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RESEARCH
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A Programming Model for the Design of Strategic-Deployment Systems

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DEPARTMENT OF DEFENSE

DEPARTMENT OF THE ARMY
OFFICE OF THE CHIEF OF RESEARCH AND DEVELOPMENT
WASHINGTON, D.C. 20310

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TO:

1. Transmitted herewith for your information and retention is (are) ____ copy (copies) of RAC-TP-211, "A Programming Model for the Design of Strategic-Deployment Systems."

2. This model was developed in the course of a broad study, begun in 1964, of the strategic objectives to be sought by the US Army to enhance further the strategic mobility of its forces. The model has been in constant use, and hence changes are occurring rapidly. The final section indicates the past applications and the current direction of the model development.

3. Recipients are cautioned to examine and fully comprehend inputs to the model. It will produce results basically dependent on inputs. These inputs are based upon representing forces not by organization but in terms of equivalent gross tonnages broken down into appropriate categories. There will be considerable variation in force structure for different environments. Consequently, the equipment for each force and the gross tonnages to be deployed will vary.

FOR THE CHIEF OF RESEARCH AND DEVELOPMENT:

Robert B Bennett

Incl
as

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Operations Research Division

LOGISTICS DEPARTMENT
TECHNICAL PAPER RAC-TP-211
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A Programming Model for the Design of Strategic-Deployment Systems

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RESEARCH ANALYSIS CORPORATION

McLEAN, VIRGINIA

FOREWORD

The background for the study effort of which the model described herein is a vital part was aptly summarized in the following excerpt from the statement of the Secretary of Defense, the Honorable Robert S. McNamara, to the House Armed Services Committee on 29 January 1964:*

> Closely related to the general purpose forces are the airlift and sealift forces required to move them promptly to wherever they might be needed. Included in the airlift forces are both MATS [now MAC] transports and the Air Force Tactical Command troop-carrier aircraft. The sealift forces include the troop ships, cargo ships, and tankers operated by the Military Sea Transport Service and the "forward floating bases."
>
> The requirements for airlift and sealift forces are not susceptible to precise calculation.
>
> First, they are subject to most of the same uncertainties which afflict the general purpose forces—the wide variety of possible contingencies, the uncertainties concerning the military strength of our opponents, etc.
>
> Second, the quick reaction capability which these forces help to provide can be achieved in a number of ways: by forward deployment of military forces, by the prepositioning of equipment and supplies either on land or in ships, and by the deployment of both men and equipment from a central reserve in the United States. Each of these alternatives, and variations of them, has certain advantages and disadvantages. And, as I pointed out last year, our present program is based on using a combination of these various methods, but we still have much to learn about the proper balance among them.

The model presented here was developed in the course of a broad study, begun in 1964, of the strategic-deployment objectives to be sought by the US Army to enhance further the strategic mobility of its forces. Since the preparation of the draft of this Technical Paper, the model has been developed further and used extensively in support of analyses of the requirements for airlift/sealift forces by the Office of the Assistant Secretary of Defense (Systems Analysis). Accomplishment of this priority work has unavoidably delayed final publication of this document.

In addition to the analysis undertaken with the model since 1964, revisions of the model and analytical techniques continue in order to provide the sophistication required in the broad deployment analyses that are being made. Past applications and the current direction that model development is taking are indicated in the final section of the paper, which deals with applications and extensions.

Lee S. Stoneback
Head, Logistics Department

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*R. S. McNamara, Secretary of Defense, "Hearings on Military Posture and H. R. 9637," before US Congress, House Committee on Armed Services (88th Congress, 2d Session) in J. H. McBride and J. I. H. Eales, Military Posture: Fourteen Issues before Congress, 1964, Center for Strategic Studies, Georgetown University, Washington, D. C., 1964.

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A Programming Model for the Design of Strategic-Deployment Systems

ABSTRACT

This paper describes a mathematical model developed by the Research Analysis Corporation for the analysis of strategic-deployment problems. The unique characteristic of the model is that it addresses the requirements of a set of contingencies in various parts of the globe simultaneously rather than those of any single or so-called "worst" case.

The model consists of a set of linear equations and inequalities that represent (or approximate) the deployment requirements of the set of contingencies and the capabilities and costs of a wide variety of deployment system components—aircraft, ships, and prepositioning sites. Using linear programming the set of equations is solved to produce the least-cost deployment system; i.e., the mix of components that is capable of meeting, at least cost, any of the set of requirements.

Following an introduction that provides a background and rationale for the development of the model, a general description of the inputs required and the outputs provided is given. The type of problem the model addresses is illustrated by a simple example.

The next two sections provide for the more technically oriented reader a detailed formulation of the model for a sample problem and an illustration and discussion of the computer solution to the problem. Also provided is an analysis of the sensitivity of the results to the assumptions and other inputs used and a description of the kinds of sensitivity analyses available.

The paper concludes with a brief section outlining applications to the analysis of strategic-deployment problems, and possible extensions of the model.

INTRODUCTION

PURPOSE

The purpose of this paper is to describe a linear programming model for the design of strategic-deployment systems that carry out global-deployment strategies while conforming to a given deployment-system policy. These terms will be defined in greater detail later in the paper but are introduced here to facilitate the ensuing discussion. In brief the strategy referred to is a comprehensive plan that specifies the level of strategic-deployment capability to be attained in each world area of potential US military involvement; the policy defines the allowable means by which the capability may be achieved; and the system is a collection of vehicles, bases, equipment stocks, and associated facilities whose primary function is to provide a capability to deploy military forces in overseas areas.

The model is a mathematical representation of the strategic-deployment system by a set of linear equations and inequalities whose terms include variables corresponding to the components of the system. By means of linear programming the set of expressions is solved by determining values of the variables such that the cost of the deployment system is minimized.

BACKGROUND

To further national objectives the US has, through treaties and otherwise, incurred military commitments around the world. To meet these commitments the US must maintain the capability to deploy its forces to virtually any area of the world. How best to provide the necessary strategic mobility is and will continue to be one of the major problems confronting the Army and the Department of Defense as a whole.

The primary concern of strategic mobility is speed. If speed were not at issue, there would hardly be a problem because a massive lift capability consisting of conventional cargo shipping is under effective US control. Several weeks or more may elapse, however, between the outbreak of war and the time when forces deployed by ship can close with the enemy in the objective area. This problem is acute in many areas in which the US has vital interests. In fact the interval between the decision to deploy forces and the time when deployment by means of conventional sealift pipeline can be established defines the critical period of possible future limited wars. If no response is made during this critical period, the military situation may well deteriorate, especially in underdeveloped areas having little capability for self-defense. In the remainder of this paper, attention is confined to each area's critical period, which ends with the establishment of a conventional sealift pipeline.

Although there is general agreement in principle about the military and deterrent value of rapid deployment, determination of the appropriate level is quite another matter. For each potential conflict area the questions of force size and deployment speed are difficult to answer. Since the threat is uncertain, there is no unique requirement in each area, and a wide range of response level in the critical period may be generally consistent with overall US objectives. The war may still be won even if the area is initially lost and must be retaken. Such a war, however, might be a great deal more costly than one in which rapid deployment permitted early stabilization and counterattack. The critical-period deployment capability corresponding to these extremes might range from no capability to the capability for the deployment of several divisions. Unfortunately the military value and the dollar cost of any deployment system appear to be incommensurable in any generally acceptable terms.

Somewhat more fortunate is the fact that the problem of how much to deploy can be studied independently of the problem of how to deploy it. The military value of the capability to follow a particular force-closure schedule is largely independent of the means by which those forces arrive in the objective area. Similarly, if a set of force-closure schedules for each potential contingency of interest is given, it is not necessary to know the military value of that strategy to select good deployment-system policies and efficient deployment systems. It is necessary, of course, to have the capacity to determine readily appropriate policies and systems for the completion of a wide variety of alternative global-deployment strategies. The ultimate goal is the selection of a global-deployment strategy and a deployment-system policy. These will imply the corresponding strategic-deployment system. That selection will involve an analysis that at least implicitly makes the military value of the strategy commensurate with the cost of the deployment system required to carry it out.

This paper is restricted to the development of a technique to facilitate the study of deployment-system policies and the design of deployment systems. Given a specific deployment strategy (set of requirements) and a policy (set of allowable components) one is still faced with the fact that there are many deployment systems that can support the strategy selected. In order to avoid a hopeless morass of additional variability, a technique must be available to generate consistent deployment systems for given combinations of deployment strategy and deployment-system policy. This function is performed by the linear-programming model described in this paper. The model guarantees the meeting of the force-closure schedule for each area as specified in the strategy, observes the restrictions established in the policy, and produces specifications for the deployment systems that minimize cost.* Minimum

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*In general, the model will produce one set of specifications from the many possible combinations of optima, or least-cost systems. The model will specify a single minimum-cost system under the constraints imposed. In practice the system is usually not unique.

system cost is the criterion invoked to select a single system from among those that are feasible, i.e., capable of meeting the demands of the deployment strategy within the policy restrictions. Notice that minimum cost is the criterion for the selection among deployment systems; it is not the criterion for selection of a global-deployment strategy or a deployment-system policy. As pointed out earlier the latter selection involves the comparison of military value and deployment-system cost and could well result in a comparatively high-cost deployment system if the resulting military value were also high.

In the following sections a general description of the model will be given in terms of its input requirements and its final output. Considerable attention will be given to representation of deployment strategies and system policies in a form appropriate for input to the model. The detailed formulation and some possible applications will then be illustrated by means of examples. Finally, several applications and additional formulations will be discussed.

GENERAL DESCRIPTION OF THE MODEL

INTRODUCTION

In the previous section the deployment model was discussed in terms of its relation to the general investigation of strategic deployment. As stated, the objective of the model is to produce specifications for a least-cost deployment system to carry out a global-deployment strategy while conforming to a given system policy. In this section the model will be described in terms of its required input and final output. In the process, detailed definitions will be given of the general terminology previously introduced. Consideration of the structure of the model will be deferred until the terminology, the function, and the purpose of the model are established.

Figure 1 illustrates diagrammatically the model, the required inputs, and the outputs. By means of linear mathematical relations the model represents both the process of acquiring a deployment system and the actual deployment operations to each of several contingency areas in order to ensure that the resulting system is capable of meeting the objectives. At the same time the model is keeping track of the cost of the system that is being acquired so that its cost can be minimized. Each of the indicated classes of input and output data is described in detail in this section.

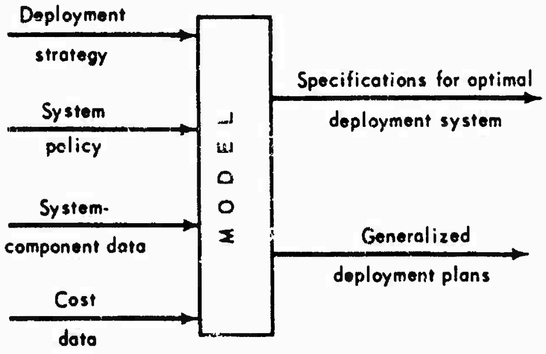

Fig. 1—Model Input and Output

MODEL INPUTS

The discussion here is in general terms and is not intended to detail the numerical values required to produce a particular system design. The intention is to identify the various classes of data required and to indicate the nature of the preparations that are involved in any meaningful application of the model. These classes of data relate to a statement of requirements to be met, allowable design approach, component technical data, and component costs.

Global-Deployment Strategies

Formulation of a global-deployment strategy begins with the identification of the areas of the world where the US has military commitments. For each contingency a skeleton scenario is constructed in which the threat and the forces to counter it are estimated in conformance with a strategy that calls for a desired level of response. This strategy may be one of early stalemate and counterattack in which the intention is to snuff out the war before it really achieves major proportions, or it may call for a perimeter defense of a restricted area from which a much later counterattack will be launched. Intermediate variations are of course possible. Within the framework of the model, each area must be considered on its own merits in view of the wide variation in threat, terrain, indigenous forces, etc., that may be expected to characterize dispersed areas of the globe. Western Europe and Southeast Asia represent extremes to be considered in attempting to design a deployment system that will function well in divergent environments. In effect, if a single system is to be developed, it must cover the full spectrum of contingencies that may occur. An appropriate strategy will include contingencies selected to cover this spectrum with regard to environment, threat, and geographical distribution. Geographical distribution is especially important if the possibility of establishing forward depots for prestocked equipment is to be considered. Although the general deployment strategy is intended to be exhaustive, its representation in terms of skeleton scenarios must be restricted to a set of typical contingencies.

In view of the variety of conditions that may be encountered, it is not considered reasonable to select a deployment system on the basis of a single contingency. In general there may be no single worst case that will imply the desired capability in all areas of the globe, and one cannot be sure of approaching the maximum economy in the system design when only one contingency is considered. The alternative components available for the construction of deployment systems vary widely in cost and scope of coverage, so that a true concept of a least-cost system based on the consideration of a single contingency cannot be attained. Further, the variability in the magnitude and timing of the deployment of forces to diverse areas make the assurance of adequacy in all cases highly dubious. Therefore it is considered preferable to begin with a set of contingencies having at least one representative from each geographical area to be covered by any deployment system. This procedure will also result in the inclusion of various environments and degrees of sophistication of the enemy that is to be faced. A requirement expressed in these terms provides the most appropriate context in which to evaluate alternative deployment-system policies and the deployment systems necessary to meet those requirements.

Representation of Forces in the Model. In establishing the force requirement for each area, the types of organization to be employed will probably be chosen. However, the model has been formulated to represent forces not by organization but in terms of equivalent gross tonnage broken down into appropriate categories.* There will be considerable variation in the structure of the forces required in different environments. An armored division would be of doubtful value in the jungle areas of Southeast Asia but could be valuable in the Middle East. The variation of force structure and size with area will result in variations in overall force weight and in the nature of the equipment that makes up the total tonnage. The transport vehicles in the deployment system are not uniformly capable of handling or lifting all types of Army equipment in an initial deployment. This fact provides the primary basis for the decomposition of the total tonnage of a force into meaningful categories with respect to the deployment system. The form of the system in the model is sensitive to the weight and equipment structure of the force to be deployed but not to its organizational structure. The organizational structure of the forces to be deployed is not completely lost in the model formulation, since the deployment time phasing is also represented, as are the sites of prepositioned supplies and materiel and, implicitly, origins of materiel within the CONUS. Organizational integrity is retained at least with regard to subdivisions of the overall critical period. The primary restriction that requires classification of the overall force tonnage will be that made necessary by the transport aircraft that will be considered in the problem. Another category might be desired on the basis that some of the equipment of the force to be deployed must of necessity be based on the CONUS, i.e., it will not be allowable to prestock this category in a forward base.

Basically, the force to be deployed to each contingency area represented in the model is originally defined in terms of its organization (type and number of divisions and support forces). An analysis is then made to determine the composition of the equipment of that force in terms of the relative tonnages that are transportable by the vehicles in the list of those available. The model allows categorization by classes of materiel size. Two classes will usually suffice if some reasonable approximations are employed. (These two classes, outsize, or O, and regular, or R, are further discussed in later subsections "Transport Vehicle Productivity" and "Tableau Detail.") By this means the force to be deployed in each area is converted to an equivalent total tonnage, which is broken down into categories or classes on the basis of transportability. This tonnage is then required to be deployed in accordance with the schedule set up for each area.

Deployment Time Phasing. The deployment requirement in each area is defined through the critical period that ends with the establishment of a conventional sealift. This sealift is presumed capable of picking up the burden of resupply and any increase in deployment rate that is desired over and above the continuing capabilities of the rapid-deployment system that has been functioning in the critical period. The length of this period is primarily a function of the geographical location of the area and will vary from one area to another. The end of the critical period is not a natural period with respect to the tactics employed in the area and does not define a tactical phase in the military strategy

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*This is not an inherent limitation of the model. Specific organizations are not explicitly denoted, primarily because of the detail that would be required and the consequent enlargement of problem size.

for the area. The force-closure schedule will presumably extend well past the end of the critical period. The plan of force deployment will change radically with the introduction of conventional sealift; however, no consideration of the deployment plan after the critical period is included in the model.

The force-closure schedule demanded by the military tactics employed in an area is independent of the natural phasing capability of the various available means of deployment. In the interest of accurately representing relative component capabilities it is desirable to construct a deployment schedule that is a natural result of the deployment-system component characteristics and at the same time is consistent with the tactical force-closure schedule. Figure 2 illustrates the distinction between the tactical schedule and the deployment schedule. The tactical force-closure schedule shown in the dotted line is a reflection of the overall strategy as applied to the specific area; the solid line indicates the deployment schedule fitted to the tactical schedule so that no shortfalls exist. The deployment schedule may at times exceed the requirements, but this is because the deployment will normally proceed at a uniform rate over the intervals shown, to minimize demands on the terminal facilities in the area. Both the tactical and the deployment schedule are specified in terms of the closure of forces in the objective area, to represent the total delay involved in the various modes of delivery available in the design of the system. If, for example, the objective area is well removed from the seaports, more time must be allowed for transportation over the land lines of communication (LOC) than for the delivery of equipment by an aircraft that can land at nearby forward airfields. This scheduling puts the competing modes of delivery into a context that permits valid comparisons.

