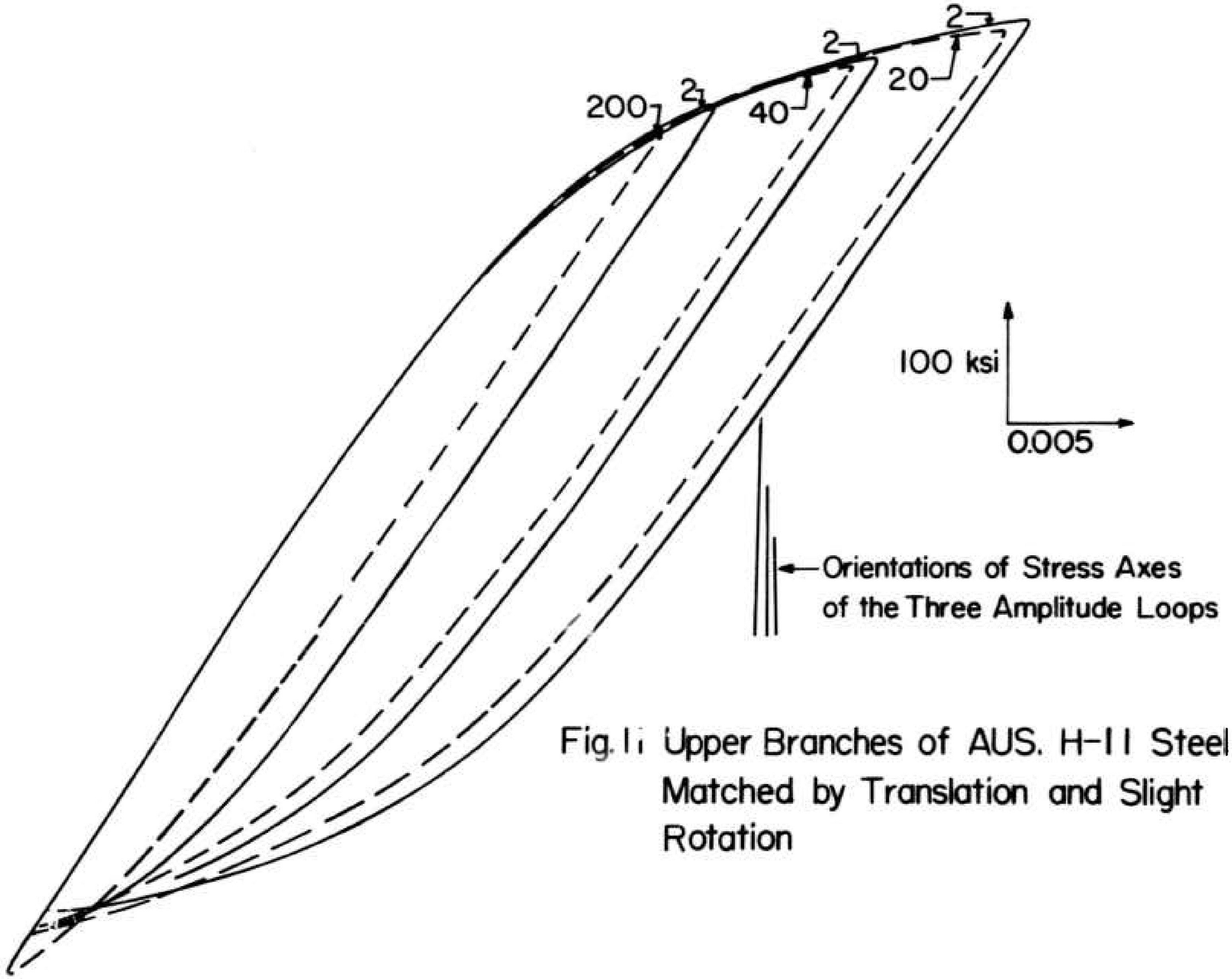


Fig. 11 Upper Branches of AUS. H-11 Steel Matched by Translation and Slight Rotation

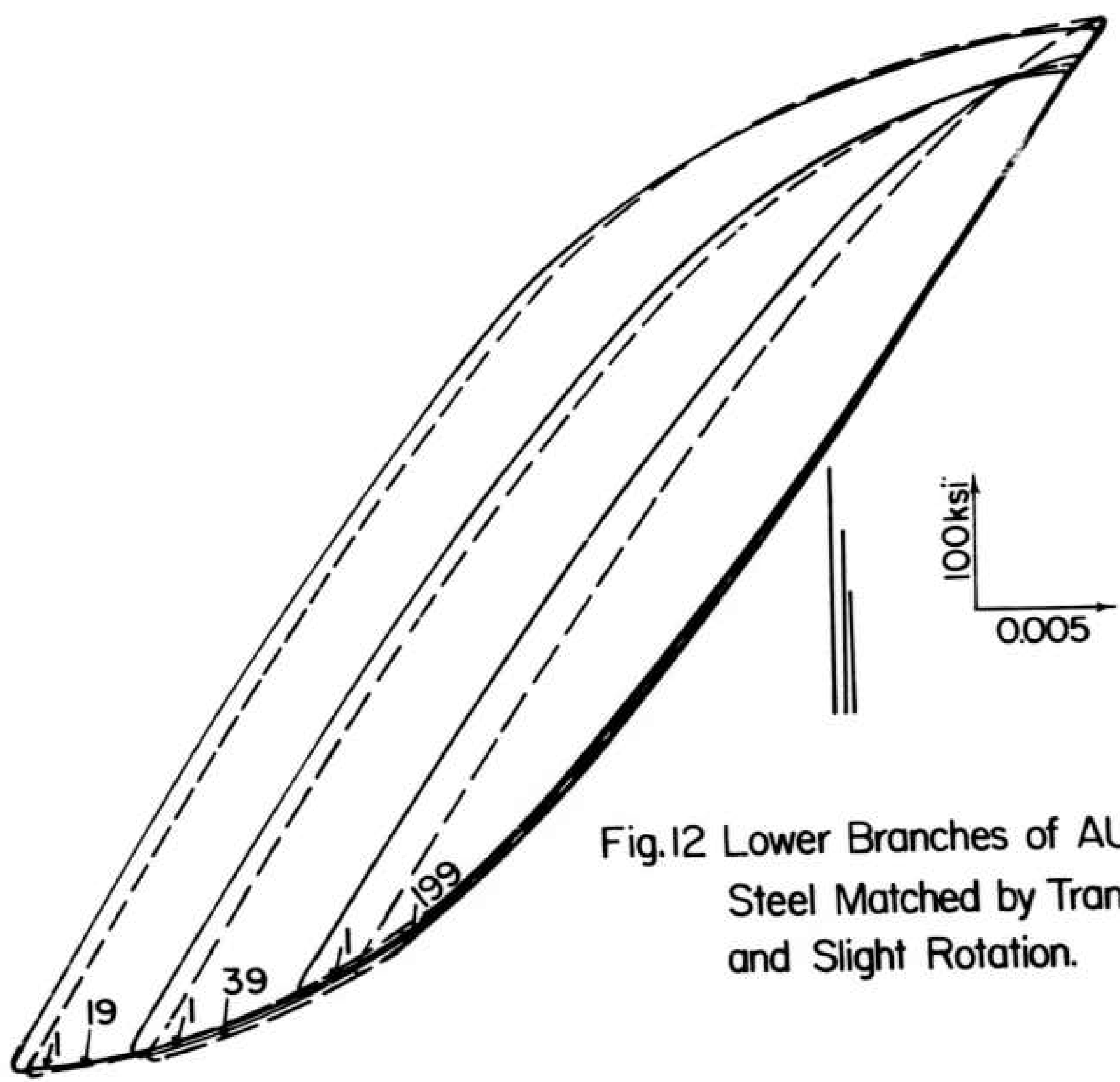


Fig. 12 Lower Branches of AUS. H-11 Steel Matched by Translation and Slight Rotation.

