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INVESTIGATION OF A
SPLIT-PATH NONLINEAR FILTER

THESIS

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INVESTIGATION OF A

SPLIT-PATH NONLINEAR FILTER

THESIS

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by

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Preface

The purpose of this study was to investigate the use of a split-path nonlinear (SPAN) filter, a concept developed by the Douglas Aircraft Company, to compensate various selected simple control systems and compare the results with those obtained by linear compensation. If favorable results were obtained with the SPAN filter, I had also planned to investigate its application to a complex control system. However, in my opinion, the results obtained with the simple control systems did not warrant the investigation of the application of the SPAN filter to a complex control system.

I would like to gratefully acknowledge the helpful comments and suggestions of Professor John J. D'Azzo, my Faculty Thesis Advisor, Captain Robert R. Rankine, my Thesis Sponsor from the Air Force Flight Dynamics Laboratory, and Mr. Bernard J. Doody from the Directorate of Systems Dynamic Analysis, Systems Engineering Group. I would further like to thank Professor D'Azzo for his thorough review of the manuscript.

John A. Tondl, Jr.

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Abstract

Use of a split-path nonlinear (SPAN) filter, a concept of the Douglas Aircraft Company, for compensating control systems is investigated by analog computer and the results are compared with those obtained with linear compensating filters. Comparisons are made of the responses of four systems to step, ramp, and sinusoidal inputs using the ITAE (integral of time multiplied by the absolute value of error) as the criterion for judging performance. Compensated system sensitivity to gain variations is also investigated. Use of the SPAN filter as a compensator showed little or no over-all improvement compared with that obtained with a less complex linear filter.

INVESTIGATION OF A SPLIT-PATH NONLINEAR FILTER

I. Introduction

Statement of the Problem

The interdependence of the phase and gain characteristics of linear filters is generally in conflict with the desires of the control engineer. The concept of the split-path nonlinear filter as developed by the Douglas Aircraft Company is to provide a device with independent control of gain and phase that may have wide application as a control system compensator. The Douglas Aircraft Company refers to this device as the SPAN filter and this notation will be used throughout this report. The purpose of this study is to investigate the use of the SPAN filter for the compensation of various control systems and to compare the results with those obtained with linear compensating filters.

Basis for Study

The basis for this study is the proprietary information contained in an unsolicited proposal submitted to the Air Force by the Douglas Aircraft Company which is listed as reference 2. Another paper written on the SPAN filter, which became available to the author after the writing of this manuscript, is listed as reference 3.

Background Information

The concept of improving the performance of control systems by the inclusion of nonlinear elements and effects originated about 1950 (Ref 4:446). Since that time, considerable effort has been expanded in this area. Several types of nonlinear compensators, which have improved the performance of certain systems, can be found in the literature. However, each of these nonlinear compensators was designed to improve the performance of a particular system which contained a specific nonlinearity such as backlash, hysteresis, and saturation. These nonlinear compensators are limited in application to a restricted number of systems in that the advantages achieved with a particular design are accompanied by certain disadvantages or limitations.

II. Theoretical Analysis of the SPAN Filter

Description of the SPAN Filter

A block diagram of the SPAN filter is shown in Figure 1. The device processes the input through two separate branches or paths and takes the product of the two resulting signals. The concept of operation of this filter is that the branch containing the bistable element controls the phase of the output signal and the branch containing the absolute value function controls the magnitude of the output signal (Ref 2:3). The output of the bistable element is +1 when its input is positive and -1 when its input is negative. As magnitude and phase are controlled in separate branches, independence of these two characteristics should result to at least some degree. This would be advantageous compared with linear filters since the magnitude attenuation

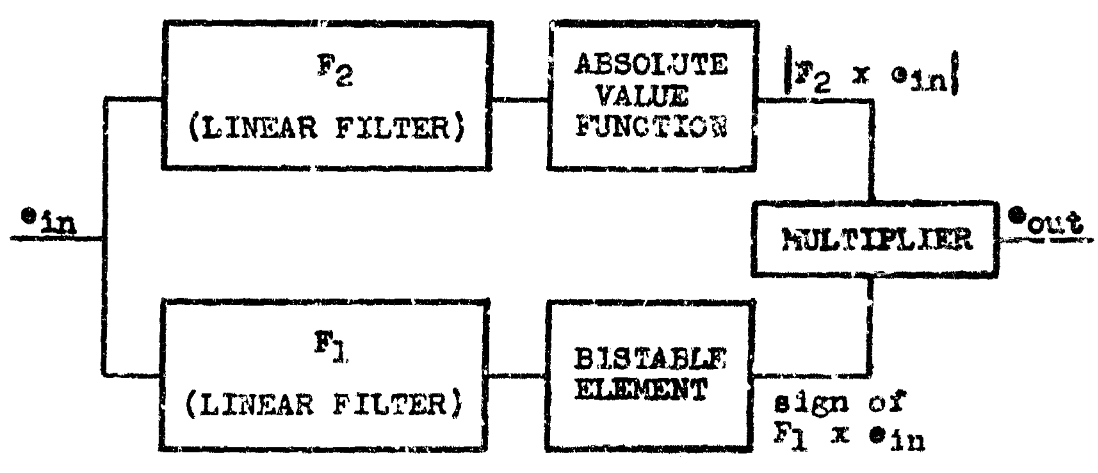


FIGURE 1

BLOCK DIAGRAM OF SPAN FILTER

over a specified frequency range provided by a linear lag network is also accompanied by a phase lag and the phase lead over a specified frequency range provided by a linear lead network is accompanied by a magnitude gain (Ref 2:3).

A better understanding of SPAN filter operation can be obtained by processing a sinusoidal wave through the filter. Consider a SPAN filter where the linear filter F_1 is a lead network and linear filter F_2 is a unity gain. The signals throughout the SPAN filter will have the form shown in Figure 2. The fundamental component of the output signal will lead the input signal. The peak amplitude of the output signal is unchanged, but the fundamental component will be attenuated slightly because of harmonic generation (Ref 2:4,5). Figure 3 shows the signals throughout the SPAN filter when F_1 is a lead network and F_2 is a lag network. It should be noted that the sinusoidal wave is "chopped" by an amount equal to the numerical sum of the phase shifts in both branches.

Analysis of the Gain and Phase Characteristics

The bistable element branch of the SPAN filter destroys all magnitude information while maintaining the phase information. The information from the absolute value branch, however, will include phase shift as well as magnitude if the linear filter F_2 is something other than a unity gain network (Ref 2:4).

Fourier analysis was used to analyze the gain and phase characteristics of the SPAN filter for a sinusoidal input. The output of the SPAN filter was expanded in a Fourier series of the form

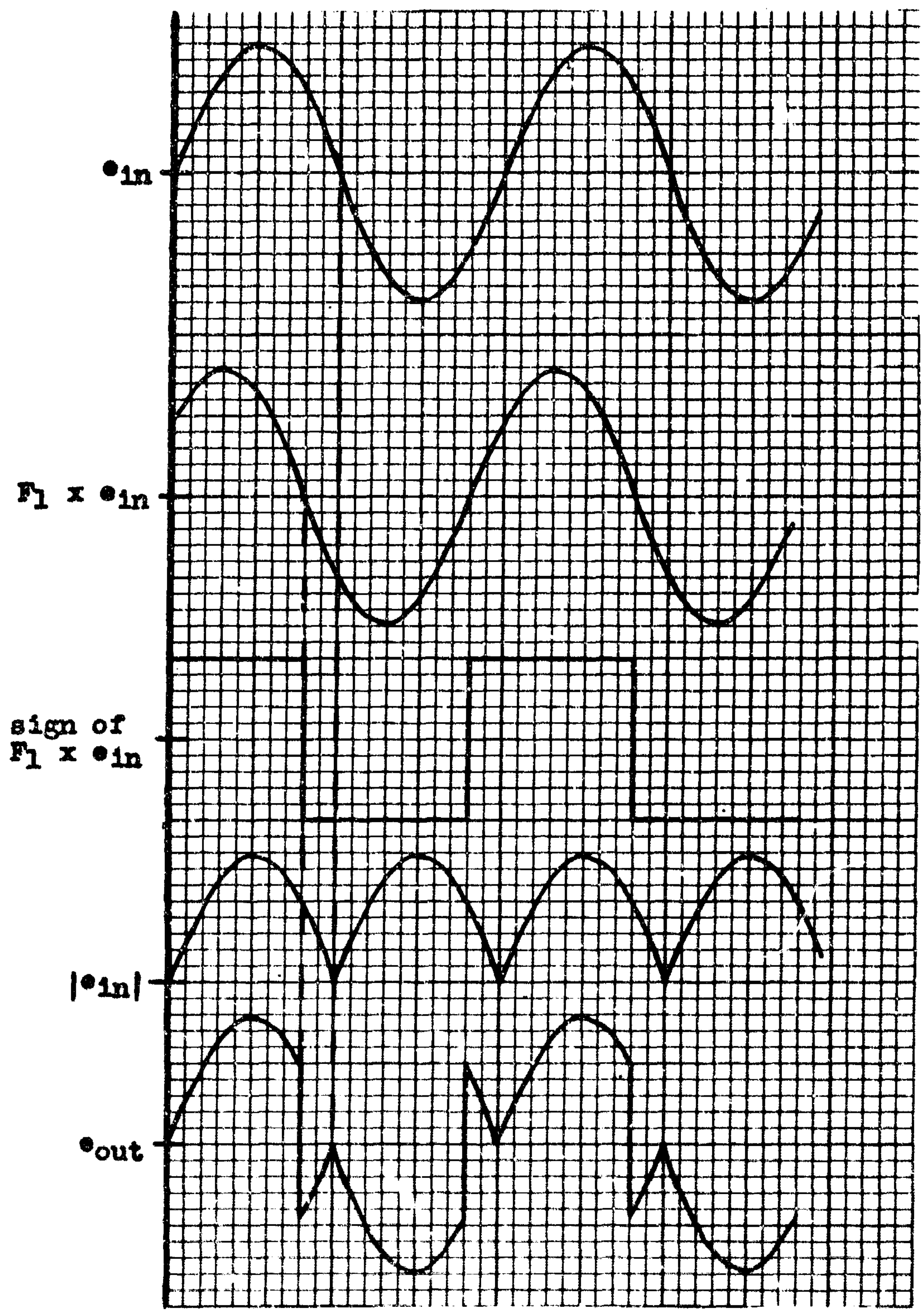


FIGURE 2

SPAN FILTER WAVEFORMS FOR F_1 PHASE LEAD & F_2 UNITY

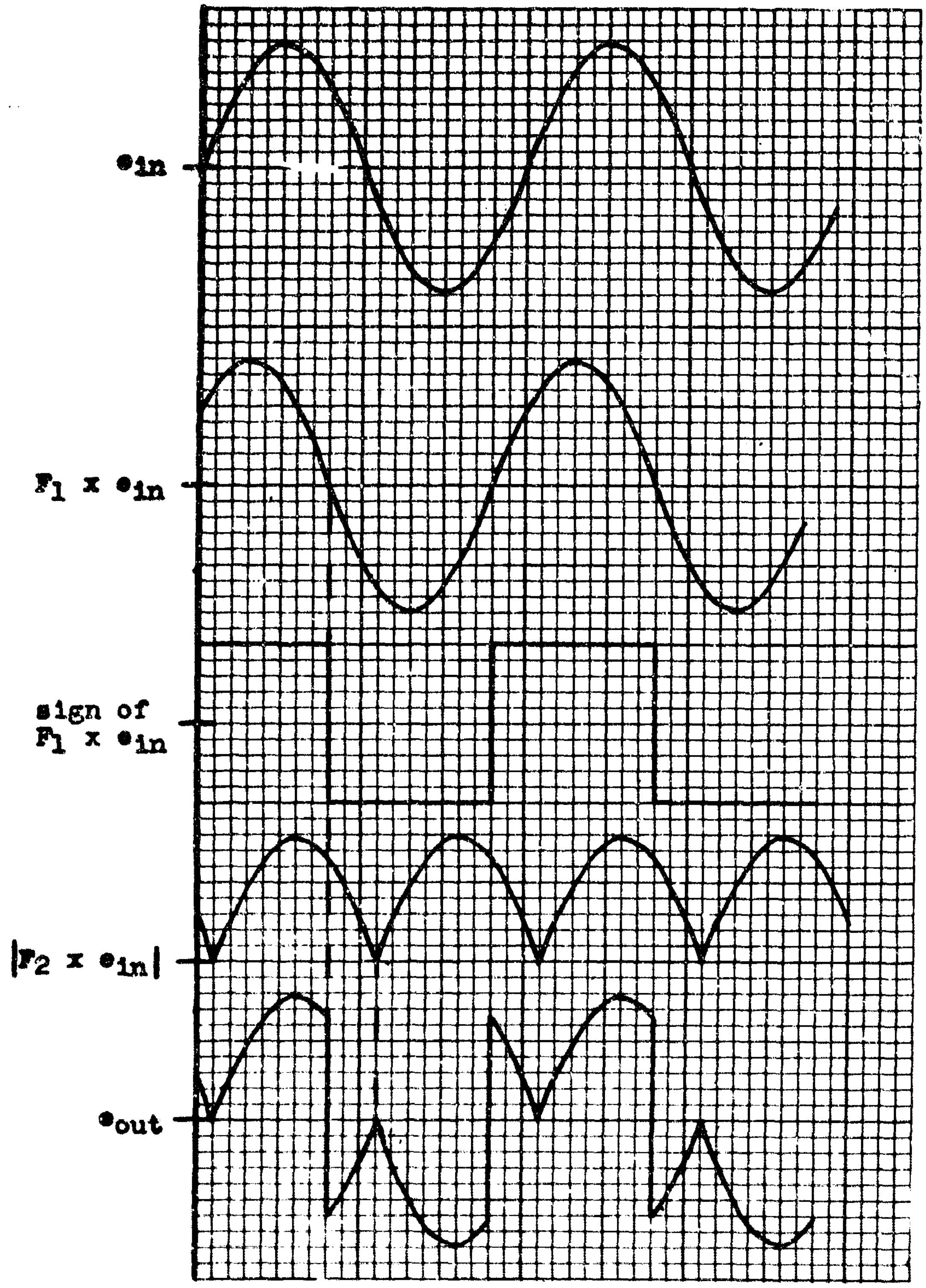


FIGURE 3

SPAN FILTER WAVEFORMS FOR F_1 PHASE LEAD & F_2 PHASE LAG

$$f(\omega t) = \sum_{n=1}^{\infty} C_n \sin(n\omega t + \phi_n) \qquad (1)$$

where C_n and ϕ_n are the amplitude and phase of the harmonic respectively. The output of the SPAN filter has half-wave symmetry and therefore contains no even harmonics (Ref 7:328).

Two configurations of the SPAN filter were examined. In the first case, F_1 was a lead network and F_2 a unity gain. In the other case, F_1 was a lead network and F_2 was a lag network. Circuit diagrams and transfer functions of the lead and lag networks used throughout this study are shown in Figure 19, Appendix B. A detailed discussion of this particular Fourier series is contained in Appendix A while amplitude and phase data for several of the harmonics are tabulated in Table I.

It should be noted that the amplitude of each harmonic is the same as long as the numerical sum of the phase shifts in both branches has the same value. However, the phase angle of each harmonic changes as different combinations of lag and lead networks are used to obtain a given numerical sum of the phase shifts in the two branches. Therefore, obtaining a given phase shift from F_1 alone produces a larger phase angle for the fundamental frequency of the output than when a given phase shift is obtained by use of filters for both F_1 and F_2. It is also important that, unlike most nonlinear compensation schemes, the phase-gain characteristics of the SPAN filter are not amplitude dependent.

