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First Quarterly Report

WIDE BANDWIDTH BASELINE RADAR STUDY

L. W. Martinson
R. P. Perry
W. I. Smith

RCA

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Rome Air Development Center
Research and Technology Division
Air Force Systems Command
Griffiss Air Force Base, New York

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FOREWORD

This report is the First Quarterly Report under Contract AF30(602)-4115, Project 6512, Task 651210. The report covers the study period of 21 March 1966 to 30 June 1966. Work was performed by the Radio Corporation of America, Missile and Surface Radar Division, Moorestown, New Jersey under contract with the Rome Air Development Center, Griffiss Air Force Base, New York. The work was performed under the direction of R. McMillan (EMASS). The report was co-authored by L.W. Martinson, R. P. Perry and W.I. Smith. The assistance of other RCA personnel, specifically J.W. Croly, H.M. Halpern and Dr. S.M. Sherman is also acknowledged.

Release of subject report to the general public is prohibited by the Strategic Trade Control Program, Mutual Defense Assistance Control List (revised 6 January 1965), published by the Department of State.

This technical report has been reviewed and is approved.

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ABSTRACT

The Wide Bandwidth Baseline Radar study was originally intended to provide the ASFIR Interferometer system with a wide bandwidth signal capability. A large part of the work described here was directed toward that goal. An additional consideration was the possible use of the future wide bandwidth radar at the Floyd Site in a wide baseline system. By RADC direction, in the latter part of this Quarter, the Floyd Site radar and its projected wide band capability was considered to be the most appropriate basis for the study, so effort was reoriented. The shift in emphasis is from broadbanding a baseline radar to that of adding a baseline capability to a broadband radar. This latter course appears more realizable. This report considers:

- Tradeoffs in the implementation of signal processing versus communication system bandwidth. An economical approach is considered.

- Processing gain achievable through the use of various postdetection smoothing techniques. Second degree polynomial smoothing is most applicable.

- The nature of data from a wide baseline system. In particular, relationships are derived for the accuracy of the location of the scatterers as a function of baseline lengths.

- Design considerations for the ASFIR wideband transmitter are provided from an early study, performed before the shift in emphasis to the Floyd Site.

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LIST OF SYMBOLS

δT_R	-	rms error in time delay to target
B	-	frequency variation of linear FM signal
E	-	total received signal energy
N_0	-	noise power per unit bandwidth
λ	-	carrier wavelength
c	-	velocity of light (3×10^{10} cm per second)
f	-	carrier frequency
E'	-	transmitted energy per pulse
KT	-	Boltzmann's constant times effective temperature
NF	-	receiver noise figure
L	-	additional loss factor
R	-	range of target
G	-	antenna gain
A_T	-	target cross section
R_1	-	monostatic range to scatterer #1 (one way)
R_1'	-	bistatic range to scatterer #1
R_2	-	monostatic range to scatterer #2 (one way)
R_2'	-	bistatic range to scatterer #2
γ	-	angle between ℓ and R_2 defined in plane of R_1, R_2 and ℓ
α	-	angle between ℓ and $(R_1' - R_1)$ defined in plane of $(R_2' - R_2)$, $(R_1' - R_1)$ and ℓ
ℓ	-	distance between scatterers #1 and #2
L	-	baseline length
θ	-	Master Site azimuth angle

LIST OF SYMBOLS
(continued)

ε	-	Master Site elevation angle
θ'	-	Slave Site azimuth angle
ε'	-	Slave Site elevation angle
ϕ	-	angle between ℓ and L
ψ	-	angle between $(R_2' - R_2)$ and R_2
ΔR	-	monostatic differential range measurement
$\Delta R'$	-	bistatic differential range measurement
S(t)	-	a time waveform representing the actual differential range variation
n(t)	-	unwanted noise due to estimation errors in range variation
h(t)	-	filter impulse response
$S_s(f)$	-	signal spectral density
S(f)	-	noise spectral density
y(t)	-	sum of signal plus noise
σ^2	-	variance of the noise
W	-	maximum tumbling rate of satellite
W'	-	1/2 pulse repetition period
n	-	order of polynomial
T	-	duration of sample
t	-	time
λ_K	-	K^{th} undetermined multiplier constant
J	-	integer
K	-	integer
β	-	prediction time

LIST OF SYMBOLS
(continued)

A	-	amplitude of sine wave
W_0	-	frequency of sine wave
ν	-	phase of sine wave
A'	-	maximum amplitude of sine wave
a	-	unknown parameter
$\epsilon(t)$	-	error
$L(\underline{X})$	-	likelihood function

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SECTION I

INTRODUCTION

The purpose of this study is to establish the equipment requirements to construct a wideband baseline radar system. The system is to be capable of utilizing relative range measurements, precise to less than one carrier wavelength, to derive the scattering point configuration of the target. The combination of wideband techniques for fine range resolution and the swept-frequency interferometer techniques for angle measurement and resolution is intended to provide superior space object identification.

The study initially called for the consideration of the ASFIR radar as the basic equipment. The ASFIR system is an interferometer having intermediate wideband capabilities, i.e. 50 MHz. The objective therefore was to provide approximately 300 MHz bandwidth for this interferometer. Recent work in updating the Floyd Site radar to provide an intermediate bandwidth capability of 250 MHz and an ultimate bandwidth of 500 MHz made it mandatory to consider the Floyd Site as a possible baseline radar in lieu of ASFIR. Therefore two months after the start of the program the RADC Project Engineer directed that emphasis be placed on the Floyd Site as a potential baseline system. The emphasis of the study was shifted by this decision from that of incorporating wide bandwidth into an interferometer to that of incorporating a baseline, bistatic capability into a wide bandwidth system. This decision has made it easier to meet the desired goals of the program, primarily because of the 60-foot dish available at the Floyd Site in comparison to a 24-foot dish available with the ASFIR.

Principal requirements for the system under study are:

a. The modified equipment must be able to determine the internal structure and motion of the target by measuring the relative positions and motions of the individual scattering points.

b. Relative range measurement must be refined to within one wavelength of the carrier frequency, constituting a coarse measure, and relative phase measurement is to be provided as a fine vernier, so that relative ranges and relative angles of scattering points of the target can be determined unambiguously.

The principal tasks are listed below in the order of performance modified for application to the Floyd Site:

a. Determination of target parameter distributions.

b. Determine of system parameters and tradeoffs.

c. Preliminary specifications and block diagrams.

d. Selection of baseline - consideration of accuracy, correlatibility, ghosting.

e. Signal processor tradeoffs.

f. Extension of data handling to slave site.

g. Range and angle track implementations.

h. Microwave link requirements and tradeoffs.

i. Data format.

j. System error analysis to prove adequacy of design.

k. Simulation of wideband baseline system.

l. System tradeoffs.

m. Final Report.

Item a	will involve estimates of the percentages of targets having particular characteristics. Classification by orbit, stability, radar properties where available.
Items b - h	will emphasize the design of the slave site and integration with developments at the Floyd Site.
Item i	will examine the system from the user's point of view.
Items j, k	will evaluate the system designed in prior items. In addition to investigating the sources of error and effects on system performance, a small scale simulation is planned. It is expected to obtain monostatic, and possibly bistatic, wideband model data. Utilizing this data, target geometry and motion will be inserted and such items as the baseline, data output, smoothing, target characteristics can be evaluated.
Item l	will utilize the results of these studies to obtain a final recommended configuration.

SECTION II

SYSTEM DESCRIPTION

1. BASELINE SYSTEM IMPLEMENTATION

A diagram of a possible Floyd Site baseline radar system is shown in Figure 1. Details of the system are omitted since the diagram is intended to delineate those functions which are required by the baseline mode. System components which are added are enclosed by dashed lines and those components changed are indicated by double line blocks.

The major transmitter receiver, signal processing and data handling units are under development for the Floyd Site radar system. The principal system characteristics of this radar are summarized.

<u>Signal Processor:</u>

Waveforms:

	Phase I	Phase II
Mode I	20-usec CW pulse	40-usec CW pulse
Mode II	20-usec x 2.5-MHz FM ramp	40-usec x 2.5-MHz FM ramp
Mode III	20-usec x 250-MHz FM ramp	40-usec x 500-MHz FM ramp
Mode IV	Selectable sequence of signals from Modes I, II, III, or any combination of these modes.	

Recording and Analysis Bandwidths:

2.5 MHz and 250 KHz (Time Expansion of 2.5 MHz)

Tracking:

Modes I and II - Acquisition and intermediate tracking using vertical polarization (transmitted).

Mode III - High-precision range tracking.

Mode III Outputs - Amplitude and Phase versus range profiles in 100-foot range window for both horizontal and vertical polarization.

Dynamic Range: 45 db minimum

Resolution: Phase I - 4 feet, Phase II - 2 feet; (for two equal velocity point targets differing by an amplitude of 34 db within range window.)

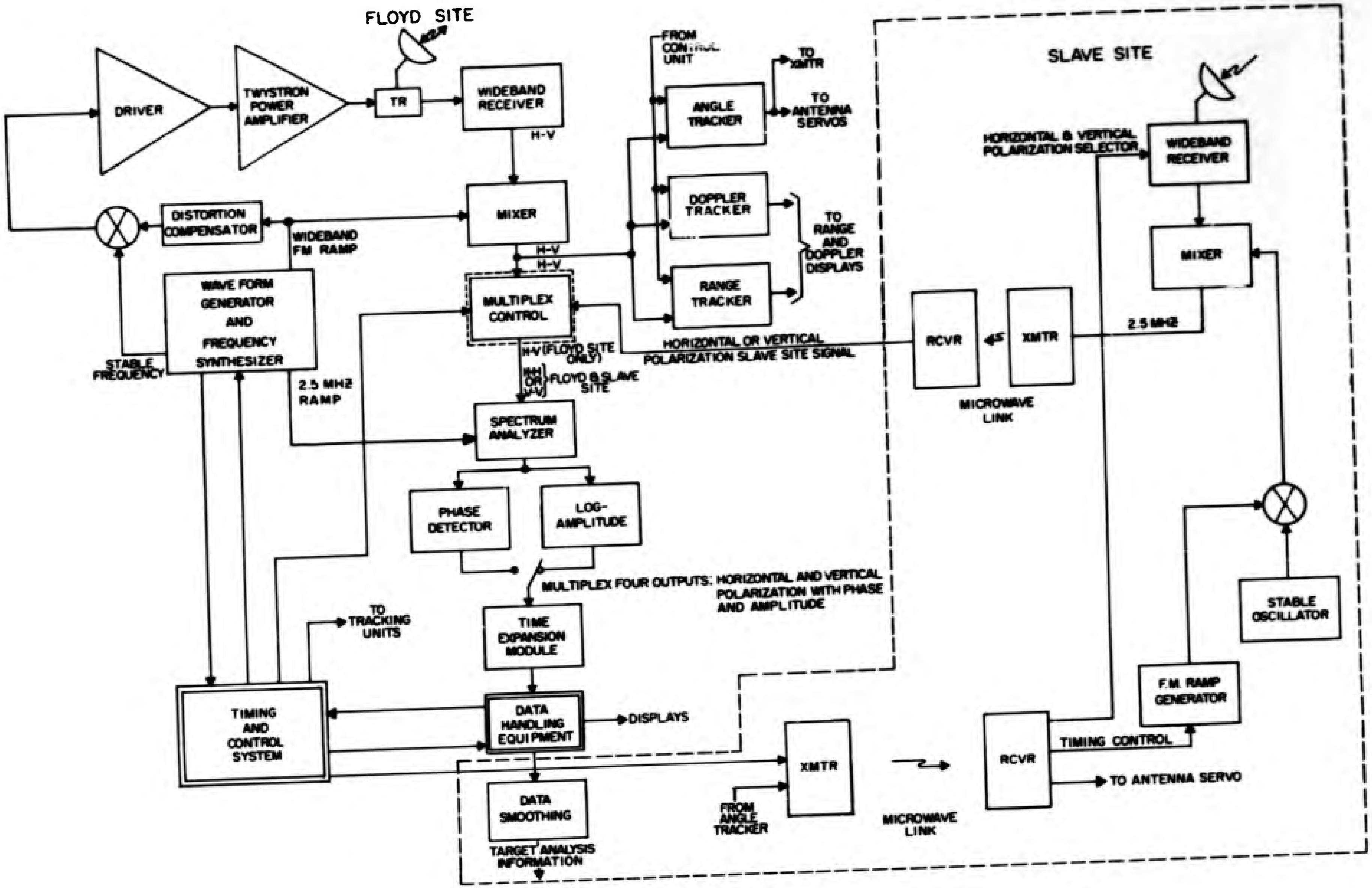

Figure 1 - PRELIMINARY FLOYD SITE BASELINE RADAR SYSTEM DESIGN

Transmitter Characteristics:

	Phase I	Phase II
Peak Power	10 Mw	10 Mw
Average Power	10 Kw	20 Kw
Center Frequency	3350 MHz	3350 MHz
Modulation Bandwidth	250 MHz	500 MHz
P.R.F.	70 pps	70 pps
Pulse Width	20 usec	40 usec
Polarization	- Transmit - vertical, Receive - simultaneous vertical and horizontal	

Antenna:

Antenna Gain:	Transmit; 53 db
	Receive; 47 db
Side Lobes:	Transmit; -15 to 20 db
	Receive; -18 db

Structural Characteristics:

60-foot diameter Cassegrain antenna with five-horn monopulse feed

Addition of a baseline capability to the Floyd Site radar requires the construction of a slave site with a duplication or sharing of receiving, signal processing and data handling equipment; the construction of a communication link between the two sites for signal data transfer and control; and the design and construction of a data smoothing facility to improve target analysis capability. The design shown in Figure 1 is dependent on the validity of the assumptions with respect to the Floyd Site radar design. (Data was not yet available on equipment to be constructed.) In the system diagram the slave site radar receiver is identical to the Floyd Site receiver with the deletion of the monopulse capability. By using a locally generated wideband FM ramp, the requirements on the bandwidth of the communication link are greatly reduced. This is practical because of the fact that only relative phase data is used in target analysis. The target parameters can be derived from monostatic range and monostatic and bistatic range changes which in turn are determined from relative received carrier phase variations. (See Section V.) Note that heterodyning preserves phase regardless of frequency and therefore it is only necessary that stable heterodyning references, not necessarily of exactly the same frequency, be provided to preserve received carrier phase through the signal processing system.

Two general requirements must be met: 1) the FM ramps must be synchronized in time to provide the same range window, and 2) the combined local oscillator and FM ramp frequency must be of sufficient stability and accuracy that the rate of change of frequency as a function of time is constant to within the measurement accuracy required. The received signal at the slave site after heterodyning with the wideband FM ramp is only 2.5 MHz. This is much less than the requirements imposed by ramp synchronization. Ramp synchronization to within about one foot should be sufficient. A communication link with a bandwidth of 50 MHz, such as is used in the ASFIR system should be adequate for this purpose. The synchronizing pulses can be time multiplexed with the doppler and tracking information.