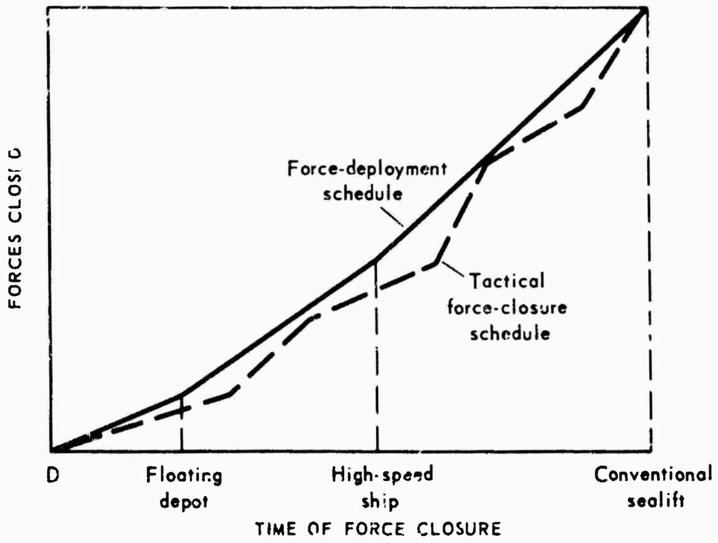

Fig. 2—Critical Period of Typical Tactical and Deployment Schedules

The overall strategy is applied to the development of the tactical schedule in the individual area and is reflected in the level of response to be made during the critical period. Thus in a given problem formulation the level of response will be consistent for all theaters. For a given area the level of response will be defined by the tactical force-closure schedule in terms of its maximum value and its shape. The time scale will not vary as a function of the strategy, since its length and division points are defined by the characteristics of the available system components and are independent of the strategy employed.

The subdivisions indicated on the time scale of Fig. 2 result from the fact that some of the system components that may appear in the optimal-deployment system cannot contribute to the deployment schedule until a later time than others. For example, high-speed ships may be effective substantially in advance of conventional sealift but not during the first two weeks because of LOC time requirements as well as the normal loading, steaming, and off-loading times. If forward floating depots are employed, they may precede high-speed ships but are still subject to the same delays and constraints, and account of them must be taken in the model. Thus although many alternatives may be available for the delivery of forces during the critical period, they are not uniformly effective throughout this period and must be defined with respect to their capability to cover the tactical force-closure-requirement schedule. With the subdivisions of the critical period defined as described, the model incorporates a three-segment linear deployment function fitted to various tactical-force-closure schedules. The overall length of the time scale and the location of the subdivision points are unique to an area and must be defined for each area. The net result of the fitting process is the statement of the cumulative tonnage that must be deployed at each subdivision point of the time scale and at the end of the overall critical period.

Deployment-System Policies

A deployment-system policy is a general statement of the ground rules to be observed in the design of a deployment system. It provides for the formulation of the restraints to be observed in defining the system. The following paragraphs define "system" and "policy" in some detail and discuss the context in which the deployment system is designed.

Definition of a Deployment System. A deployment system is a collection of transport vehicles, equipment stocks, forward bases, and associated facilities whose primary function is to provide a capability for the rapid strategic deployment of US forces. The systems of interest are those capable of meeting the requirements of any of a set of contingencies in various parts of the world. For the purposes of this paper the requirements that the systems must meet are those existing prior to the time at which forces can be delivered by conventional sealift, including the time required for sealifted forces to be moved from ports or beaches to the battle area.* Allowable components of the deployment system, hence, are those that can contribute to meeting the requirements during this early or critical period. The length of the critical period, which is primarily a function of distance, may vary from one contingency area to another.

Definition of the Deployment-System Policy. In the definition of a deployment system the components mentioned were considered without restrictions

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*Using the establishment of conventional sealift as a point in time in determining deployment requirements is arbitrary. Conventional sealift can be included in such a model as described here, if desired.

except with regard to their ability to meet requirements during the critical period. It is the function of the system policy to further delimit the use of allowable components and to identify the options that are considered available in the design of a particular system. The policy adopted will deal with and be reflected in restrictions that apply to the time frame of interest, transition or interim requirements, transport vehicle options, prepositioning options, and terminal facility options. It is desirable, in formulating the model, to include a broad range of options concerning these restrictions or ground rules and to rely on parameter adjustments to modify the model to represent a particular policy. In many cases the relevant parameters may be varied continuously so as to run through a complete range of related policies or strategies. The major categories of policy restrictions are as follows:

Time frame. The time frame in which systems will be operational will determine the components that can be considered available for inclusion in the deployment system. Normally the model will be employed to investigate systems with a capability greater than that of an existing system. Creation of an improved system can require time for the development and acquisition of new items, for the production of additional quantities of existing items, or for some other purpose. A time limit must be set for achieving the necessary capability, therefore, to ensure that a valid set of competing system components is considered.

Interim requirements and transition policy. In making the transition from the current system to a new system of greater capability, there may be an interim period in which the system capability is not stated in the deployment strategy. The strategy may specify goals for the new system but say nothing about the capability to be maintained along the way. The least-cost system configuration as ultimately defined by the model may include components that are not currently in existence and may call for the retirement of certain components of the existing system. Presumably some minimum interim capability is to be maintained during any transition that is required.

Transport vehicle options. A major element in the definition of a system policy is the determination of the kinds of vehicle that will be permitted to compete for inclusion in the least-cost deployment system. As indicated before, the time frame selected determines the vehicles that can be available. However, further restrictions may be desired. It may be of interest, for example, to examine the implications of the exclusion of one or more of the vehicle options as a means of investigating alternative system policies. Conversely it may be desirable for some reason to require the inclusion of certain components without regard to their relative economic merit. The model can be structured to permit the introduction, when desired, of various constraints on the available options.

Prepositioning options. Another major element in the definition of system policy is the specification of the allowable prepositioning posture. The extremes are the complete elimination of the prepositioning and the removal of all restrictions with regard to type, amount, and location. Intermediate policies may include restriction of the classes of tonnage, restriction of the type to shore based only or floating base only, and restrictions as to the number and location of the forward bases. In order to specify all the alternative routes available to the transport vehicles in the system, it is necessary in the definition of the policy to specify the allowable base locations. It does not follow that all the base options will be employed in the least-cost system, but when multiple contingencies are involved it is usually difficult to determine in advance those that will be selected. Since each prepositioning site is a separate system component a certain amount of discretion in the number of possible bases is required to prevent the size of the model from getting out of hand.

Retention posture. For all components of the system the question of the cost of inherited assets vs that of new components must be considered. In the model this question must be answered in terms of cost and productivity. Obviously an equally productive inherited asset will be preferable to a new component that must be purchased. Care must therefore be exercised in the definition of systems during various time periods, from current through transition to the final system, in stating what the system costs are and how they are shared between inherited assets and new materiel.

Terminal and special facility options. An aspect of system policy secondary with respect to cost but very important with respect to a genuine deployment capability is the problem of terminal and LOC capacity in the individual contingency area. Consideration must be given to deployment restrictions that may result from inadequate airfield, port, and intratheater transportation facilities. This problem is important in the consideration of strategies calling for a high-level response during the critical period. It is desirable to include provisions in the deployment model for the accounting of terminal and LOC capacity used by each system.

Numerous techniques may include this accounting. One is to consider the available capacity in each area as an environmental restriction. This approach has inherent difficulties, as when a given strategy cannot be implemented because terminal limitations render the deployment infeasible with any allowable combination of system components. Inadequacies in a single area among several in the model may prevent a feasible solution.* The model demands deployment feasibility in every area included and in a time-phased manner. A more serious problem in trying to observe existing terminal-capacity restrictions is that a system design of artifically high cost, compared with one in which augmentation of capacity is permitted, may result.

An alternative approach is to admit facility augmentations as components of the system. The addition of facilities in contingency areas would have status within the model comparable to that of forward-base construction. These components would be added to the system on a schedule consistent with the procurement and construction of other additional system components. The model formulation can be arranged to keep track of utilization of existing facilities and any required facility augmentation. Cost is assessed only for the excess above existing capacity required in each area. In this fashion the terminal-facility requirements are an integral part of the system design and exert an influence on the design in proportion to their relative cost.

A third approach is to ignore the terminal and special-facility problem within the model and to determine the facility requirements implied by each design produced. In any event the status and options with regard to facility requirements must be established within each system-policy alternative.

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*A "feasible" solution is a possible combination of systems that will solve the deployment problem as stated, regardless of cost; when no such combination of systems is possible, solution of the problem as stated is "infeasible."

Equivalent consideration must be given to any specialized facilities required in the utilization of other system components.

Summary. A virtually unlimited number of distinct system policies can be formulated by the selection of alternatives among the classes of restrictions discussed above. The model formulation has relations within its structure for the representation of each of these classes of restriction. In the formulation of a specific system policy, a position must be taken with regard to each type of restriction.

Establishment of the System Design Context. The object of defining the system policy is to establish an appropriate context for the consideration of alternative deployment systems to implement a global deployment strategy. The stage is set in terms of what the system must accomplish and in terms of the options available to the system designer. Within this context, infinite variety remains for the design of a specific system that will meet the deployment requirements in each area while observing all of the restrictions laid down in the system policy. With sufficient effort in setting the framework of the options and reasonable assumptions, the designing of the deployment system can be turned over to the linear-programming model. The objective employed in the present model is the minimization of deployment-system cost. This in no way restricts the options available at the higher or policy-making level but merely provides for the efficient implementation of any strategy-policy combination of interest and determines the corresponding implementation cost. As discussed earlier, an analysis at a higher level is still required to select the strategy and policy. Much can be learned about what constitutes an economically desirable system-design policy by a parametric investigation that is essentially independent of the strategy or deployment level that may be chosen. The deployment level may be treated as an independent variable and varied over a significant range. A good system policy will remain so over a wide range of variation, as will the composition (but not the size) of a good deployment system. That is, the direction to go in the development of strategic deployment capability may be determined to a large extent independent of the strategy to be implemented.

Vulnerability. Divergent risks are inherent in the application of such diverse system components as long-haul airlift and the dependence on shore-based prepositioned equipment. Each component that may be considered for employment in the system will operate in a somewhat different environment and will be vulnerable in varying degrees, depending on the routes and the sophistication of the enemy. An element of military value is, of course, the relative security and dependability of the overall system. The ranking of the various system components will vary with the area in which they are used.

There are at least two ways to account for vulnerability in attempting to construct systems of equal military effectiveness: certain components may be restricted from use in particular areas, or attrition may be assessed explicitly within the model. The latter approach is preferable in the sense that a vulnerable component would be driven out of the solution because of the economic impact of the heavy losses to be expected. This approach increases the data-preparation effort considerably and is a function of estimates of the relative vulnerability of various components in a wide variety of environments. An additional difficulty is that the resulting vulnerability of a particular system selection will depend on the subsequent enemy response in each area. It is therefore considered a major problem in a model of this type to reflect adequately and accurately the relative vulnerability of system-component options. If desired, however, the mechanism for including operational attrition is available within the model, although this has not been considered in applications up to this time.

It must be recognized that all systems will encounter operational degradation, and so long as there is no reliable means for distinguishing among alternative systems within the confines of a single system policy, there is no strong requirement to attempt vulnerability assessment within the model. It may of course be possible to recognize degrees of overall system vulnerability between alternative systems and system policies. A generally valid principle, however, is that the greater the variety of available means for deployment the greater the enemy's problem in effectively countering the system. (For this reason the presentation of alternative optima, or alternative least-cost solutions, may be useful in system planning.) A single-mode system might be disastrously vulnerable. A multimode system may be degraded but still be able to support the general objectives in each area.

The preceding discussion has concentrated primarily on the question of operational vulnerability during wartime deployment activity. Another concern is of course the relative peacetime security of the various components. The practice of shore-based prepositioning is often cited as a high-risk option subject to other drawbacks as well. To a large degree the attendant risks of prepositioning can be reduced to an acceptable level by appropriate selection of base sites and by other security measures. The point to be made is that the achievement of this level of security can be obtained only at a cost that must be charged to that option if it is to be included in the system. In general the costing of a component is designed to include the attendant cost of rendering it roughly equal in acceptability or in consistency of assumptions to all other available components.

Flexibility. Another criticism of various possible system components is their alleged lack of flexibility. In the approach taken here the question of system flexibility is transferred from the component to the system policy and more directly to the formulation of the overall deployment strategy. Although a single instance of equipment prepositioning in a single base is primarily restricted to local application as contrasted with the global capability of intercontinental transport aircraft, a widely dispersed collection of prepositioning bases cannot be said to lack flexibility in the geographical sense. The feasible system designs permitted under the policy and strategy requirements are by definition as flexible as the strategy itself. The model formulation can permit the duplication of any components (e.g., prepositioned materiel) that are geographically restricted, should those components be strongly compelled for reasons of economy to enter the model solution for the least-cost system. Flexibility to accommodate multiple contingencies may also be demanded of a system. A hypothetical system relying to a considerable extent on essentially unlimited prepositioned equipment stocks might well prove to have superior flexibility in this sense, as compared with a system based on a centralized system located exclusively in the CONUS. It is considered that these facets of the problem are covered in the analysis preliminary to application of the model and do not constitute an issue with respect to the individual system components.

System-Component Data

Exclusive of costs, the primary data required are transport-vehicle productivities and the terminal-facility requirements. A particular type of vehicle may be used in a variety of modes during area deployment operations, and measures of its productivity in these modes must be evaluated. Each of these data categories is discussed below.

Transport-Vehicle Productivity. In representing transport-vehicle utilization in the model, distinctions are made with regard to area, time phase, route, and class of tonnage* transported. The productivity of a lift vehicle is a function of all these factors, as well as of vehicle characteristics such as range and speed, and a distinct productivity measure is required for each combination that defines an allowable vehicle utilization. In the model, deployment bookkeeping is carried out in terms of a standard unit of tonnage (usually a kiloton). For this reason the natural measure of productivity is the number of vehicles required for the delivery of a unit of tonnage. This number is determined for each allowable vehicle utilization and is the reciprocal of the usual measure of productivity, since it has units of vehicles per kiloton.

In determining vehicle productivity over a specific route it is important to establish the actual number of days available to the vehicle for delivery operations during the overall length of the time phase being considered. For example, in the initial time phase in an area one must work backward from the end of the phase with respect to the last item from the last vehicle or vehicle sortie. If the vehicle is a heavy transport aircraft requiring an airfield with high-strength runways and such fields are only in rear areas, two or three additional travel days may be required to reach the objective area. In assessing the productivity of this aircraft in the first phase, these additional travel days must be deducted from the available working days to determine the number of aircraft required to deliver a unit of tonnage that can be closed within the time limits for that phase. Days deducted from the first phase are available for application to the productivity in the next phase, but a similar cutoff time allowance must be made with respect to the end of each phase. In this fashion the relative airfield capabilities of various aircraft may be represented. An aircraft that can utilize forward airfields and reduce subsequent travel time gets credit for some additional working days in the evaluation of its productivity. Similar considerations apply to each time phase and to each type of lift vehicle. Ships are particularly restricted in this regard, since an unloading allowance must be included in the time deduction in addition to the LOC time.

No consideration will be given here to the actual means for the computation of the number of vehicles required to deliver a unit of tonnage over a given route in a specified time. It may be as simple as determining the payload of a ship that cannot recycle, or it may be quite complex, as in the case of aircraft that may be able to cycle several times during the available working time.

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*Class of tonnage is of particular concern for aircraft, where some items are too large, or outsize, for certain aircraft. Other aircraft exhibit different productivities for regular and outsize items; these tonnage types, classes R and O, respectively, are treated as distinct types in the model

Methods for generating these data are available, including some very sophisticated aircraft loading and cycling models. For the purposes of this paper, vehicle productivities are merely part of the required input. It is worth mentioning, however, that the productivity evaluations made by the model constitute a more accurate representation of the relative strategic-deployment capabilities of vehicles than simple ton-mile evaluation does. This is particularly true of aircraft, since the effects of airfield capability, range, speed, payload, and fuselage width are all accounted for explicitly. Ton-mile capability evaluation can be quite misleading in the strategic-deployment context.

Terminal Facility Requirements. The discussion here is restricted to facilities that are shared by two or more system components. It is reasonable to consider three classes of terminal facilities in each area: airfield capacity, port or beach capacity, and a generalized LOC capacity. If the aircraft considered in the problem have varying airfield requirements, it will be desirable to split airfield capacity into forward-field capacity and heavy-field capacity in accordance with aircraft capabilities. In defining port and LOC capacity the appropriate measure is kilotons per day off-loading and port clearance capacity. Airfield capacity is more appropriately measured in terms of sorties per day, since payload variation among distinct types of aircraft is considerable, and capacity of an airfield is more directly related to the number of takeoffs and landings to be accommodated than to the tonnage involved. Tonnage delivered by air to rear-area fields of course generates LOC requirements as well as airfield-capacity requirements. In the model the required augmentation may be determined by accounting for the facility requirements generated by each mode of vehicle use and balancing them against the available capacity.

In general it is probably sufficient to assume when formulating the model that trooplift capacity is adequate by virtue of the Civil Reserve Air Fleet (CRAF) augmentation available to supplement the trooplift capacity of the transport aircraft and ships. No explicit trooplift activities appear in the model discussed in this paper. The implications of trooplift requirements could be accounted for, however, by including in the facility requirements for the deployment of equipment the concomitant requirements for troop airlift sorties to provide the troops that will marry up with their equipment in the area. It is thus possible for equipment delivered by ship to generate a corresponding airfield capacity requirement in addition to that for port and LOC capacity.