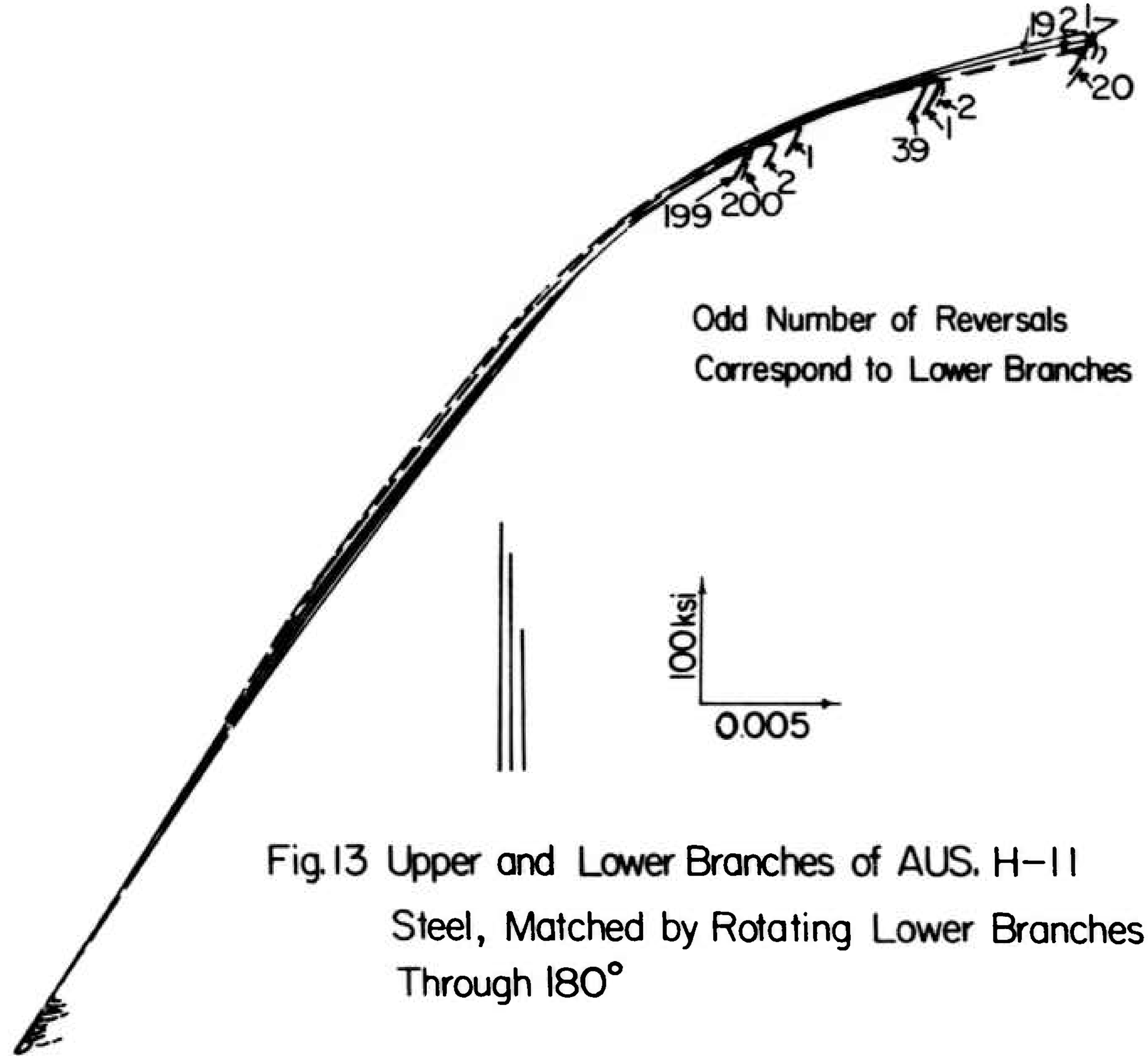


Fig. 13 Upper and Lower Branches of AUS. H-11 Steel, Matched by Rotating Lower Branches Through 180°

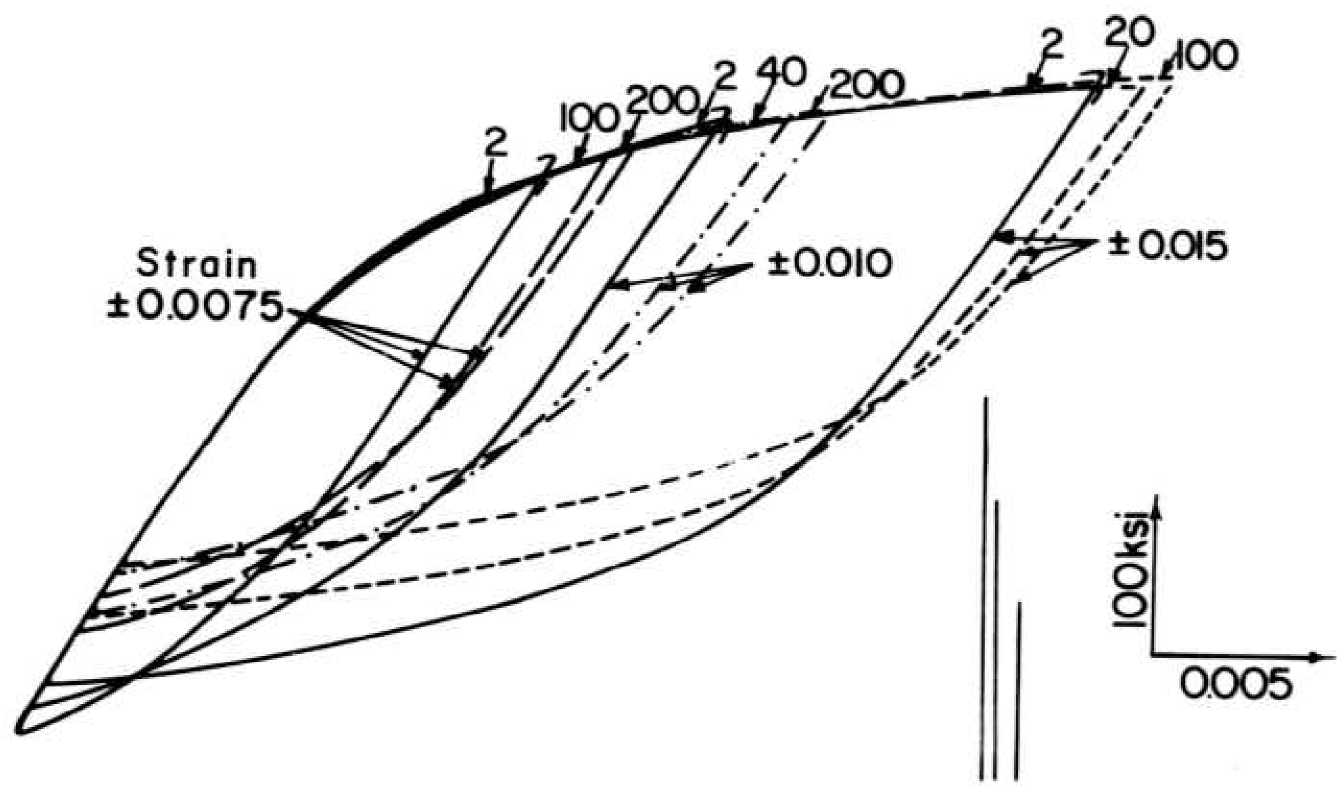


Fig. 14 Upper Branches of SAE 1045 Steel Matched by Translation and Slight Rotation.

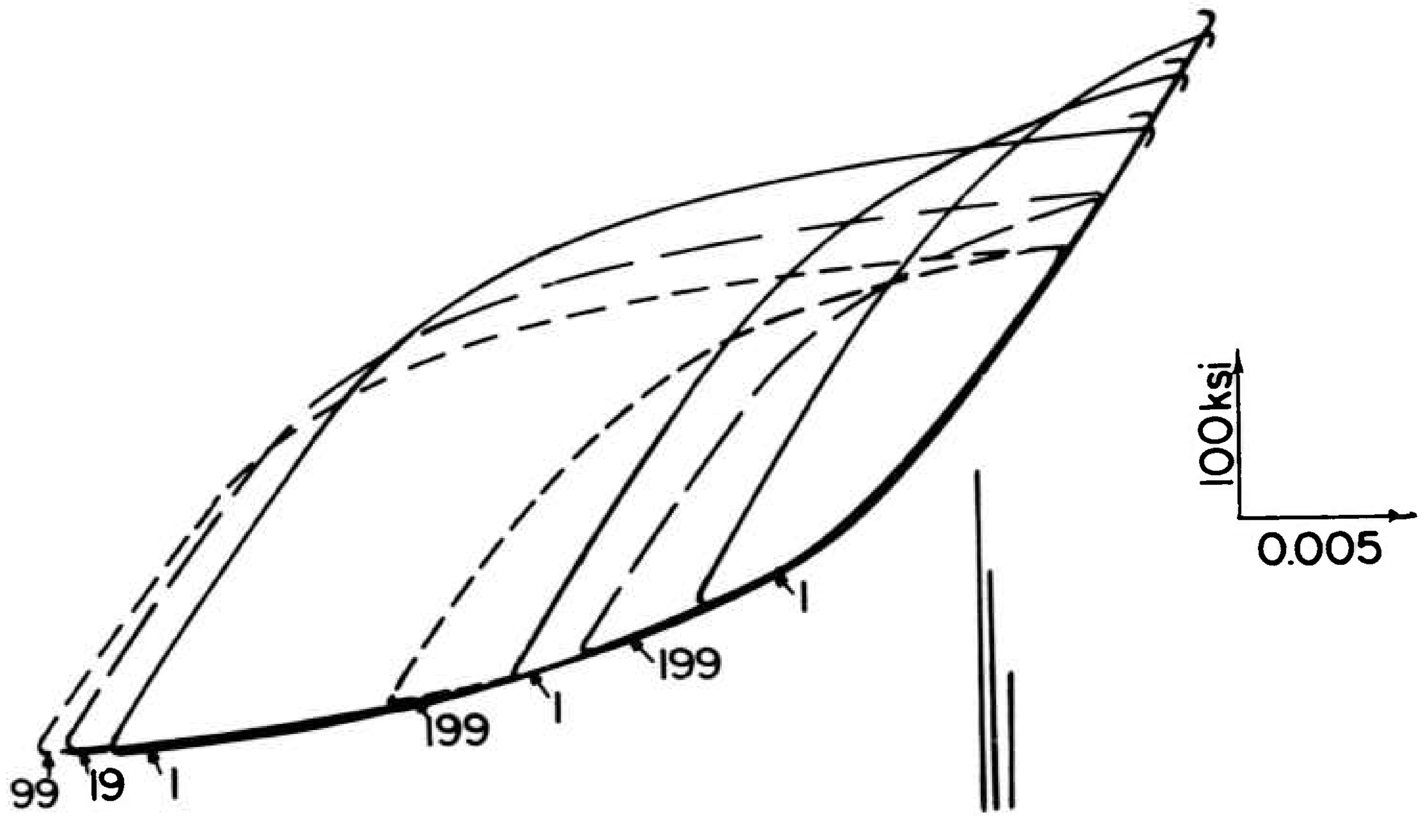


Fig.15 Lower Branches of SAE 1045 Steel Matched by Translation and Slight Rotation.

