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PRELIMINARY DEVELOPMENT OF GUST DESIGN PROCEDURES BASED ON POWER SPECTRAL TECHNIQUES

VOLUME I. Theoretical and General Considerations

JOHN C. HOUBOLT

TECHNICAL REPORT AFFDL-TR-66-58, VOL. I

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AIR FORCE FLIGHT DYNAMICS LABORATORY
RESEARCH AND TECHNOLOGY DIVISION
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

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FOREWORD

This final report was prepared by Aeronautical Research Associates of Princeton, Inc., Princeton, New Jersey, in fulfillment of USAF Contract Number AF33(615)-2144; BPSN: 5(6899-61430014-0000-600-FD); Project and Task Numbers: None. This effort was done under Laboratory Director's Funds. This project is part of a program to provide the theoretical and practical applied research necessary to maintain USAF structural design criteria for aerospace flight vehicles and to establish optimum structural requirements, flight limitations and design philosophies commensurate with vehicle structural life.

The work was administered under the direction of the Air Force Flight Dynamics Laboratory, Research and Technology Division, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, Mr. Paul L. Hasty (FDTR), Project Engineer. This investigation was administered under Project 1367 and Task 136702; all follow-on work will be funded under this task.

The work reported in this study was conducted by Aeronautical Research Associates of Princeton, Inc. with Dr. John C. Houbolt as principal investigator, and covers the period from November 1964 to March 1966. The manuscript was released by the author in March 1966 for publication as an RTD Technical Report.

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This technical report has been reviewed and is approved.

FRANCIS J. JANIK, JR.
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ABSTRACT

A review is made of the progress and findings that have been obtained in a study aimed at the development of gust design procedures based on power spectral techniques. Included is a consideration of atmospheric turbulence makeup. A corresponding discussion is given of the aircraft structural parameters that are needed in spectral evaluation and on the practical aspects of their evaluation. Emphasis is given to the spectral design procedures that have evolved from the study, and a new hypothesis for fatigue considerations is advanced, cast in the notation of gust loads design. Ease of application, interpretation and implication to design, present and future, and comparison with past design procedures are covered.

CONTENTS

ILLUSTRATIONS

SYMBOLS

a	lift curve slope
A	structural parameter; $\sigma_x = A\sigma_w$
c, c_0	wing chord
f	frequency, cps
h	altitude
$H(\omega)$	frequency response function due to sinusoidal gust
k	reduced frequency, $k = \omega c/2V$
K_g	gust alleviation factor
L	scale of turbulence
n	total number of times load level x is crossed with positive slope
n_0	total number of zero crossings with positive slope
N	number of times per second load level x is crossed with positive slope
N_0	number of zero crossings per second with positive slope
$p(\sigma)$	probability distribution function of σ
P	proportion of time spent in turbulence
s_r	reference stress as used in fatigue
S	wing area
T	total (lifetime) flight time
T_t	total time in turbulence
T_F	average duration of a mission flight
U_d	gust velocity value similar to U_{de}
U_{de}	"derived" gust velocity
V	flight velocity
V_e	equivalent airspeed

SYMBOLS (Cont'd.)

W	airplane weight
x	increment in load or response level due to gusts
$x_{L.L.}$	load level representing limit load
x_{1-g}	1 - g load level
α, β	constants
λ	wavelength
μ_g	mass ratio or mass parameter, $2W/a\rho cgS$
μ_O	mass ratio, $\mu_O = 4\mu_g$
ρ	air density
σ	r.m.s. value
σ_c	composite r.m.s. value of vertical gust velocity
σ_x	r.m.s. value of response x
σ_w	r.m.s. value of vertical gust velocity
$\phi(\Omega)$	power spectrum
ω	angular frequency
Ω	spacial frequency, ω/V

Note:

$$\Omega = \frac{\omega}{V} = \frac{2k}{c} = \frac{2\pi}{\lambda}$$

$$\phi(\Omega) = L\phi(L\Omega) = V\phi(\omega) = \frac{c}{2}\phi(k)$$

SECTION 1

INTRODUCTION

This report deals with the subject of designing aircraft for gust loads encounter on a power spectral basis. Recent studies suggest that developments on the subject during the past several years could be simplified or streamlined considerably and, in fact, that procedures could be put on a much firmer basis, while still preserving generality. It is the purpose of this report to bring out for consideration the results of these newer studies, and to indicate the associated improvements that are desired in the analysis of flight operational data. At the same time, a new hypothesis is advanced for the treatment of fatigue due to gust loading that circumvents the necessity of introducing various notions such as cumulative damage concepts and the like. The procedure for treating the fatigue problem simultaneously with the gust loads exceedance problem is shown, using a common language and notation for both.

The body of the report presents the essential results, while appendices are used for the more detailed treatments. Appendix A defines and indicates some of the practical aspects that are involved in the determination of the basic structural response parameters A and N_0; Appendix B gives a brief development of the gust design treatment advanced here, and Appendix C covers the new fatigue hypothesis.

SECTION 2

BASIC RELATIONS FOR LOAD COUNT

Gust Encounter

As a result of examining past theoretical studies and airplane operation statistics on gust loads, and on the basis of further theoretical study which is reviewed briefly in Appendix B, a newer and concise concept on the statistical treatment of gust loads has been developed. The concept is embodied in a single universal curve, Figure 1, which allows for the rather quick estimation of the expected number N of times per second given load levels are crossed during flight of an airplane. The definition and meaning of the other terms follow.

A and N_0: These are the basic airplane structural response parameters which apply; A relates in linear fashion the r.m.s. value σ_c of gust input to the r.m.s. value σ_x for the response according to the relation

$$\sigma_x = A\sigma_c$$

The parameter N_0 is a characteristic frequency parameter which denotes the number of times per second the response time history crosses the value zero with positive slope. Appendices A and B define these parameters more completely and indicate the practical aspects of their evaluation.

x: x refers to the chosen response quantity, such as bending stress, bending moment, acceleration, etc. More precisely, it is an incremental value Δx above the 1 - g load level value, and for design is expressed in terms of chosen design allowables