System-Component-Cost Data

The activities represented in the model fall into two distinct categories: those that relate to the acquisition and maintenance of the system itself and those that specify the utilization of the system in responding to each contingency covered in the global deployment strategy. Cost considerations are restricted to the peacetime system costs over a specified period of time. These costs will dominate any short-term incremental operating costs associated with a contingency that might occur during the costing interval. An attempt to assess deployment operation costs would lead to a string of untenable assumptions as to the number of wars expected to occur in the costing period and the equipment requirements to replace wartime losses. It does not appear profitable to become involved in analyses of this type. The purpose of the following discussion is to indicate some of the general principles and assumptions applied in treating system costs within the model. There is no attempt to consider the details of cost-estimation techniques or even to establish an appropriate costing period. These details will depend on the requirements of the user.

Proportionality Assumption. It is a requirement of the model that the cost of each system component be treated as if it were directly proportional to the quantity of items in the component. This assumption is generally reasonable with respect to the majority of the system components and is consistent with the level of accuracy required in a model of this generality. Operation and maintenance costs are in fact almost directly proportional to the number of vehicles in a fleet or to the size of a base. Production cost tends to be nonlinear, particularly in the early production stages of aircraft. However, for components having a production history the assumption of an average unit cost is reasonable. The unit cost of a completely new component may vary enough to warrant the checking of the number of units in a solution against the unit cost at that production level to ensure that the linear cost assumption is sufficiently accurate for the production quantity of interest. If the cost variation is considered significant, the problem may be iterated using modified costs.

The inclusion of research, development, test, and evaluation (RDTE) costs in the model is optional. These costs may be added to the system item costs in the basic formulation. Thus the question whether an undeveloped component would actually have appeared in the optimal solution if the development cost had been apportioned among the number of items of that component called for in the solution does not arise. An alternative method would be to omit RDTE costs; then, to assess the validity of the solution, a program option that establishes the range of component unit cost over which the solution remains valid would be utilized. If the unit cost of a component, adjusted for the proportionate share of development cost, remains within the allowable range, the solution remains valid and the total cost is easily determined. If not, iterative techniques may again be applied. Violation of the cost range does not imply that a component necessarily disappears from the system; it may merely appear at a reduced level with a new cost range sufficiently wide to accommodate the unit development charge. A further useful option in this type of investigation is the provision for the continuous variation of the unit cost for a single component or a group of related components. Cost data, uncertain for any component not yet developed, will in any event require cost sensitivity analysis.

The important point is the ready availability of methods for checking the validity of the solutions that are obtained. This feature renders the use of linear cost assumptions reliable and practical.

Cost Categories. The three costing categories identified in estimating component costs are operation and maintenance (O&M), initial investment, and RDTE. The development status of the component defines the applicable categories in each case. O&M costs are assessed for every component in the system over the selected costing period. Inherited components, those in production, and those for which there exists a firm commitment are assessed only O&M costs. Components for which development can be considered essentially complete, but whose production has not been previously contracted for, are assessed initial investment cost in addition to O&M. Finally, conceptual components are subject to all categories of cost. A vehicle from the inventory is considered a component distinct from a vehicle of the same kind procured for use in addition to the current or committed fleet. The productivities are identical, but the vehicles must be distinguished on the basis of cost.

Costing Period and Transition Costs. The costing period selected for the evaluation of system costs may be chosen on the basis of the user's requirements. Perhaps the simplest approach is to consider a fixed period beginning with the time of system completion. A retirement cost may be assessed against the activity of retiring any existing system component that has useful life remaining at the end of the period during which the system is being constructed. Such an approach fails to consider the system costs during its transition phase. Such information may be of interest and may even have an effect on the configuration of the final system design. If interim deployment capacity goals are set for the system, the retirement of components must be timed in accordance with the introduction of new components in the inventory. This implies that the cost of operating components for a portion of the overall costing period should be assessed against retirement cost in addition to a charge for unused life at the time of replacement. The result is that the net cost of including a component from the inventory in the final system is effectively reduced. Similarly a new component entering the inventory will be charged operating costs proportional to its active life in the system during the costing period. Other details must be considered, such as the allowance for remaining life of a component at the end of the costing period and the appropriateness of discounting of future costs. The cost approaches discussed above, together with the system completion date established in the system policy, combine to determine the form of the cost function to be incorporated in the model.

Assurance of Constant Effectiveness. The basic philosophy of system design within the confines of a given deployment-system policy is to ensure that each alternative allowed by the policy achieves an equal time-phased deployment profile. The analysis pivots on a fixed set of force-closure schedules with a fixed force composition provided in each area. This is considered to ensure equal effectiveness provided that the equipment deployed by each mode is comparable in its state of readiness and in its relative availability. For example, equipment that is prepositioned must be equivalent in all respects to equipment that might be deployed from the CONUS. It must be the type that is normally used by the troops and was used for their training. Provisions must thus be made for updating any equipment that is prestocked. This equipment must be in a comparable state of readiness and reliability to that deployed from other sources. Finally, there must be assurance that this equipment will actually be available for use in responding to a contingency. There must be provision for military and political security of prestocked equipment as well as for its maintenance in a high readiness status. The corresponding costs must be included in the annual operating costs and the initial investment required for adequate storage facilities such as dehumidified warehouses. Similar care must be exercised with respect to all allowable system components identified in the system policy.

The assurance of equal effectiveness places a significant burden on the specification of a component and the associated assessment of its cost. Only in such a context is it reasonable to turn over the system-design process to the linear-programming model, for the final selection of combinations of the available components and deployment activities for systems that will provide the specified level of military effectiveness at minimum system cost. Alternative strategies will of course result in varying levels of military value and so, perhaps, may alternative system policies, but there the variability of the system effectiveness must end if meaningful results are to be obtained from a system-design process based on a least-cost criterion.

MODEL OUTPUTS

The general outputs of the model are the specification of the optimal system and its cost, and the specification of a feasible use of the system in response to the requirements of each hypothetical contingency in the strategy to be implemented. Each class of output is discussed briefly in the following paragraphs and will be illustrated later in the paper.

Deployment System Specifications and Cost

The total system cost is determined by the model, where the search is for the least-cost combination of deployment means within the constraints and assumptions specified by the user. The system specification of the model is the complete list of components that make up the optimal system together with the quanity of each component. Included are the number of each type of transport vehicle to be retained or acquired and the location and type of each forward base, including the quantity of each class of equipment composing the total tonnage prestocked there.

In a typical solution it might be of interest to require retention of all components of an existing system and to augment this system with either more of the same components or with some new component or components. In another case elimination of any or all of the existing components may be permitted. Cost comparisons under both assumptions could be of considerable interest and could be obtained from solutions based on the two alternative deployment-system policies.

Generalized Area Deployment Plans

A secondary output of the model is a set of deployment plans, one for each contingency area. These plans are specified in terms of tonnage separated into the classes previously identified with respect to the lift capacity of the transport vehicles. The plan specifies, for each deployment, the tonnage moved or contributed by each system component together with the route over which the tonnage is moved. Implicit in the plan is the capability to assemble in the area the tonnage equivalent of the required forces in accordance with the force-closure schedule specified in the deployment strategy. A great deal of additional detailed planning would be required to convert these data into anything resembling a conventional movement plan. The generalized plans produced by the model as a part of the optimal solution could, however, provide the basis for such detailed planning. In this regard it should be noted that the tonnage plan merely indicates one way each deployment could be made. Usually the plan will not be unique. In areas in which one or more of the individual phase requirements is not limiting with respect to the overall system capacity, there exist alternative plans that could be accommodated by the system and that might prove superior from a tactical or other standpoint.

A SIMPLIFIED EXAMPLE

Before proceeding to the detailed formulation of the model, it may be profitable to examine a simplified example to illustrate some of the basic model features that have been described earlier in this section. A simple problem will be stated and formulated and the solution will be given. The example will illustrate the pitfalls of an approach that attempts to select a single worst case as a basis for designing a system or one that considers an unduly restricted set of possible solutions. The problem requires only six variables to represent system acquisition activities and deployment activities and eight relations to express the deployment strategy and the policy restrictions. In any reasonably comprehensive and realistic model, of course, the number of activities may number in the hundreds with a lesser but comparable number of relations.

Statement of the Problem

A small country, S, has important economic interests in and mutual defense agreements with two still smaller countries, S1 and S2. Countries S1 and S2 are remote from S but are relatively close to each other; S2 is closer to S than S1 is (see Fig. 3). Country S wishes to maintain a capability to deploy forces to either S1 or S2 because each is subject to the threat of invasion from neighboring countries. Simultaneous deployment to S1 and S2 is not considered

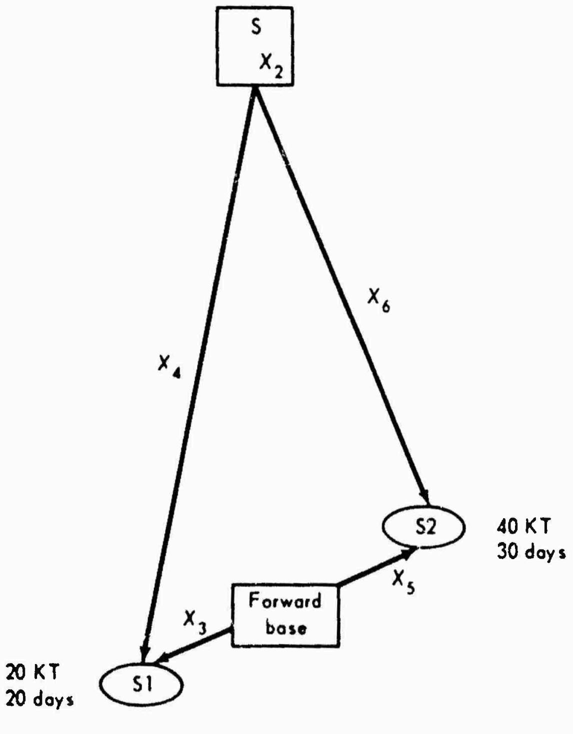


Fig. 3—Problem Diagram: Illustration of Multiple Strategic-Deployment Requirement for Country S

in this example. Military analysis of each area indicates that S1 can be effectively supported if a force whose weight is 20 KT can be deployed within a period of 20 days. A contingency in S2, on the other hand, will require 40 KT, but 30 days may be allowed for its deployment. In this example, Countries S1 and S2 represent the total strategic-deployment problem of Country S, and the schedules given above define the deployment strategy to be investigated.

Country S has on hand a fleet of 50 transport aircraft, which it received from a larger country under a military aid program. There is no investment cost for these aircraft, but the 10-yr operating cost of each is \$8 million. Country S also has the option to buy floating-depot ships and military equipment at a cost (investment + 10 yr of operation) equivalent to \$10 million per kiloton of equipment prestocked in such ships. The civil air fleet of country S has sufficient capacity to lift the required number of troops to S1 or S2.

The deployment strategy adopted by Country S sets the deployment objective as the capability to deploy 20 KT to S1 in 20 days or 40 KT to S2 in 30 days. The deployment system policy states that the allowable system components are existing transport aircraft operating from Country S and floating-depot ships that may be procured and operated from a base in the theater containing S1 and S2. Retirement of part or all the existing aircraft fleet is allowed if it should prove economical.

The problem is to define the least-cost deployment system, employing only aircraft and floating-depot ships, that can implement the selected deployment strategy.

Analytical Formulation

The following variables will be required:

X_1—Kilotons of equipment prestocked in floating-depot ships
X_2—Number of transport aircraft continued in operation
X_3—Kilotons of equipment delivered to S1 by floating-depot ship
X_4—Kilotons of equipment delivered to S1 by airlift
X_5—Kilotons of equipment delivered to S2 by floating-depot ship
X_6—Kilotons of equipment delivered to S2 by airlift

Note that the values of X_1 and X_2 will specify the composition of the deployment system. Values of X_3 and X_4 will give the deployment plan for responding to a contingency in S1 if a deployment is required. Values of X_5 and X_6 give the plan for S2.

The problem can now be formulated analytically in terms of the variables defined above:

$$X_i \geq 0 \qquad i = 1, 2, \ldots, 6$$

$$X_2 \leq 50 \tag{1}$$

$$X_3 + X_4 \geq 20 \tag{2}$$

$$X_1 - X_3 \geq 0 \tag{3}$$

$$X_2 - 2.5 X_4 \geq 0 \tag{4}$$

$$X_5 + X_6 \geq 40 \tag{5}$$

$$X_1 \qquad - X_5 \geq 0 \tag{6}$$

$$X_2 \qquad - X_6 \geq 0 \tag{7}$$

$$10X_1 + 8X_2 \qquad = \text{cost} \tag{8}$$

Relation 1 states that the number of aircraft in the deployment system cannot exceed the 50 on hand; no additional acquisition of aircraft is possible. Relation 2 states that the sum of the tonnages delivered by floating-depot ships and aircraft to S1 must equal the requirement there, which is 20 KT. Relation 3 requires that the tonnage to be delivered to S1 by ships (X_3) be no greater than the floating-depot tonnage available in the deployment system (X_1). Relation 4 requires that the number of aircraft utilized in the S1 deployment be no greater than the number of aircraft retained in the deployment system (X_2). The coefficient of X_4, 2.5, is in units of aircraft per kiloton and converts the tonnage airlifted (X_4) to the corresponding number of aircraft required. This coefficient is a function of the aircraft productivity over the route from S to S1 and the time allowed for the deployment.

Expressions 2 through 4 relate to a deployment to Country S1 in the event of a contingency there. Expressions 5 through 7 are similar and relate to a deployment to Country S2. Note, however, that in 7 the coefficient of X_6 is 1. The route from S to S2 is shorter than that from S to S1, and the deployment time is now 30 days rather than 20 days. These two factors combine to allow each aircraft to fly more sorties with a greater payload. The net result is that only one aircraft is required for each kiloton of equipment delivered by airlift to S2.

Finally, 8 expresses the 10-yr deployment-system cost for any values of X_1 and X_2 that specify the system composition. It is this function that is to be optimized (minimized) while satisfying relations 1 through 7. Any set of nonnegative values for the variables X_1 through X_6 that accomplishes this constitutes an optimal solution to the problem.

Solution and Discussion

The optimal solution to the problem is given below, with values rounded to the nearest integer:

X_1 = 7 KT (tonnage prestocked in floating-depot ships)
X_2 = 33 (number of aircraft maintained in active fleet)
X_3 = 7 KT (tonnage to S1 by floating-depot ship)
X_4 = 13 KT (tonnage to S1 by airlift)
X_5 = 7 KT (tonnage to S2 by floating-depot ship)
X_6 = 33 KT (tonnage to S2 by airlift)
\$334 million = 10-yr deployment system cost

The least-cost deployment-system design, as specified by the values of X_1 and X_2, is composed of a fleet of 33 aircraft and a sufficient number of floating-depot ships to prestock 7 KT of equipment in the theater containing countries S1 and S2. It will be economical to retire 17 aircraft from the original fleet of 50. The values of X_3 through X_6 give the tonnages to be moved by sea and air if a contingency arises in either S1 or S2 and constitute the deployment plans for each area.

The plan for Country S1 calls for the deployment of 7 KT by floating-depot ship and 13 KT by air for a total of 20 KT, as required. Since an average of 2.5 aircraft is required for each kiloton delivered to S1, the fleet of 33 aircraft will be adequate to perform its portion of the lift. In the plan for Country S2, 7 KT are again delivered by sea, but in this case the air fleet has the capability to deploy 33 KT for a total deployment of 40 KT.

It is useful to introduce at this point the term "slack variable" and to indicate the significance of "slack activities." Referring to inequality 1 of the simple problem formulation (see the previous section, "Analytical Formulation") it can be seen that this could be written in equation form as

$$X_2 + X_2' = 50$$

where X_2' is a positive variable introduced to force the equality. This variable is termed a "slack" variable, and in this case indicates the slack or nonactive portion of the aircraft fleet. The value of X_2' from the solution is 17, and is the nonactive portion of the fleet, which is retirable. Slack variables and their counterpart in deployment activities, slack activities, are often valuable in determining the utilization of the system and are generally a part of the computer output in analyzing a typical problem.

It has been shown that a mix of the available components produces an acceptable solution with a lower cost than a system having a single component. If the deployment system were composed of aircraft alone, the full fleet of 50 aircraft would be required. If only floating depot ships were used, a total of 40 KT would have to be prestocked in the S1–S2 theater. In either case the 10-yr deployment-system cost would be $400 million. The optimal system cost here is $334 million, which indicates that the mixed deployment system produces a cost saving of about 16 percent.

Just as no single class of components should be considered to the exclusion of others, no single requirement should be used as a basis for the system design. Suppose, for example, that the analysis had been based on the selection of area S2 as comprising the worst case in view of the magnitude of the tonnage requirement. A comparison of airlift and floating depots would have disclosed that 40 aircraft at a cost of $320 million could accomplish the deployment there, whereas 40 KT of floating-depot stocks would have been required at a cost of $400 million. The selection would be in favor of retaining just 40 aircraft as the preferred deployment system. This system would not be capable of meeting the deployment requirements in S1, however, since the 40 aircraft could deploy only 16 KT of equipment there in the specified 20 days. If, on the other hand, S1 had been selected as the worst case on the basis of being at the greater distance and having the more stringent time requirement, the preferred system would have been the prestockage of 20 KT in floating depots at a cost of $200 million. Fifty aircraft, costing $400 million, would otherwise be required. Again this system would be incapable of meeting the deployment requirement in S2, since the requirement there is 40 KT.