In the baseline mode simultaneous horizontal and vertical polarization is not implemented since the handling of only one polarization at a time leads to a significant simplification in the equipment. The signal processor in the Floyd Site which simultaneously processes both polarizations can be adapted to the baseline mode by addition of a multiplex control. With this addition the operator is able to select either horizontal or vertical polarization signals for processing in the system, but not both. The data handling equipment is only modified for routing signals to the data smoothing unit in the interferometer mode. Further study will be made on the question of relative performance with and without both polarizations.

The system described represents a cost effective approach to providing high definition baseline data for experimental investigation.

2. IMPLEMENTATION OF AN ABSOLUTE INTERFEROMETER

The system configuration previously described is intended to meet minimum requirements with respect to baseline phasing and provides only relative phase between point scatterers at the slave site. If an absolute interferometer utilizing absolute bistatic range is required, the system becomes more complicated, since the absolute phase of the received signals at the slave site must be known with respect to the Floyd Site. This can only be achieved if the absolute delay between the two sites is known to within a fraction of the carrier wavelength and the absolute phase of the received signals is preserved. Heterodyning at the slave site must therefore be done with a precise reference identical to, or slaved to, the reference used at the Floyd Site. Two general approaches can be considered; the wideband FM ramp signal can be relayed to the Floyd Site via a wideband communication link or a delayed FM ramp reference can be transmitted to the slave site for heterodyning via a similar link.

Microwave communication systems with the required 500 MHz bandwidth are not available as off-the-shelf items, but the components are available and systems can be fabricated for special applications. The maximum signal-to-noise ratio obtainable may limit the ability to relay the wideband received radar signal because of the dynamic range requirement. However, a lower signal-to-noise ratio is adequate for either transmitting the reference or an accurate synchronizing signal. One of the principal problems in the instrumentation of the link is the retention of wideband waveform shape. through the various dispersive elements in the chain by compensation methods. Time-varying effects due to temperature and humidity changes introduce additional complications. Further study is required to determine the feasibility of compensation. Phase and frequency control of a local oscillator can conceivably be achieved by transmitting two references from the Floyd Site and heterodyning to obtain the esired frequency. The baseline delay must be accurately measured and controlled.

The detailed requirements for a true interferometer capability will be investigated further, together with a study of possible implementation techniques.

SECTION III

SIGNAL-TO-NOISE RATIO REQUIREMENTS FOR ENVELOPE RANGE ACCURACY

This program calls for investigation and study to establish the equipment requirements for experimentally verifying the ability of a wideband baseline radar system to utilize relative range measurements with a precision equivalent to less than one carrier wavelength to derive the scattering point configuration of a target. Specifically it requires that the range rms error be less than $\pm 12°$ of carrier phase, for each baseline element, in determining relative range of two scatterers separated in range by at least 3 feet, and each having a cross-section of 0.1 square meters.

This can be accomplished by measuring the phases of the scattering point returns against some common reference. The phase measurement, however, is subject to an ambiguity of 360°. The envelope position measurement, on the other hand, is a gross measure which can be used to resolve the ambiguity. This approach requires that the signal-to-noise ratio of the received signal be such that the relative range of the scatterers can be estimated to an accuracy of less than one carrier wavelength.

If we consider the Floyd Site system, it will be shown that there is sufficient received energy for that system to achieve this without any post processing of multiple pulse returns. In the initial stages of the program when the ASFIR system was being considered, this was an important problem and considerable post processing gain was required. Problems can arise, however, even with the Floyd Site as the master station if the slave site uses a smaller antenna dish. Also, if the scatterers have a smaller radar cross-section, some post processing may have to be employed.

For a linear FM pulse signal, the rms error in the range delay estimate is[1]:

$$\delta T_R = \frac{\sqrt{3}}{\pi B(2E/N_0)^{\frac{1}{2}}} \qquad \text{(III-1)}$$

where δT_R is the rms error in seconds, B is the frequency variation, E is the signal energy and N_0 is the noise power per cycle per second.

Assuming equal radar cross-sections for two scatterers (therefore equal signal-to-noise ratios), the rms error in relative envelope position measurement is $\sqrt{2}$ times the rms error in measuring the envelope position of each scatterer. If the relative position is to be measured to $\pm 1/2$ wavelength, the position of each scatterer must be measured to $\pm \frac{1}{2\sqrt{2}}$ wavelength. Therefore defining the rms error as:

$$\delta T_R = \frac{\frac{1}{2\sqrt{2}}\lambda}{c} = \frac{1}{2\sqrt{2}\,f} \qquad \text{(III-2)}$$

where λ is the carrier wavelength, f is the carrier frequency and c is the velocity of light.

Equating (III-1) to (III-2) and substituting B = 500 MHz and f = 3000 MHz, the required signal-to-noise ratio is:

$$\frac{E}{N_0} = 16.4 \text{ db}$$

Allowing 1 db loss for a tapered weighting for side lobe reduction, the adjusted signal-to-noise ratio at the receiver is:

$$\frac{E}{N_0} = 17.4 \text{ db}$$

The signal-to-noise ratio out of the Floyd Site system in its final configuration is:

$$\left(\frac{E}{N_0}\right)_{Act} = \frac{E' G^2 A_T \lambda^2}{(4\pi)^3 KT\,\overline{NF}\,L R^4} \qquad \text{(III-3)}$$

in which:

E' = transmitted energy in each pulse

= $(20 \times 10^{-6})(7.15 \times 10^{6})$ = 143 joules

KT = 4×10^{-21} watts/Hz

$\overline{NF}$ = 5 db, or a factor of 3.2 (assumed)

L = losses = 3 db or a factor of 2 (assumed)

R = range = 500 n.m. = 9.2×10^5 meters

G = antenna gain = 52 db = 1.6×10^5

λ = wavelength = (3350 MHz) = .0895 meters

A_T = target cross-section = 0.1 meter^2

Substituting these into Equation (III-3), the actual signal-to-noise ratio is:

$$\left(\frac{E}{N_0}\right)_{Act} = 19 \text{ db}$$

This indicates that the signal-to-noise ratio per pulse is sufficient for the Floyd Site operating in its final configuration and as a monostatic radar.

The actual rms error would be (substituting into Equation (III-1)):

$$\delta T_R = 0.0965 \text{ feet}$$

(which includes the 1 db loss due to side lobe suppression). This is compared to a carrier wavelength of 0.3 feet.

For the interim Floyd Site system which has a 250 MHz bandwidth, the rms error would be 0.193 feet. If we assume the slave site does not have a 60-foot antenna but rather a 20-foot antenna, the bistatic received signal-to-noise ratio will decrease by 10 db. This will result in an rms error per pulse of 0.62 feet.

Since this error is above what can be tolerated to determine the range to within one carrier wavelength, it is necessary to perform some form of data smoothing to reduce the error to within acceptable limits.

The received signal-to-noise ratio for the ASFIR system, if we consider G = 42 db (24-foot dish) and E' = 1000 joules, would be:

$$\frac{E}{N_0} = 7.44 \text{ db}$$

which indicates the need for at least 10 db gain by post processing smoothing for the ASFIR system.

SECTION IV

SIGNAL-TO-NOISE RATIO REQUIREMENTS FOR PHASE ACCURACY

The equipment specification requires that the error due to thermal noise in measuring relative phase between two scatterers shall not exceed ± 12 degrees of carrier phase. Assuming equal radar cross-section for the two scatterers (therefore equal signal-to-noise ratios), the rms error in relative phase is $\sqrt{2}$ times the rms error in measuring the phase of each scatterer, which must therefore be $\pm 12/\sqrt{2}$ degrees.

The formula relating phase accuracy to signal-to-noise ratio (E/N_0) is[1]:

$$\Delta \phi = \frac{1}{\sqrt{2E/N_0}} \qquad \text{(IV-1)}$$

in which $\Delta \phi$ is the phase error expressed in radians.

Solving for E/N_0:

$$\frac{E}{N_0} = 13.5 \text{ db}$$

This shows that the signal-to-noise ratio (19 db) for the final Floyd Site configuration is quite adequate to meet the phase accuracy requirements.

If however there is a 10 db loss due to a smaller receiving antenna used in bistatic operation, then some sort of data smoothing is necessary to provide the equivalent of a 4.5 db increase in signal-to-noise ratio.

If the ASFIR system were used, an equivalent signal-to-noise improvement of 6 db would be required.

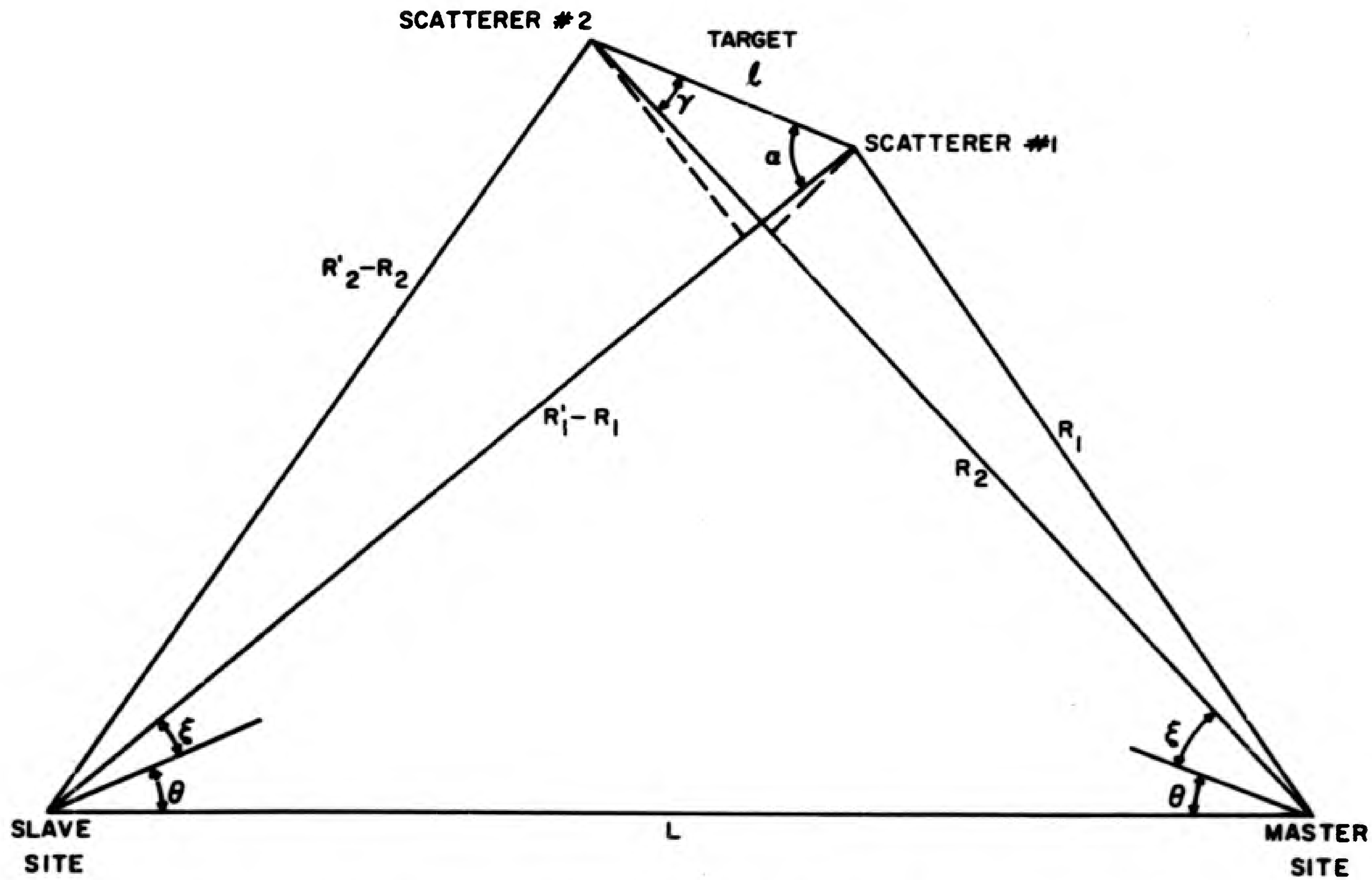

Figure 2 - GEOMETRY OF WIDE BASELINE RADAR SYSTEM

SECTION V

INFORMATION IN BISTATIC SYSTEM OUTPUT

We can construct a geometric model which can be used to reconstruct the target based on the monostatic and bistatic differential range measurements. A model of the wide baseline bistatic radar system is shown in Figure 2. This figure represents the target at any instant of time as being comprised of two discrete scatterers whose distance, ℓ, between them is constant due to the body rigidity. (Actually this distance can vary if we consider the scatterers as moving over the surface of the body.)

The monostatic differential range can be determined by projecting the target separation distance, ℓ, onto the range vector R_1 or R_2. Since the range distance is quite large with respect to the target extent this approximation is quite valid.

The angle γ is defined in the plane containing R_1, R_2 and ℓ; the angle α is defined in the plane containing R_1', R_2' and ℓ.

The monostatic range difference is:

$$\Delta R = 2(R_1 - R_2) = -2\ell\cos\gamma \qquad \text{(V-1)}$$

The bistatic range difference is:

$$\Delta R' = R_1' - R_2' = -\ell\cos\gamma + \ell\cos\alpha \qquad \text{(V-2)}$$

If we consider the special case where the master site, slave site and the target scatterers are all in the same plane, then the diagram shown in Figure 3 is applicable.

