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GENERAL ATRONICS®

General Atronics Corporation
1200 E. Mermaid Lane
Philadelphia 18,
Pennsylvania

GENERAL ATRONICS CORPORATION

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EXTENDED TARGET DETECTION WITH EMPHASIS ON ACOUSTIC SURVEILLANCE

VOLUME II: EXTENSIONS AND SPECIFIC APPLICATIONS

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Prepared by
Burton S. Abrams

ABSTRACT

Based on the procedures and results of Volume I of this report, a general equation for output signal-to-noise ratio is derived for a received signal in the presence of reverberation with a non-flat spectrum as well as ambient noise with a flat spectrum. This result is then applied to a RAKE radiometer processing a monochromatic pulse and a linearly swept FM pulse returned from an extended fluctuating target in order to obtain, with reasonable assumptions, the output signal-to-noise ratio for these waveforms in algebraic form. It was not possible to put the output signal-to-noise ratio for the doppler invariant pulse in algebraic form, so that result is given in integral form. For all three waveforms, however, simple adjustments are outlined for application of the results to simpler processors (two-filter radiometer, weighted radiometer, matched filter) and simpler targets (extended non-fluctuating target, fluctuating point target, stationary point target). Also, an expression for processing gain is derived which relies on this output signal-to-noise ratio.

VOLUME II

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GLOSSARY OF PRINCIPAL SYMBOLS

NOTE: a tilde (~) above a quantity indicates a complex envelope.

A_k = target strength of the k^{th} specular reflector, dimensions of reciprocal time

A_p = target strength of the continuous portion of the target as assumed by the processor, dimensions of reciprocal time squared

A_T = target strength of the continuous portion of the target, dimensions of reciprocal time squared

$A_{T'}$ = target strength of a point target, dimensions of reciprocal time

b = one-half the radian sweep rate of the linearly swept FM pulse

$C(\tau,f)$ = Woodward ambiguity function of the transmitted waveform

E_x = energy of returned signal

G_p = processing gain

$K(t_1,t_2)$ = quadratic processing kernel

k_p = doppler compression factor for which the processor is designed

k_T = doppler compression factor of the target echo

N_{01} = one-sided thermal noise power density spectrum

N_{R1} = maximum value of the one-sided reverberation power density spectrum

$n(t)$ = thermal noise at the system input

P_{i0} = input power in the returned signal frequency band in the absence of signal

P_{iz} = input power in the returned signal frequency band in the presence of signal

P_0 = output power with the returned signal absent

P_z = output power with the returned signal present

R	= output signal-to-noise ratio
$R_{n+r}(\tau)$	= autocorrelation function of the input thermal noise plus reverberation
$R_p(u,\tau)$	= target scatter correlation as assumed by the processor
$R_T(u,\tau)$	= actual target scatter correlation
$R_{\tilde{z}}(t_1,t_2)$	= $E\{\tilde{z}(t_1)\tilde{z}^*(t_2)\}$
$r(t)$	= reverberation at the system input
$S_{n+r}(f)$	= power density spectrum of the input thermal noise plus reveberation
$S_p(f,\tau)$	= Fourier transform on u of $R_p(u,\tau)$
$S_r(f)$	= reverberation power density spectrum
$S_T(f,\tau)$	= Fourier transform on u of $R_T(u,\tau)$
$(S/N)_i$	= input signal-to-noise ratio in the frequency band of the returned signal
T	= duration of the actual transmitted pulse
u_k	= range delay of the k^{th} specular reflector of the target
u_0	= average range delay of the target
W	= returned signal bandwidth
$w(t)$	= total received waveform
$z(t)$	= received signal
α	= sweep parameter of the transmitted doppler invariant pulse
Δ_p	= one standard deviation of the radian doppler spread of the target as assumed by the processor
Δ_T	= one standard deviation of the radian doppler spread of the actual target
σ_{10}^2	= variance of P_{10}
σ_n^2	= variance of input thermal noise in the frequency band of the returned signal
σ_P	= one standard deviation of the time equivalent of the range spread of the target as assumed by the processor
σ_r^2	= variance of the input reverberation in the frequency band of the returned signal

σ_T = one standard deviation of the time equivalent of the range spread of the actual target

ω_c = transmitted radian carrier frequency

ω_{d0} = average radian doppler shift

ω_R = one standard deviation of the radian spread of the reverberation power density spectrum

I. Introduction

The objective of the analytical work performed in this volume is to express the output signal-to-noise ratio and the processing gain in forms that fulfill the following two requirements:

1. The expressions should be directly applicable to an actual acoustic detection system.
2. The formulas should preferably be in algebraic forms, rather than the integral forms of Volume I, to facilitate any future computations that may be performed.

To meet the first requirement, the results of Volume I, in which only flat noise is considered, must be extended to include interference from non-flat reverberation as well as from flat ambient noise. The second requirement will be met by assuming Gaussian envelopes for the transmitted waveforms and the continuous portions of target scattering functions so that the integrals can be put into algebraic forms.

II. Technical Summary

A. Signal-to-Noise Ratio with Non-Flat Reverberation

The output signal-to-noise ratio in general integral form for the situation in which the noise background consists of non-flat reverberation as well as flat ambient noise is derived in Appendix H. The derivation proceeds along the lines of Appendix D, in which the noise is assumed flat, and results in an output signal-to-noise ratio of

$$R = \frac{\frac{1}{W^2}\left|\iint S_p^*(-f;\tau)S_T(-f,\tau)C^*(k_p\tau,f)C(k_T\tau,f)d\tau df\right|^2}{\{4N_{01}{}^2\iint|S_p(-f;\tau)|^2|C(k_p\tau,f)|^2 d\tau df + \\ +\iint\{2N_{01}[S_{\tilde{r}}(f_2)+S_{\tilde{r}}(f_1-f_2)]+S_{\tilde{r}}(f_2)S_{\tilde{r}}(f_1-f_2)\}\cdot \\ \cdot\left|\int S_p(-f_1;\tau)C(k_p\tau,f)e^{-j(\omega_1-\omega_2)\tau}d\tau\right|df_1df_2\}} \qquad (1)$$

In the above equation $C(\tau,f)$ is the Woodward complex signal ambiguity function defined by

$$C(\tau,f) \triangleq \int\tilde{x}(t)\tilde{x}^*(t-\tau)e^{j\omega t}\,dt, \qquad (2)$$

in which $\tilde{x}(t)$ is the complex envelope of the transmitted signal. The time compression factor due to the doppler dispersion of the target is k_T, and k_p is the time compression factor assumed by the processor. $S_T(f;\tau)$ is the Fourier transform on the delay variable u of the target scatter correlation $R_T(u;\tau)$ discussed at length in Appendix A, and $S_p(f;\tau)$ is that target function for which the processor is designed. N_{01} is the one-sided ambient noise spectral density, and $S_{\tilde{r}}(f)$ is the spectral density of the reverberation complex envelope. W is the bandwidth of the returned signal.

B. Signal-to-Noise Ratio for Specific Waveforms, Targets, and Processors

Equation (1) has been applied to specific cases arising from all combinations of variations of conditions in the following categories:

1. Targets - point stationary target, point fluctuating target, extended nonfluctuating target, and extended fluctuating target.

2. Processors - matched filter, weighted radiometer, two-filter radiometer, and RAKE radiometer.

3. Waveforms - monochromatic pulse, linearly swept FM, and doppler invariant pulse.

This analysis has been performed in Appendix G. It may be noted that the first three targets are special cases of the extended fluctuating target, and the first three processors are special cases of the RAKE radiometer. Thus, the analysis has been performed in Appendix J for a RAKE radiometer processing a return from an extended fluctuating target. The other special cases are easily obtained from adjustments in the final result.

The analysis of Appendix J makes several assumptions to simplify the calculations:

1. Only that processing channel that is matched to the returned signal in range as well as doppler shift and dispersion is considered. Thus, in Equation (1), $k_p = k_T = k = 1+(f_d/f_c)$, where f_d is the doppler shift and f_c is the carrier frequency. Parameters that may or may not be matched are the doppler spread and the target extension.

2. To facilitate the integration indicated in (1), a Gaussian-shaped scatter correlation is used with the range spread and the doppler spread uncorrelated, so that its general form is

$$R_T(u;\tau) = [A_T e^{-\tau^2/2\Delta_T^2} e^{-(u-u_0)^2/2\sigma_T^2} + \sum_k A_k \delta(u-u_k)] e^{j\omega_{d0}\tau}$$

$$S_T(f;\tau) = [\sqrt{2\pi}\sigma_T \Delta_T e^{-\tau^2/2\Delta_T^2} e^{-\sigma_T^2\omega^2/2} e^{-j\omega u_0} + \sum_k A_k e^{-j\omega u_k}] e^{j\omega_{d0}\tau} \quad (3)$$

where u_0 = the average range delay of the target,

u_k = the range delay of the k^{th} specular reflector,

σ_T = one standard deviation of the range spread,

ω_d = 2π times the average doppler shift,

Δ_T = one standard deviation of the doppler spread

The above equation considers the target as having two contributing portions, the first term representing the extended continuous portion, and the second representing a number of specular components.

3. The scatter correlation assumed by the processor contains only the continuous part, so that its general form is

$$R_p(u;\tau) = A_p e^{-\tau^2/2\Delta_p^2} e^{-(u-u_0)^2/2\sigma_p^2} e^{j\omega_{d0}\tau}$$

$$S_p(f;\tau) = \sqrt{2\pi}\sigma_p A_p e^{-\tau^2/2\Delta_p^2} e^{-\sigma_p^2\omega^2/2} e^{-j\omega u_0} e^{j\omega_{d0}\tau} \quad (4)$$

4. The physical envelope of each type of pulse is assumed to be Gaussian with energy and peak amplitude the same as with a rectangular pulse. Thus, the complex envelopes used are

a. for the monochromatic pulse

$$\tilde{x}(t) = \sqrt{2E_x/T}\, e^{-(\pi/2)(t^2/T^2)} \quad (5)$$

in which E_x is the energy in the returned signal and T is the duration of the equivalent rectangular pulse;

b. for the linearly swept FM pulse

$$\tilde{x}(t) = \sqrt{2E_x/T}\, e^{-t^2(\pi/2T^2 - jb)} \quad (6)$$

in which 2b is the sweep rate in rps/sec;

c. for the doppler invariant pulse

$$\tilde{x}(t) = \sqrt{2E_x/T}\,\exp\{-[\frac{\pi t^2}{2T^2} + \frac{j\omega_c}{\alpha}\ln(1-\alpha t) + j\omega_c t]\}, \quad (7)$$

in which ω_c is the carrier radian frequency and α controls the sweep.

5. The power density spectrum of the reverberation complex envelope also taken to be Gaussian,

$$S_{\tilde{r}}(f) = 2N_{R_1}\, e^{-\omega^2/2\omega_R^2},$$

where N_{R_1} is the maximum value of the actual one-sided reverberation power density spectrum, and ω_R defines its bandwidth. Since the reverberation power density spectrum is determined by the convolution of the signal energy density spectrum with the intrinsic power density spectrum of the medium, ω_R depends on both the medium and the signal bandwidth.

From the above assumptions, algebraic forms of Equation (1) are derived in Appendix G for the monochromatic pulse and the linearly swept FM pulse. The output signal-to-noise ratio for the monochromatic pulse is shown to be

$$R = \frac{\sqrt{\pi}\frac{E_x^2}{W^2}\Delta_p T\sqrt{2T^2+\pi\Delta_p^2}\left[\frac{\sqrt{2\pi}\,\Delta_T\sigma_T A_T}{\sqrt{(\Delta_T^2T^2+\Delta_p^2T^2+\pi\Delta_p^2\Delta_T^2)(T^2+\pi\sigma_p^2+\pi\sigma_T^2)}} + \frac{\sum_k A_k e^{-(\frac{\pi}{2})\frac{(u_k-u_0)^2}{T^2+\pi\sigma_p^2}}}{\sqrt{(T^2+\pi\Delta_p^2)(T^2+\pi\sigma_p^2)}}\right]^2}{\left\{\frac{N_{01}^2}{\sqrt{T^2+2\pi\sigma_p^2}} + \frac{4N_{01}N_{R1}\Delta_p T\omega_R \exp\left[-\frac{2\omega_{d_0}^2(T^2+2\pi\sigma_p^2)\Delta_p^2T^2}{(T^2+2\pi\sigma_p^2)(2T^2+\pi\Delta_p^2+4\Delta_p^2\omega_R^2T^2)+\pi\Delta_p^2T^2}\right]}{\sqrt{(T^2+2\pi\sigma_p^2)(2T^2+\pi\Delta_p^2+4\Delta_p^2\omega_R^2T^2)+\pi\Delta_p^2T^2}} + \frac{2N_{R1}^2\Delta_p T\omega_R^2\exp\left[-\frac{2\omega_{d_0}^2\Delta_p^2T^2}{2T^2+\pi\Delta_p^2+2\Delta_p^2\omega_R^2T^2}\right]}{\sqrt{(4\pi\omega_R^2\sigma_p^2+2\omega_R^2T^4\pi)(2T^2+\pi\Delta_p^2+2\Delta_p^2\omega_R^2T^2)}}\right\}}. \quad (8)$$

For the linearly swept FM pulse:

$$R = \frac{\left\{\sqrt{\pi}\frac{E_x^2}{W^2}k^2\Delta_p^2T^2\left[\frac{\sqrt{2\pi}A_T\sigma_T\Delta_T}{\sqrt{T^2(\Delta_p^2+\Delta_T^2)+k^2\Delta_p^2\Delta_T^2[\pi+4b^2T^2(\sigma_p^2+\sigma_T^2)]}} + \frac{\sum_i A_i \exp\left\{-\frac{2k^2\Delta_p^2b^2T^2}{T^2+\pi k^2\Delta_p^2+4k^2\Delta_p^2\sigma_p^2b^2T^2}\right\}}{\sqrt{T^2+\pi k^2\Delta_p^2+4k^2\Delta_p^2\sigma_p^2b^2T^2}}\right]^2\right\}}{\left\{\frac{k\Delta_p N_{01}^2}{\sqrt{2T^2+\pi k^2\Delta_p^2+8k^2\sigma_p^2\Delta_p^2b^2T^2}} \div \frac{2\sqrt{\pi}N_{01}N_{R1}\omega_R\exp\left[-\frac{\pi\omega_{d0}^2}{2(2b^2T^2+\pi\omega_R^2)}\right]}{bT^2\sqrt{2b^2T^2+\pi\omega_R^2}} + \frac{\pi N_{R1}^2\omega_R^2\exp\left[-\frac{\pi\omega_{d0}^2}{4b^2T^2+\pi\omega_R^2}\right]}{bT^2\sqrt{\pi\omega_R^4+4b^2T^2\omega_R^2+4\pi b^2}}\right\}} . \tag{9}$$

As previously pointed out, the above equations apply to the general case of the RAKE radiometer processing a signal returned from an extended fluctuating target. For application to other processors and targets, the following modifications may be made:

1. nonfluctuating extended target: $\Delta_T \to \infty$
2. fluctuating point target: $\sigma_T \to 0$, $\sigma_T A_T = A_T'$, $A_k = 0$
3. stationary point target: $\Delta_T \to \infty$, $\sigma_T \to 0$, $\sigma_T A_T = A_T'$, $A_k = 0$
4. two-filter radiometer: $\Delta_p \to \infty$
5. weighted radiometer: $\sigma_p \to 0$
6. matched filter: $\Delta_p \to \infty$, $\sigma_p \to 0$.

The results of Equations (8) and (9) with the above conditions applied are tabulated in Table J-1 and Table J-2, respectively, in Appendix J.

