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ABSTRACT: This report describes two computer simulation models that have been developed at the Naval Ordnance Laboratory to provide assistance to minefield planners in evaluating the performance of minefields. Provision is made for dealing with varying input parameters, and primary attention is focused on studying the effectiveness of arming delays and ship count settings in preserving the threat in the presence of influence sweeping. Use of the models is explained in detail and sample results for a hypothetical minefield are exhibited.

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This report was prepared under Naval Ordnance Systems Command Task ORD-32D-201/UF33-313-220. It describes two computer simulation models that have been developed to assist minefield planners in evaluating the performance of minefields under various assumed conditions.

The need for models of the type depicted here arose from difficulites encountered in trying to apply more complicated models that require actuation and damage data not yet fully available. These factors have been aggregated into a simple lethal radius concept, but flexibility has been provided to permit rapid exploration of the effects of other parameters on the outcomes of engagements.

The assistance of the minefield planning staff at the Mine Warfare Force in defining the problem and describing the kind of help that would be useful is gratefully acknowledged.

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By direction

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Chapter 1

INTRODUCTION

Despite the extensive work that has been done by many analysts on the modeling and simulation of minefield performance, the actual planning of specific minefields as currently carried out at the Mine Warfare Force (MWF) still depends heavily on judgmental choices for many of the settings for the individual mine control mechanisms. Furthermore, given any assigned distribution of settings for the mines in a field, the methods now available to the staff at MWF for predicting the expected performance of the field are reportedly not practicably useable. The primary reason cited for this is that the prescribed model (SAMEM, for Sustained Attrition Minefield Evaluation Model, developed at Dahlgren) that has been programed for the MWF computer requires a great deal of specific input data on actuation and damage functions, some of which are not yet available in their Firing Data Library (FIDLIB). With gaps in the data, the model simply halts and cannot proceed until the necessary numbers are supplied.

By a combination of experiment and analysis the necessary work is going forward at the Naval Ordnance Laboratory (NOL) and elsewhere to expand the knowledge of actuation and damage functions for various mines against potential target ships. As the data bank is improved and enlarged in its coverage of mine and target combinations, the planners can look forward to being able to prescribe sensitivity settings for intended applications with greater confidence. When all of the entries in the FIDLIB that are needed for any particular situation of interest become known, the SAMEM simulation model will become fully operable and can be applied as intended to evaluate the expected performance of any given minefield.

However, this full capability is not expected to be achieved for some time, and for the interim the minefield planners at MWF have expressed a strong desire to have at their disposal some simpler kind of model that would be less demanding in its requirements for firing and damage data. In particular, they would be willing to aggregate those factors into a "cookie-cutter" function in order to be able to estimate, even if fairly crudely, the effects of varying the possible distributions of arming delays and ship count settings. The goal, of course, is to be able to search efficiently for optimum combinations of these parameters.

Discussions were held between the minefield planners at MWF and analysts at NOL to try to define the scope of the problem and the nature of possible approaches. It was recognized and agreed that in the interest of expediting progress many simplifying assumptions might have to be made in the model or models, so long as the two primary parameters -- arming delay and ship count settings -- could be kept highly visible and their effects readily assessable.

This report represents a first response to try to satisfy the needs as understood and interpreted. It is most certainly not regarded as the final answer to all the needs of minefield planners, but if it can provide some interim tools of practical utility it will be well worth the effort. In the next chapter the problem that was tackled will be discussed and defined in greater depth. Following that, the models that have been generated to deal with the problem will be described in careful detail. Then some sample results obtained through exercising the models will be presented and discussed. At the present stage no formal algorithms for optimizing the combinations of settings have been developed, but the models will at least permit comparisons to be made and should thereby point the way toward better choices than could be made at random or through intuition alone.

Chapter 2

PROBLEM DESCRIPTION

As background for the problem to be treated, a few elementary observations on the purposes of minefields and measures of their effectiveness will be offered here. Since mines contain explosives and triggering mechanisms, it is clear that the endowed purpose of an individual mine as a naval weapon is to inflict damage on a target vessel. Furthermore, since mines are waiting weapons that depend on the target to approach them, it is equally clear that in order to fulfill such a purpose they must be placed where targets are expected to go. If there were just one target and its path were precisely predictable, all it should take is one mine with appropriate characteristics to inflict lethal damage to its target.