From Figure 3, the monostatic range difference is:

$$\Delta R = R_2 - R_1 = \ell\sin(90^\circ - \theta + \phi) = \ell\cos(\phi - \theta) \qquad \text{(V-3)}$$

and the bistatic range difference is:

$$\Delta R' = R_2' - R_1' = \ell\cos(\phi - \theta) + \ell\cos(\theta - \phi + \psi) \qquad \text{(V-4)}$$

where

$$\sin\psi = \frac{L\sin\theta}{R_2' - R_2} \qquad \text{(V-5)}$$

The information contained in the received signals is therefore:

Monostatic Range:

$$\Delta R = \ell\cos(\phi - \theta) \qquad \text{(V-6)}$$

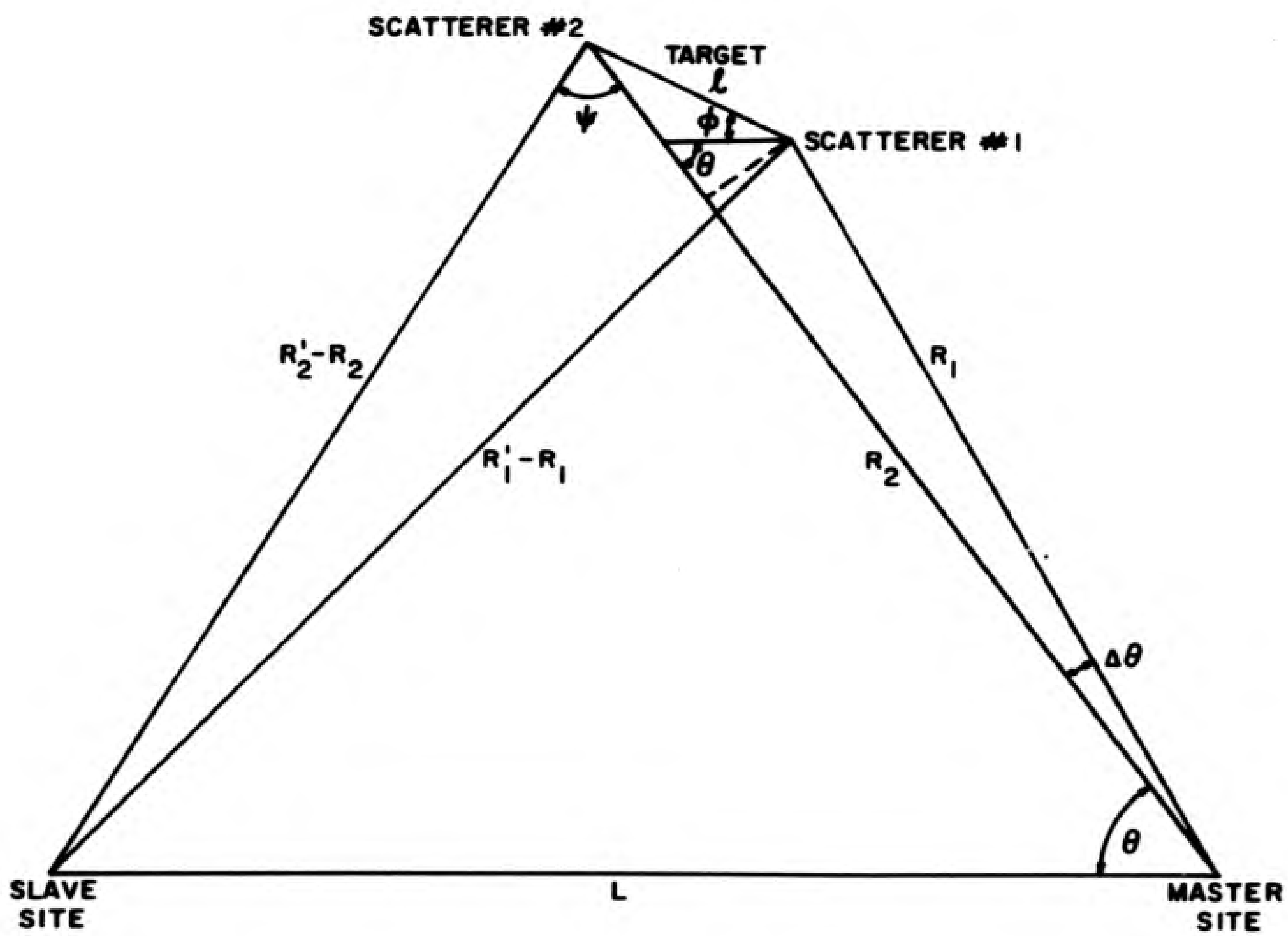

Figure 3 - GEOMETRY OF WIDE BASELINE SYSTEM FOR PLANAR CASE

Bistatic Range:

$$\Delta R' = 2\Delta R - \frac{L \ell \sin\theta}{R_2' - R_2} \sin(\theta - \phi) \qquad \text{(V-7)}$$

Thus, given: θ, ΔR, $\Delta R'$, L, $R_2' - R_2$, we can solve for ℓ and ϕ.

It can be noted that there will be no ghost problem; that is, having the order of reception of two targets differ in the monostatic and bistatic stations. This can be shown if we consider the previous bistatic and monostatic equations for the case of $\theta = 90^o$.

$$\Delta R' \text{ (bistatic)} = \Delta R \text{ (monostatic)} - \frac{\ell' L}{R_2' - R_2} \cos\phi \qquad \text{(V-8)}$$

then $\Delta R'$ and ΔR can reverse in sign only if

$$\frac{\ell' L}{R_2' - R_2} \cos\phi > \Delta R \qquad \text{(V-9)}$$

substituting:

$\phi = 0$

$\ell = 30$ ft.

$L = 7$ n.m.

$R_2' - R_2 = 200$ n.m.

$\Delta R < 1.05$ feet

which is below the range resolution capability of the system.

The measured differential ranges will change from pulse to pulse due to the motion of the target. If the target motion were sinusoidal, then the bistatic and monostatic range differences resulting from the projection components of the target will also vary sinusoidally. The specification of this study requires consideration of all targets with tumbling rates less than 2 revolutions per minute.

We could also consider several additional types of target motion as shown in Table I. The more complex the type of motion, the more difficult it is to smooth the differential range information.

Also in many cases some additional problems to consider are that a solid target will cause masking of the target scatterers for at least half a cycle of motion, and the scattering point may drift over the surface of the target.

TABLE I - TARGET MOTION

	Type	Effects on ΔR and $\Delta R'$
1.	Fast Tumbling Rate	Nearly Sinusoidal
2.	Slow Tumbling Rate	Includes Tumbling Effect and Orbital Rotation
3.	No Tumbling	Orbital Rotation
4.	Tumbling and Precessing Target	Complex Periodic Motion

SECTION VI

DATA SMOOTHING

If we assume that the error in measurement of the monostatic and bistatic range for each transmitter pulse is independent of the errors in other pulses, then we can consider this process similar to that of sampling at equal intervals, corresponding to the Nyquist rate, the sum of a deterministic signal and band limited white noise. This model is shown in Figure 4.

The time-varying signal plus noise can be reconstructed by feeding the sample data into a low pass filter.

We can consider several possible models for the type of smoothing required to increase the relative range estimation accuracy of the system. The first would be to consider S(t) as an infinite sample of an unknown stochastic process with known spectral density. (A model of which is discussed in Appendix I.) This would represent a model of a detected satellite whose tumbling rate was not known. Also this would represent a model with no shadowing of the data during each tumbling cycle. The second model we will consider is when shadowing does occur and the data representing S(t) only exists for half a cycle. The third model we will consider is when the functional form of S(t) is known; such as a section of a sine wave and the problem is then to estimate the parameters of the waveform.

Of the three models considered, the second one appears the most suitable to this particular problem. This is due to the fact that this model does not rely on knowledge of the signal waveform and the uncertainty of the exact relative scatterer motion. It will be shown that a second order polynomial smoothing filter will be quite suitable to solve the smoothing problem.

1. CASE I

In this case we assume an input signal y(t) to consist of a wanted signal S(t) added to noise n(t). Both S(t) and n(t) are taken to be sample functions from real-valued wide-sense stationary random processes with a stationary cross-correlation. We want to find a weighting function h(t) for a fixed-parameter realizable linear filter which acts on the entire past of y(t) in such a way that the output of the filter at time t is a best mean-square approximation to S(t) [2].

If the signal and noise are uncorrelated and either has zero mean, it is shown in Appendix II that the optimum filter transfer function is:

$$H(f) = \frac{S_s(f)}{S_s(f) + S_n(f)} \qquad \text{(VI-1)}$$

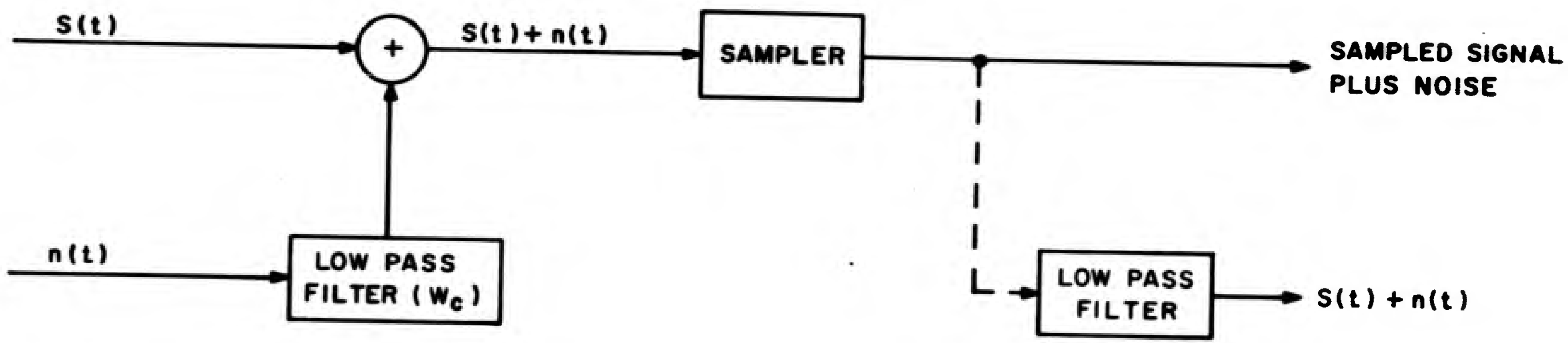

Figure 4 - MODEL OF SAMPLE DATA SYSTEM

Defining $S_n(f)$ as that of band limited white noise with noise power per unit bandwidth of $N_0/2$ and bandwidth of W′, the noise power without filtering would be:

$$\sigma_{in}^2 = \int_{-W'}^{+W'} \frac{N_0}{2} \delta f = N_0 W' \qquad \text{(VI-2)}$$

The noise power with filtering is:

$$\sigma_{out}^2 = \int_{-W}^{W} \frac{N_0}{2} \delta f = N_0 W \qquad \text{(VI-3)}$$

therefore:

$$\frac{\sigma_{out}^2}{\sigma_{in}^2} = \frac{W}{W'} \qquad \text{(VI-4)}$$

if W = maximum tumbling rate of satellite = 0.0033 rps and
W′ = ½ pulse repetition period = 25 cps.

$$\frac{\sigma_{out}^2}{\sigma_{in}^2} = 28.75 \text{ db}$$

This type of a model can only be used as an indication of the type of smoothing gain which might be expected under ideal conditions of an unknown waveform embedded in wideband white noise. Practically conditions are different. That is, we will usually see the waveform for less than one-half a cycle.

2. CASE II

Methodology has been developed where the unknown signal is represented by a n^{th} order polynomial whose coefficients are unknown over some sample duration, T. Appendix III shows a derivation of a general polynomial smoothing filter. It is shown that the optimum filter impulse response, h(t), is:

$$h(t) = \sum_{0}^{n} \lambda_K' \, t^K \qquad \text{(VI-5)}$$

where the λ_K's are determined by the solution of

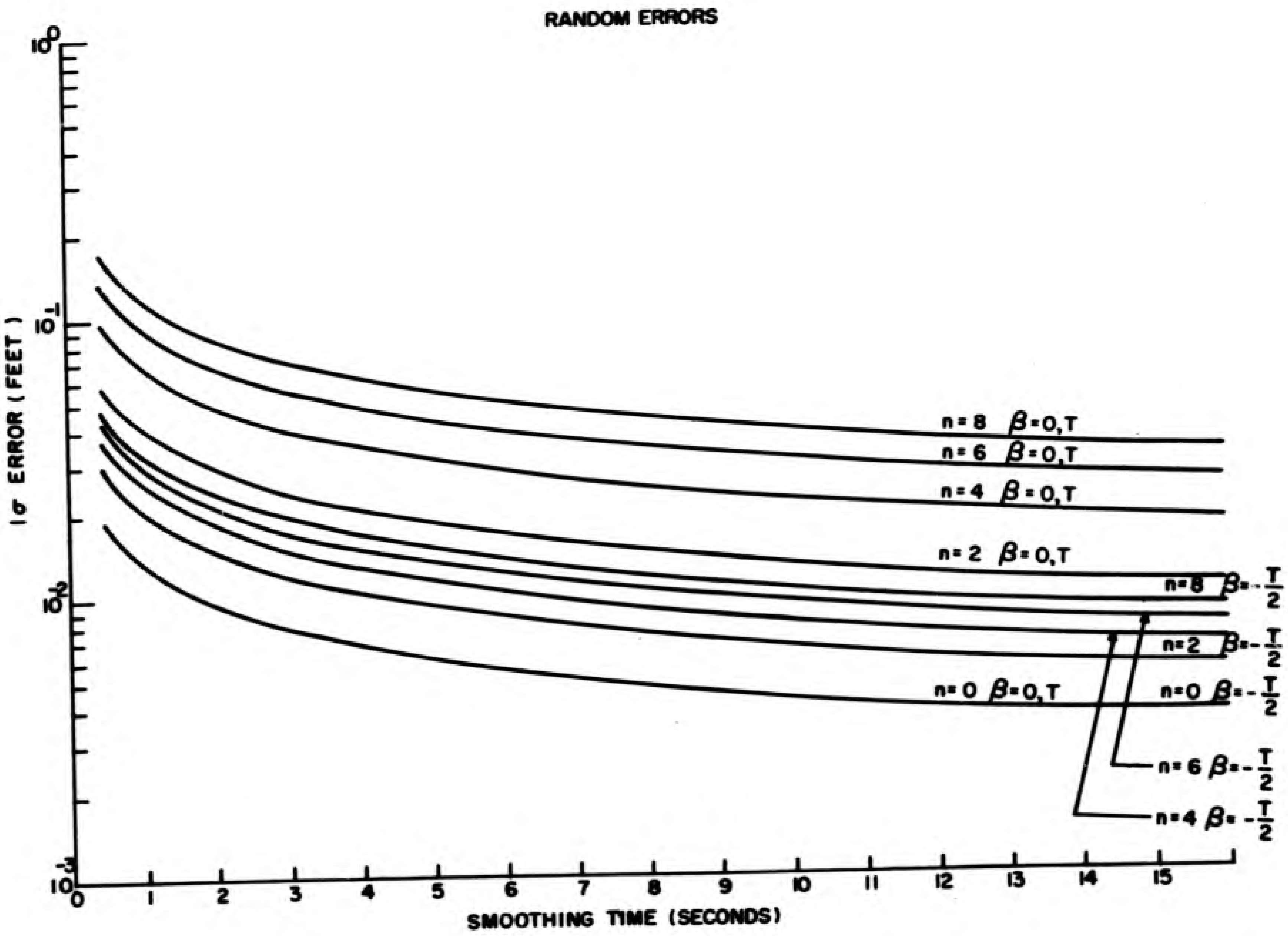

Figure 5 - RANDOM ERRORS WITH POLYNOMIAL SMOOTHING

$$\sum_{0}^{n} \lambda'_K \left(\frac{T^{(j+K+1)}}{j+K+1} \right) = (-\beta)^j \qquad K,j = 0, 1, 2, \ldots, n \quad \text{(VI-6)}$$

For a zero order smoothing (n = 0)

$$h(t) = \frac{1}{T} \qquad (0 < t < T) \qquad \text{(VI-7)}$$

For first order smoothing (n = 1)

$$h(t) = \frac{4}{T} - \frac{6}{T^2} t \quad (0 < t < T) \qquad \text{(VI-8)}$$

For second order smoothing (n = 2)

$$h(t) = \frac{9}{T} - \frac{36t}{T^2} + \frac{30}{T^3} t^2 \qquad (0 < t < T) \qquad \text{(VI-9)}$$

The noise power out of the smoothing filter can be calculated as shown in Appendix IV to determine the ratio of input to output noise power.