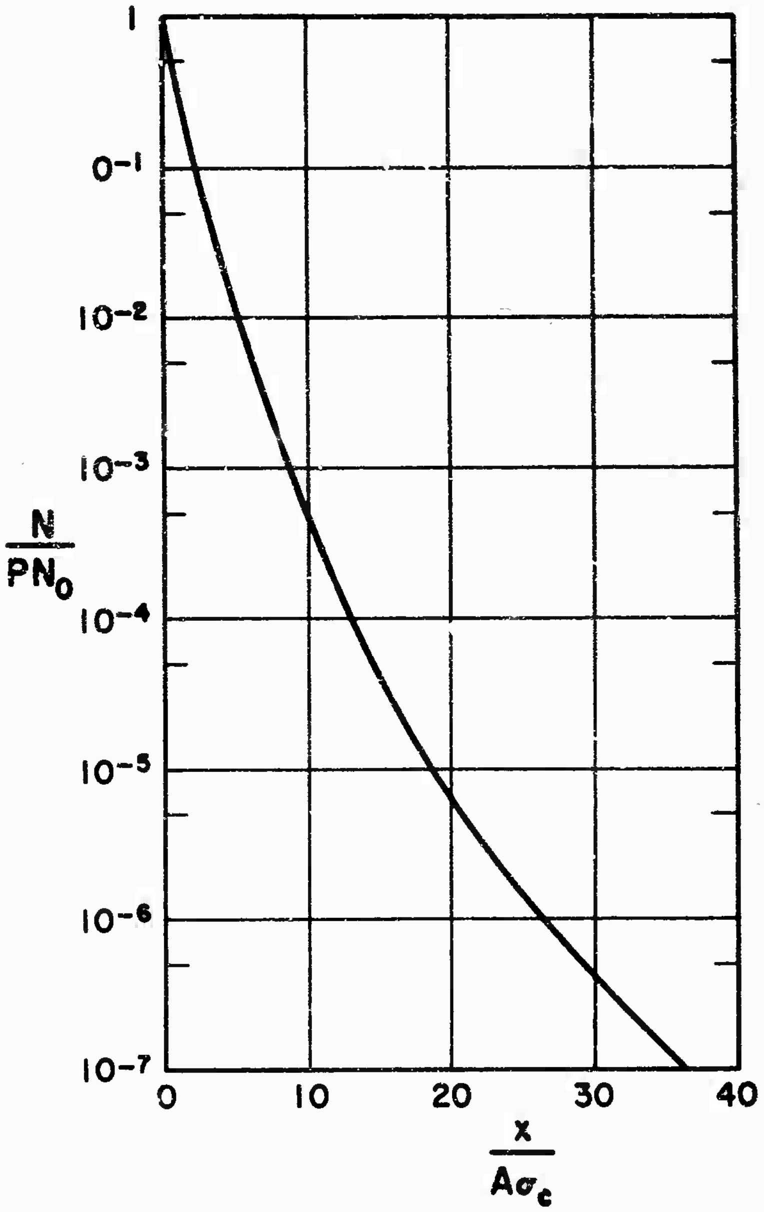

Figure 1. "Universal" Curve for Load Exceedance.

in accordance with the design philosophy being followed; for example, for design in terms of limit load, x is

$$x = \Delta x = x_{L.L.} - x_{1-g}$$

where $x_{L.L.}$ is the allowed limit load value.

σ_c and P: σ_c is a composite value denoting the r.m.s. value of gust velocity for all turbulence encounter during the lifetime of the aircraft, and P is the associated value denoting the proportion of total flight time that the airplane is in turbulence; these will be discussed in greater detail in the section on design.

The curve of Figure 1 is found (Appendix B) to be represented well by

$$\frac{N}{PN_0} = \left(\frac{3.36\ A\sigma_c}{x}\right)^{6.74} \qquad (1)$$

for $x/A\sigma_c > 9.13$; a relation applying to lower values of the abscissa is given in Appendix B as well, but since this range is not normally of interest the relation is not repeated here. The empirical constants 3.36 and 6.74 are tentative in nature, and should be updated as more and more operational data are cast in the "universal" form suggested here. The total number n of times given load levels are crossed or exceeded during the over-all flight time of the aircraft follows simply from Equation (1) as

$$n = PTN_0\left(\frac{3.36\ A\sigma_c}{x}\right)^{6.74} \qquad (2)$$

where T represents the total flight time. As shown subsequently, this relation forms the basis for the design for gusts from a number of load exceedances point of view.

Fatigue

At the same time, because of the implications of recent fatigue data for random loading conditions, and because the manner of handling and presenting data for both fatigue and gust load count can be developed along closely similar lines, a new fatigue concept is advanced here. The hypothesis is: fatigue life is a function simply of the r.m.s. stress level of the over-all time history of stress exposure, regardless of the sequence of load application. In terms of the gust loads parameters, the treatment on fatigue given in Appendix C indicates that fatigue life may be expressed by an equation of the form

$$n_0 = PTN_0 = \left(\frac{\beta x}{A\sigma_c}\right)^{\alpha} \qquad (3)$$

where n_0 refers to the total number of times the zero stress level may be crossed with positive slope before fatigue failure (on the basis of zero mean value), and where α and β are empirical factors which are associated with the material and type of construction practices used. The product βx is a reference empirical stress of the type often found in analytical representations of fatigue life, and hence β relates this reference stress to the chosen design value of x; the product $A\sigma_c$ is noted to be simply the r.m.s. value σ_x of the random stress history.

The remarkable correspondence between Equations (2) and (3) will be dealt with in the next section.

SECTION 3

DESIGN

σ_c and P Values

The application of Equation (2) requires that consideration be given to the choice of σ_c and P values. Two procedures are offered. In one, σ_c and P are simply stipulated constant values, and hence the procedure is very much akin to the discrete gust approach in which design gust velocity U_{de} of 50 fps, for example, is stipulated and no account is taken of the amount of turbulence encounter. This first procedure is felt to have considerable merit due to its simplicity.

In the second procedure, explicit consideration is given to the various turbulence intensity levels encountered and to the amount of time spent in turbulence at various altitudes. Many studies along this line have been persued in recent years. They generally have proceeded by breaking up the turbulence encounter into severity brackets, usually of two types, nonstorm and storm. Operational data are used in an attempt to deduce the statistically applicable variations with altitude of intensity and proportion of time for each of the severity brackets chosen, with the object of using these deduced values later for design purposes. It is felt that breakdowns of this type are perhaps an overrefinement or that the accuracy of the results available does not warrant this much detail. Here the use of single measures for gust intensity and proportion of time is advocated; specifically, in question form, what statistically is the over-all r.m.s. gust velocity value of the turbulence encountered at each altitude, and likewise, what is the proportion of time on the average that turbulence is encountered at each altitude?