However, because mine placement is subject to error and mines are not perfectly reliable, and also because target paths are not exactly predictable, it is customary to plant a number of mines in the general area where targets are expected to transit and hope that chance will provide the necessary conjunction. The usual way to express this chance is by a probability statement representing the threat of the minefield to a random transitor. Specifically, the threat of a minefield relative to a given transitor is the probability that in crossing the field the transitor will trigger at least one mine and will be sunk or damaged to some prescribed level. Standard methods are in general use for calculating this threat, and such a number is often used as the measure of effectiveness of a minefield. However, as so many people have so often pointed out, threat alone is not a fully satisfactory indicator of the value of a minefield. In fact, it does not necessarily provide even a good basis for comparison, because a threat level higher than a "sufficient" threshold may represent merely a waste of mines and minelaying effort rather than a truly better minefield. Unfortunately, objective criteria for sufficiency that could be routinely applied in advance by a minefield planner simply do not exist, and are not likely ever to exist owing to their preponderant dependence on scenario conditions, which cannot be reliably forecast.

Even though the "right" threat level cannot be prescribed in advance with any certainty, nevertheless some threat level must be chosen as an objective for the minefield planner to work toward since this is a primary determinant of the required density of mines to be implanted in the field. Assuming that mines are not to be scattered around purely in a spirit of casual vandalism, one must presume that a naval campaign director who calls for mining an area has valid reasons for wanting to impede marine traffic in that area. That is, he anticipates that if no impediments were put in the way the area would be used for traffic that is of value to the enemy, and he considers that mines could provide an effective impediment. It can be taken as axiomatic that if absolutely no traffic is expected, there would be no valid military purpose in mining an area.

The decision that the campaign planner must inexorably come to grips with is what threat level should be prescribed for each particular minefield that he wants to have laid. A whole host of questions must be considered in arriving at such decisions. In fact, there are so many issues involved that it is almost impossible even to enumerate them all, let alone organize them into a logical framework with appropriate weighting factors and therefrom devise a valid decision algorithm. Just to mention a few, the expected traffic (both numbers and kind) that would "normally" flow has to be postulated from incomplete evidence. This is part of a broader estimation of the intentions and requirements of the enemy, and a consideration of the alternatives that he might utilize. Such predictions involve not only an assessment of objective, tangible intelligence but also insight into intangibles having to do with the enemy's value systems and thought processes, in order to judge what degree of danger would deter him. As a matter of fact, the planner must initially decide whether deterrence or destruction is his primary objective. If it is the former perhaps only a modest threat would suffice, because there is strong evidence that mariners have exceedingly high respect for minefields and will attempt to cross them only with extreme reluctance no matter what the threat level is, so long as they believe it to be nonzero. For deterrence purposes overtness is an asset and the presence of a minefield should be highly publicized, even to the point of bluffing or using dummy loads to impress possible observers. On the other hand, if the goal is to destroy a single high value target, covertness is critically important and the threat level should be very high.

Somewhere in the decision-making and planning process consideration must be given to logistic and delivery conditions. That is, the resultant requirements for mines must be within available supply and delivery constraints. This means that the enemy's defenses and his reaction capabilities must be estimated, among other things. Also, the whole campaign area of responsibility needs to be looked at, to decide on the allocation of resources among the potential transit regions and to allow for prospective reseeding requirements. Once all the necessary higher level decisions are made regarding the areas to be mined, the types of targets to prepare for, the nature and extent of countermeasures to expect, and the level of threat desired, the detailed planning of the minefield can be undertaken. With the aforementioned guidelines, and using available information on environmental conditions and the characteristics of assignable mines, the minefield planner must generate a recommended selection of mines, by type and quantity, and a distribution of settings for their programmable controls. "Optimization" of these choices is his goal.

This immediately raises the question of what constitutes an optimum. Here is where the need for a valid measure of effectiveness comes to the fore, by means of which a planner can compare and rank the possible alternatives. Unfortunately, the earnest efforts of mining analysts have not yet succeeded in producing a clear cut answer to this need that is universally applicable. Consequently, the criteria adopted for each occasion are more or less customized to fit the scenarios under consideration. This fact of life, while perhaps regrettable in some ideal sense, need not be too distressing provided the rationale for making choices is adequately elucidated and the rating rules are well defined. To illustrate, let us consider some examples.