As far as the doppler invariant case is concerned, an argument is advanced in Appendix J that the results for the linearly swept FM pulse should well approximate results with the doppler invariant pulse. The reasons are two-fold: the first relies essentially on the fact that for the parameters of interest, the ratio of pulse bandwidth to center frequency is much much less than one; secondly, only the channel matched in doppler dispersion is considered, minimizing the effect of this phenomenon on the linearly swept FM ambiguity function. Nevertheless, because the argument presented is not iron-clad, the output signal-to-noise ratio for the doppler invariant pulse returned from an extended fluctuating target and processed by a RAKE radiometer is written in integral form as

$$R = \frac{\left\{\frac{E_x^2}{T^2W^2}\left|\iint e^{-\frac{\tau^2}{2k^2\Delta_{p_e}^2}-\frac{\sigma_p^2\omega^2}{2}}\left[\sqrt{2\pi}\sigma_T A_T e^{-\frac{\tau^2}{2k^2\Delta_{T_e}^2}-\frac{\sigma_T^2\omega^2}{2}}+\sum_i A_i e^{-j\omega(u_i-u_0)}\right]\cdot \left|\int e^{-\frac{\pi}{2T^2}[t^2+(t-\tau)^2]} e^{j\frac{\omega_c}{\alpha}\ln\left[\frac{1-\alpha\tau}{1-\alpha t+\alpha\tau}\right]} e^{j\omega\tau}dt\right|^2 d\tau df\right|^2\right\}}{\left\{N_{01}^2\iint e^{-\frac{\tau^2}{k^2\Delta_{p_e}^2}-\sigma_p^2\omega^2}\left|\int e^{-\frac{\pi}{2T^2}[t^2+(t-\tau)^2]} e^{j\frac{\omega_c}{\alpha}\ln\left[\frac{1-\alpha t}{1-\alpha t+\alpha\tau}\right]} e^{-j\omega_c\tau}e^{j\omega t}dt\right|^2\right.}$$

$$\cdot d\tau df + N_{01}N_{R1}\iint\left[e^{-\frac{\omega_2^2}{2\omega_R^2}}+e^{-\frac{(\omega_1-\omega_2)^2}{2\omega_R^2}}\right]\left|\iint e^{-\frac{\tau^2}{2k^2\Delta_{p_e}^2}-\frac{\sigma_p^2\omega_1^2}{2}}\right. \cdot$$

$$\cdot e^{-\frac{\pi}{2T^2}[t^2+(t-\tau)^2]} e^{j\frac{\omega_c}{\alpha}\ln\left[\frac{1-\alpha t}{1-\alpha t+\alpha\tau}\right]} e^{-j(\omega_1-\omega_2+\omega_c-\omega_{d0})\tau} dt d\tau\Big|^2 df_1 df_2$$

$$+N_{R1}^2\iint e^{-\frac{\omega_2^2}{2\omega_R^2}-\frac{(\omega_1-\omega_2)^2}{2\omega_R^2}}\left|\iint e^{-\frac{\tau^2}{2k^2\Delta_{p_e}^2}-\frac{\sigma_p^2\omega_1^2}{2}} e^{-\frac{\pi}{2T^2}[t^2+(t-\tau)^2]}\right. \cdot$$

$$\left.\left.\cdot e^{j\frac{\omega_c}{\alpha}\ln\left[\frac{1-\alpha t}{1-\alpha t+\alpha\tau}\right]} e^{-j(\omega_1-\omega_2+\omega_c-\omega_{d0})\tau} dt d\tau\right|^2 df_1 df_2\right\} \qquad (10)$$

It was not possible to put this in a simpler algebraic form. The same modifications for simpler targets and processors may be made in (10) as are made for the monochromatic pulse results of (8) and the linearly swept FM pulse results of (9).

C. Processing Gain

Processing gain is treated here as it is in Volume I, except for the addition of a non-flat reverberation spectrum to the flat ambient noise spectrum. The processing gain is defined as

$$G_p \triangleq \frac{\sqrt{R}}{(S/N)_i}, \tag{11}$$

in which R is the output signal-to-noise ratio, and $(S/N)_i$ is an appropriately defined input signal-to-noise ratio.

The input signal-to-noise ratio is defined in Appendix K in two ways which are shown in that appendix to be mathematically equivalent.

(a) $(S/N)_i$ is the ratio of average power in the signal complex envelope to average power in the total noise complex envelope, measured at the output of a simple filter that removes "out-of-band" noise.

(b) By sampling the squared envelope of the input after the same simple filtering to obtain a test statistic, we can define

$$(S/N)_i = \frac{\overline{P}_{iz}-\overline{P}_{io}}{\sigma_{io}} \tag{12}$$

where the sampled value of the input is P_{iz} in the presence of signal and P_{io} in its absence, σ_{io}^2 is the variance of P_{io}, and the overbar indicates an ensemble average.

The simple filter described in both definitions is used to limit the noise to the same band of frequencies occupied by the signal, that is, to remove as much interference as possible by simple filtering. For simplification of the calculations it is assumed to have a Gaussian amplitude characteristic

$$|H(f)| = e^{-\frac{(\omega-\omega_c-\omega_{d0})^2}{4\pi W^2}} + e^{-\frac{(\omega+\omega_c+\omega_{d0})^2}{4\pi W^2}}, \tag{13}$$

where ω_c is the radian carrier frequency, ω_d is the radian doppler shift, and W is the one-sided noise bandwidth in Hz. The filter than operates on the noise complex envelope is

$$|\tilde{H}(f)| = 2e^{-\frac{(\omega-\omega_{d0})^2}{4\pi W^2}} . \tag{14}$$

As in the calculation of output signal-to-noise ratio, the reverberation power density spectrum is assumed to have a Gaussian shape

$$S_r(f) = \tfrac{1}{2}N_{R1}[e^{-\frac{(\omega-\omega_c)^2}{2\omega_R^2}} + e^{-\frac{(\omega+\omega_c)^2}{2\omega_R^2}}]$$

$$S_{\tilde{r}}(f) = 2N_{R1}e^{-\frac{\omega^2}{2\omega_R^2}} \tag{15}$$

With N_{01} representing the one-sided spectral density of ambient noise and a target characterized by Equation (3), the processing gain is shown in Appendix K to be given by

$$G_p = \frac{W^2\sqrt{R}[N_{01} + \frac{\omega_R N_{R1}}{\sqrt{2\pi W^2+\omega_R^2}} \exp\{-\frac{\omega_{d0}^2}{2(2\pi W^2+\omega_R^2)}\}]}{E_x[\frac{\sqrt{2\pi}\sigma_T A_T}{\sqrt{2\pi\sigma_T^2+T^2}} + \sum_k A_k \exp\{-\frac{\pi}{T^2}(u_k-u_0)^2\}]} \tag{16}$$

In the above equation R is given by (8) for a monochromatic pulse, (9) for a linearly swept FM pulse, and (10) for a doppler invariant pulse, where the target is extended and fluctuating and the processor is a RAKE radiometer. For simpler targets and processors, the following modifications apply:

1. non-fluctuating extended target: $\Delta_T \to \infty$
2. fluctuating point target: $\sigma_T \to 0$, $\sigma_T A_T = A_T'$, $A_k = 0$
3. stationary point target: $\Delta_T \to \infty$, $\sigma_T \to 0$, $\sigma_T A_T = A_T'$, $A_k = 0$
4. two-filter radiometer: $\Delta_p \to \infty$
5. weighted radiometer: $\sigma_p \to 0$
6. matched filter: $\Delta_p \to \infty$, $\sigma_p \to 0$

III. Discussion and Conclusions

The results obtained in this volume for evaluating target detectors are primarily extensions of the general development presented in Volume I, along with the application of these extended results to specific targets, processors, and waveforms.

The extended results involve mainly the derivation of general forms for the output signal-to-noise ratio and the system processing gain when a non-flat reverberation power density spectrum is added at the input to the flat power density spectrum of white noise. The effect of these two noise contributors may be found in the denominator of the general equation for the processor output signal-to-noise ratio. The denominator consists of three terms: one involving only thermal noise, the second involving reverberation, and the third representing interaction between the two. The thermal noise term is, of course, the total denominator when reverberation is neglected.

Extension of the general formula for processing gain has required modification of the input signal-to-noise ratio as well as the output signal-to-noise ratio when reverberation is considered. The input thermal plus reverberation noise power only in the frequency band of the echo has been considered for this derivation.

In order to apply these general results to specific cases, certain approximations were made to permit the general expressions to be put into algebraic form. These approximations involve the assumption of Gaussian shapes for the continuous portion of the actual target scatter correlation as well as the scatter correlation assumed by the processor; for the shape of the reverberation power density spectrum; and for the physical envelope of the pulses considered. Furthermore, only the processing channel matched in average doppler and average delay to the returned signal has been considered.

The waveforms considered are the monochromatic pulse, the linearly swept FM pulse, and the doppler invariant pulse. The above approximations have allowed the output signal-to-noise ratio for the monochromatic pulse and for the linearly swept FM pulse to be expressed in an algebraic form, making for easy computation. The result for the doppler invariant pulse, however, had to be left in integral form.

The target types considered are the extended fluctuating target and, as special cases of it, the nonfluctuating extended target, the fluctuating point target, and the stationary point target. The processors considered are the RAKE radiometer and, as its special cases, the two-filter radiometer, the weighted radiometer, and the matched filter. These processors all have a form which may be expressed as a quadratic processing kernel.

APPENDIX H
SIGNAL-TO-NOISE RATIO WITH A NON-FLAT REVERBERATION SPECTRUM

Consideration will be given here to the problem of a non-flat spectrum of noise, specifically one arising from reverberation. The output signal-to-noise ratio, as defined in Appendix D, will be derived here for the case of flat thermal noise and non-flat reverberation. The signal-to-noise ratio is

$$R = \frac{(\overline{P}_z-\overline{P}_0)^2}{\overline{P_0^{\,2}}-\overline{P}_0^{\,2}} \tag{H-1}$$

where P_z = P with echo present,
P_0 = P with echo absent,
and the overbar indicates an ensemble average.

As in Appendix D, the effect of doppler dispersion has been neglected at the outset. A simple method for its inclusion, which is derived in Appendix G, will be applied at the end of this signal-to-noise ratio derivation.

With $\tilde{z}(t)$ the complex envelope of the returned signal, $\tilde{n}(t)$, the complex envelope of thermal noise, and $\tilde{r}(t)$, the complex envelope of reverberation, we have as the system input

$$\tilde{w}(t) = \tilde{z}(t)+\tilde{n}(t)+\tilde{r}(t) \tag{H-2}$$

when signal is present, and

$$\tilde{w}(t) = \tilde{n}(t)+\tilde{r}(t) \tag{H-3}$$

when there is no signal. Applying these definitions to Equation (C-13) gives

$$\overline{P}_z-\overline{P}_0 = \iint K(t_1,t_2)\ \overline{[\tilde{z}^*(t_1)+\tilde{n}^*(t_1)+\tilde{r}^*(t_1)][\tilde{z}(t_2)+\tilde{n}(t_2)+\tilde{r}(t_2)]}$$

$$- \overline{[\tilde{n}^*(t_1)+\tilde{r}^*(t_1)][\tilde{n}(t_2)+\tilde{r}(t_2)]}\ dt_1dt_2 \tag{H-4}$$

By making the reasonable assumption of mutual independence among $\tilde{z}(t)$, $\tilde{n}(t)$ and $\tilde{r}(t)$, we have

$$\overline{P}_z-\overline{P}_0 = \iint K(t_1,t_2)\tilde{z}^*(t_1)\tilde{z}(t_2)dt_1dt_2$$

$$= \iint K(t_1,t_2)R_{\tilde{z}}^*(t_1,t_2)dt_1dt_2 \qquad \text{(H-5)}$$

just as in Equation (D-9). This has been shown in Appendix D to be equivalent to

$$(\overline{P}_z-\overline{P}_0)^2 = \frac{1}{W^4}\left|\iint S_p^*(-f;\tau)S_T(-f;\tau)|C(\tau,f)|^2 d\tau df\right|^2, \qquad \text{(H-6)}$$

where the following definitions apply:

$$S_p(f;\tau) = S_p^*(f;-\tau) \triangleq \int_{-\infty}^{\infty} R_p(u;\tau)e^{-j\omega u}\,du$$

$$S_T(f;\tau) = S_T^*(f,-\tau) \triangleq \int_{-\infty}^{\infty} R_T(u;\tau)e^{-j\omega u}\,du, \qquad \text{(H-7)}$$

in which R_p is the scatter correlation for which the processor is designed, and R_T is the scatter correlation of the actual target;

$$C(\tau,f) \triangleq \int_{-\infty}^{\infty} \tilde{x}(t)\tilde{x}^*(t-\tau)e^{j\omega t}\,dt, \qquad \text{(H-8)}$$

the Woodward complex signal ambiguity function of the echo that would be returned from a moving point target reflecting the same amount of energy as the actual target.

The numerator of R, as given in (H-1), is expressed in (H-6). In computing the denominator $\overline{P}_0$ will first be written and its square then subtracted from $\overline{P_0^2}$.

From Equation (C-13) we have

$$\overline{P}_0 = \iint K(t_1,t_2)\overline{[\tilde{n}^*(t_1)+\tilde{r}^*(t_1)][\tilde{n}(t_2)+\tilde{r}(t_2)]}\,dt_1dt_2$$

$$= \iint K(t_1,t_2)[\overline{\tilde{n}^*(t_1)\tilde{n}(t_2)}+\overline{\tilde{r}^*(t_1)\tilde{r}(t_2)}]\,dt_1dt_2 \qquad \text{(H-9)}$$

where the two processes are assumed independent, and

$$\overline{P_0^2} = \iiiint K(t_1,t_2)K(t_3,t_4)\overline{[\tilde{n}^*(t_1)+\tilde{r}^*(t_1)][\tilde{n}(t_2)+\tilde{r}(t_2)]\cdot}$$

$$\overline{\cdot\,[\tilde{n}^*(t_3)+\tilde{r}^*(t_3)][\tilde{n}(t_4)+\tilde{r}(t_4)]}\,dt_1dt_2dt_3dt_4. \qquad \text{(H-10)}$$

In order to handle the reverberation, it will be assumed to be a bandpass process that is wide-sense stationary over any range interval larger than that corresponding to a ping duration. Furthermore, it is assumed to be zero-mean and Gaussian. Since these characteristics also apply to the thermal noise n(t), the following property may be used:*

$$\overline{[\tilde{n}^*(t_1)+\tilde{r}^*(t_1)][\tilde{n}(t_2)+\tilde{r}(t_2)][\tilde{n}^*(t_3)+\tilde{r}^*(t_3)][\tilde{n}(t_4)+\tilde{r}(t_4)]} =$$
$$= R^*_{\tilde{n}+\tilde{r}}(t_1-t_2)R^*_{\tilde{n}+\tilde{r}}(t_3-t_4)+R^*_{\tilde{n}+\tilde{r}}(t_1-t_4)R_{\tilde{n}+\tilde{r}}(t_2-t_3) \quad \text{(H-11)}$$

Inserting (H-11) into (H-10) and squaring (H-9) gives

$$\overline{P_0^2}-\bar{P}_0^2 = \iiiint K(t_1,t_2)K(t_3,t_4)R^*_{\tilde{n}+\tilde{r}}(t_1-t_4)R_{\tilde{n}+\tilde{r}}(t_2-t_3)dt_1dt_2dt_3dt_4 \quad \text{(H-12)}$$

Using Equation s (C-14) and (D-17) we have

$$\overline{P_0^2}-\bar{P}_0^2 = \iiiint K(t_1,t_2)K^*(t_4,t_3)R^*_{\tilde{n}+\tilde{r}}(t_1-t_4)R_{\tilde{n}+\tilde{r}}(t_2-t_3)dt_1dt_2dt_3dt_4$$
$$= \frac{1}{W^2}\iiiiiint \tilde{x}(t_1-u_1)\tilde{x}^*(t_2-u_1)R_p(u_1;t_1-t_2)\tilde{x}^*(t_4-u_2)\tilde{x}(t_3-u_2)\cdot$$
$$\cdot R^*_p(u_2;t_4-t_3)R^*_{\tilde{n}+\tilde{r}}(t_1-t_4)R_{\tilde{n}+\tilde{r}}(t_2-t_3)du_1du_2dt_1dt_2dt_3dt_4 \quad \text{(H-13)}$$

Employing the definitions of (H-7) and (H-8), and noting that

$$\int\tilde{x}(v)\tilde{x}^*(v-\tau)R_p(t-v,\tau)dv = \int S_p(-f;\tau)C(\tau,f)e^{-j\omega t}\,df \quad \text{(H-14)}$$

$$\int R^*_{\tilde{n}+\tilde{r}}(t)R_{\tilde{n}+\tilde{r}}(t-\tau)e^{-j\omega t}dt = \int S^*_{\tilde{n}+\tilde{r}}(-f_1)S_{\tilde{n}+\tilde{r}}(f-f_1)e^{-j(\omega-\omega_1)\tau}df_1 \quad \text{(H-15)}$$

and the assumption that

$$S_{\tilde{n}+\tilde{r}}(-f) = S_{\tilde{n}+\tilde{r}}(f), \quad \text{(H-16)}$$

allows manipulation of Equation (H-13) to obtain

*I.S. Reed, "On a moment theorem for complex Gaussian processes", IRE *Trans* Info Th, vol. IT-8, no. 3, pp. 194-195, April 1962. A.H. Nuttall, "Higher order covariance function for complex Gaussian Processes", IRE *Trans* Info Th, vol. IT-8, no. 3, pp. 255-256.

$$\overline{P_0^2}-\overline{P}_0^2 = \frac{1}{W^2}\iiiint S_p(-f_1;\tau_1)S_p^*(-f_1,\tau_2)C(\tau_1,f_1)C^*(\tau_2,f_1)e^{-j(\omega_1-\omega_2)(\tau_1-\tau_2)}$$

$$\cdot\ S_{\tilde{n}+\tilde{r}}(f_2)S_{\tilde{n}+\tilde{r}}(f_1-f_2)df_1df_2d\tau_1d\tau_2$$

$$= \frac{1}{W^2}\iint S_{\tilde{n}+\tilde{r}}(f_2)S_{\tilde{n}+\tilde{r}}(f_1-f_2)\left|\int S_p(-f_1;\tau)C(\tau,f_1)e^{-j(\omega_1-\omega_2)\tau}d\tau\right|^2$$

$$\cdot\ df_1df_2 \qquad \text{(H-17)}$$

Because the reverberation and the white thermal noise are independent,

$$S_{\tilde{n}+\tilde{r}}(f) = S_{\tilde{n}}(f) + S_{\tilde{r}}(f). \qquad \text{(H-18)}$$

Since the complex envelope processor is concerned only with a band of frequencies around DC, that is, narrow compared to the carrier frequency f_c, we can use equation (E-23) to write

$$S_{\tilde{n}+\tilde{r}}(f) = 2N_0 + S_{\tilde{r}}(f). \qquad \text{(H-19)}$$

Then (H-17) becomes

$$\overline{P_0^2}-\overline{P}_0^2 = \frac{4N_{01}^2}{W^2}\iint|S_p(-f;\tau)|^2|C(\tau,f)|^2d\tau df + \frac{1}{W^2}\iint\{2N_{01}[S_{\tilde{r}}(f_2)+$$

$$+S_{\tilde{r}}(f_1-f_2)]+S_{\tilde{r}}(f_2)S_{\tilde{r}}(f_1-f_2)\}\left|\int S_p(-f_1;\tau)C(\tau,f_1)\ \cdot\right.$$

$$\left.\cdot e^{-j(\omega_1-\omega_2)\tau}d\tau\right|^2df_1df_2. \qquad \text{(H-20)}$$

The output signal-to-noise ratio of Equation (H-1), using Equations (H-6) and (H-20), is

$$R = \frac{\frac{1}{W^2}\left|\iint S_p^*(-f;\tau)S_T(-f;\tau)|C(\tau,f)|^2d\tau df\right|^2}{\{4N_{01}^2\iint|S_p(-f;\tau)|^2|C(\tau,f)|^2d\tau df+\iint\{2N_{01}[S_{\tilde{r}}(f_2)+S_{\tilde{r}}(f_1-f_2)]+ +S_{\tilde{r}}(f_2)S_{\tilde{r}}(f_1-f_2)\}\left|\int S_p(-f_1;\tau)C(\tau,f_1)e^{-j(\omega_1-\omega_2)\tau}d\tau\right|^2df_1df_2\}}$$

(H-21)

Note that when there is no reverberation ($S_r(f) = 0$), the above equation reduces to the case of a flat noise spectrum, as given in Equation (D-27).