Figure 2 shows some tentative initial variations of σ_c and P that are estimated from the consideration of existing data and analyses. Most of the data indicate that σ_c is nearly constant with altitude and in the neighborhood of 3 to 3.5 fps; a constant value of 3.0 has been chosen here. For P the choice is more in doubt; some results suggest curve a, others curve b. (The circumstance here is quite odd, since of all quantities P should be the easiest to establish.) It is advocated therefore that the appropriate P variation curve (as well as the σ_c curve) be established from the data that exist and from those being gathered. At the moment, curve a is felt to be more appropriate, although proof of this feeling is lacking.

Thus even in the more involved second procedure discussed here, only the P variation has to be considered, since σ_c appears constant. The effective value for P applying to chosen missions, or to flight paths which are followed on the average, can quite easily be found from the P variation curve, by appropriately weighing the times at each altitude bracket. Figure 3 shows example results for the assumed flight plan variation shown also in the figure, assuming curve b of Figure 2 is the appropriate P variation; hold times can be taken into account conveniently by using an effective rate of climb or descent, and the rather low value of 300 feet per minute used here reflects this consideration. Hence, if an airplane is to be designed to fulfill an average mission covered by this figure, then the effective P to

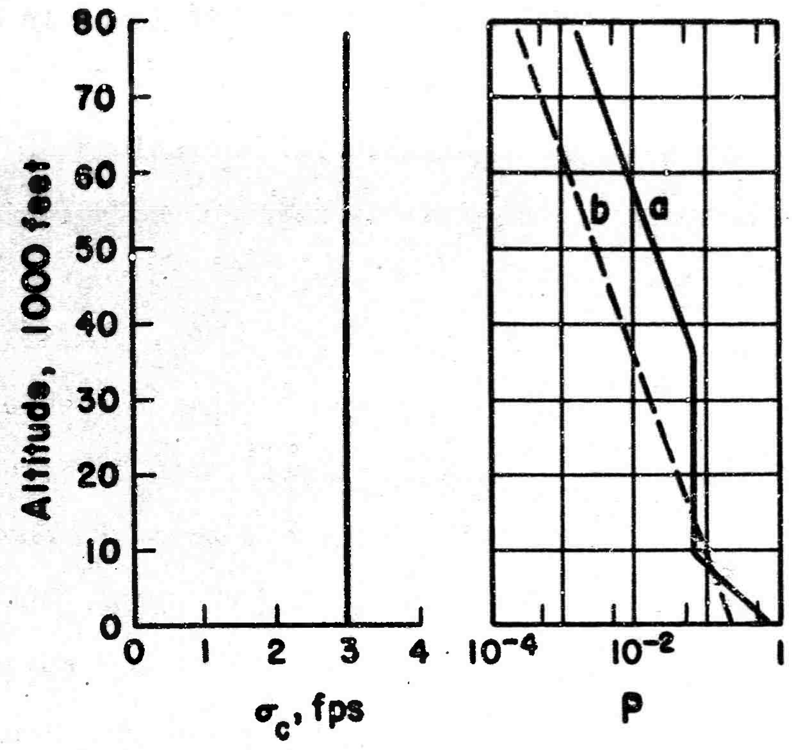

Figure 2. Tentative σ_c and P Values.

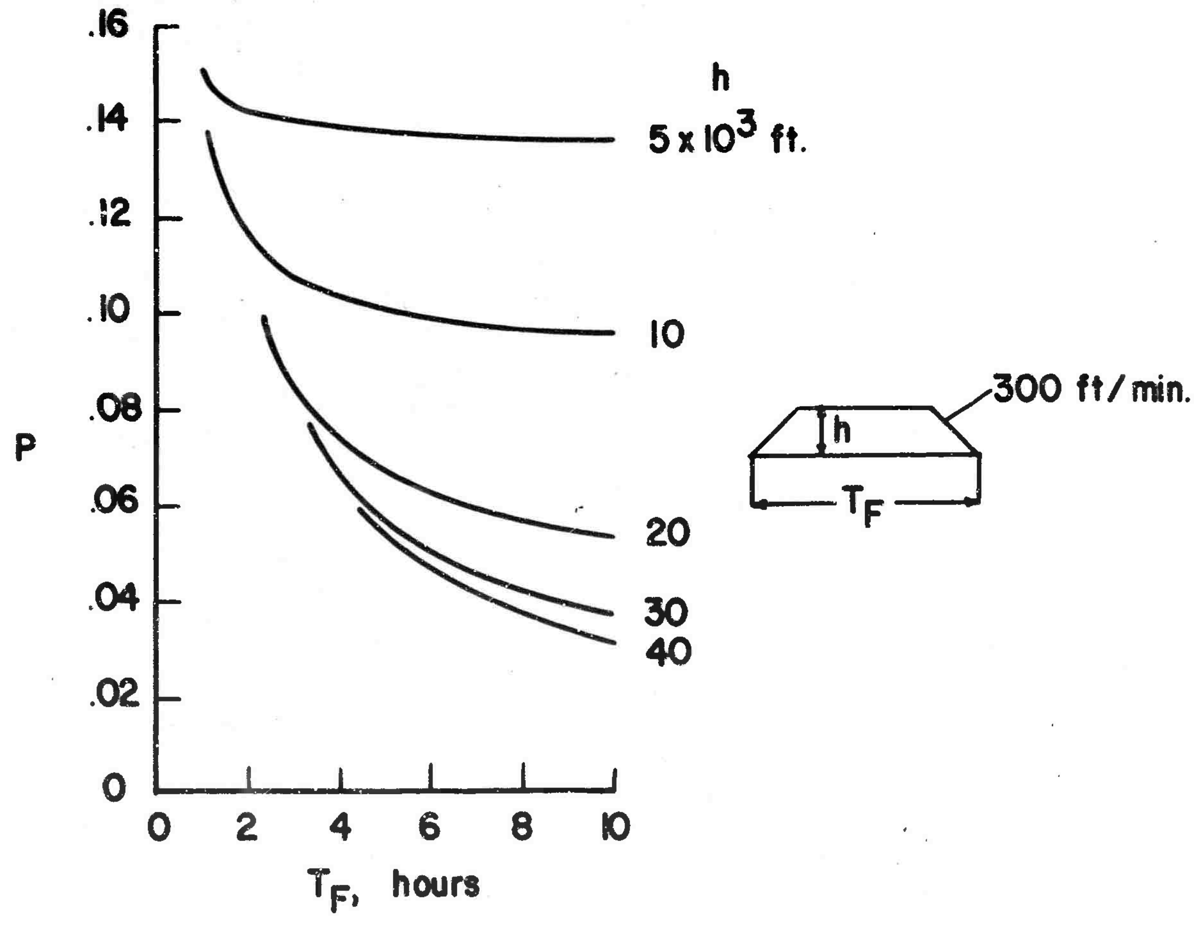

Figure 3. Proportion of Time in Turbulence for a Given Mission.

be used in design can be taken directly from the figure.