Starting with a simple case, suppose the enemy is expected to try sending one or more ships through an area that is a prospect for mining, and that he has no sweeping or other countermeasure capability available. In this case the task of the minefield planner is relatively simple. Accepting the directives from superior echelons regarding the expected targets and the desired threat level, the planner designates the most appropriate mine types available in the stock inventory, specifies the necessary sensitivity settings in accordance with handbook data, and from the resulting damage widths and size of the area to be mined calculates the number of mines required to obtain the desired threat level. Note that where no countermeasures are expected, all the mines should be set for minimum arming delay and on ship count one, since the only reason for using other values of these options is to foil influence sweeping. The timers for disarming or scuttling the mines should be set for whatever period the mines are expected to be needed.

Except for choosing the best type of mine, in the event that more than one type is available, the planner in the above situation has few decisions to make. Strictly from the point of view of effectiveness, the more ripe mines of any given type are present the greater will be the threat to transitors. Hence, in effect, the planner just keeps adding more mines until the prescribed threat level is reached. Of course, if the area is very large and the desired threat level is very high, the limits of logistic constraints might force acceptance of a reduced threat. In such instances the number of mines is simply the maximum that can be planted with the means available.

Note that in the scenario just described, the minefield planner does not have to concern himself explicitly with how to measure effectiveness. He merely has to pick suitable mines for the environment and for the expected targets and then calculate the number required to provide the specified threat. The real burden of decision rests on the campaign director who elects to mine the area, since he has to decide what will constitute a sufficient threat either to deter the enemy or to sink enough of his ships, if he attempts to push through, to have a consequential impact on the war. As the earlier discussion attempted to bring out, there is no formal algorithm based on a quantified measure of effectiveness that will cope with these criteria simultaneously. Hence, the decision is necessarily judgmental, but it does seem intuitively evident that threat, either real or perceived, is the right kind of measure to be concerned with here.

If threat is taken as an appropriate measure of effectiveness, it follows automatically that one minefield is better than another if it poses a greater threat to transitors. This implies that adding more mines to any proposed field would improve it. Of course, the incremental improvement per added mine, or the so-called rate of marginal return, diminishes as the level of threat increases. The question is, what is the best stopping point. From a theoretical cost-effectiveness viewpoint, the applicable theorem states that further investment is justified so long as the marginal return exceeds the marginal cost. While the marginal cost of sowing more mines is relatively easy to calculate, unfortunately the marginal return (in the form of reduction of enemy strength) is exceedingly difficult to estimate. Consequently, judgment has always had to take over and close the loop.

Turning next to a scenario that is more challenging to the detail planner of a minefield, consider the case where influence minesweeping is expected to be an important factor. Now the planner must not only select the proper mines, the appropriate sensitivity settings, and the number of mines needed to provide a prescribed initial threat, but he must also assign distributions of arming delays and ship count settings to the mines so as to do the best possible job of foiling the sweepers and preserving the threat. To accomplish this he clearly needs some type of measure of effectiveness, beyond initial threat, to use as a basis for comparison between possible alternatives. An approach to this problem will be proposed here.

Where a determined and capable enemy is expected to persist in using a mined waterway, he presumably will run ship transits until some stopping criterion is reached, such as a sinking or possibly two or more such events. Then he will engage in clearing and sweeping operations until he thinks it is safe to resume ship transits. These continue until he again finds it too unsafe or costly, at which point more sweeping occurs, and so on until the field is depleted or the campaign ends. It seems reasonable in this situation to declare that the goal of the miner is to minimize the number of ships that get through safely within the campaign time frame. Note that this is not necessarily equivalent to maximizing the number of ships that are sunk, although in many cases this may be a concomitant characteristic. The reason for this distinction in concepts is that one selection of counter-countermeasure (CCM) settings for arming delay and ship count may cause much more time to be spent in sweeping than another selection, and thereby reduce the number of transits without necessarily sinking more ships.