Using a digital computer the impulse response of the smoothing filter and the smoothing gain was calculated. The results are shown in Figure 5. The plots are based on the final version of the Floyd Site system which has a bandwidth of 500 mcs and a received signal-to-noise ratio of 18 db. If half the bandwidth is used (250 mcs), the curves should be raised by a factor of 2. Plots are shown for the error (in feet) at the center and at either end of the data aperture, the error being the minimum at the center.

Since it was assumed that the signal could be represented by a n^{th} order polynomial, it is also necessary to determine the systematic errors which result from this approximation. Figure 6 shows a plot of systematic errors of a tumbling rate of 0.033 cps or the maximum required for the system. A reasonable tradeoff is the selection of smoothing time so that the random errors and systematic errors are equal. A plot of these intersections are shown in Figure 7. These are shown for three different system configurations. From this figure, it is noted that a first or second polynomial is probably the most efficient with a smoothing time of two to three seconds.

3. CASE III

The third model we will consider is that of estimating the parameters of an unknown waveform whose functional form is known. That is, by knowing the parameters we can completely define the waveform. This model would consider the case when the differential

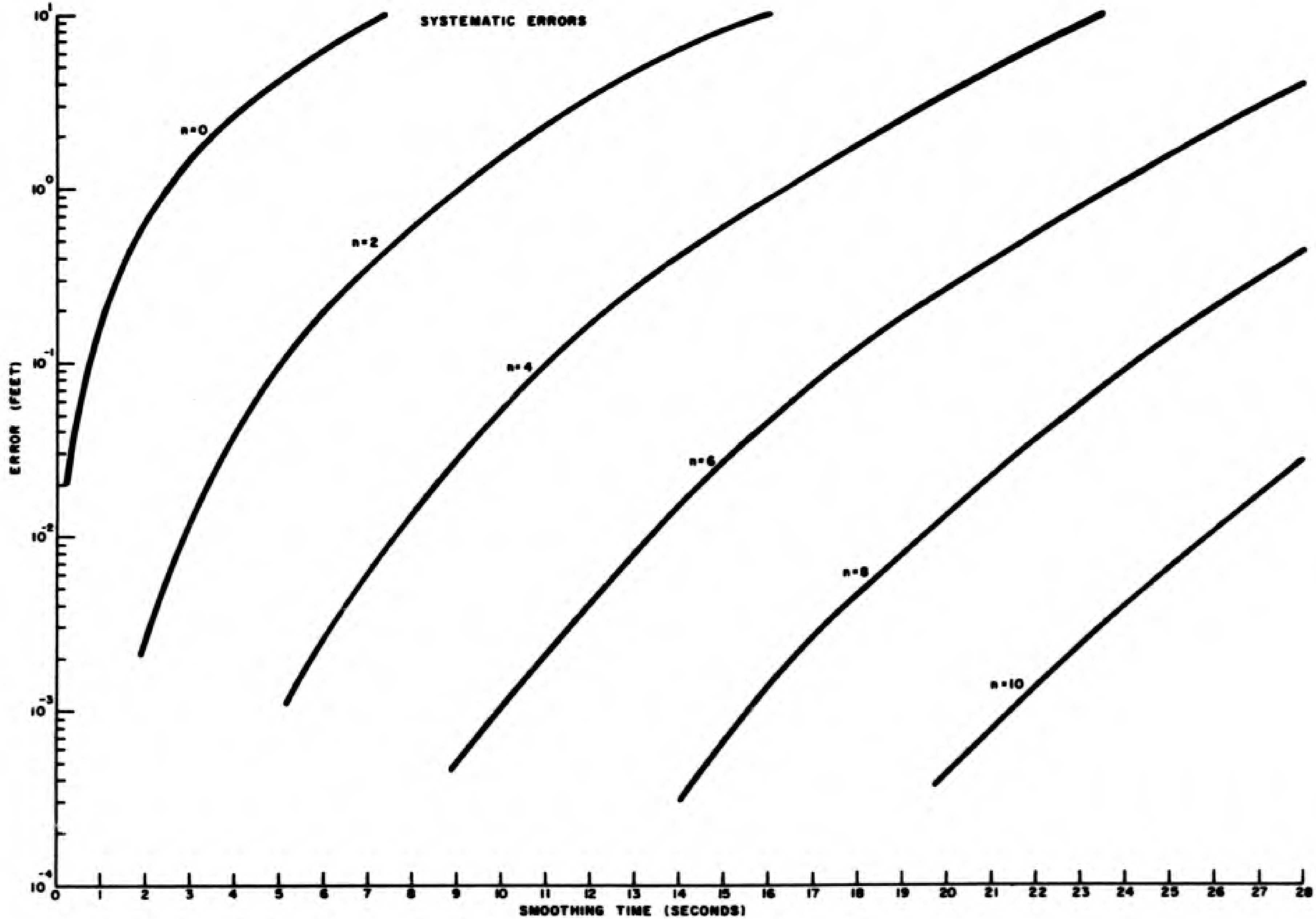

Figure 6 - SYSTEMATIC ERRORS

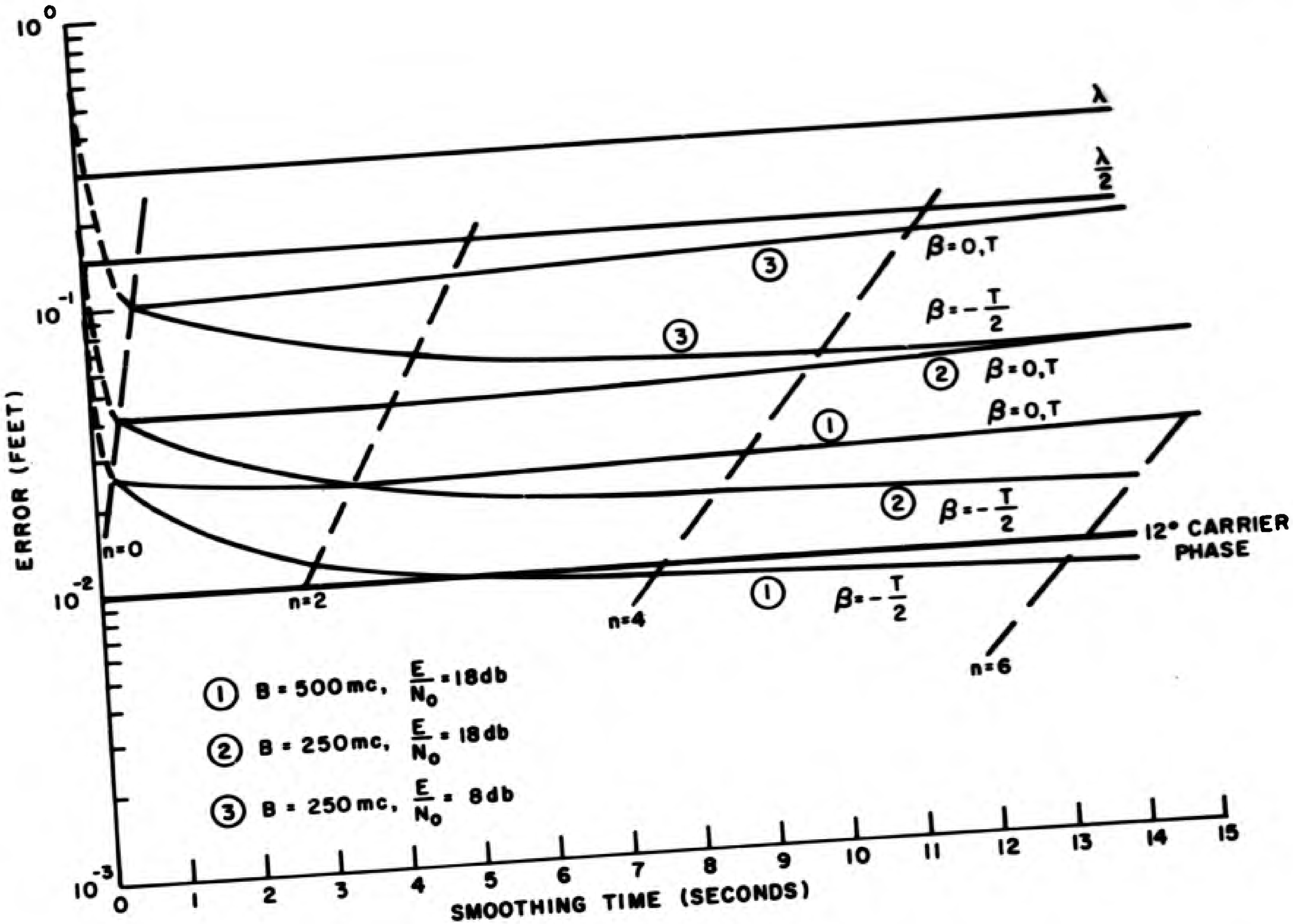

Figure 7 - OPTIMUM SMOOTHING TIME

range variations are exactly sinusoidal and there was no shifting in target location on the body. It is shown in Appendix V that, if the signal, S(t), is represented by a sine wave of unknown amplitude, then the optimum estimate of the amplitude, A, is:

$$\hat{A} = \frac{\int_{-T}^{T} X(t)\cos(W_0 t + \mu)\, dt}{T + \frac{1}{2W_0}\left[\sin(2W_0 T)\cos(2\mu)\right.} \qquad \text{(VI-10)}$$

and the variance of the estimate is:

$$\sigma^2(\hat{A}) = \frac{N_0}{T}\left(1 + \frac{\sin(2W_0 T)}{(2W_0 T)}\sin(2\mu)\right)^{-1} \qquad \text{(VI-11)}$$

For the case of:

$$W_0 = 0.033 \text{ cps}$$

$$T = 15.2 \text{ seconds}$$

$$\mu = 0$$

$$\sigma^2(\hat{A}) = \frac{N_0}{T} = \frac{\sigma^2}{760}$$

where σ^2 is the variance of the estimate in range for a single pulse.

For the final configuration Floyd Site system, the rms envelope error would be:

$$\sigma(\hat{A}) = \frac{0.0965}{27.6} = 0.00350 \text{ ft.}$$

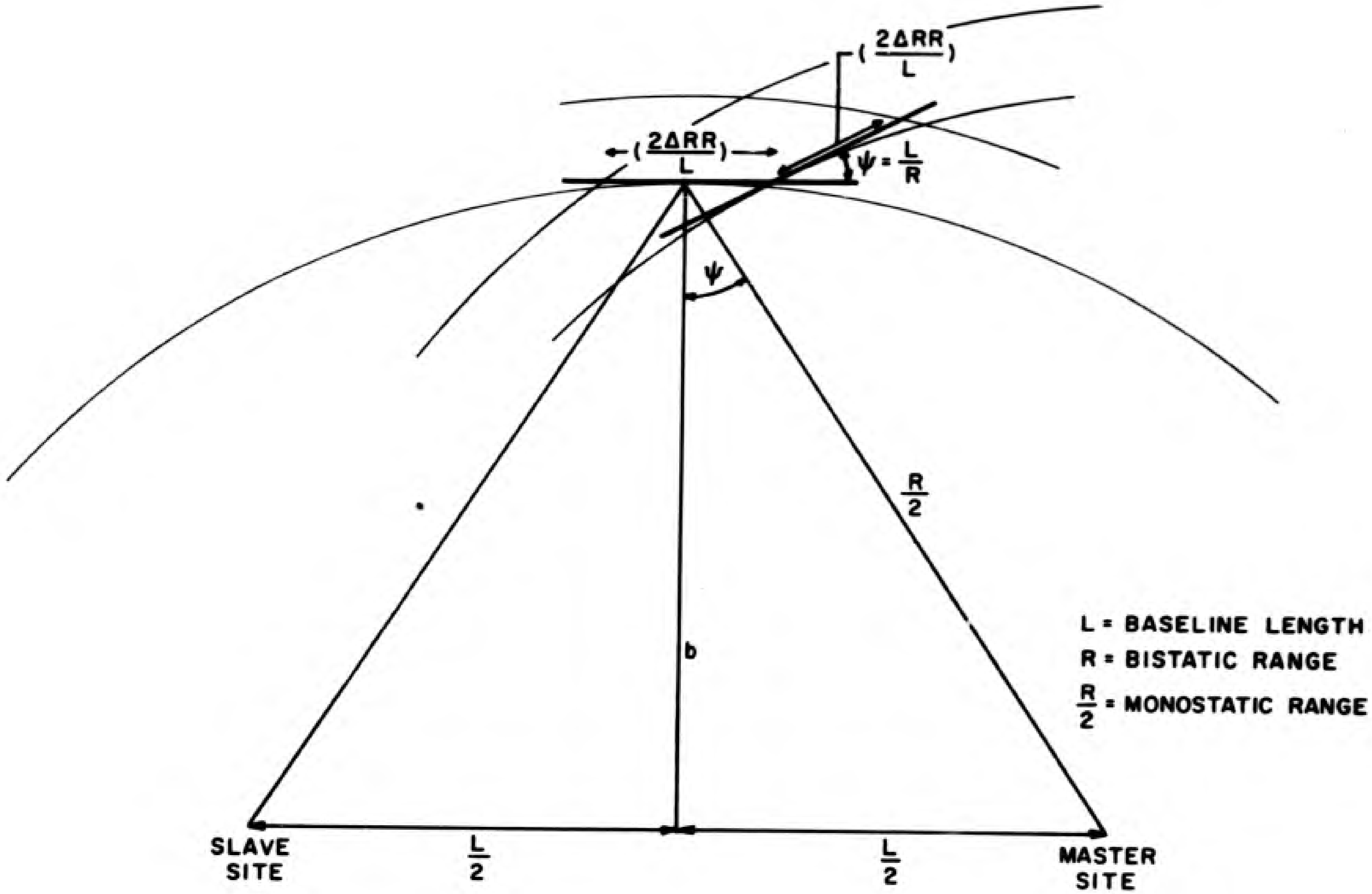

Figure 8 - SYSTEM ACCURACY

SECTION VII

BASELINE SELECTION

One of the problems to be solved as part of this study is the optimum length and direction of the baseline for the wide bandwidth, wide baseline radar system. Since the target orbital motion enables even a monostatic system to look at the target at different angles parallel to the orbital path, it is desirable to have the direction of the baseline perpendicular to the direction of travel. Although it may be difficult to establish what satellite will exist one to two years from now, it is possible to base this direction on the typical paths of satellites of interest.

The length of the baseline can be established comparing the tradeoffs between the accuracy with which the relative locations of the scatterers can be determined, the distortion effects due to the scattering points shifting on the target surface, ghosting effects, shadowing, and losses due to baseline propagation.

We can determine the effect the baseline has on the relative accuracy with which each target scatterer can be located by considering the diagram shown in Figure 8.

Contours of constant monostatic range are determine by concentric circles. Contours of constant bistatic range are determined by confocal ellipses. The special case of the target being equidistant between the two radar sites is shown in Figure 8. By considering individually the effects of measurement errors in bistatic and monostatic range, we can construct an area of error as shown in the diagram.