Direct Design

The direct design for gust loads on a load count basis and for fatigue considerations as well, follow in a straightforward way from Equations (2) and (3). These equations plot as shown in Figure 4. The use of $A\sigma_c/x$ versus PTN_0 is chosen simply on the basis of the close resemblance to the familiar S-N diagrams of fatigue. Suppose that x is associated with limit load values and that n is stipulated not to exceed 5. A design is checked then by entering the chart at the computed values of $A\sigma_c/x$ and PTN_0, and if the point falls below the $n = 5$ line, the design is judged safe. (Note, consideration of the curve of Figure 1 at the larger abscissa value indicates that, for x on a limit load basis, an n in the neighborhood of 50 corresponds to about one encounter for x presented in terms of ultimate load.)

At the same time, safety from fatigue failure can be checked. The particular material and construction used fix α and β, and hence fix the fatigue curve. The remoteness of the design point indicates then the margin of safety against fatigue failure. Note that although not explicitly stated here, the fatigue curves used should reflect the effect of the $1-g$ mean flight stresses.

Thus, this single and concise chart embodies the complete procedure advanced herein for designing aircraft for gust encounter, and allows the nearness to fatigue troubles due to gusts to be monitored as well.

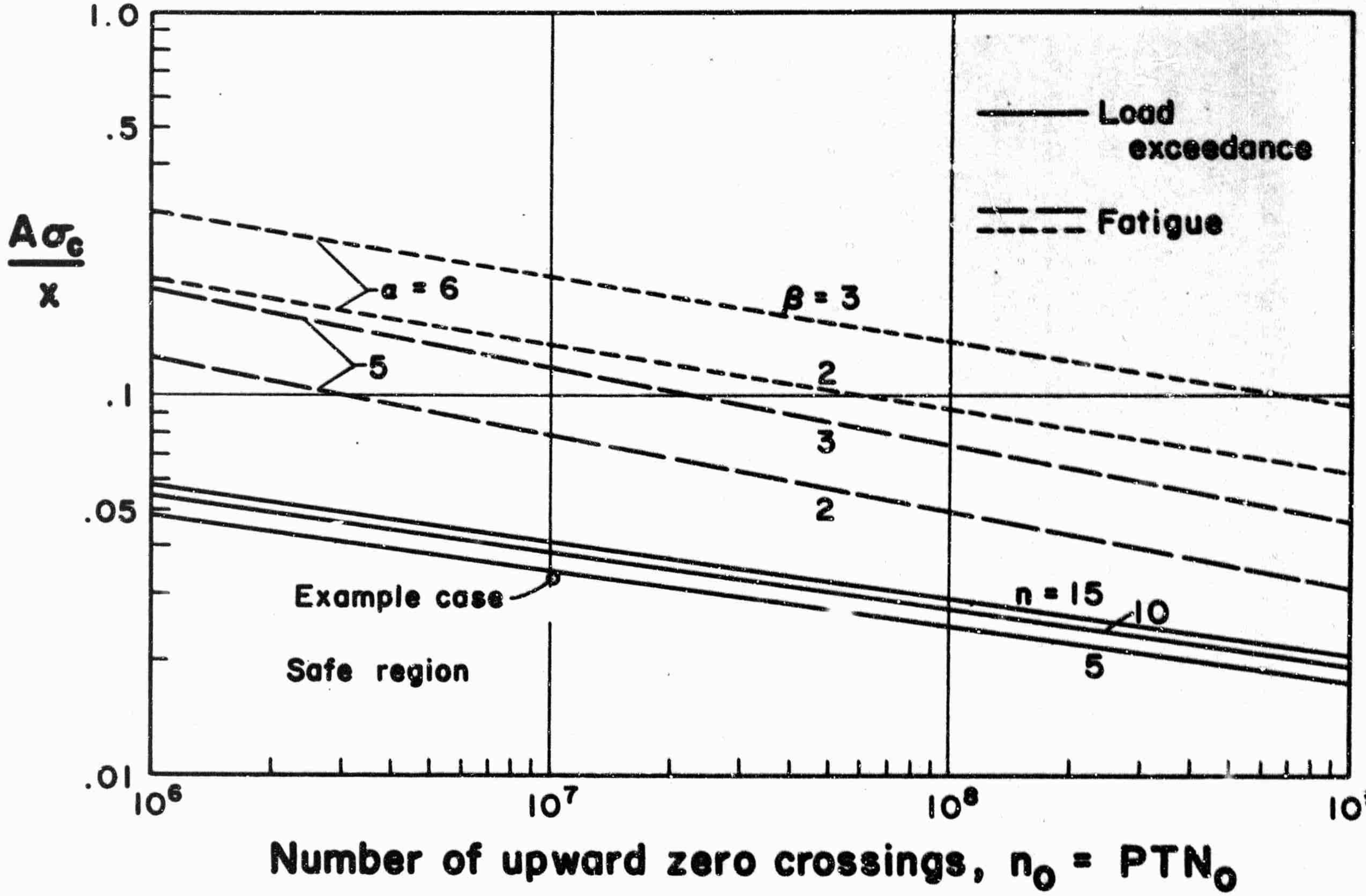

Figure 4. Master Design Chart for Load Exceedance and Fatigue Life (Note, the illustrative fatigue curves shown here do not include effect of 1 - g mean flight load.).

An illustrative example is given here to indicate the approach:

<u>Design conditions given</u>

$x = 28{,}000$ psi
$\beta = 2.5$
$T = 30{,}000$ hrs $= 10.8 \times 10^7$ sec
$n = 5$

<u>For the mission</u>

$$\left.\begin{array}{l} \sigma_c = 3 \text{ fps} \\ T_F = 3 \text{ hrs} \\ h = 20{,}000 \text{ ft} \end{array}\right\} P = .085 \text{ from Figure 3}$$

<u>From the airplane preliminary design</u>

$A = 310$ psi/fps
$N_0 = 1.1$ cps

These values yield

$$\frac{A\sigma_c}{x} = .0332$$

$$PTN_0 = 10.1 \times 10^6$$

which spots just below the n = 5 curve of Figure 4, thus indicating a safe design. The point is also noted to be far removed from the fatigue curves.