Since no algorithm exists for generating an absolute optimum, it is proposed to cast the measure of effectiveness as a relative criterion. That is, minefield A is superior to minefield B if, within a prescribed time period and under the same enemy program of transiting and sweeping operations, field A can be expected to permit fewer safe transits than field B. The phrase "same program" here means that the enemy employs identical rules for switching between transiting and sweeping in both cases. This conditional basis for comparison invokes explicit recognition of the very important fact that the effectiveness of CCM selections is dependent on the actions taken by the enemy and therefore cannot be measured on any independent scale. In fact, changing the assumptions about enemy behavior can easily cause a reordering of the preferences between alternative distributions of mine settings. Theoretically, one might attach weighting factors or probabilities to the potential response patterns of the enemy and then compare average or expected outcomes over this spectrum for alternative minefield configurations. In practice, however, this is likely to be exceedingly difficult to carry through on a comprehensive scale, so generally only limited experimentation is anticipated.

With a criterion now defined for making comparisons of the outcomes of minefield engagements, what remains is to provide the machinery for predicting the performance of any proposed configuration, with suitable flexibility available in the input parameters. Initial attempts were made to develop a wholly analytical model that would handle the requirements, but the combinatorial complexities that ensued quickly became unmanageable. Consequently, that goal was abandoned and the effort was redirected into formulating a fully computerized simulation. The resulting model is described in the next chapter.

However, the desire to inject some analytical flavor into the procedures would not quite die, so an additional effort was made to modify the first model and turn it into a hybrid that would combine computer simulation with certain elements of probability theory. The results of this endeavor, which appear to possess some useful advantages over the totally simulated approach, are described in Chapter 4. This will give the minefield planner a choice of which type of model he would prefer, or a basis for checking results if he is willing to try both.

To avoid repetitive coverage in the descriptions of the two models in the following chapters, a brief preview of their common features is now offered. The minefields considered are simple rectangular areas of width W on the entering side and length L for transit distance. The rectangle may be thought of as a whole field or as just a channel within which the action is confined. Each mine to be laid in the field is accorded a fixed range of effectiveness, r, such that if a ship passes within range r of a mine that is armed and on count one the ship is sunk and if it misses by more than r nothing happens. The programs as set up now permit only one type of mine per field, i.e., they all have the same r. If it is deemed necessary at some future time, modifications could probably be made to provide for handling mixed minefields involving two or more different values of r. It should be remembered that the dominant interest in developing the models was to provide means for studying the effects of varying arming delays and ship count settings, and not for exploring actuation and damage functions. With a simple cookie-cutter model there are no wasted firings or partial damage effects to be taken into account.

On some appropriate basis, or by pure guesswork if necessary, the model user decides how many mines he wants to try planting in the field and he then assigns to them any desired distributions of arming delays and ship count settings. This specific set of mines is next thoroughly randomized and scattered over the field. At a chosen time transit operations are initiated sequentially, at random entry points. Ship speed and field length determine how long a transit takes. Simulated clock time is maintained in the computer. By Monte Carlo processes the computer determines advances in ship count and the occurrence of sinkings. The computer continuously keeps track of the status of all mines relative to armed condition and ship count. When the criterion for cessation of transits is met, sweeping is begun. Each sweep takes a selectable interval of time and eliminates all armed mines on count one (or a fraction thereof) and also advances ship counts on other armed mines. Sweeps continue until a prescribed stopping criterion is reached, and then transits resume. This process continues for any preset total period or until the minefield is exhausted.

Any desired results of the engagement can be tabulated, such as the number of transits in each sequential group or block, the number of sweeps in each block, the number of sinkings, etc. Because each engagement is subject to fluctuations in outcome due to the Monte Carlo processes that are involved, a reasonable number of iterations should be performed to provide averages and variances. Various other statistical measures of interest can also be generated. The threat level at any given stage can be shown in the output.

The minefield planner can presumably extract useful clues from the results regarding the efficacy of his original choice of distributions for arming delay and ship count setting, and on that basis will propose new assignments for these factors. Keeping all other conditions constant, a new set of engagement results will be obtained by re-running the program. By this cut-and-try approach insight should be acquired, and hopefully a near-optimum range of recommended settings can be developed. Also, by playing with variations in the behavior imputed to the enemy, the sensitivity of the engagement results to such inputs can be examined. Any guidance available from intelligence sources regarding anticipated enemy practices in dealing with minefields should be heeded to try to achieve realism in applying the models. Even at best, however, one cannot expect to model enemy wartime behavior with precision because of the adaptive learning that takes place during operations and the improvisations that develop in the field to cope with emergency needs. Thus, in a very real sense there can be no rigorously determined optimum distributions of arming delays and ship count settings. However, by exercising the models extensively one should be able to acquire a better appreciation for parametric relationships and perhaps avoid egregiously ineffective minefield plans.