The solution to the example just cited would not pose a real difficulty. However, more complex situations may arise in which shortfalls and cost-reduction opportunities may not be readily apparent.

MODEL FORMULATION

INTRODUCTION

In this section sufficient detail is provided, using standard linear-programming terminology, to make explicit the general formulation of the model. The formulation technique and the terminology are taken from Dantzig.* The tableau form of the model will be given, except for the omission of the slack activities.

A linear-programming model is essentially a system of simultaneous linear inequalities and/or equalities. The model tableau sets out the coefficient matrix and the requirements vector together with an identification of the activity vectors and the items.

By way of review: the activities are the problem variables whose optimal levels are sought, and the items are the commodities that are produced or consumed by the program activities. Each relation (inequality or equality) in the system traces the flow of a single item into or from all the activities with which it is associated. The components of an activity vector identify every item that is produced or consumed by that activity. The tableau presents a compilation of the input and output coefficients that define the quantity of each item consumed or produced by an activity operated at the unit level. The exogenous flows are the components of the requirements vector that define the flow of items into and from the system of activities as a whole. The activity and item notation is alphanumeric and conforms to the standard format used in the SHARE LP codes for the solution of linear-programming problems.

In order to render the discussion explicit the formulation of a simplified truncated model will be described here; the general formulation will become evident in the process. A brief summary of the problem will be given, the structure of the tableau indicated, and a representative section of the model described in some detail. The remainder of the model is essentially a repetition of the same format with appropriate variations in the input-output coefficients. The example presented is hypothetical, and thus no significance is to be attached to numerical values used.

*Dantzig, George B., Linear Programming and Extensions, Princeton University Press, Princeton, N. J., 1963.

PROBLEM SUMMARY

The deployment strategy considered in this truncated example includes force-closure schedules for three contingency areas, two of which are in the same general theater and may be served by a single base. A schematic arrangement is shown in Fig. 4.

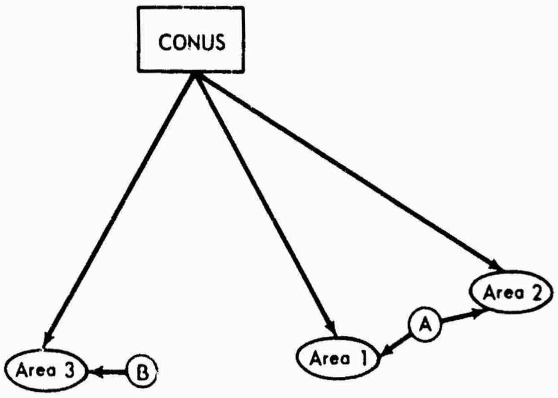


Fig. 4—Problem Diagram: Illustration of Hypothetical CONUS/ Multiple Strategic-Deployment Requirements

Force requirements are specified at three distinct points of time within the critical period in each area. Only two of the three time phases appear explicitly in the formulation since the third is included implicitly. The composition of the forces to be deployed is represented by segregation of the total tonnage into two classes with respect to its transportability by the vehicles to be considered in the deployment-system policies of interest.

The system components identified in the deployment-system policy are transport aircraft of various types, high-speed ships, forward floating depots, and sets of equipment prestocked in forward bases ashore. Only two base locations are permitted in the policy, one in each of two general theaters, and no intertheater transfer of prestocked equipment is allowed. This problem is abbreviated with respect to the scope of the deployment strategy formulated and with respect to the number of system acquisition and deployment options permitted. For example, both terminal-facility augmentation activities and terminal-capacity items have been omitted, and only a minimum of possible deployment modes are included in terms of routes and transfer options. These simplifications were introduced to facilitate the presentation of both the formulation and sample solutions. The basic pattern is repetitive, and nothing is gained by considering a more complete formulation, since specific elements to be included will depend on the special interests of the user.

TABLEAU STRUCTURE

Figure 5 indicates the basic structure of the model when presented in tableau form. As noted earlier, two distinct types of activities are included

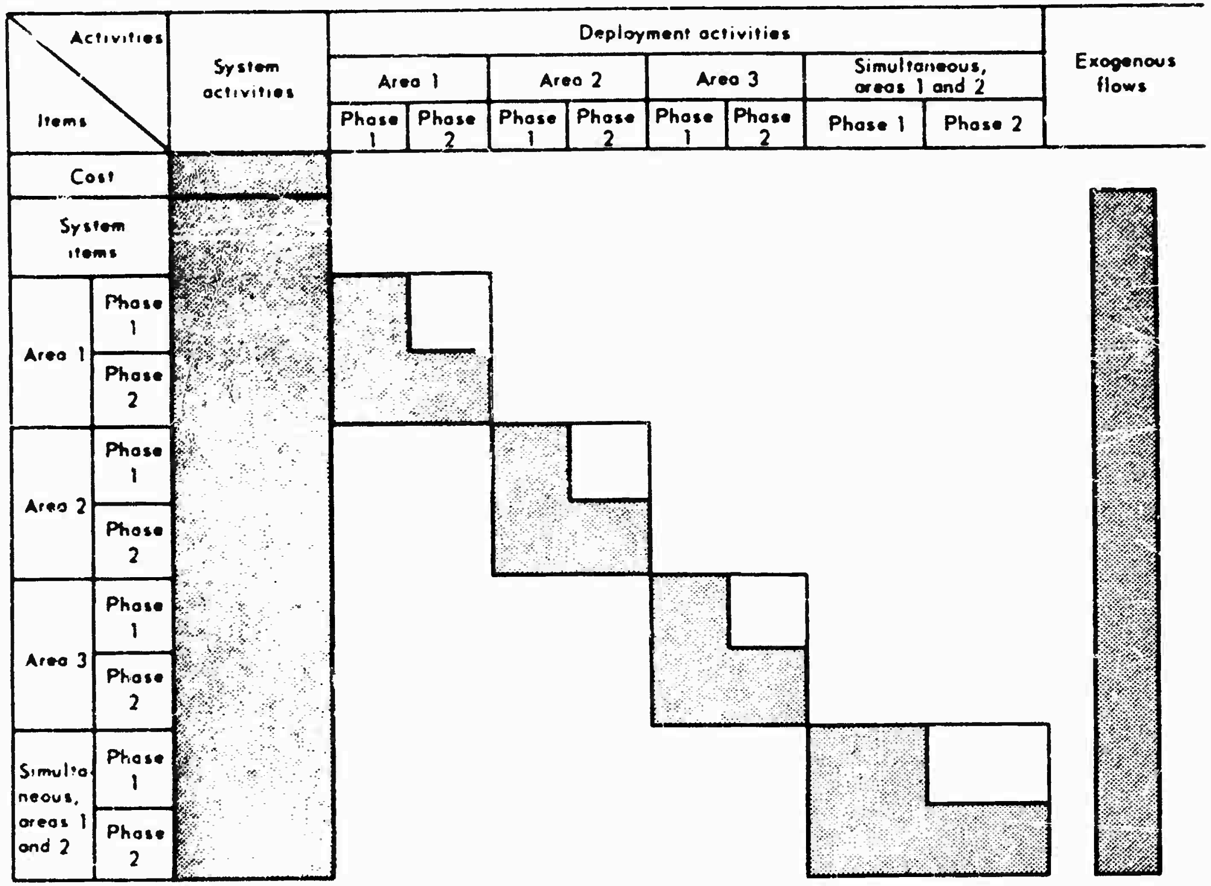


Fig. 5—Problem Structure in Tableau Form

in the model: system activities and deployment activities. The system activities refer to the actions necessary to the acquisition and maintenance of the physical-deployment system and are indicated at the left of the tableau. The deployment activities are the actions taken during a contingency response that collectively result in the deployment of forces in accordance with a schedule adequate to support the predetermined strategy in each area. Each large block along the tableau diagonal contains the coefficients relating the activities and items involved in the deployment of forces to a single area. Generally the system activities result in the provision of vehicles and equipment stocks to accomplish force deployment, whereas the deployment activities define the detailed utilization of these components in responding to the contingencies explicitly identified in the global deployment strategy. At the left of the tableau the item breakout is indicated. A set of general system items is included to permit the inclusion of constraints that apply to the deployment system as a whole. These items include inherited-component limits, component-production limits, and restrictions that specialize the available components to reflect alternative deployment-system policies. These restrictions or constraints are independent of the individual deployments to be considered. The remaining items are in groups corresponding to the deployment in which they are involved.

The shaded areas in the tableau denote the presence of input-output coefficients and indicate the interrelations among the activities and items. It can be seen that the system activities perform a linking function that ties together the otherwise independent deployment problems. The system activities define the resources that are available for use in making the individual deployments. The technique employed in the present formulation is to force the model to design its own least-cost system by identifying the system-component options available to it and stating the deployments that it must be capable of performing. The functions of deployment-system design and future-contingency planning are thus combined in a single problem. These activities obviously interact strongly, and both may be improved by considering them simultaneously.

As indicated in Fig. 5, each deployment problem is subdivided into two time phases with the activities and item identified accordingly. The shading indicates that these time phases are not independent within a deployment, because items consumed in the first-phase activities are not available in the second phase. In addition to time phases the activities within a deployment problem are differentiated with respect to transport vehicles, route utilized, and class of tonnage transported, although these are not shown in Fig. 5.

Another feature of the model indicated in the illustration of the tableau structure is the possibility of formulating the problem of multiple, simultaneous contingencies. The first three area problems are considered to involve contingencies that occur independently, and any forces and vehicles operating from a central reserve are fully available for the response to a single contingency. The question of what force deployments the system can produce in the event of simultaneous contingencies is also of interest. A capability, perhaps at a reduced level, for responding to simultaneous contingencies might well form an integral part of a comprehensive deployment strategy. The last rectangle in the diagram diagonal indicates the appearance of a subproblem in which simultaneous contingencies occur in area 1 and area 2 (Fig. 4). The formulation results essentially from the inclusion of the now simultaneous demands of the area 1 and area 2 activities on basically the same items. The same system components now must be allocated for use in two distinct deployment operations, and two distinct sets of deployment requirements must be met simultaneously. Again a simple case of simultaneous contingencies in the same general theater has been selected for the example, since a minimum of items is required. There is no restriction on the contingencies that may be considered to occur simultaneously.

Even if the strategy does not specify multiple contingencies, selected areas may be included as occurring simultaneously with the requirements set to zero during normal solution runs. These solutions will not be affected by this feature. The system capabilities may then be tested by the use of a parametric run in which the simultaneous-contingency requirements are increased until they equal some final values such as the requirements set in the original independent-contingency problem. The point at which the system cost begins to rise (the original single-contingency requirements remain at their nominal value throughout) indicates the extent of the deployment tonnage that can be supplied in the concurrently active areas by the original system that was designed on the basis of independent area requirements. Subsequent solutions at a higher level of simultaneous-deployment force requirements can be investigated to explore the cost implications of incorporating additional system capability.

The last topic to be discussed in connection with Fig. 5 is the objective or cost function, which accounts for the flow of dollars consumed by each system activity in the model. It is this function that is to be minimized while satisfying all the restrictions and constraints specified in the other item relations. The deployment activities are considered to have zero cost, as discussed previously.

Figure 6 is a reproduction of a tableau printout available from the LP code employed at RAC in the solution of linear-programming problems. It indicates in more detail the tableau structure described above. Because of reproduction limitations on legibility the simultaneous-contingency section of the model has been omitted. This section is shown in Fig. 7 in compressed form, including the system-activity interactions. These figures are presented here only to indicate the structural details already referred to. The next subsection defines the activity and item symbols that appear in these figures and describes the activities and item flows in somewhat more detail. After reading further, it may be profitable to return to these figures for a more comprehensive view of the overall model.

It should be recalled that the number of individual areas and activities that may be included has no inherent limitation. Of course, very real practical limitations in the size of the model to be employed result from the computer capacity available. It is considered that with the judicious selection of deployment strategies and representative contingency areas, 400- to 500-item relations will provide for the adequate representation of deployment-system design problems of interest. This is a reasonable limit, well within the capacity of readily available codes and computers.

TABLEAU DETAIL

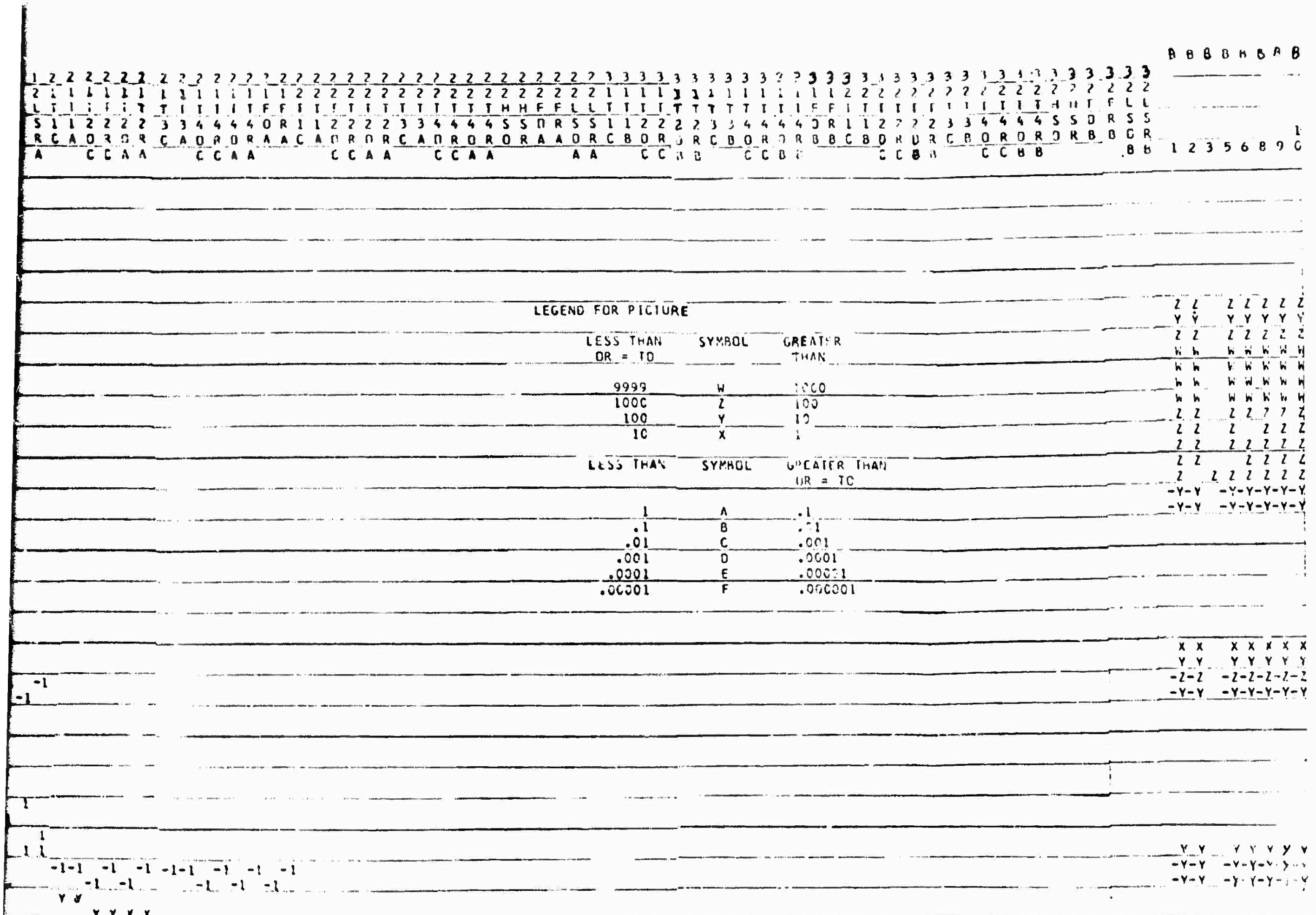
Figure 8 presents a detail of certain sections of the tableau-like printout shown in Fig. 6. To conserve space, this printout codes all numerical quantities other than ±1 by order of magnitude. The sign convention observed in the

```
                                           1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
         O R O R O B R B B F S F S F S F S 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 2 2
         T T T T T T T T H B B B B B B B B T T T T T T T T T T T T F F T T T T T T T T T T T T H H F F L L T T T T T T T T T T T T F F T T T T T T T T T T T T H
         1 1 2 2 3 3 3 4 S O O R R O O R R 1 1 2 2 2 2 3 3 4 4 4 4 O R 1 1 2 2 2 2 3 3 4 4 4 4 S S O R S S 1 1 2 2 2 2 3 3 4 4 4 4 O R 1 1 2 2 2 2 3 3 4 4 4 4 S
                           A A A A B B B B C A O R O R C A O R O R A A C A O R O R C A O R O R O R A A O R C A O R O R C A O R O R A A C A O R O R C A O R O R O
                                             C C A A     C C A A         C C A A     C C A A         A A     C C A A     C C A A         C C A A     C C A A

COST0    X   X   Y Y   Y Y Y X Y X Y X Y X
COST1    X X X X X Y X Y Y Y X Y X Y X Y X
COST2    Y X Y X Y Y X Y Y Y Y Y Y Y X Y X
COST3    Y X Y X Y Y X Y Y
COST4                      Y Y Y Y Y Y Y Y
COST5
COST6
COST7
T1L      1 1
T2L          1 1
T3L              1   1
+ BT3L             1
+ BT4L                 1
- ACAP   1   X   X X   Y
+ HSL                    1
+ FBL                      1   1   1   1
+ SBL                        1   1   1   1
+ OL                       1 1     1 1
+ RL                           1 1     1 1
+ PSL                      1 1 1 1 1 1 1 1
+11RR                                     -1-1  -1  -1-1-1  -1  -1  -1
+11OR                                         -1  -1      -1  -1  -1
+11T1   -1                                 Y Y
+11T2       -1                                 Y Y X X
+11T3           -1-1                                   Y X
+11T4                 -1                                   X X A A
+11FO                     -1                                       1
+11SO                       -1                     1           1
+11FR                         -1                                     1
+11SR                           -1           1       1   1       1
+11FOL                                                             1
+11FRL                                                               1
1TRR                                      -1-1  -1  -1-1-1  -1  -1  -1-1-1  -1  -1-1-1  -1  -1  -1  -1  -1
1TOR                                          -1  -1      -1  -1  -1      -1  -1      -1  -1  -1  -1  -1
+12T1   -1                                                             Y X
+12T2       -1                                                             Y Y X X
+12T3           -1-1                                                               Y X
+12T4                 -1                                                               X X A A
+12HS                   -1                                                                     A
+12FO                     -1                                       1                               1
+12SO                       -1                     1           1               1           1           1
+12FR                         -1                                     1                               1
+12SR                           -1           1       1   1       1       1       1   1                   1
+12LSC                                                            -A-A                        -A-A     1 1
+21RR                                                                                                     -1-1  -1  -1-1-1  -1  -1  -1
+21OR                                                                                                         -1  -1      -1  -1  -1
+21T1   -1                                                                                                 Y Y
+21T2       -1                                                                                                 Y Y X X
+21T3           -1-1                                                                                                   Y X
+21T4                 -1                                                                                                   X X A A
+21FO                     -1                                                                                                       1
+21SO                       -1                                                                                     1           1
+21FR                         -1                                                                                                     1
+21SR                           -1                                                                           1       1   1       1
+21FOL                                                                                                                             1
+21FRL                                                                                                                               1
2TRR                                                                                                      -1-1  -1  -1-1-1  -1  -1  -1-1-1  -1  -1-1-1  -1  -
2TOR                                                                                                          -1  -1      -1  -1  -1      -1  -1      -1  -1
+22T1   -1                                                                                                                             Y Y
+22T2       -1                                                                                                                             Y Y X X
+22T3           -1-1                                                                                                                               Y X
+22T4                 -1                                                                                                                               X X A
+22HS                   -1
+22FO                     -1                                                                                                       1
+22SO                       -1                                                                                     1           1               1           1
+22FR                         -1                                                                                                     1
+22SR                           -1                                                                           1       1   1       1       1       1   1
+22LSC
+31RR
+31OR
+31T1   -1
+31T2       -1
+31T3           -1-1
+31T4                 -1
+31FO                               -1
+31SO                                 -1
+31FR                                   -1
+31SR                                     -1
+31FOL
+31FRL
3TRR
3TOR
+32T1   -1
+32T2       -1
+32T3           -1-1
+32T4                 -1
+32HS                   -1
+32FO                               -1
+32SO                                 -1
+32FR                                   -1
+32SR                                     -1
+32LSC
```