<u>Comparison Design</u>

An even simpler design procedure may be used by a comparison technique. Write Equation (2) in terms of airplane 2, and also in terms of airplane 1, and divide the equations. The result is that shown in Figure 5. Suppose that airplane 1 is an older gust-critical airplane that has proven itself airworthy by many successful

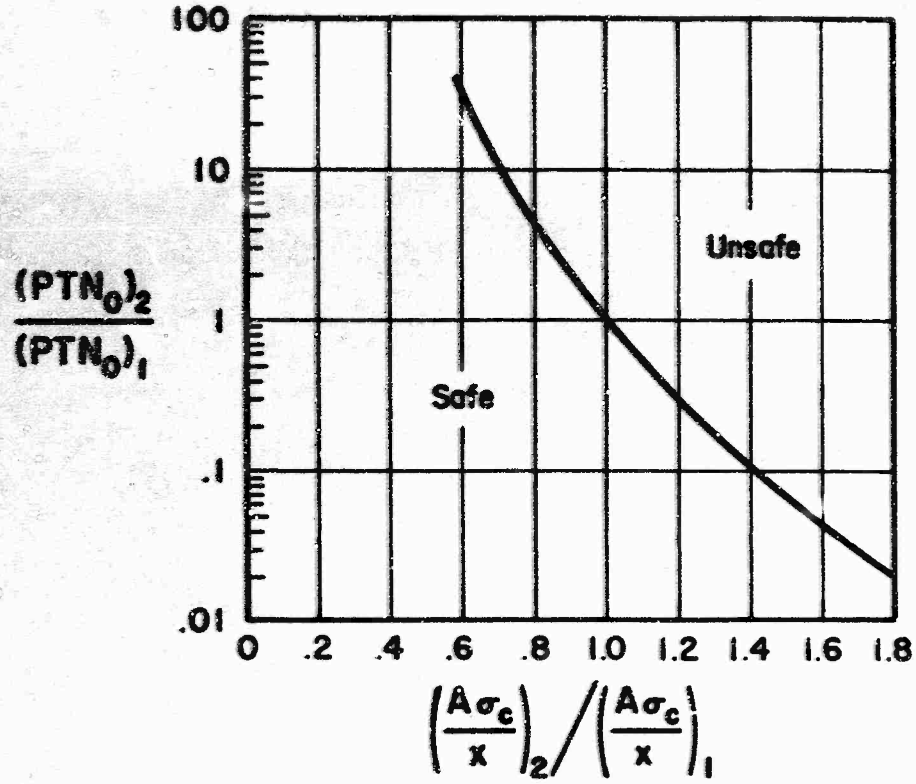

Figure 5. Gust Design by Comparison (Note, stresswise, adding metal lowers A).

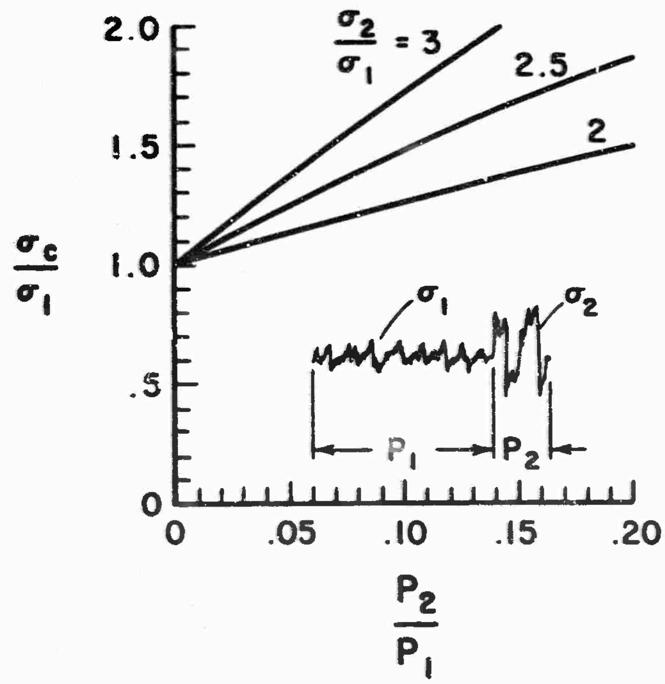

Figure 6. σ_c Increase Due to Extended Severe Turbulence Encounter.

years of flight, and suppose airplane 2 is the new airplane under design consideration. The ratios of the $A\sigma_c/x$ and PTN_0 quantities are established and the adequacy of the new design is simply established according to whether the point falls on the safe side or the unsafe side of the line. For airplanes of similar category, this appears to be an attractive approach and has the merit of good simplicity. Other advantages are also offered; for example, the use of ratios tends to minimize uncertainties that may exist in the determination of the parameters such as A and N_0, in that errors common to each determination, such as, say, in the choice of lift curve slope, tend to cancel one another.

SECTION 4

CONCLUSIONS AND RECOMMENDATIONS

Two points are discussed here. One is in reference to the σ_c and P values that are chosen for design. The values and curves presented apply basically to medium and higher altitude flights and for conditions on the average over long periods of flight time. For special purpose missions, such as low altitude flight in a high percentage of severe type turbulence, the values of σ_c and P must be selected accordingly. Figure 6 is illustrative of how σ_c might increase when operating under more severe turbulence conditions; the majority of the flight is considered to occur at the r.m.s. level σ_1, and effectively a certain percentage of the more severe level σ_2 is added; $P_1 + P_2$ becomes the total P. As an example, if σ_1 is taken as 3, then for the specific case of $P_2/P_1 = .05$ and $\sigma_2 = 7.5$ the effective σ_c is found to be 3.72.