Chapter 3

SIMULATION MODEL I

As indicated earlier, this model of a minefield engagement is based entirely on computer simulation of sequential events, with no explicit admixture of probabilistic formulations. All of the stochastic elements involved in determining the outcomes of the events are dealt with in Monte Carlo fashion by drawing random numbers from appropriate distributions. The model simply attempts to mirror real events as they might occur. The description given here explains how the model works and what it does, but the actual computer program is not included as such. This can be provided to interested users in the form of card decks and/or tapes. The program was written in Fortran and the test runs were carried out on NOL's CDC 6400 computer.

To set the model into operation the user has to make a series of initial choices of input data. The first such choice is the size of the rectangular minefield; values of W and L are chosen in nautical miles and the average lethal radius per mine r is prescribed in yards. (The computer makes all necessary conversions among units.) Next the number of mines m to be implanted is selected, and any desired combination of arming delays and ship count settings is distributed among the m mines. The total time required for the mine-laying operation is taken to be T hours. These mines with their specific settings are thoroughly scrambled and scattered randomly over the rectangular field. This is done in the computer by drawing two coordinates, x on the width axis and y on the length axis, from random number sources suitably arranged to represent uniform probability density functions over W and L. The time of water entry for each mine is randomly drawn from the interval zero to T hours. A complete inventory listing is made of all mines in the field, including for each mine its position, its initial ship count, and the time it will become armed in the water. This completes the process of setting up the minefield.

Transits occur sequentially and do not overlap in time. The starting time for the first transit coincides with the earliest time any mine becomes armed. The time allotted per transit is obtained by dividing L by an assumed transit speed. A transit is simulated by a process that amounts to overlaying a strip of width 2r at a random entry point x. The mine listing is consulted to see if that strip contains any mines and, if so, what condition they are in. If it contains no ripe mines (armed and on ship count one), the transit is labeled safe. Any armed mines in the strip that are on count two or higher are advanced one ship count, and the inventory list is properly updated. After a safe transit the next transit starts at a random time within a selectable delay interval. Again a random x is drawn, a strip of width 2r is laid down, and the inventory list is consulted to check on the presence and condition of mines in the strip.

The above transit procedures are continued in sequence as long as the transits are found to be safe, with proper updating of clock time and the mine inventory list after each event. A record is kept of the number of safe transits in each such block. Whenever a strip that is laid down is found to contain a ripe mine, the ship is registered as sunk and the first ripe mine in the strip (the one with smallest y) is deleted from the inventory list as having been expended. The list is also checked to see if any armed mines on count two or higher are in the portion of the strip preceding the mine that caused the kill. If so, these are advanced by one count and the list is properly annotated.

Once a kill has occurred the enemy is assumed to begin sweeping after a selectable fixed delay. Each full sweep of the field is credited with removing all armed mines on count one and advancing the ship count by one on all other armed mines in the field. For sweep purposes, all mines that become armed during the sweep interval are considered in effect as being armed during the entire interval. After a sweep, which takes an assignable fixed length of time, the clock is advanced and the mine inventory list is updated. The next sweep starts immediately upon completion of the preceding one. Sweeps are continued until a selectable number of consecutive sweeps fail to explode a mine, thus completing a block. When this criterion is met transits are immediately resumed, using the procedures described above. The whole process is continued until all the mines are gone or some predetermined total time has elapsed. The sequence of events is recorded as they occur, including transits, kills, sweeps, and changes in the mines' status.

The set of events just described constitutes one run of the model. For statistical purposes such runs are iterated a number of times under identical input conditions in order to allow reasonable play for the randomized stochastic influences. In these iterations the same initial field is retained and only the subsequent random influences come into play. It is possible, of course, to replant the field if desired.