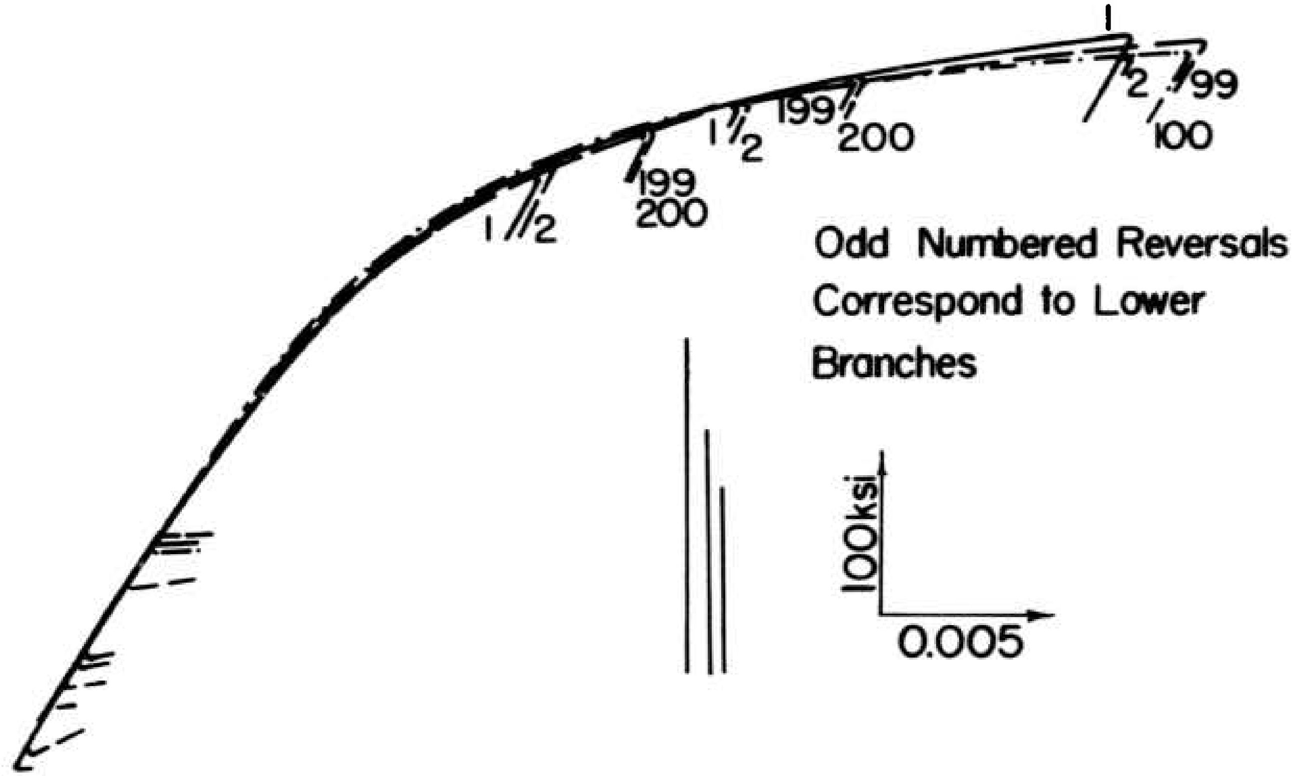


Fig. 16 Upper and Lower Branches of SAE 1045 Steel Matched by Rotating the Lower Branches by 180°.