As shown in Appendix G, accounting for the effects of doppler dispersion gives

$$R = \frac{\frac{1}{W^2}\left|\iint S_p^*(-f;\tau) S_T(-f;\tau) C^*(k_p\tau,f) C(k_T\tau,f)\, d\tau\, df\right|^2}{\left\{4N_{01}^2 \iint |S_p(-f;\tau)|^2 |C(k_p\tau,f)|^2 d\tau\, df + \iint \left\{2N_{01}[S_{\tilde{r}}(f_2) + S_{\tilde{r}}(f_1-f_2)] + S_{\tilde{r}}(f_2) S_{\tilde{r}}(f_1-f_2)\right\} \left|\int S_p(-f_1;\tau) C(k_p\tau,f_1) e^{-j(\omega_1-\omega_2)\tau} d\tau\right|^2 df_1\, df_2\right\}} \tag{H-22}$$

where k_T is the time compression factor introduced by the target and k_p is the time compression factor assumed by the processor. For the calculations to follow, it will be assumed that the processor is correctly compensated for the effect of target motion on the returned signal, in which case $k_p = k_T$.

APPENDIX J

SIGNAL-TO-NOISE RATIO FOR SPECIFIC WAVEFORMS, TARGETS AND PROCESSORS

Using the results of Appendix H, specific forms of the output signal-to-noise ratio will be written down for all combinations of target processors and waveforms. These variations will include:

1. Targets - point stationary target, point fluctuating target, extended non-fluctuating target, and extended fluctuating target.
2. Processors - matched filter, weighted radiometer, two-filter radiometer, and RAKE radiometer.
3. Waveforms - monochromatic pulse, linearly swept FM, and doppler invariant pulse

In all cases the results will apply only to that processing channel that is matched in doppler shift and doppler dispersion, as well as average time delay, to the returned signal. Doppler dispersion will be neglected initially, and then will be included using the method given in Appendix G.

The first three target types are actually special cases of the extended fluctuating target. Based on this observation, the output signal-to-noise ratio will be derived for the extended fluctuating target, and the other results will be found from this result by neglecting the appropriate contributors in the expression.

The scatter correlation for the extended fluctuating target will have two components: one arising from the extended portions of the target, and one representing the specular reflectors. For analytical simplification, both the range dependence and the doppler dependence of the extended portion of the scatter correlation will be assumed to be independently Gaussian. The scatter correlation will be

$$R_T(u;\tau) = [A_T e^{-\tau^2/2\Delta_T^2} e^{-(u-u_0)^2/2\sigma_T^2} + \sum_k A_k \delta(u-u_k)] e^{j\omega_{d0}\tau}, \quad \text{(J-1)}$$

where u_0 = the average range delay of the target

σ_T = one standard deviation of the range spread

ω_{d0} = the average radian doppler shift

Δ_T = one standard deviation of the doppler spread

u_k = the range delay of the k^{th} specular reflector.

Thus, from (J-1) and (H-7).

$$S_T(f;\tau) = [\sqrt{2\pi}\sigma_T A_T e^{-\tau^2/2\Delta_T^2} e^{-(\sigma_T^2/2)\omega^2} e^{-j\omega u_0} + \sum_k A_k e^{-j\omega u_k}] e^{j\omega_{d0}\tau} \quad \text{(J-2)}$$

A similar situation exists in the set of processors to be considered. The RAKE radiometer, being the most general one, will be used in the derivation of output signal-to-noise ratio. The scatter correlation assumed by the processor will not include any specular reflectors, so it will be taken as

$$R_p(u;\tau) = A_p e^{-\tau^2/2\Delta_p^2} e^{-(u-u_0)^2/2\sigma_p^2} e^{j\omega_{d0}\tau}, \quad \text{(J-3)}$$

and

$$S_p(f;\tau) = \sqrt{2\pi}\sigma_p A_p e^{-\tau^2/2\Delta_p^2} e^{-\sigma_p^2\omega^2/2} e^{-j\omega u_0} e^{j\omega_{d0}\tau} \quad \text{(J-4)}$$

1. Monochromatic Pulse

Although the envelope of the monochromatic pulse transmitted by the CASS system is well represented by a rectangular pulse, for analytical convenience the envelope will be taken to be Gaussian-shaped with the same peak amplitude and the same energy as its rectangular equivalent. In this case the complex envelope is given by

$$\tilde{x}(t) = \sqrt{2E_x/T}\, e^{-\frac{\pi}{2}\frac{t^2}{T^2}} \qquad \text{(J-5)}$$

where E_x is the energy in the returned signal and T is the duration of the actual rectangular pulse. Its Woodward ambiguity function, as defined in (H-8), is

$$\begin{aligned} C(\tau,f) &= 2E_x/T \int e^{-\frac{\pi}{2}\frac{t^2}{T^2}} e^{-\frac{\pi}{2T^2}(t-\tau)^2} e^{j\omega t}\, dt \\ &= 2E_x\, e^{-\frac{\pi}{4}\frac{\tau^2}{T^2}} e^{-\frac{\omega^2 T^2}{4\pi}} e^{j\frac{\omega\tau}{2}} \end{aligned} \qquad \text{(J-6)}$$

Using (J-2, (J-4) and (J-6) in H-6) yields

$$\begin{aligned} (\bar{P}_{\tilde{z}}-\bar{P}_0)^2 = \frac{1}{W^4}\Big|\iint[&4\sigma_p\sigma_T A_p A_T E_x^2 e^{-\frac{\tau^2}{2}\left(\frac{1}{\Delta_p^2}+\frac{1}{\Delta_T^2}+\frac{\pi}{T^2}\right)} e^{-\frac{\omega^2}{2}\left(\sigma_p^2+\sigma_T^2+\frac{T^2}{\pi}\right)} \\ &+ \frac{4\sigma_p A_p}{\sqrt{2\pi}} E_x{}^2 e^{-\frac{\tau^2}{2}\left(\frac{1}{\Delta_p^2}+\frac{\pi}{T^2}\right)} e^{-\frac{\omega^2}{2}\left(\sigma_p^2+\frac{T^2}{\pi}\right)} \sum_k A_k e^{j\omega(u_k-u_0)}]\, d\tau\, d\omega\Big|^2 \end{aligned} \qquad \text{(J-7)}$$

Using the relationship that

$$\int e^{-\alpha t^2} e^{j\beta t}\, dt = \sqrt{\pi/\alpha}\; e^{-\beta^2/4\alpha} \qquad \text{(J-8)}$$

Equation (J-7) can be evaluated as

$$\begin{aligned} (\bar{P}_{\tilde{z}}-\bar{P}_0)^2 = 32\pi^2\sigma_p^2 A_p^2 \Delta_p^2 \frac{T^2 E_x{}^4}{4}\Big[&\frac{\sqrt{2\pi}\,\Delta_T\,\sigma_T\,A_T}{\sqrt{(\Delta_T^2T^2+\Delta_p^2T^2+\pi\Delta_p^2\Delta_T^2)(T^2+\pi\sigma_p^2+\pi\sigma_T^2)}} \\ &+ \frac{\sum_k A_k \exp\{-\frac{\pi}{2}\frac{(u_k-u_0)}{(T^2+\pi\sigma_p{}^2)}\}}{\sqrt{(T^2+\pi\Delta_p^2)(T^2+\pi\sigma_p^2)}}\Big]^2 \end{aligned} \qquad \text{(J-9)}$$

For case of analysis, the reverberation power density spectrum resulting from a monochromatic pulse will be assumed to be Gaussian-shaped, so that

$$S_r(f) = \frac{1}{2}N_{R1}\left[e^{-(\omega-\omega_c)^2/2\omega_R^2} + e^{-(\omega+\omega_c)^2/2\omega_R^2}\right]$$

$$S_{\tilde{r}}(f) = 2N_{R1}\, e^{-\omega^2/2\omega_R^2} \qquad \text{(J-10)}$$

where f_c is the transmitted carrier frequency, and ω_R defines the spectrum spread.

Inserting Equations (J-4), (J-6) and (J-10) into Equation (H-20) gives

$$\overline{P_0{}^2} - \overline{P}_0{}^2 = \frac{16N_{01}{}^2}{W^2}\iint \sigma_p^2 A_p^2 E_x^2 e^{-\tau^2\left(\frac{1}{\Delta_p^2} + \frac{\pi}{2T^2}\right)} e^{-\omega^2\left(\sigma_p^2 + \frac{T^2}{2\pi}\right)} d\tau d\omega$$

$$+ \frac{8N_{01}N_{R1}}{\pi W^2}\iint \sigma_p^2 A_p^2 E_x^2 \left[e^{-\omega_2^2/2\omega_R^2} + e^{-(\omega_1-\omega_2)^2/2\omega_R^2}\right] e^{-\omega_1^2\left(\sigma_p^2 + \frac{T^2}{2\pi}\right)}$$

$$\cdot \left|\int e^{-\frac{\tau^2}{2}\left(\frac{1}{\Delta_p^2} + \frac{\pi}{2T^2}\right)} e^{-j\left(\frac{\omega_1}{2} - \omega_2 - \omega_{d0}\right)\tau} d\tau\right|^2 d\omega_1 d\omega_2$$

$$+ \frac{8N_{R1}{}^2}{\pi W^2}\iint \sigma_p^2 A_p^2 E_x^2 e^{-\omega_2^2/2\omega_R^2} e^{-(\omega_1-\omega_2)^2/2\omega_R^2} e^{-\omega_1^2\left(\sigma_p^2 + \frac{T^2}{2\pi}\right)}$$

$$\cdot \left|\int e^{-\frac{\tau^2}{2}\left(\frac{1}{\Delta_p^2} + \frac{\pi}{2T^2}\right)} e^{-j\left(\frac{\omega_1}{2} - \omega_2 - \omega_{d0}\right)\tau} d\tau\right|^2 d\omega_1 d\omega_2 \qquad \text{(J-11)}$$

Applying Equation (J-8) gives

$$\left|\int e^{-\frac{\tau^2}{2}\left(\frac{1}{\Delta_p^2} + \frac{\pi}{2T^2}\right)} e^{-j\left(\frac{\omega_1}{2} - \omega_2 - \omega_{d0}\right)\tau} d\tau\right|^2 = \frac{4\pi\Delta_p{}^2T^2}{2T^2 + \pi\Delta_p{}^2}$$

$$\cdot\, e^{-\frac{\Delta_p^2 T^2(\omega_1 - 2\omega_2 - 2\omega_{d0})^2}{2(2T^2 + \pi\Delta_p{}^2)}} \qquad \text{(J-12)}$$

Thus,

$$\overline{P_0^{\,2}}-\overline{P}_0^{\,2} = \frac{16N_{01}^{\,2}}{W^2}\iint \sigma_p^2 A_p^2 E_x^2 e^{-\tau^2\left(\frac{1}{\Delta_p^2}+\frac{\pi}{2T^2}\right)} e^{-\omega^2\left(\sigma_p^2+\frac{T^2}{2\pi}\right)} d\tau d\omega$$

$$+ \frac{32N_{01}N_{R1}\Delta_p^2T^2}{W^2(2T^2+\pi\Delta_p^2)}\iint \sigma_p^2 A_p^2 E_x^2 e^{-\omega_2^2/2\omega_R^2} e^{-\omega_1^2\left(\sigma_p^2+\frac{T^2}{2\pi}\right)} e^{-\frac{\Delta_p^2T^2(\omega_1-2\omega_2-2\omega_{d0})^2}{2(2T^2+\pi\Delta_p^{\,2})}} d\omega_1 d\omega_2$$

$$+ \frac{32N_{01}N_{R1}\Delta_p^2T^2}{W^2(2T^2+\pi\Delta_p^2)}\iint \sigma_p^2 A_p^2 E_x^2 e^{-\frac{(\omega_1-\omega_2)^2}{2\omega_R^{\,2}}} e^{-\omega_1^2\left(\sigma_p^2+\frac{T^2}{2\pi}\right)} e^{-\frac{\Delta_p^2T^2(\omega_1-2\omega_2-2\omega_{d0})^2}{2(2T^2+\pi\Delta_p^{\,2})}} d\omega_1 d\omega_2$$

$$+ \frac{32N_{R1}^2\Delta_p^2T^2}{W^2(2T^2+\pi\Delta_p^2)}\iint \sigma_p^2 A_p^2 E_x^2 e^{-\frac{\omega_2^2}{2\omega_R^2}} e^{-\frac{(\omega_1-\omega_2)^2}{2\omega_R^{\,2}}} e^{-\omega_1^2\left(\sigma_p^2+\frac{T^2}{2\pi}\right)}$$

$$\cdot\, e^{-\frac{\Delta_p^2T^2(\omega_1-2\omega_2-2\omega_{d0})^2}{2(2T^2+\pi\Delta_p^2)}} d\omega_1\, d\omega_2 \qquad \text{(J-13)}$$

In order to ease the evaluation of (J-13), Equation (J-8) will be used to develop two general forms. First,

$$\iint e^{-A\omega_1^2} e^{-B\omega_2^2} e^{-C(\omega_1-2\omega_2-2\omega_{d0})^2} d\omega_1 d\omega_2$$

$$= \iint e^{-A\omega_1^2} e^{-B(\omega_1-\omega_2)^2} e^{-C(\omega_1-2\omega_2-2\omega_{d0})^2} d\omega_1\, d\omega_2$$

$$= \frac{\pi}{\sqrt{AB+BC+4AC}}\, e^{-\omega_{d0}\left(\frac{4ABC}{AB+BC+4AC}\right)} \qquad \text{(J-14)}$$

Also,

$$\iint e^{-A\omega_1^2} e^{-B\omega_2^2} e^{-B(\omega_1-\omega_2)^2} e^{-C(\omega_1-2\omega_2-2\omega_{d0})^2} d\omega_1 d\omega_2$$

$$= \frac{\pi}{\sqrt{(2A+B)(B+2C)}}\, e^{-\omega_{d0}^2\left(\frac{4BC}{B+2C}\right)} \qquad \text{(J-15)}$$

Applying (J-14)and (J-15) to (J-13) gives

$$\overline{P_0^2}-\bar{P}_0^2 = \frac{32\pi\sqrt{\pi}\sigma_p^2A_p^2E_x^2\Delta_pT}{W^2\sqrt{2T^2+\pi\Delta_p^2}}\Bigg\{\frac{N_{01}^2}{\sqrt{(2\pi\sigma_p^2+T^2)}} + \frac{4N_{01}N_{R1}\Delta_pT\omega_R\exp\left[-\frac{2\omega_{d0}^2(2\pi\sigma_p^2+T^2)\Delta_p^2T^2}{(2\pi\sigma_p^2+T^2)(2T^2+\pi\Delta_p^2+4\Delta_p^2T^2\omega_R^2)+\pi\Delta_p^2T^2}\right]}{\sqrt{[(2\pi\sigma_p^2+T^2)(2T^2+\pi\Delta_p^2+4\Delta_p^2T^2\omega_R^2)+\pi\Delta_p^2T^2]}} + \frac{2N_{R1}^2\omega_R^2\Delta_pT\exp\left[-\frac{2\omega_{d0}^2\Delta_p^2T^2}{(2T^2+\pi\Delta_p^2+2\omega_R^2\Delta_p^2T^2)}\right]}{\sqrt{(4\pi\sigma_p^2\omega_R^2+2\omega_R^2T^2+\pi)(2T^2+\pi\Delta_p^2+2\omega_R^2\Delta_p^2T^2)}}\Bigg\} \qquad (J\text{-}16)$$