Using the accumulated run records, the program computes statistical summaries of the blocks of events. These include the average number of safe transits preceding each kill, the average number of sweeps following each kill, the average number of mines swept in each block of sweeps, the average initial threat at the start of each block of transits, the average threat just prior to a kill, and standard deviations about these averages. Other outputs now programed are the average total number of safe transits in a run, the average total number of sweeps in a run, the average total number of mines swept in a run, the average number of kills in a run, and the average number of mines expended per kill, along with standard deviations about these averages. Various other measures that are not a part of the currently programed output could also be produced if desired, since it is a simple matter of programing to retrieve any information that is generated in the course of the runs.

After careful examination of the results obtained with his first set of choices for arming delay and ship count distributions, the minefield planner should devise a new set that he thinks might improve the outcomes, i.e., lead to a reduction in the number of safe transits. The whole game can then be re-run and the results compared. By successive cut-and-try experiments of this sort, the goal is to arrive at the best possible assignment of arming delays and ship count settings that can be found. Of course, it must be remembered that an optimum thus derived pertains only to the postulated enemy pattern for transiting and sweeping. If time and patience permit, the planner should extend the experimentation to try variations in these parameters also.

For one who studies and understands the details of the present program, as written to represent the model that has been described here, it should be very easy to introduce any desired modifications. For example, if it is assumed that the enemy knows the minefield has been laid he might be expected to start sweeping right away, before risking any transits. The program can easily be adjusted to bypass the initial transits and start out with sweeps instead of transits. Also, changes in the stopping criteria for switching between transits and sweeps can readily be inserted. In the model described in the next chapter some flexibility of this sort has already been built into the program.

Chapter 4

SIMULATION MODEL II

This version of a minefield engagement model, in contrast to the one described in the preceding chapter, is of hybrid construction in that it involves a mixture of direct simulation of events and calculations of expected outcomes. Use of this model starts as before with selection of the dimensions of a rectangular field W x L in nautical miles and a lethal radius per mine of r yards. A value m is chosen for the number of mines to be laid, and any desired combination of arming delays and ship count settings is distributed among these mines. However, in this model the mines are not scattered in Monte Carlo fashion over the field and no specific position coordinates are assigned to individual mines. Instead, the number of mines in any random strip is assumed to follow a Poisson law. The way this assumption is employed in determining the outcome of transits will be described in subsequent paragraphs.

The laying of the mines is assumed to occur in random order over a period of T hours. This effect is simulated by adding a random number between zero and T to the arming delay previously assigned to each mine. A record is maintained of the number of armed mines on ship count i that are in the water at any time t. Call this number $M_{Ai}(t)$. The first ship transit starts at any desired time; this is inserted by adding an arbitrary constant to the earliest time at which any mine becomes armed. The time allotted per transit is equal to L divided by an assumed transit speed. The number of ripe mines present during a transit period is taken to be fixed as the value that $M_{A1}(t)$ has at the start of the transit. In other words, the program provides conservative results by ignoring the mines that might become armed during the transit interval. For convenience of notation in the next paragraph, use the generic designator m_1 for the number of ripe mines thus determined.

Since these m_1 mines are randomly scattered over the whole field, their average density is $\rho = m_1/WL$. The expected number of such mines in a random strip of width 2r running the length of the field is $E_{A1} = 2rL\rho = 2rm_1/W$. The assumption is now made that the actual number of ripe mines in the strip has a Poisson distribution about the expected value. Using the Poisson formula one obtains the following:

$P_{A1}(0) = \exp(-E_{A1})$ = probability of no ripe mine in the strip

$P_{A1}(\geq 1) = 1-\exp(-E_{A1})$ = probability of at least one ripe mine in the strip (this represents the initial threat)

A decision regarding the presence or absence of ripe mines in the strip is made by the computer in Monte Carlo fashion. To do this it chooses one of the above outcomes randomly in proportion to the probabilities $P_{A1}(0)$ and $P_{A1}(\geq 1)$. If the outcome is "no ripe mine in the strip", the transitor gets through safely. In this case a check is then made to see if other armed mines on higher ship counts should be advanced one ship count. From the records of $M_{A2}(t)$ the computer determines the number of armed mines on count two at the beginning of the transit. Call this m_2. Then the following quantities are calculated:

$E_{A2} = 2rm_2/W$ = expected no. of armed mines on count 2 in strip

$P_{A2}(0) = \exp(-E_{A2})$ = probability of no armed mines on count 2 in strip.