The harmonic content of the SPAN filter can be quite large. For example, when the amount of phase shift from both F_1 and F_2 equals

TABLE I

HARMONIC CONTENT OF SPAN FILTER OUTPUT

Phase Shift		Harmonics							
		1st		3rd		5th		7th	
F_1[a]	F_2[b]	Amp[c]	Phase[d]	Amp	Phase	Amp	Phase	Amp	Phase
10°	0°	0.99	1	0.02	110	0.02	123	0.02	137
20°	0°	0.98	4	0.07	130	0.07	157	0.06	-176
25°	0°	0.97	7	0.11	140	0.10	174	0.09	-152
30°	0°	0.96	10	0.16	150	0.14	-169	0.12	-127
40°	0°	0.91	17	0.26	170	0.21	-134	0.15	- 74
50°	0°	0.84	26	0.37	-170	0.26	- 98	0.15	- 17
60°	0°	0.77	38	0.48	-150	0.28	- 60	0.14	60
70°	0°	0.70	52	0.56	-130	0.26	- 17	0.16	130
80°	0°	0.66	70	0.62	-110	0.23	32	0.19	-151
90°	0°	0.64	90	0.64	- 90	0.21	90	0.21	- 90
20°	20°	0.91	- 3	0.26	150	0.21	-154	0.15	- 94
40°	20°	0.77	18	0.48	-170	0.28	- 80	0.14	40
60°	20°	0.66	50	0.62	-130	0.23	12	0.19	-171
80°	20°	0.66	90	0.62	- 90	0.23	128	0.19	- 49
20°	40°	0.77	- 2	0.48	170	0.28	-100	0.14	20
40°	40°	0.66	30	0.62	-150	0.23	- 8	0.19	169
60°	40°	0.66	70	0.62	-110	0.23	108	0.19	- 69
80°	40°	0.77	102	0.48	- 70	0.28	-160	0.14	80
20°	60°	0.66	10	0.62	-170	0.23	- 28	0.19	149
40°	60°	0.66	50	0.62	-130	0.23	88	0.19	- 89
60°	60°	0.77	82	0.48	- 90	0.28	180	0.14	60
80°	60°	0.91	103	0.26	- 50	0.21	-106	0.15	-166
55°	55°	0.70	73	0.56	-105	0.26	142	0.16	- 14
90°	65°	0.97	173	0.11	- 25	0.10	- 59	0.09	- 93
90°	90°	1.00	180	0.00	0	0.00	0	0.00	0

[a]Lead phase shift obtained from linear filter F_1

[b]Lag phase shift obtained from linear filter F_2

[c]Amplitudes of harmonics based on a SPAN filter output with a peak value of unity

[d]Phase angle of harmonic rounded off to nearest degree

90°, the magnitudes of the 3rd, 5th, and 7th harmonics are 1.0, 0.33, and 0.33 of the fundamental respectively. Also, the 5th harmonic is "in phase" with the fundamental. Unless the plant being compensated greatly attenuated the higher order harmonics, the control system output could be quite distorted.

Figure 4 is a plot of attenuation of the fundamental compared to the amplitude of the output signal versus the numerical sum of the phase shifts in both branches. A maximum attenuation of 3.92 db occurs at a phase shift of 90°.

From the above analysis one can conclude that the first harmonic gain and phase characteristics of the SPAN filter are almost independent when a linear filter is included only in the bistable element branch. The degree of independence is reduced when a lag network is used for F_2 as the phase shift of filter F_2 is accompanied by a magnitude attentuation. However, this degree of phase-gain independence is not without penalty in that the SPAN filter output has a high harmonic content.

Selection of a SPAN Filter Configuration

In the previous section, the SPAN filter was discussed in two configurations: (1) F_1 a lead network and F_2 unity gain, and (2) F_1 a lead network and F_2 a lag network. Two other configurations could be obtained by reversing the roles of F_1 and F_2. If F_1 were a lag network and F_2 were either a unity gain or lead network, the fundamental of the output would lag the input. This configuration may have application but was not investigated during this study. It was assumed that the response of the plants studied could be best improved

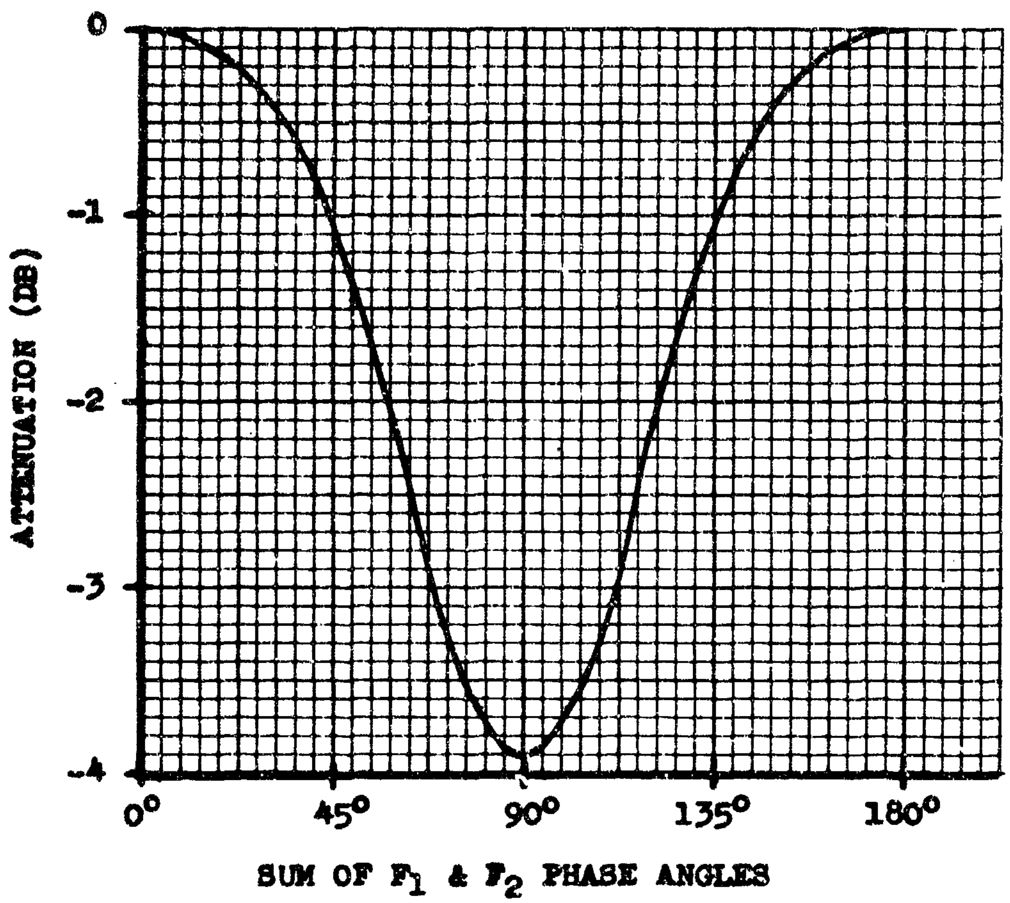


FIGURE 4

EFFECT OF F_1 & F_2 PHASE ANGLES ON THE AMPLITUDE OF THE FUNDAMENTAL FREQUENCY OUTPUT OF THE SPAN FILTER with the generation of a lead angle by the SPAN filter, that is, by moving the 180° crossover point to a higher frequency.

Other possible configurations using simple linear lead or lag networks could be obtained by using two lead or two lag networks for both F_1 and F_2. In either case, no advantage would be achieved. If F_1 and F_2 are the same type of network but not identical, the resulting phase angle of the fundamental component of the SPAN filter output will be less than that obtained when F_1 is a lead or lag network and F_2 is a unity gain. If F_1 and F_2 are the same identical networks, the output of the SPAN filter will be exactly the same as that obtained from a linear compensating filter. This was verified on the analog computer and no further investigation was made on these two configurations.

The SPAN filter with F_1 as a lead network and F_2 as either a unity gain or lag network were the only configurations chosen for analog computer study.

Describing Functions

The describing function is a linear "approximately equivalent transfer function" which is used for approximating the effect of a nonlinear element or characteristic. This approximation is made in the frequency domain by defining the describing function in terms of a Fourier series for the response of the nonlinear element to a sinusoidal input. The definition of the describing function requires that the d-c term of a Fourier series be equal to zero and that all harmonics other than the fundamental be negligible (Ref 9:139,146). Using the notation of equation (1), the describing function is given by

$$G_D(jw) = \frac{C_1 \angle \phi_1}{A \angle 0^\circ} \tag{2}$$

where the input to the nonlinear component is A sin wt.

In Table I it was shown that, for a sinusoidal input, the harmonics in the output of the SPAN filter are not negligible when the numerical sum of the phase shifts obtained from F_1 and F_2 is between 25° and 155°. This Table also shows that the phase angles of the fundamental frequency of the SPAN filter output, for the phase shifts of 25° and 155° mentioned above, are 6.7° and 173.3° respectively. In the majority of possible SPAN filter applications, the desired phase angle of the fundamental will probably lie within these limits. Consequently, a describing function calculated by the above method would be of doubtful value.

In spite of their doubtful validity, describing functions were calculated for the SPAN filter in the two configurations chosen for analog computer study. It was hoped that "ballpark" values would be obtained for the time constants of the linear lead and lag networks for the purpose of optimizing plant response using the SPAN filter.

For a given phase shift obtained from the SPAN filter, the describing function will have a magnitude and phase and will appear as a point on a log magnitude-angle diagram or Nichols chart. As the phase shift obtained by both the linear lead and lag networks is a function of frequency, the describing function was plotted as a function of wT.

The first describing function was claculated for a SPAN filter with F_1 as a lead network and F_2 as unity gain. The lead network used has a transfer function

$$G_c(jw) = \frac{1 + jwT}{1 + j0.1wT} \tag{3}$$

The maximum phase shift attainable from this particular lead network is approximately 55°. A Nichols chart of this describing function plotted as $-1/G_D$ appears in Figure 5. Each point of this curve is the stability point (psuedo 180° crossover point) for a particular value of wT. The stability point is seen to have a maximum displacement at $wT = 3$ where $-1/G_D = 1.81\text{db}\ \underline{/-211.8^{\circ}}$.

The first trial value of the time constant to give optimum performance for a particular plant of system was calculated by determining the frequency at which the uncompensated plant had a phase angle of -211.8°. It is shown in Part IV of this report that this procedure came within a factor of 2 or 3 of the desired time constant. Although not particularly accurate, it did provide a starting point for an analog computer study. This procedure has no application to a plant such as K/s^2. The final network F_1 used in the SPAN filter was adjusted by trial and error until the best system performance was obtained.

Other describing functions were calculated for the SPAN filter when F_1 was a lead network and F_2 was a lag network with a transfer function

$$G_c(jw) = \frac{1 + jwT}{1 + j\alpha wT} \tag{4}$$

where α is the pole-zero ratio and is greater than 1. These describing functions were determined to be of no value for selecting the

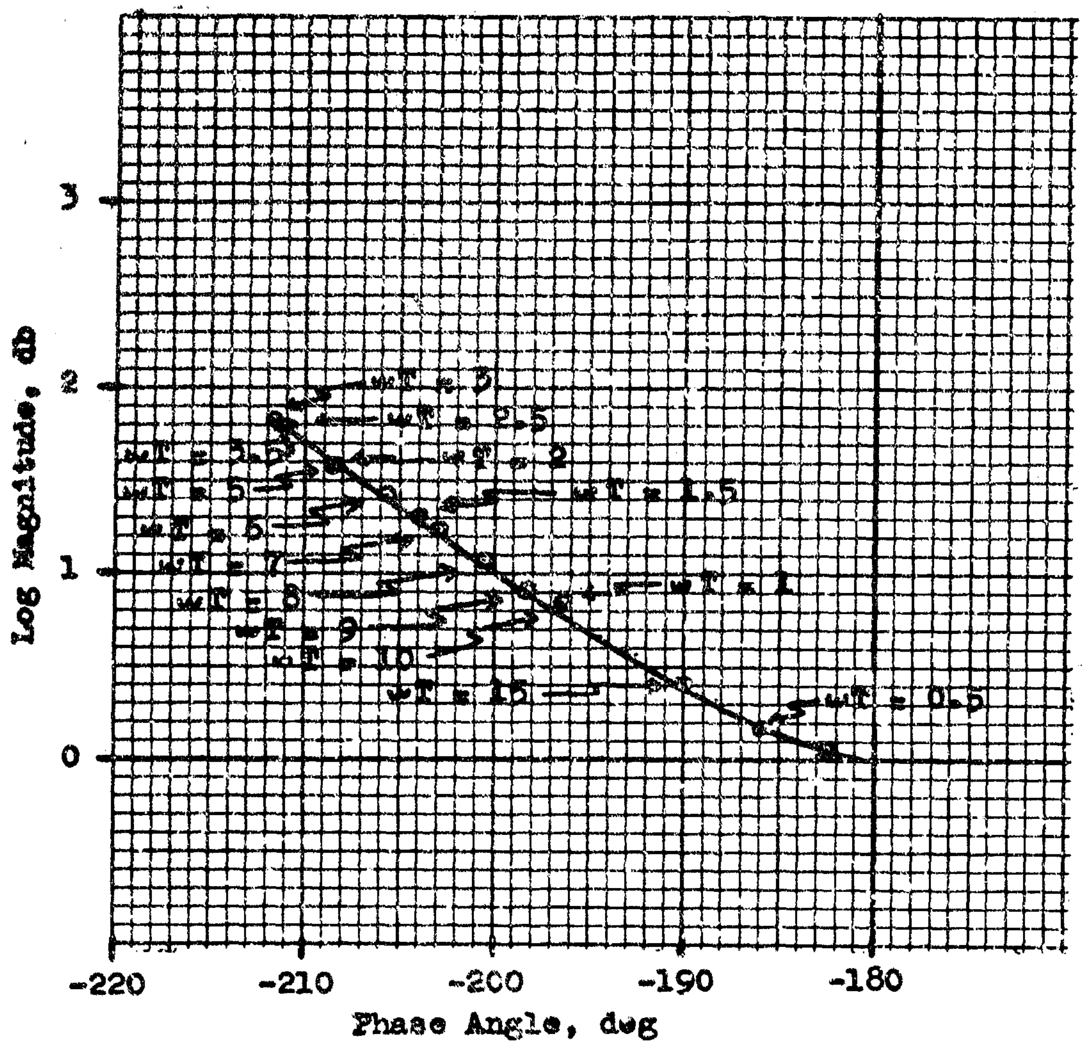


FIGURE 5

DESCRIBING FUNCTION FOR SPAN FILTER

WITH F_1 A LEAD NETWORK ($\alpha = 0.1$) AND F_2 UNITY GAIN linear filter network time constants. This result was anticipated as the harmonic content of the SPAN filter output is high for this configuration. No data for these describing functions is included in this report.

III. Analog Computer Analysis of the SPAN Filter

Method of Analysis

The purpose of this analog computer study was to obtain closed loop system response data for both SPAN filter and linear filter compensated unity feedback systems. The ITAE (integral of time multiplied by the absolute value of error) was chosen as the criterion by which to evaluate both linear and SPAN filter compensated systems for optimum performance to a step function input. It is one of the better over-all figures of merit for a zero steady-state step error system (Ref 1:587).

Ramp and sinusoidal inputs were also used to determine how well both the optimized SPAN and linear filter compensated systems followed time varying signals. The final check made on each compensated system was to determine its sensitivity to gain variations. This was accomplished by using step function inputs and obtaining the ITAE value for different values of system gain.

The linear compensating filter used during this study was a lead-lag compensator with a transfer function of

$$G_c(s) = \frac{(1 + T_1 s)(1 + T_2 s)}{(1 + 10T_1 s)(1 + 0.1T_2 s)} \tag{5}$$

This filter was chosen as it has the advantages of both the lead and lag compensators and is therefore one of the better linear compensators (Ref 1:452).

The Model 16-31R Electronic Associates, Inc., Analog Computer located in Building 57, Area B, Wright-Patterson AFB, Ohio, was used for this study.

Imposed Restrictions for Computer Analysis

The pole-zero ratios in both the lead and lag networks used in the SPAN filter were limited to a maximum value of 10. This was also the value used for the pole-zero ratio in the linear lead-lag filter. The ranges of alpha were therefore limited to $1 < \alpha_1 < 10$ and $0.1 < \alpha_2 < 1$.

The sinusoidal inputs were restricted to the range of 0.5 to 10.0 radians/second. This is a typical range of input frequencies to a mechanical control system.

The ITAE used as the optimum performance criterion is a time weighted function which increasingly penaltizes system errors as time progresses. Due to the long time constants normally associated with lag networks, a considerable amount of time can elapse before the error is reduced to zero. In order to reduce the time required for the output of the circuit simulating the ITAE function to reach some steady-state value, the input of the ITAE circuit was modified so that the integration stopped when the error signal became less than a preselected value. This value was established at less than 2% of the system steady-state output. Within this limit, it was varied slightly from system to system, but once selected for a system, it was held constant regardless of the compensating filter used.

Description of Test Plants Used

Four different plants were compensated with both the linear lead-lag and SPAN filters during the analog computer simulation. Each was sufficiently different from the other three so that the SPAN filter's application to a variety of systems might be investigated.

