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Dimensionless Treatment of Vacuum Trajectory

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DIMENSIONLESS TREATMENT OF VACUUM TRAJECTORY

ABSTRACT

By the use of dimensionless variables the initial velocity is eliminated and the initial elevation is the only parameter distinguishing one trajectory from another. All vacuum trajectories have the same maximum ordinate and time of flight.

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The vacuum trajectory as a basis of investigation can only give limiting values and indicate the parabolic shape of the trajectory and the approximate time relations. The only force acting is that of gravity which is assumed to produce an acceleration of constant magnitude g. For heavy projectiles the vacuum trajectory is sufficiently accurate for many purposes.

It has been found in the course of computing trajectories that for a reasonable approximation it is more convenient to use dimensionless variables and reduce the amount of work usually involved. All trajectories with the same initial elevation are identical if the aforementioned variables are used.

Definitions:

Let t be the time after firing, x the horizontal distance and y the vertical distance, positive upwards from the muzzle; θ the angle of the trajectory with the horizontal and θ_o the initial angle of elevation; v the magnitude of velocity and v_o the initial or muzzle velocity; g the acceleration due to gravity; and let $\dot{x} = \frac{dx}{dt}$, $\dot{y} = \frac{dy}{dt}$

Then: $\ddot{x} = o$

$$\dot{x} = \dot{x}_o = v_o \cos \theta_o$$

$$x = \dot{x}_o t$$

and $\ddot{y} = -g$

$$\dot{y} = \dot{y}_o - gt = v_o \sin \theta_o - gt$$

$$y = -\frac{1}{2}gt^2 + \dot{y}_o t$$

$$v^2 = \dot{x}^2 + \dot{y}^2 = \dot{x}_o^2 + \dot{y}_o^2 = v_o^2 - 2\dot{y}_o gt + g^2t^2$$

$$\frac{\dot{v}}{v} = \frac{-g\dot{y}}{v^2}$$

Arc length $s = \int \sqrt{v_o^2 - 2\dot{y}_o gt + g^2t^2}\,dt = \frac{1}{2g}\left[\dot{y}_o v_o - \dot{y}v + \dot{x}_o^2 \log\left(\frac{v-\dot{y}}{v_o-\dot{y}_o}\right)\right]$

where the subscript zero denotes the value when t = o

$$\frac{dv/ds}{v} = \frac{g\dot{y}}{v^3}$$

Definitions: In dimensionless variables

Let $c = \frac{v_o \sin\theta_o}{g} = \sqrt{\frac{2y\ \max.}{g}} = \frac{\dot{y}_o}{g}$,

$T = t/c$, $X = x/gc^2$, $Y = y/gc^2$

Then: $\frac{d^2X}{dT^2} = 0$

$$\frac{dX}{dT} = \frac{dX}{dt} \cdot \frac{dt}{dT} = \frac{\dot{x}}{gc^2} \cdot c = \cot\theta_o$$

$$X = T\cot\theta_o$$

and $\frac{d^2Y}{dT^2} = -1$

$$\frac{dY}{dT} = \frac{\dot{y}}{gc^2} \cdot c = 1 - T$$

$$Y = T - 1/2T^2$$

Let $V^2 = \left(\frac{dX}{dT}\right)^2 + \left(\frac{dY}{dT}\right)^2 = \cot^2\theta_o + (1-T)^2$

and $S = \int_o^T \sqrt{\cot^2\theta_o + (1-T)^2} = \int_o^T V\,dt$

Then $S = \frac{s}{gc^2} = 1/2\left[V_o \left.\frac{dY}{dT}\right|_o - V\frac{dY}{dT} + \left(\left.\frac{dX}{dT}\right|_o\right)^2 \log\frac{V - \frac{dY}{dT}}{V_o - \left(\frac{dY}{dT}\right)_o}\right]$

Now $\frac{dV}{dS} = \frac{dV}{dT} \cdot \frac{1}{V}$ and $V\frac{dV}{dT} = -(1-T)$

so $\frac{\frac{dV}{ds}}{V} = \frac{T-1}{V^3} = \frac{T-1}{\left[\cot^2\theta_o + (1-T)^2\right]^{\frac{3}{2}}}$

For any vacuum trajectory the maximum ordinate is $\frac{1}{2}$ and occurs at $T = 1$, impact occurs at $T = 2$; and the range is $2 \cot \theta_o$. Then by simple substitution one can get results in the usual independent variables.

The rate of change of the trajectory angle θ is given by $\frac{d\theta}{dS} = \frac{\cot \theta}{V^3}$

For $v^2 = (gc^2)^2 \left[(\frac{dX}{dT})^2 + (\frac{dY}{dT})^2 \right] \frac{1}{c}2 = g^2c^2V^2$

$$\frac{d\theta}{dS} = \frac{d\theta}{ds} \cdot gc^2 = \frac{-g \cos \theta}{g^2c^2V^2} \cdot gc^2 = \frac{-\cos \theta}{V^2}$$

$$\cos \theta = \frac{\dot{x}}{v} = \frac{gc^2 \frac{dX}{dT} \cdot \frac{1}{c}}{gcV} = \frac{\cot \theta_o}{V}$$

$$\therefore \frac{d\theta}{dS} = \frac{-\cot \theta_o}{V^3}$$

The points of inflection of $\frac{d\theta}{dS}$ vs S are at $T = 1 \pm \frac{\cot \theta_o}{V^3}$

In figure 1 are shown three graphs of V versus S/Sw and three graphs $\frac{\frac{dV}{ds}}{V}$ versus S/Sw. The arc length S has been normalized by dividing by the total length of the trajectory, S_W.