$P_{A2}(1) = E_{A2}\exp(-E_{A2})$ = probability of one armed mine on count 2 in strip

$P_{A2}(2) = \frac{1}{2}(E_{A2})^2 \exp(-E_{A2})$ = probability of two armed mines on count 2 in strip

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$P_{Ai}(2) = \frac{1}{i!}(E_{A2})^i \exp(-E_{A2})$ = probability of i armed mines on count 2 in strip.

The computer picks one of the above outcomes randomly in proportion to the calculated probabilities, and advances the ship count to one for that many mines. The inventory records for $M_{A1}(t)$ and $M_{A2}(t)$ are revised appropriately. The same kind of process is repeated for the mines on ship counts three, four, etc., and a fully updated inventory of mine status as of the end of the first transit is generated. At this point any desired fixed or randomized delay interval before the second transit starts can be inserted on the time axis. Using the updated inventory of mine status as of the time of the second transit, the computer proceeds through the same process as before to determine whether the second transitor gets through safely or is sunk. This procedure continues as long as successive transitors get through safely, and a record is kept of the number of safe transits.

Unless the minefield is totally ineffective it can be expected that at some stage in the Monte Carlo selections the computer will produce the outcome "at least one ripe mine in the strip." When this occurs that transitor is considered sunk, and the computer deletes one armed mine on count one from the inventory of such mines.

Since there is a chance that the "sunk" transitor might have advanced the ship count on some armed mines in its strip prior to being sunk, the computer must Monte Carlo the occurrence of such events. As before $E_{A2} = 2rm_2/W$ is calculated, using m_2 from the properly updated inventory record. Then values of $P_{A2}(i)$ for i = 0, 1, 2 --- are calculated from the Poisson formula as before, and one of the outcomes is randomly chosen proportionally to the probabilities. If the i so chosen is not zero, each of these i mines is given a probability p of being encountered before the mine that did the sinking, where p is a randomly drawn fraction of the transit distance. The computer then decides by new random drawings from zero to one, using p for a decision criterion, how many of the i mines should be advanced to ship count one and revises the inventory records accordingly. This same process is also carried out for mines on ship count three, four, etc.

After a sinking, an option is provided for either continuing with more transits or starting sweeping operations. If transits are continued the processes described above are simply repeated. Any desired "stopping rule" can be adopted, such as stopping transits after one sinking or after two sinkings, three sinkings, etc., or this number can be randomly drawn from the integers one to k. Whatever the choice, at some point in time sweeping is to be started. This time may be set as a fixed interval after the last sinking or after a randomly selected delay following the last sinking. Each sweep requires some fixed interval of time, and it may be assumed to be totally effective in triggering all mines in condition to be triggered (i.e., destroying all armed mines on count one and advancing all other armed mines by one count), or it may be assumed to have some fractional effectiveness such as 50% or 80%, for example.

If the values of $M_{Ai}(t)$ are constant within a sweep interval, those constant values are used to obtain the numbers of mines to be removed or advanced. If the sweep period extends over changes in the values of $M_{Ai}(t)$, weighted average values are calculated. Since the sweeping process is assumed to involve only influence effects, rather than mechanical removal, there is no effect on unarmed mines. The computer simulates totally effective sweeping simply by deleting from the inventory record all armed mines on count one and revising the numbers on other counts. For a sweep of fractional effectiveness F, the inventory counts are changed only by the fractional amount F instead of by the total amount corresponding to 100% effectiveness. (If multiplying by F yields non-integral values, rounding to a nearest integer is done so as to deal only with whole numbers of mines.)

Successive sweep cycles are carried out according to any desired rule for their continuance or termination. The possibilities provided are:

- sweep exactly k times, k = 1, 2, ---
- sweep a random number of times, from 1 to k

c sweep until k consecutive sweeps occur with no mine explosions, k = 1, 2, ---(the number can also be randomly drawn if desired).

When the criterion for stopping the sweeping cycles is reached, transits are resumed by reverting to the procedures described earlier and using properly updated records of mine status. Transits are continued sequentially until the criterion for stopping them is reached, and then sweeping is resumed again. The whole process runs until some preselected stopping time is reached or all the mines are gone. A record is maintained of all events as they occur.

