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THESIS

A THEORETICAL BASIS FOR THE CONCEPT OF EFFECTIVENESS

by

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A THEORETICAL BASIS FOR THE CONCEPT OF EFFECTIVENESS

by

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ABSTRACT

This thesis offers, to those concerned with analysis of modern weapons systems, a general methodology for devising appropriate and meaningful measures of effectiveness. This methodology does not include a specific model for "plugging in" system parameters and mechanically "grinding out" system effectiveness. It is intended only as a general "plan" through which the researcher can channel his own judgment and experience. The primary purpose of this plan is to guide the researcher through a logical transition from a purely subjective, and more or less vague, concept of effectiveness to a useable and more explicit formulation.

Effectiveness is modeled as that single system characteristic positioned at the apex of a characteristic "pyramid". This pyramid is constructed with "layers" of progressively fewer and more subjective characteristics. Mathematical properties of measurements appropriate to these characteristics are discussed as a function of the intended use of the effectiveness measurement. The type of measurement required to meet an analysis objective is dependent on the objective function or optimization criteria chosen. Because of this fact, the types of measurements have been classified into four scales and each scale related to a particular type of objective function.

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1. Introduction.

This thesis is principally concerned with the formulation of an expression for the relative or absolute magnitude of a system's effectiveness. This "expression of magnitude" is the essence of the concept of measurement; but before attempting any plan for obtaining such an expression, it has appeared logical to explore some of the relevant properties of measurement. The particular manner in which this expression of magnitude is accomplished depends on its ultimate use in the analysis. In consideration of this fact, it has further appeared logical to discuss and catalogue different types of analyses and objective functions and relate them to types of measurement.

The main feature of this thesis is the "characteristic pyramid" described in section 7.1. The reader may wish to refer to this section initially so as to understand better the direction of the preceding sections, or to make a brief appraisal of the model's applicability to his own area of interest. The organization of this thesis is directed to that reader interested in the general concept of system effectiveness. Those seeking a specific model for system effectiveness should appraise section 7.1 and utilize other sections as a reference to those features of interest.

1.1 Justification.

The validity, and therefore the usefulness, of operations research depends upon the skill with which projects are designed, and particularly upon the shrewdness with which criteria (such as payoff and objective functions) are selected. This criterion problem is often relatively neglected in operations research literature, and has apparently usually been "solved" in practice by assuming the first plausible payoff function that springs to mind; or if several spring to mind, by trying all of them and compromising (or letting a decision-maker compromise) among the results of alternative computations. This problem is much too important for casual treatment. Calculating quantitative solutions using the wrong criteria is equivalent to answering the wrong question. If the methods of operations research are applied to the wrong criteria, its quantitative methods may prove worse than useless to its clients.

The terminology, "effectiveness" and "system effectiveness", is used extensively by the operations researcher, and is, more often than not, included in the objective function. This terminology is also applied as a "common denominator" through which to compare alternative systems. A poor choice of a quantitative expression for effectiveness is equivalent to selecting the wrong criteria.

From the multitude of characteristics exhibited by a system, there must be selected a set of factors that completely (or at least adequately) define the system's effectiveness. Typical characteristics, widely used, involve such terms as exchange rates, operability factors, probability of kill, and so on.

All of these parameters used may be further divided into cases which are designed to cover range, altitude, speed, and the like. In studying any particular system, a particular selection of a system of characteristics must be made. The particular selection will depend on the particular situation under study. Some characteristics may be considered to be of greater importance than others, so that these measures may be "weighted" in various ways. It often requires considerable judgment to select a workable and representative system of characteristics to be used as a basic for decision.

Effectiveness is often conceived as a subjective quantity that is not ordinarily amenable to direct, physical measurement. Because of this, certain sets of characteristics that can be objectively described are chosen to indicate the subjective concept. This is, in effect, an extrapolation from the subjective concept to the objective characteristics. Unless this extrapolation is logically sound and thoroughly understood, it is equivalent to an unintentional, redefinition of the system's mission.

The effectiveness of a system depends on the mission of that system. If the mission is narrowly defined, it may be possible to measure the system's ability to achieve that mission directly through analytical models or experimental testing; however, narrow definition of systems and their missions tends to lead to a sub-optimization of the system with respect to broader, joint missions. Conversely, broader definition reduces the sub-optimization problem among systems, but increases the difficulty of obtaining a valid, analytical expression for the effectiveness. This suggests that one of the basic considerations in system definition should be an "optimization" of this measurability-sub-optimization relationship.

To be useful as an analytical tool, the concept of effectiveness must be defined in such a way so as to permit its characterization through a set of physical measurements. The choice and number of the elements in this set is important in its own right, but the functional relationship between these elements and the concept of effectiveness is the most elusive element of the analysis.

The purpose of this thesis is to focus on the concept of effectiveness, to determine the types of measure and methods of measurement that are appropriate to various objectives, and to model the relationship between the elements of a system and the system's effectiveness.

1.2 Formulating the problem

Following is a listing of tasks that must be performed, in the general order presented, in the evaluation of a system's effectiveness:

(a) Mission definition.

(b) System definition.

(c) Specification of relevant system characteristics.

(d) Choice of an objective function.

(e) Construction of a model that is consistent with the objective function.

(f) Data acquisition.

(g) "Fitting" of the data to the model.

2. Measurement.

Measurement is defined by Peter Caws[1] as "the assignment of particular mathematical characteristics to conceptual entities in such a way as to permit (1) unambiguous mathematical description of every situation involving the entity, and (2) the arrangement of all occurrences of it in a quasi-serial order." The term "quasi-serial order" is taken to mean an order that determines, for any two occurrences, either that they are equivalent with respect to the property in

[1] C. W. Churchman and P. Ratoosh, Measurement, Definition and Theories (Boston: John Whiley & Sons, Inc., 1962) p. 5

question or that one is greater than the other. This definition of measurement is applied to conceptual entities, which implies that before we can hope to measure effectiveness we must have some conceptual notion of effectiveness. A definition is some statement which sets the entity in unambiguous relation with other entities in the same or different groups.

From these two definitions we can see that it may be possible to define effectiveness without providing a measurement for it, but it is impossible to measure effectiveness without first defining the concept.

The "quasi-serial ordering" requirement can be accomplished in four (4) ways which are classified by S. S. Stevens[2] into four (4) scales of measurement. These four scales are described, along with their properties, in Table 1.