The four basic unity feedback systems chosen for this investigation have the following forward transfer functions:

$$G(s) = \frac{K}{s^2} \tag{8}$$

$$G(s) = \frac{K}{s(s + 1)(s + 2)} \tag{9}$$

$$G(s) = \frac{K}{s(s^2 + s + 4)} \tag{10}$$

$$G(s) = \frac{K(10 - s)}{s(s + 20)} \tag{11}$$

The first system listed is unstable for all values of gain. The second and third systems are stable but have unsatisfactory transient responses. Also, both systems become unstable at relatively low values of gain. The second system has its open loop poles on the real axis whereas the third system has a pair of complex poles located close to the imaginary axis. The fourth system differs from the other three in that it is a non-minimum phase system, having a zero in the right-half of the s-plane. This system is covered in reference 2 and therefore serves to correlate the results obtained in this study. The unsatisfactory system response of this system for a step function input has both an undershoot, due to the zero in the right-half plane, and an overshoot. The root locus of each of the four systems must be reshaped to improve system stability and/or transient response.

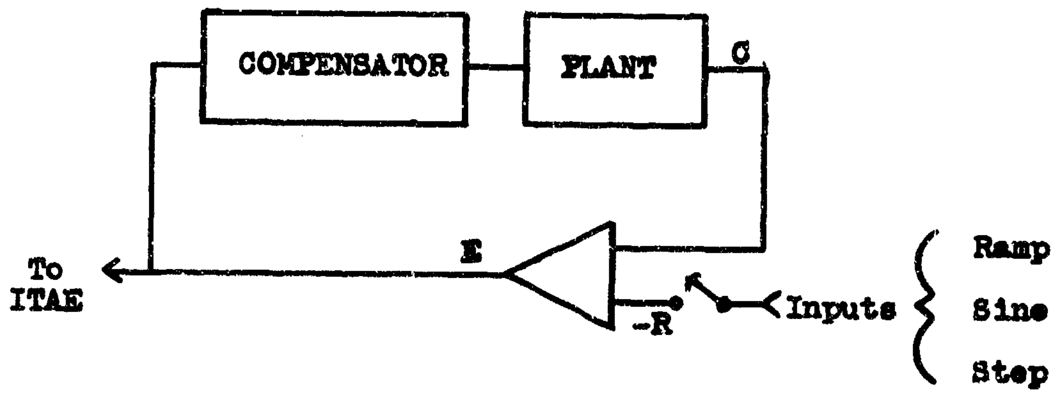


FIGURE 6

BLOCK DIAGRAM OF ANALOG COMPUTER SIMULATION FOR UNITY FEEDBACK SYSTEMS

Analog Computer Simulation of the Unity Feedback Systems

A block diagram of the analog computer simulation for the unity feedback systems is shown in Figure 6. Analog computer circuits for the four plants, the linear lead-lag filter, the SPAN filter, and the ITAE function were designed by the author and are contained in Appendix C. Design of the four plants was straightforward and will not be discussed. However, some discussion is in order for the simulation of the ITAE function, the SPAN filter, and the linear lead-lag filter.

Although the design of the ITAE circuit, shown in Figure 22, was straightforward, the modification to the circuit that was mentioned previously will be discussed. Function relay #2 was incorporated to make the error input to the multiplier zero when the error became less than 2% of the system steady-state output. The function relay logic is such that if the algebraic sum of the two relay inputs is positive, the relay is up, and if the algebraic sum is negative, the relay is down (Ref 8:35). One input to relay #2 is the output of the absolute value function circuit which is always positive and the other input is connected to the output arm of a potentiometer with a negative voltage applied. As a result, when the magnitude of the voltage output of the absolute value circuit is less than the magnitude of the potentiometer voltage, the input to one-half of the multiplier will be zero or ground.

References 5 and 6 were consulted for possible circuits that could be used for simulation of the linear lead-lag filter. The undesirable feature of the given circuits was that more than one potentiometer had to be adjusted to effect a change in one parameter. It was considered desirable to obtain a circuit in which each variable parameter could be controlled by a single potentiometer. This would simplify the process of changing a parameter value and because of this simplicity, improve accuracy. The desired circuit was achieved and is shown in Figure 23. This reduction in the number of potentiometers was also applied to the design of the lead and lag networks included in the SPAN filter simulation.

When the basic design of the SPAN filter simulation was tried on the analog computer, it was observed that the filter output was not zero for a zero input. By analyzing the circuit it was discovered that the single relay originally used to simulate the bistable element would oscillate or chatter when the error input approached zero. This relay chatter was alternately applying plus and minus 100 volts input to one-half of the multiplier which was sufficient to cause the quarter-square diode multiplier to generate a small output. To remedy this situation, two additional function relays were added to the circuit (see Figure 24) for the purpose of grounding both inputs to the multiplier when the error voltage became negligible (less than 0.1 volt). Specifications on the multiplier indicate that its output will be less than ± 2 millivolts when both inputs are zero (Ref 8:18). The logic of the circuitry incorporating relay #4 and #5 is that the output of the bistable element simulation is as follows:

$$e_{out} = \begin{cases} 100 \text{ volts for } e_{in} > 0.1 \text{ volt} \\ 0 \qquad \text{for } -0.1 < e_{in} < 0.1 \text{ volt} \\ -100 \text{ volts for } e_{in} < -0.1 \text{ volt} \end{cases}$$

<u>Procedure for Determining Filter Parameters for Optimum Performance</u>

<u>Linear Lead-lag Compensator</u>. The root locus of each plant was examined to determine the most likely time constants for the lead network portion of the linear lead-lag compensator with which to obtain optimum performance. Repeated analog computer runs were made for various values of time constants and gains in search of "the" optimum operating point.

<u>SPAN Filter</u>. System performance was optimized with both of the selected SPAN filter configurations. The first configuration used on each of the four plants had F_1 as a lead network and F_2 a unity gain. It will be referred to as configuration I for the remainder of this report. For this configuration, the describing function shown in

TABLE II

SPAN FILTER LEAD NETWORK TIME CONSTANT
FOR OPTIMUM PERFORMANCE
($\alpha_2 = 0.1$)

PLANT	METHOD	
	DESCRIBING FUNCTION	ANALOG COMPUTER
Type 2	*	0.34 seconds
Type 1 (Real Poles)	1.15 seconds	0.76 seconds
Type 1 (Complex Poles)	1.27 seconds	0.50 seconds
Non-minimum Phase	0.11 seconds	0.10 seconds

*Describing function method not applicable to K/s^2 plant

Figure 5 was used to obtain a time constant value which served as a starting point in the analog computer search for the optimum operating point. The time constant was calculated by determining the frequency at which the uncompensated plant had a phase angle of -211° and dividing this into 3. The value $wT = 3$ provides the maximum displacement of the pseudo 180° crossover point. Repeated analog computer runs were made for various values of time constants and gains. Table II is a comparison of the lead network time constants, for a pole-zero ratio of 10, that were obtained first by use of the describing function and then by optimization of the response on the analog computer. Based on the limited number of plants compensated, it can be concluded that the accuracy of selecting an optimum time constant value by use of the describing function is not only dependent upon the harmonic content of the SPAN filter output but is also dependent upon the plant being compensated.

The second configuration of the SPAN filter, hereafter referred to as configuration II, had F_1 as a lead network and F_2 as a lag network. After many analog computer runs in which both lead and lag time constants and pole-zero ratios were varied, it was concluded that the value of the time constant and pole-zero ratio selected for the lead network in configuration I were the best choices for the lead network in configuration II. No guide was found for determining time constant and pole-zero ratio values for the lag network. Optimization of plant performance was obtained through a trial and error procedure. Of the four plants compensated, only the two Type 1 plants showed improved performance with configuration II over that obtained with configuration I.

IV. Analysis of Analog Computer Results

Introduction

The ITAE was used to determine when optimum performance was achieved for a step function input using both linear lead-lag and SPAN filter compensation. After the optimum operating point was determined for each of the compensating filters, ramp and sinusoidal inputs were applied to determine the compensated system's response to time varying signals. The ramp function used had a slope of approximately 1 volt per second. Frequencies of 0.5, 1.0, 2.0, 5.0, and 10.0 radians/second were used as the sinusoidal inputs, with the amplitude of the input held constant throughout this frequency range. The final check made on plant operation for each type of compensation was system sensitivity to gain variations.

The analog computer results used in this investigation were obtained over a two month period. An effort was made to duplicate potentiometer settings for each system tested, but it is quite possible that they differed from time to time by a few hundredths of a volt. Other minor deviations in computer performance could also have occurred over this time period. The ITAE value was the most likely measurement to be sensitive to these minor variations, particularly when its value was in the order of a few hundredths of a unit. However, repeated checks of system performance from time to time indicated that the value of the ITAE for a given system operating point varied only slightly. Therefore, it is believed that the values obtained can be used in a valid comparison of SPAN filter versus linear lead-lag compensation.

The four test plants used will be referred to by a descriptive name rather than by transfer function throughout the remainder of this report. The transfer functions of these four plants were listed in equations 8 through 11, Part III, and they will be referred to respectively as a Type 2 plant, a Type 1 plant with real poles, a Type 1 plant with complex poles, and a non-minimum phase plant.

Tables summarizing the optimum performance of each plant with lead-lag and SPAN filter compensation are contained in Appendix D for step inputs, in Appendix E for ramp inputs, and in Appendix F for sinusoidal inputs. Appendix G contains graphs showing the sensitivity of each system response to variations in gain from the optimum value.

Analysis of Type 2 Plant Performance

Step Function Input. The smallest value of ITAE for the Type 2 plant was obtained with lead-lag compensation. The ITAE values obtained with lead-lag, SPAN configuration I, and SPAN configuration II compensation were 0.04, 0.08, and 0.18 respectively. For this plant, the error input to the ITAE circuit was reduced to zero when the plant output was within 99.7% of its final value.

Response of this Type 2 plant to a step function input for lead-lag and SPAN filter compensation is shown in Figure 7. The linear compensated system has a much faster rise time to first zero error and has a settling or solution time that is 80% of that obtained with the SPAN filter compensated system. The various performance characteristics such as time for the error to reach its first zero, time to reach the maximum peak overshoot, percent peak overshoot, and time for the response to settle within 5% of its final value are tabulated

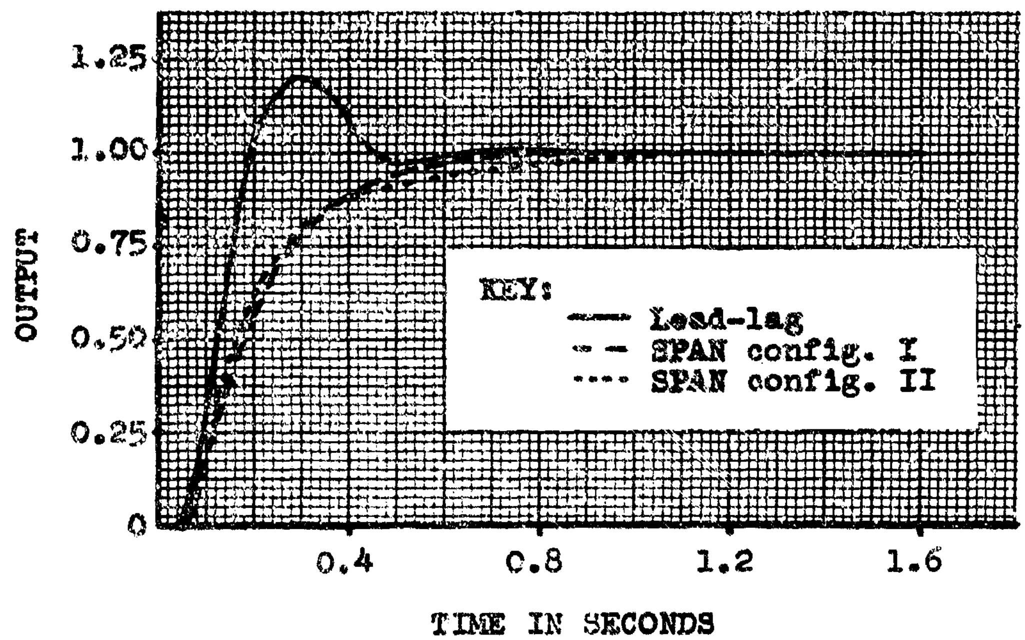


FIGURE 7

COMPENSATED TYPE 2 PLANT RESPONSE

TO STEP FUNCTION INPUT

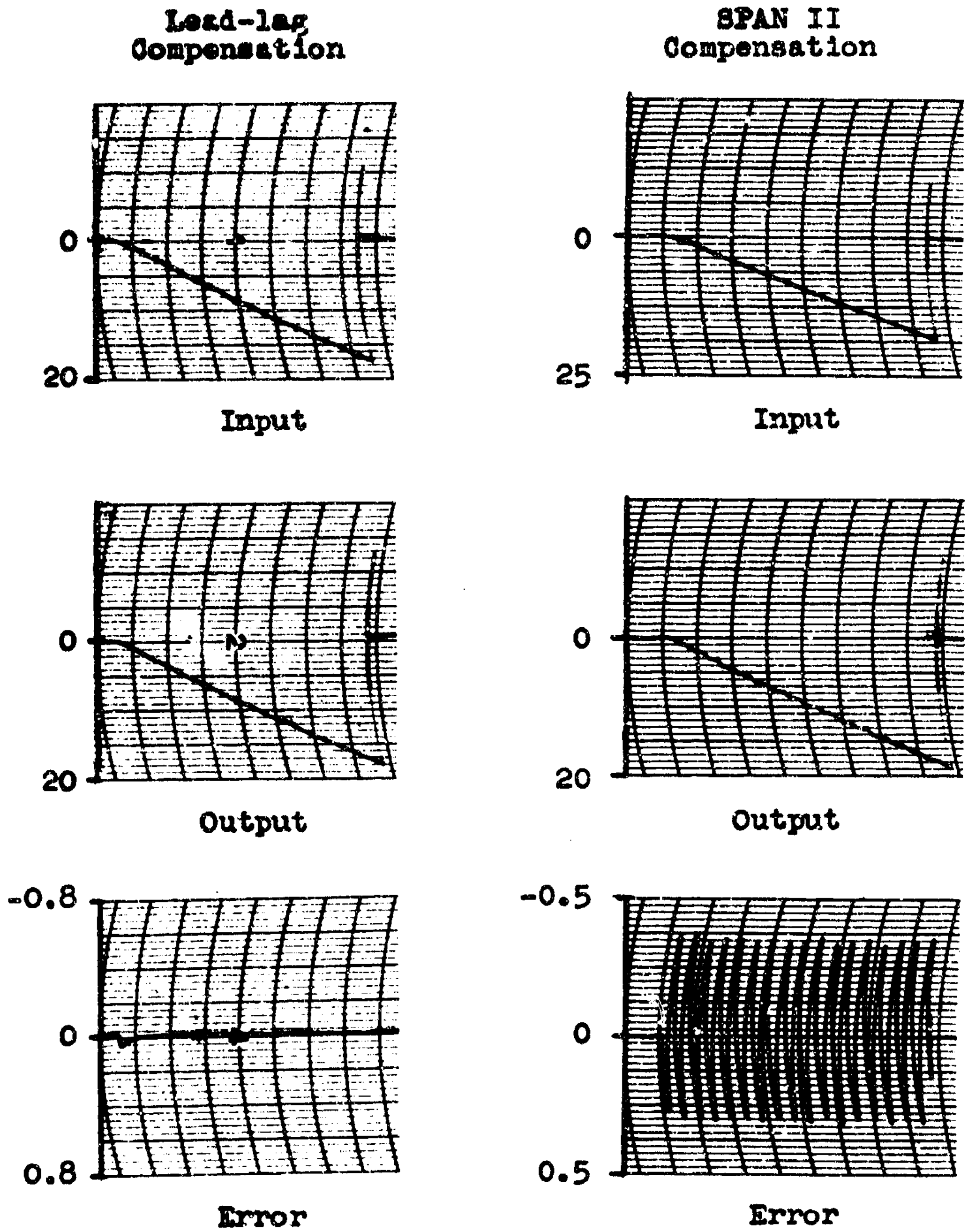
in Table III, Appendix D, for both linear and SPAN filter compensated systems.

Ramp Function Input. Table VII, Appendix E, includes response data for a ramp function input and Figure 8 shows the responses obtained with both lead-lag and SPAN filter compensated systems. The output of the Type 2 plant with SPAN filter compensation oscillated about the ramp slope line whereas the plant output with lead-lag compensation had no steady-state error.

Sinusoidal Input. Tabulated information for the sinusoidal input is contained in Table XI, Appendix F. For the range of frequencies applied to the input (0.5 to 10.0 radians/second), linear lead-lag compensation caused the least phase shift of the output. The frequency response of this plant was approximately the same for input frequencies from 0.5 to 2.0 radians/second regardless of the compensator used. However, use of the SPAN filter resulted in a smaller bandwidth compared to that obtained with lead-lag compensation. Both linear and SPAN compensated responses for inputs of 5.0 and 10.0 radians/second are shown in Figures 9 and 10.