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GLOSSARY OF SYMBOLS USED

The subscript zero on a symbol means that it is to be evaluated at the point where the initial conditions are applied.

The dot over a symbol means differentiation with respect to time.

A.

g = acceleration due to gravity

s = arc length along trajectory from muzzle

t = time after firing

θ = angle of trajectory above horizontal

v = magnitude of velocity

x = horizontal distance or range

y = vertical distance or altitude

B.

$c = (v_o \sin \theta_o)/g$

$S = s/gc^2$ = dimensionless arc length

S_W = S at impact

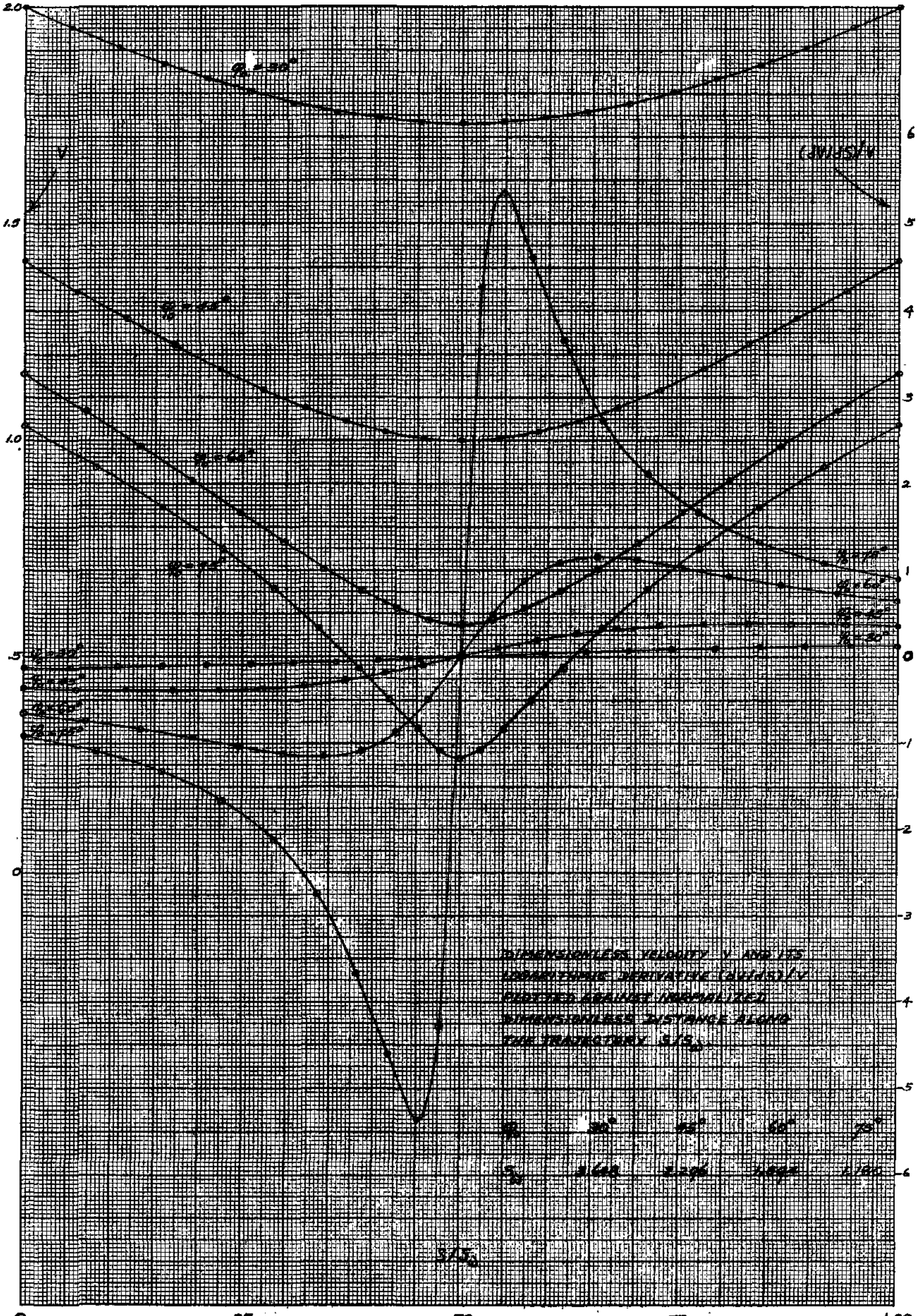
$T = t/c$

θ_o = angle of departure

$V = v/gc^2$ = dimensionless velocity

$X = x/gc^2$ = dimensionless horizontal co-ordinate

$Y = \frac{y}{gc^2}$ = dimensionless altitude



V
(dV/dS)/V
DIMENSIONLESS VELOCITY V AND ITS LOGARITHMIC DERIVATIVE (dV/dS)/V PLOTTED AGAINST NORMALIZED DIMENSIONLESS DISTANCE ALONG THE TRAJECTORY S/S0
S/S0
2.0
1.5
1.0
.5
0
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
0
.25
.50
.75
1.00
30°
60°
75°
2.296

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