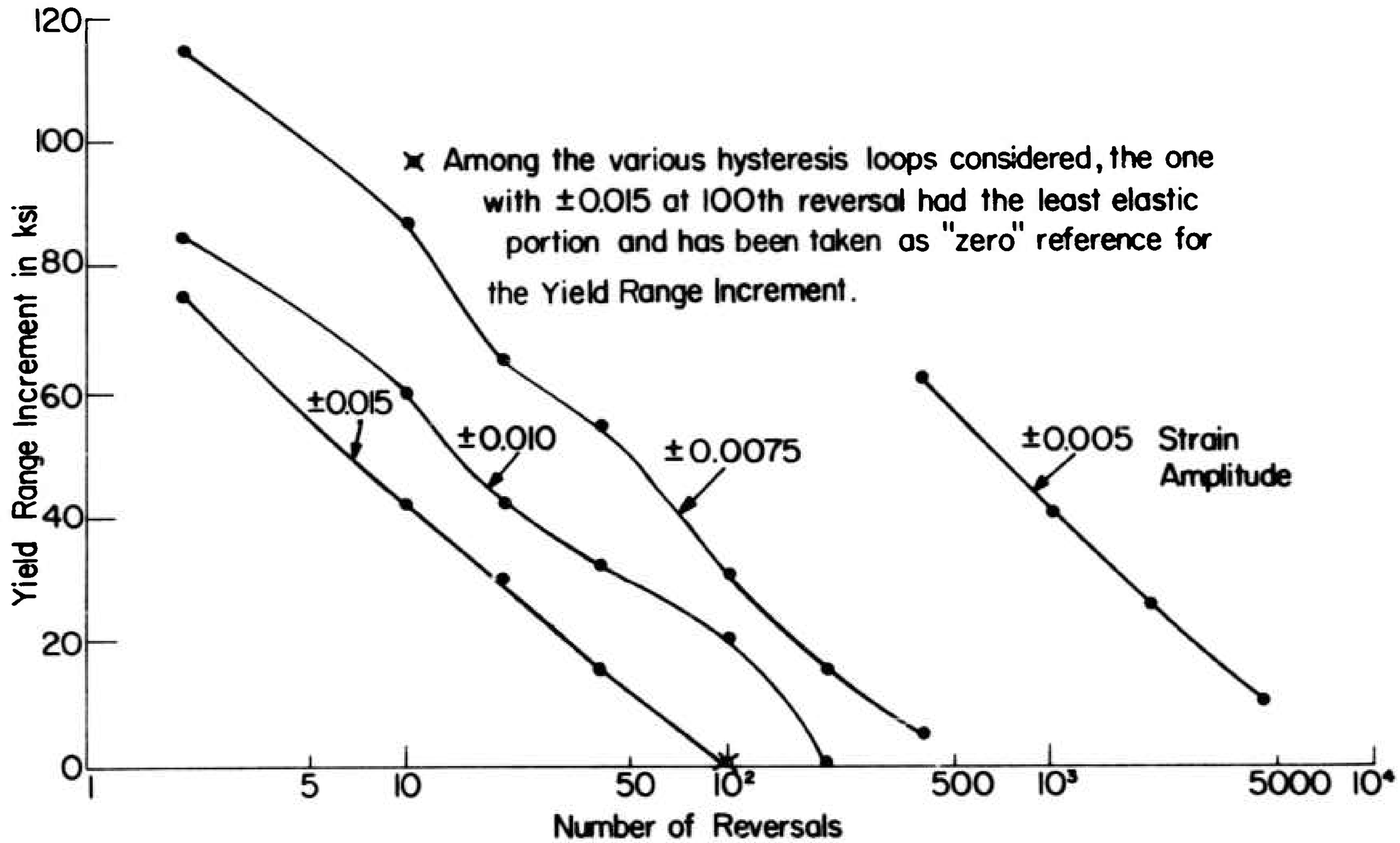


Fig.17 Variation of Yield Range Increment with Reversals for SAE 1045 Steel

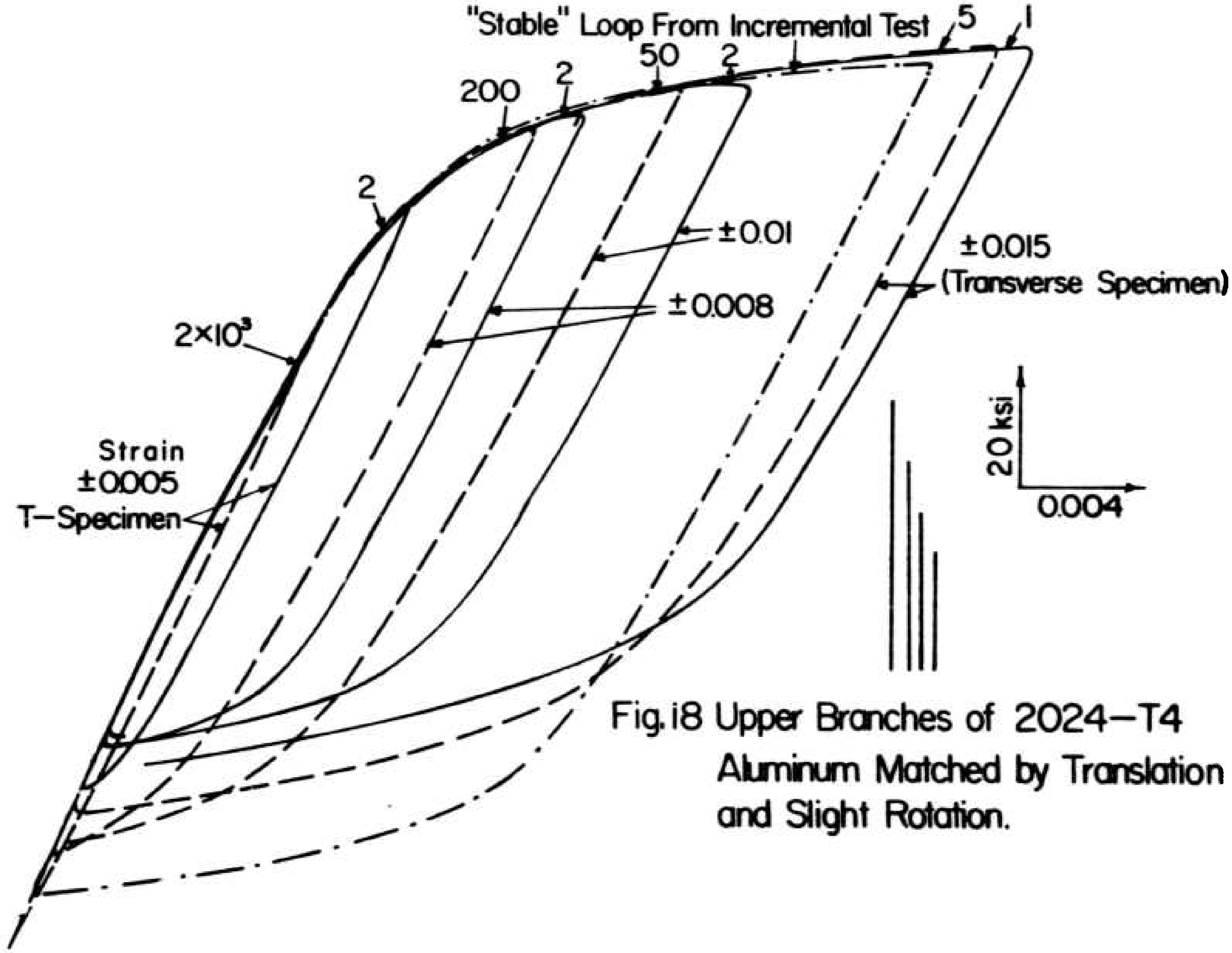


Fig. 18 Upper Branches of 2024–T4 Aluminum Matched by Translation and Slight Rotation.

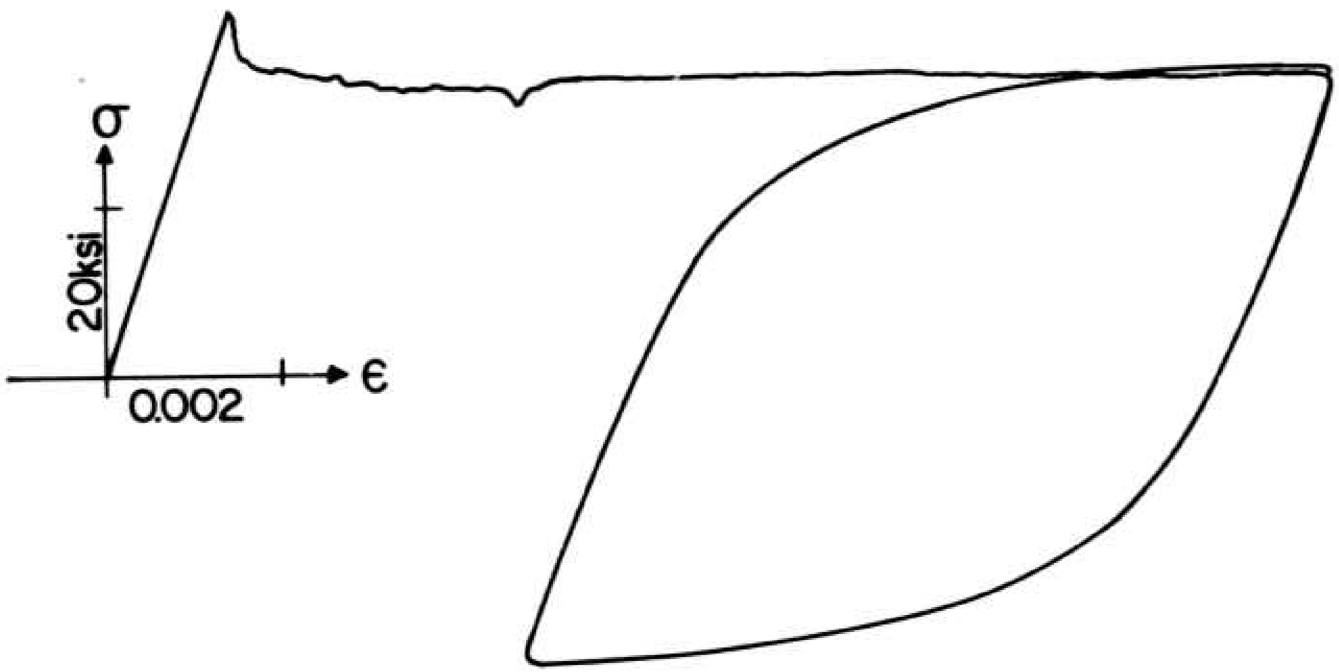


Fig. 19 Typical Start of a Non-Fully Reversed Strain Control Test of Normalized SAE 1018 Steel

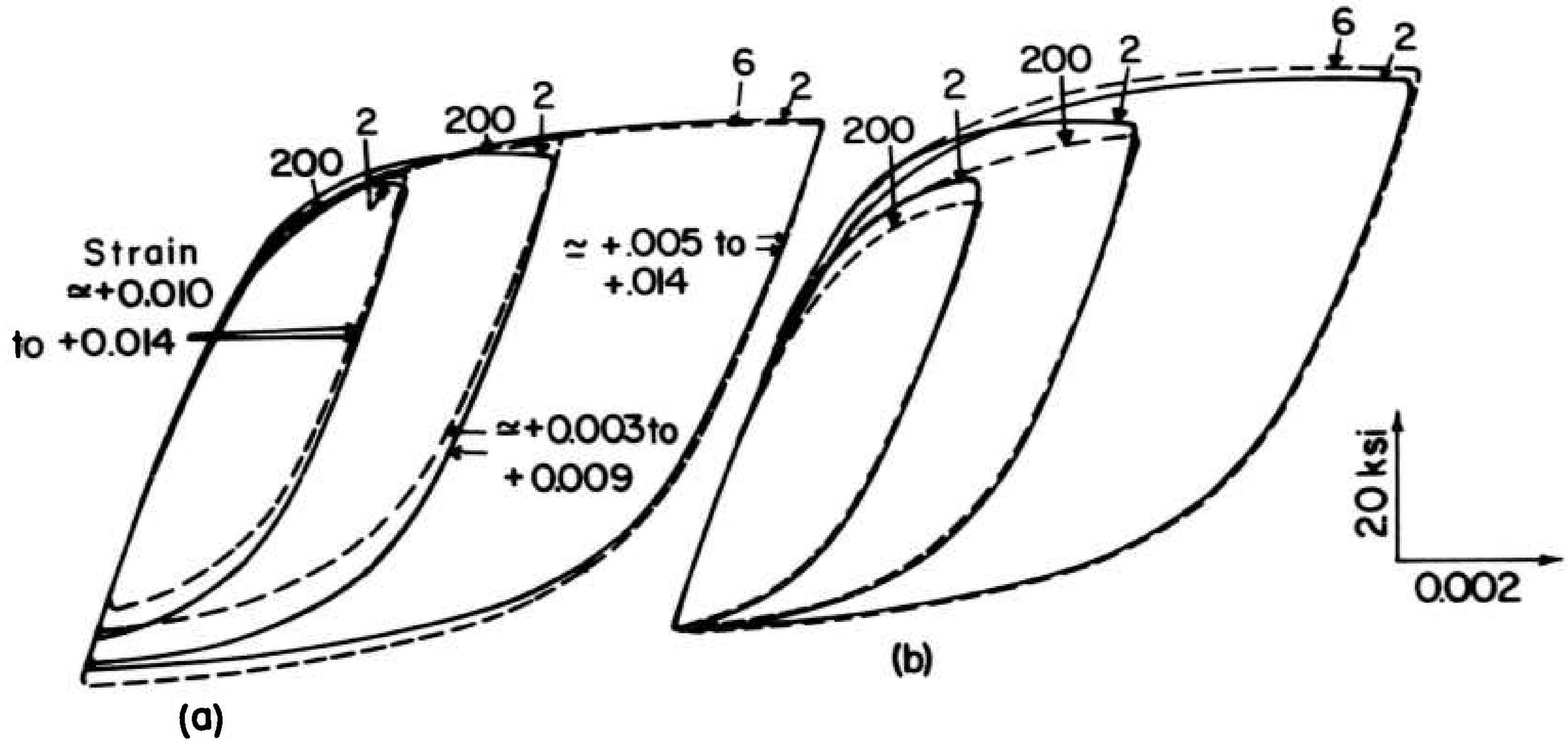


Fig. 20 Transient Hysteresis Loops of Normalized SAE 1018 Steel with (a) Matched Upper Branches and (b) Matched Lower Tips