Using (J-9) and (J-16) in the signal-to-noise ratio definition in H-1), we have

$$R = \frac{\sqrt{\pi}\Delta_pT\frac{E_x^2\sqrt{2T^2+\pi\Delta_p^2}}{W^2}\left[\frac{\sqrt{2\pi}\ \Delta_T\sigma_T\ A_T}{\sqrt{(\Delta_T^2T^2+\Delta_p^2T^2+\pi\Delta_p^2\Delta_T^2)(T^2+\pi\sigma_p^2+\pi\sigma_T^2)}} + \frac{\sum_k A_ke^{-\frac{\pi}{2}\frac{(u_k-u_0)^2}{(T^2+\pi\sigma_p^2)}}}{\sqrt{(T^2+\pi\Delta_p^2)(T^2+\pi\sigma_p^2)}}\right]^2}{\Bigg\{\frac{N_{01}^2}{\sqrt{2\pi\sigma_p^2+T^2}} + \frac{4N_{01}N_{R1}\Delta_pT\omega_R\exp\left[-\frac{2\omega_{d0}^2(2\pi\sigma_p^2+T^2)\Delta_p^2T^2}{(2\pi\sigma_p^2+T^2)(2T^2+\pi\Delta_p^2+4\Delta_p^2T^2\omega_R^2)+\pi\Delta_p^2T^2}\right]}{\sqrt{(2\pi\sigma_p^2+T^2)(2T^2+\pi\Delta_p^2+4\Delta_p^2T^2\omega_R^2)+\pi\Delta_p^2T^2}} + \frac{2N_{R1}^2\Delta_pT\omega_R^2\exp\left[-\frac{2\omega_{d0}^2\Delta_p^2T^2}{2T^2+\pi\Delta_p^2+2\omega_R^2\Delta_p^2T^2}\right]}{\sqrt{(4\pi\sigma_p^2\omega_R^2+2\omega_R^2T^2+\pi)(2T^2+\pi\Delta_p^2+2\omega_R^2\Delta_p^2\ T^2)}}\Bigg\}} \qquad (J\text{-}17)$$

as the output signal-to-noise ratio for a RAKE radiometer receiving an echo from an extended fluctuating target. For the other cases of interest, the following modifications should be made in (J-17):

1. Non-fluctuating extneded target: $\Delta_T \to \infty$
2. Fluctuating point target: $\sigma_T \to 0$, $\sigma_T A_T = A_T'$. $A_k = 0$
3. Stationary point target: $\Delta_T \to \infty$, $\sigma_T \to 0$, $\sigma_T A_T = A_T'$, $A_k = 0$
4. Two-filter radiometer: $\Delta_p \to \infty$
5. Weighted radiometer: $\sigma_p \to 0$
6. Matched filter: $\Delta_p \to \infty$, $\sigma_p \to 0$

Table J-1 gives R for these various conditions of interest. No correction need be made for doppler dispersion; its effects are negligible in the case of the monochromatic pulse.

2. Linearly Swept FM

Just as in the case of the monochromatic pulse, the envelope (as seen by a noncoherent envelope detector) will be taken to be Gaussian-shaped with the same peak amplitude and the same energy as the rectangular envelope that it supplants. The complex envelope, then, is given by

$$\tilde{x}(t) = \sqrt{2E_x/T}\, e^{-t^2\left(\frac{\pi}{2T^2} - jb\right)} \tag{J-18}$$

where E_x is the energy in the returned signal, T is the duration of the actual rectangular pulse, and 2b is the sweep rate in (rps)/sec. Its Woodward ambiguity function, as defined in (H-8), is

$$C(\tau,f) = \frac{2E_x}{T} \int e^{-t^2\left(\frac{\pi}{2T^2} - jb\right)} e^{-(t-\tau)^2\left(\frac{\pi}{2T^2} - jb\right)} e^{j\omega t}\, dt$$

$$= 2E_x\, e^{-\tau^2 \frac{T^2}{\pi}\left(\frac{\pi}{4T^4} + b^2\right)} e^{-\frac{\omega^2 T^2}{4\pi}} e^{-\frac{\omega\tau T^2 b}{\pi}} e^{j\frac{\omega\tau}{2}} \tag{J-19}$$

Using (J-2), (J-4), and (J-19) in (H-6) yields

$$(\bar{P}_z-\bar{P}_0)^2 = \frac{1}{W^4}\Big|\iint\Big[4\sigma_p\sigma_T A_p A_T E_x^2 e^{-\frac{\tau^2}{2}\left(\frac{1}{\Delta_p^2} + \frac{1}{\Delta_T^2} + \frac{\pi}{T^2} + \frac{4T^2b^2}{\pi}\right)}$$

$$\cdot\, e^{-\frac{\omega^2}{2}\left(\sigma_p^2+\sigma_T^2+\frac{T^2}{\pi}\right)} e^{-\frac{2\omega\tau T^2 b}{\pi}} + \frac{4\sigma_p A_p}{\sqrt{2\pi}} E_x^2 e^{-\frac{\tau^2}{2}\left(\frac{1}{\Delta_p^2} + \frac{\pi}{T^2} + \frac{4T^2b^2}{\pi}\right)}$$

$$\cdot\, e^{-\frac{\omega^2}{2}\left(\sigma_p^2+\frac{T^2}{\pi}\right)} e^{-\frac{2\omega\tau T^2 b}{\pi}} \sum_k A_k e^{j\omega(u_k-u_0)}\Big]\, d\tau\, d\omega\Big|^2. \tag{J-20}$$

Applying (J-8) leads to

$$(\bar{F}_z-\bar{F}_0)^2 = 32\pi^2\sigma_p^2A_p^2\Delta_p^2\frac{T^2E_x^4}{W^4}\left[\frac{\sqrt{2\pi}\,A_T\,\sigma_T\,\Delta_T}{\sqrt{(\Delta_p^2T^2+\Delta_T^2T^2+\pi\Delta_p^2\Delta_T^2)(T^2+\pi\sigma_p^2+\pi\sigma_T^2)+4\Delta_T^2\Delta_p^2(\sigma_T^2+\sigma_p^2)b^2T^4}}\right.$$

$$\left.+\frac{\sum_k A_k\exp\{-\frac{(u_k-u_0)^2(\pi T^2+\pi^2\Delta_p^2+4T^4b^2\Delta_p^2)}{2[(T^2+\pi\sigma_p^2)(T^2+\pi\Delta_p^2)+4\sigma_p^2\Delta_p^2b^2T^4]}\}}{\sqrt{(T^2+\pi\Delta_p^2)(T^2+\pi\sigma_p^2)+4\Delta_p^2\sigma_p^2b^2T^4}}\right]^2. \qquad \text{(J-21)}$$

The energy density spectrum of the swept FM waveform with a Gaussian envelope is also Gaussian in shape,* with its radian $\sigma^2 \approx \frac{2b^2T^2}{\pi}$ when $bT^2 >> \pi/2$. It is reasonable, then, to assume that the reverberation spectrum, which is proportional to the convolution of the signal energy density spectrum with the intrinsic reverberation spectrum of the medium is Gaussian. As in the case of the monochromatic pulse, then, the reverberation spectrum will be taken to be

$$S_r(f) = \tfrac{1}{2}N_{R1}\left[e^{-\frac{(\omega-\omega_c)^2}{2\omega_R^2}} + e^{-\frac{(\omega+\omega_c)^2}{2\omega_R^2}}\right]$$

$$S_{\bar{r}}(f) = 2N_{R1}\,e^{-\frac{\omega^2}{2\omega_R^2}} \qquad \text{(J-22)}$$

where $\omega_R^2 \geq \frac{2b^2T^2}{\pi}$.

Inserting Equations (J-4), (J-19) and (J-22) into (H-20) gives

*H. Van Trees, "Optimum Signal Design and Processing for Reverberation-Limited Environment", IEEE *Trans* on Military Electronics, vol. MIL-9, no. 2, July 1965, pp. 212-229.

$$\overline{P_0{}^2}-\bar{P}_0{}^2 = \frac{16N_{01}^2}{W^2}\iint \sigma_p^2 A_p^2 E_x^2 e^{-\tau^2\left(\frac{1}{\Delta_p^2}+\frac{\pi}{2T^2}+\frac{2b^2T^2}{\pi}\right)} e^{-\omega^2\left(\sigma_p^2+\frac{T^2}{2\pi}\right)} e^{-2\omega\frac{\tau T^2 b}{\pi}} d\tau d\omega$$

$$+ \frac{8N_{01}N_{R1}}{\pi W^2}\iint \sigma_p^2 A_p^2 E_x^2 \left[e^{-\frac{\omega_2^2}{2\omega_R^2}} + e^{-\frac{(\omega_1-\omega_2)^2}{2\omega_R^2}}\right] e^{-\omega_1^2\left(\sigma_p^2+\frac{T^2}{2\pi}\right)}$$

$$\cdot \left|\int e^{-\frac{\tau^2}{2}\left(\frac{1}{\Delta_p^2}+\frac{\pi}{2T^2}+\frac{2b^2T^2}{\pi}\right)} e^{-\frac{\omega_1 \tau T^2 b}{\pi}} e^{-j\left(\frac{\omega_1}{2}-\omega_2-\omega_{d0}\right)\tau} d\tau\right|^2 d\omega_2 d\omega_1$$

$$+ \frac{8N_{R1}^2}{\pi W^2}\iint \sigma_p^2 A_p^2 E_x^2 e^{-\frac{\omega_2^2}{2\omega_R^2}} e^{-\frac{(\omega_1-\omega_2)^2}{2\omega_R^2}} e^{-\omega_1^2\left(\sigma_p^2+\frac{T^2}{2\pi}\right)}$$

$$\cdot \left|\int e^{-\frac{\tau^2}{2}\left(\frac{1}{\Delta_p^2}+\frac{\pi}{2T^2}+\frac{2b^2T^2}{\pi}\right)} e^{-\frac{\omega_1 \tau T^2 b}{\pi}} e^{-j\left(\frac{\omega_1}{2}-\omega_2-\omega_{d0}\right)\tau} d\tau\right|^2 d\omega_2 d\omega_1 . \qquad \text{(J-23)}$$

Application of Equation (J-8) gives

$$\left|\int e^{-\frac{\tau^2}{2}\left(\frac{1}{\Delta_p^2}+\frac{\pi}{2T^2}+\frac{2b^2T^2}{\pi}\right)} e^{-\frac{\omega_1 \tau T^2 b}{\pi}} e^{-j\left(\frac{\omega_1}{2}-\omega_2-\omega_{d0}\right)\tau} d\tau\right|^2$$

$$= \frac{4\pi^2 \Delta_p{}^2 T^2}{2\pi T^2+\pi^2\Delta_p{}^2+4b^2\Delta_p{}^2 T^4} \exp\left\{-\frac{\pi\Delta_p{}^2T^2\left[(\omega_1-2\omega_2-2\omega_{d0})^2-\frac{4T^4b^2}{\pi^2}\omega_1{}^2\right]}{4(2\pi T^2+\pi^2\Delta_p{}^2+4b^2T^4\Delta_p{}^2)}\right\} \qquad \text{(J-24)}$$

At this point it is useful to delete certain terms that will not affect the signal-to-noise ratio for the parameters of interest. First we note that, in the linear FM pulse, T = 1 sec., and

$$2bT^2 \geq 200\pi \text{ rps} \qquad \text{(J-25)}$$

Also, for a 300-foot target the spread in time delay is 0.12 seconds, so that

$$\pi\sigma_{P,T}^2 \leq T^2 \qquad \text{(J-26)}$$

Thirdly, the maximum doppler spread, at a center frequency of 10 kHz, is at most 2 Hz, so that $1/\Delta_{p,T} < 4\pi$ and

$$4b^2T^2\Delta_p^{\ 2} >> 2\pi. \qquad \text{(J-27)}$$

Then (J-24) becomes

$$|\int e^{-\frac{\tau^2}{2}\left(\frac{1}{\Delta_p^2} + \frac{\pi}{2T^2} + \frac{2b^2T^2}{\pi}\right)} e^{-\frac{\omega_1 \tau T^2 b}{\pi}} e^{-j\left(\frac{\omega_1}{2} - \omega_2 - \omega_{d0}\right)\tau} d\tau|^2$$

$$\approx \frac{\pi^2}{b^2T^2} \exp\left\{-\frac{\pi[(\omega_1 - 2\omega_2 - 2\omega_{d0})^2 - \frac{4T^2b^2}{\pi^2}\omega_1^{\ 2}]}{16b^2T^2}\right\}, \qquad \text{(J-28)}$$

and (J-23) simplifies to

$$\overline{P_0^{\ 2}} - \overline{P}_0^{\ 2} = \frac{16N_{01}^2}{W^2}\int\int \sigma_p^2 A_p^2 E_x^2 e^{-\tau^2\left(\frac{2\pi T^2 + \pi^2\Delta_p^2 + 4b^2\Delta_p^2 T^4}{2\pi\Delta_p^2 T^2}\right)} e^{-\omega^2\left(\frac{2\pi\sigma_p^2 + T^2}{2\pi}\right)}$$

$$\cdot e^{-2\omega\frac{\tau T^2 b}{\pi}} d\tau d\omega + \frac{8N_{01}N_{R1}}{\pi W^2}\int\int \sigma_p^2 A_p^2 E_x^2 [e^{-\frac{\omega_2^2}{2\omega_R^2}} + e^{-\frac{(\omega_1-\omega_2)^2}{2\omega_R^{\ 2}}}]$$

$$\cdot e^{-\frac{\omega_1^2 T^2}{2\pi}} \left(\frac{\pi^2}{b^2T^2}\right) \exp\left\{-\frac{\pi[(\omega_1 - 2\omega_2 - 2\omega_{d0})^2 - \frac{4T_4 b^2\omega_1^2}{\pi^2}]}{16b^2T^2}\right\}$$

$$+ \frac{8N_{R1}^2}{\pi W^2}\int\int \sigma_p^2 A_p^2 E_x^2 e^{-\frac{\omega_2^2}{2\omega_R^2}} e^{-\frac{(\omega_1-\omega_2)^2}{2\omega_R^{\ 2}}} e^{-\frac{\omega_1^2 T^2}{2\pi}} \left(\frac{\pi^2}{b^2T^2}\right)$$

$$\cdot \exp\left\{-\frac{\pi[(\omega_1 - 2\omega_2 - 2\omega_{d0})^2 - \frac{4T^4b^2\omega_1^{\ 2}}{\pi^2}]}{16b^2T^2}\right\}. \qquad \text{(J-29)}$$

Using (J-8) and the general forms of (J-14) and (J-15), (J-29) becomes

$$\overline{P_0^2} - \bar{P}_0^{\,2} = \frac{4\sigma_p^2}{W^2} A_p^2 E_x^2 \left[\frac{8\pi\sqrt{\pi}\,\Delta_p T N_{01}^2}{\sqrt{(2T^2+\pi\Delta_p^2)(T^2+2\pi\sigma_p^2)+8\sigma_p^2\Delta_p^2 b^2 T^2}} + \frac{16\pi^2 N_{01} N_{R1}\omega_R}{bT^2\sqrt{2b^2T^2+\pi\omega_R^2}} \cdot \exp\left\{-\frac{\pi\omega_{d0}^2}{2(2b^2T^2+\pi\omega_R^2)}\right\} + \frac{8\pi^2\sqrt{\pi}N_{R1}^2\omega_R^2 \exp -\left\{\frac{\pi\omega_{d0}^2}{4b^2T^2+\pi\omega_R^2}\right\}}{bT^2\sqrt{\pi\omega_R^4+4b^2T^2\omega_R^2+4\pi b^2}}\right]. \qquad (J\text{-}30)$$