If we consider the case where the range is 100 nautical miles and the baseline length is 10 nautical miles, each leg of the diagram would correspond to 0.2 feet and ψ would be 5.74°. The distance between each set of lines would be 0.02 feet. The error will increase linearly with range and decrease inversely with the baseline length.

SECTION VIII

WIDEBAND TRANSMITTERS (300 MHz) for an ASFIR WIDE BANDWIDTH SYSTEM

1. SUMMARY

The Floyd Site radar transmitter now under RADC contract provides one important alternative for a wideband S-band transmitter suitable for the system being considered. This approach exploits the best features of the Twystron-type tube in a short pulse (20 - 40 us) system. An alternative crossed-field amplifier type transmitter offers unusually excellent spectral purity of chirped output for pulse durations in the order of one millisecond by combining modern developments including the Linear Beam Switch Tube, a high power amplitron, floating deck keyer circuits and a low bandwidth phase correction servo. The attributes of these tubes and techniques mate uniquely well, to answer the need for long pulse wideband systems.

A decision to employ the Floyd Site Transmitter for this program resulted in terminating effort on the long pulse system design prior to overall cost estimating and detailed analytical effort.

2. SYSTEM ALTERNATIVES

The objective of this effort was to explore transmitter alternatives for use in the Wide Bandwidth Baseline Radar system. Two types of final tubes suited to the application were considered to be within the state of the art. The first is a crossed-field type amplifier, such as an amplitron, and the second a hybrid TWT/klystron (Twystron) type tube, exclusively manufactured by the Varian Corporation.

The required combination of broad instantaneous bandwidth and high power are not presently available in other types of tubes.

A broad tube specification was prepared to assist in selecting tubes and was distributed to all major tube suppliers. A copy of this specification is shown in Appendix VI.

Raytheon can supply a suitable amplitron final amplifier. Litton Industries can supply a nearly ideal type switch tube to pulse the amplitron. Varian can supply a suitable Twystron, and can also supply TWT's which can drive either the amplitron or Twystron final tubes.

Discussions were held with each of the vendors, Raytheon, Varian and Litton.

Budgetary quotes were solicited.

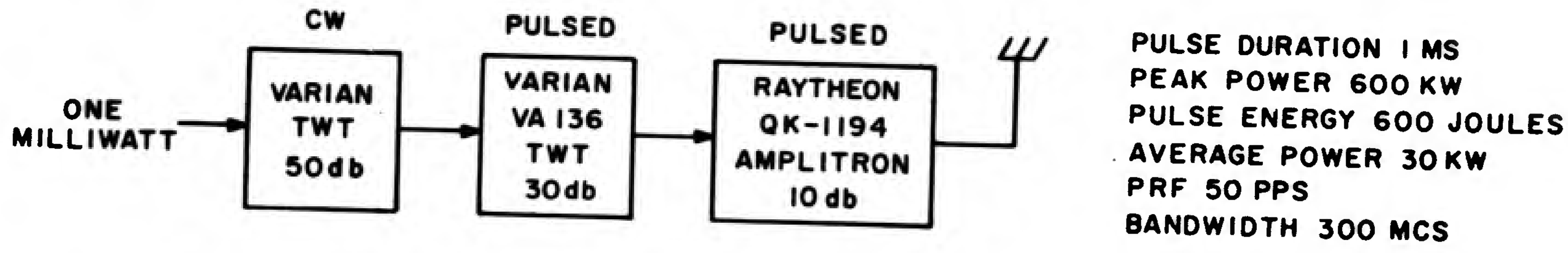

FIGURE 9

STATE OF THE ART TUBE LINEUP

FOR THE CROSSED FIELD FINAL AMPLIFIER ALTERNATIVE

The Twystron investigation revealed that the state of the art in 1967 will be represented by the 40 us pulse, 7.2 megawatt tube (288 joules/pulse) and is now under development by Varian for RADC. An extension of the art, even if development is undertaken, could look to a maximum 100 us pulse tube at 3 megawatts peak power (still 300 joules/pulse). Extension of the pulse duration to 100 us probably does not offer any significant tradeoff advantage to system parameter design, as it is in the same technique range appropriate to 40 us operation. Hence, additional Twystron development effort does not seem advantageous.

Since a transmitter for the twystron tube is already under development by Westinghouse Corporation for RADC within the Signal Processing Test Facility program, continued effort in exploring this type transmitter seems unwarranted.

The Westinghouse Twystron transmitter design, or perhaps the actual Floyd Site radar, could be available for the purposes of this program.

The Floyd Site Twystron transmitter under development has the following characteristics:

Peak power	7.2 megawatts
Pulse repetition frequency	70 pps
Pulse duration	20 to 40 us
Frequency	3000 to 3500 mcs, linear chirp
Average power	10 to 20 kilowatts
Joules/pulse	288

The alternative crossed-field amplifier type transmitter could use the following RF tube lineup:

The advantages attributed to the amplitron transmitter are:

1. High efficiency - 65-70%
2. High percentage bandwidth - 7 - 13%
3. Good phase linearity - 2-3° across the band into a flat load
4. Low phase pushing factor - 10% current changes
5. Versatility - duplexing at the input power level and direct gainless feed through the "cold" final amplifier in variable power systems.

The disadvantages are:

1. Small gain per tube - 10 - 16 db
2. Spurious generation - adjacent frequency filtering required
3. Good input-output isolation required
4. The requirement for RF excitation to precede the cathode pulse
5. Variation of operating impedance across the frequency band

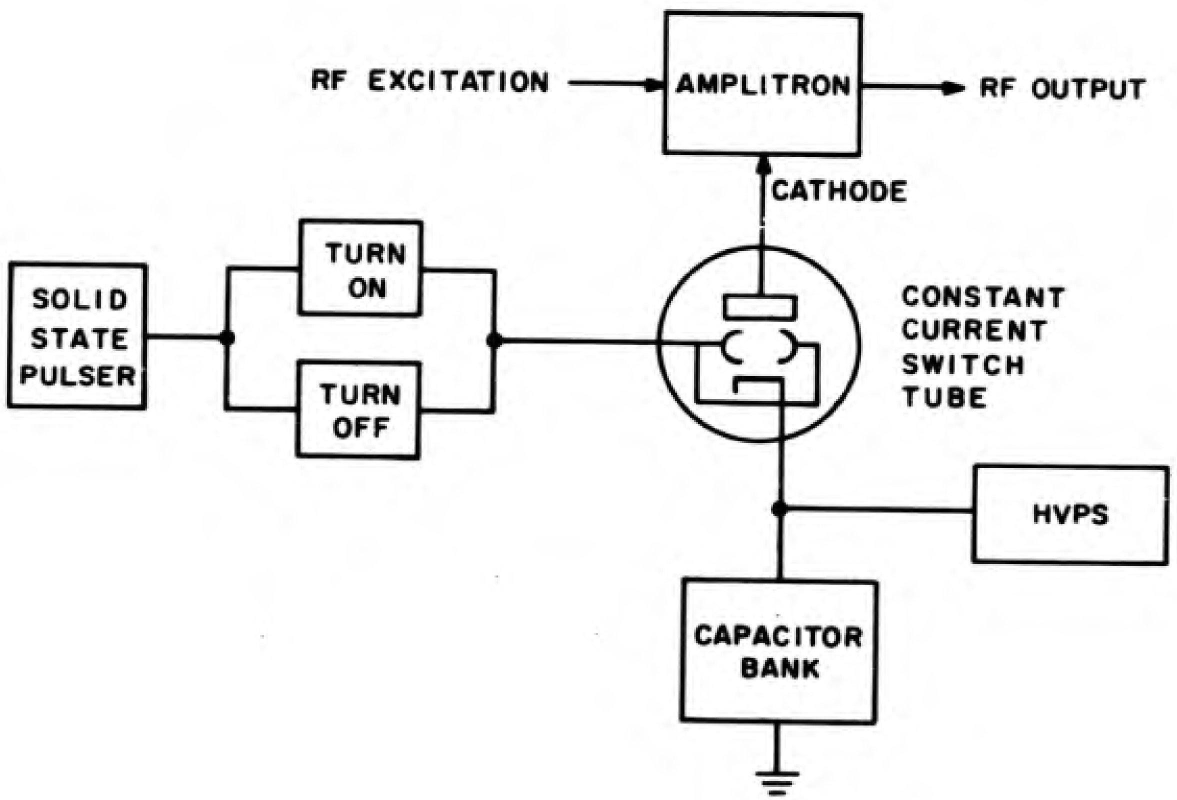

Figure 10 - CONSTANT CURRENT FLOATING DECK PULSER

Figure 10 shows a pulsing technique that is ideally suited to operation of an amplitron in a chirped system.

The switch tube (see Figure 13) is a minor modification to the linear beam switch tube designed by Litton Industries for RCA during the TRADEX program. With the voltage at the modulating anode fixed, the current is constant, independent of load impedance variation and power supply voltage droop. This allows a smaller capacitor bank than normal. It also keeps the amplitron operating current at an optimum, fixed level, across the chirped bandwidth, despite the variation of amplitron impedance. Finally, it eliminates phase pushing effects in the amplitron since it maintains constant intra-pulse current. For pulses as long as one millisecond, it is relatively easy to keep the modanode voltage constant (as is necessary) using a "floating deck" type pulser nearly identical to that designed for the TRADEX radar transmitter.

Pulse linearity disturbances due to mismatches in the overall transmitter system can be reduced, using a closed loop feedback system as in Figure 11.

A suitable phase bridge has been developed by RCA, for comparing dynamically (during the chirped pulse) the relative phase variations between transmitter input and output signals. It derives an instantaneous phase error analog signal which can be amplified and fed to the helix of the first TWT as a phase correction signal. This corrects for phase linearity errors in all three transmitter amplifier stages, when the correct amount of linear time delay is maintained in the bridge reference input arm.

When the pulse duration over which the 300 megacycle chirp occurs is as long as one millisecond, the phase error correction rates are easily handled in a moderate bandwidth servo amplifier system.

Figure 12 shows phase linearity disturbances contributed by the amplitron, as measured in a tube in the same family as the proposed QKS-1194.

The driver TWT features a modanode, which is pulsed with a floating deck pulser very similar to that employed with the linear beam switch tube. RCA has achieved excellent pulse spectral characteristics in the TRADEX program using this type of circuit arrangement. Although the chirp bandwidth is much greater here, the slow phase error rates generated as a result of the long (one millisecond) pulse duration during which chirp occurs, are servo-reduced relatively easily, as in the amplitron stage.

Figure 14 is a block diagram of the overall transmitter design described above.