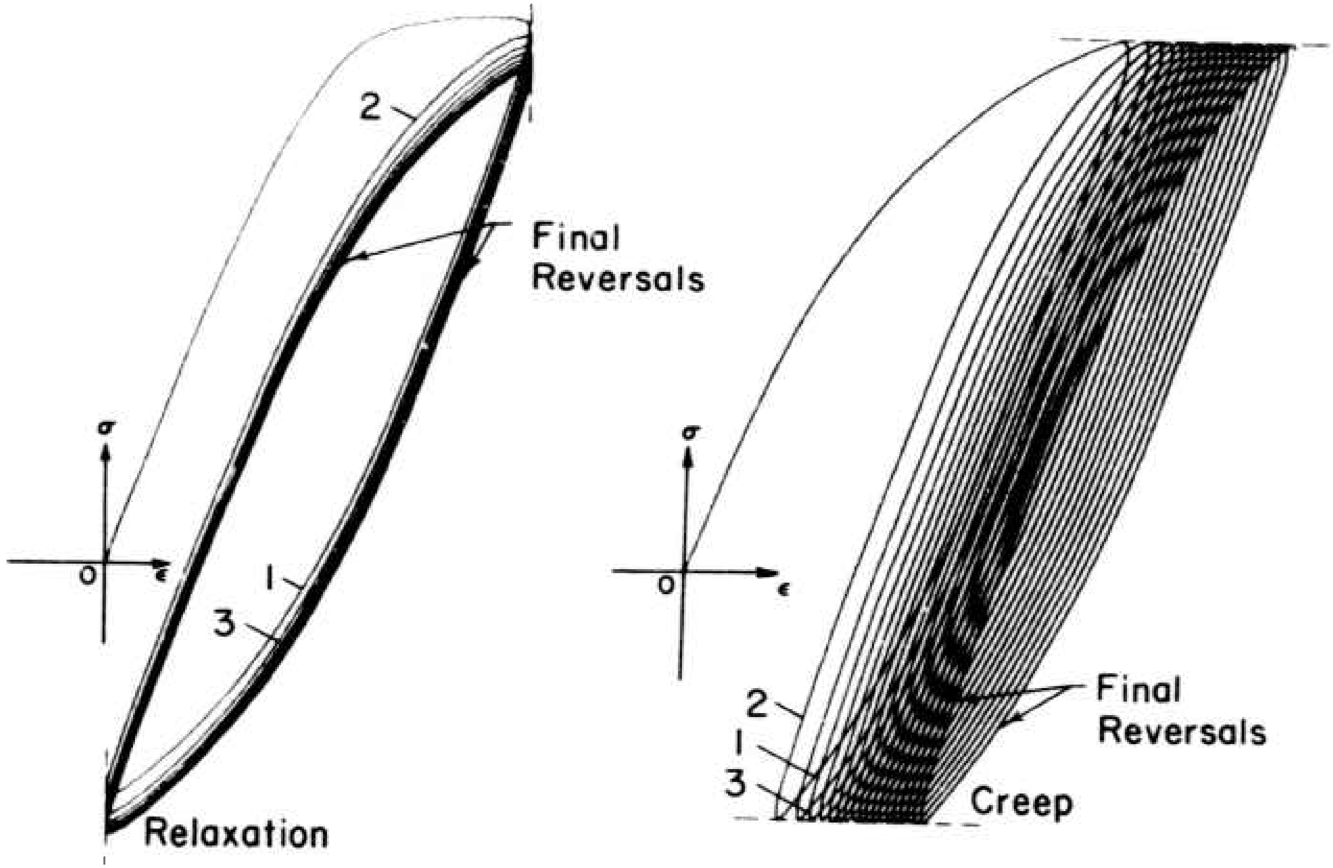


Fig. 21a Hysteretic Behavior of SAE 1018 Steel in Cyclic Relaxation and Cyclic Creep

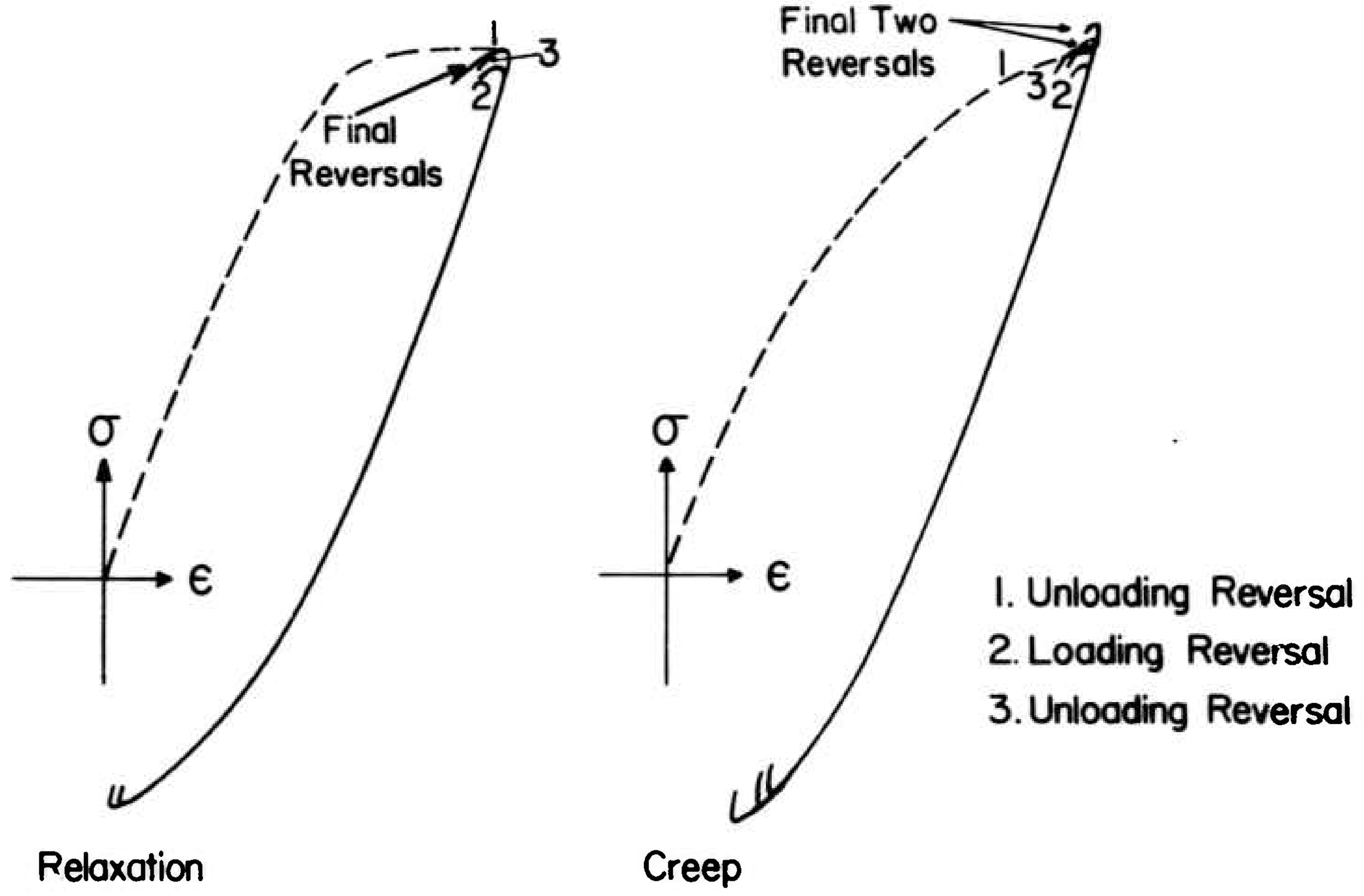


Fig. 21b Comparison of Hysteresis Branches in Relaxation and Creep of SAE 1018 Steel from Fig. 21a