Final simplifications from (J-25), (J-26) and (J-27) lead to a signal-to-noise ratio, from (J-21) and (J-30) in (H-1),

$$R = \frac{\left\{\sqrt{\pi}E_x^2 A_p^2 T^2 \left[\frac{\sqrt{2\pi}\, A_T\, \sigma_T\, \Delta_T}{\sqrt{T^2(\Delta_p^2+\Delta_T^2)+\Delta_p^2\Delta_T^2[\pi+4b^2T^2(\sigma_p^2+\sigma_T^2)]}} + \frac{\sum_k A_k \exp\left\{-\frac{2\Delta_p^2T^2b^2}{T^2+\pi\Delta_p^2+4\Delta_p^2\sigma_p^2b^2T^2}\right\}}{\sqrt{T^2+\pi\Delta_p^2+4\Delta_p^2\sigma_p^2b^2T^2}} \right]^2\right\}}{\left\{W^2\left[\frac{\Delta_p N_{01}^2}{\sqrt{2T^2+\pi\Delta_p^2+8\sigma_p^2\Delta_p^2b^2T^2}} + \frac{2\sqrt{\pi}N_{01}N_{R1}\omega_R\exp\left\{-\frac{\pi\omega_{d0}^2}{2(2b^2T^2+\pi\omega_R^2)}\right\}}{bT^2\sqrt{2b^2T^2+\pi\omega_R^2}} + \frac{\pi N_{R1}^2\omega_R^2\exp\left\{-\frac{\pi\omega_{d0}^2}{4b^2T^2+\pi\omega_R^2}\right\}}{bT^2\sqrt{\pi\omega_R^4+4b^2T^2\omega_R^2+4\pi b^2}}\right]\right\}} \qquad (J\text{-}31)$$

The above result can easily be modified to include the doppler dispersion effect. From (H-21) the general form of the signal-to-noise ratio, as modified for doppler dispersion with $k = 1 + (\omega_d/\omega_c)$ as the time compression factor, is

$$R = \frac{\frac{1}{W^2}\left|\iint S_p^*(-f;\tau)S_T(-f;\tau)|C(k\tau,f)|^2 d\tau df\right|^2}{\left\{4N_{01}^2\iint|S_p(-f;\tau)|^2|C(k\tau,f)|^2 d\tau df + \iint\{2N_{01}[S_{\tilde{r}}(f_2)+S_{\tilde{r}}(f_1-f_2)]+S_{\tilde{r}}(f_2)S_{\tilde{r}}(f_1-f_2)\}\left|\int S_p(-f;\tau)C(k\tau,f_1)e^{-j(\omega_1-\omega_2)\tau} d\tau\right| df_1 df_2\right\}} \qquad (J\text{-}32)$$

or alternatively

$$R = \frac{\frac{1}{W^2}\left|\iint S_p^*(-f;\tau/k)S_T(-f;\tau/k)|C(\tau,f)|^2 d\tau df\right|^2}{\{4N_{01}^2\iint|S_p(-f;\tau/k)|^2|C(\tau,f)|^2 d\tau df + \iint\{2N_{01}[S_{\tilde{r}}(f_2)+S_{\tilde{r}}(f_1-f_2)]+S_{\tilde{r}}(f_2)S_{\tilde{r}}(f_1-f_2)\}\left|\int S_p(-f_1;\tau/k)C(\tau,f_1)e^{-j(\omega_1-\omega_2)\tau}d\tau\right| df_1 df_2\}} \qquad \text{(J-33)}$$

The modifications to (J-31) then are:

Δ_T becomes $k\Delta_T$

Δ_p becomes $k\Delta_p$

Thus,

$$R = \frac{\left\{\frac{\sqrt{\pi}}{W^2}E_x^2k^2\Delta_p^2T^2\left[\frac{\sqrt{2\pi}\,A_T\sigma_T\Delta_T}{\sqrt{T^2(\Delta_p^2+\Delta_T^2)+k^2\Delta_p^2\Delta_T^2[\pi+4b^2T^2(\sigma_p^2+\sigma_T^2)]}} + \frac{\sum_i A_i\exp\left\{-\frac{2k^2\Delta_p^2T^2b^2}{T^2+\pi k^2\Delta_p^2+4k^2\Delta_p^2\sigma_p^2b^2T^2}\right\}}{\sqrt{T^2+\pi k^2\Delta_p^2+4k^2\Delta_p^2\sigma_p^2b^2T^2}}\right]^2\right\}}{\left\{\left[\frac{k\Delta_pN_{01}^2}{\sqrt{2T^2+\pi k^2\Delta_p^2+8k^2\sigma_p^2\Delta_p^2b^2T^2}} + \frac{2\sqrt{\pi}N_{01}N_{R1}\omega_R\exp\left\{-\frac{\pi\omega_{do}^2}{2(2b^2T^2+\pi\omega_R^2)}\right\}}{bT^2\sqrt{2b^2T^2+\pi\omega_R^2}} + \frac{\pi N_{R1}^2\omega_R^2\exp\left\{-\frac{\pi\omega_{do}^2}{4b^2T^2+\pi\omega_R^2}\right\}}{bT^2\sqrt{\pi\omega_R^4+4b^2T^2\omega_R^2+4\pi b^2}}\right]\right\}} \qquad \text{(J-34)}$$

Equation (J-34) applies to a RAKE radiometer receiving an echo from an extended fluctuating target. For the other cases of interest, the following modifications should be made in (J-17):

1. Non-fluctuating extended target: $\Delta_T \to \infty$
2. Fluctuating point target: $\sigma_T \to 0$, $\sigma_TA_T = A_T'$, $A_k = 0$
3. Stationary point target: $\Delta_T \to \infty$, $\sigma_T \to 0$, $\sigma_TA_T = A_T'$, $A_k = 0$
4. Two-filter radiometer: $\Delta_p \to \infty$
5. Weighted radiometer: $\sigma_p \to 0$
6. Matched filter: $\Delta_p \to \infty$, $\sigma_p \to 0$.

Table J-2 gives R for these various conditions of interest.

3. Doppler Invariant Pulse

Because of the complicated form of the doppler invariant waveform, it is not possible to write the output signal-to-noise ratio in a simple form. Since only the channel matched in average doppler and average time delay to the returned pulse, and since the ratio of pulse bandwidth to center frequency in the CASS is so much less than 1 (0.06 at a maximum), it is suspected that the results with a doppler invariant waveform will be very similar to those using linear swept FM. Nevertheless, the output signal-to-noise ratio will be written down in its integral form.

To be consistent with the analyses done for the monochromatic pulse and the linearly swept FM, the physical envelope will once again be taken to be Gaussian-shaped with the same peak amplitude and the same energy as the rectangular envelope that it supplants. The complex envelope is given by

$$\tilde{x}(t) = \sqrt{2E_x/T}\, e^{-\left[\frac{\pi t^2}{2T^2} + j\frac{\omega_c}{\alpha}\ln(1-\alpha t) + j\omega_c t\right]} \qquad \text{(J-35)}$$

where E_x is the energy in the signal, T is the duration of the rectangular pulse, ω_c is the radian carrier frequency, and α is the sweep parameter. Its ambiguity function, as defined in (H-8), is

$$C(\tau,f) = \frac{2E_x}{T}\int e^{-\frac{\pi}{2T^2}[t^2+(t-\tau)^2]}\, e^{j\frac{\omega_c}{\alpha}\ln\left[\frac{1-\alpha t}{1-\alpha t+\alpha\tau}\right]}\, e^{-j\omega_c\tau}\, e^{j\omega t}\, dt. \qquad \text{(J-36)}$$

As in the preceding cases, the reverberation spectrum is assumed to be Gaussian, and given by (J-22). Thus, using (J-2), (J-4), (J-22), and (J-36) in (H-21) gives

$$R = \frac{\left\{\frac{E_x^2}{W^2T^2}\left|\iint e^{-\frac{\tau^2}{2k^2\Delta_p^2}} e^{-\frac{\sigma_p^2\omega^2}{2}}\left[\sqrt{2\pi}\,\sigma_T A_T e^{-\frac{\tau^2}{2k^2\Delta_T^2}} e^{-\frac{\sigma_T^2\omega^2}{2}} + \sum_i A_i e^{-j\omega(u_k-u_0)}\right]\left|\int e^{-\frac{\pi}{2T^2}[t^2+(t-\tau)^2]} e^{j\frac{\omega_c}{\alpha}\ln\left[\frac{1-\alpha t}{1-\alpha t+\alpha\tau}\right]} e^{j\omega t}dt\right|^2 d\tau df\right|^2\right\}}{\begin{aligned}&\left\{N_{01}^2\iint e^{-\frac{\tau^2}{k^2\Delta_p^2}} e^{-\sigma_p^2\omega^2}\left|\int e^{-\frac{\pi}{2T^2}[t^2+(t-\tau)^2]} e^{j\frac{\omega_c}{\alpha}\ln\left[\frac{1-\alpha t}{1-\alpha t+\alpha\tau}\right]} e^{-j\omega_c\tau} e^{j\omega t}dt\right|^2 d\tau df\right.\\ &+N_{01}N_{R1}\iint\left[e^{-\frac{\omega_2^2}{2\omega_R^2}} + e^{-\frac{(\omega_1-\omega_2)^2}{2\omega_R^2}}\right]\left|\iint e^{-\frac{\tau^2}{2k^2\Delta_p^2}} e^{-\frac{\sigma_p^2}{2}\omega_1^2} e^{-\frac{\pi}{2T^2}[t^2+(t-\tau)^2]}\right.\\ &\left.\cdot e^{j\frac{\omega_c}{\alpha}\ln\left[\frac{1-\alpha t}{1-\alpha t+\alpha\tau}\right]} e^{-j(\omega_1-\omega_2+\omega_c-\omega_d)\tau} dt d\tau\right|^2 df_1 df_2\\ &+N_{R1}^2\iint e^{-\frac{\omega_2^2}{2\omega_R}} e^{-\frac{(\omega_1-\omega_2)^2}{2\omega_R^2}}\left|\iint e^{-\frac{\tau^2}{2k^2\Delta_p^2}} e^{-\frac{\sigma_p^2}{2}\omega_1^2} e^{-\frac{\pi}{2T^2}[t^2+(t-\tau)^2]}\right.\\ &\left.\left.\cdot e^{j\frac{\omega_c}{\alpha}\ln\left[\frac{1-\alpha t}{1-\alpha t+\alpha\tau}\right]} e^{-j(\omega_1-\omega_2+\omega_c-\omega_d)\tau} dt d\tau\right|^2 df_1 df_2\right\}\end{aligned}} \qquad \text{(J-37)}$$

Doppler dispersion has been accounted for in (J-37) in the same manner as for linear FM in (J-34), through the parameter $k = 1 + (\omega_d/\omega_c)$. As with the other two waveforms, the output signal-to-noise ratio equation for the RAKE radiometer with a deep fluctuating target given in (J-37) may be adjusted for special cases in the following manner:

1. Non-fluctuating extended target: $\Delta_T \to \infty$
2. Fluctuating point target: $\sigma_T \to 0$, $\sigma_T A_T = A_T'$, $A_k = 0$
3. Stationary point target: $\Delta_T \to \infty$, $\sigma_T \to 0$, $\sigma_T A_T = A_T'$, $A_k = 0$
4. Two-filter radiometer: $\Delta_p \to \infty$
5. Weighted radiometer: $\sigma_p \to 0$
6. Matched filter: $\Delta_p \to \infty$, $\sigma_p \to 0$.

To support the contention that using a doppler invariant pulse gives results very similar to those with a linearly swept FM pulse, let us rewrite (J-35) as

$$\tilde{x}(t) = \sqrt{2E_x/T}\; e^{-[\frac{\pi t^2}{2T^2} + j\omega_c t - j\frac{\omega_c}{\alpha}\sum_{i=1}^{\infty}(\alpha t)^i]}$$

$$= \sqrt{2E_x/T}\; e^{-[\frac{\pi t^2}{2T^2} - j\omega_c \alpha t^2 - j\omega_c \alpha^2 t^3 - \ldots]} \qquad \text{(J-38)}$$

where $|\alpha t| \leq |\alpha| T < 1$.

It may be seen by comparison with (J-18) that the imaginary t^2 term in the exponent is the linear sweep term, with $b = \omega_c \alpha$. The ratio of the magnitude of the t^3 term to that of the t^2 term $|\alpha t|$ satisfies the inequality:

$$|\alpha t| \leq |\alpha| T = \frac{b}{\omega_c} T. \qquad \text{(J-39)}$$

Since the sweep rates of interest are less than 400 Hz/sec.,

$$b \leq \frac{400}{2}(2\pi) \text{ rps/sec} = 400\pi \text{ rps/sec} \qquad \text{(J-40)}$$

Also, $T = 1$ sec. and $\omega_c \geq 6.5(2\pi)k$ rps, so that the ratio of the t^3 term to the t^2 term in the doppler invariant exponent of (J-39) is bounded by

$$|\alpha t| \leq 0.031. \qquad \text{(J-41)}$$

This small ratio would apparently allow the t^3 and higher order terms to be neglected, supporting the approximation of the doppler invariant pulse by a linearly swept FM pulse.

Furthermore, since consideration is given here only to that channel matched in doppler dispersion to the returned signal, the length of the backbone of the linearly swept FM pulse ambiguity function does not exhibit any shrinkage, a characteristic shared by the doppler invariant pulse.

TABLE J-1

MONOCHROMATIC PULSE OUTPUT SI

MATCHED FILTER

STATIONARY POINT TARGET	$$\frac{2\pi E_x^2 A_{T'}^2/W^2}{N_{01}^2 + \frac{2\sqrt{2}N_{01}N_{R1}\omega_R T}{\sqrt{\pi+2\omega_R^2T^2}}\exp[-\frac{\omega_{d0}^2T^2}{\pi+2\omega_R^2T^2}] + \frac{2N_{R1}^2\omega_R^2T^2}{\pi+2\omega_R^2T^2}\exp[-\frac{2\omega_{d0}^2T^2}{\pi+2\omega_R^2T^2}]}$$
FLUCTUATING POINT TARGET	$$\frac{2\pi^2\frac{E_x^2A_{T'}^2\Delta_T^2}{W^2(T^2+\pi\Delta_T^2)}}{N_{01}^2 + \frac{2\sqrt{2}N_{01}N_{R1}\omega_R T}{\sqrt{\pi+2\omega_R^2T^2}}\exp[-\frac{\omega_{d0}^2T^2}{\pi+2\omega_R^2T^2}] + \frac{2N_{R1}^2\omega_R^2T^2}{\pi+2\omega_R^2T^2}\exp[-\frac{2\omega_{d0}^2T^2}{\pi+2\omega_R^2T^2}]}$$
NONFLUCTUATING EXTENDED TARGET	$$\frac{\frac{E_x^2}{W^2}[\frac{\sqrt{2\pi}A_T\sigma_T T}{\sqrt{T^2+\pi\sigma_T^2}} + \sum_k A_k\exp\{-\frac{\pi(u_k-u_0)^2}{2T^2}\}]^2}{N_{01}^2 + \frac{2\sqrt{2}N_{01}N_{R1}\omega_R T}{\sqrt{\pi+2\omega_R^2T^2}}\exp[-\frac{\omega_{d0}^2T^2}{\pi+2\omega_R^2T^2}] + \frac{2N_{R1}^2\omega_R^2T^2}{\pi+2\omega_R^2T^2}\exp[-\frac{2\omega_{d0}^2T^2}{\pi+2\omega_R^2T^2}]}$$
FLUCTUATING EXTENDED TARGET	$$\frac{\frac{E_x^2}{W^2}[\frac{\pi\sqrt{2}A_T\sigma_T\Delta_T T}{\sqrt{(T^2+\pi\Delta_T^2)(T^2+\pi\sigma_T^2)}} + \sum_k A_k\exp\{-\frac{\pi(u_k-u_0)^2}{2T^2}\}]^2}{N_{01}^2 + \frac{2\sqrt{2}N_{01}N_{R1}\omega_R T}{\sqrt{\pi+2\omega_R^2T^2}}\exp[-\frac{\omega_{d0}^2T^2}{\pi+2\omega_R^2T^2}] + \frac{2N_{R1}^2\omega_R^2T^2}{\pi+2\omega_R^2T^2}\exp[-\frac{2\omega_{d0}^2T^2}{\pi+2\omega_R^2T^2}]}$$