This classification narrows our definition of measurement somewhat, to those relationships between conceptual entities for which some property of the real number system can serve as a useful model. This restriction is implied if we say that a measurement is the assignment of numerals to aspects of those entities according to a rule. It is the particular property of the real number system, which we choose to serve as a model, that determines the properties and applicability of the measurement. Some "conceptual entities" can be measured on one type of scale and not on another, but the objective of the measurement may be consistent with only certain types of scales.

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[2] Ibid, p. 25

TABLE I

CLASSIFICATION OF SCALES OF MEASUREMENT

SCALE	Empirical Operation Necessary to Create the Scale <u>or</u> Which can be Determined From the Scale.	Mathematical Structure of the Scale	Group Structure	Typical Examples
NOMINAL	Determination of Equality	$x' = f(x)$ where $f(x)$ is any one-to-one Substitution	Rermutation Group	"Numbering" of Football Players
ORDINAL	Determination of Equality, Greater, or Less	$x' = f(x)$ where $f(x)$ is any monotonic increasing Function	Isotonic Group	Street Numbers, Grades of Lumber, Hardness of Minerals
INTERVAL	Determination of the Equality of Intervals or of Differences	$x' = ax + b$ $a \quad 0$	Linear	Time (Calendar) Temperature (F,C) Potential Energy
RATIO	Determination of the Equality of Ratios	$x' = cx$	Similarity Group	Numerosity, Length. Mass

NOTE: Any Number (x) on a Scale Can be Replaced By Another (x') Without Affecting the Measurement

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2.1 Classification of effectiveness measurement objectives.

When an attempt is made to find an expression, or measure, of a system's effectiveness, some specific use is planned for this measure. The nature of this "use" or objective determines the type of scale required to meet the objective.

2.1.1. Comparison of several alternative systems or configurations to determine the least cost for a given effectiveness, i.e.,

Minimize Cost
so that
Effectiveness = Constant

Since there is no requirement for a knowledge of the absolute or relative magnitude of the effectiveness, the most general scale that will satisfy the objective is the <u>nominal</u>.

2.1.2 Comparison of a discrete set of alternative systems or configurations to determine the greatest effectiveness for a cost less than or equal to some given amount, or conversely, i.e.,

$$\begin{array}{ccc} \text{Maximize Effectiveness} & & \text{Minimize Cost} \\ \text{so that} & \text{or} & \text{so that} \\ \text{Cost} \leq \text{Constant} & & \text{Effectiveness} \geq \text{Constant} \end{array}$$

There is no requirement for a knowledge of the absolute magnitude of the effectiveness, but we must be able to order a set of discrete alternatives. The most general scale that will satisfy this objective is the <u>ordinal</u>.

2.1.3 Comparison of a "continum" of system configurations to determine the greatest effectiveness for a cost less than or equal

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to some constant, or conversely, i.e.,

Maximize Effectiveness so that Cost $\leqslant$ Constant	<u>or</u>	Minimize Cost so that Effectiveness $\geqslant$ Constant.

This objective is essentially the same as 2.1.2, but we are required to maximize a continuous function as opposed to choosing a maximum from a set of discrete quantities.

2.1.4 An objective that falls naturally into this class is the determination of the <u>rate</u> of increase (or decrease) in effectiveness as a function of cost. There is no requirement for a knowledge of the absolute magnitude of the effectiveness, but we must be able to compare amounts of increase in effectiveness as a function of the costs of those increases. The most general scale that will satisfy this objective is the <u>interval scale</u>.

2.1.5 Determination of the absolute effectiveness of a system, so as to make utility comparisons of cost-effectiveness combinations.

Compare: X Effectiveness and Y Cost
with X' Effectiveness and Y' Cost,
where $X' > X$ and $Y' > Y$, and make the "best" choice.

This objective requires a measurement that provides a clear concept of the absolute magnitude of the effectiveness, and can be provided only by the <u>ratio scale</u>.

3. Cost.

All of the objectives discussed and classified above concern some type of cost-effectiveness trade-off. The term "cost" is not restricted to the narrow definition of "dollar cost", but includes the general idea of resource requirements. Almost all system analysis is performed using the basic concept of some sort of trade-off between resource and effectiveness. To accomplish this trade-off, these two system parameters (cost and effectiveness) are separated, sometimes rather artificially, and envisioned as separate entities. This distinction is not, however, theoretically necessary. We could define the mission so as to include resource requirements and define the single resultant entity to be the system effectiveness. This concept of a system's effectiveness would then imply only a single, unambiguous criterion for choice, namely, maximize effectiveness. This is not, however, the approach taken in this thesis, and the usual distinction between cost and effectiveness will be retained.

4. A hypothetical example of the scales of measurement appropriate to various objectives.

System definition:	The warhead to be utilized on an existing missile.
Mission:	To produce a specified "overpressure distribution" over a circular area centered on the detonation point.

Suppose that, originally, the only information available concerning the blast effects of various alternatives is that some produce identical distributions and others produce different distributions. From this primitive information we can place the effectiveness of the warheads on a <u>nominal scale</u>. If we establish a criterion of choosing the warhead with the least cost subject to its having an effectiveness <u>equal to</u> that of some arbitrarily selected warhead, we can use this scale for selection of the best warhead.

Suppose that through testing and/or analytical methods we now learn that the values of this distribution are related to some measurable parameter of the warhead through an "increasing, monotonic function". If this is the only information available concerning this function, we can place the effectiveness of different designs on an ordinal scale. If we establish a criterion of choosing the warhead having the greatest effectiveness for a cost less than or equal to a set amount (or conversely), we can pick the best warhead.

Through further testing or analytical investigation we may learn the functional relationship existing between the design parameter and the effectiveness to within a multiplicative factor and an additive constant.

This would be expressed as:

Effectiveness = U·f (design parameter) + b,

where f is known, and U and b are unknown constants.

This scale is the most widely used for effectiveness because it requires no knowledge of the absolute magnitude of the effectiveness and the choice of units (U) is completely arbitrary. Potential energy offers an example of a physical quantity measured on this scale.

If we are given the potential energy of object A as 10 "units" we must be furnished additional information concerning the reference point and the nature of the units before the statement is useful in itself. If, in addition, we are given the potential energy of object B as 20 "units", and told that the reference points and units of the two measurements are equal, we can make meaningful comparisons between the potential energy of the two objects.