Compensation of the plant with configuration I of the SPAN filter gave a better ITAE value than that obtained with configuration II. Its use also resulted in a better response for sinusoidal inputs as some distortion of the output was observed when configuration II compensation was used.

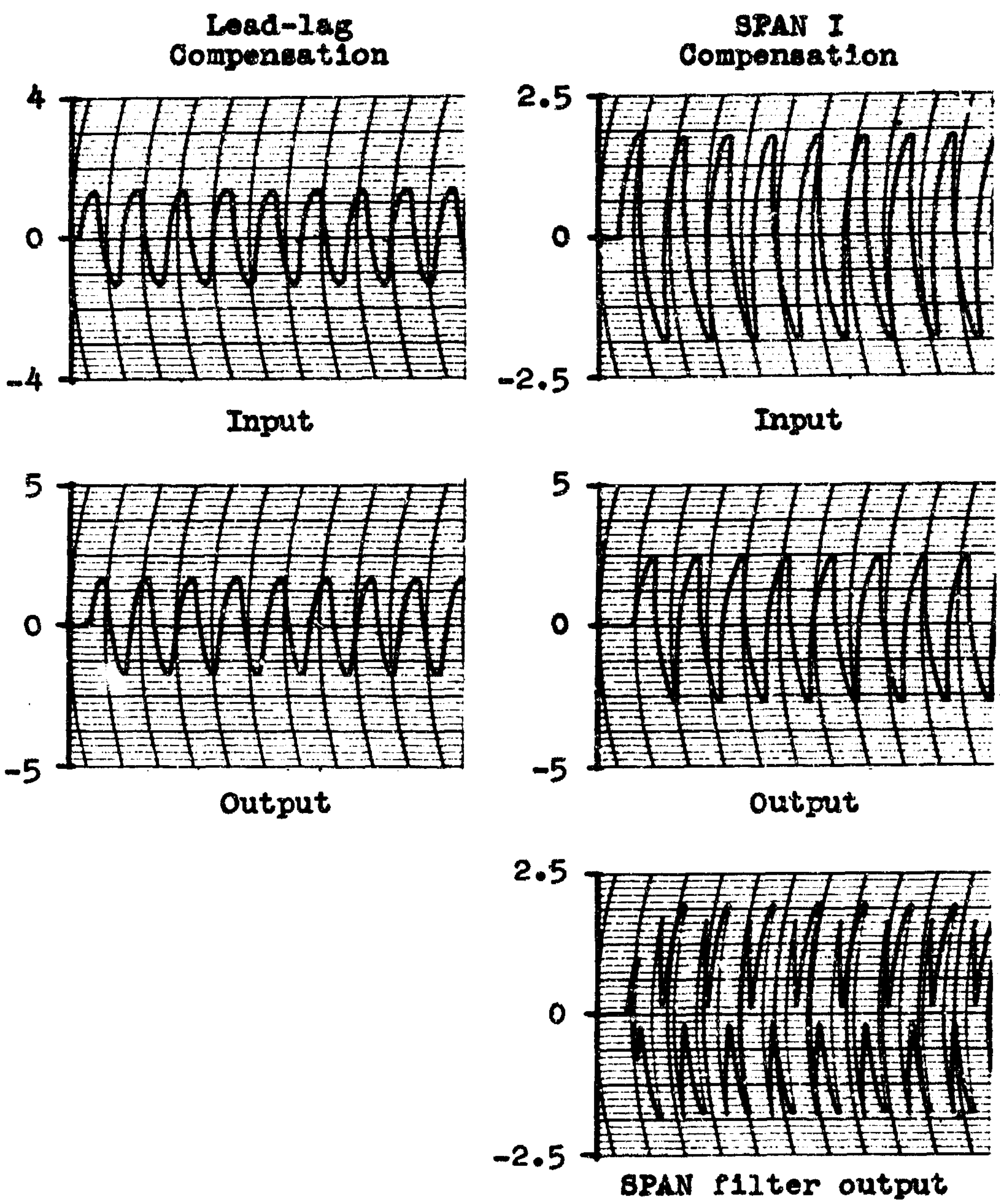
Sensitivity of System Response to Gain Variations. Figure 25, Appendix G, is a plot of ITAE versus percent change of gain from the optimum value. From this figure it can be seen that, regardless of



Recorder speed is 2 mm/sec

FIGURE 8

TYPE 2 PLANT RESPONSE FOR RAMP FUNCTION INPUT

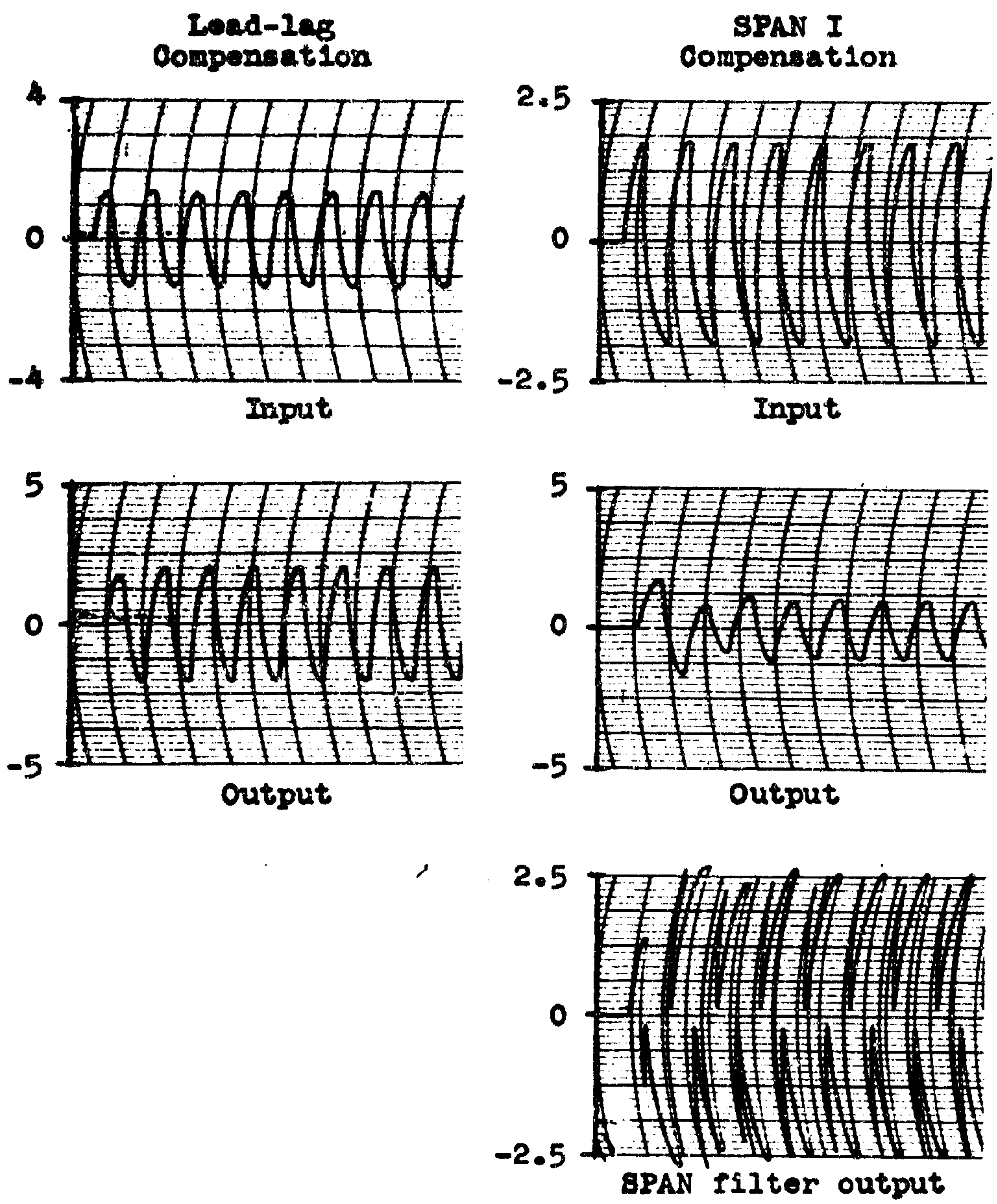


Recorder speed is 5 mm/sec

FIGURE 9

TYPE 2 PLANT RESPONSE

FOR SINUSOIDAL INPUT OF 5 RADIANS/SECOND



Recorder speed is 10 mm/sec

FIGURE 10

TYPE 2 PLANT RESPONSE

FOR SINUSOIDAL INPUT OF 10 RADIANS/SECOND

the type of compensation used, the ITAE changed very little for changes of gain from a 75% decrease from optimum to a 100% increase above optimum. In fact, for configuration II and lead-lag compensation there are any number of optimum points. For gains less than 75% below optimum, the system compensated with configuration II of the SPAN filter was the least sensitive to gain variations.

Analysis of Performance of Type 1 Plant with Real Poles

Step Function Input. Use of configuration II of the SPAN filter to compensate the Type 1 plant with real poles resulted in a lower value of ITAE than obtained by either configuration I or the lead-lag compensator. The minimum ITAE value obtained with configuration II was 2.0 whereas ITAE values for lead-lag and configuration I compensation were 2.4 and 6.3 respectively. Error input to the ITAE circuit was reduced to zero for this system when the plant output was within 99% of its final value. An examination of Figure 11 and the information tabulated in Table IV, Appendix D, shows that although rise time to first zero error was longer with either configuration of the SPAN filter than with the lead-lag compensator (2.1 seconds versus 1.3 seconds), compensation with configuration II resulted in a settling time that was 74% of that required with lead-lag compensation. The oscillatory transient was considerably reduced in amplitude by the use of configuration II.

Ramp Function Input. For a ramp function input, the plant output with SPAN filter compensation is a ramp function superimposed with a damped oscillation whereas the output for lead-lag compensation had no oscillation. However, the least steady-state error was obtained with

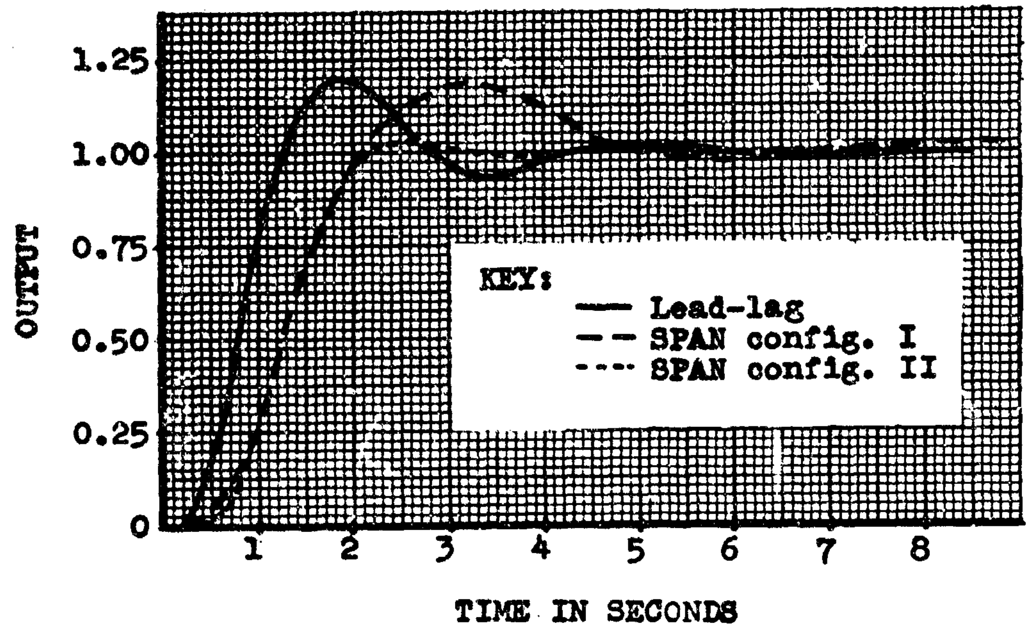
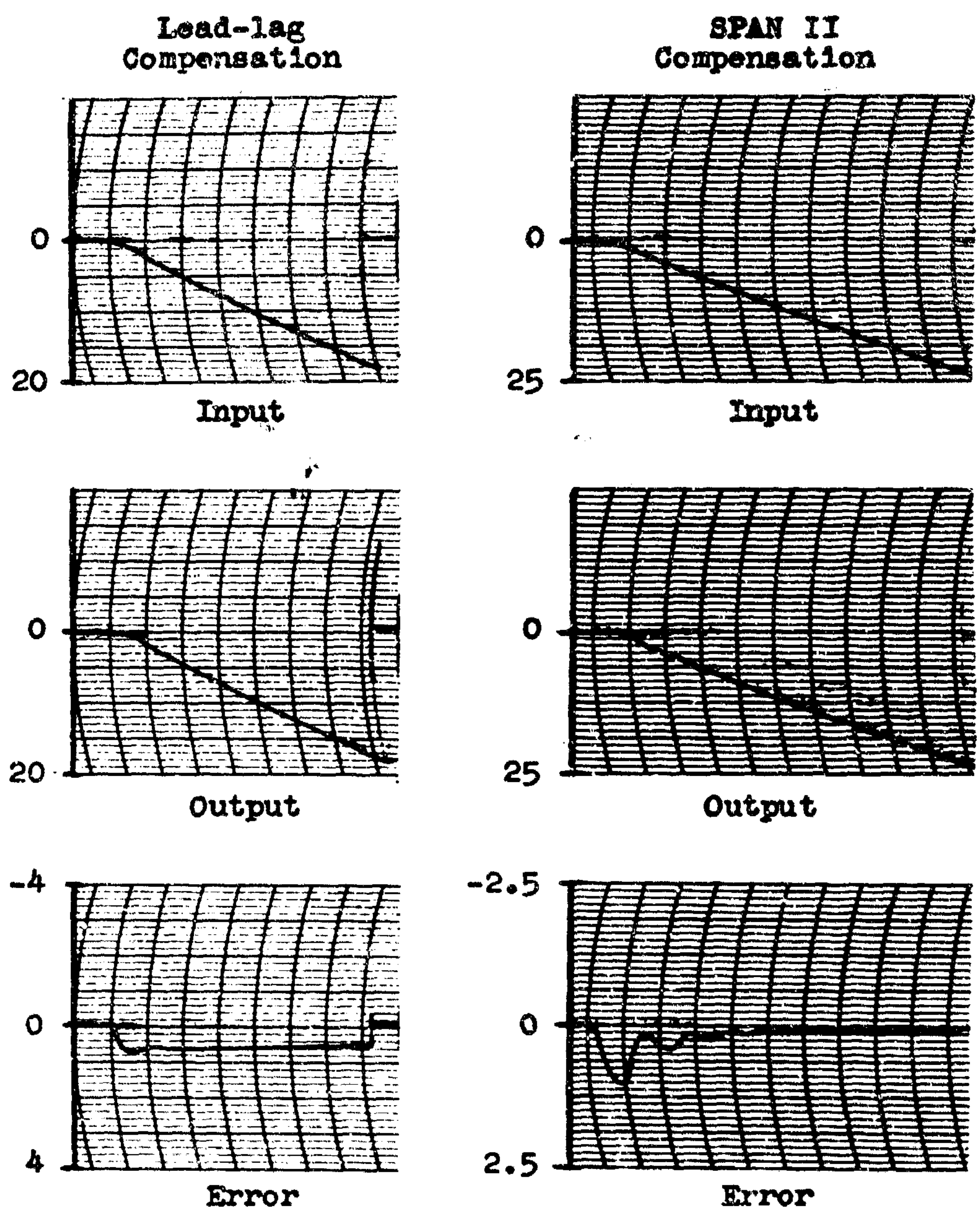