Fig. 6—Computer Printout of Tableau for Single-Theater Contingencies

A



```
LEGEND FOR PICTURE

   LESS THAN      SYMBOL     GREATER
    OR = TO                   THAN

      9999           W        1000
      1000           Z        100
       100           Y        10
        10           X        1

   LESS THAN      SYMBOL     GREATER THAN
                               OR = TO

         1           A        .1
        .1           B        .01
       .01           C        .001
      .001           D        .0001
     .0001           E        .00001
    .00001           F        .000001
```

ut of Tableau for Single-Theater Contingencies in Sample Problem

3

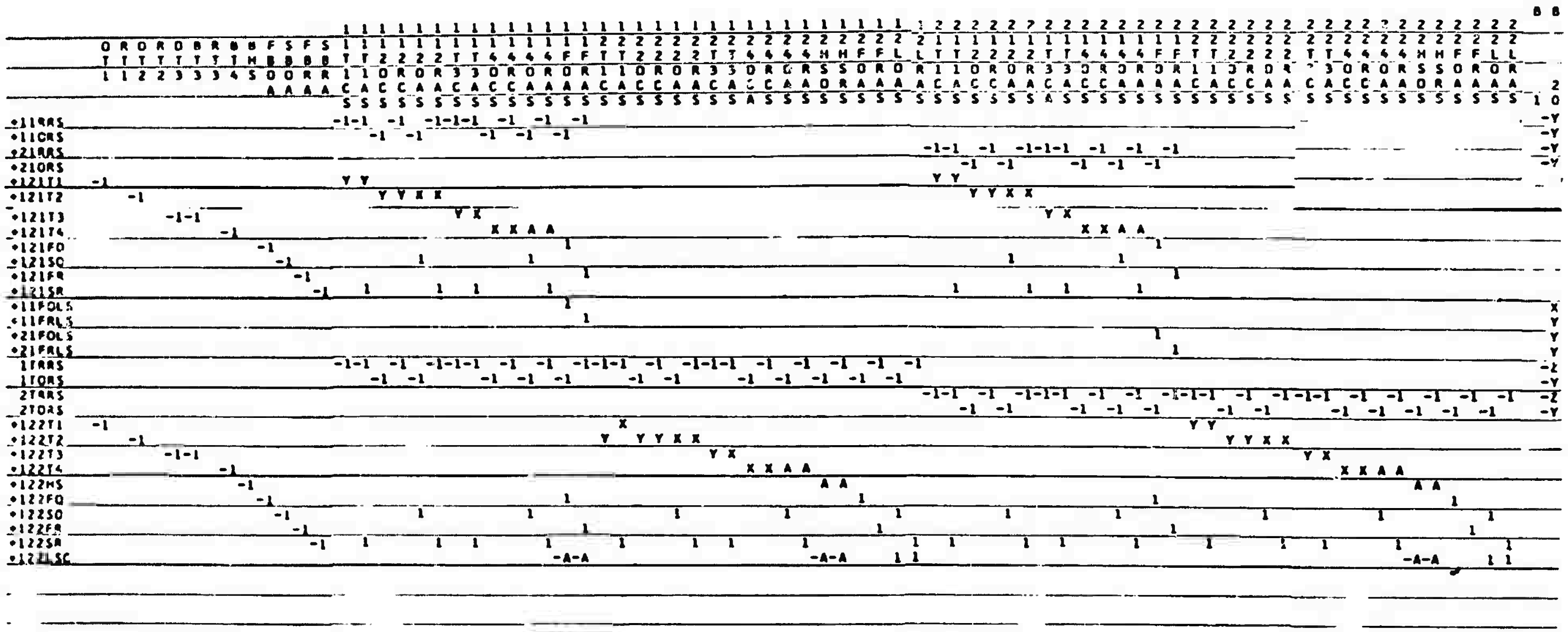
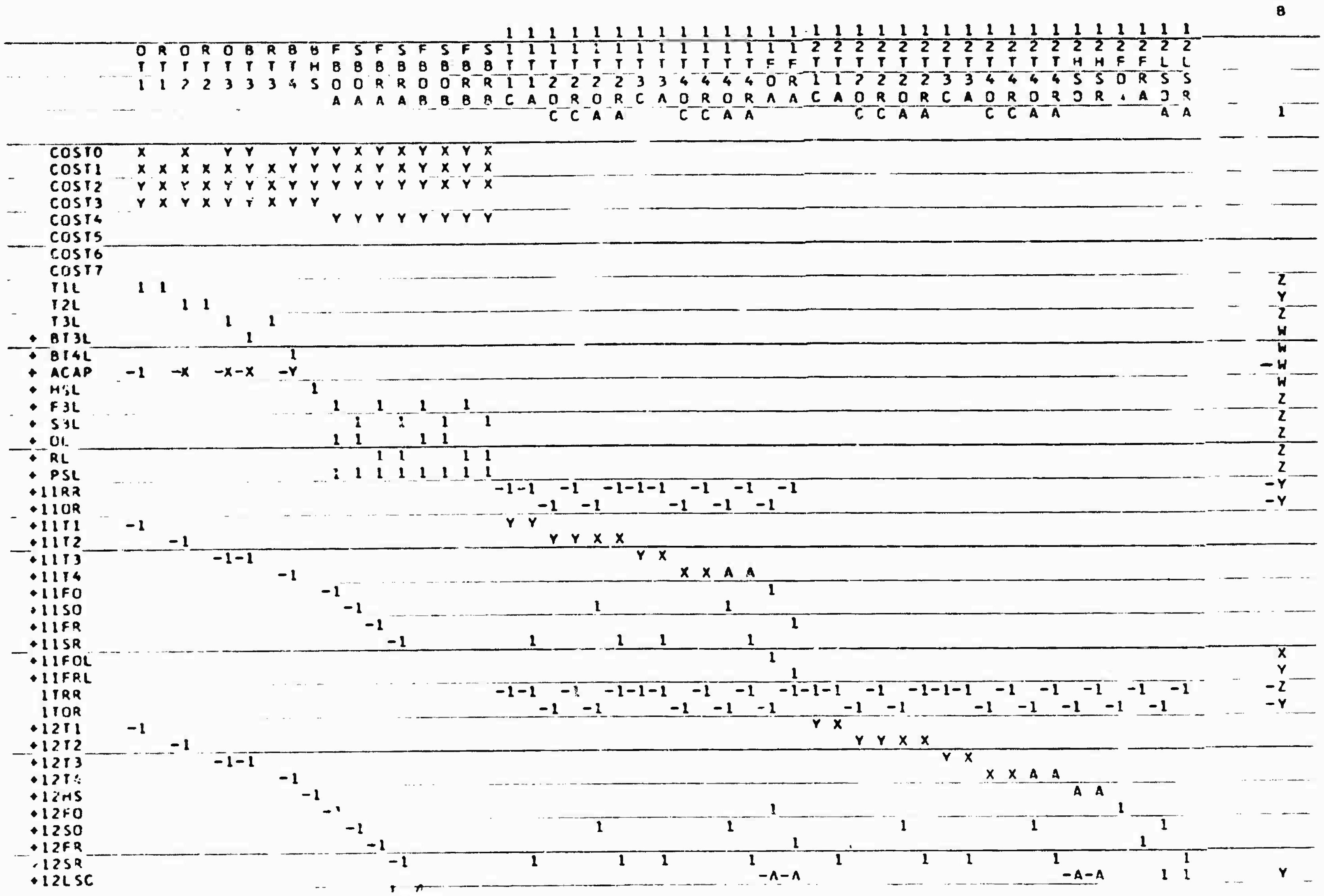


Fig. 7—Computer Printout of Tableau for Simultaneous Contingencies in Theater 1 and 2 in Sample Problem



```
                                               1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1       8
             O R O R O B R B B F S F S F S F S 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
             T T T T T T T T H B B B B B B B B T T T T T T T T T T T T F F T T T T T T T T T T T T H H F F L L
             1 1 2 2 3 3 3 4 S O O R R O O R R 1 1 2 2 2 2 3 3 4 4 4 4 O R 1 1 2 2 2 2 3 3 4 4 4 4 S S O R S S
                               A A A A B B B B C A O R O R C A O R O R A A C A O R O R C A O R O R O R . A O R
                                                   C C A A     C C A A         C C A A     C C A A         A A       1

  COST0      X   X   Y Y   Y Y Y X Y X Y X Y X
  COST1      X X X X X Y X Y Y Y X Y X Y X Y X
  COST2      Y X Y X Y Y X Y Y Y Y Y Y Y X Y X
  COST3      Y X Y X Y Y X Y Y
  COST4                        Y Y Y Y Y Y Y Y
  COST5
  COST6
  COST7
  T1L        1 1                                                                                                     Z
  T2L            1 1                                                                                                 Y
  T3L                1   1                                                                                           Z
◆ BT3L                 1                                                                                             W
◆ BT4L                     1                                                                                         W
◆ ACAP      -1  -X  -X-X  -Y                                                                                        -W
◆ HSL                        1                                                                                       W
◆ F3L                          1   1   1   1                                                                         Z
◆ S3L                            1   1   1   1                                                                       Z
◆ OL                           1 1     1 1                                                                           Z
◆ RL                               1 1     1 1                                                                       Z
◆ PSL                          1 1 1 1 1 1 1 1                                                                       Z
◆11RR                                         -1-1  -1  -1-1-1  -1  -1  -1                                          -Y
◆11OR                                             -1  -1      -1  -1  -1                                            -Y
◆11T1       -1                                 Y Y
◆11T2           -1                                 Y Y X X
◆11T3               -1-1                                   Y X
◆11T4                     -1                                   X X A A
◆11FO                         -1                                       1
◆11SO                           -1                         1           1
◆11FR                             -1                                     1
◆11SR                               -1           1       1   1       1
◆11FOL                                                                 1                                             X
◆11FRL                                                                   1                                           Y
  1TRR                                        -1-1  -1  -1-1-1  -1  -1  -1-1-1  -1  -1-1-1  -1  -1  -1  -1  -1      -Z
  1TOR                                            -1  -1      -1  -1  -1      -1  -1      -1  -1  -1  -1  -1        -Y
◆12T1       -1                                                             Y X
◆12T2           -1                                                             Y Y X X
◆12T3               -1-1                                                               Y X
◆12T4                     -1                                                               X X A A
◆12HS                       -1                                                                     A A
◆12FO                         -1                                       1                             1
◆12SO                           -1                         1           1               1           1           1
◆12FR                             -1                                     1                             1
◆12SR                               -1           1       1   1       1       1       1   1       1           1
◆12LSC                               '                              -A-A                        -A-A     1 1       Y
```

Fig. 8—Computer Printout of Tableau Detail for One Theater in Sample Problem

formulation represents all activity inputs as positive quantities, whereas all outputs are negative. In the notation, up to six characters are available for the designation of activities and the naming of items. At first glance this notation appears to be cumbersome, but it is actually quite convenient, since mnemonic codes may be utilized to permit immediate recognition of an activity or item in the solution of a problem. This is a considerable convenience since the number of activities and items is large, and use of mnemonic-code names eliminates the necessity of referring to a dictionary of variable definitions in the interpretation of results. The consistent use of a specific order of identification of activity characteristics further facilitates the reading of activity and item names.

Some of the general notational principles employed here will be mentioned before turning to the actual definition of the symbols. All system activities have the first space blank in their symbols as do the item names, which refer to the system in general. Deployment activities always begin with a two-digit notation. The first digit indicates the area and the second digit the time phase within the overall critical period in which the activity occurs. Subsequent symbols identify the vehicle involved, the class of tonnage carried, and the route origin, in that order. Deployment item names follow a similar pattern with respect to the area and time-phase designation.

Notation

System Activities. The system activities included in this example are listed across the top and at the left of Fig. 8, and each is read from top to bottom as indicated in the definitions given below:

OT1	operate and maintain transport aircraft T1 over the costing period
RT1	retire T1 aircraft
OT2	operate and maintain transport aircraft T2
RT2	retire T2 aircraft
OT3	operate and maintain T3 aircraft
BT3	procure, operate, and maintain T3 aircraft in addition to those in the inventory
RT3	retire T3 aircraft
BT4	develop, procure, operate, and maintain T4 aircraft
BHS	develop, procure, operate, and maintain specialized high-speed ships
FBOA	establish a floating-base depot stocked with class O tonnage in base location A
SBOA	establish a shore base stocked with class O tonnage in location A
FBRA	establish a floating depot stocked with class R tonnage in location A
SBRA	establish a shore base stocked with class R tonnage in location A
FBOB, etc.	identical with above activities except in base location B

Transport aircraft T1 and T2 are in the inventory at the beginning of the period but are no longer in production, so that additional aircraft of these types are not considered available. Any quantity up to the limit of availability may be retained in the system; any not retained may be retired from service. T3 aircraft are still in production, so that three options are available with respect to these aircraft. Those in the inventory and committed to production may be retained or retired, and additional aircraft of this type may be procured. Aircraft T4 is in the concept-design stage and may be developed and procured if such action proves economically desirable. A similar situation exists with respect to the high-speed ships. In both these cases the cost of RDTE and procurement must be included in addition to O&M costs. The BT3 cost includes procurement plus O&M costs, since these are above the initial committed procurement level. The floating- and shore-base options are each divided into two distinct activities on the basis of the class of tonnage stocked. There would be, in reality, only a single floating-base group, and each floating-depot ship would combine both classes of tonnage, since tactical loading would probably be employed. A similar situation applies to shore bases. The distinction is made only to ensure equipment availability in the proper proportions to outfit the forces to be deployed. Bases in locations A and B are of course physically distinct since they are located in widely separated theaters.