Returning to our warhead example, we can make a direct measurement of the tons of explosives contained in the warhead. Assuming that f is the cube-root function, we construct an expression for the effectiveness of the warhead in the form:

$$\text{Effectiveness} = U \cdot f\,(\text{tons of explosive})^{1/3} + b.$$

If design A has effectiveness $= U \cdot f(x) + b$, and cost Y dollars
design B has effectiveness $= U \cdot f(x') + b$, and cost Z dollars
design C has effectiveness $= U . f(x'') + b$, and cost W dollars,
we can make a statement of the form:

For (Z-Y) dollars we can increase the effectiveness by $U \cdot f(x') - U \cdot f(x)$ above the effectiveness of design A, and,
For (W-Y) dollars we can increase the effectiveness by

$$(U \cdot f(x'') - U \cdot f(x))$$

above that of design A.

This statement provides the rate of increase of effectiveness as a function of the cost of that increase, and we may be able to make a choice between designs A, B, and C on the basic of this information.

Finally, we may be able to obtain the exact value of the overpressure as some function of design parameters. Since the mission of the warhead was defined "to produce a specified overpressure", we can obtain the overpressure of a particular design, compare it to the specified overpressure, and make a statement of the form: The effectiveness of the design is X% (or warhead A is X% effective). An extremely important point to bear in mind, in connection with this example and in an actual analysis, is that we are expressing the ability of the warhead to achieve its expressed mission and nothing else. If the mission of the warhead were intended as the accomplishment of certain damage to the target, there is no guarantee that the ability to achieve X% of the overpressure specified will, in fact, produce X% of the damage which would have resulted from the specified overpressure. This is a measurement that can be placed on the ratio scale and requires that the units of the quantity measured be identical (or differ by a known multiplicative factor) with the units expressing the mission attainment. In addition, it requires a common reference point which, in our example, is assumed to be the following:

Zero overpressure corresponds to zero effectiveness.

Having a measurement of effectiveness on this scale permits a criterion of "maximum utility" for alternative cost-effectiveness combinations.

5. Units for Effectiveness.

Karl Menger[3] has shown that the variables of physics such as work, heat, energy, accelleration, etc., cannot be regarded as the class of numbers; nor can they be regarded as the class of physical entities. They must be regarded as the class of pairs (n, E) such that one element of the pair is a real number and the other is an element from the class of physical entities.

Effectiveness can be considered as a physical entity. In fact, it is a characteristic of a physical system in this discussion. It cannot be expressed as only a number any more than it can be expressed

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[3] Menger, Karl, On Variables in Mathematics and Natural Sciences, British Journal for the Philosophy of Science, Vol. no. 18, 1954, p. 135.

as only a physical entity, if the expression is to be useful in quantitative analysis.

Three commonly used expressions of effectiveness may appear as contradictions to this idea. These are, (1) Probability, (2) Reliability, and (3) Ratios. All of these expressions imply the "unit" of complete or total effectiveness and they must include a lucid description of this "unit" to be meaningful. For a proposed measure of effectiveness to be placed on the ratio scale and expressed in units of U (say p.s.i), the mission of the system must be expressable in U and, in addition, the number of these units U necessary to achieve the mission must be known.

Probability is commonly understood to represent the frequency of some specified event in relation to the number of opportunities for this event to occur. The statement that the probability of obtaining "heads" on any one toss of a coin is 1/2 means that the frequency with which this event will occur (over a long series of trials) is 1/2 of the opportunities afforded. If probability is used as a measure of effectiveness we are saying that the system will exhibit complete effectiveness with the stated frequency. It is necessary to understand exactly how this "complete effectiveness" event is characterized before the probability statement can assume any meaning. For example, suppose "probability of kill" is used as the measure of effectiveness for some system. The implicit assumption is that each time the system exhibits the "kill event" it is completely effective for that trial, and further, if it exhibits the kill event each and every time over a long series of trials, it is completely effective (can accomplish its mission with certainty). The unit of effectiveness is the "kill event" and the number of these units necessary for complete effectiveness must equal the number of trials.

Reliability is just that terminology applied to the probability of accomplishing a mission, and has the same interpretation as a probability statement.

A ratio can be used in the same manner as probability, except that it is usually an "a posteriori" statement rather than an "a priori" statement. A ratio can also be interpreted as the magnitude of some system characteristic relative to some standard magnitude. Used as a measure of effectiveness, this implies that the "standard magnitude" of the characteristic will endow complete effectiveness.

6. Measurability and Sub-Optimization.

The usual task of quantitative analysis is the improvement of decisions at relatively low levels (efficiency "in the small"). Optimum decisions at low levels do not, however, imply an optimal solution to higher level structural decisions. If the Navy attempts to optimize the design of an aircraft carrier, taking the inclusion of a nuclear power plant as given, then it runs the risk of sub-optimization within a frame work in which the type of power plant is not assumed, but is a variable of the analysis. Suppose we have three sub-systems, A, B, and C, which are components of a larger system S. The effectiveness of system S is a function of the effectiveness of the sub-systems. Each of the sub-systems has a mission which, <u>in combination</u>, contribute to the mission of S. Optimizing the design of the sub-systems, according to some criterion that seems appropriate to their individual missions will not, in general, optimize the criterion appropriate to the composite system S.

Because of the sub-optimization problem, there has been a marked effort within the Defense Department to define systems in a broader, more inclusive manner. If the optimization process is applied correctly to the broader systems, sub-optimization among components is (in theory) avoided. It might seem that this problem could be entirely eliminated by defining the "system" as the complete defense establishment, and performing the optimization directly on this "system". There are any number of obvious reasons why this would be an impossible task; but there is one, somewhat more subtle, difficulty that is not immediately obvious. This difficulty tends to dominate the process even at relatively low levels, and has to do with expressing the effectiveness element of the optimization criterion.

6.1 Optimization.

Any optimization process implies an operation on two or more characteristics of the system. If we optimize the mix of sand and cement to obtain the strongest concrete, the two elements are the "mix" and the strength of the concrete. A further requirement is that some functional relationship exist between these elements. Here, the strength of the concrete is a function of its "mix". Alternatively, we can optimize the strength of the concrete subject to some cost constraint. If both the cost and strength of the concrete are functions of the mix, then this "mix" can be the parameter which relates cost to strength. In military systems analysis the two elements of the optimization process are generally the mix of elements (or design) and the effectiveness; or else the cost and effectiveness. It would be difficult to imagine an optimization function that did not include the concept of effectiveness (although the terminology might be different). The concept of effectiveness contains the purpose of conceiving or building the system, and optimization implies an operation concerned with the purpose of the system. The first step in planning an optimization criterion is the selection of the elements that the process is to operate on.