FIGURE 11

RESPONSE OF COMPENSATED TYPE 1 PLANT WITH REAL POLES TO STEP FUNCTION INPUT



Recorder speed is 2 mm/sec

FIGURE 12

RESPONSE OF TYPE 1 PLANT WITH REAL POLES

FOR RAMP FUNCTION INPUT

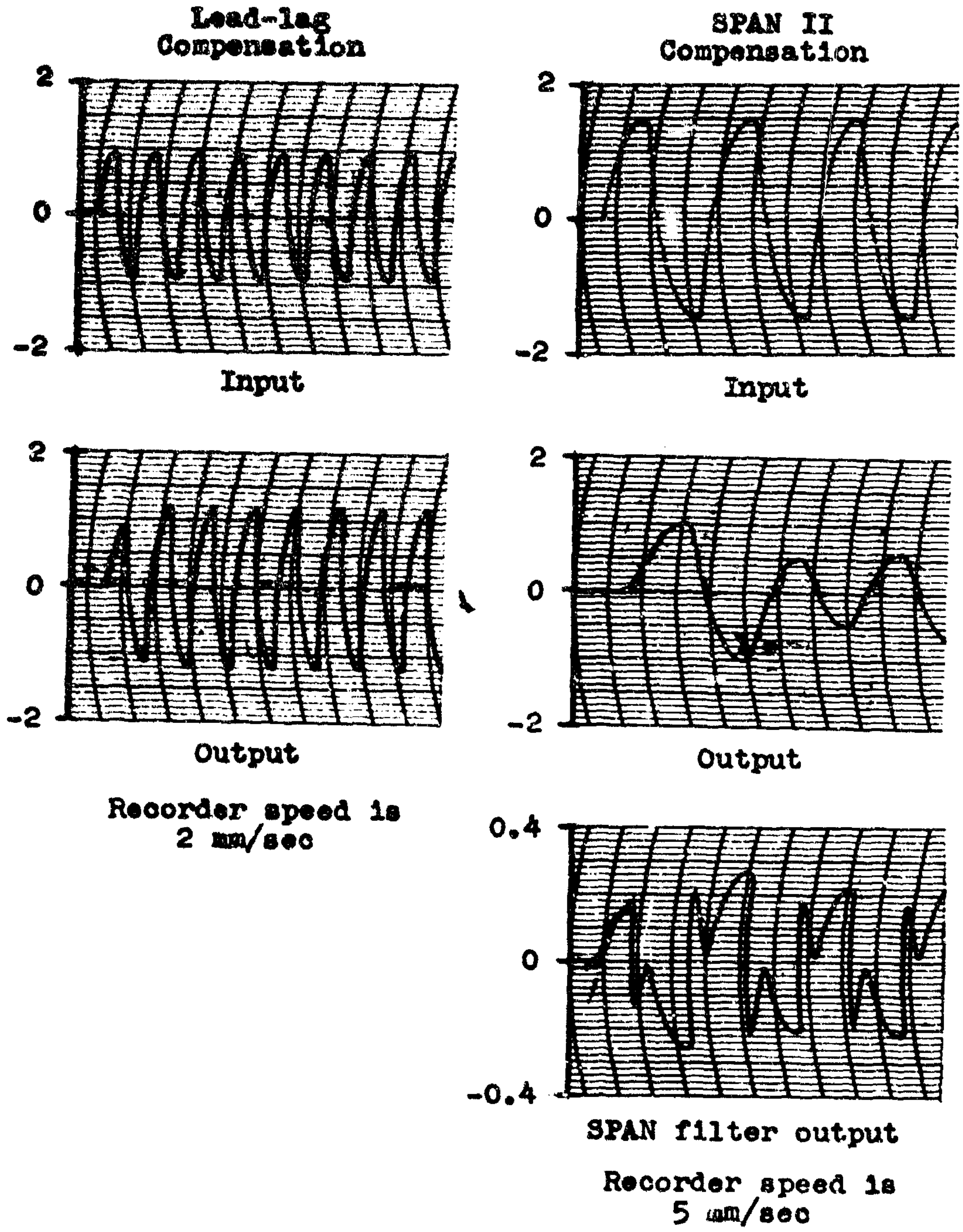


FIGURE 13

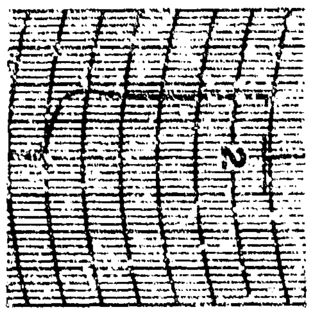
RESPONSE OF TYPE 1 PLANT WITH REAL POLES

FOR SINUSOIDAL INPUT OF 2 RADIANS/SECOND

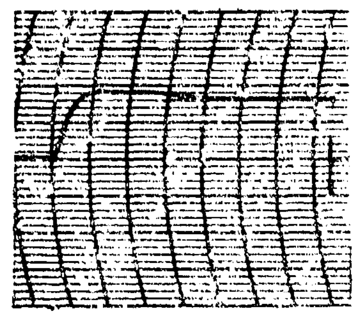
configuration II compensation. Graphical and tabulated ramp function response data are given in Figure 12 and Table VIII, Appendix E.

Sinusoidal Input. The plant output for SPAN filter compensation had less phase shift at 0.5 radians/second but had a greater phase shift for frequency inputs greater than or equal to 2 radians/second than that obtained with lead-lag compensation as shown in Table XII, Appendix F. The data contained in this table also indicates that the SPAN filter compensated plant has a smaller bandwidth than the linear compensated plant. System response for both lead-lag and SPAN filter compensation to an input frequency of 2.0 radians/second is shown in Figure 13.

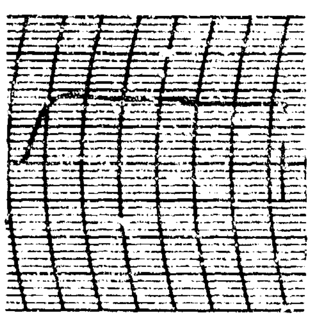
Sensitivity of System Response to Gain Variations. Figure 26, Appendix G, shows the effect of gain variations on the ITAE for this system. The lead-lag compensated system was the least sensitive to gain increases from optimum whereas the SPAN filter compensated system was the least sensitive to gain decreases. The system compensated with configuration I of the SPAN filter appears to be less sensitive than the other compensators although its ITAE value is the largest of the three. No explanation was found for the peak in the configuration II curve; however, a thorough investigation was not conducted in this area. Study of the system responses for gains in the region indicated that the plant output decayed at a slower rate from its peak overshoot value than it did for lesser or greater gains. Figure 14 contains computer traces of system responses for different values of gain in this region.



K = 48



K = 49



K = 50

K = 51

FIGURE 14

RESPONSES TO STEP INPUTS FOR VALUES OF GAIN
THAT PRODUCED A PEAK IN ITAE VERSUS % CHANGE OF GAIN
CURVE FOR THE TYPE 1 PLANT WITH REAL POLES

Analysis of Performance of Type 1 Plant with Complex Poles

Step Function Input. As in the other Type 1 system, the lowest value for ITAE was obtained with configuration II of the SPAN filter as the compensator. The ITAE value for configuration II was 5.1 as compared with 8.4 and 15.3 for configuration I and lead-lag compensation respectively. For the Type 1 plant with complex poles, the error input to the ITAE circuit was reduced to zero when the plant output was within 98% of its final value. A larger bias value was used for this system as the transient response decayed at a slower rate than in the other systems.

Step responses for this system are shown in Figure 15. Settling time for the configuration II compensated plant was less than one-half of that required with lead-lag compensation. Again, the SPAN filter compensated plant required more time to reach first zero error than for lead-lag compensation, but the times required only differed by 7%. Tabulated step function input data is provided in Table V, Appendix D.

Ramp Function Input. Response of the SPAN filter compensated system to a ramp function input was not as good as that obtained with lead-lag compensation. The steady-state error for SPAN filter compensation was at least 40% greater than that obtained with lead-lag compensation. Tabulated data for this plant is contained in Table IX, Appendix E.

Sinusoidal Input. The SPAN filter compensated system also had a poorer response for sinusoidal inputs compared to that obtained with lead-lag compensation, as indicated in Table XIII, Appendix F.

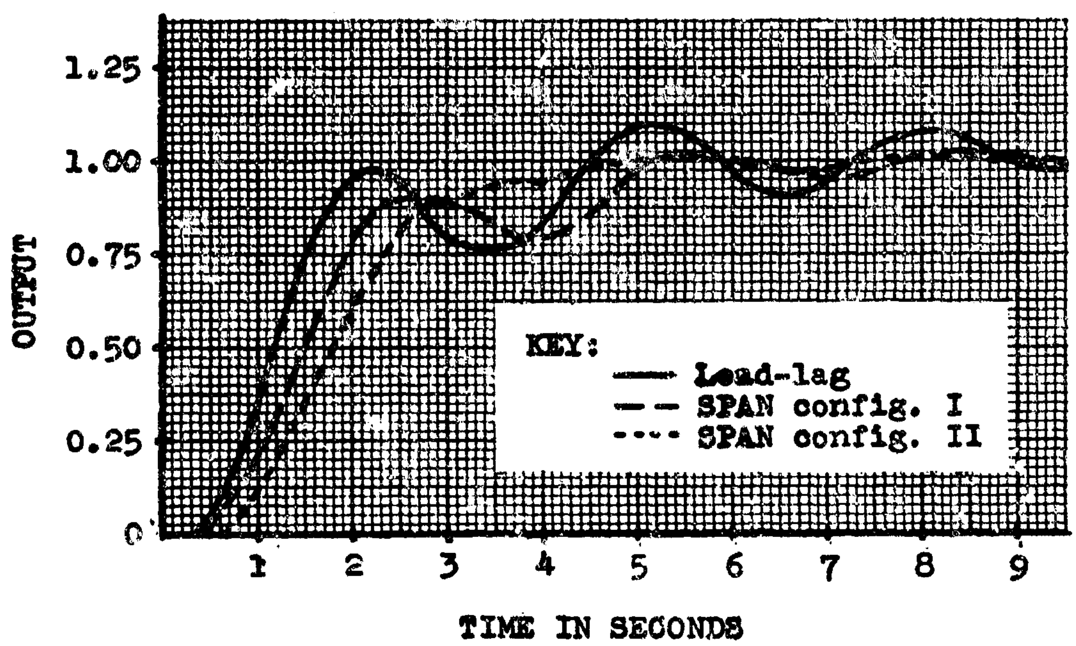


FIGURE 15

RESPONSE OF COMPENSATED TYPE 1 PLANT WITH COMPLEX POLES TO STEP FUNCTION INPUT

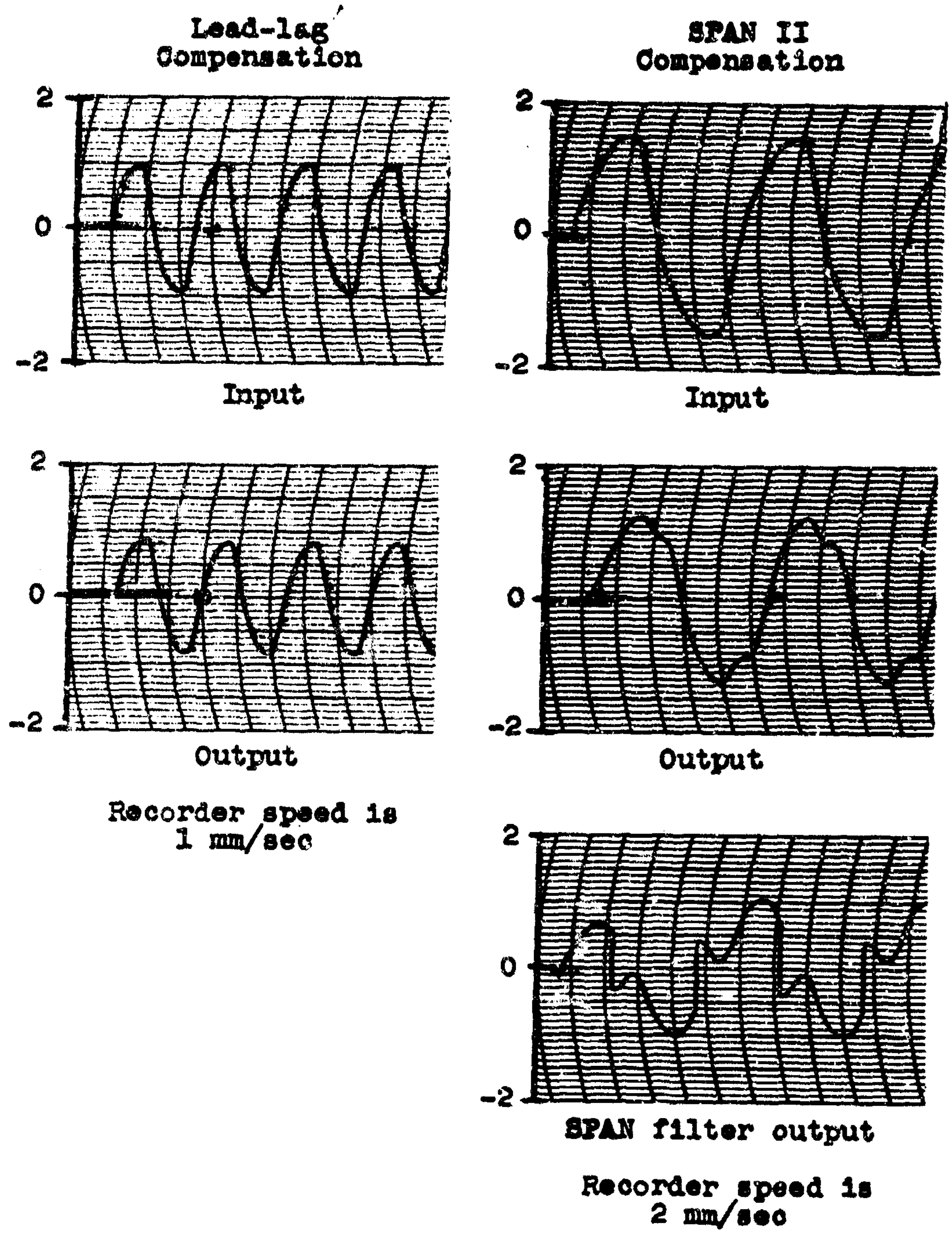


FIGURE 16

RESPONSE OF TYPE 1 PLANT WITH COMPLEX POLES

FOR SINUSOIDAL INPUT OF 0.5 RADIANS/SECOND

Throughout the frequency range of 0.5 to 10.0 radians/second, the output of the SPAN compensated system had greater phase shift, reduced bandwidth, and some distortion. The responses of both SPAN and lead-lag compensated plants to a sinusoidal input of 0.5 radians/second are shown in Figure 16.

Sensitivity of System Response to Gain Variations. Figure 27, Appendix G, shows the effect of gain variations on the ITAE for this system. The output of the plant with SPAN filter compensation was less sensitive to gain variations than with lead-lag compensation. Configuration II compensation produced the least sensitive system, particularly to gain increases.

Analysis of Non-minimum Phase Plant Performance

Step Function Input. Approximately 0.05 was obtained as the ITAE value for the non-minimum phase plant regardless of the type of compensation used. For this plant the error input to the ITAE circuit was reduced to zero when the plant output was within 99.5% of its final value.

Figure 17 and Table VI, Appendix D, contain the response information of this plant. A comparison of this data reveals that regardless of the compensation used, the system response had approximately the same settling time and had little or no overshoot. The primary difference in the responses existed in the amount of undershoot. Use of the lead-lag compensator gave a response with a larger but earlier occurring undershoot than that obtained with SPAN filter compensation. During the trial and error procedure of adjusting the parameters of configuration II of the SPAN filter in search of the system optimum

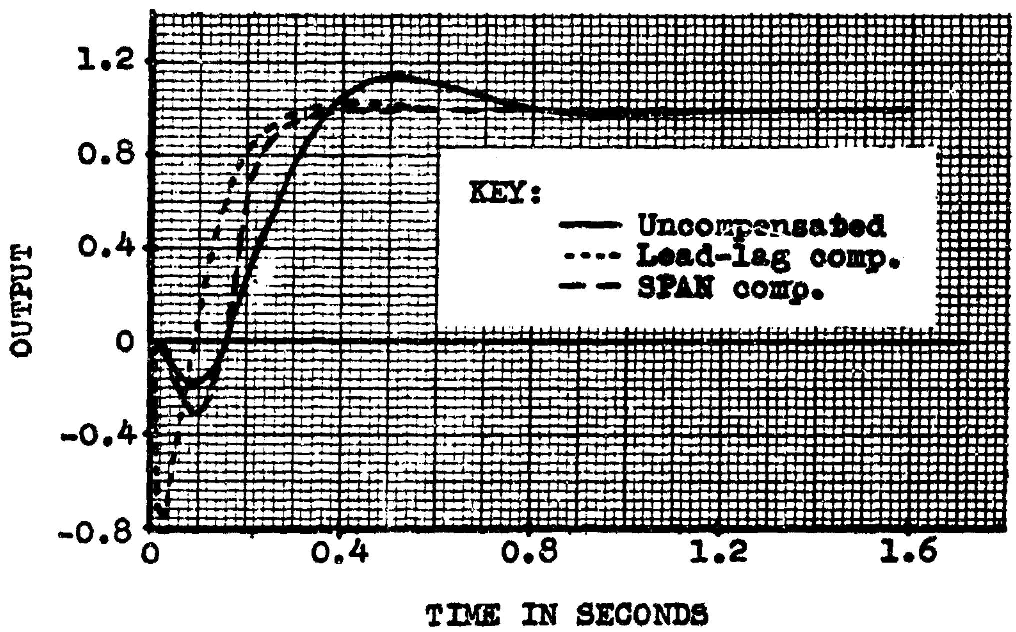


FIGURE 17

NON-MINIMUM PHASE PLANT RESPONSE

TO STEP FUNCTION INPUT

operating point, it was determined that the optimum point was approached as the pole-zero ratio of F_2, the lag network, approached unity (or when F_2 approached a unity gain network as in configuration I). The responses obtained from both configurations differed slightly so data for both is contained in Table VI, Appendix D. Slight variations in potentiometer settings on the analog computer probably account for this difference. However, because of the similarity of the responses, only the response for configuration I is given in Figure 17.