The process described above constitutes one sample run of the model. To allow suitable play for the random processes in the Monte Carlo drawings and to gather statistical evidence, the runs are iterated a number of times under the same input conditions. Using run results accumulated in memory, the computer produces statistical summaries of the blocks of events. These are identical in form with those of Model I, but for the reader's convenience they will be repeated here.

The printed results include the average number of safe transits preceding each kill, the average number of sweeps following each kill, the average number of mines swept in each block of sweeps, the average initial threat at the start of each block of transits, the average threat just prior to a kill, and standard deviations about these averages. Other outputs are the average total number of safe transits in a run, the average total number of sweeps in a run, the average total number of mines swept in a run, the average number of kills in a run, and the average number of mines expended per kill, along with standard deviations about these averages. Other measures can be produced if desired by programing the computer to retrieve any information that is generated in the course of the runs.

As discussed previously, the user should study the results obtained for his first selection of distributions for arming delay and ship count settings and attempt to devise a better combination. After a number of experiments, hopefully he can approach an optimum, although there is no formal way to establish when an optimum has been found or how sensitive it might be to variations in enemy behavior patterns.

Chapter 5

SAMPLE RESULTS

To test the programs that were written to implement the previously described models a purely hypothetical minefield was set up with the following characteristics:

width = 3 nm

length = 6 nm

no. of mines = 450

lethal radius = 35 yds

laying time = 24 hrs

transit time = 0.6 hrs

sweep time = 12 hrs

The distributions of arming delay and ship count settings are as shown in the following table, where the entries indicate the number of mines in each initial condition.

		Arming Delay (hours)							
		1	3	24	72	168	336	840	Totals
Ship Count	1	6	5	11	7	5	10	46	90
	2	6	5	12	6	5	10	46	90
	3	6	5	12	6	5	10	46	90
	4	6	5	12	6	6	9	46	90
	5	7	5	11	6	6	10	45	90
Totals		31	25	58	31	27	49	229	450

Other input conditions can be described as follows. The time delay between consecutive transits is a randomly drawn number between 6 and 9 hours. A block of transits terminates when one kill occurs, and sweeping begins after a fixed delay of 36 hours following a kill. There is no delay between consecutive sweeps, and sweeping terminates after three consecutive sweeps occur with no mine blasts. Transits resume immediately after a block of sweeps terminates. A run terminates after 1000 hours have elapsed or all the mines are wiped out, whichever occurs sooner. Runs are iterated 100 times to obtain statistical results on the outcomes.

It turned out for both models that the first transit started at a computer clock time of 1.2 hours, and that there were always three blocks of transits and sweeps before the mines were all wiped out. Specific results obtained from Model I were as follows:

	No. of Transits		No. of Sweeps		No. Mines Swept	
	Mean	Sigma	Mean	Sigma	Mean	Sigma
Block 1	6.56	3.67	15.89	2.49	171.00	.00
Block 2	15.57	6.52	8.00	.00	48.00	.00
Block 3	40.11	6.84	8.00	.00	228.00	.00
Per Iter.	62.24	4.07	31.89	2.49	447.00	.00

The threat at the beginning of each block of transits turned out to be zero in each case, but the threat at the time of the last (fatal) transit in each block provided the following statistics:

	Threat at Kill	
	Mean	Sigma
Block 1	.20	.07
Block 2	.11	.02
Block 3	.36	.10

The corresponding results obtained by running Model II with the same inputs were very similar to those for Model I, as shown in the following:

	No. of Transits		No. of Sweeps		No. Mines Swept	
	Mean	Sigma	Mean	Sigma	Mean	Sigma
Block 1	6.34	3.30	15.73	2.32	170.73	2.69
Block 2	16.45	7.66	8.00	.00	48.27	2.69
Block 3	39.92	7.84	8.00	.00	228.00	.00
Per Iter.	62.71	3.98	31.73	2.32	447.00	.00

	Threat at Kill	
	Mean	Sigma
Block 1	.20	.07
Block 2	.11	.03
Block 3	.35	.10

The comparability between the various computed outcomes for the two models is sufficiently close to give a high degree of confidence that both models validly represent the engagements as depicted. It is hoped that these models will be put to use by others, and that the users will provide feedback to the authors regarding any modifications and extensions that could enhance the usefulness of the present versions.