6.2 Measurability

As the system becomes more broadly defined, envisioning the mission as a unique entity becomes more difficult. It may be necessary to state the mission in broad subjective terms, or even as a set of "sub", or alternative, missions. Examples of broadly defined missions are:

(1) Provide a deterrent against enemy aggression,

(2) Provide a retaliatory capability,

(3) Provide a strategic nuclear delivery capability <u>and</u> an aerospace research vehicle.

These missions suggest no easily conceived, unambiguous, entity which can be used as the second element of the measurement set. To obtain this element, and a measurement that can be included in the optimization process, some characteristic of the system is chosen that <u>contributes</u> to the achievement of its mission. Unfortunately, the effectiveness of the system usually derives from additional characteristics in combination with the one measured. The difficulty of combining this set of characteristics into a single element that can be logically included in the optimization process increases as the system becomes more broadly defined.

6.3. Sub-Optimization of Characteristics in Broadly Defined Systems

When the effectiveness of a system is represented by a measurement of a contributing characteristic, rather than by a measurement on the set of all contributing characteristics, and this measurement is included in the optimization process, we are actually sub-optimizing between characteristics just as we sub-optimized between components in narrowly defined systems.

7. System Model.

A system can be modeled in the following way to indicate the manner in which effectiveness is derived from the basic resources employed in the system. A hypothetical missile system is used to provide representative examples for the notational elements.

Let $x_1, x_2, x_3, \cdots, x_r$, represent the basic resources used in the system. These can be single, initial, requirements or a flow per unit time.

For example:

x_1 = tons of steel plate

x_2 = gallons of propellent

x_3 = number of personnel

x_4 = megatons of explosives

x_5 = number of electronic components

. . .

. . .

. . .

x_r = etc.

Let $a_{i,j}$, $i = 1,2,3, \cdots, s$, $j = 1,2,3, \cdots, r$ represent the j^{th} production process operating on the i^{th} resource.

Let $(a_{i,j})$ be the matrix of the $a_{i,j}$'s

For example:

$a_{1,1}$ = cut the steel plate to dimension D.

$a_{1,2}$ = form the steel plate into cylinder C.

$a_{1,3}$ = apply red paint to the steel plate.

. . .

. . .

. . .

$a_{1,s}$ = etc.

Now if the x_i's, $i = 1, 2, 3 \cdots, r$ are considered to constitute a row vector $\bar{X}$, and we form the combination[4];

$$\bar{X}\,(a_{i,j}) = \bar{Y}$$

then $\bar{Y}$ is a column vector having components:

$y_1, y_2, y_3, \cdots, y_s$,

where y_k is a basic component (or unit) of the system.

For example:

y_1 = missile airframe

y_2 = guidance system

---

[4] The elements of $\bar{X}$ are related to those of $(a_{i,j})$ just as in ordinary vector multiplication, but these elements are not combined through ordinary multiplication. The element of $(a_{i,j})$ "operates on" its corresponding element in $\bar{X}$ to form an element of $\bar{Y}$. The nature of this operation will be evident in context.

y_3 = propulsion system

y_4 = warhead

. . .

. . .

. . .

y_s = etc.

Let $b_{u,v}$, $u = 1,2,3, \cdots, s$, $v = 1,2,3, \cdots, t$ represent the v^{th} tactic, employment, or environmental condition operating on the u^{th} basic component. and $(b_{u,v})$ be the matrix of the $b_{u,v}$'s.

For example:

$b_{1,1}$ = use two warheads on each airframe.

$b_{1,2}$ = protect each missile with a silo.

$b_{1,3}$ = representative characteristic of enemy antimissile defense.

. . .

. . .

. . .

$b_{1,t}$ = etc,

Now since $\overline{Y}$ is a column vector, form the combination;

$$Y^T (b_{u,v}) = \overline{C}$$

Then $\overline{C}$ is a column vector having components;

$c_1, c_2, c_3, \cdots, c_t$

where c_h is the h^{th} "basic" system characteristic.

For example:

c_1 = probability that a missile reaches the antimissile defense area, given that it is launched.

c_2 = range of early warning radars

c_3 = reliability of guidance system

c_4 = warheads damage radius.

. . .

. . .

. . .

c_t = etc.

It is assumed that the c_h's are such that adequate, quantitative expression can be obtained for them.

7.1 System Characteristics.

In relating the c_h's, $h = 1,2,3, \cdots, t$, to a single expression for effectiveness, it is helpful to utilize set notation.

Let the set of all c_h',s, $h = 1,2,3 \cdots, t$, be denoted by $C^{(1)}$ where the superscript refers to the "level" of the elements. Upon adding the superscript to the elements we have:

$$C^{(1)} = c_1^{(1)}, c_2^{(1)}, c_3^{(1)}, \cdots, c_t^{(1)}$$

Now there will exist e sub-sets of $C^{(1)}$ where $e < t$ that have the property of completely describing the effectiveness of the system, if suitable functions are defined on these sets.

Let $S_j^{(1)}$, $j = 1,2,3,\cdots, e$, denote the j^{th} such sub-set of $C^{(1)}$.

Let $C^{(2)} = c_1^{(2)}, c_2^{(2)}, c_3^{(2)}, \cdots, c^{(2)}$ represent the set of "second level" characteristics whose elements are functions of the $S_j^{(1)}$'s.

This "second level" set of characteristics represents a manner of describing the system's effectiveness through a smaller, and more general, set of parameters. The lowest, or first, level of characteristics are all those numerous, and distinct attributes of a system contributing to its effectiveness. The elements of the "second level" set are combinations of these basic attributes that describe a slightly more general system characteristic. For example; suppose two basic (or first level) characteristics of a missile system are speed, and burn time. One element of the "second level" set would be the range of the missile as a function of these two basic characteristics. (i.e., $C_j^{(2)}$ = (speed) (burn time))

Then:

$c_j^{(2)} = f_{j,1}(S_j^{(1)}), \quad j = 1,2,3, \cdots, e$, where

$f_{j,1}$ is the function that relates the j^{th} set of first level characteristics to the j^{th} second level characteristic.