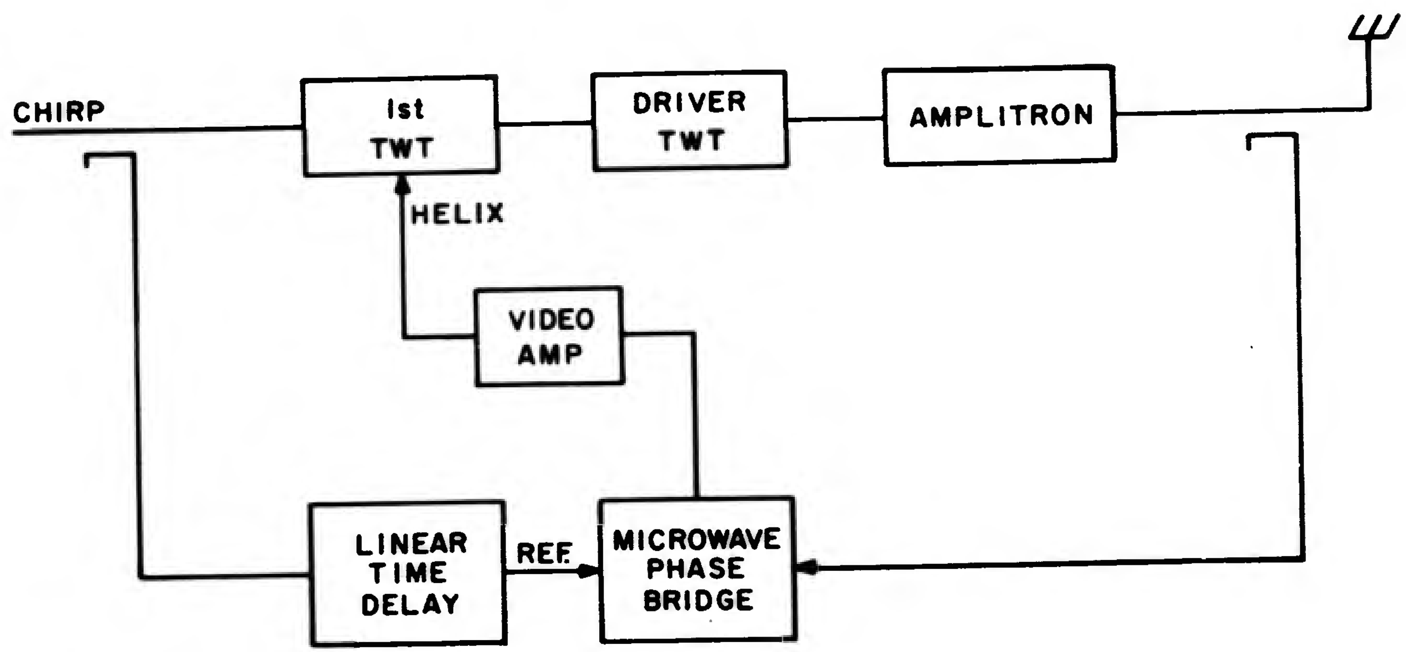

Figure 11 - CLOSED LOOP PHASE LINEARIZER CIRCUIT

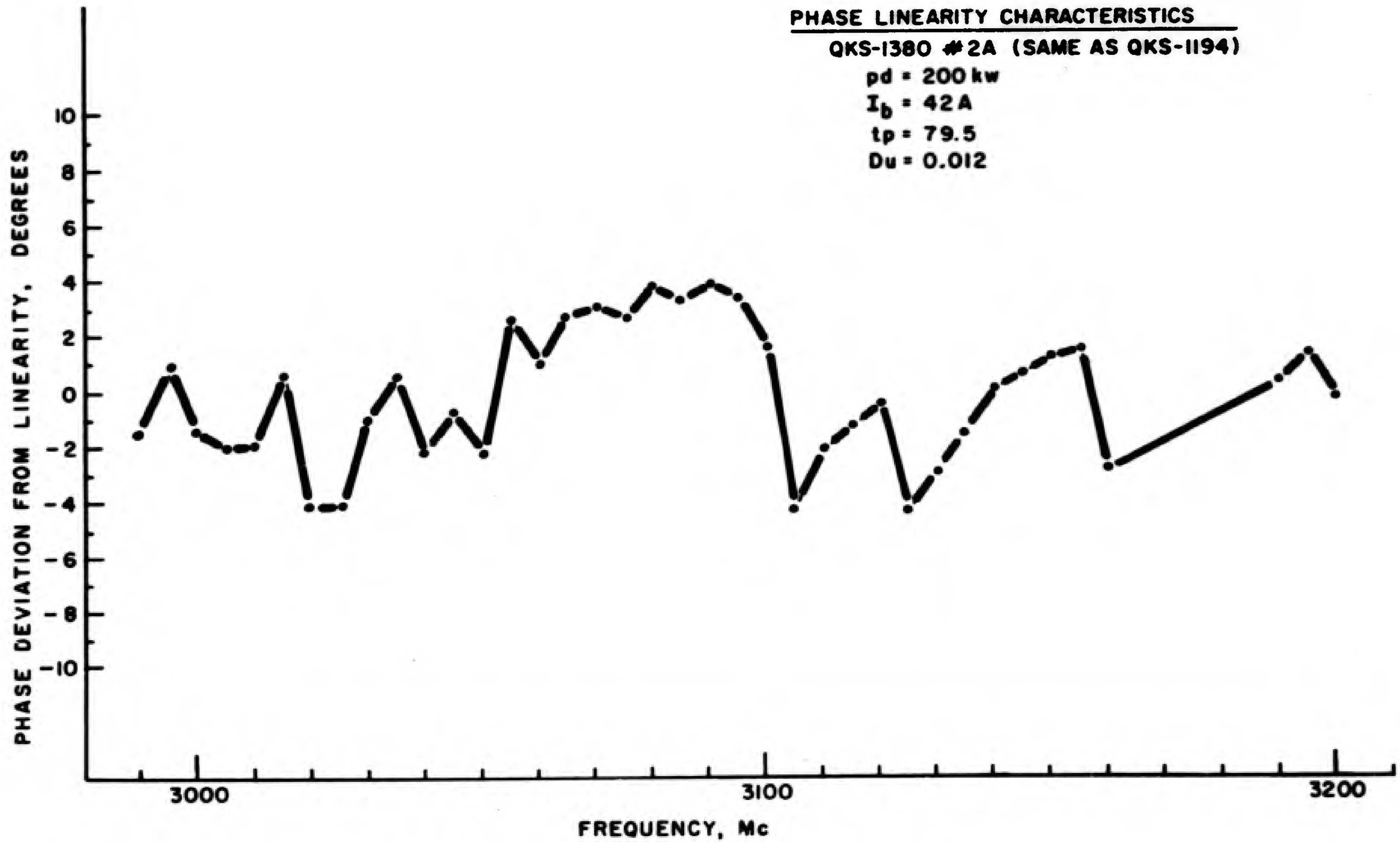

Figure 12 - PHASE LINEARITY PLOT OF AN AMPLITRON OF THE SAME FAMILY AS THE QKS-1194

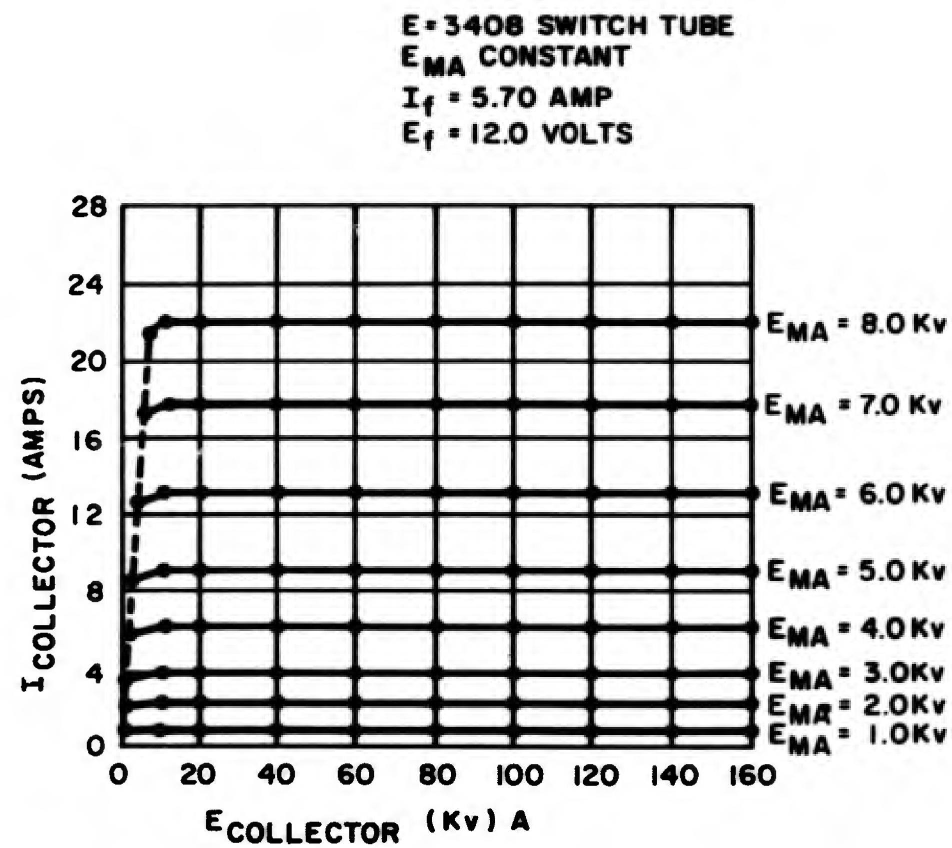

Figure 13 - TYPICAL BEAM SWITCH TUBE i_p e_p CHARACTERISTICS

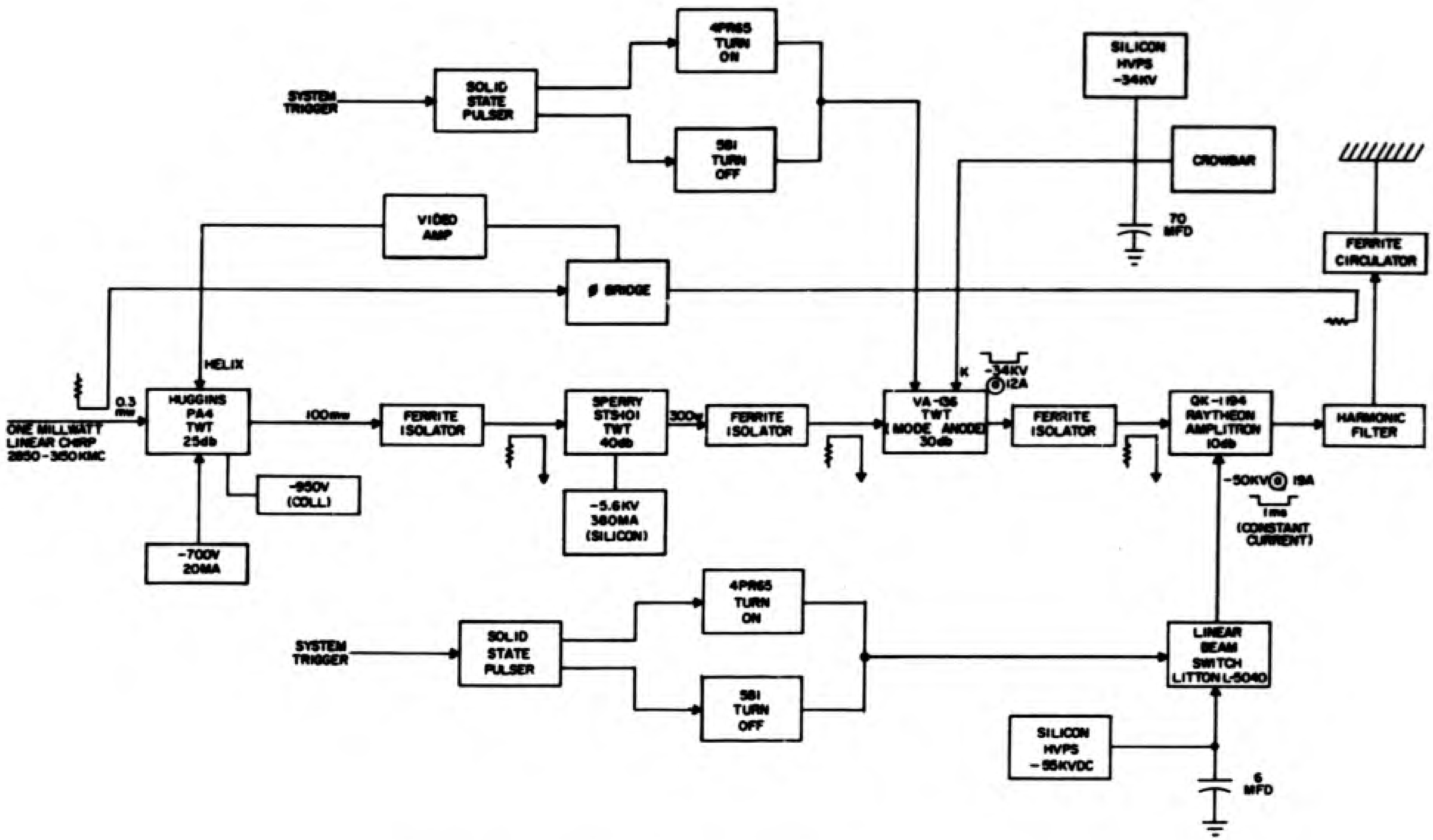

Figure 14 - PHASE LINEAR TRANSMITTER

SECTION IX

SUMMARY AND CONCLUSIONS

By direction, the Floyd Site radar has been used as the basis for the Wide Bandwidth Baseline system instead of the ASFIR system. Initial analysis shows that the bandwidth of the interconnecting microwave link can probably be kept below 50 MHz, provinding good path stability is maintained and the system is used primarily for measurement of relationship between scatterers rather than absolute range. If the system is to be used as an absolute interferometer, it will become somewhat more complicated due to the increased accuracy in the synchronization between the master site and slave site.

The Floyd Site system will have a sufficient received signal-to-noise ratio to determine the envelope position of a single pulse to an accuracy better than one carrier wavelength. In addition, it is capable of determining carrier phase better than the required 12 degree accuracy for this program. The change from the ASFIR to the Floyd Site radar provided considerable improvement in signal-to-noise ratio. The improvement is due to the 60-foot dish available at the Floyd Site. It may not be economical to put such a dish at the slave site.

In the case that a smaller dish is used or it is desired to exceed the specification, post detection processing, or data smoothing, would be necessary.

Three smoothing models have been developed in this report. They are:

1) Where the desired information is an infinite sample of an unknown stochastic process with known spectral density,

2) Where the desired information is a short finite sample of an unknown stochastic process,

3) Where the desired information can be determined when a few specific parameters are known.

Of these three models, the second was chosen as most suitable and was analyzed most completely. It is shown that, if we consider polynomial smoothing, a second order approximation would probably be the most economical and efficient. Although higher orders will give slightly better performance, the mechanization complexity would be unwarranted.

An initial investigation of the tradeoffs in baseline selection has been discussed. Primary emphasis has been on the accuracy in target definition which can be obtained by increasing the baseline length. Based on these calculations, it is suggested that the baseline need be no greater than 10 miles.

SECTION X

PROGRAM FOR NEXT INTERVAL

Performance specifications will be developed for new components or components that must be modified.

Baseline recommendations will be made and justification provided.

Error analysis will be performed on recommended systems to ensure specification compatibility.

Dynamic simulation of the system using cross-section data for wide band signals will be considered, and a determination will be made of what can be done in this vein within the scope of the program.

APPENDIX I

A STOCHASTIC MODEL OF A TUMBLING TARGET

If we consider tumbling targets only, $S(t)$, the signal can be represented as a sample from an ensemble of functions.

$$S(t) = A \cos (w_o t + \mu) \qquad \text{(A.I-1)}$$

where A, w_o, and μ are independent random variables which have uniform probability distributions as follows:

$$P(\mu) = \frac{1}{2\pi} \qquad 0 < \mu < 2\pi \qquad \text{(A.I-2)}$$

$$P(w_o) = \frac{1}{2W} \qquad -W < w_o < W \qquad \text{(A.I-3)}$$

$$P(A) = \frac{1}{A} \qquad 0 < a < A \qquad \text{(A.I-4)}$$

This model is stationary as can be seen by computing the autocorrelation functions of $S(t)$, $R(t, t+\tau)$.

$$R(t, t+\tau) = E\left[S(t)S(t+\tau)\right] = E\left[A^2 \cos(w_o t + \tau) \cos(w_o(t+\tau) + \mu)\right] \qquad \text{(A.I-5)}$$

$$= E\left(\frac{A^2}{2}\right)E(\cos w_o \tau) + E\left(\frac{A^2}{2}\right)E(\cos 2\mu)E(\cos(2w_o t + w_o\tau))$$

$$- E\left(\frac{A^2}{2}\right)E(\sin 2\mu)E(\sin(2w_o t + w_o\tau)) \qquad \text{(A.I-6)}$$

where

$$E\left(\frac{A^2}{2}\right) = \frac{1}{2}\int_{-\infty}^{\infty} A^2\, P(A)\, dA = \frac{1}{2A}\int_0^A A^2\, da \qquad \text{(A.I-7)}$$

$$E(\cos w_o\tau) = \int_{-\infty}^{\infty} \cos w_o\tau\, P(w_o)\, dw_o = \frac{1}{2W}\int_{-W}^{W} \cos w_o\tau\, dw_o \qquad \text{(A.I-8)}$$

$$= \frac{\sin W\tau}{W\tau} \qquad \text{(A.I-9)}$$

$$E(\cos 2\mu) = \frac{1}{2\pi}\int_0^{2\pi} \cos 2\mu \, d\mu = 0 \qquad \text{(A.I-10)}$$

$$E(\sin 2\mu) = 0 \qquad \text{(A.I-11)}$$

Substituting back into (A.I-6)

$$R(t, t+\tau) = \left(\frac{A'^2}{6}\right)\left(\frac{\sin W\tau}{W\tau}\right) \qquad \text{(A.I-]}$$

which indicates S(t) represents a stationary random function.

The spectral density of S(t) is:

$$S(w) = \frac{1}{\pi}\int_{-\infty}^{\infty} R(\tau)\, e^{-jw\tau}\, d\tau \qquad \text{(A.I-13)}$$

$$S(w) = \frac{A'^2}{12W} \quad -W < w < +W \qquad \text{(A.I-14)}$$

APPENDIX II

OPTIMUM WIENER FILTER ASSUMING INFINITE SMOOTHING TIME

Assuming the input signal to the filter to be:

$$y(t) = S(t) + n(t) \qquad \text{(A.II-1)}$$

where S(t) is a wanted signal and n(t) is additive noise. (Both S(t) and n(t) are taken to be sample functions from real-valued wide-sense stationary random processes with a stationary cross-correlation.)