The second point is relative to the scale of turbulence L which appears in the expression representing the power spectrum for the input gust velocities, Equation (A1) in Appendix A. This quantity was not touched upon in the preceding discussion, but comment is merited. In general, the structural parameter A is found to be dependent on L, and therefore the value of L selected for use is quite important. The evaluation of L has been of much concern, and estimates range all the way from 500 to 5,000 feet. The primary reason why its evaluation is illusive at the moment is due to the fact that the time history records of vertical gust velocity that have been deduced from flight studies generally show uncertain or unreliable low frequency--or long wavelength--content. The long wavelength components present lead to a large evaluated scale value. It is suspected, however, that these longer wavelengths arise because of instrumentation limitation or inaccuracies, and thus the scale values deduced are in question. The essential point to be made here is that more attention should be paid to the instrumentation used in flight tests and what can be expected of it. A simple evaluation procedure that should be used, for example, but which does not seem to have been followed, is the following. Flight tests of the instrumented airplane should be made in still air, both for the conditions of level flight and with deliberate pull-up and push-down maneuvers; these flights should be of at least 4 minutes duration. The records should then be evaluated in the same manner as if turbulence had been encountered, to see if the vertical velocity time history evaluates to zero, as it should. Any residual trace that appears serves to indicate instrumentation limitations, and possibly what low frequency contaminations are likely to be found in the gust records. A guide would then be available to select appropriate low frequency filtering devices in the record evaluation procedures.

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2. S.O. Rice. "Mathematical Analysis of Random Noise". Parts I and II. Bell Syst. Tech. Journ., Vol. XXIII, No. 3, July 1944; Parts III and IV, Vol. XXIV, No. 1, January 1945.

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APPENDIX A

STRUCTURAL PARAMETERS

This appendix serves simply to review the definition of two basic structural parameters, A and N_O, that are significant in the spectral determination of gust loads (see Reference 1 for a fuller treatment). The basic relation between the input gust spectrum and the output response spectrum is

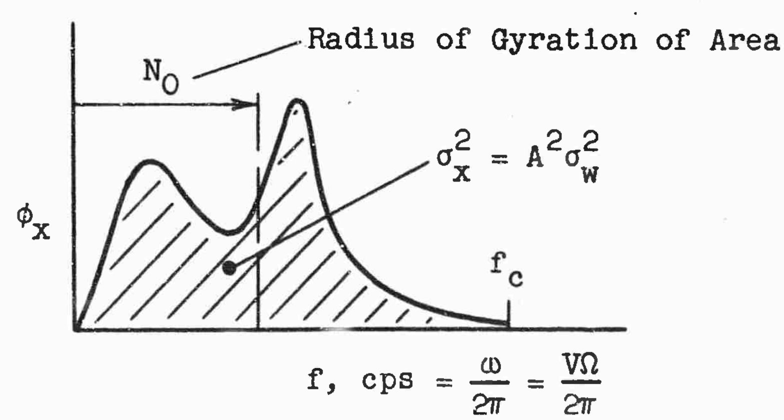
$$\phi_x(\omega) = |H(\omega)|^2 \, \phi_w(\omega)$$

where x is any dynamic response quantity of concern and H is the frequency response function for x due to unit sinusoidal gusts. For ϕ_w, an expression of growing favor and which exhibits an $\omega^{-5/3}$ fall off at large frequencies is

$$\phi_w(\Omega) = \frac{\sigma_w^2 L}{\pi} \; \frac{1 + \frac{8}{3}(1.339\, L\Omega)^2}{\left[1 + (1.339\, L\Omega)^2\right]^{\frac{11}{6}}}$$

where σ_w, the r.m.s. gust velocity, and L, the turbulence scale, characterize the turbulence being encountered, and where $\Omega = \omega/V$.

The basic structural parameters are derived from the output spectrum in accordance with the following sketch and equations.



$$A = \frac{\int_0^{\omega_c} \phi_x(\omega)\,d\omega}{\sigma_w}$$

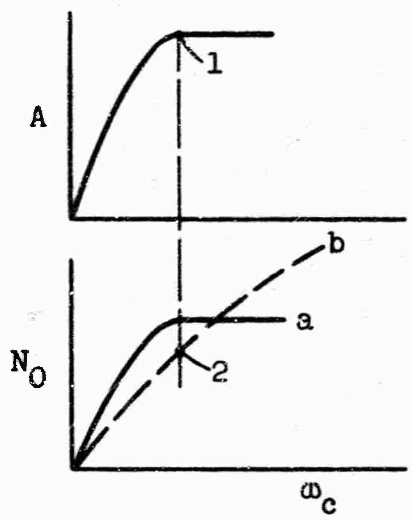
$$N_0 = \frac{1}{2\pi}\left[\frac{\int_0^{\omega_c} \omega^2 \phi_x(\omega)\,d\omega}{\int_0^{\omega_c} \phi_x(\omega)\,d\omega}\right]^{\frac{1}{2}}$$

The parameter A relates the r.m.s. value of gust input in linear fashion to the r.m.s. value of output according to the relation

$$\sigma_x = A\sigma_w \tag{A1}$$

and N_0 is an over-all characteristic frequency of response of the airplane and specifically denotes the number of times per second that the response time history crosses the value zero with positive slope.

Instead of using a theoretical upper limit of infinity in the integration, a cut off frequency ω_c is introduced in practice, otherwise N_0 would appear unrealistically unbounded in some cases. The value of ω_c to use in practice is of much concern, but a rule of thumb that holds much promise is that brought out in the following sketches showing the variation of A and N_0 with ω_c.



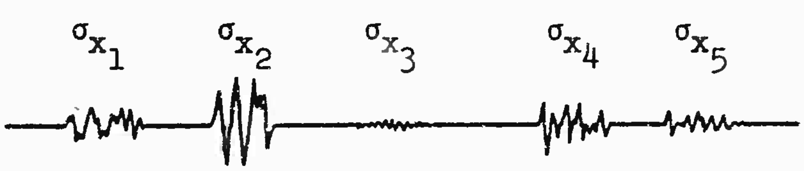
In the determination of the parameter A there is not much difficulty, for a plateau is usually reached quite sharply. For N_O, no difficulty is encountered in some cases such as case a illustrated. For case b, however, the difficulty of ever increasing N_O arises; the rule of thumb here is to take as ω_c the point where A has become essentially flat, point 1, and read off N_O at this frequency, point 2. Thus, since the contribution to the total power or to A of spectral information beyond ω_c is essentially negligible, the contribution to N_O beyond ω_c is essentially hash or noise type information, and hence should be disregarded in arriving at level crossing which has meaning from a practical point of view.