Reviewing the notational scheme we have:

$c_j^{(k)}$ = the j^{th}, k-level, characteristic of a system.

$C^{(k)}$ = the set of k-level characteristics completely describing the system's effectiveness.

$S_j^{(k)}$ = the j^{th} sub-set of $C^{(k)}$.

$f_{j,k}$ = the function relating the j^{th} sub-set of k-level characteristics to the j^{th} $(k + 1)$-level characteristic.

Then:

$$c_j^{(k+1)} = f_{j,k}(S_j^{(k)}), \quad j = 1,2,3,\cdots, e$$

where e is the general "termination" of the index j.

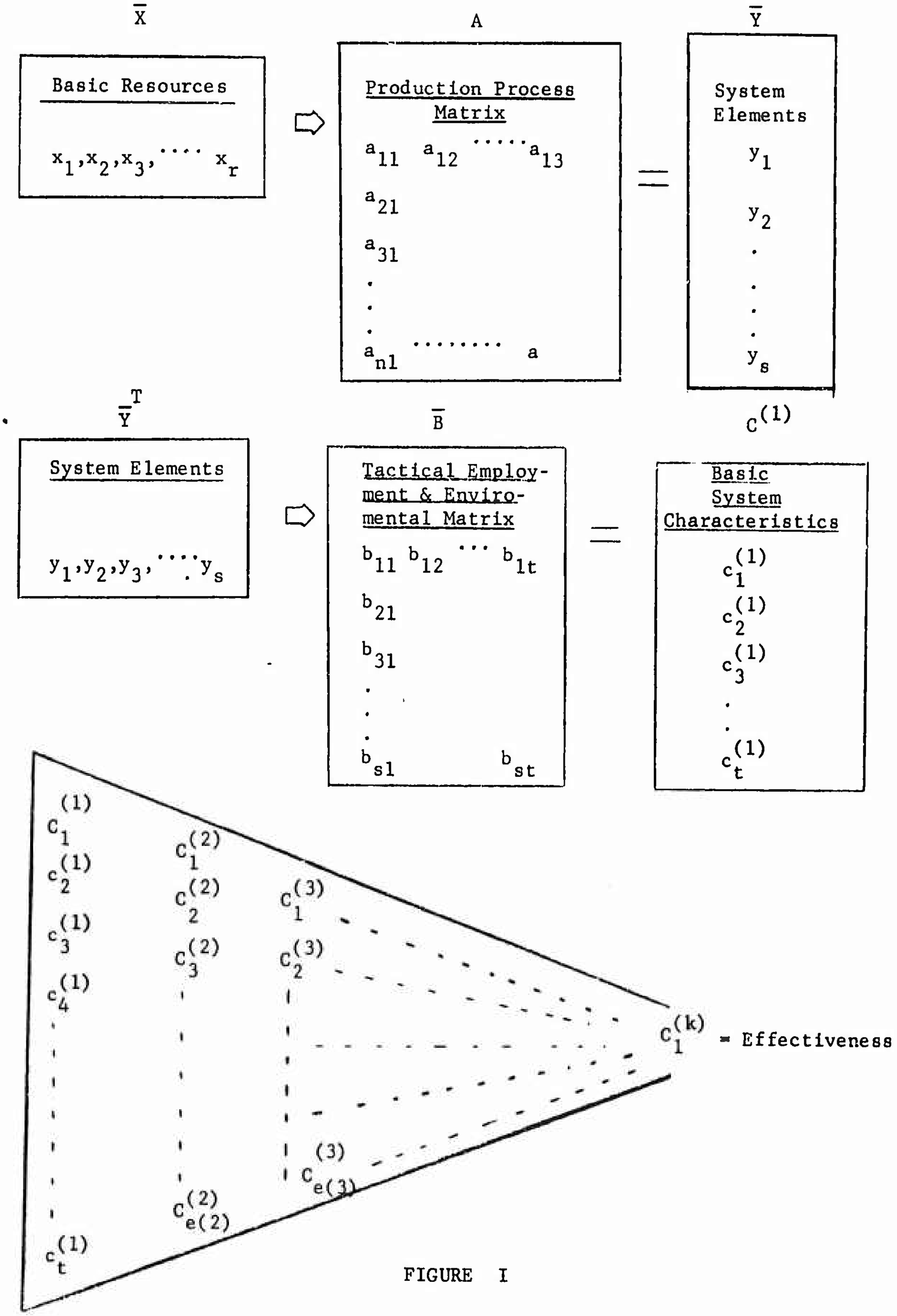
Now e is a "strictly decreasing" step function of k, i.e., as we express the effectiveness of a system in terms of higher level characteristics, the number of these characteristics necessary to describe fully the effectiveness decreases. Finally, there will exist some k for which $e = 1$. This single characteristic, then, fully describes the effectiveness of the system and, if suitable restrictions are imposed on the $f_{j,k}$'s and $c_j^{(1)}$'s, is a measurement of the effectiveness. These restrictions will be discussed later.

In common with many notational schemes or models, the notation here is much more difficult than the idea that the model is intended to convey. The following diagrammatic representation of the mathematical model is presented to augment the notation. (Figure 1)

7.2 Properties of the $c_j^{(k)}$'s and $f_{j,k}$'s.

The properties that must be possessed by the $c_j^{(k)}$'s and $f_{j,k}$'s depend on the objective of the effectiveness measurement. (see section 2.1) These properties are listed below in connection with analysis objectives.

(1). If the measure of effectiveness is required only for the determination of equality; the $c_j^{(1)}$, $j = 1,2,3,\cdots, t$ can be any measurement suitable for the nominal scale, and the set of $f_{j,k}$'s are required only to provide a one-to-one substitution.


$\overline{X}$
Basic Resources
$x_1, x_2, x_3, \cdots\ x_r$
A
Production Process Matrix
$a_{11}\ a_{12} \cdots a_{13}$
a_{21}
a_{31}
$a_{n1} \cdots\cdots a$
=
$\overline{Y}$
System Elements
y_1
y_2
y_s
$\overline{Y}^T$
System Elements
$y_1, y_2, y_3, \cdots y_s$
$\overline{B}$
Tactical Employment & Enviromental Matrix
$b_{11}\ b_{12} \cdots b_{1t}$
b_{21}
b_{31}
$b_{s1}\ \ b_{st}$
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$C^{(1)}$
Basic System Characteristics
$c_1^{(1)}$
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$c_3^{(1)}$
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$c_1^{(1)}$
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$c_4^{(1)}$
$c_t^{(1)}$
$c_1^{(2)}$
$c_2^{(2)}$
$c_3^{(2)}$
$c_{e(2)}^{(2)}$
$c_1^{(3)}$
$c_2^{(3)}$
$c_{e(3)}^{(3)}$
$c_1^{(k)}$ = Effectiveness

FIGURE I

(2) If the measure of effectiveness is required only for an "ordering" among alternatives; the $c_j^{(1)}$'s can be any measurement suitable for the ordinal scale, and the $f_{j,k}$'s can be any monotonic increasing functions.