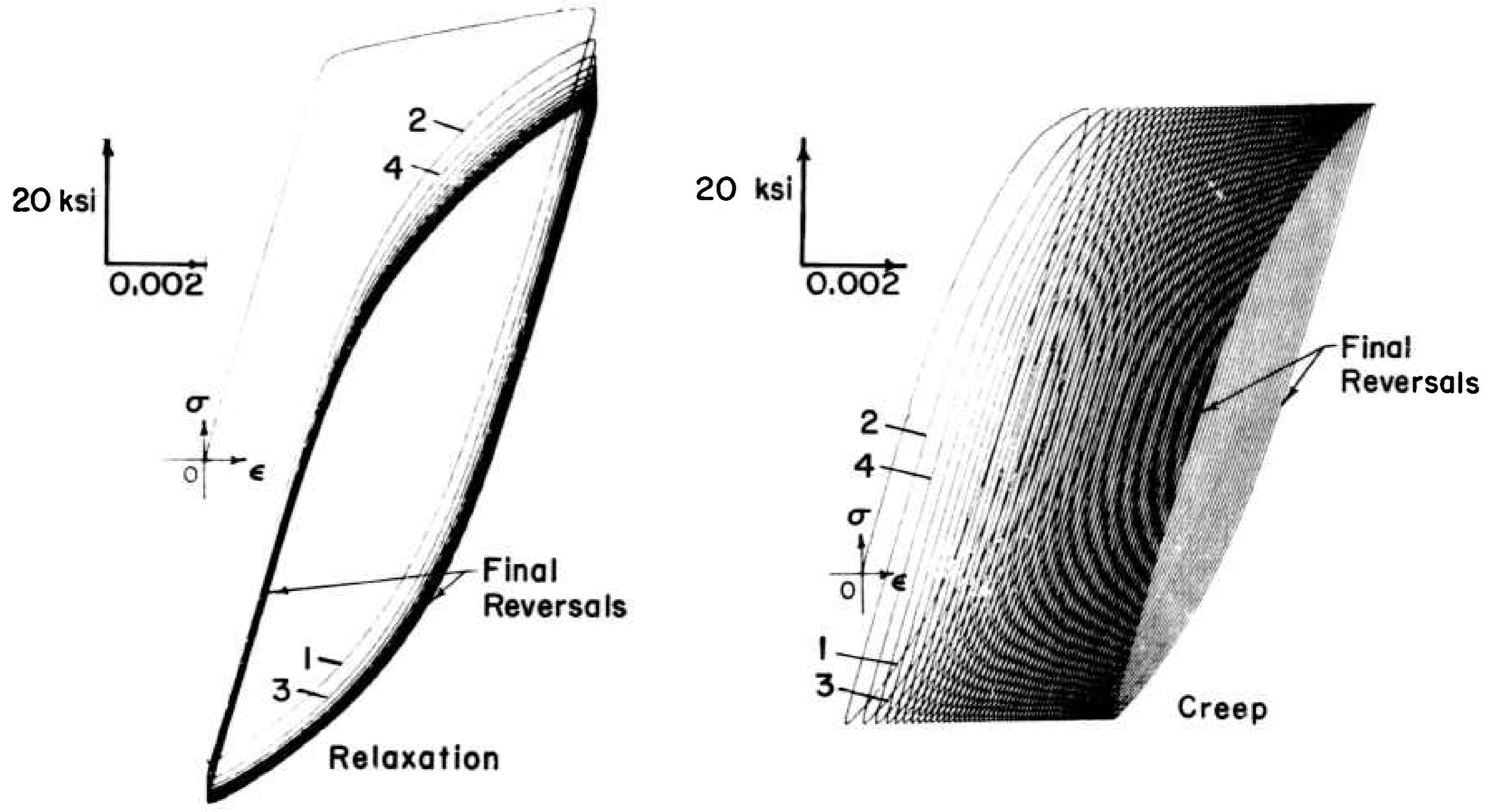


Fig. 22a Hysteretic Behavior of SAE 1045 Steel in Cyclic Relaxation and Cyclic Creep

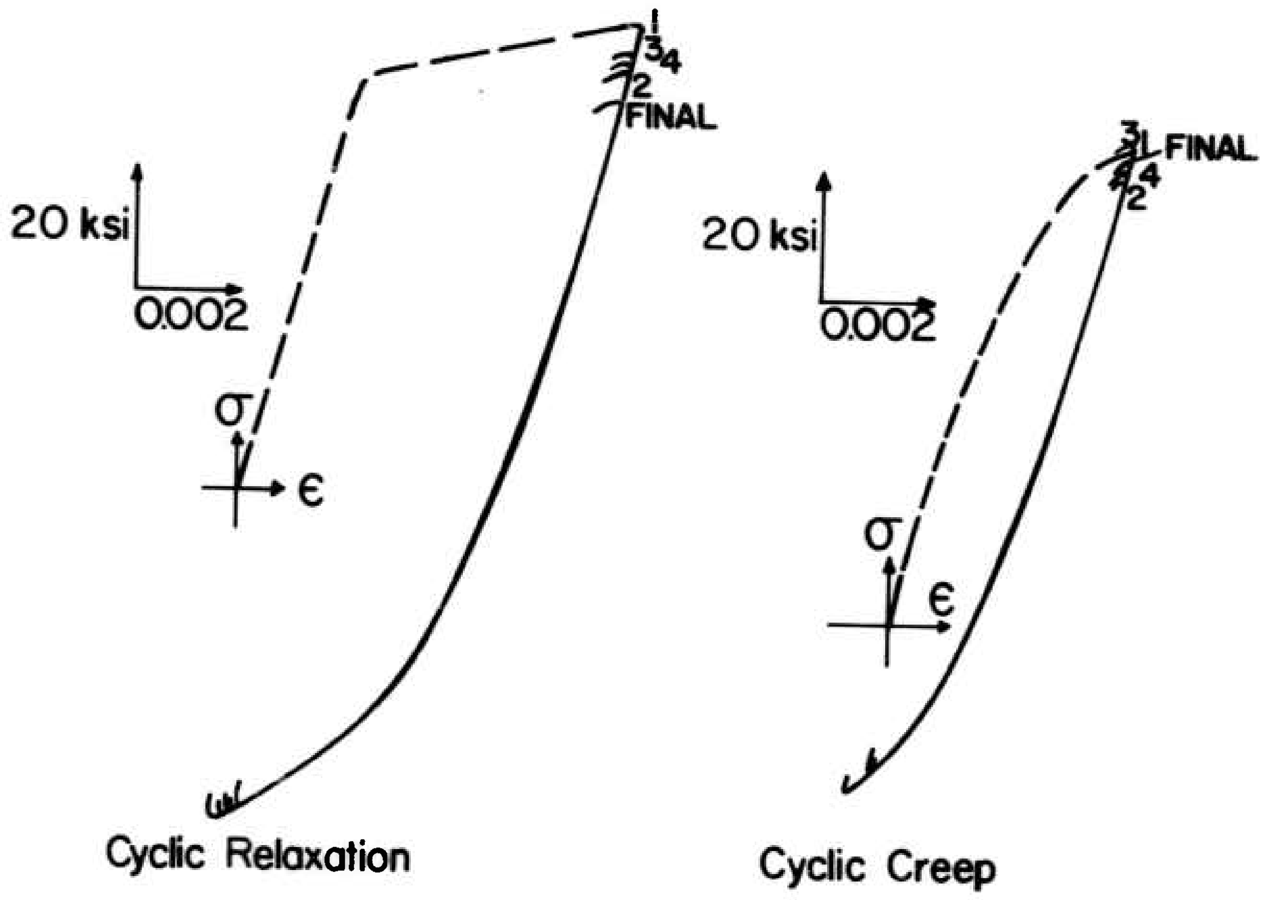


Fig. 22b Comparison of Hysteresis Branches in Relaxation and Creep of SAE 1045 Steel from Fig. 22a

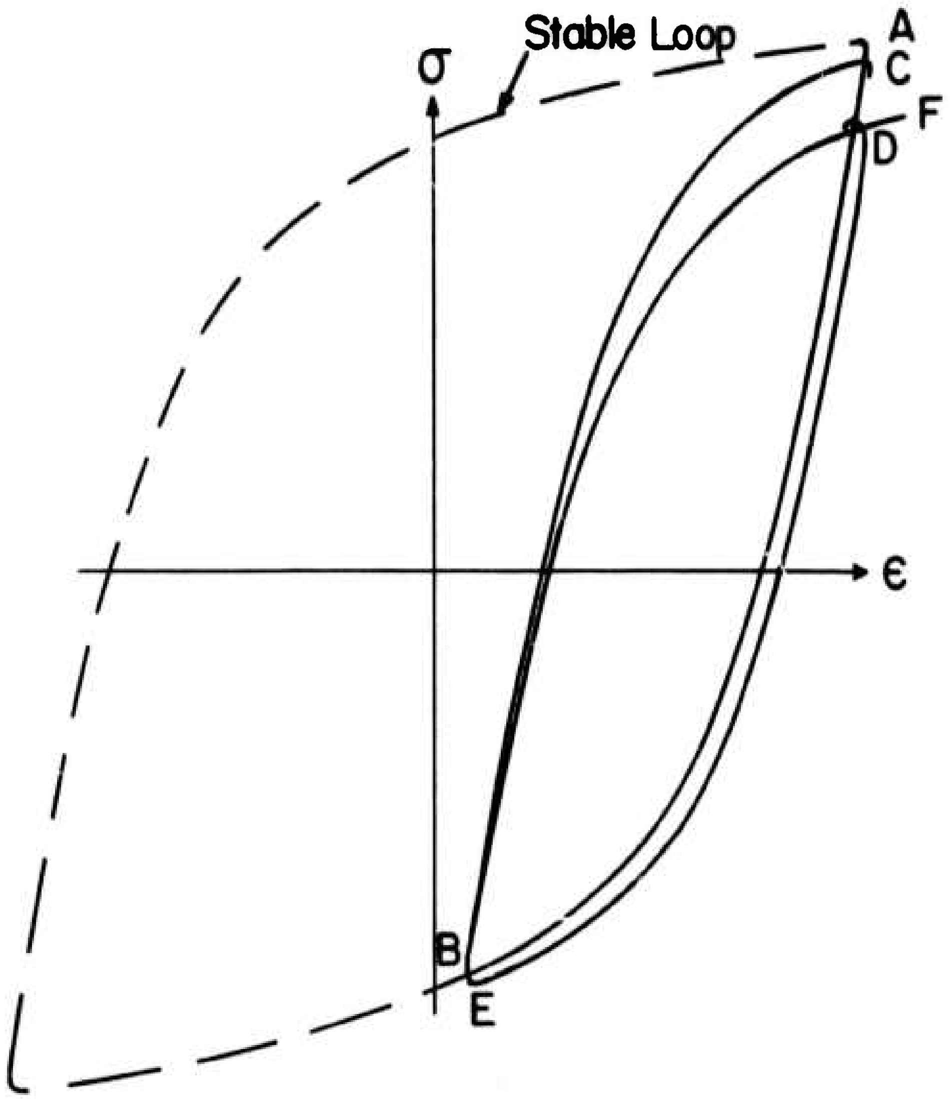


Fig. 23a Initial and Final Hysteresis Loops of a Normalized SAE 1018 Steel in a Relaxation Test.

