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WEALTH EQUIVALENTS, RISK AVERSION, AND THE MARGINAL BENEFIT FROM INCREASED SAFETY

Michael K. Block, et al

Naval Postgraduate School
Monterey, California

May 1974

NAVAL POSTGRADUATE SCHOOL
Monterey, California

Rear Admiral Mason Freeman
Superintendent

Jack R. Borsting
Provost

The work reported herein was supported in part by the Foundation Research Program of the Naval Postgraduate School with funds provided by the Chief of Naval Research.

This report was prepared by:

MICHAEL K. BLOCK
Assistant Professor of
Operations Research and
Administrative Sciences

ROBERT C. LIND
Associate Professor of Urban Management
Graduate School of Business
Stanford University, Stanford, CA 94305

Reviewed by:

D. A. SCHRADY, Chairman
Department of Operations Research
and Administrative Sciences

Released by:

JOHN M. WOZENCRAFT
Dean of Research

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

AD-781375

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NPS-55Xb74051	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) WEALTH EQUIVALENTS, RISK AVERSION, AND THE MARGINAL BENEFIT FROM INCREASED SAFETY		5. TYPE OF REPORT & PERIOD COVERED Technical Report
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Michael K. Block Robert C. Lind		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940 Code 55Xb		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61151N;RR000-01, RR000-01-50; PO 3-0045
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research, Arlington, Va. 22217		12. REPORT DATE May 1974
		13. NUMBER OF PAGES 20
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release;distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Professor Block's research was supported in part by the Office of Naval Research Foundation Research Program, Naval Postgraduate School.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Wealth equivalents
Safety decisions
Risk aversion

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In this paper we show that if a wealth equivalent to an undesirable event exists, then the assumption of risk aversion has strong and interesting implications for safety decisions. In addition, we set forth sufficient conditions for the existence of a wealth equivalent to a nonmonetary and undesirable event.

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S/N 0102-014-6601

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1. Introduction

Individuals and collections of individuals face a wide range of decisions that involve the choice of spending resources to reduce the probability of some uncertain and undesirable event. Investments in locks and police protection to reduce the probability of being burgled, investments in preventative health measures, expenditures to reduce the risk of accidents, and expenditures on national defense all fall into this category. In this paper we shall refer to all decisions of this type as safety expenditures. The characteristic of these expenditures is that they involve committing current resources, which can be measured in dollars, to alter a gamble involving consequences of a partially nonmonetary nature. The consequences are nonmonetary in the sense that some attributes of the consequences cannot be bought and sold in any market. For example, if you were in an automobile accident, your car might be ruined and you might be injured and possibly disabled for life. In this case, you could be compensated for your car, your medical expenses, and possible income lost because of your disability, but there is no market in which you can buy back your health. It is the latter aspect of this outcome which we call nonmonetary.

While the nonmonetary aspects of an outcome present an interesting and vexing valuation problem, decision analysts are quick to point out that it is irrelevant for most safety decisions. Since such decisions usually involve determination of how much an individual would be willing to pay for some small reduction in the probability of an unfavorable outcome, it is not necessary to know the value of reducing the probability from one to zero. Although this is true for most purposes, as we show, it is possible to obtain much more powerful results concerning safety decisions if all consequences can be converted to monetary equivalents. In particular, one can draw strong conclusions about the marginal willingness to pay for increments of safety, as a function of the level of safety, from the assumption of risk aversion alone in this case.

In the first section of this paper we define the concept of a wealth equivalent to an outcome with some nonmonetary attributes. We then demonstrate that, under the assumption that such a wealth equivalent exists, risk aversion implies that the marginal willingness-to-pay for an increment in safety (measured by the reduction in the probability of an unfavorable outcome) decreases as the initial probability of an unfavorable outcome increases. For example, an individual would be willing to pay less to reduce the probability of being mugged from .40 to .39 than from .10 to .09. This result, which seems counter intuitive, is shown to have a number of interesting behavioral implications.