(3) If the measure of effectiveness is required for the determination of marginal rates of increase; the $c_j^{(1)}$'s can be any measurement suitable for the interval scale, and the $f_{j,k}$'s can be any linear functions whose constants need not be known.

(4) If the measure of effectiveness is required as one of the elements in a cost-effectiveness combination that is to be compared for maximum utility; the $c_j^{(1)}$'s must be "absolute" measures on a ratio scale, and the $f_{j,k}$'s must be linear with all constants known. This linearity does not refer to the individual variables, but rather to the set of variables over which the function is defined.

8. Measurability of the $c^{(k)}$'s

We assume that the basic characteristics of the system, which are denoted $c^{(1)}$, are easily conceived, conceptual entities. Now there are two alternative methods of obtaining the next highest level set of characteristics: (1) direct physical measurement through some type of experimental testing, and (2) analytical modelling of the set of functions $f_{j,1}$, $j = 1,2,3,\cdots, e$. Both of these tasks become more difficult as the level (k) of the characteristics increases. Because of this increase in difficulty, some level (k) is reached such that it is either impossible, or we are unwilling, to express the (k+1)-level of characteristics with any degree of certainty. Since the termination of the index j (e) is a function of k, there will exist $e(k)$ k-level characteristics that contain all information contributing to the effectiveness of the system. We are now presented with the problem of finding some function E, defined over the e(k) elements of $C^{(k)}$, that will provide a single, quantitative expression for the effectiveness of the system. The difficulty of finding or formulating this function will increase as the number of variables (e(k)) increases. This "trade-off" between measurability of individual system characteristics, and difficulty of obtaining the effectiveness function defined over this set (of characteristics) is encountered in every system analysis. The ability to resolve successfully this trade-off will also depend on the generality of the system definition and mission; and this in turn, affects the sub-optimization problem between systems. This interdependence of concepts is indicated in Figure II.

From the preceding discussion (and Figure II), it can be seen that an important consideration in the system and mission definition phase should be a satisfactory "trade-off" between the sub-optimization among systems, and that among the characteristics of these systems.

9. The Effectiveness Function.

Let E represent the effectiveness of a system.

Then:

$$E = E(c_1^{(k)}, c_2^{(k)}, c_3^{(k)}, \ldots, c_e^{(k)}), \text{ where:}$$

k is the highest level of measurable characteristics and e is the number of these characteristics necessary to completely describe the effectiveness.

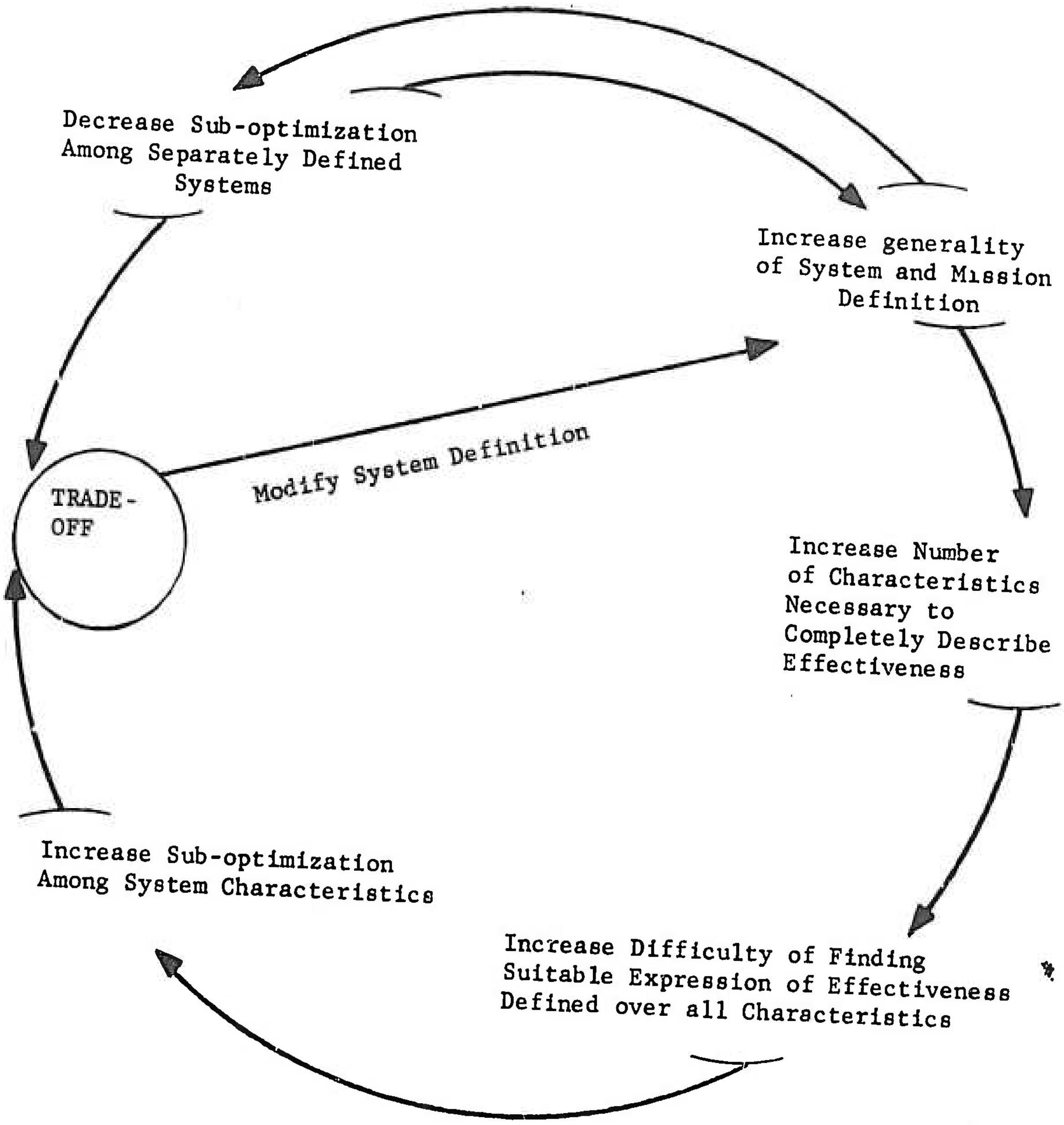
Decrease Sub-optimization Among Separately Defined Systems
Increase generality of System and Mission Definition
TRADE-OFF
Modify System Definition
Increase Number of Characteristics Necessary to Completely Describe Effectiveness
Increase Sub-optimization Among System Characteristics
Increase Difficulty of Finding Suitable Expression of Effectiveness Defined over all Characteristics