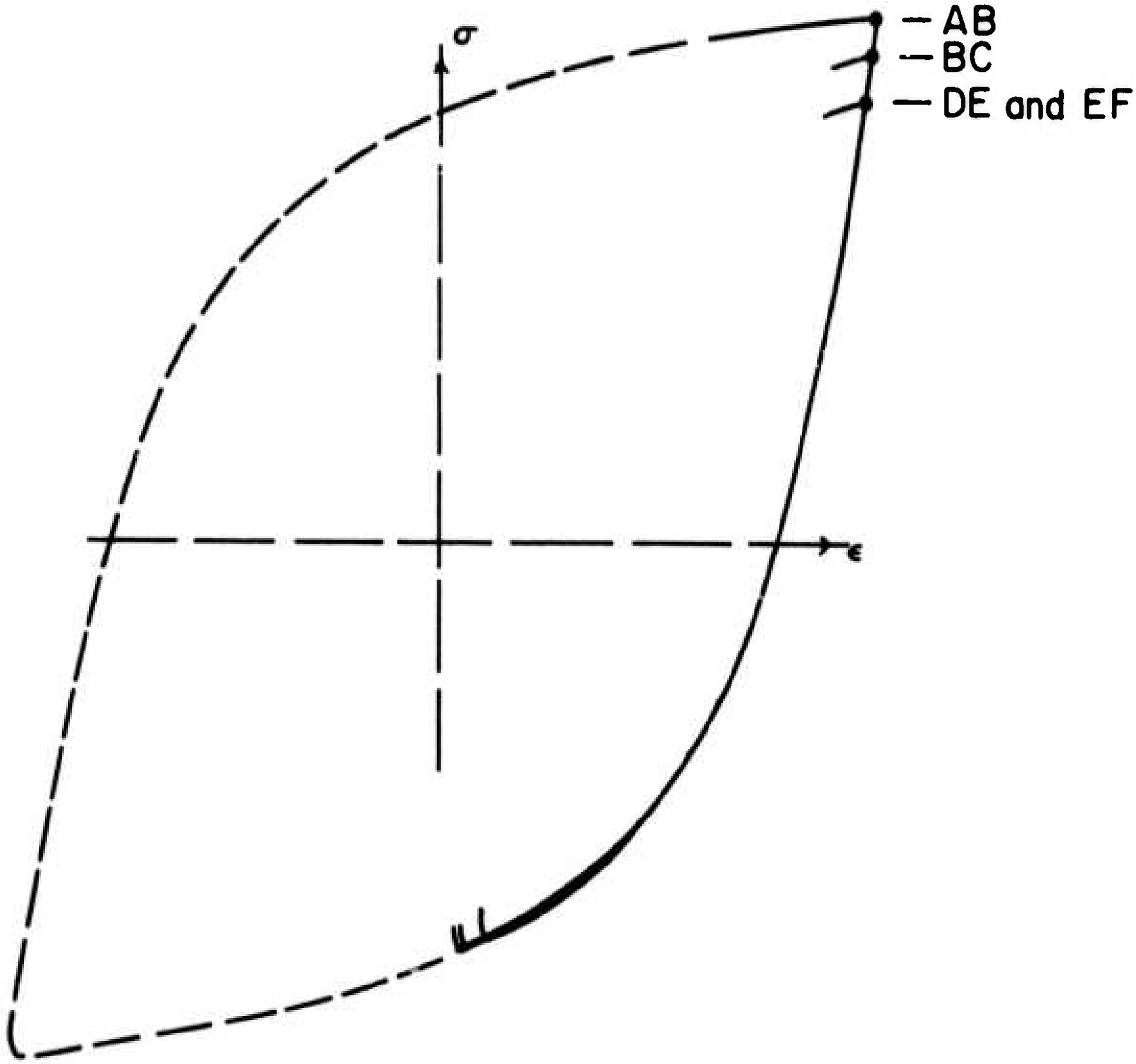


Fig. 23b Comparison of Initial and Final Hysteresis Branches of Normalized SAE 1018 Steel in Relaxation from Fig. 23a

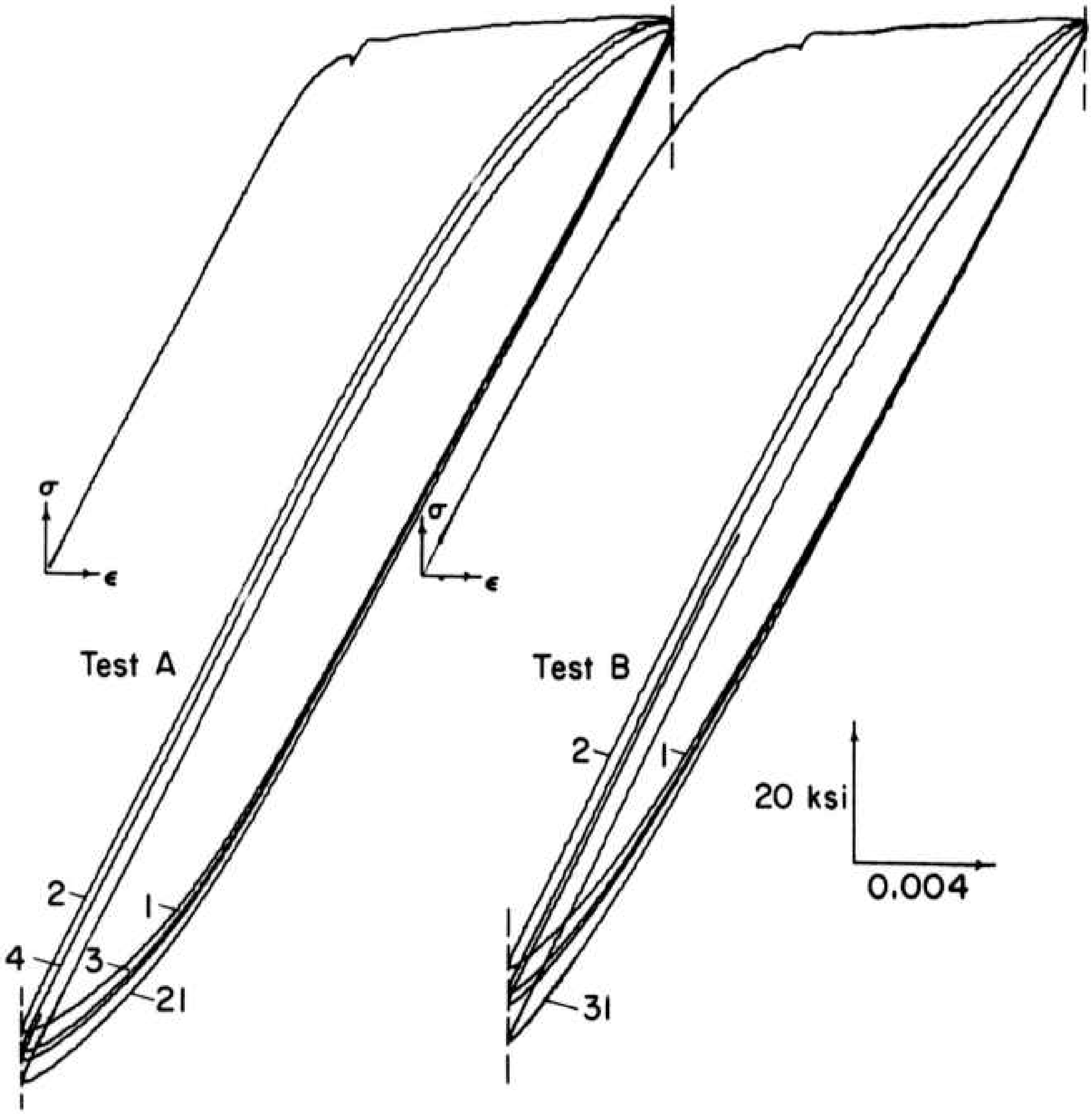


Fig. 24a Hysteretic Behavior of 7075-T6 Aluminum in Cyclic Relaxation at Two Strain Amplitudes