As the response of this non-minimum phase plant may not be as familiar to the reader as the other plants, the uncompensated response is also shown in Figure 17. It should be noticed in this figure that not only did both the lead-lag and SPAN filter compensation improve the rise time to first zero error and reduce the overshoot, but also they both increased the amount of undershoot. If this undershoot is objectionable in the basic plant, then some type of compensation other than those discussed in this paper must be used to eliminate or reduce it.

Ramp Function Input. Tabulated data for this plant's response to a ramp function input are contained in Table X, Appendix E. The steady-state error in the output was approximately the same for either compensator, with configuration II having the least error of the three.

Sinusoidal Input. The amplitude of the plant output remained quite constant for sinusoidal frequency inputs from 0.5 to 10.0 radians/second regardless of the type of compensator used. Configuration I of the SPAN filter caused less phase shift in the output for

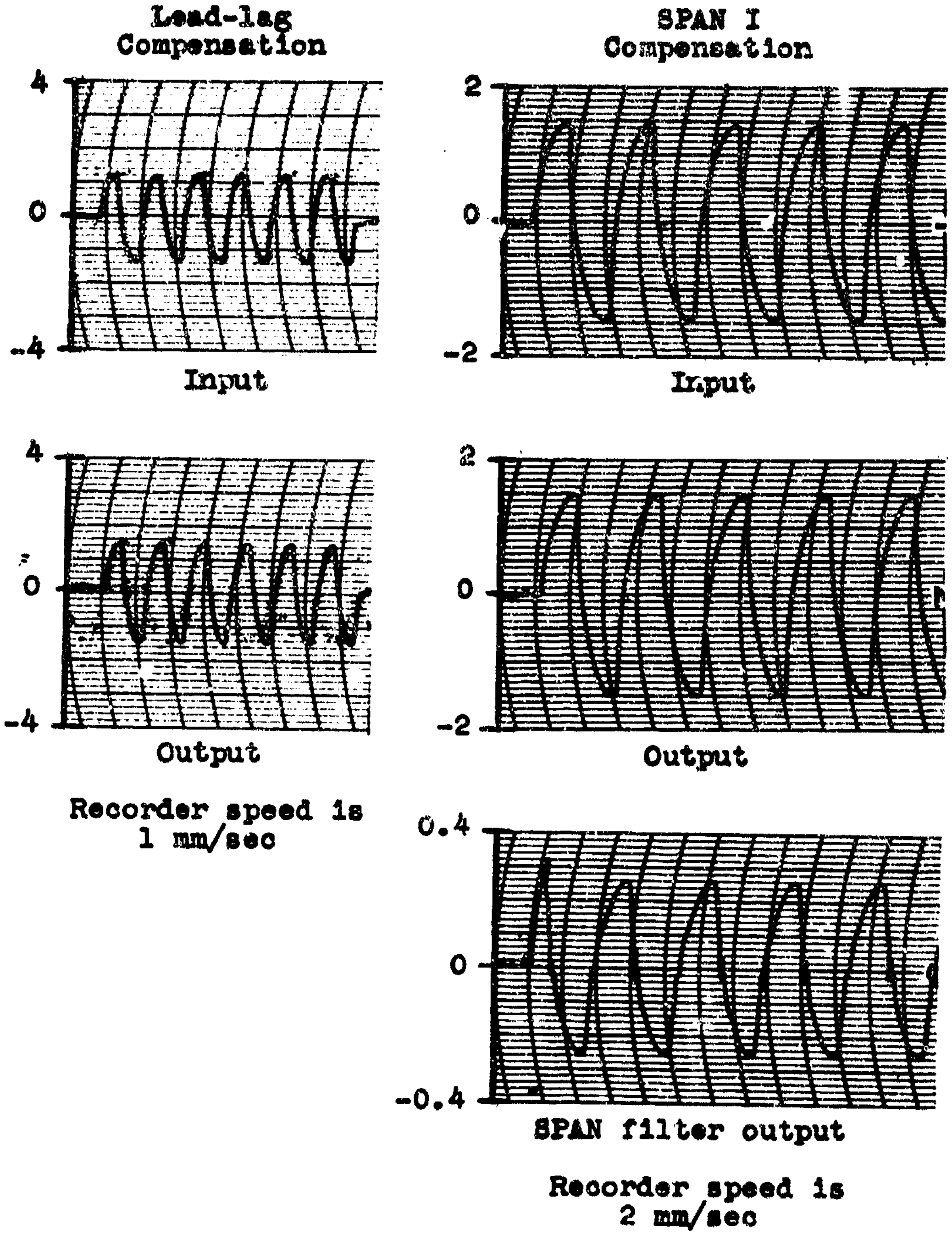


FIGURE 18

NON-MINIMUM PHASE PLANT RESPONSE

FOR SINUSOIDAL INPUT OF 1 RADIAN/SECOND

all input frequencies except at 10.0 radians/second. Information on plant response to sinusoidal inputs is contained in Table XIV, Appendix F. Figure 18 shows the compensated response for an input frequency of 1.0 radian/second.

Sensitivity of System Response to Gain Variations. The effect of gain variations on the ITAE for this system is shown in Figure 28, Appendix G. As previously mentioned, SPAN filter configurations I and II were very similar. As expected, gain variation data for both configurations was also similar and therefore only one curve is plotted for the SPAN filter in Figure 28. The difference in the degree of sensitivity between lead-lag and SPAN filter compensated systems is slight with the SPAN compensated system being the least sensitive.

V. Conclusions

For the four plants used during this investigation, use of the SPAN filter as a compensator showed little or no over-all improvement compared with that obtained with the less complex linear lead-lag compensator. Based on the ITAE, the SPAN filter did improve the step response of both Type 1 plants used in this study over that obtained with lead-lag compensation. The SPAN filter also reduced system sensitivity to gain variations for the Type 1 plant with complex poles. However, better ramp and sinusoidal responses from both Type 1 plants were obtained with lead-lag compensation. For the other two plants, use of the SPAN filter slightly improved over-all performance of the non-minimum phase plant but provided no improvement in the Type 2 plant performance over that obtained with linear lead-lag compensation.

Based on the results obtained in the area of investigation covered by this study, it is concluded that the present concept of the SPAN filter will not result in a practical device with general application which will provide improved compensated response over that obtained with linear compensation. This is particularly true for those systems which have inputs that are not solely restricted to a step function input. As with other nonlinear compensators, there may be some type of system with which the SPAN filter could be used to provide some desirable effect not attainable with linear compensators. The independent control of gain and phase is a desirable feature but in the present concept of the SPAN filter, this advantage is outweighed by its disadvantages.

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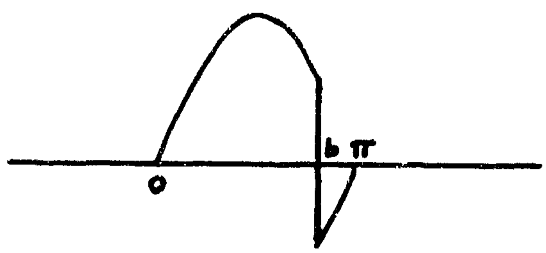
Appendix A

Discussion of Fourier Series

For a sinusoidal input, the SPAN filter output is shown in Figures 2 and 3. One-half period of the SPAN filter output is shown here for convenience.



The Fourier series for this output can be expressed in the form

$$f(wt) = \sum_{n=1}^{\infty} A_n \cos nwt + \sum_{n=1}^{\infty} B_n \sin nwt \qquad (12)$$

Consider the case where F_2 is a unity gain network. Let f be the phase shift produced by the lead network F_1, then the bistable element switches at the angle b which is equal to $\pi - f$. The coefficients of the Fourier series are found to be as follows:

for n = 1

$$A_1 = \frac{2}{\pi} \sin^2 b \qquad (13)$$

$$B_1 = \frac{1}{\pi} (2b - \pi - \sin 2b) \qquad (14)$$

for n > 1

$$A_n = \frac{1}{\pi} \left\{ \left[\frac{2-2\cos(n+1)b}{n+1} \right] - \left[\frac{2-2\cos(n-1)b}{n-1} \right] \right\} \quad (15)$$

$$B_n = \frac{2}{\pi} \left[\frac{\sin(n-1)b}{n-1} - \frac{\sin(n+1)b}{n+1} \right] \quad (16)$$

The Fourier series can also be expressed as

$$f(wt) = \sum_{n=1}^{\infty} C_n \sin(nwt + \phi_n) \quad (17)$$

where

$$C_n = \sqrt{A_n^{\,2} + B_n^{\,2}} \quad (18)$$

$$\phi_n = \tan^{-1} \frac{A_n}{B_n} \quad (19)$$

For the case where F_1 is a lag network, let f be the numerical sum of the phase shifts obtained in both the lead and lag networks. Then the above formulae can be used with the exception that

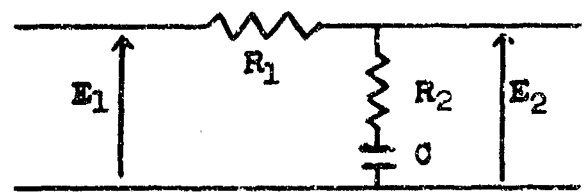
$$\phi_n = -e + \tan^{-1} \frac{A_n}{B_n} \quad (20)$$

where e is the phase shift in the lag network.

Appendix B

Lag Network

Circuit



Transfer Function

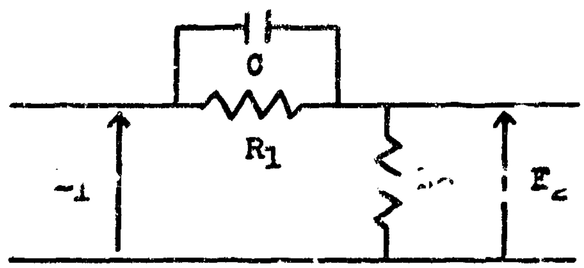
$$G(s) = \frac{E_2(s)}{E_1(s)} = \frac{1 + T_1 s}{1 + \alpha_1 T_1 s}$$

where $T_1 = R_2 C$

and $\alpha_1 = \frac{R_1 + R_2}{R_2}$

Lead Network

Circuit



Transfer Function

$$G(s) = \frac{E_2(s)}{E_1(s)} = \alpha_2 \frac{1 + T_2 s}{1 + \alpha_2 T_2 s}$$

where $T_2 = R_1 C$

and $\alpha_2 = \frac{R_2}{R_1 + R_2}$

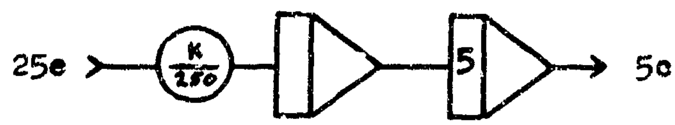
FIGURE 19

LINEAR LAG AND LEAD NETWORKS

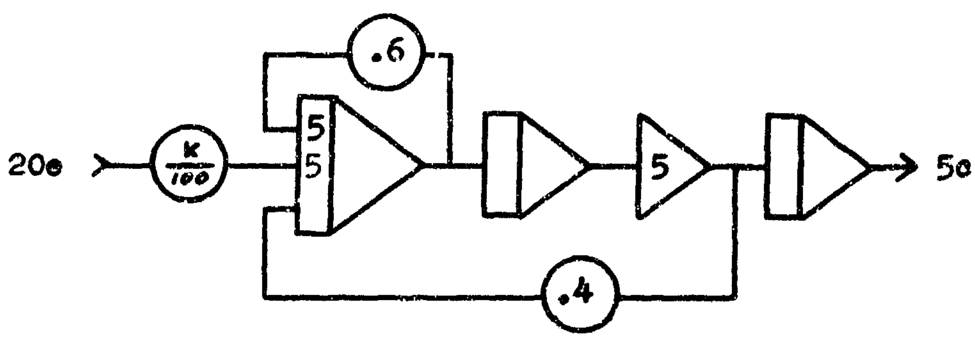
Appendix C

Analog Computer Circuits

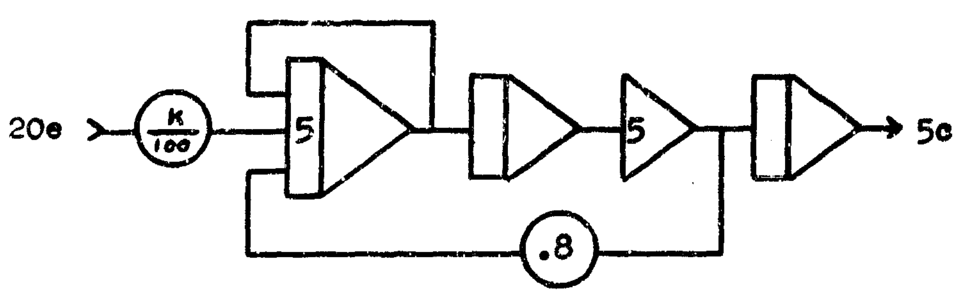
Amplifier gains are equal to 1 when they are not marked.