TABLE J-1

OUTPUT SIGNAL-TO-NOISE RATIOS

WEIGHTED RADIOMETER

	$\dfrac{2\pi\sqrt{\pi}E_x^2 A_{T1}^2 \dfrac{\Delta_p\sqrt{2T^2+\pi\Delta_p^2}}{W^2(T^2+\pi\Delta_p^2)}}{N_{01}^2 + \dfrac{2\sqrt{2}N_{01}N_{R1}\Delta_p\omega_R T\exp[-\dfrac{\omega_{d0}^2\Delta_p^2T^2}{T^2+\pi\Delta_p^2+2\Delta_p^2\omega_R^2T^2}]}{\sqrt{T^2+\pi\Delta_p^2+2\Delta_p^2\omega_R^2T^2}} + \dfrac{2N_{R1}^2\Delta_p\omega_R^2T^2\exp[-\dfrac{2\omega_{d0}^2\Delta_p^2T^2}{2T^2+\pi\Delta_p^2+2\Delta_p^2\omega_R^2T^2}]}{\sqrt{(2\omega_R^2T^2+\pi)(2T^2+\pi\Delta_p^2+2\Delta_p^2\omega_R^2T^2)}}}$
	$\dfrac{2\pi\sqrt{\pi}E_x^2 \dfrac{A_{T1}^2\Delta_T^2\Delta_p\sqrt{2T^2+\pi\Delta_p^2}}{W^2[(\Delta_T^2+\Delta_p^2)T^2+\pi\Delta_p^2\Delta_T^2]}}{N_{01}^2 + \dfrac{2\sqrt{2}N_{01}N_{R1}\Delta_p\omega_R T\exp[-\dfrac{\omega_{d0}^2\Delta_p^2T^2}{T^2+\pi\Delta_p^2+2\Delta_p^2\omega_R^2T^2}]}{\sqrt{T^2+\pi\Delta_p^2+2\Delta_p^2\omega_R^2T^2}} + \dfrac{2N_{R1}^2\Delta_p\omega_R^2T^2\exp[-\dfrac{2\omega_{d0}^2\Delta_p^2T^2}{2T^2+\pi\Delta_p^2+2\Delta_p^2\omega_R^2T^2}]}{\sqrt{(2\omega_R^2T^2+\pi)(2T^2+\pi\Delta_p^2+2\Delta_p^2\omega_R^2T^2)}}}$
	$\dfrac{\sqrt{\pi}\dfrac{E_x^2\Delta_p\sqrt{2T^2+\pi\Delta_p^2}}{W^2(T^2+\pi\Delta_p^2)}[\dfrac{\sqrt{2\pi}\sigma_T A_T T}{\sqrt{T^2+\pi\sigma_T^2}} + \sum_k A_k\exp\{-\dfrac{\pi(u_k-u_0)^2}{2T^2}\}]^2}{N_{01}^2 + \dfrac{2\sqrt{2}N_{01}N_{R1}\Delta_p\omega_R T\exp[-\dfrac{\omega_{d0}^2\Delta_p^2T^2}{T^2+\pi\Delta_p^2+2\Delta_p^2\omega_R^2T^2}]}{\sqrt{T^2+\pi\Delta_p^2+2\Delta_p^2\omega_R^2T^2}} + \dfrac{2N_{R1}^2\Delta_p\omega_R^2T^2\exp[-\dfrac{2\omega_{d0}^2\Delta_p^2T^2}{2T^2+\pi\Delta_p^2+2\Delta_p^2\omega_R^2T^2}]}{\sqrt{(2\omega_R^2T^2+\pi)(2T^2+\pi\Delta_p^2+2\Delta_p^2\omega_R^2T^2)}}}$
	$\dfrac{\sqrt{\pi}E_x^2\Delta_p\dfrac{\sqrt{2T^2+\pi\Delta_p^2}}{W^2}[\dfrac{\sqrt{2\pi}\sigma_T A_T\Delta_T T}{\sqrt{(\Delta_T^2T^2+\Delta_p^2T^2+\pi\Delta_p^2\Delta_T^2)(T^2+\pi\sigma_T^2)}} + \dfrac{\sum_k A_k\exp -\dfrac{\pi(u_k-u_0)^2}{2T^2}}{\sqrt{T^2+\pi\Delta_p^2}}]^2}{N_{01}^2 + \dfrac{2\sqrt{2}N_{01}N_{R1}\Delta_p\omega_R T\exp[-\dfrac{\omega_{d0}^2\Delta_p^2T^2}{T^2+\pi\Delta_p^2+2\Delta_p^2\omega_R^2T^2}]}{\sqrt{T^2+\pi\Delta_p^2+2\Delta_p^2\omega_R^2T^2}} + \dfrac{2N_{R1}^2\Delta_p\omega_R^2T^2\exp[-\dfrac{2\omega_{d0}^2\Delta_p^2T^2}{2T^2+\pi\Delta_p^2+2\Delta_p^2\omega_R^2T^2}]}{\sqrt{(2\omega_R^2T^2+\pi)(2T^2+\pi\Delta_p^2+2\Delta_p^2\omega_R^2T^2)}}}$

TABLE J-1 [contin

TWO-FILTER RADIOMETER

STATIONARY POINT TARGET	$\dfrac{2\pi E_x{}^2 T A_{T1}^2/[(T^2+\pi\sigma_p{}^2)W^2]}{\dfrac{N_{01}^2}{\sqrt{2\pi\sigma_p^2+T^2}} + \dfrac{4N_{01}R_{01}\omega_R T\exp[-\frac{2\omega_{d0}^2T^2(2\pi\sigma_p^2+T^2)}{(2\pi\sigma_p^2+T^2)(\pi+4\omega_R^2T^2+\pi T^2}]}{\sqrt{(2\pi\sigma_p^2+T^2)(\pi+4\omega_R^2T^2)+\pi T^2}} + \dfrac{2R_{01}^2\omega_R^2T\exp[-\frac{2\omega_{d0}{}^2T^2}{\pi+2\omega_R^2T^2}]}{\sqrt{(4\pi\sigma_p^2\omega_R^2+2\omega_R^2T^2+\pi)(\pi+2\omega_R^2T^2)}}}$	[illegible]
FLUCTUATING POINT TARGET	$\dfrac{\dfrac{2\pi^2E_x{}^2T\Delta_T{}^2A_{t1}^2}{(T^2+\pi\Delta_T{}^2)(T^2+\pi\sigma_p{}^2)W^2}}{\dfrac{N_{01}^2}{\sqrt{2\pi\sigma_p^2+T^2}} + \dfrac{4N_{01}R_{01}\omega_R T\exp[-\frac{2\omega_{d0}^2T^2(2\pi\sigma_p^2+T^2)}{(2\pi\sigma_p^2+T^2)(\pi+4\omega_R^2T^2)+\pi T^2}]}{\sqrt{(2\pi\sigma_p^2+T^2)(\pi+4\omega_R^2T^2)+\pi T^2}} + \dfrac{2R_{01}^2\omega_R^2T\exp[-\frac{2\omega_{d0}{}^2T^2}{\pi+2\omega_R^2T^2}]}{\sqrt{(4\pi\sigma_p^2\omega_R^2+2\omega_R^2T^2+\pi)(\pi+2\omega_R^2T^2)}}}$	[illegible]
NONFLUCTUATING EXTENDED TARGET	$\dfrac{\dfrac{E_x{}^2T}{W^2}[\dfrac{\sqrt{2\pi}\sigma_TA_T}{\sqrt{T^2+\pi\sigma_p^2+\pi\sigma_T^2}} + \dfrac{\sum_k A_k\exp -\frac{\pi(u_k-u_0)^2}{2(T^2+\pi\sigma_p^2)}}{\sqrt{T^2+\pi\sigma_p{}^2}}]^2}{\dfrac{N_{01}{}^2}{\sqrt{2\pi\sigma_p^2+T^2}} + \dfrac{4N_{01}R_{01}\ {}_RT\exp[-\frac{2\omega_{d0}^2T^2(2\pi\sigma_p^2+T^2)}{(2\pi\sigma_p^2+T^2)(\pi+4\omega_R^2T^2)+\pi T^2}]}{\sqrt{(2\pi\sigma_p^2+T^2)(\pi+4\omega_R{}^2T^2)+\pi T^2}} + \dfrac{2R_{01}^2\omega_R^2T\exp[-\frac{2\omega_{d0}{}^2T^2}{\pi+2\omega_R^2T^2}]}{\sqrt{(4\pi\sigma_p^2\omega_R^2+2\omega_R^2T^2+\pi)(\pi+2\omega_R^2T^2)}}}$	[illegible]
EXTENDED FLUCTUATING TARGET	$\dfrac{\dfrac{E_x{}^2T}{W^2}[\dfrac{\sqrt{2}\pi\Delta_T\sigma_TA_T}{\sqrt{(T^2+\pi\Delta_T^2)(T^2+\pi\sigma_p^2+\pi\sigma_T^2)}} + \dfrac{\sum_k A_k\exp\{-\frac{\pi(u_k-u_0)^2}{2(T^2+\pi\sigma_p^2)}\}}{\sqrt{T^2+\pi\sigma_p{}^2}}]^2}{\dfrac{N_{01}{}^2}{\sqrt{2\pi\sigma_p^2+T^2}} + \dfrac{4N_{01}R_{01}\omega_RT\exp[-\frac{2\omega_{d0}^2T^2(2\pi\sigma_p^2+T^2)}{(2\pi\sigma_p^2+T^2)(\pi+4\omega_R^2T^2)+\pi T^2}]}{\sqrt{(2\pi\sigma_p^2+T^2)(\pi+4\omega_R^2T^2)+\pi T^2}} + \dfrac{2R_{01}^2\omega_R^2T\exp[-\frac{2\omega_{d0}{}^2T^2}{\pi+2\omega_R^2T^2}]}{\sqrt{(4\pi\sigma_p^2\omega_R^2+2\omega_R^2T^2+\pi)(\pi+2\omega_R^2T^2)}}}$	[illegible]

J-1 [continued]

RAKE RADIOMETER

$\ldots{}_R T^2)$	$$\frac{\dfrac{2\pi\sqrt{\pi}E_x{}^2 T\Delta_p\sqrt{2T^2+\pi\Delta_p^2}A_{T1}^2}{W^2(T^2+\pi\Delta_p^2)(T^2+\pi\sigma_p^2)}}{\dfrac{N_{01}^2}{\sqrt{2\pi\sigma_p^2+T^2}}+\dfrac{4N_{01}R_{01}\Delta_p\omega_R T\exp[-\dfrac{2\omega_{d0}^2(2\pi\sigma_p^2+T^2)\Delta_p^2T^2}{(2\pi\sigma_p^2+T^2)(2T^2+\pi\Delta_p^2+4\Delta_p^2\omega_R^2T^2)+\pi\Delta_p^2T^2}]}{\sqrt{(2\pi\sigma_p^2+T^2)(2T^2+\pi\Delta_p^2+4\Delta_p^2\omega_R^2T^2)+\pi\Delta_p^2T^2}}+\dfrac{2R_{01}^2\Delta_p\omega_R^2T\exp[-\dfrac{2\omega_{d0}^2\Delta_p^2T^2}{2T^2+\pi\Delta_p^2+2\omega_R^2T^2\Delta_p^2}]}{\sqrt{(4\pi\sigma_p^2\omega_R^2+2\omega_R^2T^2+\pi)(2T^2+\pi\Delta_p^2+2\Delta_p^2\omega_R^2T^2)}}}$$
$\ldots\omega_R^2T^2)$	$$\frac{\dfrac{2\pi\sqrt{\pi}E_x{}^2 T\Delta_p\sqrt{2T^2+\pi\Delta_p^2}\,\Delta_T^2A_{T1}^2}{W^2(\Delta_T^2T^2+\Delta_p^2T^2+\pi\Delta_p^2\Delta_T^2)(T^2+\pi\sigma_p^2)}}{\dfrac{N_{01}^2}{\sqrt{2\pi\sigma_p^2+T^2}}+\dfrac{4N_{01}R_{01}\Delta_p\omega_R T\exp[-\dfrac{2\omega_d^2(2\pi\sigma_p^2+T^2)\Delta_p^2T^2}{(2\pi\sigma_p^2+T^2)(2T^2+\pi\Delta_p^2+4\Delta_p^2\omega_R^2T^2)+\pi\Delta_p^2T^2}]}{\sqrt{(2\pi\sigma_p^2+T^2)(2T^2+\pi\Delta_p^2+4\Delta_p^2\omega_R^2T^2)+\pi\Delta_p^2T^2}}+\dfrac{2R_{01}^2\Delta_p\omega_R^2T\exp[-\dfrac{2\omega_d^2\Delta_p^2T^2}{2T^2+\pi\Delta_p^2+2\omega_R^2T^2\Delta_p^2}]}{\sqrt{(4\pi\sigma_p^2\omega_R^2+2\omega_R^2T^2+\pi)(2T^2+\pi\Delta_p^2+2\Delta_p^2\omega_R^2T^2}}}$$
$\ldots\omega_R^2T^2)$	$$\frac{\dfrac{\sqrt{\pi}\Delta_p TE_x{}^2\sqrt{2T^2+\pi\Delta_p^2}}{(T^2+\pi\Delta_p{}^2)W^2}\left[\dfrac{\sqrt{2\pi}\sigma_TA_T}{\sqrt{T^2+\pi\sigma_T^2+\pi\sigma_p^2}}+\dfrac{\sum_k A_k\exp\{-\dfrac{\pi(u_k-u_0)^2}{2(T^2+\pi\sigma_p^2)}\}}{\sqrt{T^2+\pi\sigma_p^2}}\right]^2}{\dfrac{N_{01}^2}{\sqrt{2\pi\sigma_p^2+T^2}}+\dfrac{4N_{01}R_{01}\Delta_p\omega_R T\exp[-\dfrac{2\omega_d^2(2\pi\sigma_p^2+T^2)\Delta_p^2T^2}{(2\pi\sigma_p^2+T^2)(2T^2+\pi\Delta_p^2+4\Delta_p^2\omega_R^2T^2)+\pi\Delta_p^2T^2}]}{\sqrt{(2\pi\sigma_p^2+T^2)(2T^2+\pi\Delta_p^2+4\Delta_p^2\omega_R^2T^2)+\pi\Delta_p^2T^2}}+\dfrac{2R_{01}^2\Delta_p\omega_R^2T\exp[-\dfrac{2\omega_d^2\Delta_p^2T^2}{2T^2+\pi\Delta_p^2+2\omega_R^2T^2\Delta_p^2}]}{\sqrt{4\pi\sigma_p^2\omega_R^2+2\omega_R^2T^2+\pi)(2T^2+\pi\Delta_p^2+2\Delta_p^2\omega_R^2T^2)}}}$$
$\ldots{}_R T^2)$	$$\frac{\sqrt{\pi}\Delta_p TE_x^2\dfrac{\sqrt{2T^2+\pi\Delta_p^2}}{W^2}\left[\dfrac{\sqrt{2\pi}\Delta_T\sigma_TA_T}{\sqrt{(\Delta_T^2T^2+\Delta_p^2T^2+\pi\Delta_p^2\Delta_T^2)(T^2+\pi\sigma_T^2+\pi\sigma_p^2)}}+\dfrac{\sum_k A_k\exp\{-\dfrac{\pi(u_k-u_0)^2}{2(T^2+\pi\sigma_p^2)}\}}{\sqrt{(T^2+\pi\Delta_p^2)(T^2+\pi\sigma_p^2)}}\right]^2}{\dfrac{N_{01}^2}{\sqrt{2\pi\sigma_p^2+T^2}}+\dfrac{4N_{01}R_{01}\Delta_p\omega_R T\exp[-\dfrac{2\omega_d^2(2\pi\sigma_p^2+T^2)\Delta_p^2T^2}{(2\pi\sigma_p^2+T^2)(2T^2+\pi\Delta_p^2+4\Delta_p^2\omega_R^2T^2)+\pi\Delta_p^2T^2}]}{\sqrt{(2\pi\sigma_p^2+T^2)(2T^2+\pi\Delta_p^2+4\Delta_p^2\omega_R^2T^2)+\pi\Delta_p^2T^2}}+\dfrac{2R_{01}^2\Delta_p\omega_R^2T\exp[-\dfrac{2\omega_d^2\Delta_p^2T^2}{2T^2+\pi\Delta_p^2+2\omega_R^2T^2\Delta_p^2}]}{\sqrt{(4\pi\sigma_p^2\omega_R^2+2\omega_R^2T^2+\pi)(2T^2+\pi\Delta_p^2+2\Delta_p^2\omega_R^2T^2)}}}$$

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MATCHED FILTER

STATIONARY POINT TARGET	$\dfrac{2\pi E_x{}^2 T^2 A_{T'}^2/W^2}{N_{01}^2 + \dfrac{2\pi N_{01}N_{R1}\omega_R \exp\left[-\dfrac{\pi\omega_{do}^2}{2(2b^2T^2+\pi\omega_R^2)}\right]}{bT^2\sqrt{2b^2T^2+\pi\omega_R^2}} + \dfrac{\pi\sqrt{\pi}N_{R1}^2\omega_R^2\exp\left[-\dfrac{\pi\omega_{do}^2}{4b^2T^2+\pi\cdots}\right.}{bT^2\sqrt{\pi\omega_R^4+4b^2T^2\omega_R^2+4\pi b^2\cdots}}}$
FLUCTUATING POINT TARGET	$\dfrac{\dfrac{2\pi^2 E_x{}^2 T^2 A_{T'}^2 k^2 \Delta_T{}^2}{(T^2+\pi k^2\Delta_T{}^2)W^2}}{N_{01}{}^2 + \dfrac{2\pi N_{01}N_{R1}\omega_R \exp\left[-\dfrac{\pi\omega_{do}^2}{2(2b^2T^2+\pi\omega_R^2)}\right]}{bT^2\sqrt{2b^2T^2+\pi\omega_R{}^2}} + \dfrac{\pi\sqrt{\pi}N_{R1}\omega_R^2\exp\left[-\dfrac{\pi\omega_{d0}^2}{4b^2T^2+\cdots}\right.}{bT^2\sqrt{\pi\omega_R^4+4b^2T^2\omega_R^2+4\pi b^2\cdots}}}$
NONFLUCTUATING EXTENDED TARGET	$\dfrac{\dfrac{E_x{}^2T^2}{W^2}\left[\dfrac{\pi\sqrt{2}A_T\sigma_T}{\sqrt{\pi+4b^2T^2\sigma_T{}^2}} + \sum_i A_i\exp\{-\frac{2}{\pi}b^2T^2\}\right]^2}{N_{01}^2 + \dfrac{2\pi N_{01}N_{R1}\omega_R \exp\left[-\dfrac{\pi\omega_{d0}^2}{2(2b^2T^2+\pi\omega_R^2)}\right]}{bT^2\sqrt{2b^2T^2+\pi\omega_R^2}} + \dfrac{\pi\sqrt{\pi}N_{R1}^2\omega_R^2\exp\left[-\dfrac{\pi\omega_{d0}^2}{4b^2T^2+\pi\cdots}\right.}{bT^2\sqrt{\pi\omega_R^4+4b^2T^2\omega_R^2+4\pi b^2\cdots}}}$
EXTENDED FLUCTUATING TARGET	$\dfrac{\dfrac{E_x{}^2T^2}{W^2}\left[\dfrac{\pi\sqrt{2}A_T\sigma_T k\Delta_T}{\sqrt{T^2+k^2\Delta_T^2(\pi+4b^2T^2\sigma_T^2)}} + \sum_i A_i\exp\{-\frac{2}{\pi}b^2T^2\}\right]^2}{N_{01}^2 + \dfrac{2\pi N_{01}N_{R1}\omega_R \exp\left[-\dfrac{\pi\omega_{do}^2}{2(2b^2T^2+\pi\omega_R^2)}\right]}{bT^2\sqrt{2b^2T^2+\pi\omega_R{}^2}} + \dfrac{\pi\sqrt{\pi}N_{R1}^2\omega_R^2\exp\left[-\dfrac{\pi\omega_{do}^2}{4b^2T^2+\cdots}\right.}{bT^2\sqrt{\pi\omega_R^4+4b^2T^2\omega_R^2+4\pi b\cdots}}}$