A more general problem formulation would include other system options than those in the present example. Activities covering terminal facility augmentation in each area could be included for forward and rear airfields, ports, and surface LOC facilities in each area. Other base locations and vehicle options would also be appropriate. Only a sufficient number of system activities is included in the example to illustrate the form that a more comprehensive model would take.

Costs and System Items. Before continuing with the definition of the deployment activities, it is appropriate here to define the system items that refer to the system as a whole. These system items are listed in the left-hand column of Fig. 8 and are defined below:

T1L	limit on T1 aircraft available from the current inventory
T2L	limit on T2 aircraft available from the current inventory
T3L	limit on number of T3 aircraft in inventory or previously committed to production
BT3L	production limit on number of T3 aircraft produced by system completion date
BT4L	production limit on T4 aircraft produced by system completion date
ACAP	airlift capacity required to be maintained in any system configuration
HSL	production limit on high-speed ships by system completion date
FBL	limit on tonnage prestocked in forward floating bases
SBL	limit on tonnage prestocked ashore
OL	limit on class O tonnage prestocked
RL	limit on class R tonnage prestocked
PSL	limit on total tonnage prestocked, independent of class

Multiple cost functions shown on the first eight rows of Fig. 8 are included to permit the convenient testing of alternative costing approaches in a single computer run. Special cost functions are also required in setting up for a sequence of solutions based on the continuous variation of selected-cost parameters. The aircraft limits are self-explanatory; the item labeled ACAP is a general airlift-capacity requirement that specifies the lower limit of general airlift capacity to be permitted in the system policy. This level of capacity may be set to any value desired through the appropriate adjustment of the exogenous flow of this item. The limits on the type of forward base, the types of tonnage stocked, and the total prestockage limit are incorporated to allow investigation of alternative deployment-system policies. For example, it might be desired to examine the effect of eliminating prestockage as an option or of restricting the amount or types of tonnage prestocked. Similarly the type of base permitted may be restricted to floating depots or to shore-based installations exclusively. In general the options that are available in the system-design process may be defined and appropriately restricted by specifying that the flow of these system items should reflect the intent of a specific system policy. Other system items of a similar nature may be introduced to define additional or more detailed overall system limitations or requirements.

Deployment Activities. The deployment activity notation shown at the top of Fig. 8 is largely self-explanatory once certain symbols and the order of their appearance are understood. The basic symbols other than the aircraft designations are as follows:

C	route origin is the CONUS
A	route origin is base location A
B	route origin is location B
O,R	tonnage class (outsize or regular)
F	floating depot
HS	high-speed ship
LS	local shipping

The first digit in the activity name indicates the deployment area and the second indicates the time phase within the critical period. The next symbol indicates the vehicle or means of delivery, and the final symbol defines the origin of the route over which the equipment is transported. A few examples will make the notation clear.

The activity name 11T1C indicates the delivery of tonnage to area 1 in the first time phase by T1-type aircraft over the route originating in the CONUS. There is no designation of the tonnage class since it is understood that T1 aircraft are capable of transporting class R tonnage only.

Aircraft T2, on the other hand, can accommodate either class of tonnage, and the aircraft's usage must be differentiated in this respect. For example, activity 12T2OA is the shipment of class O tonnage from base location A to area 1 during phase 2 by means of T2-type aircraft.

No route origin appears in the designation of high-speed ship activities, because it is understood that they originate from the CONUS. This restriction is peculiar to this example and is not inherent in the concept. Note that in this example there is no HS activity in phase 1 because phase 2 is defined as beginning with the initiation of high-speed ship service.

As a final example the last activity listed in Fig. 8 calls for the delivery of class R tonnage to area 1 during phase 2 by means of local shipping commandeered in the area of base location A. It is understood that the equipment delivered by this activity was drawn from the shore-based prestocked equipment at base A. Similar sets of activities (not shown in Fig. 8) are identified for the remaining areas explicitly represented in the model. They differ with respect to the area and phase designations and will be easily recognized in Fig. 6.

Deployment Items. There are two basic types of deployment items, viz, the tonnage of each class that is to be deployed and the resources that are utilized in deploying that tonnage. With a few exceptions the item names should be self-explantory in view of the preceding discussion. The two items at the beginning of each phase group indicate the tonnage required in that period by class. Thus item 11RR traces the deployment of class R tonnage during the first phase in area 1 and 11OR does the same for class O. The relative proportion of each tonnage class reflects the composition of the forces to be deployed in each phase of the critical period. A slight departure is made with respect to specifying the second-phase tonnage. Rather than identify this explicitly, i.e., 12RR and 12OR, the total tonnage requirement for the combined phase 1 and phase 2 periods for each class is designated by 1TRR and 1TOR. This specific formulation is employed in order to permit the deployment schedule to exceed the closure schedule for the first time phase if such an option would result in system economy. The remaining items of the tableau keep track similarly of the transport vehicles and base stocks that are utilized in the deployment activities.

Special mention is required for the items labeled 11FOL and 11FRL. These items produce the equivalent of a third subinterval in the overall critical period. Equipment delivered by means of floating-depot ships will be delayed relative to aircraft deliveries in the first phase as explicitly represented in the model formulation. If a portion of the requirement in the first phase must actually be closed prior to the arrival of floating-depot equipment, the burden of that deployment falls on the aircraft in the system. The total that may be deployed by floating depots in phase 1 is limited to an amount less than the total for phase 1, and this limit must reflect the composition of the forces involved. This restriction is enforced by limits on the quantity of tonnage that may be deployed by floating-depot ships in the initial time phase in each area. One final example of the basic resource items will complete the discussion of notation. Item 12SO concerns the flow of class O tonnage prestocked at the forward base (understood to be location A, which serves area 1) during phase 2 of the area 1 deployment.

Input-Output Coefficients

The coefficients shown or indicated by a letter code in Fig. 8 may be interpreted as the quantity of each item either produced or consumed by the operation of each activity at the unit level. Thus a single column in the tableau shows all the interactions of a specific activity at the top of the tableau with each associated item in the left-hand column of the tableau. The absence of an interaction is indicated by a blank (zero) at the intersection of the activity column and the item row. As stated earlier, the sign convention represents all activity outputs as negative quantities and all inputs as positive. A few examples will indicate the logic of the formulation. The aircraft made available by the corresponding system activities may be used during each time phase in each area, i.e., throughout each deployment critical period. For example, the operation and maintenance of one T1 aircraft during the system costing period (the units of this activity are numbers of aircraft) consumes X or Y dollars (depending on the cost function used), consumes one aircraft from the available inventory (T1L), contributes one unit of ton-mile capability to the system airlift capability (ACAP), and finally, produces one aircraft that is available for use in each phase of each deployment operation (e.g., 11T1, 12T1, 21T1).

Activity BHS at the unit level (one ship) consumes Y dollars, uses one unit of production capacity (HSL) and produces one ship available for use in the second phase of each area deployment (e.g., 12HS, 22HS).*

The prestockage of 1 KT of class R tonnage in the shore base at location A (SBRA) consumes X or Y dollars, one unit of the shore-based tonnage limit (SBL), one unit of the class R tonnage limit (RL), and one unit of the total prestockage limit (PSL); and produces 1 KT of class R tonnage available at location A for use in either phase 1 or phase 2 of the deployments to area 1 or area 2 (11SR or 12SR and 21SR or 22SR). This activity produces an output available only in area 1 and area 2 deployments, since intertheater transfer of prestocked equipment is not permitted in the system policy represented in this example.

The bookkeeping associated with the deployment activities ensures that a kiloton drawn from the base in phase 1 of a deployment operation is also withdrawn from the stock considered available in the second phase. Thus the system prestockage activity never gets double credit for a unit of tonnage prestocked in a forward base and used in deployments to the same area.

Turning next to a few deployment-activity examples, consider first the activity designated 11T20A. This activity consumes no dollars but produces 1 KT of class O tonnage in the area 1 objective area in phase 1 (11OR), and also contributes thereby 1 KT to the total requirement for class O tonnage (1TOR). The unit level of every deployment activity is 1 KT, since this simplifies the bookkeeping process with respect to deployment requirements. The items consumed by the operation of 11T20A at unit level are X aircraft from those available (11T2), 1 KT of shore-based class O tonnage available for phase 1 use (11SO), and also 1 KT from that available for later use in phase 2 (12SO). This somewhat tortured formulation ensures that the phase 1 activities get the first choice of any equipment that is prestocked. This activity, 11T20A, is but one of four distinct modes of employment possible for the T2 aircraft in each time phase. Each of these modes requires the designation of a separate activity to define the utilization of the aircraft that have been made available by a single system activity (OT2). The remainder of the aircraft activities are similar.

The activities in which floating-depot ships and high-speed ships are employed also take account of possible second trips within the critical period for these vehicles operating in a shuttle mode from the forward base. Some fraction of these vehicles may have time for such a trip, and, when they do, they become equivalent to local shipping. Activity 11FRA illustrates this feature. At the unit level this activity produces 1 KT of class R tonnage during phase 1 and contributes a like amount to the total R requirement, but in addition it produces A kilotons of local shipping capacity that may be utilized in phase 2. The items consumed by this activity are also indicated in and may be read from the figure.

Item Flows

The input-output coefficients were described above as organized by activity. They may also be interpreted as tracing the flow of the items into and

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*For illustrative purposes, only the area 1 portion of the tableau is given in Fig. 8. Entries for areas 2 and 3 are analogous to those for area 1.

from the activities with which the items interact. This interpretation may be used to write the model in the relation form (equalities and inequalities). The form of each relation is indicated to the left of the item name. A blank indicates an equality relation, a "+" indicates a "≤" relation, and a "−" designates a "≥" relation. The constant term in each of these relations (one corresponding to each item) is given in the column headed B at the extreme right of Fig. 8. These are the exogenous item flows that are produced or consumed by the system of activities as a whole and are external to the system. The tonnage delivered is subtracted from the existing requirements, whereas the existing resources are available to be consumed by the system. Blanks in the exogenous flow column indicate that the item in question is internal to the model. An example of this situation is found in the flow of item 11T3, for example. This is an internal bookkeeping item and ensures that the number of aircraft utilized in phase 1 of the area 1 deployment does not exceed the number that has been made available through the total of activities OT3 and BT3.

The phase 1 deployment-requirement-item flows, 11RR and 11OR, indicate that the negative of the total contributions from the activities involved must be less than or equal to the negative of the phase 1 tonnage requirements in area 1. This is the equivalent of requiring that the activity contributions equal or exceed this requirement. The negative equality form of the relation results from the input-output sign convention. Note that, in the formulation used, equality is demanded for the total requirements of class O and R tonnages, 1TRR and 1TOR, but a less-than-or-equal-to relation may also be used if desired.

SAMPLE SOLUTIONS AND SENSITIVITY ANALYSIS

Two sample solutions to the formulated model are given here together with a discussion of sensitivity analysis and parametric programming as applied to the results. Since this problem was formulated in terms of fictitious components and data, the interpretations given are illustrative only. This problem was constructed to test the feasibility of the formulation and is used here only to indicate the kinds of information to be derived from such a model when operating on real data.

The strategy adopted and the composition of the forces involved are represented in the model by the amounts of class R and O tonnages specified in the area-requirement items for each deployment phase, and the system policy is implicit in the basic formulation. The basic solution printout gives the system cost and the levels of each of the activities in the optimal program for acquiring and utilizing the deployment system corresponding to the given strategy and policy.

BASIC SOLUTIONS

Figure 9 is a reproduction of a solution to an example based on the formulation discussed in the previous section. At the top of the printout the system cost is given opposite the item name, COST2, which was the specific cost function used in obtaining the solution shown. The column headed "J(H)" identifies the activities in the optimal solution, and the second column, "BETA(H)," specifies the activity level. The system activities are presented first and define the composition of the optimal system. This is the output of the system-design portion of the model. In this solution the optimal system contains 160 T3 and 110 T4 aircraft (after rounding), 38 high-speed ships, and various tonnages of equipment prestocked ashore in forward bases. All of the T1 and T2 aircraft have been retired, and the model has rejected the use of forward floating depots in designing the least-cost system. These were allowable system components under the deployment-system policy formulated, but their cost has prevented their inclusion in the optimal system.

The remainder of the activity printout specifies a set of feasible deployment plans for each area, broken down by time phase within the critical periods. In this solution all phase 1 deployments are accomplished by air, with a considerable proportion of the tonnage originating from the forward-base stocks. The shore-based stocks have been allocated among the two base locations as indicated by the system activities, and these stocks are exhausted during the

```
TOTAL  NO. ETA  ROW      CURRENT  CHOSEN    VECTR   RHS C/V  CURRENT O/J
ITERS ETAS REC IDENT.     VALUE   VECTOR    REMVD   NO. NO.   THETA/PHI  UNSCRAMBLED SOL'N.
  364  115   C  COST2 -13484.958                         8        • • • •

  J(H)           BETA(H)        ROW(I)         PI(I)               B(I)
0 00000    10C78.79646555-      COST0           .                   .
0 00000     9C00.12418492-      COST1           .                   .
0 00000    13484.95792392-      COST2       1.00000000              .
0 00000    12193.50617008-      COST3           .                   .
0 00000     2538.29371711-      COST4           .                   .
0 00000           .             COST5           .                   .
0 00000           .             COST6           .                   .
0 00000           .             COST7           .                   .
   RT1         5C0.00000000      T1L        3.50000000-      500.00000000
   RT2          50.00000000      T2L        5.00000000-       50.00000000
   OT3         16C.00000000      T3L        6.71246220-      160.00000000
   BT4         1C9.72852994    + BT3L           .            9000.00000000
   BHS          38.46936015    + BT4L           .            9000.00000000
   FBOA           .            - ACAP           .            1400.00000000
   SBOA           .73577539    + HSL            .            9000.00000000
   SBRA         72.31317746    + FBL            .             900.00000000
   FBOB           .            + SBL            .             900.00000000
   SBOB          9.64945985    + OL             .             900.00000000
   SBRB         67.30158730    + RL             .             900.00000000
  11T1C           .            + PSL       31.50513918        150.00000000
  11T2RC          .            +11RR            .              70.00000000-
  11T3C          9.75609756    +11OR            .              20.00000000-
  11T4OC        19.26422461    +11T1            .                   .
  11T4OA          .73577539    +11T2            .                   .
  11T4RA        72.31317746    +11T3            .                   .
  12T1C           .            +11T4            .                   .
  12T2RC          .            +11FO       10.91558784              .
  12T3C         10.00000000    +11SO            .                   .
  12T4RC        26.12584046    +11FR            .                   .
  12HSO         50.00000000    +11SR            .                   .
  12HSR         91.80488452    +11FOL           .              10.00000000
  12FOA           .            +11FRL           .              35.00000000
  12FRA           .             1TRR            .             210.00000000-
  12LSOA          .             1TOR            .              70.00000000-
  21T1A           .            +12T1            .                   .
  21T2OA          .            +12T2            .                   .
  21T3A         56.33802817    +12T3            .                   .
  21T4OC        29.26422461    +12T4            .                   .
  21T4RC         2.68682254    +12HS            .                   .
  21T4OA          .73577539    +12FO            .                   .
  21T4RA        15.97514929    +12SO            .                   .
  21FOA           .            +12FR            .                   .
  21FRA           .            +12SR            .                   .
  22T1C           .            +12LSC           .              30.00000000
  22T2OC          .            +21RR       30.58955134         75.00000000-
  22T3C         10.81081081    +21OR       30.58955134         30.00000000-
  22T4RC        30.31174860    +21T1         .82413970              .
  22HSO         55.00000000    +21T2        1.86770003              .
  22HSR         98.87744059    +21T3        3.65648316              .
  22FOA           .            +21T4       16.74905188              .
  22FRA           .            +21FO       30.58955134              .
  22LSOA          .            +21SO       20.20513918              .
  31T1B           .            +21FR       30.58955134              .
  31T2RB          .            +21SR            .                   .
  31T3C         12.69841270    +21FOL           .              15.00000000
  31T4OC        25.35054C15    +21FRL           .              38.00000000
  31T4OB         9.64945985     2TRR       21.50000000        215.00000000-
  31T4RB        67.30158730     2TOR       21.50000000         85.00000000-
  31FRB           .            +22T1         .32379518              .
  32T1C           .            +22T2         .83984375              .
  32T2RC          .            +22T3        1.45270270              .
  32T3C         13.22314050    +22T4        5.93922652              .
  32T4RC        35.62614608    +22HS       86.00000000              .
  32HSO         60.00000000    +22FO       21.50000000              .
  32HSR         86.15071342    +22SO       21.50000000              .
  32FOB           .            +22FR       21.50000000              .
  32FRB           .            +22SR       41.70513918              .
  32LSOB          .            +22LSC           .              30.00000000
+ BT3L        9C00.00000000    +31RR       47.60723441         80.00000000-
+ BT4L        8890.27147C06    +31OR       47.60723441         35.00000000-
- ACAP        1571.75618853    +31T1         .83352734              .
+ HSL         8961.53063985    +31T2        1.89917621              .
+ FBL          5C0.00000000    +31T3        3.77835194              .
+ SBL          750.00000000    +31T4       17.31172160              .
+ OL           889.61476476    +31FO       53.00513918              .
+ RL           760.38523524    +31SO       38.60513918              .
+11RR           12.06921502    +31FR       47.60723441              .
+11FR             .            +31SR            .                   .
+11SR             .            +31FOL           .              20.00000000
+11FOL          10.00000000    +31FRL           .              40.00000000
+11FRL          35.0000C000     3TRR            .             215.00000000-
+12HS            3.01813902     3TOR            .              95.00000000-
+12LSC          72.54146535    +32T1            .                   .
+21SR             .            +32T2            .                   .
+21FOL          15.0000C000    +32T3            .                   .
+21FRL          38.00000000    +32T4            .                   .
+22LSC          53.08162409    +32HS            .                   .
+31SR             .            +32FO            .                   .
+31FOL          2C.0000C000    +32SO            .                   .
+31FRL          40.00000000    +32FR            .                   .
+32HS            1.03168179    +32SR       38.60513918              .
+32LSC          63.84521402    +32LSC           .              20.00000000
```