Next we examine the relationship between behavior towards risk, the relative marginal utility of wealth given a favorable as opposed to unfavorable outcome, and the existence of a wealth equivalent . In this section we derive a set of sufficient conditions for the existence of a wealth equivalent to an outcome with some nonmonetary attributes.

2. The Case of Wealth Equivalents

Consider the case of an individual with wealth w who faces a gamble where he may or may not incur some undesirable consequence, denoted by v. Then his utility is a function of his wealth and whether or not v occurs. Formally, we define his utility by $U(w,v)$ where v is 1 or 0 depending on whether or not v occurs. In addition we assume that $U(w,v)$ satisfies the expected utility theorem and is twice differentiable in w. Finally, we posit that $U_w > 0$, $U_{ww} < 0$, and $U(w,1) < U(w,0)$, i.e, whether or not v occurs, an individual values increases in wealth and is risk averse, and if v occurs, he is worse off at all levels of wealth.

Now consider the cost of incurring the undesirable consequences represented by v. This cost or loss can be unambiguously defined in terms of wealth if there exists an amount of money L such that

$$U(w,1) = U(w-L,0), \tag{1}$$

i.e., the individual is indifferent between a loss of L dollars and having his initial wealth w and suffering the consequences represented by v. The amount L is the wealth equivalent of suffering the consequences represented by v. L includes both any direct monetary losses associated with v such as the value of a demolished car or stolen goods plus the value the individual places on nonmonetary attributes of v, such as pain and anguish. Clearly, L also represents the maximum amount that an individual would pay to avoid suffering v in a certain world.

Now suppose an individual has wealth, w, and that there is a probability p that he will suffer the consequences represented by v. Using (1), his expected utility is given by

$$pU(w,1) + (1-p)\,U(w,0) = pU(w-L,0) + (1-p)\,U(w,0), \tag{2}$$

and thus for simplicity we can rewrite the equation for the individual's expected utility as

$$pU(w-L) + (1-p)\,U(w). \tag{3}$$

We can now define the cost to the individual of a probability of suffering the consequences v by the equation

$$(4) \qquad U(w-c(p,w,L)) = pU(w-L) + (1-p)\,U(w).$$

In equation (4) $c(p,w,L)$ represents the amount one would willingly pay to eliminate the risk associated with a probability p of a loss L given wealth w. The derivative of c with respect to p represents the individual's marginal willingness to pay for reductions in the probability of incurring the loss of L or equivalently the consequences of v.

Clearly, $c_p(p,L,w)$ is positive (this follows directly from the assumption that $U_w > 0$). Further, if we totally differentiate (4) twice with respect to p we obtain

$$(5) \quad -c_{pp}(p,L,w)\,U_w(w-c(p,L,w)) + (c_p(p,L,w))^2 U_{ww}(w-c(p,L,w)) = 0.$$

From (5) and the previous assumptions that $U_w > 0$ and $U_{ww} < 0$, it follows that $c_{pp}(p,L,w) < 0$.[2] Therefore, in the case where a wealth equivalent exists the marginal willingness to pay for a reduction in the threat of suffering the consequences of v, as measured by p, decreases as a function of p for a risk-averse individual. For example, what a risk-averse individual would be willing to pay for a one percentage point

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[2] Obviously, if we had assumed risk preference ($U_{ww} > 0$), the sign of (5) would be reversed.

reduction in the probability of being the victim of a crime would be less the higher the crime threat.[3]

Given this result, it is of interest to explore the implications of risk aversion for the response of the cost function, c, to marginal changes in L. Clearly, $c_L > 0$ and if we twice differentiate (4) with respect to L, we obtain

$$(6) \qquad -c_{LL}U_w(w-c(p,L,w)) + c_L^2U_{ww}(w-c(p,L,w)) = pU_{ww}(w-L)$$

Rearranging terms we get

$$(7) \qquad c_{LL}(p,L,w) = \frac{c_L^2(p,L,w)\ U_{ww}(w-c(p,L,w))-pU_{ww}(w-L)}{U_w(w-c(p,L,w))}$$