TABLE J-2

ARLY SWEPT FM PULSE OUTPUT SIGNAL-TO-NOISE RATIOS

	WEIGHTED RADIOMETER
$\ldots{}^2_R\exp[-\frac{\pi\omega_{d0}^2}{4b^2T^2+\pi\omega_R^2}]$ / $\ldots+4b^2T^2\omega_R^2+4\pi b^2$	$$\frac{\frac{2\pi\sqrt{\pi}E_x^2k^2\Delta_p^2T^2A_{T'}^2}{(T^2+\pi k^2\Delta_p^2)W^2}}{\frac{k\Delta_pN_{01}^2}{\sqrt{2T^2+\pi k^2\Delta_p^2}}+\frac{2\sqrt{\pi}N_{01}N_{R1}\omega_R\exp[-\frac{\pi\omega_{d0}^2}{2(2b^2T^2+\pi\omega_R^2)}]}{bT^2\sqrt{2b^2T^2+\pi\omega_R^2}}+\frac{\pi N_{R1}^2\omega_R^2\exp[-\frac{\pi\omega_{d0}^2}{4b^2T^2+\pi\omega_R^2}]}{bT^2\sqrt{\pi\omega_R^4+4b^2T^2\omega_R^2+4\pi b^2}}}$$
$\ldots\omega_R^2\exp[-\frac{\pi\omega_{d0}^2}{4b^2T^2+\pi\omega_R^2}]$ / $\ldots{}_R^4+4b^2T^2\omega_R^2+4\pi b^2$	$$\frac{\frac{2\pi\sqrt{\pi}E_x^2k^2\Delta_p^2T^2A_{T'}^2\Delta_T^2}{[T^2(\Delta_p^2+\Delta_T^2)+\pi k^2\Delta_p^2\Delta_T^2]W^2}}{\frac{k\Delta_pN_{01}^2}{\sqrt{2T^2+\pi k^2\Delta_p^2}}+\frac{2\sqrt{\pi}N_{01}N_{R1}\omega_R\exp[-\frac{\pi\omega_{d0}^2}{2(2b^2T^2+\pi\omega_R^2)}]}{bT^2\sqrt{2b^2T^2+\pi\omega_R^2}}+\frac{\pi N_{R1}^2\omega_R^2\exp[-\frac{\pi\omega_{d0}^2}{4b^2T^2+\pi\omega_R^2}]}{bT^2\sqrt{\pi\omega_R^4+4b^2T^2\omega_R^2+4\pi b^2}}}$$
$\ldots\}]^2$ / $\ldots\exp[-\frac{\pi\omega_{d0}^2}{4b^2T^2+\pi\omega_R^2}]$ / $\ldots+4b^2T^2\omega_R^2+4\pi b^2$	$$\frac{\sqrt{\pi}E_x^2\frac{k^2\Delta_p^2T^2}{W^2}\left[\frac{\sqrt{2\pi}\,A_T\sigma_T}{\sqrt{T^2+k^2\Delta_p^2(\pi+4b^2T^2\sigma_T^2)}}+\frac{\sum_i A_i\exp\{-\frac{2k^2\Delta_p^2T^2b^2}{T^2+\pi k^2\Delta_p^2}\}}{\sqrt{T^2+\pi k^2\Delta_p^2}}\right]^2}{\frac{k\Delta_pN_{01}^2}{\sqrt{2T^2+\pi k^2\Delta_p^2}}+\frac{2\sqrt{\pi}N_{01}N_{R1}\omega_R\exp[-\frac{\pi\omega_{d0}^2}{2(2b^2T^2+\pi\omega_R^2)}]}{bT^2\sqrt{2b^2T^2+\pi\omega_R^2}}+\frac{\pi N_{R1}^2\omega_R^2\exp[-\frac{\pi\omega_{d0}^2}{4b^2T^2+\pi\omega_R^2}]}{bT^2\sqrt{\pi\omega_R^4+4b^2T^2\omega_R^2+4\pi b^2}}}$$
$\ldots\{-\frac{2}{\pi}b^2T^2\}]^2$ / $\ldots\exp[-\frac{\pi\omega_{d0}^2}{4b^2T^2+\pi\omega_R^2}]$ / $\ldots+4b^2T^2\omega_R^2+4\pi b^2$	$$\frac{\sqrt{\pi}E_x^2\frac{k^2\Delta_p^2T^2}{W^2}\left[\frac{\sqrt{2\pi}\,A_T\sigma_T\Delta_T}{\sqrt{T^2(\Delta_p^2+\Delta_T^2)+k^2\Delta_p^2\Delta_T^2(\pi+4b^2T^2\sigma_T^2)}}+\frac{\sum_i A_i\exp-\frac{2k^2\Delta_p^2T^2b^2}{T^2+\pi k^2\Delta_p^2}}{\sqrt{T^2+\pi k^2\Delta_p^2}}\right]^2}{\frac{k\Delta_pN_{01}^2}{\sqrt{2T^2+\pi k^2\Delta_p^2}}+\frac{2\sqrt{\pi}N_{01}N_{R1}\omega_R\exp[-\frac{\pi\omega_{d0}^2}{2(2b^2T^2+\pi\omega_R^2)}]}{bT^2\sqrt{2b^2T^2+\pi\omega_R^2}}+\frac{\pi N_{R1}^2\omega_R^2\exp[-\frac{\pi\omega_{d0}^2}{4b^2T^2+\pi\omega_R^2}]}{bT^2\sqrt{\pi\omega_R^4+4b^2T^2\omega_R^2+4\pi b^2}}}$$

TABLE J-2

TWO-FILTER RADIOMETER

STATIONARY POINT TARGET	$$\frac{\dfrac{2\pi\sqrt{\pi}E_x^2T^2A_{T'}^2}{(\pi+4b^2T^2\sigma_p^2)W^2}}{\dfrac{N_{01}^2}{\sqrt{\pi+8\sigma_p^2b^2T^2}}+\dfrac{2\sqrt{\pi}N_{01}N_{R1}\omega_R\exp[-\frac{\pi\omega_{d0}^2}{2(2b^2T^2+\pi\omega_R^2)}]}{bT^2\sqrt{2b^2T^2+\pi\omega_R^2}}+\dfrac{\pi N_{R1}^2\omega_R^2\exp[-\frac{\pi\omega_{d0}^2}{4b^2T^2+\pi\omega_R^2}}{bT^2\sqrt{\pi\omega_R^4+4b^2T^2\omega_R^2+4\pi b^2}}}$$
FLUCTUATING POINT TARGET	$$\frac{\dfrac{2\pi\sqrt{\pi}E_x^2T^2k^2\Delta_T^2A_{T'}^2}{[T^2+k^2\Delta_T^2(\pi+4b^2T^2\sigma_p^2)]W^2}}{\dfrac{N_{01}^2}{\sqrt{\pi+8\sigma_p^2b^2T^2}}+\dfrac{2\sqrt{\pi}N_{01}N_{R1}\omega_R\exp[-\frac{\pi\omega_{d0}^2}{2(2b^2T^2+\pi\omega_R^2)}]}{bT^2\sqrt{2b^2T^2+\pi\omega_R^2}}+\dfrac{\pi N_{R1}^2\omega_R^2\exp[-\frac{\pi\omega_{d0}^2}{4b^2T^2+\pi\omega_R^2}]}{bT^2\sqrt{\pi\omega_R^4+4b^2T^2\omega_R^2+4\pi b^2}}}$$
NONFLUCTUATING EXTENDED TARGET	$$\frac{\sqrt{\pi}\dfrac{E_x^2T^2}{W^2}\left[\dfrac{\sqrt{2\pi}\,A_T\sigma_T}{\sqrt{\pi+4b^2T^2(\sigma_p^2+\sigma_T^2)}}+\dfrac{\sum_i A_i\exp\{-\frac{2b^2T^2}{\pi+4\sigma_p^2b^2T^2}\}}{\sqrt{\pi+4\sigma_p^2b^2T^2}}\right]^2}{\dfrac{N_{01}^2}{\sqrt{\pi+8\sigma_p^2b^2T^2}}+\dfrac{2\sqrt{\pi}N_{01}N_{R1}\omega_R\exp[-\frac{\pi\omega_{d0}^2}{2(2b^2T^2+\pi\omega_R^2)}]}{bT^2\sqrt{2b^2T^2+\pi\omega_R^2}}+\dfrac{\pi N_{R1}^2\omega_R^2\exp[-\frac{\pi\omega_{d0}^2}{4b^2T^2+\pi\omega_R^2}]}{bT^2\sqrt{\pi\omega_R^4+4b^2T^2\omega_R^2+4\pi b^2}}}$$
EXTENDED FLUCTUATING TARGET	$$\frac{\sqrt{\pi}\dfrac{E_x^2T^2}{W^2}\left[\dfrac{\sqrt{2\pi}A_T\sigma_Tk\Delta_T}{\sqrt{T^2+k^2\Delta_T^2[\pi+4b^2T^2(\sigma_p^2+\sigma_T^2)]}}+\dfrac{\sum_i A_i\exp\{-\frac{2b^2T^2}{\pi+4\sigma_p^2b^2T^2}\}}{\sqrt{\pi+4\sigma_p^2b^2T^2}}\right]^2}{\dfrac{N_{01}^2}{\sqrt{\pi+8\sigma_p^2b^2T^2}}+\dfrac{2\sqrt{\pi}N_{01}N_{R1}\omega_R\exp[-\frac{\pi\omega_{d0}^2}{2(2b^2T^2+\pi\omega_R^2)}]}{bT^2\sqrt{2b^2T^2+\pi\omega_R^2}}+\dfrac{\pi N_{R1}^2\omega_R^2\exp[-\frac{\pi\omega_{d0}^2}{4b^2T^2+\pi\omega_R^2}}{bT^2\sqrt{\pi\omega_R^4+4b^2T^2\omega_R^2+4\pi b^2}}}$$

TABLE J-2 [continued]

	RAKE RADIOMETER
$\ldots\omega_R^2\exp[-\frac{\pi\omega_{d0}^2}{4b^2T^2+\pi\omega_R^2}]$ $\ldots\sqrt{\pi\omega_R^4+4b^2T^2\omega_R^2+4\pi b^2}$	$$\frac{\dfrac{2\pi\sqrt{\pi}E_x{}^2k^2\Delta_p^2T^2A_{T'}^2}{W^2[T^2+k^2\Delta_p^2(\pi+4b^2T^2\sigma_p^2)]}}{\dfrac{k\Delta_pN_{01}^2}{\sqrt{2T^2+\pi k^2\Delta_p^2+8k^2\Delta_p^2\sigma_p^2b^2T^2}}+\dfrac{2\sqrt{\pi}N_{01}N_{R1}\omega_R\exp[-\frac{\pi\omega_{d0}^2}{2(2b^2T^2+\pi\omega_R^2)}]}{bT^2\sqrt{2b^2T^2+\pi\omega_R^2}}+\dfrac{\pi N_{R1}^2\omega_R^2\exp[-\frac{\pi\omega_{d0}^2}{4b^2T^2+\pi\omega_R^2}]}{bT^2\sqrt{\pi\omega_R^4+4b^2T^2\omega_R^2+4\pi b^2}}}$$
$\ldots{}_R^2\exp[-\frac{\pi\omega_{d0}^2}{4b^2T^2+\pi\omega_R^2}]$ $\ldots\omega_R^4+4b^2T^2\omega_R^2+4\pi b^2$	$$\frac{\dfrac{2\pi\sqrt{\pi}E_x{}^2k^2\Delta_p^2T^2\Delta_T^2A_{T'}^2}{[T^2(\Delta_p^2+\Delta_T^2)+k^2\Delta_p^2\Delta_T^2(\pi+4b^2T^2\sigma_p^2)]W^2}}{\dfrac{k\Delta_pN_{01}^2}{\sqrt{2T^2+\pi k^2\Delta_p^2+8k^2\sigma_p^2\Delta_p^2b^2T^2}}+\dfrac{2\sqrt{\pi}N_{01}N_{R1}\omega_R\exp[-\frac{\pi\omega_{d0}^2}{2(2b^2T^2+\pi\omega_R^2)}]}{bT^2\sqrt{2b^2T^2+\pi\omega_R^2}}+\dfrac{\pi N_{R1}^2\omega_R^2\exp[-\frac{\pi\omega_{d0}^2}{4b^2T^2+\pi\omega_R^2}]}{bT^2\sqrt{\pi\omega_R^2+4b^2T^2\omega_R^2+4\pi b^2}}}$$
$\ldots{}^2\}$ $\ldots]^2$ $\ldots{}_R^2\exp[-\frac{\pi\omega_{d0}^2}{4b^2T^2+\pi\omega_R^2}]$ $\ldots\sqrt{\pi\omega_R^4+4b^2T^2\omega_R^2+4\pi b^2}$	$$\frac{\sqrt{\pi}E_x\dfrac{2k^2\Delta_p^2T^2}{W^2}\Big[\dfrac{\sqrt{2\pi}\,A_T\sigma_T}{\sqrt{T^2+k^2\Delta_p^2[\pi+4b^2T^2(\sigma_p^2+\sigma_T^2)]}}+\dfrac{\sum_i A_i\exp\{-\frac{2k^2\Delta_p^2T^2b^2}{T^2+\pi k^2\Delta_p^2+4k^2\Delta_p^2\sigma_p^2b^2T^2}\}}{\sqrt{T^2+\pi k^2\Delta_p^2+4k^2\Delta_p^2\sigma_p^2b^2T^2}}\Big]^2}{\dfrac{k\Delta_pN_{01}^2}{\sqrt{2T^2+\pi k^2\Delta_p^2+8k^2\sigma_p^2\Delta_p^2b^2T^2}}+\dfrac{2\sqrt{\pi}N_{01}N_{R1}\omega_R\exp[-\frac{\pi\omega_{d0}^2}{2(b^2T^2+\pi\omega_R^2)}]}{bT^2\sqrt{2b^2T^2+\pi\omega_R^2}}+\dfrac{\pi N_{R1}^2\omega_R^2\exp[-\frac{\pi\omega_{d0}^2}{4b^2T^2+\pi\omega_R^2}]}{bT^2\sqrt{\pi\omega_R{}^4+4b^2T^2\omega_R^2+4\pi b^2}}}$$
$\ldots\frac{2b^2T^2}{\ldots+4\sigma_p^2b^2T^2}\}$ $\ldots]^2$ $\ldots b^2T^2$ $\ldots{}_R^2\exp[-\frac{\pi\omega_{d0}^2}{4b^2T^2+\pi\omega_R^2}]$ $\ldots\pi\omega_R^4+4b^2T^2\omega_R^2+4\pi b^2$	$$\frac{\sqrt{\pi}E_x{}^2\,\dfrac{k^2\Delta_p^2T^2}{W^2}\Big[\dfrac{\sqrt{2\pi}\,A_T\sigma_T\Delta_T}{\sqrt{T^2(\Delta_p^2+\Delta_T^2)+k^2\Delta_p^2\Delta_T^2[\pi+4b^2T^2(\sigma_p^2+\sigma_T^2)]}}+\dfrac{\sum_i A_i\exp\{-\frac{2k^2\Delta_p^2T^2b^2}{T^2+\pi k^2\Delta_p^2+4k^2\Delta_p^2\sigma_p^2b^2T^2}\}}{\sqrt{T^2+\pi k^2\Delta_p^2+4k^2\Delta_p^2\sigma_p^2b^2T^2}}\Big]^2}{\dfrac{k\Delta_pN_{01}^2}{\sqrt{2T^2+\pi k^2\Delta_p^2+8k^2\sigma_p^2\Delta_p^2b^2T^2}}+\dfrac{2\sqrt{\pi}N_{01}N_{R1}\omega_R\exp[-\frac{\pi\omega_{d0}^2}{2(2b^2T^2+\pi\omega_R^2)}]}{bT^2\sqrt{2b^2T^2+\pi\omega_R^2}}+\dfrac{\pi N_{R1}^2\omega_R^2\exp[-\frac{\pi\omega_{d0}^2}{4b^2T^2+\pi\omega_R^2}]}{bT^2\sqrt{\pi\omega_R^4+4b^2T^2\omega_R^2+4\pi b^2}}}$$

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APPENDIX K

PROCESSING GAIN

A formula for processing gain when the noise background includes non-flat reverberation will be given here by following the development of Appendix F. The processing gain will be defined as

$$G_p = \frac{\sqrt{R}}{(S/N)_i}, \qquad \text{(K-1)}$$

where R is the output signal-to-noise ratio and where $(S/N)_i$ is the input signal-to-noise ratio, which we define in one of two ways.