Fig. 9—Solution Printout of Sample Problem with Prestockage Limited to 150 KT (PSL ≤ 150)

```
TOTAL  NO. ETA  ROW       CURRENT  CHOSEN    VECTR    RHS C/V  CURRENT D/J
ITERS ETAS REC IDENT.      VALUE   VECTOR    REMVD    NO. NO.   THETA/PHI  UNSCRAMBLED SOL*N.
  381  132   C  COST2 -11147.579                        9        . . . .

   J(H)          BETA(H)          ROW(I)          PI(I)              B(I)
0 00000      7829.40752451-       COST0            .                  .
0 00000      8410.29454200-       COST1            .                  .
0 00000     11147.57864905-       COST2       1.00000000              .
0 00000      8558.70841[illegible]-       COST3            .                  .
0 00000      5088.13592814-       COST4            .                  .
0 00000            .              COST5            .                  .
0 00000            .              COST6            .                  .
0 00000            .              COST7            .                  .
   RT1        500.00000000        T1L         3.50000000-     500.00000000
   PT2         50.00000000        T2L         5.00000000-      50.00000000
   CT3        160.00000000        T3L         6.68450153-     160.00000000
   BT4         30.17042253     +  BT3L             .         9000.00000000
   BHS         33.20804091     +  BT4L             .         9000.00000000
   FBOA             .          -  ACAP             .         1400.00000000
   SBOA        30.00000000     +  HSL              .         9000.00000000
   SBRA       118.02265469     +  FBL              .          900.00000000
   FBOB             .          +  SBL              .          900.00000000
   SBOB        35.00000000     +  DL               .          900.00000000
   SBRB       116.97734531     +  RL               .          900.00000000
  11T1A             .          +  PSL         2.10000000      300.00000000
  11T2RC            .          +11RR               .           70.00000000-
  11T3A        68.37606838     +11OR               .           20.00000000-
  11T4OC        5.86619901     +11T1               .                .
  11T4OA       14.13380099     +11T2               .                .
  11T4RA        1.62393162     +11T3               .                .
  11FRA             .          +11T4               .                .
  12T1A             .          +11FO        3.06619463              .
  12T2RC            .          +11SO               .                .
  12T3C         3.39688498     +11FR               .                .
  12T3A        48.02265469     +11SR               .                .
  12T4RC        7.18343394     +11FOL              .           10.00000000
  12HSO        34.13380099     +11FRL              .           35.00000000
  12HSR        81.39702639      1TRR               .          210.00000000-
  12FOA             .           1TOR               .           70.00000000-
  12LSCA       15.86619901     +12T1               .                .
  21T1A             .          +12T2               .                .
  21T2OA            .          +12T3               .                .
  21T3A        56.33802817     +12T4               .                .
  21T4OA       30.00000000     +12HS               .                .
  21T4RA       18.66197183     +12FO               .                .
  21FOA             .          +12SO               .                .
  21FRA             .          +12FR               .                .
  22T1C             .          +12SR               .                .
  22T2OC            .          +12LSC              .           30.00000000
  22T3C        10.81081081     +21RR       18.23380537         75.00000000-
  22T4RC        8.33437087     +21OR       15.23380537         30.00000000-
  22HSC        55.00000000     +21T1        1.44712741              .
  22HSR        77.83216363     +21T2        3.27946140              .
  22FOA             .          +21T3        6.42035400              .
  22FRA             .          +21T4       29.40936350              .
  22LSCA            .          +21FO       18.23380537              .
  22LSRA       43.02265469     +21SO               .                .
  31T1B             .          +21FR       18.23380537              .
  31T2RB            .          +21SR               .                .
  31T3C         2.17175883     +21FOL              .           15.00000000
  31T3B        54.80819783     +21FRL              .           38.00000000
  31T4OB       35.00000000      2TRR       12.30000000        215.00000000-
  31T4RB       23.02004334      2TOR       12.30000000         85.00000000-
  31FRB             .          +22T1         .18524096              .
  32T1C             .          +22T2         .48046875              .
  32T2RC            .          +22T3         .83108108              .
  32T3C        13.22314050     +22T4        3.39779006              .
  32T4RC        9.79559173     +22HS       49.20000000              .
  32HSO        60.00000000     +22FO       12.30000000              .
  32HSR        72.83216363     +22SO       12.30000000              .
  32FOB             .          +22FR       12.30000000              .
  32FRB             .          +22SR       12.30000000              .
  32LSOB            .          +22LSC              .           30.00000000
  32LSRB       39.14910414     +31RR        2.18703340         80.00000000-
+ BT3L       9000.00000000     +31OR        2.18703340         35.00000000-
+ BT4L       8569.82957747     +31T1         .20250309              .
- ACAP        141.91971830     +31T2         .46139945              .
+ HSL        8566.79195909     +31T3         .90373281              .
+ FBL         500.00000000     +31T4        4.20583346              .
+ SBL         800.00000000     +31FO       14.40000000              .
+ DL          835.00000000     +31SO               .                .
+ RL          865.00000000     +31FR        2.18703340              .
+11SC          15.86619901     +31SR               .                .
+11FR               .          +31FOL              .           20.00000000
+11SR          48.02265469     +31FRL              .           40.00000000
+11FOL         10.00000000      3TRR        9.20000000        215.00000000-
+11FRL         35.00000000      3TOR        9.20000000         95.00000000-
+12HS           4.32533406     +32T1         .15181518              .
+12LSC         48.79304921     +32T2         .40174672              .
+21SR          43.02265469     +32T3         .76033058              .
+21FOL         15.00000000     +32T4        2.98701299              .
+21FRL         38.00000000     +32HS       36.80000000              .
+22LSC          6.90216985     +32FO        9.20000000              .
+31SR          39.14910414     +32SO        9.20000000              .
+31FOL         20.00000000     +32FR        9.20000000              .
+31FRL         40.00000000     +32SR        9.20000000
+32LSC         20.70054495     +32LSC              .           20.00000000
```

Fig. 10—Solution Printout of Sample Problem with Prestockage Limited to 300 KT (PSL ≤ 300)

phase 1 deployment operations. In phase 2 of each deployment the bulk of the tonnage requirement is moved by high-speed ships with the remainder shipped from the CONUS by air.

Returning to the remaining columns in the basic solution printout, the center column, "Row(I)," is a listing of the item names used in the original formulation. These item names are required for the interpretation of the next columns. The column "PI(I)" contains the shadow prices or marginal values (the latter term will be used here) of corresponding items in column "Row (I)." Their use will be discussed later in the general consideration of sensitivity analysis. The final column, "B(I)," is simply a compilation of the external constraints utilized in the solution presented. They too are identified by the item names appearing in the column "Row (I)."

To indicate the effect of a change in system policy with respect to the total prestockage allowed, Fig. 10 presents the solution to the problem just described but with the prestockage limit raised to 300 KT. With the previous prestockage limit of 150 KT, a large number of T4 aircraft was included in the fleet. However, doubling the prestocking capability results in a very significant reduction in the system cost, attributable primarily to the reduction in the requirement for T4 aircraft. Other changes in the system include a small reduction in the number of high-speed ships and of course an increase in the tonnage prestocked at each base location. Some corresponding changes in the deployment activities will also be noted. Sufficient tonnage is now allowed in the forward bases to permit stockage for use in the second phase of the deployment operations. It is seen that some local shipping capacity can now be used in the transfer of tonnage from the bases to the deployment area, in addition to some second-phase airlift from these same locations. These comparisons indicate that the system cost and composition is sensitive to the deployment policy on allowable prestockage. A more informative approach in a situation of this sort is the use of parametric programming in which the prestockage limit is varied continuously, generating a complete family of optimal solutions. This technique will be covered more fully later in this section, since there are several opportunities for its use in the investigation of strategy and policy.

SENSITIVITY ANALYSES

A considerable amount of information is available from a linear-programming model beyond that contained in a particular solution, and several quantities related to the solution are readily obtained. For each activity identified in the system policy, it is possible to obtain information about the sensitivity of the solution to the cost of that activity and to determine the range of validity for the solution obtained. In a problem area such as strategic deployment where there are no firm requirements or policy restrictions the examination of a continuous range of the dominant-problem parameters is virtually mandatory.

Marginal Values

The marginal values were referred to previously and appear in column "PI(I)" of the basic solution printout. These values have a simple interpretation in the present model. The marginal value is the decrease in system cost that would result if the external-constraints value were to be increased by 1 unit. These values are strictly local in the sense that they apply only to values close to those listed. The result cannot be reliably extrapolated to values significantly different from those on which the solution was based. Furthermore care must be exercised in the interpretation of the signs associated with these values. For example, in Fig. 9 the first marginal value, corresponding to item T1L, shows that if one additional aircraft of this type were in the inventory, the system cost would increase by 3.5 units. The negative sign indicates that the cost would decrease in the negative sense, which is equivalent to an increase. The logic of this result is that it would be necessary to retire one additional T1 aircraft at a retirement cost of 3.5 units per aircraft.

As another example consider the prestockage limit. If this limit were increased by 1 KT, a reduction in system cost of 31.5 units would result. The implication of this result is that the system cost is sensitive to this prestockage limit, and that a system policy that relaxed this limit would save additional money. This result is confirmed by the solution in Fig. 10, in which the prestockage limit was raised to 300 KT. The marginal value associated with this item in Fig. 10 is only 2.1 units. The implication of this result is that most of the possible economy to be extracted from prepositioning has already been derived from a policy that permits a total of 300 KT to be stocked in forward bases. What is of more interest, however, is the specific level of prepositioning at which little further saving is possible. This type of information is extremely valuable in the identification of good deployment-system policies and can be obtained by the continuous variation of this parameter. The determination of the appropriate level of prepositioning will be considered further in the discussion of parametric programming.

Of particular interest in the consideration of deployment strategies is the marginal value of the class O and R tonnage requirements in each area. Since the requirement flows are entered in the model as negative values (system outputs), an increase in the algebraic value corresponds to a reduction in the requirement. A positive marginal value for such an item indicates that the deployment requirement represented is binding on the solution, i.e., the stated requirement cannot be exceeded except at additional cost. A zero value indicates that a greater requirement could have been accommodated at no increase in system cost. With these indicators the balance of the deployment strategy may be examined. If, for example, only a single requirement is binding, it might be possible to revise that requirement downward to permit cost reduction without affecting the overall strategy significantly. In the example given here it will be noted that the area 1 requirements are not binding. Area 2 requirements for both deployment phases are binding, but only the phase 1 requirement in area 3 has positive marginal value. The deployment in area 1 could be increased by a small amount without increasing the system cost, indicating that the system required to meet the demands of area 2 could accomplish more than is demanded of it in area 1. The phase 2 requirement in area 3 could also be raised slightly without added systems cost. In Fig. 10, however, it is seen that only the area 1 requirements can be increased. The changes in the system composition brought about by increasing the allowed prestockage have resulted in a system that cannot increase its contribution in phase 2 of area 3 without additional expenditure. The marginal values of 3TRR and 3TOR are no longer zero as they were in the solution shown in Fig. 9.

The marginal values can be useful in the interpretation of the optimal solution, primarily in the identification of areas in which further investigation would be profitable.

Cost Ranges and Reduced Cost Factors

It is important in the interpretation of solutions to know the range over which the cost of each activity can vary without affecting the validity of the solution. This kind of information is essential with regard to the cost of system components that have not yet been developed. It will be recalled that in one costing approach the RDTE cost may not be included in the cost function but may be added to the unit cost of any activity involving an undeveloped component. In this approach, increment in unit cost associated with development cannot be determined until the quantity of that component is specified in the solution. The solution to a problem in which such a component appears therefore remains conditional until a check is made to determine if the adjusted cost remains within the range of solution validity for that component.

Cost Ranges. Figure 11 shows the cost ranges for each activity appearing in the optimal solution presented in Fig. 9. The solution of Fig. 9, defined by the number and type of system items, remains valid so long as no activity cost falls outside the lower (Limit 1) and upper (Limit 2) bounds given for it. The final two columns indicate the activity that would enter the optimal solution if the actual cost for each activity violated the respective limits shown. In some cases, cost ranges will appear for activities that are at the zero level (i.e., not used in the optimal system). In such a case, Limit 1 represents the cost at which that activity might become competitive. For example, activity FBOA (prestockage of class O tonnage in floating depots at location A) would be a contender in the system if its cost were 20.58 or less, as compared with a cost of 31.5 originally estimated for that activity.

Checking the upper limit on the T4 aircraft unit cost shows that a substantial margin remains for the accommodation of RDTE cost. Since the activity level calls for the production of 110 aircraft and the development costs are distributed over the quantity procured, the RDTE cost per aircraft could be as great as 59.67 − 40, or almost 20 units. At this production level it is almost certain that the development cost per aircraft will be less than one-half the cost for the initial investment plus O&M for a period of several years represented by the 40.0-unit cost per aircraft used in the problem. In this case it may be safely inferred that the solution is valid. The cost range varies with the problem conditions, however, and must be checked in each case. The cost range for the high-speed-ship procurement activity is also very broad, so that there is little question about the solution validity in this specific example. Cost coefficients for development items are subject to considerable uncertainty and must be given close scrutiny in any event. A sensitivity analysis with continuous variation of the cost parameters also may be desirable.

Reduced Cost Factors. The cost ranges available in the form shown in Fig. 11 are restricted to the activities that are included in the optimal solution (including those at the zero level). Cost data are also available for the activities that are not in the optimal fleet mix. Figure 12 shows such information for the system activities from the same basic problem solution. The quantities shown indicate the amount by which the unit cost for the activity would have to

```
                         COST RANGES

BASIS                    COST IN                                   INCOMING VECTOR
VECTOR   BETA VALUE      PROBLEM      LIMIT 1      LIMIT 2     AT LIM 1      AT LIM 2


NOW STARTING CLASS   SYST
  RT1     500.00000     3.5000000    * * * *      8.5185177   UNBOUNDED          OT1
  RT2     50.000000     5.0000000    * * * *      10.393280   UNBOUNDED          OT2
  OT3     160.00000     15.600000    * * * *      16.687538   UNBOUNDED          RT3
  FBOA                  31.500000    20.584412     * * * *       11FOA     UNBOUNDED
  SBOA    .73577539     10.200000    10.200000    10.200000      11T4RC       11T20C
  BT4     109.72853     40.000000    35.017702    59,677255       RT3           FBRB
  BHS     38.469360     86.000000    .00000095    146.99767      +22T1         +21SO
  SBRA    72.313177     10.200000    10.200000    10.200000      11T20C       11T4RC
  SBOB    9.6494598     7.1000000    7.1000000    7.8435259      31T4RC       31T3B
  FBOB                  21.500000    16.102095     * * * *       31FOB     UNBOUNDED
  SBRB    67.301587     7.1000000    6.3631788    7.1000000      31T3B        31T4RC
```

Fig. 11—Computer Printout of Cost Ranges for Activities in the Basis of Solution to Sample Problem

```
     OT2          5.39328001          RT2           .
     RT3          1.08753780          BT4           .
     SBOA          .                  FBRA       10.91558784
     SBOB          .                  FBRB        5.39790477
     OT1          5.01851779          RT1           .
     OT3           .                  BT3        12.71246220
     BHS           .                  FBOA          .
     SBRA          .                  FBOB          .
     SBRB          .
```

Fig. 12—Computer Printout Listing Unit-Cost Reduction Required To Make Competitive Those Items Not Part of Least-Cost System in Sample Problem

be reduced to make that activity competitive. If, for example, the cost per kiloton for class R tonnage stocked in floating depots at location A (FBRA) were reduced by 10.92 units or more, this activity would be competitive in the optimal solution.

Parametric Programming

All the sensitivity data discussed so far have common limitations: the variation is restricted to a single activity or item, and the indicated result cannot be extrapolated with reliability. Their primary value then is in the indication of critical parameters that should be given closer examination by other techniques. With respect to cost coefficients and flow values, the restrictions cited above can be eliminated through the application of parametric programming.

The specific methods of setting up the problem for parametric programming vary with the specific code and computer employed and are readily obtained from the appropriate user's manual. Two specific examples utilizing the LP40 program will be treated, and several other possibilities will be mentioned.