FIGURE II

M. C. Heuston and G. Ogawa[5] have listed 5 properties that this function must possess, and that will provide an ordering relationship comparable to the ordinal scale.

(1) Its domain of definition is that part of euclidean e-space satisfying the condition that a point is in that set when all its components (i.e., the $c_j^{(k)}$'s) are all non-negative. This non-negative requirement can always be satisfied through a simple translation of the origin.

(2) Its range (i.e., the value of the function) is also non-negative.

(3) When it is set equal to some constant, the resulting contour will define a hypersurface.

(4) Its first partial derivatives are all positive. This means that as any one characteristic is increased (all others remaining constant) the effectiveness of the system will increase.

(5) It is strictly quasi-concave- a property that can be shown to be equivalent to the "law of diminishing marginal returns". This means that as any one characteristic is increased (all others remaining constant) the effectiveness will increase, but at a decreasing rate.

These properties may provide some insight into the nature of this function, but are of little help in its construction. This task is usually (and necessarily) left to a decision maker who constructs the function in the form of his personal utility function. Assuming

[5] Heuston, M. C. and Ogawa, G., Observations on the Theoretical Basis of Cost-Effectiveness, Journal of the Operations Research Society of America, Vol. 14, No. 2, p. 242.

that the decision-maker's objectives are consistent with national objectives, the analyst's only concern is that the number and nature of characteristics presented to the decision maker are such that a well-formed, consistent utility function can be defined over this set (of characteristics). Whether E is considered to be some mathematical relationship or a utility function, the difficulty in obtaining a "good" E is dependent on the number of elements in $C^{(k)}$.

9.1 A Mathematical Model of E .

A common model that is found in a wide range of "effectiveness literature" possesses all but number 5 (quasi-concave) of the properties proposed above. This model provides a framework on which to apply relatively simple utility functions of only one variable.

$$E = \sum_{i=1}^{i=e} \left(W_i C_i^{(k)} \right) \qquad \text{(equation \#2}$$

where the w_i's are arbitrarily assigned "weighting factors" that represent the utility of the individual $c_i^{(k)}$'s.

This model has the appeal of simplicity, but possesses serious shortcomings when applied over a wide range of characteristic values. Its primary application is envisioned as "ordering" the effectiveness of several systems possessing identical <u>types</u> of characteristics. Unless this expression is envisioned as absolute, no suggestion is given concerning the comparison of systems not possessing identical type characteristics.

10. Conclusion.

From this analysis of the concept of system effectiveness, the following list of tasks, considerations, and decisions should be followed in progressing from system definition to a quantitative criterion of its value.

1. The mission should be defined as broadly as possible, consistent with some concept of how its ability to achieve this mission can be expressed quantitatively.

2. The system designed or envisioned to accomplish this mission should be explicitly defined out to some "boundary". This boundary must separate the system from its environment, and contributions to this mission from other elements or systems (outside this boundary) is incidental and outside of their own missions.

3. A criterion for judging the value of the system, or for choosing between alternative systems or designs must be formulated, and/or,

4. a method of optimizing the design or choice of the system must be devised.

5. Based on the optimization method chosen, certain types of measurements must be obtained for a complete set of characteristics at the highest level possible.

6. Depending on the number of elements in this set, a method of expressing the effectiveness of the system as a function of these elements must be designed, <u>or</u> if this cannot be obtained with the

desired confidence of its correctness,

7. the mission must be re-defined in such a manner that the effectiveness can be more confidently expressed.

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APPENDIX I

A Simplified, hypothetical missile system.

Description:	100 ballistic missiles, silo protected, with associated early-warning radar system and command and control system.
Mission:	To deter the enemy from a premptive strike.

Resources:

x_1 = steel plate

x_2 = propellent

x_3 = explosives

x_4 = electronic components

x_5 = concrete

x_6 = labor

x_7 = operating personnel

Production processes:

a_{11} = form steel plate into cylinder C.

a_{21} = mix propellent into formula F

a_{31} = 0

a_{41} = assemble guidance systems G

a_{51} = 0

a_{61} = assembly processes

a_{71} = 0

Basic system element:

$$y_1 = \sum_{i=1}^{i=7} (x_i a_{i,1}) \quad = \text{missile airframe (complete except for warhead)}$$

Production processes:

a_{12} = form shape B

a_{22} = 0

a_{32} = shape explosives into shape S

a_{42} = assemble fusing mechanism

a_{52} = 0

a_{62} = assembly

a_{72} = 0

Basic system element:

$$y_2 = \sum_{i=1}^{i=7} (x_i a_{i,2}) \quad = \text{warhead assembly}$$

Production processes:

a_{13} = form radar reflector R

a_{23} = 0

a_{33} = 0

a_{43} = assemble radar transceiver

a_{53} = build radar sites

a_{63} = assembly and construction

a_{73} = staff with operating crew

Basic system element:

$$y_3 = \sum_{i=1}^{i=7} (X_i a_{i,3}) = \text{radar early-warning system}$$

Production processes:

$a_{14} = a_{24} = a_{34} = a_{54} = 0$

a_{44} = assemble communication equipment

a_{64} = assembly and construction

a_{74} = staff command & control system with operating crew

Basic system element:

$$y_4 = \sum_{i=1}^{i=7} (X_i a_{i,4}) = \text{command and control subsystem}$$

Production processes:

a_{15} = form silo hardware

a_{25} = 0

a_{35} = 0

a_{45} = 0

a_{55} = build silos

a_{65} = assembly and construction

a_{75} = staff silo with launch crews

Basic system element:

$$y_5 = \sum_{i=1}^{i=7} (X_i a_{i,5}) = \text{silo}$$

Then:

$$\left((x_1, x_2, x_3, x_4, x_5, x_6, x_7)\right) \begin{bmatrix} a_{11} & a_{12} & \cdots, a_{15} \\ a_{21} & & \\ a_{31} & & \\ \cdot & & \\ \cdot & & \\ a_{71} & \cdots\cdots & , a_{75} \end{bmatrix} = \begin{bmatrix} y_1 = \text{airframe} \\ y_2 = \text{warhead} \\ y_3 = \text{early-warning system} \\ y_4 = \text{command \& control} \\ y_5 = \text{silos} \end{bmatrix}$$

Tactics and environmental matrix.