The sign of the right hand term is not determined by the assumption of risk aversion alone. Put differently, as the size of the loss, L, increases, the cost of the threat increases, but the assumption of risk aversion alone does not imply that the cost increases at a decreasing rate as one might expect.[4]

While risk aversion alone does not leave a strong implication for the sign of c_{LL} the expression in (6) can be employed to analyze an extremely interesting problem. Using (6) we can explore the effect on the cost of the threat, c, of simultaneously changing the nature of undesirable

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[3]This result is not at all obvious. Becker [1968], in his pioneering work on the optimal level of law enforcement, assumes as a fundamental tenet of his model that essentially the opposite is true, namely that marginal harm from crime rises with the crime rate.

[4]Again drawing on the literature on the cost of the threat of crime, Stigler (1970) asserts that risk aversion implies that the cost of crime increases at an increasing rate with increases in the size of loss. As the results above indicate, this is not necessarily true for the cost of the threat of crime.

outcome and the probability of occurrence while leaving the expected value of the equivalent loss, $E(L) = pL$, constant. It can be shown that risk aversion in wealth implies that an increase in p accompanied by a decrease in L such that pL remains constant reduces the cost of the threat.

To proceed note that from the assumption that $E(L)$ is constant it follows that

$$\text{(8)} \qquad \frac{dE(L)}{dp} = L + p\frac{dL}{dp} = 0.$$

Then by differentiating both sides of (4) with respect to p we get

$$\text{(9)} \qquad U_w(w-c(p,L(p),w))[-\frac{d}{dp}\, c(p,L(p),w)] =$$

$$U(w-L(p)) - U(w) + Lp\; U_w(w-L(p)).$$

Applying the mean value theorem to the right side of equation (9) and dividing by $-U_w(w-c(p,L(p),w))$ we get

$$\text{(10)} \qquad \frac{d}{dp}\, c(p,L(p),w) = \frac{L[U_w(w^*) - U_w(w-L(p))]}{U_w(w-c(p,L(p),w))}$$

where $w-L(p) < w^* < w$. Risk aversion implies that the numerator on the right side of (10) is negative. Therefore, for a risk-averse individual in the case where a wealth equivalent exists, increasing the probability of an adverse outcome and simultaneously reducing its severity so that the expected value of the equivalent loss, pL, remains constant reduces the cost of the threat. Thus, policies which reduce the magnitude of losses even though they may be compensated by a higher incidence of occurrence will generate a net benefit.

Now consider the effect on the cost of a specific threat of changes in wealth. Clearly, in most cases the wealth equivalent L will depend upon the level of wealth, w, and if we are to consider changes in the cost of a threat with changes in w, we must specify the cost of the threat by $c(p,L(w),w)$. Although no general statement can be made concerning the sign of c_w, two special cases are of interest. If $L(w)$ is a constant as in the case of a fixed monetary loss, then the assumption of decreasing absolute risk aversion as measured by the Arrow-Pratt coefficient implies that the cost of the threat decreases with increases in wealth. If, on the other hand, L is a constant proportion of wealth, then Arrow's hypothesis of increasing relative risk aversion would imply that the cost of the threat would be an increasing function of wealth.