$$G(s) = \frac{K}{s^2}$$



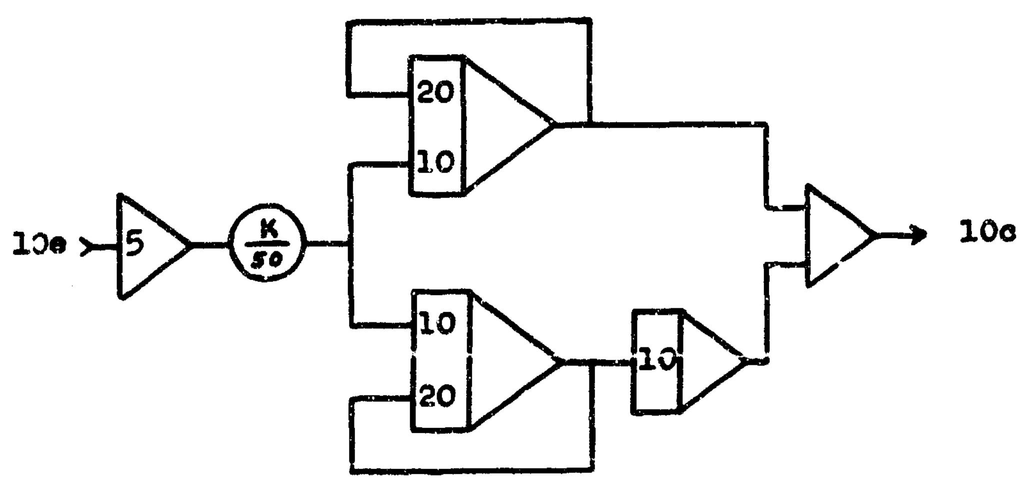
$$G(s) = \frac{K}{S(S + 1)(S + 2)}$$



$$G(s) = \frac{K}{S(S^2 + S + 4)}$$

FIGURE 20

ANALOG COMPUTER SIMULATION OF MINIMUM PHASE PLANTS



$$G(s) = \frac{K(10 - S)}{S(S + 20)}$$

FIGURE 21

ANALOG COMPUTER SIMULATION OF NON-MINIMUM PHASE PLANT

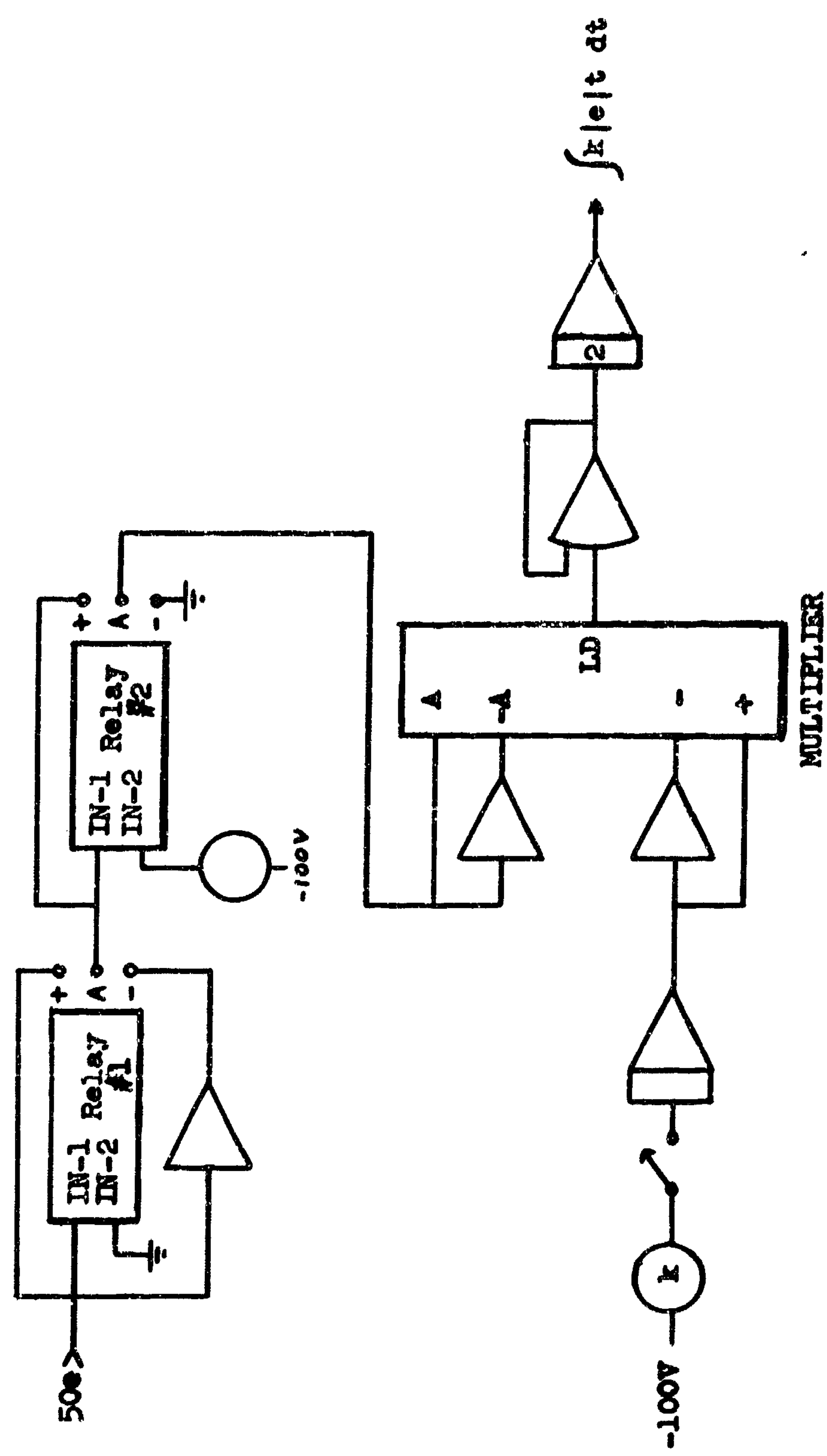


FIGURE 22

ANALOG COMPUTER SIMULATION OF ITAE

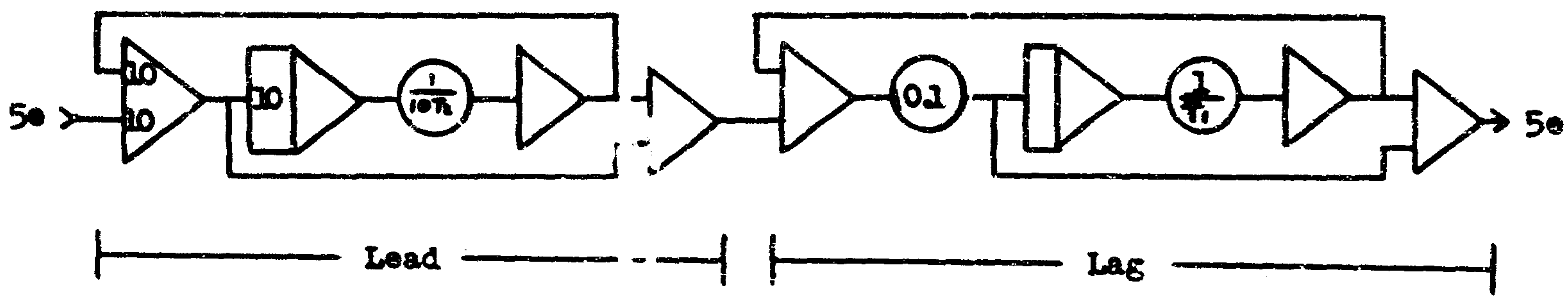


FIGURE 23

ANALOG COMPUTER SIMULATION OF LEAD-LAG COMPENSATOR

$$G_c(s) = \frac{(1 + T_2S)(1 + T_1S)}{(1 + 0.1T_2S)(1 + 10T_1S)}$$

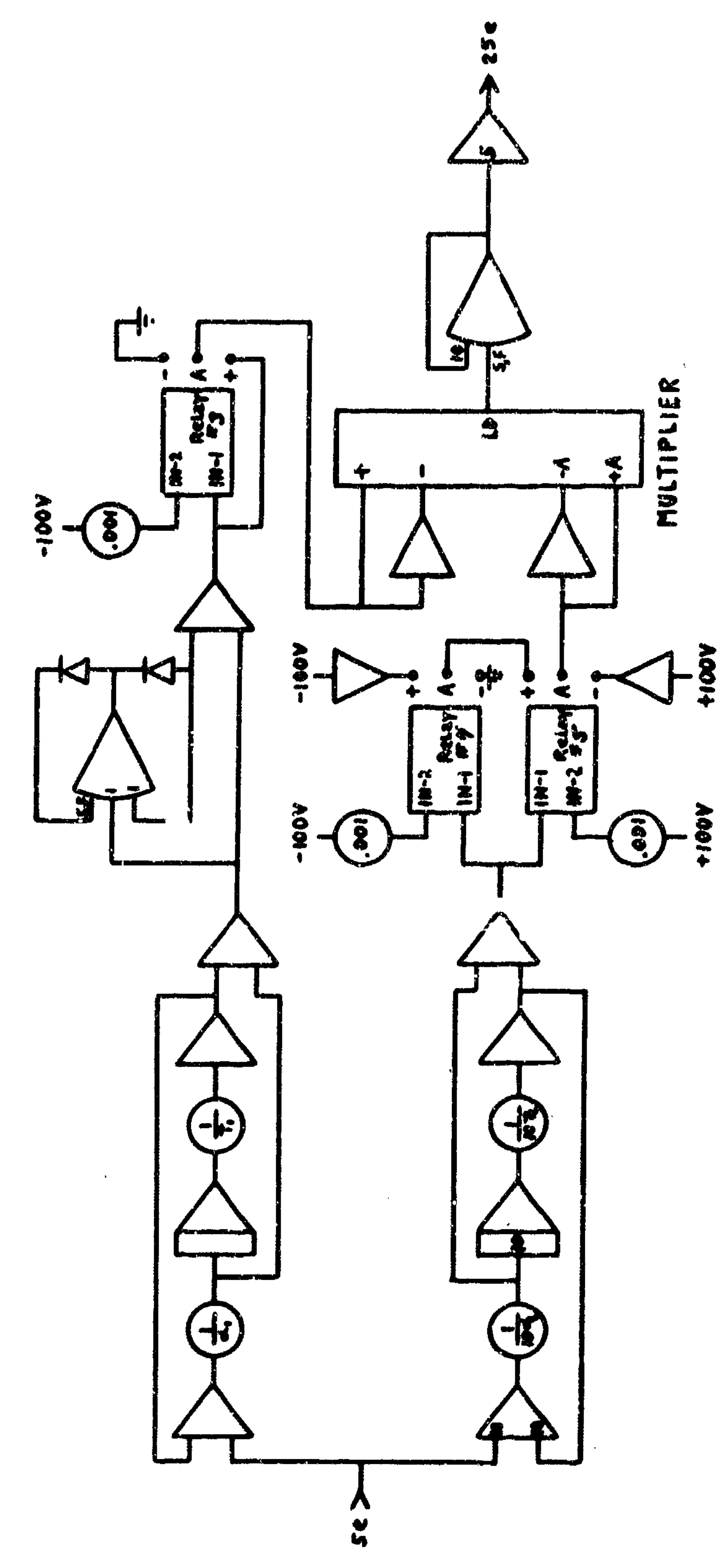


FIGURE 29
ANALOG COMPUTER SIMULATION OF SPAN FILTER

Appendix D

Tables for Plant Performance for Step Function Input

TABLE III

TYPE 2 PLANT PERFORMANCE FOR STEP FUNCTION INPUT

NOTE: All times g[illegible]n in seconds

Filter	ITAE	Parameter Values for Optimum Performance					Rise Time to First Zero Error	Time to Peak Overshoot	% Peak Over-shoot	Settling Time (5%)
		K	T_1	α_1	T_2	α_2				
Lead-lag	0.04	250	50	10	0.69	0.1	0.19	0.28	19	0.42
SPAN I	0.08	50			0.34	0.1	0.80			0.52
SPAN II	0.18	250	3	4	0.34	0.1	1.08			0.70

TABLE IV

PERFORMANCE OF TYPE 1 PLANT WITH REAL POLES FOR A STEP FUNCTION INPUT

NOTE: All times given in seconds

Filter	IAE	Parameter Values for Optimum Performance					Rise Time to First Zero Error	Time to Peak Overshoot	% Peak Over-shoot	Settling Time (5%)
		K	T_1	α_1	T_2	α_2				
Lead-lag	2.4	30	50	10	1.50	0.1	1.3	1.9	20	2.7
SPAN I	6.3	4			0.76	0.1	2.1	3.2	19	4.5
SPAN II	2.0	40	7	10	0.76	0.1	2.1	2.6	2	2.0

TABLE V

PERFORMANCE OF TYPE 1 PLANT WITH COMPLEX POLES

FOR A STEP FUNCTION INPUT

NOTE: All times given in seconds

Filter	ITAE	Parameter Values for Optimum Performance					Rise Time to First Zero Error	Time to Peak Overshoot	% Peak Over-shoot	Settling Time (5%)
		K	T_1	α_1	T_2	α_2				
Lead-lag	15.3	26	50	10	0.26	0.1	4.5	5.2	10	8.6
SPAN I	8.4	2			0.50	0.1	5.1	5.5	2	7.2
SPAN II	5.1	2.5	1	2	0.50	0.1	4.9	5.4	1	4.0

TABLE VI

NON-MINIMUM PHASE PLANT PERFORMANCE

FOR STEP FUNCTION INPUT

NOTE: All times given in seconds

Filter	ITAE	Parameter Values for Optimum Performance					Rise Time to First Zero Error	Time to Peak Overshoot	% Peak Over-shoot	Settling Time (5%)
		K	T_1	α_1	T_2	α_2				
Uncomp	0.10	9.0					0.34	0.50	17	0.60
Lead-lag	0.05	118.0	50	10	0.045	0.1	0.32	0.44	3	0.27
SPAN I	0.05	11.4			0.1	0.1	0.36			0.28
SPAN II	0.05	12.0	0.05	1	0.1	0.1	0.32	0.44	3	0.28

Appendix E

Tables for Plant Performance for Ramp Function Input

TABLE VII

TYPE 2 PLANT PERFORMANCE FOR RAMP FUNCTION INPUT

Filter	Ramp Slope (volts/sec.)	Steady State Error (volts)
Lead-lag	1.00	0.00
SPAN I	1.04	0.07
SPAN II	1.00	0.35

TABLE VIII

PERFORMANCE OF TYPE 1 PLANT WITH REAL POLES

FOR A RAMP FUNCTION INPUT

Filter	Ramp Slope (volts/sec.)	Steady State Error (volts)
Lead-lag	1.07	0.75
SPAN I	1.03	0.50
SPAN II	1.02	0.12

TABLE IX

PERFORMANCE OF TYPE 1 PLANT WITH COMPLEX POLES

FOR A RAMP FUNCTION INPUT

Filter	Ramp Slope (volts/sec.)	Steady State Error (volts)
Lead-lag	1.06	1.2
SPAN I	1.01	2.0
SPAN II	1.07	1.7

TABLE X

NON-MINIMUM PHASE PLANT PERFORMANCE

FOR RAMP FUNCTION INPUT

Filter	Ramp Slope (volts/sec.)	Steady State Error (volts)
Lead-lag	1.02	0.18
SPAN I	1.02	0.16
SPAN II	1.01	0.15

Appendix F

Tables for Plant Performance for Sinusoidal Input

TABLE XI

TYPE 2 PLANT PERFORMANCE FOR SINUSOIDAL INPUT

Filter	Frequency (rad/sec)	$\frac{e_{out}}{e_{in}}$	Phase Lag (degrees)	Error (% of input)
Lead-lag	0.5	1.07	0.0	1.1
	1.0	1.10	0.0	3.6
	2.0	1.14	0.0	11.4
	5.0	1.26	5.8	37.4
	10.0	1.48	5.2	107.0
SPAN I	0.5	1.03	0.0	2.1
	1.0	1.04	0.0	4.3
	2.0	1.10	0.0	11.9
	5.0	1.45	11.6	109.2
	10.0	0.57	12.6	147.0
SPAN II	0.5	1.06	2.4	8.4
	1.0	1.07	0.0	11.7
	2.0	1.07	0.0	15.4
	5.0	1.12	5.8	78.0
	10.0	0.78	11.4	164.0

TABLE XII

PERFORMANCE OF TYPE 1 PLANT WITH REAL POLES

FOR SINUSOIDAL INPUT

Filter	Frequency (rad/sec)	$\frac{\theta_{out}}{\theta_{in}}$	Phase Lag (degrees)	Error (% of input)
Lead-lag	0.5	1.04	6.7	16.1
	1.0	1.17	10.4	70.0
	2.0	1.30	18.5	185.0
	5.0	0.16	20.8	117.6
	10.0	0.02	28.7	109.2
SPAN I	0.5	1.15	5.7	37.9
	1.0	1.92	10.4	170.0
	2.0	0.38	37.2	143.0
	5.0	0.02	23.2	104.0
	10.0			103.0
SPAN II	0.5	1.07	3.8	48.3
	1.0	1.96	11.0	165.4
	2.0	0.40	35.5	141.2
	5.0	0.03	24.5	102.8
	10.0			102.8

TABLE XIII

PERFORMANCE OF TYPE 1 PLANT WITH COMPLEX POLES

FOR SINUSOIDAL INPUT

Filter	Frequency (rad/sec)	$\frac{\theta_{out}}{\theta_{in}}$	Phase Lag (degrees)	Error (% of input)
Lead-lag	0.5	0.85	13.4	63.2
	1.0	0.75	11.5	87.0
	2.0	1.74	24.2	241.0
	5.0	0.04	46.1	108.0
	10.0	0.01		108.0
SPAN I	0.5	0.76	16.4	75.6
	1.0	0.61	12.0	96.2
	2.0	0.80	30.8	175.0
	5.0	0.02	50.1	104.4
	10.0			104.4
SPAN II	0.5	0.82	12.6	96.6
	1.0	0.49	16.8	105.3
	2.0	0.39	32.3	138.0
	5.0	0.01	47.0	103.4
	10.0			102.2

TABLE XIV

NON-MINIMUM PHASE PLANT PERFORMANCE

FOR SINUSOIDAL INPUT

Filter	Frequency (rad/sec)	$\frac{e_{out}}{e_{in}}$	Phase Lag (degrees)	Error (% of input)
Lead-lag	0.5	1.10	00.0	10.0
	1.0	1.12	2.3	19.3
	2.0	1.13	3.5	37.8
	5.0	1.13	10.4	93.0
	10.0	1.36	6.9	171.0
SPAN I	0.5	1.03	0.0	8.3
	1.0	1.02	1.2	16.9
	2.0	1.03	2.8	40.0
	5.0	1.12	8.7	106.0
	10.0	1.27	10.9	200.0
SPAN II	0.5	1.01	1.9	7.7
	1.0	1.04	2.3	15.4
	2.0	1.02	4.6	37.2
	5.0	1.15	10.4	102.2
	10.0	1.06	10.9	212.5