Enumerating the elements of $(b_{u,v})$ is not enlightning because of the generality necessary. This matrix imposes tactics, procedures, and environmental conditions on the system elements producing the basic system characteristics.

Then: $Y^T B$ = $c_1^{(1)}$ = burn time of propulsion system.

$c_2^{(1)}$ = flight profile parameters.

$c_3^{(1)}$ = accuracy of guidance system.

$c_4^{(1)}$ = probability of penetrating anti-missile defense.

$c_5^{(1)}$ = reliability of airframe.

$c_6^{(1)}$ = reliability of guidance system.

$c_7^{(1)}$ = range of early warning radar.

$c_8^{(1)}$ = reaction time of silo launch crew.

$c_9^{(1)}$ = reaction time of command and control system.

$c^{(1)}_{10}$ = reliability of fusing mechanism.

$c^{(1)}_{11}$ = fusing height.

$c^{(1)}_{12}$ = overpressure distribution of warhead.

$c^{(1)}_{13}$ = blast resistence of silo.

Second level set of characteristics.

Sub-sets of the set $C^{(1)}$ are now chosen to represent a set of heigher level characteristics.

$S^{(1)}_1 = (c^{(1)}_1, c^{(1)}_2, c^{(1)}_3)$ = (burn time, flight profile, accuracy of guidance system)

Then: $c^{(2)}_1 = f_{1,1}(S^{(1)}_1)$ = C.E.P of missile at specified targets

$S^{(1)}_2 = (c^{(1)}_4, c^{(1)}_5, c^{(1)}_6)$ = (probability of penetration, airframe reliability, guidance system reliability)

Then: $c^{(2)}_2 = f_{2,1}(S^{(1)}_2)$ = probability of missile reaching target.

$S^{(1)}_3 = (c^{(1)}_7, c^{(1)}_8, c^{(1)}_9)$ = (radar range, silo launch time, c&c launch time.)

Then: $c^{(2)}_3 = f_{3,1}(S^{(1)}_3)$ = probability that missile can be launched if attacked.

$S^{(1)}_4 = (c^{(1)}_{10}, c^{(1)}_{11}, c^{(1)}_{12})$ = (fusing reliability, fusing height, overpressure)

Then: $c^{(2)}_4 = f_{4,1}(S^{(1)}_4)$ = damage radius centered on explosion point.

$S^{(1)}_5 = (c^{(1)}_{13})$ = (blast resistance of silo.)

Then: $c^{(2)}_5 = f_{5,1}(S^{(1)}_5)$ = damage radius necessary for attacking missile.

All the elements in $C^{(2)}$ can be obtained, either through experimental testing or by obtaining $f_{j,1}$ analytically.

Third level set of characteristics.

Sub-sets of $C^{(2)}$ can now be chosen to represent the set, $(C^{(3)})$ of third level characteristics.

$S_1^{(2)} = (c_1^{(2)}, c_4^{(2)}) =$ (C.E.P. damage radius)

Then: $c_1^{(3)} = f_{1,2}(S_1^{(2)}) =$ (expected damage at specified targets.)

$S_2^{(2)} = (c_2^{(2)}, c_3^{(2)}, c_4^{(2)}) =$ (probability of missile reaching target, probability that missile can be launched, damage radius necessary for attacking missile)

Then: $c_2^{(3)} = f_{2,2}(S_2^{(2)}) =$ probability that missile will reach target area.

These elements of $C^{(3)}$ cannot be obtained through experimental testing, but the functions $(f_{1,2}, f_{2,2})$ can be constructed with reasonable confidence.

Effectiveness function.

We now have a set of characteristics containing only two, well defined, elements which completely describes the effectiveness of the system.

$$E = E (c_1^{(3)}, c_2^{(3)})$$

If we believe, as has been assumed here, that the ability of this missile system to deter the enemy from conducting a premptive strike is a function of these two elements only, any information concerning

E will contribute to obtaining a measurement for the effectiveness. Since $c_1^{(3)}$ and $c_2^{(3)}$ are both measurements on a ratio scale, it is possible to express the effectiveness of the system in absolute terms. If $c_1^{(3)} < a$ and/or $c_2^{(3)} < b$, where a and b are some minimum value of the characteristics, then we can say that the effectiveness will be zero. We may be able to establish two other constants p, and q such that if; $c_1^{(3)}$ p and $c_2^{(3)}$ q the effectiveness will be complete or 100%. These constants establish the end points of a ratio scale and we require the effectiveness as a function of $c_1^{(3)}$ and $c_2^{(3)}$ when:

$$a < c_1^{(3)} < p \qquad \text{and} \qquad b < c_2^{(3)} < q \quad .$$

From this point we can assign the problem to a decision maker's utility function or apply some model such as eq. 2 .

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13. ABSTRACT

This thesis offers, to those concerned with analysis of modern weapons systems, a general methodology for devising appropriate and meaningful measures of effectiveness. This methodology does not include a specific model for "plugging in" system parameters and mechanically "grinding out" system effectiveness. It is intended only as a general "plan" through which the researcher can channel his own judgment and experience. The primary purpose of this plan is to guide the researcher through a logical transition from a purely subjective, and more or less vague, concept of effectiveness to a useable and more explicit formulation.

Effectiveness is modeled as that single system characteristic positioned at the apex of a characteristic "pyramid". This pyramid is constructed with "layers" of progressively fewer and more subjective characteristics. Mathematical properties of measurements appropriate to these characteristics are discussed as a function of the intended use of the effectiveness measurement. The type of measurement required to meet an analysis objective is dependent on the objective function or optimization criteria chosen. Because of this fact, the types of measurements have been classified into four scales and each scale related to a particular type of objective function.

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