The discussion of the case where a wealth equivalent exists can be summarized as follows: If there exists a wealth loss that is equivalent to the occurrence of a specific undesirable event, then the cost to an individual of the probability p that this event will occur is identical to adding to his wealth a random component with probability p of being $-L$ and probability $(1-p)$ of being 0 with an expected value of $-pL$. The cost of adding this random component is

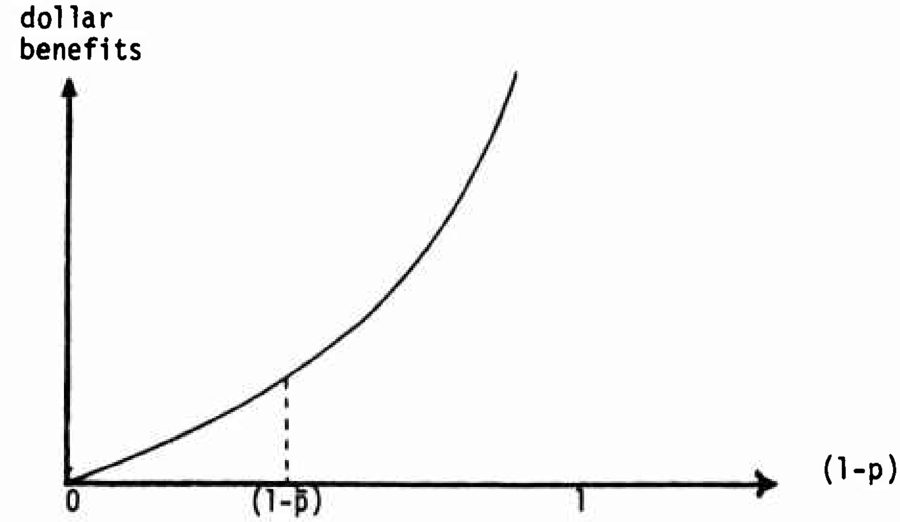
$$c(p,L,w) = pL + r(p,L,w) \tag{11}$$

where pL is the expected value of the loss and $r(p,L,w)$ is the cost of risk bearing. For risk-averse individuals, the value of the function r is positive.

Since pL is linear in p, it follows from $c_{pp} < 0$ that $r_{pp} < 0$, that is, the cost of risk bearing does not increase proportionately as p increases. Similarly, the sign of c_{LL} and r_{LL} will be the same; however, whether the cost of risk bearing increases at an increasing or decreasing rate as the size of the loss increases is not determined by risk aversion alone.

The important implication of the foregoing analysis is that if the income effects of the intra-marginal investments in safety are negligible, then the marginal benefits from a reduction in the threat of harm, measured by p , is increasing as pictured in Figure 1.

Figure 1



The implication of this is that if, given an initial level of hazard $\bar{p}$, we choose to reduce $\bar{p}$ to $\bar{p}-\varepsilon$ at a given cost, then we should be willing to spend more for each successive increment of safety. This implies that investments in safety should be pushed to some point beyond which diseconomies of scale occur with regard to reductions in p. If, for example, one spends ten million to reduce by ten percentage points the chance of nuclear pollution because the benefits are determined to exceed the costs, then risk aversion and the assumption that a wealth equivalent to the effects of such pollution exist implies that the benefits from a further ten percentage point reduction are even greater and therefore would justify a greater expenditure.[5]

[5]Clearly, if the first increment of safety required a substantial amount of total resources, the income effect could negate this result. This may be particularly important in the case of national defense.

3. Risk Aversion, The Marginal Utility of Wealth, and the Existence of Wealth Equivalents

It is important to remember that an individual's initial wealth, w, as defined above, is the discounted value of his expected lifetime income. Note that this sets a limit on L because $w-L$ cannot drop below some minimum $\bar{w}$ which is the level of wealth required to sustain an individual over his lifetime. For example, suppose that v represented death and that $U(w,v) < U(\bar{w},0)$, i.e., life at subsistence is preferable to death with a legacy equal to lifetime wealth.[6] In this case, the individual would be willing to pay an amount equal to $(w-\bar{w})$ in order to save his life, but if he were able to buy his life for this amount, this would be strictly preferable to death. Therefore, there is no feasible reduction in his wealth that is equivalent to death. In this case we say a wealth equivalent does not exist in the sense just defined.