APPENDIX B

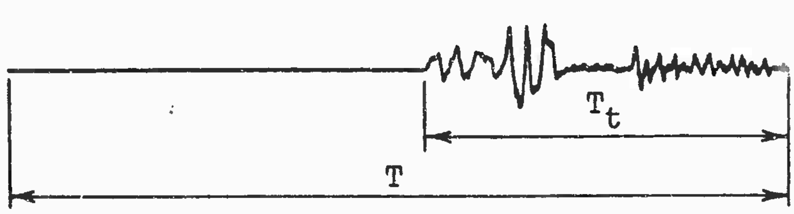
LOAD LEVELS DUE TO GUST ENCOUNTER

We derive here a fundamental curve for load level crossings that forms the basis for the complete design of aircraft due to gusts on a power spectral basis. A remarkable closeness to fatigue strength considerations appears and so it is fitting to advance a new and analogous procedure for fatigue design as well.

We start from a slightly different viewpoint from that considered in other studies, such as in Reference 1, but show that similar end results are obtained. Consider the timewise variation of gust encounter of an airplane during its lifetime. A time history record of the gust velocities encountered, or of some resulting structural response quantity, might appear for example as shown in the following sketch



For convenience, let us now lump in link fashion the quiescent periods together, and likewise the turbulence patches, so that an equivalent time history record might appear



As has been verified in a gross sense, let us assume that each patch of turbulence encountered is locally stationary and Gaussian in character. Then from the relation due to Rice (Reference 2)

$$N = N_0 e^{-\frac{x^2}{2\sigma^2}}$$

giving the number of times per second that a given level x is crossed with positive slope, and with Equation (A1), we can write the number of times per second a given load or response level is crossed as the integrated result

$$N = \frac{1}{T}\int_0^{T_t} N_0 e^{-\frac{x^2}{2A^2\sigma^2}} dt \qquad \text{(B1)}$$

where N_0, A, and σ are in general functions of time. Note σ refers here to the r.m.s. value of gust velocity σ_w, but for brevity in notation we drop the subscript w. Now N_0 varies only slightly with time and so we regard it constant. For generality, we could consider the timewise variation of the product $A\sigma$, but since the variation of A is not large, we also choose A constant; specifically, the r.m.s. value A_c is suggested as the logical choice in accordance with the approximation

$$\int A^2\sigma^2 dt = A_c^2 \int \sigma^2 dt$$

Equation (B1) thus may be written

$$N = \frac{T_t}{T} N_0 \int_0^1 e^{-\frac{x^2}{2A_c^2\sigma^2}} d\left(\frac{t}{T_t}\right)$$

$$= PN_0 \int_0^1 e^{-\frac{x^2}{2A_c^2\sigma_c^2}\frac{\sigma_c^2}{\sigma^2}} d\left(\frac{t}{T_t}\right) \tag{B2}$$

where P denotes the proportion of total flight time spent in discernible turbulence and σ_c is the r.m.s. value of σ, defined by

$$\sigma_c^2 = \frac{1}{T_t}\int_0^{T_t} \sigma^2 dt = \int_0^1 \sigma^2 d\left(\frac{t}{T_t}\right)$$

The σ_c value may also be written

$$\sigma_c^2 = \int_0^\infty \sigma^2 p(\sigma) d\sigma$$

where $p(\sigma)$ represents the probability distribution function of the r.m.s. value σ of gust velocity that applies during the entire flight experience. These equivalent relations imply then that

$$d\left(\frac{t}{T_t}\right) = p(\sigma) d\sigma \tag{B3}$$

or

$$\frac{t}{T_t} = \int_0^\infty p(\sigma)d\sigma = 1$$

and that for the limits

$$t = 0, \qquad \sigma = 0$$

$$t = T_t, \qquad \sigma = \infty$$

With Equation (B3), Equation (B2) may be expressed as

$$N = PN_0 \int_0^\infty \sigma_c p\left(\frac{\sigma}{\sigma_c}\right) e^{-\frac{x^2}{2A_c^2\sigma_c^2}\frac{\sigma_c^2}{\sigma^2}} d\left(\frac{\sigma}{\sigma_c}\right) \tag{B4}$$

which agrees with Equation (52) of Reference 1, thus showing the correspondence of the approach used herein with that used in Reference 1.

Equations (B2) or (B4) imply then a universal curve relating N/PN_0 to $x/A\sigma_c$. (Henceforth A is to be regarded as the composite value A_c, but the subscript will be dropped for brevity of notation.) The consideration of much operational flight data in this light has led to the tentative universal curve shown in Figure 1. It is found that in the range of $x/A\sigma_c > 10$, the curve is expressed well by

$$\frac{N}{PN_0} = \left(\frac{aA\sigma_c}{x}\right)^b \tag{B5a}$$

and that for $x/A\sigma_c < 10$, the relation

$$\frac{N}{PN_0} = e^{-\frac{b}{ae}\frac{x}{A\sigma_c}} \qquad \text{(B5b)}$$

applies with reasonable accuracy. The joining point, with equal height and slope values is given by

$$\left(\frac{x}{A\sigma_c}\right)_1 = ae$$

$$\left(\frac{N}{PN_0}\right)_1 = e$$

For the curve shown in Figure 1, values of a and b are found to be $a = 3.36$, $b = 6.74$, so that

$$\frac{N}{PN_0} = \left(\frac{3.36\ A\sigma_c}{x}\right)^{6.74}, \qquad \frac{x}{A\sigma_c} > 9.13 \qquad \text{(B6a)}$$

$$= e^{-.738\frac{x}{A\sigma_c}}, \qquad \frac{x}{A\sigma_c} < 9.13 \qquad \text{(B6b)}$$

With A and N_0 given for a particular airplane, and σ_c and P stipulated, the curve of Figure 1 defines completely the expected number of times per second a given response level x is crossed. The number of times n the level x is crossed in the total flight of the aircraft follows simply as

$$n = NT$$

For N given by Figure 1 and specifically by Equation (B6a), this number becomes

$$n = PTN_0\left(\frac{3.36\ Ac_c}{x}\right)^{6.74} \tag{B7}$$

This equation, or Figure 1, thus embodies a concise and succinct statement for the design of aircraft due to gust encounter, as is discussed in the body of the report. As a reiteration, the values of $a = 3.36$ and $b = 6.74$ are tentative, and should be updated as more and more operational data are cast in the "universal" form suggested here.