Appendix G

Effect of Gain Variation on the ITAE

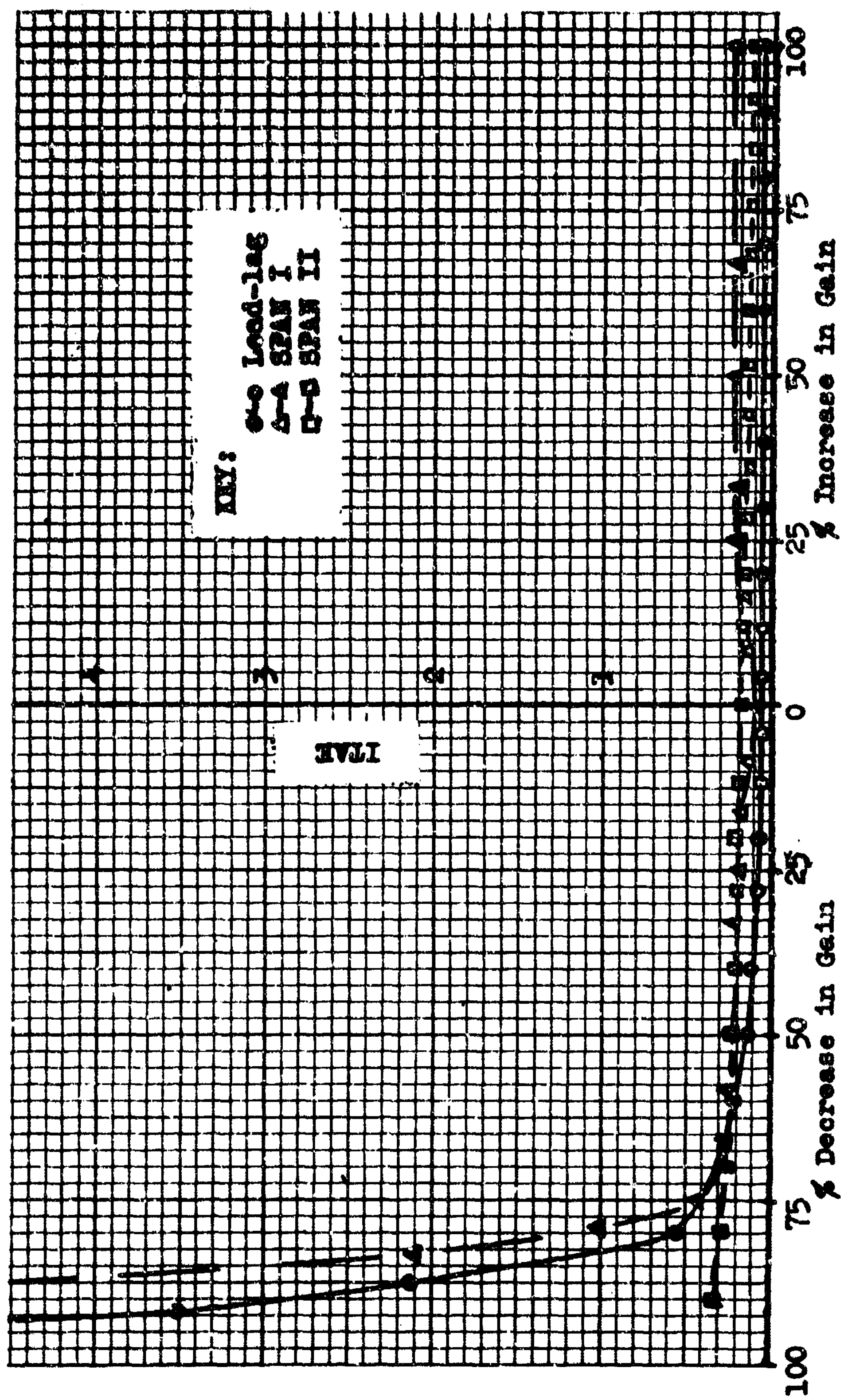


FIGURE 25

EFFECT OF GAIN VARIATION ON THE ITAE FOR COMPENSATED TYPE 2 PLANT

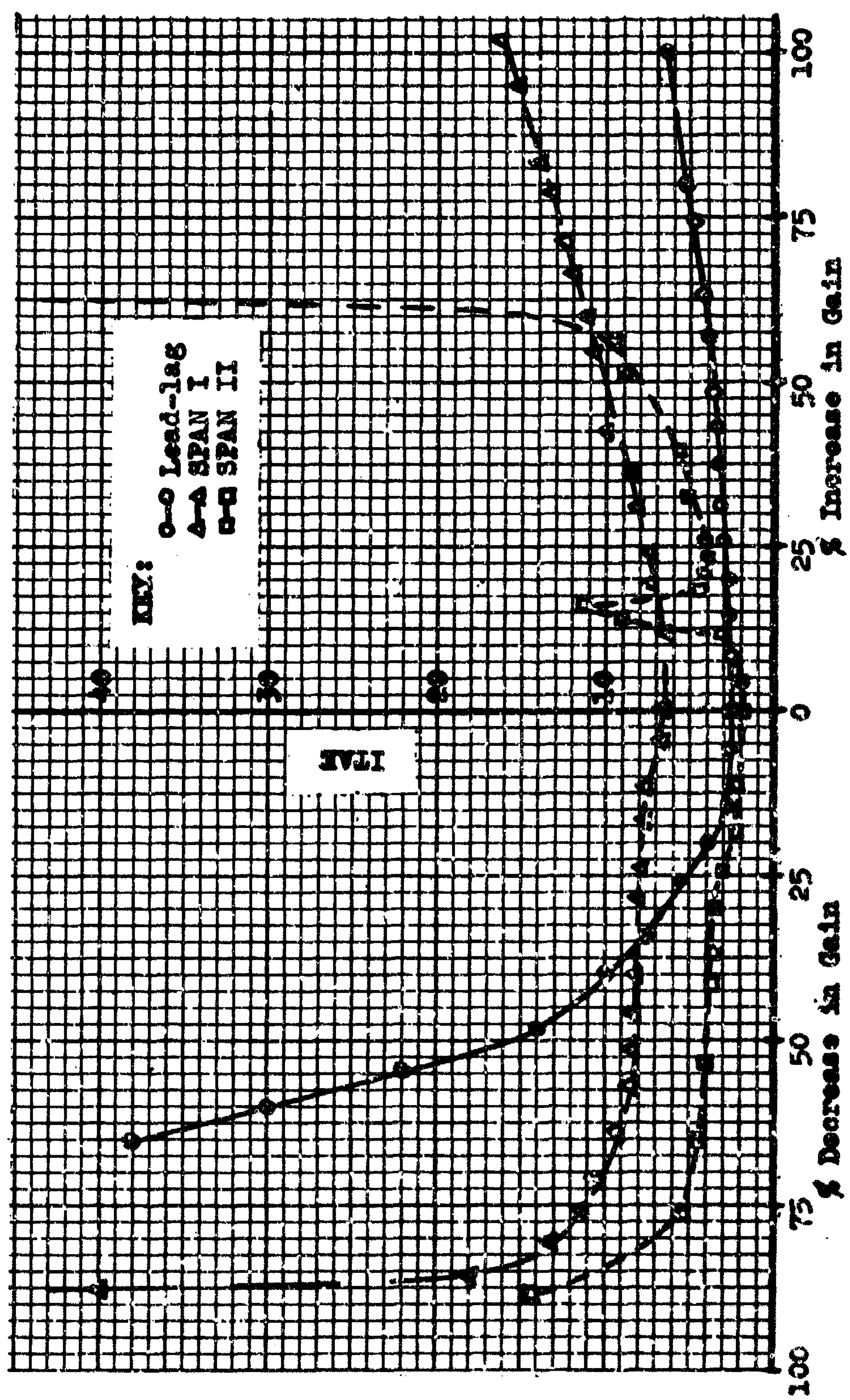


FIGURE 26

EFFECT OF GAIN VARIATION ON THE ITAE FOR COMPENSATED TYPE 1 PLANT WITH REAL POLES

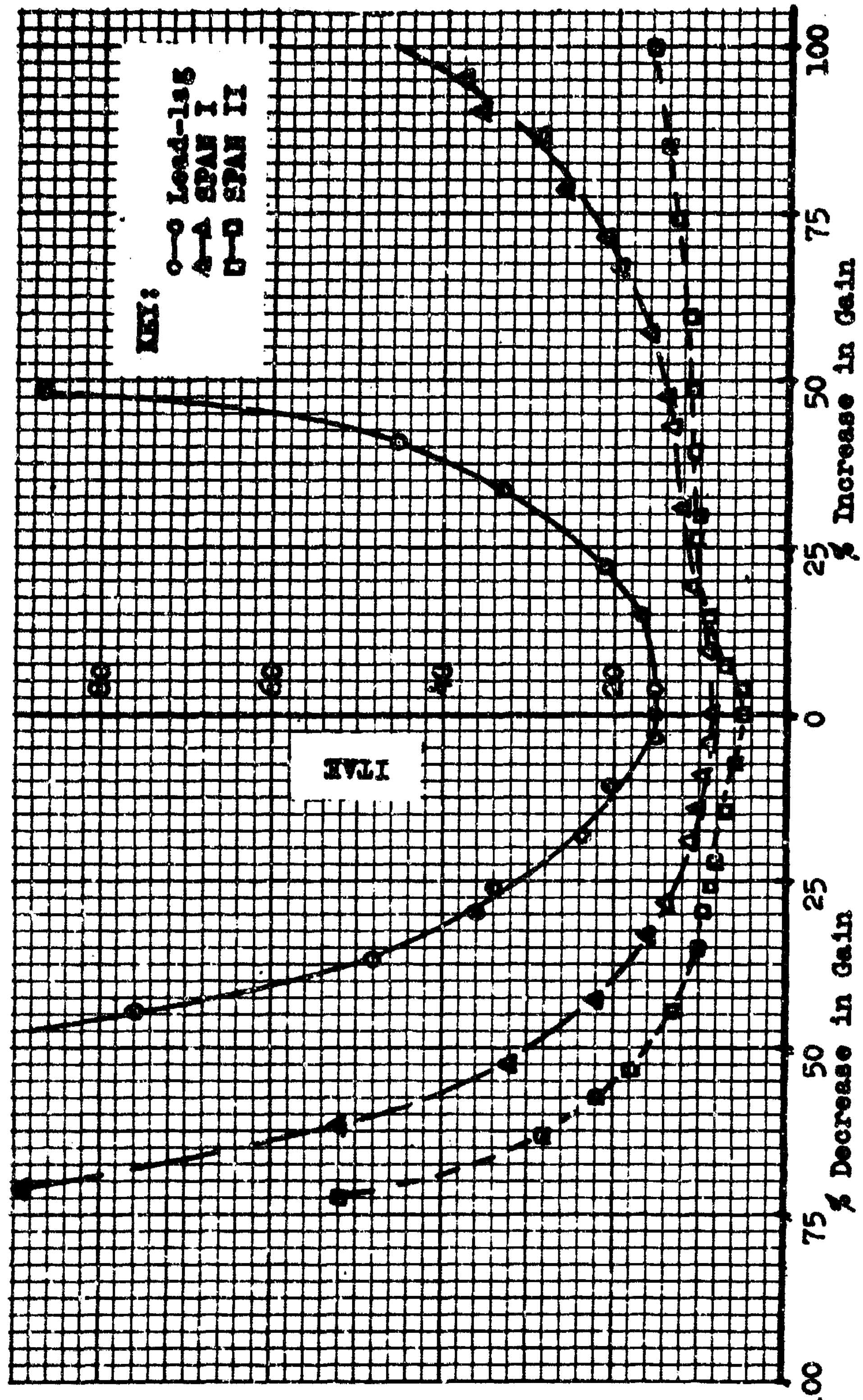


FIGURE 27

EFFECT OF GAIN VARIATION ON THE ITAE FOR COMPENSATED TYPE 1 PLANT WITH COMPLEX POLES

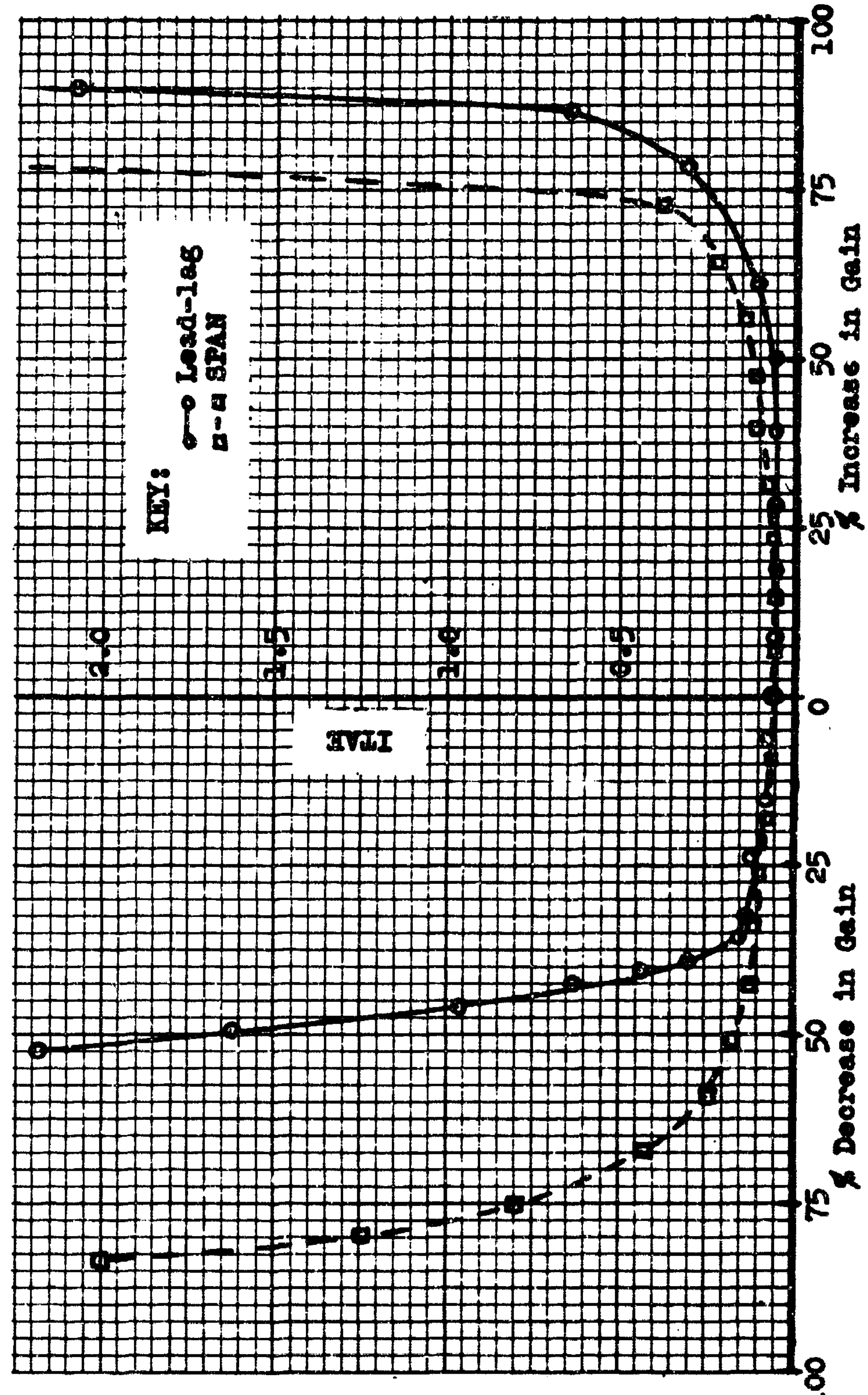


FIGURE 28

EFFECT OF GAIN VARIATION ON THE ITAE FOR COMPENSATED NON-MINIMUM PHASE PLANT

Vita

John A. Tondl, Jr. was born on 6 February 1930 in Ravenna, Nebraska, the son of John Tondl and Anna Tondl. After graduating from Schuyler High School, Schuyler, Nebraska, in 1947, he enlisted in the United States Air Force. While serving as an enlisted man from 1947 to 1954, his duties included tours as an instructor in both the USAF Radar Mechanics School and the USAF Radiological Defense School. He received a commission as a 2nd Lieutenant in June 1954 after graduating from the USAF Officers' Candidate School. From 1954 to 1957 he was assigned as both an instructor and liaison officer with the Armed Forces Special Weapons Project. In February 1960 he received his Bachelor of Science in Electrical Engineering from the University of New Mexico where he attended as an AFIT student. From February 1960 up to his assignment to the Air Force Institute of Technology, he was assigned in AFLC with duty stations at Chateauroux AS, France, and Kirtland AFB, New Mexico.

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This thesis was typed by Mrs. Anna L. Lloyd.