(a) $(S/N)_i$ is the ratio of average power in the signal complex envelope to average power in the total noise complex envelope, measured at the output of a simple filter to be described shortly.

(b) By sampling the squared envelope of the input after the same simple filtering to obtain a test statistic, we can define

$$(S/N)_i = \frac{\bar{P}_{iz} - \bar{P}_{i0}}{\sigma_{i0}}, \qquad \text{(K-2)}$$

where the sampled value of the input is P_{iz} in the presence of signal and P_{i0} in its absence. The variance of P_{i0} is σ_{i0}^2.

In both of these definitions a bandpass filter is needed preceding the input signal-to-noise ratio measurement to limit the noise to the same frequency band as that of the signal.

In definition (b), the numerator is given by the average difference of input power with signal and without signal. Under the condition of independence between signal and noise, this is simply the average signal power. Thus, the numerators of the two definitions are equivalent, and are given as in Appendix F by $R_z(t,t)$. The most reasonable time to evaluate the average signal power is at the time corresponding to the delay of the midpoint of the target, $t = u_0$.

Now σ_{1o}^2 is the variance of the squared magnitude of the total input noise complex envelope. It can be found, by substituting $\tilde{n}(t)+\tilde{r}(t)$ for $\tilde{n}(t)$ in (E-25), to be

$$\sigma_{10}^2 = \sigma_{\tilde{n}+\tilde{r}}^4$$

or

$$\sigma_{10} = \sigma_{\tilde{n}+\tilde{r}}^2 = \sigma_{\tilde{n}}^2 + \sigma_{\tilde{r}}^2 \qquad \text{(K-3)}$$

where $\tilde{n}(t)$, the ambient noise complex envelope, and $\tilde{r}(t)$, the reverberation noise complex envelope, are considered to be independent and zero mean. As given in (K-3), σ_{10} is simply the average power in the total noise complex envelope. Thus, the denominators of the two definitions are equivalent, and are given by $\sigma_{\tilde{n}}^2+\sigma_{\tilde{r}}^2$.

If we define W as the noise bandwidth of the previously described bandpass filter, one whose passband is the band of frequencies occupied by the spectrum of the returned signal, then from (E-25),

$$\sigma_{\tilde{n}}^2 = 2N_{01}W \qquad \text{(K-4)}$$

For simplification of the computation of $\sigma_{\tilde{r}}^2$, the filter amplitude characteristic will be assumed Gaussian.

$$|H(f)| = e^{-\frac{(\omega-\omega_c-\omega_{d0})^2}{4\pi W^2}} + e^{-\frac{(\omega+\omega_c+\omega_{d0})^2}{4\pi W^2}} \qquad \text{(K-5)}$$

where ω_c is the carrier radian frequency and ω_{d0} is the doppler shift. The filter that operates on the complex envelope of noise is, then,

$$|\tilde{H}(f)| = 2e^{-\frac{(\omega-\omega_{d0})^2}{4\pi W^2}} \qquad \text{(K-6)}$$

With the reverberation power density spectrum taken as

$$S_r(f) = \frac{1}{2} N_{R1}[e^{-\frac{(\omega-\omega_c)^2}{2W_R^2}} + e^{-\frac{(\omega+\omega_c)^2}{2W_R^2}}], \qquad \text{(K-7)}$$

the spectrum of the complex envelope of the reverberation is

$$S_{\tilde{r}}(f) = 2N_{R1}\, e^{-\omega^2/2\omega_R^2} \qquad \text{(K-8)}$$

The power in the reverberation complex envelope is

$$\sigma_{\tilde{r}}^2 = \frac{1}{4}\int |\tilde{H}(f)|^2 S_{\tilde{r}}(f)\, df$$

$$\sigma_{\tilde{r}}^2 = \frac{2\omega_R\, W\, N_{R1}}{\sqrt{2\pi W^2+\omega_R^2}}\, e^{-\frac{\omega_{d0}^2}{2(2\pi W^2+\omega_R^2)}}, \qquad \text{(K-9)}$$

where (K-6) and (K-8) have been used.

Thus, the input signal-to-noise ratio, using (K-2), (K-3), (K-4) and (K-9), is

$$(S/N)_i = \frac{R_{\tilde{z}}(u_0,u_0)}{2N_{01}W + \frac{2\omega_R W N_{R1}}{\sqrt{2\pi W^2+\omega_R^2}} \exp\{-\frac{\omega_{d0}^2}{2(2\pi W^2+\omega_R^2)}\}}. \qquad \text{(K-10)}$$

As in Appendix F

$$R_{\tilde{z}}(u_0,u_0) = \frac{1}{W}\int C(0,f) S_T(-f;0) e^{j\omega u_0}\, df, \qquad \text{(K-12)}$$

giving

$$(S/N)_i = \frac{\frac{1}{2W^2}\int C(0,f) S_T(-f;0) e^{j\omega u_0}\, df}{N_{01} + \frac{\omega_R N_{R1}}{\sqrt{2\pi W^2+\omega_R^2}} \exp - \frac{\omega_{d0}^2}{2(2\pi W^2+\omega_R^2)}}. \qquad \text{(K-13)}$$

Using the ambiguity function definition of (H-8), it can be seen that

$$C(0,f) = \int |\tilde{x}(t)|^2 e^{j\omega t}\, dt, \qquad \text{(K-14)}$$

where $|\tilde{x}(t)|$ is the physical envelope of the signal. For computational ease, this has been taken to be

$$|\tilde{x}(t)| = \sqrt{2E_x/T}\, e^{-\frac{\pi}{2}\frac{t^2}{T^2}} , \tag{K-15}$$

giving

$$C(0,f) = 2E_x e^{-\omega^2 T^2/4\pi} , \tag{K-16}$$

independent of whether the waveform is a monochromatic pulse, linear FM, or doppler invariant. Using (J-2) for $S_T(f;\tau)$ with (K-16) in (K-13) gives

$$(S/N)_i = \frac{E_x\left[\frac{\sqrt{2\pi}\sigma_T A_T}{\sqrt{2\pi\sigma_T^2+T^2}} + \sum_k A_k\, e^{-\frac{\pi}{T^2}(u_k-u_0)^2}\right]}{W^2\left[N_{01} + \frac{\omega_R N_{R1}}{\sqrt{2\pi W^2+\omega_R^2}} \exp\left\{-\frac{\omega_{d0}^2}{2(2\pi W^2+\omega_R^2)}\right\}\right.} . \tag{K-17}$$

From (K-1) and (K-17), the processing gain is

$$G_p = \frac{W^2\sqrt{R}\left[N_{01} + \frac{\omega_R N_{R1}}{\sqrt{2\pi W^2+\omega_R^2}} \exp\left\{-\frac{\omega_{d0}^2}{2(2\pi W^2+\omega_R^2)}\right\}\right]}{E_x\left[\frac{\sqrt{2\pi}\sigma_T A_T}{\sqrt{2\pi\sigma_T^2+T^2}} + \sum_k A_k e^{-\frac{\pi}{T^2}(u_k-u_0)^2}\right]} \tag{K-18}$$

where R is given by (J-17) for a monochromatic pulse, by (J-31) for a linearly swept FM waveform, or by (J-37) for a doppler invariant pulse.

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B. Abrams, "Extended Target Detection with Emphasis on Acoustic Surveillance. Volume II: Extensions and Specific Applications," Report 1584-2041-1, General Atronics Corporation, Contract N62269-67-C-0028, 15 January 1967

p. iii — In the definition of E_x, "...returned signal..." should be replaced by "...transmitted signal..."

p. iv — "Δ_p" should be replaced by "Δ_p^{-1}"
"Δ_T" should be replaced by "Δ_T^{-1}"
The definition of "W" should be "the transmitted signal bandwidth." After the definition of "W" insert:
"W_a = bandwidth of the processing channel input or input noise bandwidth"

p. 2 — Equation (1): the integrand of the numerator should be
"$S_p^*(-k_T f;\tau)S_T(-k_p f;\tau)C^*(k_p\tau,k_T f)C(k_T\tau,k_p f)$"
The integrand of the first term of the denominator should be
"$|S_p(-k_p f;\tau)|^2|C(k_p\tau;k_p f)|^2$"
In the second term of the denominator the τ integration should be
"$|\int S_p(-k_p f_1;\tau)C(k_p\tau,k_p f_1)e^{-j(\omega_1-\omega_2)\tau}d\tau|^2$"

p. 4 — In the definitions following Equation (3), "ω_d" should read "ω_{d0}".
The next line, which reads "Δ_T = ...of the doppler spread" should read "Δ_T^{-1} = ... of the doppler spread in radians/sec"
In the line following Equation (5), "...returned signal..." should read "...transmitted signal..."

p. 6 — Equation (9): remove both "k^2" and "T^2" from the factors multiplying both numerator terms. In the second and third terms of the denominator, replace the product "b^2T^2" by "$k^2b^2T^2$" in the four places that it occurs.

p. 8 — Equation (13): replace "W" by "W_a" in both exponentials.

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p. 9 In the second line change "W" to "W_a". At the end of the second line add "of the processing channel".
Equation (14): Replace "W" by "W_a".
Equation (16): The first factor should be "WW_a" in the numerator, instead of "W^2". Everywhere else replace "W" by "W_a".

p. H-4 Equation (H-19): "N_0" should read "N_{01}"

p. H-5 Equation (H-22): The integrand of the numerator should be "$S_p^*(-k_T f;\tau)S_T(-k_p f;\tau)C^*(k_p\tau,k_T f)C(k_T\tau,k_p f)$"
In the denominator the integrand of the first term should be "$|S_p(-k_p f;\tau)|^2|C(k_p\tau,k_p f)|^2$"
The integrand of the τ integration in the second term of the denominator should be
"$S_p(-k_p f_1;\tau)C(k_p\tau,k_p f_1)\ e^{-j(\omega_1-\omega_2)\tau}$"

p. J-2 In the definitions following Equation (J-1), "Δ_T = ... of the doppler spread" should be changed to "Δ_T^{-1} = ... of the doppler spread in radians/sec"

p. J-3 In the line following Equation (J-5) "...returned signal..." should read "...transmitted signal..."
Equation (J-9): In the second term "(u_k-u_0)" should be changed to "$(u_k-u_0)^2$"

p. J-5 Equation (J-14): In the exponent of the last line of the equation, "ω_{d0}" should be changed to "ω_{d0}^2"

p. J-7 In the line following Equation (J-18), "...returned signal..." should read "...transmitted signal..."

p. J-9 Equation (J-25), the sign "$\leq$" should be changed to "$\ll$".

p. J-10 The third line of Equation (J-29) should have the differentials "$d\omega_1 d\omega_2$" at the end of the line. Furthermore, the last term of the numerator of the exponential on that line should have "T_4" replaced by "T^4".
The last line of Equation (J-29) should have the differentials "$d\omega_1 d\omega_2$" at the end of the line.

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p. J-11 Equation (J-30): The last symbol in the denominator of the first term should be "T^4" instead of "T^2".

Equation (J-31): Of the factors multiplying both terms of the numerator, "A_p^2" should be changed to "Δ_p^2" and "T^2" should be removed.

Equation (J-32): The integrand of the numerator should be changed from

"$S_p^*(-f;\tau)S_T(-f;\tau)|C(k\tau,f)|^2$" to "$S_p^*(-kf;\tau)S_T(-kf;\tau)|C(k\tau,kf)|^2$"

The integrand of the first term of the denominator should be changed from

"$|S_p(-f;\tau)|^2|C(k\tau,f)|^2$" to "$|S_p(-kf;\tau)|^2|C(k\tau,kf)|^2$"

In the second term of the denominator the integration should be changed from

"$|\int S_p(-f;\tau)C(k\tau,f_1)e^{-j(\omega_1-\omega_2)\tau}d\tau|$" to

"$|\int S_p(-kf_1;\tau)C(k\tau,kf_1)e^{-j(\omega_1-\omega_2)\tau}d\tau|^2$"

p. J-12 Equation (J-33): In the first term of the denominator "N_{01}^2" should be replaced by "$k^2N_{01}^2$".

The second term of the denominator should be rewritten as

"$\iint\{2N_{01}[S_{\tilde{r}}(\frac{f_2}{k})+S_{\tilde{r}}(\frac{f_1-f_2}{k})]+S_{\tilde{r}}(\frac{f_2}{k})S_{\tilde{r}}(\frac{f_1-f_2}{k})\}$

$|\int S_p(-f;\frac{\tau}{k})C(\tau,f_1)e^{-j(\omega_1-\omega_2)\tau/k^2}d\tau|^2df_1df_2$"

instead of

"$\iint\{2N_{01}[S_{\tilde{r}}(f_2)+S_{\tilde{r}}(f_1-f_2)]+S_{\tilde{r}}(f_2)S_{\tilde{r}}(f_1-f_2)\}$

$|\int S_p(-f_1;\frac{\tau}{k})C(\tau,f_1)e^{-j(\omega_1-\omega_2)\tau}d\tau|^2df_1df_2$".

Insert the following before Equation (J-34):

"ω_R becomes $k\omega_R$

ω_{d0} becomes $k\omega_{d0}$

In the last two terms of the denominator b becomes k^2b.

These two terms are multiplied by k^4."

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p. J-12 [cont] Equation (J-34): Both "T^2" and "k^2" should be removed from the factors multiplying both terms of the numerator. In the second and third terms of the denominator, wherever the product "b^2T^2" appears, substitute "$k^2b^2T^2$".

p. J-13 In the line following Equation (J-35), "...energy in the signal..." should be changed to "...energy in the transmitted signal..."

Table J-1, first page: In the expressions for the weighted radiometer with a stationary point target and with a fluctuating point target, the numerator factor "A_{T1}^2" should be changed to "$A_{T'}^2$".

Table J-1, second page: The second and third terms of the denominator of every expression on the page has "R_{01}" as a multiplying factor. This should be changed to "N_{R1}". In the expressions for the two-filter radiometer with a stationary point target and for the RAKE radiometer with a fluctuating point target, the numerator factor "A_{T1}^2" should be changed to "$A_{T'}^2$". In the expression for the two-filter radiometer with a fluctuating point target, the numerator factor "A_{t1}^2" should be changed to "$A_{T'}^2$".

Table J-2, both pages: The numerator factor "T^2" should be removed from every expression on both pages. Each expression should be multiplied by "$1/k^2$". In the second and third terms of the denominator of every entry, the product "b^2T^2" should be changed to "$k^2b^2T^2$" in the four places where it occurs. Add this note to Table J-2: "In considering targets with velocities less than 30 knots, $k = 1 + 2v/c$ is bounded by $0.98 \leq k \leq 1.02$. Thus, it may be neglected everywhere except in exponents."

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p. K-2 Four lines above Equation (K-4) use the symbol "W_a" instead of "W".
Equation (K-4): Replace "W" by "W_a".
Equation (K-5): Replace "W" bu "W_a" in both exponentials.
Equation (K-6): Replace "W" by "W_a".

p. K-3 Equation (K-7): Replace "W_R" by "ω_R" in both exponentials.
Equation (K-9): Replace "W" by "W_a" throughout the equation.
Equation (K-10): Replace "W" by "W_a" throughout the equation.
Equation (K-13): In the numerator, replace "$1/2W^2$" by "$1/2WW_a$". In the denominator, replace "W^2" by "W_a^2".

p. K-4 Equation (K-17), denominator: The factor multiplying both terms, "W^2",should be replaced by "WW_a". Everywhere else "W^2" should be replaced by "W_a^2". Also, the denominator should be ended by a right-hand square bracket.
Equation (K-18), numerator: The factor multiplying both terms, "W^2", should be replaced by "WW_a". Everywhere else "W^2" should be replaced by "W_a^2".

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p. J-7 Equation (J-19), first line. In the second exponential, replace "-jb" by "+jb".
Equation (J-19), second line. In the first exponential change "$\frac{\pi}{4T^4}$" to "$\frac{\pi^2}{4T^4}$".