APPENDIX C

FATIGUE CONSIDERATIONS

Recent fatigue studies of axially loaded specimens under random loading (Reference 3) indicate that the fatigue life appears to be governed almost wholly by the r.m.s. value of the applied stress, independent of the spectral shape of the loading. Typical results are shown in Figure 7. These results are considered significant and remarkable and suggest a new way of considering the fatigue life of aircraft structures. Instead of considering different loading cycles of different magnitudes or of trying to consider fatigue through use of cumulative damage concepts, we advance here the following hypothesis: for random loading, as encountered by aircraft, fatigue life is a function simply of the r.m.s. stress level. The r.m.s. level may be changing with time, as large or low level load sequences are encountered, but it is the over-all r.m.s. level that is effective and important. With a little stretch in thought, it is conceivable that fatigue due to all loads sources, whether taxiing, ground-to-air cycle, maneuvering, or gusts may be treated in this way; hence the r.m.s. stress value of the lumped time history of all these load sources is postulated as being the primary value governing fatigue life.

The results shown in Figure 7 are fitted well by the expression

$$n_0 = \left(\frac{70,000}{\sigma}\right)^{5.27}$$

where n_0 is the total number of zero crossings with positive

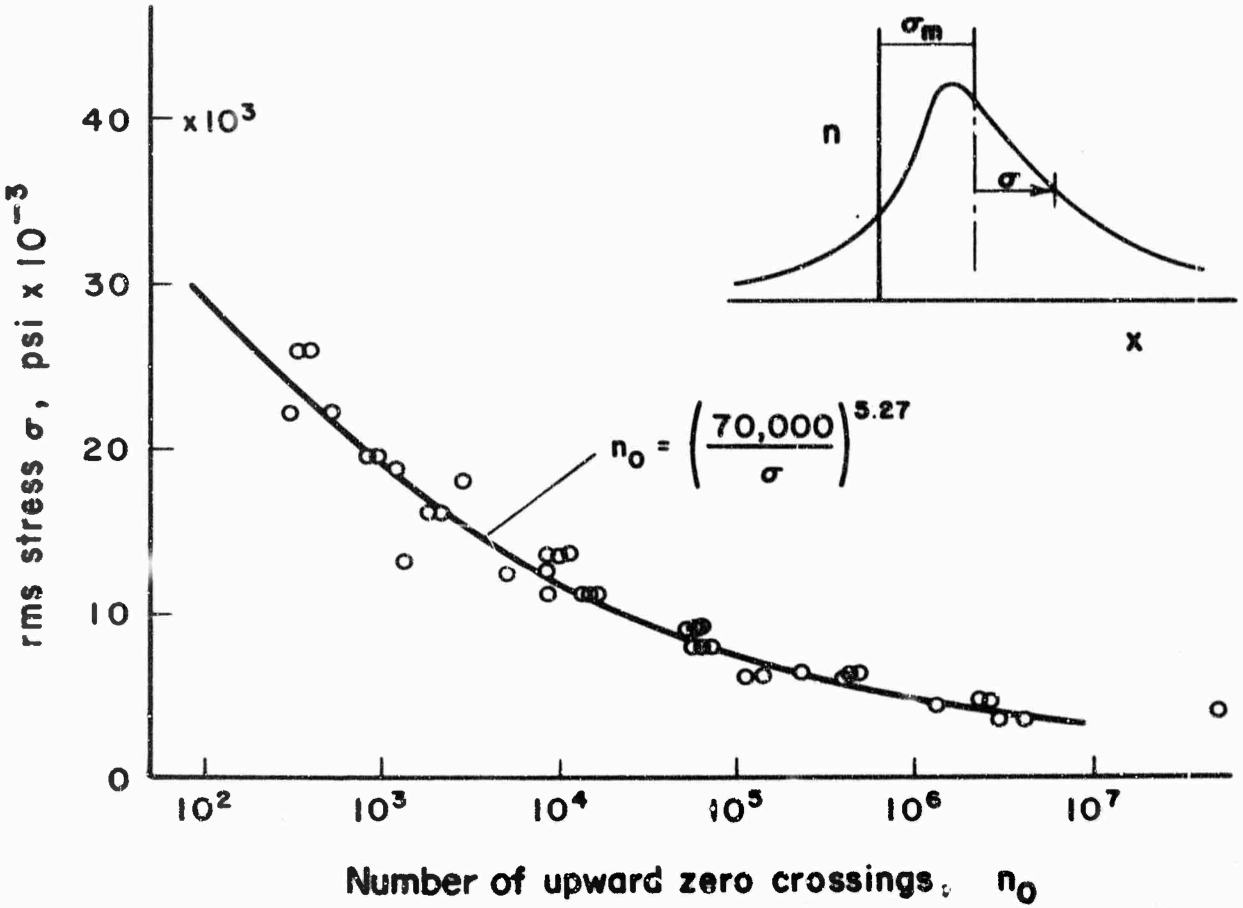


Figure 7. Fatigue Life for Random Loading of 2024-T4 Specimens. (Results are for $\sigma_n = 0$ and a Gaussian distribution for x.).

slope necessary to produce fatigue failure. The equation suggests the general form

$$n_0 = \left(\frac{s_r}{\sigma}\right)^{\alpha} \tag{C1}$$

where s_r is some empirical reference stress. Consider that this reference stress is expressed in terms of the design stress level x according to the relation

$$s_r = \beta x \tag{C2}$$

and recall that from Appendix A σ_x is given by (with σ_w replaced by the composite value σ_c)

$$\sigma_x = \bar{A}\sigma_c$$

Equation (C1) would thus appear

$$n_0 = \left(\frac{\beta x}{\bar{A}\sigma_c}\right)^{\alpha} \tag{C3}$$

In application to aircraft flight through turbulence, n_0 would be given by $n_0 = N_0 T_t = PTN_0$; hence Equation (C3) becomes

$$PTN_0 = \left(\frac{\beta x}{\bar{A}\sigma_c}\right)^{\alpha} \tag{C4}$$

The remarkable correspondence between this equation for fatigue and Equation (B7) is dealt with in the body of the report.

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13. ABSTRACT

A review is made of the progress and findings that have been obtained in a study aimed at the development of gust design procedures based on power spectral techniques. Included is a consideration of atmospheric turbulence makeup. A corresponding discussion is given of the aircraft structural parameters that are needed in spectral evaluation and on the practical aspects of their evaluation. Emphasis is given to the spectral design procedures that have evolved from the study, and a new hypothesis for fatigue considerations is advanced, case in the notation of gust loads design. Ease of application, interpretation and implication to design, present and future, and comparison with past design procedures are covered.

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