The output of any filter weighting function h(t) is:

$$\int_{-\infty}^{\infty} h(t-\tau)y(\tau)\,d\tau = \int_{-\infty}^{\infty} h(\tau)y(t-\tau)\,d\tau \qquad \text{(A.II-2)}$$

where $h(t) = 0,\ t < 0$

The average squared error, $\mathcal{E}$, is:

$$\mathcal{E} = E\left\{\left[S(t+\beta) - \int_{-\infty}^{\infty} h(\tau)y(t-\tau)\,d\tau\right]^2\right\} \qquad \text{(A.II-3)}$$

if $\beta = 0$

$$\mathcal{E} = E\left[S^2(t)\right] - 2\int_{-\infty}^{\infty} h(\tau)E\left[S(t)y(t-\tau)\right]d\tau + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\tau)h(u)E\left[y(t-\tau)y(t-u)\right]d\tau\,du \qquad \text{(A.II-4)}$$

$$= R_s(0) - 2\int_{-\infty}^{\infty} h(\tau)R_{sy}(\tau)\,d\tau + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\tau)h(u)R_y(\tau-u)\,d\tau\,du \qquad \text{(A.II-5)}$$

Using the calculus of variations, it can be shown that the minimum rms error occurs when

$$R_{sy}(\tau) = \int_{0}^{\infty} h(u)R_y(\tau-u)\,du \qquad \tau \geq 0 \qquad \text{(A.II-6)}$$

Taking Fourier transforms of both sides

$$\int_{-\infty}^{\infty} R_{sy}(\tau)e^{-j\omega\tau}\,d\tau = S_{sy}(f) \qquad \text{(A.II-7)}$$

$$\int_{-\infty}^{\infty}\int_{0}^{\infty} h(u)R_y(\tau - u)\, e^{-j\omega\tau}\, d\tau = \int_{-\infty}^{\infty} h(u)\, e^{-j\omega u}\, du \int_{-\infty}^{\infty} R_y(\xi)\, e^{-j\omega\xi}\, d\xi \qquad \text{(A.II-8)}$$

then:

$$S_{sy}(f) = H(f)S_y(f) \qquad \text{(A.II-9)}$$

$$H(f) = \frac{S_{sy}(f)}{S_y(f)} \qquad \text{(A.II-10)}$$

If the signal and noise are uncorrelated and either signal has zero mean.

$$H(f) = \frac{S_s(f)}{S_s(f) + S_n(f)} \qquad \text{(A.II-11)}$$

APPENDIX III

OPTIMUM POLYNOMIAL SMOOTHING

In this case it is assumed that S(t) has a certain functional form which can be expressed as a polynomial. We want to design a filter with an impulse response h(t) to minimize the error in $S(t + \beta)$.

The filter is constrained so that its output represents $S(t + \beta)$ when only S(t) is applied.

The filter output is:

$$y(t) = \int_{-\infty}^{\infty} h(t_1) S(t - t_1)\, d t_1 \qquad \text{(A.III-1)}$$

Expanding $S(t - t_1)$ in a Taylor series:

$$S(t - t_1) = S(t) - t_1 S'(t) + \tfrac{1}{2} t_1{}^2 S''(t) + \ldots + (-1)^n \frac{t_1}{n!} S^{(n)}(t)$$

(A.III-2)

we can also expand $\hat{S}(t + \beta)$ in a Taylor series:

$$\hat{S}(t + \beta) = S(t) + \beta S'(t) + \tfrac{1}{2} \beta^2 S''(t) + \ldots + \frac{1 \beta^n}{n!} S^{(n)}(t) \qquad \text{(A.III-3)}$$

$$S(t - t_1) = \sum_{k=0}^{k=n} \frac{(-1)^K}{K!} S^K(t)\, t_1{}^K \qquad \text{(A.III-4)}$$

$$\hat{S}(t + \beta) = \sum_{k=0}^{k=n} \frac{\beta^K}{K!} S^K(t) \qquad \text{(A.III-5)}$$

$$y(t) = \hat{S}(t + \beta) = \int_{-\infty}^{\infty} h(t_1) S(t - t_1)\, d t_1 \qquad \text{(A.III-6)}$$

$$\sum_{k=0}^{k=n} \frac{\beta^K}{K!} S^K(t) = \sum_{k=0}^{k=n} \frac{(-1)^K}{K!} S^{(K)}(t) \int_{-\infty}^{\infty} t_1{}^K h(t_1)\, d t_1 \qquad \text{(A.III-7)}$$

Equating coefficient of the derivatives

$$\int_{-\infty}^{\infty} t_1^K\, h(t_1)\, d t_1 = (-\beta)^K \qquad \text{for } K = 0, 1, 2, \ldots, n \qquad \text{(A.III-8)}$$

$$H(S) = \int_{-\infty}^{\infty} h(t_1)\, e^{-St_1}\, d t_1 \qquad \text{(A.III-9)}$$

$$H'(S) = \int_{-\infty}^{\infty} (-t_1) h(t_1)\, e^{-St_1}\, d t_1 \qquad \text{(A.III-10)}$$

$$H^{(K)}(S) = \int_{-\infty}^{\infty} (-t_1)^K h(t_1)\, e^{-St_1}\, d t_1 \qquad \text{(A.III-11)}$$

where

$$S = jw$$

The error $\overline{\varepsilon^2}$ in the output of the filter is:

$$\overline{\varepsilon^2} = E\left\{\left[S(t + n) - \int_{-\infty}^{\infty} h(t_1) y(t - t_1)\, d t_1\right]^2\right\} \qquad \text{(A.III-12)}$$

where $y(t) = S(t) + n(t)$ (A.III-13)

$$\overline{\varepsilon^2} = E\left\{\left[S(t + \beta) - \int_{-\infty}^{\infty} h(t_1) S(t - t_1)\, d t_1 - \int_{-\infty}^{\infty} h(t_1) n(t - t_1)\, d t_1\right]^2\right\} \qquad \text{(A.III-14)}$$

Since the filter is constrained so that

$$S(t + \beta) = \int_{-\infty} h(t_1) S(t - t_1)\, d t_1 \qquad \text{(A.III-15)}$$

then:

$$\overline{\varepsilon^2} = E\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(t_1) h(t_2) n(t - t_1) n(t - t_2)\, d t_1\, d t_2\right. \qquad \text{(A.III-16)}$$

$$\overline{\varepsilon^2} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(t_1) h(t_2) R_n(t_2 - t_1)\, d t_1\, d t_2 \qquad \text{(A.III-17)}$$

the minimum error is when $d(\overline{\varepsilon^2}) = 0$.

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[h(t_1)\delta h(t_2) + h(t_2)\delta h(t_1)\right] R_n(t_2 - t_1)\, dt_1 dt_2 = 0 \qquad \text{(A.III-18)}$$

This is true when (by symmetry)

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(t_2)\delta h(t_1) R_n(t_2 - t_1)\, dt_1 dt_2 = 0 \qquad \text{(A.III-19)}$$

$$\int_{-\infty}^{\infty} \delta h(t_1) \int_{-\infty}^{\infty} h(t_2) R_n(t_2 - t_1)\, dt_1 dt_2 = 0 \qquad \text{(A.III-20)}$$

also setting:

$$\delta(-\beta)^K = 0 = \int_{-\infty}^{\infty} \delta h(t_1) t_1^K dt_1 \quad \text{for } K = 0, 1, 2, \ldots, n \qquad \text{(A.III-21)}$$

If we have a finite time of observation, we have the following (n + 2) number of equations to solve.

$$\int_0^T \delta h(t_1) t_1^K dt_1 = 0 \quad \text{for } K = 0, 1, 2, \ldots, n \qquad \text{(A III-22)}$$

$$\int_0^T \delta h(t_1) \int_0^T h(t_2) R_n(t_2 - t_1)\, dt_2 dt_1 = 0 \qquad \text{(A.III-23)}$$

Using method of undetermined multipliers

$$\int_0^T h(t_2) R_n(t_2 - t_1)\, dt_2 = \sum_0^n \lambda_K t^K \qquad \text{(A.III-24)}$$

if n(t) is white noise, $R_n(\tau) = N_o \delta(\tau)$ (A.III-25)

$$N_0 \int_0^T h(t_2)\delta(t_2 - t_1)\, dt_2 = \sum_0^n \lambda_K t_1^K \qquad \text{(A.III-26)}$$

therefore

$$h(t_1) = \sum_0^n \lambda'_K t_1^K \qquad \text{(A.III-27)}$$

where the N_0 is included in λ'_K

Substituting into the following equation:

$$\int_{-\infty}^{\infty} t_1^{\,j} h(t_1)\, d t_1 = (-\beta)^j \qquad j = 0, 1, 2, \ldots, n \tag{A.III-28}$$

$$\int_{-\infty}^{\infty} t_1^{\,j} \sum_{0}^{n} \lambda'_K \, t^K = (-\beta)^j \tag{A.III-29}$$

Therefore:

$$\sum_{0}^{n} \lambda'_K \frac{T^{j+K+1}}{j+K+1} = (-\beta)^j \qquad K, j = 0, 1, 2, \ldots, n \tag{A.III-30}$$

APPENDIX IV

[illegible]SE POWER OUT OF POLYNOMIAL SMOOTHING FILTERS

The impulse response of the zero order predictor has been determined to be:

$$h(t) = \frac{1}{T} \qquad 0 < t < T \qquad \text{(A.IV-1)}$$

and the filter transformation is

$$H(f) = \int_{-\infty}^{\infty} h(t)\, e^{-j2\pi ft}\, dt = \frac{1}{T}\int_{0}^{T} e^{-j2\pi ft}\, dt \qquad \text{(A.IV-2)}$$

and

$$\left|H(f)\right| = \frac{\sin \pi fT}{\pi fT} \qquad \text{(A.IV-3)}$$

The noise power out of the filter is:

$$\sigma^2_{out} = \int_{-\infty}^{\infty} S(f) \left|H(f)\right|^2 df \qquad \text{(A.IV-4)}$$

$$\sigma^2_{out} = \frac{N_0}{2} \int_{-W_c}^{W_c} \left(\frac{\sin \pi fT}{\pi fT}\right)^2 df \qquad \text{(A.IV-5)}$$

changing variables of integration

$$\sigma^2_{out} = \frac{N_0}{2\pi T} \int_{-\pi W_c T}^{\pi W_c T} \frac{\sin^2 x}{x^2}\, dx \qquad \text{(A.IV-6)}$$

$$\frac{\sigma^2_{out}}{\sigma^2_{in}} = \frac{1}{2\pi W_c T}\left[- \frac{2 \sin^2 \pi W_c T}{\pi W_c T} + \int_{-2\pi W_c T}^{2\pi W_c T} \frac{\sin y}{y}\, dy \right] \qquad \text{(A.IV-7)}$$

if $W_c = 25$ cps and $T = 15.2$ sec.

$$\frac{\sigma^2_{out}}{\sigma^2_{in}} = \frac{1}{2W_c T} = -28.8 \text{ db}$$

If we assume that W_c is larger than the filter bandwidth, the noise power can be also computed using the following equation:

$$\sigma^2_{out} = \int_0^T \int_0^T h(t_1)h(t_2)R_n(t_2 - t_1)\, dt_2\, dt_1 \qquad \text{(A.IV-8)}$$

and if, $R_n(\tau) \approx \frac{N_0}{2} \delta(\tau)$ (A.IV-9)

then

$$\sigma^2_{out} = \frac{N_0}{2} \int_0^T \int_0^T h(t_1)h(t_2)\delta(t_2 - t_1)\, dt_1\, dt_2 \qquad \text{(A.IV-10)}$$

$$\sigma^2_{out} = \frac{N_0}{2} \int_0^T h^2(t)\, dt \qquad \text{(A.IV-11)}$$

for the zero order predictor ($\beta = 0$)

$$\frac{\sigma^2_{out}}{\sigma^2_{in}} = \frac{1}{2W_c} \int_0^T h^2(t)\, dt \qquad \text{(A.IV-12)}$$

$$= \frac{1}{2W_c} \int_0^T \frac{1}{T^2}\, dt \qquad \text{(A.IV-13)}$$

$$= \frac{1}{2W_c T} \qquad \text{(A.IV-14)}$$

$= -28.8$ db

for the first order predictor ($\beta = 0$)

$$\frac{\sigma^2_{out}}{\sigma^2_{in}} = \frac{1}{2W_c} \int_0^T \left(\frac{4}{T} - \frac{6}{T^2} t \right)^2 dt \qquad \text{(A.IV-15)}$$

$= -22.8$ db

for second order predictor ($\beta = 0$)

$$\frac{\sigma_{out}^2}{\sigma_{in}^2} = \frac{1}{2W_c} \int_0^T \left(\frac{9}{T} - \frac{36t}{T^2} + \frac{30}{T^3} t^2 \right)^2 dt \qquad \text{(A.IV-16)}$$

$$= -19.3 \text{ db}$$

APPENDIX V

PARAMETER ESTIMATION

An illustration of the basic problem and the method of solution of signal parameter estimation is given below. Suppose the received signal is of the form

$$X(t) = S(t;a) + \epsilon(t) \quad ; -T \leq t \leq T \qquad \text{(A.V-1)}$$

where $\epsilon(t)$ has known probability distributions. The parameter a is unknown, but the function S(t;a) is of known form, so that for any prescribed t and a, we know what S is.

Approximating X(t) by an n-term Karhunen-Loeve expansion, the resulting expansion coefficients are:

$$X_i = S_i + \epsilon_i \quad ; i = 1, 2, \ldots, n \qquad \text{(A.V-2)}$$

This notation might also refer to a set of n time-samples of the received signal plus noise. If the noise probability distributions are known, the likelihood function corresponding to the data vector $\underline{X} = (X_1, X_2, \ldots, X_n)$ is:

$$L(\underline{X}) = p(X_1 - S_1, X_2 - S_2, \ldots, X_n - S_n) \qquad \text{(A.V-3)}$$

where p is an n-dimensional function of known form depending on the unknown parameter a. Using Fisher's method of maximum likelihood(3), the optimum estimate s of the unknown parameter a is determined by solving the equation

$$\frac{\partial L}{\partial a} = 0 \qquad \text{(A.V-4)}$$

for a.

From the Cramér-Rao inequality, the variance of <u>any</u> estimator $\hat{a}$ of the unknown parameter a is bounded below as follows:

$$\sigma^2(\hat{a}) \geq \frac{\left[\frac{\partial E(a)}{\partial a}\right]^2}{E\left[\left(\frac{\partial \log L}{\partial a}\right)^2\right]} \qquad \text{(A.V.-5)}$$

If the expected value $E(\hat{a})$ equals a, the estimator is said to be "unbiased", in which case

$$\sigma^2(\hat{a}) \geq \frac{1}{E\left[\left(\frac{\partial \log L}{\partial a}\right)^2\right]} \qquad \text{(A.V.6)}$$

Those estimates $\hat{a}$ whose variance equals the lower bound in (A.V-6) are called "efficient" estimates. Furthermore, if an efficient estimate $\hat{a}$ exists, it will satisfy equation (A.V-4); that is, it will be a maximum likelihood estimator. Under certain regularity conditions, as the bandwidth observation time product increases, the maximum-likelihood estimates become asymptotically efficient. The significance of these estimates for small random samples of size n is given by Cramer(3). The asymptotic efficiency of maximum likelihood estimators based on random processes is proven by Grenander(4) in a manner similar to Cramer.