Variation of the Prestockage Limit. Earlier in this section it was indicated that the system design was sensitive to the value of the exogenous flow corresponding to the overall limit on the prestockage of tonnage in forward bases. Figure 13 shows the variation in system composition and cost as this limit is varied continuously over the range 100 to 400 KT, tracing the levels of the system activities in the optimal system as the system policy is varied. The point of primary interest here is that the system cost stabilizes at a prestockage level of slightly more than 200 KT. The use of additional prestockage remains slightly advantageous, but the saving achieved beyond a limit of about 215 KT is no longer very significant. Since prepositioning of equipment has an associated risk, it may not be desirable to utilize this activity beyond the value at which significant reduction in system cost is possible. Thus a good policy with respect to the allowable prestockage, in the problem formulated, would set the limit somewhere in the vicinity of the breakpoint in the cost function. After this point a continuing tradeoff of prepositioning for ships occurs, but little system-cost change is involved. The system composition has stabilized at a prestockage limit of about 340 KT. Beyond this point there is no further change in the optimal system configuration with regard to this parameter.

It must be recalled, however, that the result in an analysis of this type is contingent on the strategy requirements assumed. The prestockage limit indicated here is largely a function of the phase 1 requirements in each area. Thus it might be of interest to investigate the effect of a change in the phase 1 requirements with the prestockage limit also varied in the same proportion. Thus what appears to be a good policy feature may be checked against a range of strategy requirements. Analysis of this sort is a valuable capability of most of the standard LP codes.

Variation of Forward-Base Costs. The costs of forward bases may be uncertain, but there may be good reason to believe that their cost will not vary greatly with respect to location and that the ratio of floating-base to shore-base cost will be constant. It thus becomes of interest to assess the effect of a variation in prepositioning cost in which all such costs are varied continuously

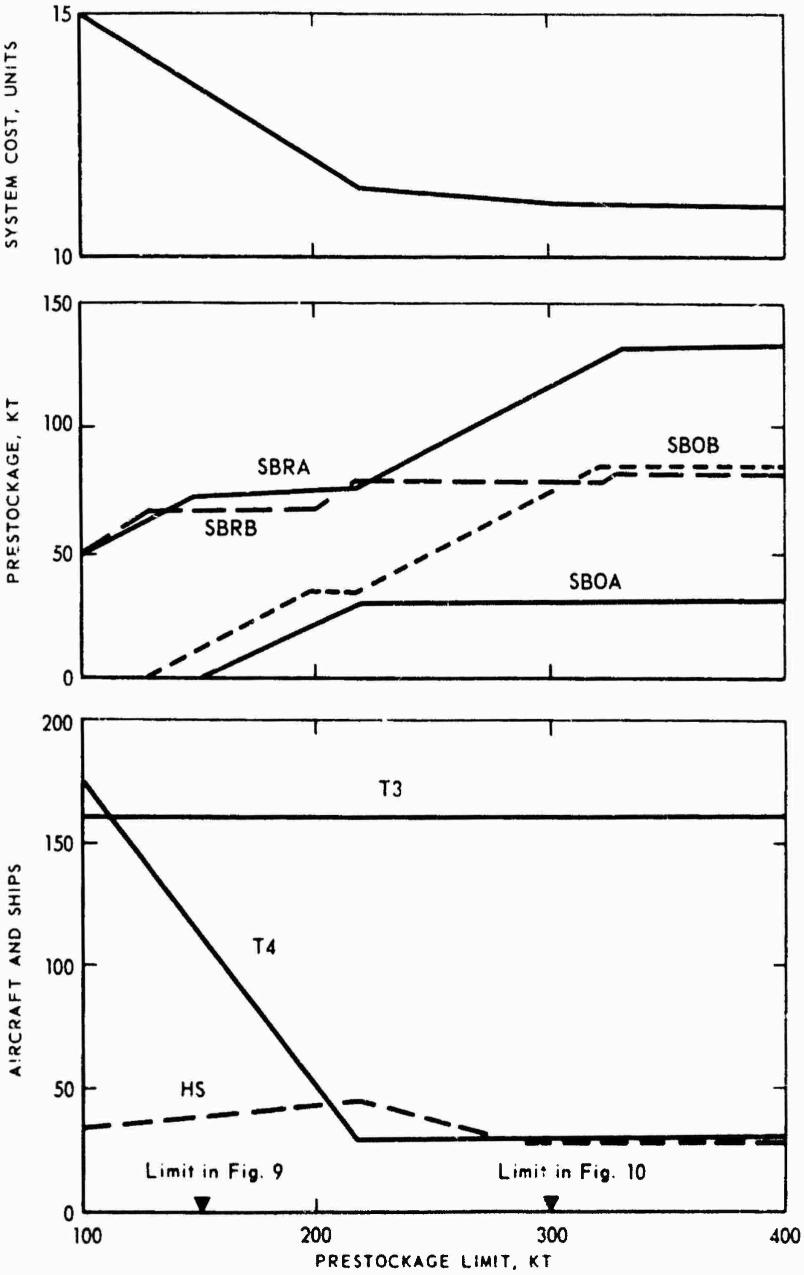

Fig. 13—System Cost and Composition as a Function of Prestockage Limit

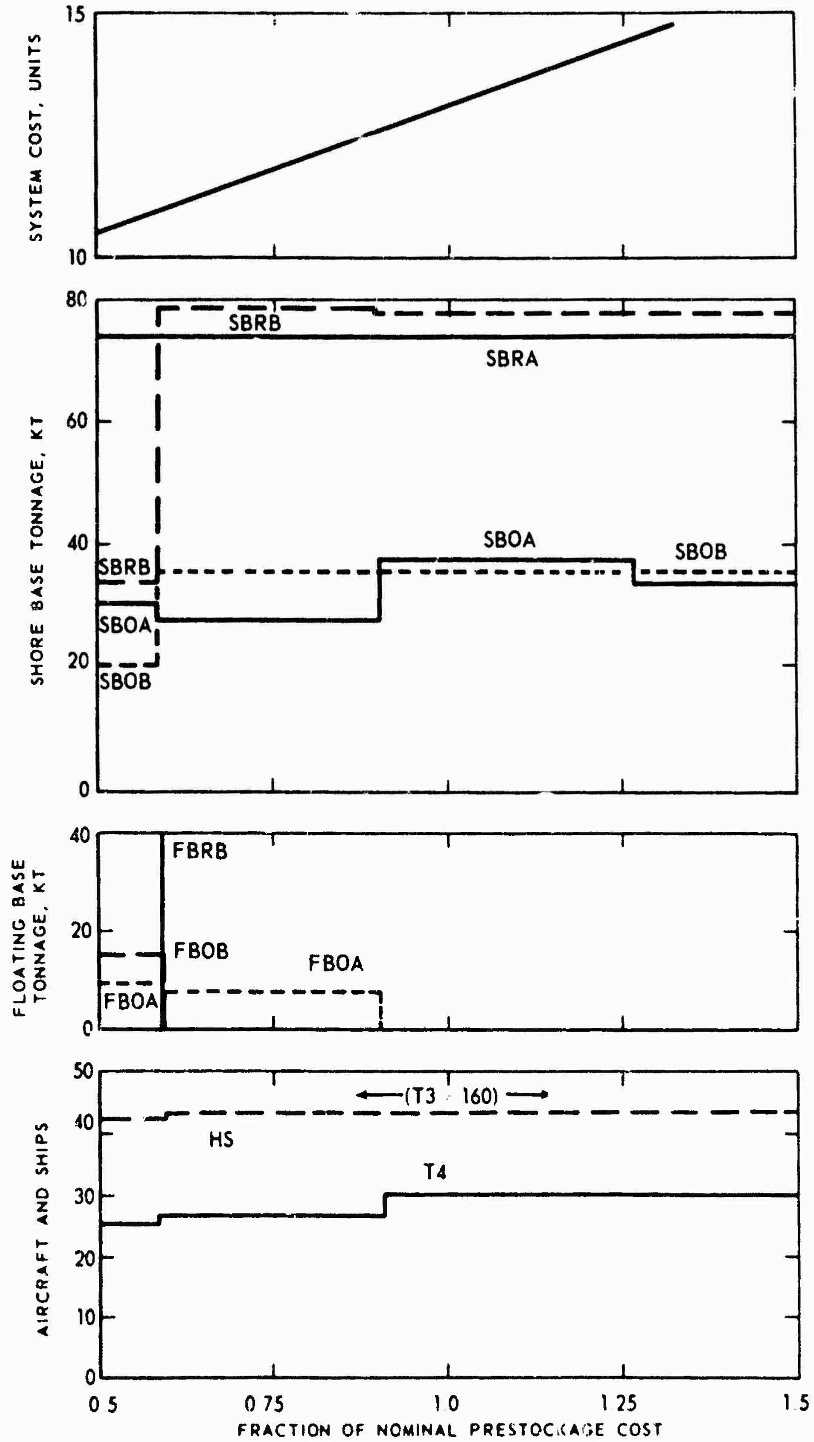

Fig. 14—System Cost and Composition as a Function of Prestockage Cost

and in the same relative proportions. The result of such an analysis is shown in Fig. 14. The cost axis represents the prepositioning cost as a fraction of the nominal value utilized in the solutions presented in Figs. 9 and 10. It will be noted that whereas the system cost varies continuously the changes in system activities are discontinuous. The problem requirements are constant throughout so that changes in the activities occur only when the prepositioning costs reach critical values. At these points, drastic revision in the system composition may result. At a low cost level, all forms of prepositioning are utilized in the optimal system. Floating-base activities drop out quite early in the competition, however, except for FBOA, which expires at a cost level only slightly lower than it had in the basic solutions presented earlier in this section. Shore-based prepositioning, on the other hand, is a hardy component whose level is quite stable over a wide cost range. A system containing such components is therefore likely to be optimal even in the face of highly inaccurate cost estimation. This type of information is valuable in the search for good objectives in the design of deployment systems and in the formulation of deployment-system policy.

Variation of Other Parameters. During the testing of the sample problem formulation presented here a number of other parameters were varied to reflect strategy and policy alternatives. In view of the uncertainty in the requirements for strategic deployment the prime contenders for such study are the global-strategy requirement levels in each contingency area. Since the individual requirement levels are almost equally binding on the solution, the significant variation is in the overall response level of the strategy.

A related problem is that of the distribution with respect to time phasing of the overall requirements in each area. Test runs were conducted in which the total in each area was held constant but the phase 1 requirement was varied from zero to one-half the total requirement. Time phasing, as might be expected, exerts a strong influence on system design and cost, since only a limited number of activities are possible in the very early stages of a deployment.

A third problem, the capacity of the system to accommodate simultaneous contingencies, has been investigated parametrically. This was accomplished by the continuous variation of the simultaneous deployment requirements from zero to the sum of two single contingency values. The result of this variation is that the previously determined optimal solution is not affected until the requirements are approximately 50 percent of the maximum total. At this stage the cost begins to rise and the composition changes to cover the new demands.

MODEL APPLICATIONS AND EXTENSIONS

The specific example discussed in the preceding section was necessarily limited in scope and completeness. Although a complete formulation was covered in the general description of the model, the formulation was restricted to two theaters. In actual application, five theaters are generally considered; however, the extension from two to five theaters presents no difficulty in formulation or problem size. Some of the more significant applications of the five-theater model will be reviewed here, together with an indication of the direction of extensions of the model.

MODEL APPLICATIONS

The strategic-deployment linear-programming model that has been used most extensively is a 400-equation set of constraints relating five worldwide military commitments of the US to the means that will be available for strategic deployment in the post-1970 decade. The model has been used to investigate several different types of problems thus far. Although the major use has been the evaluation of deployment systems, this has been by no means the only application. Some of the more interesting applications are noted below.

The primary problem addressed, evaluation of deployment systems, has included extensive analysis for the C-5A aircraft and fast-deployment-logistics (FDL) ship. Analyses of both systems have included the generation of sensitivity data that indicate solution (optimal system mix and cost) sensitivity to unit aircraft and/or ship costs, to the availability of limited numbers of each vehicle, and to the use of the vehicles in various modes of operation.

The economic impact of altering the US worldwide prepositioning posture is readily investigated in terms of the model, since all prepositioning sites are defined in the model. The removal of prepositioned materiel from site to site and the transfer from shore-based to ship-based locations are definable, and analysis can be conducted to determine cost and system mix consequences.

The question of changes in logistics characteristics in theaters of interest can be considered in the model. For example, losses of ports or ground LOC are assessable in terms of economic impact on the optimal system. It should be noted that thus far the RAC strategic-deployment linear-programming model addresses only intertheater deployment; thus the situation of changes in intratheater logistic characteristics is not explicitly included in the model.

One of the more interesting problems that can be investigated by the model is the value of extraterritorial sites to the US so far as strategic deployment is concerned. A large number of airlift and sealift en route bases are defined in the model, in terms of vehicle productivities, and thus can be assessed in terms of value in strategic deployment. The determination of a dollar value for any of numerous sites is possible again with the qualification that the value is that for strategic deployment.

A final application that is readily undertaken is determining the capability of fixed systems to move tonnage in time increments and to specific areas definable in terms of the model. Such analyses are possible simply by using an alternative objective function where tonnage to be moved is the objective function.

The foregoing are all examples of types of applications that have been made with the model. However, questions of additional realism, of additional systems, and of problems interacting with strategic deployment are of increasing interest, and extensions of the model, described in the following section, are constantly being pursued.

MODEL EXTENSIONS

The model extensions discussed here fall into two classes, additional realism and larger problems that can answer questions related to strategic deployment without necessarily including additional realism in model details.

Considering factors affecting realism in the simple problem and in applications, perhaps the most unrealistic deletions are the terminal-facility-augmentation activities. The deployment items corresponding to the various classes of terminal capacity in each area were also omitted. These activities and items were discussed earlier, but their importance deserves further emphasis. Terminal capacities are limiting in any large-scale deployment operation, particularly in underdeveloped areas. Realistic future-contingency planning demands the recognition of existing limitations and the provision for their elimination. The exogenous flows of the terminal-capacity items may account for the existing airfield, port, and LOC facilities. The levels of the augmentation activities will account for any additional facilities that may be required. These are specified in terms of the class of facility and the area location. Including a rough estimate of the facility-augmentation cost in the system cost function will prevent unrealistic system designs that would imply exorbitant demands for a particular type of facility, e.g., large fleets of aircraft to overcome port throughput limitations. Although facility-augmentation costs may not be significant individually, their cumulative effect over several deployment areas may be of importance in shaping the optimal system composition. At the very least, terminal-facility-augmentation activities serve to quantify the terminal-facility requirements associated with a given strategy and policy and will indicate the magnitude of the effort needed to eliminate shortfalls. Existing facilities are properly considered part of the system environment, but additional facilities must be treated as an integral part of the overall deployment system.

In the area of deployment activities there are some additional possibilities of interest. The representation of the CONUS as a single route origin leaves much to be desired with respect to realism, since at present the forces to be deployed are widely dispersed. The incorporation of multiple route origin within the CONUS and the use of items representing the forces available at each of these origins would be a refinement of some interest. The possibility of consolidation of the central reserve forces could also be investigated by means of appropriate system-activity options. This would of course require the careful estimation of the costs associated with such an operation. One could also investigate the desirability of some further deployment options such as intertheater shipment of prestocked materiel (at least during multiple contingencies) and forward-base stock replenishment by high-speed ships. Stock replenishment would permit final delivery to the objective area by aircraft having forward-field landing capability. Greater refinement in the representation of the forces to be deployed may be obtained through the introduction of additional tonnage classes and the use of distinct item flows corresponding to each.

One final extension of the model will be mentioned here. In the present formulation, only a single system design is produced, and this system is tied to a specified completion date. Interim requirements are covered in the specification of the system-design policy, which predetermines the ground rules for transition to the final system. However, it is possible to formulate the model to produce a sequence of system designs covering distinct costing periods. It is necessary, however, to repeat the deployment subproblems for each area in each costing period. System activities would be keyed to the time periods in which they were available, and system components selected in one costing period would become available resources for possible use in subsequent periods. Retirement activities would be included to cover transition phases, and a single cost function would assess the total system cost over all periods covered. In this fashion an optimal sequence of systems could be determined. Since the model size is directly proportional to the number of costing periods in the sequence, additional study will be required to determine if any real advantage is to be gained.

Model size is a major problem, particularly in the second area of extending the model to include other questions. This area includes such additional problems as longer times for initial deployment, finer time intervals, resupply for very long periods of time, and peacetime deployment in conjunction with strategic-deployment systems.

The computer running time involved in solving a problem increases as the third power of the ratio of problem size; thus the consequence of doubling the size of a 1-hr problem is obvious. And the inclusion of such additional problems as noted does in fact double problem size.

The solution to the larger problems to handle the additional questions of interest thus far seems to lie in the direction of developing subproblems that can be investigated either singly or in combination with other subproblems. The general sequence of solution is to solve the subproblems (standard terminology for these problems is "module") and then combine the individual module solutions to obtain the overall problem solution. This approach to decomposing the strategic-deployment linear-programming problem has not thus far been fully demonstrated but is thought to offer reasonable assurance of success until larger and faster computing machines are available to investigate the problems that are being developed.

SUPPLEMENTARY

INFORMATION

IDENTIFICATION	FORMER STATEMENT	NEW STATEMENT	AUTHORITY
AD-804 292 Research Analysis Corp., McLean, Va. Technical paper. Rept. no. RAC-TP-211 Sep 66 Contract DA-44-188-ARO-1	No Foreign without approval of Headquarters, Department of the Army, Attn: Office of the Chief of Research and Development, Washington, D. C.	No limitation	CRD, D/A ltr, 16 Dec 69