This situation raises an interesting question. Specifically, under what general conditions can it be shown that a wealth equivalent to the hazard, v, must exist? It is to this point we now turn by considering the relationships among wealth equivalents, risk aversion, and the relative magnitudes of the marginal utility of income with and without the occurrence of v. In particular, we show that the assumptions of risk aversion in wealth and $U_w(w,0) > U_w(w,1)$ imply the existence of a wealth equivalent to v.

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[6]Because much of most people's wealth is in the form of human capital, one would need to assume that an individual had a life insurance policy for an amount equal to the discounted value of his lifetime earnings.

To proceed, consider an undesirable event or hazard, v , such that

(12) $$U(w,1) < U(\bar{w},0) ,$$

i.e., an individual would be willing to reduce his income to the minimum level required to sustain his life rather than incur the event v. We continue to assume that wealth is valued positively whether or not v occurs so that

(13) $$U(0,1) < U(w,1) < U(\bar{w},0) < U(w,0)$$ [7]

From (12) it follows that there is no loss L such that $(w-L,0)$ can be substituted for $(w,1)$ and therefore, the cost function c cannot be defined as in (2). Because the occurrence of v causes greater disutility than reducing one's wealth to bare subsistence, the individual in this sense values the avoidance of v more than his wealth, although because his budget constraint is $(w-\bar{w})$, this is the maximum amount that he would (could) pay to avert v.

It is easy to show that there is some minimum probability, $\bar{p}$, such that for p, $\bar{p}<p<1$, this individual would willingly pay $(w-\bar{w})$ to reduce the probability of v to zero and such that for $p < \bar{p}$ he would pay less. We now define a new cost function associated with the threat of v, $c^*(p,w)$, for p , $0 \leqq p \leqq \bar{p}$ as before by

(14) $$U(w-c^*(p,w),0) = pU(w,1) + (1-p)\,U(w,0).$$

By definition, the range of $c^*(p,w)$ is constrained to be less than $(w-\bar{w})$ and for $p < \bar{p}$, the individual's marginal willingness to pay for a

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[7] If v represented death, then (13) would imply that an individual would prefer life at subsistance to death and his initial wealth, but given the prospect of death, would prefer to have a legacy.

reduction in the threat of v is simply $c^*_p(p,w)$. However, for $p > \bar{p}$, the amounts that an individual would pay to eliminate the chance of v, given probabilities of occurrence of p and $p-\varepsilon$ are the same. Nevertheless, it can be shown that, at the same time, the individual would pay some positive amount to reduce the probability from 1 to $(1-\varepsilon)$. The marginal willingness to pay for decrements in p for $p > \bar{p}$ is not zero and it is the measure that is relevant to decisions to invest in safety.

We formalize this willingness to pay by defining the function $m(p,\varepsilon)$ as the amount that an individual with wealth, w, facing a probability p of the occurrence of v would pay to reduce this probability to $p-\varepsilon$. Therefore,

$$(15) \qquad pU(w,1) + (1-p)\, U(w,0) = (p-\varepsilon)\, U(w-m(p,\varepsilon),1) + (1-p+\varepsilon)\, U(w-m(p,\varepsilon),0).$$

For $p < \bar{p}$ it is easily shown that

$$m(p,\varepsilon) = c^*(p,w) - c^*(p-\varepsilon,w).$$

We now demonstrate that if $U_w(w,1) < U_w(w,0)$, then for small ε, $m_p(p,\varepsilon) > 0$ and that for $0 \leq p \leq \bar{p}$, $c^*_{pp}(p,w) > 0$, i.e., the marginal benefits from a decrease in the probability of v increases with the level of the threat. On the other hand, if $U_w(w,1) > U_w(w,0)$, then, $m_p(p,\varepsilon) < 0$ for small ε and $c^*_{pp}(p,w) < 0$.

To demonstrate these results we scale the utility function so that $U(w,1) = 0$ and $U(w,0) = 1$.[8] From (15) it follows that

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[8]This demonstration was suggested by the work of Raiffa [1969].