If the additive noise ϵ is Gaussian with known correlation function $R(t,t')$, the K-L expansion coefficients ϵ_i will be statistically independent with known variances equal to the Eigenvalues σ_i^2. In this case, the likelihood function in (A.V-3) becomes

$$L(\underline{X}) = \prod_{i=1}^{n} \frac{1}{\sigma_i \sqrt{2\pi}} \exp \frac{-(X_i - S_i)^2}{2\sigma_i^2} \qquad \text{(A.V-7)}$$

Since

$$\frac{\partial L}{\partial a} = \frac{-L}{2} \frac{\partial(-2 \log L)}{\partial a} \qquad \text{(A.V-8)}$$

the equation

$$\frac{\partial(-2 \log L)}{\partial a} = 0 \qquad \text{(A.V-9)}$$

can be used in calculating $\hat{a}$, rather than (A.V-4).

$$-2 \log L = C + \sum_{i=1}^{n} \frac{(X_i - S_i)^2}{\sigma_i^2} \qquad \text{(A.V-10)}$$

Therefore

$$\sum_{i=1}^{n} \frac{\partial S_i}{\partial a} \left(\frac{X_i - S_i}{\sigma_i^2} \right) = 0 \qquad \text{(A.V-11)}$$

This is the basic equation which must be solved for a. From (A.V-10),

$$\frac{\partial \log L}{\partial a} = \sum_{i=1}^{n} \frac{\partial S_i}{\partial a} \left(\frac{X_i - S_i}{\sigma_i^2} \right) \qquad \text{(A.V-12)}$$

so that

$$E\left[\left(\frac{\log L}{a}\right)^2\right] = \sum_{i=1}^{n} \frac{1}{\sigma_i^2}\left(\frac{\partial S_i}{\partial a}\right)^2 \qquad \text{(A.V-13)}$$

The reciprocal of this quantity is the lower bound on the variance of $\hat{a}$ in (A.V-6).

In general, the calculation of these quantities requires the solution of the basic K-L integral equation. However, in certain special cases of practical interest, these equations can be re-expressed in terms of the original receiver input $X(t)$, and the signal function $S(t,a)$. For example, suppose

$$S(t) = A \cos (W_0 t + \mu) \qquad \text{(A.V-14)}$$

The parameter a could be the amplitude A, the carrier frequency W_0, or the phase angle μ. Suppose in addition that $\epsilon(t)$ is white noise with spectral density N_0. From the K-L integral equation, there is only one Eigenvalue, $\sigma^2 = N_0$. Therefore any complete set of functions $\Phi_i(t)$ orthonormal on the observation interval can be used as Eigenfunctions. Therefore,

$$S(t;a) = \sum_i S_i \Phi_i(t)$$

$$\frac{S(t;a)}{a} = \sum_i \frac{\partial S_i}{\partial a} \Phi_i(t) \qquad \text{(A.V-15)}$$

$$X(t) = \sum_i X_i \Phi_i(t)$$

It follows from the above, and (A.V-12) that

$$\frac{\partial \log L}{\partial a} = \frac{1}{N_0} \int_{-T}^{T} \frac{\partial S(t;a)}{\partial a}\left[X(t) - S(t;a)\right] \partial t \qquad \text{(A.V-16)}$$

Applying the white noise assumption to (A.V-16)

$$E\left[\left(\frac{\log L}{a}\right)^2\right] = \frac{1}{N_0} \int_{-T}^{T} \left(\frac{\partial S(t;a)}{\partial a}\right)^2 \partial t \qquad \text{(A.V-17)}$$

If the signal is sinusoidal, as in (A.V-14)

$$\frac{\partial S(t;a)}{\partial a} = \begin{cases} \cos (W_0 t + \mu); & a = A \\ -At \sin (at + \mu); & a = W_0 \\ -A \sin (W_0 t) + a); & a = \mu \end{cases} \tag{A.V-18}$$

Setting the integral in (A.V-16) equal to zero and solving for a = A gives the estimator

$$\hat{A} = \frac{\int_{-T}^{T} x(t) \cos (W_0 t + \mu) \partial t}{T + \frac{1}{2W_0} \left[\sin (2W_0 T) \cos (2\mu) \right]} \tag{A.V-19}$$

From (A.V-17) and (A.V-18),

$$E\left[\left(\frac{\partial \log L}{\partial A}\right)^2\right] = \frac{1}{N_0} \int_{-T}^{T} \cos^2(W_0 t + \mu) \partial t$$

$$= \frac{1}{N_0} \left\{ T + \frac{1}{2W_0} \left[\sin(2W_0 T) \cos(2\mu) \right] \right\} \tag{A.V-20}$$

From (A.V-19), the expected value of $\hat{A}$ is:

$$E(\hat{A}) = A \tag{A.V-21}$$

Since S(t;A) is a linear function A, $\hat{A}$ is an efficient estimate, so that from (A.V-6) and (A.V-20),

$$\sigma^2(\hat{A}) = \frac{N_0}{T} \left\{ 1 + \frac{\sin (2W_0 T)}{(2W_0 T)} \sin (2\mu) \right\}^{-1} \tag{A.V-22}$$

This can be verified by noting that

$$\hat{A} - A = \frac{\int_{-T}^{T} \epsilon(t) \cos (W_0 t + \mu) \partial t}{\int_{-T}^{T} \cos^2(W_o t + \mu) \partial t} \tag{A.V-23}$$

and by applying the white noise assumption in calculating the variance of $\hat{A}$, using (A.V-23).

When a = W_0, it follows from (A.V-16) and (A.V-18) that the maximum likelihood estimator W_0 is the solution of the equation

$$2 \int_{-T}^{T} t \sin(at + \mu) X(t)\, dt = A \int_{-T}^{T} t \sin(2at + 2\mu)\, dt \tag{A.V-24}$$

When $a = \theta$, it follows that θ is the solution of

$$2 \int_{-T}^{T} \sin(W_0 t + a) X(t)\, dt = A \int_{-T}^{T} \sin(2W_0 t + 2a)\, dt \tag{A.V-25}$$

These equations can be used to construct a feedback system for the estimation of W_0 and μ .

The variance of $\hat{W}_0$ is bounded below by the inequality

$$\sigma^2(\hat{W}_0) \geq \frac{\left[\frac{\partial E(W_0)}{\partial W_0}\right]^2}{\left(\frac{E}{N_0}\right) g(W_0,T)} \tag{A.V-26}$$

where E is the signal energy, A^2T;

where,

$$g(W_0,T) = T^2 \left\{ \frac{1}{3} - \cos 2\mu \left[\frac{\sin(2W_0T)}{(2W_0T)} + \frac{\cos(2W_0T)}{2(W_0T)^2} - \frac{\sin(2W_0T)}{4(W_0T)^3} \right] \right\} \tag{A.V-27}$$

Since the equation defining W_0 in (A.V-24) is nonlinear, the mean value function $E(W_0)$ is not readily available, so that the numerator of (A.V-26) is not easily calculated. In interpreting the meaning of the numerator of (A.V-5) and (A.V-26), it is useful to write

$$E(\hat{a}) = a + b(a) \tag{A.V-28}$$

where a is the true value of the parameter and b(a) is the bias error. Therefore,

$$\frac{\partial E(\hat{a})}{\partial a} = 1 + \frac{\partial b(a)}{\partial a} \tag{A.V-29}$$

In the case of $\hat{W}_0$ and $\hat{\theta}$, it is clear from (A.V-24) that b(a) is a nonlinear function of a, whose form is not explicitly available.

From (A.V-5), (A.V-17) and (A.V-18),

$$\sigma^2(\hat{\mu}) \geq \frac{\left[\frac{E(\hat{\mu})}{\partial \mu}\right]^2}{\left(\frac{E}{N_0}\right) h(\theta, T)} \qquad \text{(A.V-30)}$$

where

$$h(\mu, T) = 1 - \frac{\sin(2W_0T)}{(2W_0T)} \cos(2\mu) \qquad \text{(A.V-31)}$$

The ratio of the square of the mean value of the estimate to its variance is a measure of "output signal-to-noise" ratio of the parameter measurement process. From the Cramer-Rao inequality in (A.V-5), we can write this output S/N as:

$$\left(\frac{S}{N}\right)_{o,a} = \frac{[E(\hat{a})]^2}{\sigma^2(\hat{a})} \leq \left[\frac{E(\hat{a})}{\frac{\partial E(\hat{a})}{\partial a}}\right]^2 \frac{1}{E\left[\left(\frac{\partial \log L}{\partial a}\right)^2\right]} \qquad \text{(A.V-32)}$$

For the amplitude parameter measurement process, in the above example, it follows from (A.V-21) and (A.V-22) that

$$\left(\frac{S}{N}\right)_{o,a} = \frac{E}{N_0}\left[1 + \frac{\sin(2W_0T)}{(2W_0T)} \cos 2\mu\right]^{-1} \qquad \text{(A.V.-33)}$$

For the frequency measurement process, it follows from (A.V-26) that,

$$\left(\frac{S}{N}\right)_{o,W_0} \leq \left[\frac{E(\hat{W}_0)}{\frac{\partial E(\hat{W}_0)}{\partial W_0}}\right]^2 \left(\frac{E}{N_0}\right) g(W_0, T) \qquad \text{(A.V-34)}$$

where the function g(x,y) is defined in (A.V-27). For the phase measurement process, it follows from (A.V-30) that

$$\left(\frac{S}{N}\right)_{o,\mu} \leq \left[\frac{E(\hat{\mu})}{\frac{\partial E(\mu)}{\partial \mu}}\right]^2 \left(\frac{E}{N_0}\right) h(\mu, T) \qquad \text{(A.V-35)}$$

If the input signal is of the form $S(t;\underline{a})$ where $\underline{a}$ is a vector of r unknown signal parameters, $a_1, a_2, \ldots, a_r$, the likelihood function

$$L(\underline{X}) = L(a_1, a_2, \ldots, a_r) \qquad \text{(A.V-36)}$$

Fisher's method of maximum likelihood involves solving the r simultaneous equations

$$\frac{\partial L}{\partial a_i} = 0 \quad ; i = 1, 2, \ldots, r \qquad \text{(A.V-37)}$$

for the parameter estimates a_i.

Cramer(3) discusses this method of estimation on page 500. On page 493, he states the generalization of the Cramer-Rao inequality (A.V-5) for the 2-parameter case, and on page 495, for the k-parameter case. Wilks(5) discusses this problem in section 12.6, entitled "multi-dimensional point-estimation". Kendall(6) treats this on pages 36 - 40, in a section entitled "simultaneous estimation of several parameters". The Cramer-Rao inequality has been called by Savage (7) the "information inequality". This is discussed in detail by Kullback(8) in his book on "Information Theory and Statistics", in Chapter 3 entitled "Inequalities of Information Theory".

APPENDIX VI

SPECIFICATION WIS 500 for A 50 KW, S-BAND, KILOJOULE PULSE, 300 MC BANDWIDTH, LINEAR PHASE AMPLIFIER

1. INTRODUCTION

An amplifier tube lineup is needed for a unit quantity of a future, developmental, radar system. Two proposals are of interest. One is for a system employing tubes which can be purchased to firm specifications, based on minimum type changes to existing tubes. The second may include development effort which can be completed within a one-year development cycle. Both must be available on a fixed-price basis, the latter may include minimum and target specifications. Cost is an important consideration.

Deviations from the above requirements will be considered where economy or availability are greatly enhanced.

Further desirable specifications are indicated below:

a. Frequency band center 3 KMC

b. Preferred pulse duration 1.2 milliseconds

c. Preferred peak power 800 kW

d. Pulse repetition frequency 50 pps

Pulse duration and peak power may be traded. Pulse energy as low as 300 joules and average power as low as 15 kW will be considered. The indicated bandwidth is instantaneous electronic bandwidth. Bandwidths as low as 150 Mc/s will be considered.

Operation is intended in the Continental U. S. and is ground-based. Detailed information requested in the tables is desired to aid in evaluation. Either all stages or any one stage from any vendor is of interest and should be included in the format supplied. Additional information or notes may be included.

2. Tubes available with firm specifications at fixed price (minor modifications to existing tubes, if necessary):

	Driver	IPA	Final Amp
Manufacturer	Raytheon	Raytheon	Raytheon
Tube Classification	QKW-1269	QKW-750B	QKS-1194
Mfr's Type #	TWT	TWT	Amplitron
Center Frequency	3000 Mc	3000 Mc	3000 Mc
Peak Power Output	2 kW	60 kW	750 kW
Maximum Pulse Duration			1200 usec
Average Power Output			20 kW
Nominal Efficiency		20%	70%
1 db Bandwidth		500	450
3 db Bandwidth		600	500
Phase Linearity thru Bandwidth			136°
Into 1.1 VSWR			
Number of cycles across Band		10	.375
Peak-to-Peak Phase Excursion from Linearity		±25°	±5°
Input to Output Time Delay			
Phase Pushing Sensitivity °/A or °/V: Cathode, Grid, Helix, Modanode		8° for 1% voltage	1° for 1% Amp.
Nominal Operating Voltage		35 kV	50 kV
Nominal Operating Current		12 amps	23 amps
Power Gain (db)		20 db	12 db

	Driver	IPA	Final Amp.
Check: Saturated		20 db	
Check: Unsaturated		30 db	
Pulse Modulation Check: Cathode, Modanode, Grid		cathode	cathode
Type of Cooling		Water Ethylene Glycol	Water Ethylene Glycol
Type of Focusing		Solenoid	Permanent Magnet
Approx. Nominal Spurious Output		40 db	25 - 30 db
Frequency of Spurious Signals A			
B			
C			
Harmonic Signal Output			-45 db

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13. ABSTRACT The Wide Bandwidth Baseline Radar study was originally intended to provide the ASFIR Interferometer system with a wide bandwidth signal capability. A large part of the work described here was directed toward that goal. An additional consideration was the possible use of the future wide bandwidth radar at the Floyd Site in a wide baseline system. By RADC direction, in the latter part of this Quarter, the Floyd Site radar and its projected wide band capability was considered to be the most appropriate basis for the study, so effort was reoriented. The shift in emphasis is from broadbanding a baseline radar to that of adding a baseline capability to a broadband radar. This latter course appears more realizable. This report considers:

- Tradeoffs in the implementation of signal processing versus communication system bandwidth. An economical approach is considered.
- Processing gain achievable through the use of various postdetection smoothing techniques. Second degree polynomial smoothing is most applicable.
- The nature of data from a wide baseline system. In particular, relationships are derived for the accuracy of the location of the scatterers as a function of baseline lengths.
- Design considerations for the ASFIR wideband transmitter are provided from an early study, performed before the shift in emphasis to the Floyd Site.

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14. KEY WORDS	LINK A ROLE	LINK A WT	LINK B ROLE	LINK B WT	LINK C ROLE	LINK C WT
Bistatic Radar Pulse Compression Phase Measurement Radar Echo Areas						

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