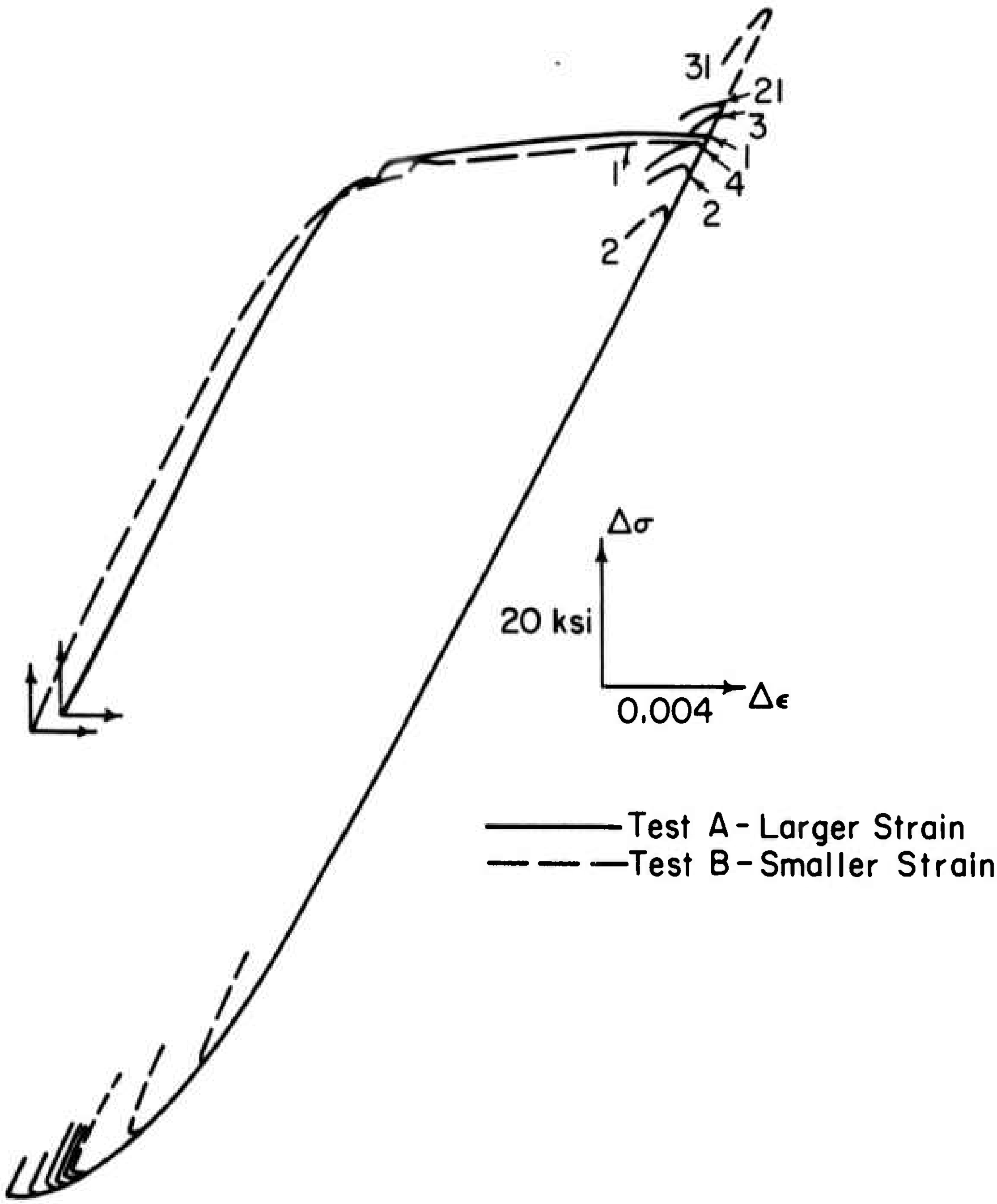
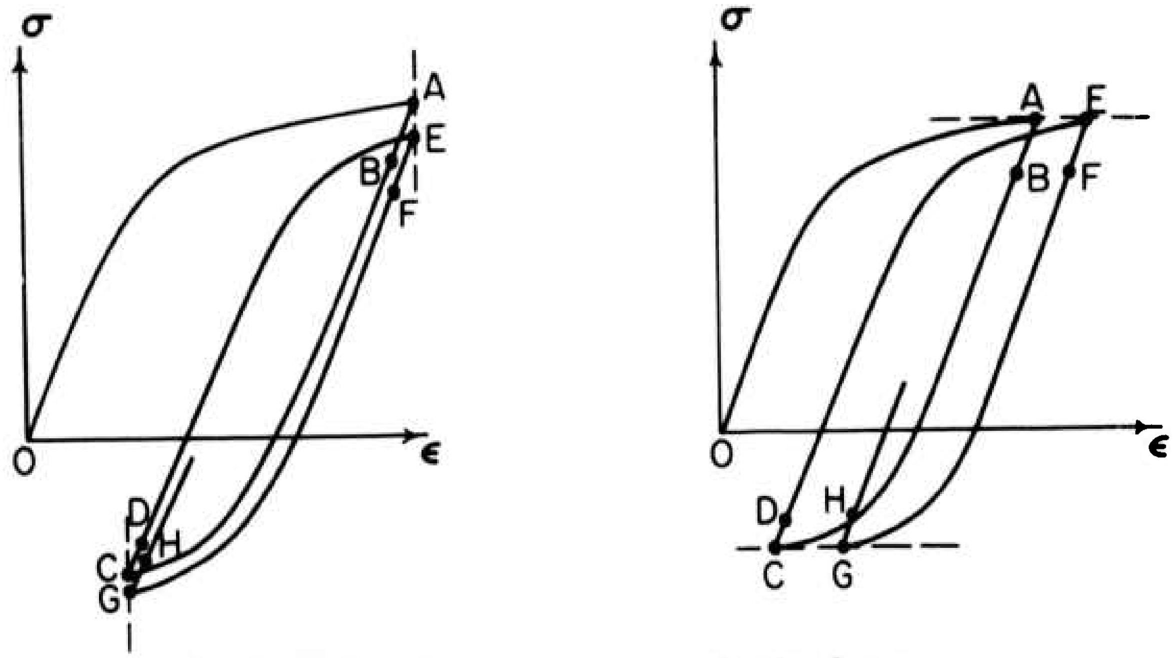


Fig. 24b Comparison of Hysteresis Branches of 7075-T6 Aluminum in Cyclic Relaxation from Fig. 24a



AB, CD, EF and GH are Yield Range Increments (YRI)

1. Assume Previous Plastic Strain Range to Govern YRI

   Because $\Delta\epsilon_p(OA) > \Delta\epsilon_p(AC)$ → AB>CD, and

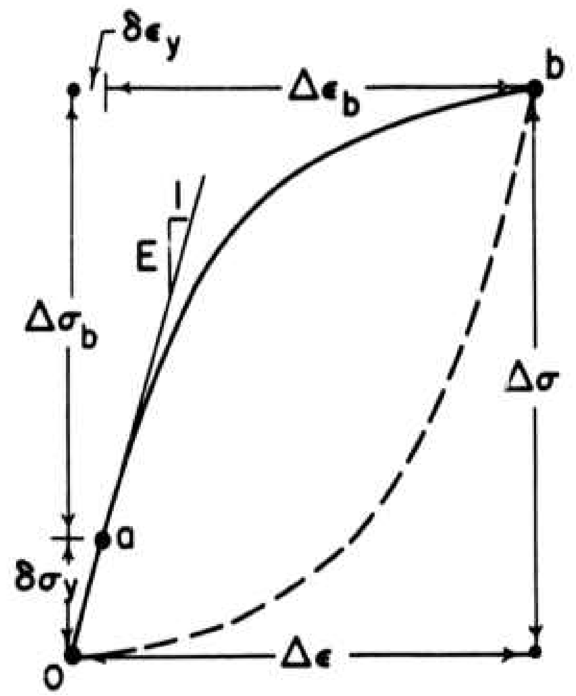
   Because $\Delta\epsilon_p(CE) > \Delta\epsilon_p(EG)$ → EF>GH

2. Assume Absolute Initial Stress to Govern YRI

   Because $|\sigma_A| > |\sigma_C|$ → AB>CD, and

   Because $|\sigma_E| > |\sigma_G|$ → EF>GH

Fig. 25 Variables Governing Relaxation and Creep.



Portion oa is "Yield Range Increment".
Portion ab is "Basic Hysteresis Curve".

<u>Mathematical Model</u>

$$\Delta\epsilon = \Delta\sigma/E + \left(\frac{\Delta\sigma - \delta\sigma_y}{K}\right)^{1/n} \quad \text{for } \Delta\sigma > \delta\sigma_y$$

$$\Delta\epsilon = \Delta\sigma/E \quad \text{for } \Delta\sigma \leq \delta\sigma_y$$

E, K and n are history independent material parameters.

$\delta\sigma_y$ is the history dependent "elastic" parameter.

FIG. 26 Analytical Description Of A Hysteresis Branch