(16) $$(1-p) = (p-\varepsilon)\, U(w-m(p,\varepsilon),1) + (1-p+\varepsilon)\, U(w-m(p,\varepsilon),0)$$

Differentiating both sides of the equation with respect to p and re-arranging terms, we get

(17) $$m_p(p,\varepsilon) = \frac{1-[U(w-m(p,\varepsilon),0) - U(w-m(p,\varepsilon),1]}{[(p-\varepsilon)U_w(w-m(p,\varepsilon),1) + (1-p+\varepsilon)U_w(w-m(p,\varepsilon),0]}$$

Therefore, $m_p(p,\varepsilon)$ is positive or negative depending on whether

(18) $$[U(w-m(p,\varepsilon),0) - U(w-m(p,\varepsilon),1)]$$

is less than or greater than 1. Since $U(w,0) - U(w,1) = 1$ by construction, it follows that if $U_w(w,0) > U_w(w,1)$, the difference in (18) will be less than 1 for sufficiently small values of ε. Alternatively, if $U_w(w,0) \leqq U_w(w,1)$, then the difference in (18) will be more than or equal to 1. Therefore, $m(p,\varepsilon)$ is an increasing function of p if $U_w(w,0) > U_w(w,1)$ and a decreasing function of p if this inequality is reversed.

It is important to re-emphasize that this result is based on the assumption that no wealth equivalent exists and that individuals are risk averse in wealth. Under these conditions whether the marginal benefits from reducing the threat are an increasing or decreasing function of p depends on the relative magnitude of the marginal utility of wealth given that v does or does not occur. For example, suppose v represents death, there is no wealth equivalent to death, and the marginal utility of wealth to an individual is higher alive than dead, then the amount he would willingly pay for a decrement in threat of death would be greater the higher the value of p. This is just the opposite of the result for the case where there was a wealth equivalent.

Now by contradiction we can establish that risk aversion and $U_w(w,0) > U_w(w,1)$ imply the existence of a wealth equivalent to the undesirable consequence v. It has been demonstrated above that given risk aversion and $U_w(w,0) > U_w(w,1)$, $m_p(p,\varepsilon) > 0$. We note, moreover, that $m(p,\varepsilon) = c^*(p,w) - c^*(p-\varepsilon,w)$ for $p \leqq \bar{p}$ and it is straightforward to show that $m_p(p,\varepsilon) > 0$ implies $c^*_{pp}(p,w) > 0$. However, if we differentiate equation (14) which defines c^* twice with respect to p, we get the result that for $p \leqq \bar{p}$, $c^*_{pp}(p,w) < 0$. This follows from risk aversion by exactly the same derivation as in equation (5). Since there exists some $\bar{p} > 0$ such that c^* is defined on the domain $[0,\bar{p}]$ and since $m_p(p,\varepsilon) > 0$ on the interval $[0,\bar{p}]$ implies that $c^*_{pp}(w,\varepsilon) > 0$, there is a contradiction. Therefore, the nonexistence of a wealth equivalent, risk aversion, and $U_w(w,0) > U_w(w,1)$ cannot all hold simultaneously. Only two of the three can hold. Therefore, if an individual is risk averse in wealth and if $U_w(w,0) > U_w(w,1)$, it follows that a wealth equivalent to v must exist.

This conclusion is particularly striking in the previous example where v is death. For a risk-averse individual with a life insurance policy equal to his lifetime wealth and for whom the marginal utility of additional wealth is greater alive than dead, there is some feasible reduction of his wealth such that faced with death or this wealth reduction, he would choose death with a legacy of his present wealth to life with reduced wealth. Alternatively, if an individual claims that he does not have a wealth equivalent to death and that the marginal utility of wealth is greater in life than death, then this individual must have a preference for risk.

In order to present a straightforward geometric interpretation of our analysis, we consider a two-state model where the states are simply v or not v . Given this interpretation U(w,1) and U(w,0) are the utility function in wealth in states v or not v. The cases where wealth equivalents do and do not exist are pictured in Figures 2a and 2b.

Figure 2a

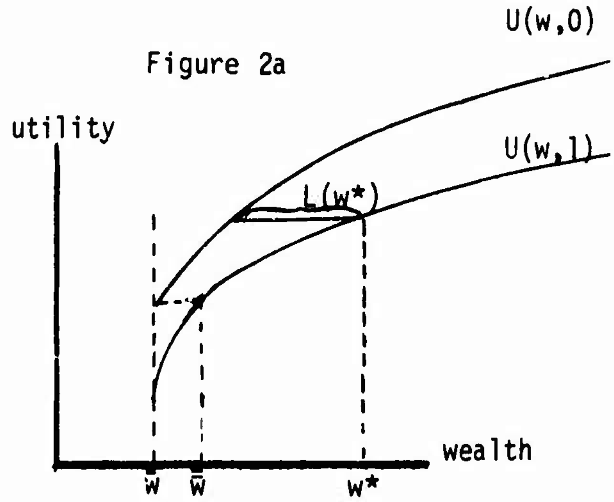
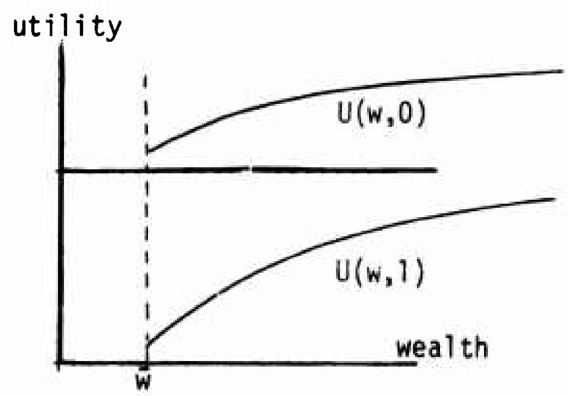


Figure 2b



First, U(w,0) is only defined for $w \geq \bar{w}$ and this is also true for most undesirable states represented by v except for death, where U(w,1) would be defined for $w \geq 0$. Second, the existence of a wealth equivalent means that you take a point on the curve U(w,1), say w* in Figure 2a, and move toward the vertical axis parallel to the horizontal axis, then you will intersect U(w,0) at a point $(w-L(w^*)) > \bar{w}$. Further, because we have assumed that $U(\bar{w},0) > U(\bar{w},1)$, it follows that there is some interval $[\bar{w},\bar{\bar{w}}]$ in which no wealth equivalent exists. $\bar{\bar{w}}$ may however be ∞ , in which case no wealth equivalent to v exists for any value of w. This is pictured in Figure 2b. It follows from all previous results that $U_w(w,1) > U_w(w,0)$ on the interval $[\bar{w},\bar{\bar{w}}]$.

In the case where $\bar{\bar{w}}$ is finite and a wealth equivalent exists for some values of w, it exists for all values of $w > \bar{\bar{w}}$. Therefore if there exists a wealth equivalent to v for some value w^* of w, then a wealth equivalent to w exists for all $w > w^*$. This is illustrated in Figure 2a. Further, for $w > \bar{w}$, $U_w(w,0) > U_w(w,1)$.

4. Summary

The line of analysis above shows that the assumption of risk aversion when coupled with assumptions about the existence of wealth equivalents or the relative magnitude of the marginal utility of wealth in different states has strong and interesting implications for the structure of utility functions. We have used this structure to analyze the relationship between the marginal willingness to pay for safety as a function of the level of safety and have set forth sufficient conditions for the existence of a wealth equivalent to an undesirable event. As a point of interest, we demonstrated that to assume that a wealth equivalent to death does not exist, one must also assume either that the marginal utility of wealth is greater in death than life or that an individual has a preference for risk.

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